Properties of the Combinatorial Clock Auction*

Jonathan Levin and Andrzej Skrzypacz†

March 2016

Abstract

The combinatorial clock auction is becoming increasingly popular for large-scale spectrum awards and other uses, replacing more traditional ascending or clock auctions. We describe some surprising properties of the auction, including a wide range of ex post equilibria with demand expansion, demand reduction and predation. Our results obtain in a standard homogenous good setting where bidders have well-behaved linear demand curves, and suggest some practical difficulties with dynamic implementations of the Vickrey auction.

*We thank Chiara Farronato, Maarten Janssen, Paul Klemperer, Erik Madsen, David Salant, Ernesto Wandeler and Bob Wilson for useful comments and discussions. The authors have advised bidders and the U.S. Federal Communications Commission on radio spectrum auctions.

†Levin: Department of Economics, Stanford University, and NBER, jdlevin@stanford.edu; Skrzypacz: Graduate School of Business, Stanford University, skrz@stanford.edu.

1 Introduction

In this paper we study some properties of a new auction design, the combinatorial clock auction (or CCA). The CCA was proposed by Ausubel, Cramton and Milgrom (2006). It is essentially a dynamic Vickrey auction. The Vickrey auction is central to economic theory as the unique auction that provides truthful incentives while achieving an efficient allocation. Yet it is often viewed as impractical for real-world applications because it requires bidders to submit bids for many possible packages of items.[1] Economists think of dynamic auctions as having an advantage in this regard because bidders can discover gradually how their demands fit together — what Paul Milgrom has called the "package discovery" problem.

The CCA combines an initial clock phase, during which prices rise and bidders state their demands in response to the current prices, with a final round in which bidders submit sealed package bids. The seller uses the final bids to compute the highest value allocation and the corresponding Vickrey payments.[2] Ideally, bidders demand their most desired package at every stated price in the clock phase, allowing for information revelation. Then in the final round, they bid their true preferences, leading to an efficient allocation with truthful Vickrey prices. The question we address is whether this is the likely equilibrium outcome of the CCA; that is, whether the desirable incentive properties of the Vickrey auction are retained.

The practical motivation for our study is the recent and widespread adoption of CCA bidding to sell radio spectrum licenses.[3] Spectrum auctions have provided the motivation for some important recent innovations in auction design, starting with the simultaneous ascending auction pioneered by the FCC in the early 1990s and subsequently adopted in many other countries (Klemperer, 2004; Milgrom, 2004). The FCC design allows for gradual information revelation, but it does not easily accommodate package bidding, and it creates incentives for demand reduction because winners pay their bids (Cramton, 2013). In principle, the CCA addresses both of these issues.

As it turns out, the CCA does have an equilibrium in which bidding is truthful and the outcome

[1] Ausubel and Milgrom (2006) offer additional reasons why sealed-bid Vickrey auctions have not caught on in practice, including vulnerability to collusion even by losing bidders, incentives for shill bidding, and the potential for low revenue.

[2] The original CCA of Ausubel, Cramton and Milgrom (2006) and the rules used in practice actually call for a slight modification of Vickrey pricing, where the Vickrey prices are adjusted upwards if the outcome is outside the core. This "core-adjustment" will not be relevant in our model, but in general it means the truth-telling properties of the Vickrey auction may not apply. For papers on core-adjustment see for example Day and Raghavan (2007), Day and Milgrom (2008), Edril and Klemperer (2009), Ausubel and Baranov (2010), Beck and Ott (2013) and Goeree and Lien (2013).

[3] Countries that have used CCAs to sell radio spectrum licenses include Australia, Austria, Canada, Denmark, Ireland, the Netherlands, Slovenia, Slovakia, Switzerland, and the UK. For more background on spectrum auctions, see Cramton (2013) or Loertscher, Marx and Wilkening (2014).

is efficient. But it is a rather tenuous equilibrium, and we identify two distinct reasons to doubt it. The first arises from the fact that CCA bidders are asked to submit their demands twice: during the clock phase and then in the final round. These demands are linked by activity rules, which are essential in a dynamic auction to make early bids meaningful and prevent bidders from holding back like eBay snipers (Ausubel, Cramton and Milgrom, 2006). The CCA activity rules have the feature that the clock phase bids can pin down exactly the allocation of items (Ausubel and Cramton, 2011; Ausubel and Baranov, 2013). Then the final bids determine the payments. But with Vickrey pricing, a bidder cannot affect her own payment unless she changes the allocation. So each bidder may be completely indifferent across her permissible final bids, despite the choice affecting the prices paid by rivals and hence incentives in the clock phase. To support the truthful and efficient equilibrium, bidders must maximally raise their final bids. But because bidders may adopt different strategic postures in their final bids, there are also a wide range of other, inefficient, ex post equilibria.

The second issue we investigate involves predatory bidding. With a Vickrey pricing rule, bidders have a strict incentive to bid truthfully only if their bid has a positive probability of winning. Yet in a CCA, bidders may have the opportunity in the clock phase to exaggerate their demand with essentially no risk of winning. By doing so, they can relax the activity rule constraints on their final bids and raise rival payments. A bidder who anticipates this type of predation has an incentive to reduce demand to avoid paying predatory prices, again leading to inefficient outcomes. While the "two demands" problem is rather specific to the CCA rules, the potential for predation seems likely to arise in any dynamic implementation of the Vickrey auction. In the CCA case, the upshot is that bidders must behave "just right" to support the truthful and efficient equilibrium, raising their final bids to the limit allowed by the activity constraints, but taking no action in the clock phase to purposefully relax these constraints.

We develop these points in a series of simple models. We start in Section 2 by describing the CCA rules and providing an example of how they work. We then focus on a standard allocation problem where bidders have linear downward sloping demand curves. In Section 3, we show how different strategic postures in the final round each give rise to a range of ex post and typically inefficient equilibria. The multiplicity arises even though we focus on (linear) proxy strategies in which bidders do not condition on rival bidding behavior. Section 4 then considers the possibility that bidders may prefer to drive up rival prices if they can do so without reducing their own payoff. We characterize an ex post equilibrium in which one bidder is predatory in the clock phase while the other restricts attention to (linear) proxy strategies and reduces his demand to keep the price down. The Appendix also considers a version of the model in which both bidders are able to relax the final bid activity constraints, and equilibrium outcomes involve demand reduction in the clock phase and are again inefficient.

The models suggest that CCA rules permit a wide range of plausible behaviors and outcomes. In Section 5, we provide some evidence from recent auctions for radio spectrum. We find that even large and sophisticated bidders have adopted widely varying strategic postures in their CCA final bids. And in at least one high-stakes auction, bidders appear to have taken actions in their clock phase that served to relax final bid constraints and raise rival prices.

Our paper relates to an extensive literature on multi-item auctions (e.g. Milgrom, 2004, 2007). One of its central messages is that multi-item auction design involves hard trade-offs. The standard auctions that have been considered - simultaneous ascending and clock auctions, pay-as-bid combinatorial auctions - have well-documented limitations. In this sense, it is hardly surprising that the CCA also appears to have some drawbacks from a strategic perspective. In fact, one point we emphasize in the conclusion is that our analysis highlights a serious challenge for any attempt to implement a dynamic Vickrey auction, namely that it may be difficult to provide incentives for bidders to submit truthful bids for packages that they are very unlikely to win, despite these bids potentially being important for setting rival prices.

More narrowly, the novelty of the CCA design means there are not many directly related papers. The closest are Janssen and Karamychev (2013) and Janssen and Kasberger (2015). The first of these papers analyzes bidding incentives in a CCA with discrete quantities and multiple products. It shows that when bidders have preferences for raising rivals' costs (modeled similarly to what we do in Section 4), bidders have an incentive to submit large final round bids and bid aggressively in the clock phase. It also considers the implications of bidder budget constraints. The second paper extends our analysis by characterizing equilibria in non-linear proxy strategies when bidders prefer to raise rivals' costs. A nice insight is that in some cases, it is possible to construct equilibria where bidders engage in predation but the outcome is nonetheless efficient. However, under some conditions this is not possible and all symmetric proxy strategy equilibria are inefficient. Finally, Salant (2013) provides a broader review of practical auction design that covers the CCA as well as competing formats.[4]

[4] Additionally, Knapek and Wambach (2012) discuss why bidding in a CCA may be strategically complicated; Bichler et.al. (2013) present experimental results on the CCA.

2 The Combinatorial Clock Auction

The combinatorial clock auction can be used with multiple bidders and multiple products in different quantities. For concreteness, we will assume there are two bidders, $i = 1, 2$, and a single product. The product is perfectly divisible and there is unit supply.

The auction consists of an initial *clock phase* and a *final bid round*. In the clock phase, the seller gradually increases the price p. In response, the bidders announce demands x_1, x_2. A bidder may reduce her demand, but not increase it, as the auction proceeds. The price increases until there is no excess demand. If at this point $\sum_i x_i = 1$, we say there is *market clearing*. Alternatively if $\sum_i x_i < 1$, there is *excess supply*. Our analysis will focus on the case where the starting price is low and bidders reduce their demands continuously, so there is never excess supply.

After the clock phase, the bidders submit final (sealed) bids $S_1(x_1)$ and $S_2(x_2)$ that express valuations for all possible quantities (i.e. each bid S_i is a function). The seller computes the allocation and winner payments based on these final bids. She selects the allocation that maximizes $\sum_i S_i(x_i)$ subject to the feasibility constraint that $\sum_i x_i \leq 1$. She then computes the Vickrey payment for each bidder. Bidder i's Vickrey payment is $\max_{x \in [0,1]} S_j(x) - S_j(x_j^*)$, where x_j^* is j's winning quantity.[5] If S_j is increasing, i pays $S_j(1) - S_j(x_j^*)$.

The final bids are constrained by activity rules that tie them to bids in the clock phase. The activity rules we describe correspond to those used in recent CCA auctions.[6] There are three parts. First, bids in the clock phase are binding. If at price p, bidder i demanded x, then bidder i's final bid for x units must be at least px, i.e. $S_i(x) \geq px$ for any p that was quoted in the clock phase. Second, final bids must satisfy revealed preference with respect to the last clock bids. If the clock phase ends at a price p^*, with i demanding x_i^*, then for any $x \neq x^*$, $S_i(x) - p^* x \leq S_i(x^*) - p^* x^*$. Third, for quantities $x \geq x^*$, i's final bids must satisfy an additional local form of revealed preference. If p was the highest price at which i demanded x or more units, then i cannot express an incremental value greater than p for obtaining slightly more than x, i.e. $\lim_{\varepsilon \to 0^+} \frac{S_i(x+\varepsilon) - S_i(x)}{\varepsilon} \leq p$.[7]

We illustrate these activity rules with an example. For this example, we assume that during

[5] We do not consider reserve prices, but they can be incorporated by adding a "bidder 0" that bids $S_0(x)$ where $S_0(1) - S_0(x)$ is the required revenue to sell $1 - x$ units.

[6] It is exactly the rule used in Canada, and equivalent to the rules used in Switzerland, Ireland, Netherlands, and UK if those auctions had been run for only a single category of licenses. The exact rules used in these auctions vary somewhat in how they handle multiple license categories.

[7] An obvious candidate for an activity rule would be to require global revealed preference. That is, if i demanded x at price p, then for any $z \neq x$, final bids must satisfy $S_i(x) - px \geq S_i(z) - pz$. In our model, this rule would *impose* what we later will call consistent bidding. However, as noted by Ausubel and Cramton (2011), such an approach seems unworkable with multiple categories because it can lead to "dead ends". In response to this, Ausubel and Baranov (2013) have suggested a global approach based on GARP. We discuss their proposal in the conclusion.

the clock phase bidder 2 behaves as if she has a value for x_2 units equal to $V_2(x_2) = 2x_2 - \frac{1}{2}x_2^2$, and a diminishing marginal value $v_2(x_2) = 2 - x_2$.[8] So when the price is p, she demands $x_2(p) = 2 - p$ units irrespective of bidder 1's behavior. The quantity $1 - x_2(p)$ is the residual supply available to bidder 1 at price p. The clock price starts at $p = 1$. It rises so long as bidder 1 demands $x_1(p) > 1 - x_2(p)$, and stops as soon as $x_1(p) \leq 1 - x_2(p)$. Suppose the clock phase ends at a price p^* with bidder 2 demanding $x_2^* = 2 - p^*$, and bidder 1 demanding $x_1^* = 1 - x_2^*$.

What are the permissible final bids for bidder 2? Despite the activity rules, she retains considerable flexibility. Figure 1 illustrates the possibilities. The lower curve shows the bids that bidder 2 submitted during the clock phase. As the price rose, bidder 2 reduced her demand continuously from 1 to x_2^*. For quantities below x_2^*, bidder 2 has not recorded any bid during the clock phase, so her bid is zero. For quantities above x_2^*, bidder 2 demanded x_2 units when the price was $p = v_2(x_2)$. When she did this, she submitted a bid of $x_2 v_2(x_2)$ for x_2 units. Of course this is less than the value function $V_2(x_2)$ that guided bidder 2's clock bidding because the auctioneer records the *revenue* from bidder 2's bid at price $p = v_2(x_2)$, and does not include her consumer surplus.

The activity rules state that bidder 2's final bid S_2 must lie everywhere above the bids she recorded during the clock phase. Also, above x_2^* the final bid function cannot rise more steeply than V_2. It must satisfy the local revealed preference restriction: $S_2'(x_2) \leq v_2(x_2)$. So at the upper extreme, bidder 2 can raise her bids to the dashed curve, which rises at a slope $v_2(x_2)$ from her final clock bid.[9] The shaded area in Figure 1 shows that space in which the final bid function must lie, for quantities above x_2^*. Below x_2^*, bidder 2 may submit any non-negative bid S_2 so long as it lies below the dotted line with slope p^* that runs straight from the origin to bidder 2's final clock bid. These bids, however, are not important for pricing.[10]

Two types of final bids will be important later. We say that bidder 2 is *consistent* if she raises her final bid above x_2^* to its maximum amount. When she does this, she expresses marginal values that correspond exactly to her (inverse) demand in the clock phase. That is, her demands are consistent in the two parts of the auction. In contrast, we say that bidder 2 is *quiet* if she does not raise her final bids at all, so that $S_2(x_2) = xv_2(x_2)$. Under quiet bidding, S_2 corresponds to the

[8] For this example it is not important whether V_2 is bidder 2's actual valuation. In our equilibrium analysis below, bidders generally will not bid truthfully in the clock phase.

[9] Our discussion presumes that bidder 2 does not increase her clock bid for x_2^*, i.e. sets $S_2(x_2^*) = x_2^* v_2(x_2^*)$. We will assume this throughout for simplicity. If bidder 2 raises her bid for x_2^* by Δ she is permitted to translate all her bids up by this same amount, so there is an analogue of quiet and consistent bidding for any choice of Δ. Of course, if bidder 2 raises her bid for x_2^*, but does not raise her other bids, then bidder 1 could end up paying very little, and in fact pays zero if S_2 achieves its maximum at x_2^*. This creates the possibility that bidder 2's final bids might be chosen in a way that drives down bidder 1's payment relative to quiet bidding, but we will not focus on it.

[10] That is because the activity rules prevent bidder 2 from claiming to value $x_2 < x_2^*$ more than she values x_2^*. So the maximum of S_2 will be achieved at a quantity $x_2 \geq x_2^*$.

revenue generated by bidder 2's clock phase demand, and the slope of S_2 is the *marginal revenue* associated with assigning more units to bidder 2, rather than the *marginal values* implicit in bidder 2's clock demand. In addition to consistent and quiet bidding, there are intermediate cases as well, since in the range $x_2 \geq x_2^*$ (Region II in the figure), bidder 2 can select any $S_2(x_2)$ that lies in the shaded area and rises less steeply than $V_2(x_2)$.

Now we turn to an important implication of the activity rules that holds whether bidders are consistent, quiet or something intermediate. Suppose the clock phase ends with market clearing. Then the final clock demands will be value-maximizing for *any* permissible final bids. To see why, let x_1^*, x_2^* be the final clock demands. Consider an alternative assignment where each bidder i receives $x_i = x_i^* + \varepsilon_i$, with $\sum_i \varepsilon_i \leq 0$ required for feasibility. From the second activity rule requirement,

$$\sum_i S_i(x_i) \leq \sum_i S_i(x_i^*) - p^* \cdot (x_i^* - x_i) = \sum_i S_i(x_i^*) + p^* \cdot \sum_i \varepsilon_i \leq \sum_i S_i(x_i^*). \tag{1}$$

This has the following consequence noted by Ausubel and Cramton (2011). If the clock phase ends with market clearing and ties are resolved in favor of the clock phase allocation, bidder i's quantity *and* payment do not depend on her final bids. The payment part follows from Vickrey pricing: fixing i's quantity, her payment depends only on the bids of others. Therefore if a bidder is maximizing her own individual profit she will be *completely indifferent* across all permissible final bids. However, these bids are very important for prices paid by the other bidder. To see this, assume for simplicity that S_2 is increasing. Then bidder 1 must pay $S_2(1) - S_2(x_2^*)$ — bidder 2's bid for all units minus her bid for the units she wins. This amount is higher if bidder 2 is consistent than if she is quiet.

3 Bidding in the CCA: Discounts and Demand Expansion

In this section and the following ones, the environment is the same. There is a single divisible unit to be allocated. Each bidder $i = 1, 2$ has marginal value for an xth unit given by $u_i(x) = a_i - b_i x$, where $a_i \geq b_i > 0$. Bidder i's total value for x units is $U_i(x) = \int_0^x u_i(z)\,dz = a_i x - \frac{1}{2} b_i x^2$. Throughout, we use lower case to denote marginal values and upper case to denote total values. We assume the a_i's are private information and take values between $[\underline{a}_i, \overline{a}_i]$, while the b_i's are common knowledge. To avoid messy corner solutions, we assume $\overline{a}_i - b_i < \underline{a}_j$, so that efficient allocation is interior and satisfies $u_1(x_1) = u_2(1 - x_1)$.

A *proxy strategy* for bidder i consists of two functions: marginal values $v_i(x)$ for the clock phase (assumed to be decreasing and continuous), and marginal values $s_i(x)$ for the final bid.[11] In the clock phase, bidder i demands x when $p = v_i(x)$ (or $x = 1$ if $p < v_i(1)$). This expresses a bid $xv_i(x)$ for x units. If the clock phase ends at a price p^*, with i demanding x_i^*, then in the final round, she bids $S_i(x) = x_i^* p^* + \int_{x_i^*}^{x} s_i(z)\,dz$ for quantities $x \geq x_i^*$. To satisfy the activity rules, the proxy values must satisfy, for all x: (i) $S_i(x) \geq xv_i(x)$, and (ii) $s_i(x) \leq v_i(x)$. The latter condition ensures revealed preference with respect to the final clock bid for $x \geq x_i^*$: it implies that $s_i(x) \leq p^* = v_i(x_i^*)$. For quantities $x < x_i^*$, bidder i can set $S_i(x) = 0$ — any bids that satisfy the activity rules in this region cannot affect allocation or pricing.

We restrict bidders to use proxy strategies and focus on equilibria in *linear* proxy strategies, in which each bidder bids a linear demand curve in the clock phase, and adopts a mixture of quiet and consistent behavior in the final round. Formally, bidder i specifies a linear demand for the clock phase

$$v_i(x) = A_i - B_i x, \tag{2}$$

with $A_i \geq B_i > 0$, and associated marginal revenue curve

$$m_i(x) = \frac{d}{dx} xv_i(x) = A_i - 2B_i x. \tag{3}$$

Bidder i also specifies a linear demand for the final bid round:

$$\begin{aligned} s_i(x) &= (1-\gamma_i)\, v_i(x) + \gamma_i m_i(x) \\ &= A_i - (1+\gamma_i)\, B_i x \end{aligned} \tag{4}$$

The parameter $\gamma_i \in [0,1]$ captures the extent to which bidder i is consistent ($\gamma = 0$) versus quiet ($\gamma = 1$). Higher values of γ_i mean lower marginal prices for bidder j, because i is expressing less value for any particular unit. It is easy to see that for any value of γ_i, $s_i(x)$ satisfies the activity rules.

A linear proxy strategy requires the choice of three parameters: A_i, B_i and γ_i. Provided the clock price starts sufficiently low, the clock phase will end with market clearing. So for any A_j, B_j, γ_j, bidder i's payoff will depend only on her choice of A_i, B_i, and will be independent of her choice of

[11] We call these proxy strategies because bidders express preferences (i.e. marginal values) that get transformed into demands in the auction. This is the same approach and terminology used by, Ausubel and Milgrom (2002) in their analysis of ascending auctions with package bidding. We focus on proxy strategies in order to emphasize issues that are specific to the CCA. The CCA also admits a rich set of contingent or bootstrapped equilibria, in which bidder j makes a certain demand in equilibrium because she believes that if she doesn't, bidder i will punish her as the auction proceeds. However, these types of dynamics are familiar from other dynamic auctions and are not specific to the CCA.

γ_i. Therefore to characterize equilibria, we fix γ_1 and γ_2 as parameters, and solve for equilibrium choices of A_1, A_2, B_1, B_2. We focus on equilibria that are ex post, in that for any a_i, the strategy adopted by bidder i is a best response to j's strategy for every value of $a_j \in [\underline{a}_j, \overline{a}_j]$.

3.1 Proxy Best Responses

We start by deriving best responses for bidder 1. We show that if bidder 2 uses a linear proxy strategy, where A_2 varies with her private information a_2, but B_2 and γ_2 do not, then bidder 1 always has an ex post best response that involves using a linear proxy strategy in which A_1 is the only parameter to vary with a_1 (so that iterating on the best-response correspondence keeps us within this class of strategies).

Suppose bidder 2 bids according to $v_2(x) = A_2 - B_2 x$. If bidder 1 bids according to $v_1(x)$, and the clock allocation is interior, then bidder 1 obtains quantity x_1 such that

$$v_1(x_1) = v_2(1 - x_1). \tag{5}$$

Then after the final bid round, bidder 1 will pay $S_2(1) - S_2(1 - x_1)$. His final payoff will be:

$$U_1(x_1) - [S_2(1) - S_2(1 - x_1)]. \tag{6}$$

A necessary condition for bidder 1's strategy to be ex post optimal is that knowing v_2, he does not prefer to purchase a slightly larger or smaller quantity than x. The marginal benefit of additional quantity is $u_1(x)$, and the marginal price is $s_2(1 - x)$. So a necessary condition for ex post optimality is that:

$$u_1(x) = s_2(1 - x). \tag{7}$$

This condition is also sufficient for ex post optimality if it holds for all v_2, x that satisfy (5).[12]

Substituting (5) into (7), and using the fact that bidder 2 is playing the linear proxy strategy $s_2(1 - x) = v_2(1 - x) - \gamma_2 B_2(1 - x)$, we obtain a best response for bidder 1:

$$v_1(x) = u_1(x) + \gamma_2 B_2(1 - x). \tag{8}$$

[12] Note that our derivation assumes for notational convenience that $S_2(x)$ is maximized at $x = 1$, but does depend on this. To see that condition (7) is sufficient, note that given v_2, bidder 1's global best response problem is first to choose x_1 that maximizes $U_1(x) - \{\max_y S_2(y) - S_2(x)\}$, which is a concave problem, and then choose some decreasing v_1 such that $v_1(x_1) = v_2(1 - x_1)$. By this reasoning, the strategies we characterize remain best responses even if we remove the restriction to continuously-decreasing proxy strategies. For example, even if bidders could drop demand discontinuously to end the clock phase with excess supply, they would not find it profitable.

Therefore bidder 1 can follow the linear proxy strategy $v_1(x) = A_1 - B_1 x$, with

$$A_1 = a_1 + \gamma_2 B_2 \qquad \text{and} \qquad B_1 = b_1 + \gamma_2 B_2, \tag{9}$$

and this is a best response for every a_2.

Note that in general the best response of bidder 1 deviates from truth-telling and the optimal deviation depends on the behavior of bidder 2 in the final bid round. The closer is bidder 2 to a quiet strategy in the final round, the more aggressive is the best response of bidder 1. To gain intuition, recall that the clock phase determines allocations and the final bid determines prices. Bidder 1's price for his final unit equals bidder 2's final marginal bid for this unit, $s_2(x_2^*)$. Under consistent bidding $s_2(x_2) = v_2(x_2)$. Under quiet bidding $s_2(x_2) = \frac{d}{dx_2} x_2 v_2(x_2)$. Consistent bidding means that bidder 1 must pay the clock price at which bidder 2 gave up the final unit. Quiet bidding means that bidder 1 must pay only the (smaller) marginal revenue reduction.

Figure 2 shows the best response problem for bidder 1, and how it depends on bidder 2's behavior. The x-axis shows the allocation: as we move to the right, we increase x_1 and decrease x_2. The y-axis is dollars. The solid green line plots bidder 2's clock demand $v_2(x_2)$. If bidder 2 is consistent in the final round, this is also her final bid, and the marginal prices faced by bidder 1, so bidder 1's best response is to purchase the quantity x_1', at which $u_1(x_1') = v_2(1 - x_1')$. He can do this by bidding truthfully. The dotted green line shows the marginal revenue $m_2(x_2)$ associated with v_2. If bidder 2 is quiet in the final round, this line represents bidder 2's final bids. Bidder 1's best response is to purchase the quantity x_1'' at which $u_1(x_1'') = m_2(1 - x_1'')$. To do this, he needs to inflate his clock round bid, so that $v_1(x_1'') = v_2(1 - x_1'')$, as shown in the picture.

The figure shows the optimization problem for bidder 1 for a single value of a_2. However, bidder 1 wants the clock phase to end with his marginal benefit for additional quantity $u_1(x_1)$ just equal to the marginal price $s_2(x_2)$ for each realization of a_2. To make this happen, bidder 1 needs to have $v_1(x_1) = v_2(x_2)$ at the relevant x_1, x_2. Solving the optimization problem for each a_2 traces out bidder 1's best-response demand curve $v_1(x_1)$. This is illustrated in Figure 3.

The best-response derivation highlights a key strategic issue in the CCA. On the one hand, if a bidder cares only about his own payoff, he is completely indifferent across permissible final bids (any γ_i is a best response). On the other hand, the way the indifference is resolved is very important for determining rival incentives in the clock phase. A bidder will want to bid truthfully in the clock phase if his rival uses a consistent final bid strategy, but overstate his clock demand if he anticipates a quiet final bid strategy.

3.2 Proxy Equilibria

We now solve for an ex post equilibrium in linear proxy strategies. To do this, we combine the best response conditions (9) for bidders 1 and 2. Then, so long as γ_1 and γ_2 are not both equal to one,

$$A_1 = a_1 + \frac{\gamma_2 (\gamma_1 b_1 + b_2)}{1 - \gamma_1 \gamma_2} \equiv a_1 + \lambda_1, \tag{10}$$

and

$$B_1 = b_1 + \frac{\gamma_2 (\gamma_1 b_1 + b_2)}{1 - \gamma_1 \gamma_2} \equiv b_1 + \lambda_1. \tag{11}$$

Proposition 1 *Fix any $\gamma_1, \gamma_2 \in [0, 1]$ with $\gamma_1 \gamma_2 < 1$. The CCA has an ex post equilibrium in linear proxy strategies, in which bidder i bids according to $v_i(x)$ in the clock phase and $s_i(x)$ in the final round, with*

$$\begin{aligned} v_i(x) &= u_i(x) + \lambda_i (1 - x), \\ \textit{and } s_i(x) &= v_i(x) - \lambda_j x, \end{aligned}$$

where $\lambda_i = \frac{\gamma_j(\gamma_i b_i + b_j)}{1 - \gamma_i \gamma_j}$.

Remark 1 *The above proposition describes equilibria for $\gamma_1 \gamma_2 < 1$. What if both bidders are completely quiet? Then the best responses (9) imply $B_1 = b_1 + B_2$, and $B_2 = b_2 + B_1$. The system "explodes" as $\gamma_1 = \gamma_2 \to 1$ and there is no equilibrium. This non-existence is a consequence of assuming 2 bidders and is familiar from other models with linear demands (e.g., Kyle 1989 or Vives 2011). For example, adding to the model a small non-strategic third bidder would allow us to construct linear proxy equilibria even for $\gamma_1 = \gamma_2 = 1$.*

Figure 4 illustrates the equilibrium. It shows the equilibrium bids $v_1(x_1)$ and $v_1(x_2)$ for a particular realization of a_1, a_2, along with the final bids $s_1(x_1)$ and $s_1(x_2)$. As the clock price rises, the bidders reduce demand along their proxy demand curves $v_1(x_1)$ and $v_2(x_2)$ until reaching market clearing at p^*. At the end of the clock phase, the final allocation is determined to be x_1^*, x_2^*. Then the final bids are made. Bidder 1 pays $S_2(1) - S_2(x_2^*) = \int_{x_2^*}^{1} s_2(x_2)\, dx_2$, which is the shaded area in the figure. The price bidder 1 pays for his last unit is $s_2(x_2^*)$.[13]

[13] We have described equilibria in the game in which the players choose proxy strategies. A natural question is whether the same behavior could be supported as equilibria if players choose their clock demands strategically in response to a continuously increasing price. This is relatively easy to establish if the bidders only observe the price and not their opponent's current demand. In the latter case, formalizing the game in continuous time is cumbersome. However, we can sketch how off-path equilibrium beliefs might be chosen to support the behavior above. If j initially bids according to the proposed strategy of some type $a_j \in [\underline{a}, \overline{a}]$, but then deviates, one can use "renewal beliefs"

3.3 Properties of Equilibria

Bidding Behavior. The equilibrium involves bidders engaging in *demand expansion* during the clock phase because they perceive that their true marginal prices will be *discounted* from the clock price if their opponent is less than consistent, $\gamma_2 > 0$. The residual supply available to bidder 1 will be x when the clock price is $p = v_2(1-x)$. Yet bidder 1's true marginal price for buying the xth unit is only $s_2(1-x) = p - \lambda_1(1-x)$. The discount is greatest if bidder 1 is buying a small quantity, and smaller for large quantities. So unless bidder 2 is consistent, the equilibrium response is to engage in demand expansion.

Allocation and Revenue. The equilibrium allocation is generally not efficient and the revenue differs from what would occur in a Vickrey auction with truthful bidding (unless both bidders are truthful and consistent, in which case allocation is efficient, and each bidder pays its truthful Vickrey price).

We can obtain a fairly sharp characterization for symmetric equilibria in which $\gamma_1 = \gamma_2 = \gamma$ and $b_1 = b_2 = b$. In this case, bidder i's clock phase strategy is $v_i(x) = u_i(x) + \frac{\gamma}{1-\gamma} b(1-x)$. Provided that $\gamma > 0$, the equilibrium allocation will be distorted toward 1/2 because a low-value bidder inflates her clock phase demand more than a high-value bidder. In particular, the efficient allocation (where $u_1(x_1) = u_2(1-x_1)$) occurs at $x_1^e = \frac{1}{2} + \frac{a_1 - a_2}{2b}$. The equilibrium outcome x_1^* solves $v_1(x_1) = v_2(1-x_1)$, which means

$$x_1^* = \frac{1}{2} + \frac{a_1 - a_2}{2b}(1-\gamma) = \frac{1}{2}\gamma + (1-\gamma)x_1^e. \tag{12}$$

It is also possible to show for the case of symmetric equilibria that the CCA revenue is decreasing in γ. Since the outcome with $\gamma = 0$ corresponds to a Vickrey auction with truthful bidding, this means that every symmetric equilibrium that involves any degree of quiet final round bidding ($\gamma > 0$) generates lower expected revenue than a truthful Vickrey auction. The derivation of this result requires some calculations that we go through in the Appendix. The Appendix also shows an example demonstrating that the revenue ranking is ambiguous for asymmetric equilibria.

where i assumes that from demand x_j at price p, j will bid according to the linear strategy of the type a_j that would have bid x_j at p (these are called renewal beliefs because they are applied regardless of j's prior behavior up to p). To make this work for all deviations, one can expand the interval of possible bidder j types by adding zero probability types so that for any x_j chosen at p, there is some a_j that would have chosen x_j under the linear strategy described above.

4 Predatory Behavior

The previous section emphasized that each bidder is indifferent between alternative final bids, yet her choice matters for how her opponent should behave in the clock phase. The result is that even if we restrict attention to relatively non-strategic proxy strategies, there are many equilibria involving varying amounts of demand expansion in the clock phase, to compensate for the price reductions offered in the final bid round. The equilibrium is generally not efficient, unless both bidders raise their bids fully in the final round.

In practice, bidders may not be truly indifferent. A bidder might benefit from raising rival costs, or from looking good relative to opponents. This possibility is discussed by Morgan, Steiglitz and Reis (2003), Janssen and Karamychev (2013) and Janssen and Kasberger (2015), among others. Such a bidder will want to be consistent in her final bid. However, she may also have an incentive to relax her activity constraints by exaggerating demand in the clock phase. Figure 5 illustrates the potential for this type of predatory bidding. In this figure, we start from the equilibrium in which both bidders are truthful and consistent. The solid area shows bidder 1's payment. If instead bidder 2 overstates her clock demand, by demanding $x_2 = 1$ until p^*, she does not change the allocation or her own payment. However, by bidding consistently with this inflated clock demand in the final round, she forces bidder 1 to pay the clock price p^* for all of his units. As we now show, if bidder 1 anticipates this predatory behavior, he will want to engage in demand reduction, leading to an inefficient outcome where the predatory bidder obtains an advantage.

4.1 Proxy Bidding and Predatory Best Responses

We now consider a version of the model in which bidder 2 first maximizes her own profit, and second attempts to make her rival pay more. We assume that bidder 1 bids some proxy strategy $v_1(x)$ in the clock phase, and then bids consistently in the final round, $s_1(x) = v_1(x)$.[14] We allow bidder 2 to use any bidding strategy so long as she does not create excess supply. To model this, we allow bidder 2 to drop her demand discretely, say from x_2' to x_2, at a given price p, but assume that if she does this, she offers to buy any intermediate quantity at that price. The last assumption ensures there will be market clearing.[15]

[14] The equilibrium we derive below is consistent with bidder 1 also having a lexicographic preference for raising bidder 2's payment, so long as either bidder 1 restricts attention to linear proxy strategies, or restricts attention to a continuous proxy strategy and there is sufficiently rich support on the possible equilibrium allocations. See Janssen and Kasberger (2015) for a follow-up to this paper which offers a more complete analysis of the case when both bidders have lexicographic preferences.

[15] In particular, if bidder 1 is demanding x_1 at p and $1 - x_1$ is strictly between x_2' and x_2, then when bidder 2 drops her demand from x_2' to x_2, bidder 2 will be assigned $1 - x_1$.

Bidder 2's most effective strategy is to keep her demand at 1 until the price p and residual supply $1-x_1$ reach a level at which

$$u_2(1-x_1) = p = v_1(x_1), \tag{13}$$

and then reduce her demand to $1-x_1$ ending the auction. She can then submit her maximal final bid of $S_2(z) = pz$ for $z \geq 1-x_1$, and make bidder 1 pay the clock price for all x_1 units.

Why is this optimal for bidder 2? First, consider bidder 2's problem of maximizing her own profit. To buy x_2 units and obtain value $U_2(x_2)$ she must pay $S_1(1) - S_1(1-x_2) = V_1(1) - V_1(1-x_2)$.[16] So she wants to choose a quantity x_2 that maximizes:

$$U_2(x_2) - [V_1(1) - V_1(1-x_2)]. \tag{14}$$

Therefore, her ex post optimal quantity is the unique solution to $u_2(x_2) = v_1(1-x_2)$. Moreover, conditional on buying x_2 units and ending the auction with market clearing, the most she can possibly make bidder 1 pay for his $x_1 = 1-x_2$ units is $x_1 v_1(x_1)$, which she achieves.

Remark 2 *If we allowed bidder 2 to create excess supply at the end of the clock phase, she could increase bidder 1's payment even more. For example, suppose bidder 1 follows a proxy strategy $v_1(x) = 1-x$. Then, player 2 with valuation $u_2 = 1-x$ could demand $x_2 = 1$ until the price reaches (almost) 1 and then drop demand to $x_2 = \frac{1}{2}$. She would then submit a final round bid with $S_2(1) = 1$ and $S_2\left(\frac{1}{2}\right) = \frac{5}{8}$. In this way the final allocation would be $\left(\frac{1}{2}, \frac{1}{2}\right)$ as in the best response we described above, but player 1 would end up paying his full value: $S_2(1) - S_2\left(\frac{1}{2}\right) = \frac{3}{8} = V_1\left(\frac{1}{2}\right)$. Such extreme predatory behavior is even more difficult to execute and even more risky for player 2 than what we describe. Moreover, analyzing equilibria in this case is difficult, so we maintain the assumption that player 2 is not allowed to create excess supply in the clock phase.*

4.2 Ex Post Equilibrium with a Predatory Player

How does bidder 2's predatory behavior affect the auction? Because bidder 1 pays the full market clearing clock prices, rather than the Vickrey payment, he optimally responds by reducing demand.

For bidder 1 to purchase x, he must pay $xu_2(1-x)$. So bidder 1's ex post best response solves

$$\max_x U_1(x) - xu_2(1-x). \tag{15}$$

[16] This formula assumes that $v_1(x) > 0$ for all x. If $v_1(x)$ is negative for large x, then bidder 2 pays $\max_x V_1(x) - V_1(1-x_2)$. Either way, bidder 2's best response is to select x_2 such that $u_2(x_2) = v_1(1-x_2)$.

The unique optimal x satisfies

$$u_1(x) = u_2(1-x) - xu_2'(1-x) = u_2(1-x) + b_2 x. \tag{16}$$

This implies demand reduction: $-xu_2'(1-x) > 0$, so in equilibrium player 1 gets a quantity that is smaller than efficient (efficiency is where $u_1(x) = u_2(1-x)$).

To implement the optimal x, bidder 1 needs the clock phase to end when $v_1(x) = u_2(1-x)$. Therefore the following linear proxy strategy is ex post optimal against any u_2 parameterized by a_2 :

$$v_1(x) = u_1(x) - b_2 x = a_1 - (b_1 + b_2)x. \tag{17}$$

Proposition 2 *Suppose bidder 2 has a lexicographic preference for making its rival pay more. Then there is an ex post equilibrium in which bidder 1 uses the proxy strategy $v_1(x) = u_1(x) - b_2 x$ in the clock phase, and bidder 2 maintains demand of 1 until dropping demand immediately to $1-x$ when $p = v_1(x) = u_2(1-x)$, and then both bidders are consistent in the final round.*

We emphasize that the cause of the inefficiency in the above equilibrium is distinct from what we identified in Proposition 1. In the previous section, inefficient ex post equilibria arise because bidders, in a situation of indifference, understate their final demand relative to their clock bidding. Here both bidders are consistent in their final round bidding. The inefficiency arises because bidder 2 is able to inflate her inframarginal clock demand and manipulate bidder 1's payment with no direct consequence for her own allocation or payment.

4.3 Properties of the Equilibrium with Predatory Player

Allocation. In the equilibrium we described the allocation is skewed inefficiently in favor of player 2. In particular, the equilibrium allocation is:

$$x_1^* = \frac{a_1 - a_2 + b_2}{b_1 + 2b_2} = x_1^e \frac{b_1 + b_2}{b_1 + 2b_2} < x_1^e, \tag{18}$$

where x_1^e is the efficient allocation. Here the intuition is straightforward: given the demand submitted by the player 1, the predatory player 2 wants to choose the same quantity as if she was bidding truthfully, whereas player 1 engages in defensive demand reduction.

Revenue. There are two opposing effects. First, bidder 1 reduces her demand relative to truthful bidding in the clock phase. Second, bidder 2 forces bidder 1 to pay the final clock price for all her units. With some algebra, one can show that revenue may be above or below the truthful Vickrey revenue. For example, if $b_1 = b_2$ and $a_1 = a_2$, the demand reduction effect dominates and revenue is lower than in the truthful equilibrium. If $b_1 = b_2$ but $a_1 \gg a_2$ (so bidder 1 gets almost all units), the predatory effect dominates and revenues are higher than in the truthful equilibrium.

Mutual Predation. The model we have described has a single predatory bidder. What happens if both bidders are predatory and attempt to push rival payments up toward the final clock prices? In the Appendix, we develop a version of the model in which each bidder has the ability to relax its final bid constraint, and increase its rival's cost. We show that this leads bidders in the clock phase to engage in demand reduction, which again creates inefficiency in the allocation. The modeling approach we adopt in the Appendix is motivated by features of CCA sales with multiple categories, where in practice bidders have a fair amount of flexibility to relax the final round revealed preference constraints. We will discuss an example of this below.

5 Evidence on CCA Bidding Behavior

Given the ambiguous nature of incentives in the CCA, a natural question is whether data from past auctions can tell us more about bidder behavior. Either bidding data or summary reports are publicly available for several CCA sales of radio spectrum licenses. This evidence suggests a striking degree of heterogeneity across bidders and across auctions. Some bidders have submitted minimal final round bids, as in the quiet strategy described above. Some have submitted final bids that express clear valuation increments and resemble the consistent strategy described above. Others seem to have followed strategies designed to make rivals pay prices that are close to the linear prices at the end of the clock phase, as in our predatory bidding example.[17]

5.1 Early UK Auctions (2008)

The United Kingdom held two early CCA sales in 2008: for spectrum licenses in the 10-40 GHz range and then for L Band licenses. The auctions had combined revenue of around £10 million. We have information on these sales from reports released by the UK government (Cramton, 2008a,b;

[17] We discuss evidence from five past auctions below. There is also some recent data available from the Canadian 700 MHz auction conducted in early 2014. Our analysis of that data further supports the claim that there can be a great deal of heterogeneity in bidder behavior. Of the three most active bidders, two (Bell Canada and Telus) submitted final bids for a large number of different license packages (close to 500) at essentially the maximum amount allowed by the activity rules, whereas the third (Rogers), which ended up paying much more for the licenses it won, submitted only a single final round bid, with which it increased its bid for its winning package.

Jewitt and Li, 2008).[18]

In the 10-40 GHz auction, there were ten bidders competing for 27 available licenses. There were 2.2 bids on average across licenses in the initial round of the clock phase, and it took 17 rounds to reach market clearing. However, there was relatively little activity in the final bid round, despite all ten bidders winning licenses. Only two bidders submitted final round bids on large numbers of packages. In Cramton's (2008a) description of the auction, the others "simply increased their clock bids, and added a handful of supplementary [i.e. final] bids on packages closely related to their bids in the latter part of the clock stage." Moreover, as Jewitt and Li (2008) explain, "all but one of the bidders made their highest supplementary bid either on their final clock package, or on a subset of it." This is an extreme form of quiet bidding. Most bidders expressed zero (or in fact negative) value for incremental spectrum beyond what they actually won!

In the L Band auction, there were eight bidders and 17 licenses for sale. Bidders could demand arbitrary packages of these licenses. Again, there was a fair amount of competition in the clock phase. The average demand for the licenses in the first round was 3.8 and the market cleared after 32 rounds. Again, however, there was little activity in the final bid round. Six of the bidders submitted final bids on just zero, one or two packages (Cramton, 2008b). Only two bidders submitted significant numbers of final bids. So again, the behavior of most bidders could be described as quiet, and there seems to have been some of the same behavior flagged by Jewitt and Li above. For example, Cramton (2008b) writes: "It is difficult to understand why WorldSpace [which entered only two new final bids] did not enter a more complete set of supplementary bids. Based on its bidding in the clock stage, it would appear to value nearly any set of three small lots at its upper limit of 2,614."

5.2 UK 4G Auction (2013)

The United Kingdom's subsequent auction for 800 and 2600 MHz spectrum involved much more valuable licenses, with the auction generating over £2 billion in revenue. There were four 10 MHz licenses and a 20 MHz license available in the 800 MHz band. There were also multiple licenses available at 2600 MHz.

Table 1 shows the number of distinct packages bid on by each bidder in the 52 clock rounds, and subsequently in the final round.[19] If a bidder bid for the same package in multiple rounds, we

[18] The UK government also published a note explaining how the bids in the 10-40 GHz auction determined the winner payments: http://stakeholders.ofcom.org.uk/binaries/spectrum/spectrum-awards/completed-awards/10-28-32-40-ghz-awards/baseprices.pdf

[19] The numbers in this section come from our own analysis of the bidding data, which is available at: http://stakeholders.ofcom.org.uk/spectrum/spectrum-awards/awards-archive/completed-awards/800mhz-

count it just once in the first column; if a bidder bid for a package in the clock phase and raised the bid in the final round, it counts in both columns.

	Packages Bid		MHz Won		
Bidder	Clock	Final	800	2600	Payment
EE	6	48	10	70	£589M
Niche (BT)	7	89	-	20	£186M
H3G	7	12	10	-	£225M
MLL	8	8	-	-	-
HKT	8	8	-	-	-
Telefonica	7	6	20	-	£550M
Vodafone	11	94	20	65	£791M

Table 1. Bidding in the UK 800/2600 Auction

Two of the bidders, MLL and HKT, dropped their demands to zero during the clock phase. Two other bidders, Telefonica and H3G, were active throughout the clock phase but submitted just a few final round bids. In contrast, EE, Niche and Vodafone bid for large numbers of packages in the final round.

Figure 6 shows the full set of package bids submitted by Vodafone. The bars represent the amount of spectrum demanded in different bands, and the line above shows the amount of each bid. Vodafone submitted bids for essentially all combinations of licenses that involved 20 MHz of low-frequency spectrum (the most it was allowed to bid for) and the most desirable high-frequency spectrum. The bids are highly systematic. Vodafone expressed a value for the 20 MHz low-frequency block nearly equivalent to its value for two 10 MHz blocks, and appears to have expressed clear incremental values for the high-frequency blocks.

The incremental values expressed in these final bids are consistent with Vodafone's demand reductions in the clock phase. For instance, in clock round 37, when the price was £87.6 million for each 10 MHz license, Vodafone reduced demand from 4 to 3 C band licenses. Later in its final round bidding, it expressed an incremental value of £87.6 million for a fourth C band license. Toward the end of the clock phase, Vodafone was bidding for 20 MHz at 800 MHz and 30 MHz in band C, and reduced its demand from 7 to 5 to 4 to 3 and then to 0 licenses in band E. The prices at which it made the reductions are consistent with the final bids shown in Figure 6. In this sense, Vodafone

2.6ghz/auction-data/

appears to have bid in a way that approximates fairly closely the consistent behavior described above.

Telefonica, which bid for similar amounts of spectrum during the clock phase, behaved very differently in its final round bidding. Figure 7 shows its complete set of package bids. Telefonica bid for 7 different packages in the clock rounds. In the final round, it added four new packages (bids 1-4), raised its bid for two packages from the clock phase (bids 5-6), and left five clock phase bids unchanged (bids 7-11). The bids it left unchanged are dominated. They could not have been winning bids or mattered for rival prices as each is for a larger package than bid 4, and offers less money.

Telefonica's five meaningful bids were quite similar in terms of spectrum demanded and amount offered. Bid 2 was Telefonica's winning bid. So Telefonica expressed very little value for packages larger than what it won — the incremental values expressed in its final bids are much lower than the prices at which it reduced demand in the clock rounds. In this sense, its bidding behavior was much closer to the quiet strategy described above than to consistent bidding.

5.3 Austrian 4G Auction (2013)

The Austrian 4G auction involved the sale of high-value spectrum licenses in the 800, 900 and 1800 MHz bands. The only bidders were the three major wireless companies in Austria. Each was limited to bidding on no more than 50% of the available licenses. This still allowed any two bidders to submit a combined bid for all licenses in the auction, i.e. it did not imply that winners automatically received some spectrum at reserve prices.

The auction yielded revenue of just over €2 billion, which far exceeded forecasts.[20] A report released by the regulatory authority after the auction cited aggressive final round bids as a key factor in the high prices paid by the winners:[21] The report reads:

> During [the final bid] stage every bidder was allowed to submit as many as 3,000 supplementary bids. (...) The three bidders actually submitted a total of more than 4,000 supplementary bids. More than 65% of these supplementary bids were submitted for the largest permissible combinations of frequency blocks, with a share of some 50% of available frequencies. In addition, the bidders utilised almost to the full the price limits that had applied to these large packages during the sealed-bid [i.e. final] stage.

[20] Press coverage after the auction quoted one industry CEO as saying that the high prices were "a bitter pill to swallow," and another as claiming that the outcome was "a disaster for the industry as a whole." (http://www.fiercewireless.com/europe/story/austrian-operators-file-complaints-over-spectrum-auction-800-mhz-900-mhz-an/2013-11-27).

[21] The report, titled "Result of the 2013 multiband auction driven by consistently offensive bidding strategy on the part of all three contenders" is available at https://www.rtr.at/en/pr/PI28102013TK, along with a presentation containing the numbers quoted below.

> (...) These supplementary bids submitted on large frequency packages had a significant effect on the prices offered by the other bidders. At the same time, such bids generally only have a marginal likelihood of winning out in the end. If these bids for very large numbers of frequencies had been ignored when determining the winners and prices, the revenue from the auction would have settled at a level of about EUR 1 billion.

A remarkable feature of the Austrian auction is that the final revenue ended up quite close to the total license prices at the end of the clock phase, which were €2.07 billion. Had the bidders submitted no final round bids (i.e. been quiet), the winners would have paid €765 million. Instead they paid €2.01 billion. If bidding in both stages of the auction was truthful, average license prices under the Vickrey formula only would be as high as prices at the end of the clock phase if bidders were willing to pay for all their incremental spectrum at the same rate as for a marginal license. It seems very likely therefore that in this auction bidders took steps to relax their final bid activity constraints. The Appendix describes a version of the model with this feature, in which due to mutually aggressive behavior both bidders can end up paying nearly full clock prices for every unit.

6 Conclusion

Our analysis highlights two properties of the combinatorial clock auction. First, the activity rules used to encourage truthful bidding mean that a bidder's final round bids may have no effect at all on her own payoff. Yet if bidders do not increase their final bids to levels consistent with their expressed demand in the clock phase (and they have no strict incentive to do so), this leads to price discounts and incentives for demand expansion in the initial clock phase. The result is a wide range of ex post equilibria, with no guarantee of an efficient allocation or truthful Vickrey prices.

Second, the auction provides bidders with the opportunity to raise rival prices with little or no risk to their own payoff by relaxing the constraints on their final bids. We have illustrated how this can lead not just to higher payments, but to distorted incentives in the clock phase and inefficient allocations. In Section 4, a single predatory bidder maintains high demand during the clock phase before dropping demand to clear the market, leading to an equilibrium in which the second bidder reduces demand to avoid high payments. Janssen and Kasberger (2015) expand this analysis by showing that in our model, if both bidders have lexicographic preferences to raise rival costs, an efficient equilibrium in proxy strategies may not exist at all.

A loose way to summarize these points is that in order to support a truthful equilibrium as we expect in a Vickrey auction, the CCA relies on bidders behaving "just right": raising their final round bids maximally so that the revealed preference activity constraints bind, but not taking actions in the clock phase to purposely relax these constraints. Our examples show how, if bidders are not sufficiently aggressive, or are overly aggressive, incentives for demand expansion and/or reduction appear and outcomes need not be efficient, even if behavior is completely understood and bidders play minimally strategic ex post equilibria.

Our analysis makes several simplifying assumptions. We mostly restrict attention to proxy strategies in which bidders do not condition their bidding on rival behavior. With contingent strategies, our model admits many more equilibria. These include highly collusive equilibria in which bidders split the market and drive each other's prices to zero, using the threat of aggressive final bids to punish deviations. However, these types of equilibria also arise in traditional clock auctions and are not special to the CCA.[22] Our assumptions also implied that the clock phase ends with market clearing. If bidders can drop demand discontinuously, the clock phase may end with some units unallocated. This can be useful for package bidders who want to avoid exposure problems — a potentially important benefit of the CCA that is not captured in our model[23] — but also creates new strategic possibilities.[24]

Another important point is that while our analysis shows some limitations of the CCA, other multi-item auction designs have their own drawbacks. Ausubel et al. (2014) have shown in great generality that uniform price auctions create incentives for demand reduction and inefficiency. Ausubel and Milgrom (2006) have catalogued problems with the sealed-bid Vickrey auction, such as incentives for collusion, and the fact that Vickrey outcomes may lie outside the core. Our analysis points to a further problem with the Vickrey auction that we view as equally serious. If bidders understand that the allocation will almost certainly lie in a particular range (the relevant situation in most radio spectrum auctions), their incentives to bid truthfully outside of this range may be very weak despite these bids potentially being crucial for pricing. Of course, in a static Vickrey auction, bidding truthfully is still weakly dominant.[25] However in a dynamic implementation of Vickrey pricing, the dominant strategy property is lost. In our model of Section 4, bidder 1's strict best response to bidder 2's predation is to engage in demand reduction, leading to inefficiency.

From a practical standpoint, an auction designer choosing between a CCA and a uniform price

[22] In fact, Riedel and Wolfstetter (2006) show that the simultaneous multiple round auction with complete information has an essentially unique subgame perfect equilibrium in which the bidders immediately demand their efficient allocation and the auction ends.

[23] See Bulow, Levin and Milgrom (2009) and Cramton (2013) for discussions of the exposure problem faced by package bidders in traditional clock auctions.

[24] If there are unallocated units in the final round of the CCA, bidders will have an incentive to bid truthfully for these units, but not necessarily for units that they cannot possibly win. The ability to create excess supply also creates new opportunities for predation, as a predatory bidder can potentially drop its demand to zero and subsequently raise its rival's payment as illustrated in Remark 2.

[25] The Vickrey auction also has many equilibria in weakly dominated strategies (see, e.g. Blume et al., 2009).

auction (e.g. a simultaneous multi-round or clock auction) faces a set of trade-offs. The need for flexible package bidding favors a CCA design. So does the potential for highly inefficient demand reduction. In the other direction, the CCA is arguably a more complicated design and can create situations where there is considerable ambiguity about the prices a bidder faces at any point in the auction. Dealing with this potentially requires a high level of bidder sophistication. As we have seen in the paper, there is also the possibility for widely varying prices within an auction depending on the strategic postures adopted by bidders.[26]

A final question is whether different CCA activity rules could resolve some of the issues we have flagged, and lead to a successful dynamic Vickrey implementation. A recent and very interesting paper by Ausubel and Baranov (2013) suggests one possibility, which is to require that clock phase bids satisfy GARP, and then to use an algorithm based on the Afriat inequalities to fill in the final bids. In our setting, this would amount to requiring global rather than local revealed preference. This would resolve the quiet bidding problems illustrated in Section 3, but not the predatory bidding problems illustrated in Section 4.[27] Nevertheless, this proposal and others seem to merit further investigation.

7 References

Ausubel, Lawrence M. (2004), "An Efficient Ascending-Bid Auction for Multiple Objects," *American Economic Review*, 94(5), 1452-1475.

Ausubel, Lawrence M. and Oleg Baranov (2010) "Core-Selecting Auctions with Incomplete Information," University of Maryland Working Paper.

Ausubel, Lawrence M. and Oleg Baranov (2013) "An Enhanced Combinatorial Clock Auction," University of Maryland Working Paper.

Ausubel, Lawrence M. and Peter Cramton (2011), "Activity Rules for the Combinatorial Clock Auction," University of Maryland Working Paper.

[26]There are several examples in past spectrum CCAs of bidders paying disparate prices for similar amounts of spectrum. In the Canadian 700 MHz auction in 2014, Telus paid roughly twice the amount of Bell Canada ($1.14 billion CAN versus $0.57 billion CAN) for roughly similar amounts of spectrum. In the 2012 Switzerland auction, Sunrise paid 33% more than Swisscom (482 million CHF versus 360 million CHF) for a smaller package of spectrum. Of course with Vickrey pricing such anomalies can occur even with truthful bidding. However, given the particular details of the auctions (for instance Bell Canada and Telus run very comparable business operations), it seems likely that differences in strategic posture, such as those described in the paper, were also a factor.

[27]A similar point about incentives for aggressive bidding also would apply in the Ausubel (2004) clinching auction, which in our setting would be an alternative dynamic VCG implementation. Ausubel, Cramton and Milgrom (2006) also proposed a relaxed version of revealed preference, which forces bidders to increase their final clock rounds bids in order to "guarantee" their package from the last clock round. However, this rule also doesn't appear to provide strong incentives to submit "correct" bids for losing packages.

Ausubel, Lawrence M., Peter Cramton and Paul Milgrom (2006), "The Clock-Proxy Auction: A Practical Combinatorial Auction Design," in Peter Cramton, Yoav Shoham and Richard Steinberg (eds.) *Combinatorial Auctions*, MIT Press, Chapter 5, pp. 115-138.

Ausubel, Lawrence M., Peter Cramton, Marek Pycia, Marzena Rostek and Marek Weretka (2014), "Demand Reduction and Inefficiency in Multi-Unit Auctions," *Review of Economic Studies*, 81(4), pp. 1366-1400.

Ausubel, Lawrence M. and Paul Milgrom (2002), "Ascending Auctions with Package Bidding," *Frontiers of Theoretical Economics*, 1(1), Article 1.

Ausubel, Lawrence M. and Paul Milgrom (2006), "The Lovely but Lonely Vickrey Auction," in Peter Cramton, Yoav Shoham and Richard Steinberg (eds.) *Combinatorial Auctions*, MIT Press, Chapter 1, pp. 17-40.

Beck, Marissa and Marion Ott (2013) "Incentives for Overbidding in Minimum-Revenue Core-Selecting Auctions," Stanford University Working Paper.

Bichler, Martin, Pasha Shabalin, and Jürgen Wolf (2013), "Do Core-Selecting Combinatorial Clock Auctions Always Lead to High Efficiency? An Experimental Analysis of Spectrum Auction Designs," *Experimental Economics*, 16(4), pp. 511-545.

Bulow, Jeremy, Jonathan Levin and Paul Milgrom (2009), "Winning Play in Spectrum Auctions," Stanford University Working Paper.

Cramton, Peter (2008a), "A Review of the 10-40 GHz Auction," Office of Communications, United Kingdom.

Cramton, Peter (2008b), "A Review of the L Band Auction," Office of Communications, United Kingdom.

Cramton, Peter (2013), "Spectrum Auction Design," *Review of Industrial Organization*, 42(2), pp. 161-190.

Day, Robert and Paul Milgrom (2008), "Core-Selecting Package Auctions," *International Journal of Game Theory*, 36, pp. 393-407.

Day, Robert. and S. Raghavan (2007) "Fair Payments for Efficient Allocations in Public Sector Combinatorial Auctions," *Management Science,* 53, pp. 1389-1406.

Erdil, Aytek and Paul Klemperer (2009). "A New Payment Rule for Core-Selecting Auctions," *Journal of the European Economic Association*, 8, 537-547.

Goeree, Jacob and Yuanchuan Lien (2013). "On the Impossibility of Core-Selecting Auctions," forthcoming in *Theoretical Economics.*

Janssen, Maarten and Vladimir Karamychev (2013), "Gaming in Combinatorial Clock Auctions," Tinbergen Institute Discussion Paper 13-027/VII.

Janssen, Maarten and Bernhard Kasberger (2015), "On the Clock of the Combinatorial Clock Auction," mimeo, University of Vienna.

Jewitt, Ian and Zhiyun Li (2008), "Report on the 2008 UK 10-40 GHz Spectrum Auction," Office of Communications, United Kingdom.

Klemperer, Paul (2004), *Auctions: Theory and Practice*, Princeton University Press.

Knapek, Stephan and Achim Wambach (2012), "Strategic Complexities in the Combinatorial Clock Auction," CESifo Working Paper 3983.

Kyle, Albert S. (1989), "Informed Speculation With Imperfect Competition," *Review of Economic Studies*, 56(3), 317–355.

Loertscher, Simon, Leslie Marx and Tom Wilkening (2014), "A Long Way Coming: Designing Centralized Markets with Privately Informed Buyers and Sellers," Duke University Working Paper.

Milgrom, Paul (2004), *Putting Auction Theory to Work*, Cambridge University Press.

Milgrom, Paul (2007), "Package Auctions and Exchanges," *Econometrica 75: 935-965.*

Morgan, John, Ken Steiglitz and George Reis (2003), "The Spite Motive and Equilibrium Behavior in Auctions," *Contributions to Economic Analysis & Policy*, 2(1), Article 5.

Riedel, Frank and Elmar Wolfstetter (2006), "Immediate Demand Reduction in Simultaneous Ascending-Bid Auctions: A Uniqueness Result," *Economic Theory*, 29(3), pp. 721-726.

Salant, David (2013), *A Primer on Auction Design, Management, and Strategy,* mimeo, Toulouse School of Economics.

Vives, Xavier. (2011), "Strategic Supply Function Competition With Private Information," *Econometrica*, 79: 1919–1966

Figure 1. Activity Rule and Final Bid Options

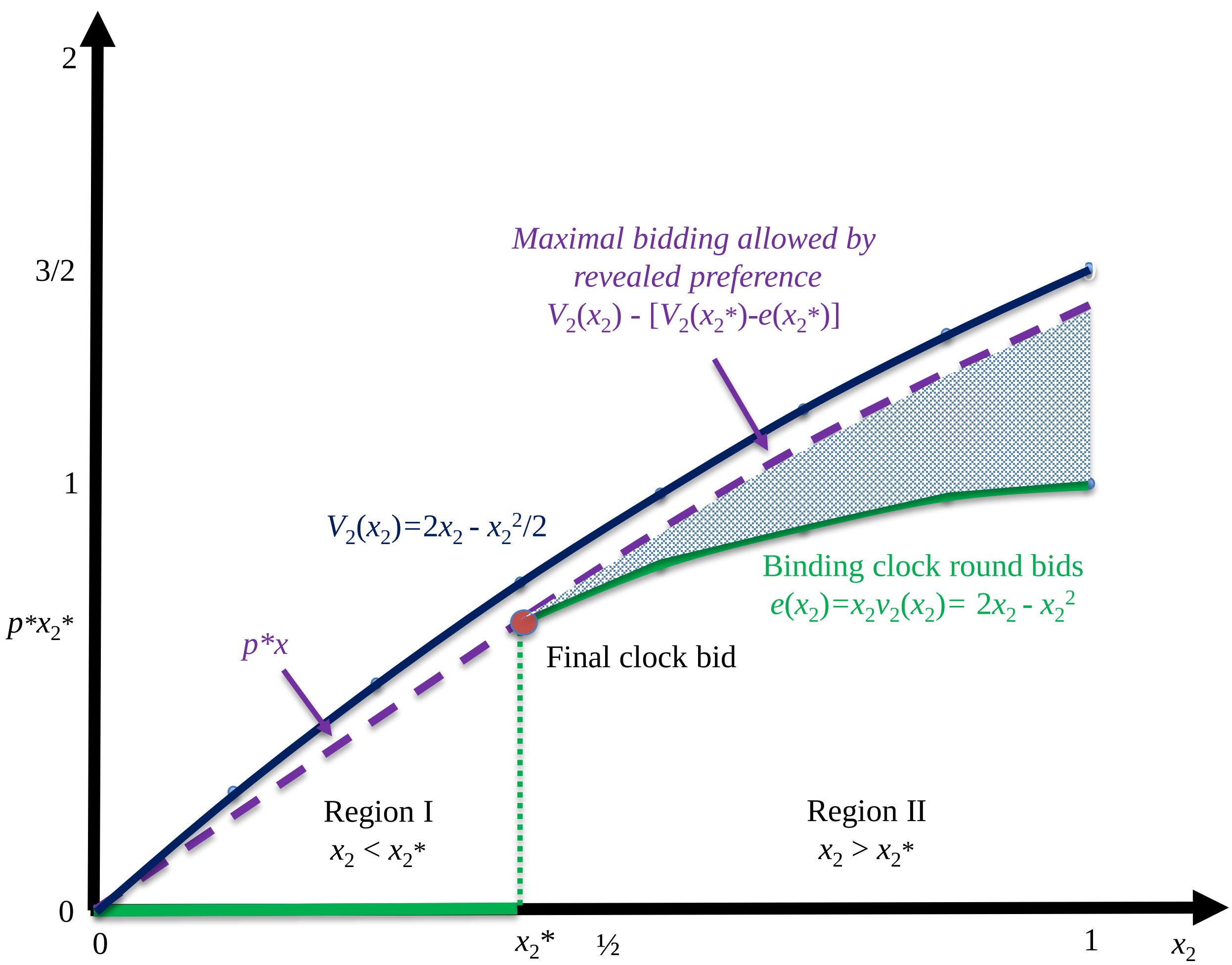

Figure shows the flexibility that bidder 2 has in choosing final bids in the final bid round. The lower curve shows bidder 2's clock phase bids, which place a lower bound on her final bids. The dashed curve is the upper bound for final bids (assuming bidder 2 leaves her last clock bid unchanged). The top curve is the valuation that guides bidder 2's clock round bidding. The dashed upper bound is parallel to this curve for quantities above x_2^*.

Figure 2. Best Responses to Quiet and Consistent Bidding

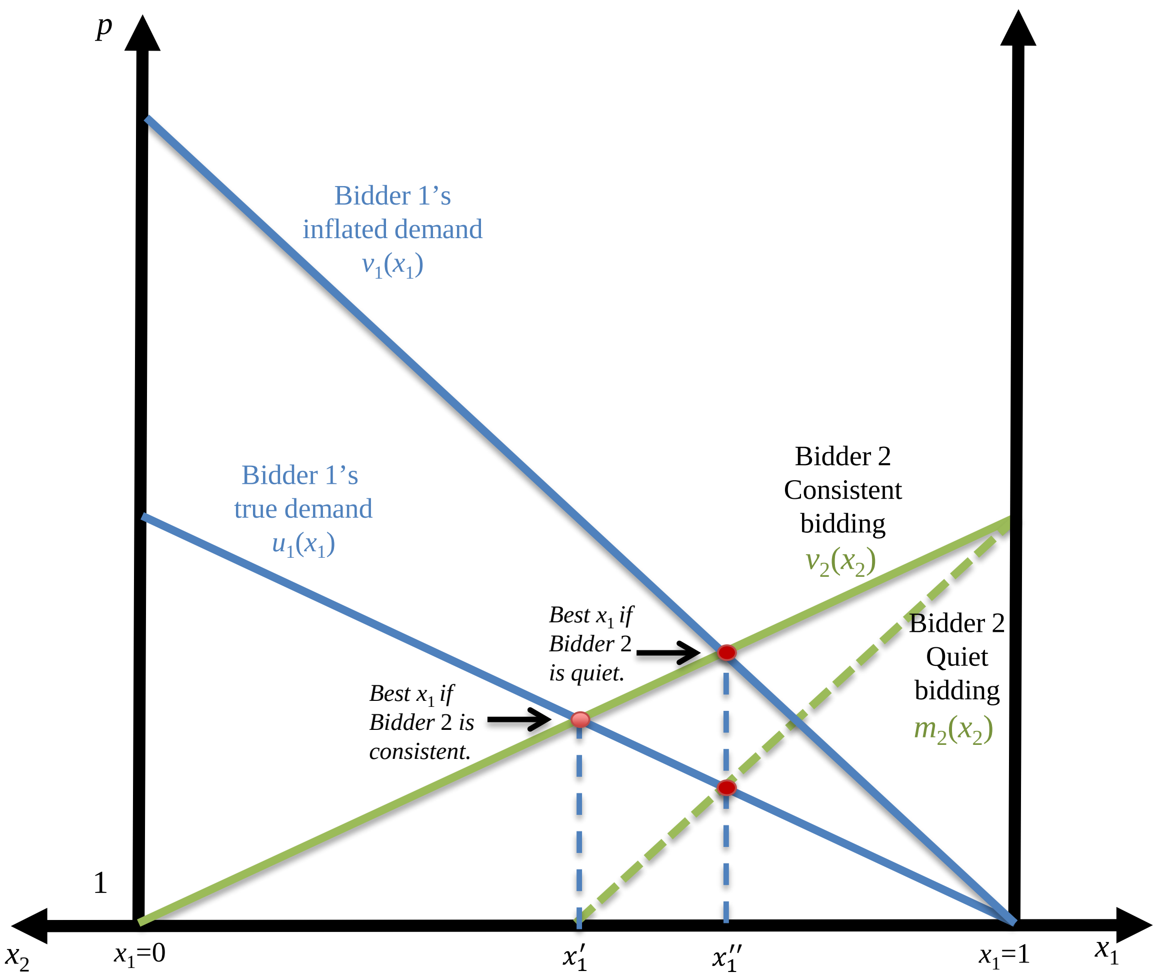

Figure shows possible best responses for bidder 1 in response to bidder 2 using a consistent or quiet strategy. If bidder 2 is consistent, bidder 1 optimally intersects his clock demand with bidder 2's clock demand. If bidder 2 is quiet, bidder 1's marginal price is lower than the clock price. The best response is to purchase the quantity at which bidder 1's marginal value equals bidder 2's marginal revenue, which can be done by inflating demand in the clock phase, as shown in the figure.

Figure 3. Identifying Ex Post Best Responses

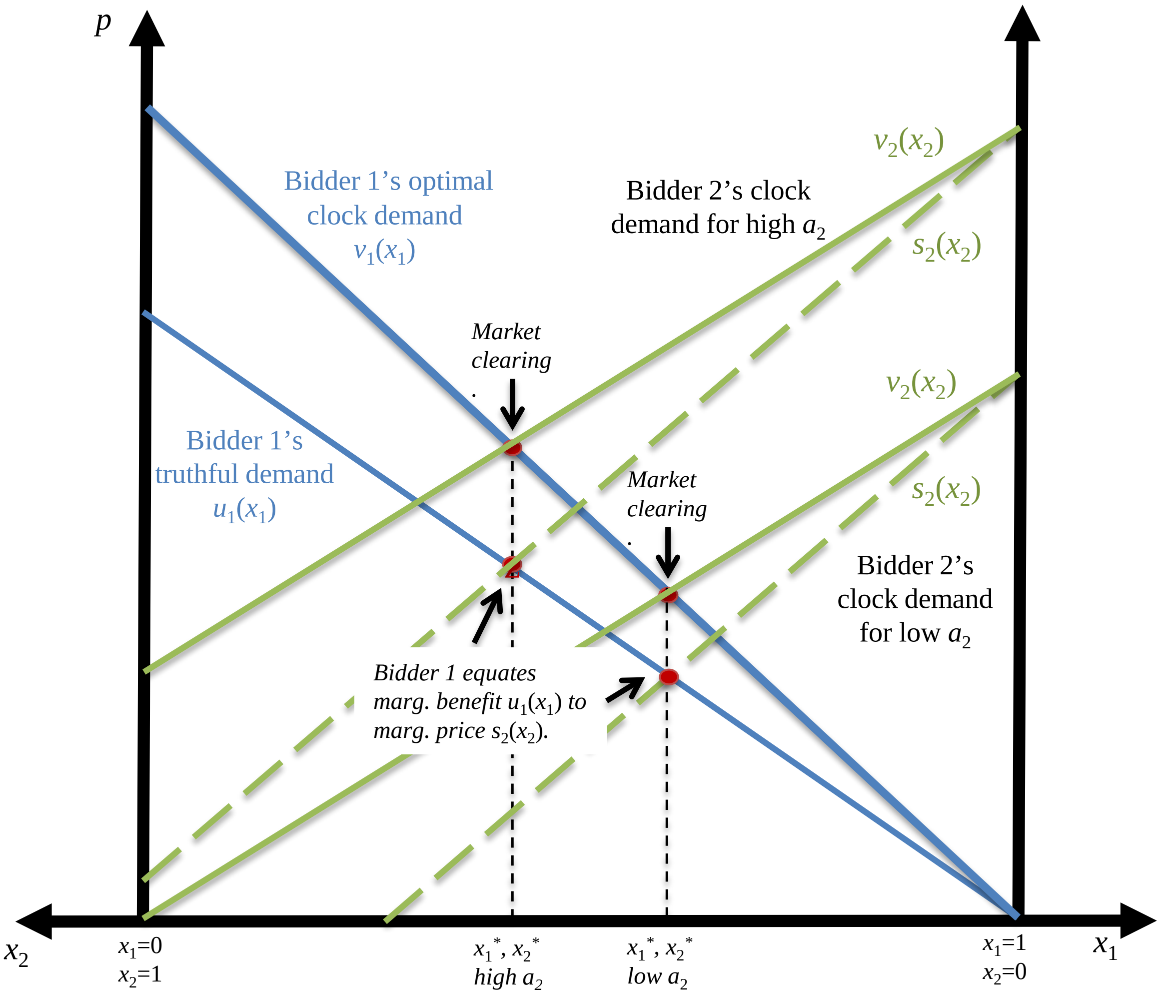

Figure shows the derivation of bidder 1's ex post best response. Bidder 1's optimal clock demand is chosen so that for any realization of bidder 2's demand, the clock phase ends (market clearing) at the allocation where bidder 1's true marginal valuation just equals his marginal price, which he correctly anticipates will be set by bidder 2's final bid. Bidder 1's clock demand inflates his true demand.

Figure 4. Equilibrium in the CCA Auction

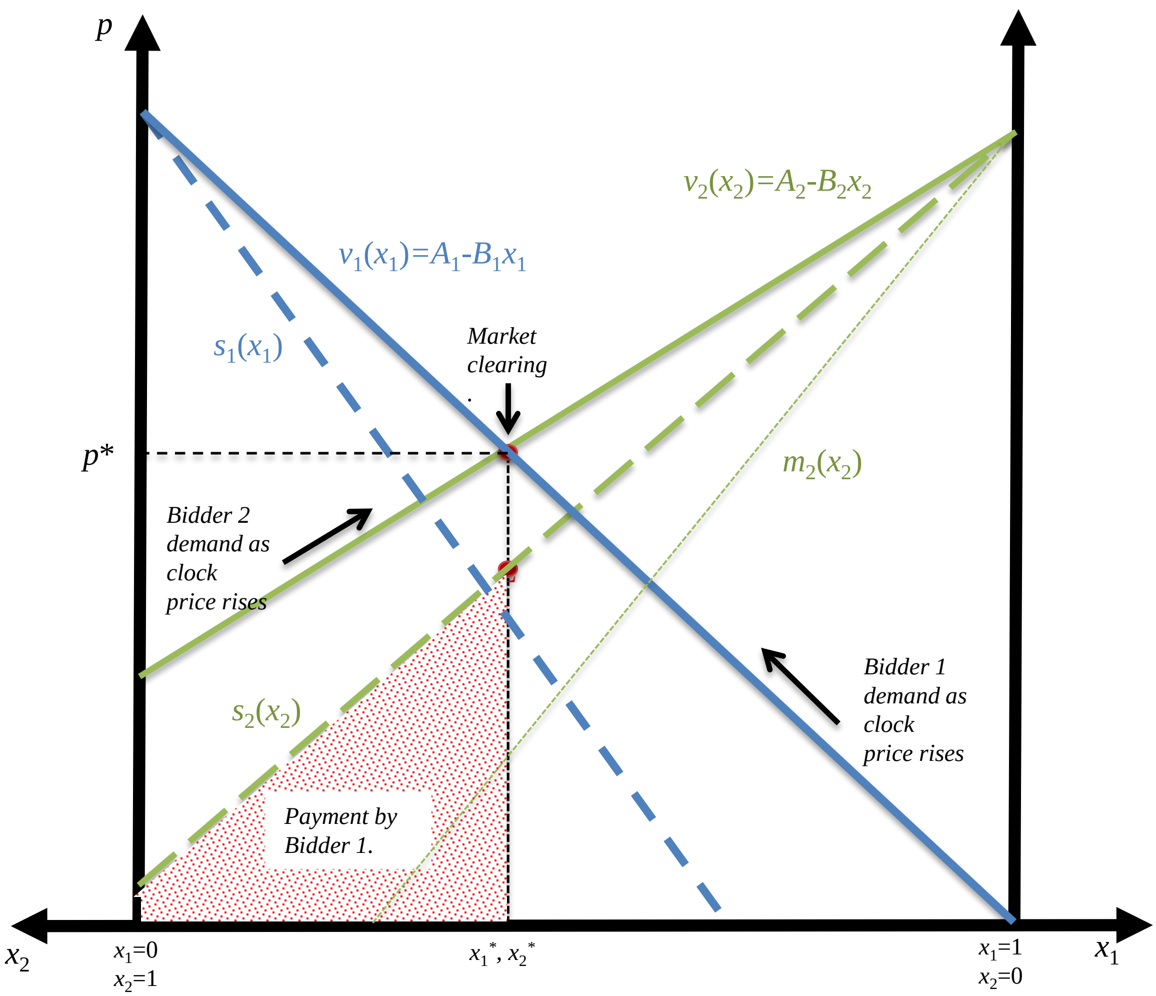

Figure shows equilibrium behavior for a single realization of values. The solid lines represent the equilibrium clock demand curves of the bidders; the dashed lines the final round demand curves. Bidder 2's clock round demand is intermediate between her clock demand and marginal revenue; she bids partway between quiet and consistent.

Figure 5. Predatory Clock Phase Bidding to Raise Rival Prices

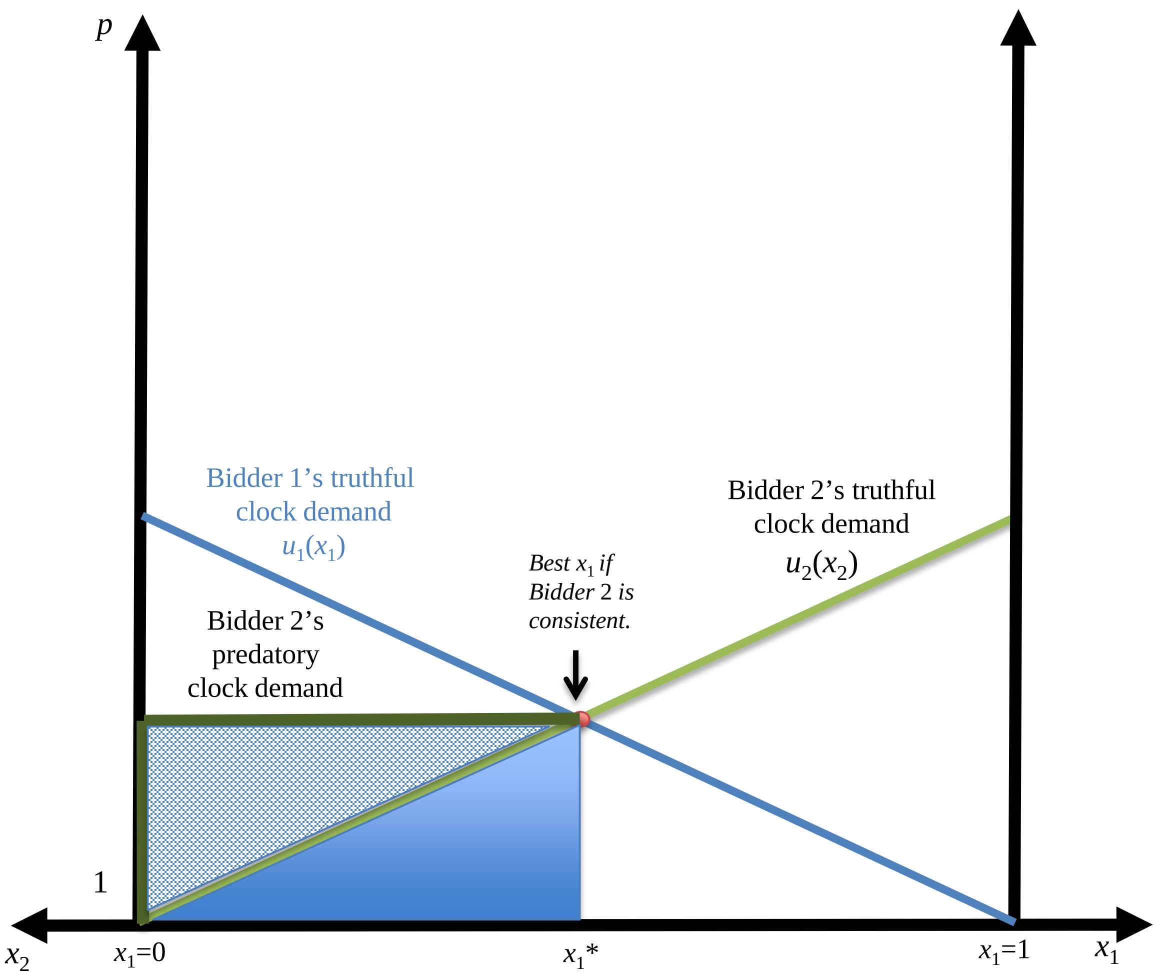

Figure shows truthful clock demands with solid shaded area representing bidder 1's payment. If bidder 2 maintains a higher (predatory) clock demand and submits final bids consistent with this higher demand, the allocation is unchanged but bidder 1 pay the shaded and dotted areas.

Figure 6. Final Bids by Vodafone in the UK 800/2600 MHz Auction

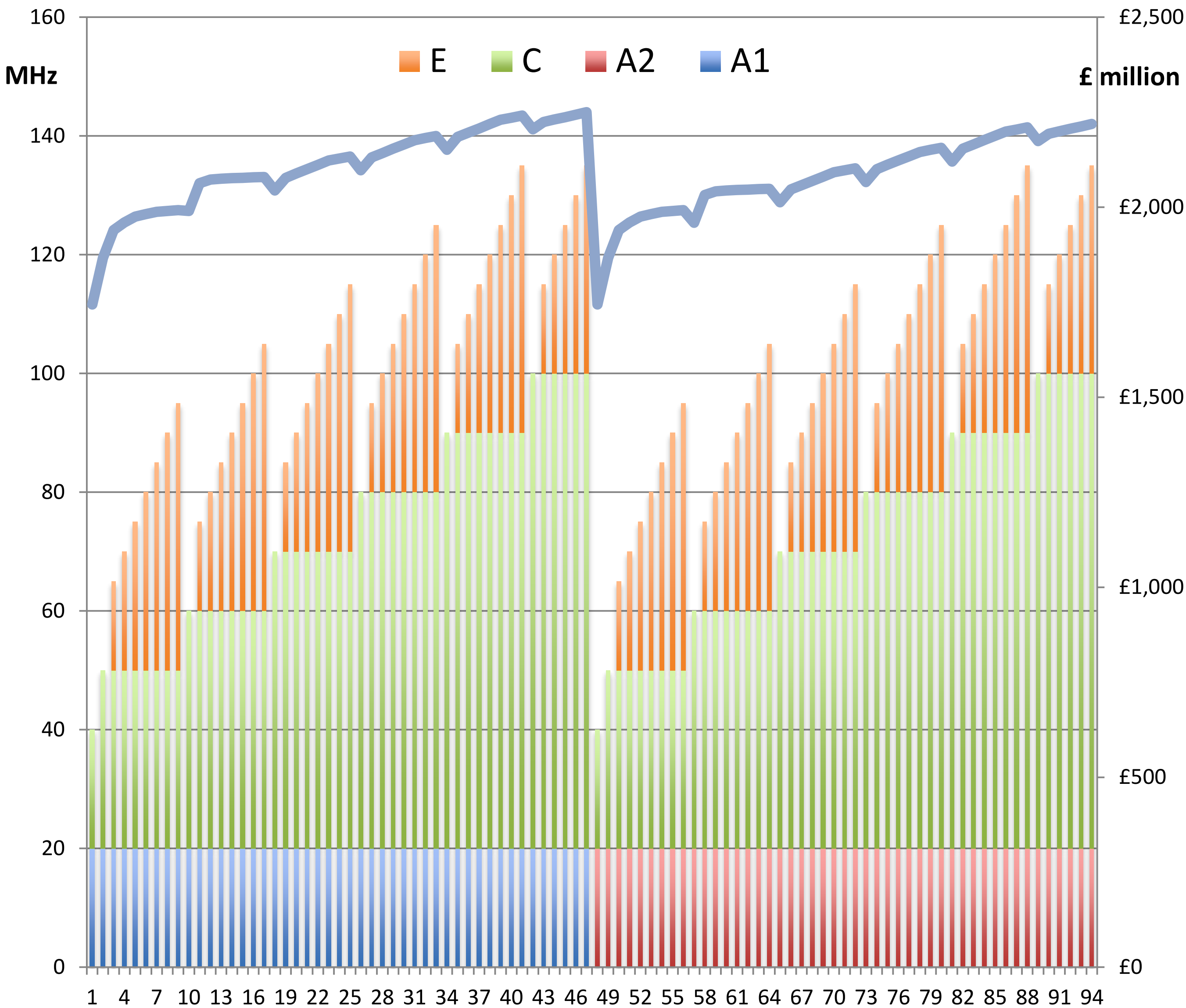

Figure shows all of Vodafone's final bids in the UK 800/2600 auction. The solid bars show the composition of each bid in terms of the MHz demanded in each of the four color-coded bands (E, C, A1 and A2). The solid line above shows the value of the bid in GBP (£). Vodafone's bids place consistent value on spectrum increments corresponding to clock behavior.

Figure 7. Final Bids by Telefonica in the UK 800/2600 MHz Auction

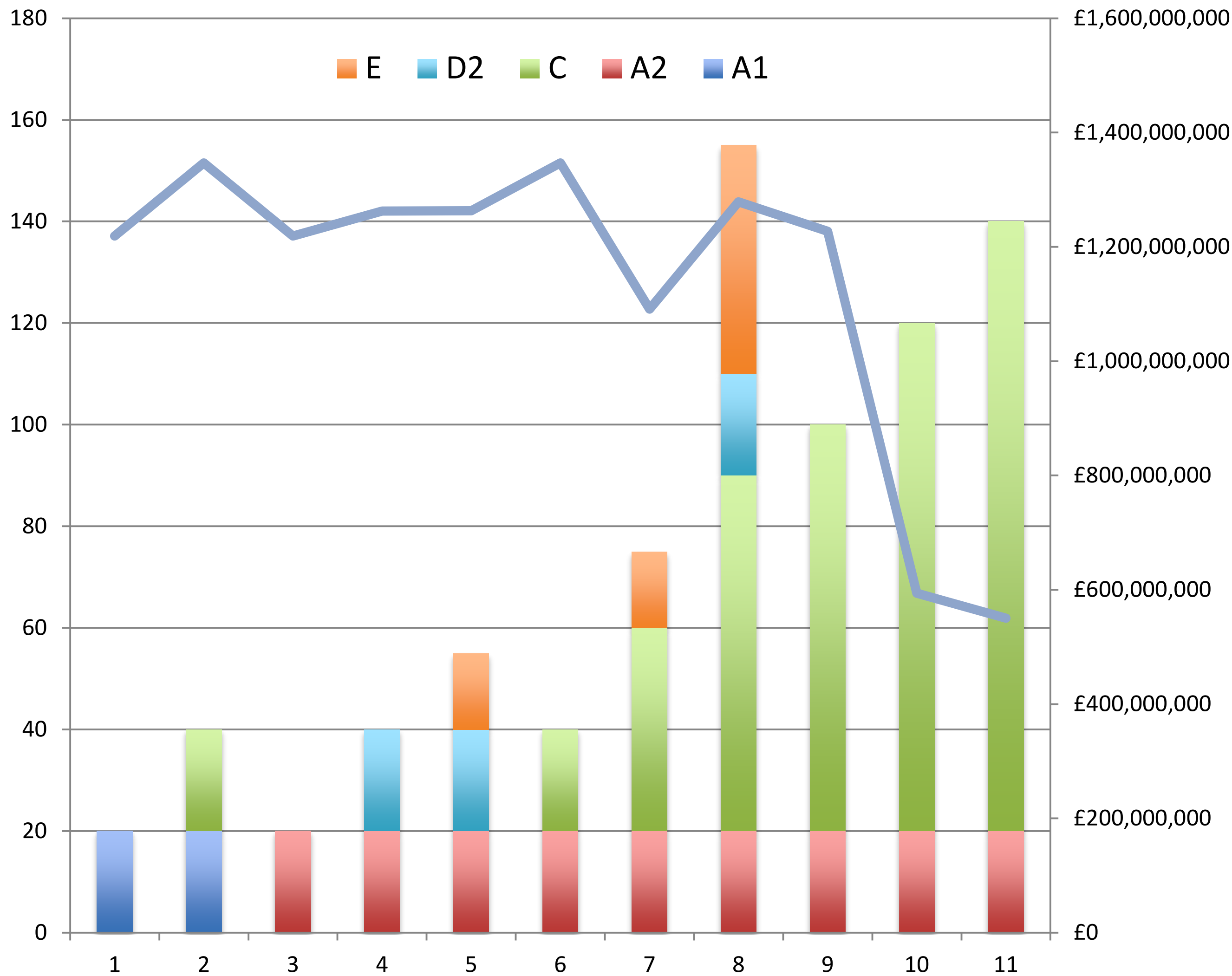

Figure shows all of Telefonica's final bids in the UK 800/2600 auction. The solid bars show the composition of each bid in terms of the MHz demanded in each of the four color-coded bands (E, C, A1 and A2). The solid line above shows the value of the bid in GBP (£). Telefonica submitted very few serious bids in the sealed bid round, with much smaller incremental valuations than it revealed during the clock phase, closer to a quiet strategy.