The Big Picture

Announcements

- Problem Set 9 due right now. We'll release solutions right after lecture.
  - ***Congratulations – you're done with CS103 problem sets!***
- ***Please evaluate this course on Axess!*** Your feedback really does make a difference.
- There's a fun and completely optional handout "Timeline of CS103 Results" up on the CS103 website. It details the history of all the results from this course.

Final Exam Logistics

- Final exam is Wednesday, December 9 from 3:30PM – 6:30PM in Cemex Auditorium.
- There's a practice exam up online. Solutions are now available in hardcopy in Gates.
- Please feel free to ask us questions on Piazza over the next couple of days. We want you to understand this material!

The Big Picture

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Cantor's Theorem: $|S| < |\wp(S)|$

Corollary: Unsolvable problems exist.

What problems can
be solved by computers?

First, we need to learn how to prove results with certainty.

Otherwise, how can we know for sure that we're right about anything?

We also should be sure we have some rules about reasoning itself.

Let's add some logic into the mix.

Let's study a few common discrete structures.

That way, we know how to model connected structures and relationships.

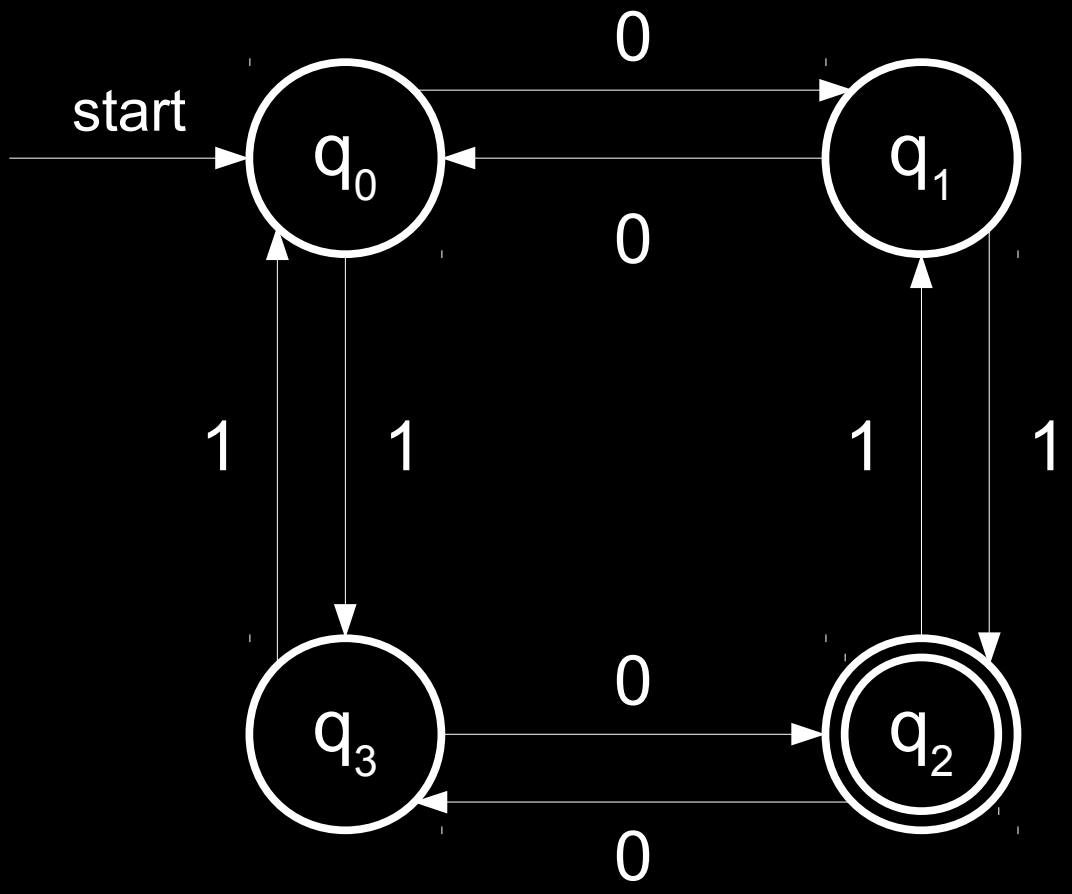
We also need to prove things about processes that proceed step-by-step.

So let's learn induction.

Okay! So now we're ready to go!

What problems are unsolvable?

Well, first we need a
definition of a computer!

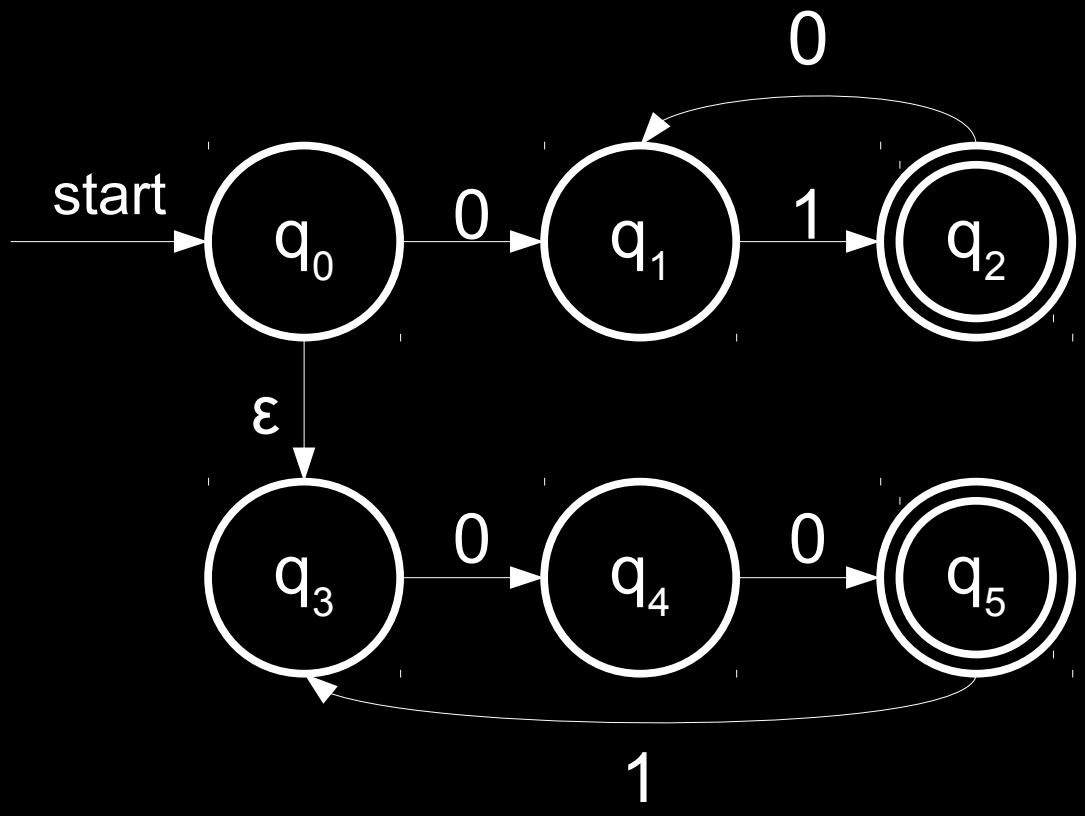

start
q_0
q_1
q_2
q_3
0
0
1
1
1
1
0
0

Cool! Now we have a model of a computer!

We're not quite sure what we can solve at this point, but that's okay for now.

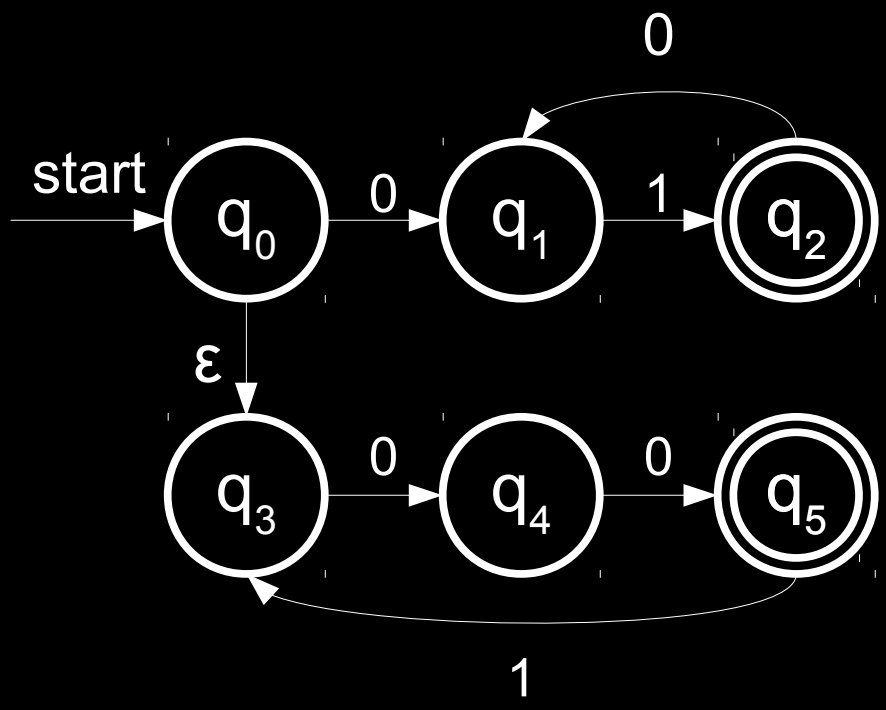
Let's call the languages we can capture this way the ***regular languages***.

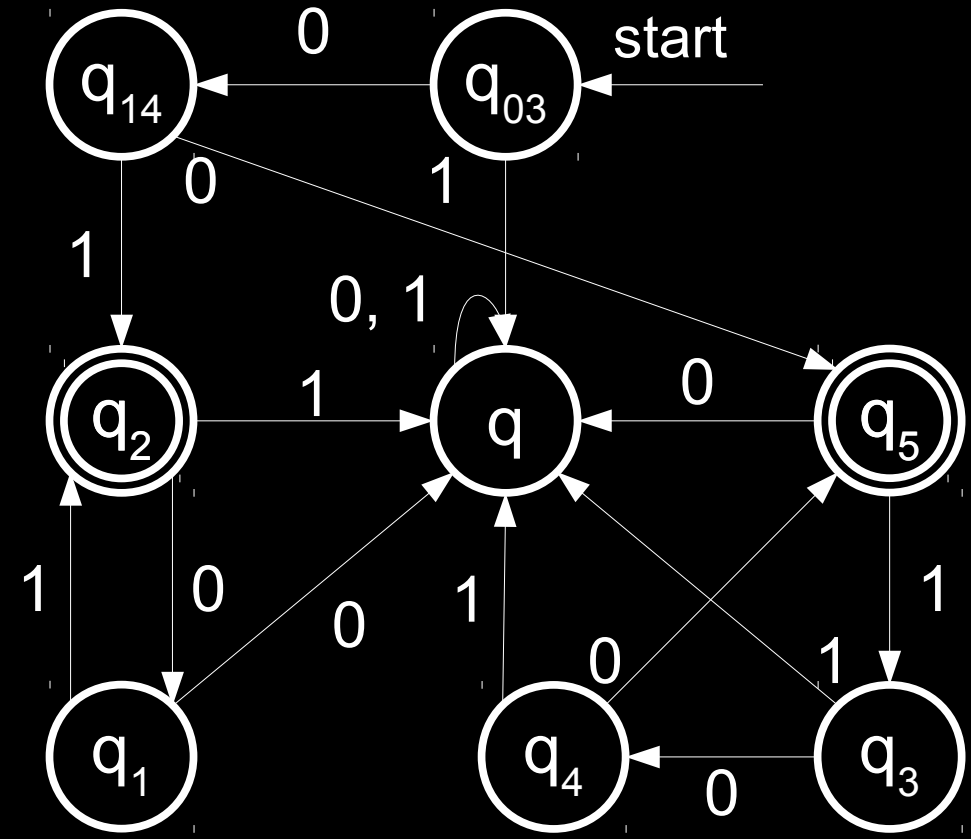
I wonder what other
machines we can make?


0
start
q_0
0
q_1
1
q_2
ε
q_3
0
q_4
0
q_5
1

Wow! Those new machines are way cooler than our old ones!

I wonder if they're more powerful?

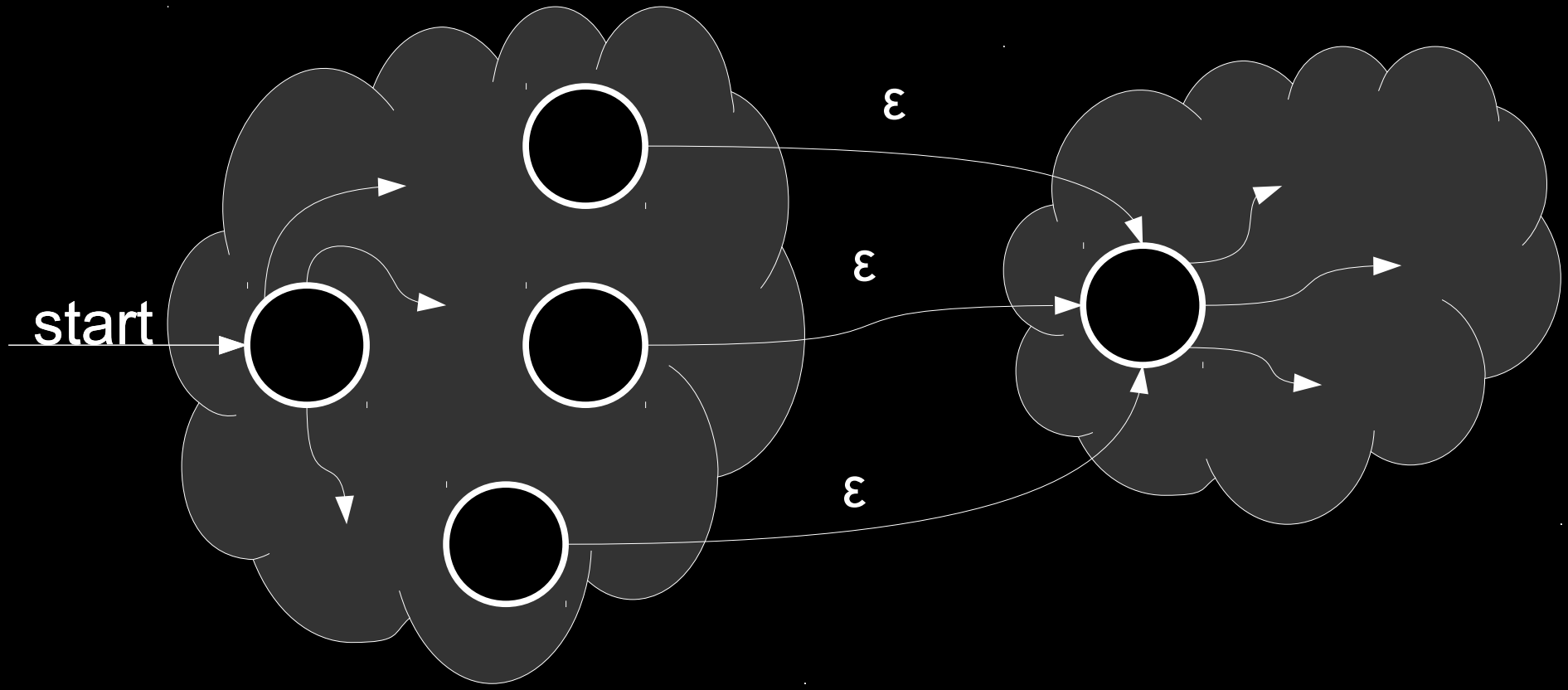

start
q_0
q_1
q_2
q_3
q_4
q_5
0
1
ε


start
q_{14}
q_{03}
q_2
q
q_5
q_1
q_4
q_3
0
1
0, 1

Wow! I guess not. That's surprising!

So now we have a new way of modeling computers with finite memory!

I wonder how we can combine these machines together?


start
ε
ε
ε

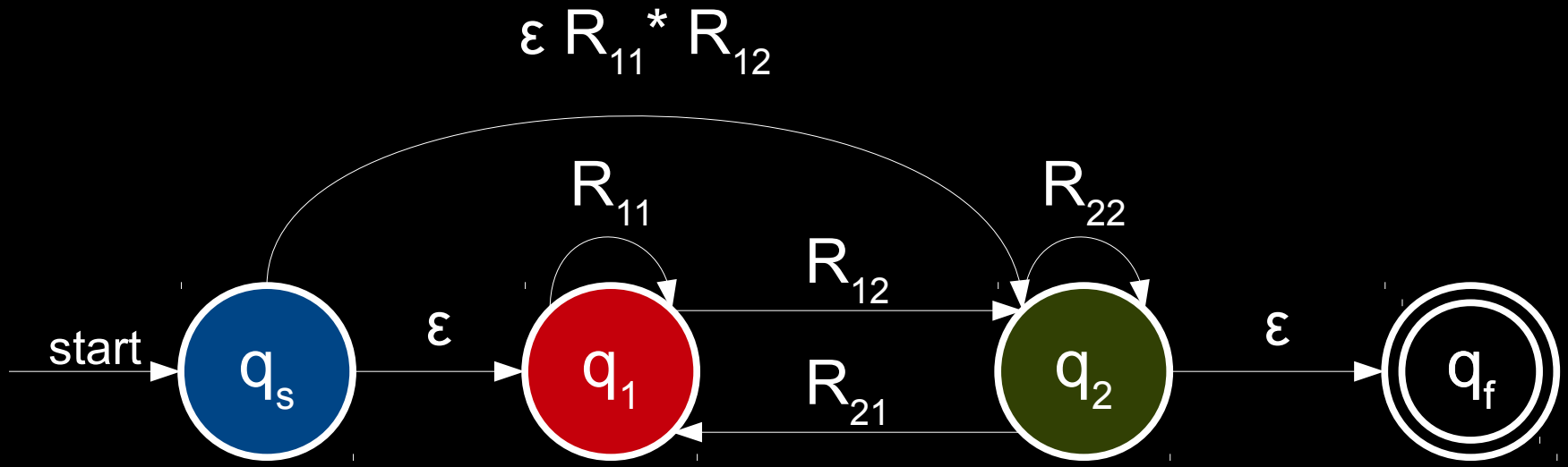
Cool! Since we can glue machines together, we can glue languages together as well.

How are we going to do that?

$$a^{+}(.a^{+})^{*}@a^{+}(.a^{+})^{+}$$

Wow! We've got a new way
of describing languages.

So what sorts of languages
can we describe this way?


ε R11* R12
R11
R22
R12
start
ε
ε
R21
qs
q1
q2
qf

Awesome! We got back the
exact same class of languages.

It seems like all our models give us the same power! Did we get every language?

$$xw \in L$$
$$yw \notin L$$

Wow, I guess not.

But we did learn something cool:

We have just explored what problems can be solved with finite memory.

So what else is out there?

Can we describe languages another way?

$$S \rightarrow aX$$
$$X \rightarrow b \mid C$$
$$C \rightarrow Cc \mid \varepsilon$$

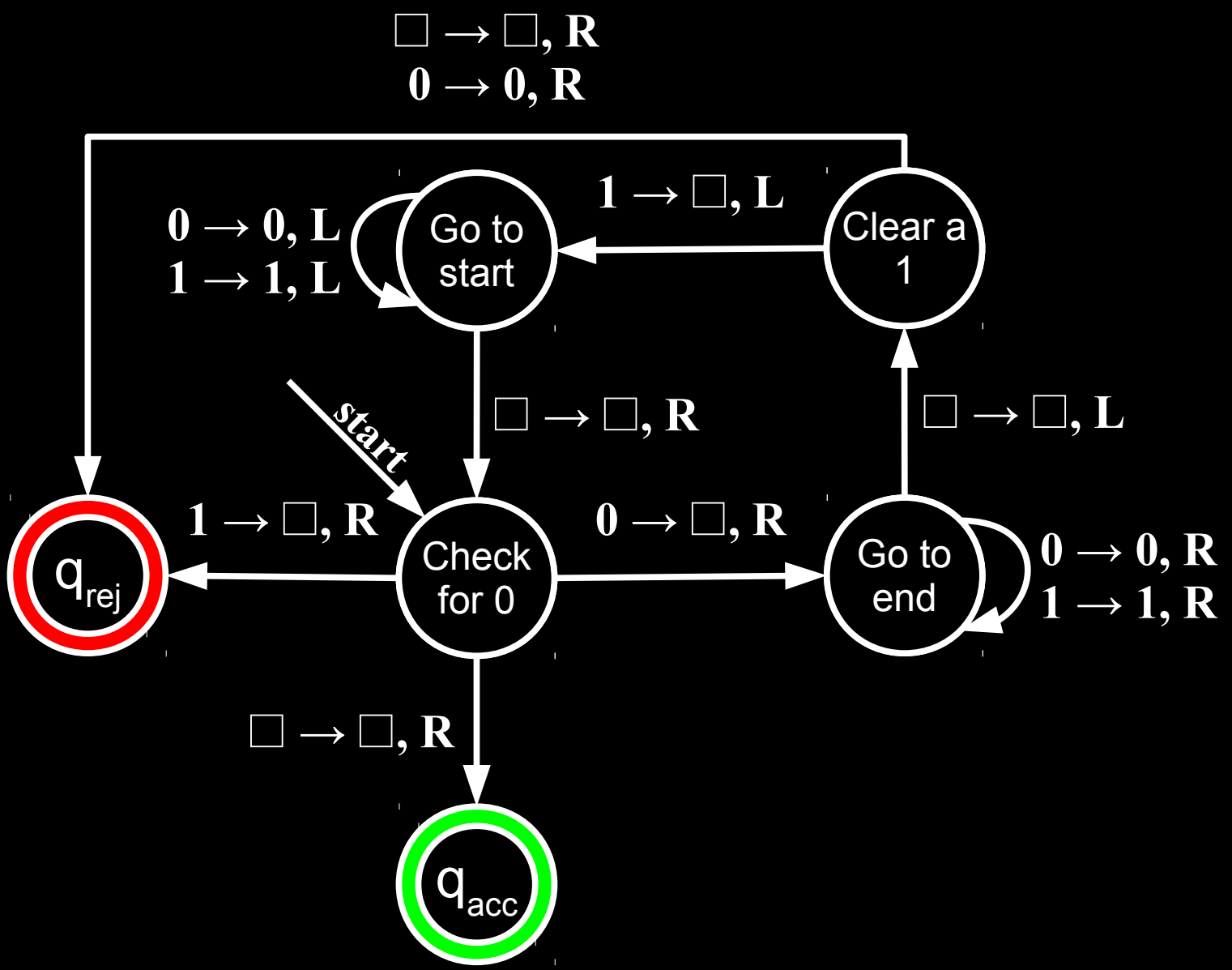
Awesome!

So, did we get every language yet?

$$|\Sigma^*| < |\wp(\Sigma^*)|$$

Hmmm... guess not.

So what if we make our memory a little better?


□ → □, R
0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
1 → □, R
0 → □, R
Check for 0
Go to end
0 → 0, R
1 → 1, R
q_rej
□ → □, R
q_acc

Cool! Can we make these more powerful?

Wow! Looks like we can't
get any more powerful.

(The ***Church-Turing thesis*** says
that this is not a coincidence!)

So why is that?

U_{TM} = "On input $\langle M, w \rangle$, where M is a TM and $w \in \Sigma^*$:

Set up the initial configuration of M running on w.

```
while (true) {
```

If M accepted w, then U_{TM} accepts $\langle M, w \rangle$.

If M rejected w, then U_{TM} rejects $\langle M, w \rangle$.

Otherwise, simulate one more step of M on w.

```
}
```

"

Wow! Our machines can simulate one another!

This is a theoretical justification for why all these models are equivalent to one another.

So... can we solve everything yet?

```cpp
#include <iostream>
#include <string>
#include <vector>
using namespace std;

const vector<string> kToPrint = {
  /* … */
};

void printProgramInQuotes() {
  for (string line: kToPrint) {
    cout << "  \"";
    for (char ch: line) {
      if (ch == '\"') cout << "\\\"";
      else if (ch == '\\') cout << "\\\\";
      else cout << ch;
    }
    cout << "\"," << endl;
  }
}

int main() {
  for (string line: kToPrint) {
    if (line == "@") printProgramInQuotes();
    else cout << line << endl;
  }
}
```

Weird! Programs can gain access
to their own source code!

Why does that matter?

```
int main() {
   string me = mySource();
   string input = getInput();

   if (willAccept(me, input)) {
      reject();
   } else {
      accept();
   }
}
```

Crazy! The power of self-reference immediately limits what TMs can do!

What if we think about solving problems in a different way?

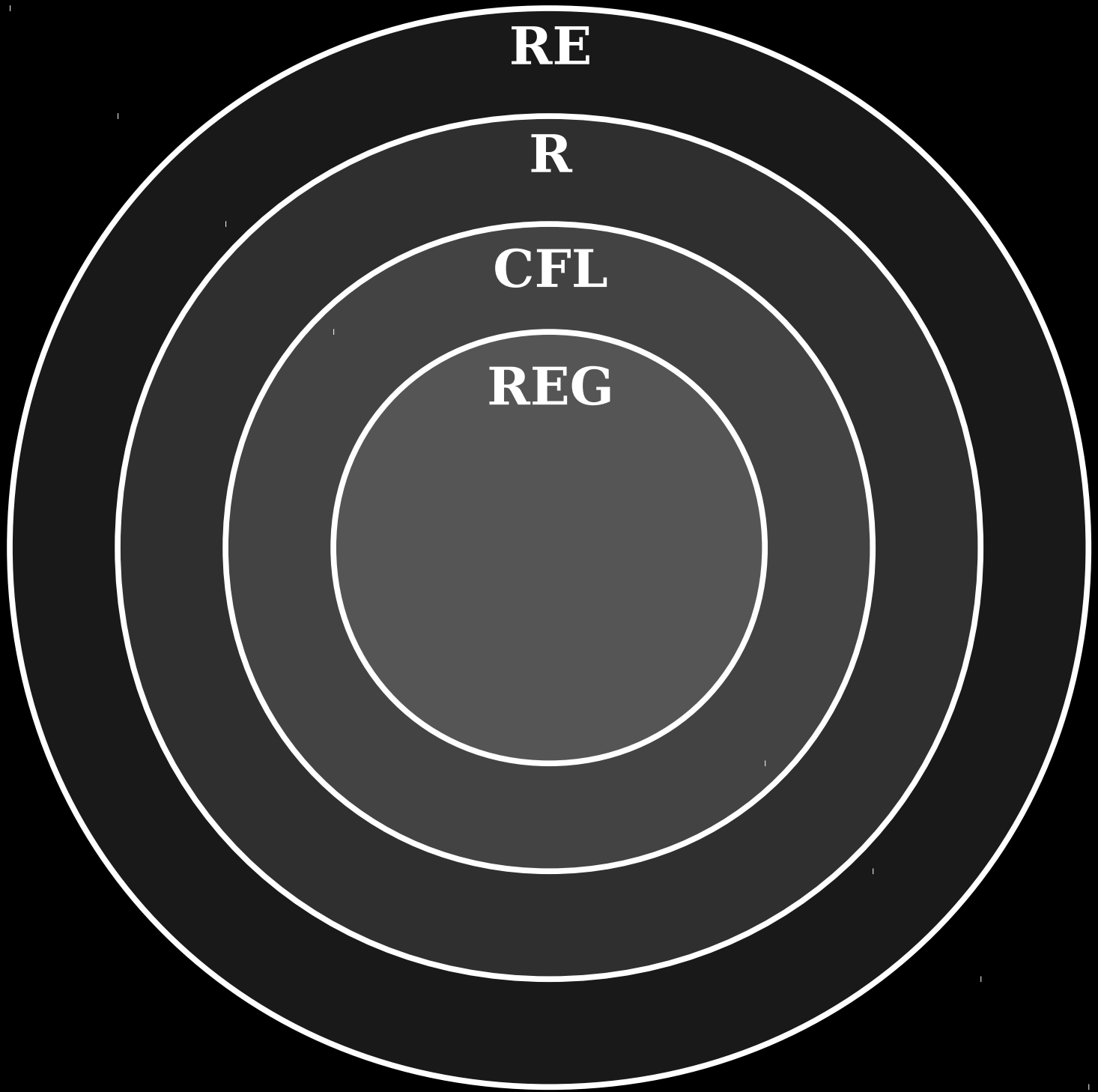
```
int main() {
   string me = mySource();
   string input = getInput();

   for (each string c) {
      if (imConvincedWillLoop(me, input, c) {
         accept();
      }
   }
}
```

	$\langle M_0 \rangle$	$\langle M_1 \rangle$	$\langle M_2 \rangle$	$\langle M_3 \rangle$	$\langle M_4 \rangle$	$\langle M_5 \rangle$	…
M_0	Acc	No	No	Acc	Acc	No	…
M_1	Acc	Acc	Acc	Acc	Acc	Acc	…
M_2	Acc	Acc	Acc	Acc	Acc	Acc	…
M_3	No	Acc	Acc	No	Acc	Acc	…
M_4	Acc	No	Acc	No	Acc	No	…
M_5	No	No	Acc	Acc	No	No	…
…	…	…	…	…	…	…	…

No	No	No	Acc	No	Acc	…

Oh great. Some problems
are impossible to solve.

But look what we learned along the way!


RE
R
CFL
REG

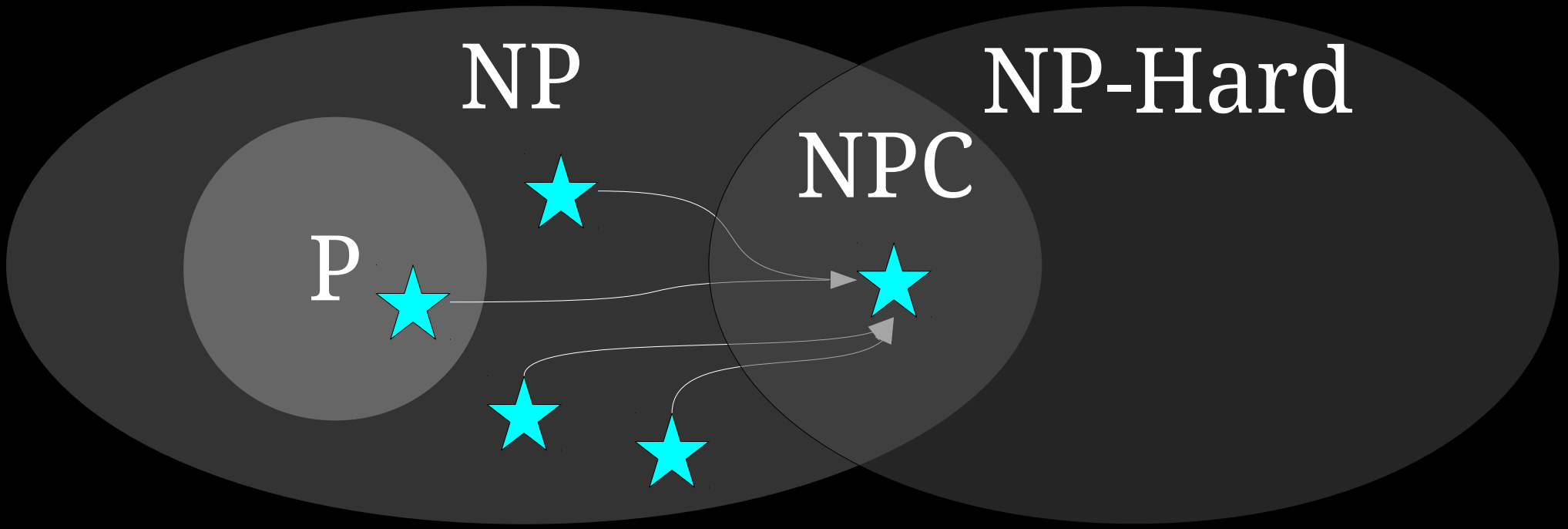
Wow. That's pretty deep.

So... what can we do efficiently?

So... how are you two related again?

No clue.

But what do we know about them?

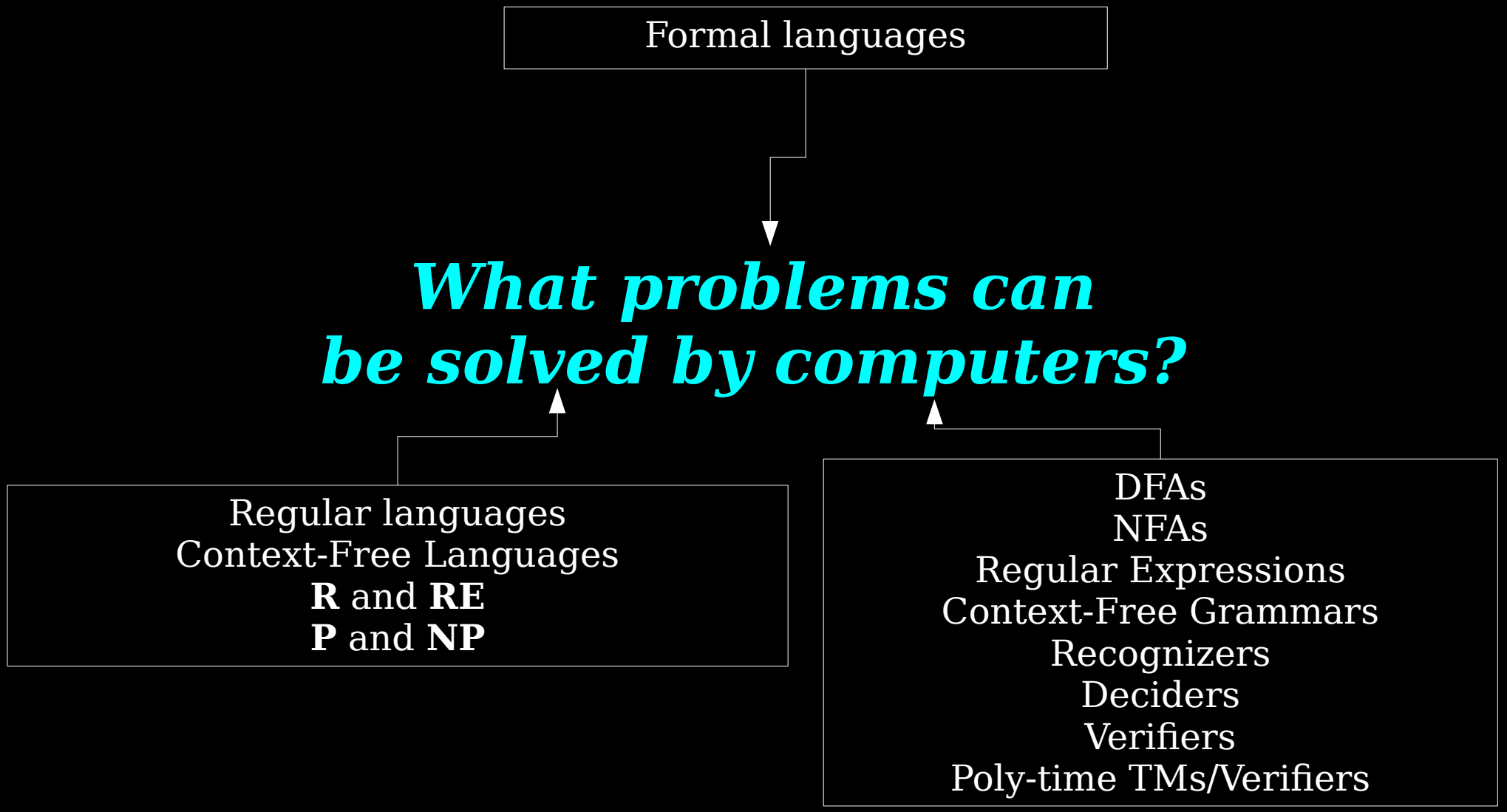

NP
NP-Hard
NPC
P

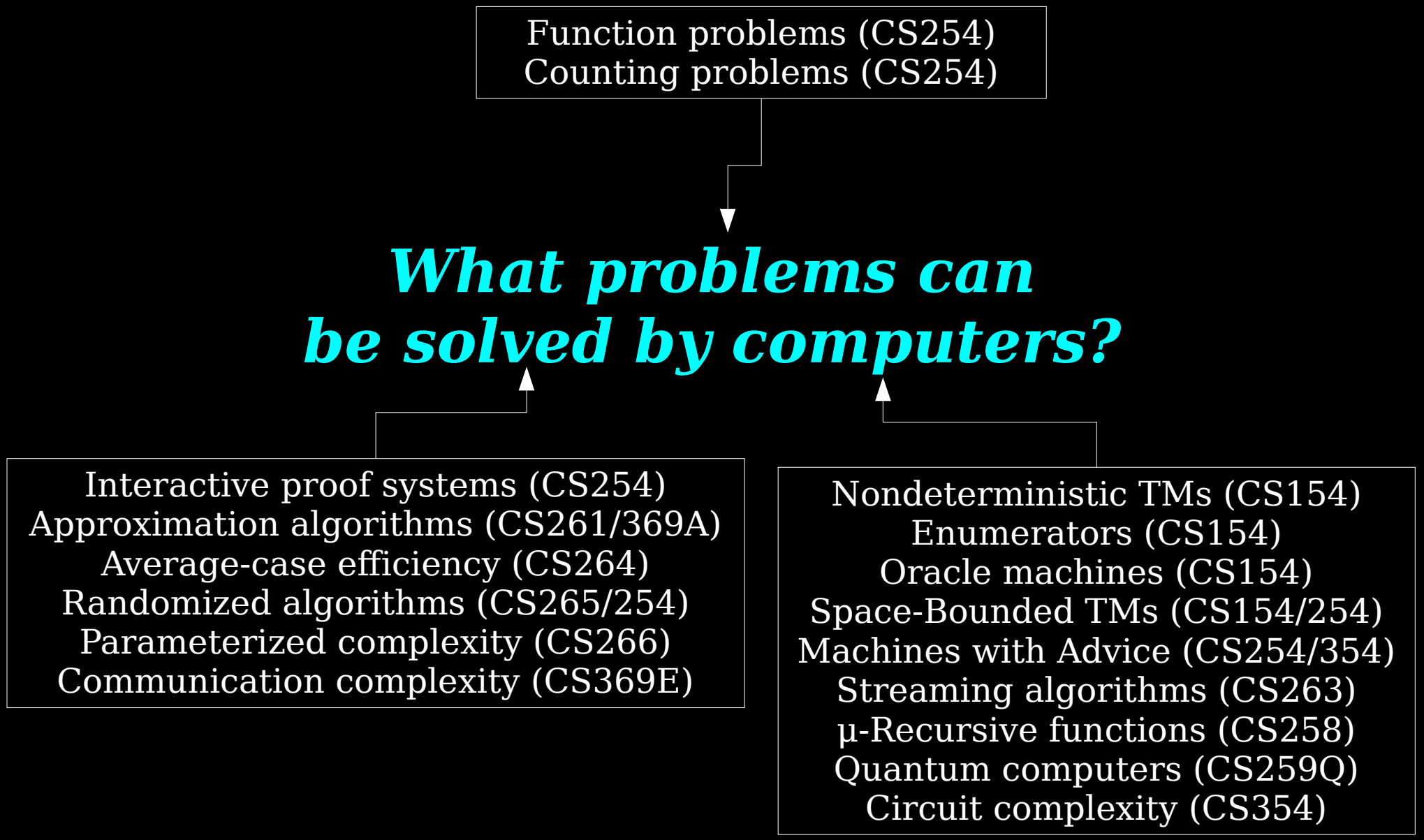
We've gone to the absolute limits of computing.

We've probed the limits of efficient computation.

Congratulations on making it this far!

What's next in CS theory?


Formal languages
What problems can be solved by computers?
Regular languages
Context-Free Languages
R and RE
P and NP
DFAs
NFAs
Regular Expressions
Context-Free Grammars
Recognizers
Deciders
Verifiers
Poly-time TMs/Verifiers


Function problems (CS254)
Counting problems (CS254)
What problems can be solved by computers?
Interactive proof systems (CS254)
Approximation algorithms (CS261/369A)
Average-case efficiency (CS264)
Randomized algorithms (CS265/254)
Parameterized complexity (CS266)
Communication complexity (CS369E)
Nondeterministic TMs (CS154)
Enumerators (CS154)
Oracle machines (CS154)
Space-Bounded TMs (CS154/254)
Machines with Advice (CS254/354)
Streaming algorithms (CS263)
μ-Recursive functions (CS258)
Quantum computers (CS259Q)
Circuit complexity (CS354)

How do we actually get the computer to effectively solve problems?

DFA design intuitions
Guess-and-check
Massive parallelism
Myhill-Nerode lower bounds
Verification
Polynomial-time reductions

How do we actually get the computer to effectively solve problems?

Algorithm design (CS161)
Efficient data structures (CS166)
Modern algorithmic techniques (CS168)
Approximation algorithms (CS261/CS369A)
Average-case efficient algorithms (CS264)
Randomized algorithms (CS265)
Parameterized algorithms (CS266)
Geometric algorithms (CS268)
Game-theoretic algorithms (CS364A/B)

What mathematical structures arise in computer science?

Sets
Propositional and First-Order Logic
Equivalence Relations
Strict Orders
Functions
Injections, Surjections, Bijections
Graphs
Planar and Bipartite Graphs
Polynomial-Time Reductions

What mathematical structures arise in computer science?

Groups, Rings, and Fields (Math 120, CS255)
Trees (Math 108, CS161)
Hash Functions (CS109, CS161, CS255)
Permutations (Math 120, CS255)
Monoids (CS149)
Lattices and Semilattices (CS143)
Control-Flow Graphs (CS143)
Vectors and Matrices (Math 113, CS205A)
Markov Decision Processes (CS221, CS229)
Modal Logic (Phil 154, CS224M)
Mapping Reductions (CS154)

Where does CS theory meet CS practice?

Finite state machines
Regular expressions
CFGs and programming languages
Password-checking
Autograding
“This program is not responding”
Polynomial-time reducibility
NP-hardness and **NP**-completeness

Where does CS theory meet CS practice?

Compilers (CS143)
Computational logic (CS157)
Program optimization (CS243)
Data mining (CS246)
Cryptography (CS255)
Programming languages (CS258)
Network protocol analysis (CS259)
Techniques in big data (CS263)
Graph algorithms (CS267)
Computational geometry (CS268)
Algorithmic game theory (CS364)

A Whole World of Theory Awaits!

What's being done here at Stanford?

Hardness results for easy problems
(Virginia Williams)

Algorithms ∩ Game theory
(Tim Roughgarden)

Learning patterns in randomness
(Greg Valiant)

Approximating NP-Hard Problems
(Moses Charikar)

Optimizing programs... randomly
(Alex Aiken)

Computing on encrypted data

(Dan Boneh)

Interpreting structure from shape
(Leonidas Guibas)

Lower bounds from upper bounds
(Ryan Williams)

So many options – what to do next?

Really enjoyed this class?

Give CS154 a try!

Interested in trying out CS?

Continue on to CS109!

Want to see this material come to life?

Check out CS143!

Want to tame infinity?

Dive into Math 161!

Like discrete structures?

Try Math 108!

Want to just go write code?

Take CS107!

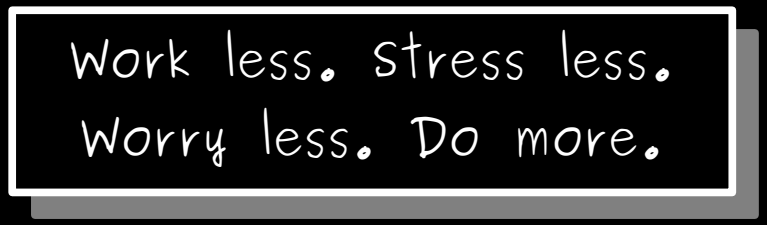
Keep on exploring! There's so much more to learn!

A Final “Your Questions”

“Hi Keith! We've been talking a lot about this million dollar question of whether or not **P = NP** and I was wondering if you ever took a shot at trying to solve this. If so how did you approach it?”

I did! I had the idea that you could give a TM a black box that has exactly one unlock code and that you could use that to show that there's a problem in **NP** that isn't in **P**. Turns out that doesn't work - there's no way to express a “black box” as input to a TM - but it did introduce me to the idea of “promise problems,” which are an active area of research!

“What will be your New Year's resolution?”

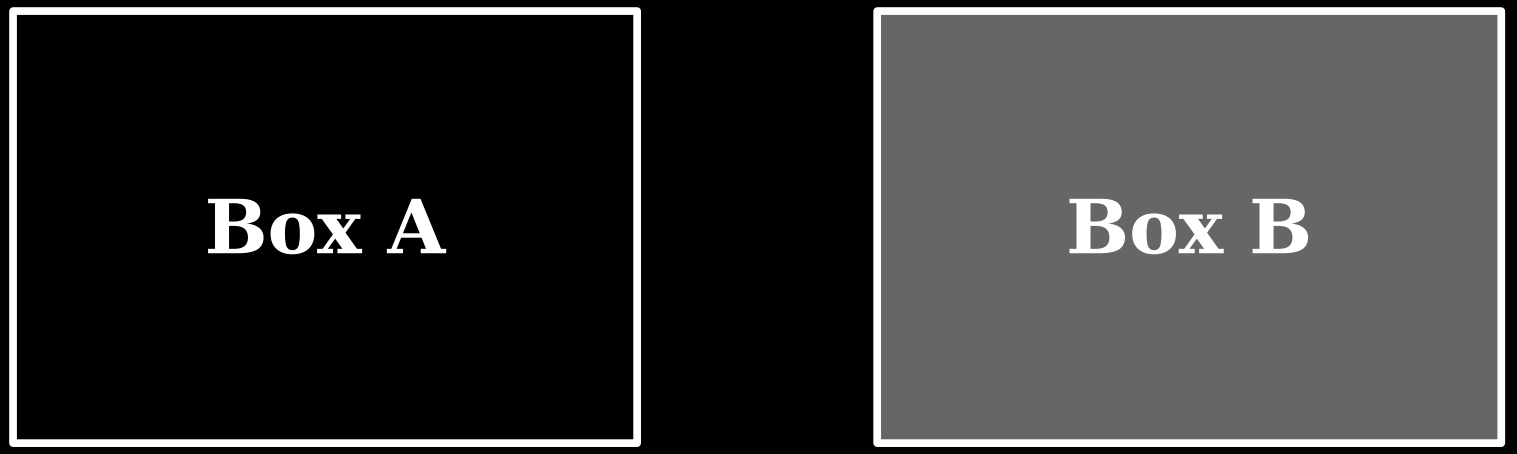


“What's your favorite topic from 103? Why?”

For me, it's the interplay between undecidability, unrecognizability, and the nature of mathematical truth.

“Are you a one-boxer or a two-boxer for Newcomb's problem?”

So let's talk about Newcomb's paradox…



Box A always has $1,000 in it.

You need to decide whether to take the contents of box A, or the contents of box A and box B.

If the ***Predictor*** guesses that you'll take just box B, then she puts $1,000,000 into box B before the game begins.

If the ***Predictor*** guesses that you'll take just box B, then she puts nothing into box B before the game begins.

Question: Do you choose to take both boxes, or just box B?

Anything else?

Final Thoughts

There are more problems to solve than there are programs capable of solving them.

There is so much more to explore and so many big questions to ask – ***many of which haven't been asked yet!***


Theory
Practice

You now know what problems we can solve, what problems we can't solve, and what problems we believe we can't solve efficiently.

My questions to you:

What problems will you ***choose*** to solve?
Why do those problems matter to you?
And how are you going to solve them?