Timeline of CS103 Results

This handout provides a timeline of some of the major results from CS103, hopefully offering you a glimpse of how everything we've explored this quarter was developed and discovered. I'm aware that this timeline is incomplete and possibly incorrect, so please feel free to reach out to me with any corrections or updates to this timeline!

- **c. 500 BCE:** A member of Pythagoras's cult, possibly Hippasus of Metapontum, discovers that the square root of two is irrational.
- **1735:** Leonhard Euler lays the foundations for graph theory and Eulerian graphs in his work on whether a particular series of bridges can be walked across once and exactly once.
- **1834:** Peter Dirichlet gives the first instance of the use of the pigeonhole principle. Dirichlet referred to this principle as the "box principle" or "drawer principle," and so some mathematicians still refer to it as "Dirichlet's box principle." *(A note to readers: I'm having trouble tracking down the name of the paper this first appeared in – let me know if you can find it!)*
- **1852:** Francis Guthrie, a mathematician and botanist, poses the Four-Color Conjecture (later the Four-Color Theorem) after noticing that he could always color a map of the counties of England with only four colors, never assigning two adjacent counties the same color.
- **1854:** George Boole publishes *An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities*, outlining a mathematical system for formal logic. His approach was based on representing logical notation as arithmetical statements involving addition and multiplication and ultimately gave rise (after some later modifications) to propositional logic.
- **1874:** Georg Cantor proves that $|\mathbb{N}| \neq |\mathbb{R}|$ in his paper "Über eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen." Interestingly, this paper did not primarily focus on cardinality arguments and instead looked at the algebraic irrational numbers (irrational numbers that are the roots of polynomials with integer coefficients) and the transcendental irrational numbers (irrational numbers that are not algebraic). The proof Cantor outlined in this paper is not his famous diagonal argument, but rather a more technical argument involving infinite sequences.
- **1879:** Gottlob Frege publishes *Begriffsschrift* ("Concept Script"), a formal notation for logical reasoning that is considered the precursor of first-order logic and other modern logical systems. Frege's system was based on implications and negations only and contained one of the earliest references to quantifiers.
- **1887:** Richard Dedekind finds a proof of the Cantor-Bernstein-Schröder theorem, but doesn't publish it. The theorem would first be published by Bernstein and Schröder in 1895, with Cantor's name added later on (Cantor stated the theorem without proof in 1885.)

- **1891:** Georg Cantor introduces the diagonal argument in his paper "Über eine elementare Frage der Mannigfaltigkeitslehre." His paper proved that there are more functions from a set S to the set $\{0, 1\}$ than there are elements of S. Although Cantor didn't state it as such, this proof is essentially the proof of Cantor's theorem (treat each function $f : S \to \{0, 1\}$ as an "indicator function" describing a particular subset of S: elements mapping to 0 are not part of the subset of S and elements mapping to 1 are a part of the subset of S.)
- **1917:** David Hilbert distinguishes first-order logic from other sorts of logical systems and presents it in a class at the University of Göttingen.
- **1928:** David Hilbert poses the Entscheidungsproblem ("decision problem,") asking whether there is a mechanical procedure by which mathematical theorems can be automatically proven or disproven, setting up a challenge to define what a "mechanical procedure" means and how one might build such a procedure for automated theorem proving.
- **1930:** Frank Ramsey publishes "On a Problem in Formal Logic," giving an algorithm for a restricted case of the Entscheidungsproblem. A key step in his result was a theorem that says that for any number of colors n and any numbers c_1, c_2, …, c_n, there is some k such that any k-clique where the edges are given one of k colors must have a c_1-clique of the first color, a c_2-clique of the second color, …, or a c_n-clique of the nth color. This theorem, later called Ramsey's Theorem, contains the Theorem on Friends and Strangers as a special case and launched much of modern combinatorics.
- **1936:** *Oh, what a year...*
  - Alonzo Church publishes "An Unsolvable Problem of Elementary Number Theory," outlining the λ-calculus (a formalism for describing computation through recursive functions) as a model of computation and using a diagonal argument to find a particular function that cannot be computed by any λ-calculus formula. Church concluded that the Entscheidungsproblem is therefore undecidable.
  - Alan Turing publishes his landmark paper "On Computable Numbers with Applications to the Entscheidungsproblem" introducing the Turing machine, the notion of TM encodings, the universal Turing machine, the undecidability of the halting problem, the equivalence of λ-calculus and Turing machines, and the undecidability of the Entscheidungsproblem.
- **1937:** Lothar Collatz poses the Collatz Conjecture.
- **1938:** Stephen Cole Kleene publishes "On Notation for Ordinal Numbers," which included as a technical lemma the result that would later be called Kleene's Second Recursion Theorem.
- **1945:** Hugo Hadwiger publishes "Überdeckung des euklidischen Raumes durch kongruente Mengen" ("Coverage of Euclidean space by matching amounts"), outlining what later will become known as the Hadwiger-Nelson Problem.
- **1951:** Stephen Cole Kleene invents the regular languages in an internal paper at the RAND corporation (this paper was republished in 1956 as "Representation of Events in Nerve Nets and Finite Automata"). This paper introduced the Kleene star operator, the regular languages, the regular expressions, and a theorem that proved that any type of finite automaton whose behavior depends only on its previous state must be equivalent in power to the regular expressions. (Note that at this point, our notions of DFAs and NFAs had not yet been invented.)

- **1953:** H. G. Landau, a mathematical sociologist, publishes "On dominance relations and the structure of animal societies: (III) The condition for a score structure," introducing the concept of a tournament winner and proving that every tournament has a tournament winner.
- **1957:** Noam Chomsky invents context-free grammars in his book "Syntactic Structures." At the time, he called them *phrase-structure grammars* and explored them in the context of modeling the syntax of natural languages.
- **1958:** Anil Nerode publishes "Linear Automaton Transformations," outlining what is now called the Myhill-Nerode theorem. His result was relative to a different class of automata than the ones we saw (DFAs and NFAs had not yet been published yet).
- **1959:** Michael Rabin and Dana Scott publish "Finite Automata and their Decision Problems," outlining the fundamentals of finite automata. Their paper invented DFAs and NFAs (and, in fact, invented the entire idea of nondeterministic computation), proved the Myhill-Nerode theorem (giving credit to Anil Nerode for his 1958 paper and to John Myhill for developing the more modern version of the theorem), demonstrated the equivalence of DFAs and NFAs via the subset construction, explored the equivalence of finite automata and regular expressions, and gave several decidable properties of finite automata. Rabin and Scott would later receive the Turing Award, considered to be the equivalent of the Nobel Prize for computer science, for their contributions.
- **1963:** Janusz Brzozowski and Edward McCluskey publish "Signal Flow Graph Techniques for Sequential Circuit State Diagrams," introducing the state elimination algorithm.
- **1965:** Three major papers were published that formed the basis for complexity theory:
  - Alan Cobham publishes "The Intrinsic Computational Difficulty of Functions," outlining polynomial-time decidability as a criterion for efficient computation.
  - Jack Edmonds publishes "Paths, Trees, and Flowers," giving a polynomial-time algorithm for maximum matching and arguing that efficient computation means polynomial-time computation. (This paper and the previous paper are the reason for the terminology "Cobham-Edmonds thesis.")
  - Juris Hartmanis and Richard Stearns publish "On the Computational Complexity of Algorithms," formally defining time complexity for Turing machines and proving the time hierarchy theorem.
- **1968:** Ken Thompson publishes "Regular Expression Search Algorithm," describing a construction for converting regular expressions into NFAs. This paper is one of the first to describe efficient regular expression matching algorithms. (Ken Thompson is more famous for co-creating UNIX along with Dennis Ritchie, the inventor of the C programming language.)
- **1971:** Stephen Cook publishes "The Complexity of Theorem-Proving Procedures," giving the first proof that SAT is **NP**-complete. Cook's proof uses a slightly different definition of reducibility (the Turing reduction) than the type of reduction typically used nowadays for polynomial-time reducibility (the mapping reduction). Technically, Cook proved that the problem of checking whether a formula is a tautology is **NP**-complete with respect to Turing reductions. (The tautology problem is known to be *co*-**NP** complete using the standard definition of **NP**-completeness, and it's an open problem whether it's **NP**-complete or not.) Cook also proved that another problem, subgraph isomorphism, is **NP**-complete. Stephen Cook later won a Turing Award for this work.

- **1972:** Richard Karp publishes “Reducibility among Combinatorial Problems,” giving 21 examples of “natural” **NP**-complete problems and launching the **P** versus **NP** problem. Karp's paper is actually quite accessible if you have a background in the definitions of **P**, **NP**, and **NP**-completeness, and I'd recommend reading over it if you get the chance. Karp later won a Turing Award, in part because of this paper.
- **1973:** Working in the Soviet Union, Leonid Levin publishes Универсальные задачи перебора, proving that SAT, exact cover, subgraph isomorphism, and a few other problems are **NP**-complete. He also proved that these problems have an interesting property: for each problem, there is a single optimal algorithm that can only be improved upon by a constant factor, even though it's unclear exactly what those algorithms are. Although Levin's work was *published* in 1973, he apparently had worked out many of the results several years earlier. The theorem that SAT is **NP**-complete is typically attributed to both Cook and Levin, even though neither worked with one another or was aware that the other was working on similar problems.
- **1975:** Two important papers are published about the $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$ question:
  - Richard Ladner publishes “On the Structure of Polynomial-Time Reducibility,” proving that if $\mathbf{P} \neq \mathbf{NP}$, then **NP** problems exist that are neither in **P** nor **NP**-complete.
  - Theodore Baker, John Gill, and Robert Solovay publish “Relativizations of the $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$ Question,” proving a result implying that proofs based purely on universality and self-reference cannot resolve $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$.
- **1976:** Kenneth Appel and Wolfgang Haken publish the first computer proof of the Four-Color Theorem, the first instance of a nontrivial theorem to be proven by a computer program.
- **1979:** Douglas Hofstadter publishes “Gödel, Escher, Bach: An Eternal Golden Braid,” coining the term “Quine” to describe self-referential programs. This book also is the source of the **MU** puzzle we saw when exploring induction for the first time.
- **1993:** Stephen Yablo publishes “Paradox without Self-Reference,” introducing what is now called Yablo's Paradox.
- **2000:** The Clay Mathematics Institute offers the Millenium Prizes for seven open mathematical problems, starting at $1,000,000 bounty for resolving the $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$ problem.
- **2015:** Laszlo Babai presents a proof that the graph isomorphism problem, a longtime candidate for an **NP**-intermediate problem, can be solved in “quasipolynomial time” (still bigger than a polynomial, but not by much). The purported proof is exciting news among the CS theory community and might lead to some major advances in the field.