Malthus to Solow

By Gary D. Hansen and Edward C. Prescott*

Prior to 1800, living standards in world economies were roughly constant over the very long run: per capita wage income, output, and consumption did not grow. Modern industrial economies, on the other hand, enjoy unprecedented and seemingly endless growth in living standards. In this paper, we provide a model in which the transition from constant to growing living standards is inevitable given positive rates of total factor productivity growth and involves no change in the structure of the economy (parameters describing preferences, technology, and policy).[1] In particular, the transition from stagnant to growing living standards occurs when profit-maximizing firms, in response to technological progress, begin employing a less land-intensive production process that, although available throughout history, was not previously profitable to operate. In addition, this transition appears to be consistent with features of development during and following the industrial revolution.

The pioneering macroeconomics textbook, Merton H. Miller and Charles W. Upton (1974), models the preindustrial period as using a land-intensive technology, where land is a fixed factor and there are decreasing returns to labor. The modern era, on the other hand, is modeled as employing a constant-returns-to-scale technology with labor and capital as inputs. A bothersome feature of this classical approach is that different technologies are used for each period. In this paper, we unify these theories by having both production functions available at all time periods in a standard general-equilibrium growth model (the model of Peter A. Diamond [1965]). Both processes produce the same good, and total factor productivity grows exogenously. We denote the land-intensive technology the *Malthus technology,* and the other, the *Solow technology.*

We show that along the equilibrium growth path, only the Malthus technology is used in the early stages of development when the stock of usable knowledge is small. Operating the Solow production process given the prevailing factor prices would necessarily earn negative profits. The absence of sustained growth in living standards in this Malthusian era follows from our assumption that the population growth rate is increasing in per capita consumption when living standards are low.[2] Eventually, as usable knowledge grows, it becomes profitable to begin assigning some labor and capital to the Solow technology. At this point, since there is no fixed factor in the Solow production function, population growth has less influence on the growth rate of per capita income and living standards begin to improve. In the limit, the economy behaves like a standard Solow growth

* Hansen: Department of Economics, UCLA, Los Angeles, CA 90095; Prescott: Department of Economics, University of Minnesota, Minneapolis, MN 55455, and Federal Reserve Bank of Minneapolis. We have benefited from excellent research assistance from Igor Livshits, Antoine Martin, and Daria Zakharova. We are grateful to Gregory Clark for useful comments and for providing us with some of the data used in Section I, and to Michele Boldrin, Jeremy Greenwood, Tim Kehoe, Steve Parente, Nancy Stokey, and three anonymous referees for helpful discussion and comments. We acknowledge support from the UCLA Council on Research (Hansen) and the National Science Foundation (Prescott). The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

[1] This paper contributes to a recent literature on modeling the transition from Malthusian stagnation to modern growth in a single unified model. Notable examples include Jasmina Arifovic et al. (1997), Charles I. Jones (1999), and Oded Galor and David N. Weil (2000). Our approach differs from the existing literature by focusing on the changing role of land in production and, in particular, the decline in land's share following the industrial revolution.

[2] In our model, this leads to a constant rate of population growth prior to the adoption of the Solow technology. This result is consistent with population data from Michael Kremer (1993), where the growth rate of population fluctuates around a small constant throughout most of the Malthusian period (from 4000 B.C. to A.D. 1650).

model, which displays many of the secular features of modern industrial economies.[3]

We interpret the decline of land's share predicted by our theory as occurring when goods produced in the industrial sector (capital) are substituted for land in production. History indicates that this was particularly important in the production of usable energy, a crucial intermediate input in producing final output. For example, railroads and farm machinery were substituted for horses, which required land for grazing. Machinery can run on fossil fuels, which requires less land to produce than grain. Another example is that better ships, produced in the industrial sector, allowed whale oil to be substituted for tallow (hard animal fat) as fuel for lighting. Tallow, like animal power, is relatively land-intensive to produce.

The existing theoretical literature on the transition from stagnation to growth has focused mostly on the role played by endogenous technological progress and/or human-capital accumulation rather than the role of land in production.[4] For example, human-capital accumulation and fertility choices play a central role in Lucas (1998), which builds on work by Becker et al. (1990). Depending on the value of a parameter governing the private return to human-capital accumulation, Lucas's model can exhibit either Malthusian or modern features. Hence, a transition from an economy with stable to growing living standards requires an exogenous change in the return to human-capital accumulation.

As we do in this paper, Jones (1999) and Galor and Weil (2000) study models where the transition from Malthusian stagnation to modern growth is a feature of the equilibrium growth path, although their approaches differ from ours by incorporating endogenous technological progress and fertility choice.[5] Living standards are initially constant in these models due to the presence of a fixed factor in production and because population growth is increasing in living standards at this stage of development. In Galor and Weil (2000), growing population, through its assumed effect on the growth rate of skill-biased technological progress, causes the rate of return to human-capital accumulation to increase. This ultimately leads to sustained growth in per capita income. In Jones (1999), increasing returns to accumulative factors (usable knowledge and labor) cause growth rates of population and technological progress to accelerate over time, and eventually, this permits an escape from Malthusian stagnation.[6]

The rest of this paper is organized as follows. In the next section, we discuss some empirical facts concerning preindustrial and postindustrial economies. In Section II, the model economy is described, and an equilibrium is defined and characterized. The development path implied by our model is studied in Section III. We provide sufficient conditions guaranteeing that the So-

[3] John Laitner (2000) uses a similar model to explain why savings rates tend to increase as an economy develops. The two production processes, however, produce different goods in his model. As a result, the transition away from the land-intensive technology requires that living standards grow prior to the transition. Hence, Laitner's model does not display Malthusian stagnation in the early stages of development. Nancy L. Stokey (2001) uses a multisector model like Laitner's to model the British industrial revolution.

[4] Examples include Gary S. Becker et al. (1990), Kremer (1993), Marvin Goodfriend and John McDermott (1995), Robert E. Lucas, Jr. (1998), Tamura (1998), Jones (1999), and Galor and Weil (2000).

[5] Another way of modeling the transition from stagnation to growth is explored by Arifovic et al. (1997). In their approach, if agents engage in adaptive learning, the economy can eventually escape from a stagnant (low income) steady state and transition to a steady state with sustained growth.

[6] Relative to the theory presented in these two papers, the particular mechanism generating technological progress is less important in our approach. What is important is that total factor productivity ultimately grows to the critical level that makes the Solow technology profitable. Since we study the consequences rather than the sources of technological progress, we treat technological advance as exogenous. Of course, this assumption implies that our theory is silent as to why usable knowledge grows at all, let alone why technological progress reached the critical threshold in England in the century surrounding 1800. Similarly, because we abstract from fertility choice, we follow Kremer (1993) and simply assume a hump-shaped relationship between population growth and living standards. Hence, our model displays a demographic transition by construction. Although the assumption that population growth increases with living standards is key to our model exhibiting Malthusian stagnation, the transition to modern growth would occur in our model even if there were no demographic transition.

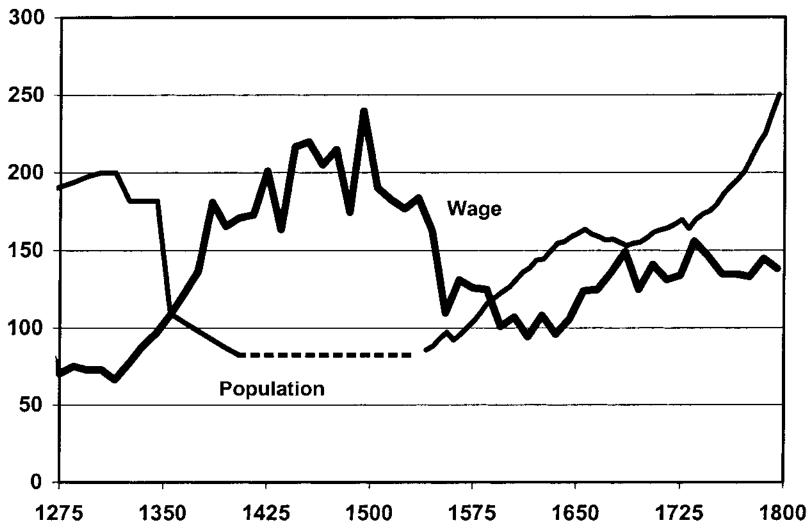

FIGURE 1. POPULATION AND REAL FARM WAGE

low technology will eventually be adopted, but we must use numerical simulations to study the transition to modern growth. Some concluding comments are provided in Section IV.

I. The English Economy From 1250 to the Present

A. *The Period 1275–1800*

The behavior of the English economy from the second half of the 13th century until nearly 1800 is described well by the Malthusian model. Real wages and, more generally, the standard of living display little or no trend. This is illustrated in Figure 1, which shows the real farm wage and population for the period 1275–1800.[7] During this period, there was a large exogenous shock, the Black Death, which reduced the population significantly below trend for an extended period of time. This dip in population, which bottomed out sometime during the century surrounding 1500, was accompanied by an increase in the real wage. Once population began to recover, the real wage fell. This observation is in conformity with the Malthusian theory, which predicts that a drop in the population due to factors such as plague will result in a high labor marginal product, and therefore real wage, until the population recovers.

Another prediction of Malthusian theory is that land rents rise and fall with population. Figure 2 plots real land rents and population for England over the same 1275–1800 period as in Figure 1.[8] Consistent with the theory, when population was falling in the first half of the sample, land rents fell. When population increased, land rents also increased until near the end of the sample when the industrial revolution had already begun.

[7] The English population series is from Gregory Clark (1998a) for 1265–1535 (data from parish records in 1405–1535 are unavailable, so we use Clark's estimate that population remained roughly constant during this period) and from E. A. Wrigley et al. (1997) for 1545–1800. The nominal farm wage series is from Clark (1998b), and the price index used to construct the real wage series is from Henry Phelps-Brown and Sheila V. Hopkins (1956). We have chosen units for the population and real wage data so that two series can be shown on the same plot.

[8] The English population series and the price index used to construct the real land rent series are the same as in Figure 1. The nominal land rent series is from Clark (1998a).

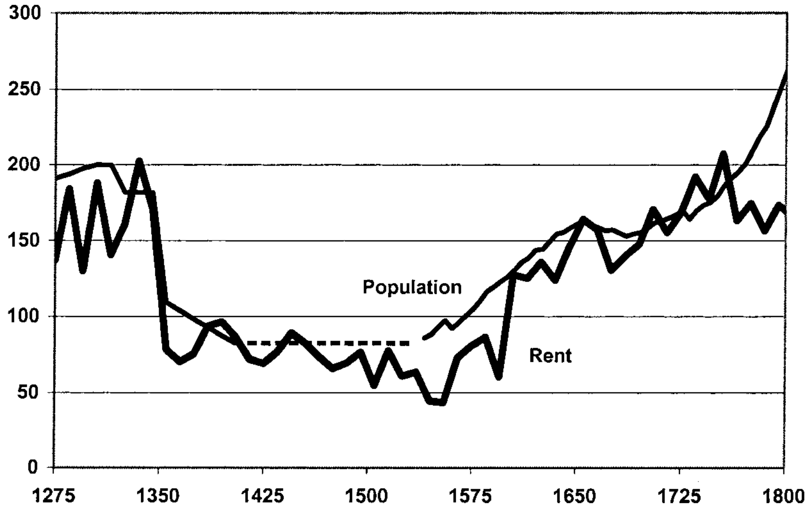

FIGURE 2. POPULATION AND REAL LAND RENT

B. *The Period 1800–1989*

Subsequent to 1800, the English economy no longer behaves according to the Malthusian theory. Both labor productivity, which moves closely with the real wage, and population grew at higher rates than in the previous era. Population increases did not lead to falling living standards as the Malthusian theory predicts. This is documented in Table 1, which reports U.K. labor productivity and population for selected years. The striking observation is that labor productivity increased by a factor of 22 between 1780 and 1989.[9] In addition, after 1870 there is no discernable relationship between population growth and labor productivity growth, which is consistent with the predictions of the Solow growth model.

A transition from Malthus to Solow implies that land has become less important as a factor of production. Indeed, the value of farmland relative to the value of gross national product (GNP) has declined dramatically in the past two centuries. Table 2 reports this ratio for the

TABLE 1—U.K. PRODUCTIVITY LEVELS

	GDP/hour[a]		Population[b]	
Year	1985 $US	Growth rate[c]	Millions	Growth rate[c]
1700	0.82		8.4	
1760			11.1	0.47
1780	1.02	0.27		
1820	1.21	0.43	21.2	1.08
1870	2.15	1.16	31.4	0.79
1890	2.86	1.44	37.5	0.89
1913	3.63	1.04	45.6	0.85
1929	4.58	1.46	45.7	0.01
1938	4.97	0.91	47.5	0.43
1960	8.15	2.27	52.4	0.45
1989	18.55	2.88	57.2	0.30

Notes: We added 5 percent to numbers for the years 1700, 1780, and 1820 to adjust for the fact that all of Ireland is included in these earlier data. The motivation for using 5 percent is that for the years 1870, 1890, and 1913, Maddison (1991) reports data with and without Southern Ireland. U.K. labor productivity without Southern Ireland was 1.05 times the U.K. labor productivity with Southern Ireland.

[a] *Source:* Angus Maddison (1991 pp. 274–76).

[b] *Source:* Maddison (1991 pp. 227, 230–39).

[c] Percentage annual growth rate.

[9] Most likely the increase in the real wage was larger than this number due to difficulties in incorporating improvements in quality and the introduction of new products in the cost of living index. For example, using lumens as a measure of lighting, William D. Nordhaus (1997) finds that the price of lighting fell 1,000 times more than conventional lighting price indexes find. Lighting in the 19th century was almost 10 percent of total household consumption expenditures. Nordhaus (1997) also finds that the price of lighting was essentially constant between 1265 and 1800.

TABLE 2—U.S. FARMLAND VALUE RELATIVE TO GNP

Year	Percentage
1870	88
1900	78
1929	37
1950	20
1990	9

Notes: The 1870 value of land is obtained by taking 88 percent of the value of land plus farm buildings, not including residences. In 1900, the value of agriculture land was 88 percent of the value of farmland plus structures.
Sources: U.S. Bureau of the Census (1975). Farmland values for 1990 are provided by Ken Erickson (online: ⟨erickson@mailbox.econ.ag.gov⟩).

United States since 1870, the first year the needed census data are available. The value of farmland relative to annual GNP has fallen from 88 percent in 1870 to less than 5 percent in 1990.[10]

II. The Model Economy

A. *Technology*

We study a one-good, two-sector version of Diamond's (1965) overlapping-generations model.[11] In the first production sector, which we call the *Malthus sector,* capital, labor, and land are combined to produce output. In the second sector, which we call the *Solow sector,* just capital and labor are used to produce the same good. The production functions for the two sectors are as follows:

$$(1) \qquad Y_{Mt} = A_{Mt} K_{Mt}^{\phi} N_{Mt}^{\mu} L_{Mt}^{1-\phi-\mu}$$

$$(2) \qquad Y_{St} = A_{St} K_{St}^{\theta} N_{St}^{1-\theta}.$$

Here, the subscript M denotes the Malthus sector and S denotes the Solow sector. The variables A_j, Y_j, K_j, N_j, and L_j (j = M, S) refer to total factor productivity, output produced, capital, labor, and land employed in sector j. In addition, $\{A_{jt}\}_{t=t_0}^{\infty}$, j = M, S, are given sequences of positive numbers.[12]

Land in this economy is in fixed supply: it cannot be produced and does not depreciate. We normalize the total quantity of land to be 1. In addition, land has no alternative use aside from production in the Malthus sector, so $L_{Mt} = 1$ in equilibrium.

Implicit behind these aggregate production functions are technologies for individual production units where, given factor prices, the optimal unit size is small relative to the size of the economy and both entry and exit are permitted. Total factor productivity is assumed to be exogenous to these individual profit centers. The Malthus production unit is one that is relatively land-intensive, like an old-fashioned family farm, because it is dependent on land-intensive sources of energy, such as animal power. The Solow production unit, on the other hand, is capital-intensive rather than land-intensive and could correspond to a factory. Consistent with this interpretation, we assume that $\theta > \phi$. Land, at least when interpreted as a fixed factor, does not enter the Solow technology at all.[13]

[10] The decline since 1929 would certainly have been greater if large agriculture subsidies had not been instituted. The appropriate number from the point of view of our theory, where value is the present value of marginal products, is probably less than 5 percent in 1990.

[11] Although we found it convenient to study an overlapping-generations model in this paper, our results should carry over to an infinite-horizon context like that used in much of the growth literature.

[12] Although there are two production processes available, there is only one aggregate production technology because this is a one-good economy. The aggregate production function is the maximal amount of output that can be produced from a given quantity of inputs. That is,

$$F(K, N, L)$$

$$\equiv \max_{\substack{0 \le K_S \le K \\ 0 \le N_S \le N}} \{A_M(K - K_S)\phi(N - N_S)^{\mu} L^{1-\phi-\mu} + A_S K_S^{\theta} N_S^{1-\theta}\}.$$

This function is not a member of the constant elasticity of substitution class that is usually assumed in applied growth theory. We were led to relax the constant elasticity assumption because Malthusian stagnation requires that the elasticity of substitution between land and labor be less than or equal to 1, while the falling land share observed after the industrial revolution requires this elasticity to be greater than 1.

[13] We have made this assumption to keep the model as simple as possible. Our results require that land's share in

Output from either sector can be used for consumption or investment in capital. Capital is assumed to depreciate fully at the end of each period.[14] Hence, the resource constraint for the economy is given by

$$C_t + K_{t+1} = Y_{Mt} + Y_{St}. \tag{3}$$

Since the production functions exhibit constant returns to scale, we assume, for analytical convenience, that there is just one competitive firm operating in each sector. Given a value for A_j, a wage rate (w), a rental rate for capital (r_K), and a rental rate for land (r_L), the firm in sector j solves the following problem:

$$\max\{Y_j - wN_j - r_K K_j - r_L L_j\} \qquad j = \mathrm{M, S} \tag{4}$$

subject to the production functions (1) and (2).

B. *Preferences and Demographic Structure*

Households live for two periods and have preferences that depend on consumption in each period of life. In particular, a young household born in period t has preferences summarized by the following utility function:

$$U(c_{1t}, c_{2,t+1}) = \log c_{1t} + \beta \log c_{2,t+1}. \tag{5}$$

Here, c_{1t} is consumption of a young household in period t and c_{2t} is consumption of an old household born in period $t - 1$.

The number of households born in period t is denoted by N_t, where

$$N_{t+1} = g(c_{1t})N_t \tag{6}$$

and the functional form of $g(\cdot)$ is given. Following Thomas R. Malthus (1798), and the more recent work of Kremer (1993), we assume that the population growth rate depends on the living standard, which we measure using consumption of a young household. In addition, we assume that this function is defined on the interval $[c_{\mathrm{MIN}}, \infty)$ and is continuous, differentiable, and single-peaked, and that $g'(c_{\mathrm{MIN}}) > 0$. The precise form of this function will be given in Section III.[15]

The initial old (period t_0) in this economy are endowed with K_{t_0}/N_{t_0-1} units of capital and $L = 1/N_{t_0-1}$ units of land. Old agents rent the land and capital to firms and, at the end of the period, sell their land to the young. Each young household is endowed with one unit of labor. Labor income is used to finance consumption and the purchase of capital and land, the return from which will finance consumption when households are old. That is, the young households maximize (5) subject to the following budget constraints:

$$c_{1t} + k_{t+1} + q_t l_{t+1} = w_t \tag{7}$$

$$c_{2,t+1} = r_{K,t+1}k_{t+1} + (r_{L,t+1} + q_{t+1})l_{t+1}.$$

The notation employed here is to use lowercase k and l to denote the capital and land owned by a particular household and uppercase K and L ($L = 1$) to denote the total stock of capital and land available in the economy. The letter q denotes the price of land.

C. *Competitive Equilibrium*

Given N_{t_0}, k_{t_0}, and l_{t_0} (where $N_{t_0-1}l_{t_0} = 1$), a competitive equilibrium in this economy consists of sequences for $t \geq t_0$ of prices, $\{q_t, w_t, r_{Kt}, r_{Lt}\}$; firm allocations, $\{K_{Mt}, K_{St}, N_{Mt}, N_{St}, Y_{Mt}, Y_{St}\}$; and household allocations, $\{c_{1t}, c_{2,t+1}, k_{t+1}, l_{t+1}\}$, such that the following are true:

the Solow technology be sufficiently small, but do not require that land's share be zero.

[14] Later, we will interpret a period in our model to be 35 years. Hence, the assumption of 100-percent depreciation is not implausible.

[15] A simple way to motivate a law of motion of this form is to allow young households to choose how many children they have. Let n_{t+1} be the number of children chosen by a young household in period t, and suppose that the utility function of a household is given by $U(c_{1t}, n_{t+1}) + \beta V(c_{2,t+1})$, where U is increasing and concave in both arguments. In addition, suppose that n_{t+1} does not affect the budget constraint of the household. In this case, the optimality condition determining n_{t+1} is $U_2(c_{1t}, n_{t+1}) = 0$. This equation can be solved to obtain $n_{t+1} = g(c_{1t})$, which implies that $N_{t+1} \equiv N_t n_{t+1} = g(c_{1t})N_t$. We have found it convenient to model g as an exogenous function, since we plan to calibrate the population dynamics to match historical data.

1. Given the sequence of prices, the firm allocation solves the problems specified in equation (4).
2. Given the sequence of prices, the household allocation maximizes (5) subject to (7).
3. Markets clear:
   $K_{Mt} + K_{St} = N_{t-1}k_t$
   $N_{Mt} + N_{St} = N_t$
   $N_{t-1}l_t = 1$
   $Y_{Mt} + Y_{St} = N_t c_{1t} + N_{t-1}c_{2t} + N_t k_{t+1}.$
4. $N_{t+1} = g(c_{1t})N_t$.

In characterizing an equilibrium, we make use of the following results:

PROPOSITION 1: *For any wage rate w and capital rental rate r_K, it is profitable to operate the Malthus sector. That is, $Y_{Mt} > 0$ for all t.*

PROOF:
Given w and r_K, when problem (4) is solved for the Malthus sector, maximum profits are equal to

$$\Pi_M(w, r_K) = A_{Mt}^{1/(1-\phi-\mu)}(1-\phi-\mu) \times \left(\frac{\phi}{r_K}\right)^{\phi/(1-\phi-\mu)}\left(\frac{\mu}{w}\right)^{\mu/(1-\phi-\mu)}$$

which is clearly positive for all t.

A similar argument applied to the Solow sector gives the following result:

PROPOSITION 2: *Given a wage rate w and capital rental rate r_K, maximized profit per unit of output in the Solow sector is positive if and only if*

$$A_{St} > \left(\frac{r_K}{\theta}\right)^{\theta}\left(\frac{w}{1-\theta}\right)^{1-\theta}. \tag{8}$$

If, in some period t, only the Malthus production process is employed, the equilibrium wage and rental rate of capital are

$$w_t = \mu A_{Mt}K_t^{\phi}N_t^{\mu-1} \tag{9}$$
$$r_{Kt} = \phi A_{Mt}K_t^{\phi-1}N_t^{\mu}.$$

COROLLARY: *Both the Malthus and Solow sectors will be operated in period t if and only if equation (8) is satisfied at the factor prices obtained by evaluating equation (9) at the period-t values of A_M, A_S, K, and N.*

If both sectors are operated, (9) is not the equilibrium factor prices. Instead, resources are allocated efficiently across the two sectors as guaranteed by the First Welfare Theorem. Hence, total output is uniquely determined by the following well-behaved maximization problem:[16]

$$Y(A_M, A_S, K, N) = \max_{\substack{0\le K_S\le K\\ 0\le N_S\le N}} \{A_M(K-K_S)^{\phi}(N-N_S)^{\mu} + A_S K_S^{\theta}N_S^{1-\theta}\}. \tag{10}$$

The equilibrium wage and rental rates are

$$w_t = \mu A_{Mt}K_{Mt}^{\phi}N_{Mt}^{\mu-1} = (1-\theta)A_{St}K_{St}^{\theta}N_{St}^{-\theta} \tag{11}$$
$$r_{Kt} = \phi A_{Mt}K_{Mt}^{\phi-1}N_{Mt}^{\mu} = \theta A_{St}K_{St}^{\theta-1}N_{St}^{1-\theta}$$
$$r_{Lt} = (1-\phi-\mu)A_{Mt}K_{Mt}^{\phi}N_{Mt}^{\mu}.$$

The first-order condition for the household's optimization problem can be arranged to yield the following expressions:

$$c_{1t} = \frac{w_t}{1+\beta} \tag{12}$$

$$q_{t+1} = q_t r_{K,t+1} - r_{L,t+1}. \tag{13}$$

In addition, the budget constraints and market-clearing conditions imply that

$$K_{t+1} = N_t(w_t - c_{1t}) - q_t. \tag{14}$$

[16] Of course, factor allocations solve this maximization problem whether or not both sectors are operated.

Given sequences $\{A_{Mt}, A_{St}\}_{t=t_0}^{t_n}$, initial conditions K_{t_0} and N_{t_0}, and an initial price of land, q_{t_0}, equations (6) and (11)–(14) determine an equilibrium sequence of prices and quantities,

$$\{w_t, r_{Kt}, r_{Lt}, q_{t+1}, c_{1t}, N_{t+1}, K_{t+1}\}_{t=t_0}^{t_n}.$$

The value of q_{t_0} is also determined by the equilibrium conditions of the model, but cannot be solved for analytically. A numerical shooting algorithm can be used to compute this initial price.

III. The Equilibrium Development Path

We choose initial conditions (K_{t_0} and N_{t_0}) and the sequences $\{A_{Mt}, A_{St}\}_{t=t_0}^{t_n}$ so that the economy is initially using only the Malthus technology [equation (8) is not satisfied] and then study how the economy develops over time. We state sufficient conditions guaranteeing that the Solow technology will eventually be adopted, but without further restrictions on the parameters of the model, there may or may not be a transition to the Solow economy with land being of minor importance in production. We use data from the Malthusian era and from the last half of the 20th century to restrict the values of the parameters and to compute the equilibrium path of the resulting economy.

The initial value N_{t_0} is set equal to 1, and the initial capital stock, K_{t_0}, is set so that it lies on the asymptotic growth path of a version of the model economy with only the Malthus technology. To characterize this growth path, we make two additional assumptions about total factor productivity and population growth. First, we assume that A_{Mt} grows at a constant rate: $A_{Mt} = \gamma_M^t$, where $\gamma_M \geq 1$. Second, we assume that $g'(c_{1M}) > 0$, where c_{1M} is defined by $g(c_{1M}) = \gamma_M^{1/(1-\phi-\mu)}$. This assumption guarantees that the Malthus-only asymptotic growth path has the Malthusian feature that per capita income is constant.[17]

Given our choice of initial conditions, as long as the Solow technology has not yet been adopted, the population growth factor is equal to $\gamma_M^{1/(1-\mu-\phi)}$ and the consumption of young individuals is constant, $c_{1t} = c_{1M}$. Aggregate output, capital, total consumption, the price of land, and the rental rate of land grow at the same rate as population. The wage and capital rental rates are constant. This implies that the expression on the right-hand side of equation (8) is also a constant, which we denote by $\hat{A}$. In this case, productivity growth translates directly into population growth, and there is no improvement in household living standards. This mimics the long-run growth path (abstracting from plagues and other disturbances) that actual economies experienced for centuries prior to the industrial revolution.

If $\{A_{St}\}_{t=t_0}^{\infty}$ grows at some positive rate, eventually A_{St} will exceed $\hat{A}$. Proposition 2 guarantees that at this point capital and labor will be allocated to the Solow technology. However, whether or not the economy will transition to a Solow economy with land being an unimportant factor of production and, if so, how long the transition will take are quantitative questions depending upon the parameters of the model.

A. *The Quantitative Exercise*

We have designed our quantitative exercise so that the economy is initially in a Malthusian steady state, and then we simulate the equilibrium path until essentially all the available capital and labor are employed in the Solow sector.[18] We interpret one model period to be 35 years. To keep this exercise simple, we assume that for all t, $A_{St} = \gamma_S^t$, where $\gamma_S > 1$.[19] The

[17] If $\gamma_M^{1/(1-\mu-\phi)}$ is larger than the maximum value of $g(c_1)$, there will be sustained growth in per capita consumption in periods when only the Malthus technology is employed.

[18] Proposition 1 implies that some fraction of total resources will always be employed in the Malthus sector, although this fraction can (and does in our simulations) converge to zero in the limit.

[19] This implies that total factor productivity in the Solow sector is growing at the same rate prior to the adoption of this technology as it is after. We do not take this assumption literally, and it is not required for our results. Although technological advancement clearly did not begin with the industrial revolution, once the Solow technology began to be used, the advantages of "learning by doing" and more immediate economic payoff almost certainly increased the rate of technological growth.

TABLE 3—PARAMETER VALUES

Parameter	Definition	Value	Comments
γ_M	Growth factor in Malthus technology	1.032	Consistent with population growth in Malthus era (doubles every 230 years or 6.57 model periods).
γ_S	Growth factor in Solow technology	1.518	Consistent with growth rate of per capita GNP in postwar United States.
ϕ	Capital share in Malthus technology	0.1	Similar to value reported in Philip T. Hoffman (1996) and Clark (1998a).
μ	Labor share in Malthus technology	0.6	Labor's share is set equal to 0.6 in both technologies.
θ	Capital share in Solow technology	0.4	Based on data for factor shares in postwar United States.
β	Discount factor	1.0	Implies annual return on capital varying from 2 percent in Malthus era to 4–4.5 percent in periods when Solow technology is heavily used.

model is calibrated so that (1) the initial Malthusian era is consistent with the growth facts describing the English economy prior to 1800, (2) the Solow-only economy matches the growth facts describing post-World War II industrialized economies, (3) the population growth rate reacts to changing living standards as reported in Kremer (1993) and Lucas (1998), and (4) the implied annual rate of return on capital is reasonable given available data. These criteria lead us to the parameter values shown in Table 3.

In addition, we use data in Lucas (1998) on population growth rates and per capita GNP for various regions of the world from 1750 to the present to calibrate the population growth function, $g(c_1)$. Population growth rates appear to increase linearly in living standards (c_1 in our model) from the Malthusian level to the level where population is doubling each period (every 35 years). Over this range, living standards double from the Malthusian level. After this, the population growth rate decreases linearly until living standards are approximately 18 times what they were in the Malthus steady state. We assume that population is constant as c_1 grows beyond this point. This gives us the following function $g(c_1)$:

$$g(c_1) = \begin{cases} \gamma_M^{1/(1-\mu-\phi)}\left(2 - \dfrac{c_1}{c_{1M}}\right) + 2\left(\dfrac{c_1}{c_{1M}} - 1\right) & \text{for } c_1 < 2c_{1M} \\ 2 - \dfrac{c_1 - 2c_{1M}}{16c_{1M}} & \text{for } 2c_{1M} \leq c_1 \leq 18c_{1M} \\ 1 & \text{for } c_1 > 18c_{1M}. \end{cases} \tag{15}$$

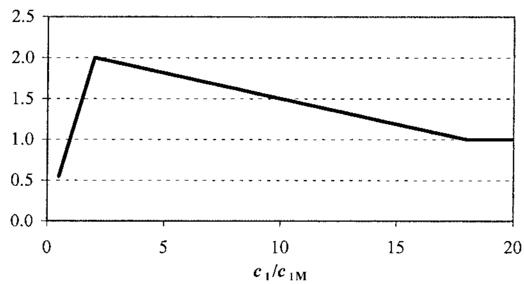

FIGURE 3. POPULATION GROWTH FUNCTION: $g(c_1/c_{1M})$

Figure 3 graphs the function $g(c_1)$ against values of c_1/c_{1M}.[20]

We simulated the economy beginning with period $t_0 = -5$ for 11 periods, at which point the transition to the Solow technology was effectively complete.[21] Figure 4 shows the

[20] Since the peak of our function $g(c_1)$ is so much larger than the population growth rate in the Malthusian era, sustained growth will not occur prior to the industrial revolution in any reasonably calibrated version of this economy. In addition, we mentioned earlier that land could be an input in the Solow production function and still obtain sustained growth following the industrial revolution, as long as land's share is not too large. Given our calibration of $g(c_1)$ and γ_S, we would require that land's share in the Solow technology exceed 0.6022 for Malthusian stagnation to occur after this technology had been adopted. We believe this to be an implausibly large value for land's share in the modern industrial period.

[21] An iterative *shooting algorithm* was used to determine the equilibrium initial price of land. As long as an equilibrium exists, which can be established using standard arguments, our computation procedure is able to approximate the equilibrium to the accuracy of the computer. In addition, because of the exhaustive nature of our one-dimensional search, we are able to establish that the equilibrium is

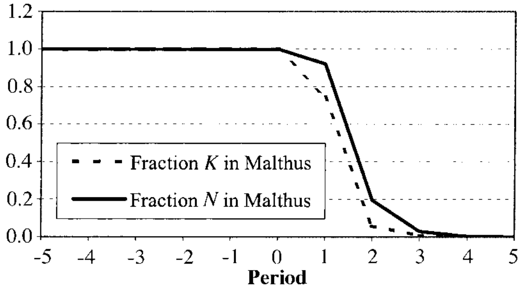

FIGURE 4. FRACTION OF INPUTS EMPLOYED IN MALTHUS SECTOR (K = CAPITAL; N = LABOR)

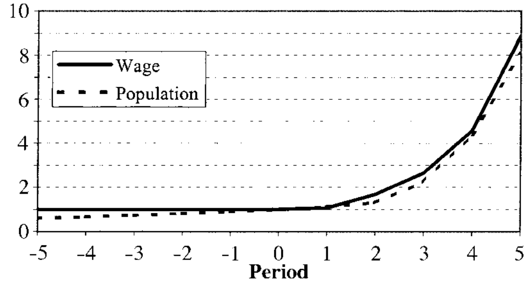

FIGURE 5. WAGE AND POPULATION

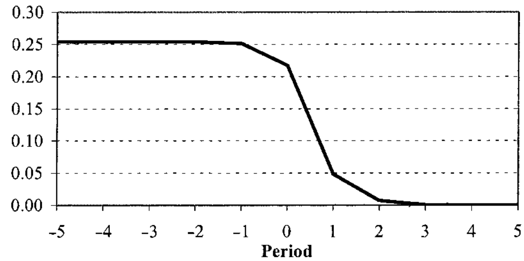

FIGURE 6. VALUE OF LAND RELATIVE TO OUTPUT

fraction of productive inputs (capital and labor) employed in the Malthus sector each period. The transition takes three generations (105 years) from the point at which the Solow technology is first used until over 99 percent of the resources are allocated to the Solow sector. As in the English industrial revolution, the transition to a modern industrial economy is not instantaneous, but takes generations to achieve.[22]

Only the Malthus technology is used from period −5 to period 0. During this time, output per worker remains constant. Once the industrial revolution begins in period 1, output per worker grows at increasingly higher rates. In particular, there is very little growth in per capita output in the first 35-year generation after the Solow technology is adopted (annualized growth rate of 0.16 percent). The economy grows at 1.3 percent annually in the next two generations. The growth rate continues to increase in subsequent generations, eventually converging to the growth rate of a Solow-only economy (2 percent annual growth in our calibration) from above.

Our simulated growth path has several features in common with the historical data shown in Figure 1 and Tables 1 and 2. During the periods when only the Malthus technology is being used, population grows at the same rate as output, and the wage stays constant. After period 0, population growth increases, and the real wage increases as well. This is shown in Figure 5, where the wage has been normalized to equal one in the Malthus steady state.

Figure 6 shows that the value of land relative to output decreases after the Solow technology is adopted. This is roughly consistent with the pattern seen in Table 2.

IV. Conclusion

Until very recently, the literature on economic growth focused on explaining features of modern industrial economies while being inconsistent with the growth facts describing preindustrial economies. This includes both models based on exogenous technical progress, such as Robert M. Solow (1957), and more recent models with endogenous growth, such as Paul M. Romer (1986) and Lucas (1988). But sustained growth has existed for at most the past two centuries, while the millennia prior have been characterized by stagnation with no significant

unique. Details on our solution procedure are available from the authors upon request.

[22] Although a protracted industrial revolution is an implication of our theory, data limitations prevent us from computing the transition path implied by the historical A_M and A_S sequences. If this were possible, more precise theoretical predictions about the shape of the historical transition path could be obtained.

permanent growth in living standards. This paper contributes to a recent literature describing unified growth models that can account for the basic growth facts of both eras, as well as the transition between the two. In particular, our theory predicts that land's share in production should fall endogenously over time, as observed historically, and that there will be an escape from Malthusian stagnation and a transition to modern growth in the sense of Solow.

Some caveats are in order. Although it has become popular in the growth literature to model the accumulation of nonrivalrous knowledge as an endogenous feature of the model economy studied, we have chosen to abstract from this and assume exogenous technological progress. We made this choice both because it simplifies our analysis and because we do not believe that there yet exists a theory of knowledge accumulation with the same level of acceptance that is accorded to the standard theory of capital accumulation. For those who disagree, we believe that endogenous growth features can be easily incorporated into our theory in a way that does not alter our main findings.[23]

In addition, we have not explored how policy and institutions, by discouraging or preventing the invention and adoption of new ideas, might play an important role in determining when the Solow technology is first used and how quickly the transition from Malthus to Solow is completed. Jones (1999), for example, emphasizes the role that policy and institutions, by affecting the rate of compensation for inventive activity, might play in determining the timing of the industrial revolution. In addition, Stephen L. Parente and Prescott (1997) have studied how policy can affect the level of the total factor productivity parameter in the Solow technology. By keeping this parameter small, policy can affect when equation (8) is satisfied and, hence, when (if ever) the industrial revolution begins. The fact that the industrial revolution happened first in England in the early 19th century rather than contemporaneously or earlier in China, where the stock of usable knowledge may have actually been higher, is perhaps due to the institutions and policies in place in these two countries.

In our theory, the transition from a land-intensive to a modern industrial economy requires that the rate of total factor productivity growth in the Solow sector be positive in periods prior to the adoption of this technology. The technology must improve sufficiently so that it ultimately becomes profitable to shift resources into this previously unused sector. Consistent with this idea, Joel Mokyr (1990 p. 6), who documents technological progress over the past 25 centuries, notes "much growth ... is derived from the deployment of previously available information rather than the generation of altogether new knowledge." Of course, some technological advancements, the wheelbarrow, for example, increased total factor productivity in both sectors, yet was employed long before the industrial revolution. Another invention, the steam engine, perhaps the quintessential invention of the industrial revolution, depended heavily on developments that occurred well within the Malthusian era (see Mokyr, 1990 p. 84). A cost-effective method for converting thermal energy into kinetic energy appears to have been a crucial precondition for the Solow technology to be profitable.

Finally, in contrast to some of the recent papers modeling the transition from stagnation to sustained growth, our theory is silent as to why population growth rates are increasing in living standards in the early stages of development and then become decreasing in living standards at more advanced stages (the demographic transition). Some economists (e.g., Galor and Weil, 2000), following Becker (1960), have argued that this may be related to a quantity-quality trade-off between the number of children a family produces versus the amount of human capital invested in each child. Other possibilities, perhaps more relevant in our context, include that the shift from the Malthus to the Solow technology involves households choosing to leave a home production sector, where children are economic assets, in order to enter a market sector, where they are not (see e.g., Mark S. Rosenzweig and Robert E.

[23] As discussed in the Introduction, Jones (1999) and Galor and Weil (2000) provide theories where there is endogenous accumulation of knowledge that eventually permits escape from Malthusian stagnation.

Evenson, 1977; Karine S. Moe, 1998).[24] We leave it to future work to incorporate these ideas into the theory studied in this paper.

REFERENCES

Arifovic, Jasmina; Bullard, James and Duffy, John. "The Transition from Stagnation to Growth: An Adaptive Learning Approach." *Journal of Economic Growth*, June 1997, *2*(2), pp. 185–209.

Becker, Gary S. "An Economic Analysis of Fertility," in Richard Easterlin, ed., *Demographic and economic change in developed countries: A conference of the Universities-National Bureau Committee for Economics.* Princeton, NJ: Princeton University Press, 1960, pp. 209–40.

Becker, Gary S.; Murphy, Kevin M. and Tamura, Robert. "Human Capital, Fertility, and Economic Growth." *Journal of Political Economy*, October 1990, Pt. 2, *98*(5), pp. S12–37.

Clark, Gregory. "Microbes and Markets: Was the Black Death an Economic Revolution?" Unpublished manuscript, University of California, Davis, 1998a.

———. "Nominal and Real Male Agricultural Wages in England, 1250–1850." Unpublished manuscript, University of California, Davis, 1998b.

Diamond, Peter A. "National Debt in a Neoclassical Growth Model." *American Economic Review*, December 1965, *55*(5), pp. 1126–50.

Galor, Oded and Weil, David N. "The Gender Gap, Fertility, and Growth." *American Economic Review*, June 1996, *86*(3), pp. 374–87.

———. "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond." *American Economic Review*, September 2000, *90*(4), pp. 806–28.

Goodfriend, Marvin and McDermott, John. "Early Development." *American Economic Review*, March 1995, *85*(1), pp. 116–33.

Hoffman, Philip T. *Growth in a traditional society: The French countryside, 1450–1815.* Princeton, NJ: Princeton University Press, 1996.

Jones, Charles I. "Was the Industrial Revolution Inevitable? Economic Growth Over the Very Long Run." Unpublished manuscript, Stanford University, 1999.

Kremer, Michael. "Population Growth and Technological Change: One Million B.C. to 1990." *Quarterly Journal of Economics*, August 1993, *108*(3), pp. 681–716.

Laitner, John. "Structural Change and Economic Growth." *Review of Economic Studies*, July 2000, *67*(3), pp. 545–61.

Lucas, Robert E., Jr. "On the Mechanics of Economic Development." *Journal of Monetary Economics*, July 1988, *22*(1), pp. 3–42.

———. *The industrial revolution: Past and future.* Unpublished manuscript, University of Chicago, 1998.

Maddison, Angus. *Dynamic forces in capitalist development: A long run comparative view.* Oxford: Oxford University Press, 1991.

Malthus, Thomas R. *An essay on the principle of population.* Oxford: Oxford University Press (1993 printing), 1798.

Miller, Merton H. and Upton, Charles W. *Macroeconomics: A neoclassical introduction.* Homewood, IL: Richard D. Irwin, Inc., 1974.

Moe, Karine S. "Fertility, Time Use, and Economic Development." *Review of Economic Dynamics*, July 1998, *1*(3), pp. 699–718.

Mokyr, Joel. *The lever of riches: Technological creativity and economic progress.* Oxford: Oxford University Press, 1990.

Nordhaus, William D. "Do Real-Output and Real-Wage Measures Capture Reality? The History of Lighting Suggests Not," in Timothy F. Bresnahan and Robert J. Gordon, eds., *The economics of new goods.* NBER Studies in Income and Wealth, Vol. 58. Chicago: University of Chicago Press, 1997, pp. 29–66.

Parente, Stephen L. and Prescott, Edward C. "Monopoly Rights: A Barrier to Riches." Research Department Staff Report 236, Federal Reserve Bank of Minneapolis, 1997.

Phelps-Brown, Henry and Hopkins, Sheila V. "Seven Centuries of the Prices of Consumables, Compared With Builders' Wage-Rates." *Economica*, November 1956, (23), pp. 296–315.

[24] A related possibility, that the reduction in fertility comes about because of an increase in the relative wage of women as capital per worker increases, is explored in Galor and Weil (1996).

Romer, Paul M. "Increasing Returns and Long-Run Growth." *Journal of Political Economy*, October 1986, *94*(5), pp. 1002–37.

Rosenzweig, Mark S. and Evenson, Robert E. "Fertility, Schooling, and the Economic Contribution of Children in Rural India: An Econometric Analysis." *Econometrica*, July 1977, *45*(5), pp. 1065–79.

Solow, Robert M. "Technical Change and the Aggregate Production Function." *Review of Economics and Statistics*, August 1957, *39*(3), pp. 312–20.

Stokey, Nancy L. "A Quantitative Model of the British Industrial Revolution, 1780–1850." Unpublished manuscript, University of Chicago, 2001.

Tamura, Robert. "From Agriculture to Industry: Human Capital and Specialization." Unpublished manuscript, Clemson University, 1998.

U.S. Bureau of the Census. *Historical statistics of the United States, colonial times to 1970.* 2 Vols. Washington, DC: U.S. Government Printing Office, 1975.

Wrigley, E. A.; Davis, R. S.; Oeppen, J. E. and Schofield, R. S., eds. *English population history from family reconstitution 1580–1837.* New York: Cambridge University Press, 1997.