Extra Practice Problems 2

Here's another set of practice problems you can work through to help prepare for the midterm. We'll release solutions to these problems on Friday. If you have any questions about them, please feel free to stop by office hours!

Problem One: Translating Out Of Logic

For each first-order statement below, write a short English sentence that describes what that sentence says. While you technically *can* literally translate these statements back into English, you'll probably have better luck translating them if you try to think about what they really mean. Then, determine whether the statement is true or false. No proofs are necessary.

- $\exists S. \ (Set(S) \land \forall x. \ x \notin S)$
- $\forall x. \ \exists S. \ (Set(S) \land x \notin S)$
- $\forall S. \ (Set(S) \to \exists x. \ x \notin S)$
- $\forall S. \ (Set(S) \land \exists x. \ x \notin S)$
- $\exists S. \ (Set(S) \land \exists x. \ x \notin S)$
- $\exists S. \ (Set(S) \to \forall x. \ x \in S)$
- $\exists S. \ (Set(S) \land \forall x. \ x \notin S \land \forall \mathrm{T}. \ (Set(T) \land S \neq T \to \exists x. \ x \in T))$
- $\exists S. \ (Set(S) \land \forall x. \ x \notin S \land \exists \mathrm{T}. \ (Set(T) \land \forall x. \ x \notin T \land S \neq T))$
- $\exists S. \ (Set(S) \land \forall x. \ x \notin S) \land \exists \mathrm{T}. \ (Set(T) \land \forall x. \ x \notin T))$

Problem Two: Binary Relations

We formally defined strict orders to be binary relations that are irreflexive, asymmetric, and transitive. In lecture, we proved that a binary relation is a strict order if and only if it is asymmetric and transitive. In Problem Set Three, you proved that a binary relation is a strict order if and only if it is irreflexive and transitive.

Prove or disprove: a binary relation R over a set A is a strict order if and only if it's irreflexive and asymmetric.

Problem Three: Functions

Let $f : A \to B$ and $g : B \to C$ be functions. Notice that f's codomain is B, which happens to be the domain of C. That means that we can apply g to the output of f.

i. Prove that if f *is* surjective and g is *not* a bijection, then $g \circ f$ is *not* a bijection.

ii. Prove that if f is *not* a bijection and g *is* injective, then $g \circ f$ is *not* a bijection.

iii. Find examples of functions f and g where neither f nor g is bijective, but $g \circ f$ is a bijection.

As hints for each of the above problems: we highly recommend starting off by drawing pictures to get a sense for how these properties interact, then turning some of your insights from those pictures into actual proofs.