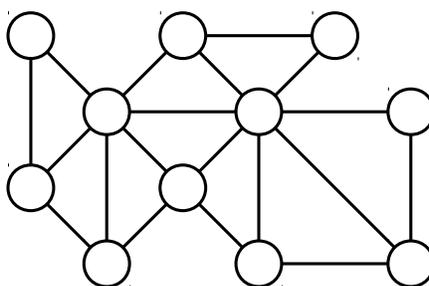


Extra Practice Problems 5

Here are some more review problems you can use to practice for the midterm. We'll release solutions on Friday, along with one final set of practice problems.

Graph Theory

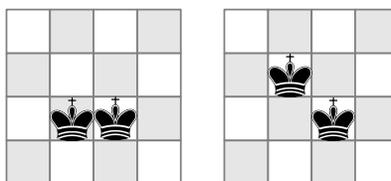
Consider the following graph:



- Prove that this graph is 4-colorable without actually finding a coloring for it.
- Prove that this graph is 3-colorable without actually finding a coloring for it.
- Prove that this graph is not 2-colorable.

A Clash of Kings

Chess is a game played on an 8×8 grid with a variety of pieces. In chess, no two king pieces can ever occupy two squares that are immediately adjacent to one another horizontally, vertically, or diagonally. For example, the following positions are illegal:



Prove that it is impossible to legally place 17 kings onto a chessboard.

Induction and Strict Orders

Let A be a set and $<_A$ be a strict order over A . Recall from Problem Set Four that a **chain in $<_A$** is a series of elements x_1, \dots, x_n drawn from A such that

$$x_1 <_A x_2 <_A \dots <_A x_n.$$

Prove, by induction, that if x_1, \dots, x_n is a chain in $<_A$ with $n \geq 2$ elements, then $x_1 <_A x_n$.

DFAs, NFAs, and Regular Expressions

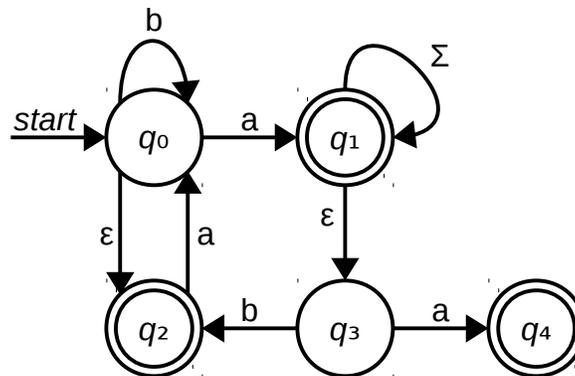
Here's a slightly tricky set of DFA, NFA, and regex design problems.

Let $\Sigma = \{ a, b \}$ and let $L = \{ w \in \Sigma^* \mid \text{the length of } w \text{ is a multiple of four and } w \text{ contains an even number of b's} \}$.

- i. Design a DFA for L .
- ii. Design an NFA for the *complement* of L .
- iii. Write a regular expression for the *complement* of L .

Automata Transformations

Consider the following NFA:



We'd like you to convert this into both a DFA and a regular expression using the standard algorithms we've discussed so far.

- i. Apply the subset construction to this NFA to convert it to a DFA.
- ii. Apply the state elimination algorithm to this NFA to convert it to a regular expression.