Online Appendix for
Population and Welfare:
Measuring Growth when Life is Worth Living

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A. The role of birth and death rates

This section of the appendix provides details on the exercise discussed in Section 4.4 of the paper. The goal is to quantify the contributions of fertility (birth rates) and longevity (death rates) to population growth.

Notation. In a given year t, total population $N(t)$ is the sum of population of different ages:

$$N(t) = \sum_{a=0}^{A} N_a(t).$$

The law of motion for $N_a(t)$ is given by:

$$N_a(t) = \begin{cases} N_{a-1}(t-1) + M_a(t) - D_a(t) & \text{if } a > 0 \\ B(t) + M_a(t) - D_a(t) & \text{if } a = 0, \end{cases}$$

where $M_a(t)$ is the net inflow of migrants of age a in year t, $D_a(t)$ is the total number of deaths at age a in year t, and $B(t)$ is the total number of births. It is useful to rewrite the law of motion in terms of death rate:

$$d_a(t) := \frac{D_a(t)}{N_a(t)} \implies N_a(t) = \begin{cases} \frac{N_{a-1}(t-1)+M_a(t)}{1+d_a(t)} & \text{if } a > 0 \\ \frac{B(t)+M_a(t)}{1+d_a(t)} & \text{if } a = 0, \end{cases}$$

Methodology. To isolate the contribution of longevity, we consider a counterfactual where we fix the death rates by age. Specifically, we start with the total population and age distribution as of 1960, and simulate the evolution of population assuming the death rates by age remained constant at their 1960 levels, but births and migration by age evolved as in the data:

$$N^{\text{sim}}(t) = \sum_{a=0}^{A} N_a^{\text{sym}}(t) \quad \text{where} \quad N_a^{\text{sim}}(t) = \begin{cases} N_a(0) & \text{if } t = 0 \\ \frac{B(t)+M_a(t)}{1+d_a(0)} & \text{if } t > 0 \text{ and } a = 0 \\ \frac{N_{a-1}^{\text{synth}}(t-1)+M_a(t)}{1+d_a(0)} & \text{if } t > 0 \text{ and } a > 0. \end{cases}$$

We refer to the growth rate of population in this simulation as the counterfactual population growth rate - the one that would have prevailed had death rates by age remained constant at 1960 levels. The gap between this counterfactual growth rate and the actual reflects the contribution of longevity (falling death rate by age) to population growth.

Data. We implement the exercise using annual data on $N_a(t)$, $D_a(t)$ and $B(t)$ from the Human Mortality Database for 24 countries: Australia, Austria, Belgium, Canada, Czechia, Denmark, Finland, France, Luxembourg, Norway, Spain, UK, Italy, Japan, Netherlands, Sweden, Switzerland, Iceland, USA, Portugal, Israel, Hong Kong, Croatia, and South Korea. For all except the last four of these countries, the data start in 1960. Table 1 shows the results of this exercise, contrasting actual and average population growth for each of the countries.

Table 1: Population Growth Holding Longevity Constant

Country	Start	End	g_N	g_N^{sim}
Australia	1960	2019	1.5%	1.4%
Austria	1960	2019	0.4%	0.2%
Belgium	1960	2019	0.4%	0.2%
Canada	1960	2019	1.3%	1.1%
Switzerland	1960	2019	0.8%	0.6%
Czechia	1960	2019	0.2%	0.0%
Denmark	1960	2019	0.4%	0.3%
Spain	1960	2019	0.7%	0.5%
Finland	1960	2019	0.4%	0.2%
France	1960	2019	0.6%	0.4%
UK	1960	2019	0.4%	0.2%
Hong Kong	1986	2019	0.9%	0.8%
Croatia	2001	2019	-0.3%	-0.4%
Iceland	1960	2019	1.2%	1.1%
Israel	1983	2016	2.3%	2.1%
Italy	1960	2019	0.3%	0.1%
Japan	1960	2019	0.5%	0.1%
Korea	2003	2019	0.4%	0.2%
Luxembourg	1960	2019	1.1%	1.0%
Netherlands	1960	2019	0.7%	0.6%
Norway	1960	2019	0.7%	0.6%
Portugal	1960	2019	0.3%	0.0%
Sweden	1960	2019	0.5%	0.4%
USA	1960	2019	1.0%	0.9%
All countries - pop weighted			0.72%	0.53%

B. Population- instead of Consumption-Equivalent units

Figure 1: Plot of PE welfare growth against CE welfare growth, 1960-2019

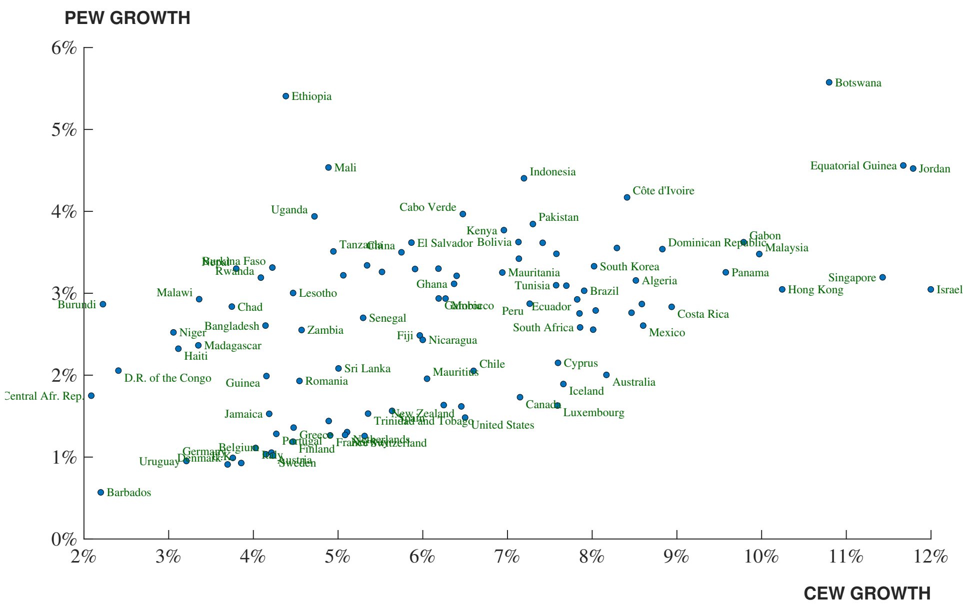

Figure 2: Plot of PE welfare growth against consumption growth, 1960-2019

C. Details for “Beyond Consumption” calculations

This section of the appendix provides details on how we implement our calculations in Section 5 of the paper.

Using parents’ first order conditions (equations 11, 12, and 13 in the paper) and the expression for growth in consumption-equivalent welfare (equation 10 in the paper), we can write:

$$
\begin{aligned}
g_{\lambda_t} &= \pi_t^p \cdot v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) \cdot \frac{dN_t^p}{N_t^p} + \pi_t^k \cdot \tilde{v}(c_t^k) \cdot \frac{dN_t^k}{N_t^k} \\
&+ \pi_t^p \cdot \frac{dc_t^p}{c_t^p} + (1-\pi_t^p) \cdot \frac{dc_t^k}{c_t^k} \\
&+ \pi_t^p \cdot (1+\alpha b_t^\theta) \cdot \frac{l_t}{l_{ct}} \cdot \frac{dl_t}{l_t} \\
&+ \pi_t^p \cdot \left(\alpha b_t^\theta + (1 + \alpha b_t^\theta) \cdot \frac{b_t \cdot e_t}{l_{ct}}\right) \cdot \frac{db_t}{b_t} \\
&+ \pi_t^p \cdot (1+\alpha b_t^\theta) \cdot \frac{b_t \cdot e_t}{l_{ct}} \cdot \frac{1}{\eta} \cdot \frac{dh_t^k}{h_t^k}.
\end{aligned}
$$

where

$$
\pi_t^p = \frac{N_t^p}{(1+\alpha b_t^\theta) N_t^p + N_t^k} \quad ; \quad \pi_t^k = \frac{N_t^k}{(1+\alpha b_t^\theta) N_t^p + N_t^k} \quad ;
$$

$$
v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) = \frac{u\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)}{u_{c^p}\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) \cdot c_t^p} \quad ; \quad \tilde{v}(c_t^k) = \frac{\tilde{u}(c_t^k)}{\tilde{u}'(c_t^k) \cdot c_t^k}.
$$

To implement these calculations, in addition to parameter values for α, θ, and η (calibration discussed in section 5.2 of the paper), we need:

1. data for N_t^p (# of adults), N_t^k (# of kids), $b_t = \frac{N_t^k}{N_t^p}$ (# of kids per adult), c_t^p (adult’s consumption), c_t^k (kid’s consumption), l_t (adult’s leisure time), l_{c_t} (adult’s work time), and $b_t e_t$ (adult’s childcare time).

2. estimates for $\frac{dh_t^k}{h_t^k}$ (growth in kid’s human capital)

3. estimates for $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$ and $\tilde{v}(c_t^k)$

C.1 Data Sources

The data inputs are:

- Total population (N_t), consumption (c_t), average hours worked, and number of employed (emp) from the Penn World Tables;[1]
- Population 0-19 year old (N_t^k) from the World Bank;
- Population 20-65 from the World Bank (used to calculate hours worked per adult l_{ct} as described below);
- Total childcare $b_t \cdot e_t$ from time-use surveys. For each country, we keep all respondents who are 20 years or older. Whenever the time-survey is not available at annual frequency, we annualize total childcare assuming a constant annual growth rate between two consecutive time use surveys.

We calculate the remaining variables using the following relationships:

$$
\begin{aligned}
N_t^p &= N_t - N_t^k \\
b_t &= \frac{N_t^k}{N_t^p} \\
l_{c_t} &= \frac{\text{average hours worked} \times \text{number of employed}}{\text{Population 20-65 years old}} \\
l_t &= 16 \text{ hours/day} - l_{c_t} - b_t e_t \\
c_t^p &= \frac{1 + b_t}{1 + \alpha b_t^\theta} \cdot c_t \\
c_t^k &= \alpha b_t^{\theta - 1} \cdot c_t^p \,.
\end{aligned}
$$

The last two expressions combine the accounting identity

$$
c_t := \frac{N_t^p}{N_t^p + N_t^k} \cdot c_t^p \; + \; \frac{N_t^k}{N_t^p + N_t^k} \cdot c_t^k \,,
$$

with the parent's optimality conditions.

[1] The specific data series we use from PWT are **pop** for N_t, **avh** for average hours worked, and **emp** for number of employed. For consumption per capita, we use the same definition from our baseline calculation.

C.2 Growth in Human Capital

We calibrate the growth rate in kid's human capital, $\frac{dh_t^k}{h_t^k}$, as follows. We first assume that growth rate is constant within a country in our sample. That implies:

$$\frac{dh_t^k}{h_t^k} = \frac{dh^k}{h^k} = \frac{dh}{h} := \text{ average of } \frac{dh_t}{h_t} \ .$$

That is, the average growth rate in kid's human capital can be captured by its growth rate for the adult population.

To measure $\frac{dh}{h}$, we assume that growth in labor productivity reflects equal contributions from growth in human capital and growth in productivity per unit of human capital (growth in w_t). The latter growth would reflect growth in TFP and physical capital. Note labor productivity can expressed in term of consumption per working hour as:

$$w_t h_t = \frac{(1+\alpha b_t^\theta) c_t^p}{l_{c_t}} \ .$$

So attributing half of productivity growth to human capital growth implies:

$$\frac{dh_t}{h_t} = \frac{1}{2} \frac{d\left(\frac{(1+\alpha b_{t+1}^\theta) c_{t+1}^p}{l_{ct+1}}\right)}{\frac{(1+\alpha b_{t+1}^\theta) c_{t+1}^p}{l_{ct+1}}} \ .$$

C.3 Value of a year of life relative to consumption

The last step in order to be able to implement our calculation is to have $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$ and $\tilde{v}(c_t^k)$ for every country-year in our sample.

Our calibration remains hinged on the U.S. in 2006, and we target a VSLY of \$185,000 for both adults and kids (consistent with EPA guidelines). To obtain $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$ and $\tilde{v}(c_t^k)$, we divide this VSLY by c_t^p and c_t^k respectively, noting that:

$$c_{\text{US, 2006}}^p = \frac{1 + b_{\text{US, 2006}}}{1 + \alpha b_{\text{US, 2006}}^\theta} \cdot c_{\text{US, 2006}} \qquad \text{and} \qquad c_{\text{US, 2006}}^k = \alpha b_t^{\theta - 1} \cdot c_{\text{US, 2006}}^p \ .$$

This yields, for the U.S. in 2006, $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) = 4.61$ and $\tilde{v}(c_t^k) = 5.71$.

Given our assumed utility function for kid's utility,

$$\tilde{v}(c_t^k) = \bar{u}_k + \log(c_t^k) \ ,$$

targeting $\tilde{v}(c_t^k) = 5.71$ for the U.S. in 2006 yields

$$\bar{u}_k \ = \ 5.87 \ .$$

For the parents, since we do not fully parameterize the utility function, we proceed as follows:

- conditional on establishing $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$ in a specific country for a base year, we chain weight to obtain its values in other years. Specifically, because of the log assumption on utility from consumption, we have:

$$v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) = \frac{u\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)}{u_{c_t^p}\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) \cdot c_t^p} = u\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$$

$$\Longrightarrow \qquad \boxed{v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) \ = \ v\left(c_{t-1}^p, l_{t-1}, c_{t-1}^k, h_{t-1}^k, b_{t-1}\right) + dU_t} \ , \qquad (1)$$

  where we get dU_t using a first order approximation:

$$dU_t = u_{c_t^p} \cdot c_t^p \cdot \frac{dc_t^p}{c_t^p} + u_{c_t^k} \cdot c_t^k \cdot \frac{dc_t^k}{c_t^k} + u_{l_t} \cdot l_t \cdot \frac{dl_t}{l_t} + u_{h_t^k} \cdot h_t^k \cdot \frac{dh_t^k}{h_t^k} + u_{b_t} \cdot b_t \cdot \frac{db_t}{b_t} \ .$$

  This reflects that, compared to our baseline treatment, the mapping of $v\left(c_t^p, c_t^k, \overrightarrow{x_t}\right)$ through time and across countries reflects, not only parent's consumption growth, but also growth in kid's consumption, leisure, and the number and quality of kids.

  Using parent's optimality conditions and budget constraint:

$$\begin{aligned} dU_t = & \frac{dc_t^p}{c_t^p} + \alpha b_t^\theta \cdot \frac{dc_t^k}{c_t^k} + \left(1 + \alpha b_t^\theta\right) \frac{l_t}{l_{c_t}} \cdot \frac{dl_t}{l_t} \\ & + \left(1 + \alpha b_t^\theta\right) \frac{b_t e_t}{l_{c_t}} \cdot \frac{1}{\eta} \frac{dh_t^k}{h_t^k} + \left[\left(1 + \alpha b_t^\theta\right) \frac{b_t\left(\phi + e_t\right)}{l_{c_t}} + \alpha b_t^\theta\right] \cdot \frac{db_t}{b_t} \ . \end{aligned} \qquad (2)$$

So once we have $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$ for a country in a base year, we can obtain $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$ in other years for that country by using equations (1) and (2).

- For all countries, we choose 2006 as the base year.
- For the U.S. in 2006, our calibration strategy (described above) yields This yields, for the U.S. in 2006, $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) = 4.61$.
- To get $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$ for the remaining countries in 2006 (base year), we use chain-weighting across countries with the U.S. acting as the "base country". To do this chain weighting, we first rank the six countries based on their per capita consumptions in 2006. We then use equations (1) and (2) to calculate the change (percent differential) in $v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)$ across any two "consecutive" countries. In calculating that percent differential:
  - We use arc growth rates:

$$\frac{dx}{x} = \frac{x_i - x_{i-1}}{1/2 \cdot x_{i-1} + 1/2 \cdot x_i}\,.$$

  - We employ Tornqvist weights — that is, weights in equation 2 are the average of the corresponding values in the 2 consecutive countries;
  - For the g_h^k terms: We assume $g_h^k = g_h$; we then back out g_h from the budget constraint (income accounting), assuming one-half of labor productivity differences across the two consecutive countries in 2006 reflect human capital differences.