# Finite Automata

Part Two

# Recap from Last Time

# Strings

- An ***alphabet*** is a finite set of symbols called ***characters***.
  - Typically, we use the symbol **Σ** to refer to an alphabet.
- A ***string over an alphabet Σ*** is a finite sequence of characters drawn from Σ.
- Example: If Σ = {**a**, **b**}, some valid strings over Σ include

**a**

**aabaaabbabaaabaaaabbb**

**abbababba**

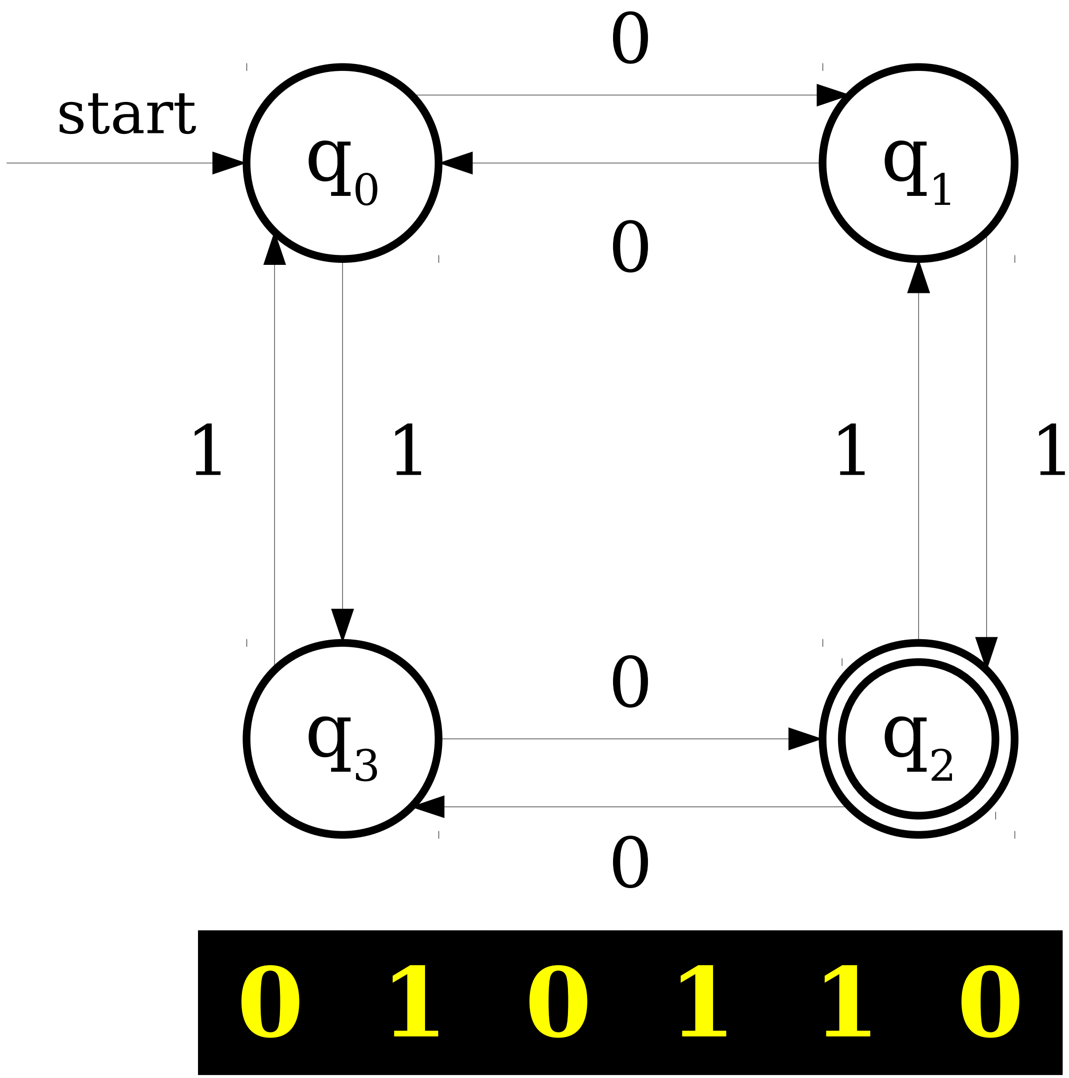
- The ***empty string*** contains no characters and is denoted **ε**.

# Languages

- A ***formal language*** is a set of strings.
- We say that $L$ is a ***language over Σ*** if it is a set of strings over Σ.
- Example: The language of palindromes over Σ = {**a**, **b**, **c**} is the set

  {ε, **a**, **b**, **c**, **aa**, **bb**, **cc**, **aaa**, **aba**, **aca**, **bab**, … }
- The set of all strings composed from letters in Σ is denoted **Σ***.
- Formally: $L$ is a language over Σ iff $L \subseteq \Sigma^*$.

# A Simple Finite Automaton



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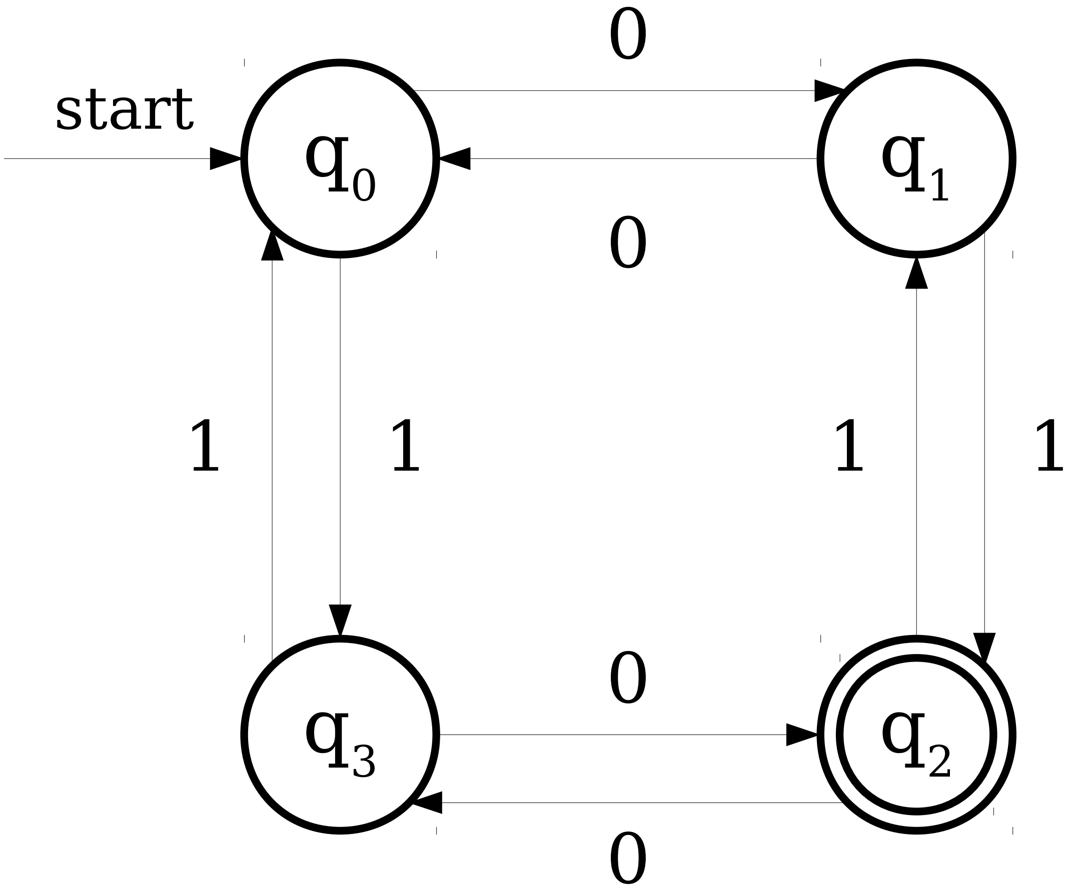



| | | | | | |
|---|---|---|---|---|---|
| **1** | **0** | **1** | **0** | **0** | **0** |

The ***language of an automaton*** is the set of strings that it accepts.

If $D$ is an automaton, we denote the language of $D$ as $\mathscr{L}(\boldsymbol{D})$.

$$\mathscr{L}(D) = \{\ w \in \Sigma^* \mid D \text{ accepts } w\ \}$$

# DFAs

- A **DFA** is a
  - **D**eterministic
  - **F**inite
  - **A**utomaton
- DFAs are the simplest type of automaton that we will see in this course.

# DFAs, Informally

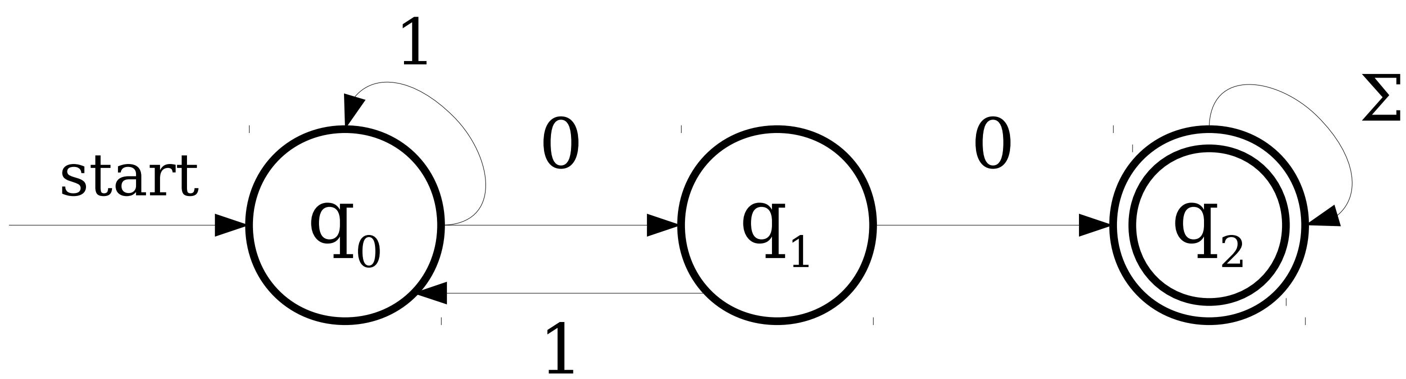
- A DFA is defined relative to some alphabet Σ.
- For each state in the DFA, there must be ***exactly one*** transition defined for each symbol in Σ.
  - This is the "deterministic" part of DFA.
- There is a unique start state.
- There are zero or more accepting states.

# Designing DFAs

- At each point in its execution, the DFA can only remember what state it is in.
- **DFA Design Tip:** Build each state to correspond to some piece of information you need to remember.
  - Each state acts as a “memento” of what you're supposed to do next.
  - Only finitely many different states ≈ only finitely many different things the machine can remember.

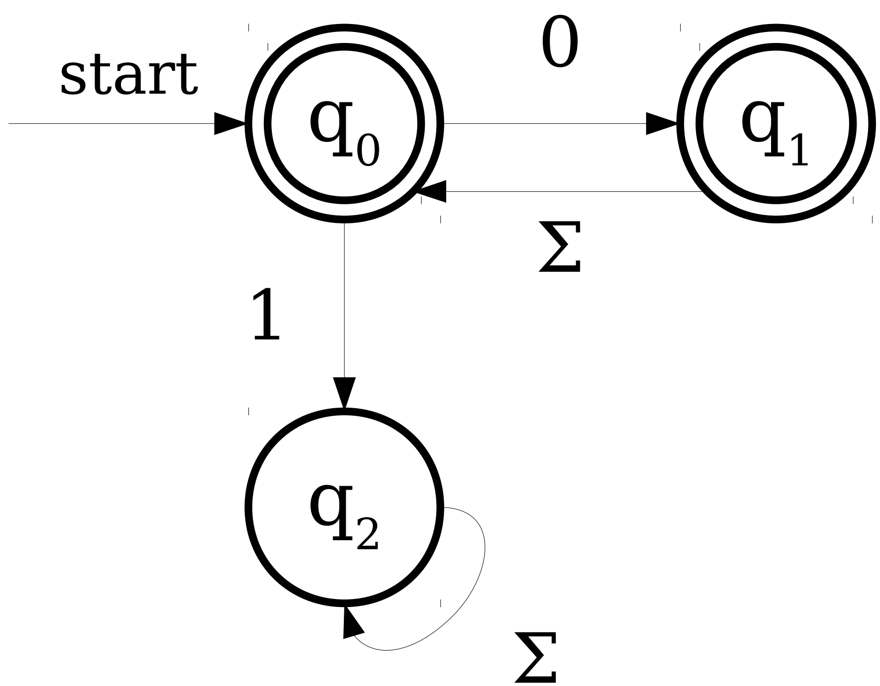
# Recognizing Languages with DFAs

$L = \{ w \in \{0, 1\}^* \mid w \text{ contains } 00 \text{ as a substring } \}$



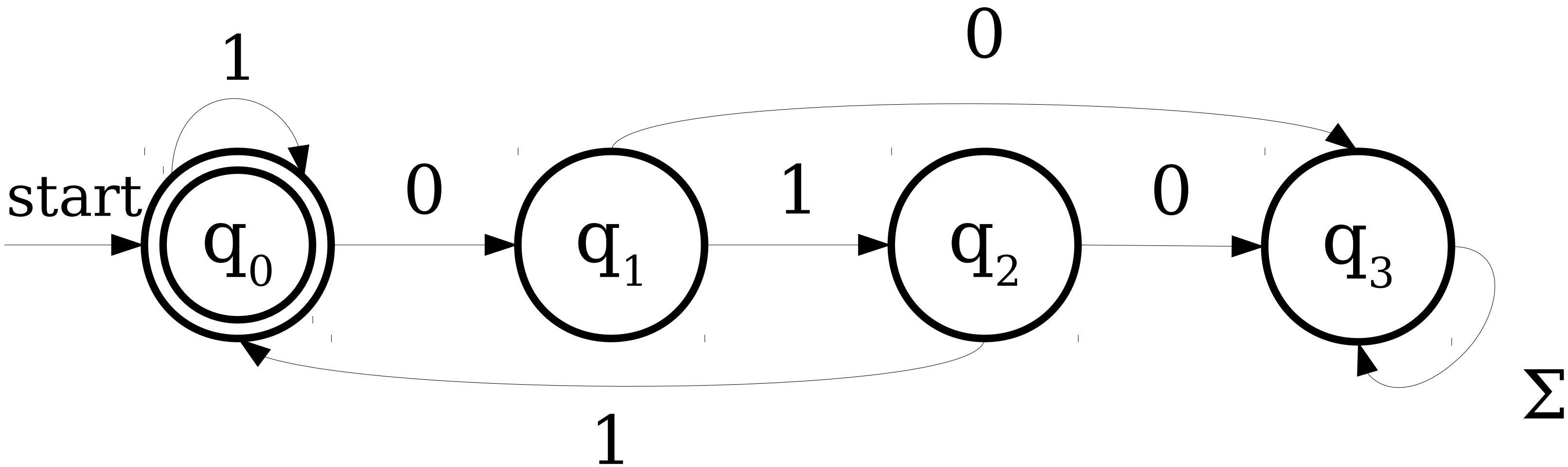
# Recognizing Languages with DFAs

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid$ every other character of $w$, starting with the first character, is $\texttt{0}$ $\}$



# New Stuff!

# Tabular DFAs



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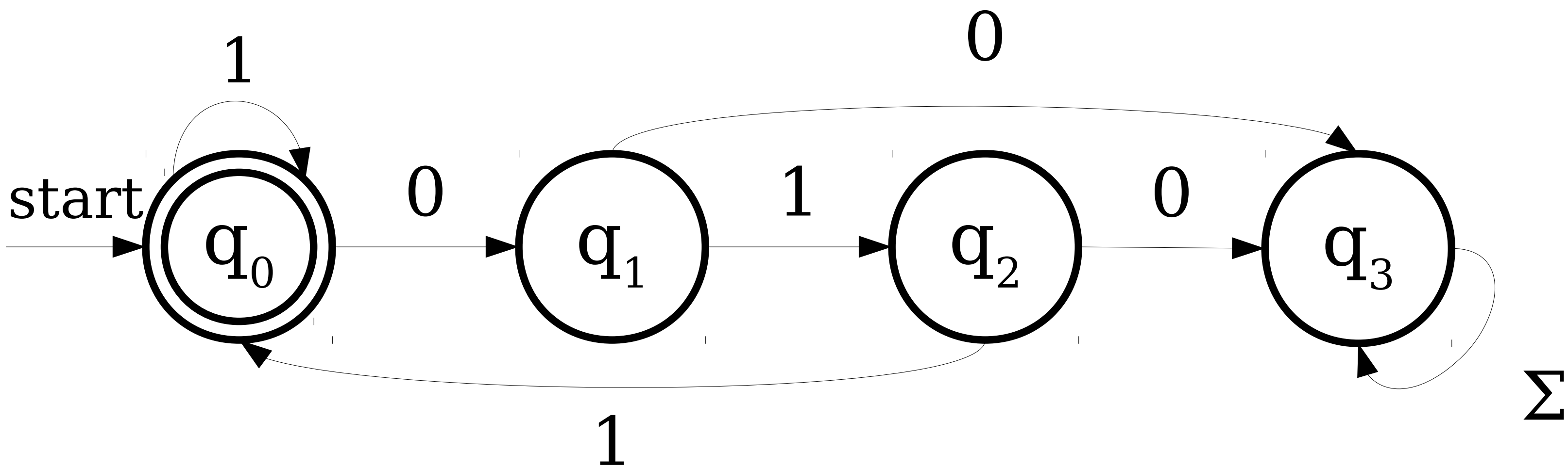
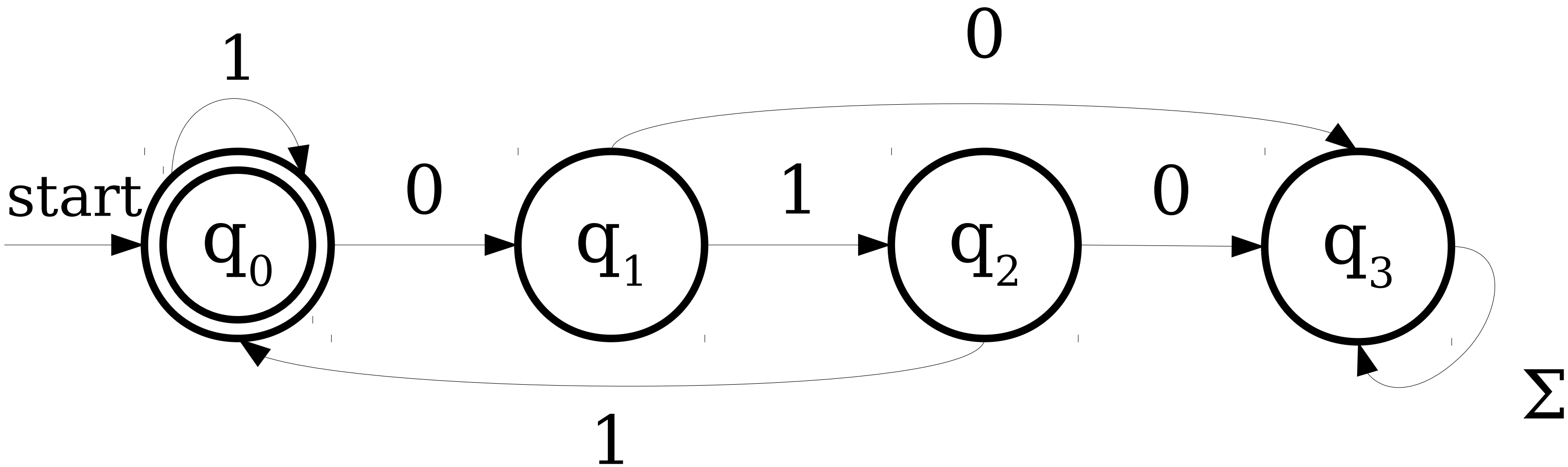



| | 0 | 1 |
|---|---|---|
| $q_0$ | | |
| $q_1$ | | |
| $q_2$ | | |
| $q_3$ | | |

# Tabular DFAs




| | 0 | 1 |
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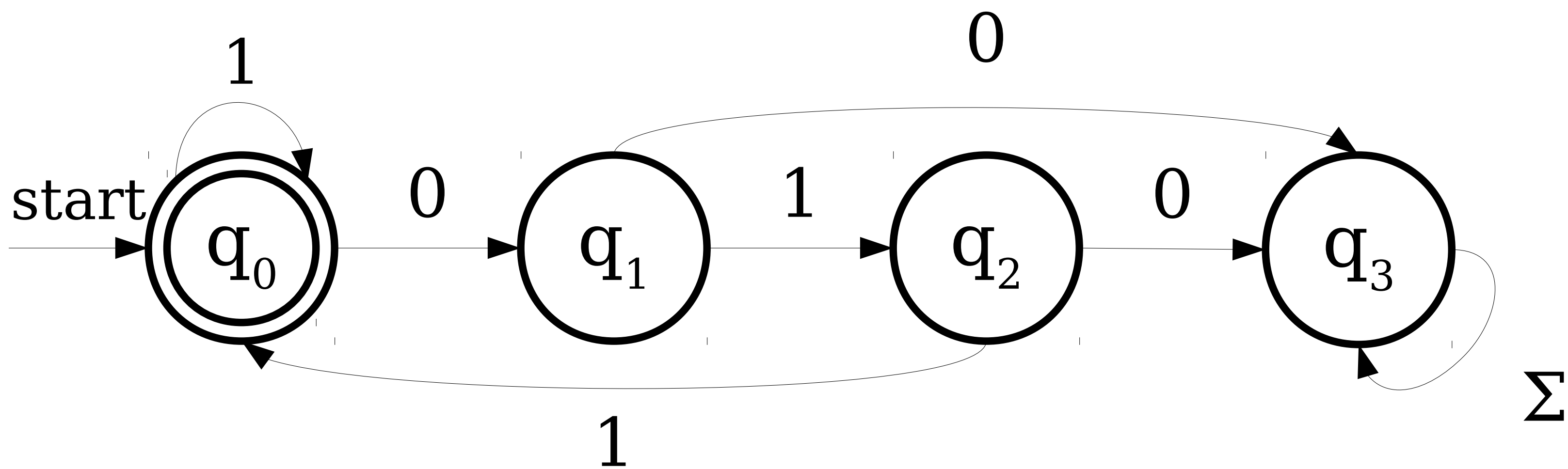


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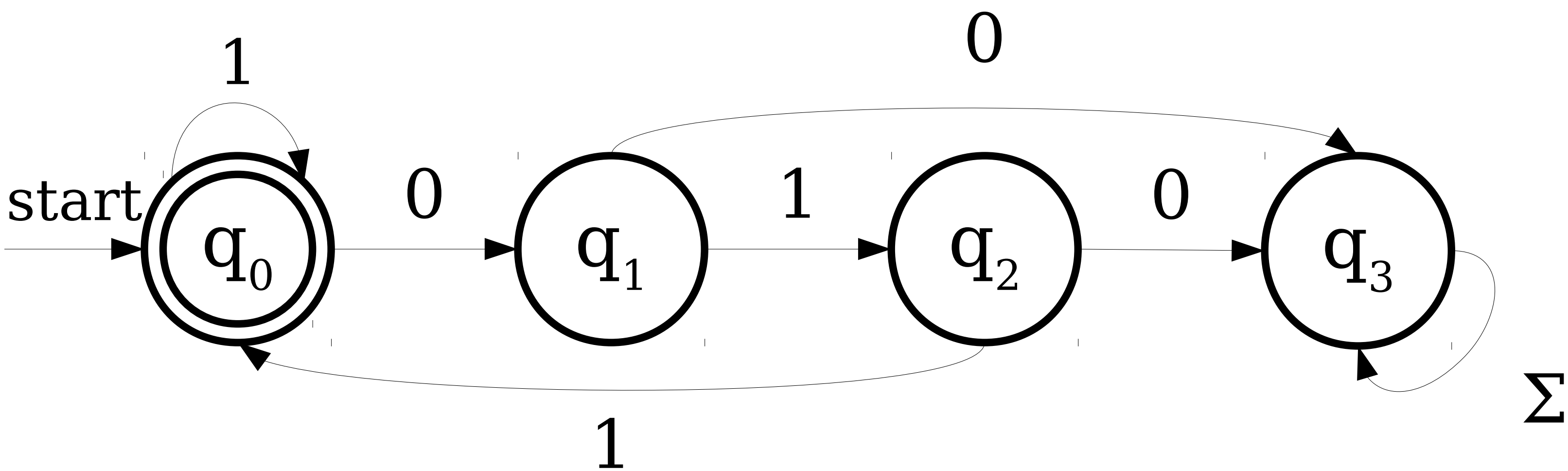


| | 0 | 1 |
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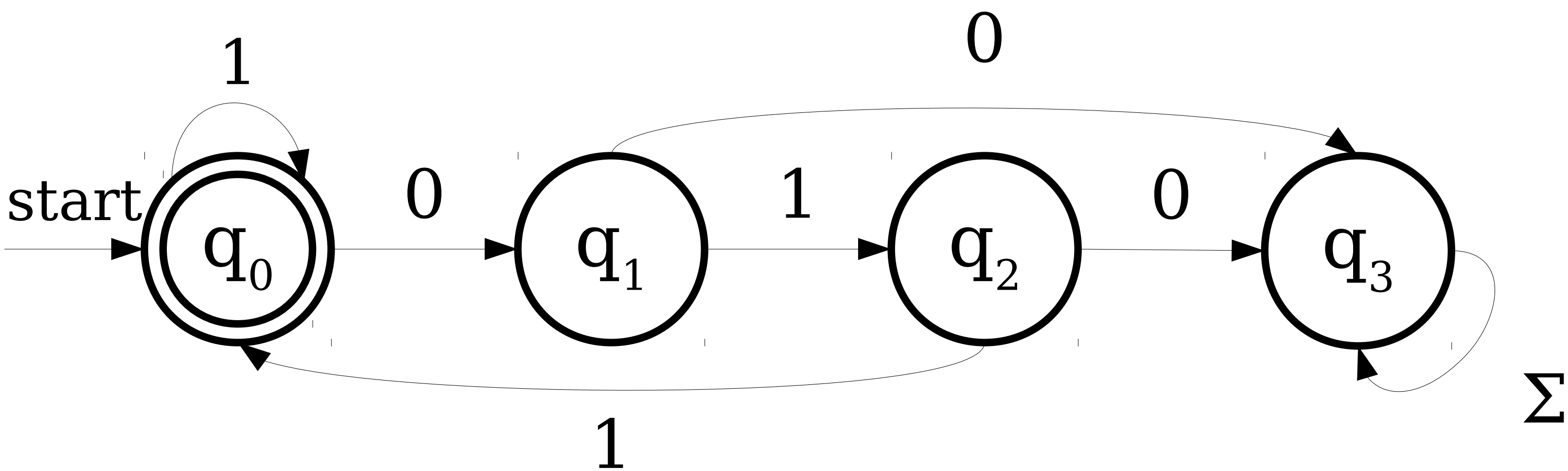


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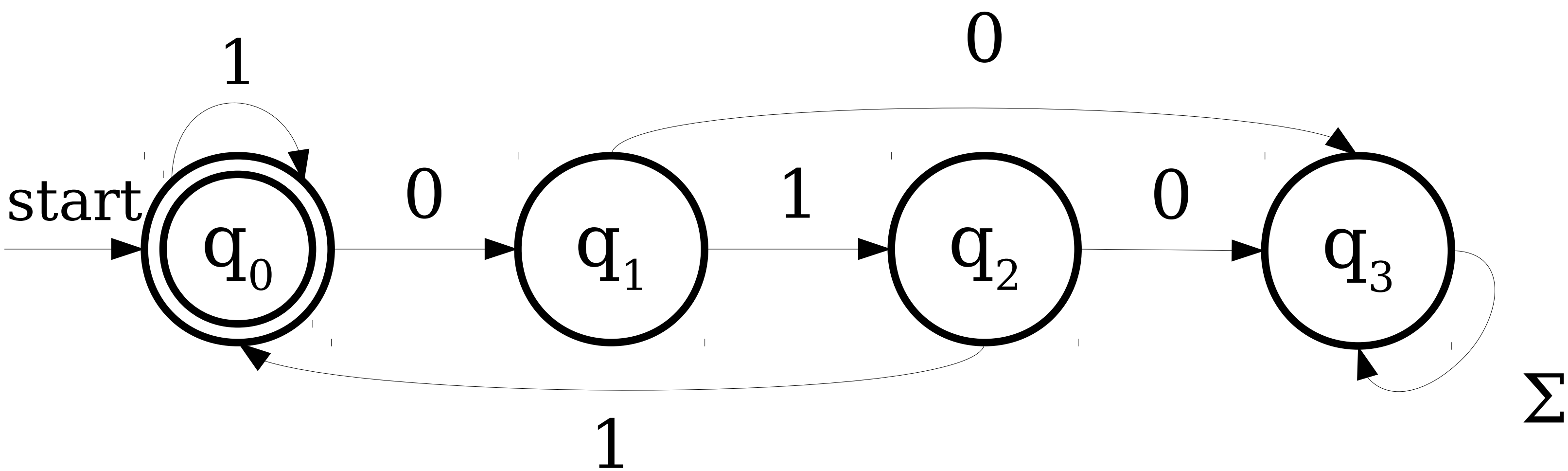


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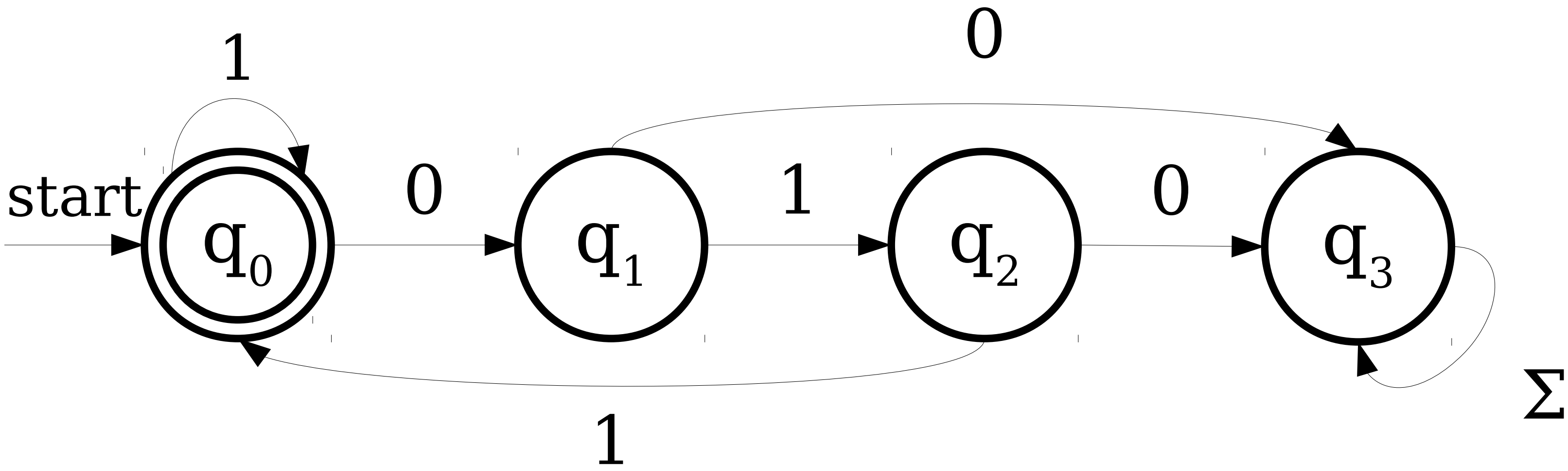


| | 0 | 1 |
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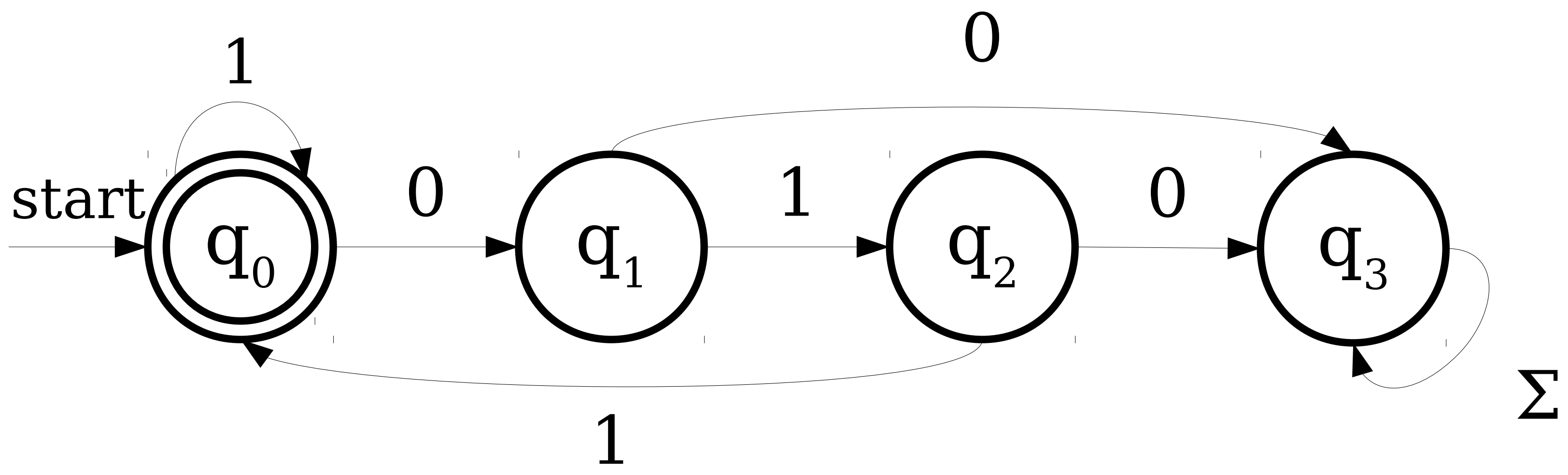


| | 0 | 1 |
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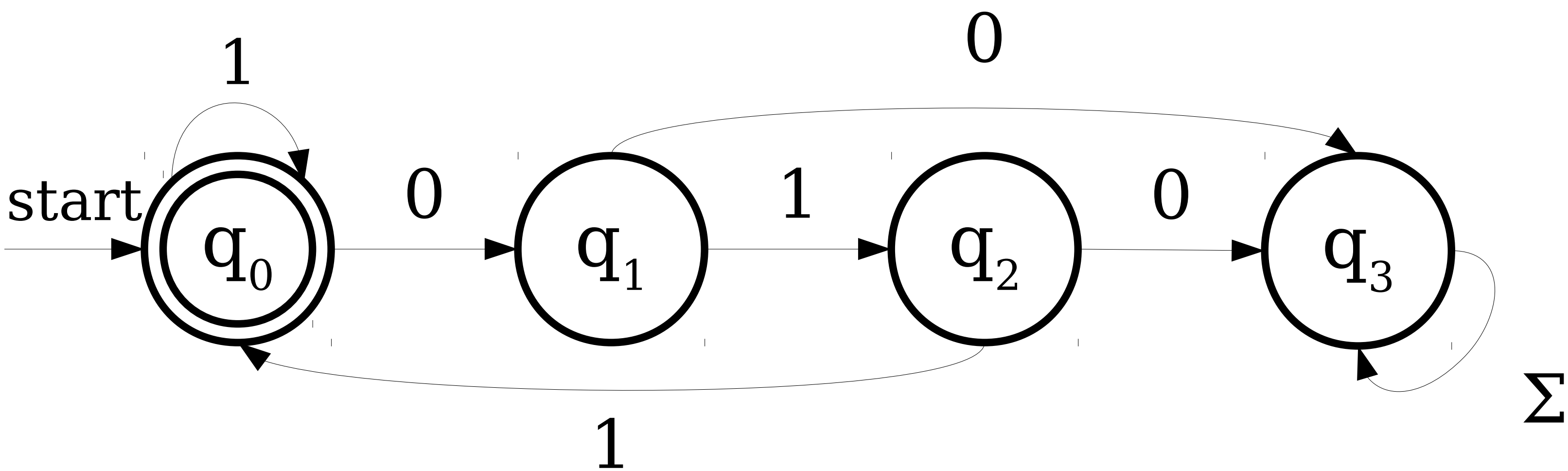


| | 0 | 1 |
|---|---|---|
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| $q_1$ | $q_3$ | $q_2$ |
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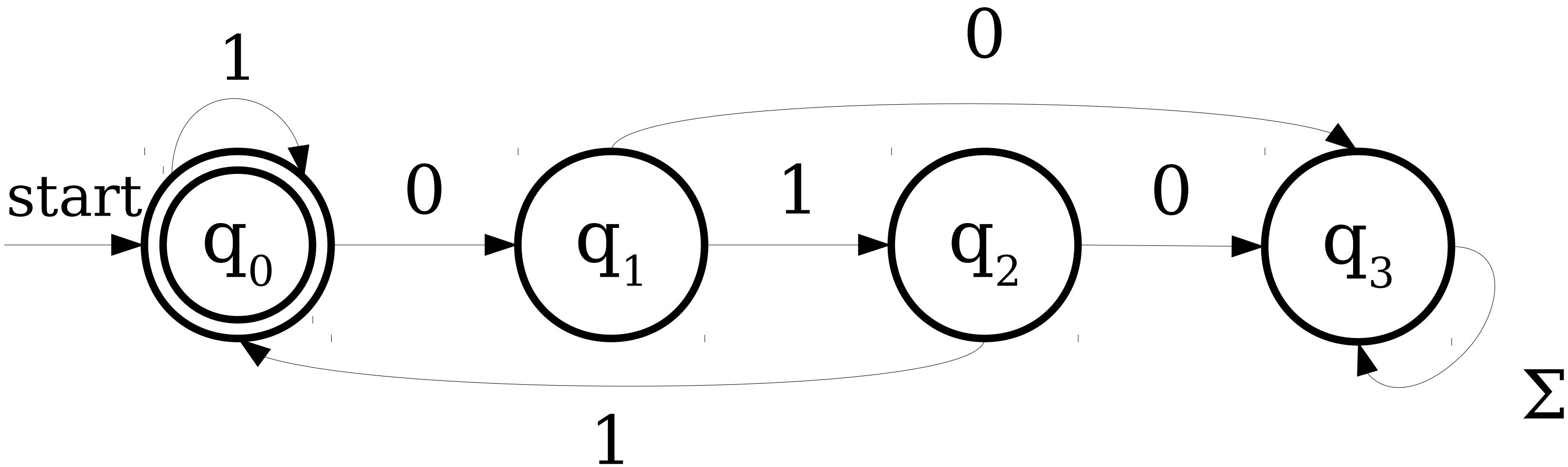


| | 0 | 1 |
|---|---|---|
| $*q_0$ | $q_1$ | $q_0$ |
| $q_1$ | $q_3$ | $q_2$ |
| $q_2$ | $q_3$ | $q_0$ |
| $q_3$ | $q_3$ | $q_3$ |

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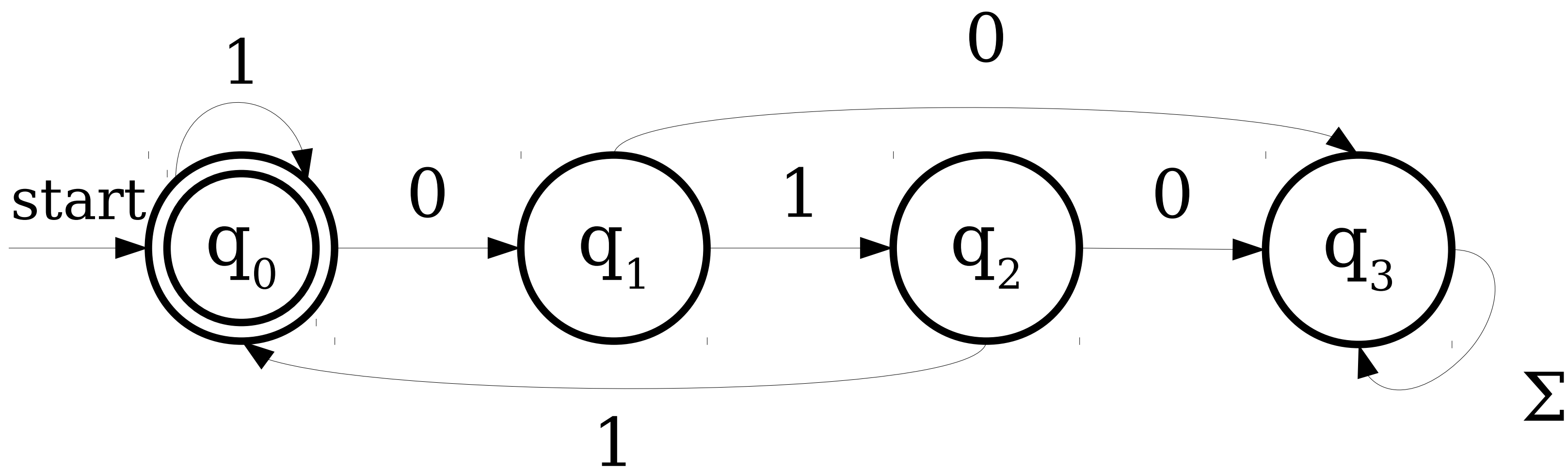


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|---|---|---|
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| $q_1$ | $q_3$ | $q_2$ |
| $q_2$ | $q_3$ | $q_0$ |
| $q_3$ | $q_3$ | $q_3$ |

The star indicates that this is an accepting state.

# Code? In a Theory Course?

```
int kTransitionTable[kNumStates][kNumSymbols] = {
     {0, 0, 1, 3, 7, 1, …},
      …
};
bool kAcceptTable[kNumStates] = {
    false,
    true,
    true,
    …
};
bool SimulateDFA(string input) {
    int state = 0;
    for (char ch: input)
        state = kTransitionTable[state][ch];
    return kAcceptTable[state];
}
```

# The Regular Languages

A language $L$ is called a ***regular language***
if there exists a DFA $D$ such that $\mathscr{L}(D) = L$.

# The Complement of a Language

- Given a language $L \subseteq \Sigma^*$, the ***complement*** of that language (denoted $\overline{L}$) is the language of all strings in $\Sigma^*$ not in $L$.
- Formally:

$$\overline{L} = \{ w \mid w \in \Sigma^* \land w \notin L \}$$

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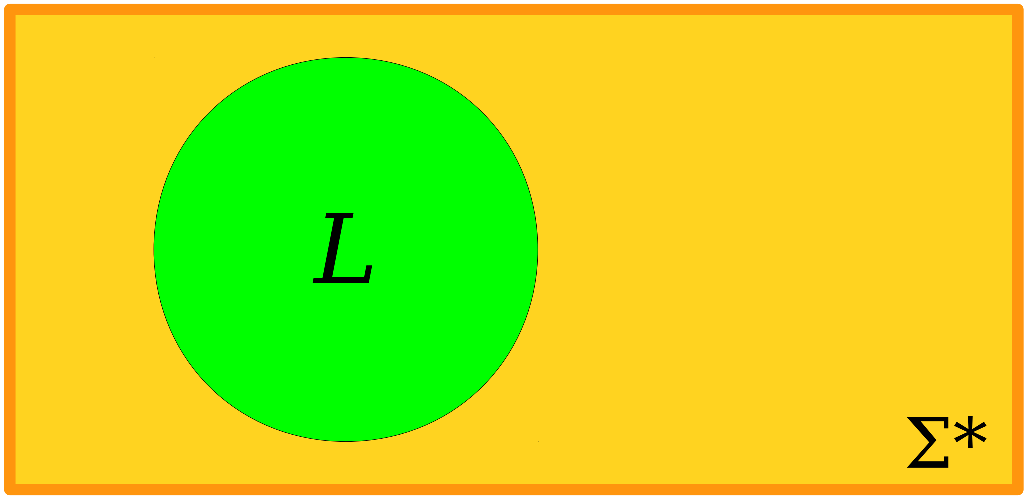

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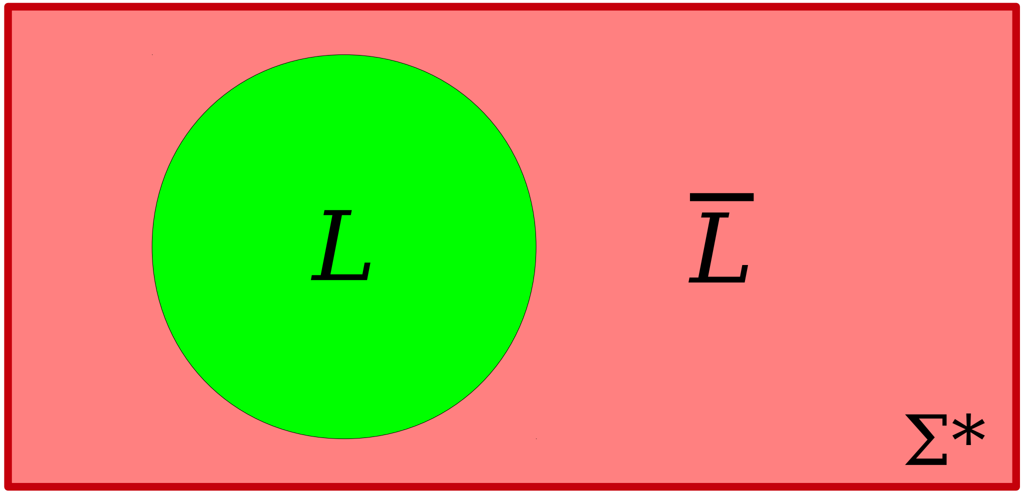
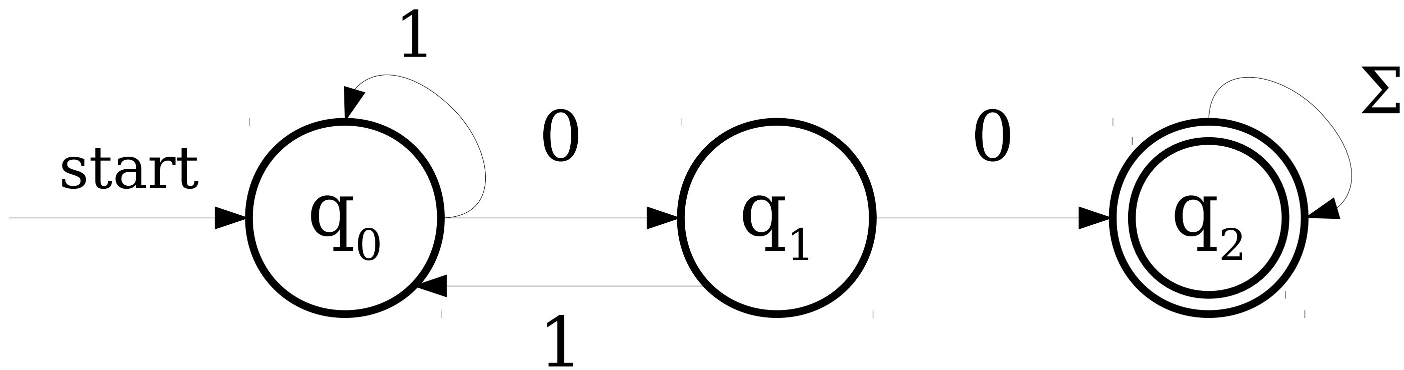


# Complementing Regular Languages

- Recall: A ***regular language*** is a language accepted by some DFA.
- **Question:** If $L$ is a regular language, is $\overline{L}$ a regular language?
- If the answer is "yes," then there must be some way to construct a DFA for $\overline{L}$.
- If the answer is "no," then some language $L$ can be accepted by a DFA, but $\overline{L}$ cannot be accepted by any DFA.

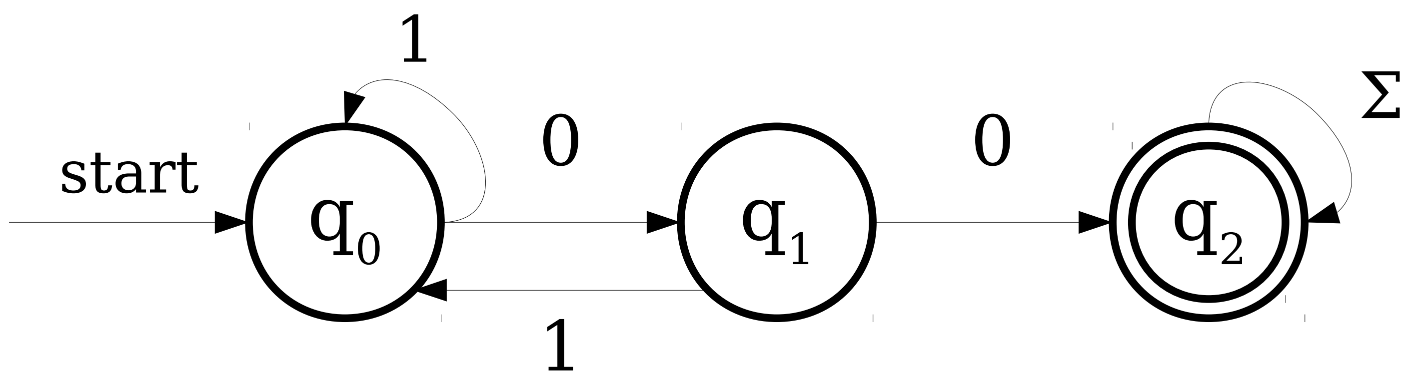
# Complementing Regular Languages

$L = \{\ w \in \{0, 1\}^* \mid w \text{ contains } 00 \text{ as a substring }\}$



# Complementing Regular Languages

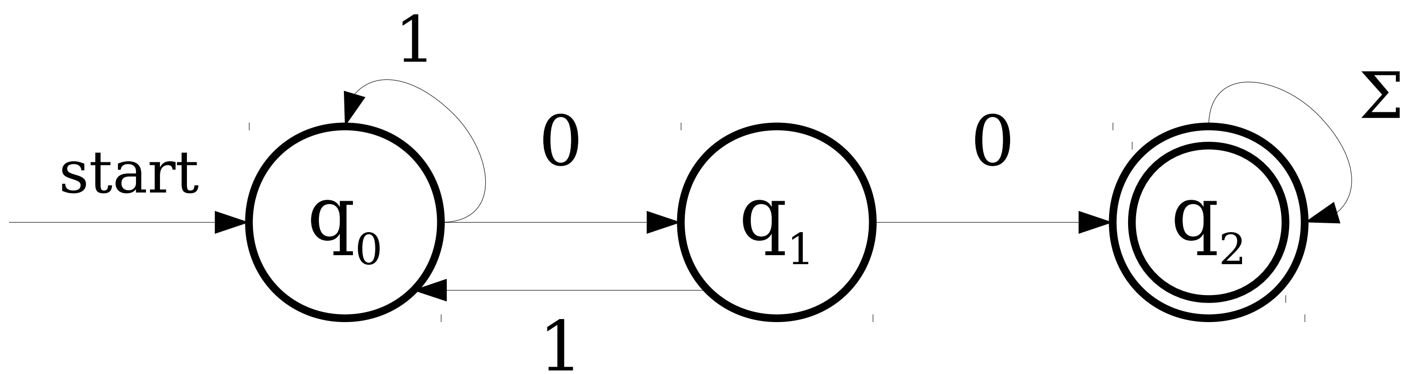
$L = \{\ w \in \{\texttt{0}, \texttt{1}\}^* \mid w$ contains $\texttt{00}$ as a substring $\}$



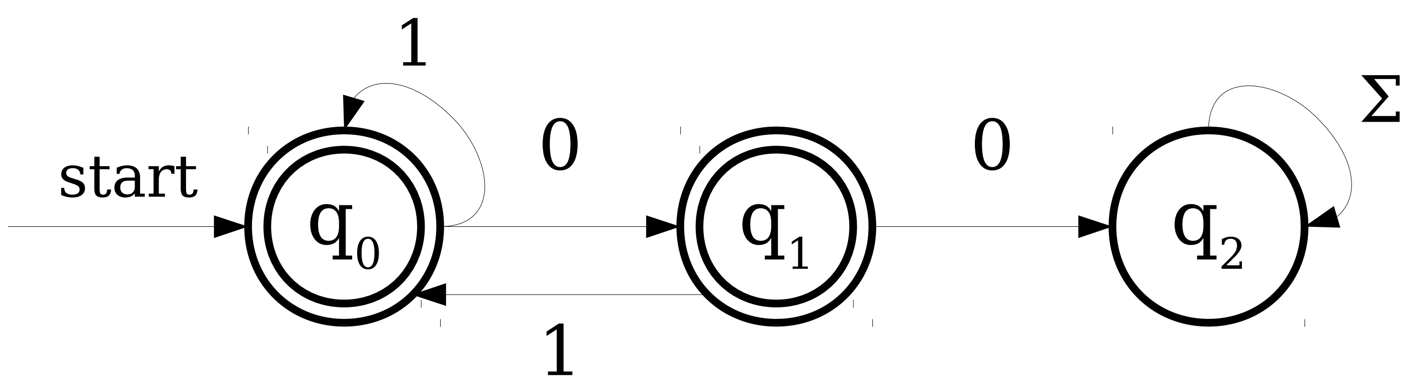

$\overline{L} = \{\ w \in \{\texttt{0}, \texttt{1}\}^* \mid w$ **does not** contain $\texttt{00}$ as a substring $\}$

# Complementing Regular Languages

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ contains } \texttt{00} \text{ as a substring } \}$

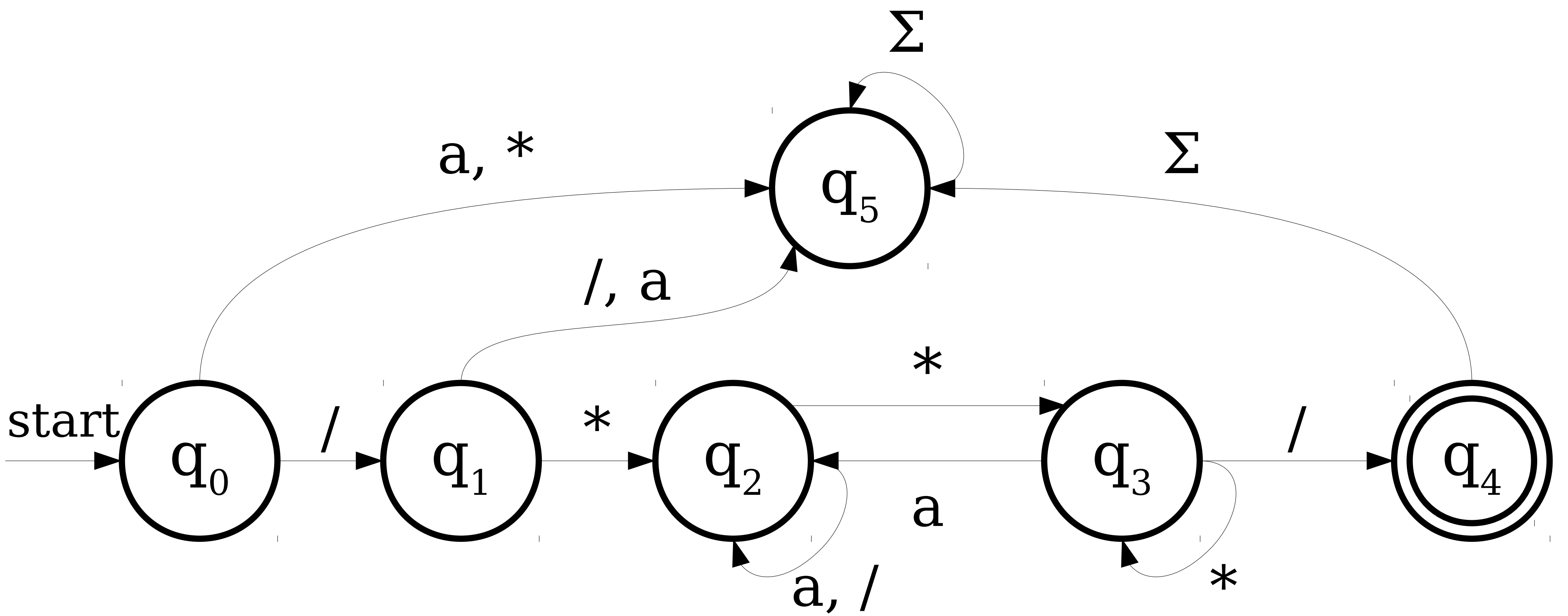



$\overline{L} = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \textbf{ does not} \text{ contain } \texttt{00} \text{ as a substring } \}$



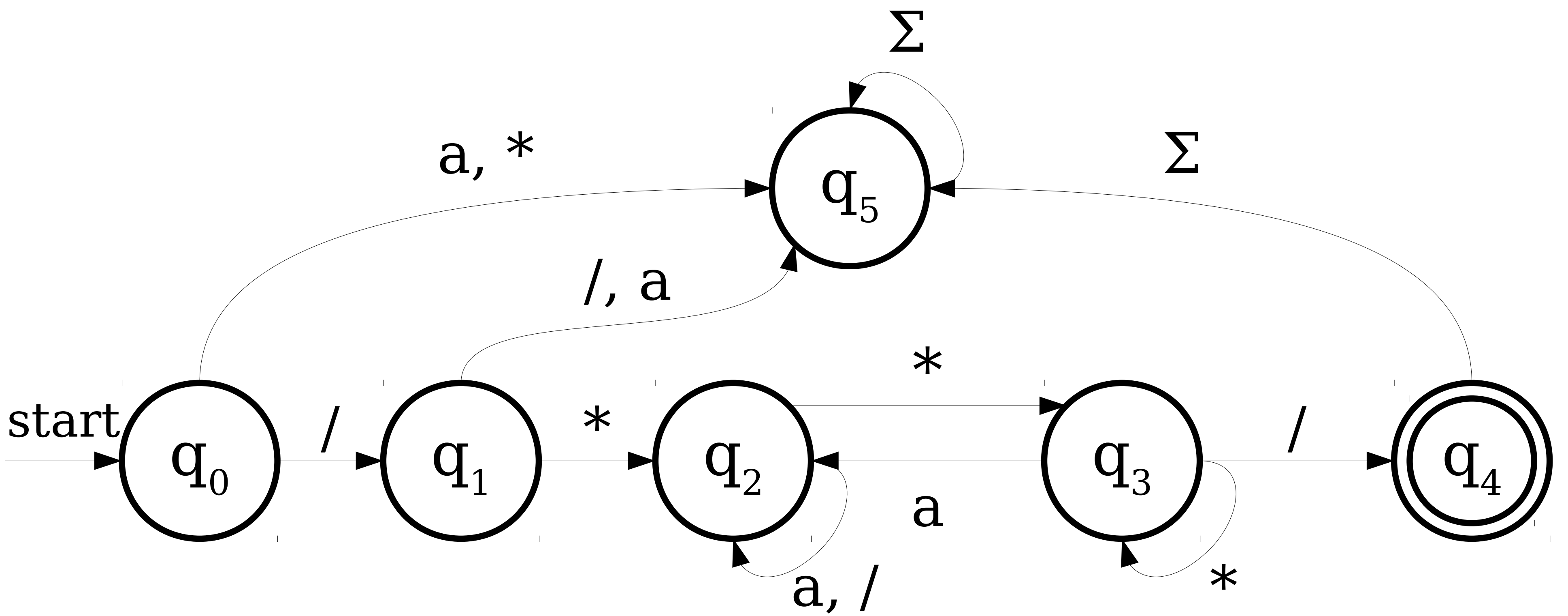
# More Elaborate DFAs

$L = \{ \, w \mid w \text{ is a C-style comment } \}$



# More Elaborate DFAs

$\overline{L}$ = { $w$ | $w$ is **not** a C-style comment }



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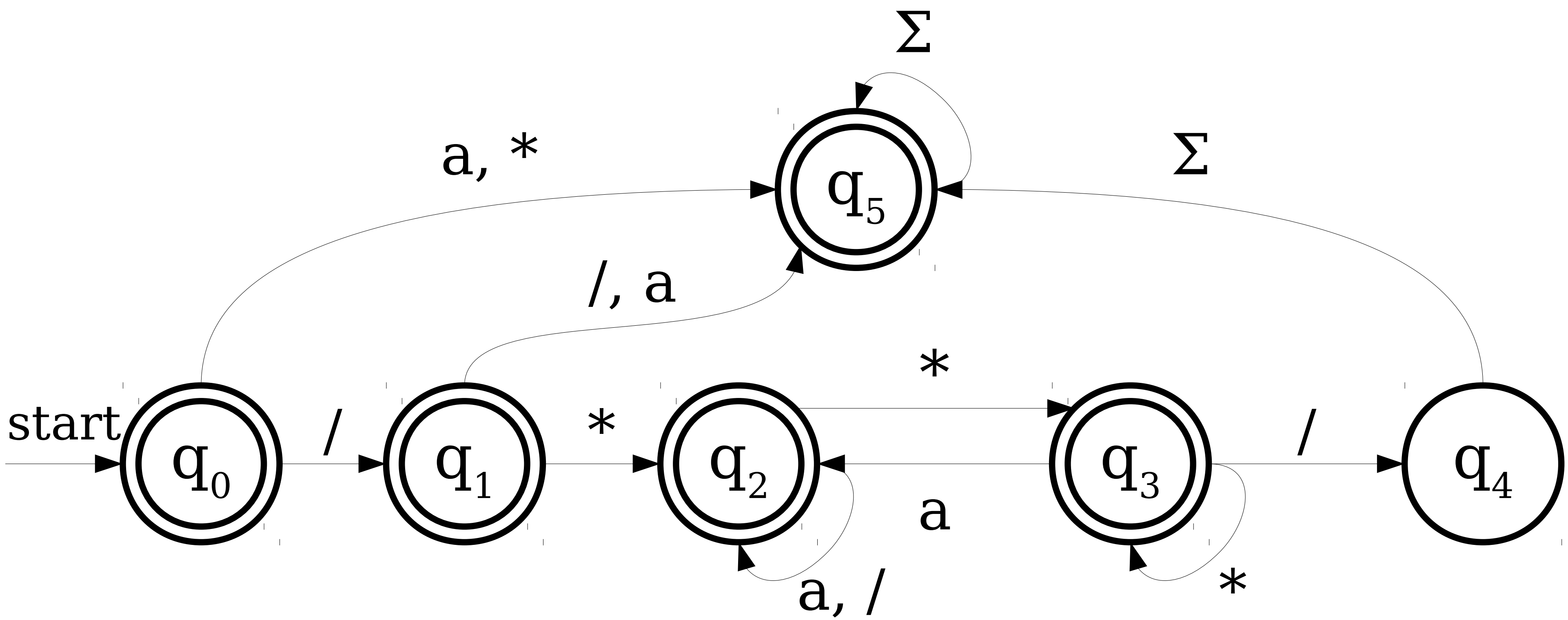
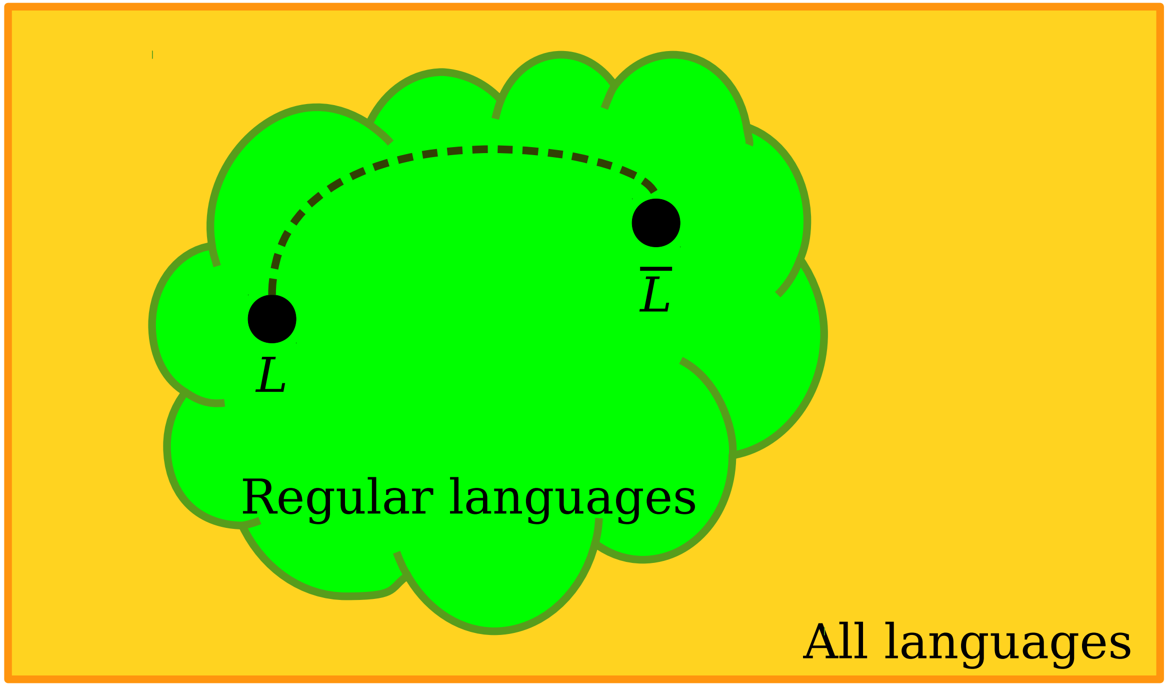


# Closure Properties

- ***Theorem:*** If $L$ is a regular language, then $\overline{L}$ is also a regular language.
- As a result, we say that the regular languages are ***closed under complementation***.



# Time-Out For Announcements!

# A Point of Clarification

***Theorem:*** If $\mathscr{U}$ is the universal set, then $|\wp(\mathscr{U})| \leq |\mathscr{U}|$

***Proof:*** The universal set $\mathscr{U}$ contains all objects. Therefore, $\wp(\mathscr{U}) \in \mathscr{U}$. Consequently, $|\wp(\mathscr{U})| \leq |\mathscr{U}|$. ■

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Therefore, $\wp(\mathscr{U}) \in \mathscr{U}$. Consequently, $|\wp(\mathscr{U})| \leq |\mathscr{U}|$. ■

Does this reasoning work?

***Theorem:*** If $\mathscr{U}$ is the universal set, then $|\wp(\mathscr{U})| \leq |\mathscr{U}|$

***Proof:*** The universal set $\mathscr{U}$ contains all objects. Therefore, every element of $\wp(\mathscr{U})$ is an element of $\mathscr{U}$. Accordingly, $\wp(\mathscr{U}) \subseteq \mathscr{U}$, so $|\wp(\mathscr{U})| \leq |\mathscr{U}|$. ■

Remember that $\in$ and $\subseteq$
are different concepts!

# Midterm Logistics

- Midterm is this Thursday from 7PM – 10PM. Locations divvied up by last (family) name:
  - Aba – Mes: Go to **Annenberg Auditorium**.
  - Mex – Zoc: Go to **Cubberly Auditorium**.
- Closed-book, closed-computer, open one double-sided 8.5” × 11” sheet of notes.
- Covers material up through and including graphs (Lectures 00 – 08) and material from PS1 – PS3.

# Your Questions

"You seem to really LOVE cute animals, in your opinion, what is the cutest animal in this world? And what is your opinion on quantum computing? Will it provide a completely different view on computability problems we are learning in this course?"

I don't think there's a single cutest animal in the world, though I'm willing to be proven wrong. ☺

As for quantum computing – it will have little to do with computability, probably, but a **lot** to do with complexity. We're still working out the details!

“We hear about a lot of different "forefront" CS topics in the popular media: IoT, The Singularity, quantum computing, etc...What are you most excited about, and what is the most misrepresented?”

I'm probably not the right person to ask this to. I'm most excited about technology being used to create jobs in the developing world and help lift more people into the middle class. I generally am pretty skeptical about “the singularity,” but that's just me.

Back to CS103!

# NFAs

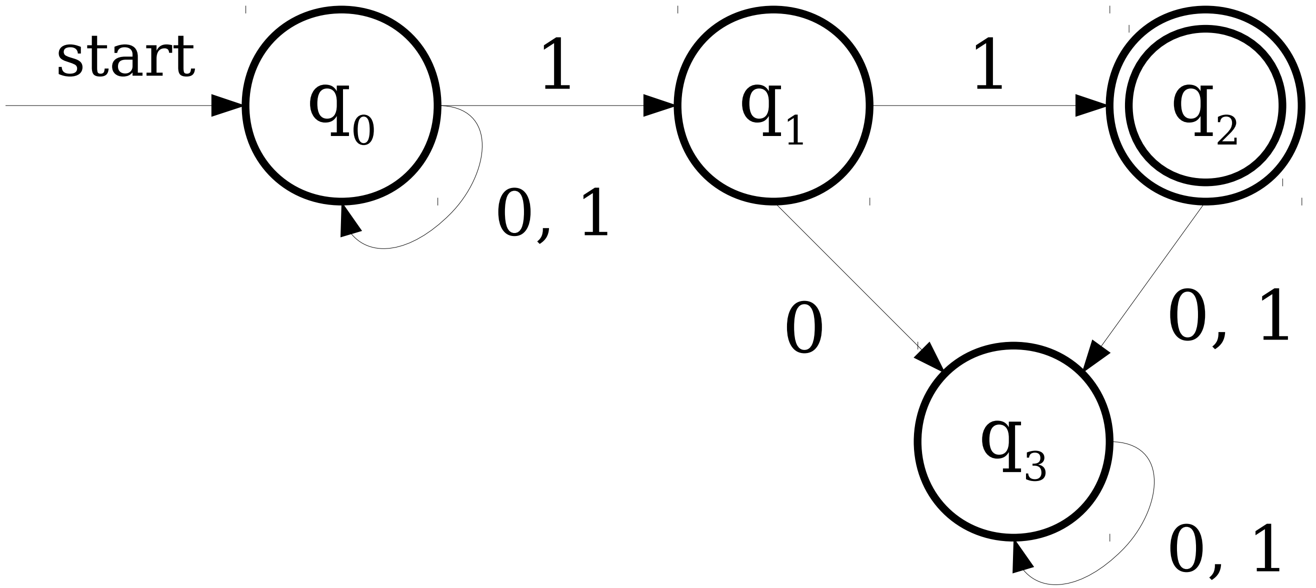
# NFAs

- An **NFA** is a
  - **N**ondeterministic
  - **F**inite
  - **A**utomaton
- Structurally similar to a DFA, but represents a fundamental shift in how we'll think about computation.

# (Non)determinism

- A model of computation is ***deterministic*** if at every point in the computation, there is exactly one choice that can make.
- The machine accepts if that series of choices leads to an accepting state.
- A model of computation is ***nondeterministic*** if the computing machine may have multiple decisions that it can make at one point.
- The machine accepts if ***any*** series of choices leads to an accepting state.

# A Simple NFA



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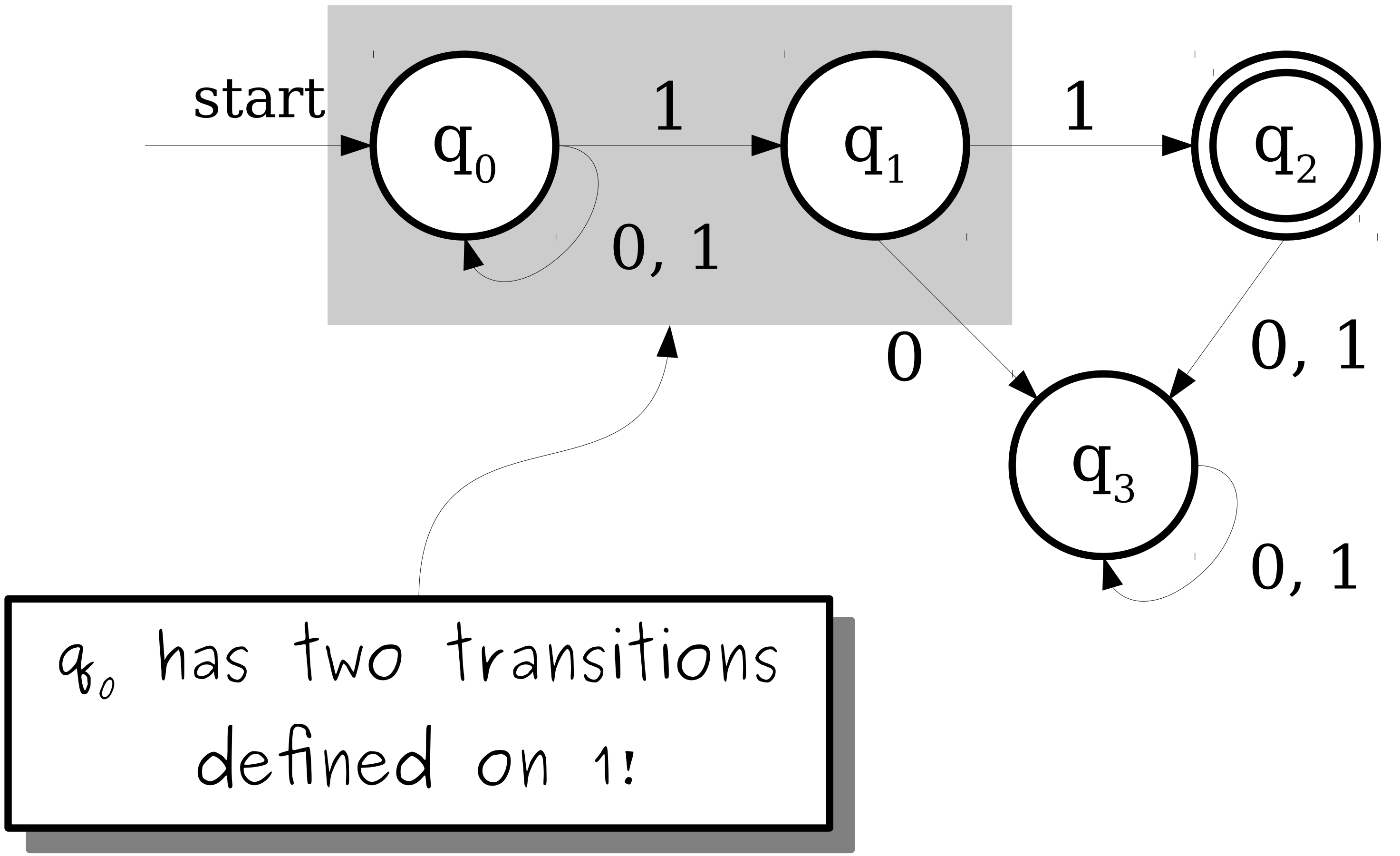
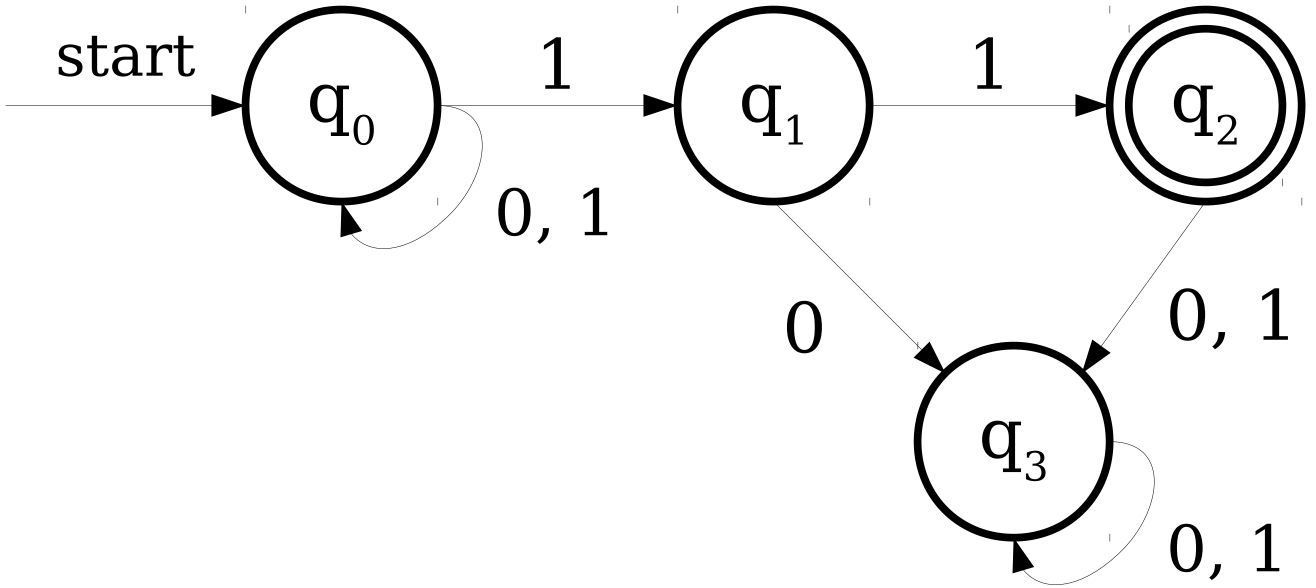
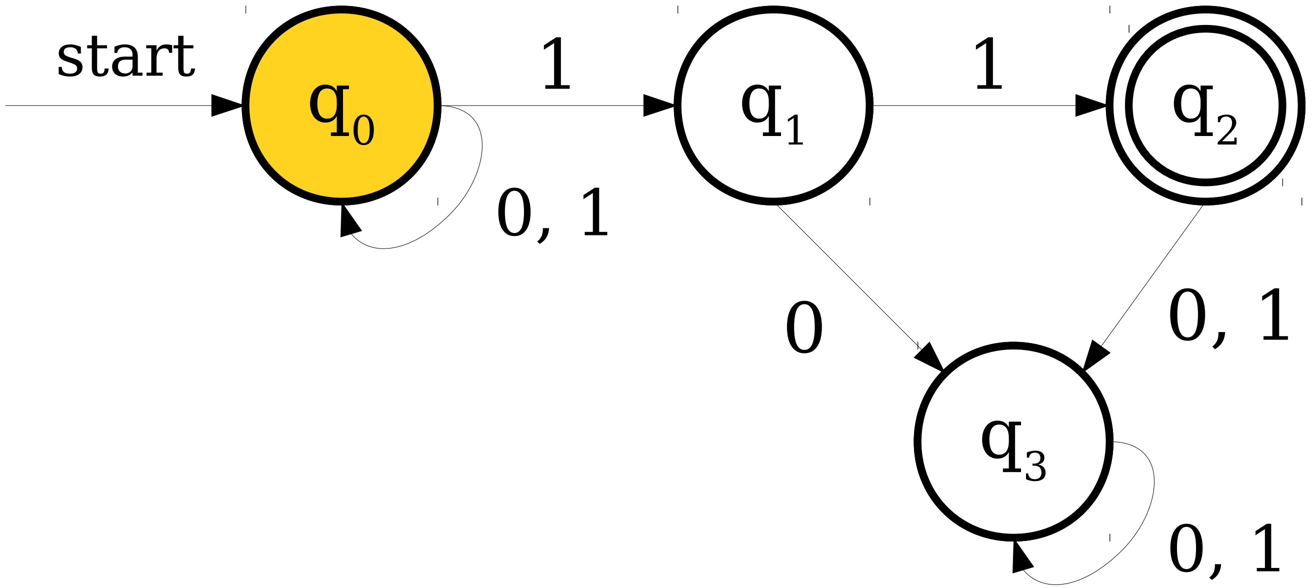

start
q0
q1
q2
q3
1
1
0, 1
0
0, 1
0, 1
q0 has two transitions
defined on 1!

# A Simple NFA




**0 1 0 1 1**

# A Simple NFA

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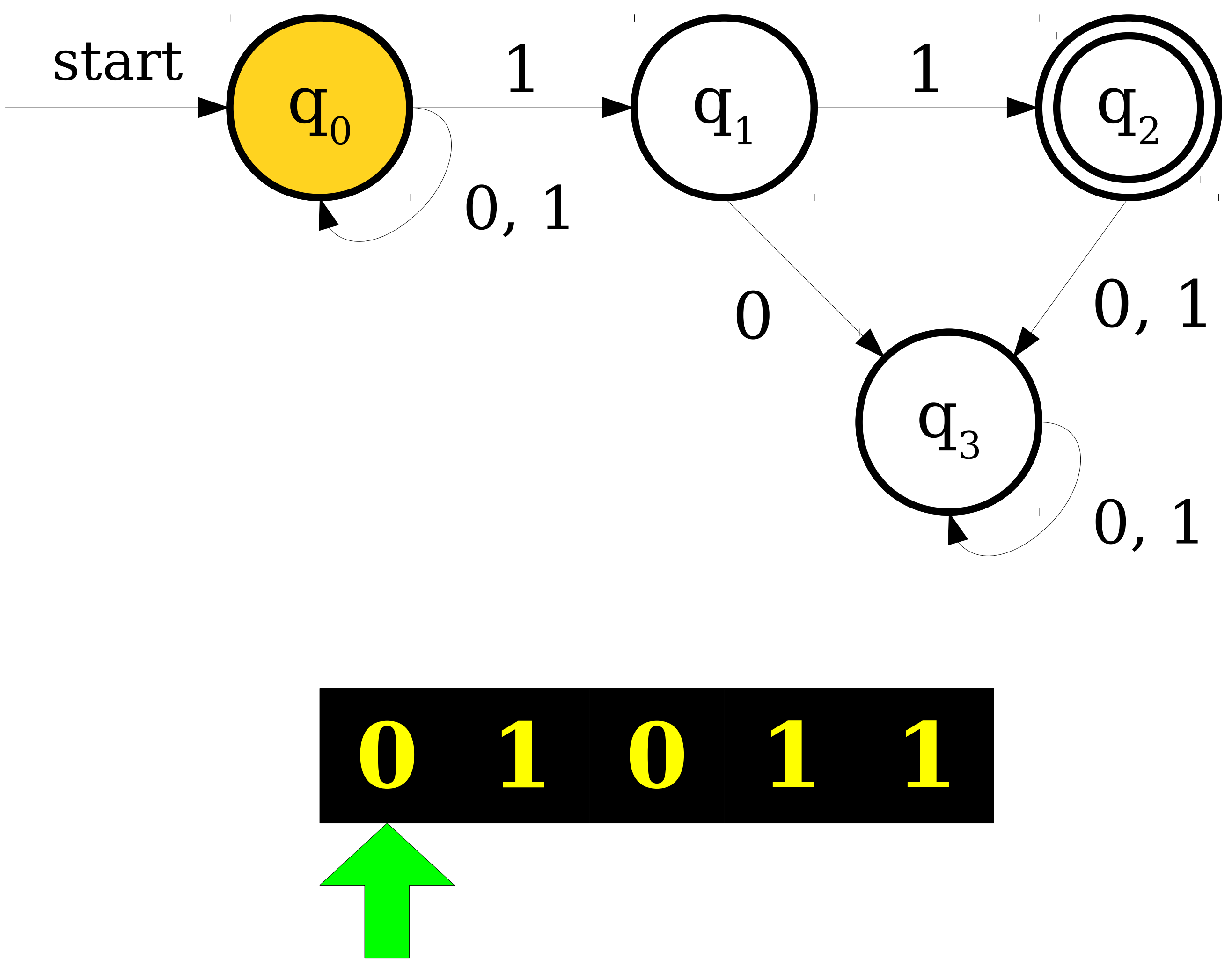

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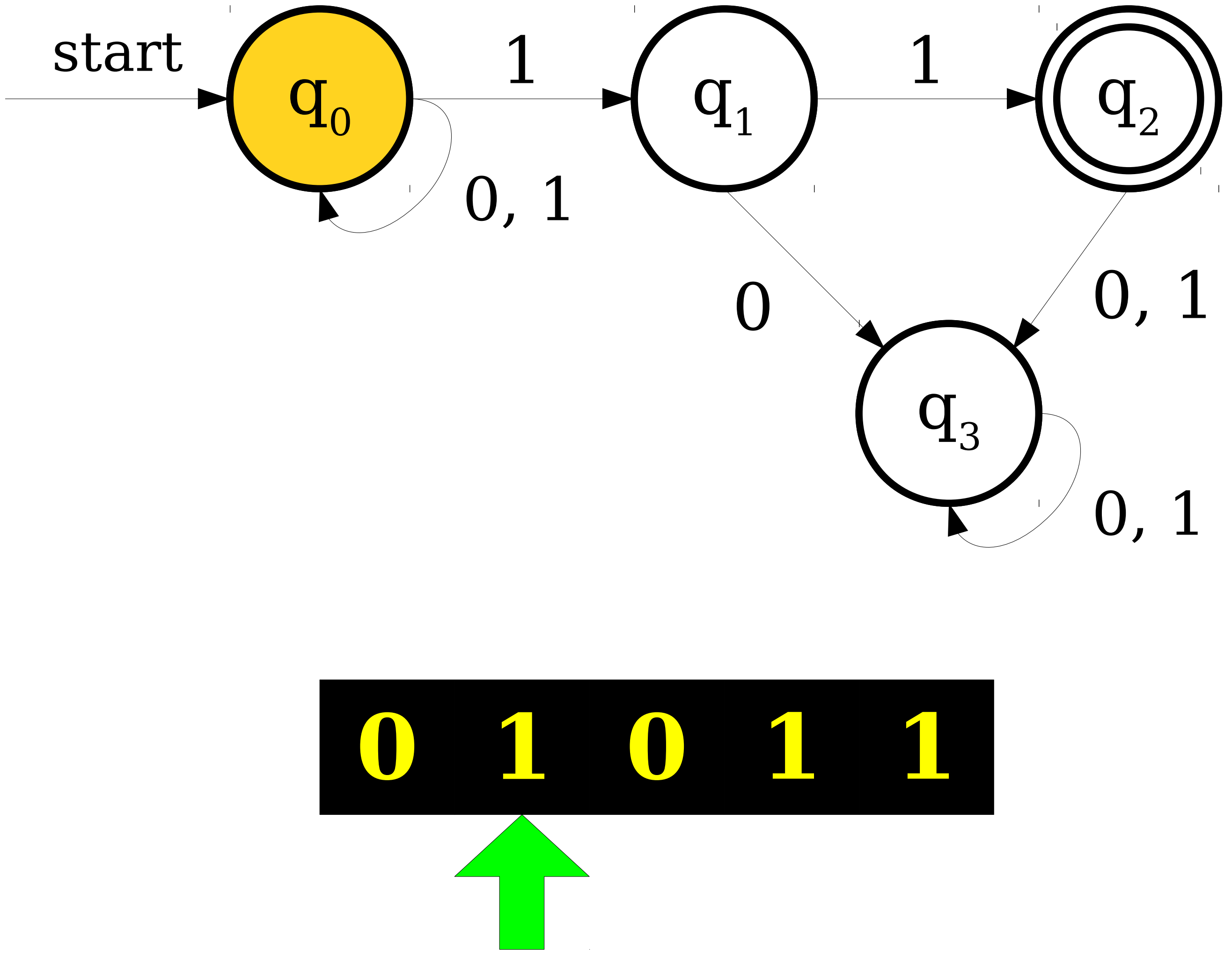

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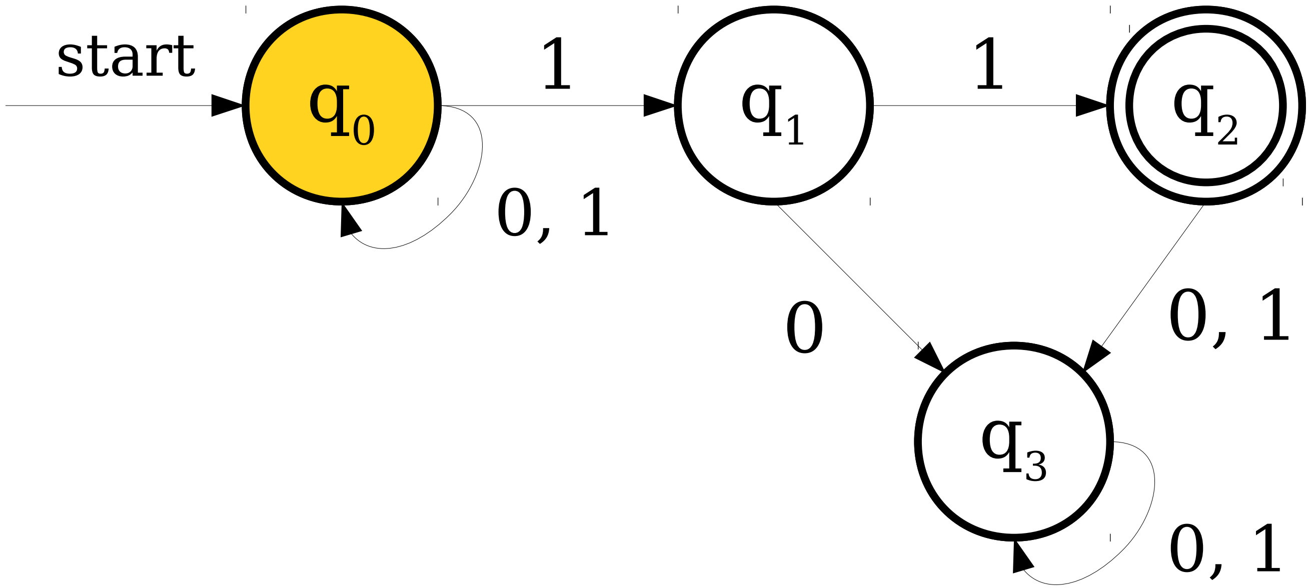


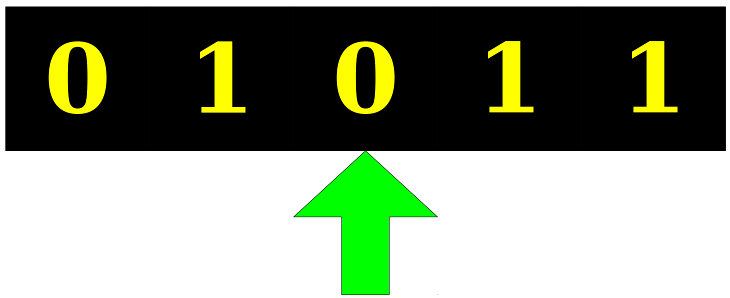

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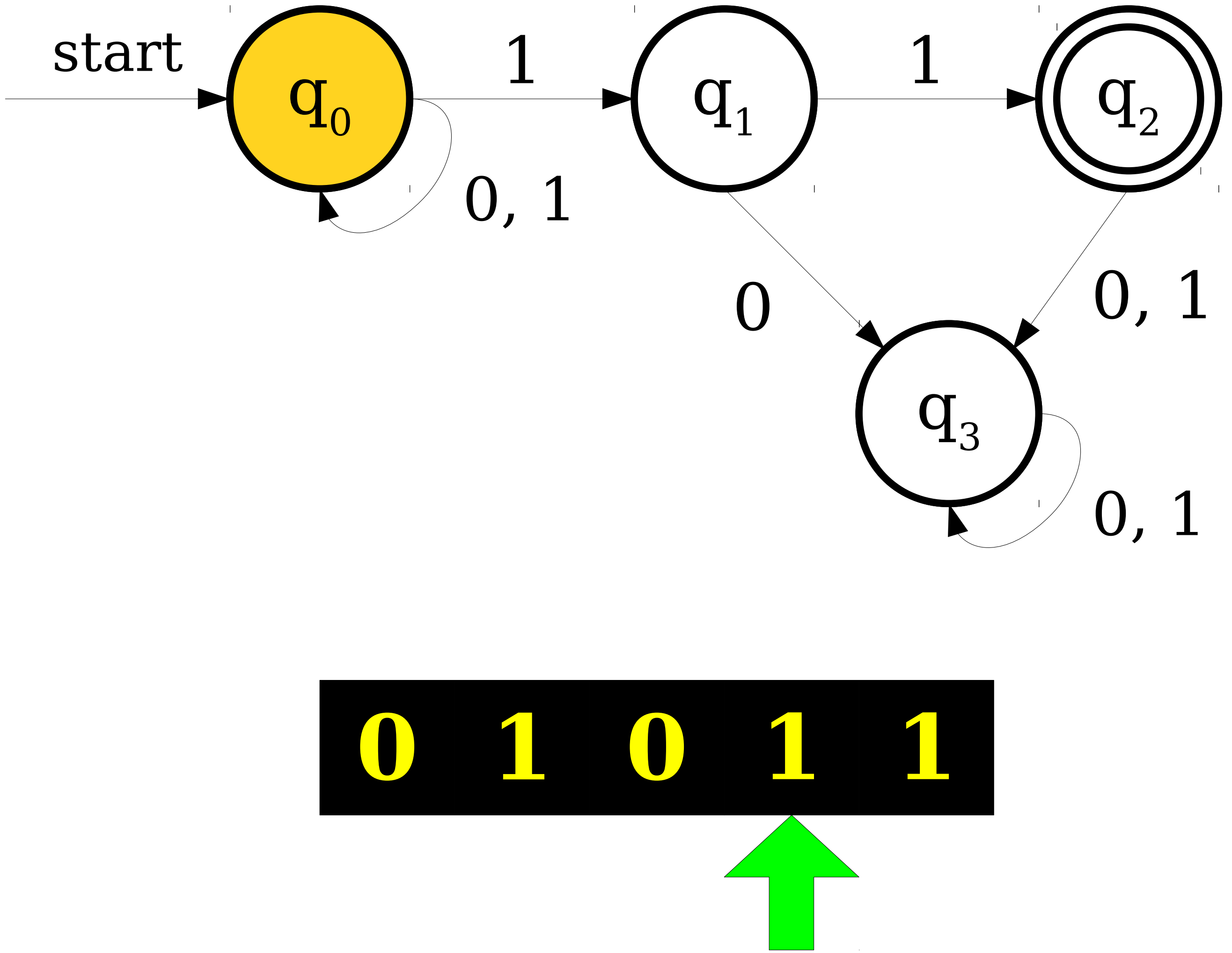

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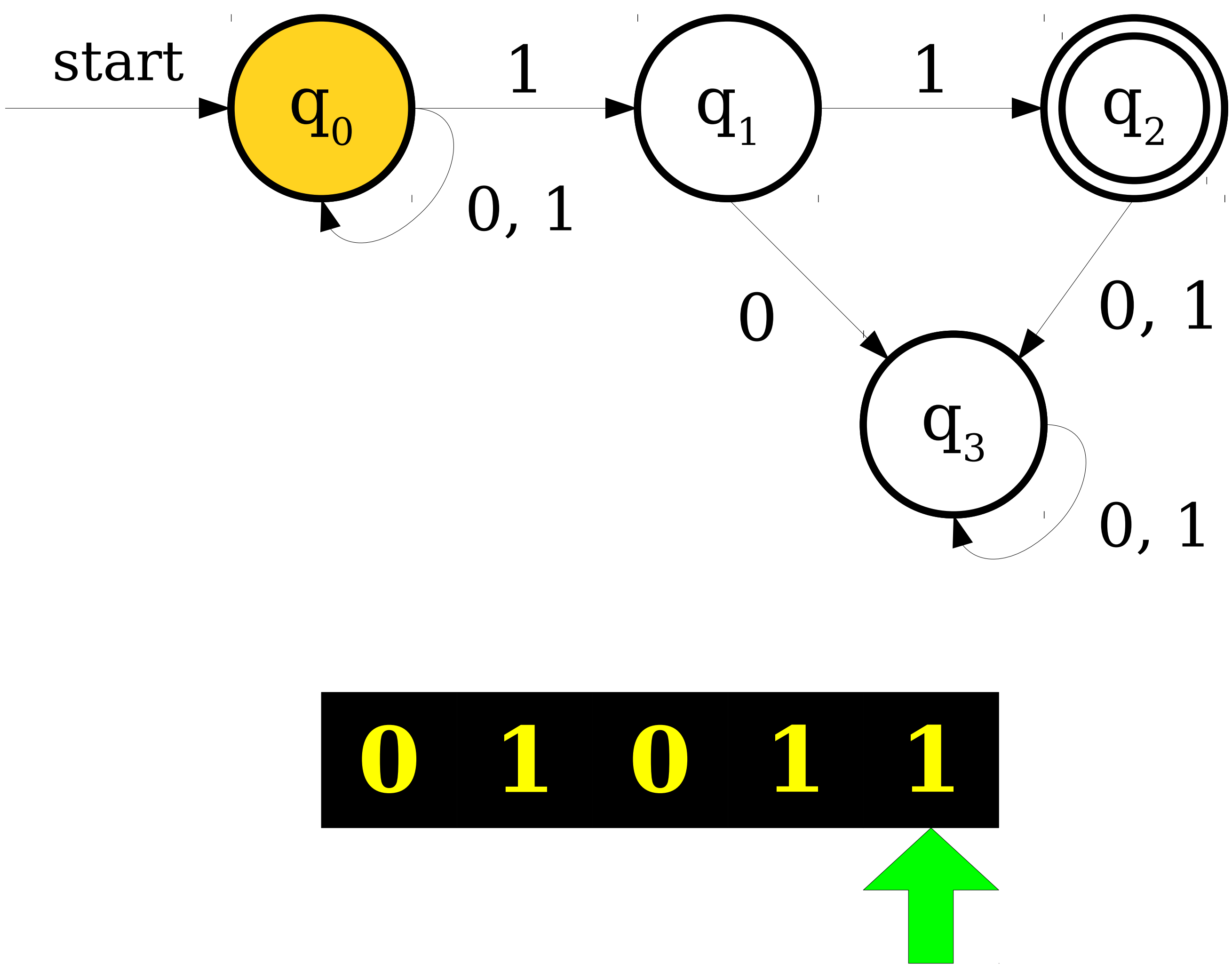

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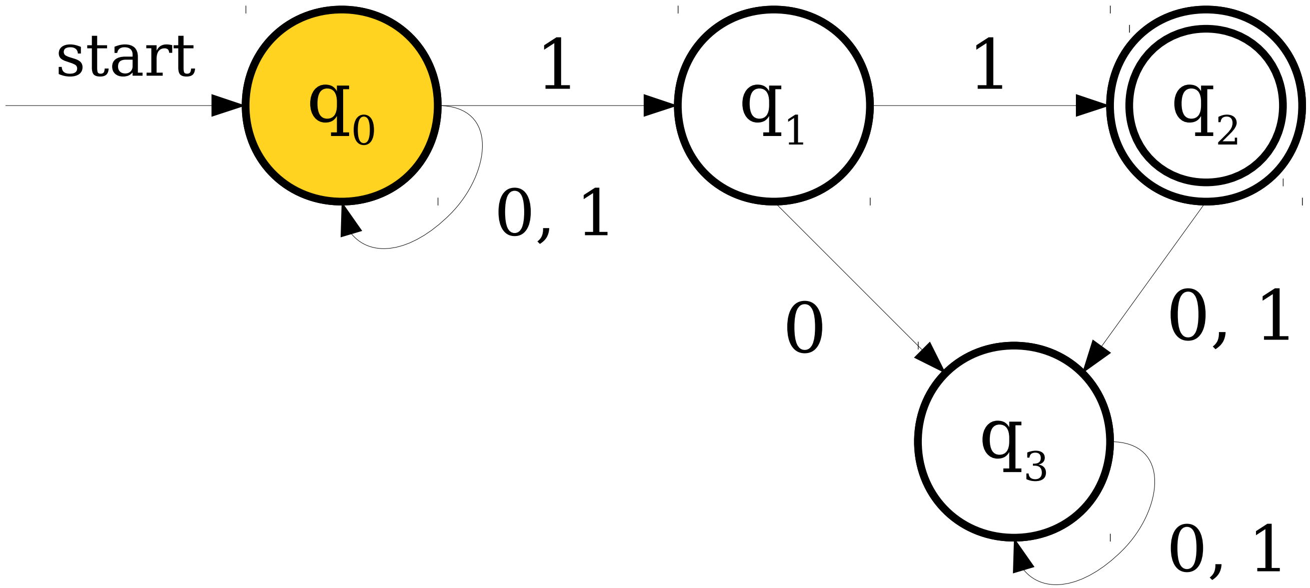
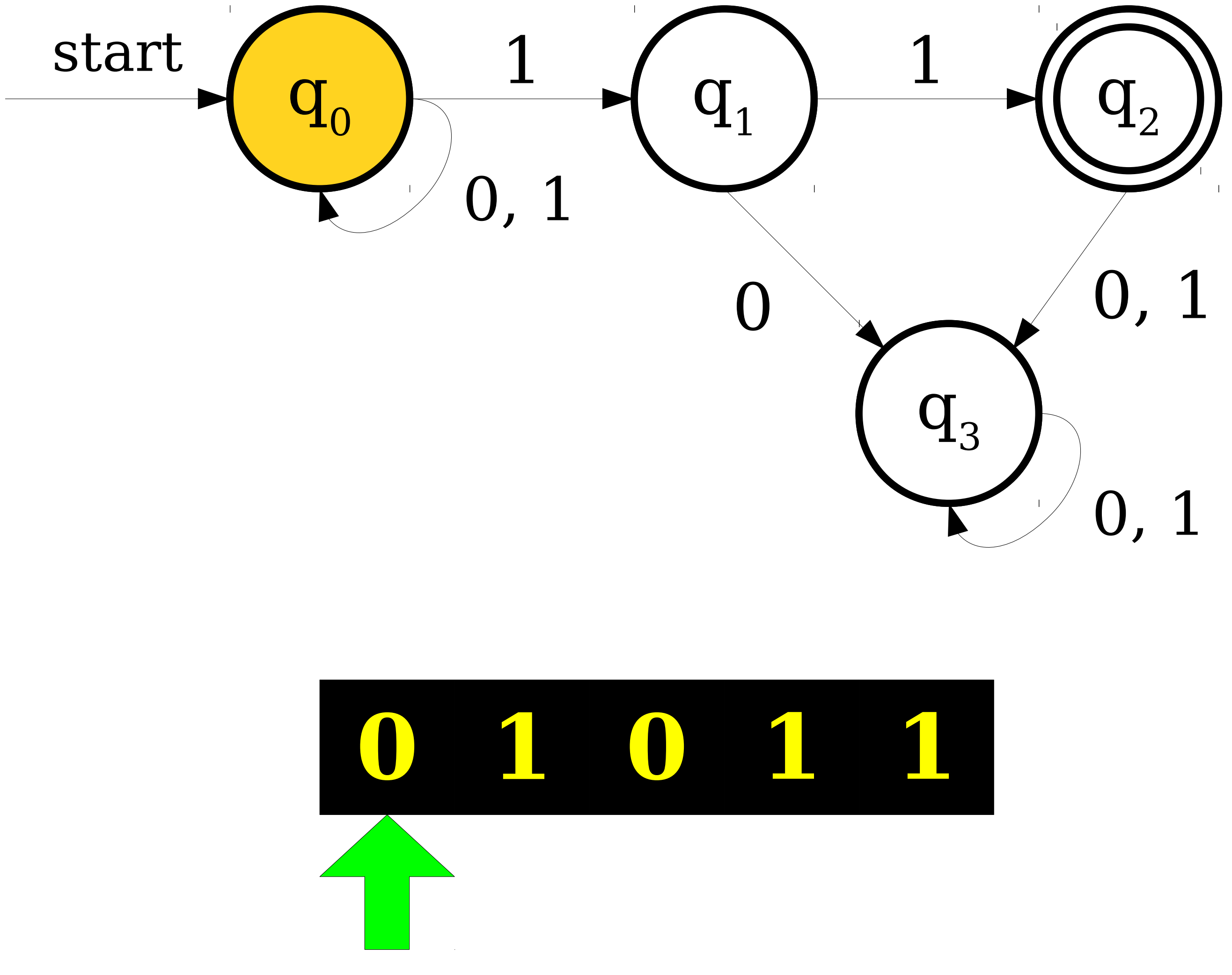



**0 1 0 1 1**

# A Simple NFA



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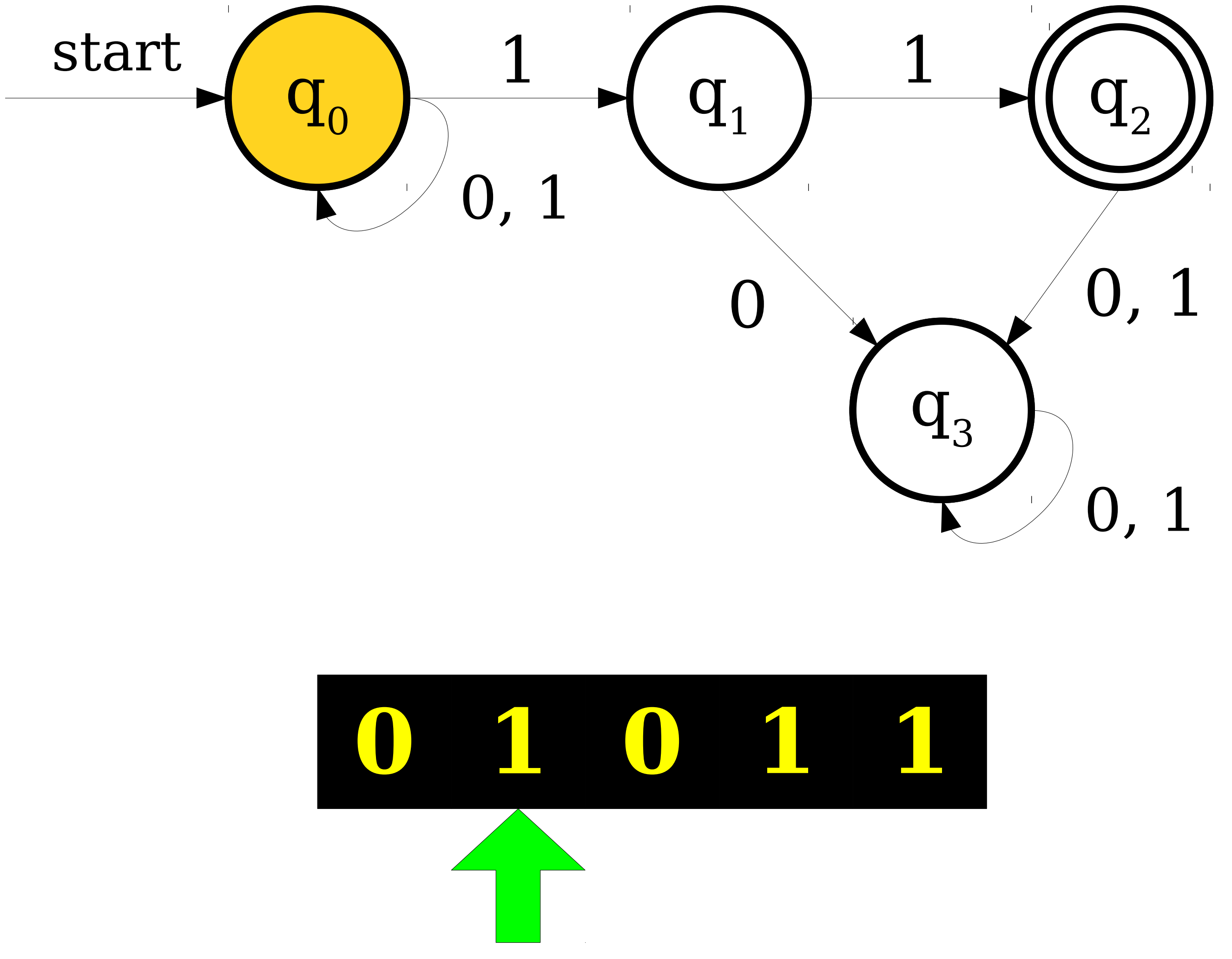

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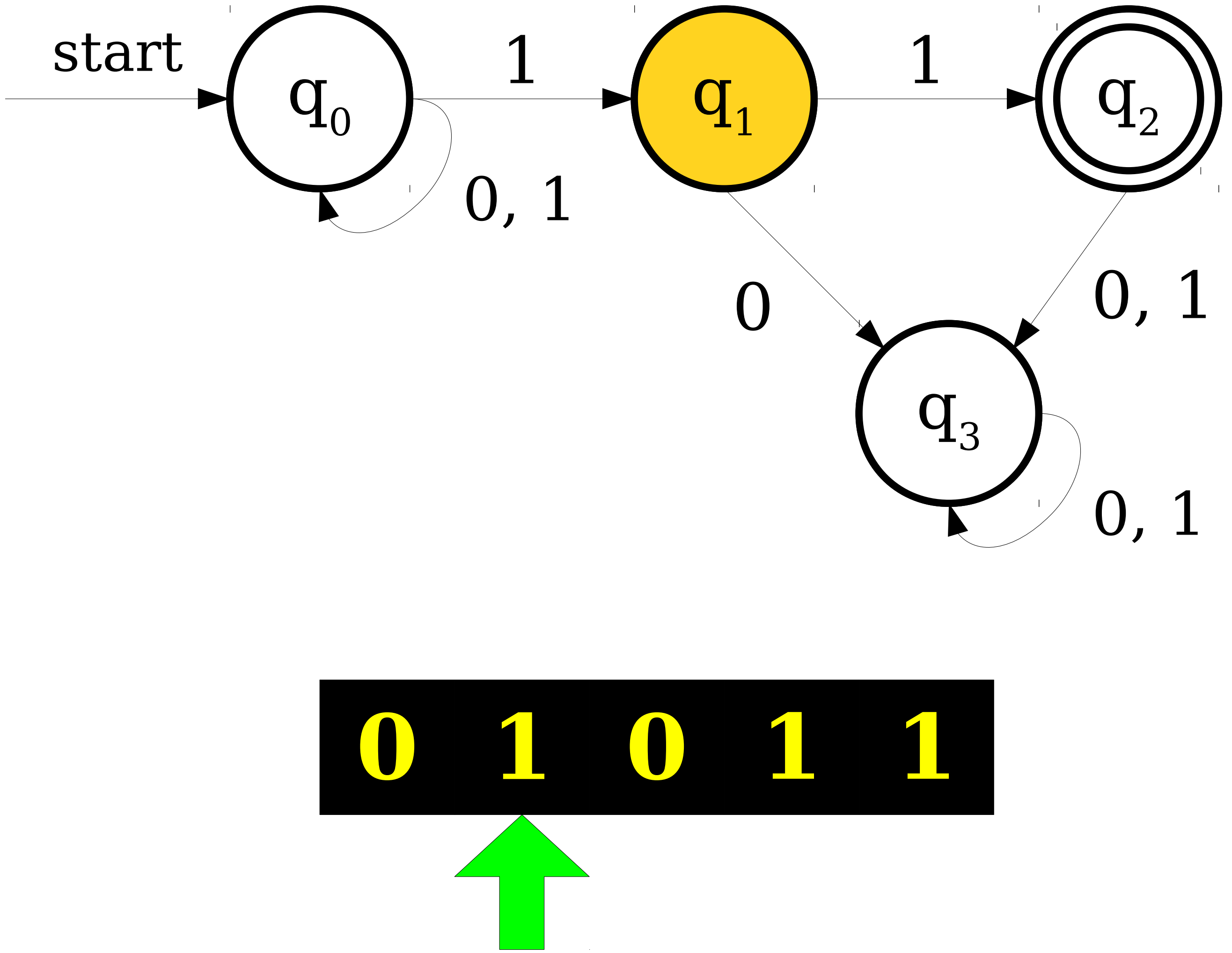

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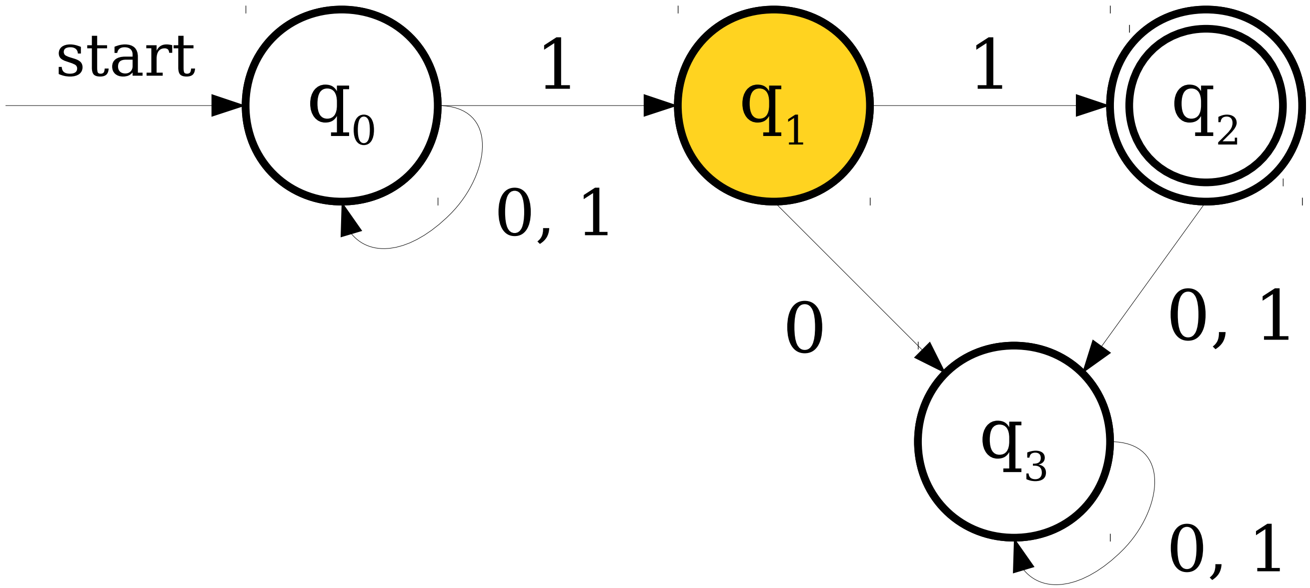


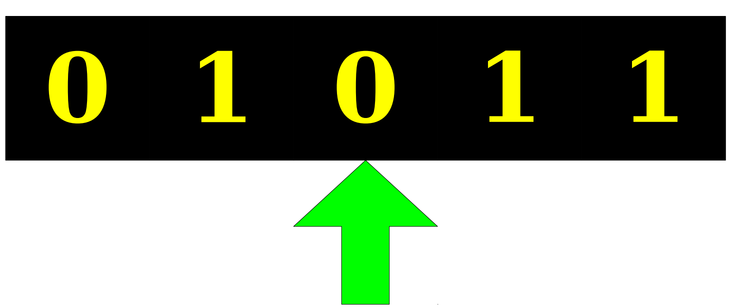

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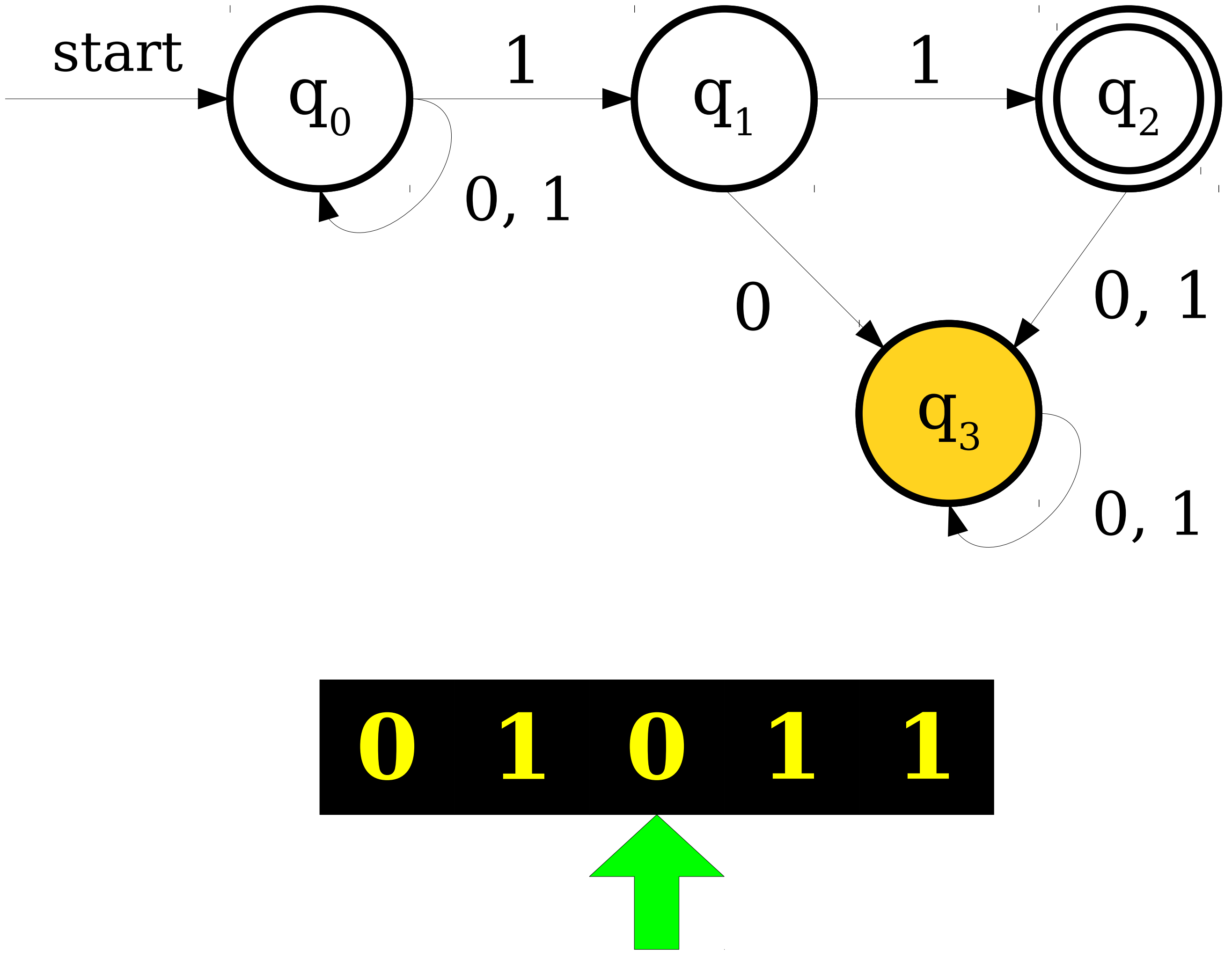

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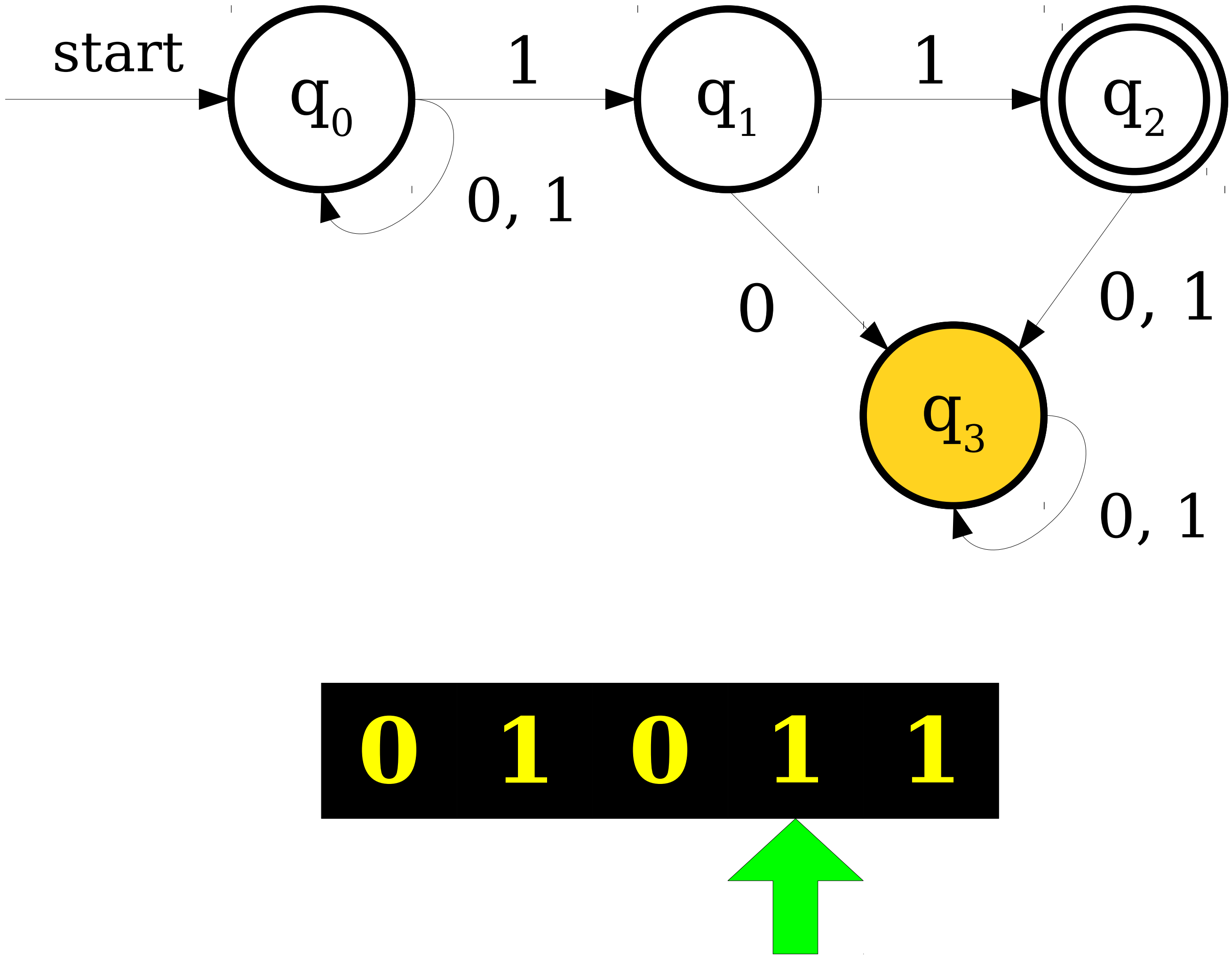

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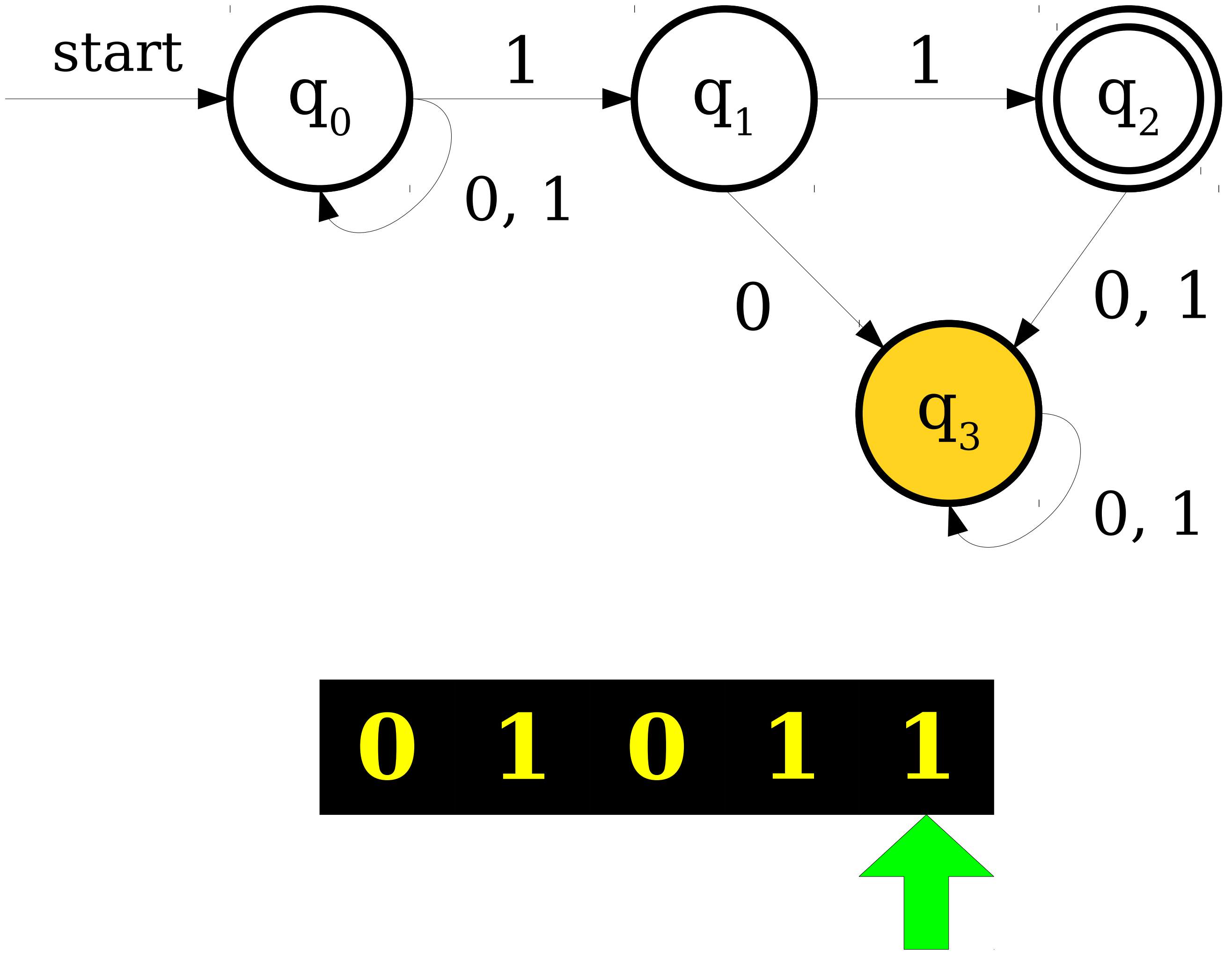

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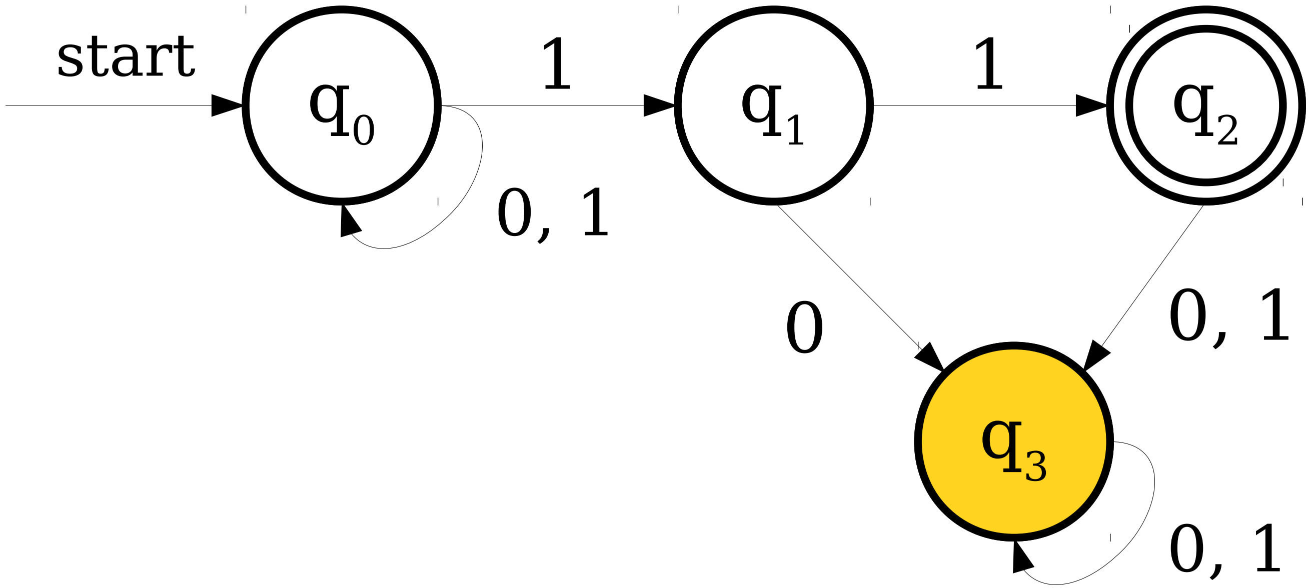


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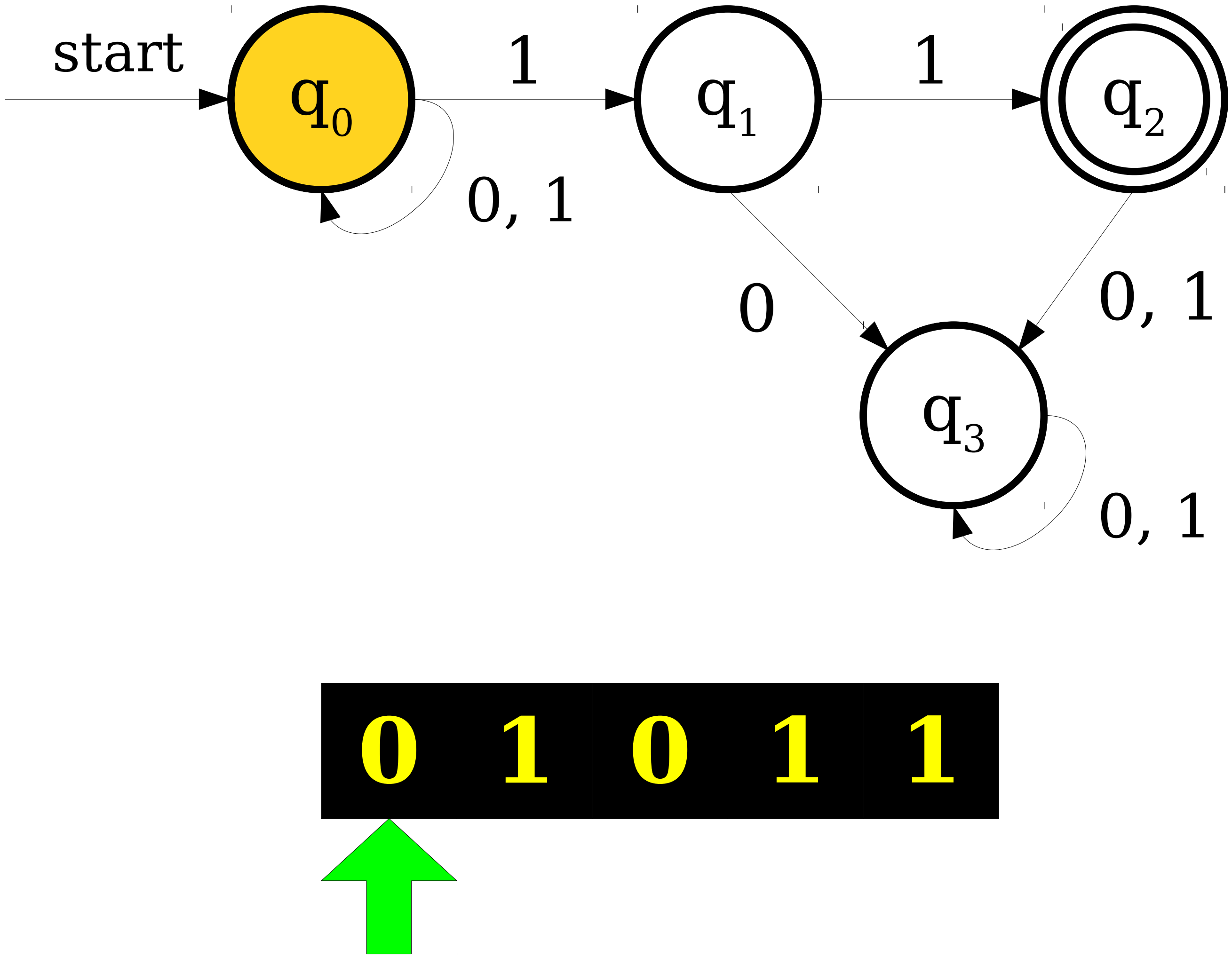

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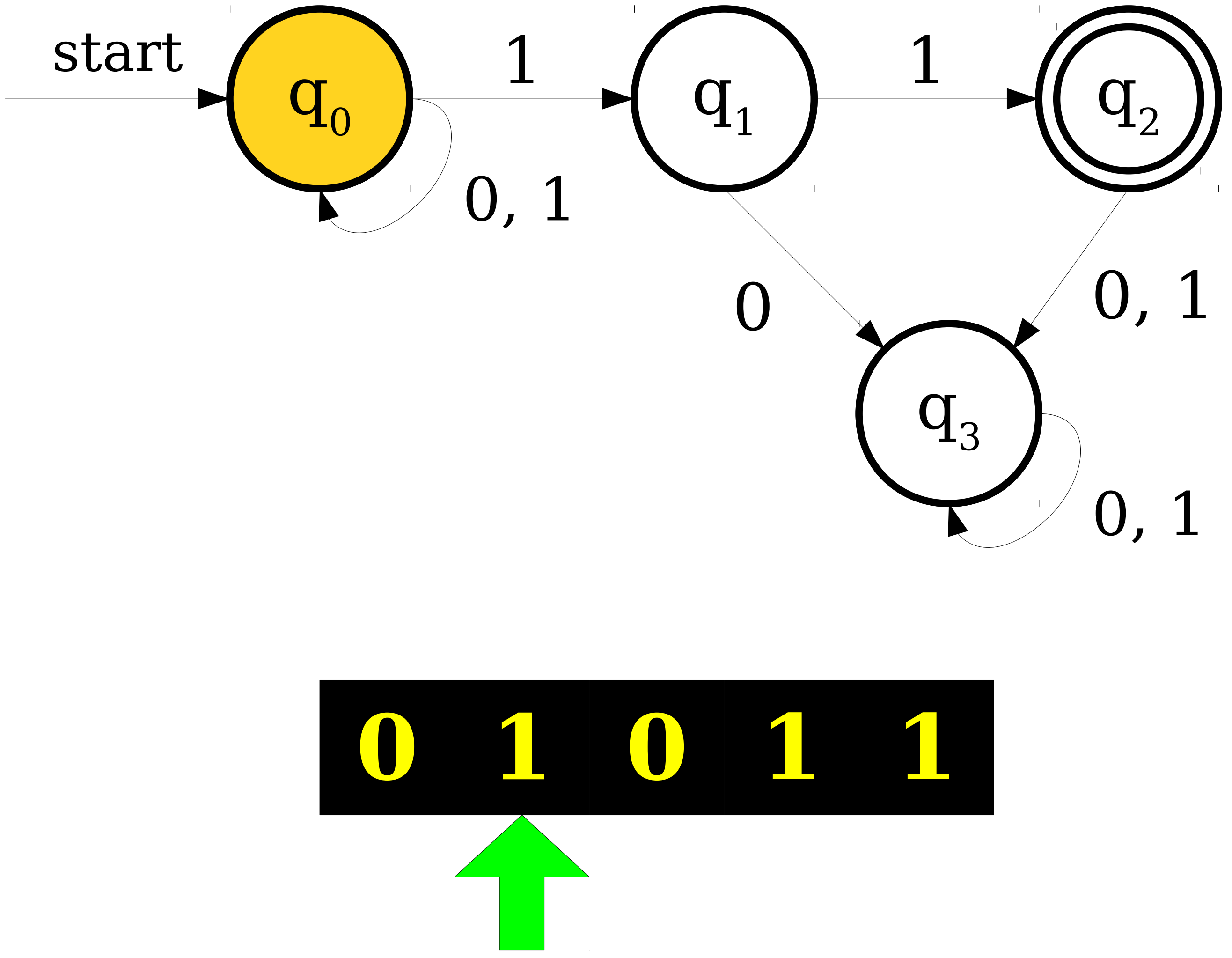

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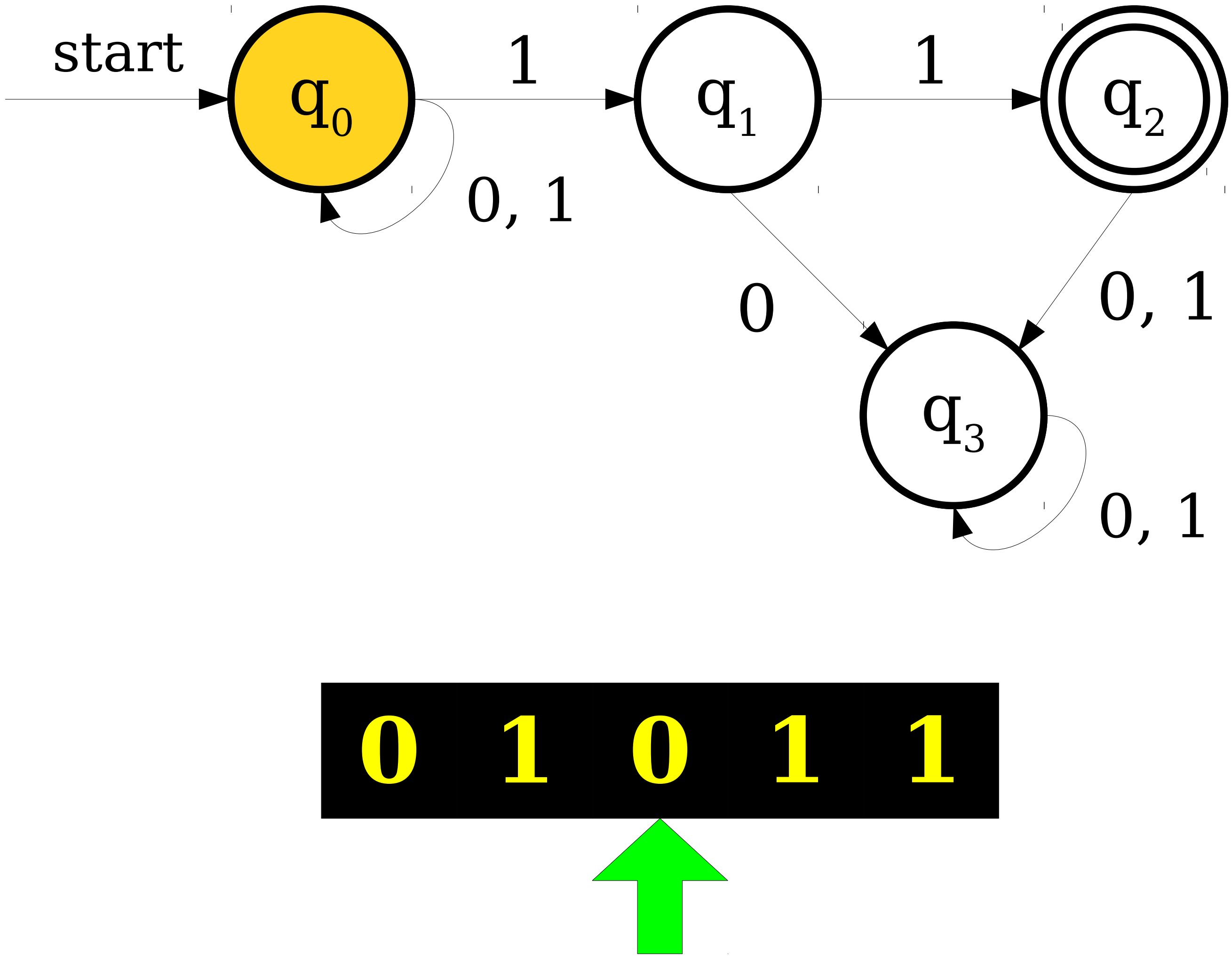

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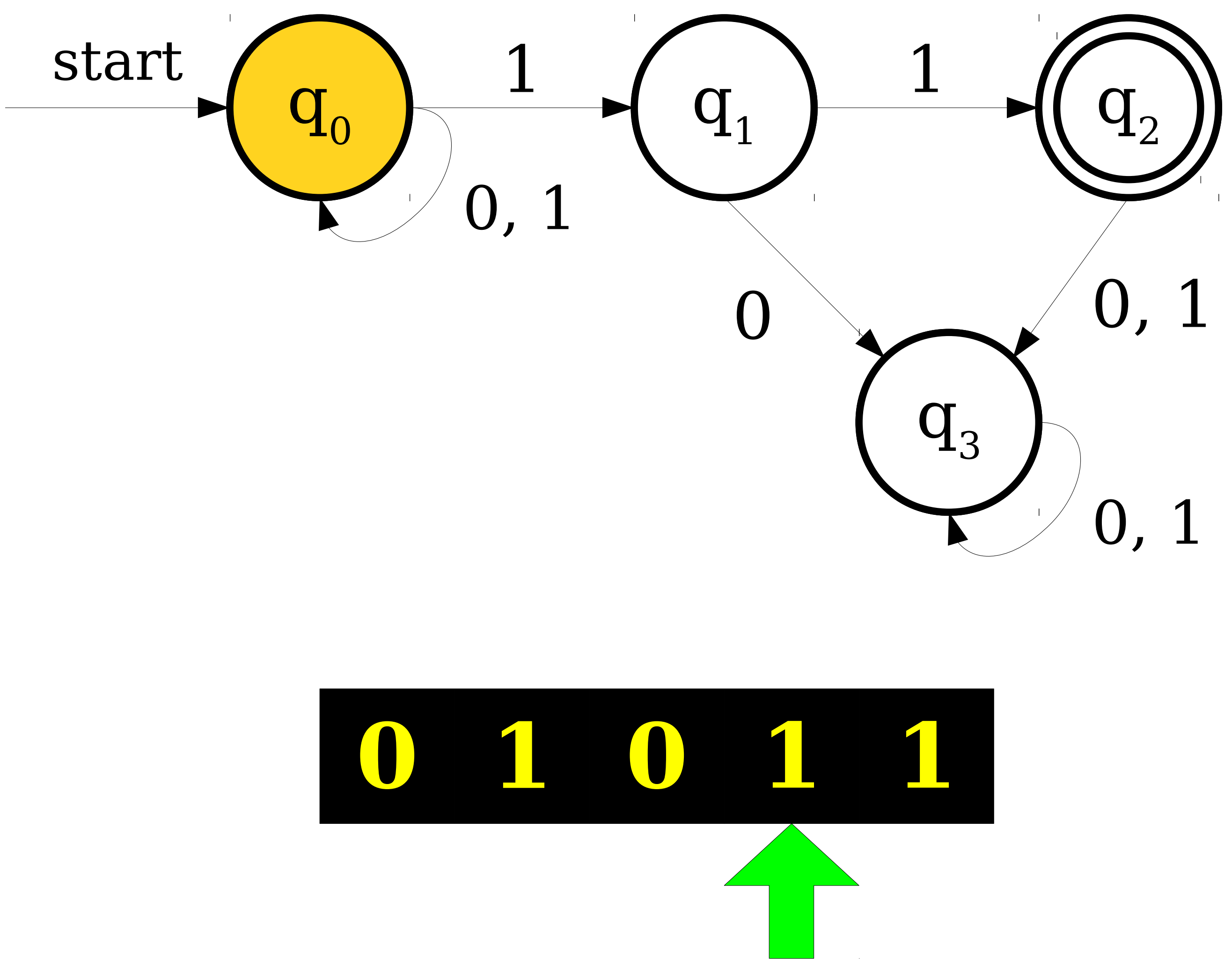

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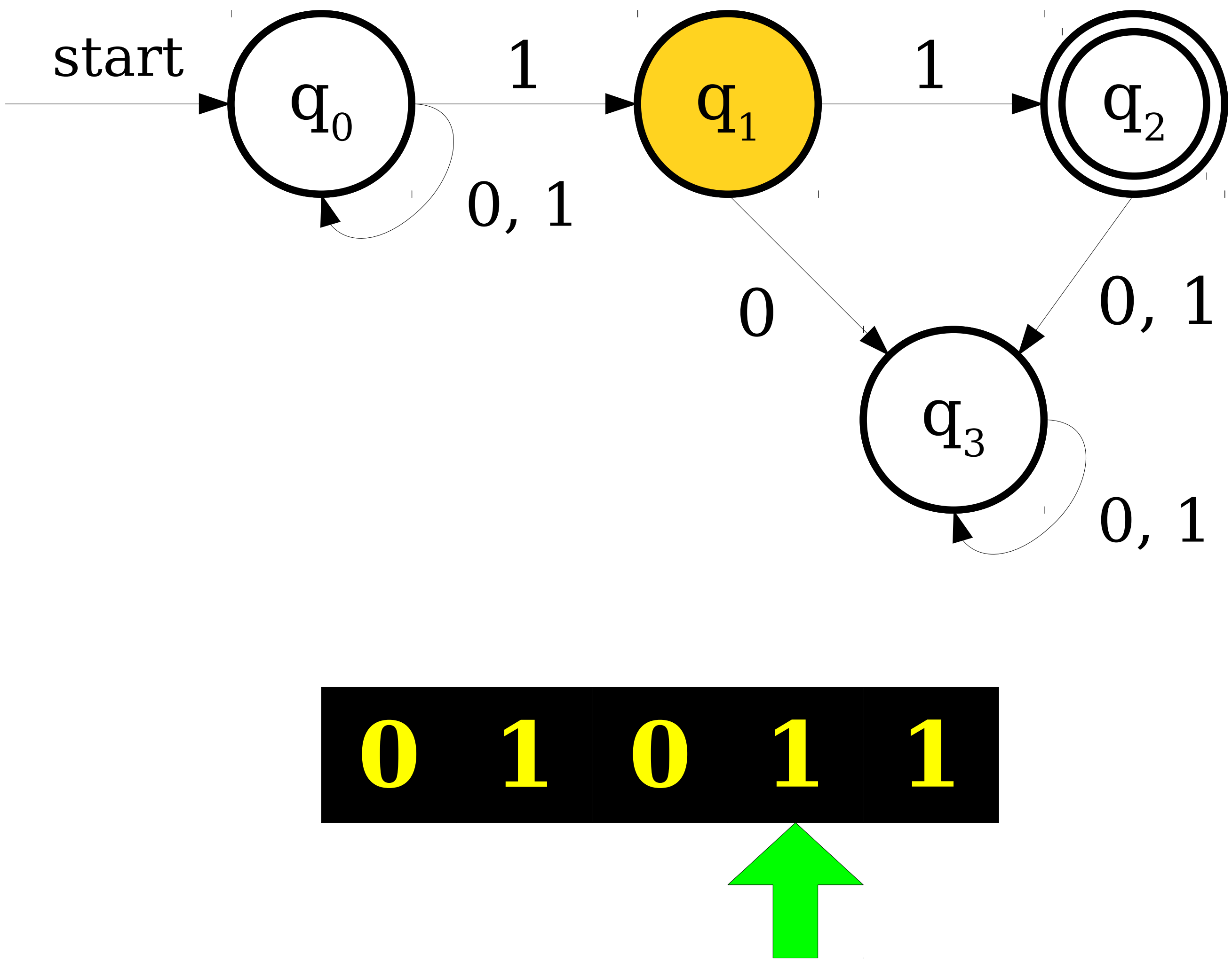

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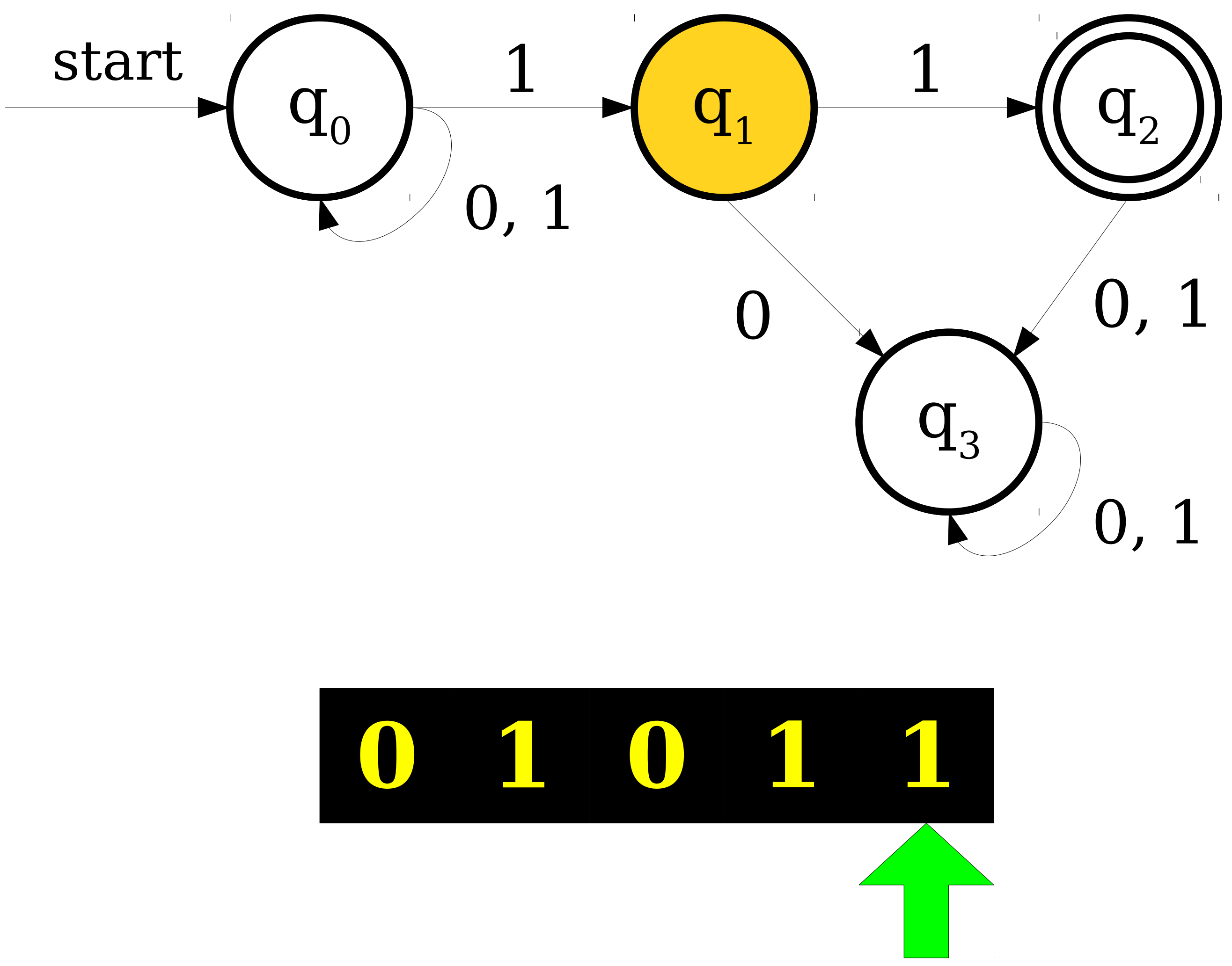

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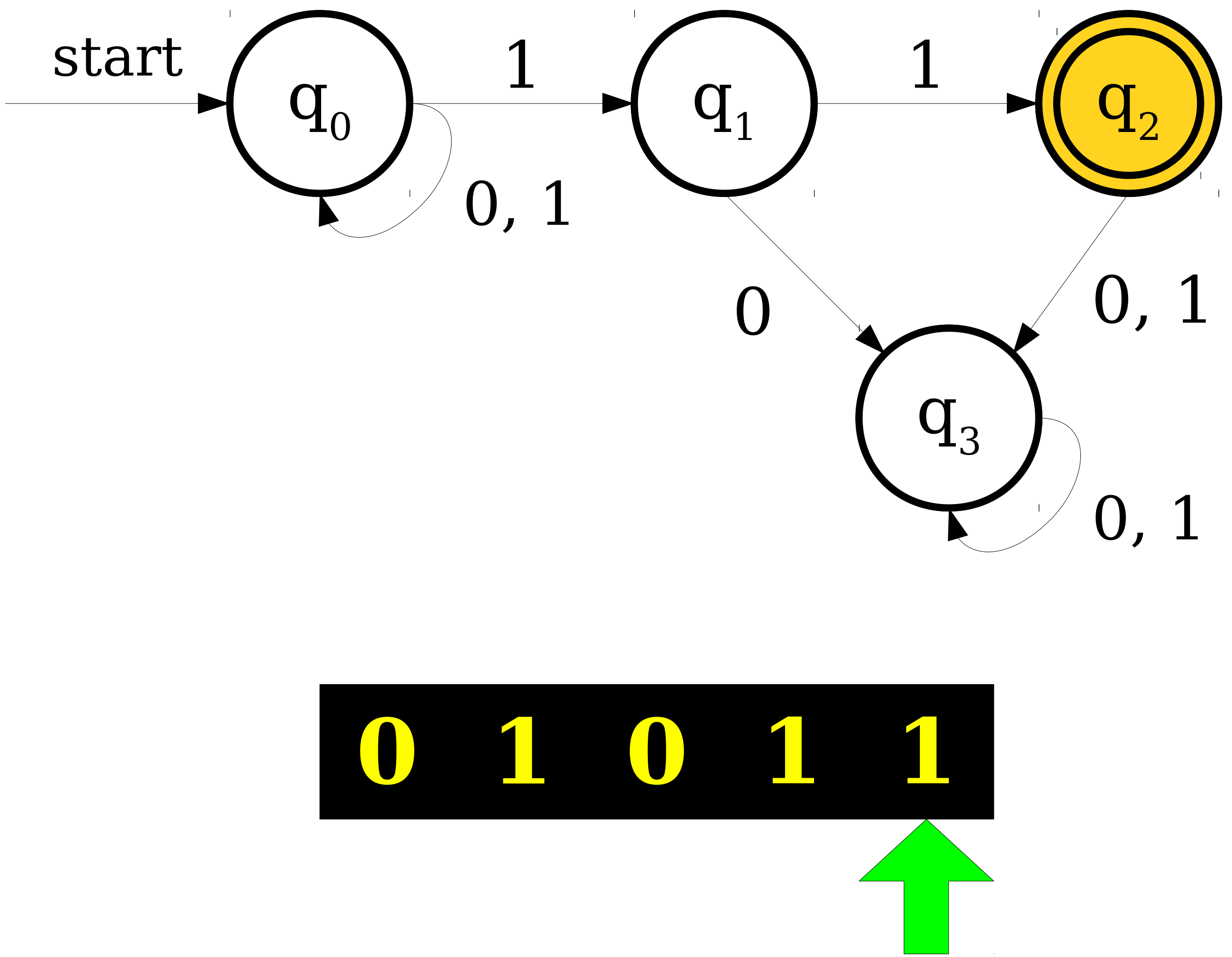

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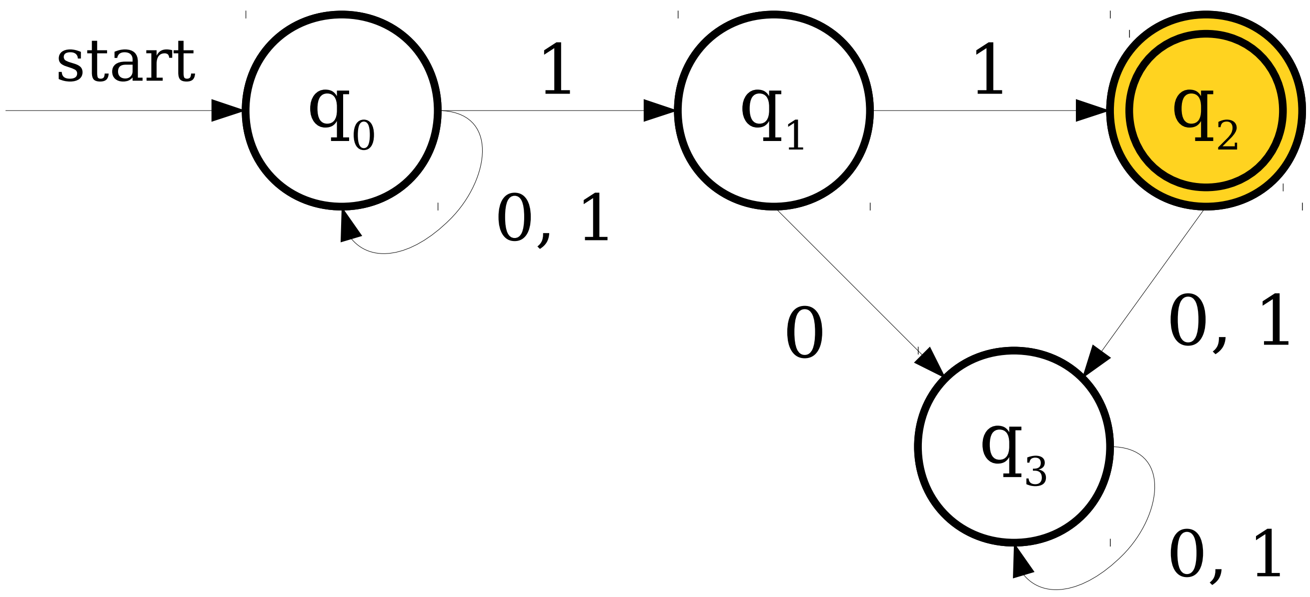


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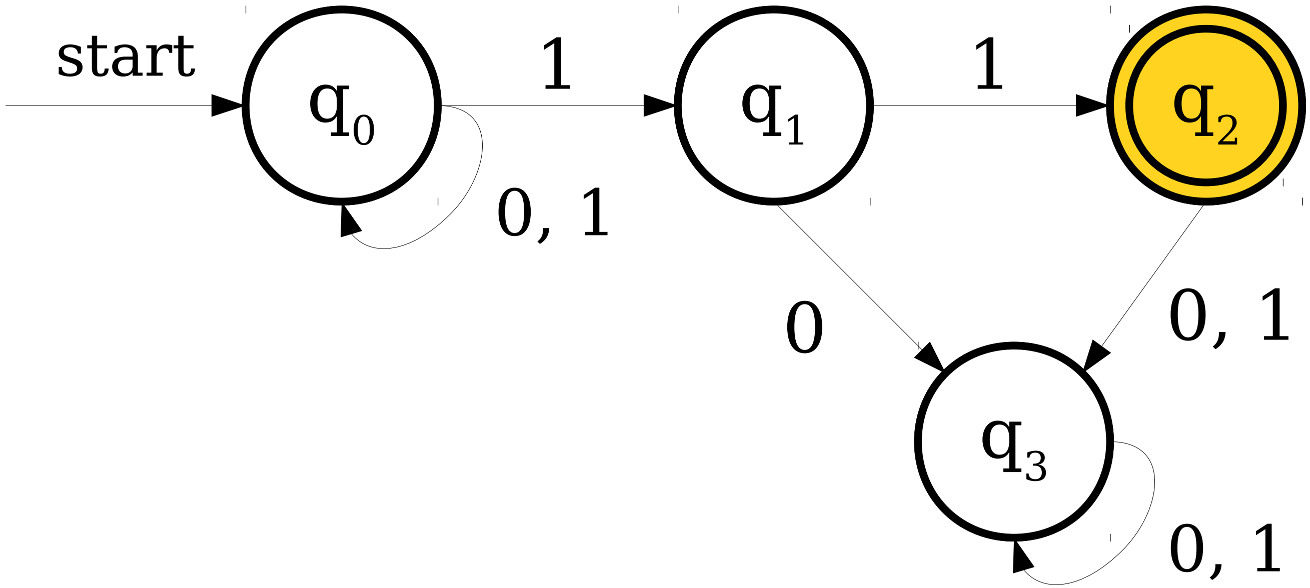


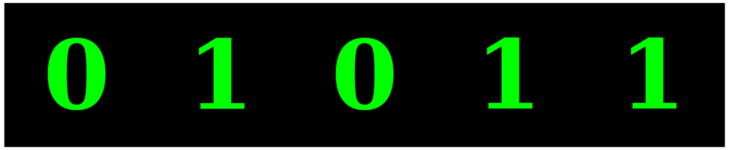

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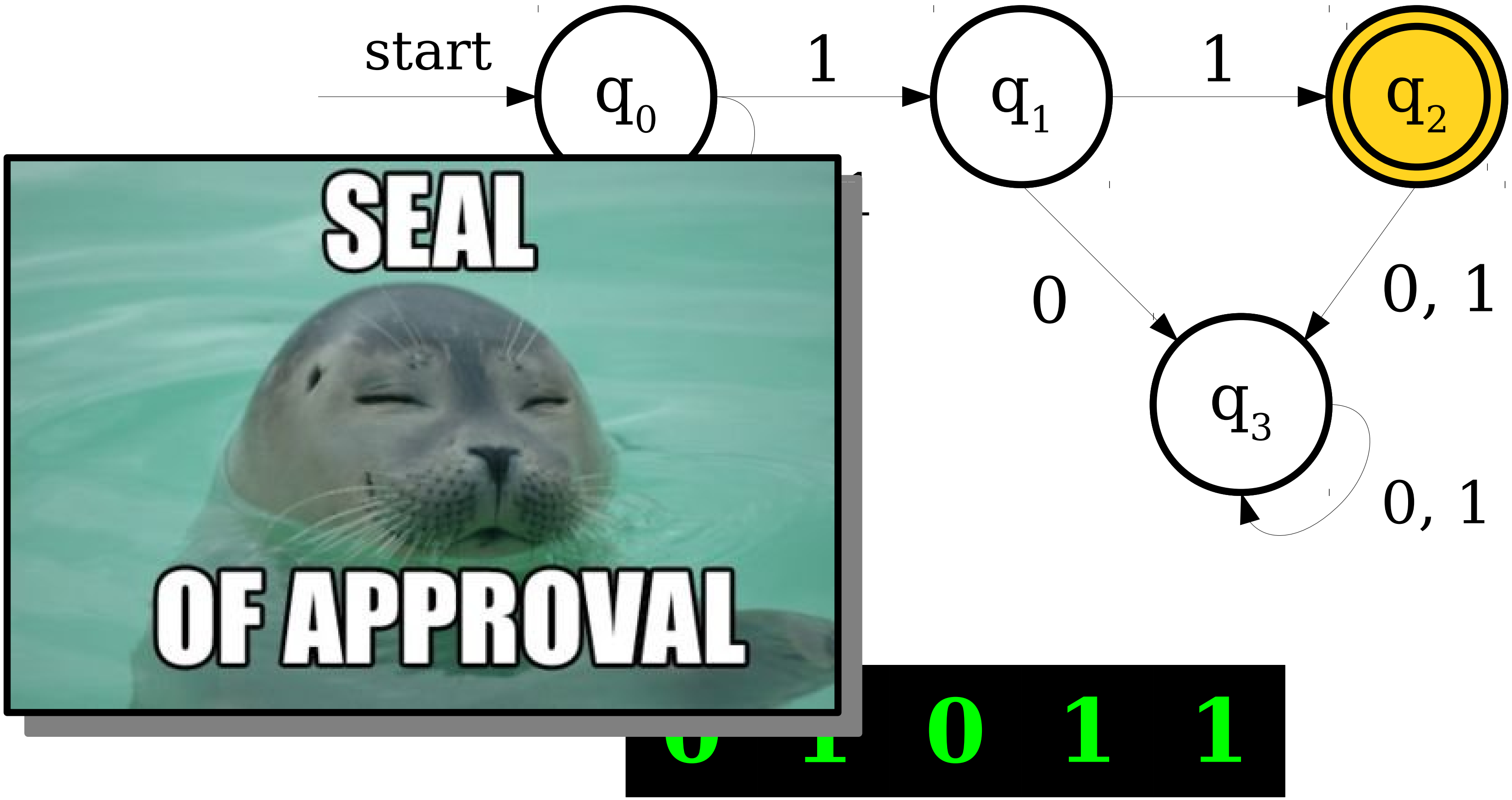

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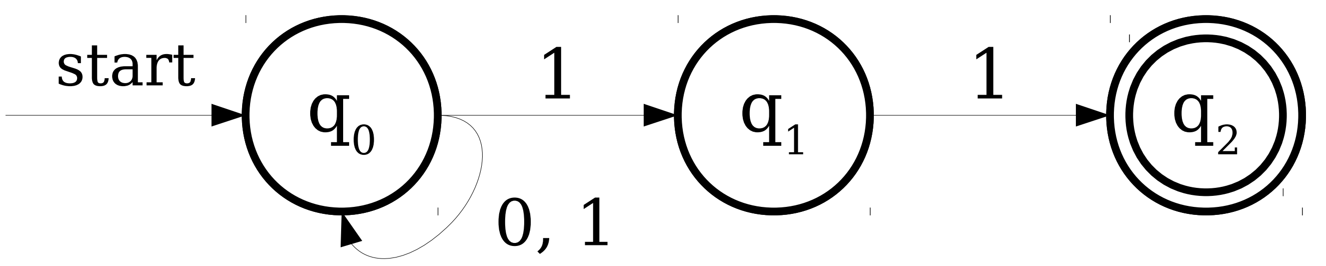

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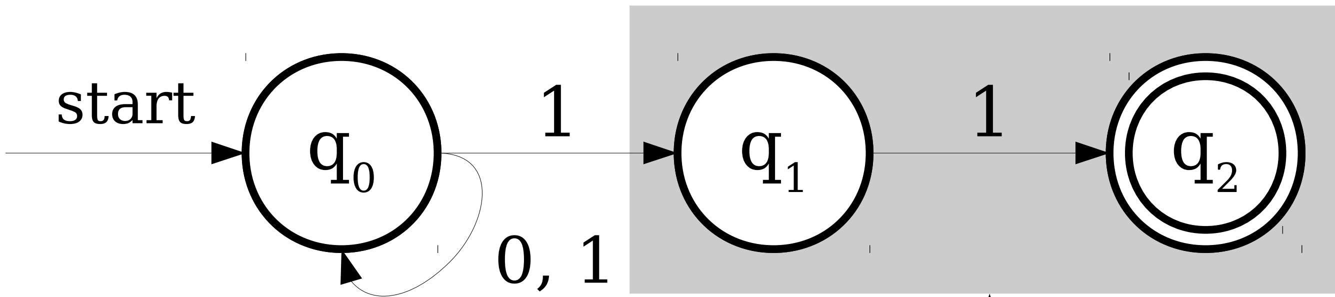


If a NFA needs to make a transition when no transition exists, the automaton **dies** and that particular path rejects.

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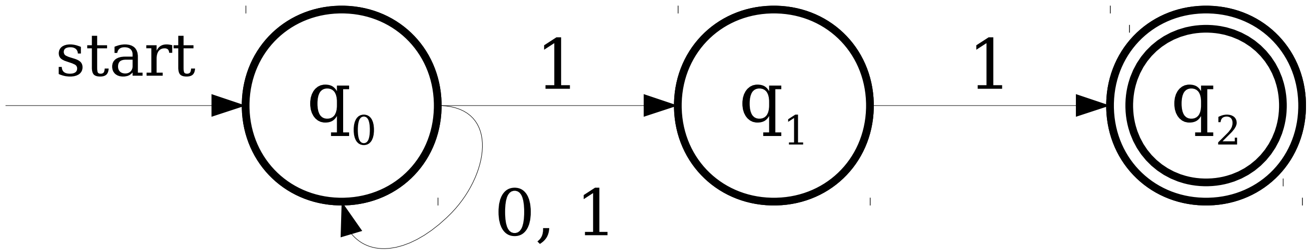


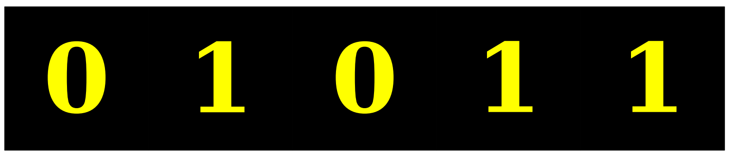

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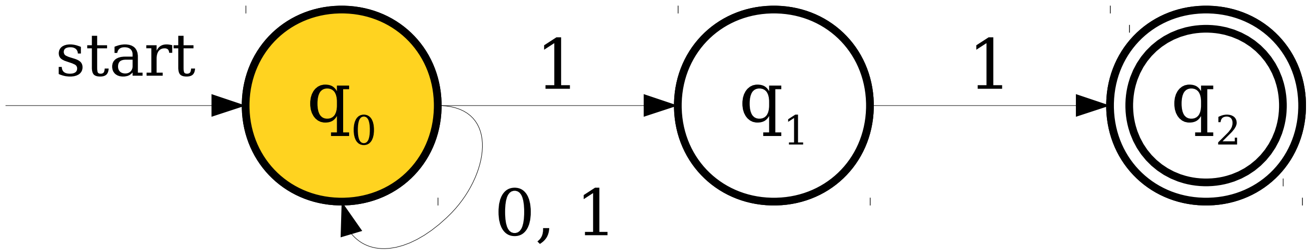


**0 1 0 1 1**

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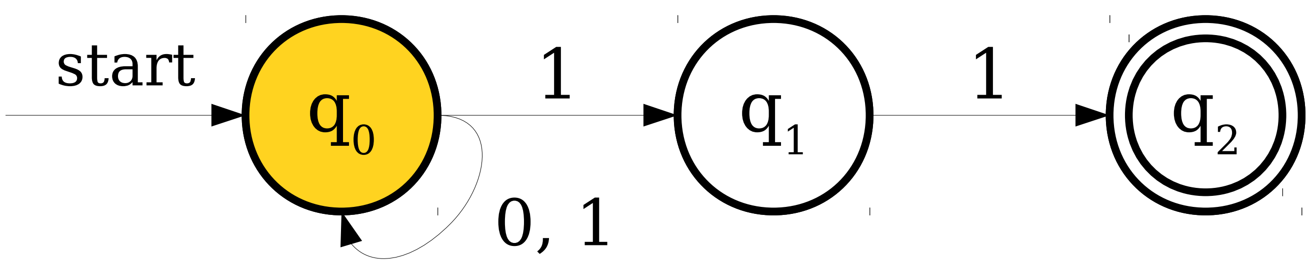


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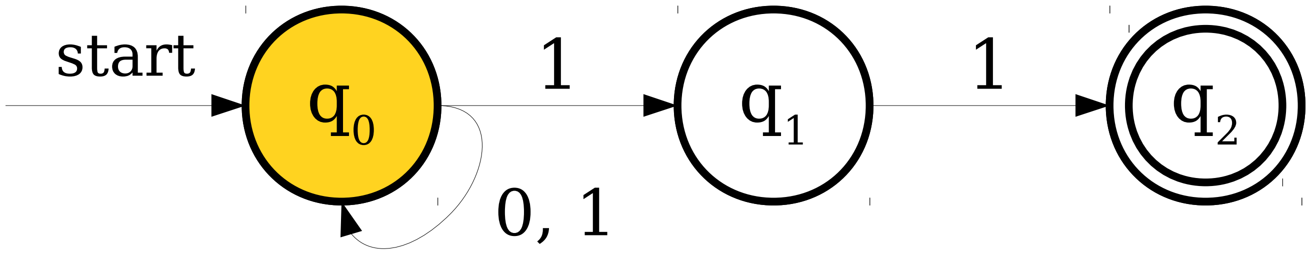


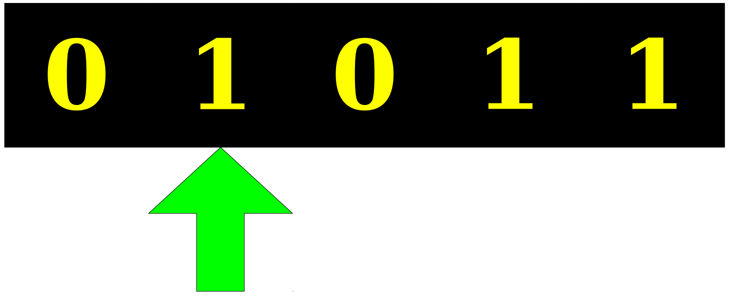

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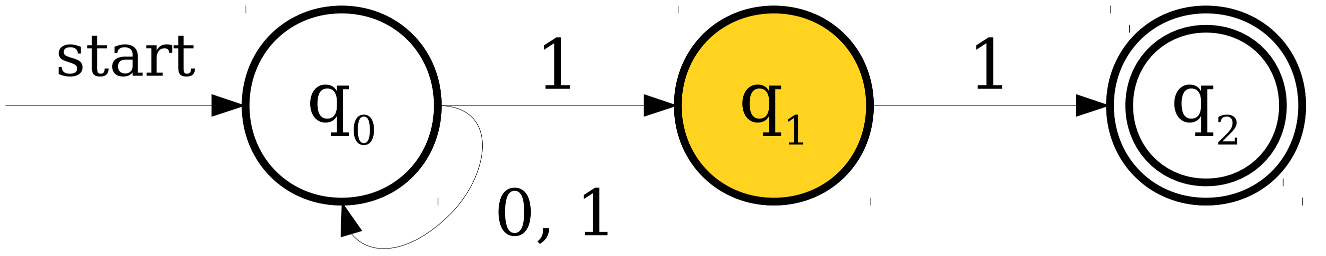


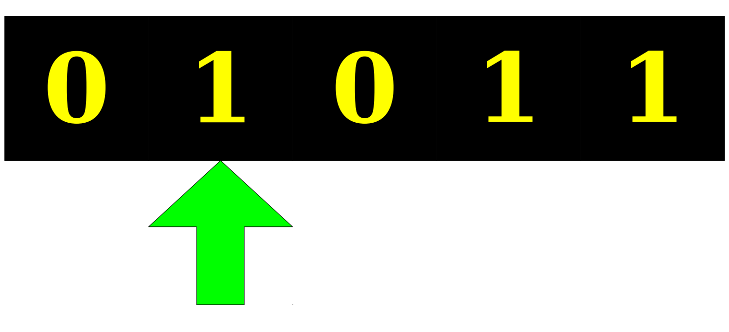

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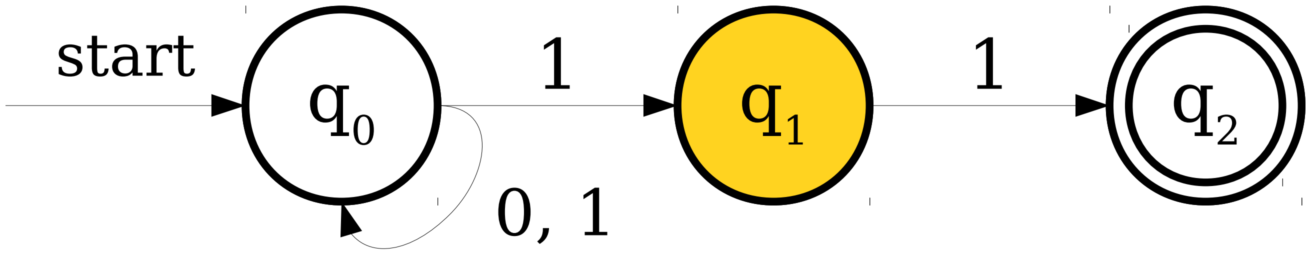


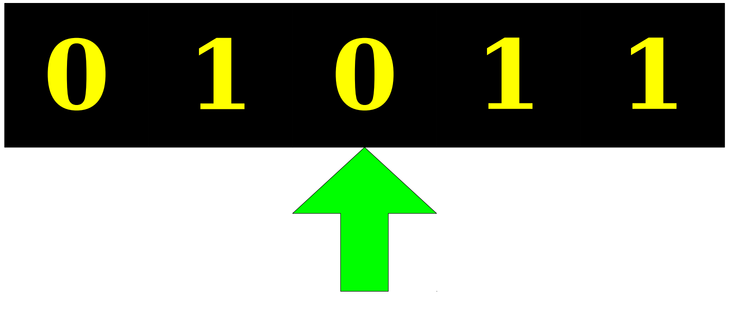

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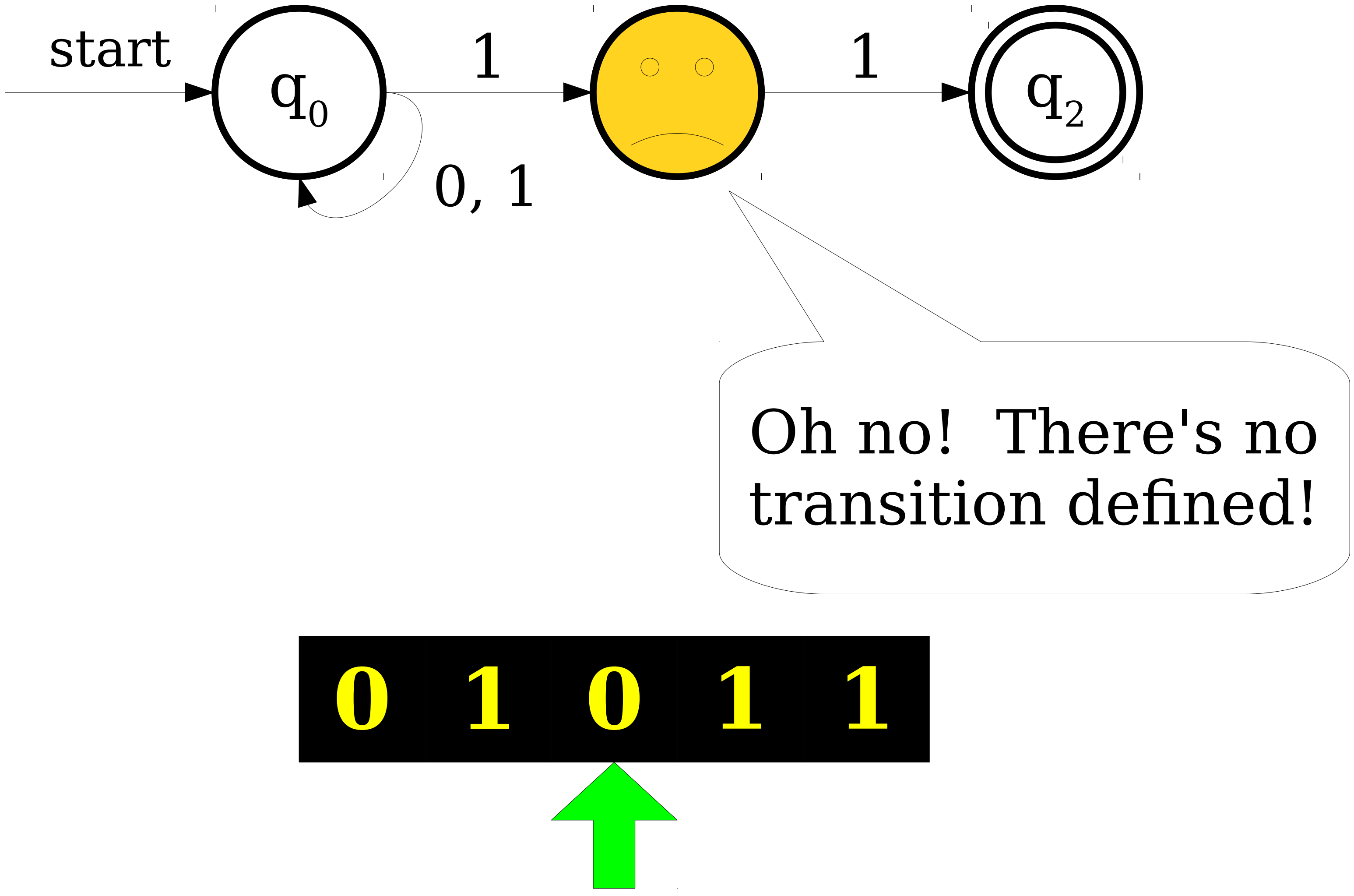

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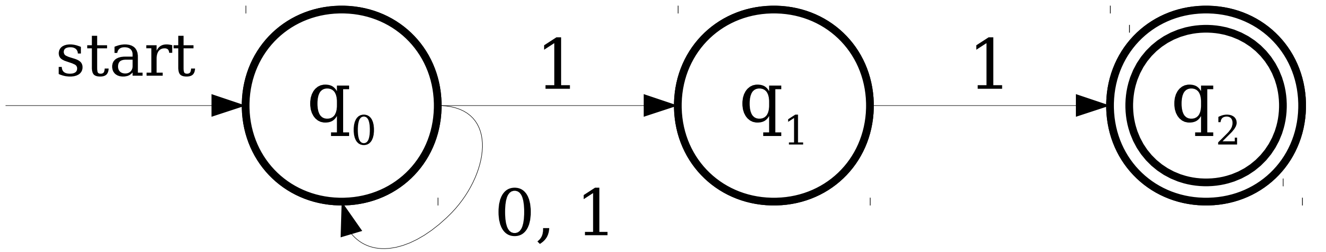


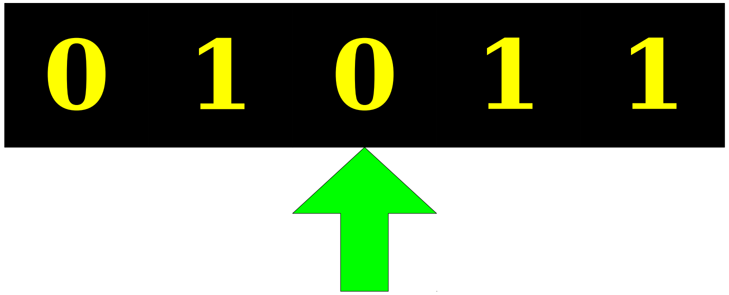

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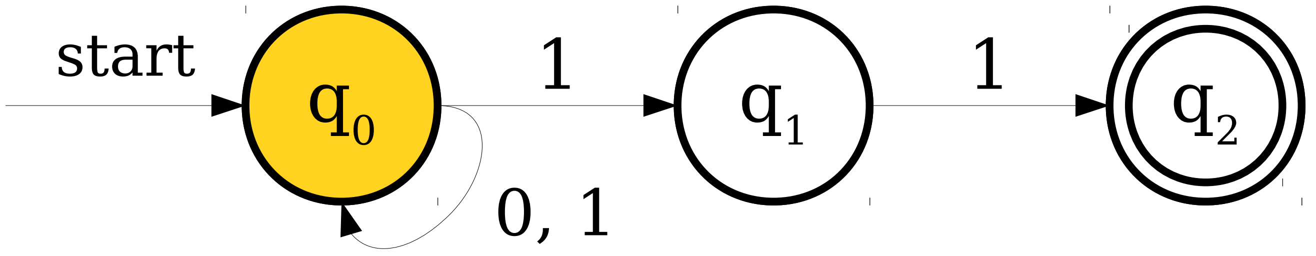


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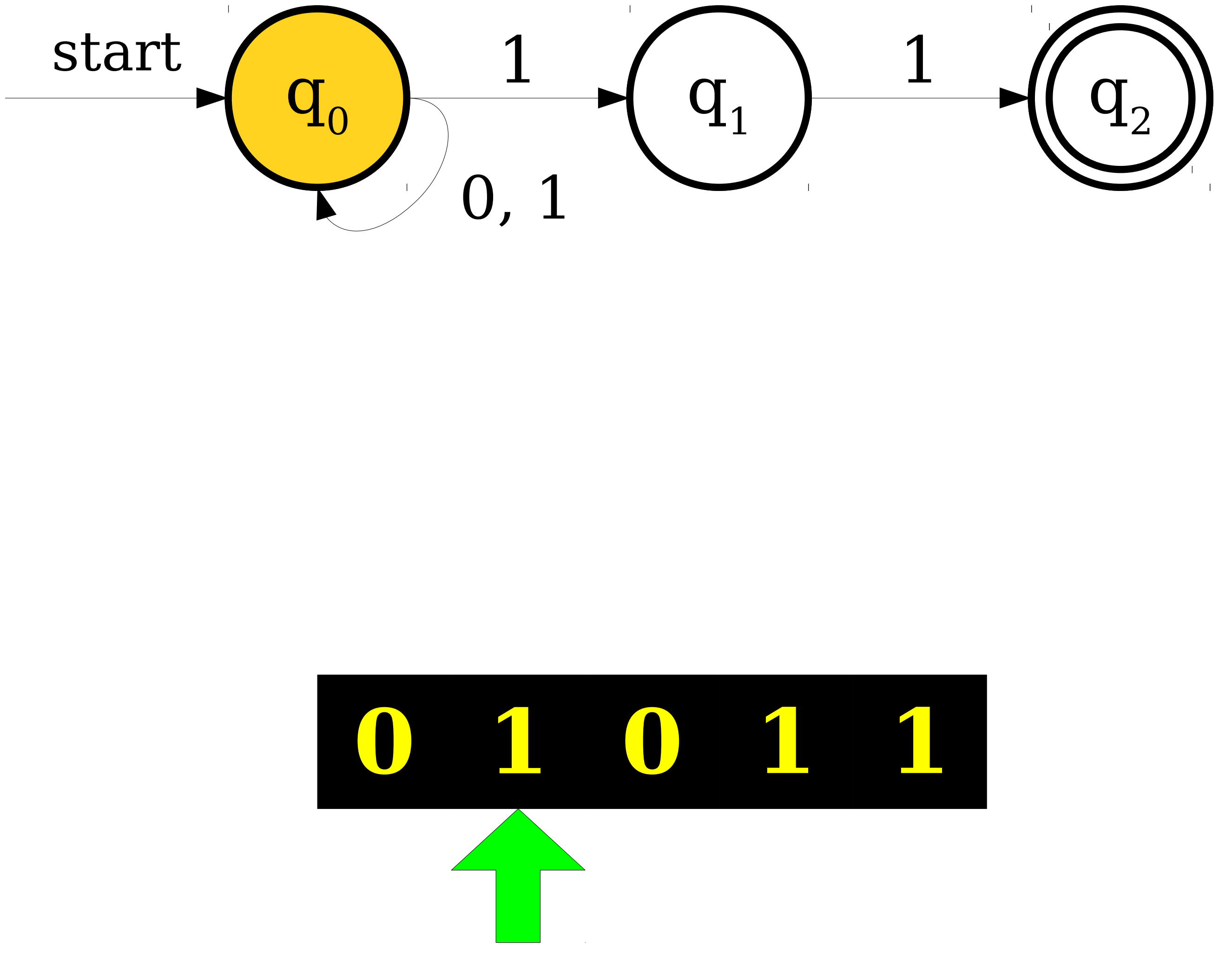

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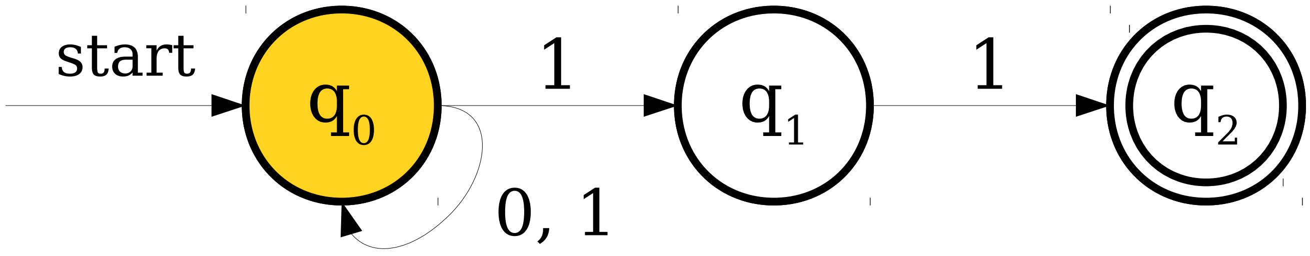


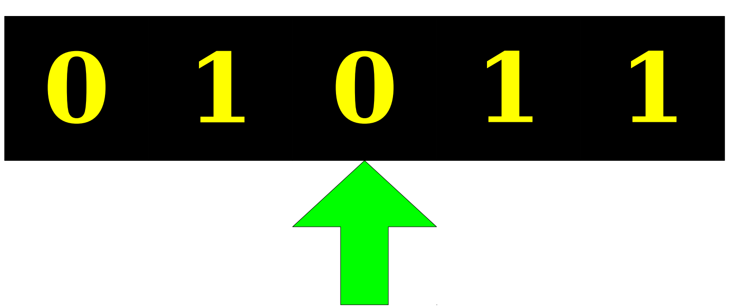

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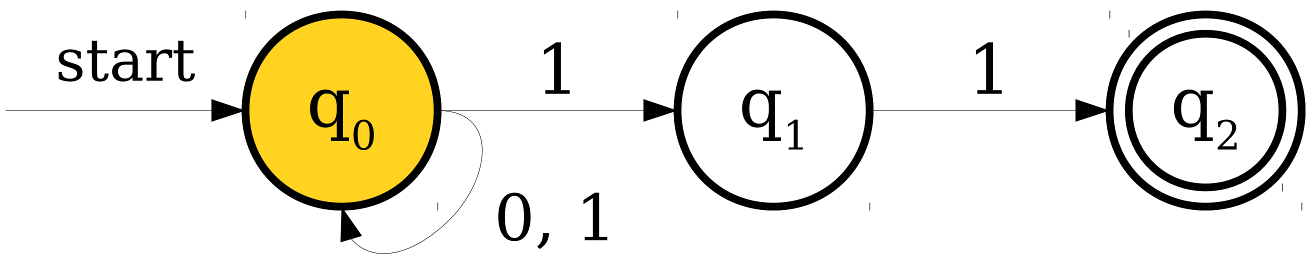


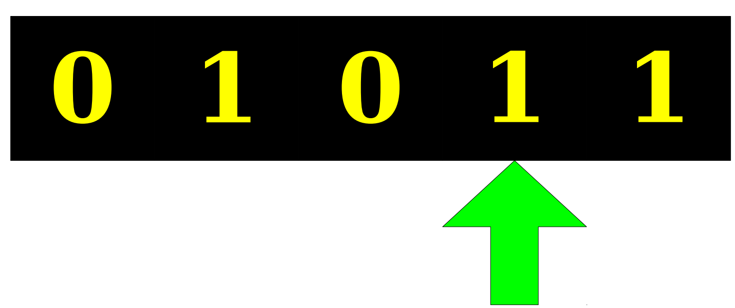

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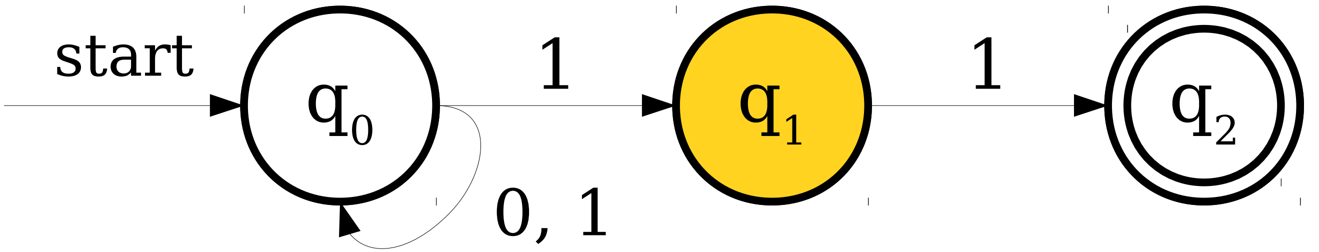


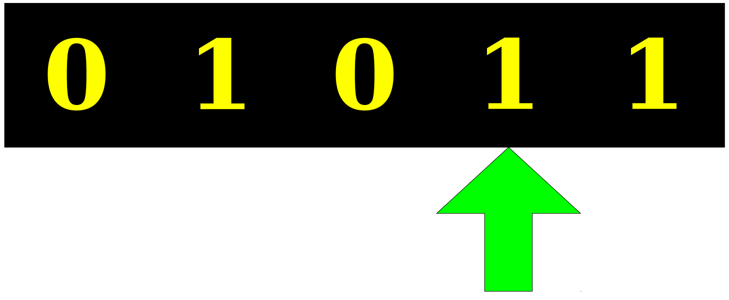

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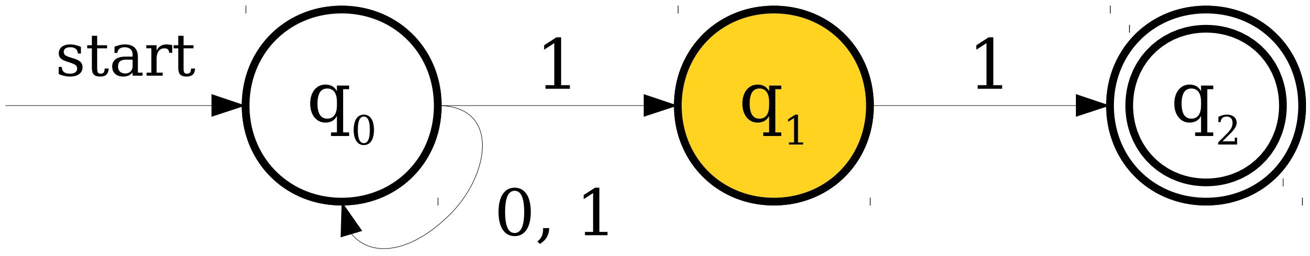


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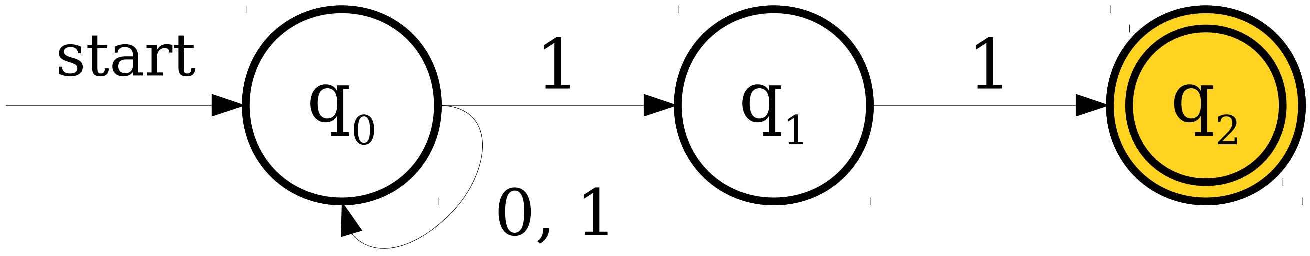


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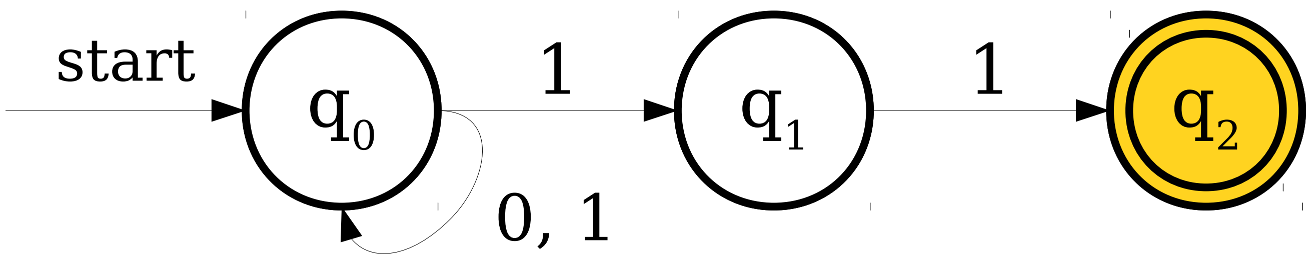


**0 1 0 1 1**

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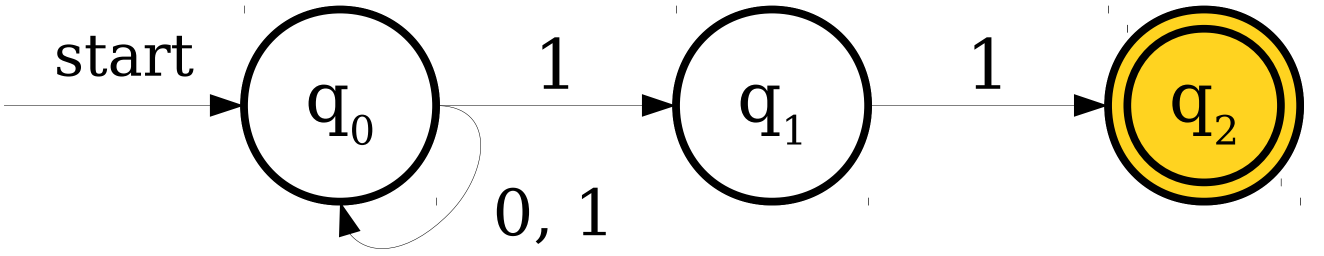


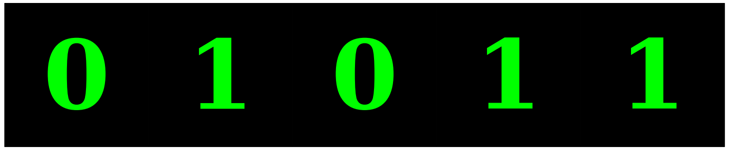

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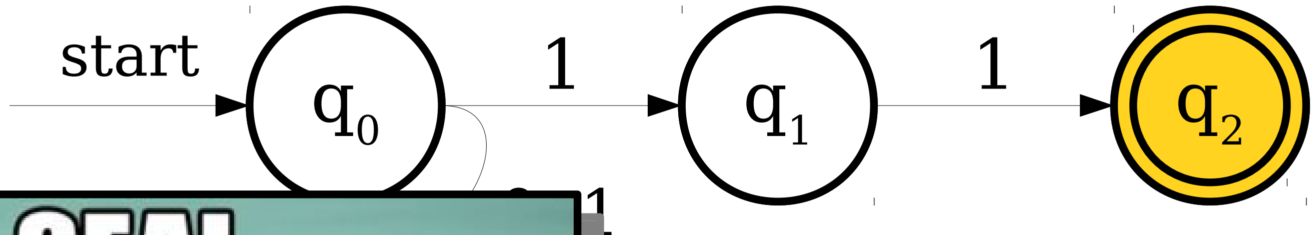



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# ε-Transitions

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- An NFA may follow any number of ε-transitions at any time without consuming any input.

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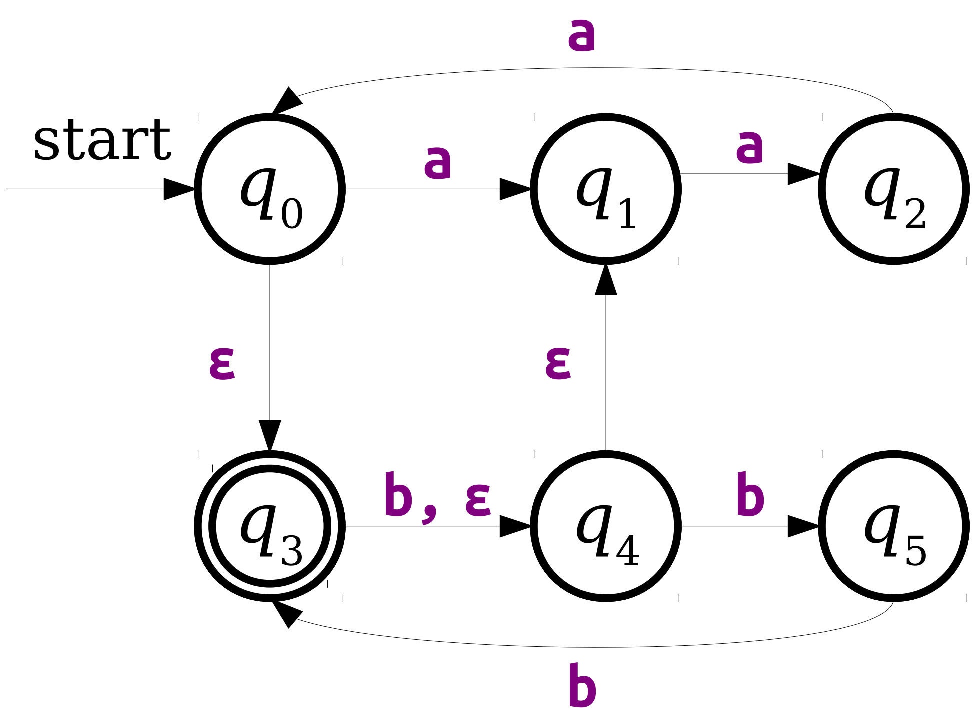

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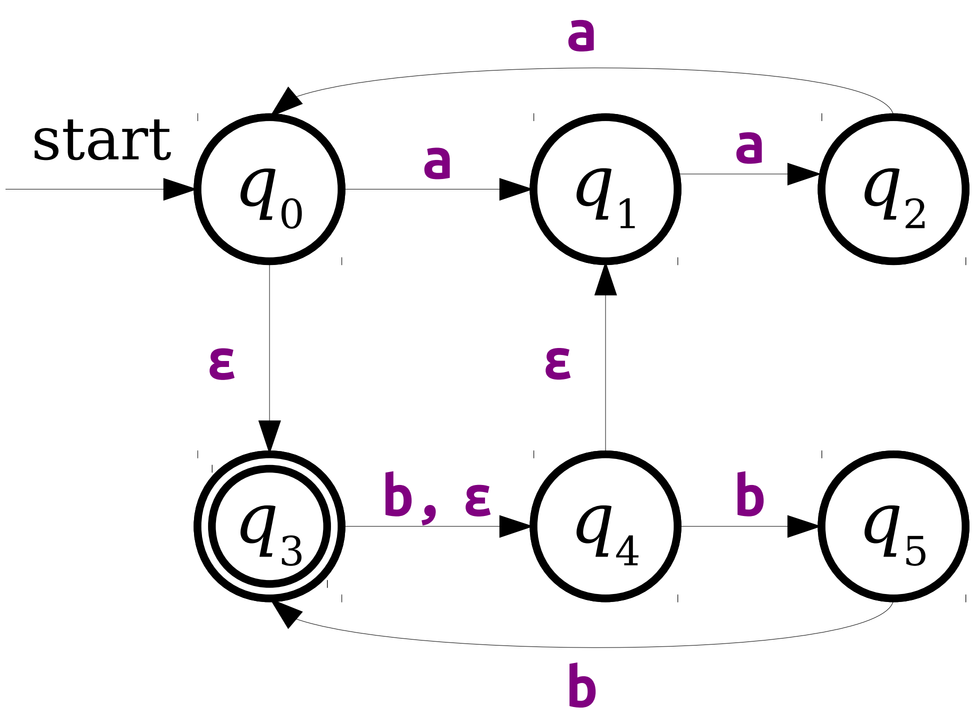


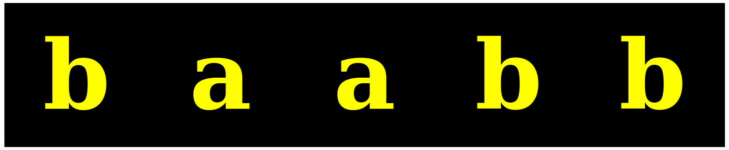

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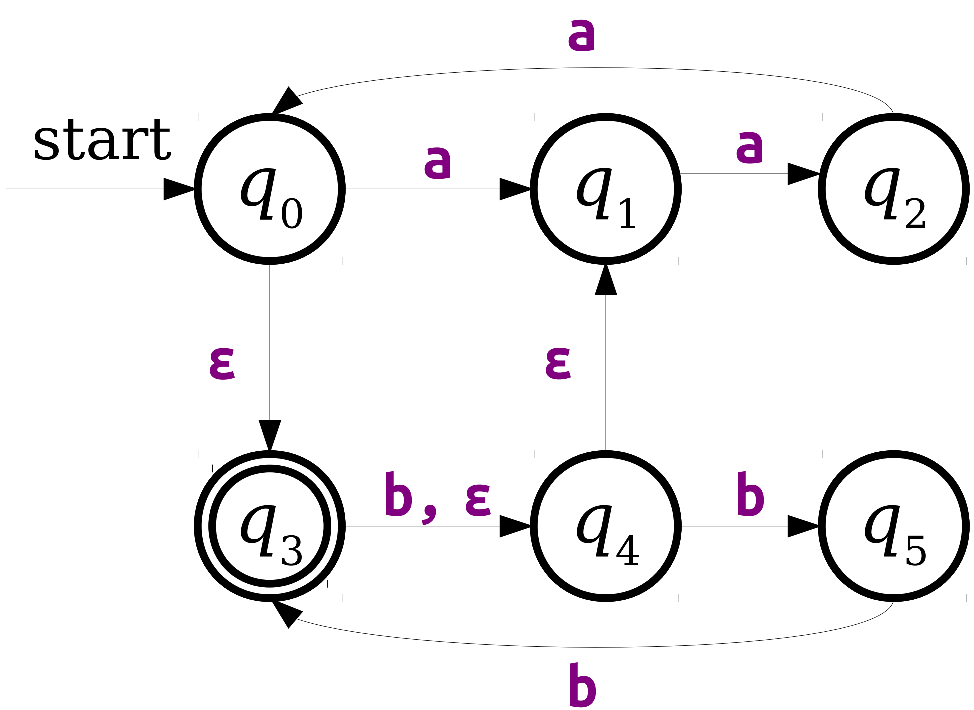


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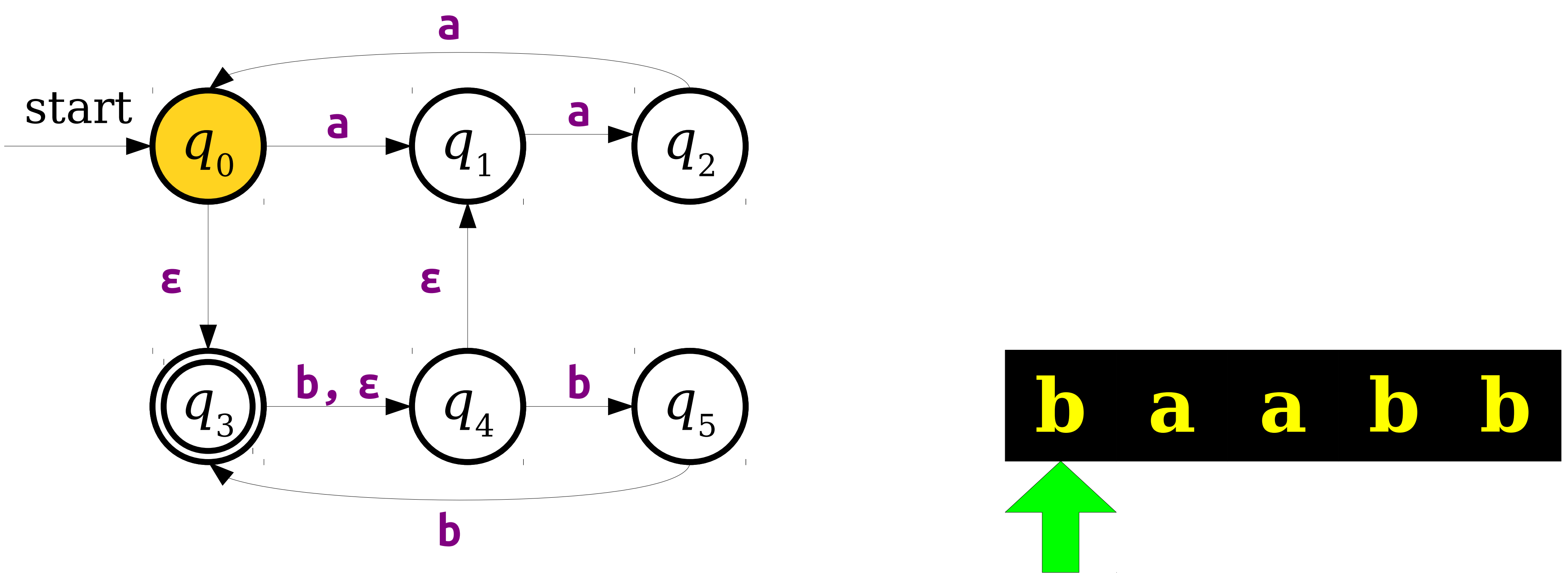

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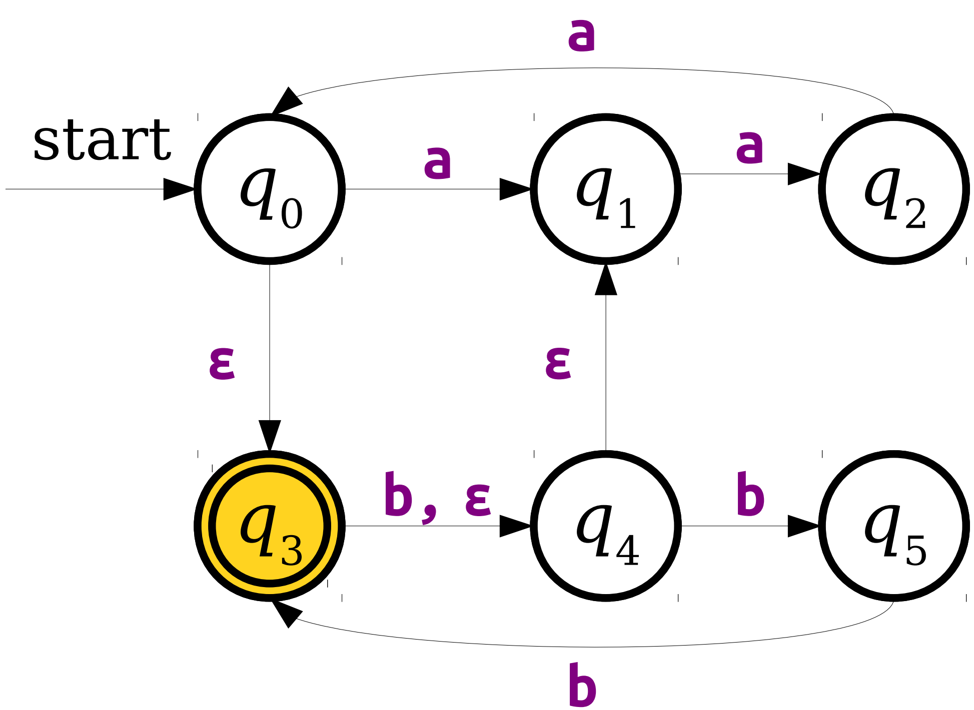


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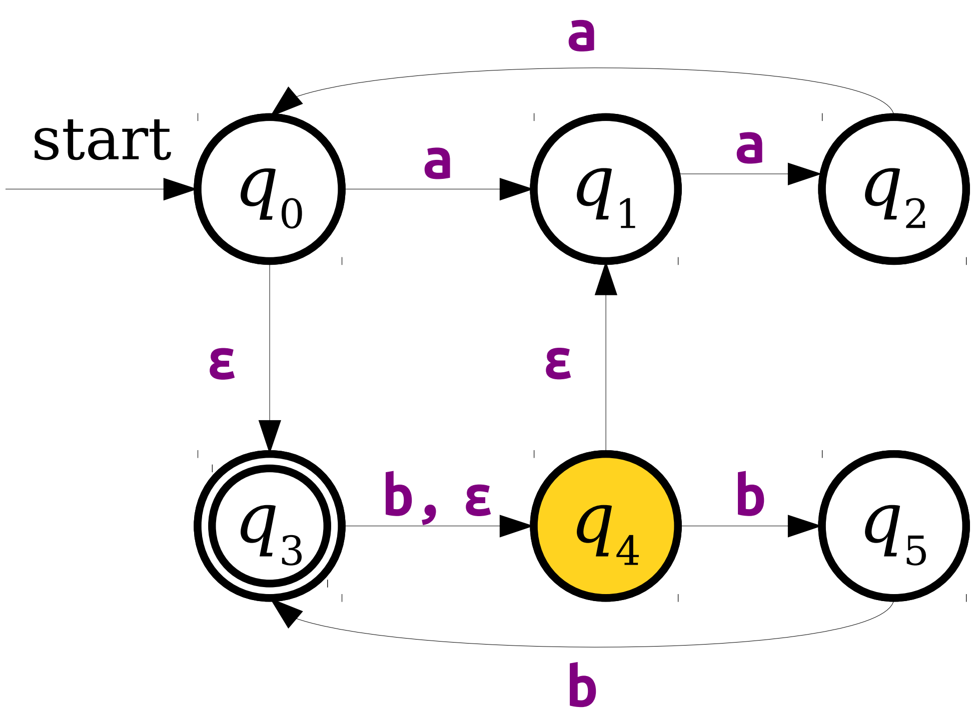


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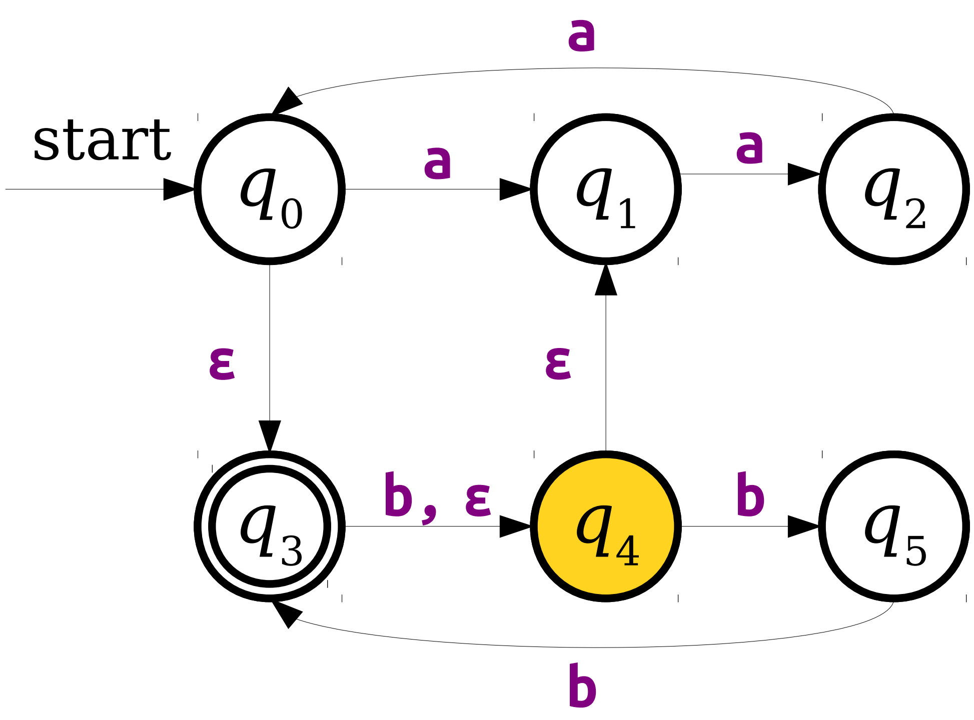


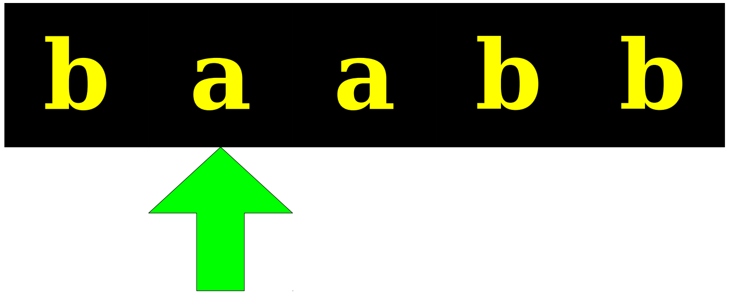

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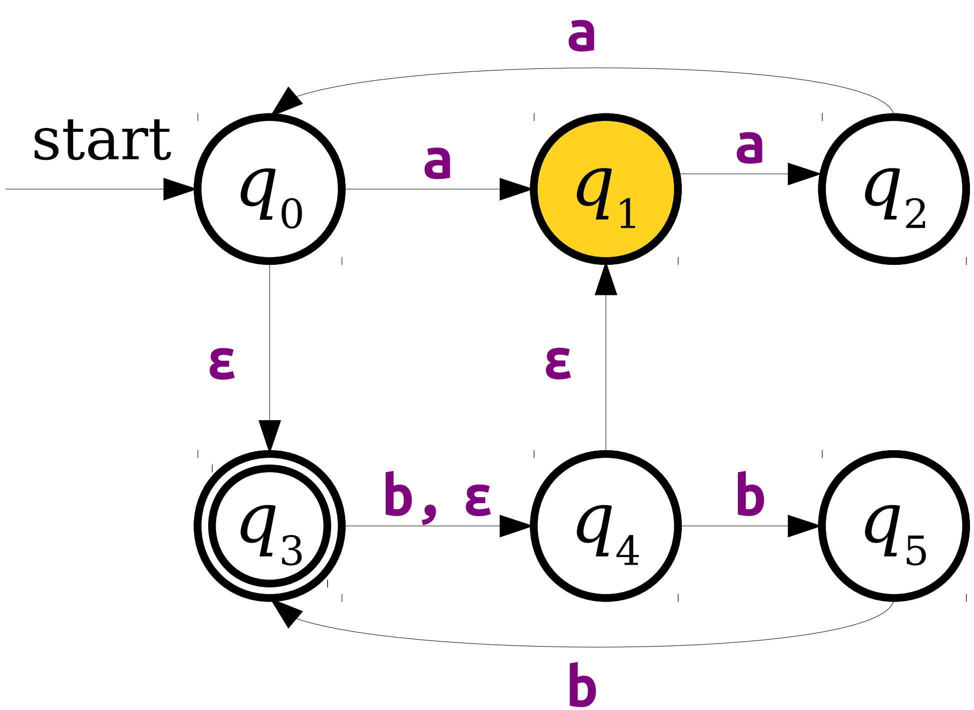


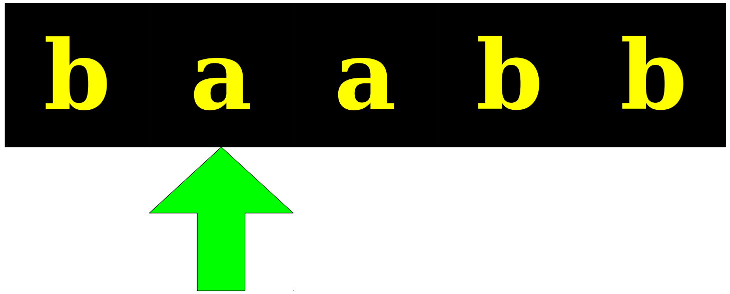

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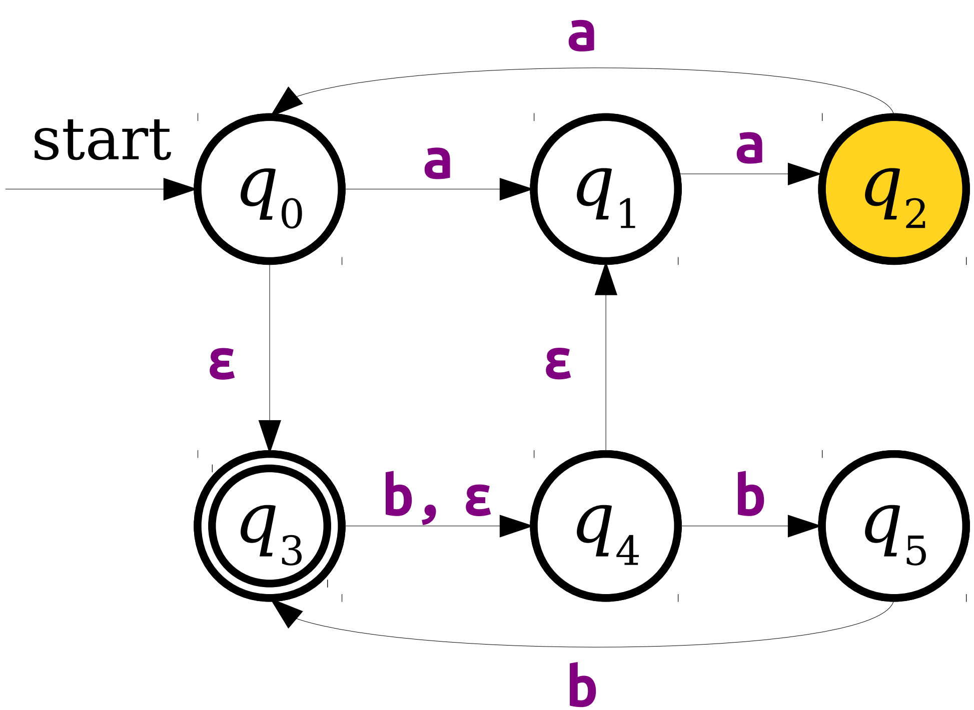


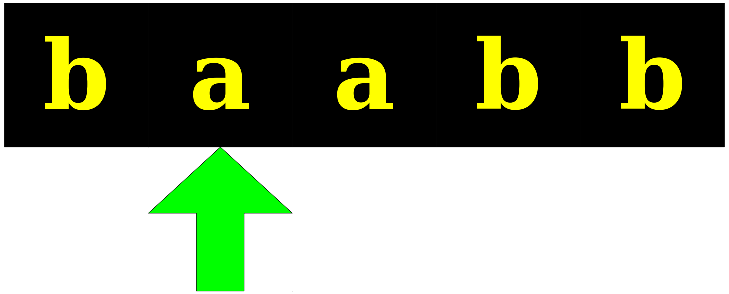

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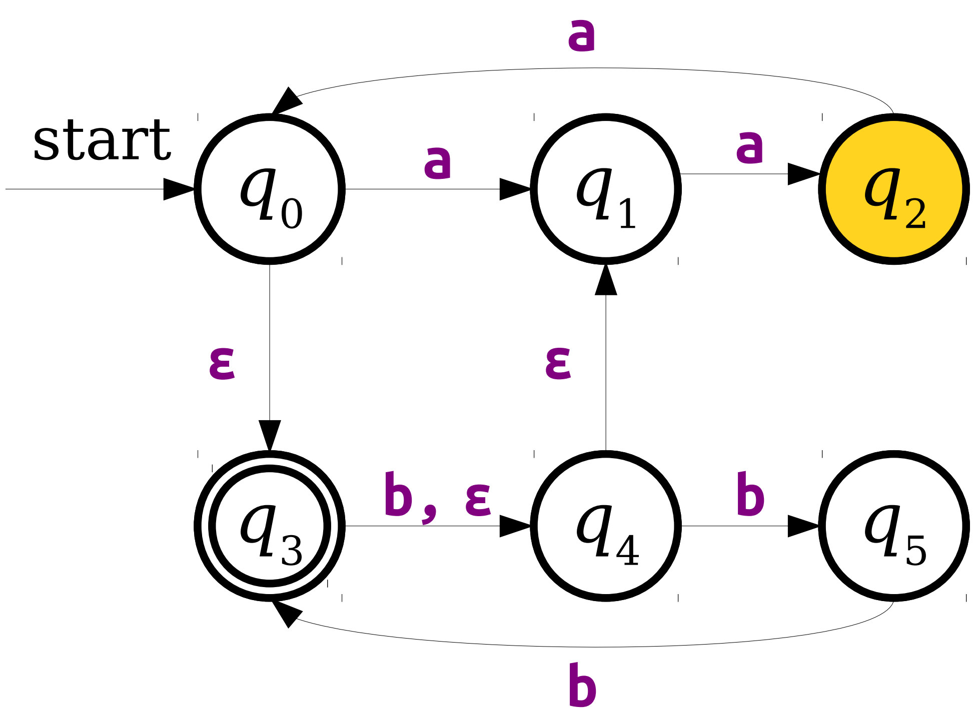


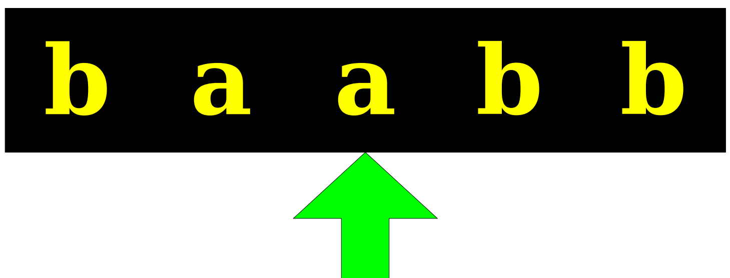

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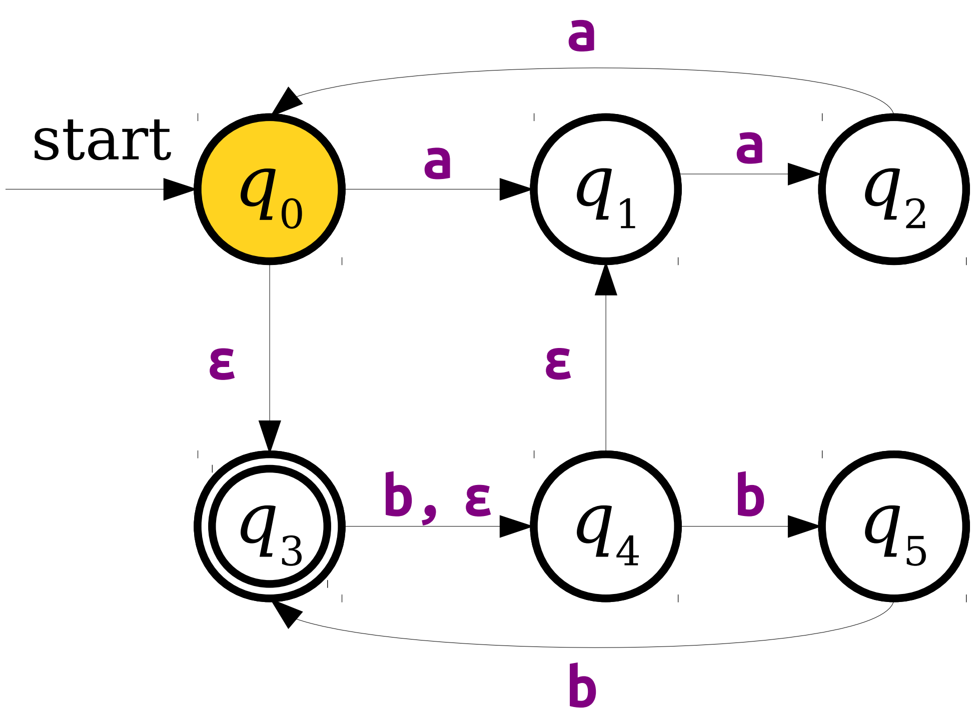


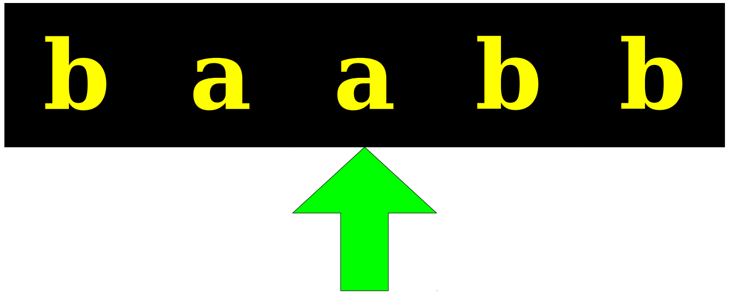

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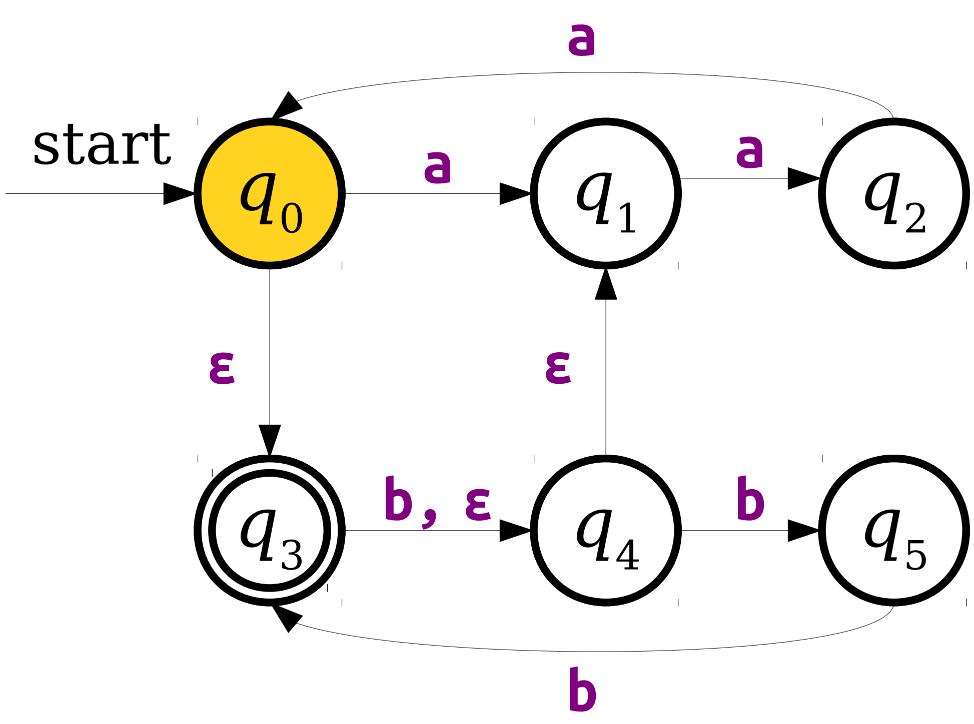


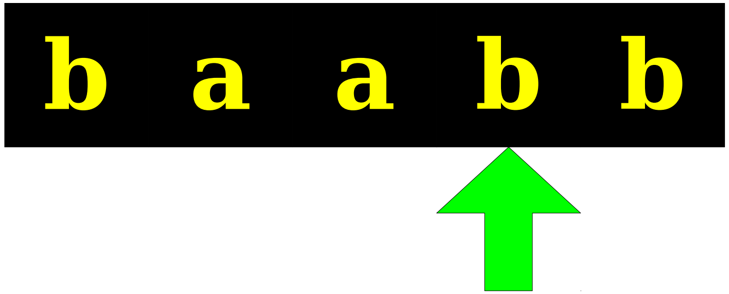

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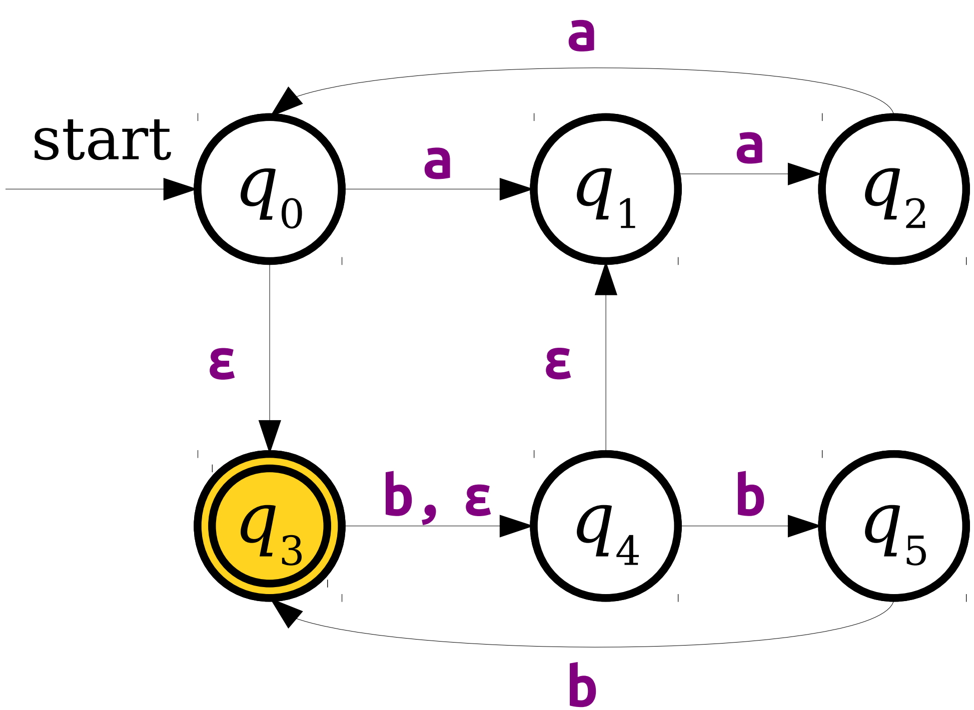


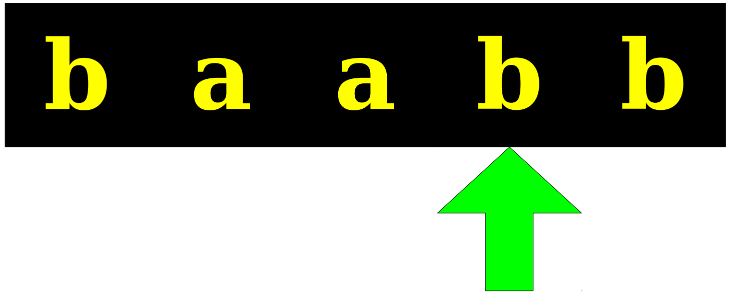

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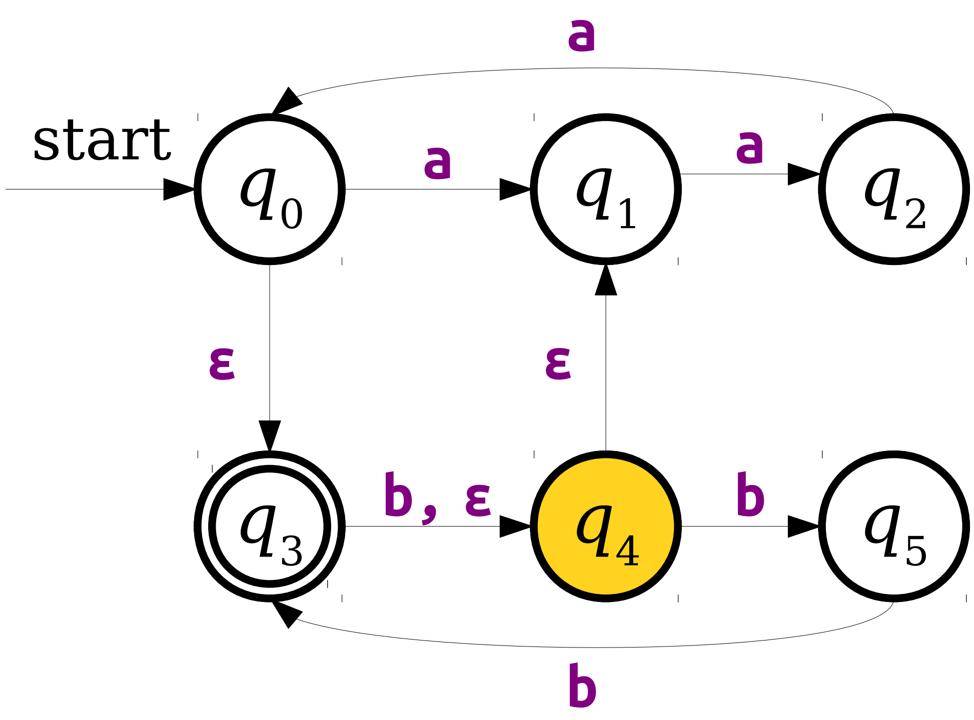


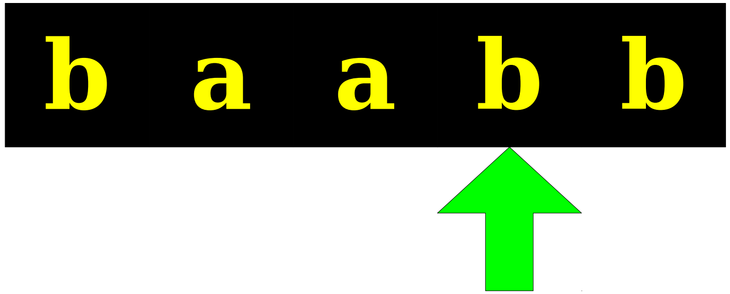

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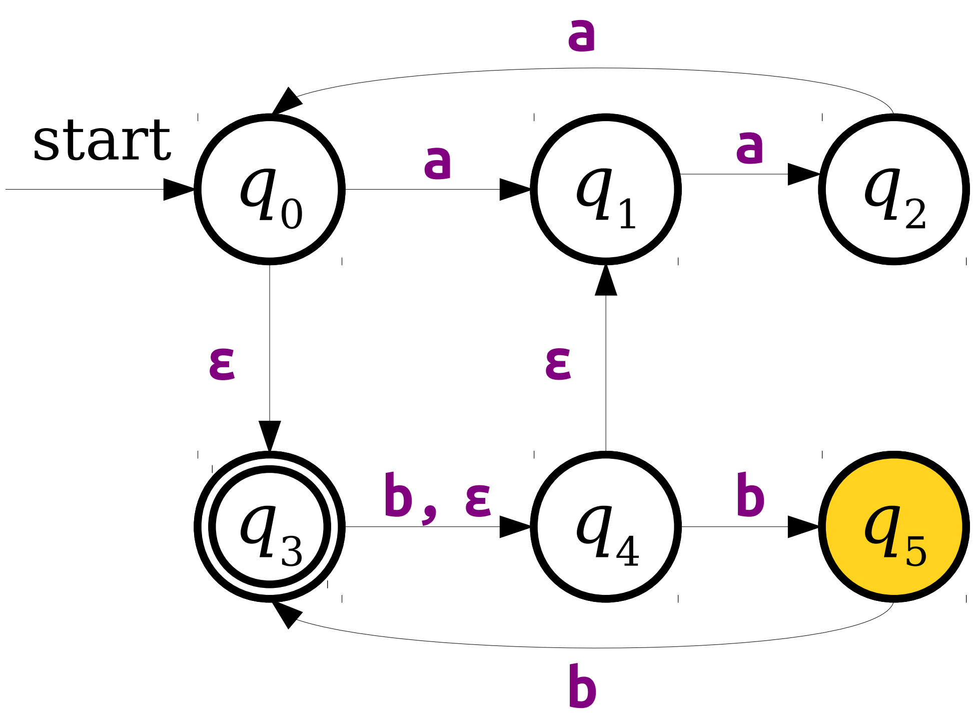


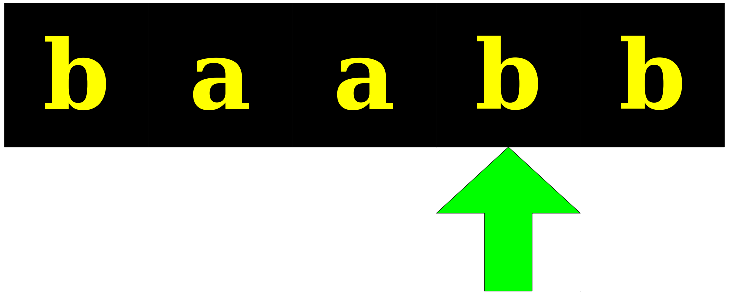

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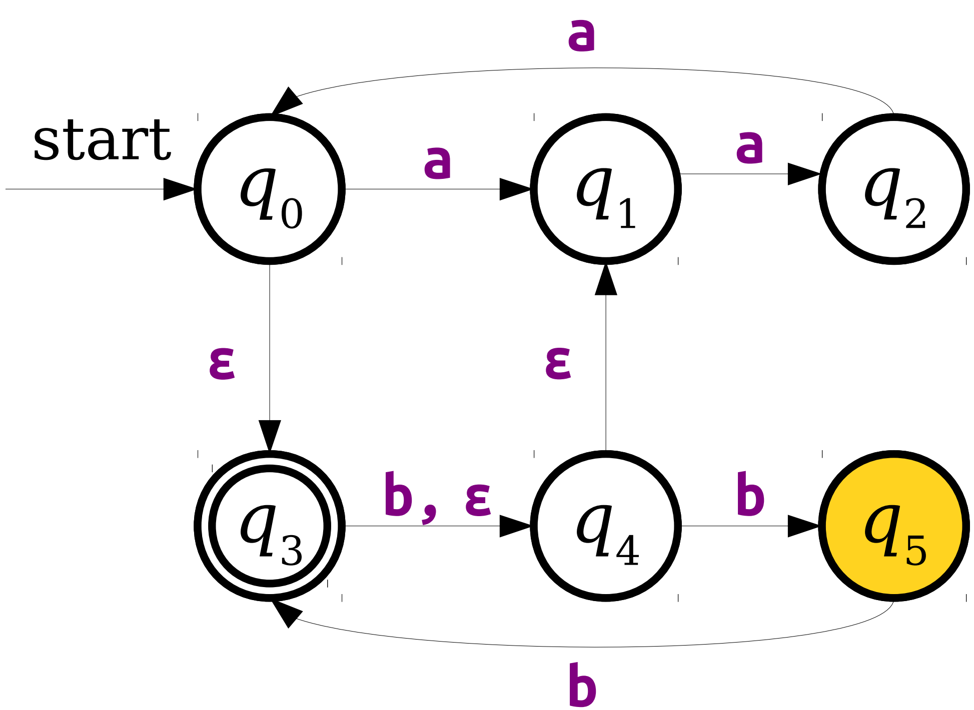


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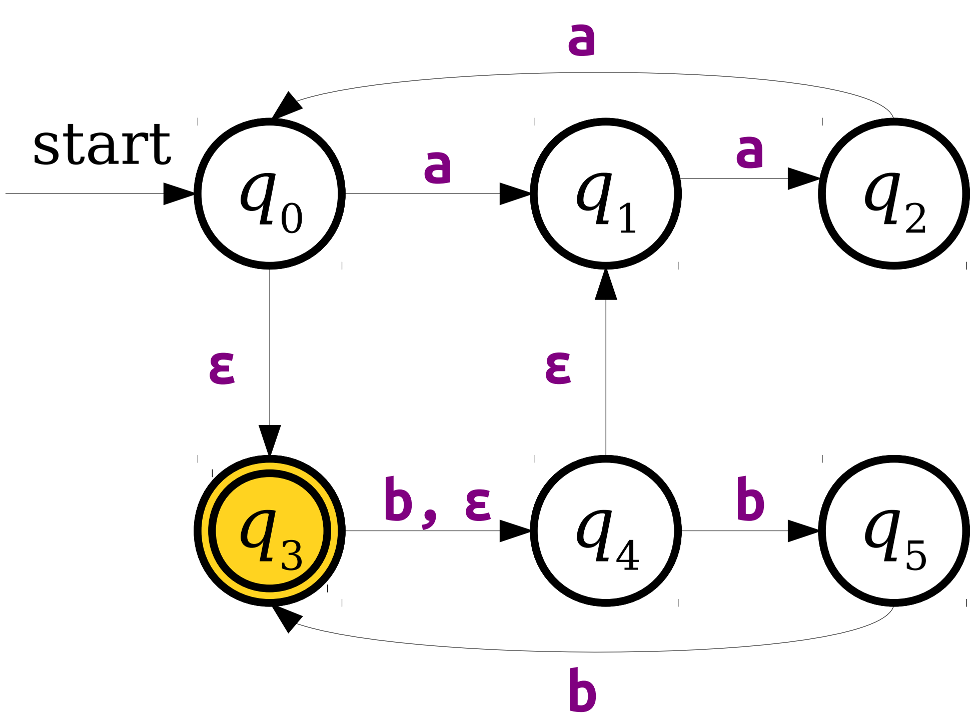


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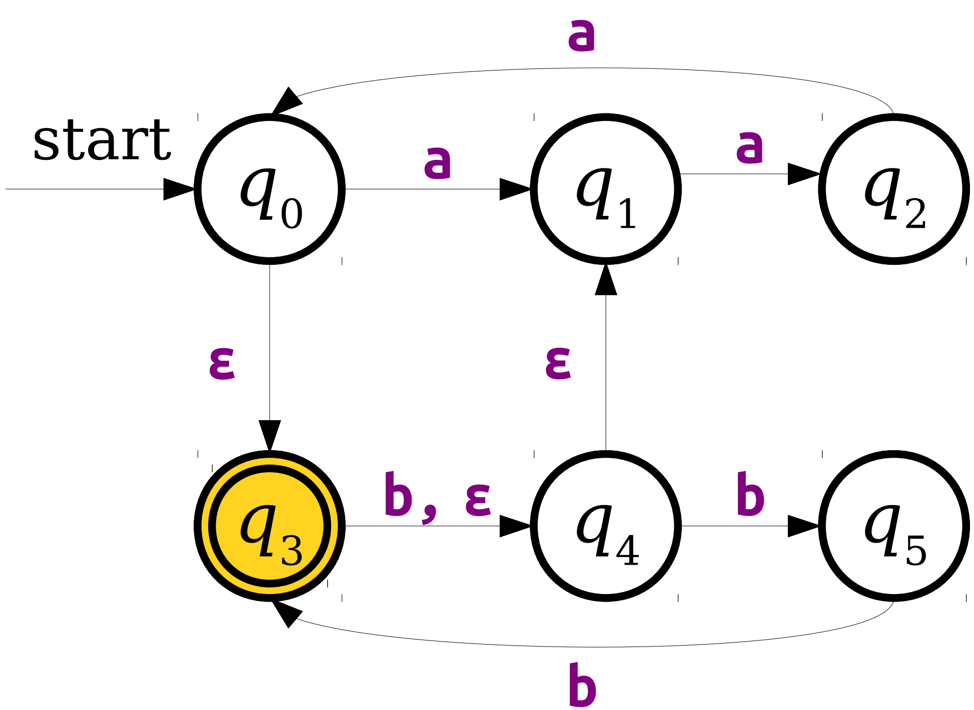
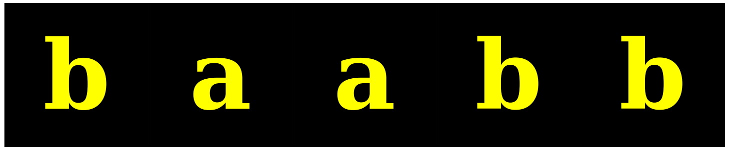


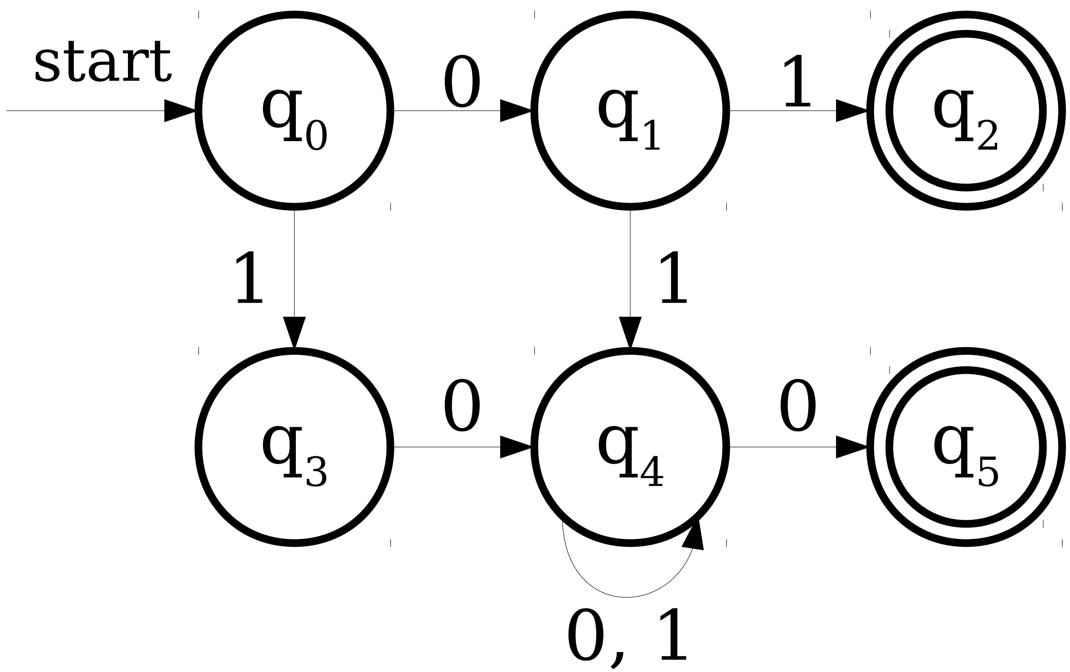
# ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.
- NFAs are not *required* to follow ε-transitions. It's simply another option at the machine's disposal.

# Intuiting Nondeterminism

- Nondeterministic machines are a serious departure from physical computers.
- How can we build up an intuition for them?
- Three approaches:
  - **Tree computation**
  - **Perfect guessing**
  - **Massive parallelism**

# Tree Computation



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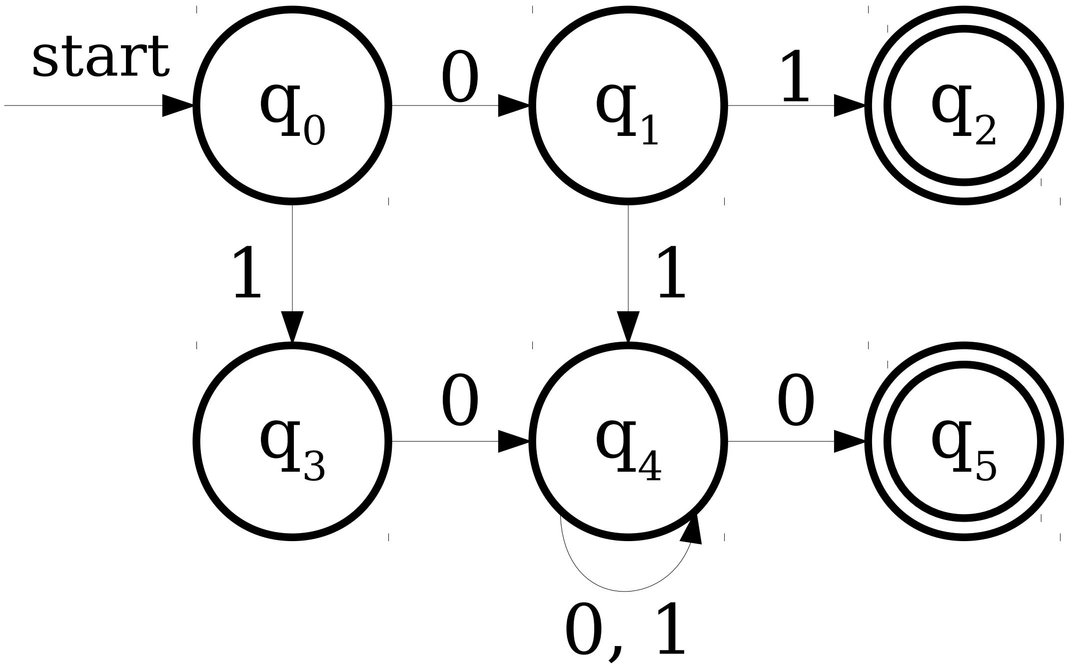


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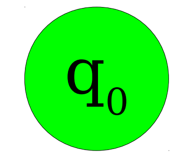


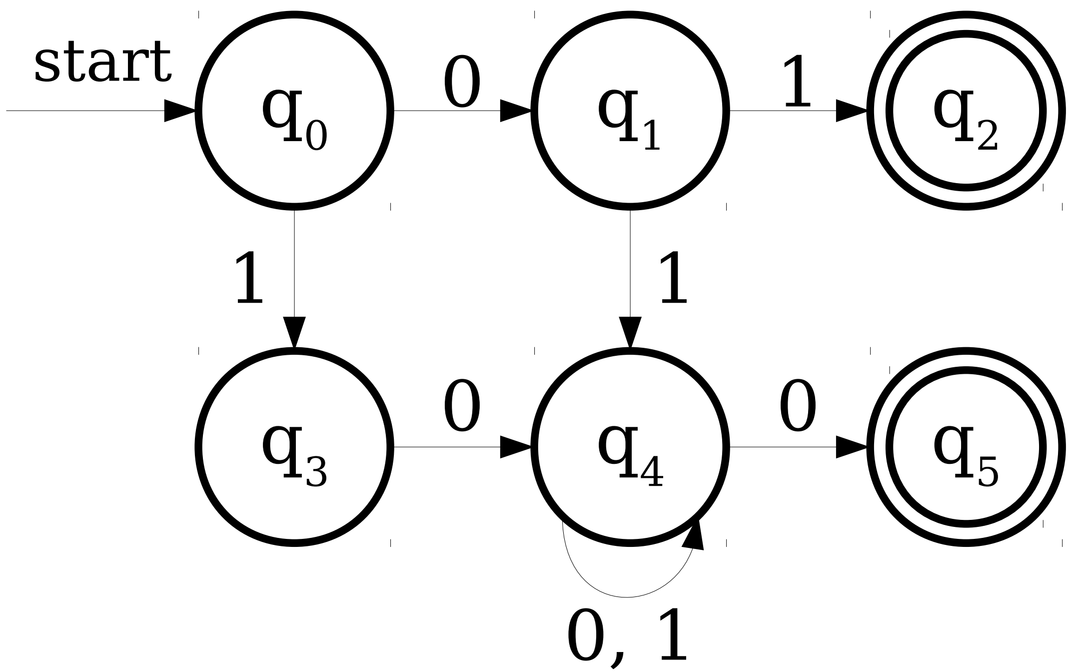


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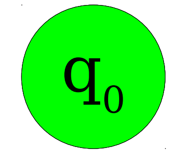


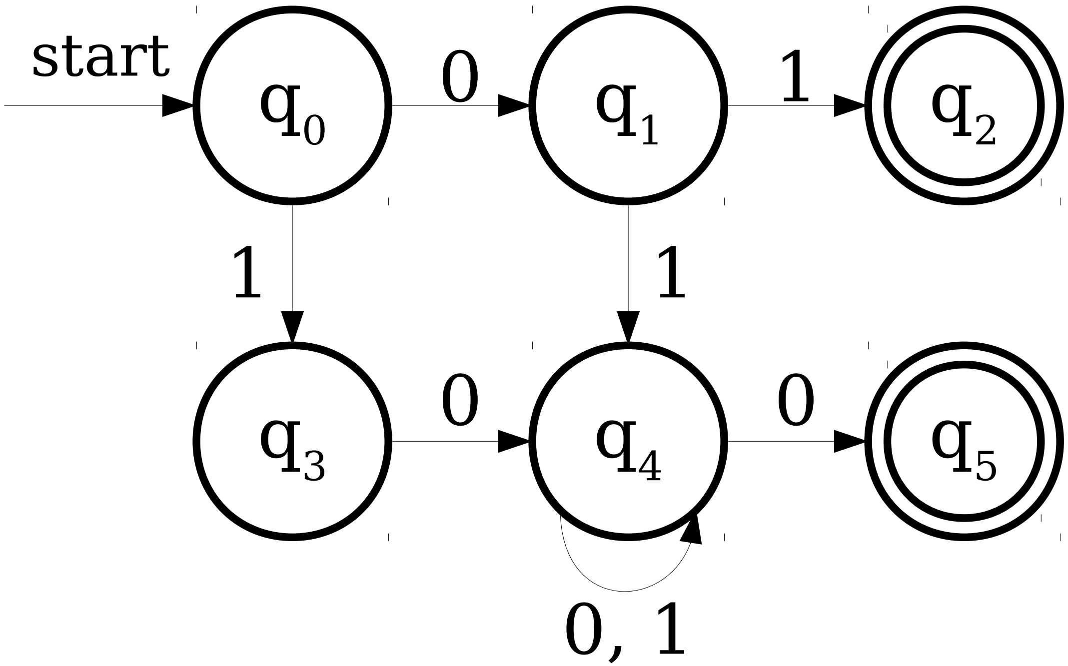


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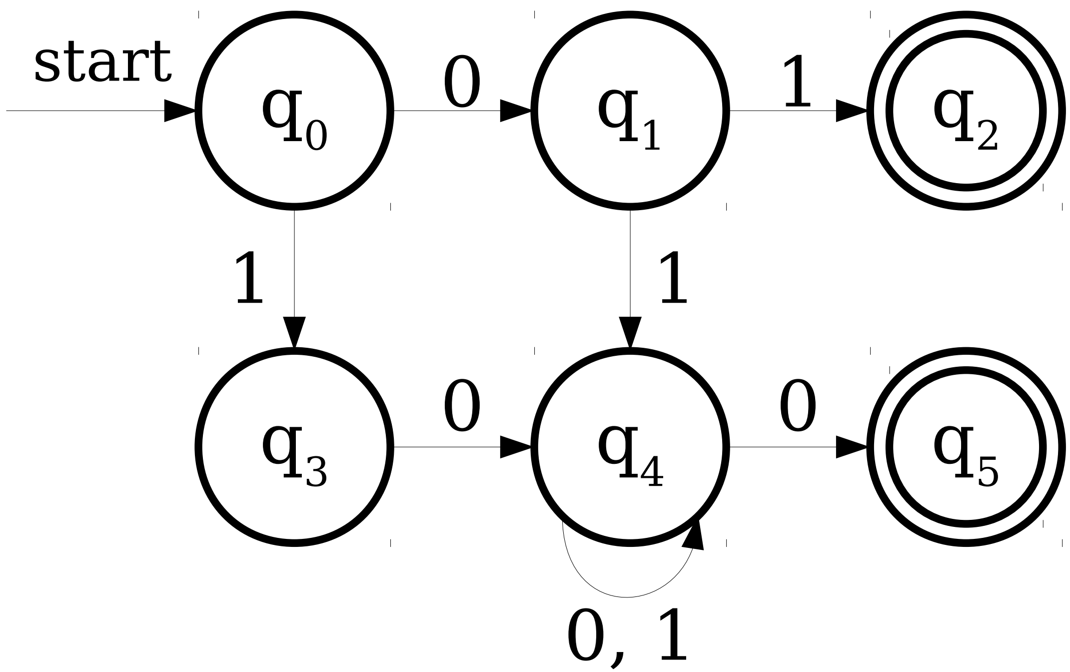


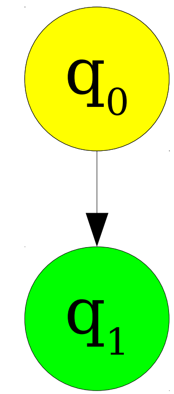


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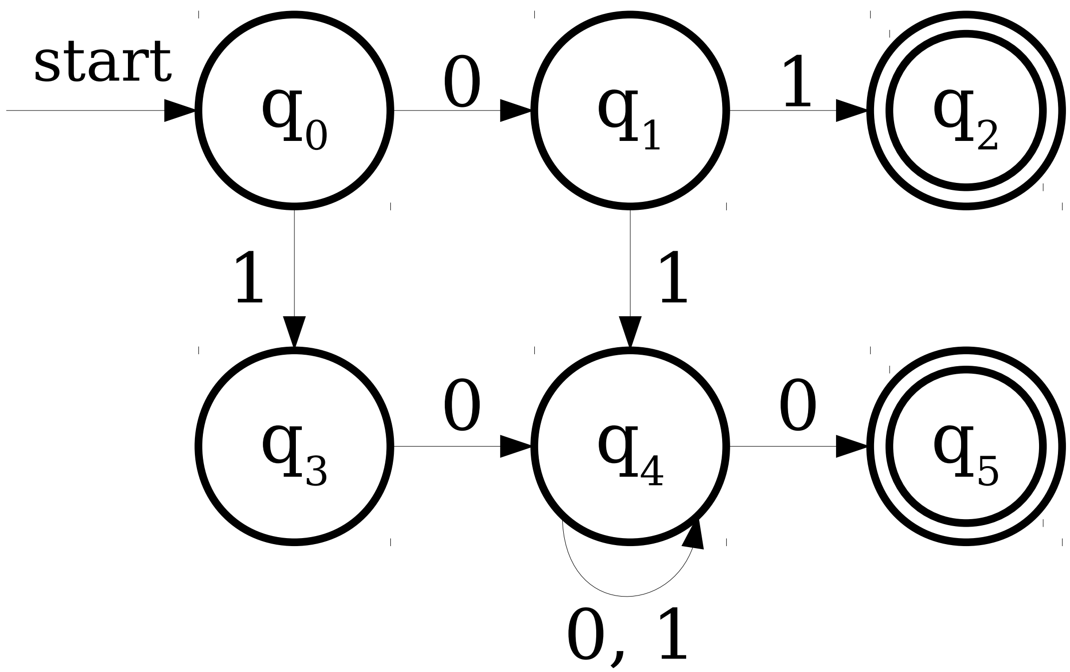


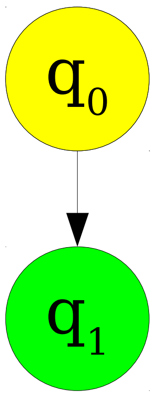


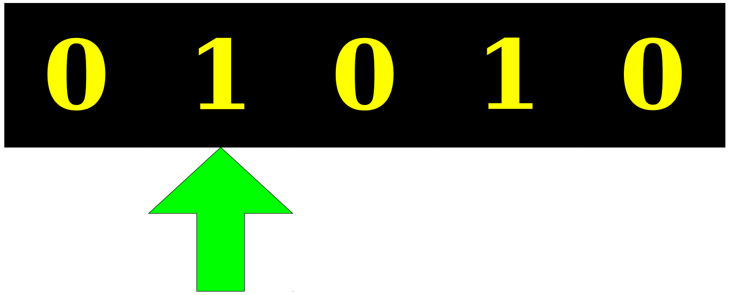

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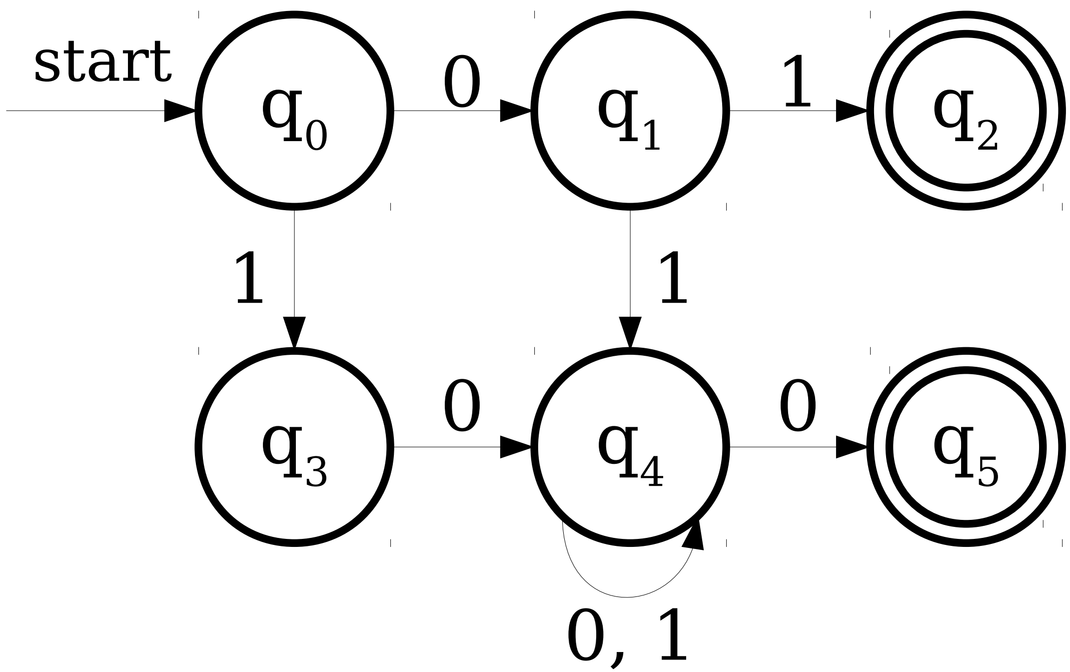


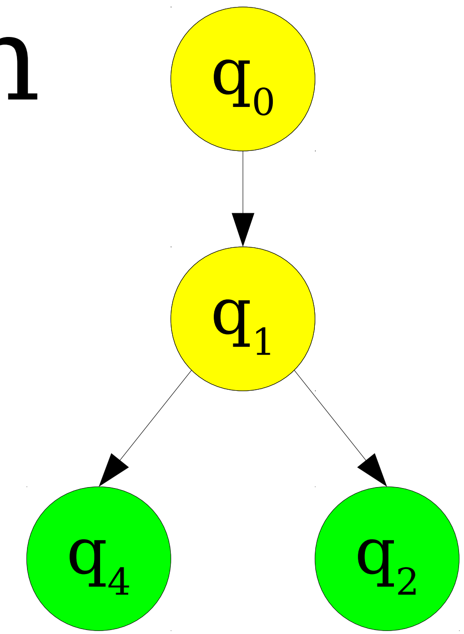


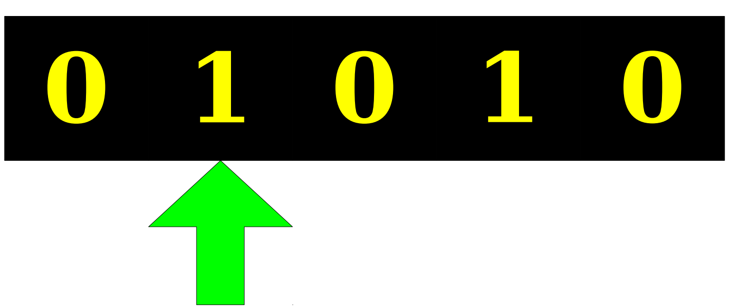

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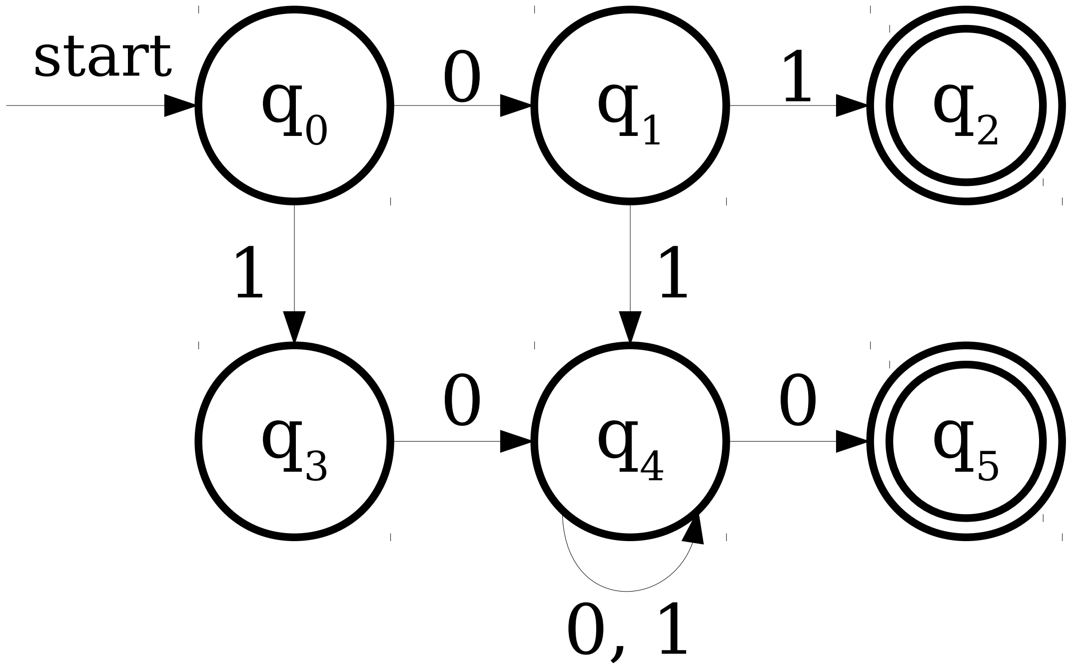


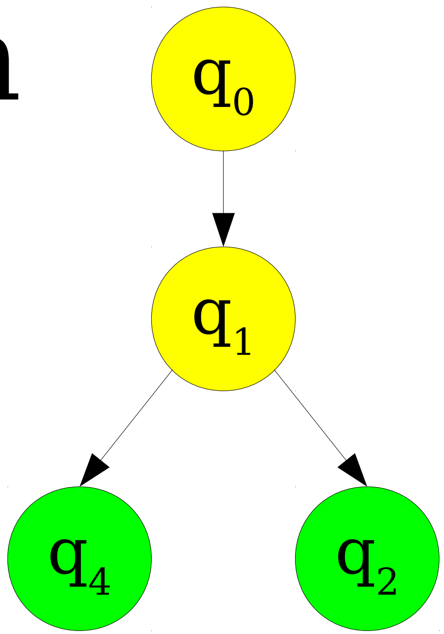


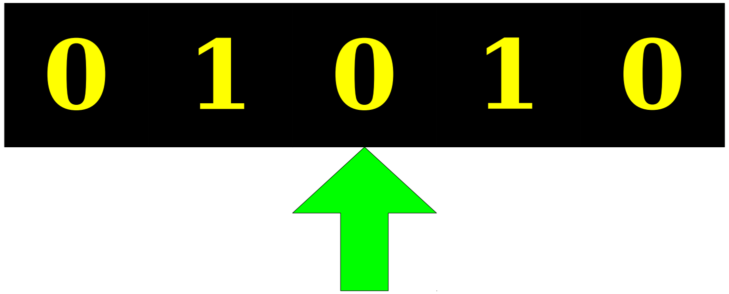

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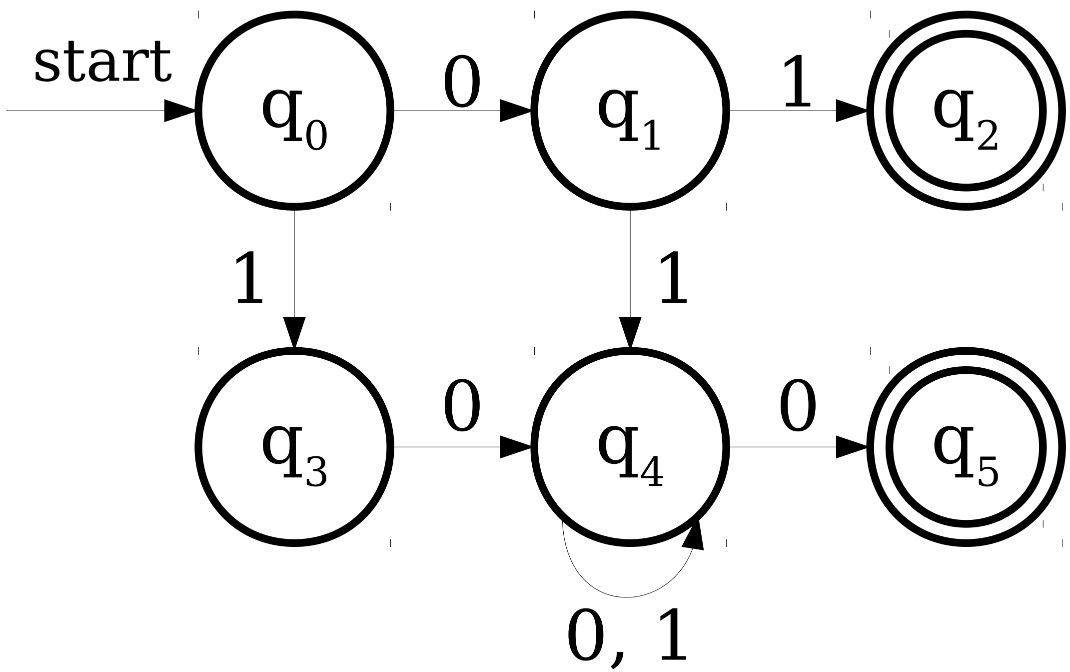


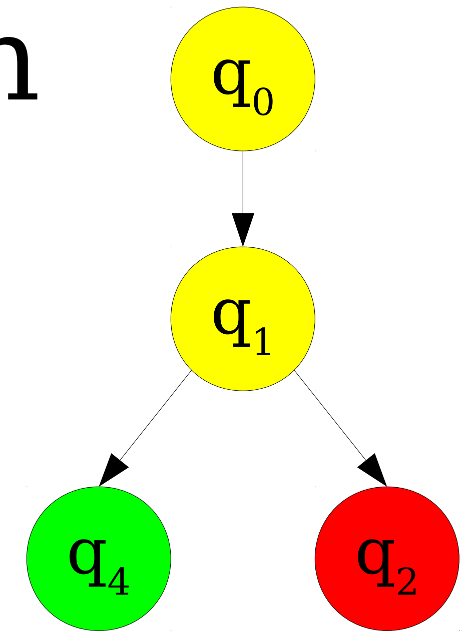


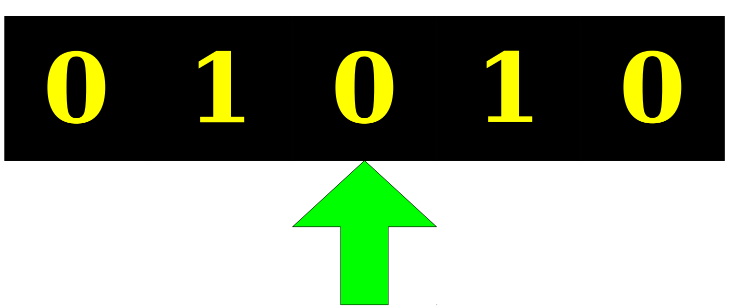

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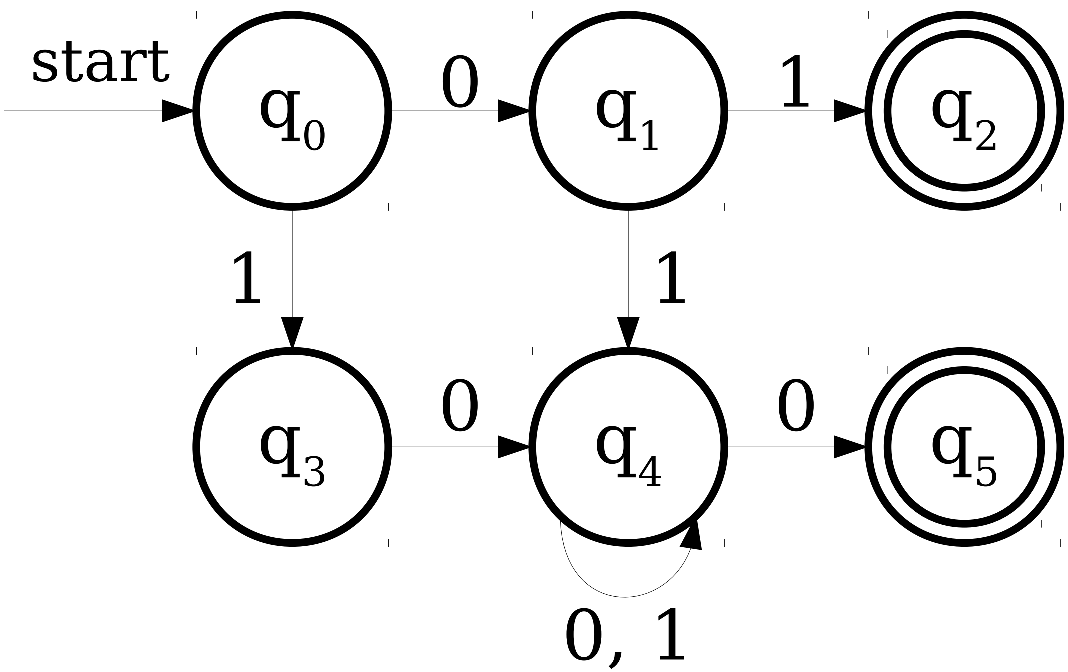


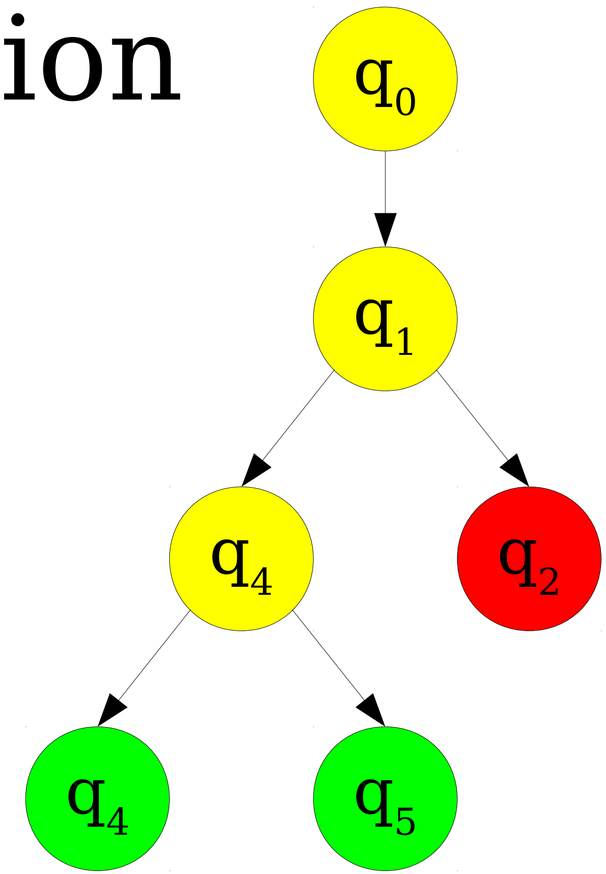


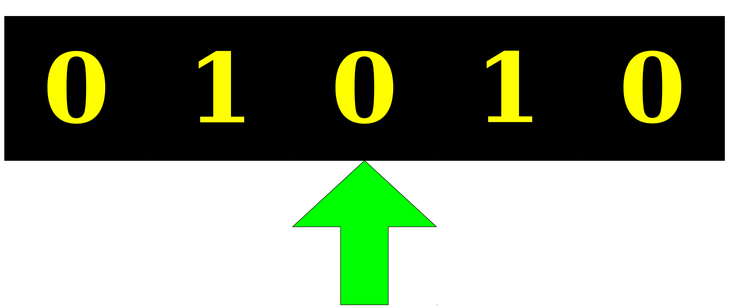

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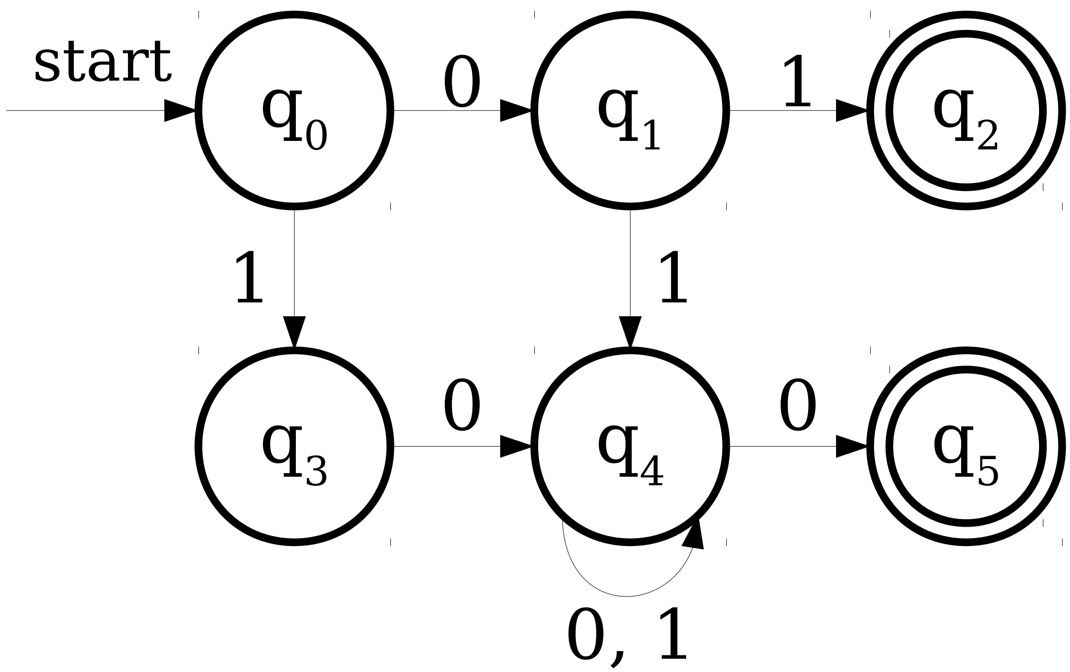


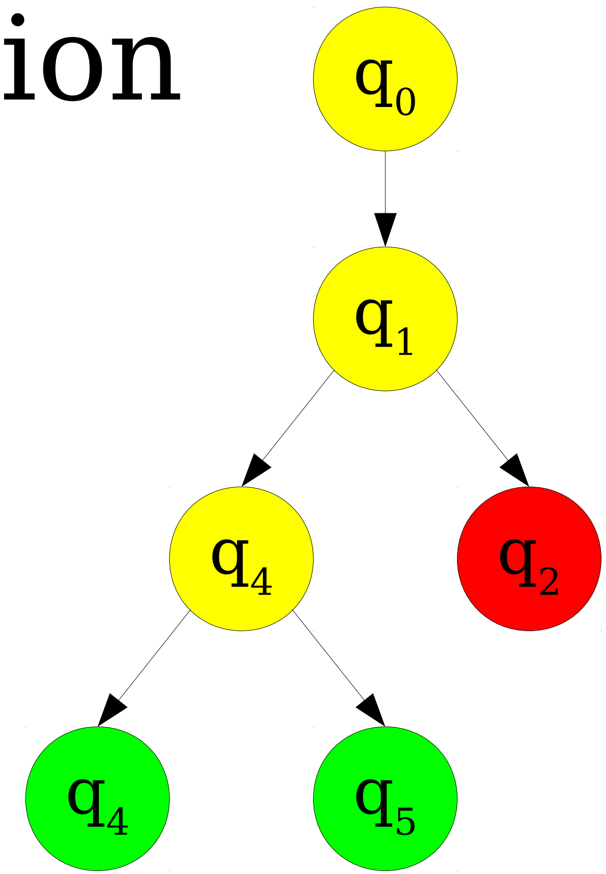


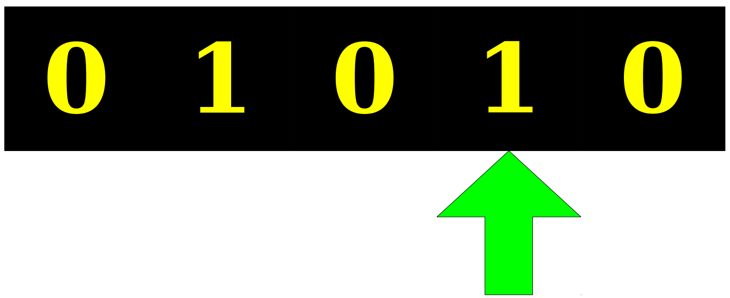

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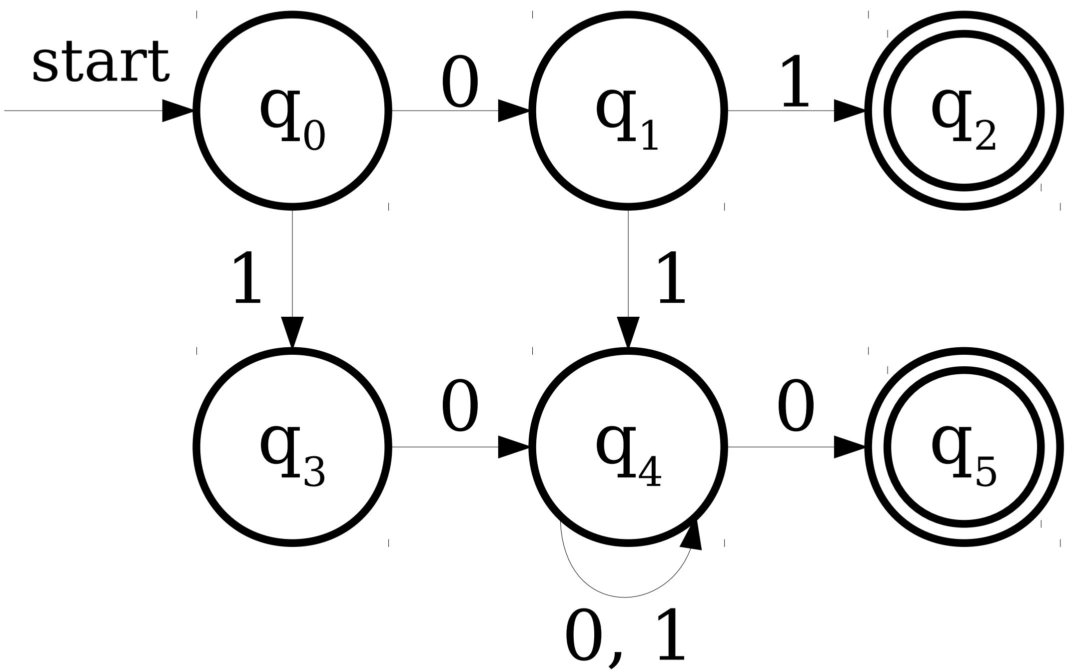


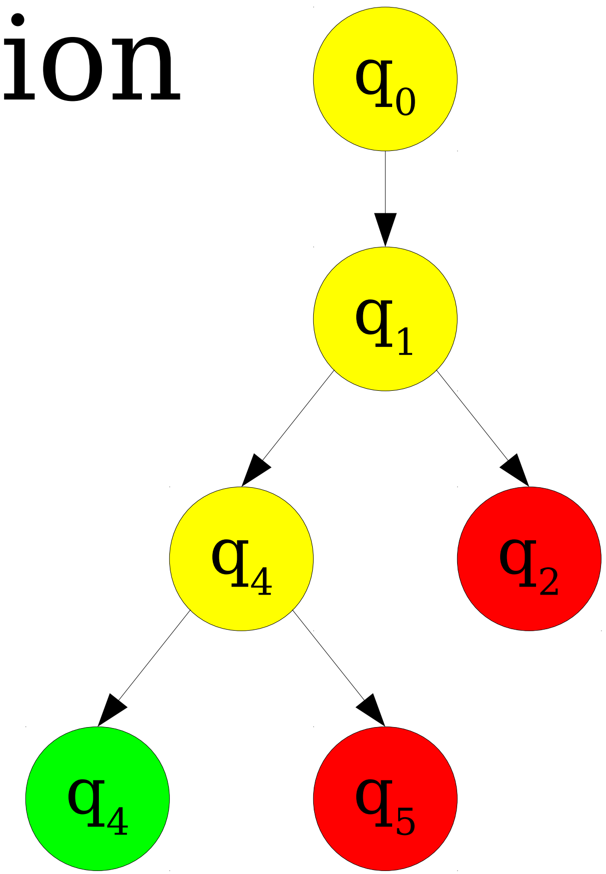


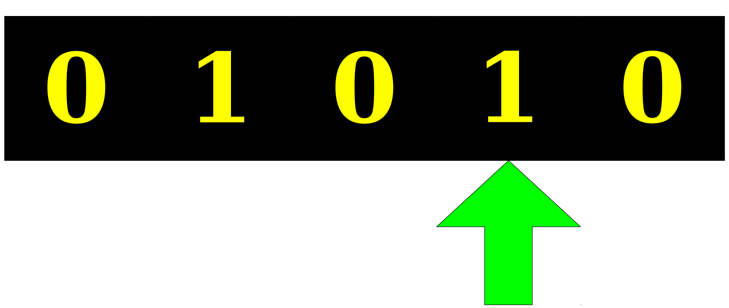

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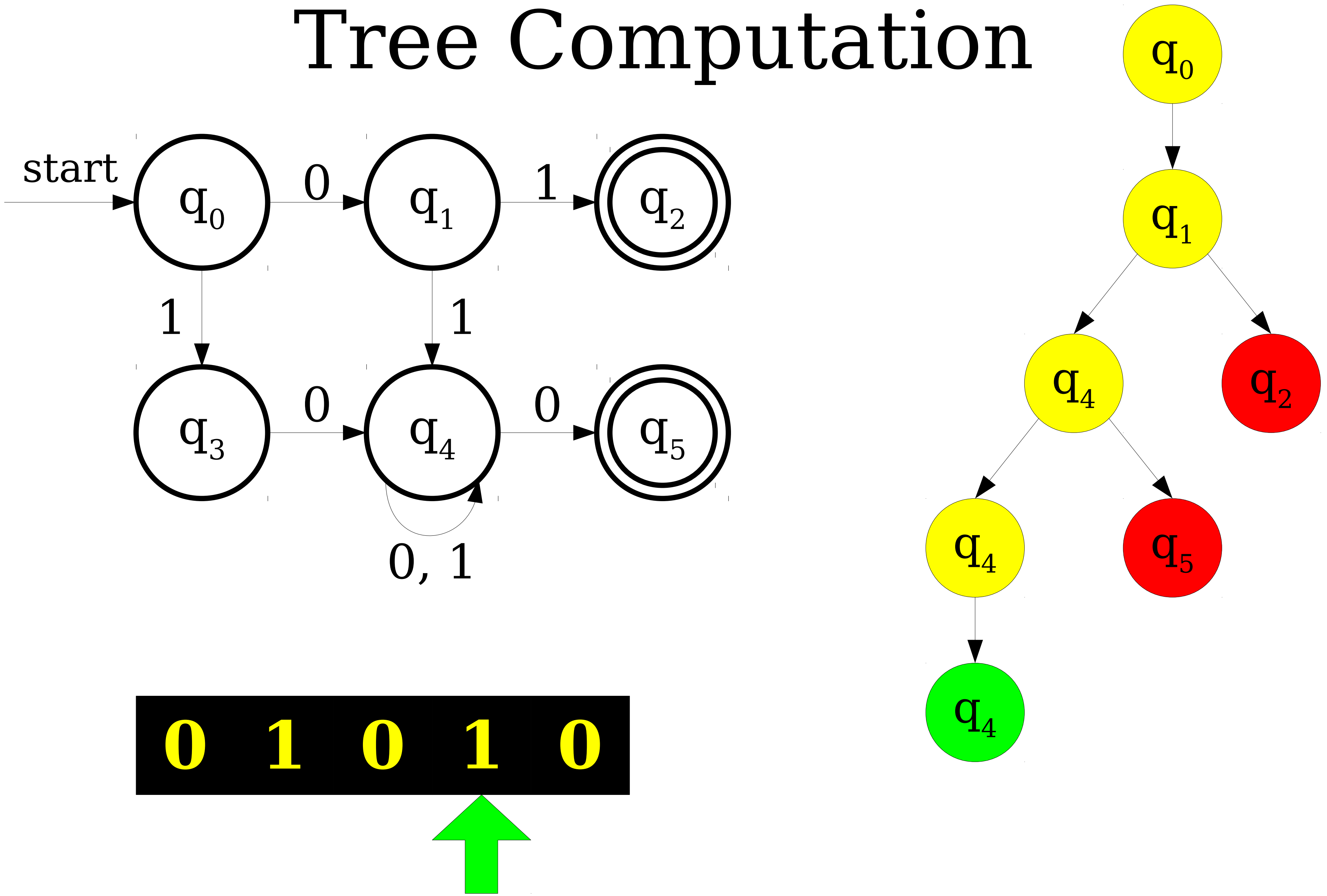

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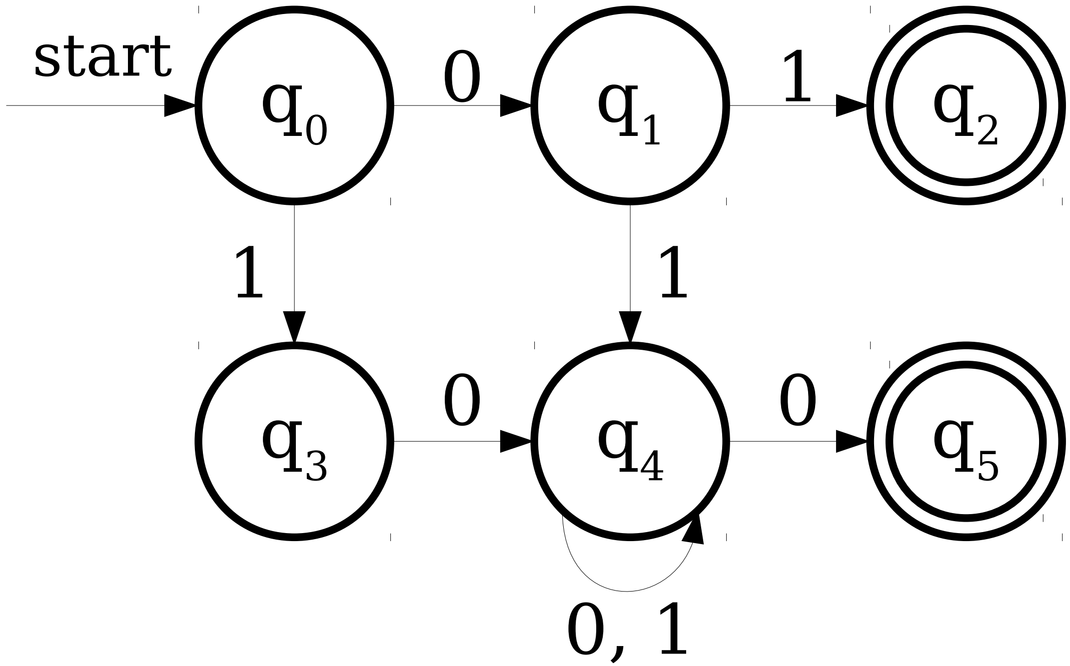



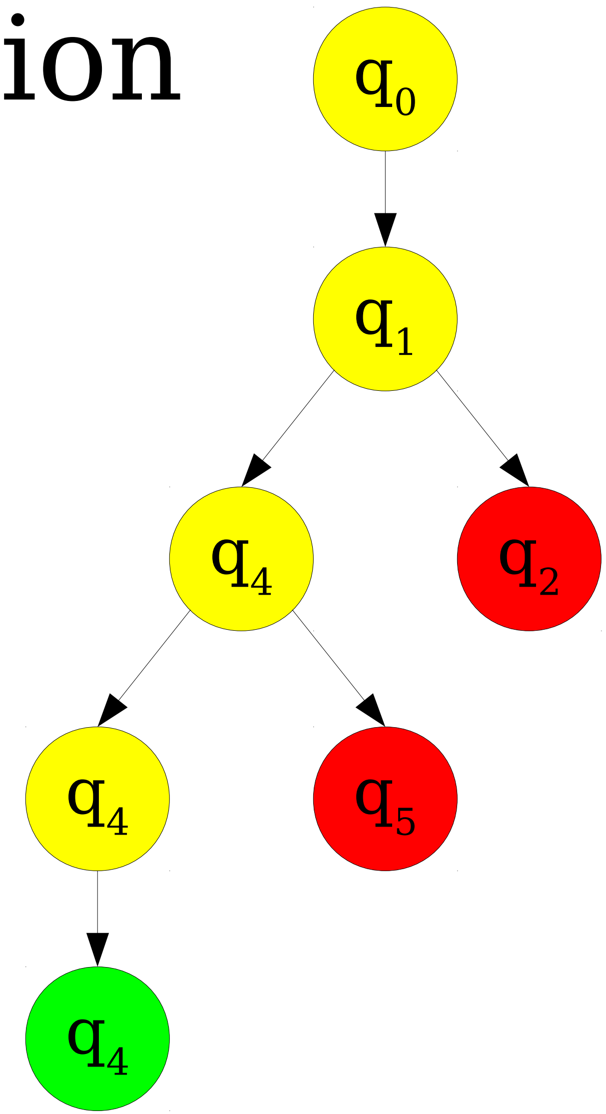

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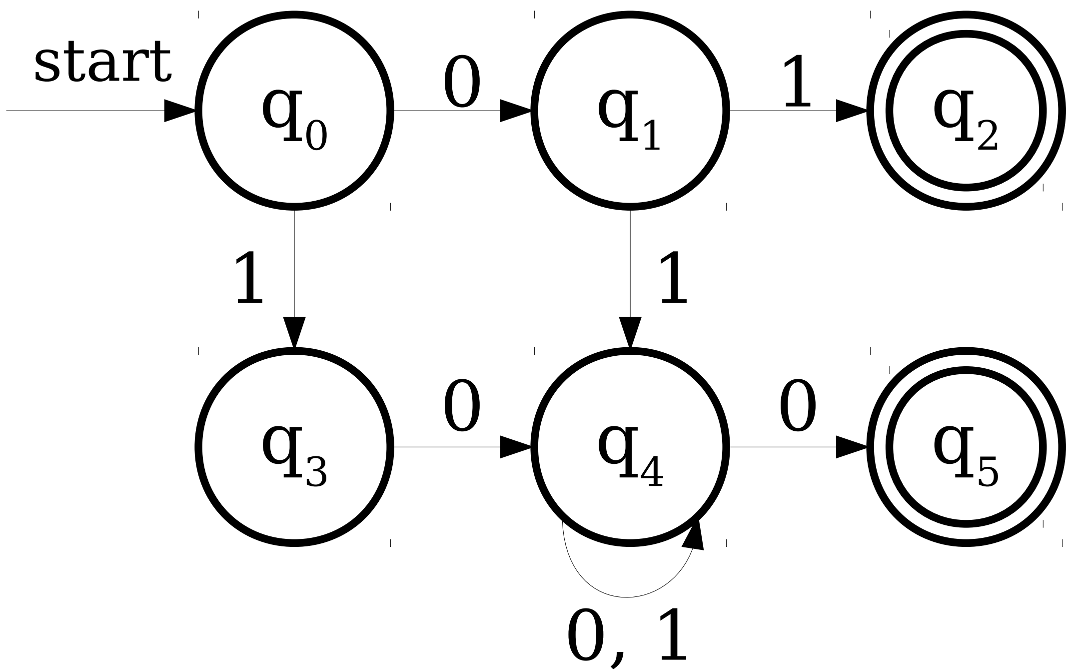



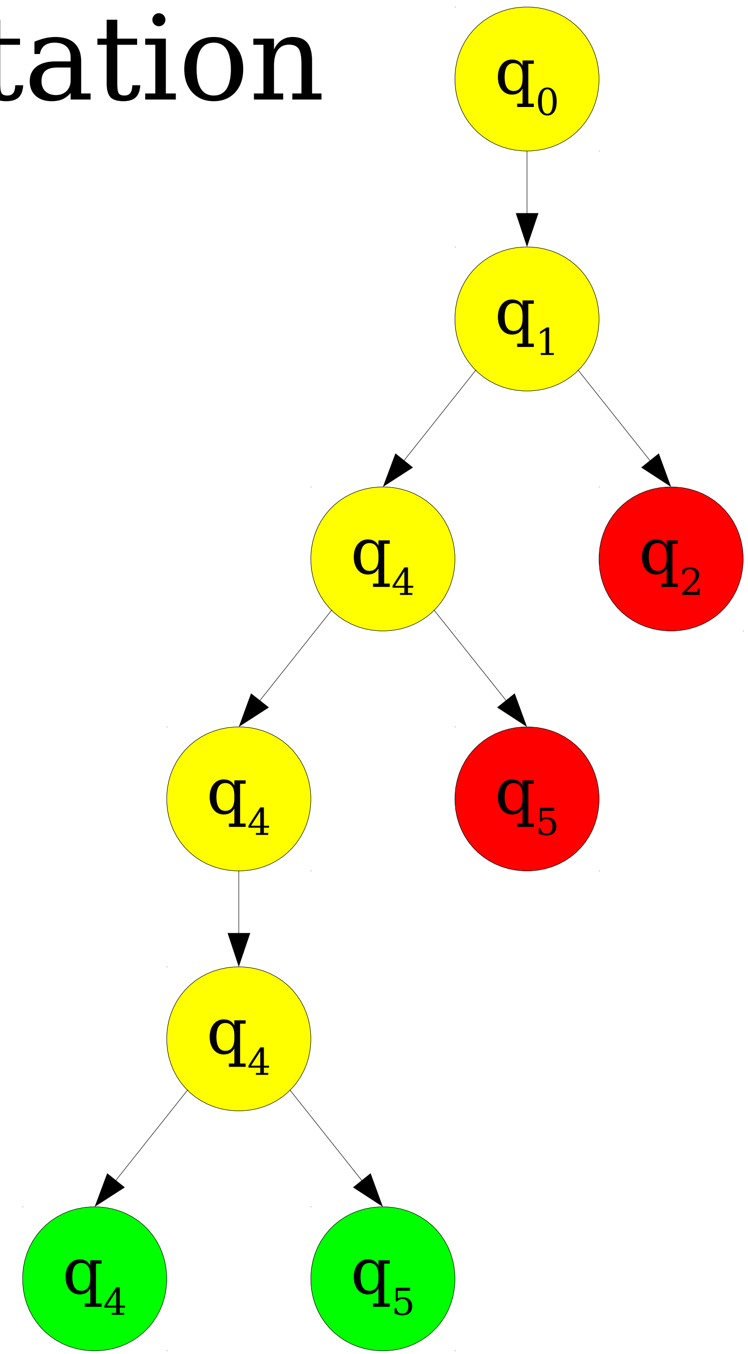

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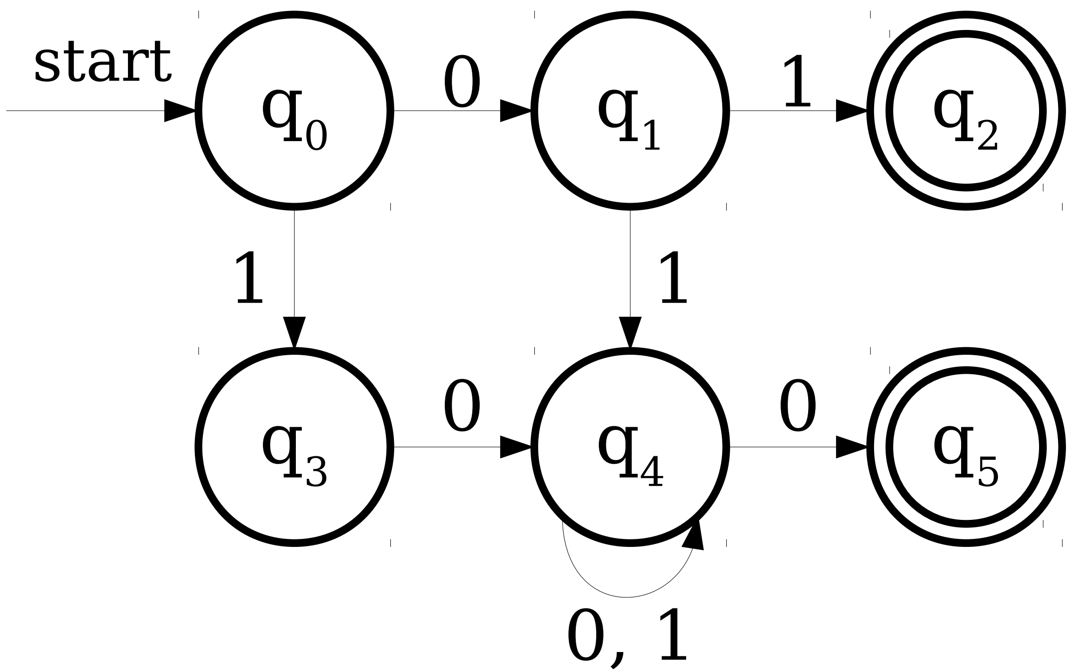


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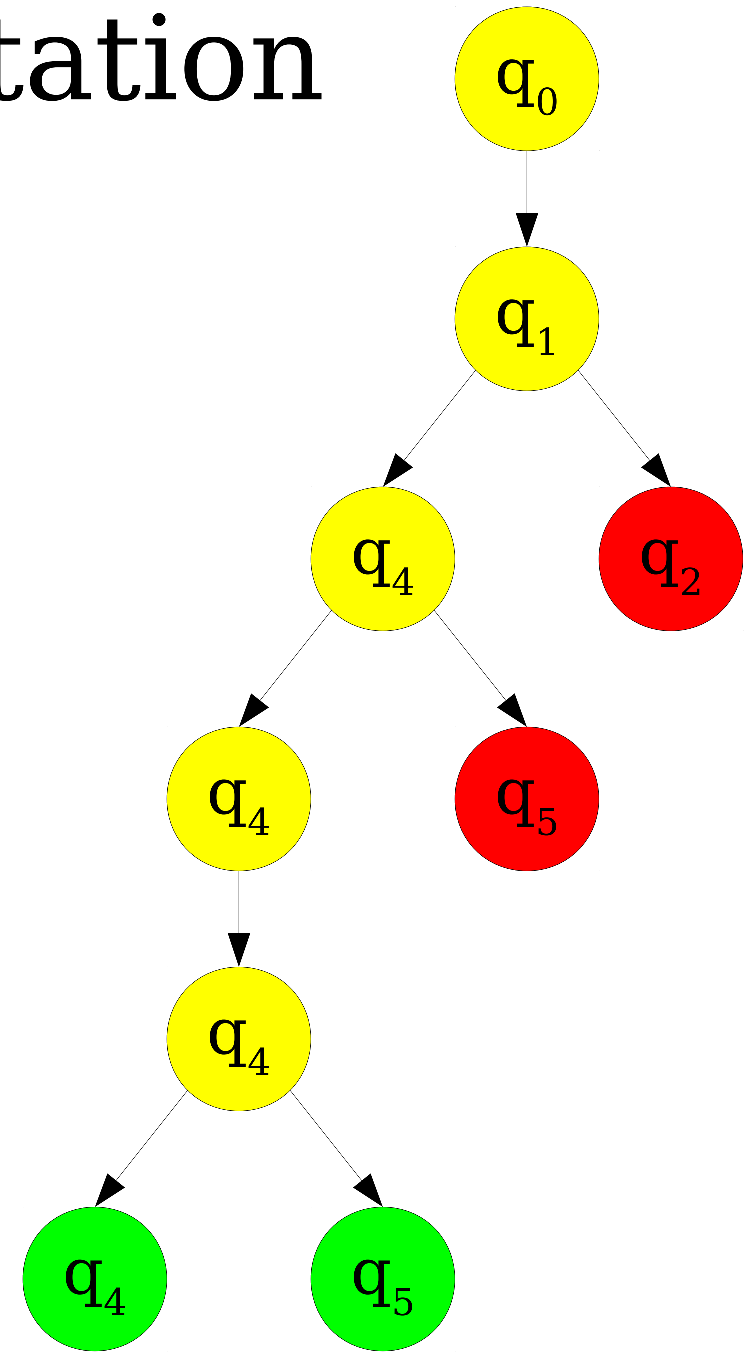

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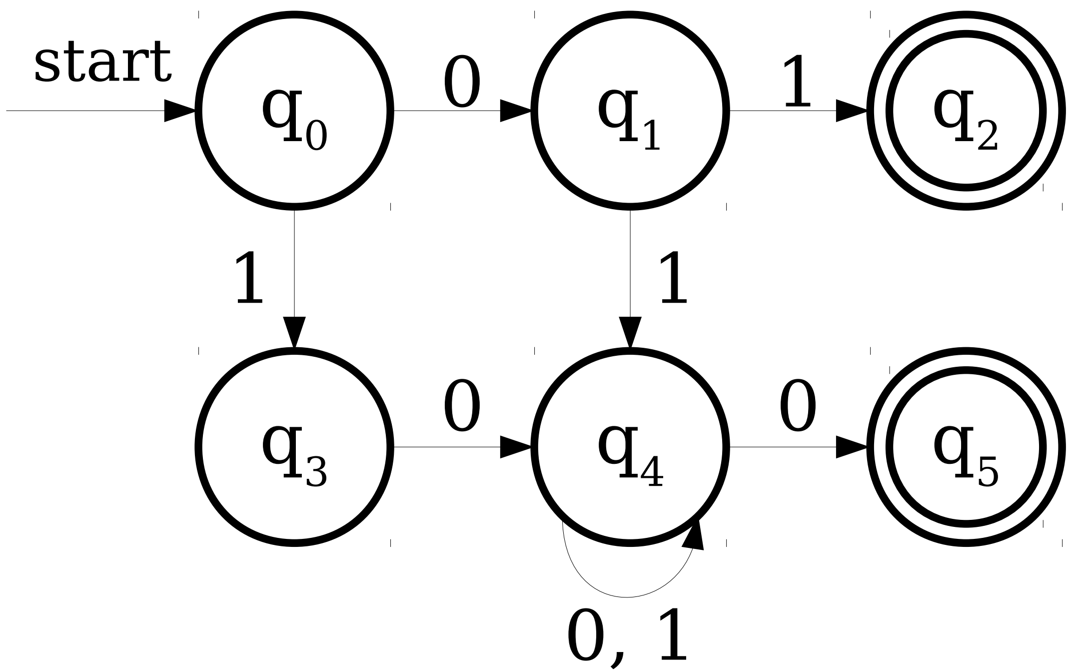


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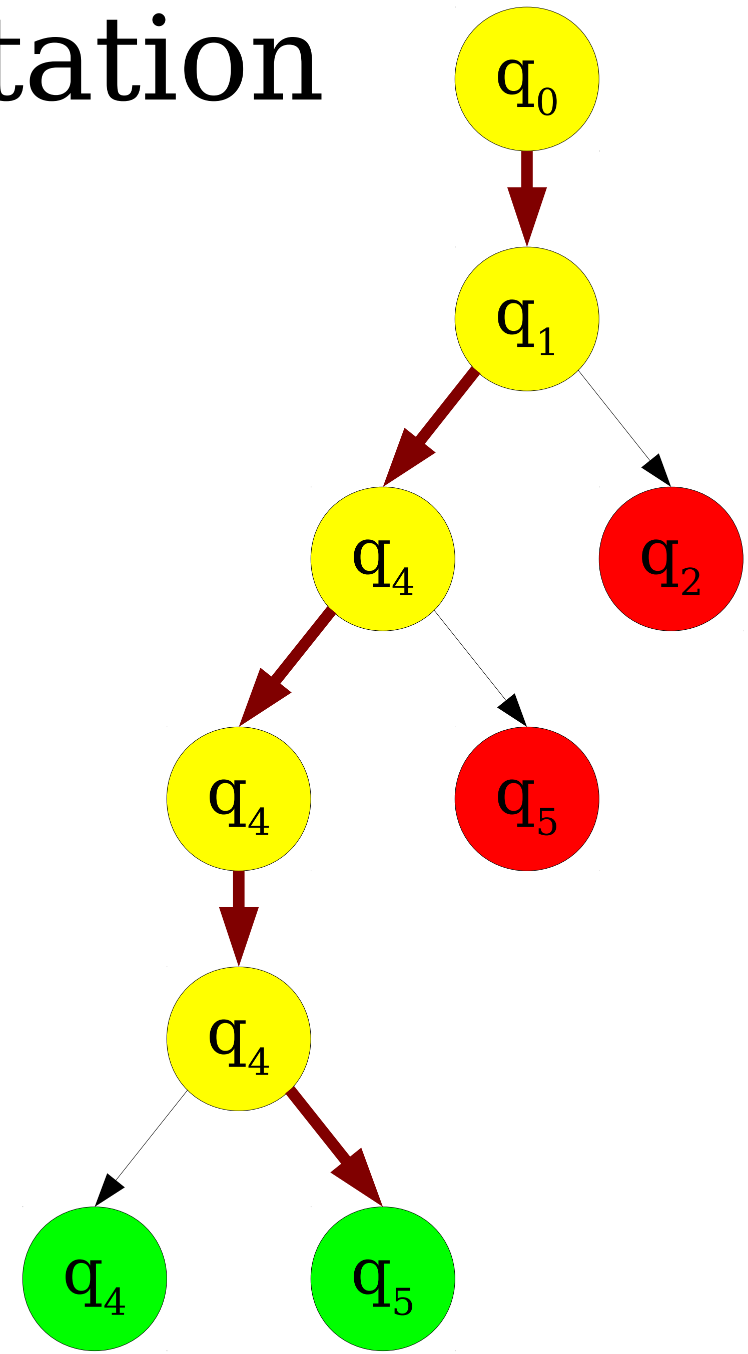

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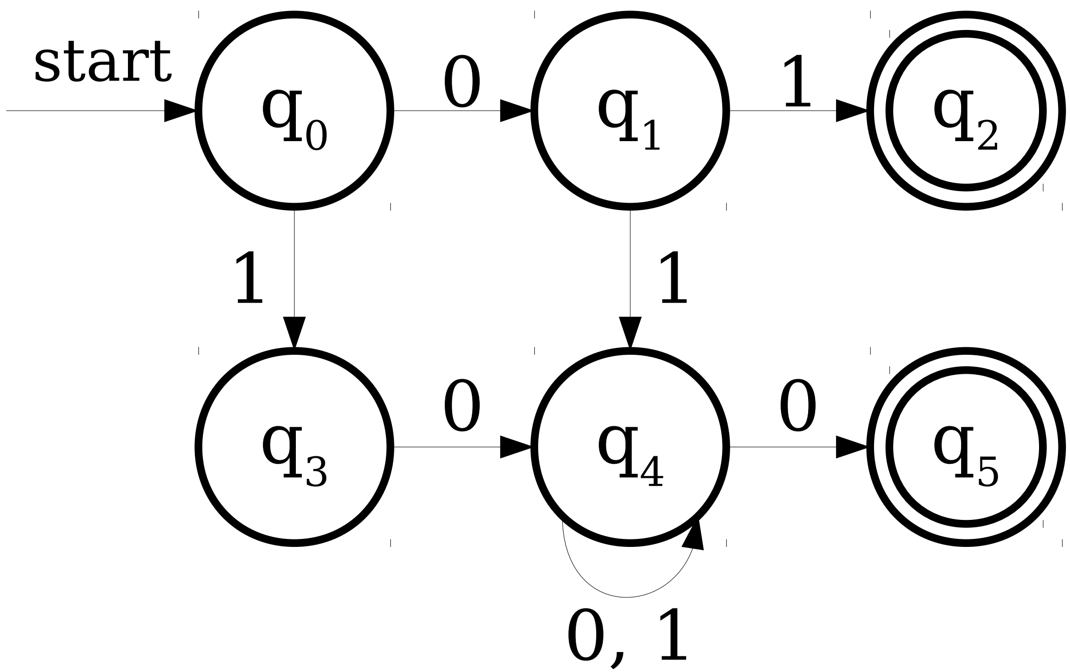


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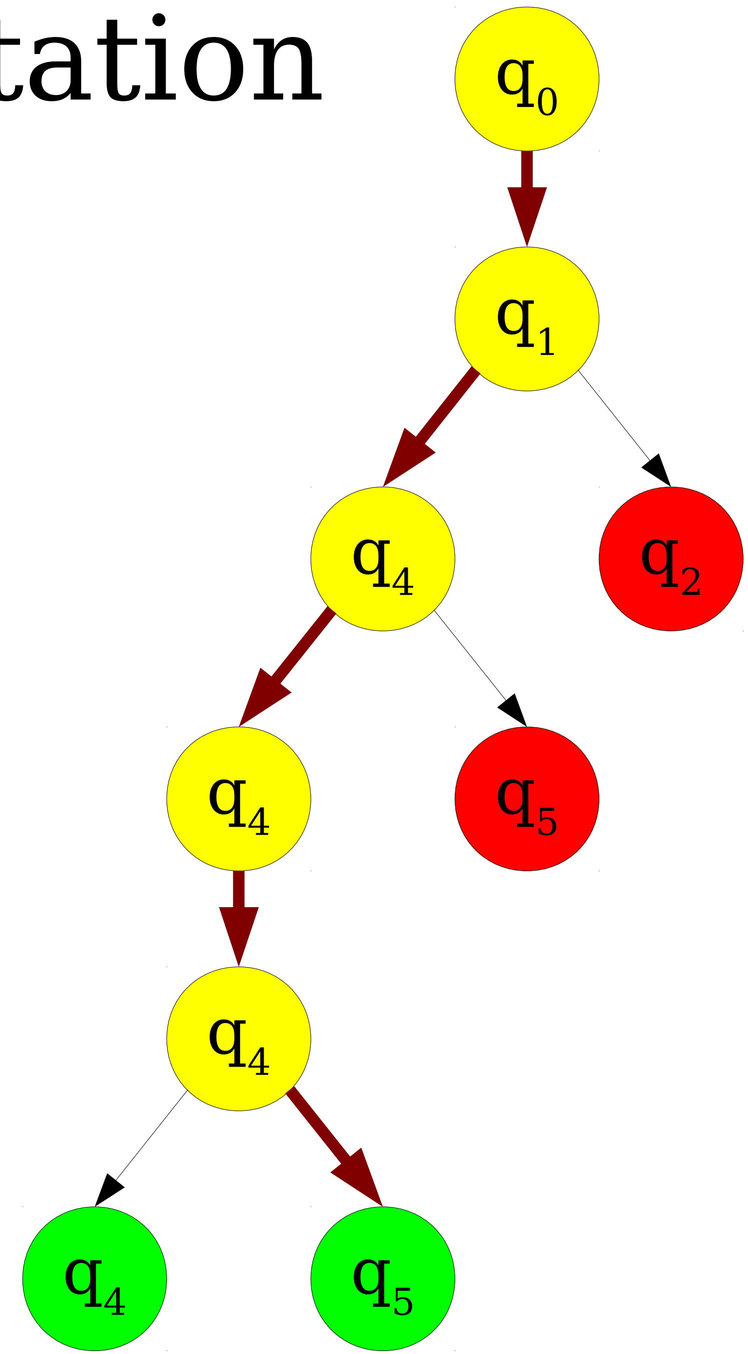
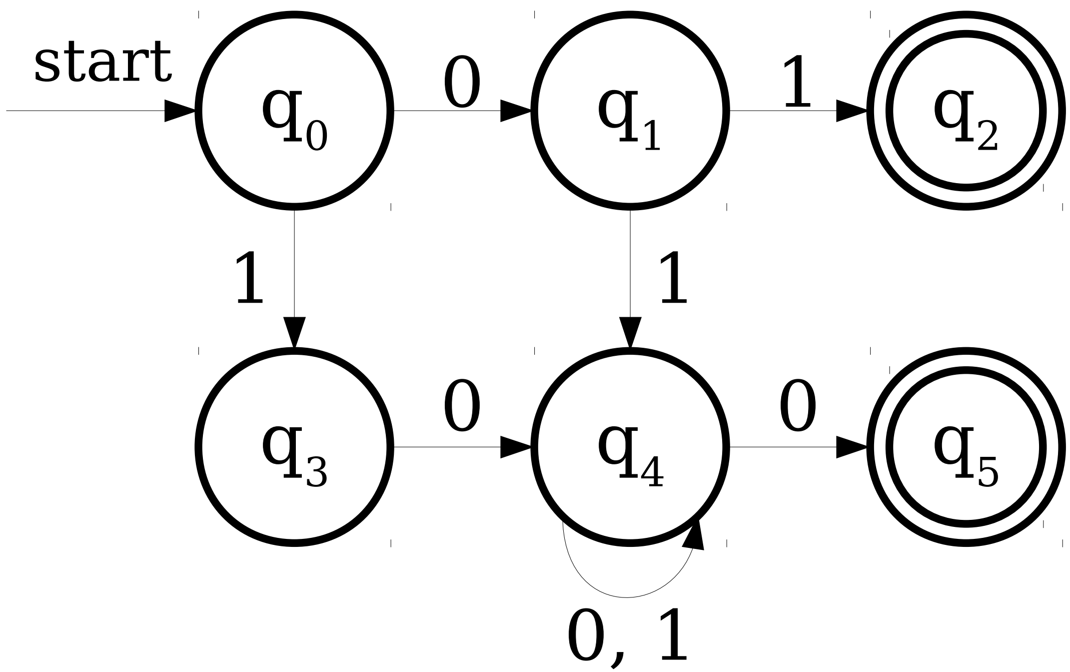


# Nondeterminism as a Tree

- At each decision point, the automaton clones itself for each possible decision.
- The series of choices forms a directed, rooted tree.
- At the end, if any active accepting states remain, we accept.

# Perfect Guessing



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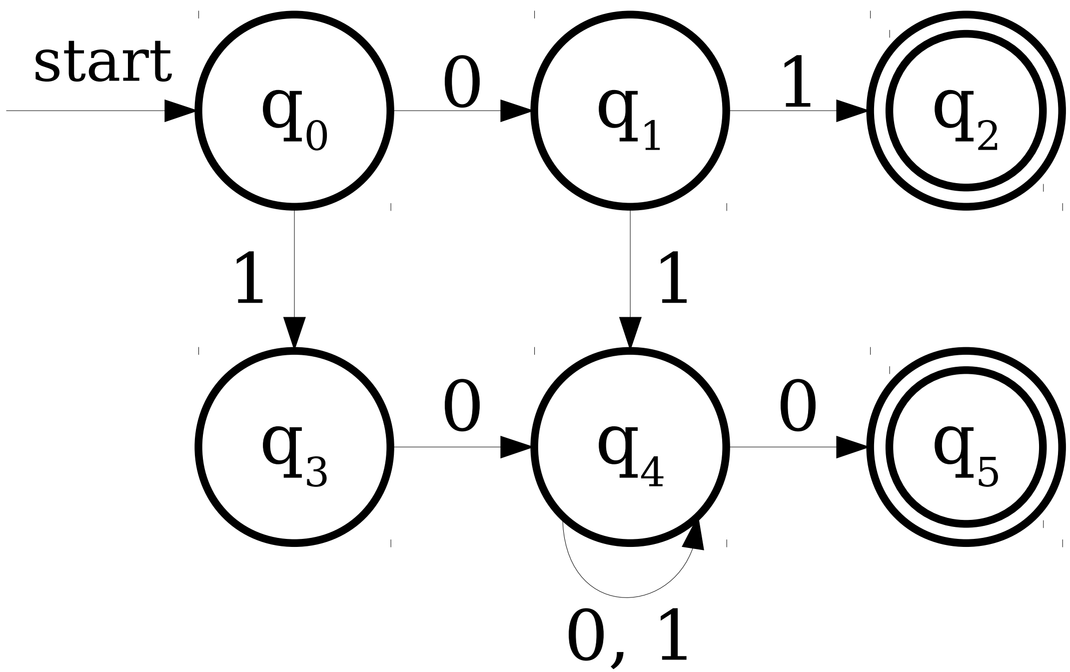


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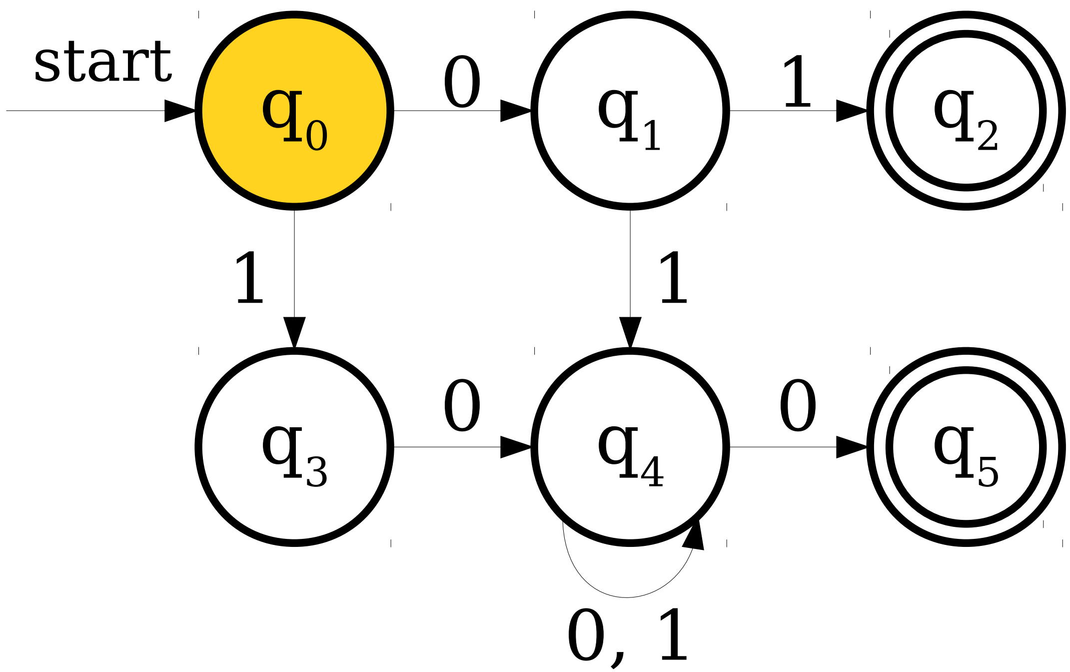


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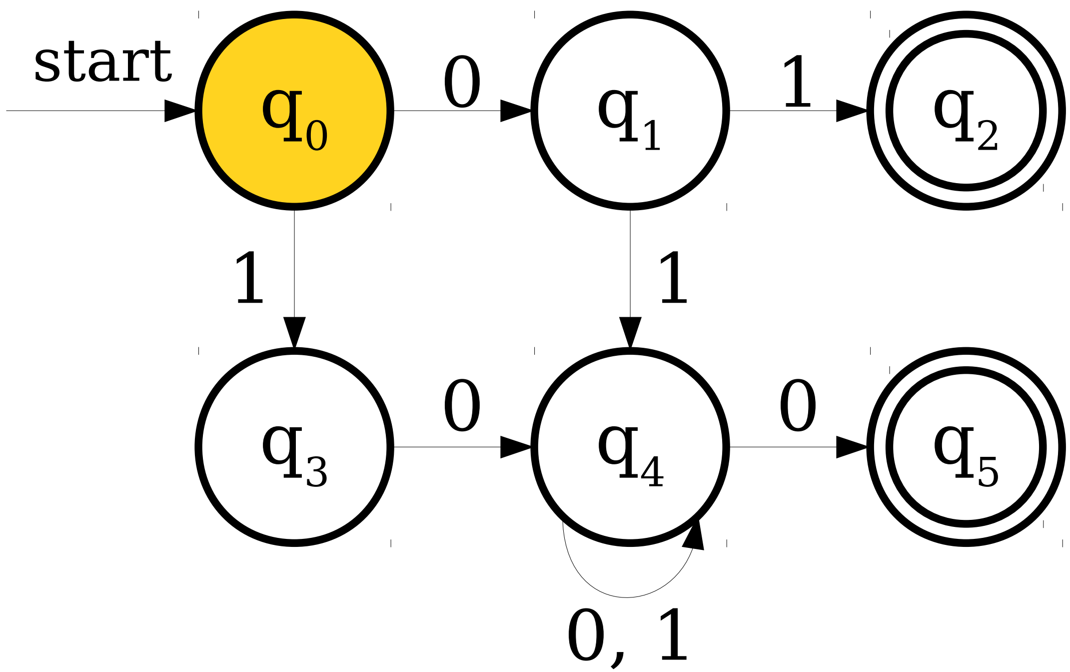


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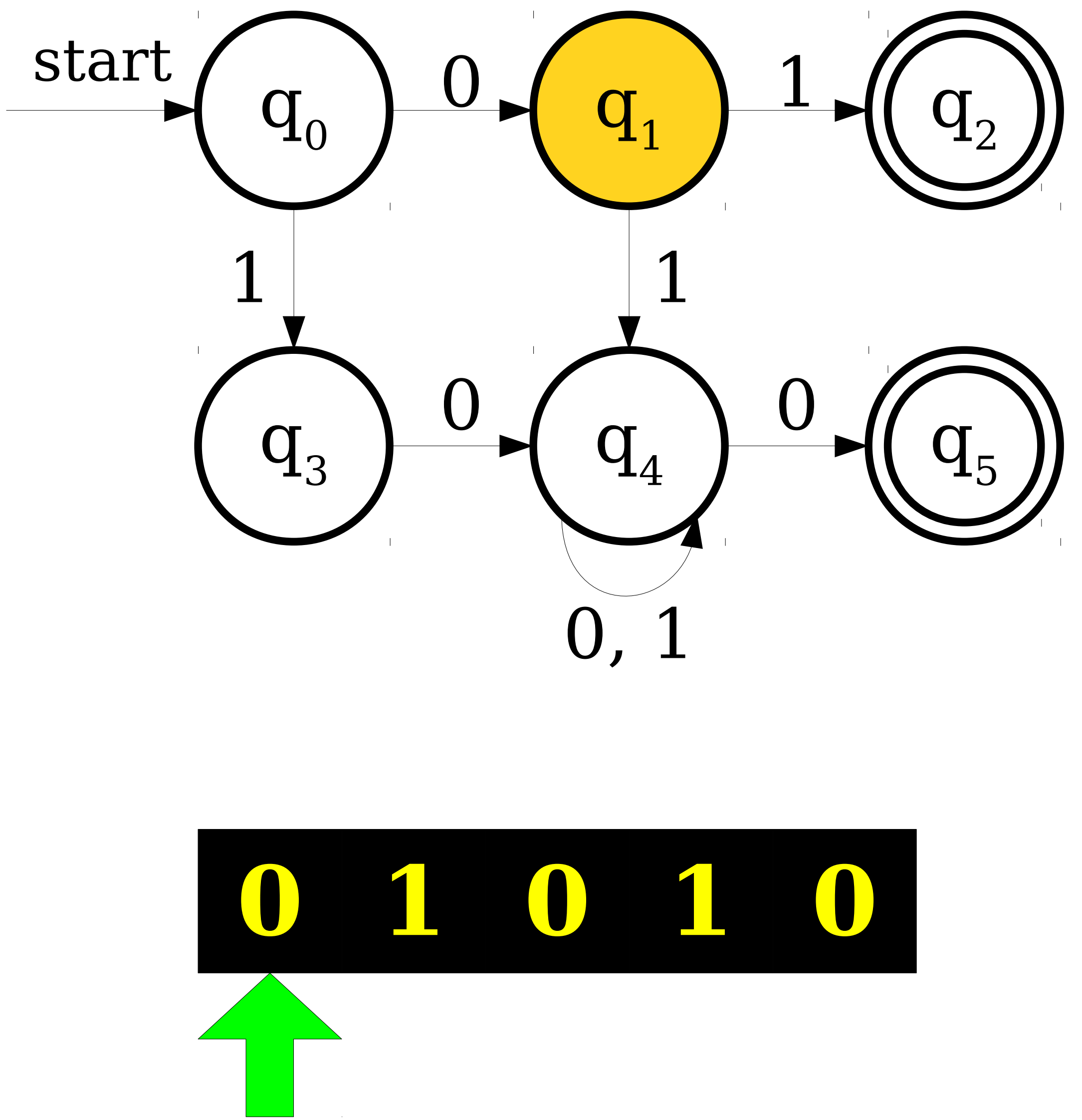

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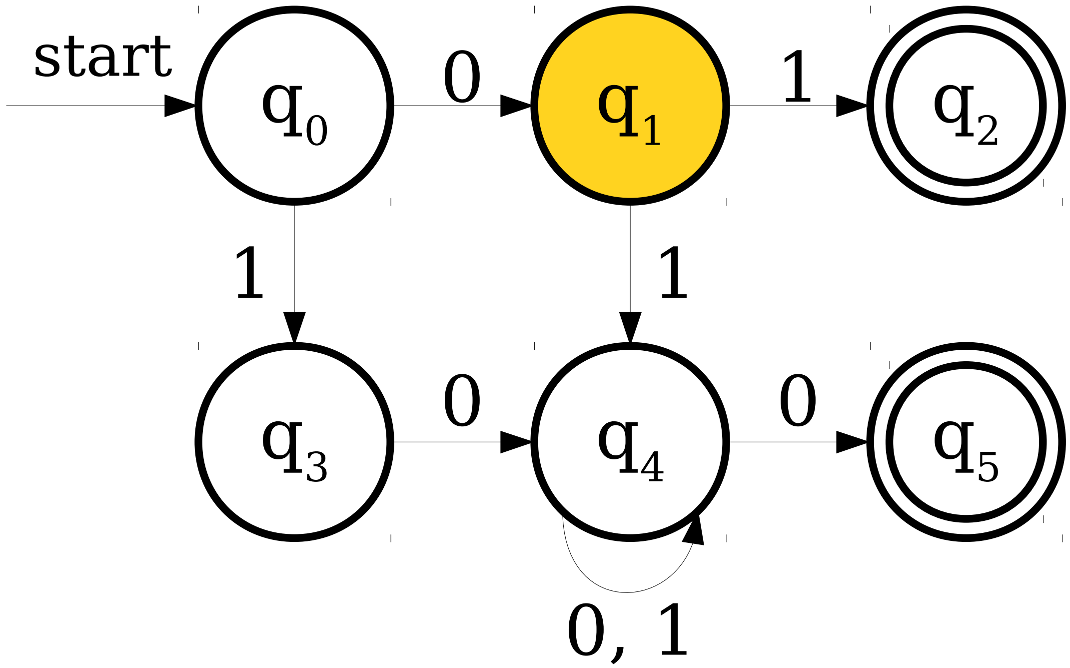


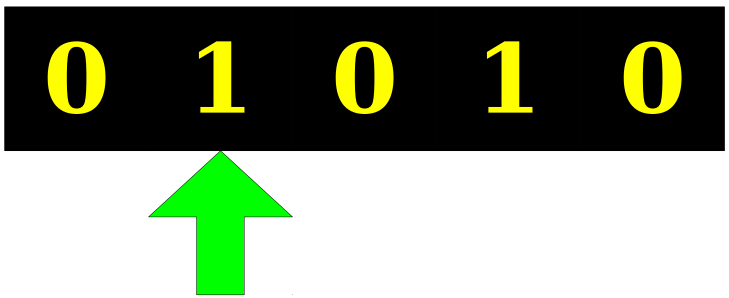

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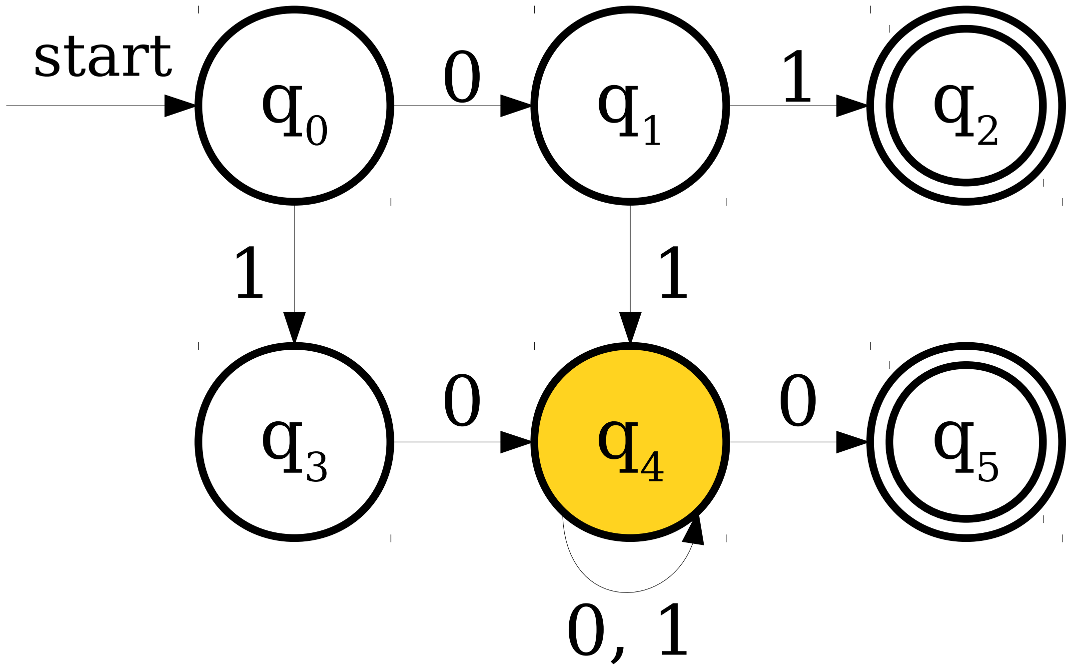


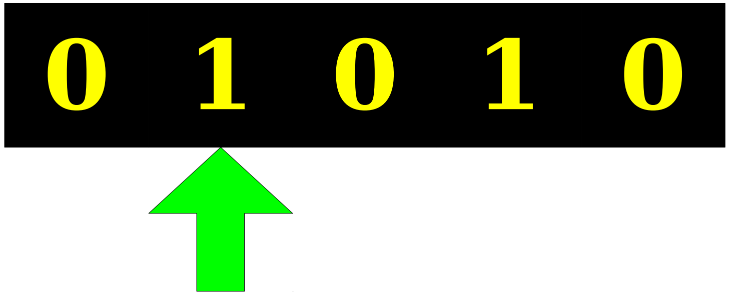

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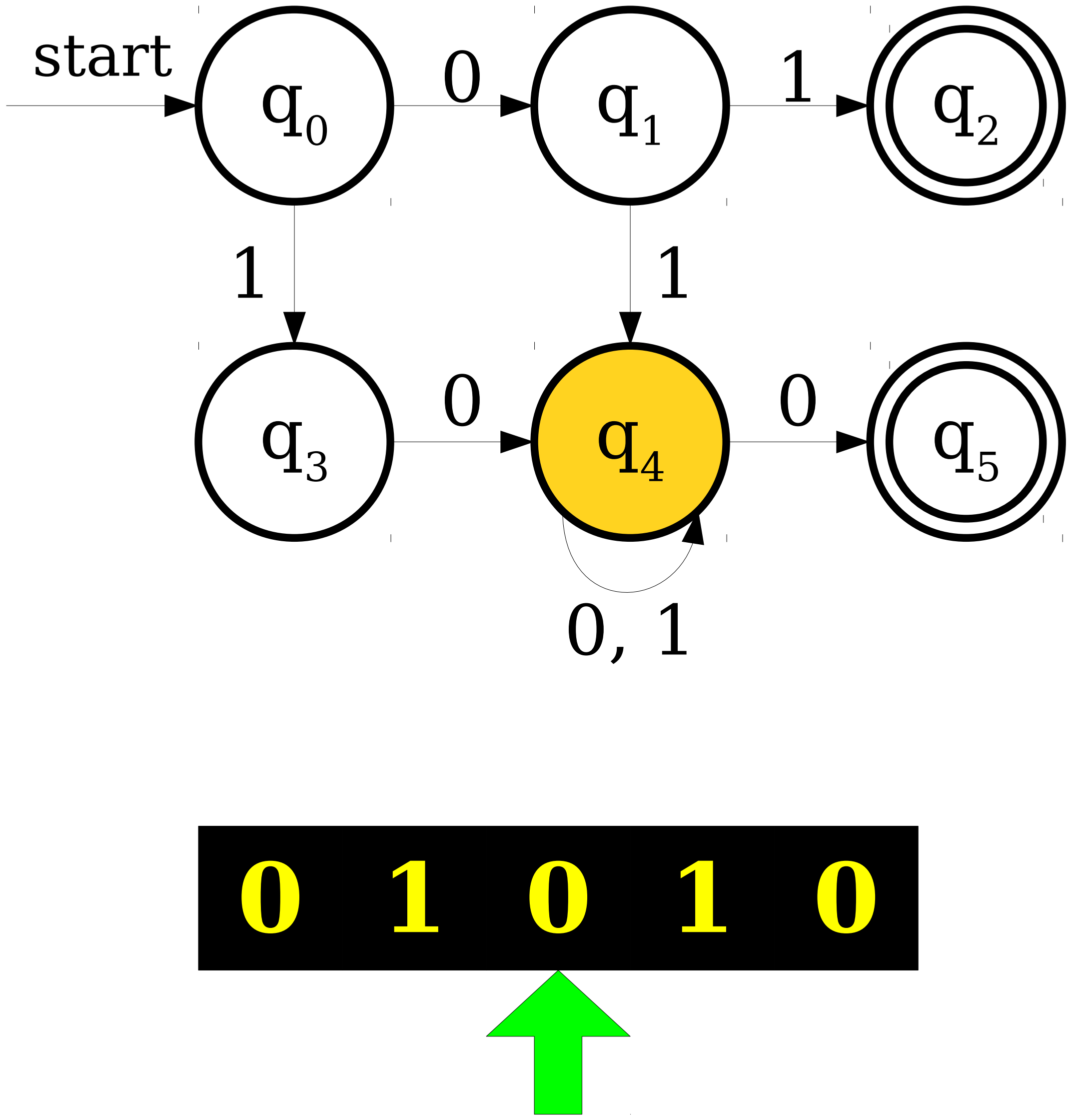

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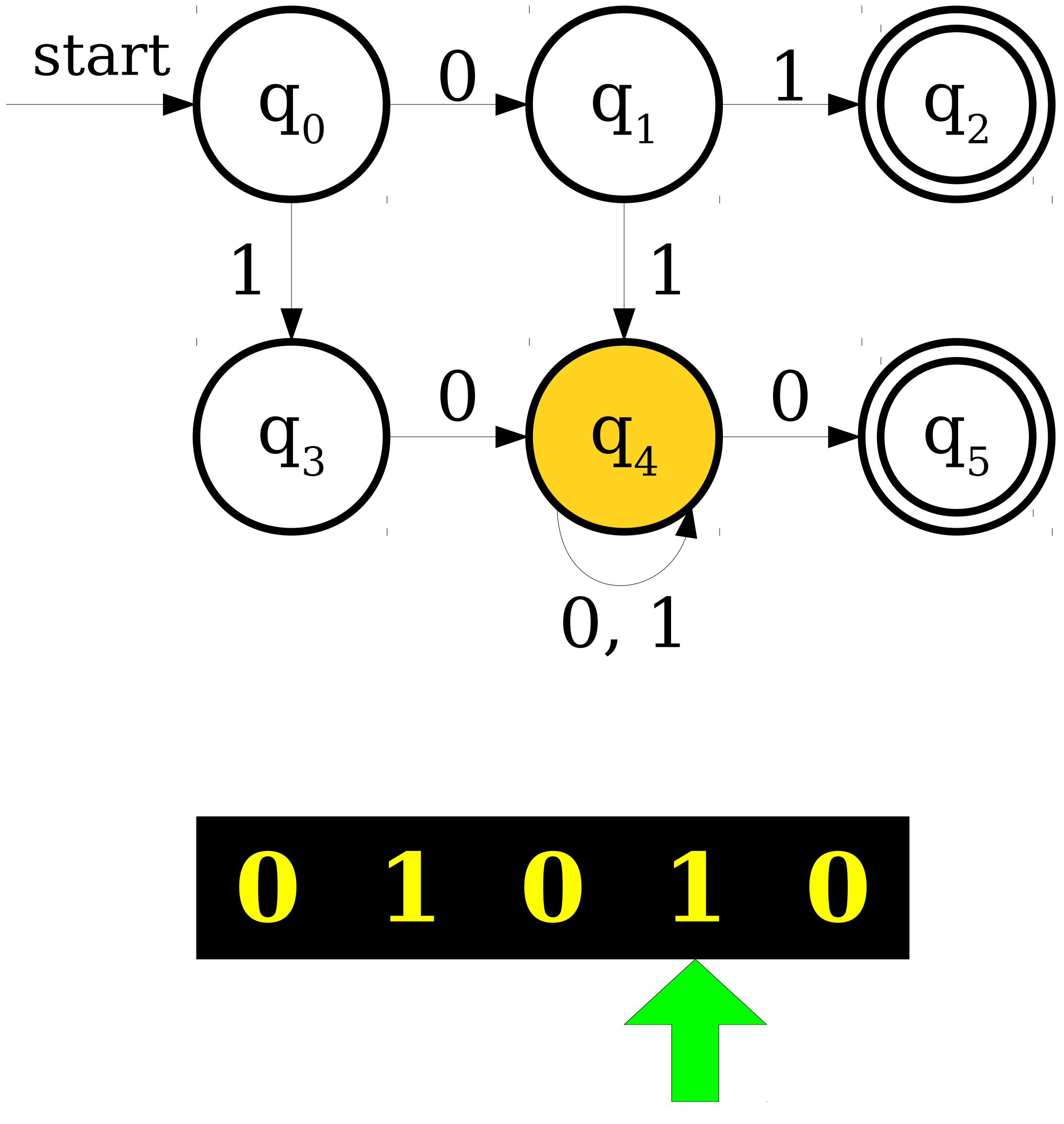

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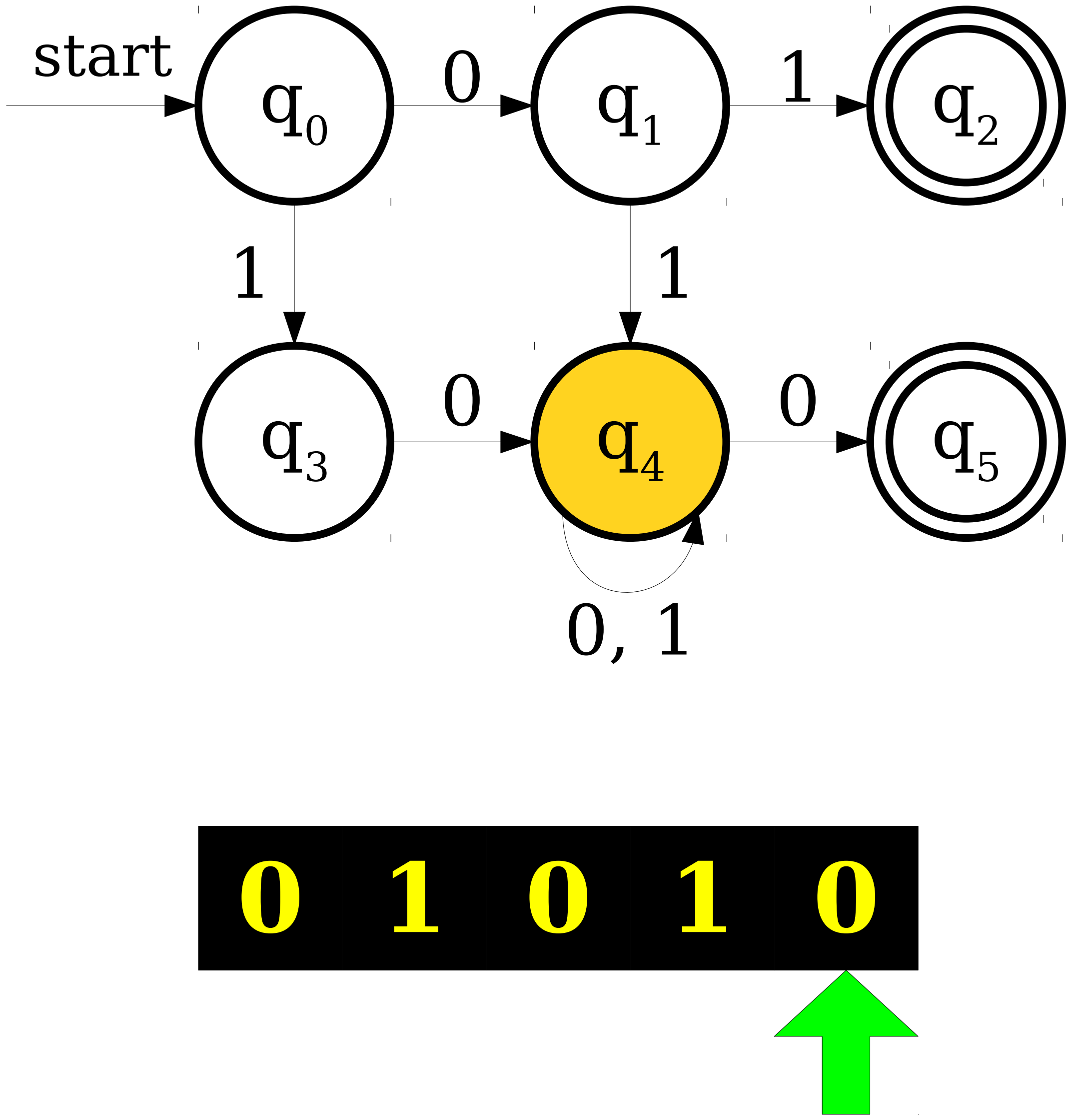

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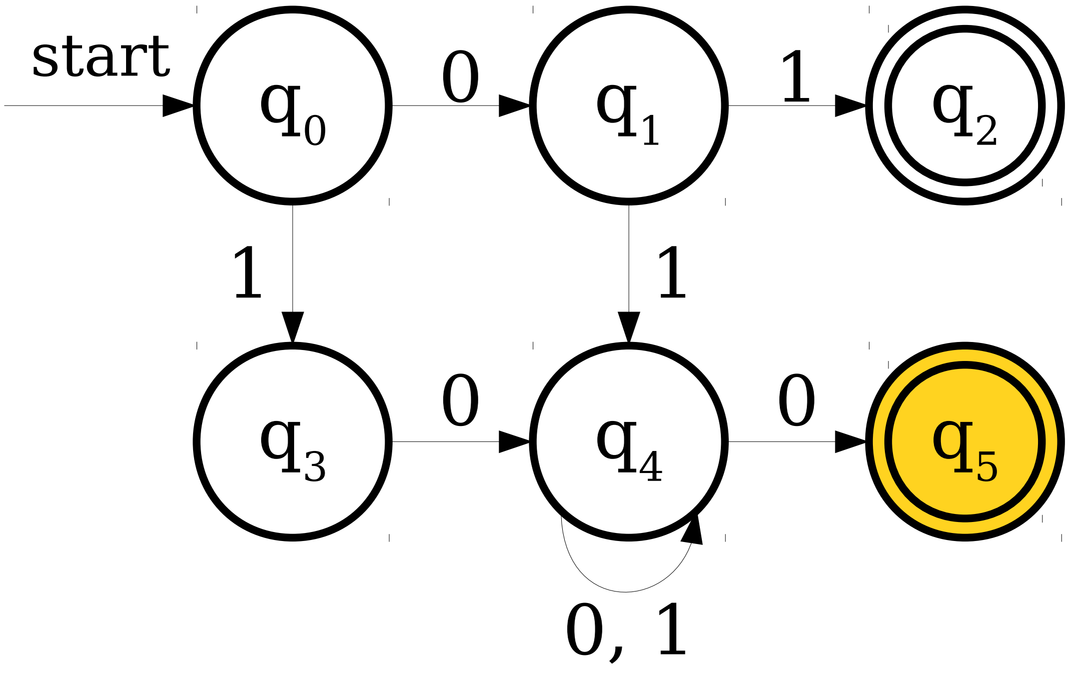


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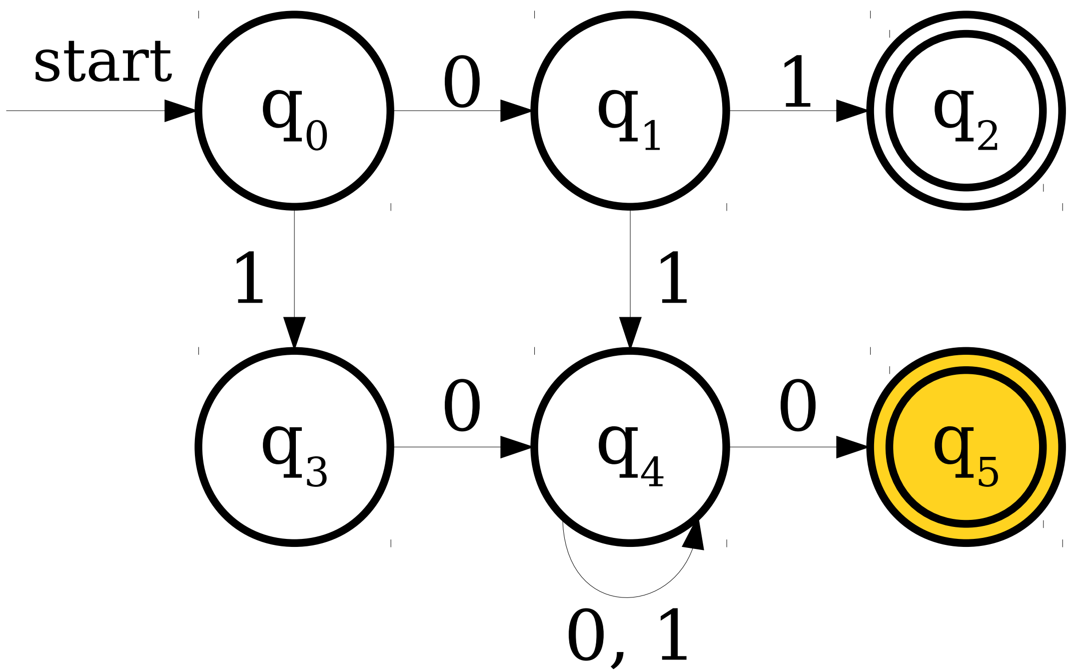


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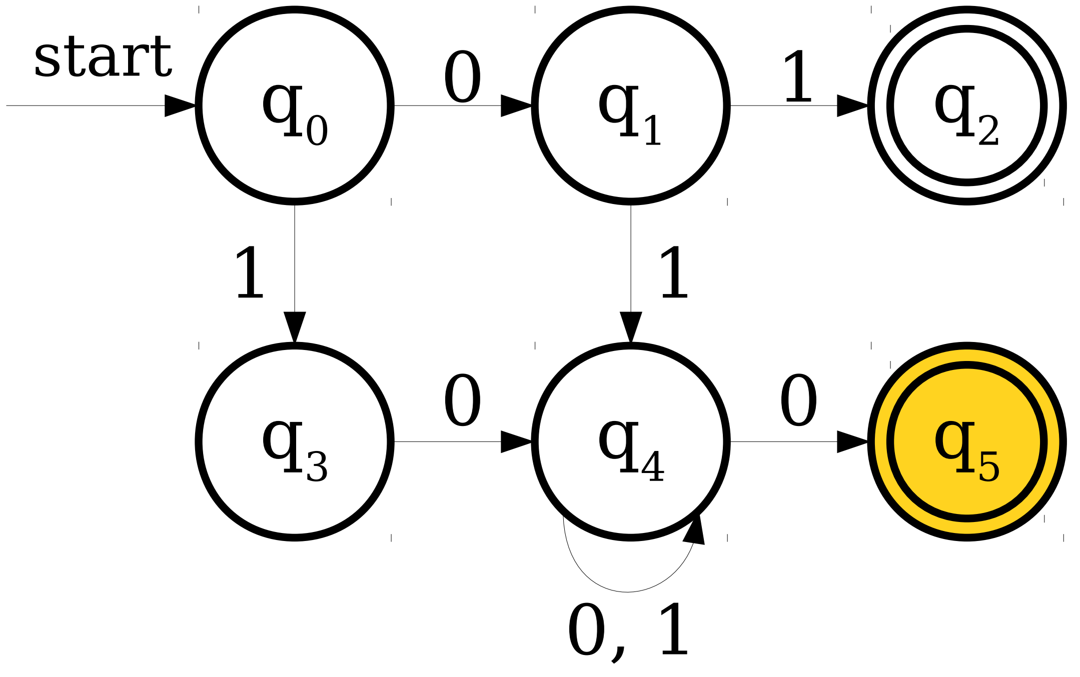
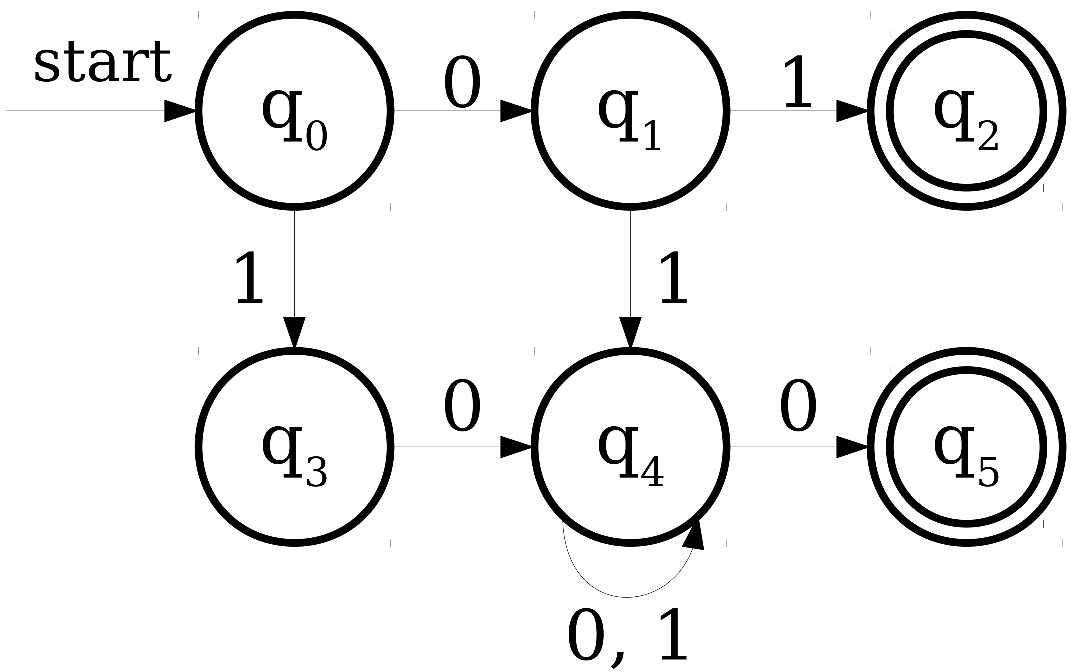


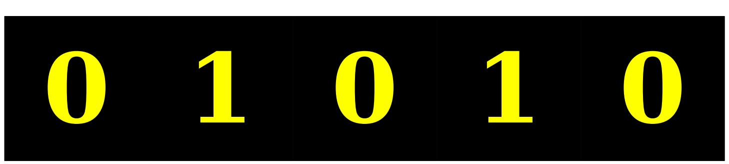

**0 1 0 1 0**

# Perfect Guessing

- We can view nondeterministic machines as having ***Magic Superpowers*** that enable them to guess the correct choice of moves to make.
  - If there is at least one choice that leads to an accepting state, the machine will guess it.
  - If there are no choices, the machine guesses any one of the wrong answers.
- No known physical analog for this style of computation – this is totally new!

# Massive Parallelism



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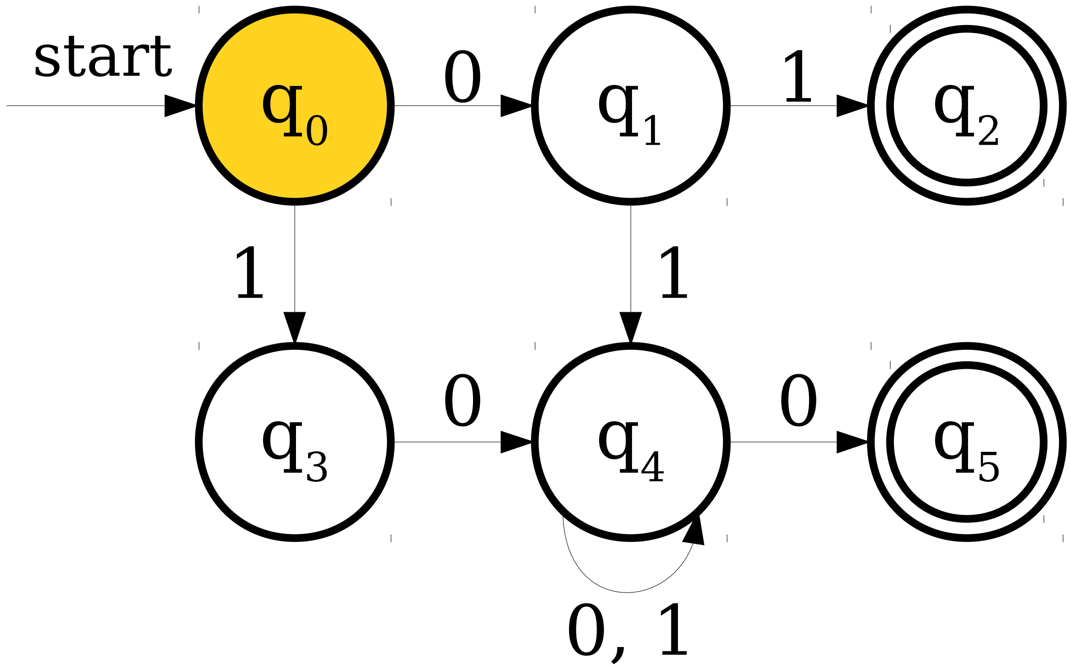


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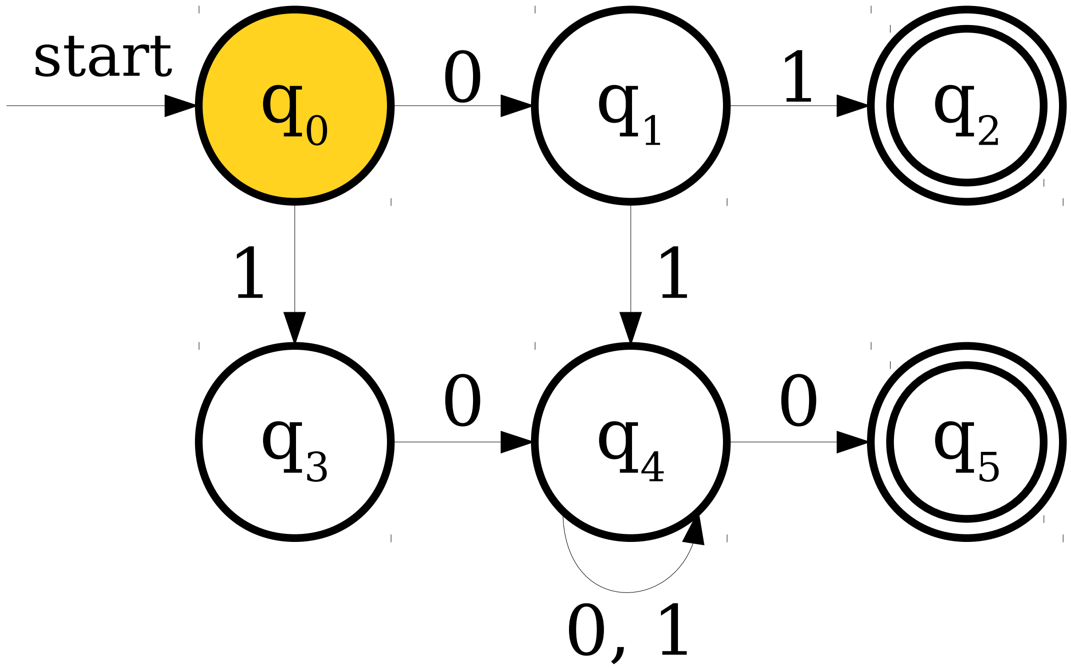
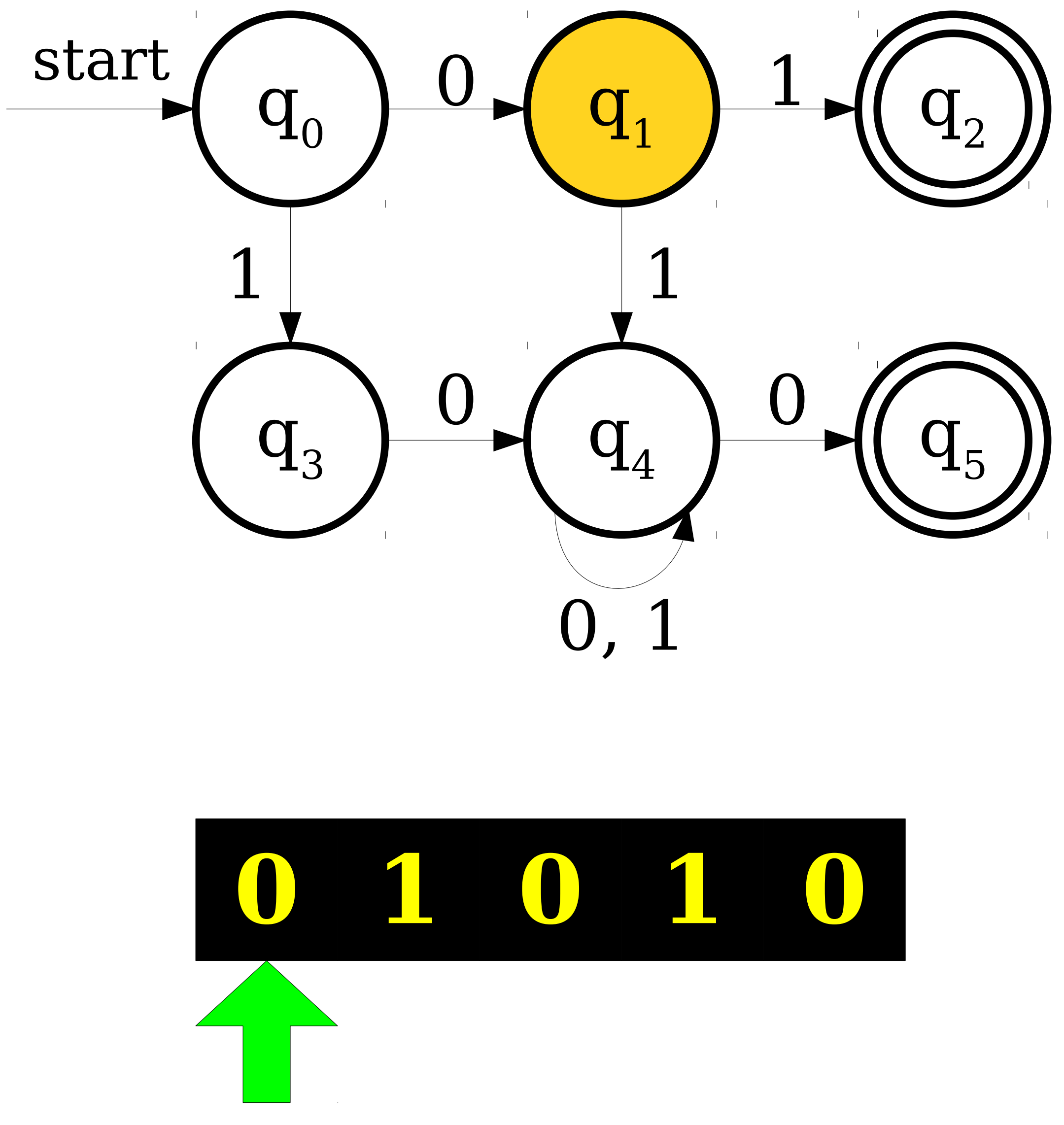



# Massive Parallelism



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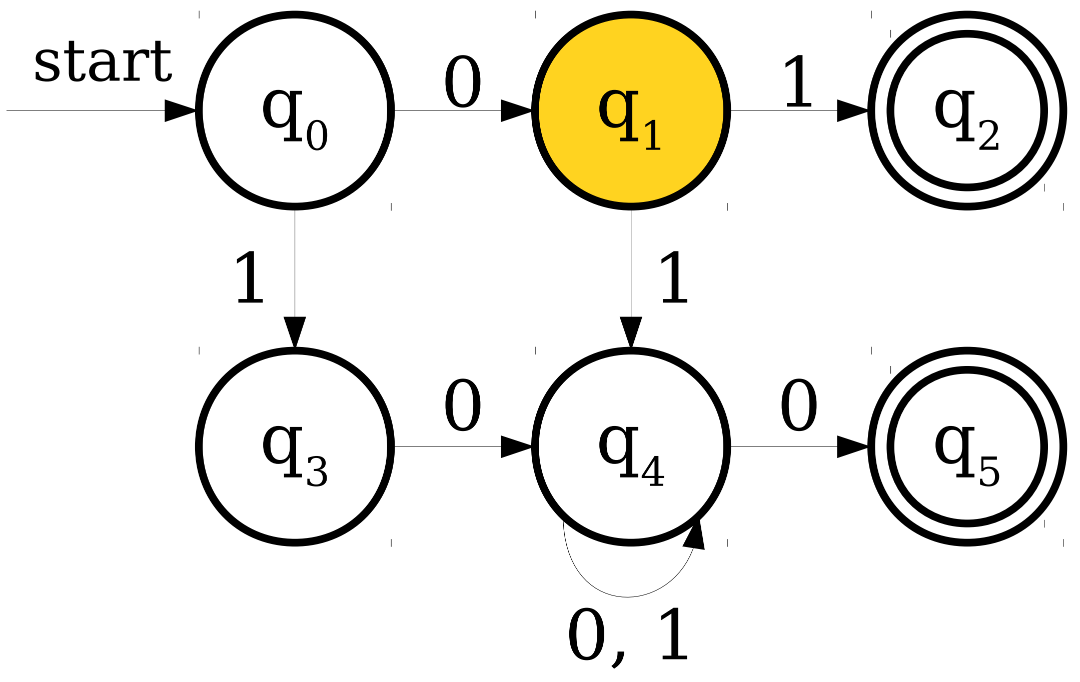


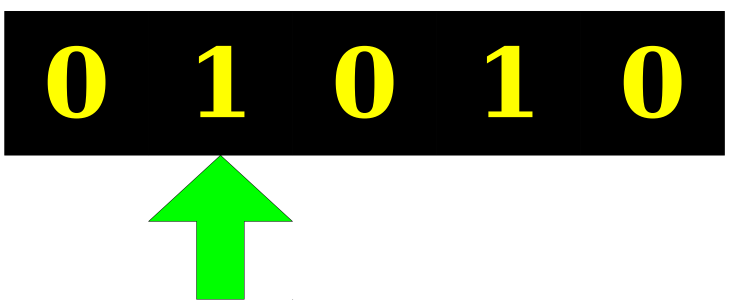

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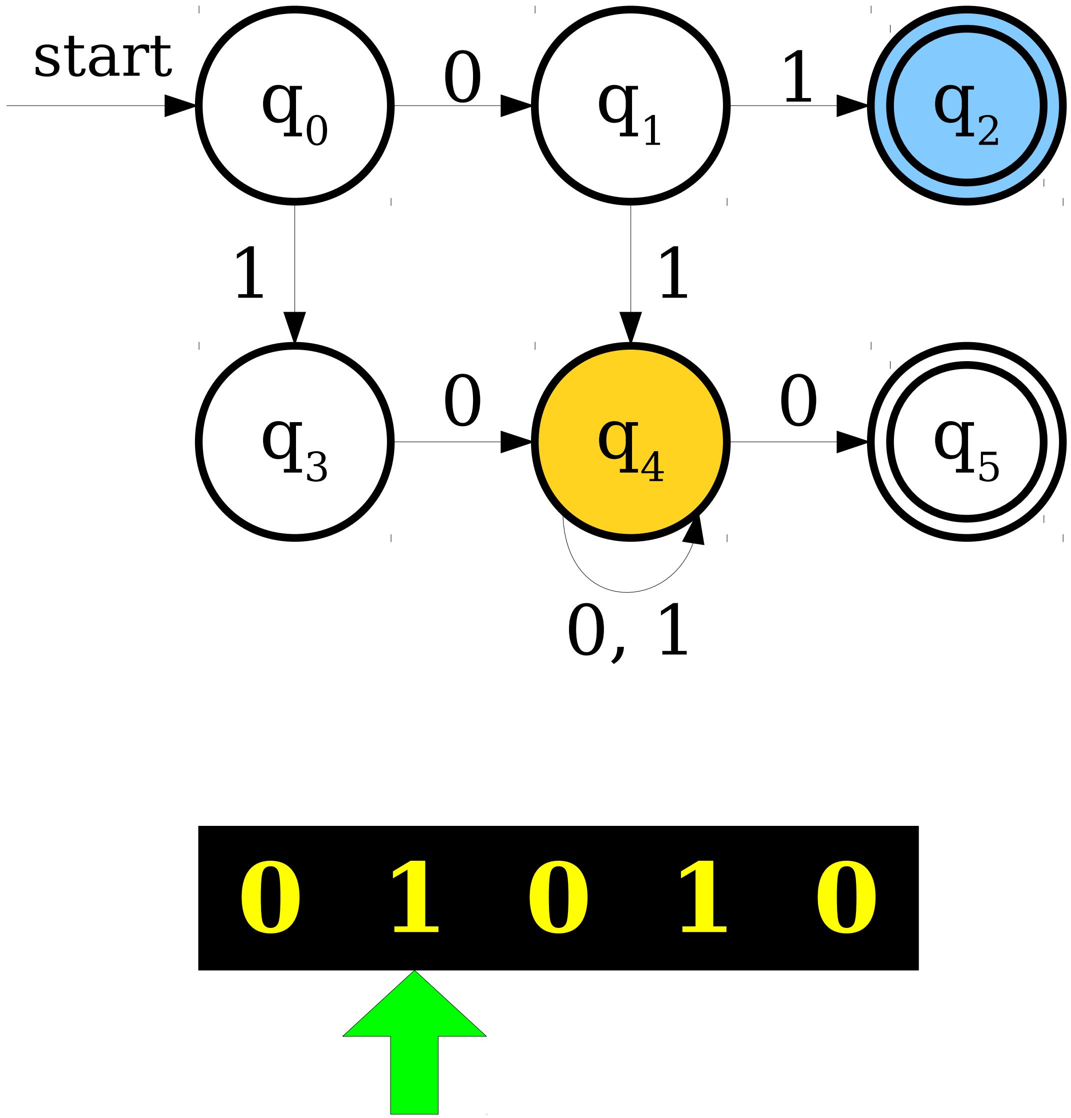

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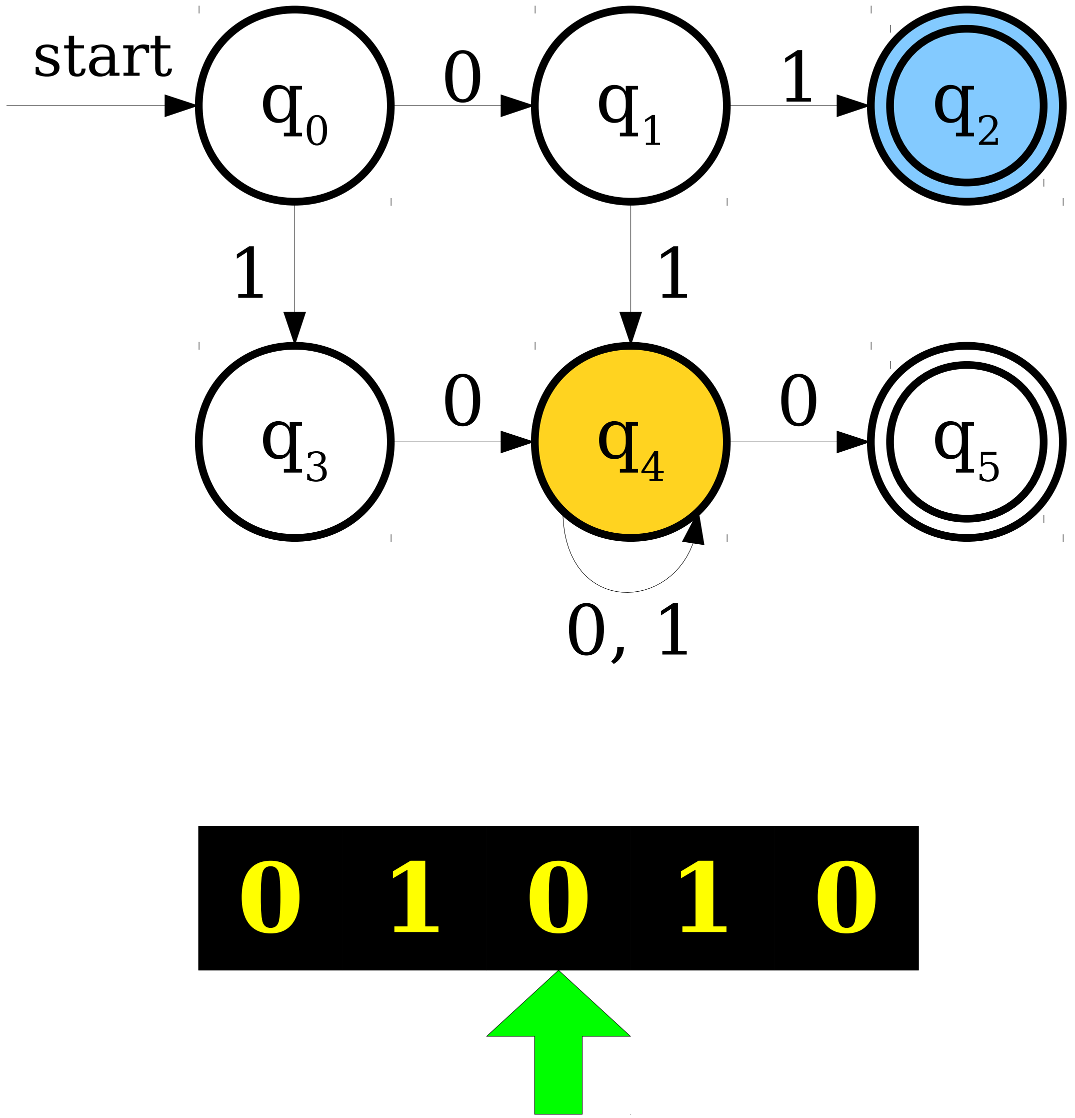

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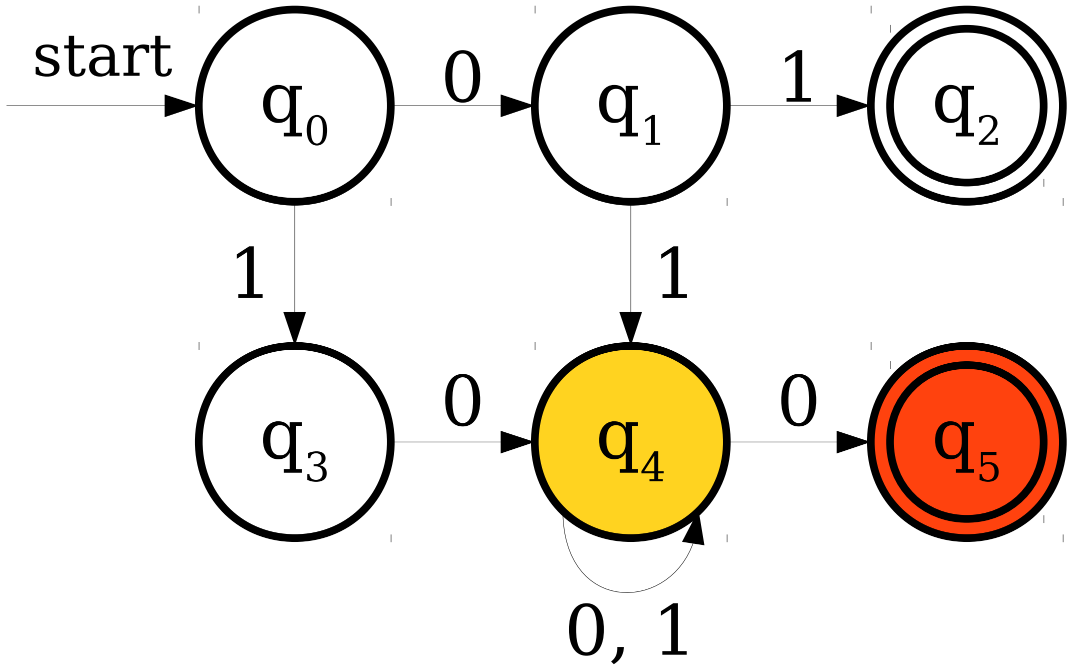


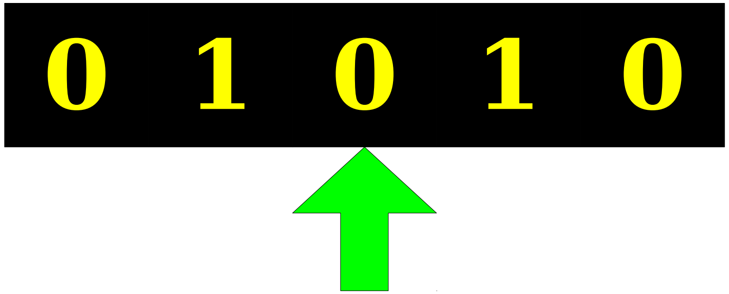

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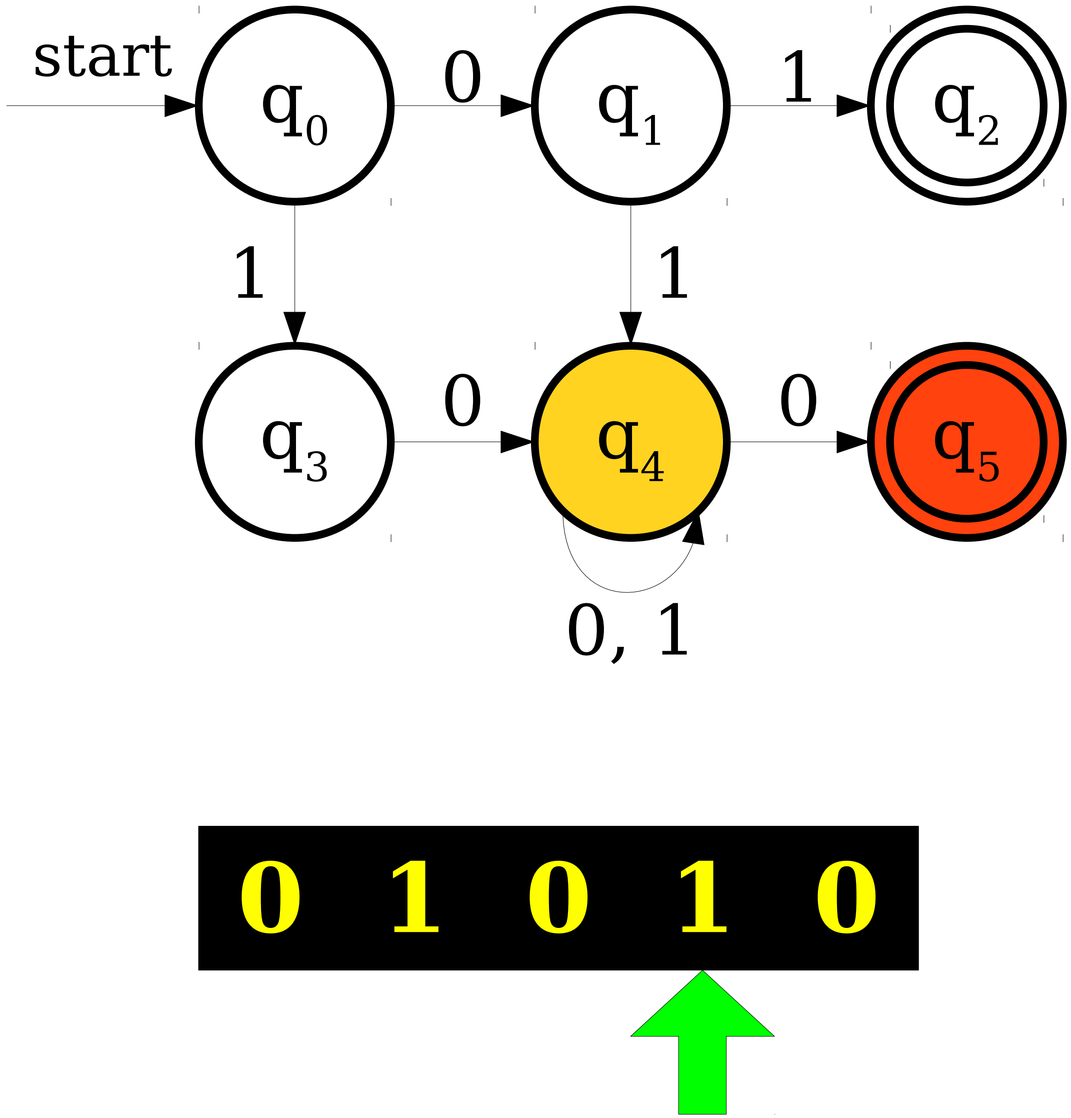

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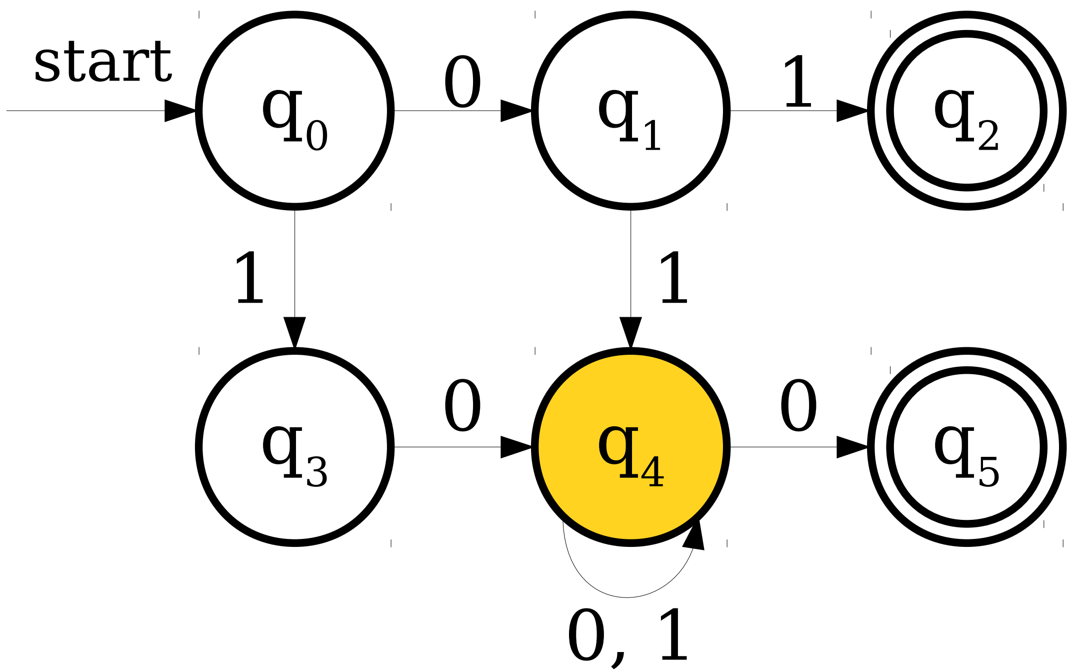


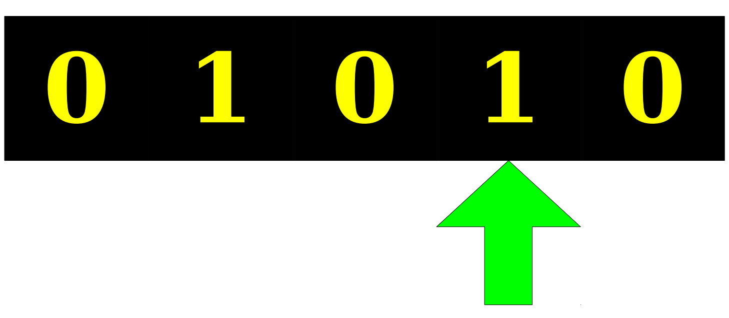

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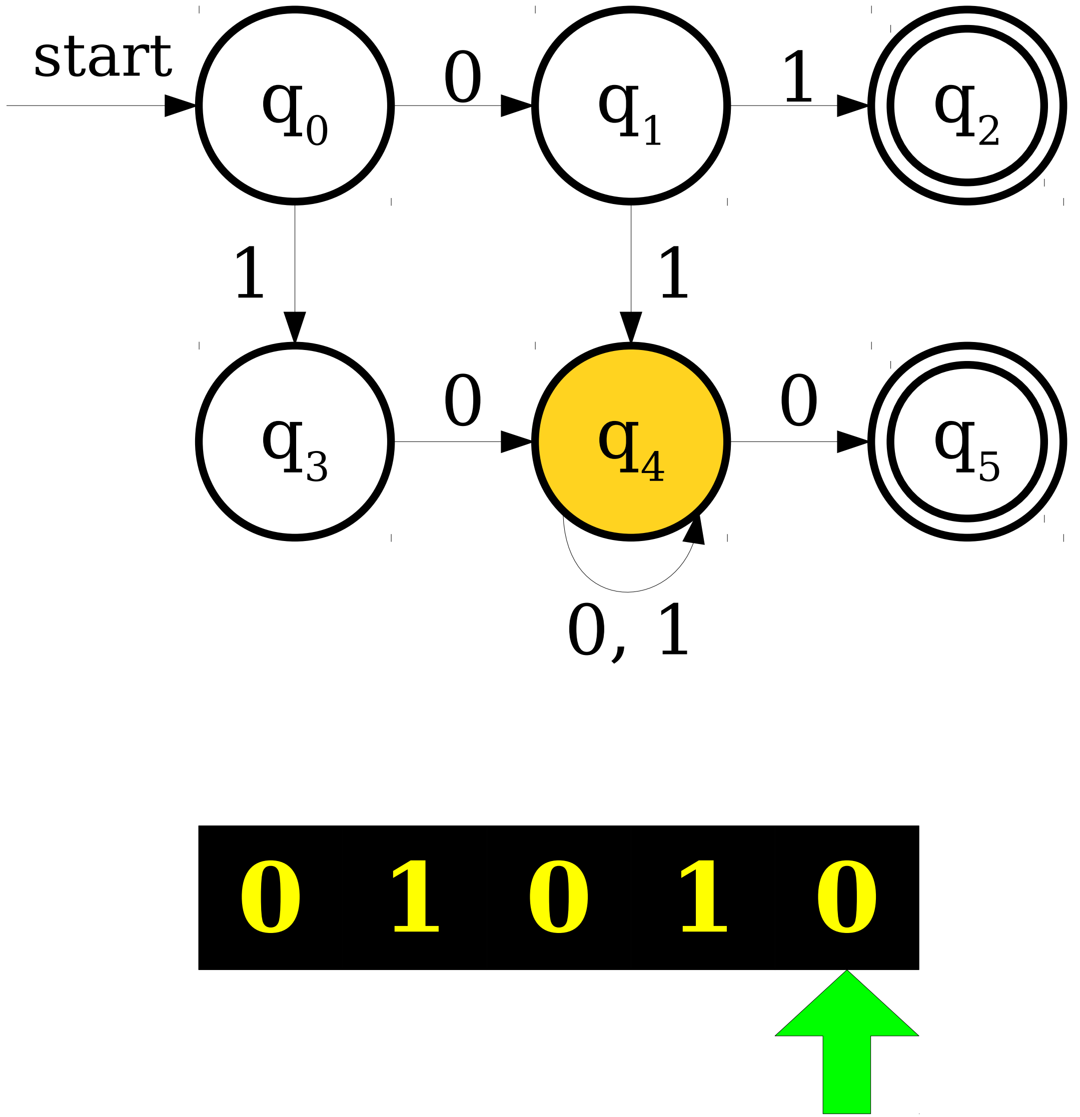

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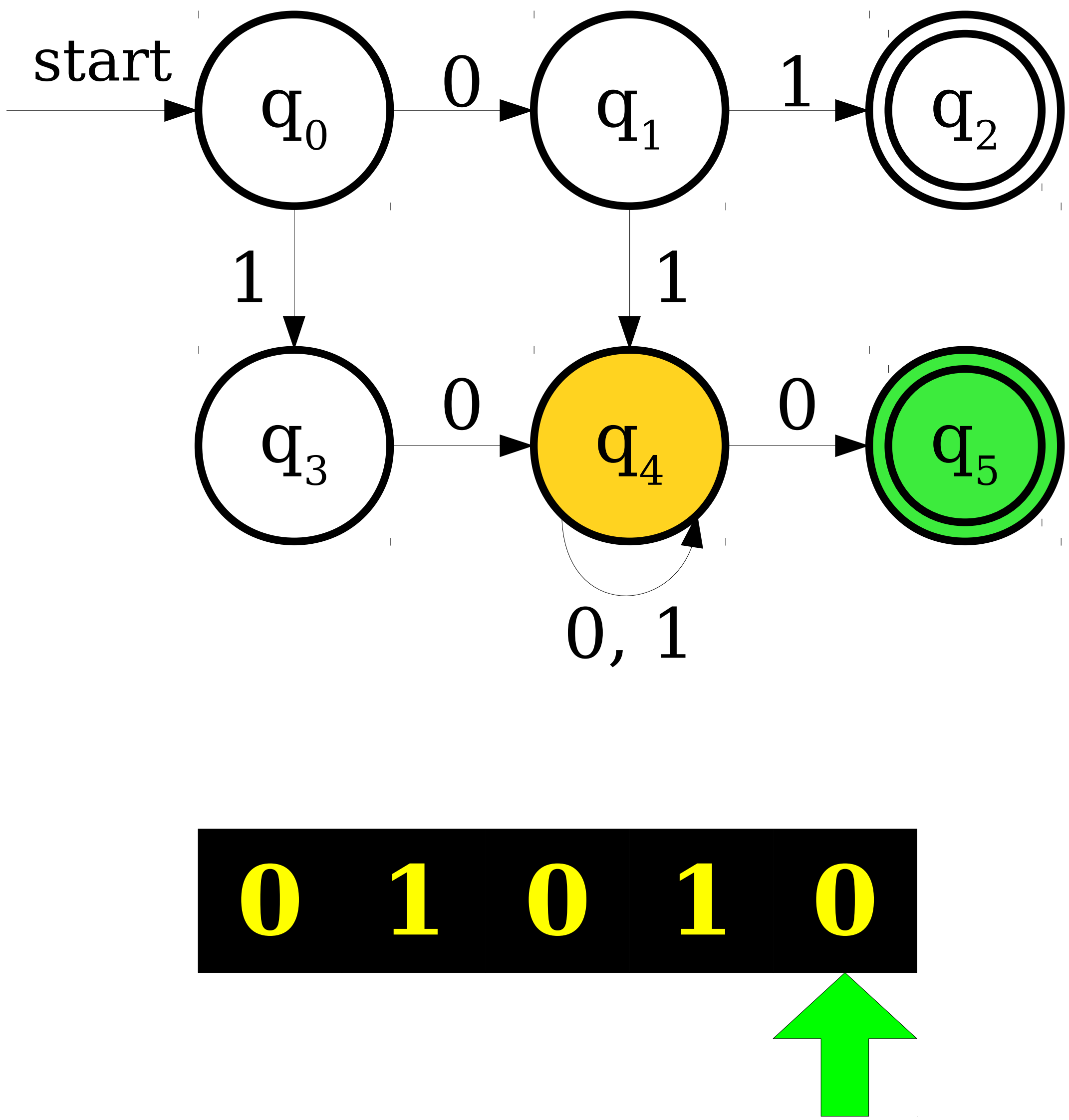

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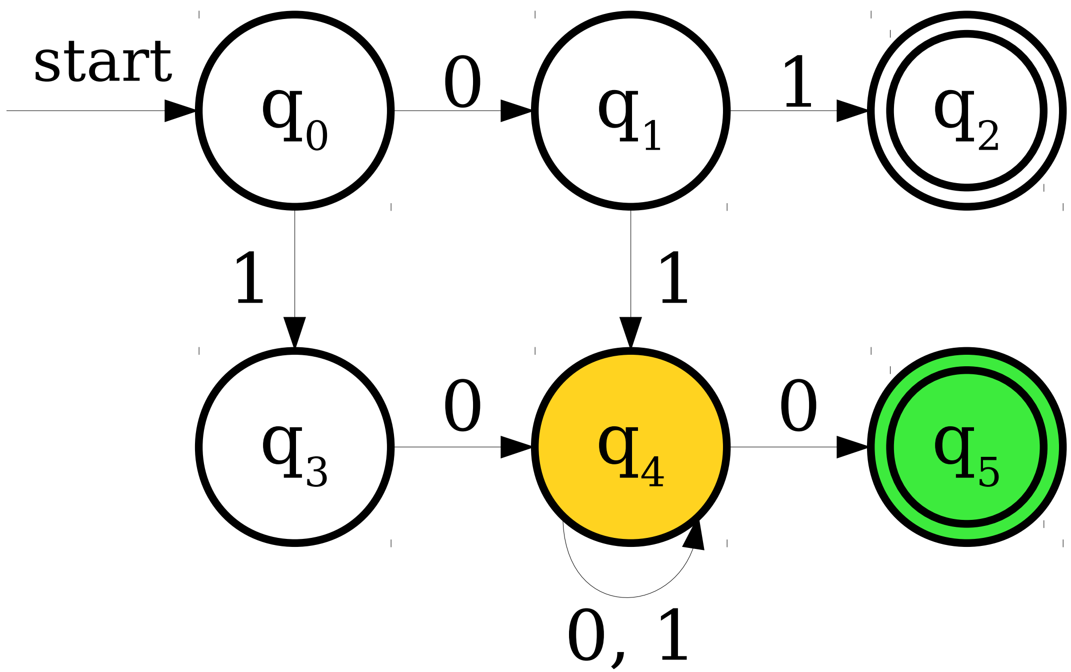


**0 1 0 1 0**

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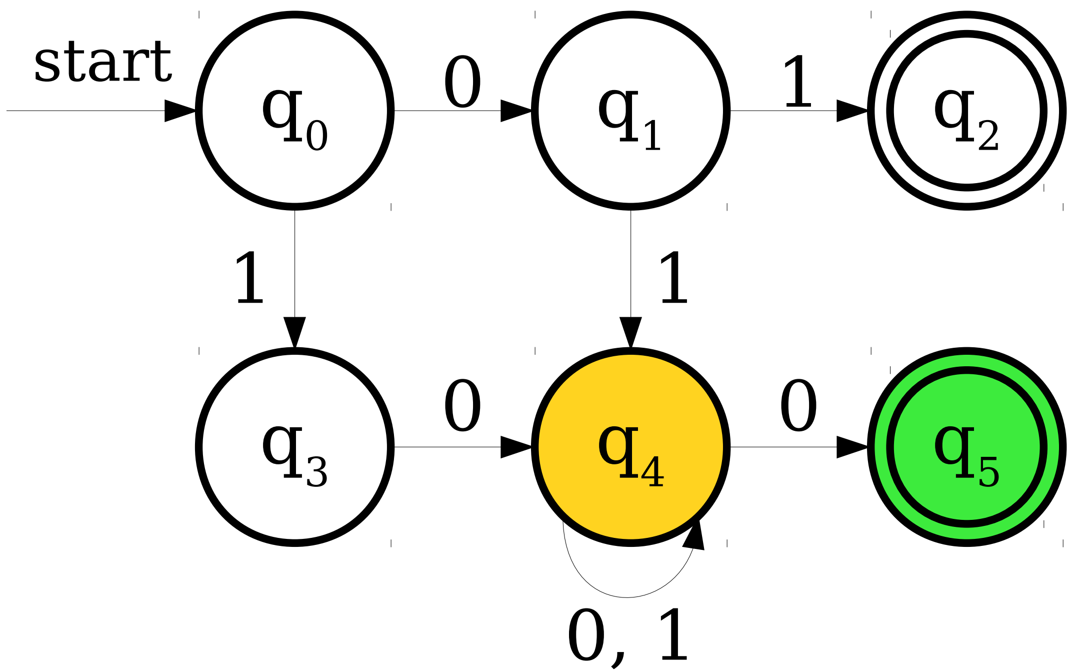


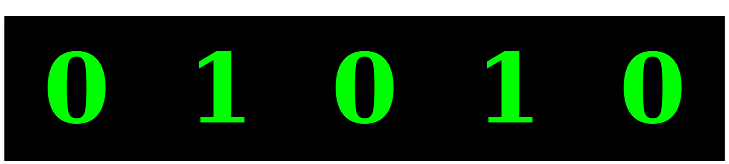

# Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- Each symbol read causes a transition on every active state into each potential state that could be visited.
- Nondeterministic machines can be thought of as machines that can try any number of options in parallel.

# So What?

- We will turn to these three intuitions for nondeterminism more later in the quarter.
- Nondeterministic machines may not be feasible, but they give a great basis for interesting questions:
  - Can any problem that can be solved by a nondeterministic machine be solved by a deterministic machine?
  - Can any problem that can be solved by a nondeterministic machine be solved ***efficiently*** by a deterministic machine?
- The answers vary from automaton to automaton.

# Designing NFAs

# Designing NFAs

- When designing NFAs, *embrace the nondeterminism!*
- Good model: ***Guess-and-check***:
  - Is there some information that you'd really like to have? Have the machine *nondeterministically guess* that information.
  - Then, have the machine *deterministically check* that the choice was correct.
- The *guess* phase corresponds to trying lots of different options.
- The *check* phase corresponds to filtering out bad guesses or wrong options.

# Guess-and-Check

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ ends in } \texttt{010} \text{ or } \texttt{101} \}$

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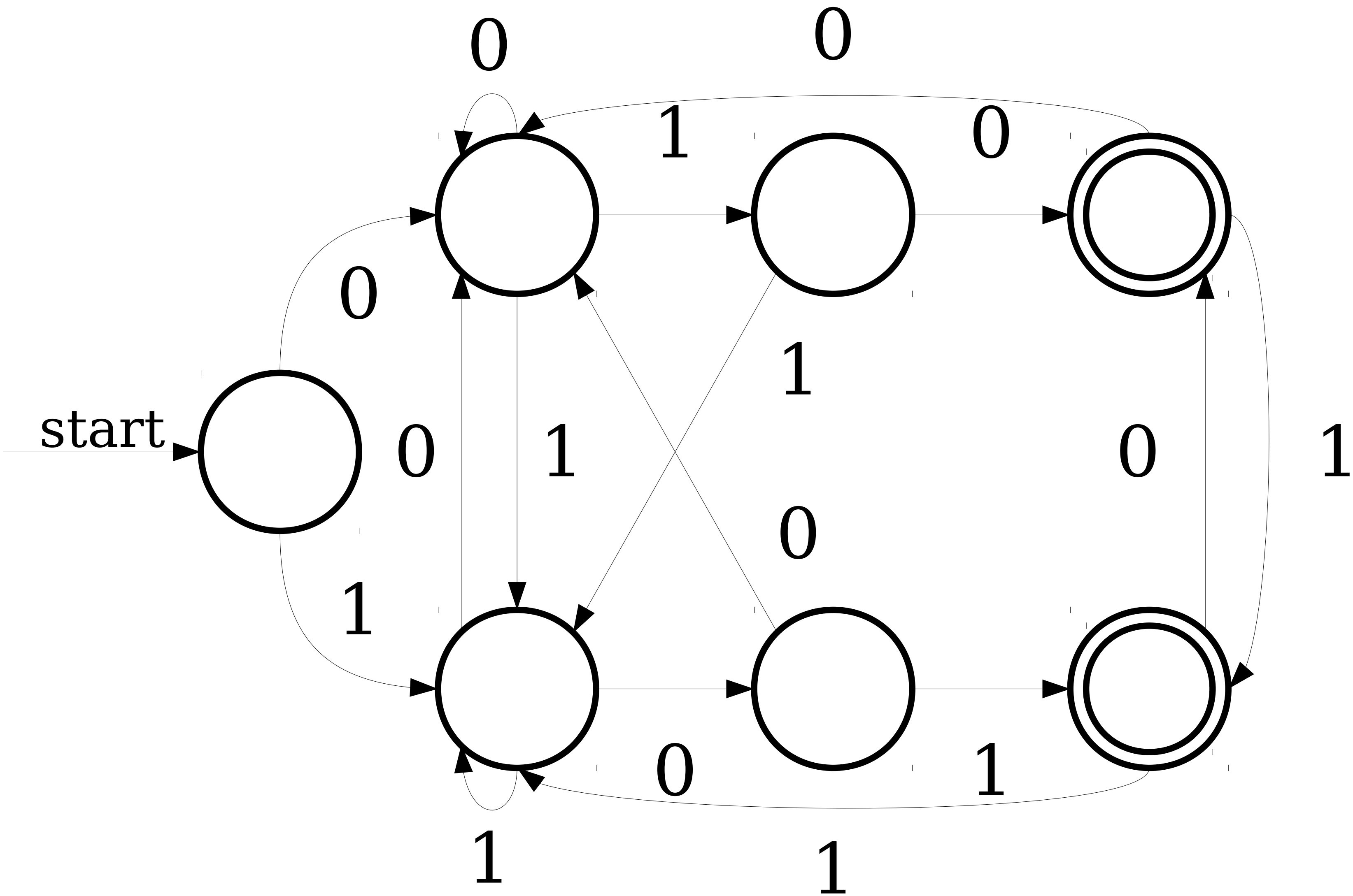

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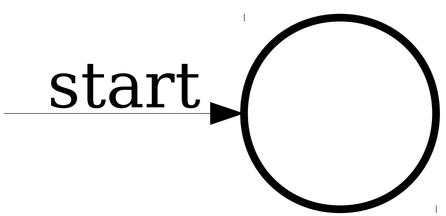

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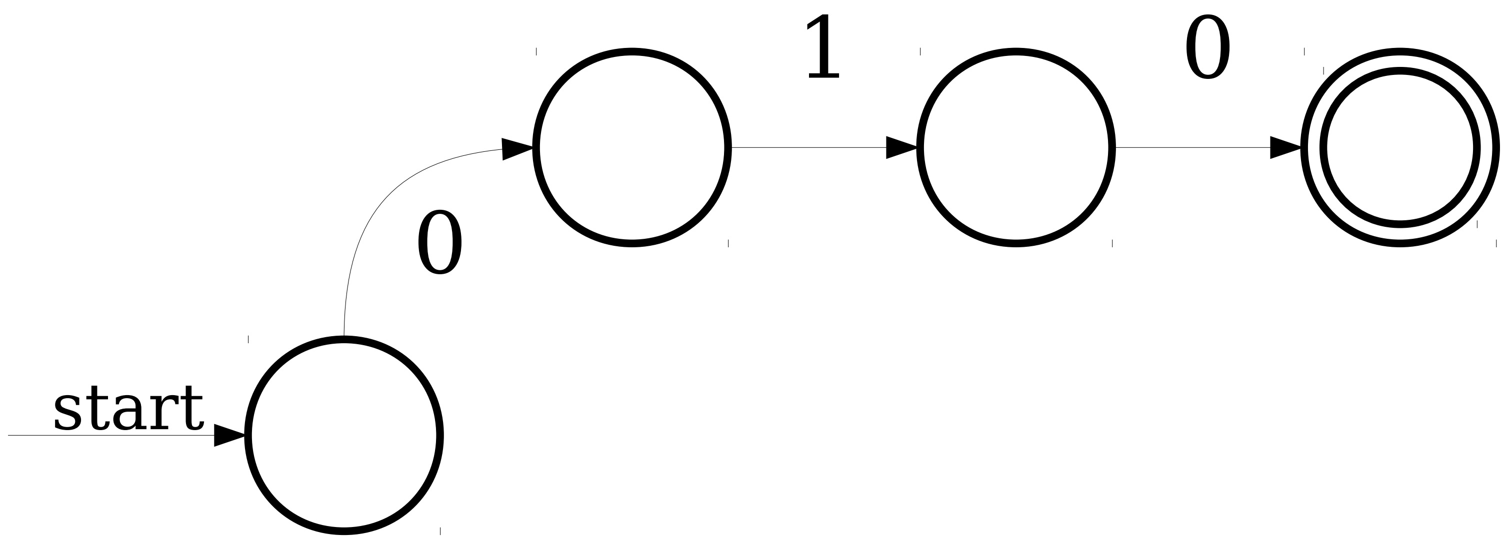

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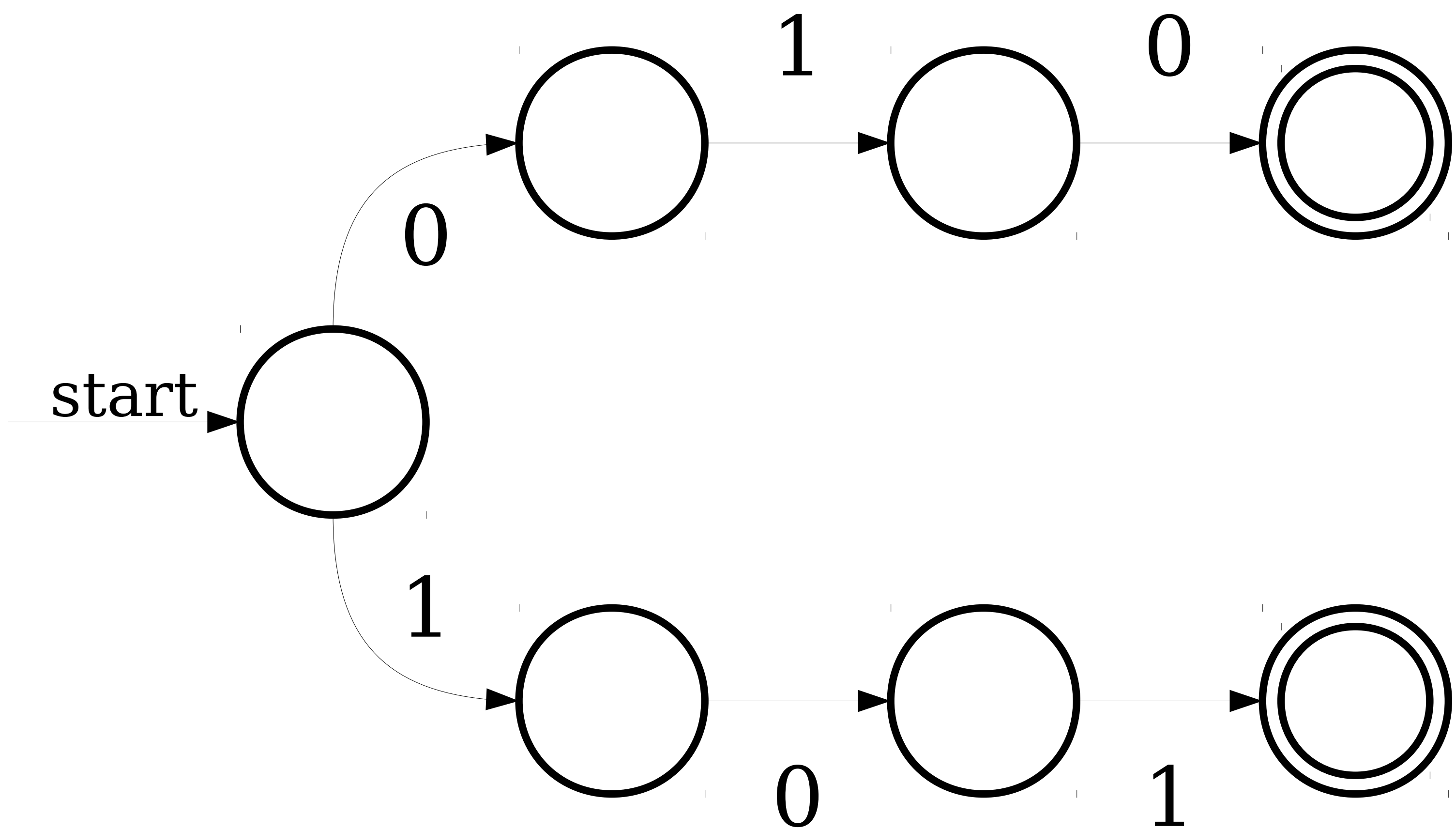

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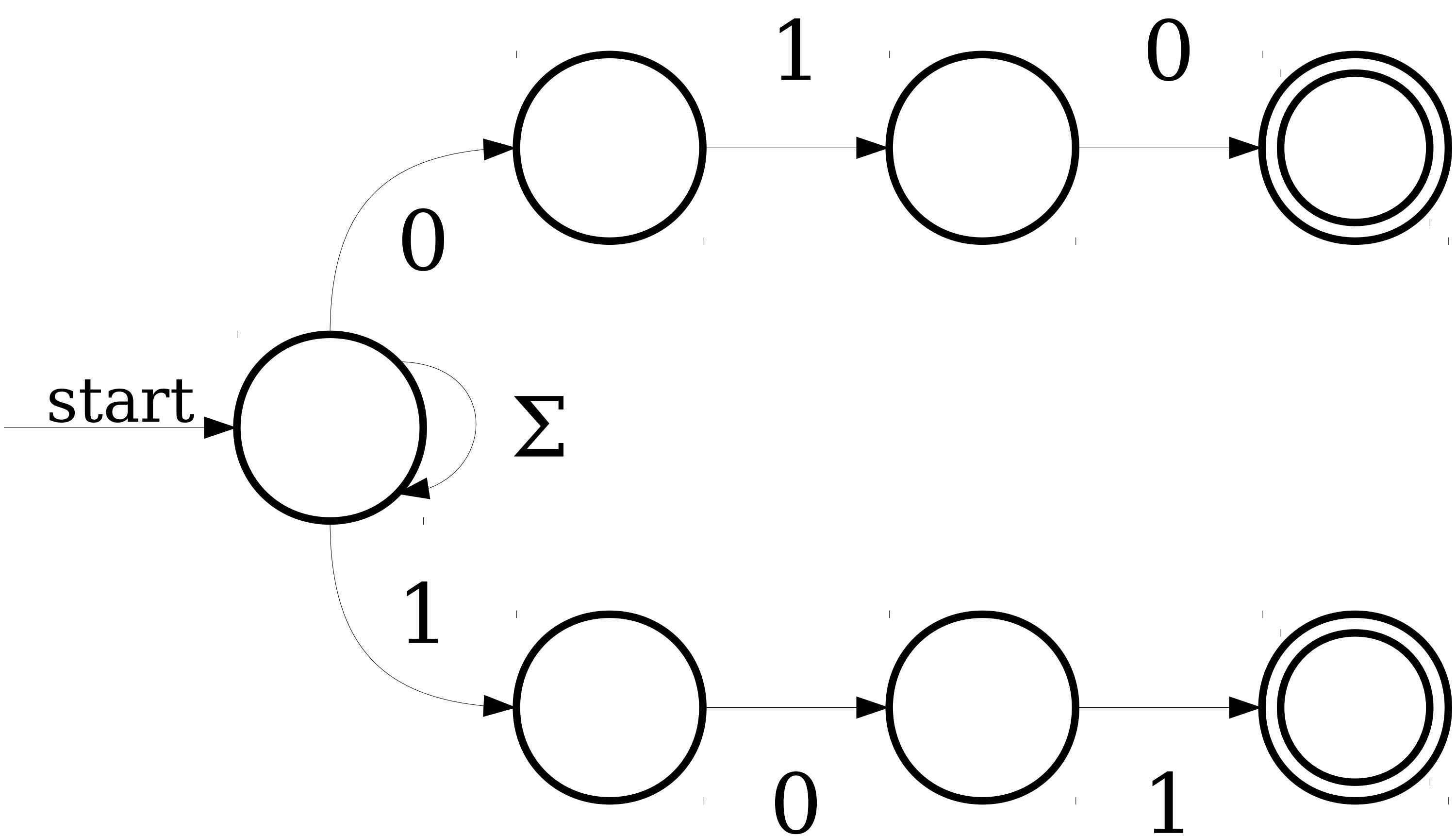

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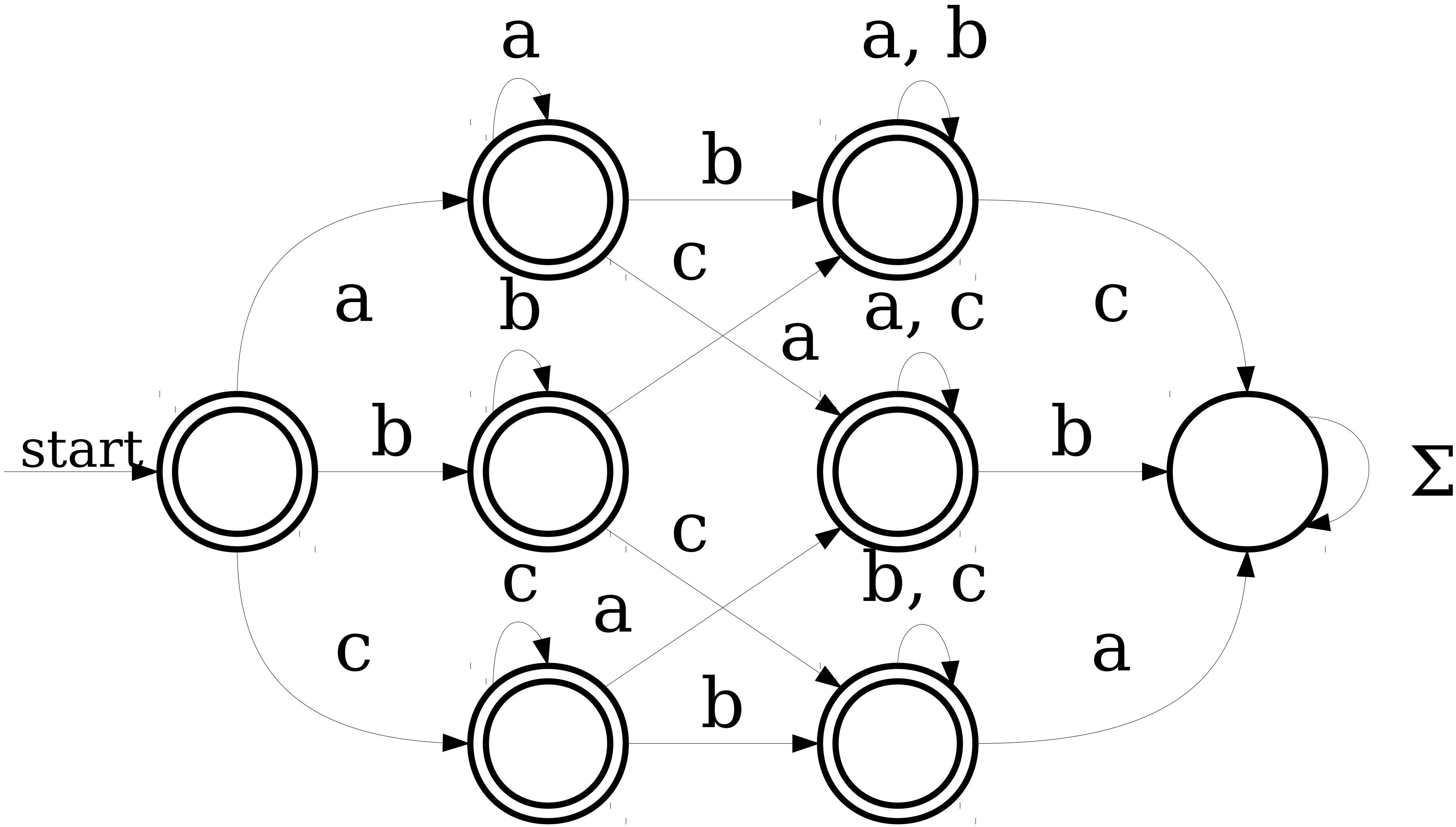

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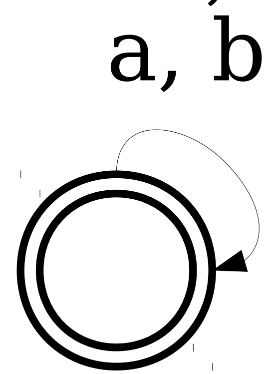

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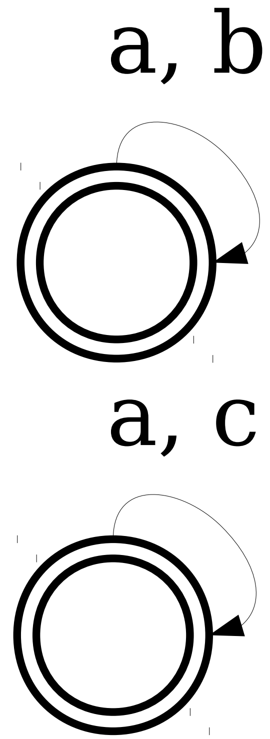

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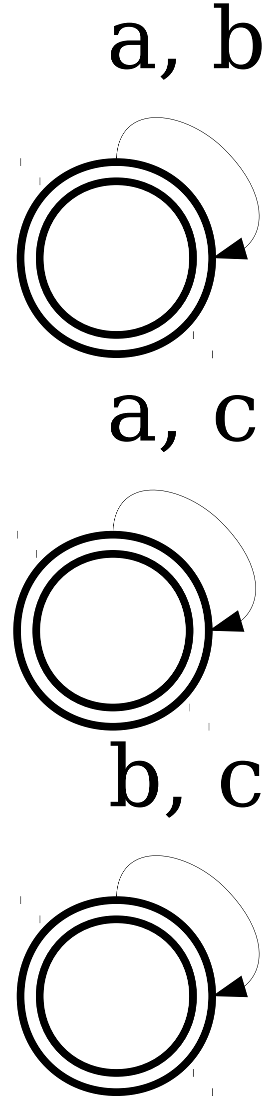

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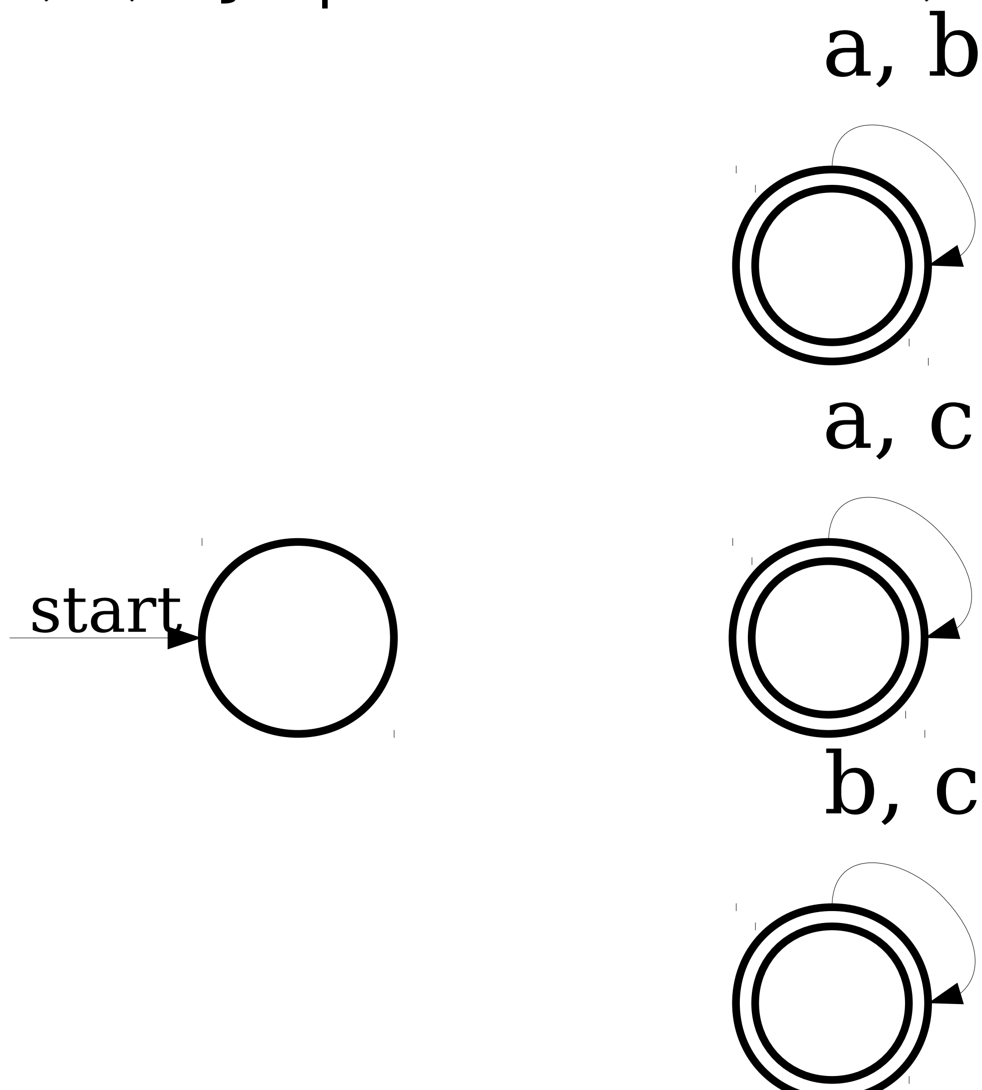

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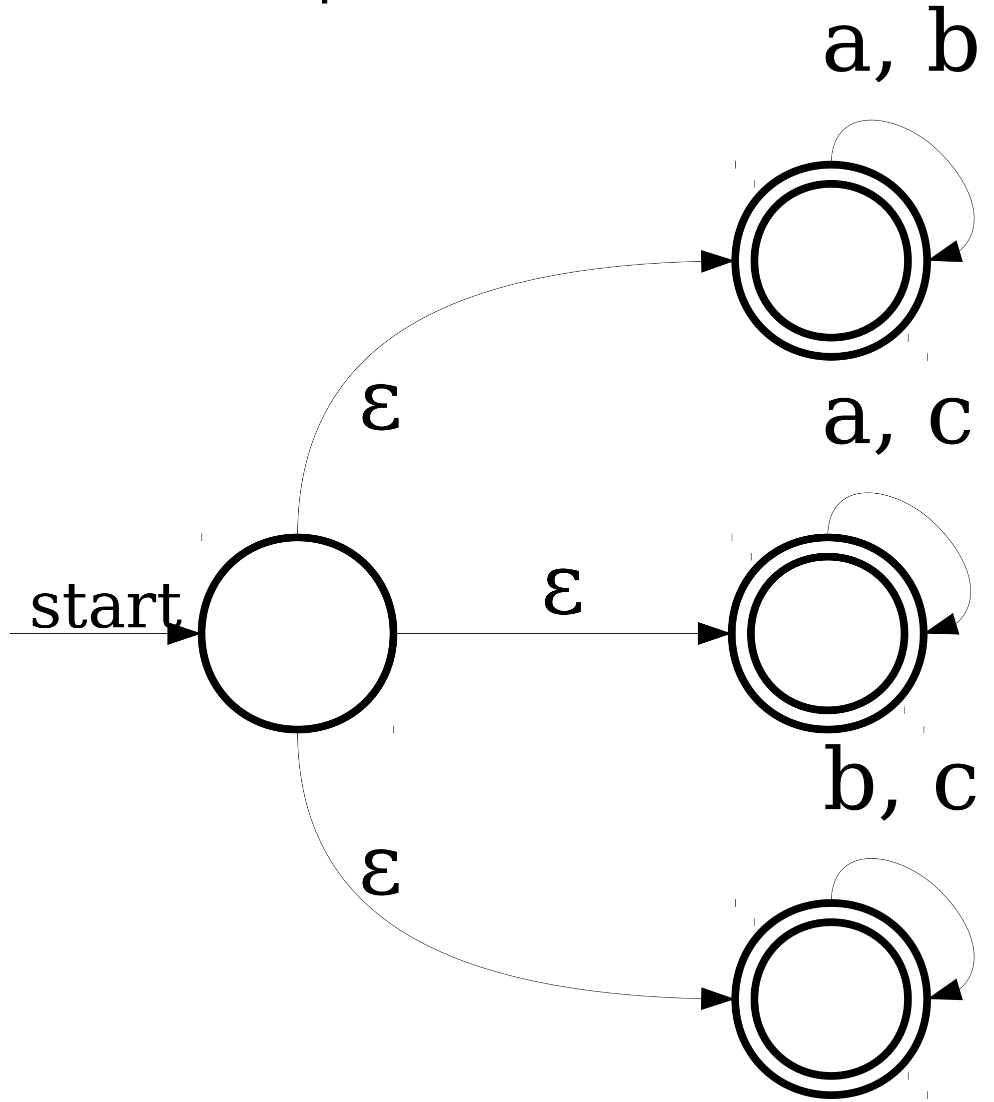

# Next Time

- **NFAs and DFAs**
  - Are NFAs more powerful than DFAs?
- **Closure Properties of Regular Languages**
  - More ways of transforming regular languages.