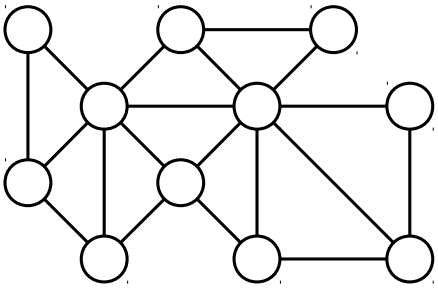
Extra Practice Problems 5

Here are some more review problems you can use to practice for the midterm. We'll release solutions on Friday, along with one final set of practice problems.

Graph Theory

Consider the following graph:



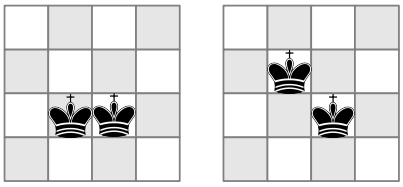
i. Prove that this graph is 4-colorable without actually finding a coloring for it.

ii. Prove that this graph is 3-colorable without actually finding a coloring for it.

iii. Prove that this graph is not 2-colorable.

A Clash of Kings

Chess is a game played on an 8 × 8 grid with a variety of pieces. In chess, no two king pieces can ever occupy two squares that are immediately adjacent to one another horizontally, vertically, or diagonally. For example, the following positions are illegal:



Prove that it is impossible to legally place 17 kings onto a chessboard.

Fibonacci Numbers

Recall that the Fibonacci numbers are defined with the following recurrence relation:

$$F_0 = 0 \qquad F_1 = 1 \qquad F_{n+2} = F_n + F_{n+1}$$

The first few Fibonacci numbers are

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$$

On Problem Set Five, you found a formula for the sum $F_0^2 + F_1^2 + \ldots + F_n^2$. Find a formula for the summation $F_0 + F_1 + \ldots + F_n$, then prove it correct using induction.

DFAs, NFAs, and Regular Expressions

Here's a slightly tricky set of DFA, NFA, and regex design problems.

Let $\Sigma = \{ a, b \}$ and let $L = \{ w \in \Sigma^* \mid$ the length of w is a multiple of four and w contains an even number of b's $\}$.

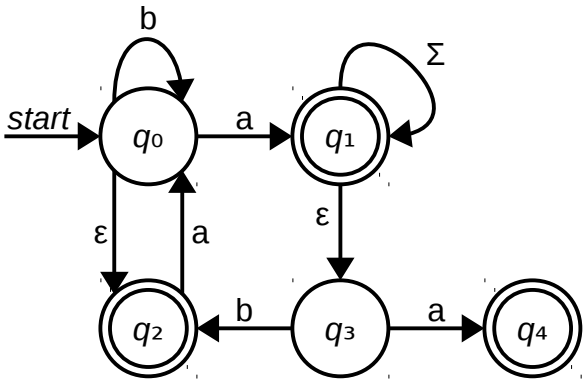
i. Design a DFA for L.

ii. Design an NFA for the *complement* of L.

iii. Write a regular expression for the *complement* of L.

Automata Transformations

Consider the following NFA:



We'd like you to convert this into both a DFA and a regular expression using the standard algorithms we've discussed so far.

i. Apply the subset construction to this NFA to convert it to a DFA.

ii. Apply the state elimination algorithm to this NFA to convert it to a regular expression.