Finite Automata

Part Two

Recap from Last Time

Strings

- An ***alphabet*** is a finite, nonempty set of symbols called ***characters***.
  - Typically, we use the symbol **Σ** to refer to an alphabet.
- A ***string over an alphabet Σ*** is a finite sequence of characters drawn from Σ.
- Example: If Σ = {**a**, **b**}, here are some valid strings over Σ:

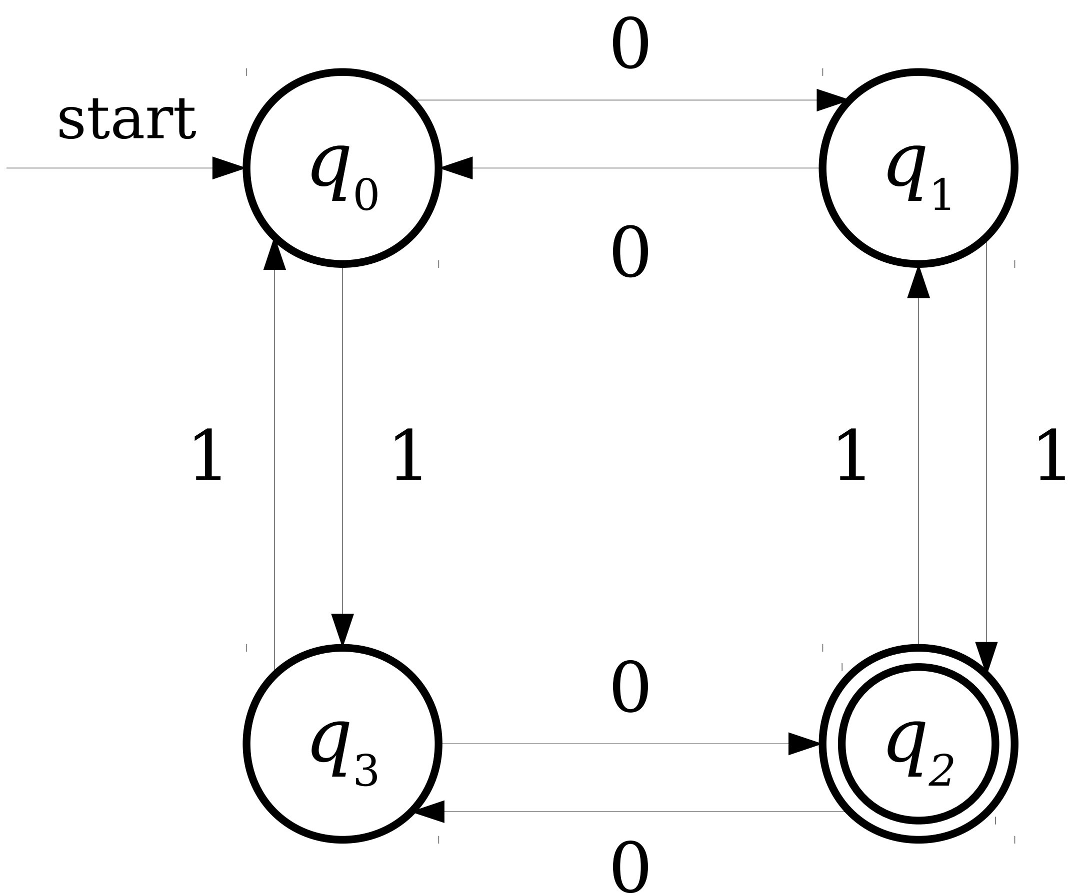
  a   **aabaaabbabaaabaaaabbb**   **abbababba**

- The ***empty string*** has no characters and is denoted **ε**.
- Calling attention to an earlier point: since all strings are finite sequences of characters from Σ, you cannot have a string of infinite length.

Languages

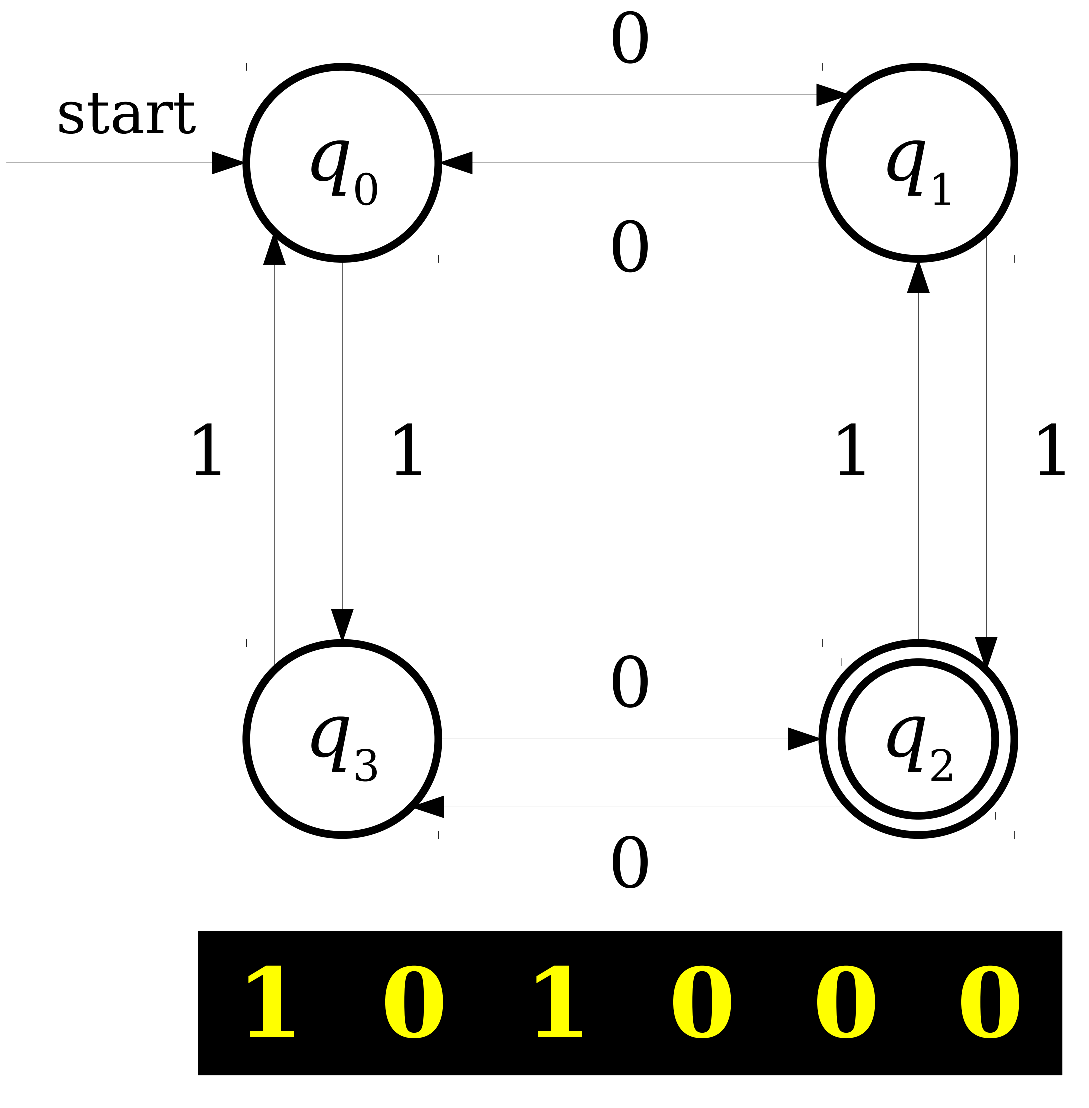
- A ***formal language*** is a set of strings.
- We say that L is a ***language over Σ*** if it is a set of strings over Σ.
- Example: The language of palindromes over Σ = {**a**, **b**, **c**} is the set
  - {ε, **a**, **b**, **c**, **aa**, **bb**, **cc**, **aaa**, **aba**, **aca**, **bab**, … }
- The set of all strings composed from letters in Σ is denoted **Σ***.
- Formally, we say that L is a language over Σ if $L \subseteq \Sigma^*$.

A Simple Finite Automaton



0 1 0 1 1 0

A Simple Finite Automaton



The ***language of an automaton*** is the set of strings that it accepts.

If D is an automaton, we denote the language of D as $\mathscr{L}(\boldsymbol{D})$.

$$\mathscr{L}(D) = \{ \ w \in \Sigma^* \mid D \text{ accepts } w \ \}$$

DFAs

- A **DFA** is a
  - **D**eterministic
  - **F**inite
  - **A**utomaton
- DFAs are the simplest type of automaton that we will see in this course.

DFAs, Informally

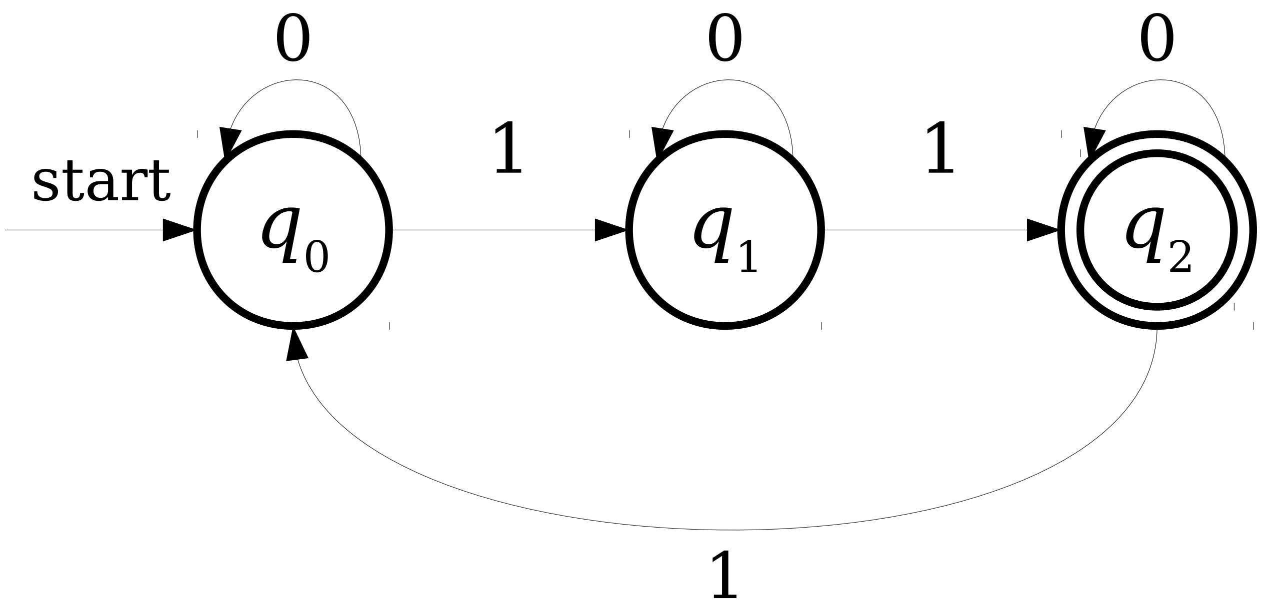
- A DFA is defined relative to some alphabet Σ.
- For each state in the DFA, there must be ***exactly one*** transition defined for each symbol in Σ.
  - This is the "deterministic" part of DFA.
- There is a unique start state.
- There are zero or more accepting states.

Designing DFAs

- At each point in its execution, the DFA can only remember what state it is in.
- **DFA Design Tip:** Build each state to correspond to some piece of information you need to remember.
  - Each state acts as a “memento” of what you're supposed to do next.
  - Only finitely many different states ≈ only finitely many different things the machine can remember.

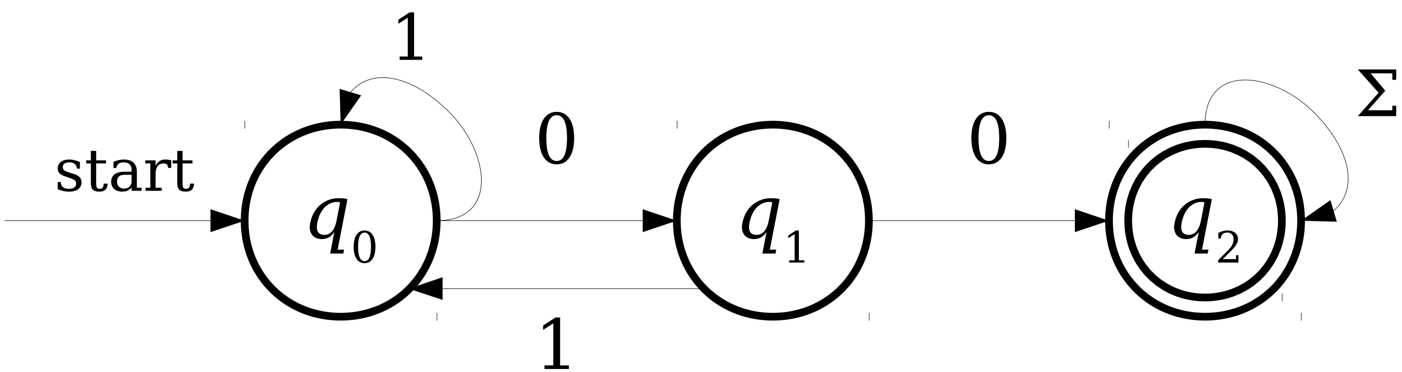
Recognizing Languages with DFAs

$L = \{ w \in \{\mathtt{0}, \mathtt{1}\}^* \mid$ the number of $\mathtt{1}$'s in w is congruent to two modulo three $\}$



Recognizing Languages with DFAs

$L = \{\ w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ contains } \mathtt{00} \text{ as a substring } \}$



More Elaborate DFAs

$L = \{ w \in \{\mathtt{a}, \mathtt{*}, \mathtt{/}\}^* \mid w \text{ represents a C-style comment } \}$

Suppose the alphabet is

$\Sigma = \{ \mathtt{a}, \mathtt{*}, \mathtt{/} \}$

Try designing a DFA for comments! Some test cases:

ACCEPTED	REJECTED
`/*a*/`	`/**`
`/**/`	`/**/a/*aa*/`
`/***/`	`aaa/**/`
`/*aaa*aaa*/`	`/*/`
`/*a/a*/`	`/**a/`

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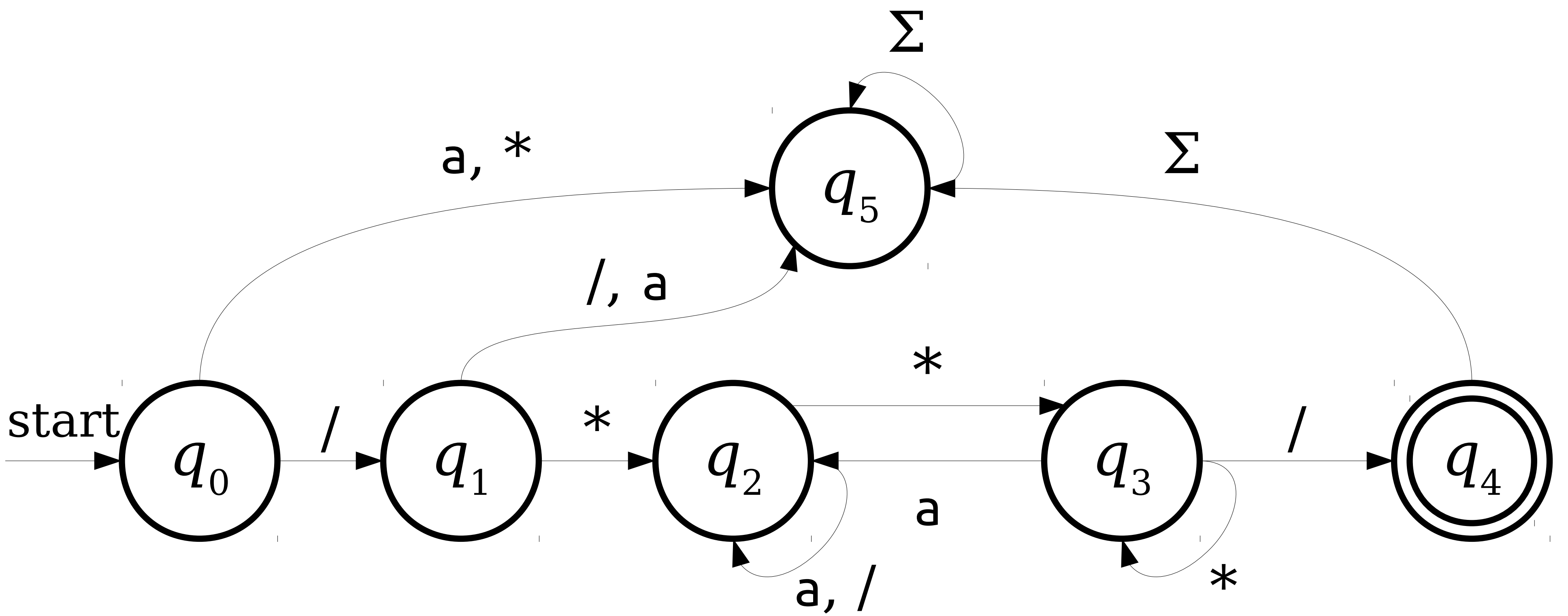

New Stuff!

More Elaborate DFAs

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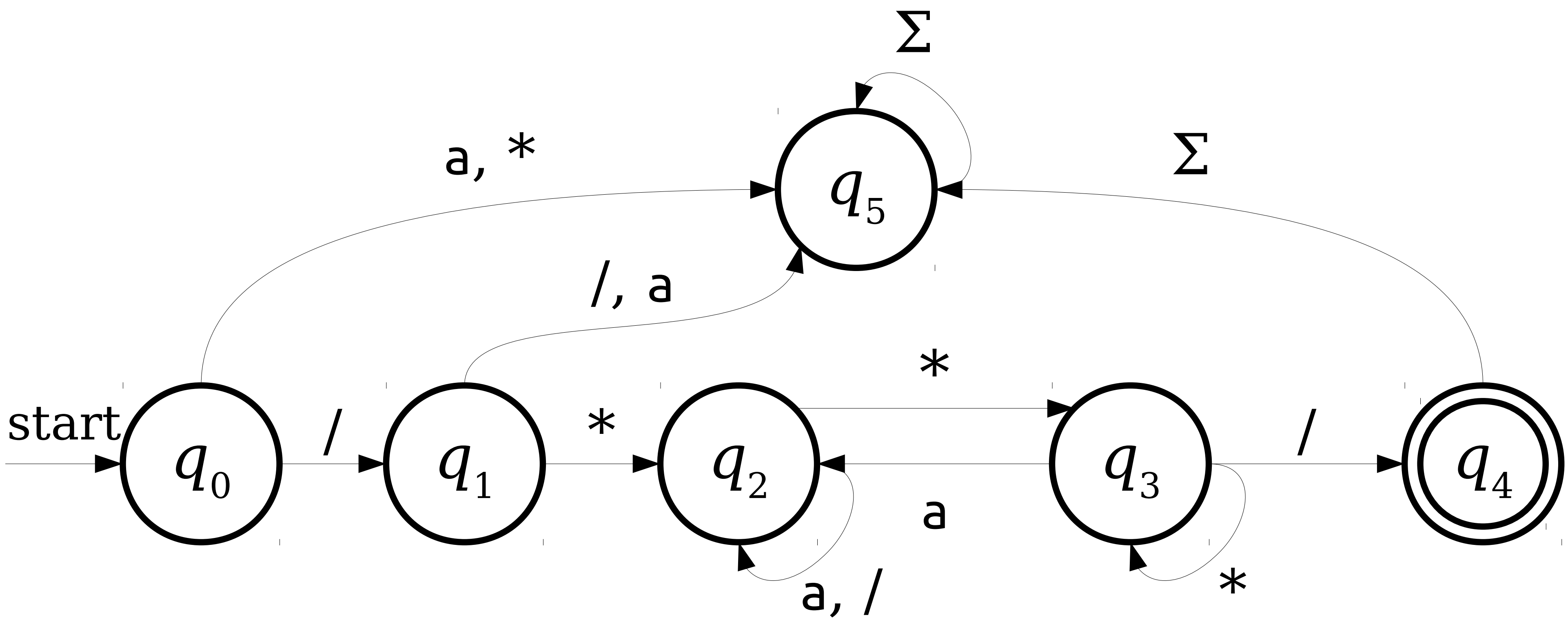
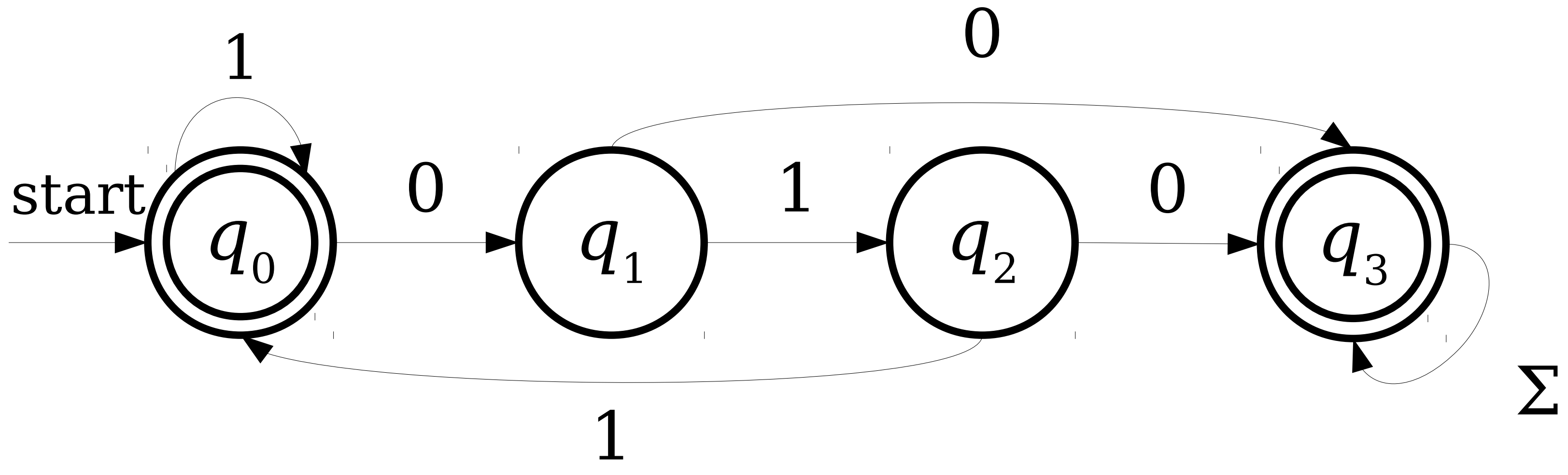


Tabular DFAs



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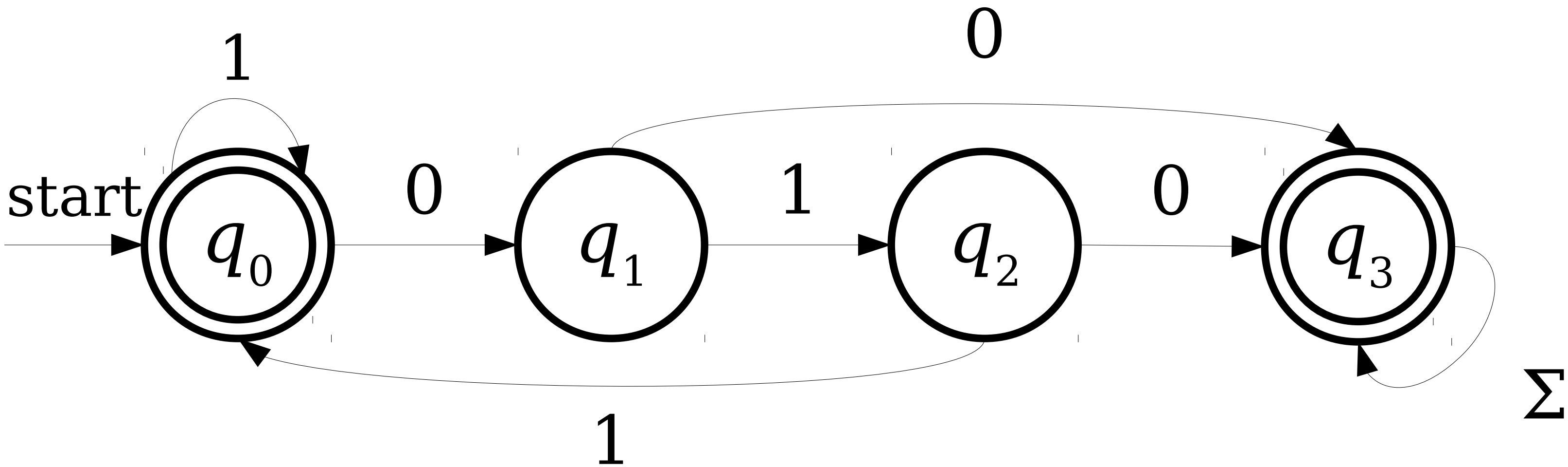
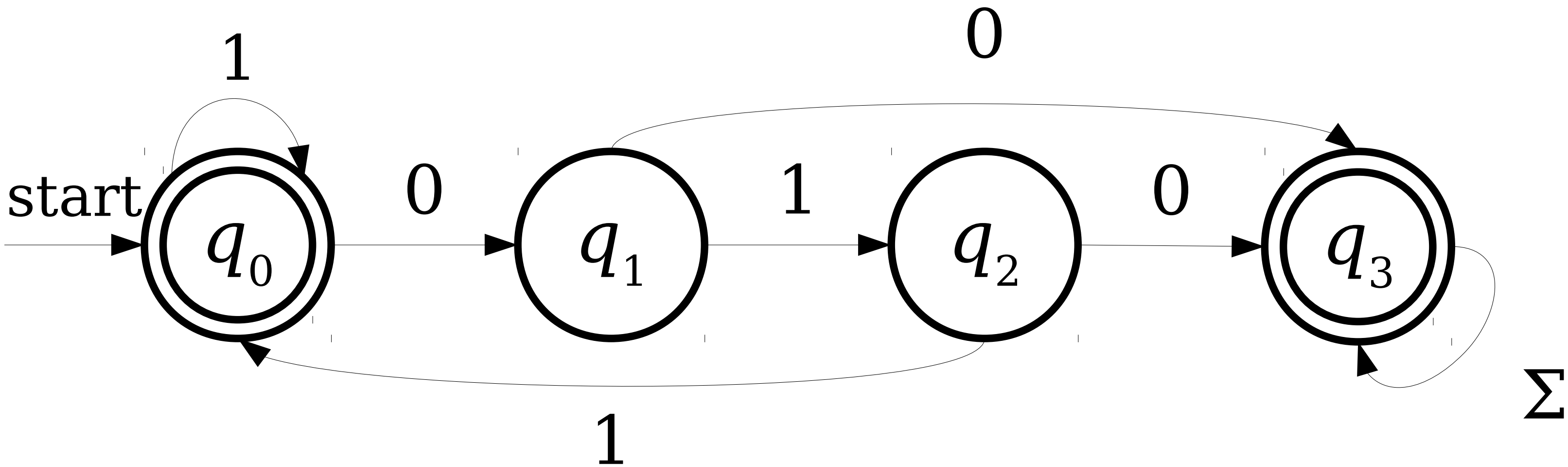


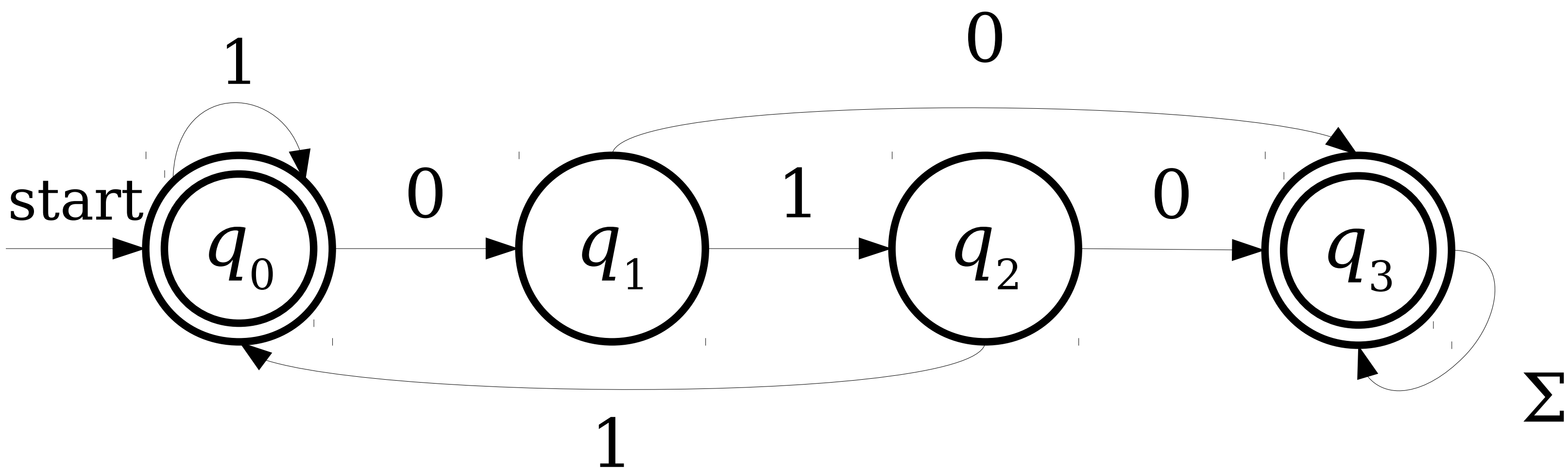
	0	1
q_0		
q_1		
q_2		
q_3		

Tabular DFAs



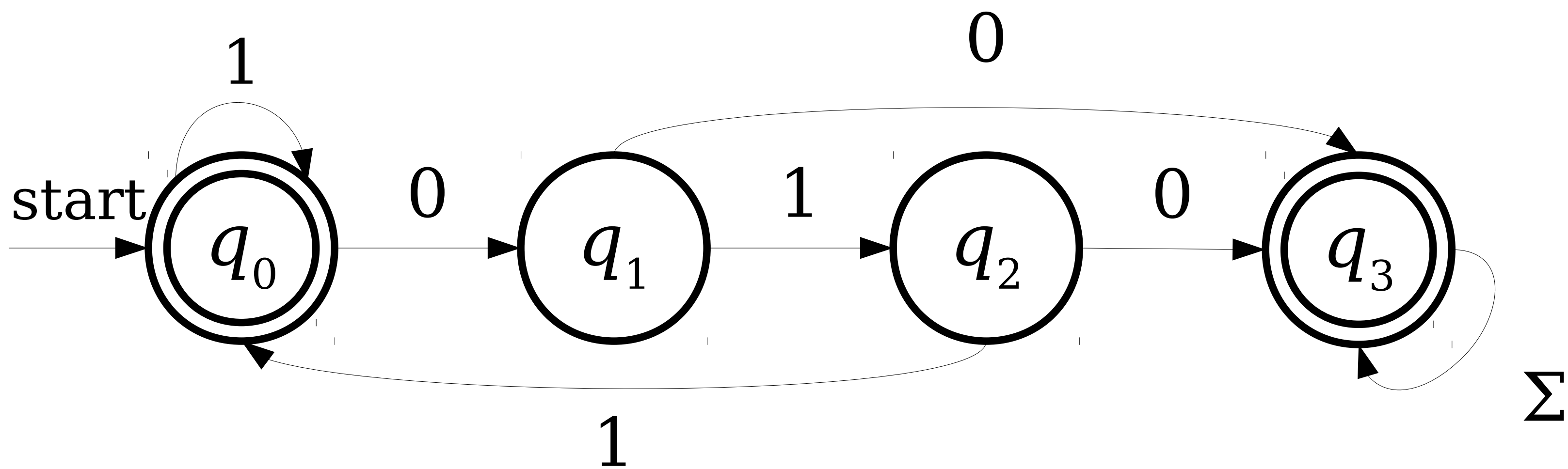
	0	1
q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
q_3	q_3	q_3

Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

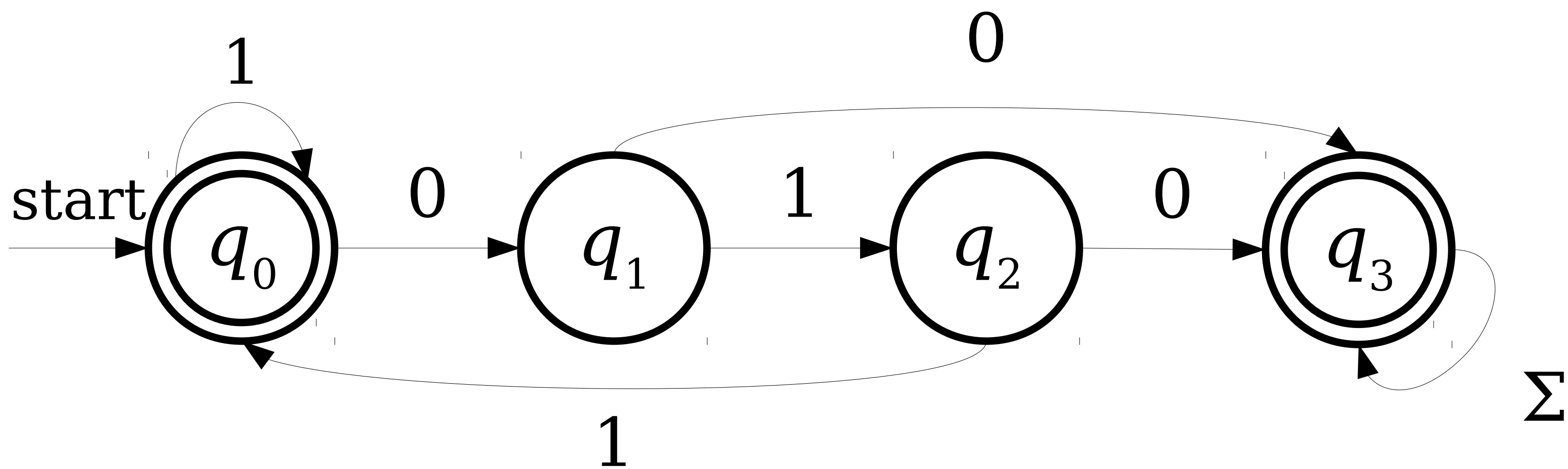
Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

These stars indicate accepting states.

Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

Since this is the first row, it's the start state.

Code? In a Theory Course?

```
int kTransitionTable[kNumStates][kNumSymbols] = {
     {0, 0, 1, 3, 7, 1, …},
      …
};
bool kAcceptTable[kNumStates] = {
    false,
    true,
    true,
    …
};
bool SimulateDFA(string input) {
    int state = 0;
    for (char ch: input)
        state = kTransitionTable[state][ch];
    return kAcceptTable[state];
}
```

The Regular Languages

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

The Complement of a Language

- Given a language $L \subseteq \Sigma^*$, the ***complement*** of that language (denoted $\overline{L}$) is the language of all strings in Σ^* that aren't in L.
- Formally:

$$\overline{L} = \{ w \mid w \in \Sigma^* \land w \notin L \}$$

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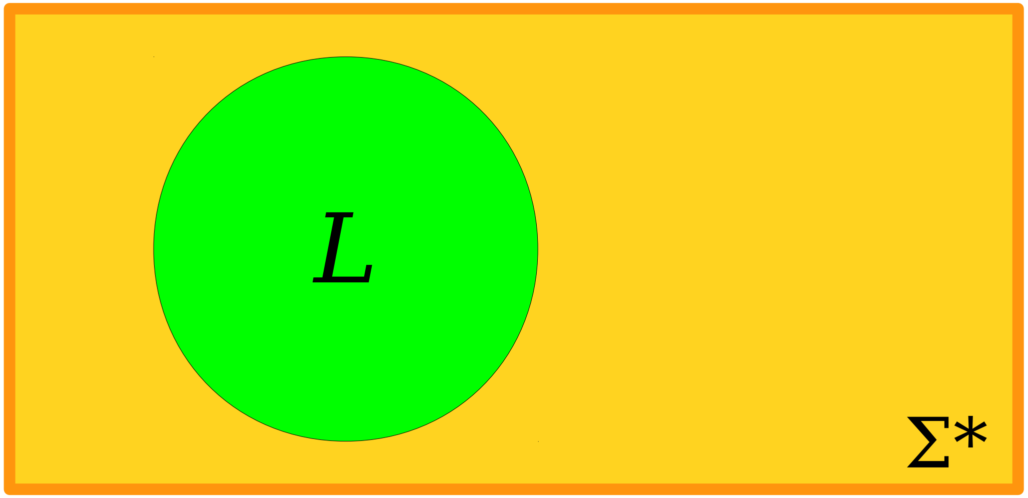

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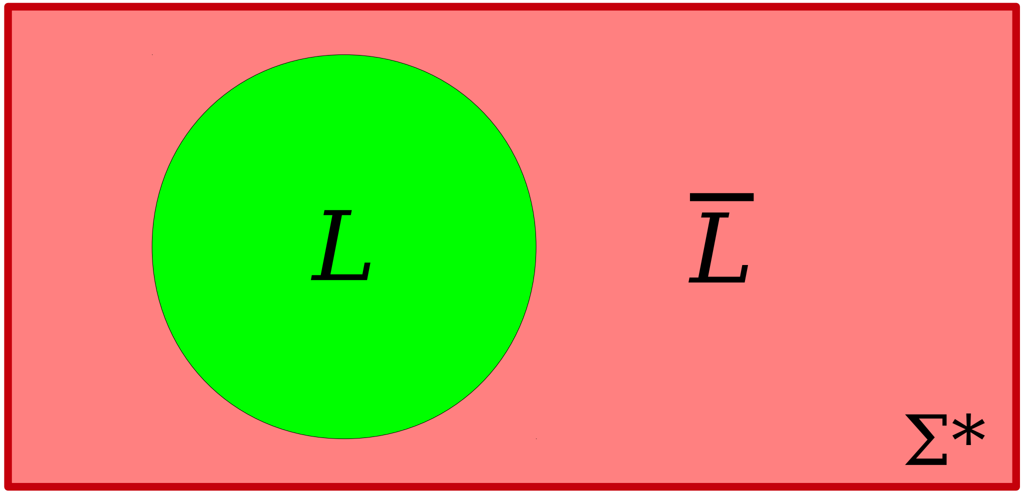
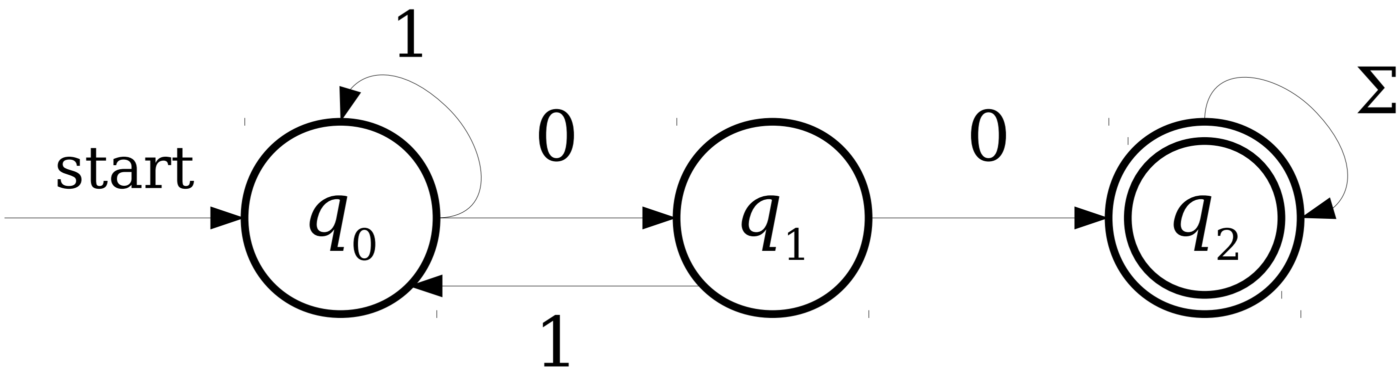


Complements of Regular Languages

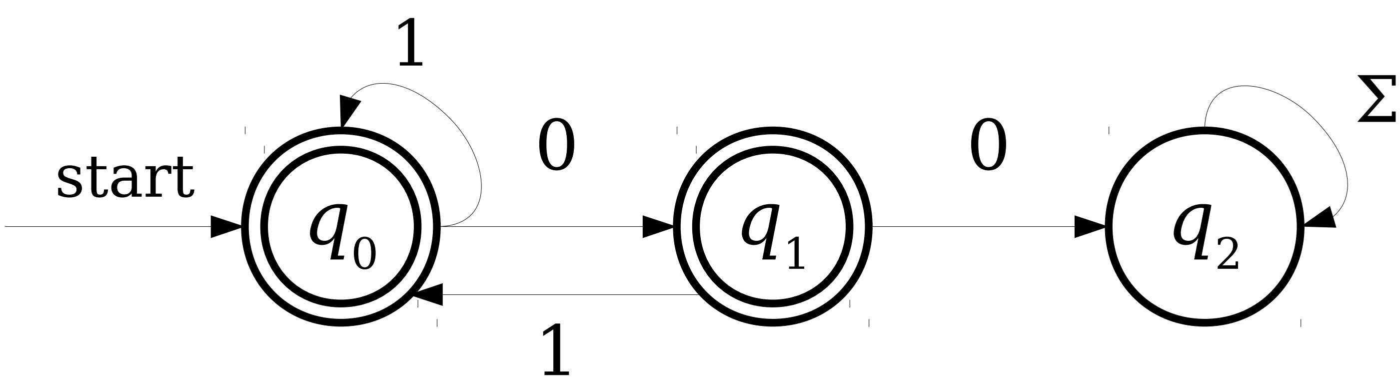
- As we saw a few minutes ago, a ***regular language*** is a language accepted by some DFA.
- ***Question:*** If L is a regular language, is $\overline{L}$ necessarily a regular language?
- If the answer is "yes," then if there is a way to construct a DFA for L, there must be some way to construct a DFA for $\overline{L}$.
- If the answer is "no," then some language L can be accepted by some DFA, but $\overline{L}$ cannot be accepted by any DFA.

Complementing Regular Languages

$L = \{ w \in \{0, 1\}^* \mid w \text{ contains } 00 \text{ as a substring } \}$

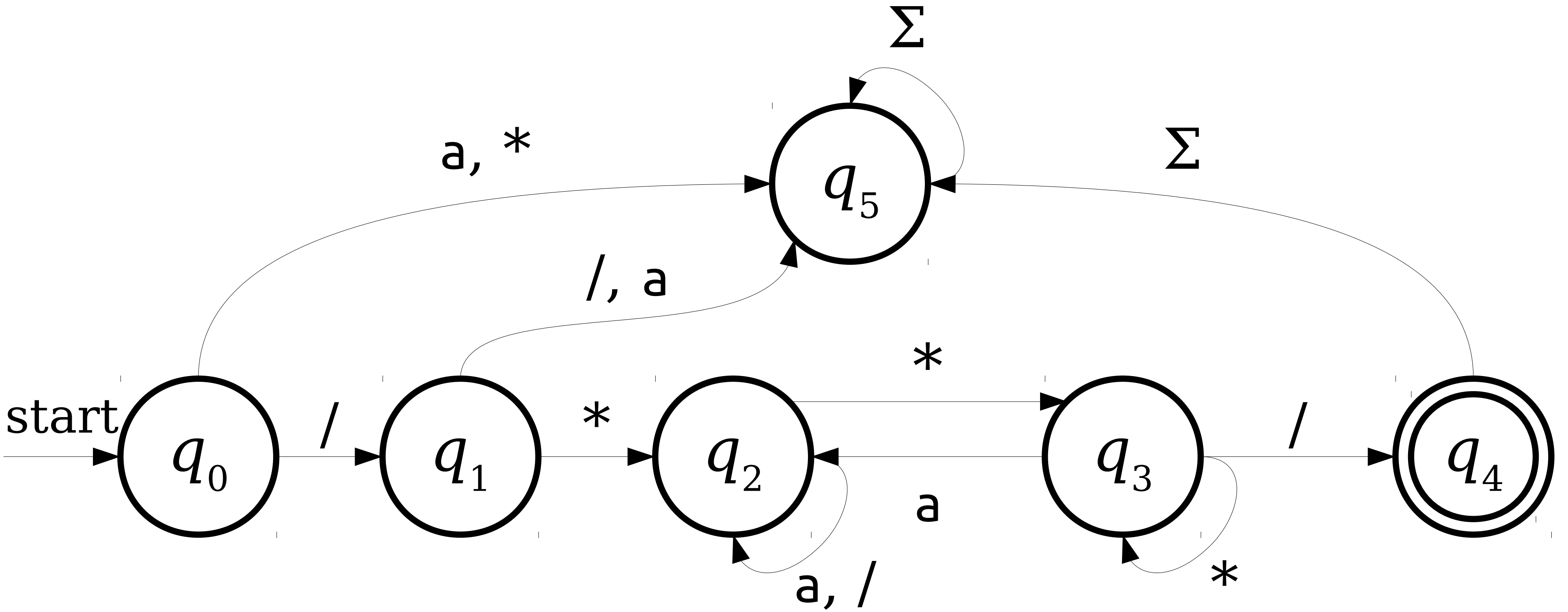


$\overline{L} = \{ w \in \{0, 1\}^* \mid w \textbf{\textit{ does not}} \text{ contain } 00 \text{ as a substring } \}$



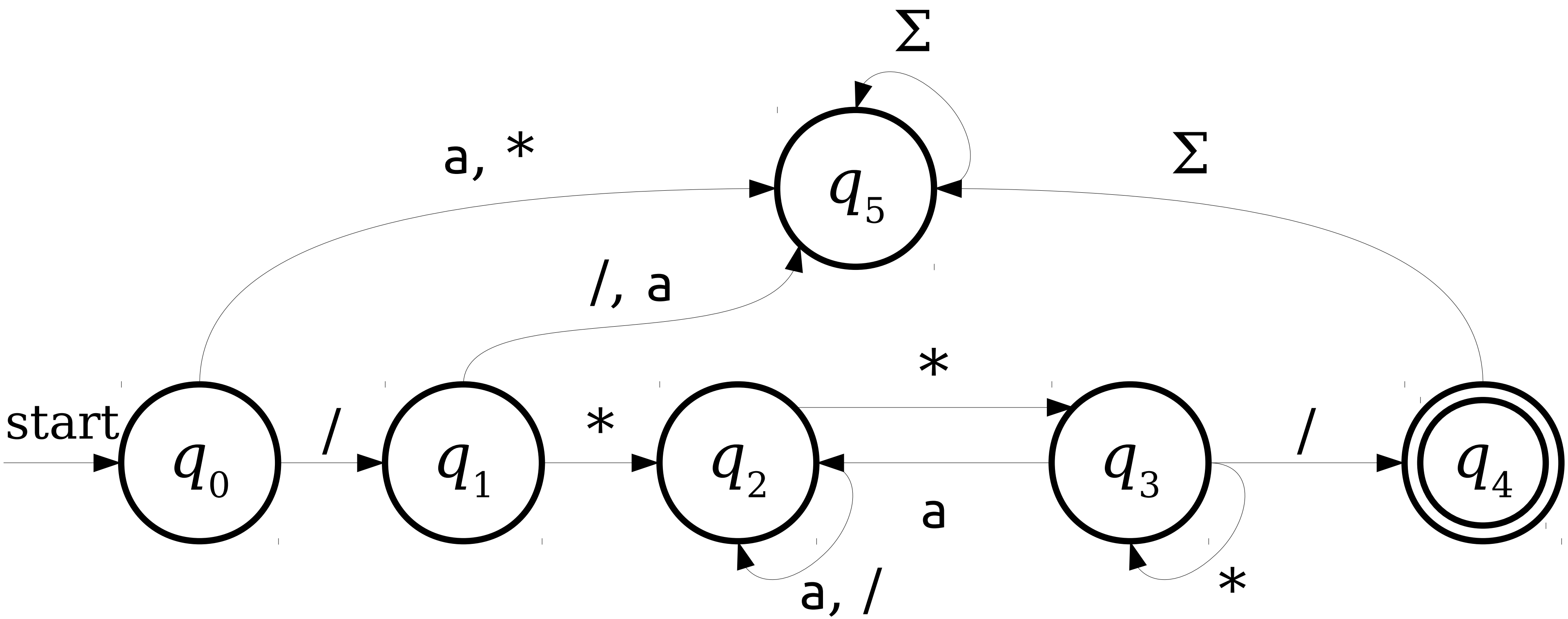
More Elaborate DFAs

$L = \{ w \in \{$a, *, /$\}^* \mid w$ represents a C-style comment $\}$



More Elaborate DFAs

$\overline{L}$ = { w ∈ {a, *, /}* | *w doesn't* represent a C-style comment }



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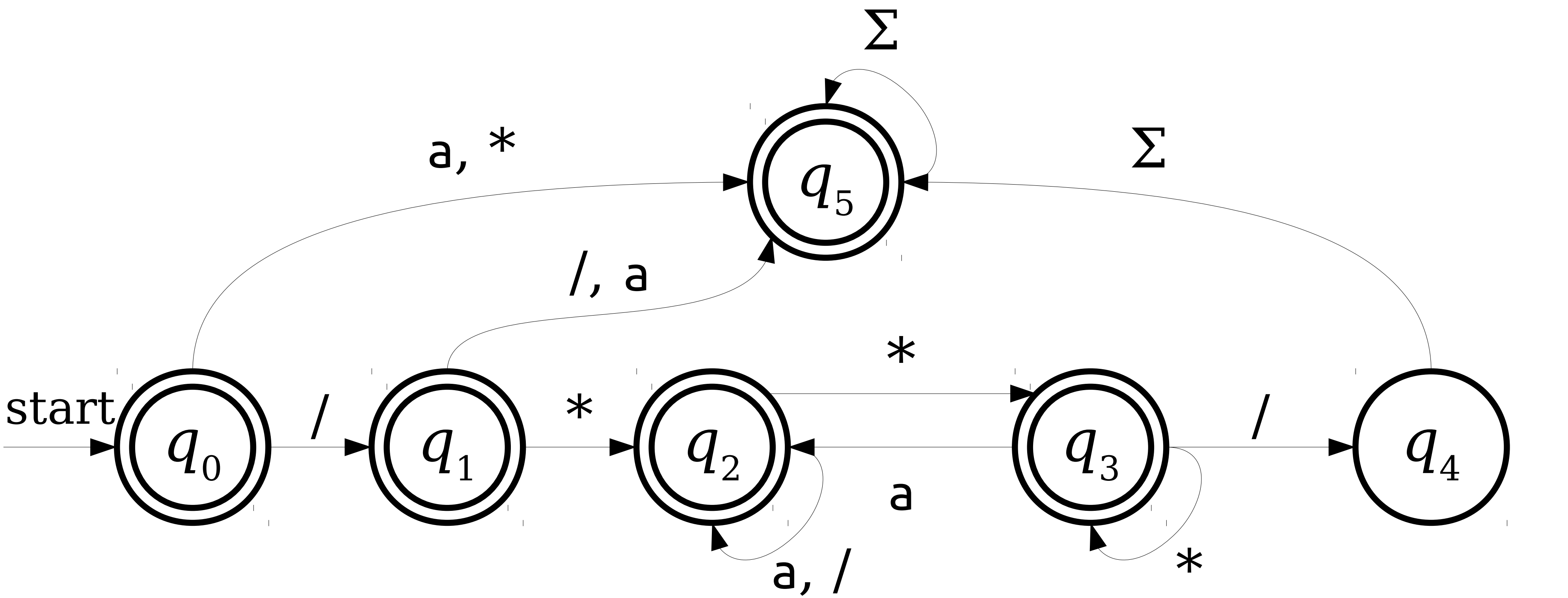
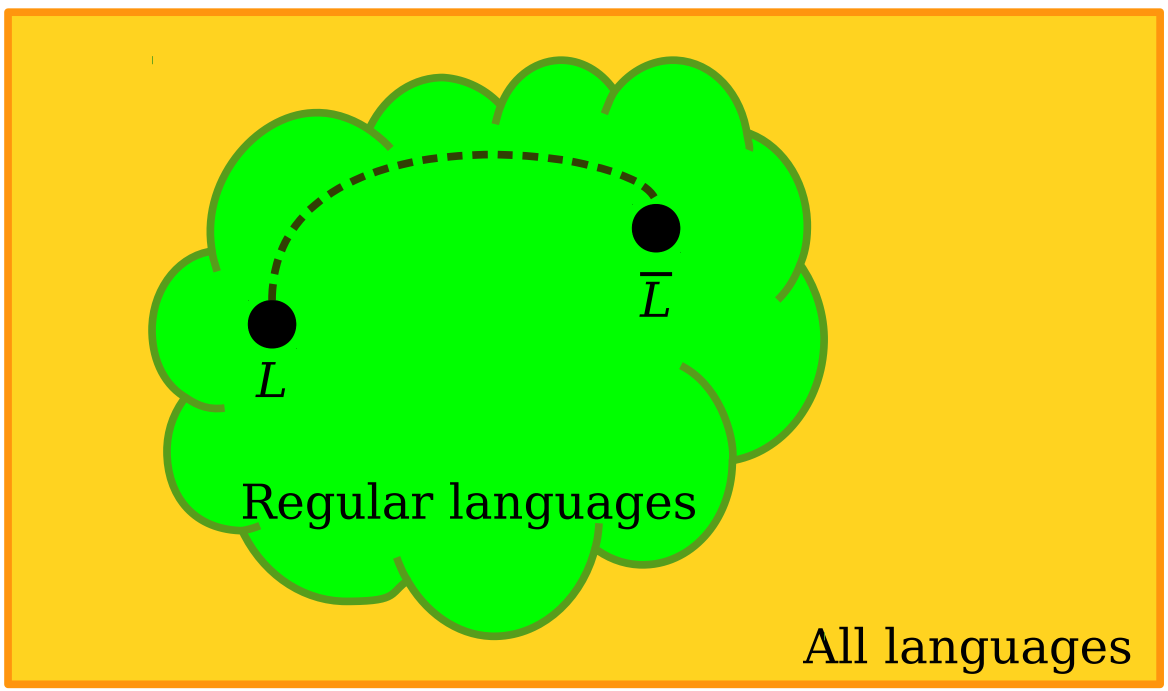


Closure Properties

- ***Theorem:*** If L is a regular language, then $\overline{L}$ is also a regular language.
- As a result, we say that the regular languages are ***closed under complementation***.



Time-Out For Announcements!

Midterm Logistics

- Our first midterm is tonight from 7PM – 10PM.
- Room locations divvied up by last (family) name:
  - `Abd – Lin`: Go to **Cubberly Auditorium**.
  - `Liu – Raj`: Go to **370-370**.
  - `Ram – Zhu`: Go to **420-040**.
- Closed-book, closed-computer, limited-notes.
  - You can have a double-sided 8.5” × 11” sheet of notes when you take the exam.
- Topic coverage is PS1 – PS3 and Lectures 00–08.

Solution Sets

- We've moved solution sets for everything midterm-related down to the basement of Gates. They're still there for pickup between now and the exam.
- ***We will be recycling all unclaimed solution sets on Wednesday***. If you'd like copies of anything, please take them soon!

Your Questions

“Any areas of CS that have application in theater, film, art and literature?”

Did you see Stanford's production of *Hairspray* last year? Matt Lathrop and his team did a phenomenal job putting together these crazy cool LED boards that they used to make the scene evolve and change in real-time.

About a year ago, two students put together a program that made it easier to compare multiple translations of a piece of literature side-by-side. About three years ago a student did a large-scale analysis of writings about cities over hundreds of years to learn how cities looked and sounded through the years and how people described them.

Also a few years back, someone analyzed the works of a famous writer (I *think* it was Virginia Woolf, but I might be wrong) and figured out that the conventional explanation of how she explored new writing styles was wrong. That came as a real shock to a lot of people.

Also, did you see *Frozen* or *Inside Out*? I rest my case. ☺

Back to CS103!

NFAs

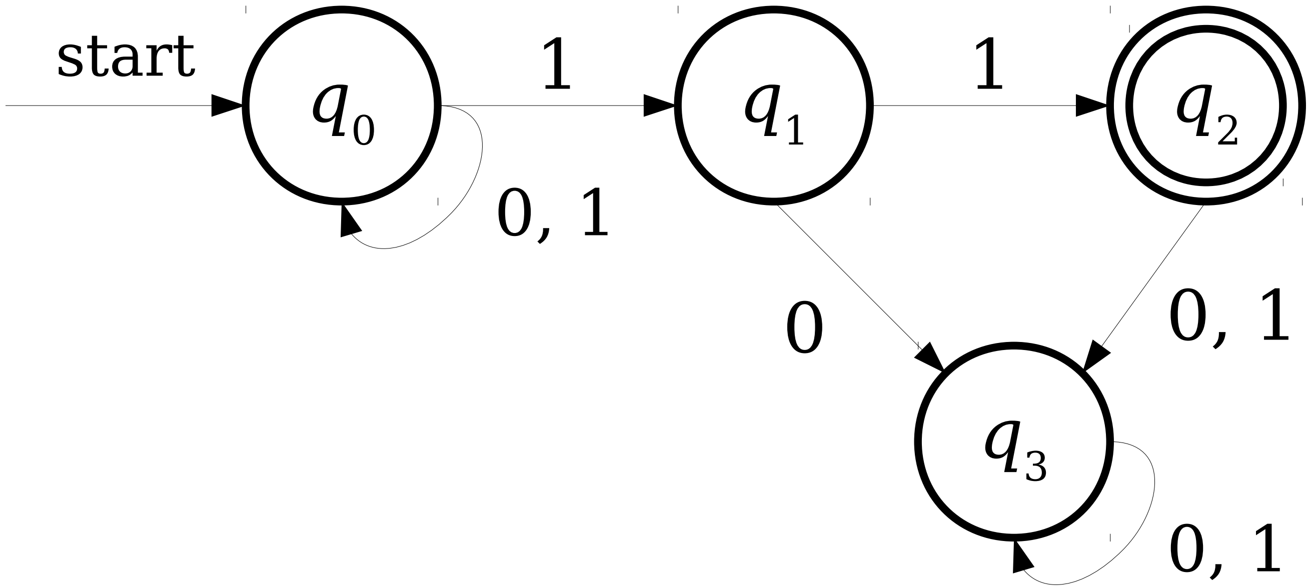
NFAs

- An **NFA** is a
  - **N**ondeterministic
  - **F**inite
  - **A**utomaton
- Structurally similar to a DFA, but represents a fundamental shift in how we'll think about computation.

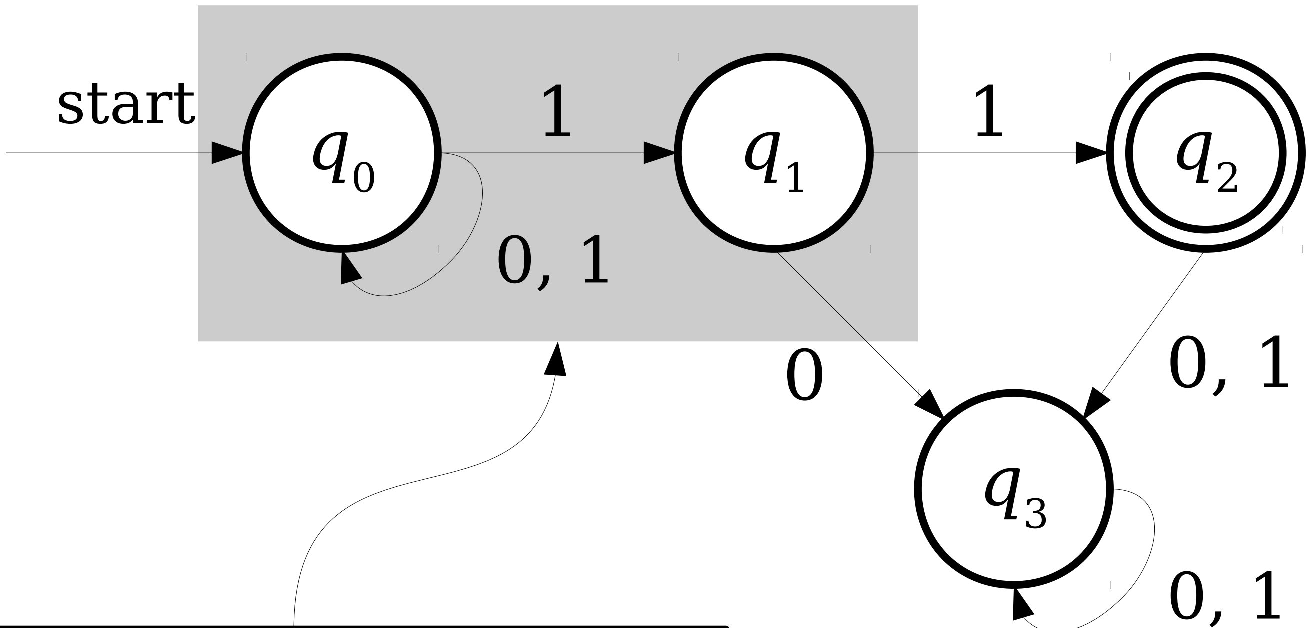
(Non)determinism

- A model of computation is ***deterministic*** if at every point in the computation, there is exactly one choice that can make.
- The machine accepts if that series of choices leads to an accepting state.
- A model of computation is ***nondeterministic*** if the computing machine may have multiple decisions that it can make at one point.
- The machine accepts if ***any*** series of choices leads to an accepting state.

A Simple NFA

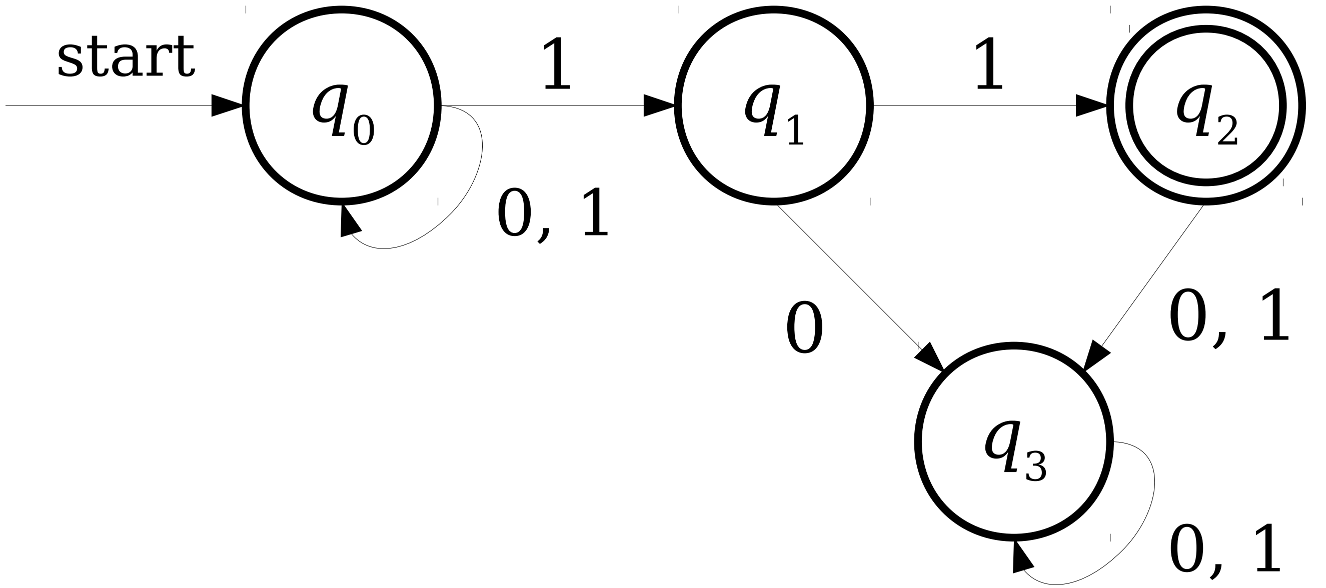


A Simple NFA



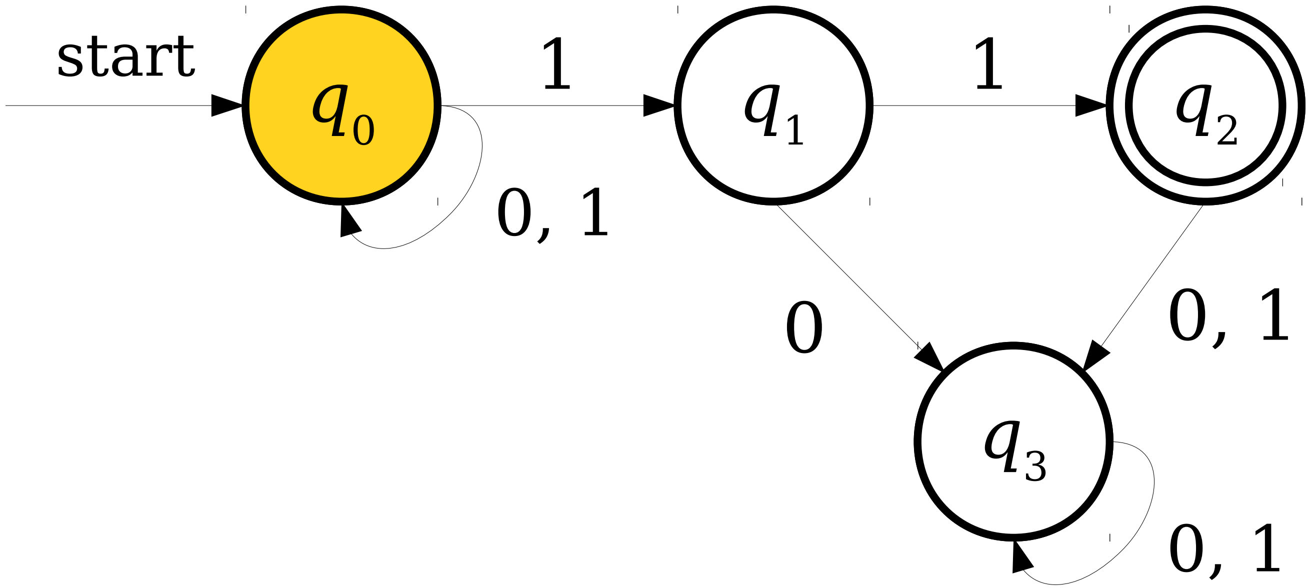
q_0 has two transitions defined on 1!

A Simple NFA



0 1 0 1 1

A Simple NFA



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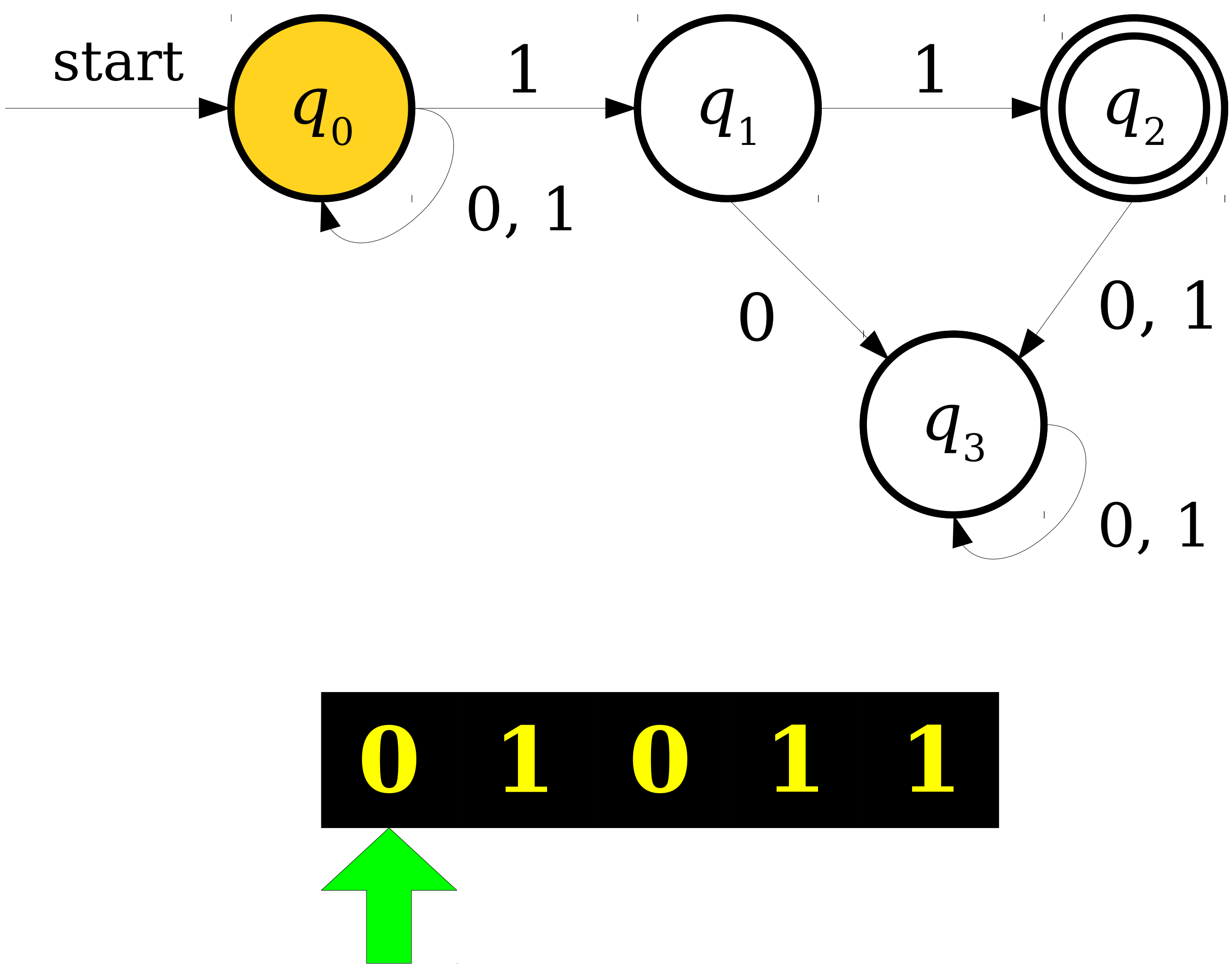

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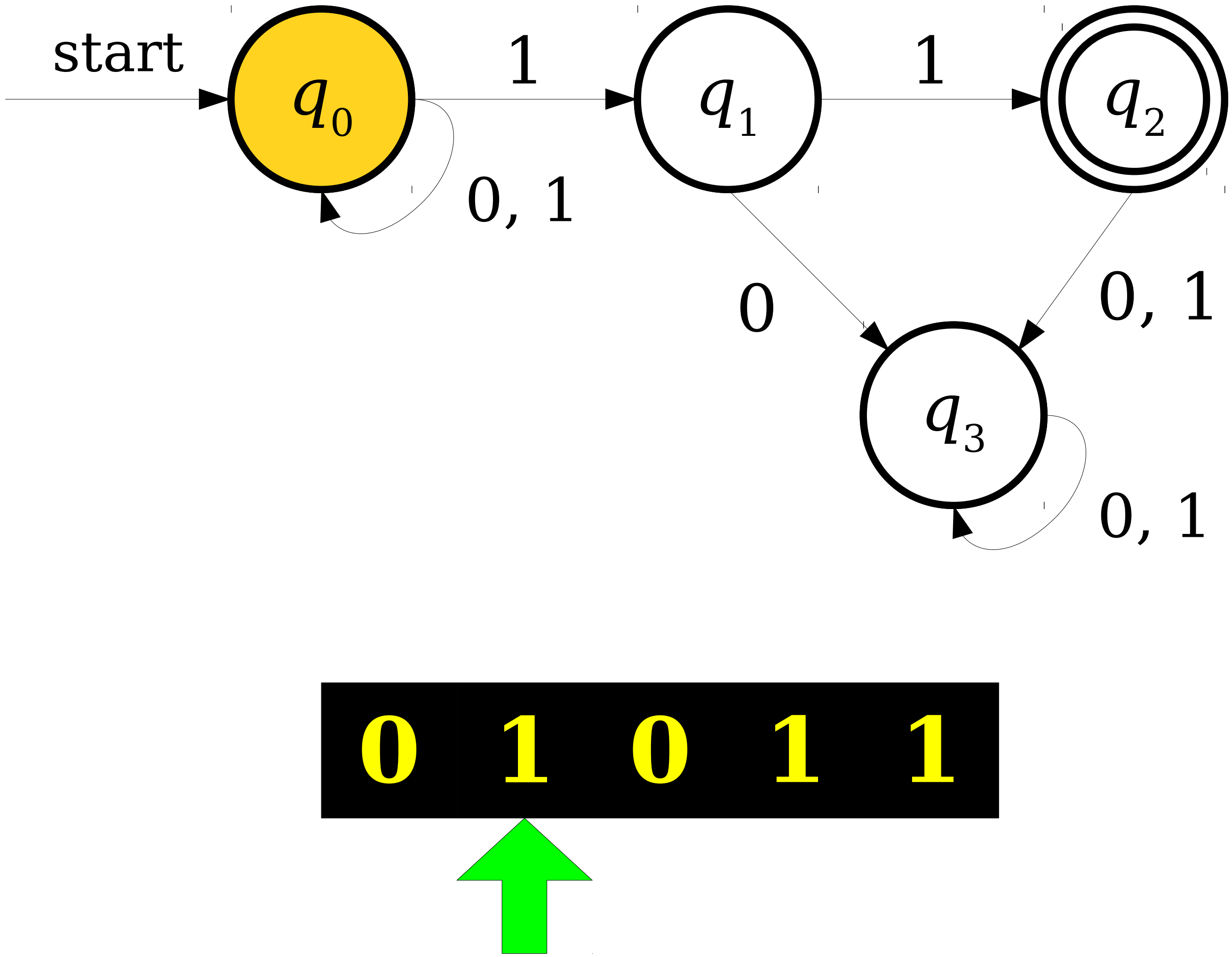

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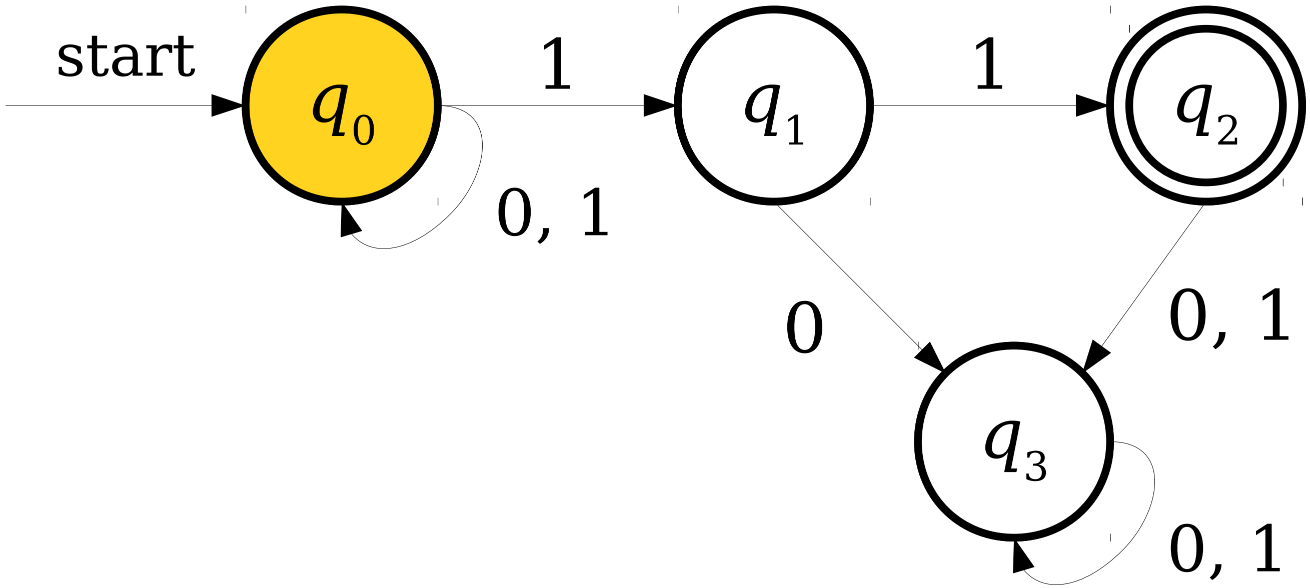

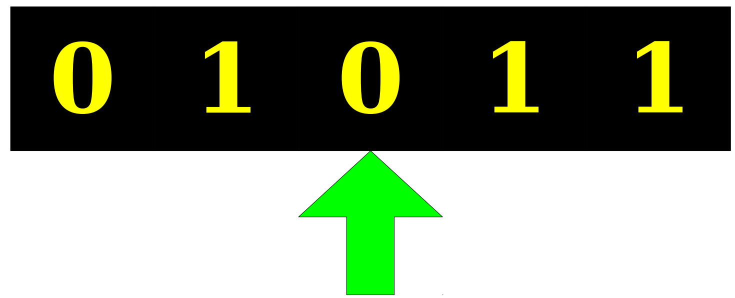

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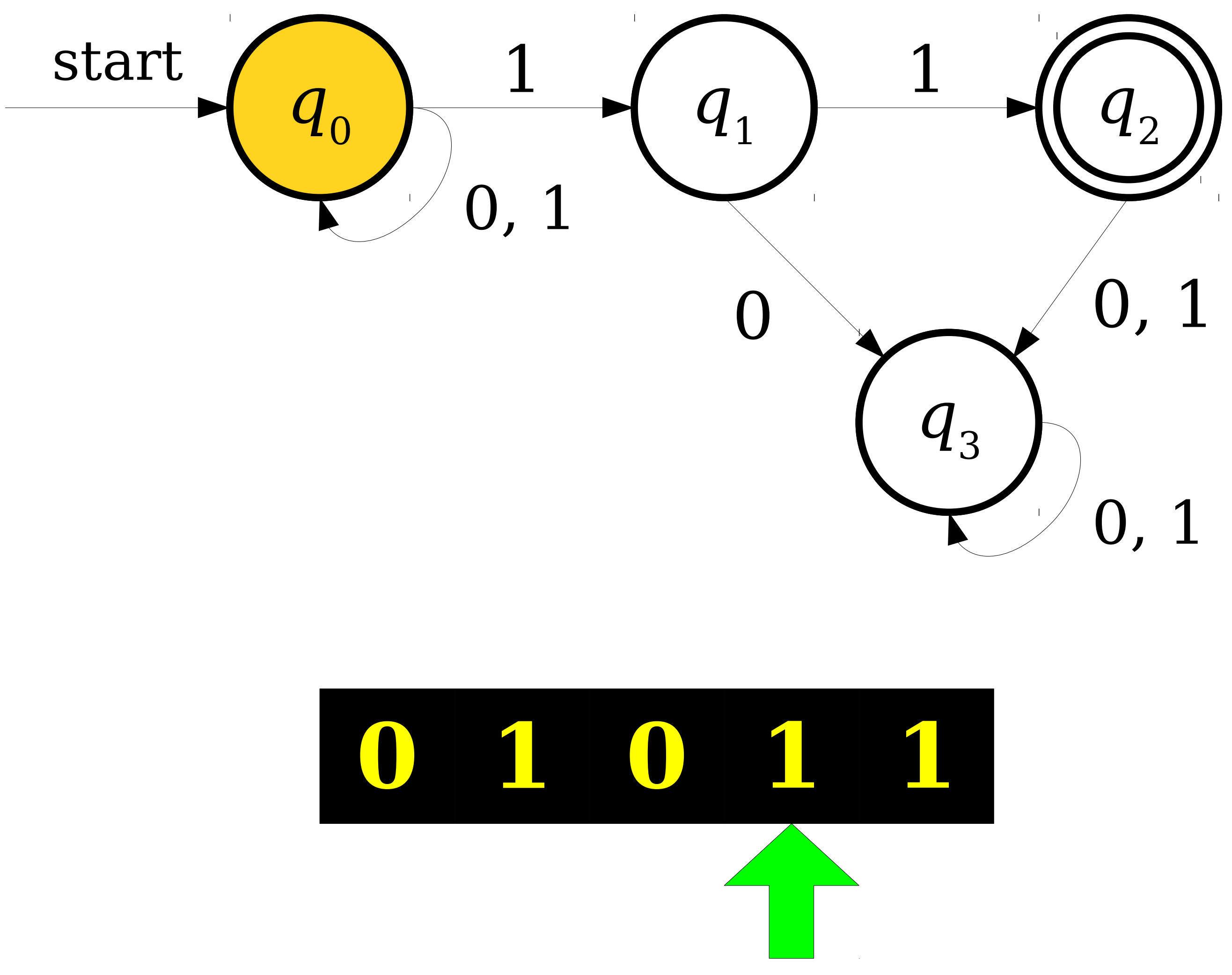

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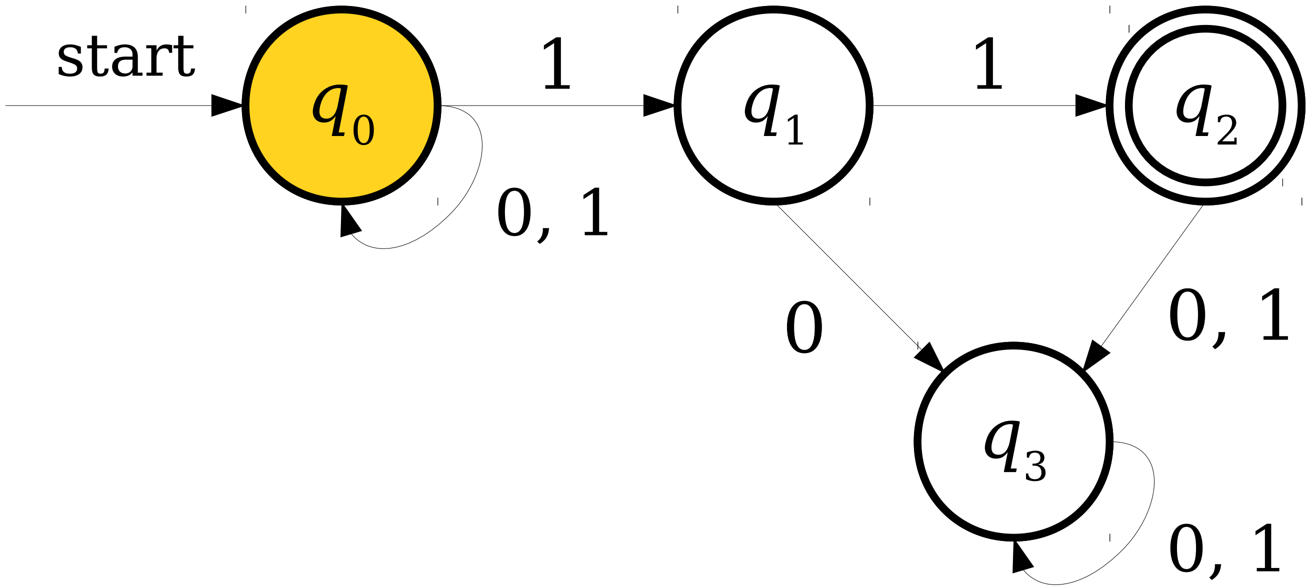

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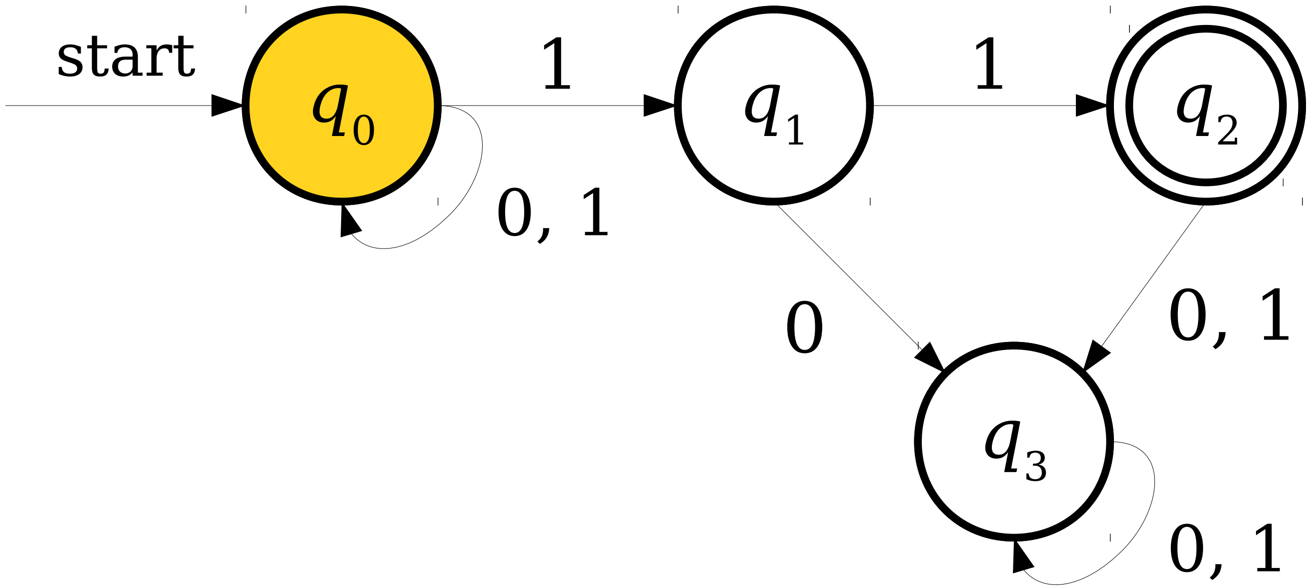

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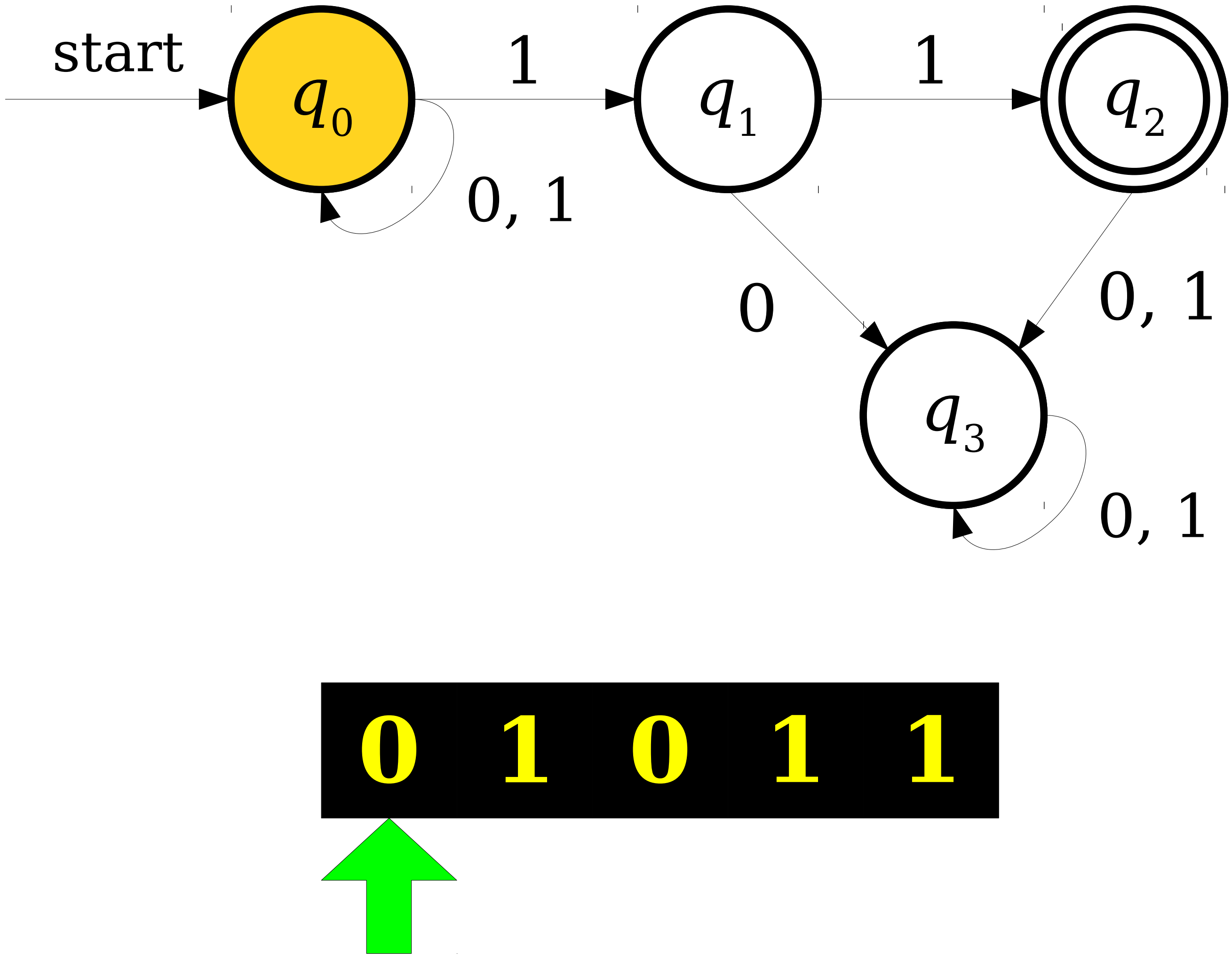

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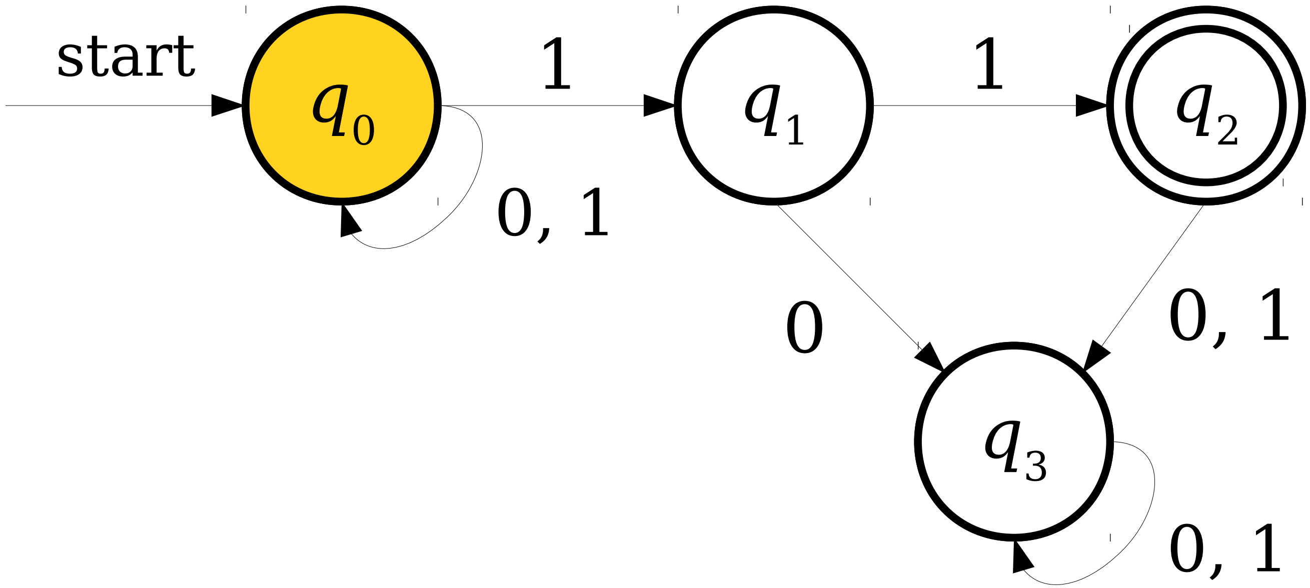

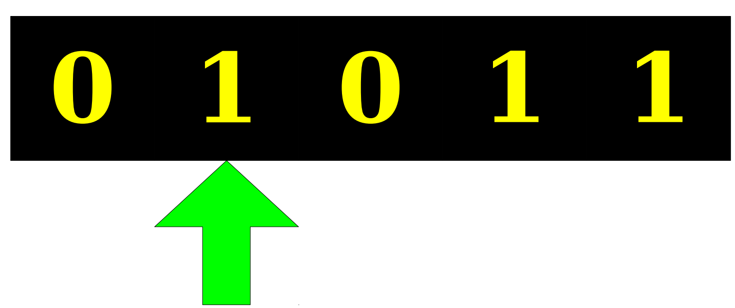

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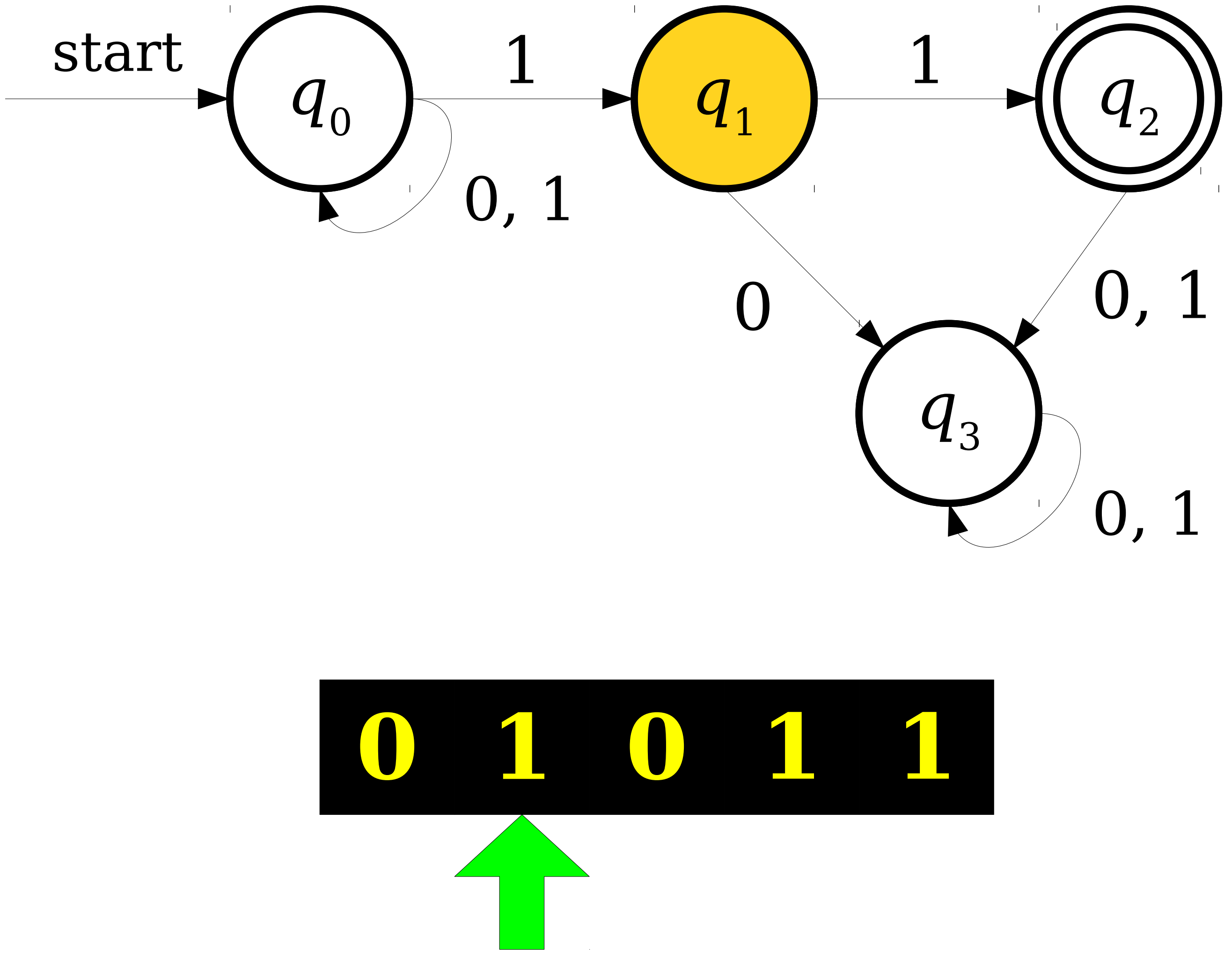

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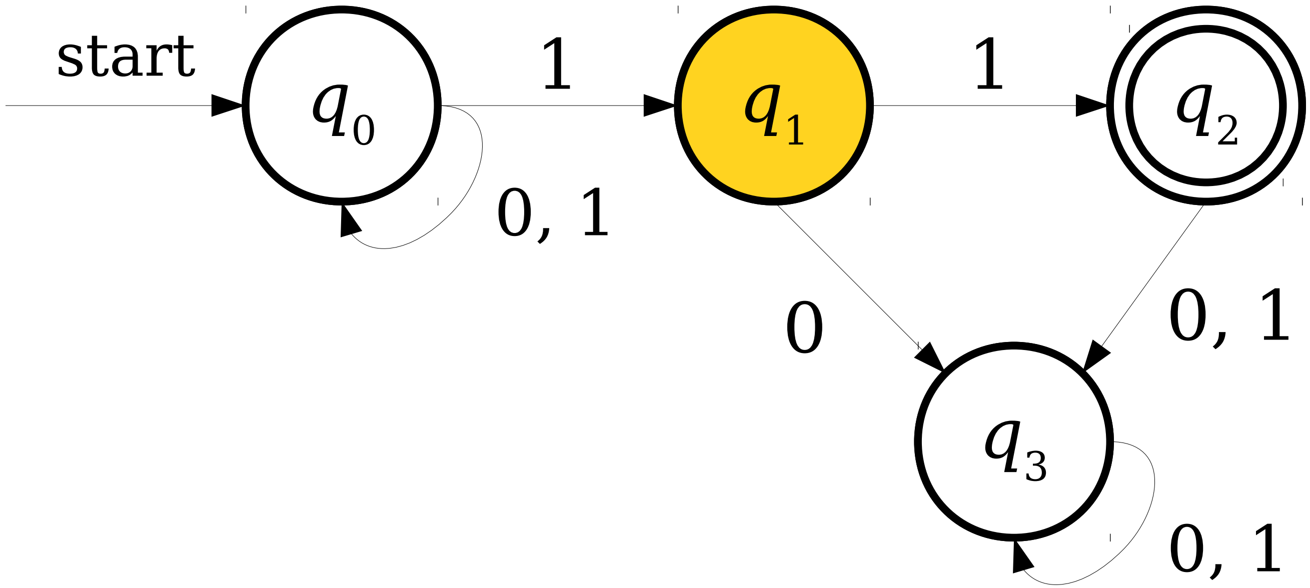

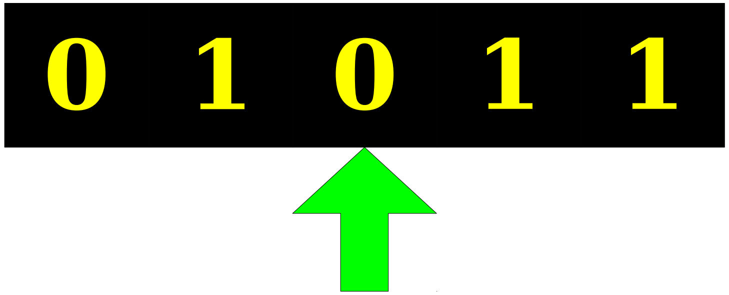

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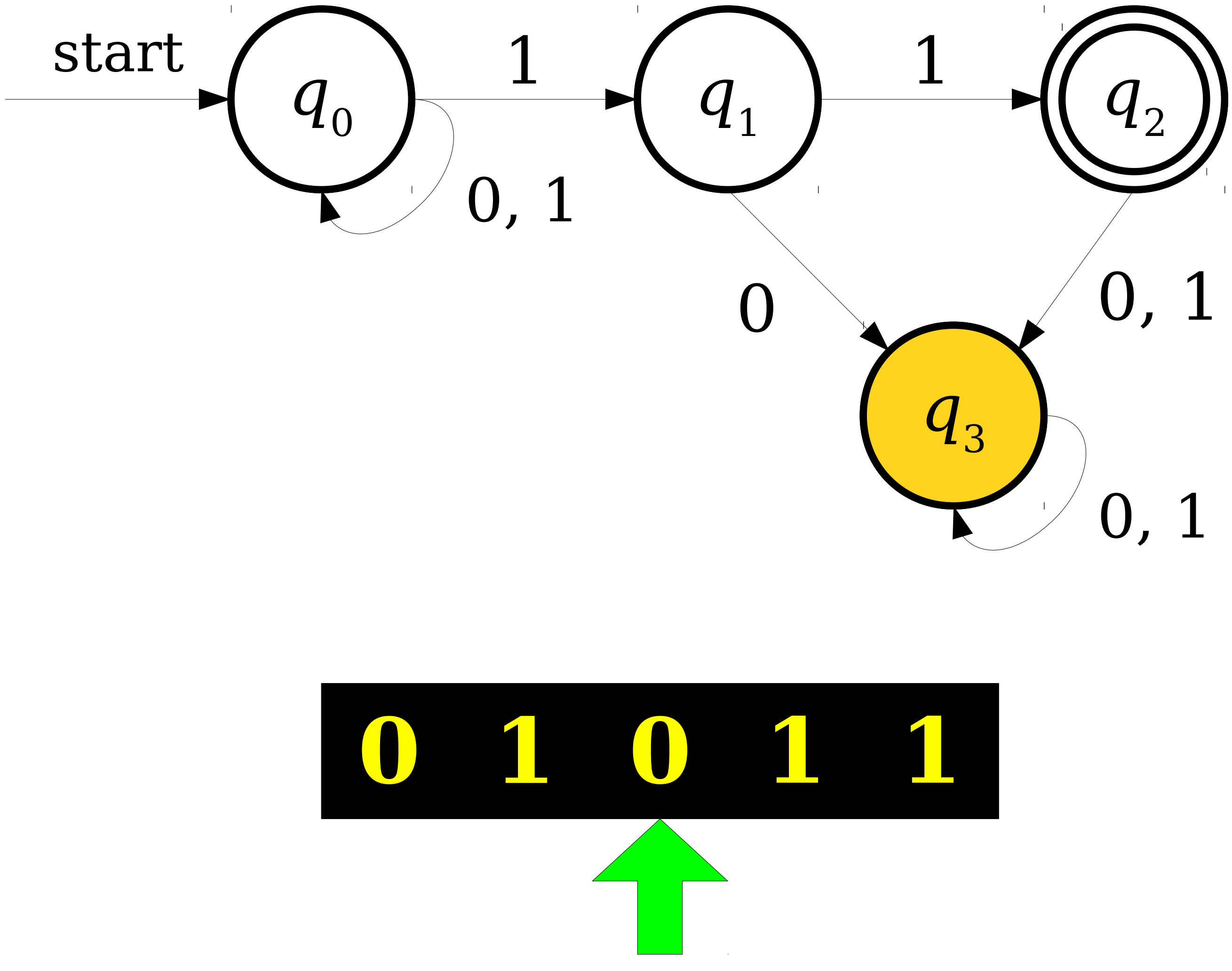

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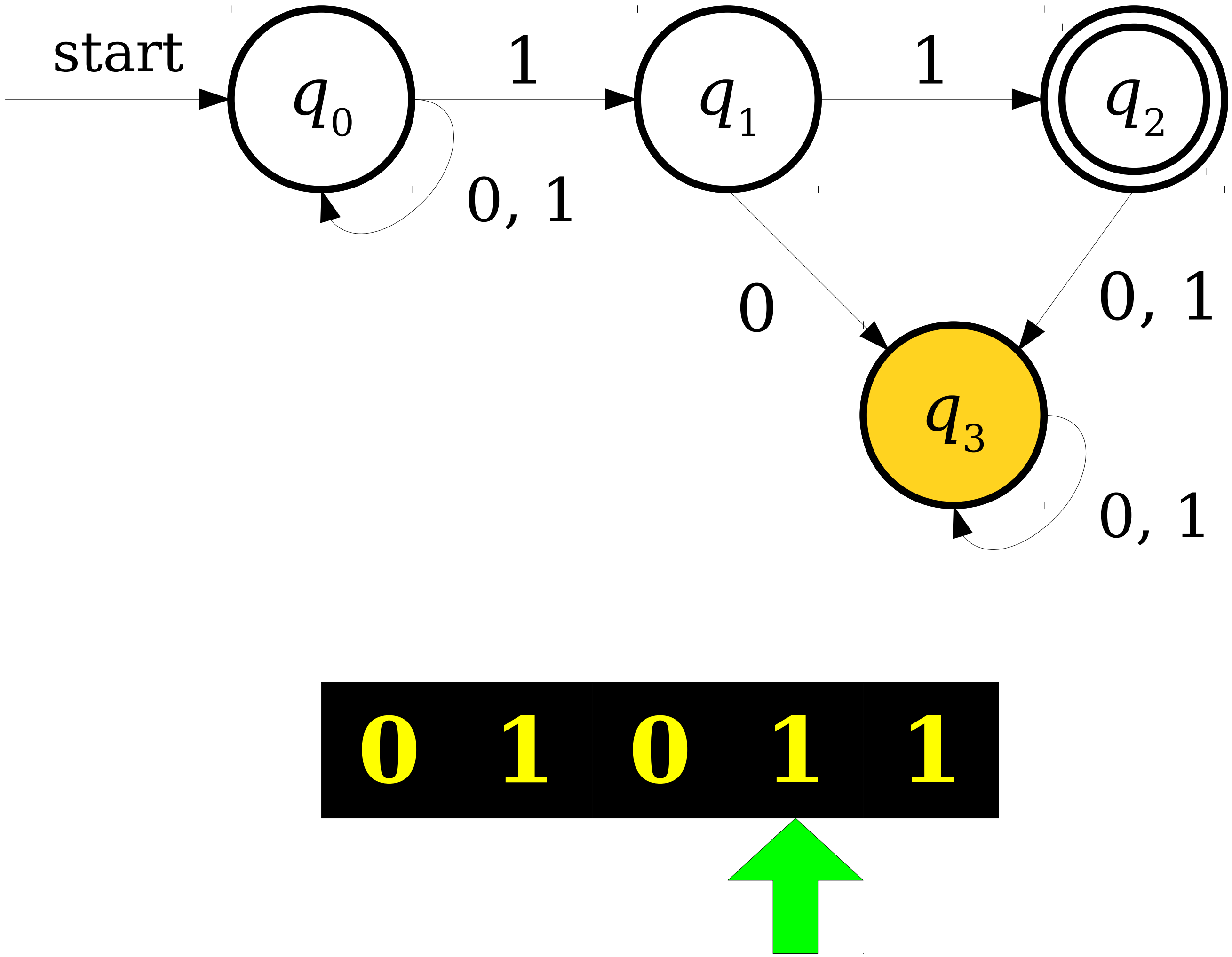

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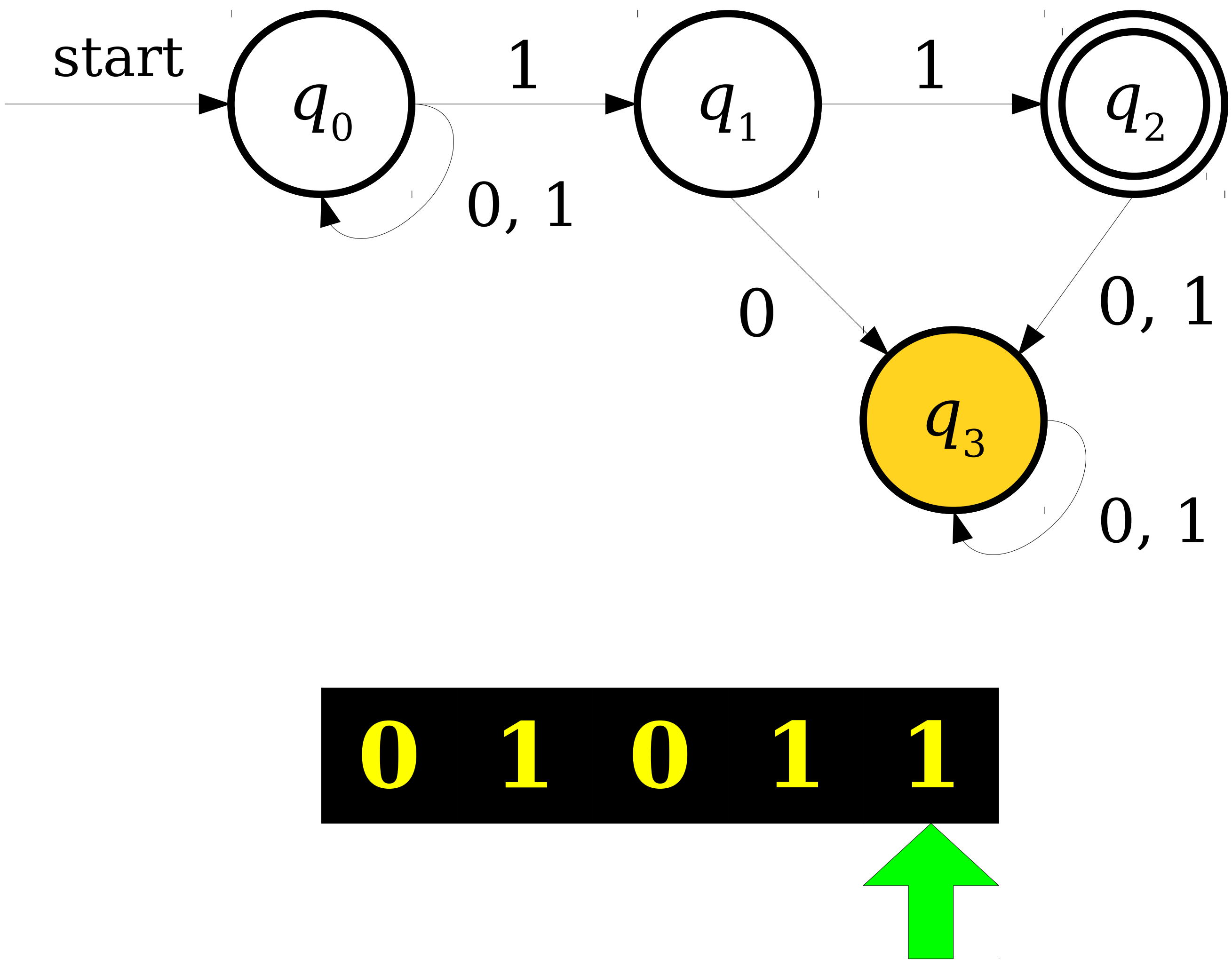

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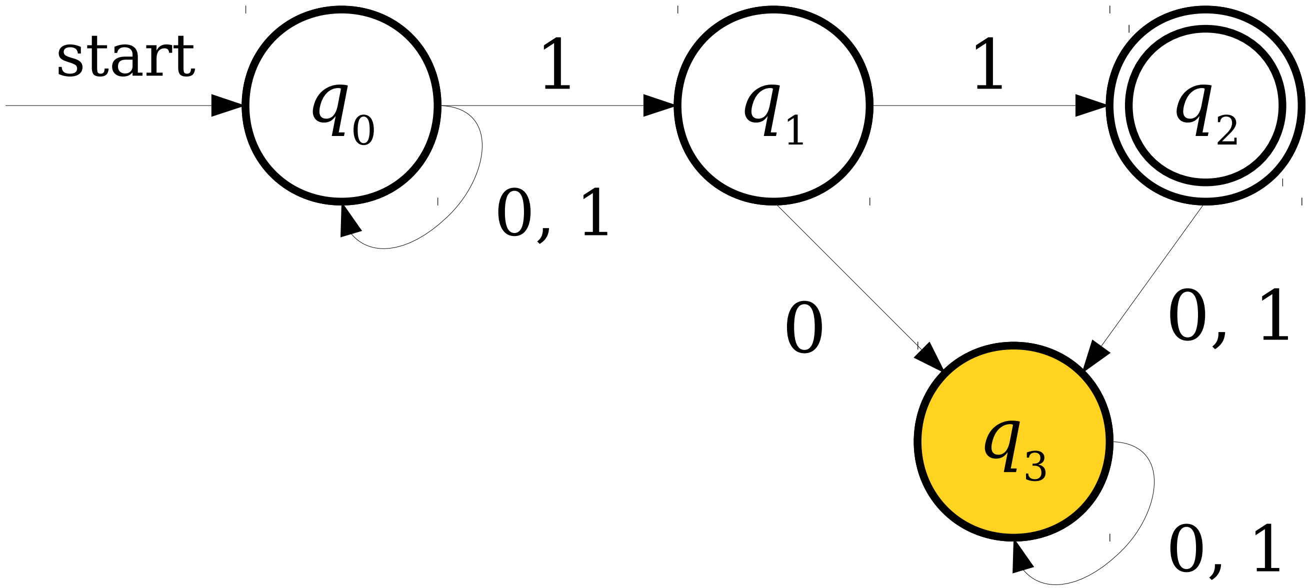

0 1 0 1 1

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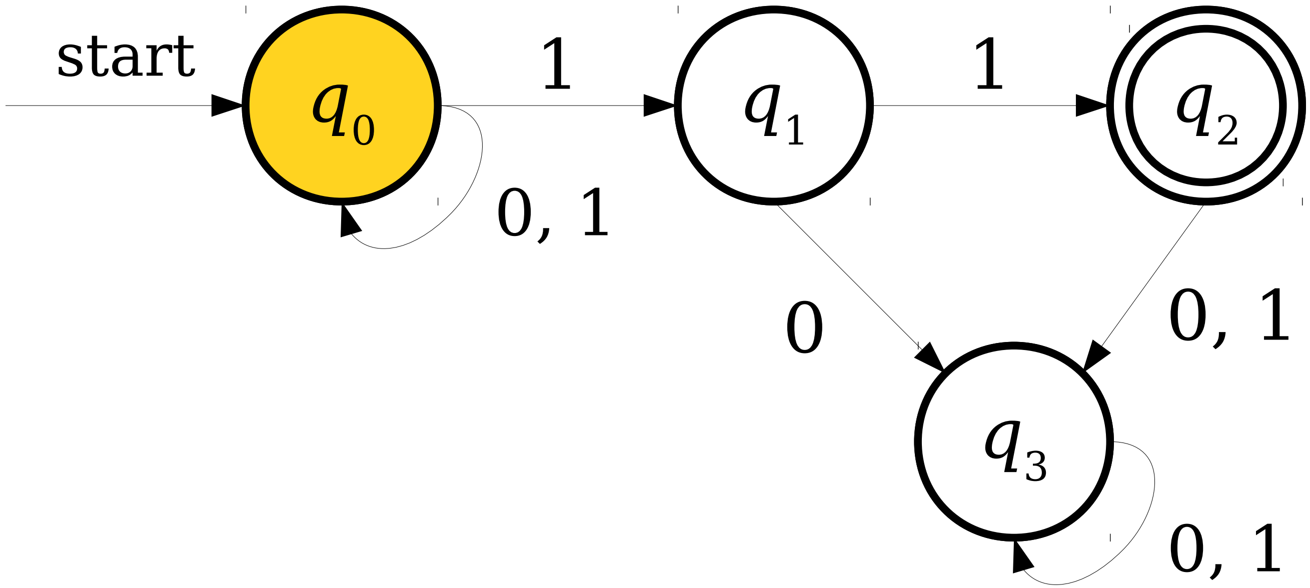

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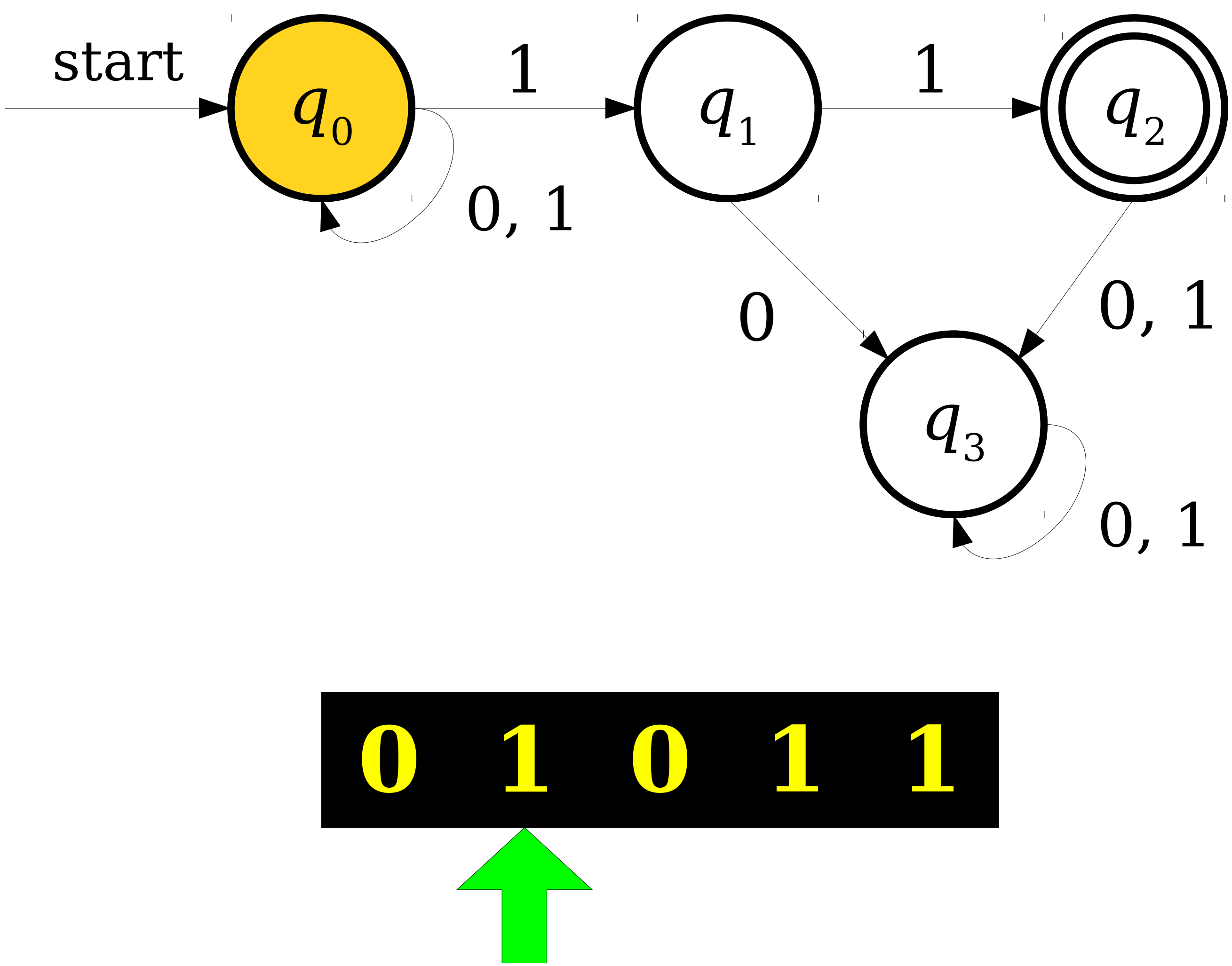

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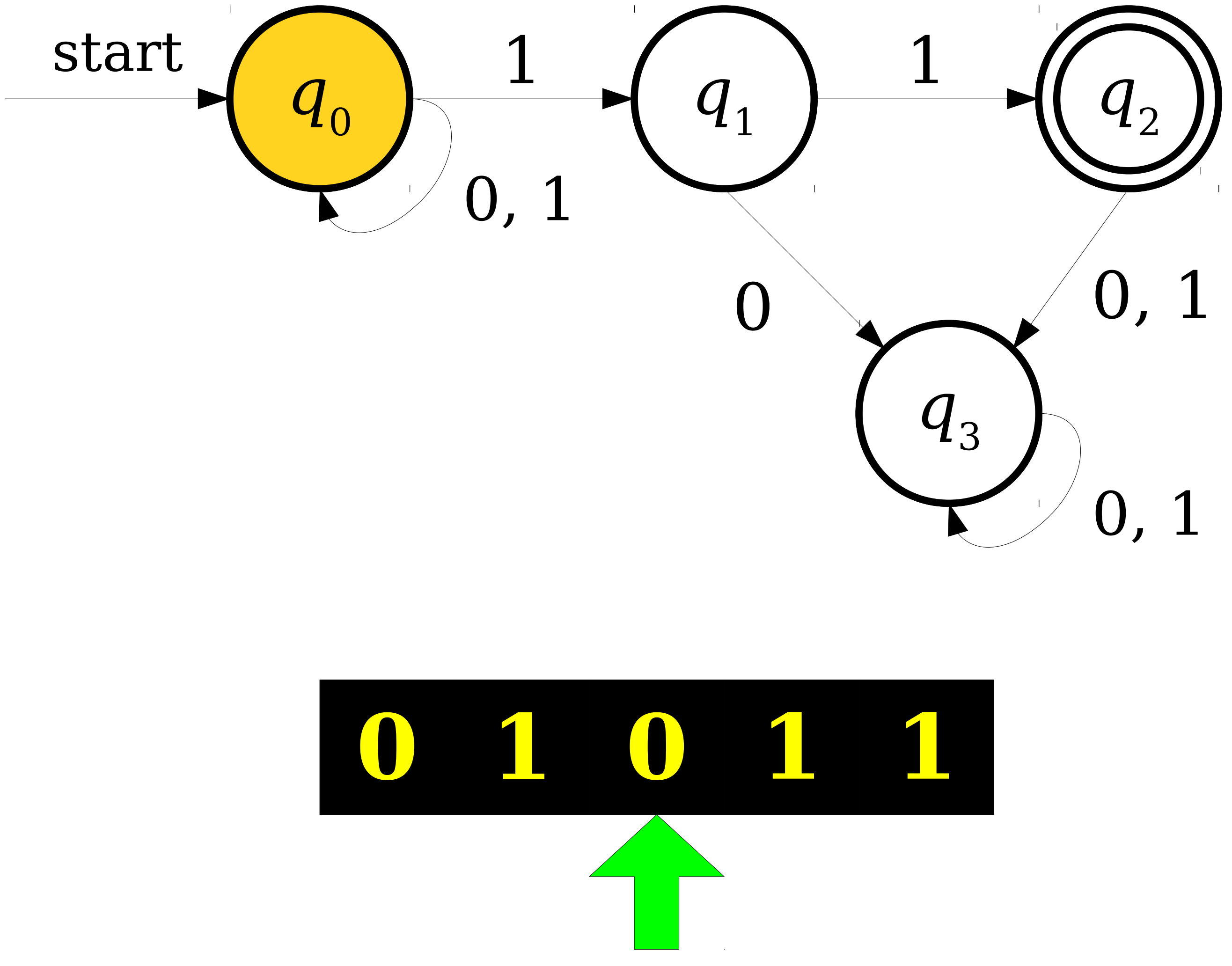

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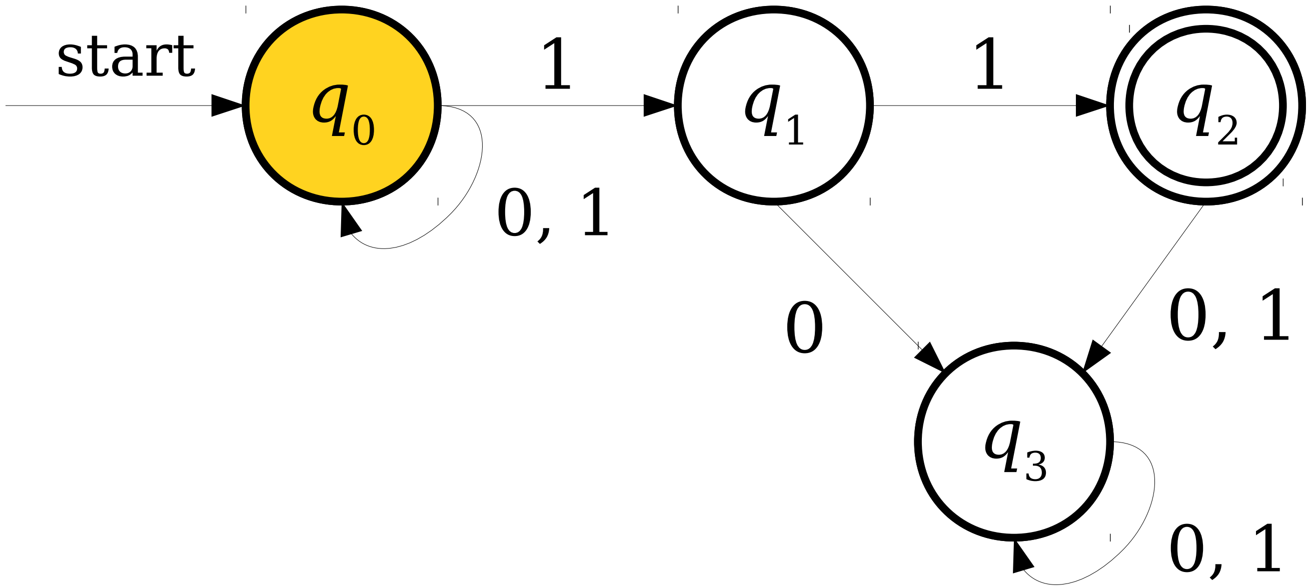

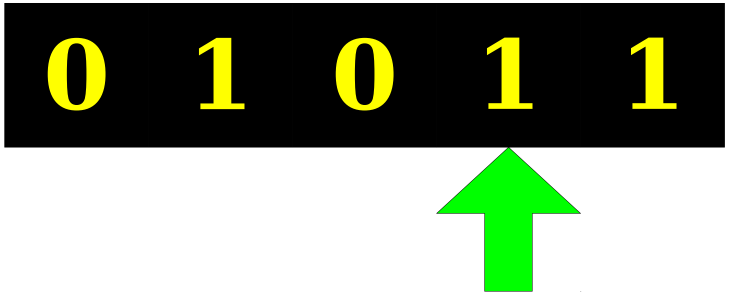

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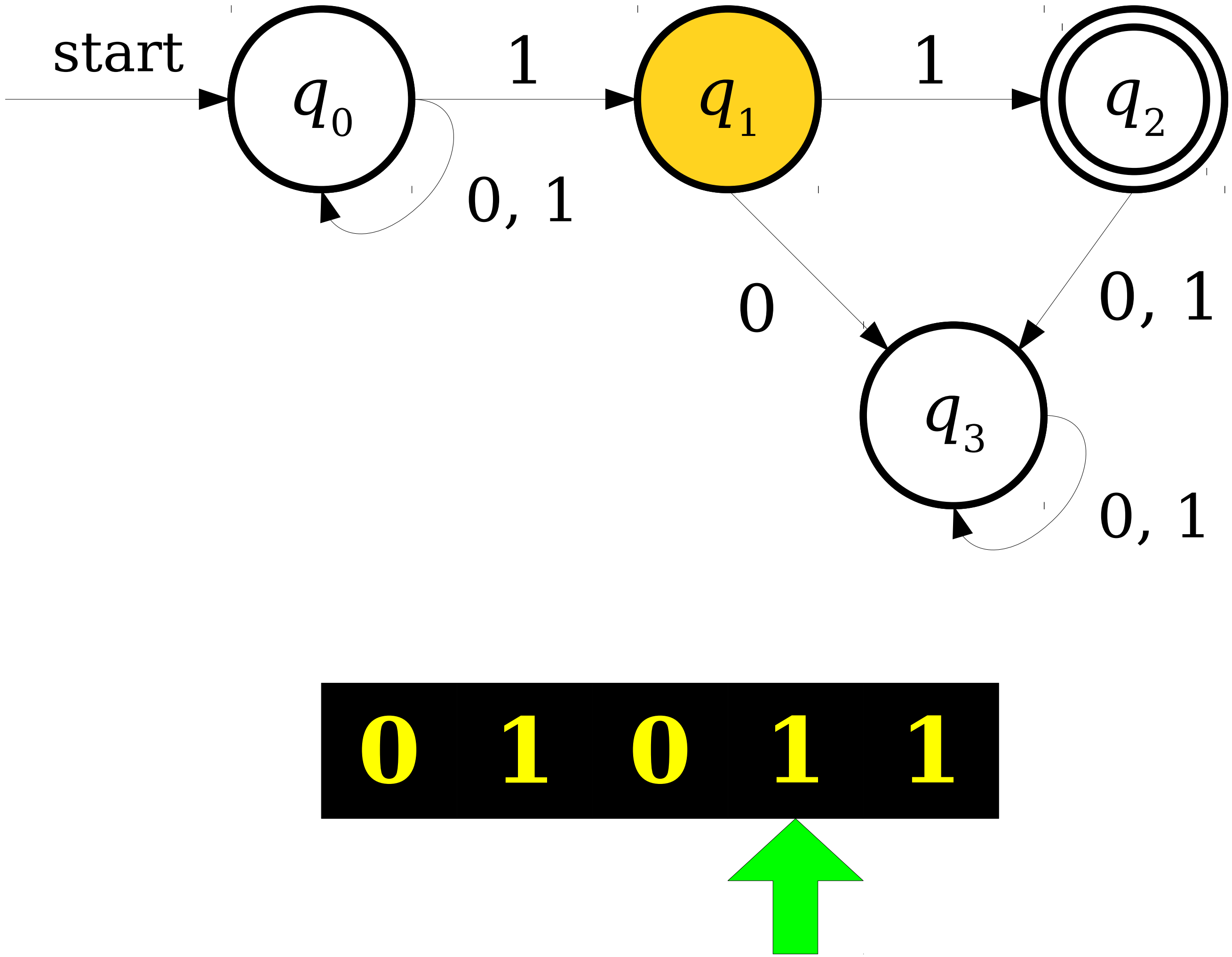

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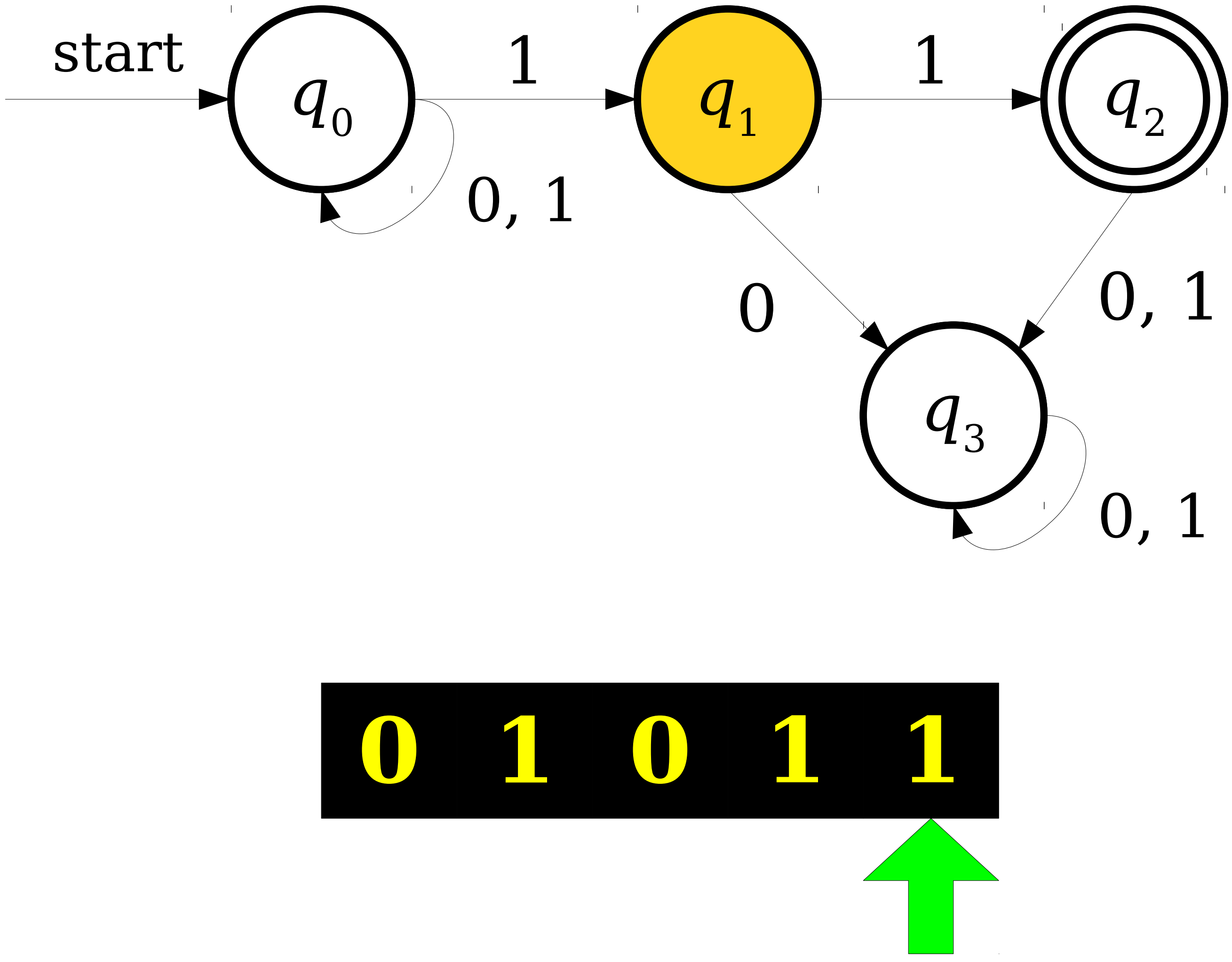

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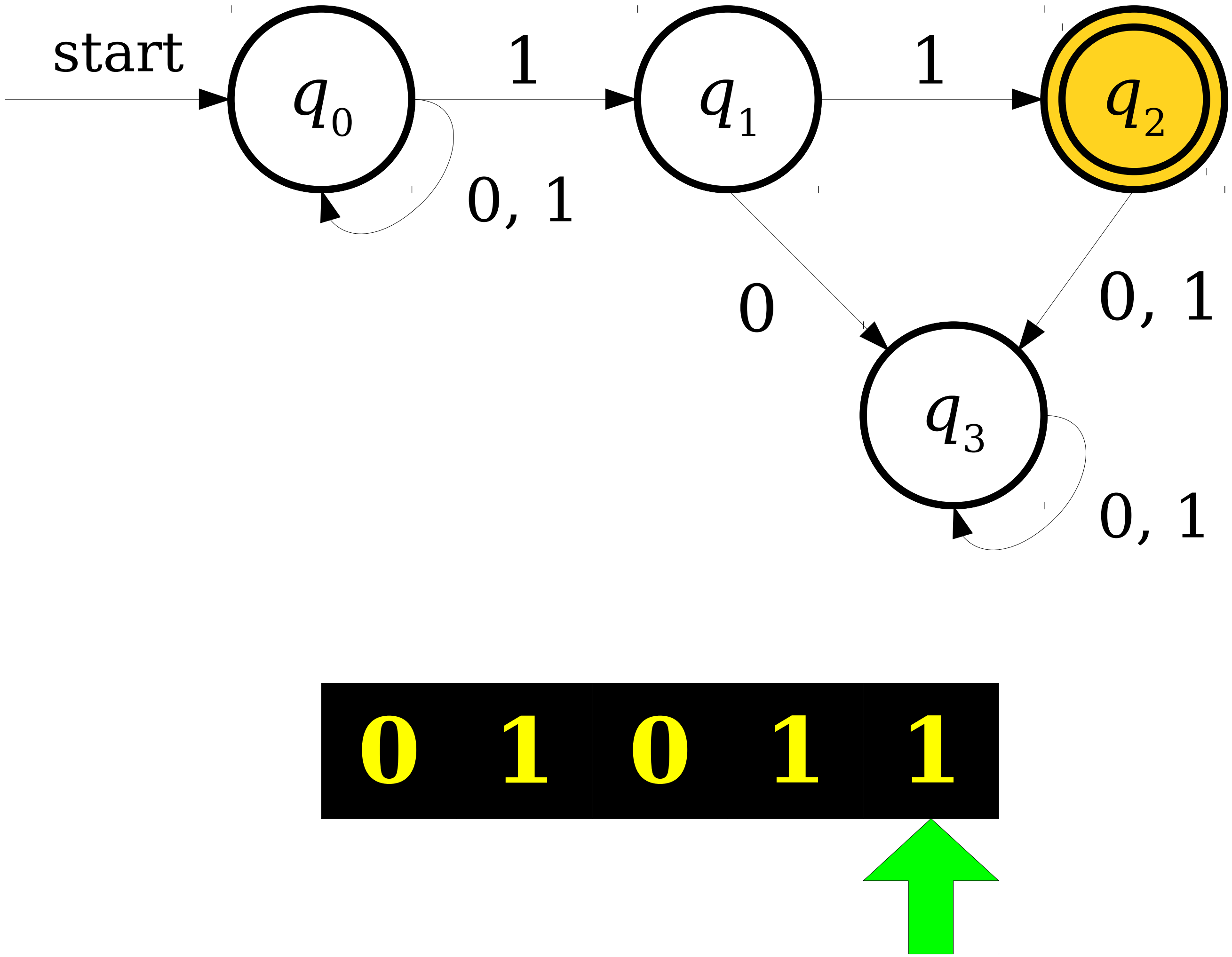

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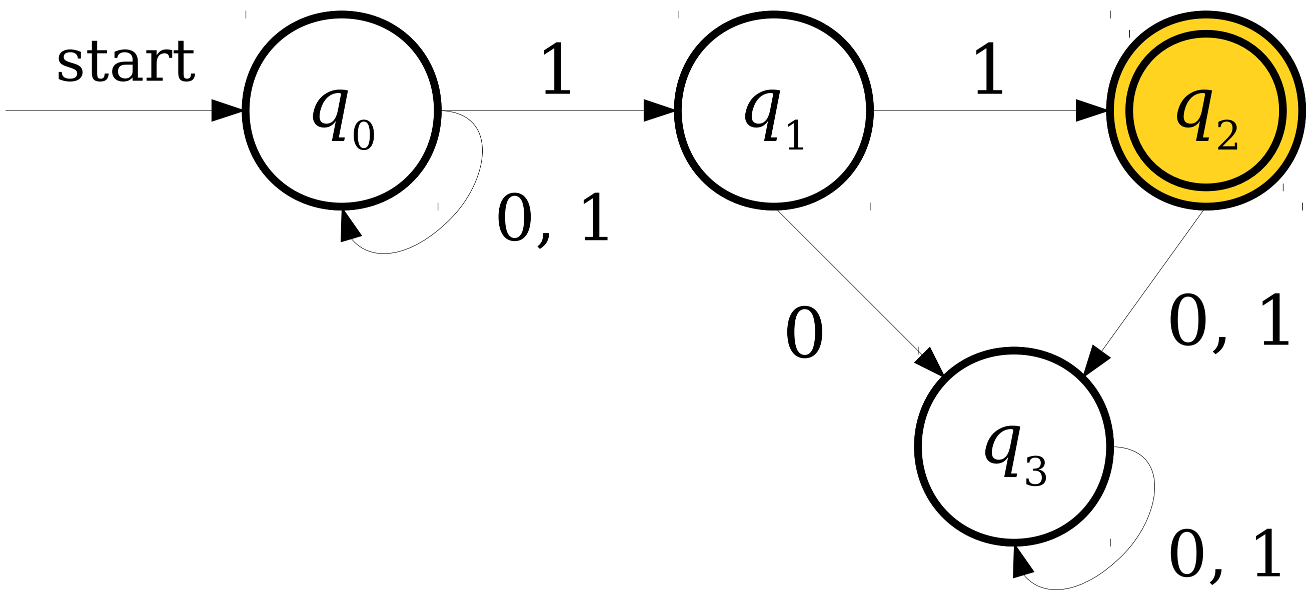

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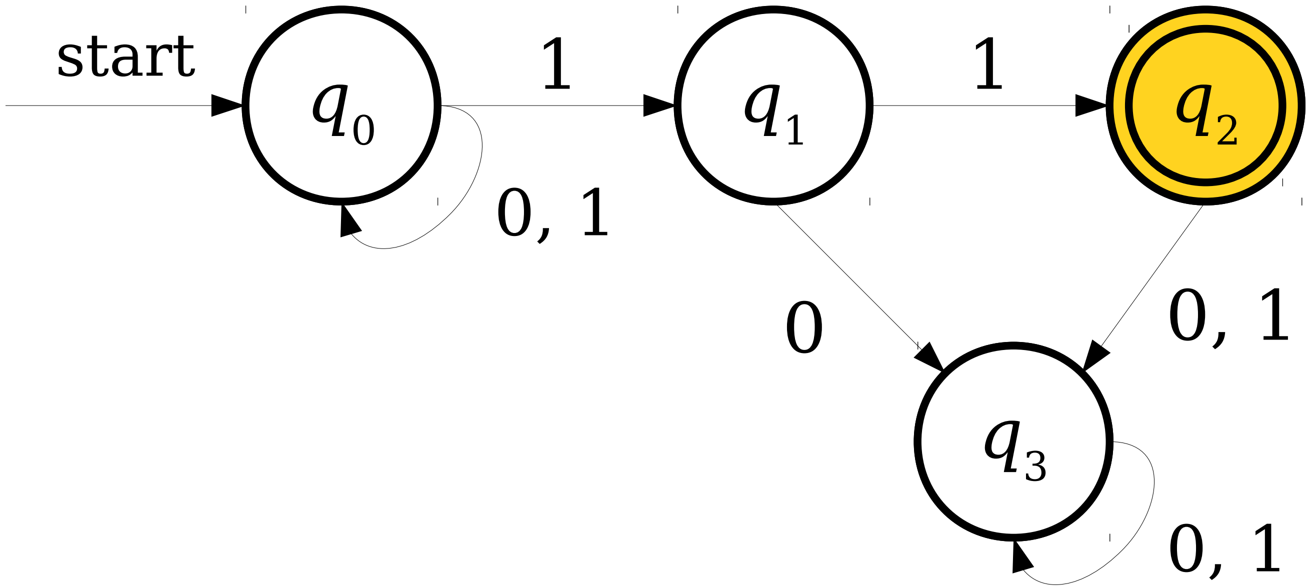

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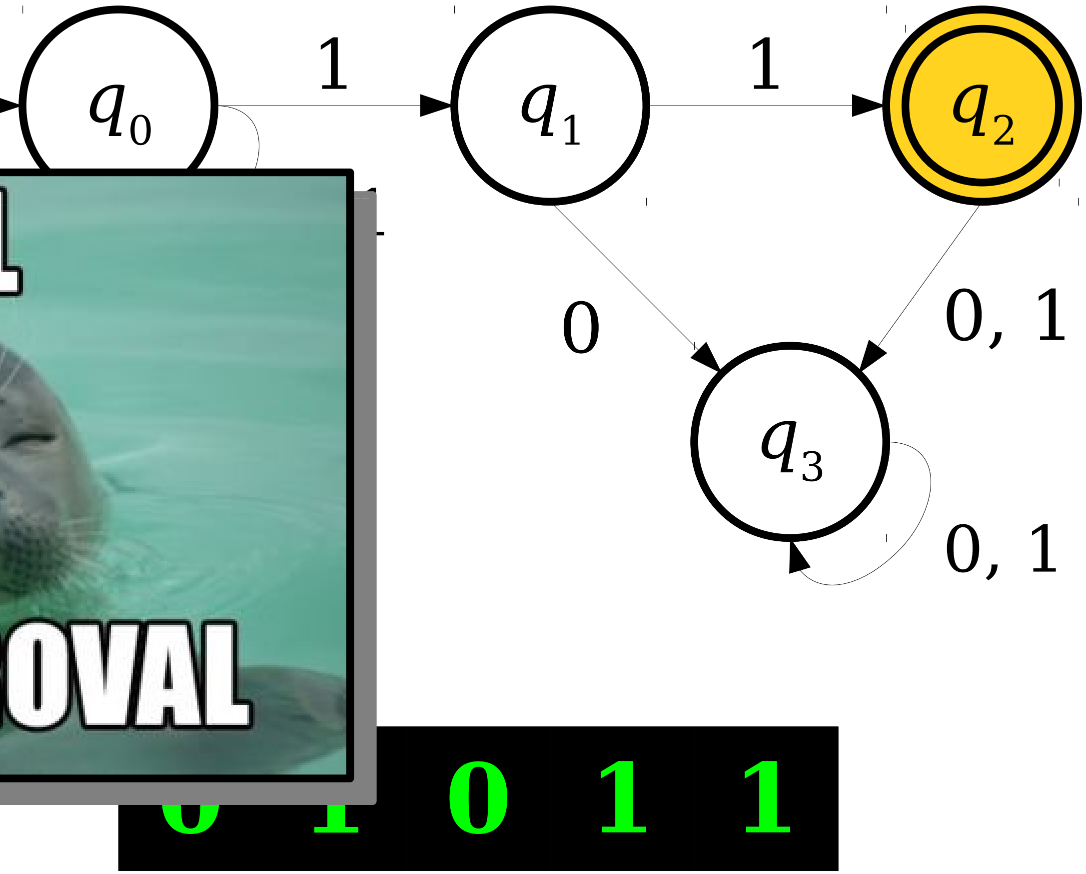

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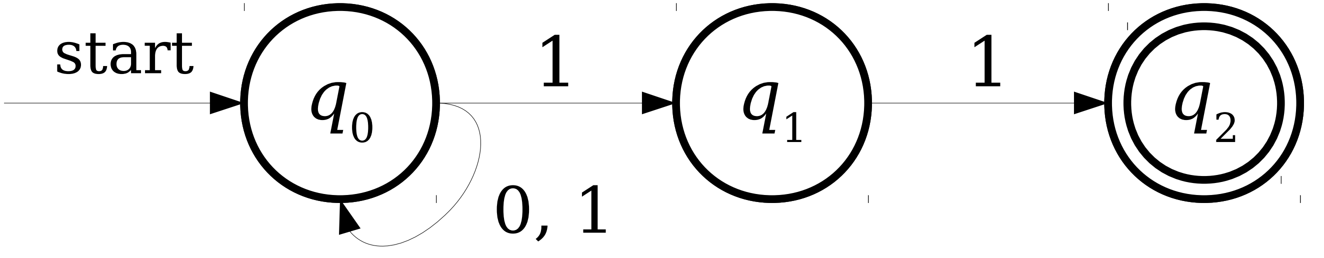

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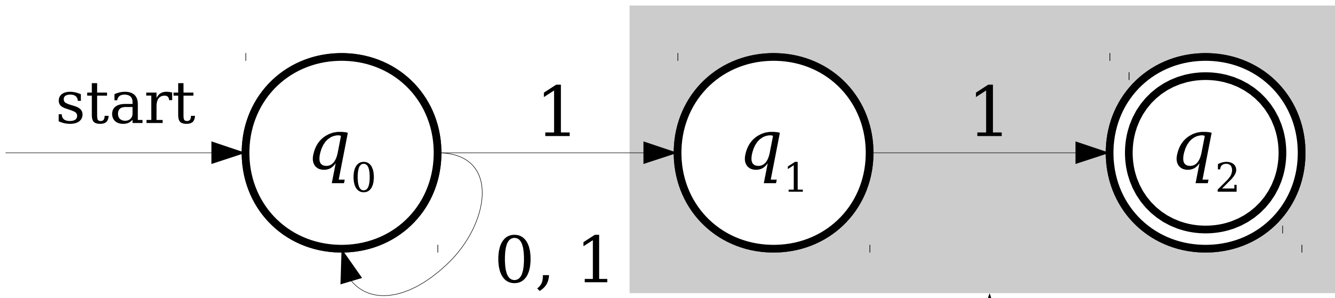
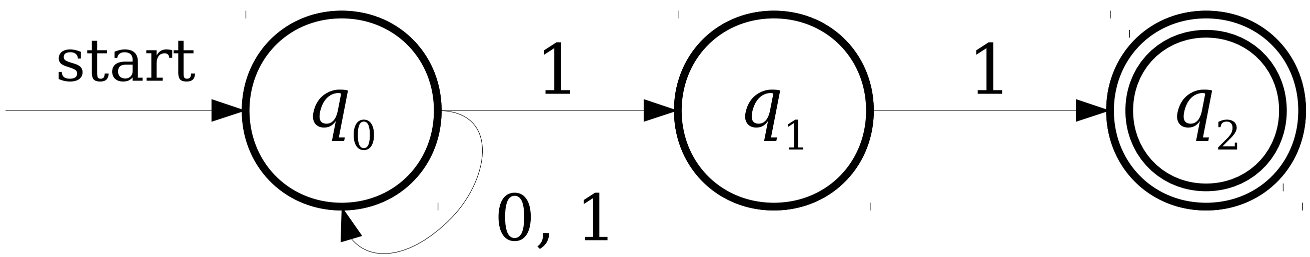


If a NFA needs to make a transition when no transition exists, the automaton dies and that particular path rejects.

A More Complex NFA



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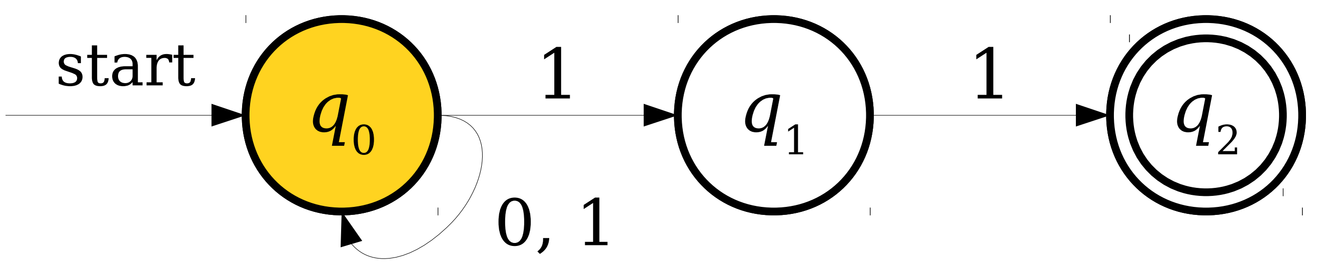

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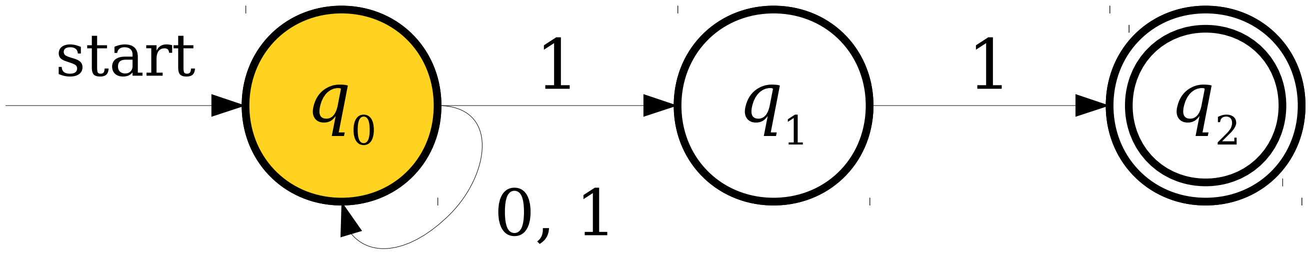

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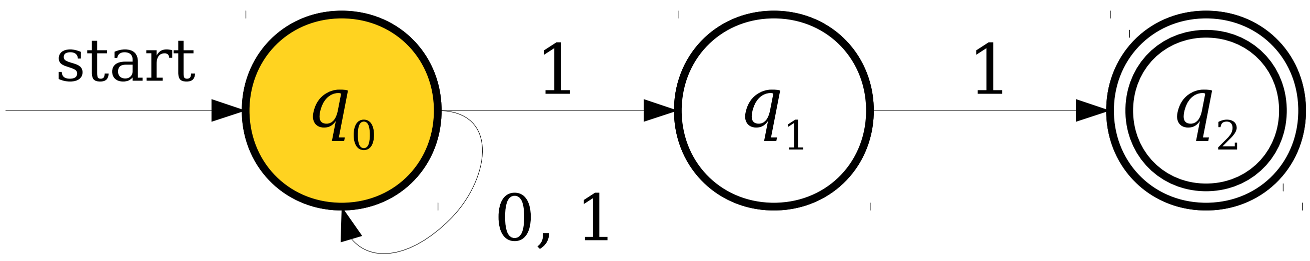

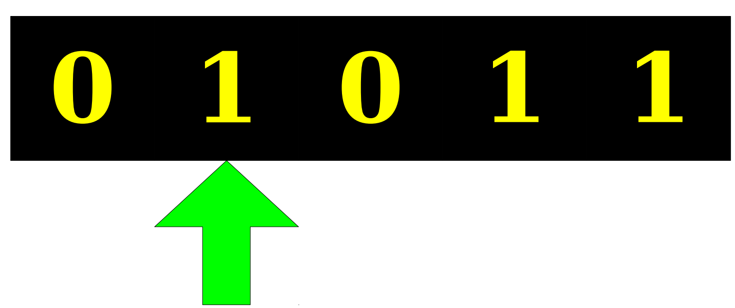

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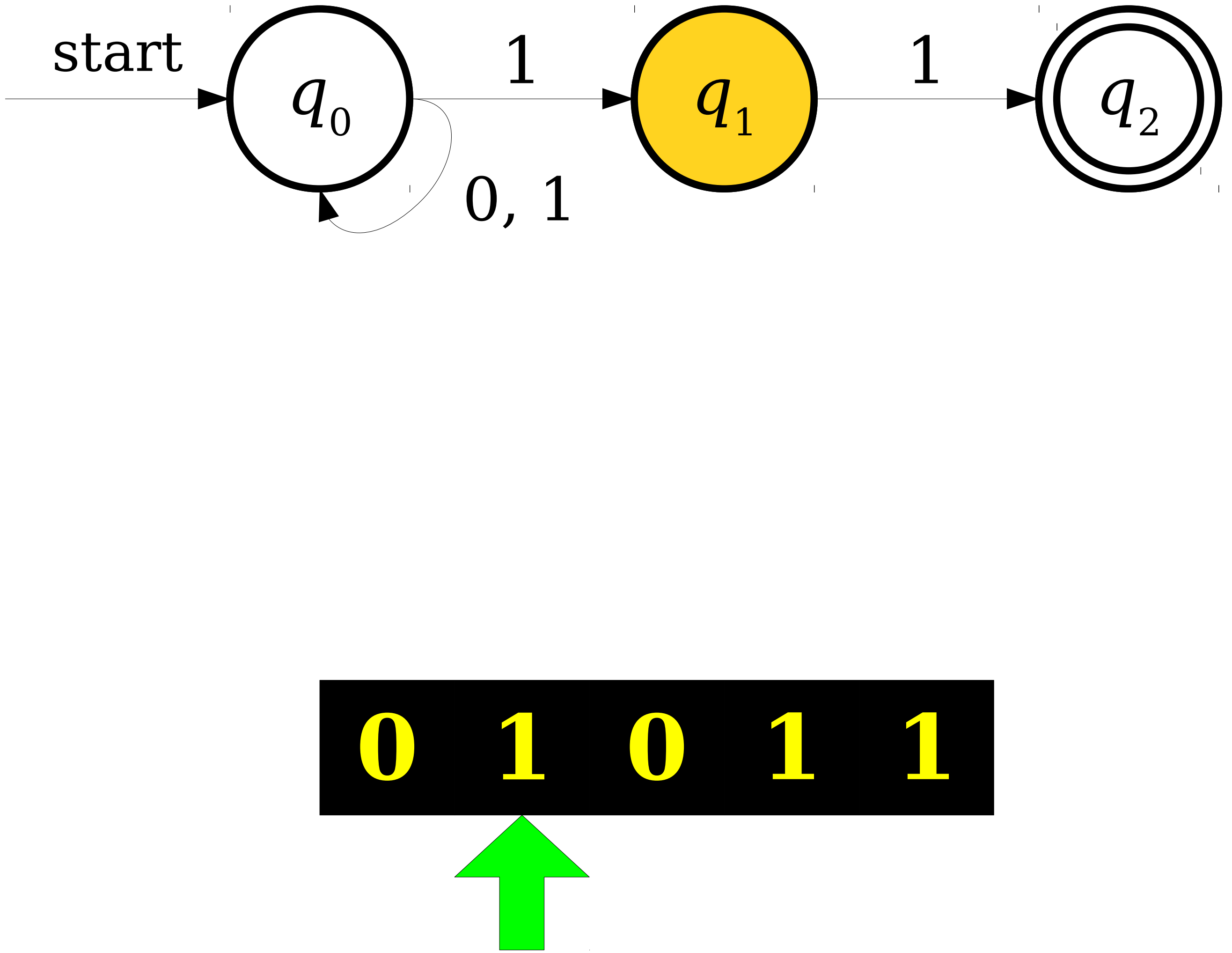

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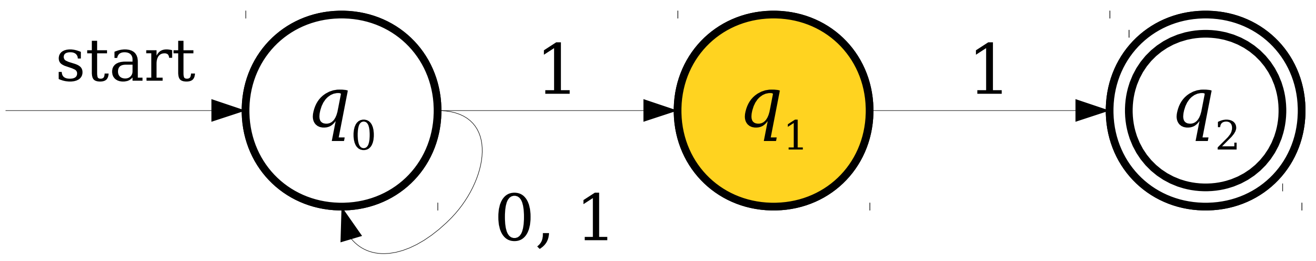

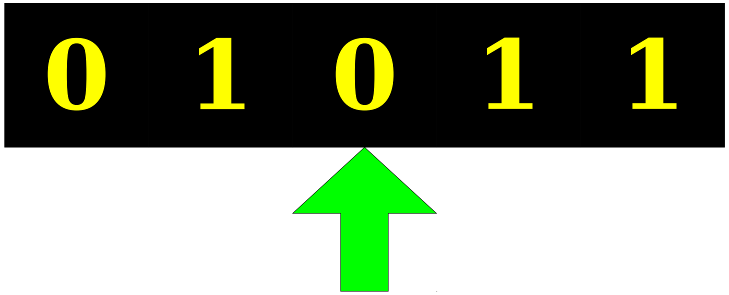

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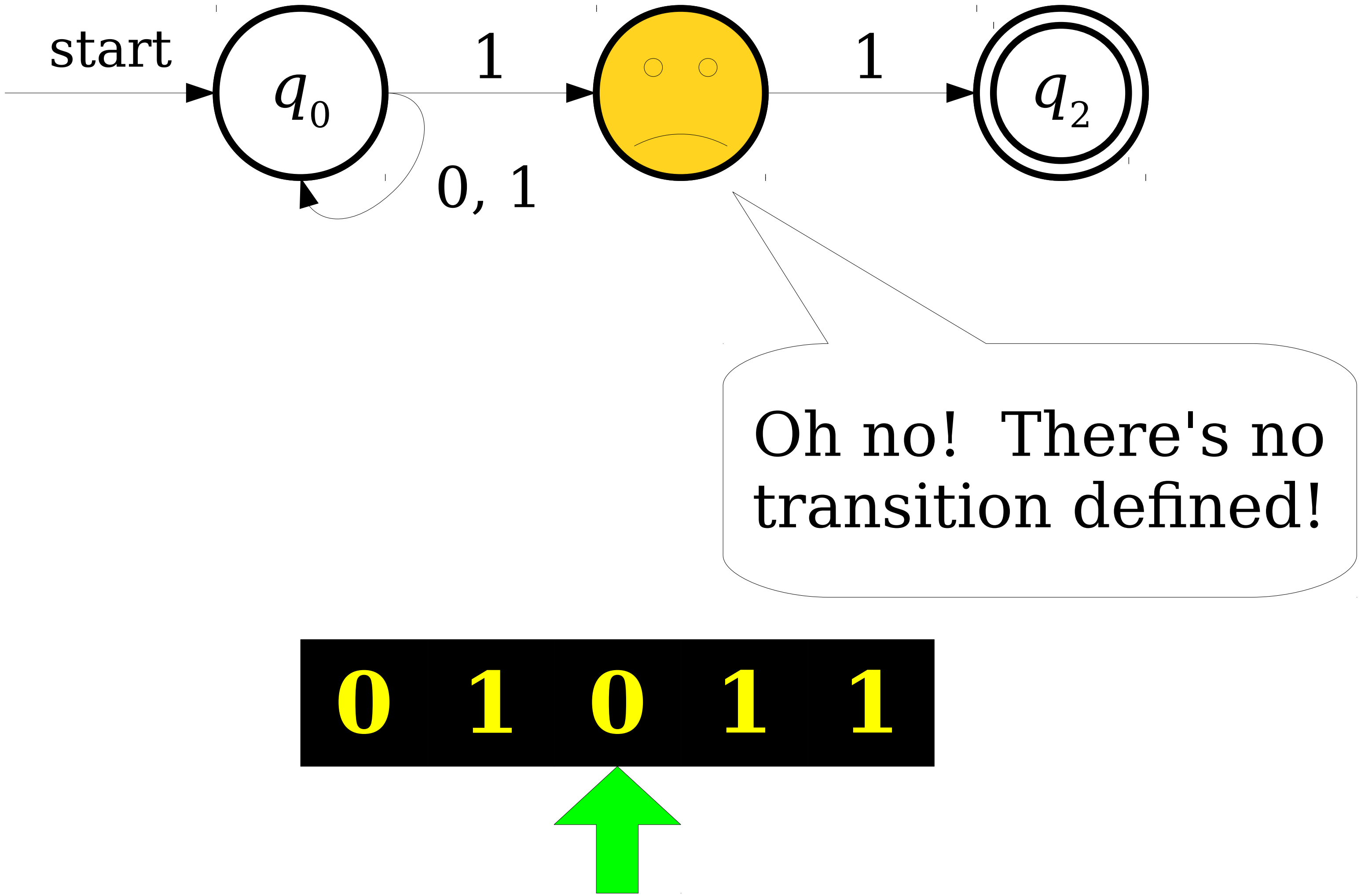

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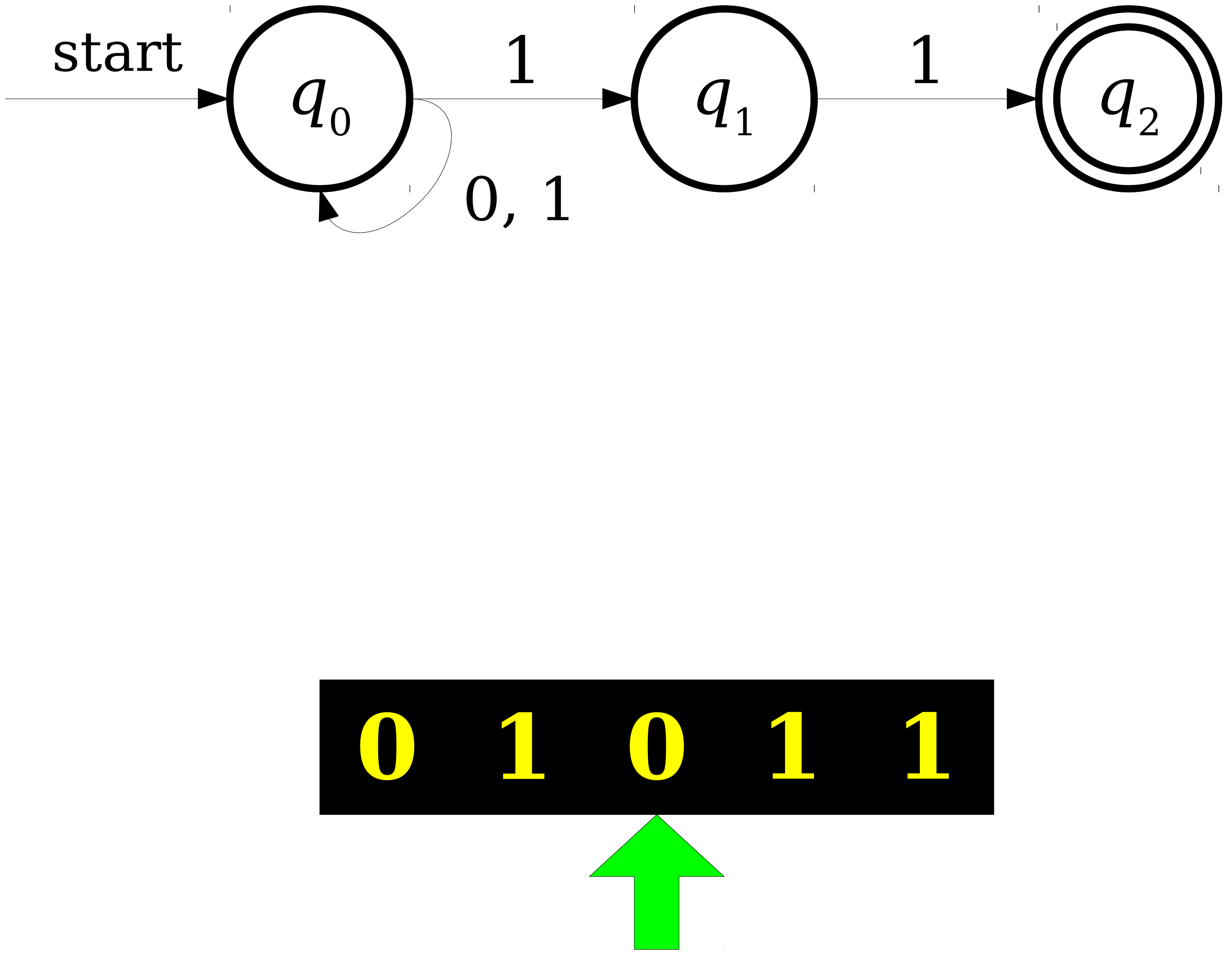

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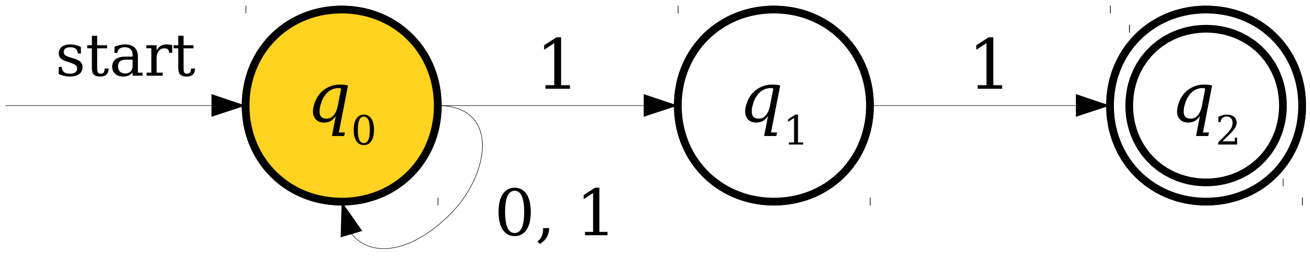

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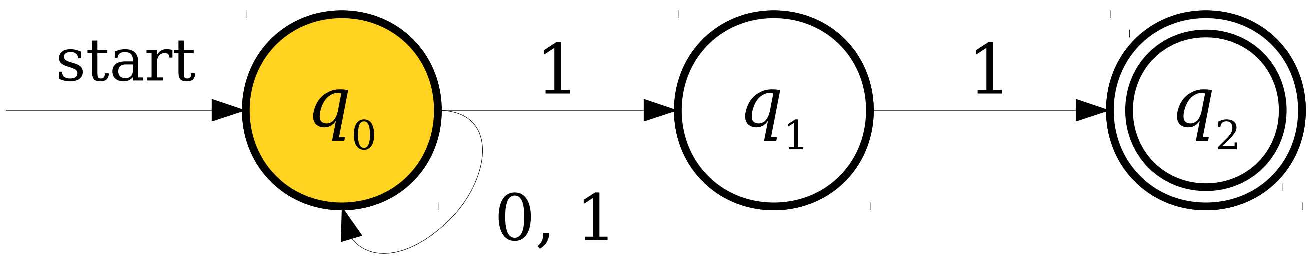

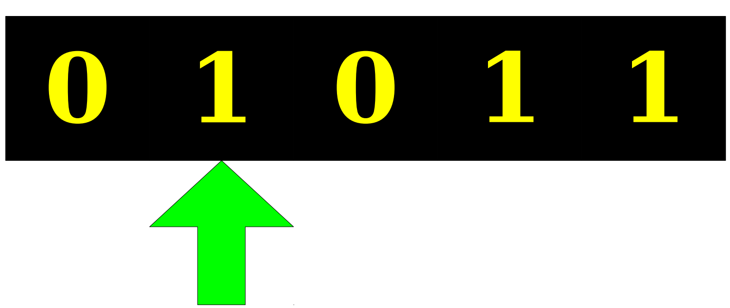

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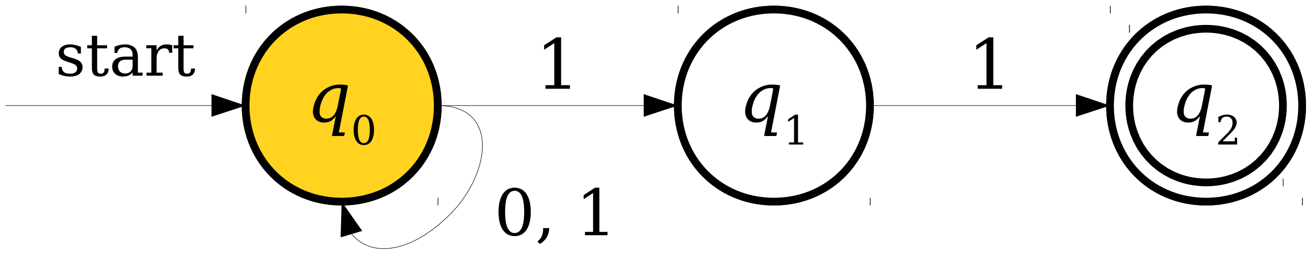

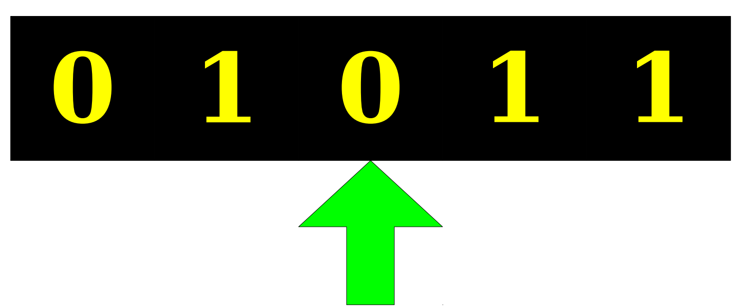

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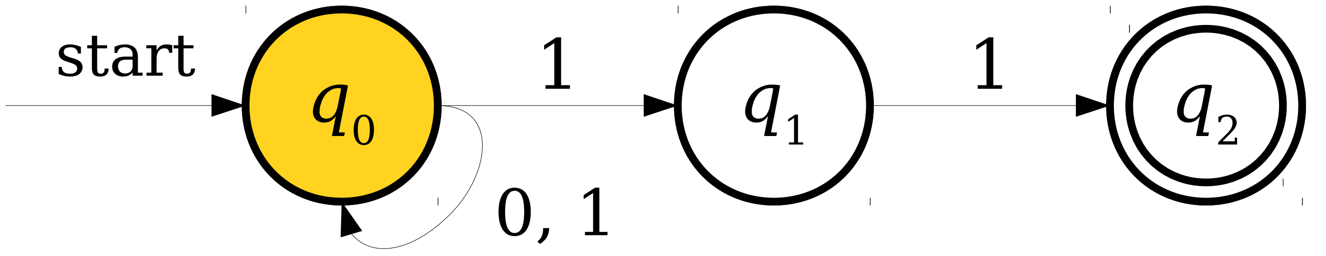

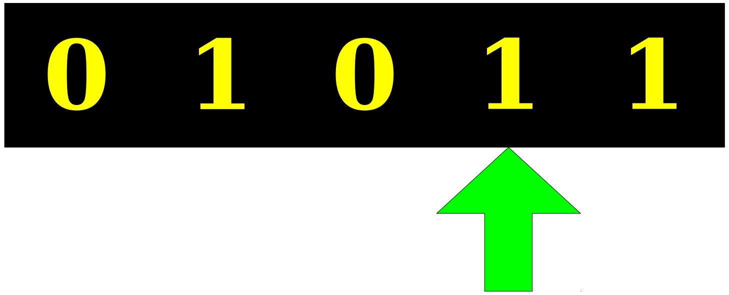

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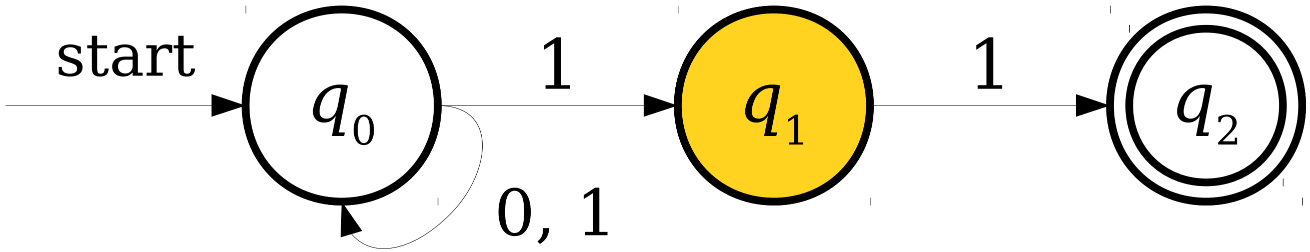

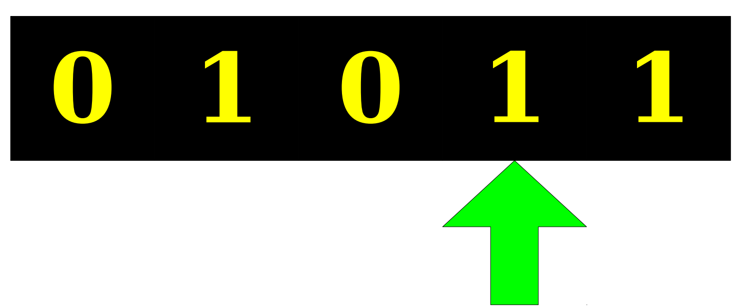

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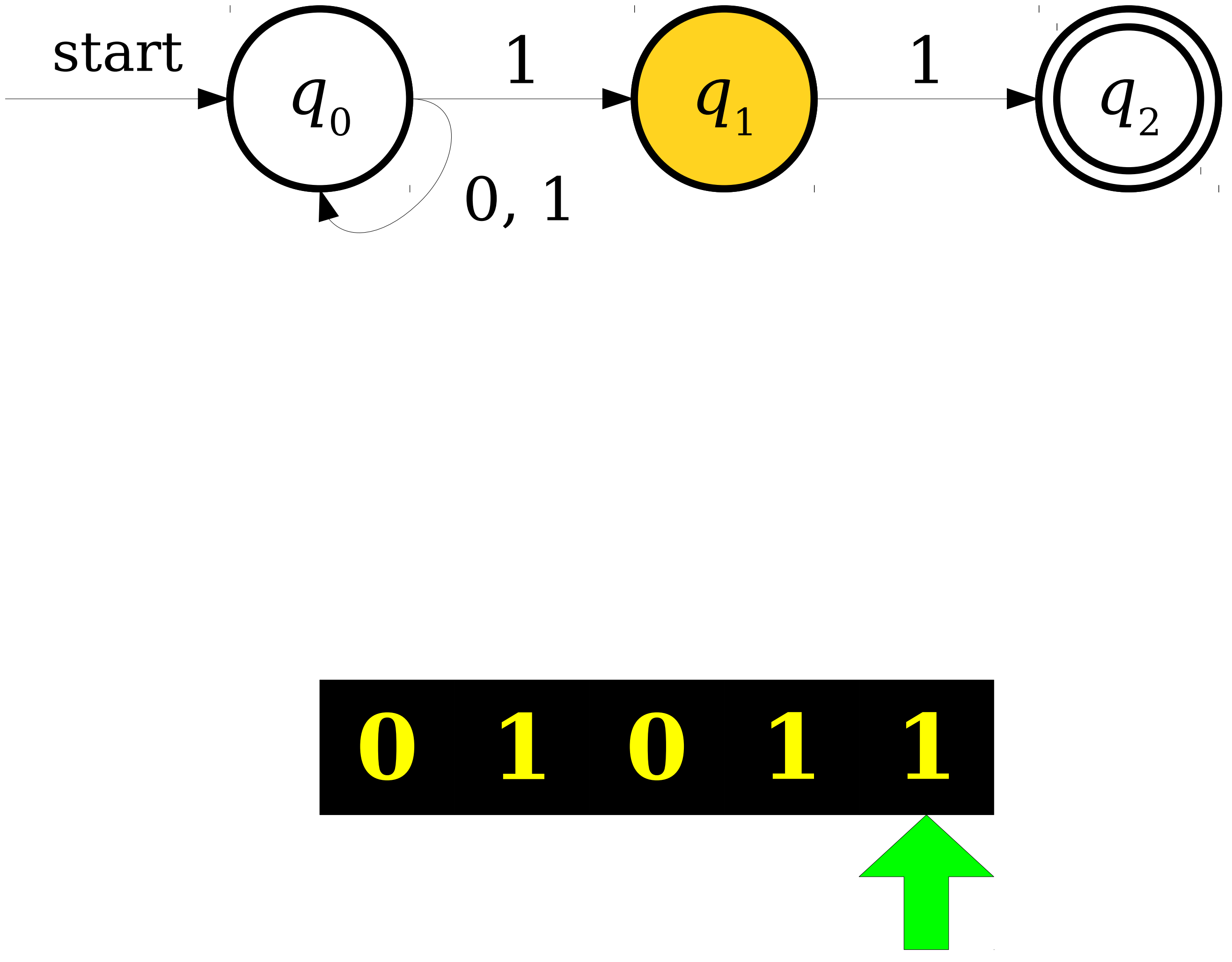

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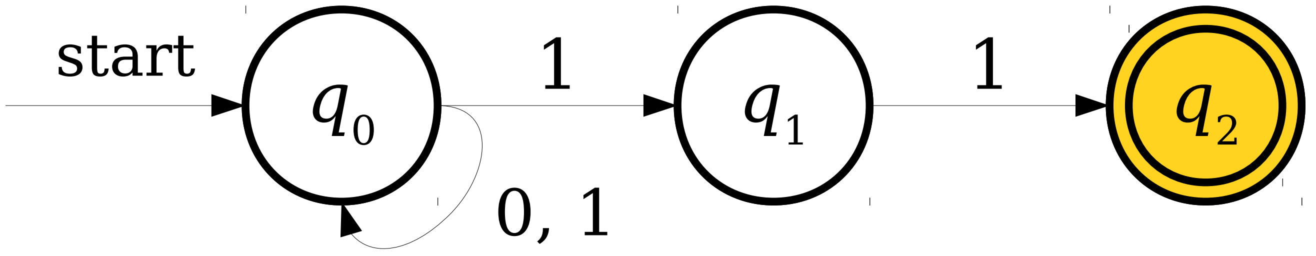

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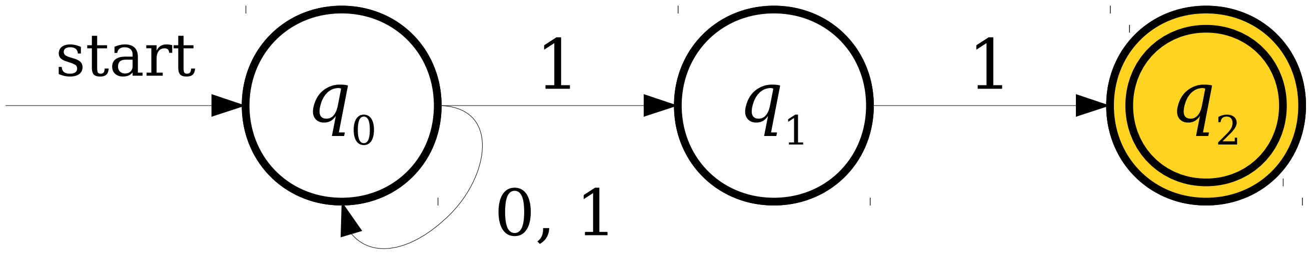

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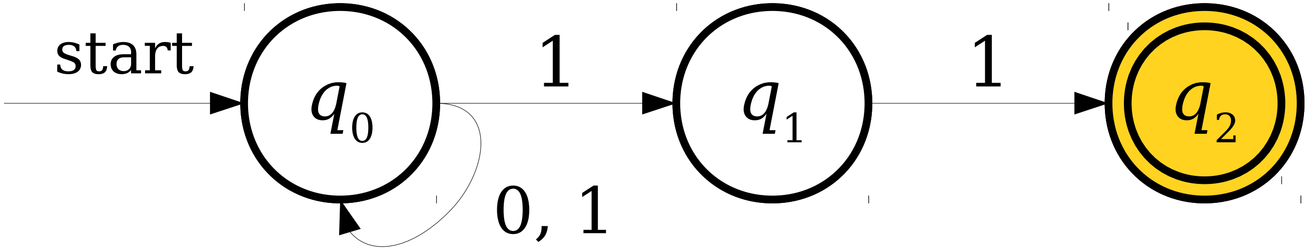

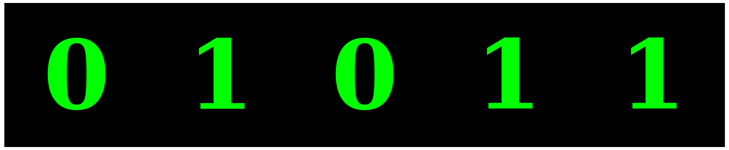

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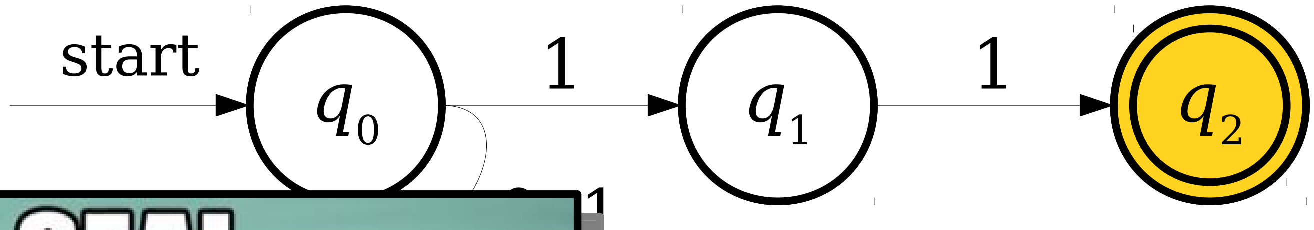

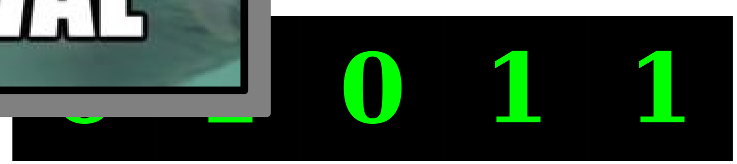

NFA Acceptance

- An NFA N accepts a string w if there is some series of choices that lead to an accepting state.
- Let $LeadsToAccept(N, c, w)$ mean "the series of choices c takes N into an accept state when run on w."
- Then

$$\boldsymbol{N \text{ accepts } w \leftrightarrow \exists c.\ LeadsToAccept(N, c, w)}$$

- Consequently,

$$\boldsymbol{N \text{ rejects } w \leftrightarrow \forall c.\ \neg LeadsToAccept(N, c, w)}$$

ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.

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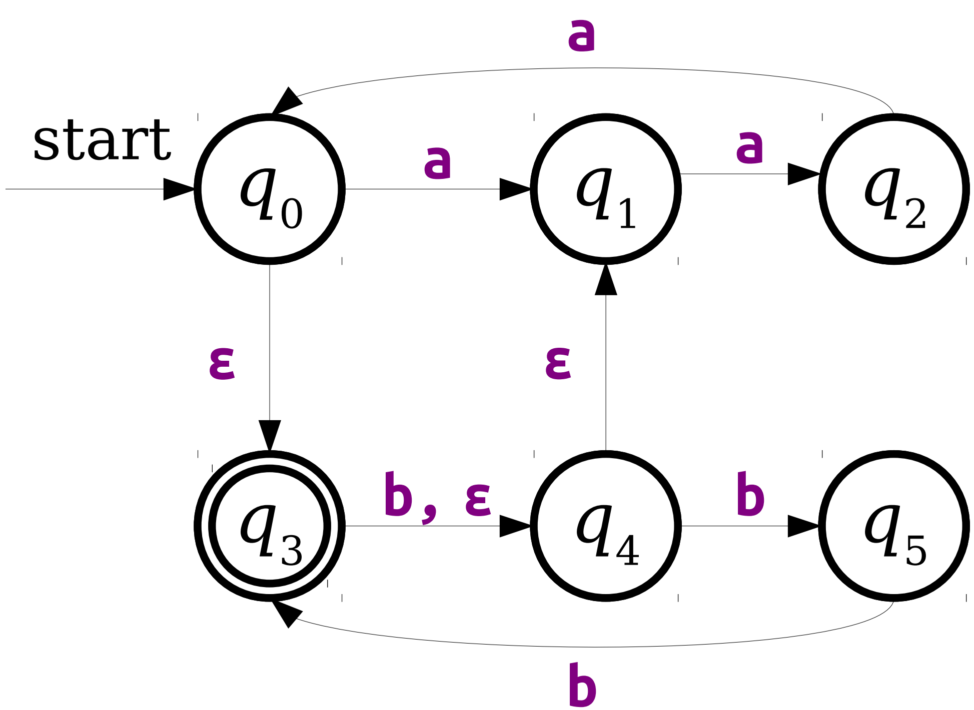

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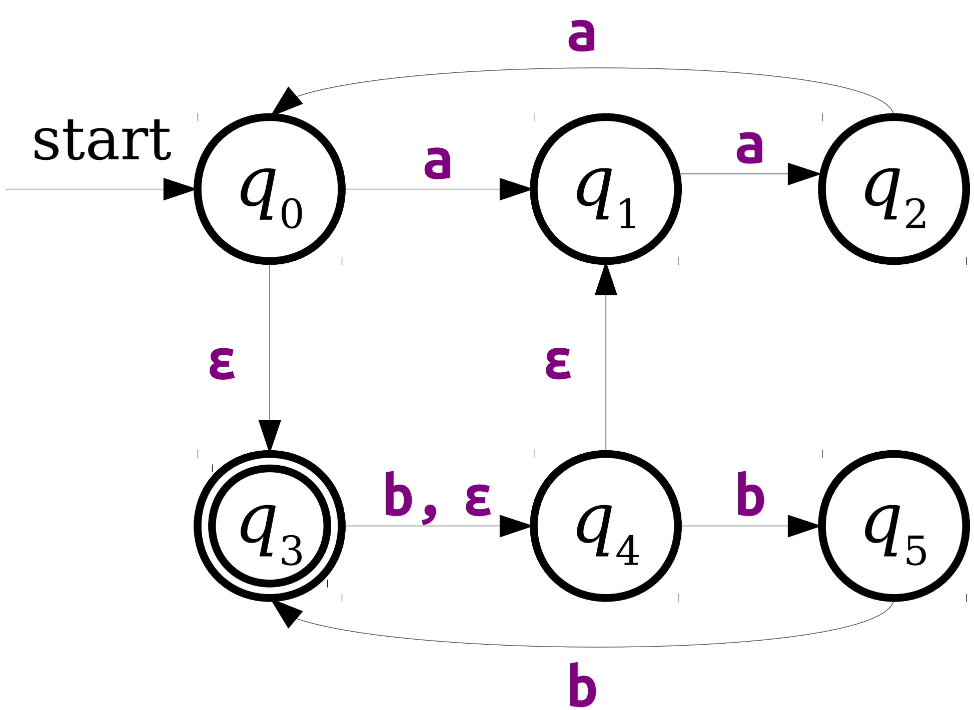

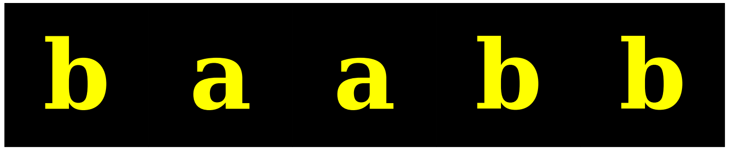

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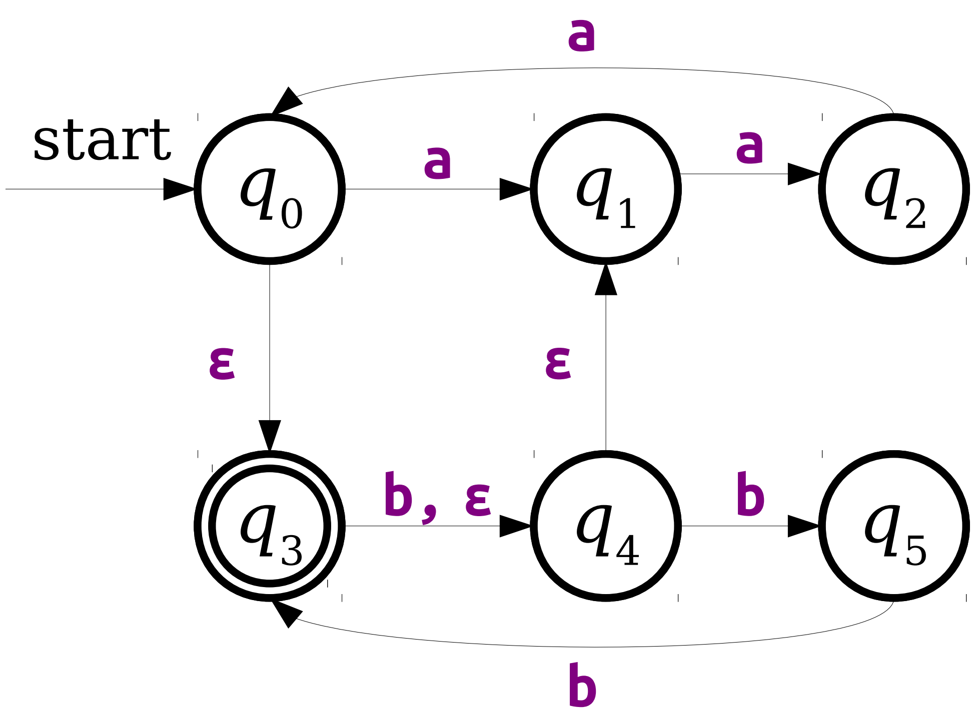

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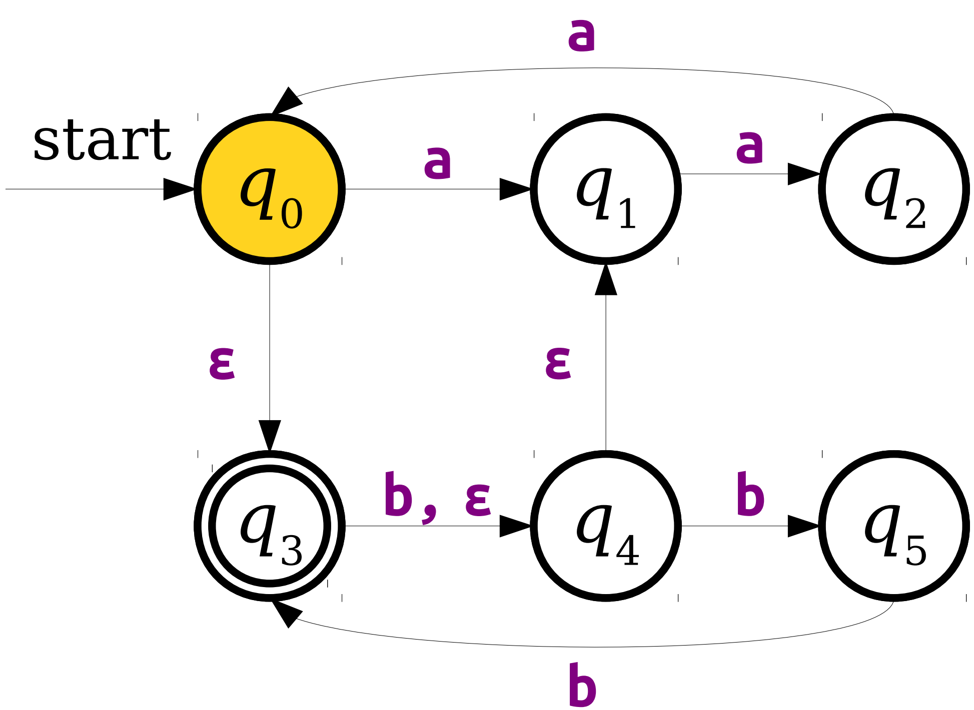

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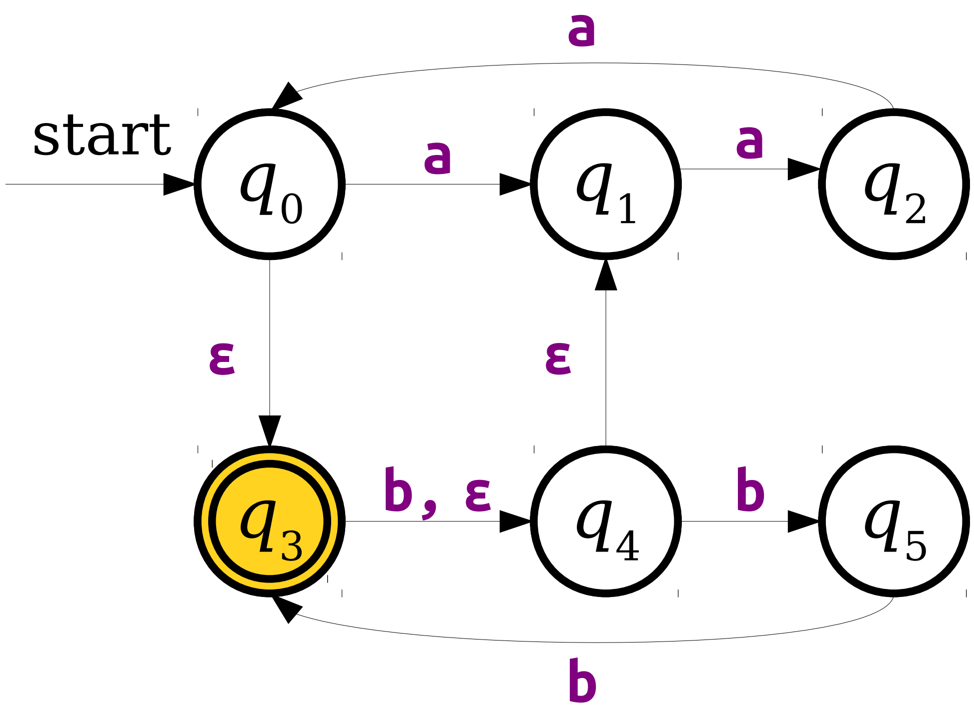

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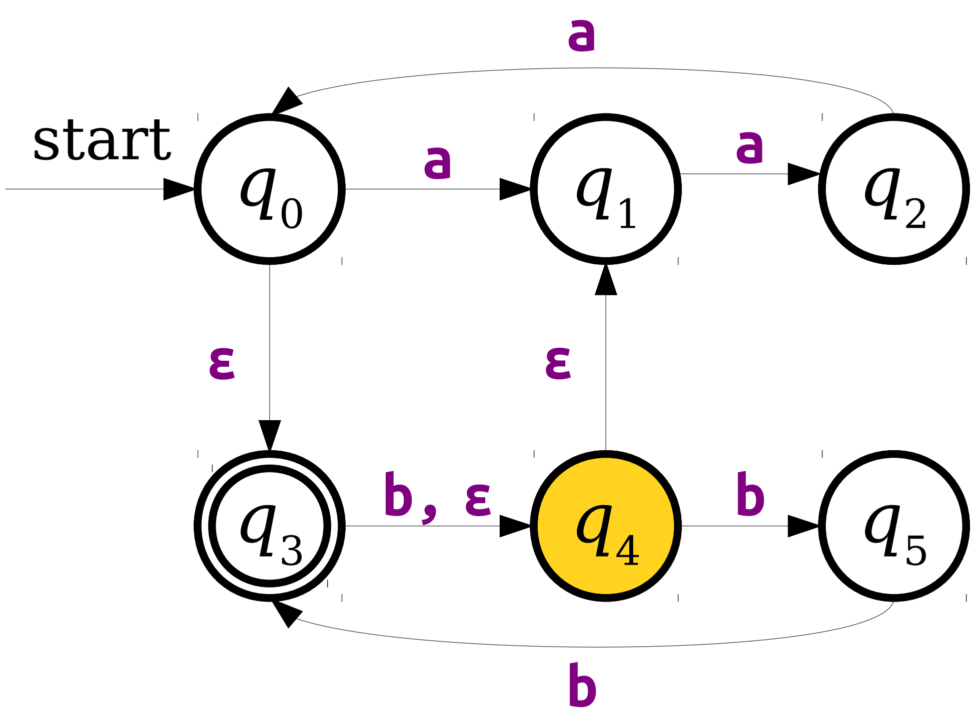

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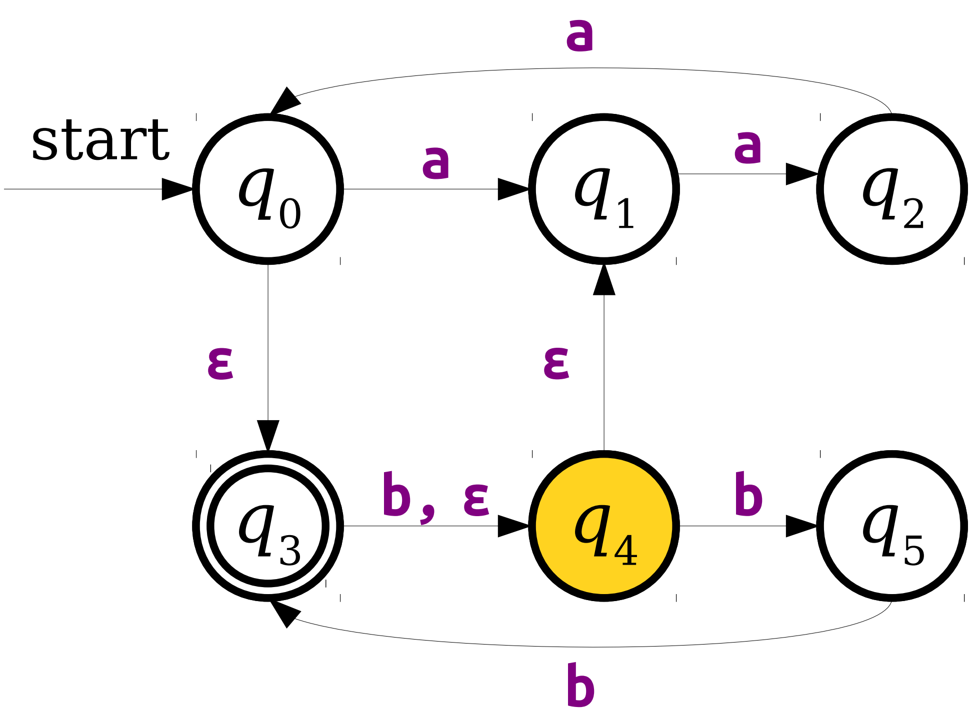

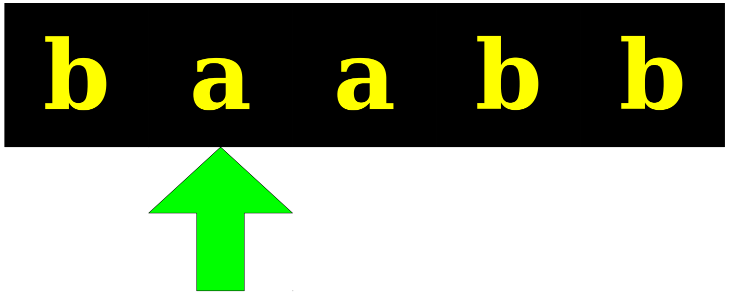

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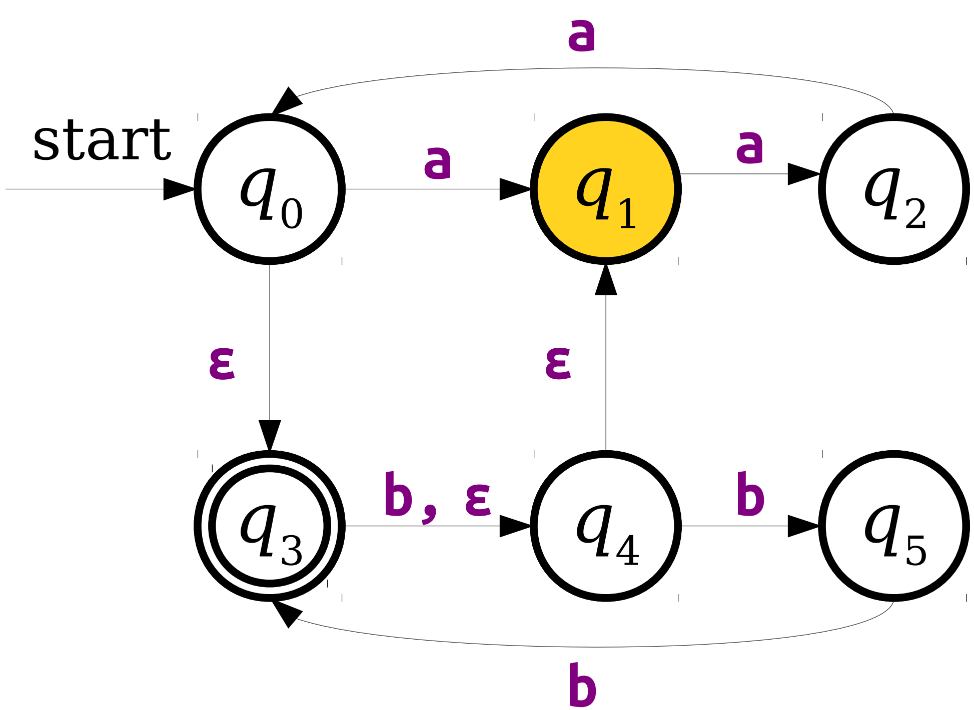

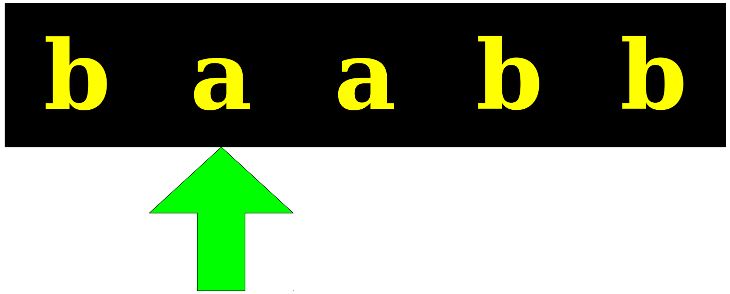

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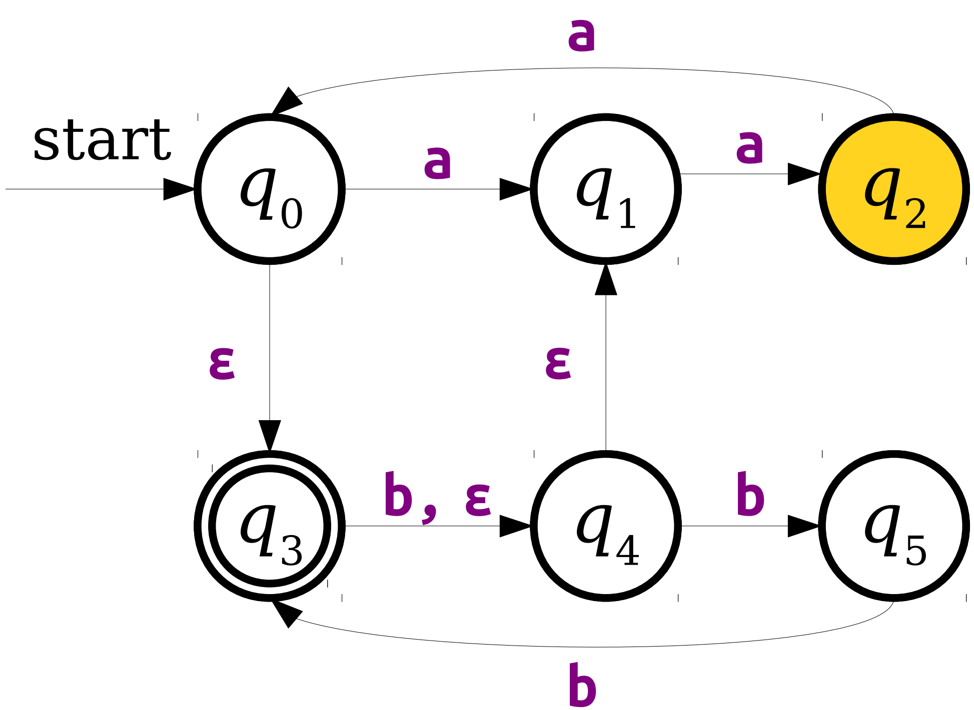

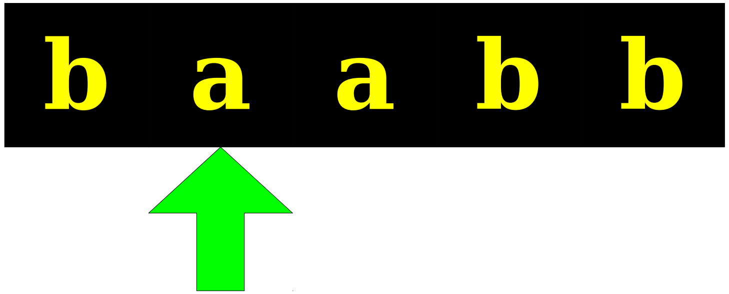

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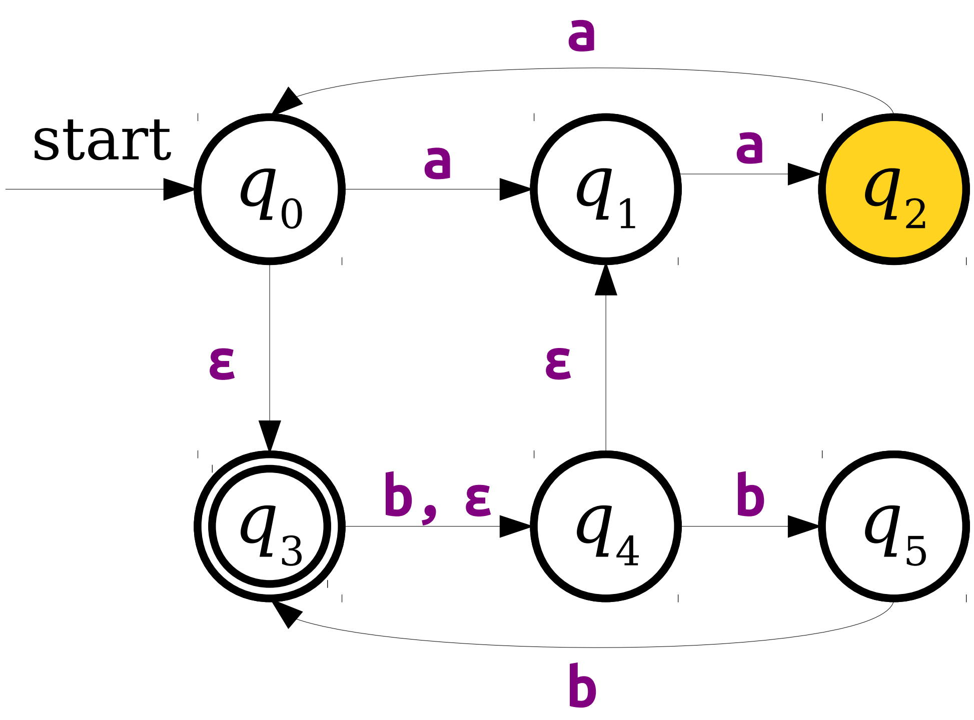

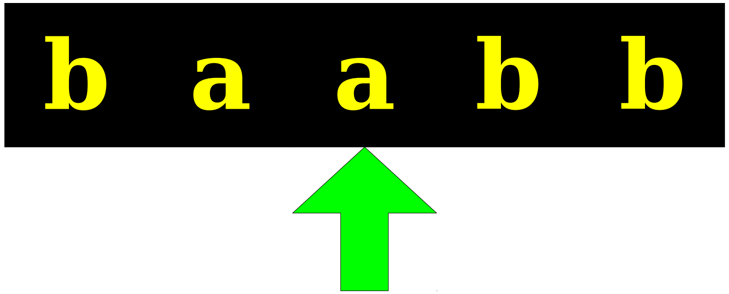

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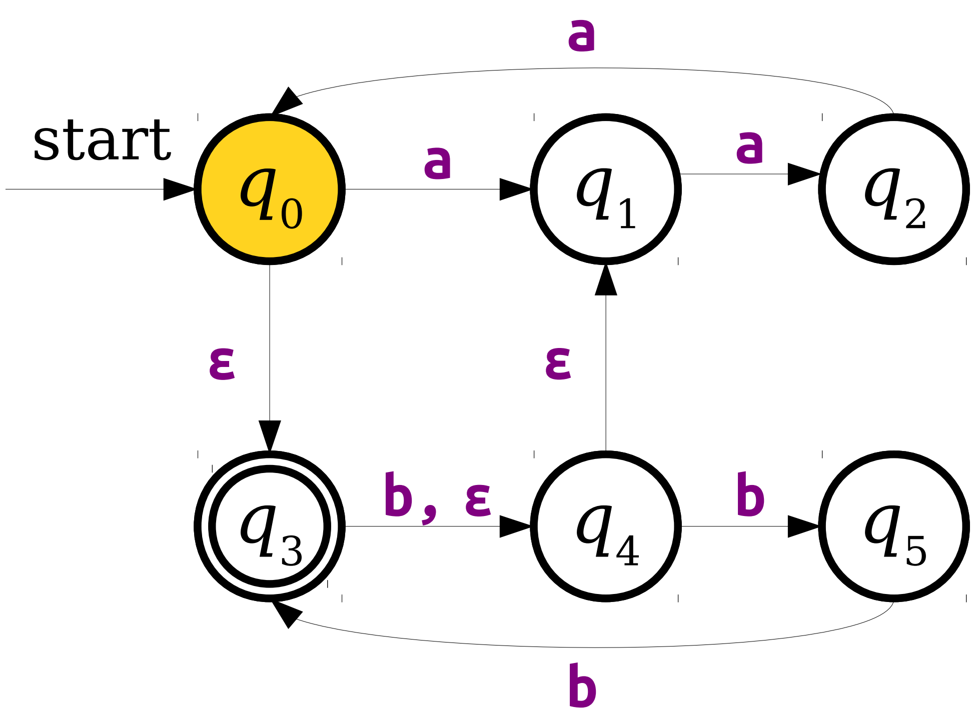

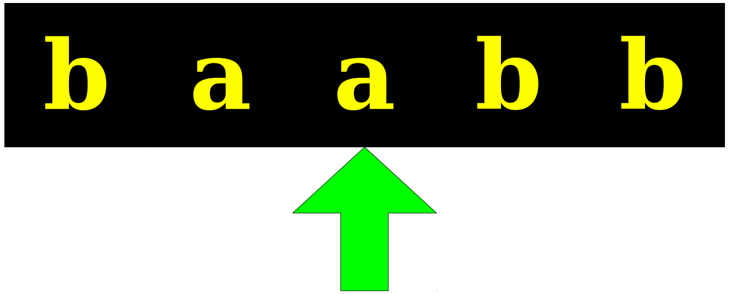

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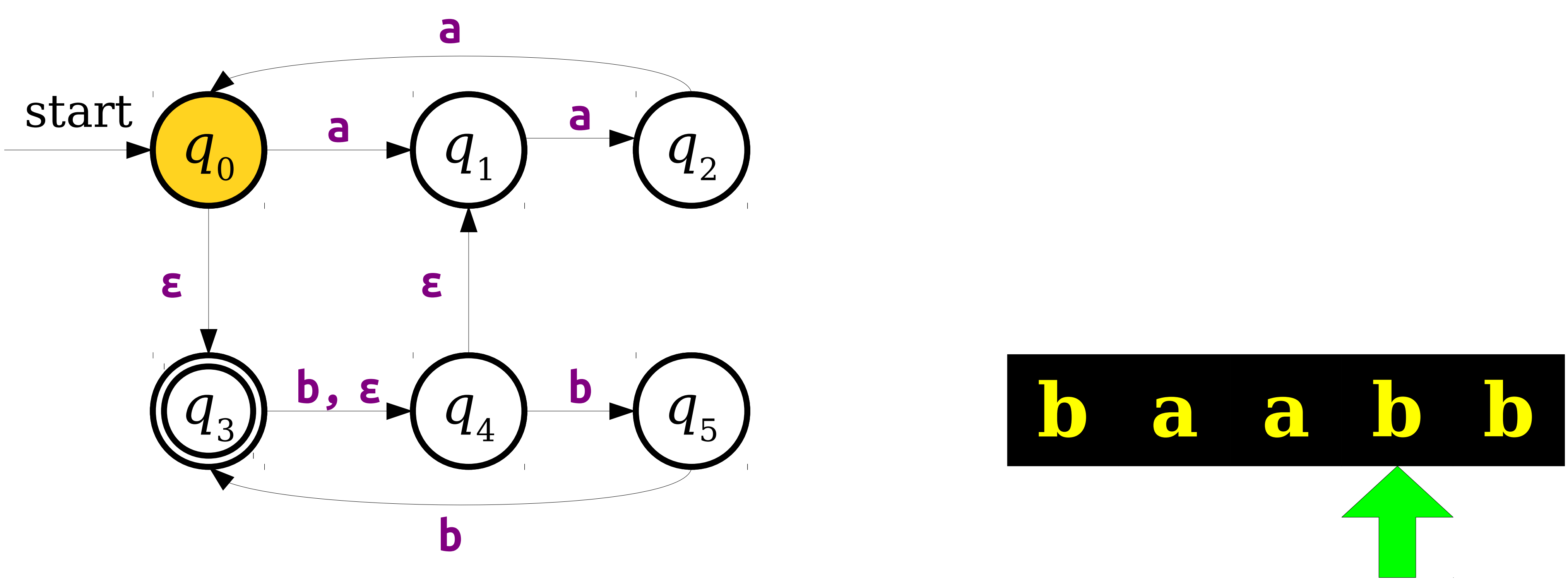

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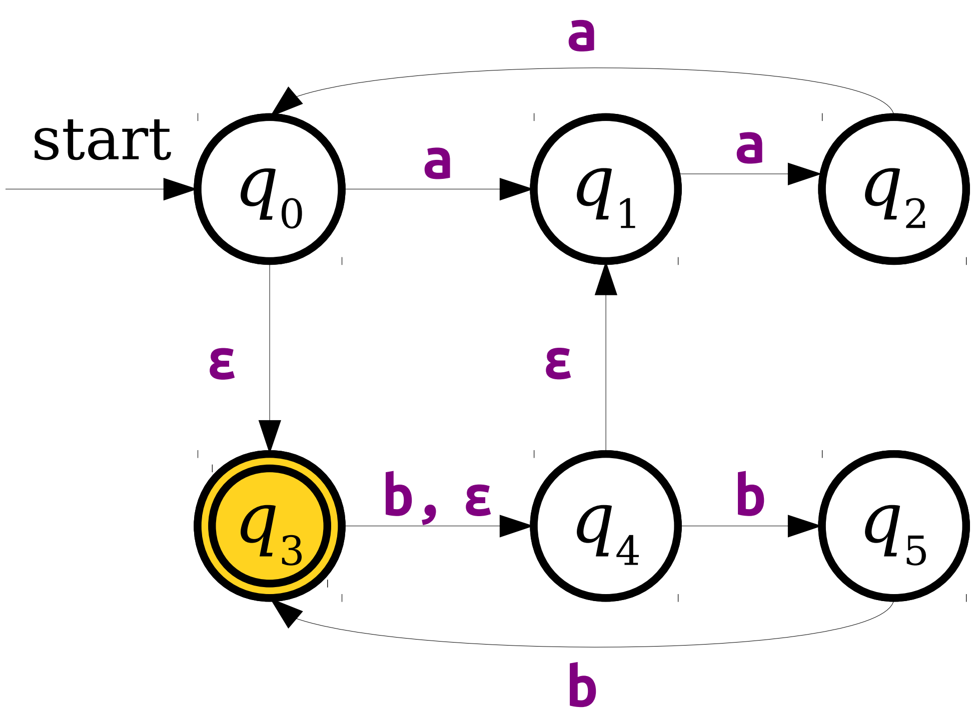

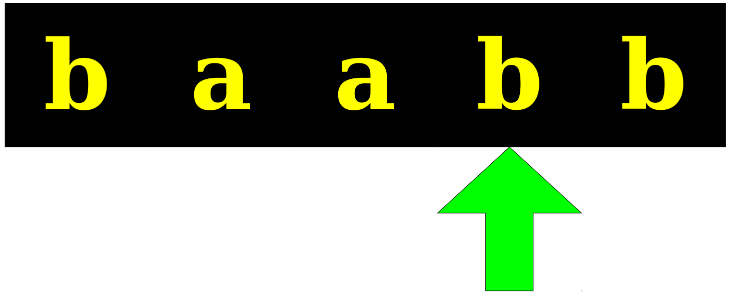

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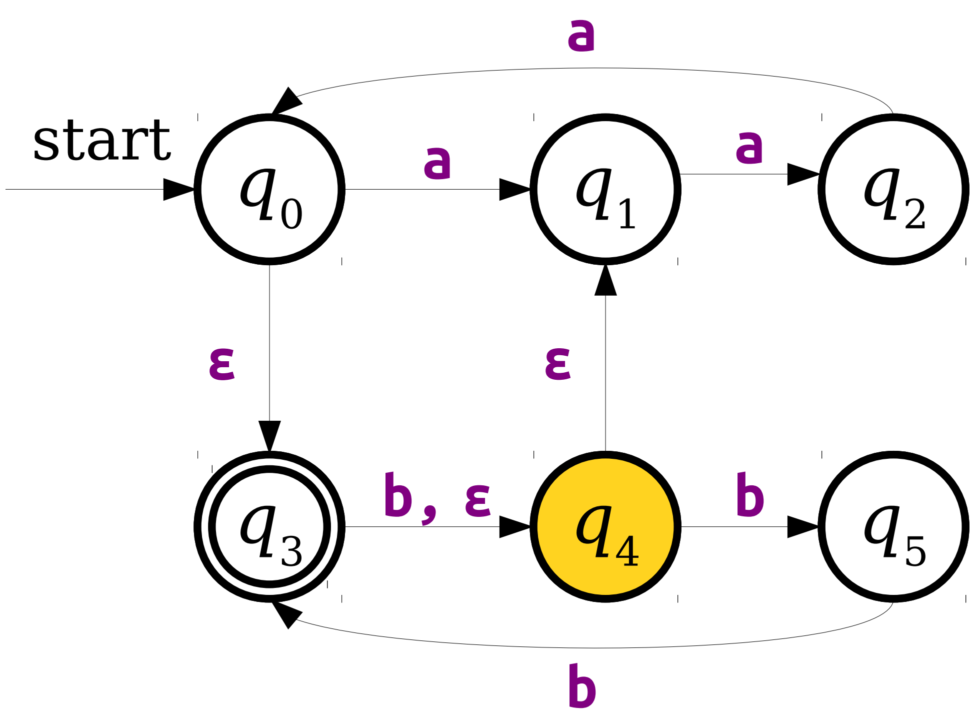

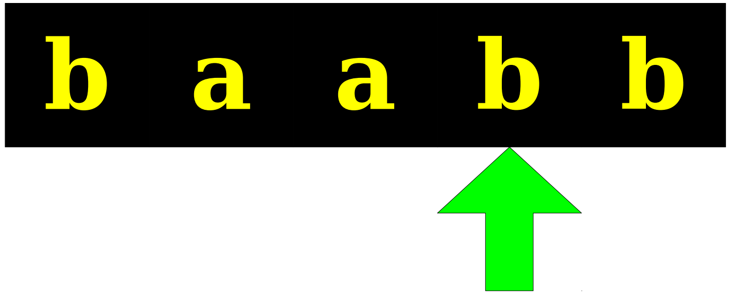

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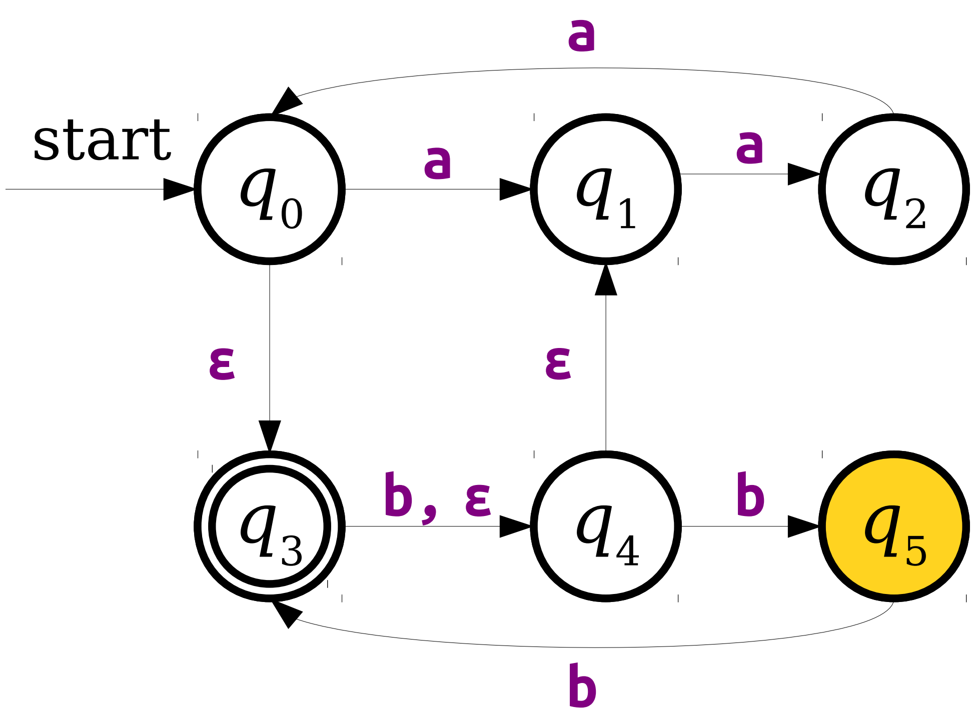

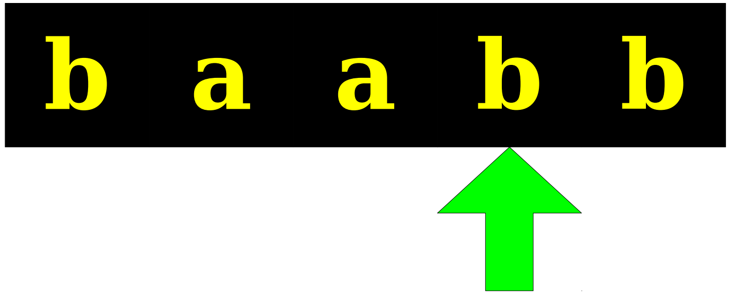

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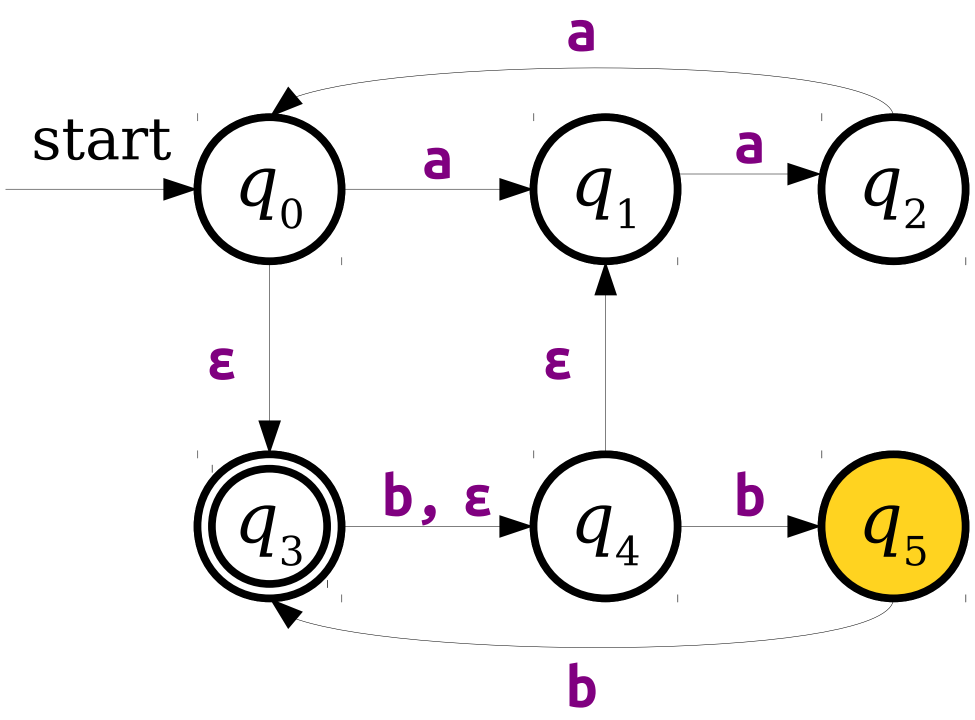

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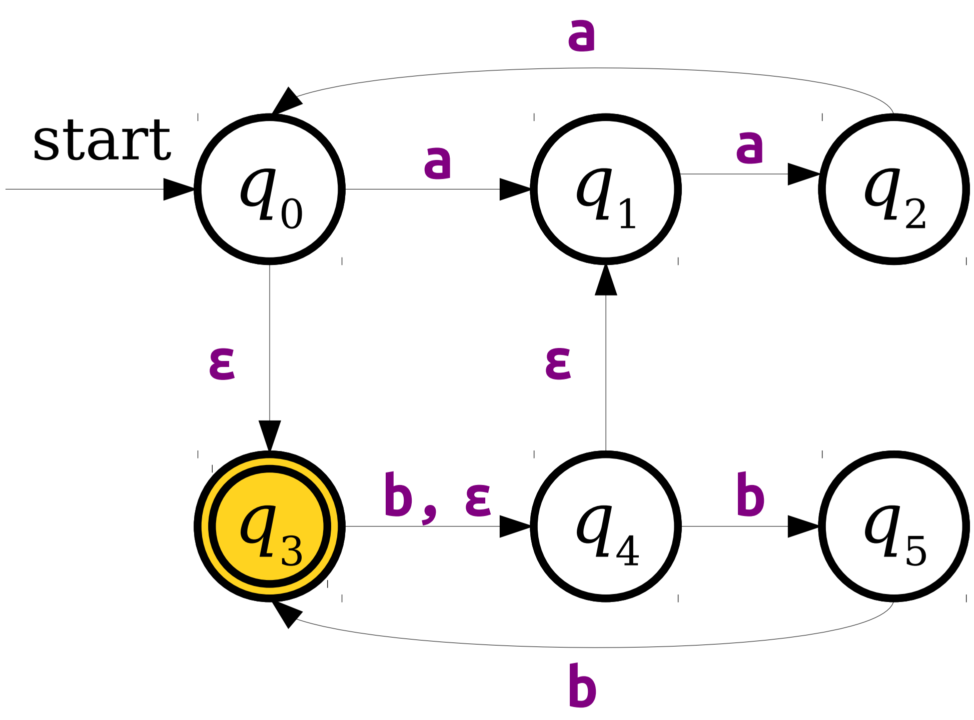

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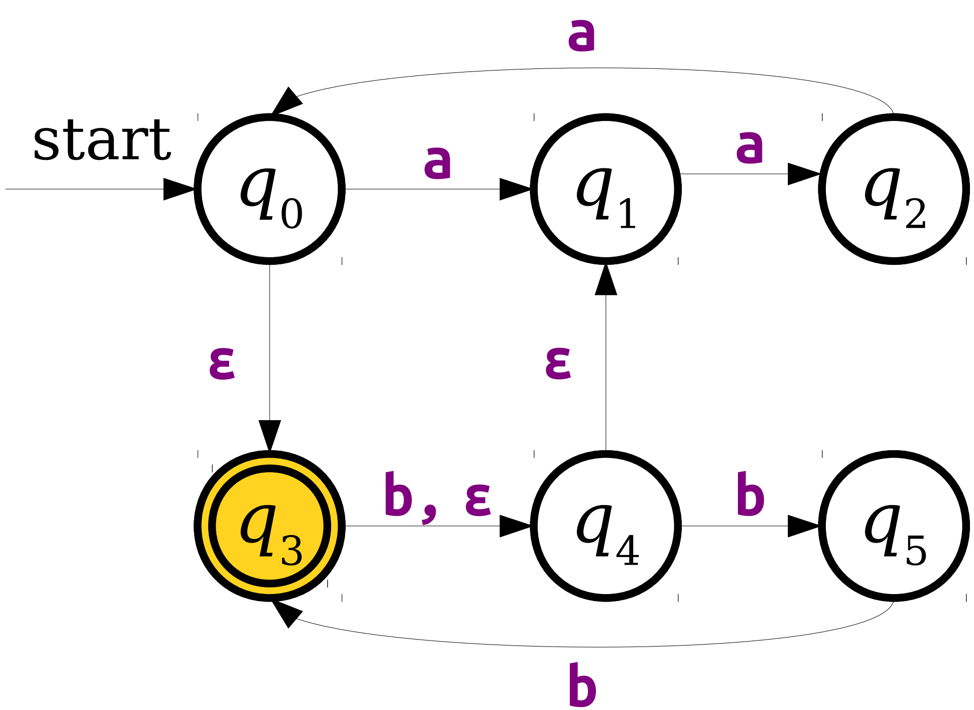

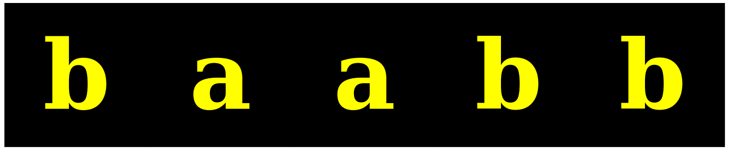

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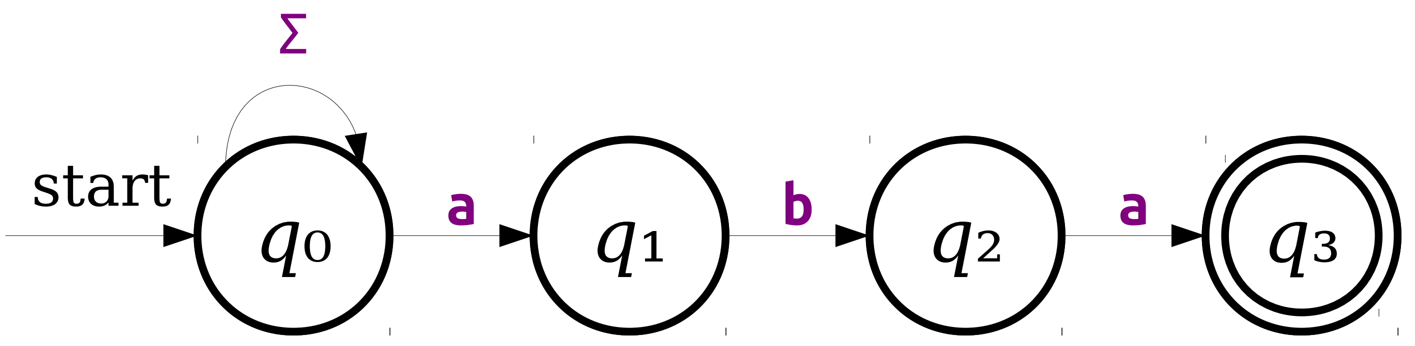
- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.
- NFAs are not *required* to follow ε-transitions. It's simply another option at the machine's disposal.

Intuiting Nondeterminism

- Nondeterministic machines are a serious departure from physical computers. How can we build up an intuition for them?
- There are two particularly useful frameworks for interpreting nondeterminism:
  - **Perfect guessing**
  - **Massive parallelism**

Perfect Guessing



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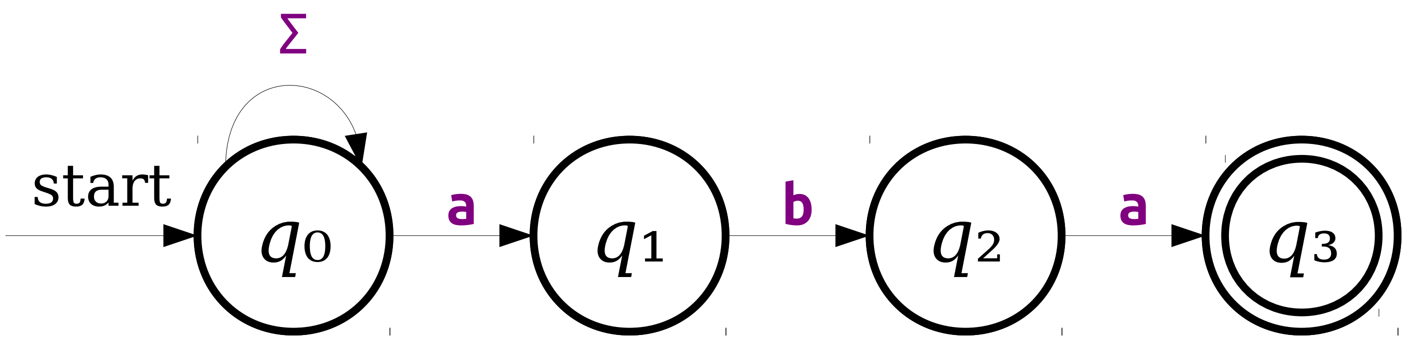
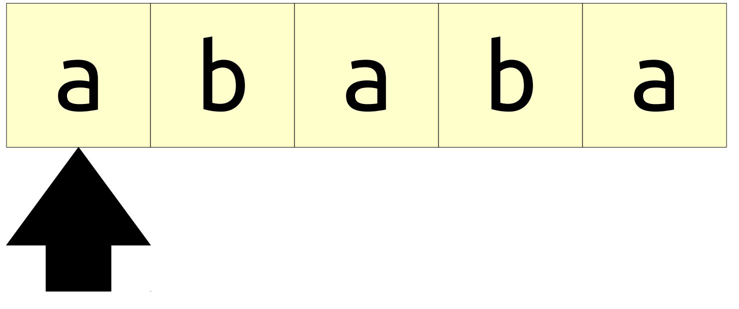
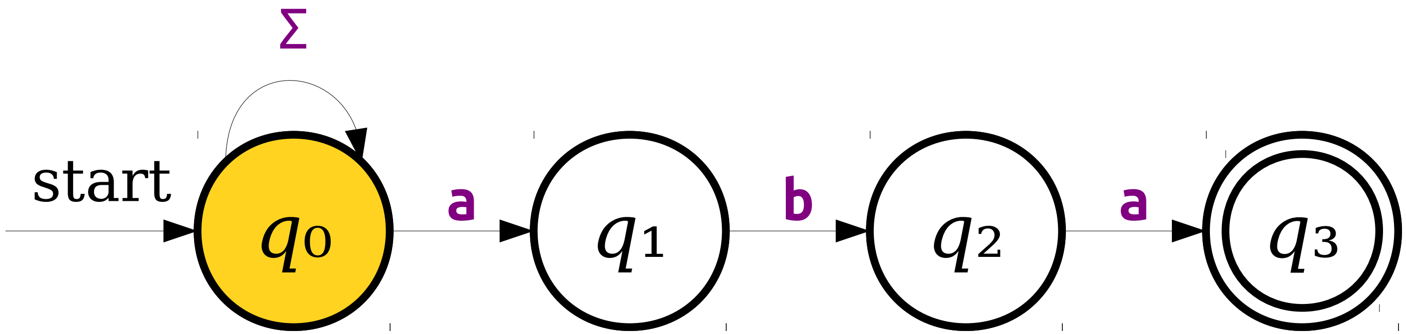
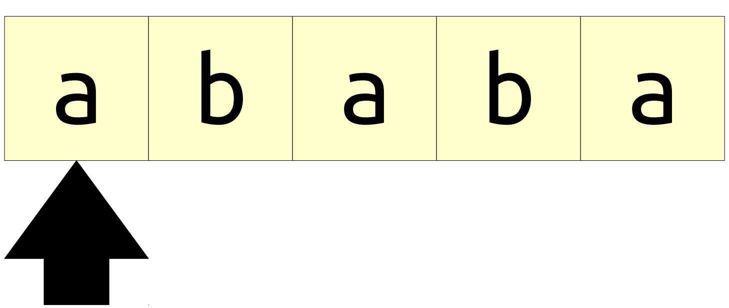




Perfect Guessing





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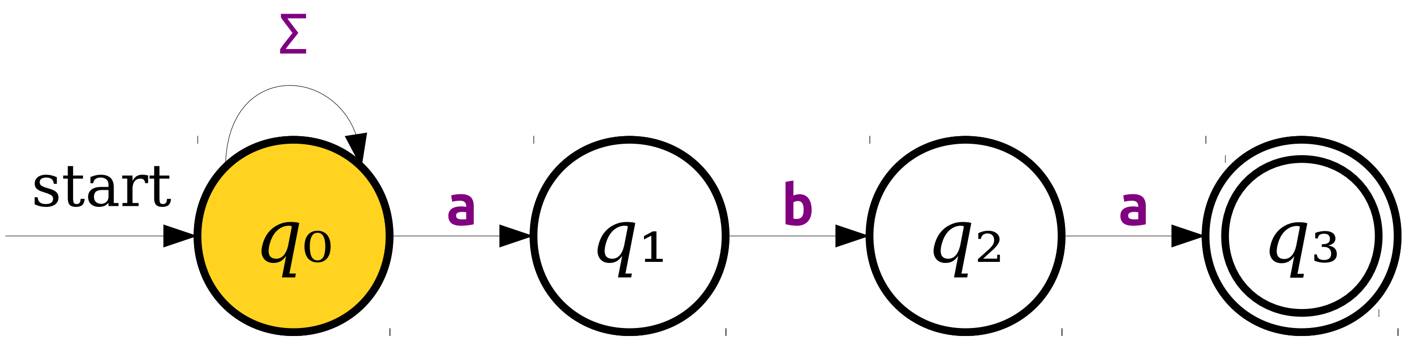

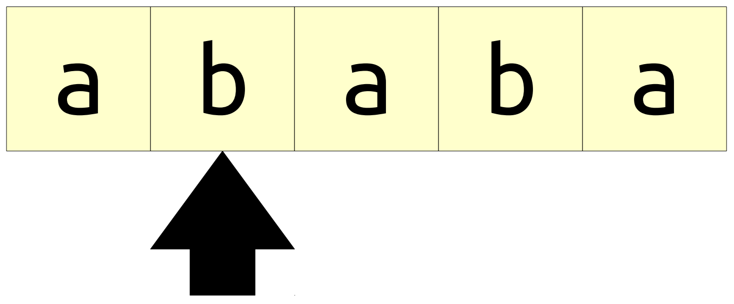

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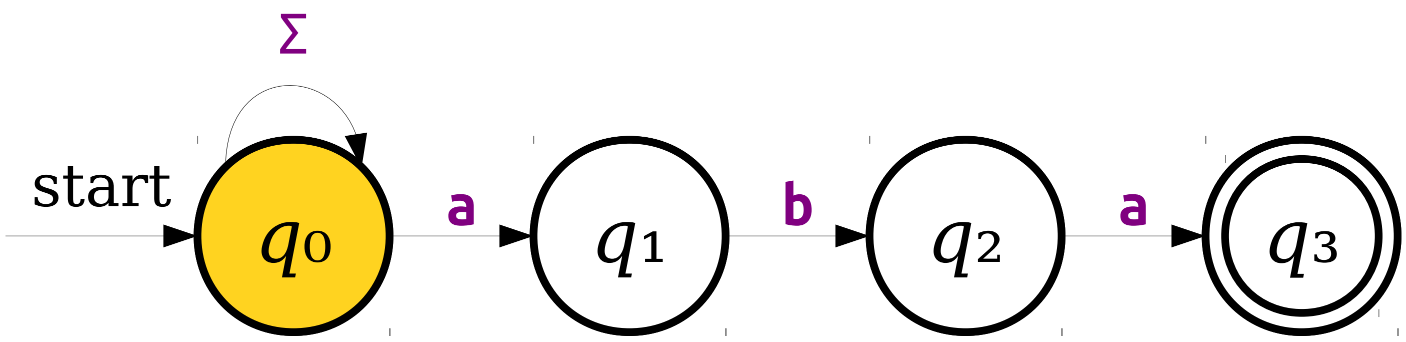

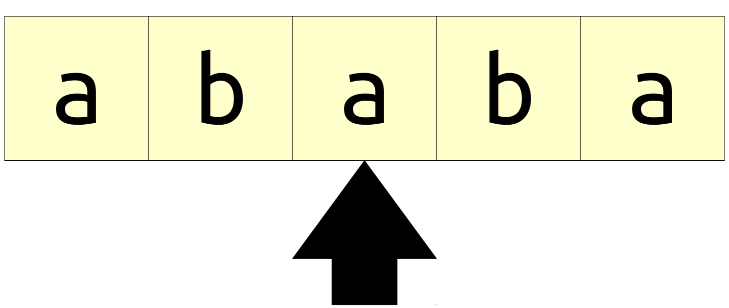

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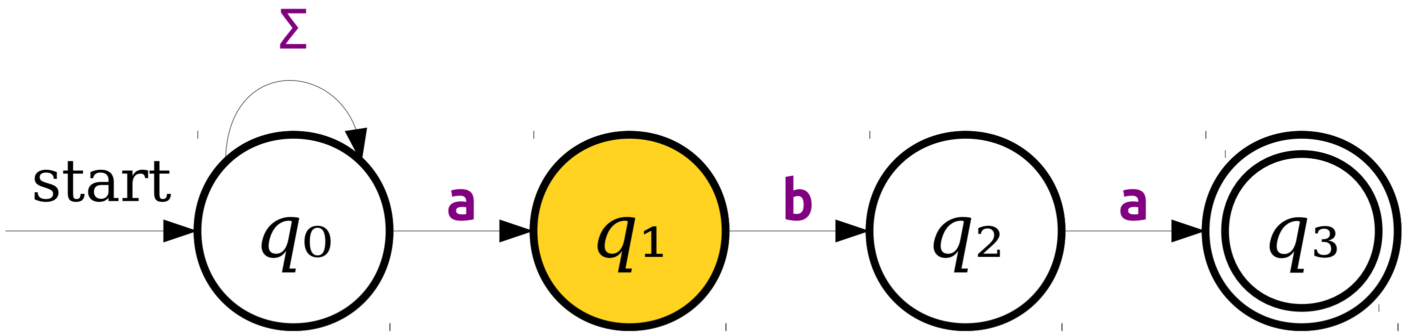

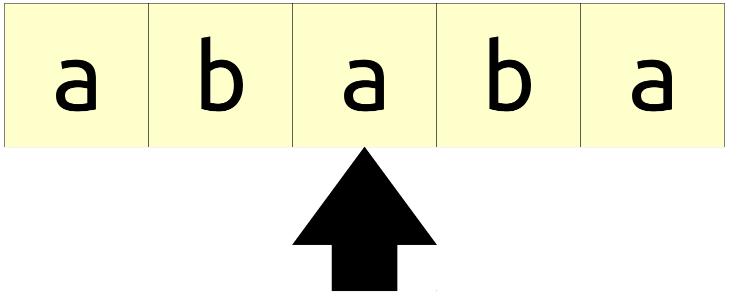

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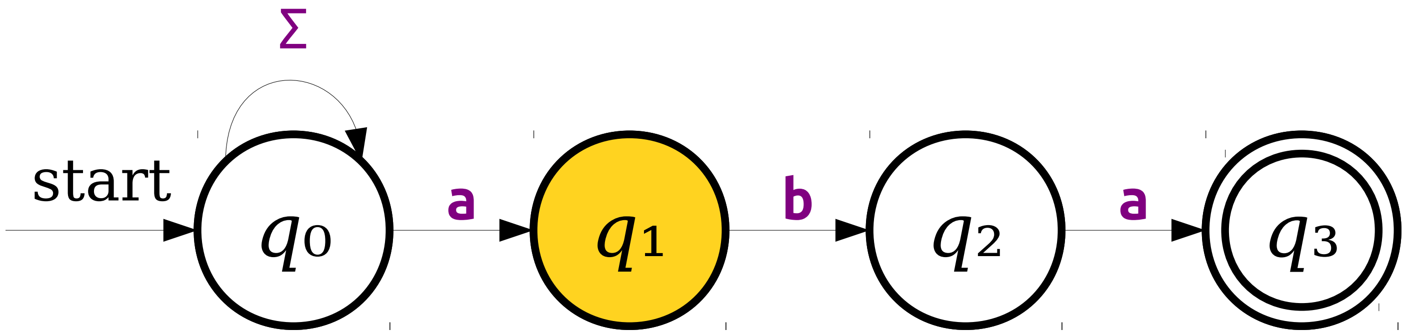

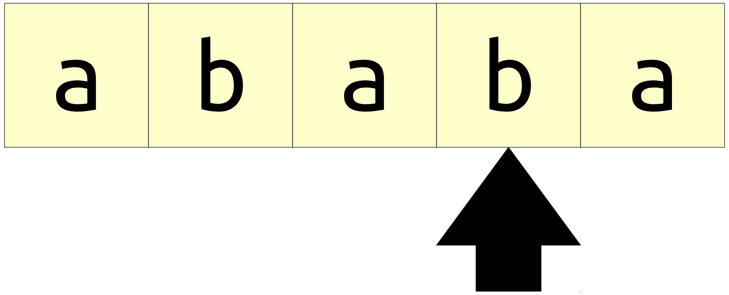

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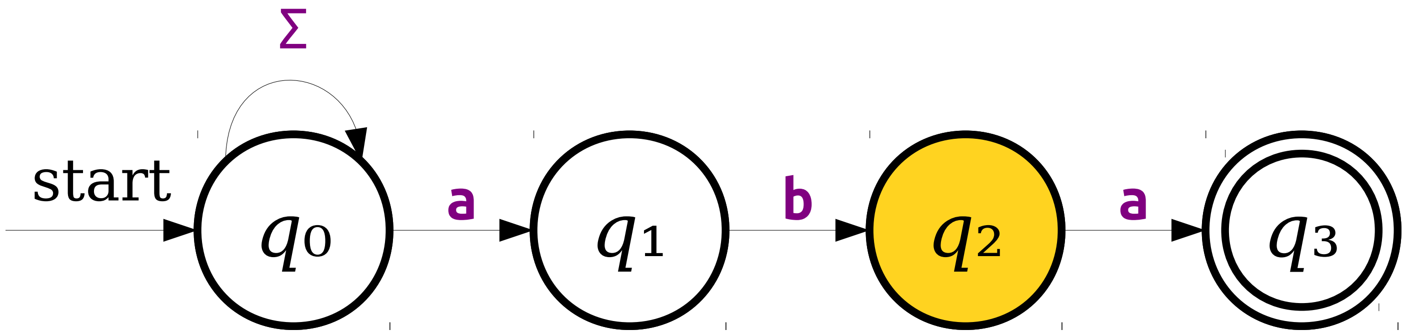

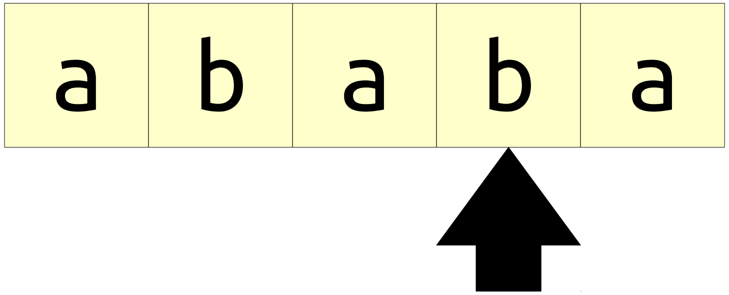

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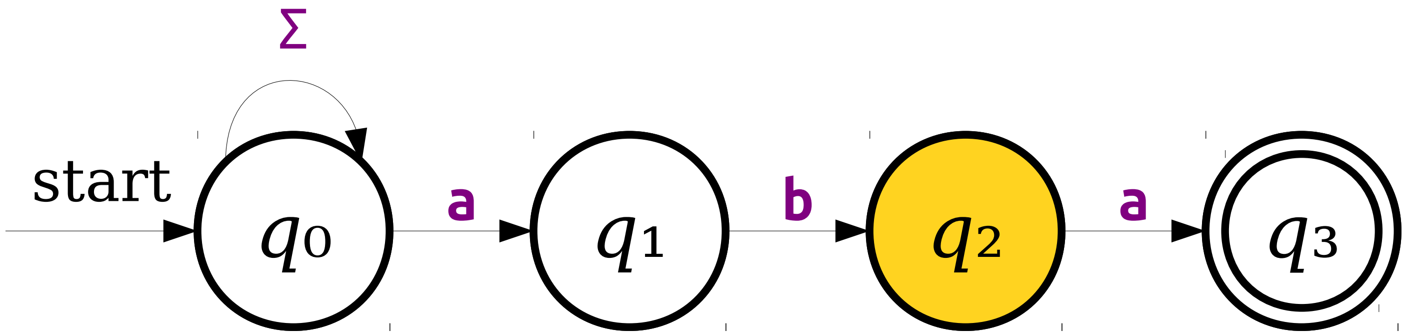

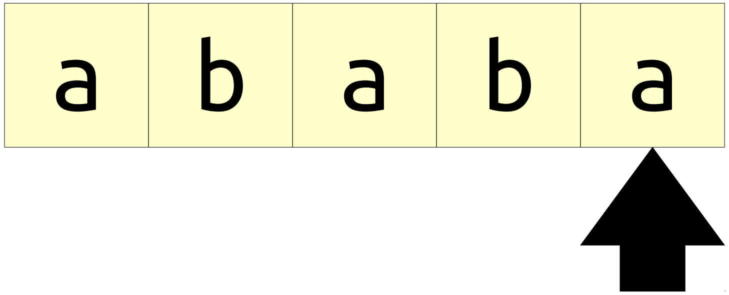

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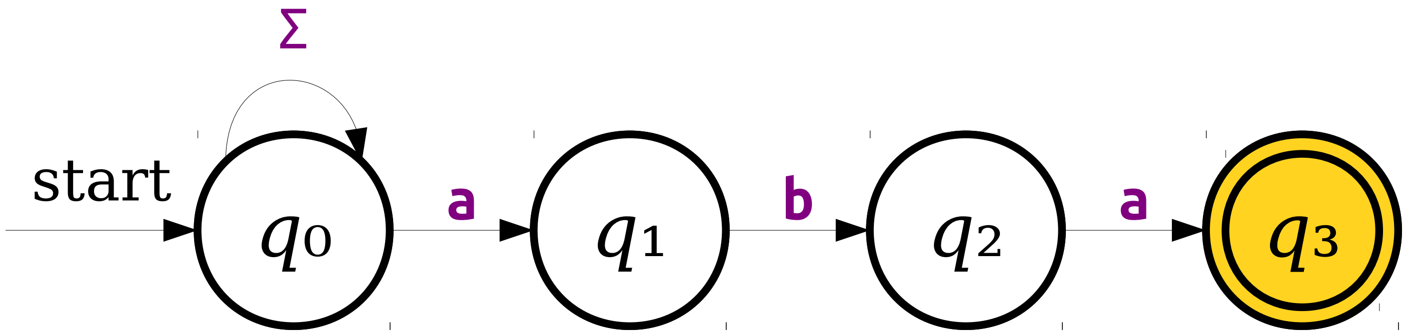

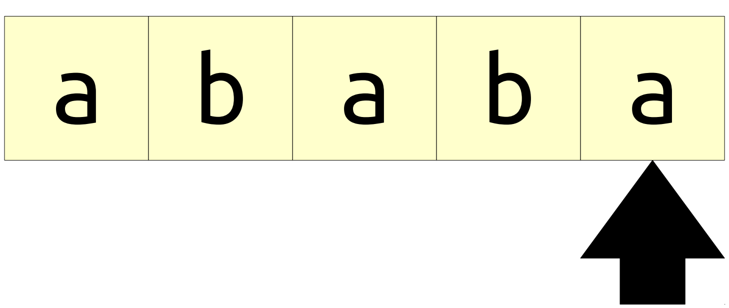

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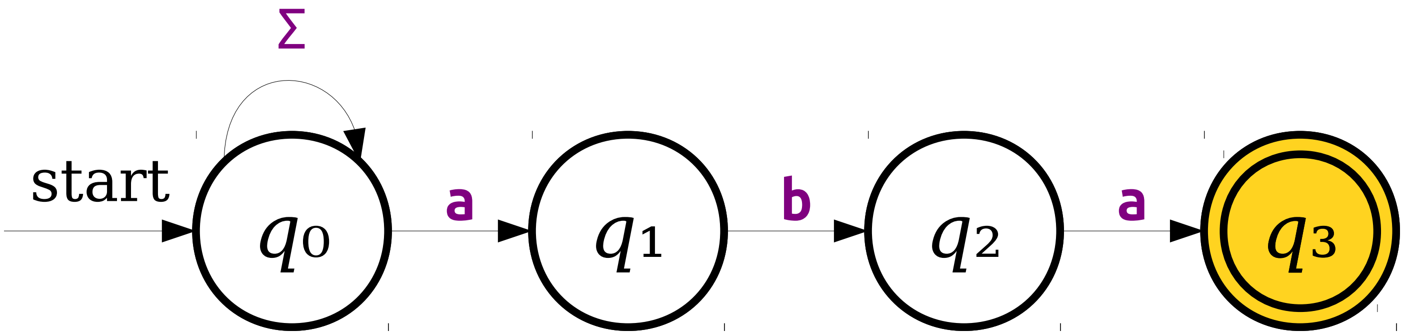
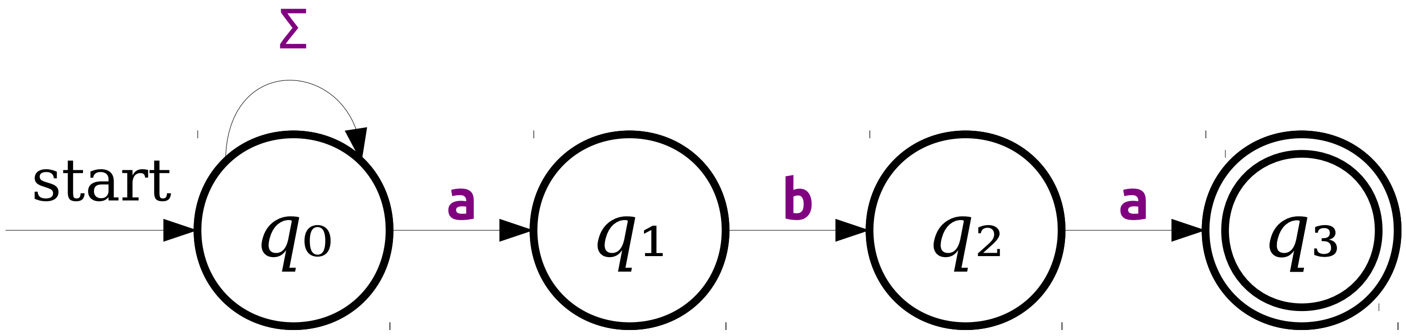


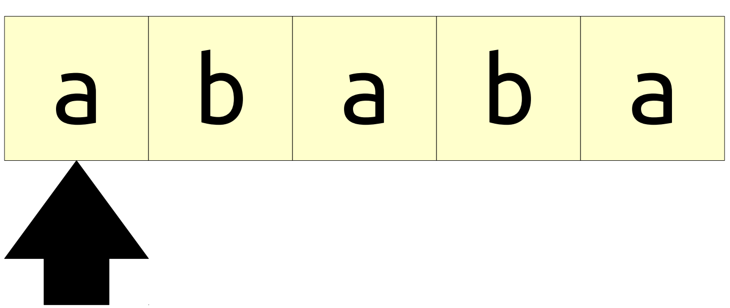
a	b	a	b	a

Perfect Guessing

- We can view nondeterministic machines as having ***Magic Superpowers*** that enable them to guess the correct choice of moves to make.
  - If there is at least one choice that leads to an accepting state, the machine will guess it.
  - If there are no choices, the machine guesses any one of the wrong guesses.
- No known physical analog for this style of computation – this is totally new!

Massive Parallelism





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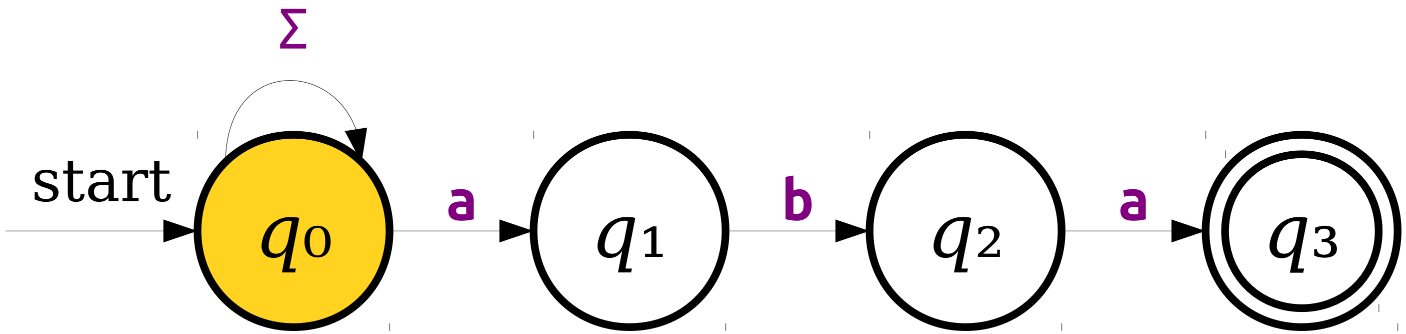

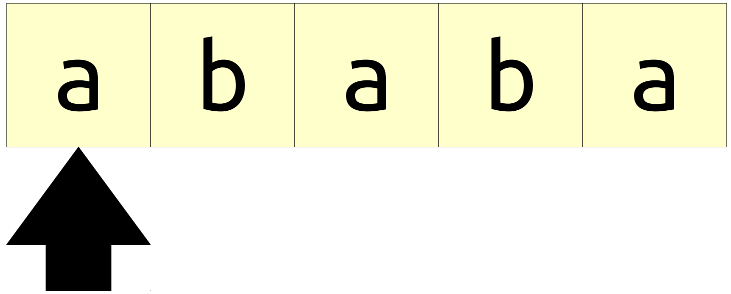

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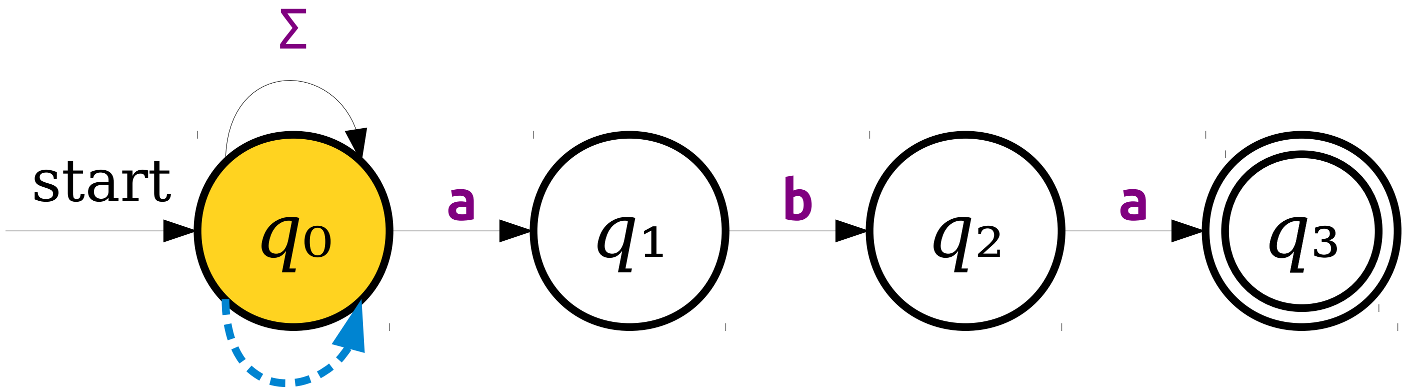

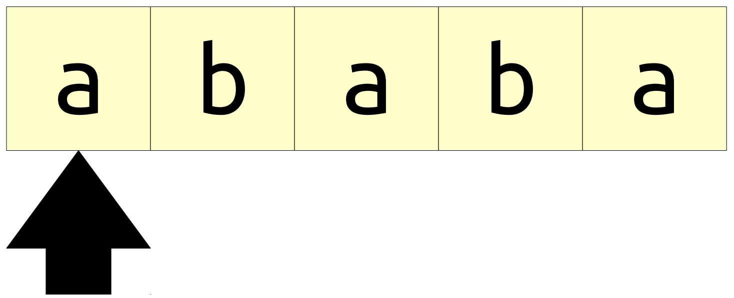

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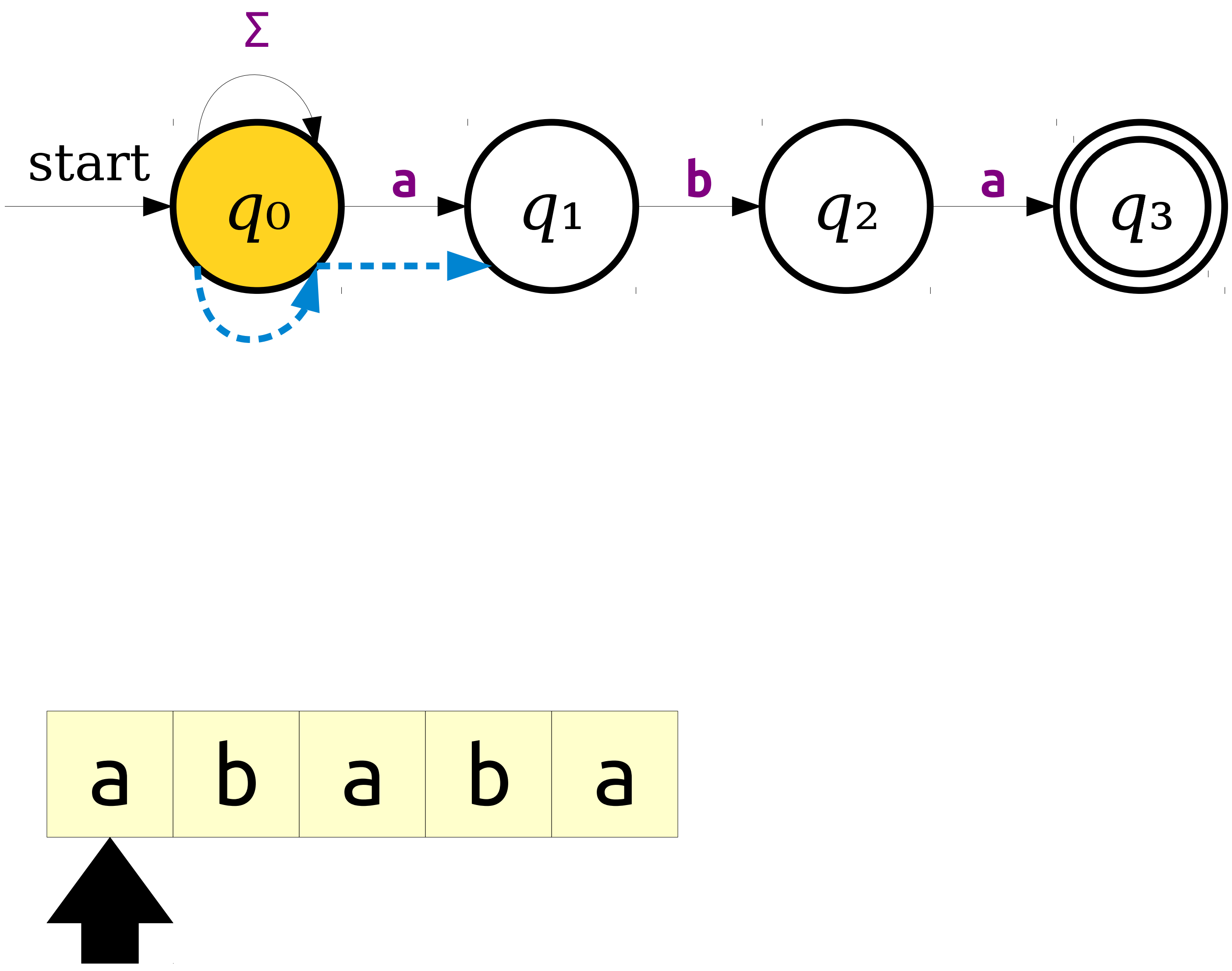

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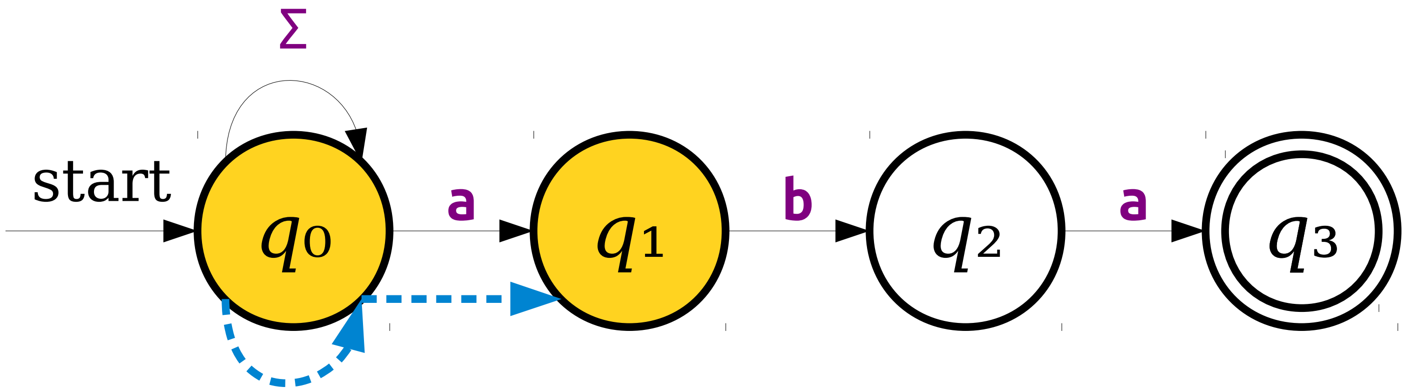

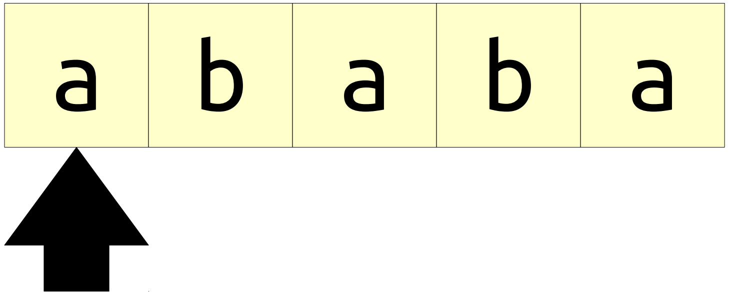

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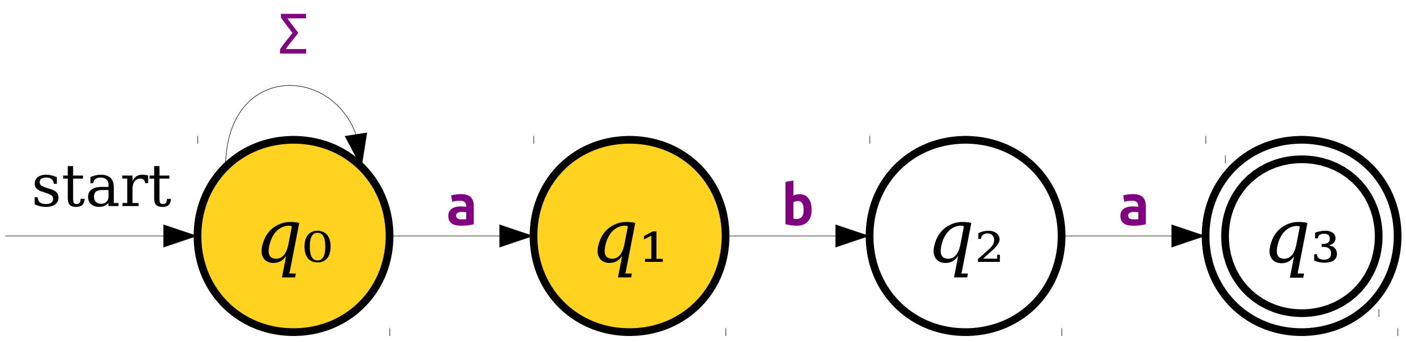

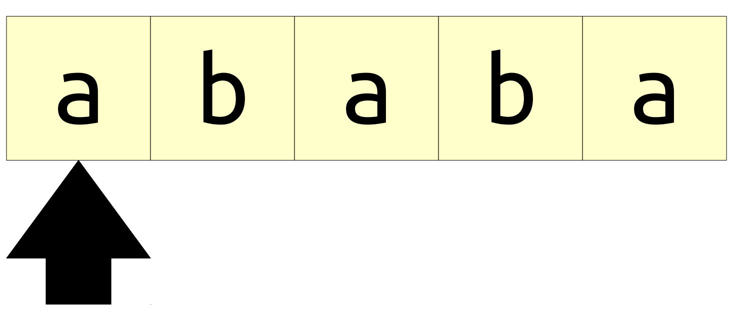

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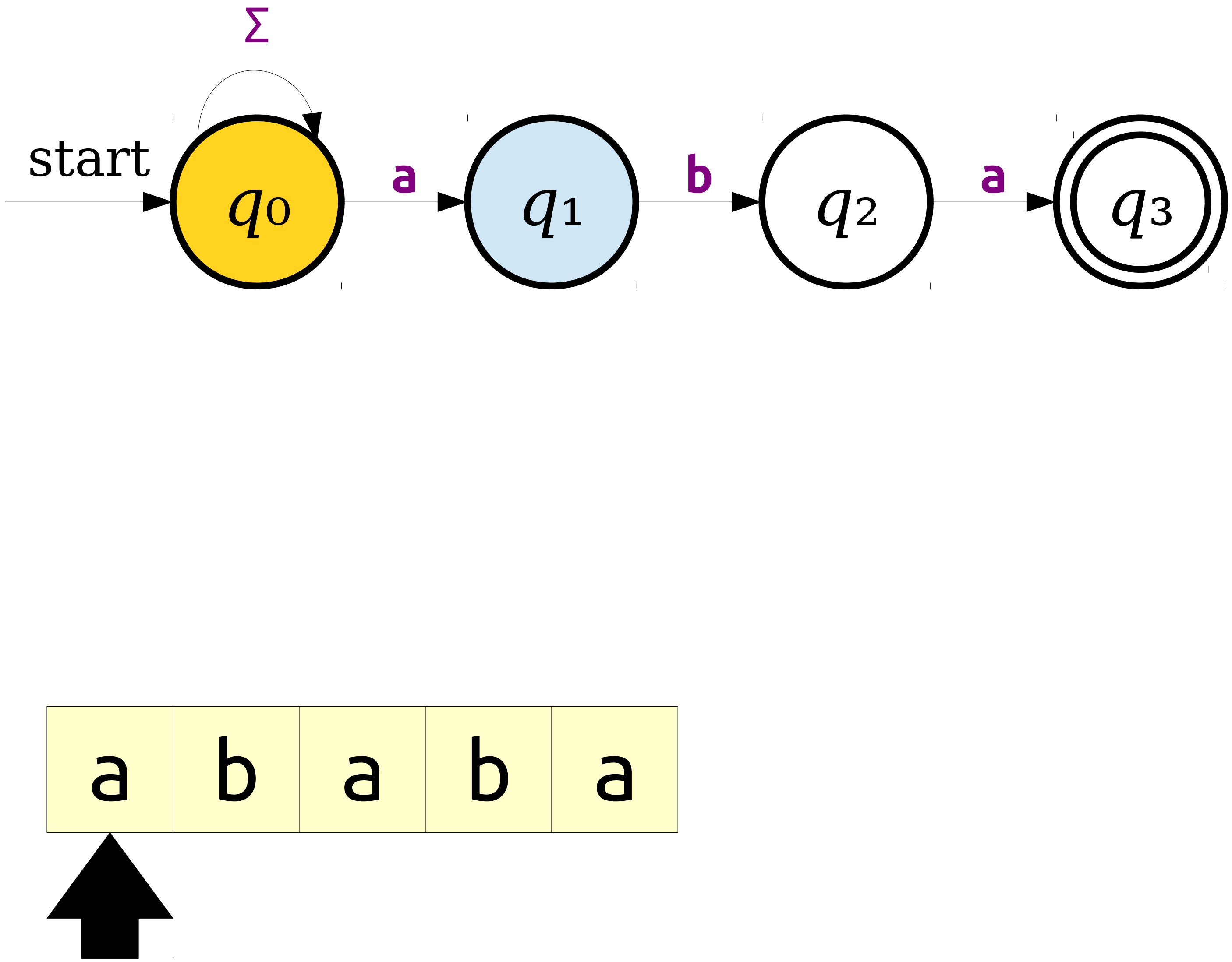

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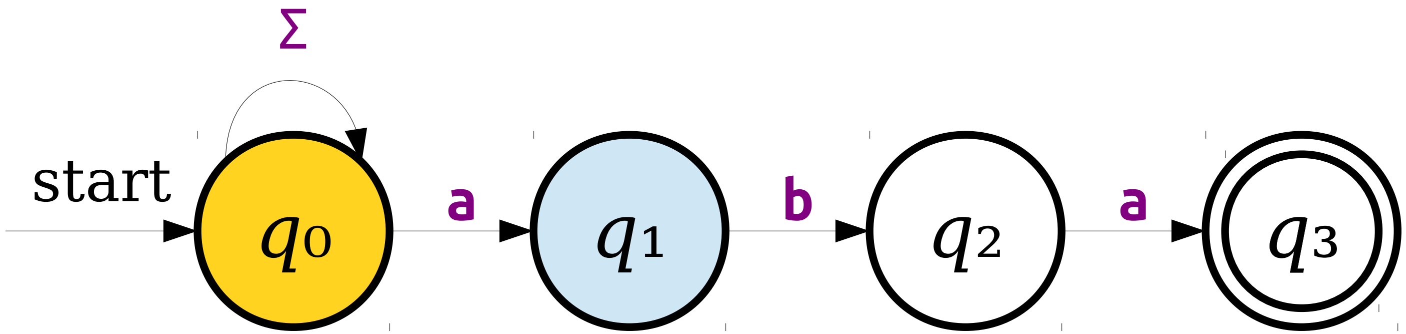

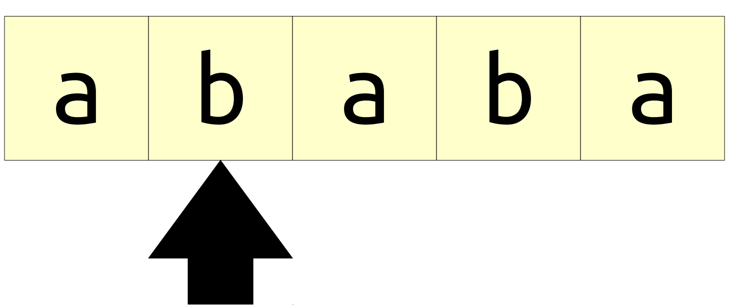

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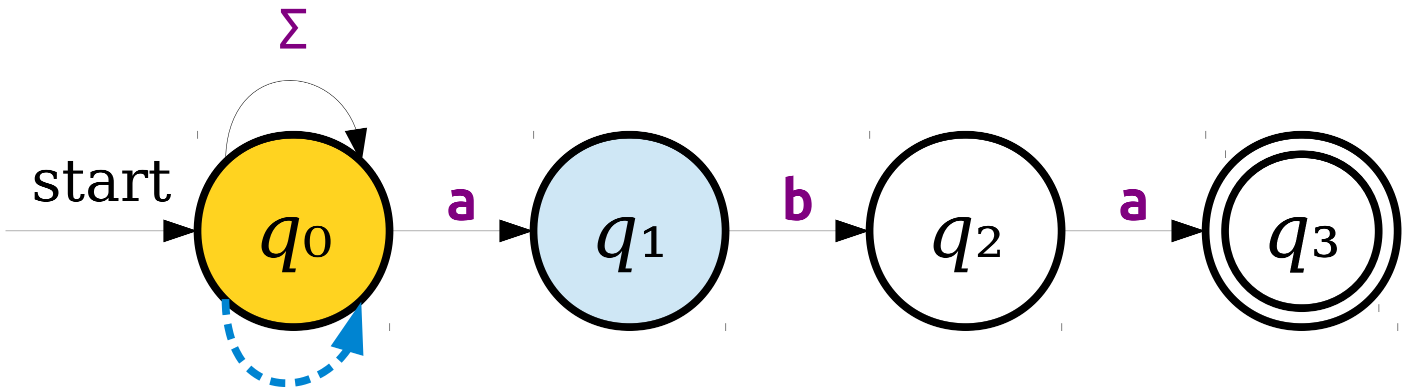

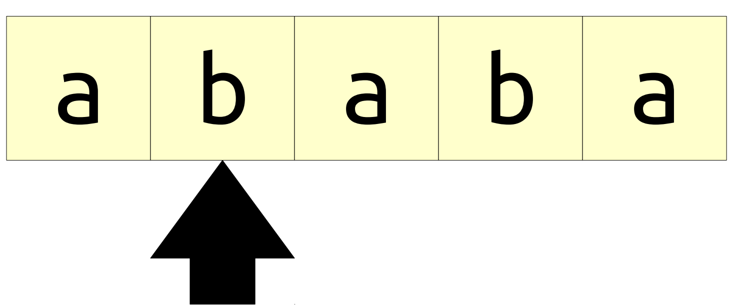

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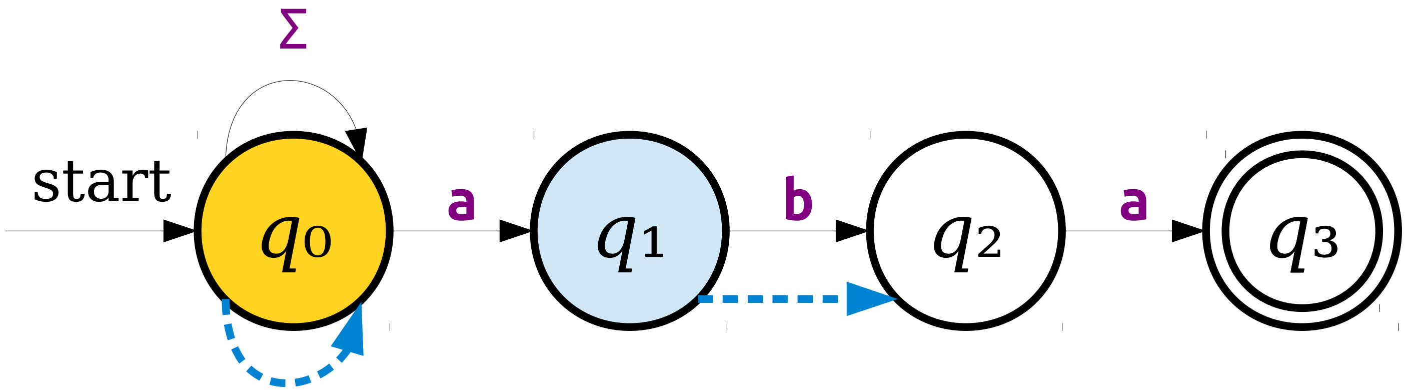

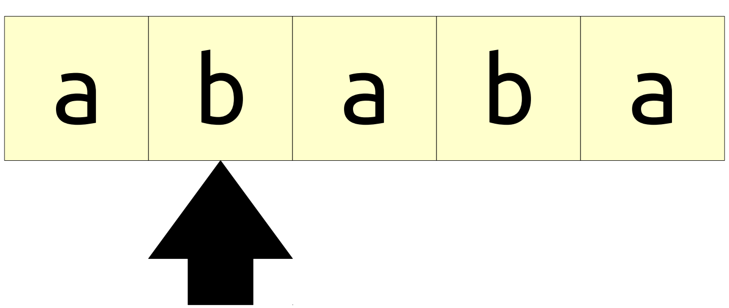

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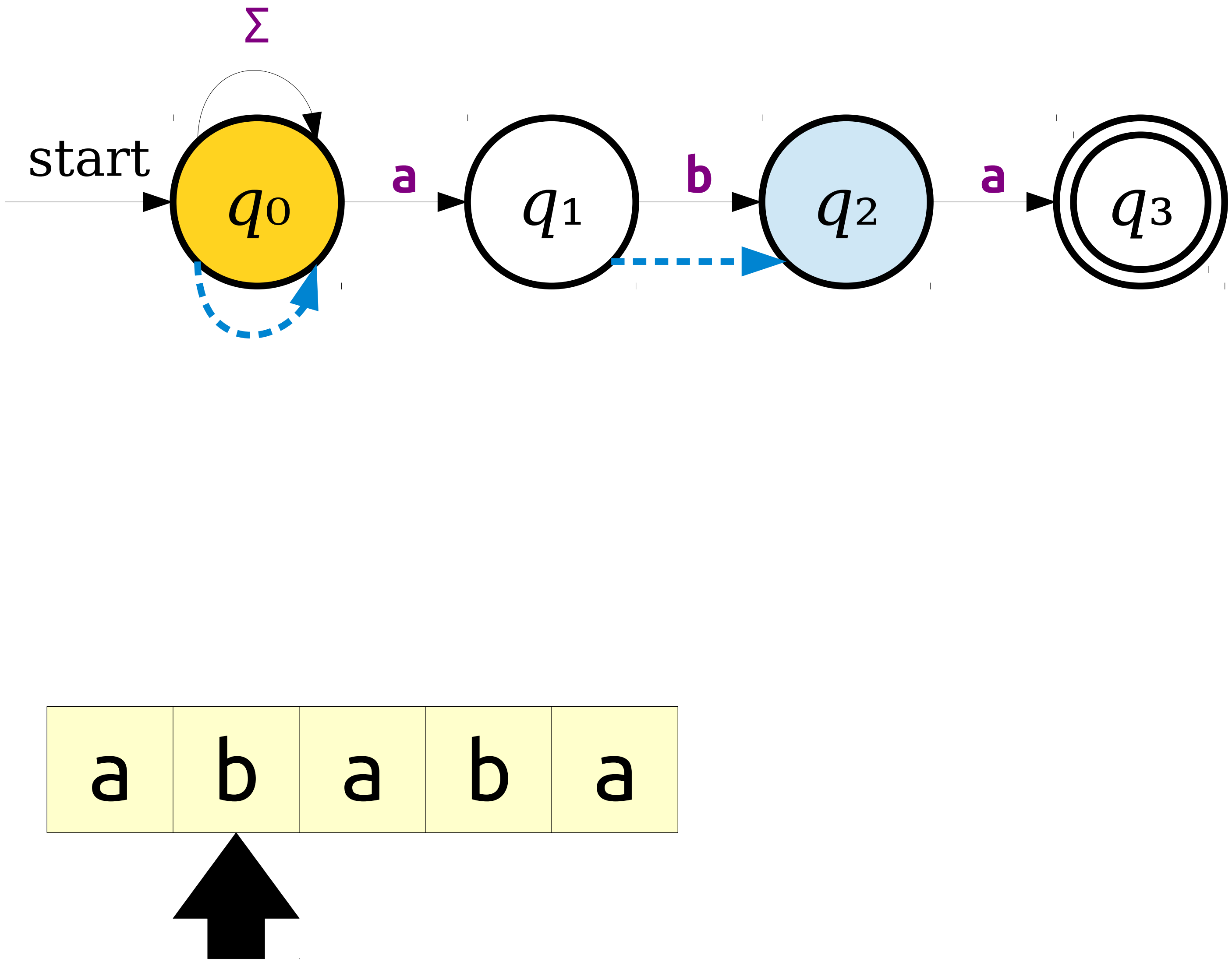

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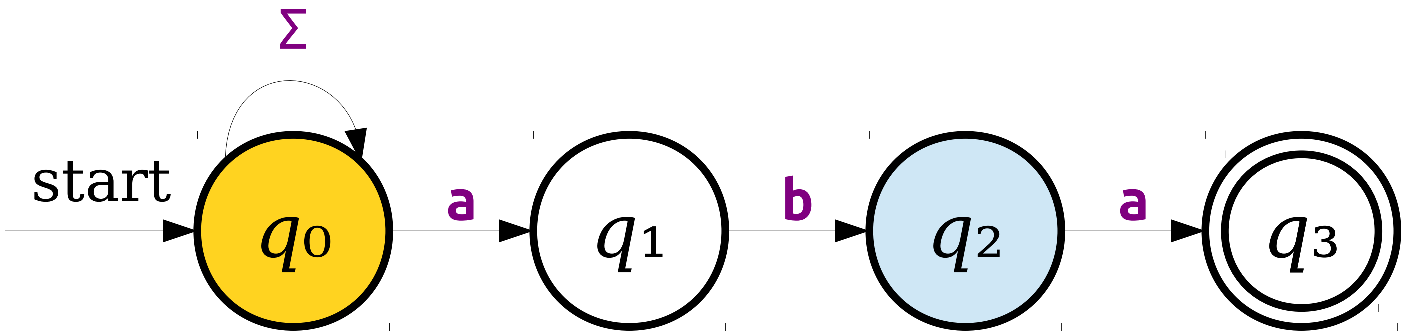

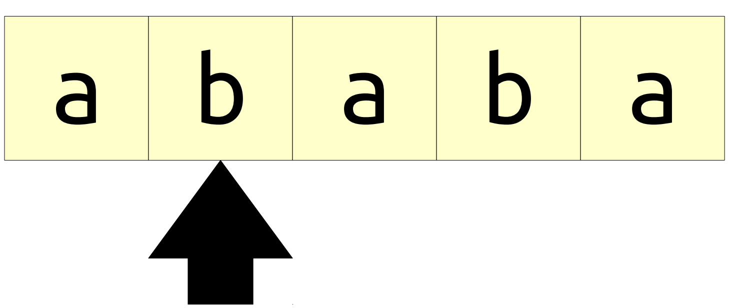

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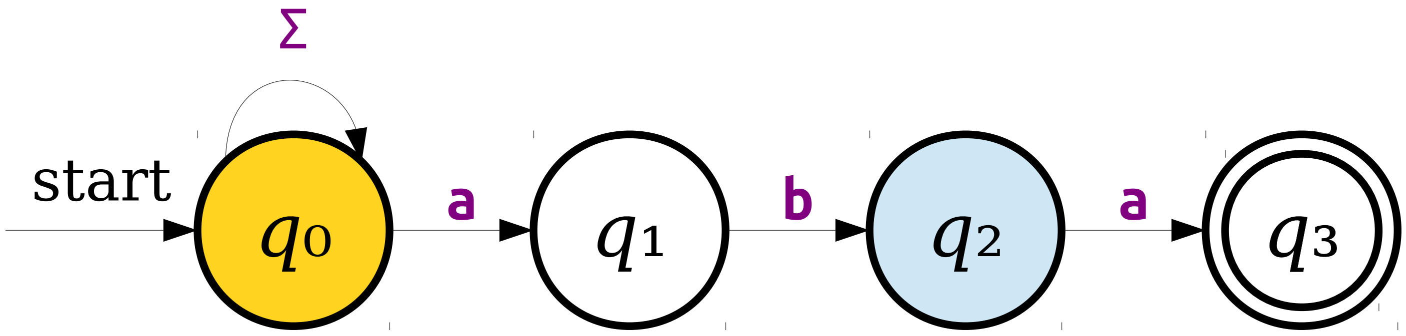

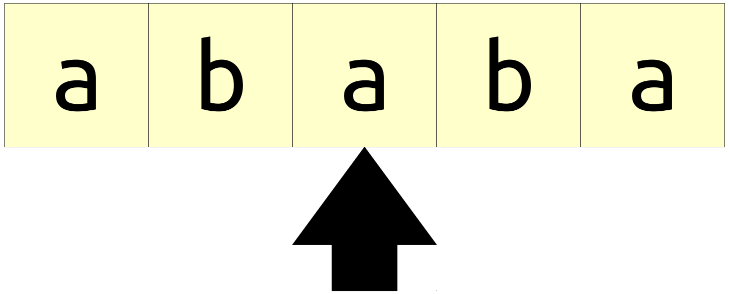

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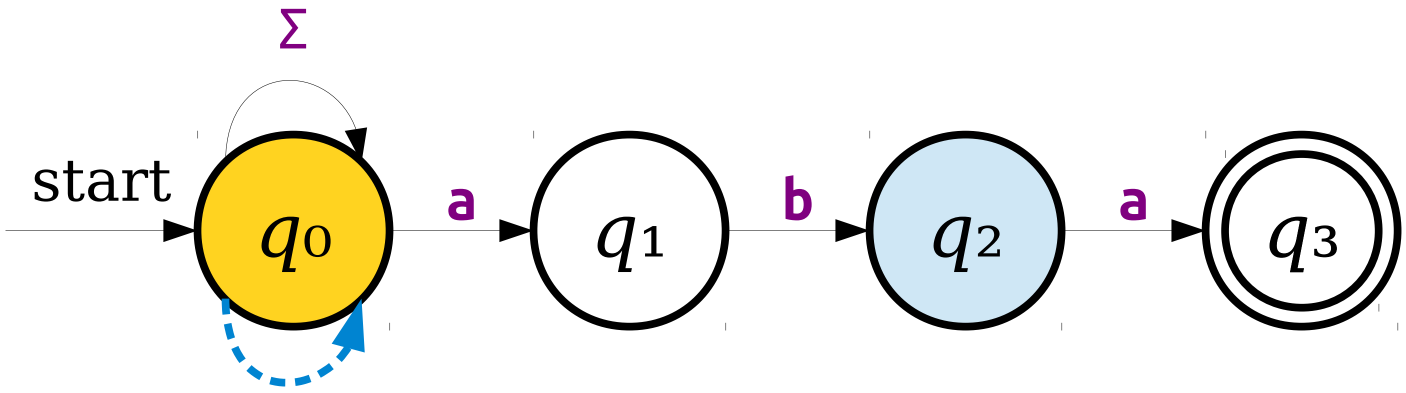

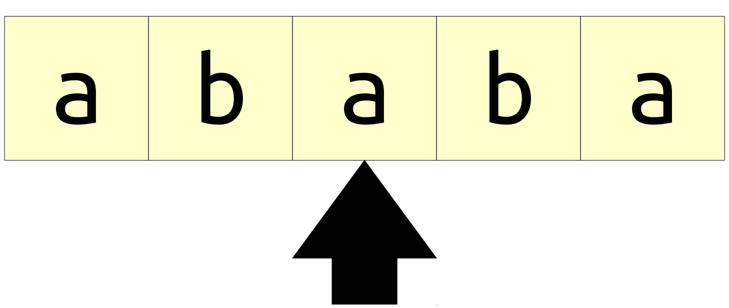

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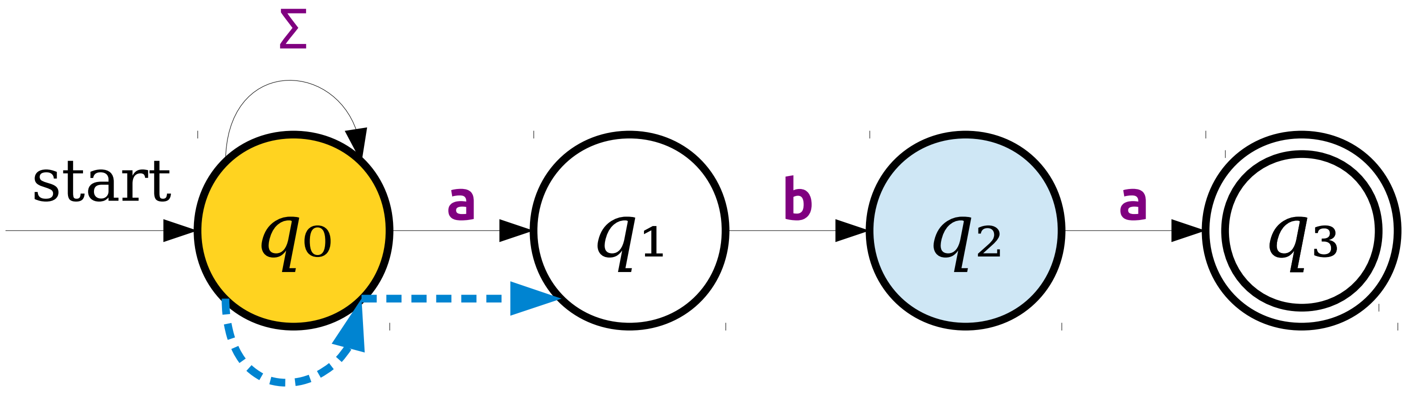

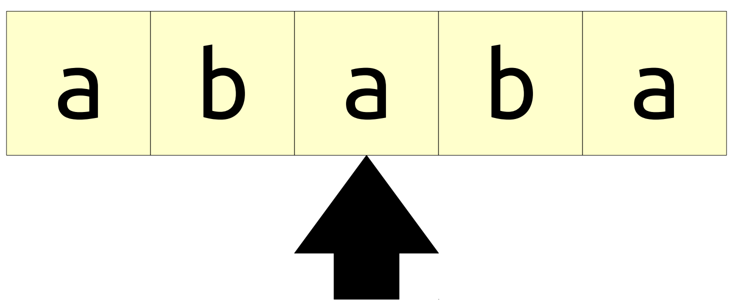

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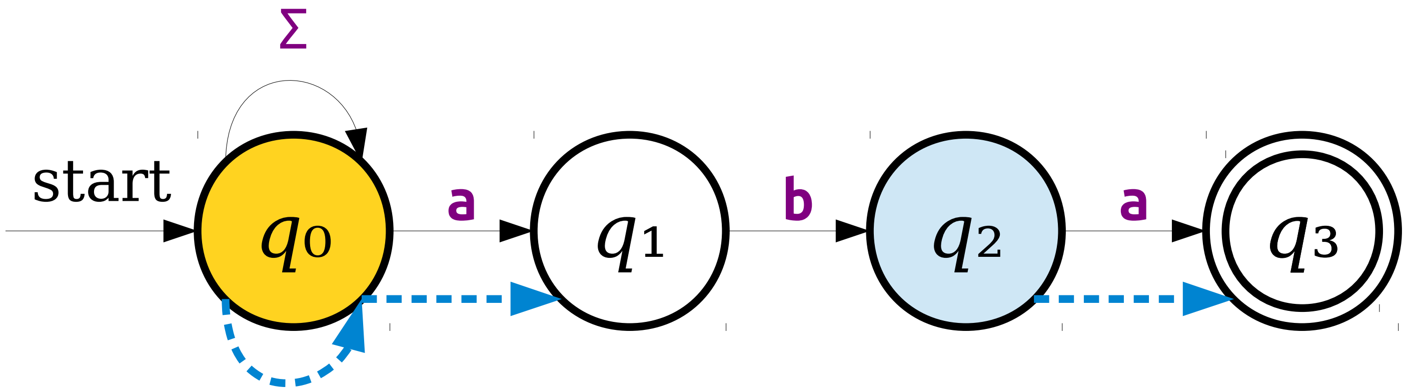

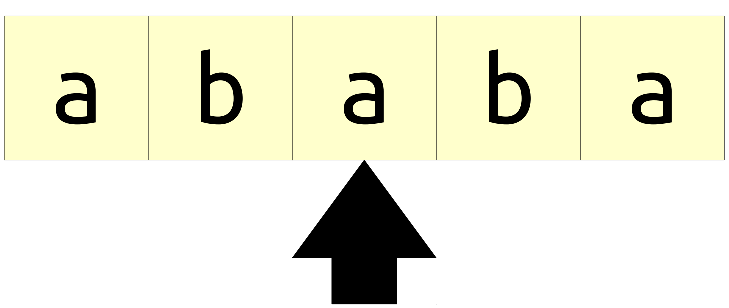

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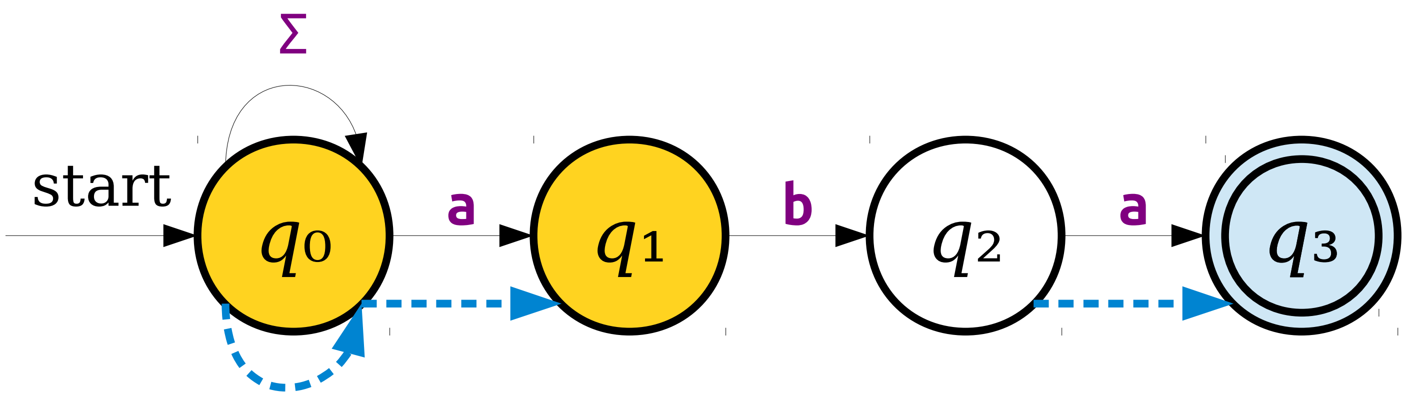

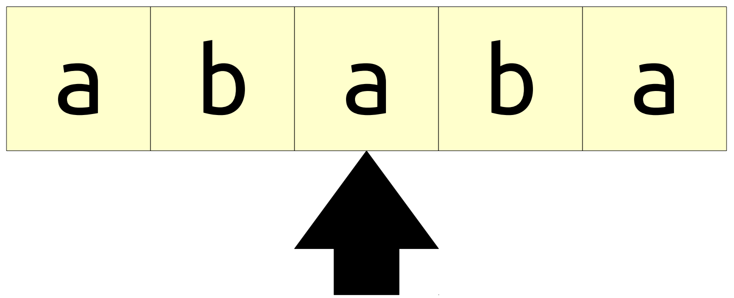

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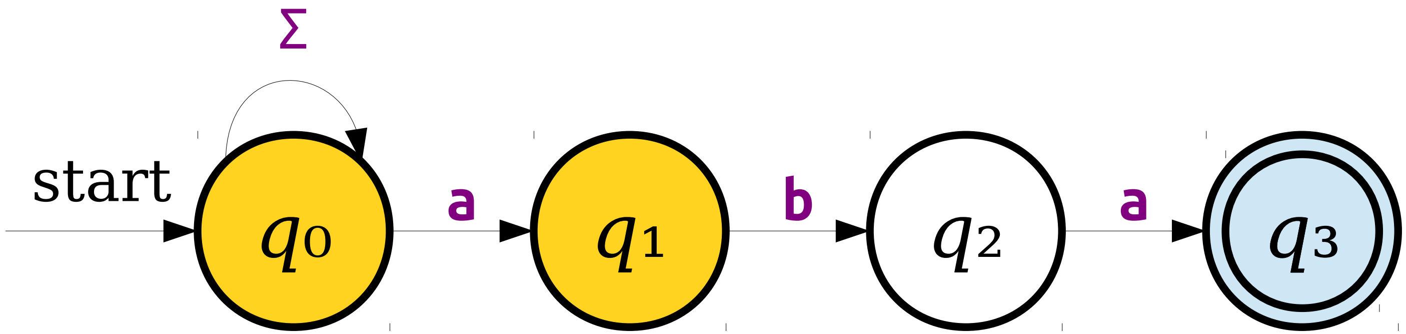

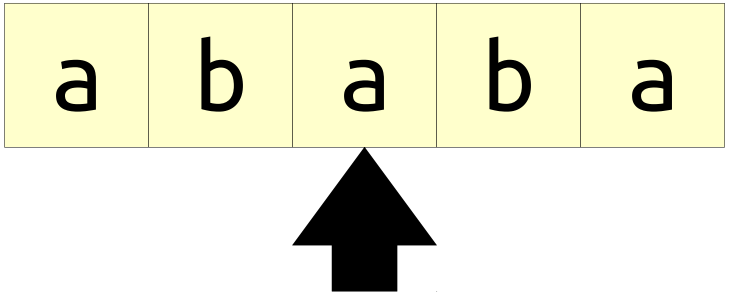

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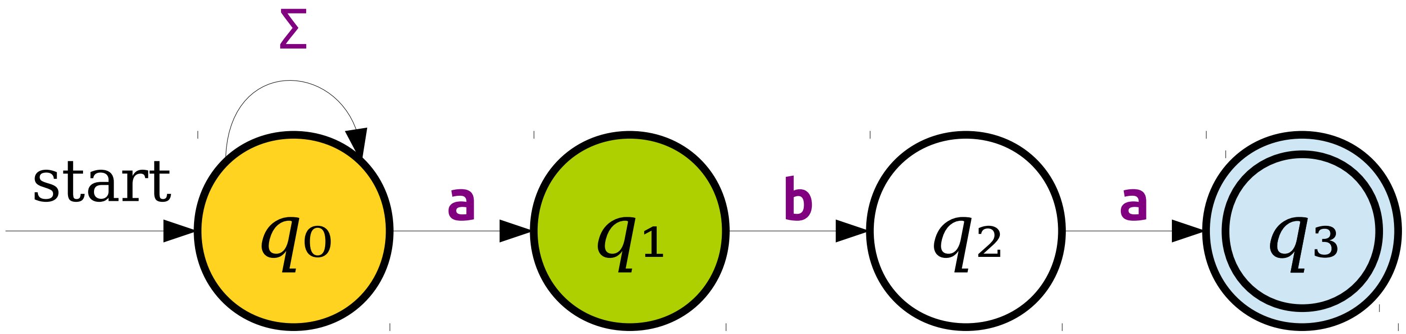

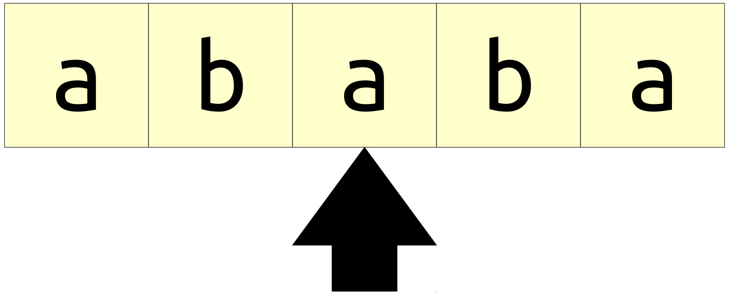

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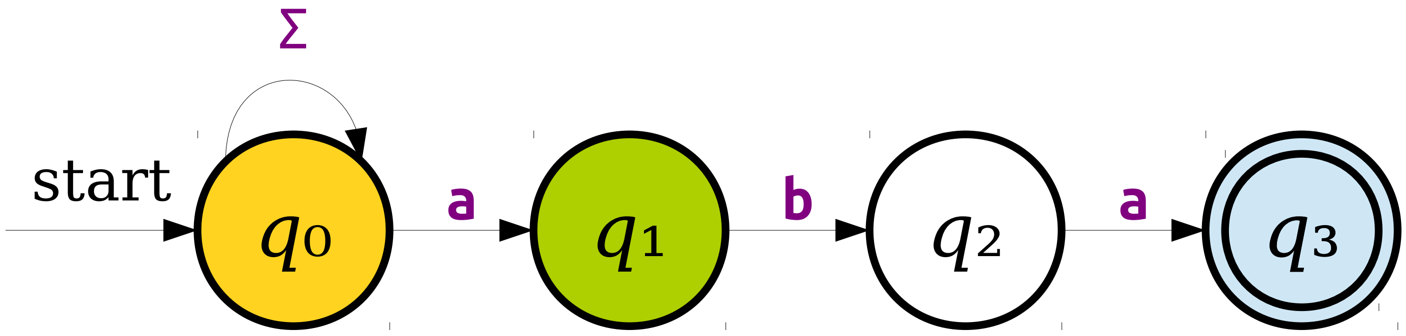

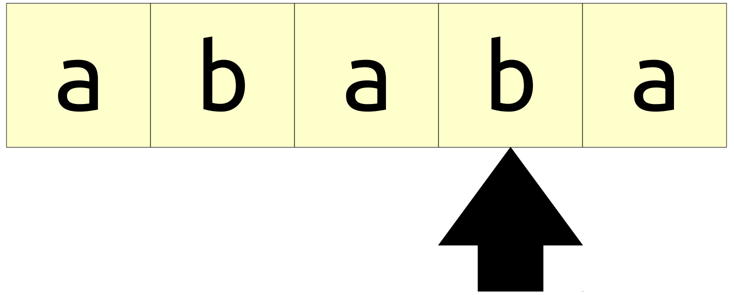

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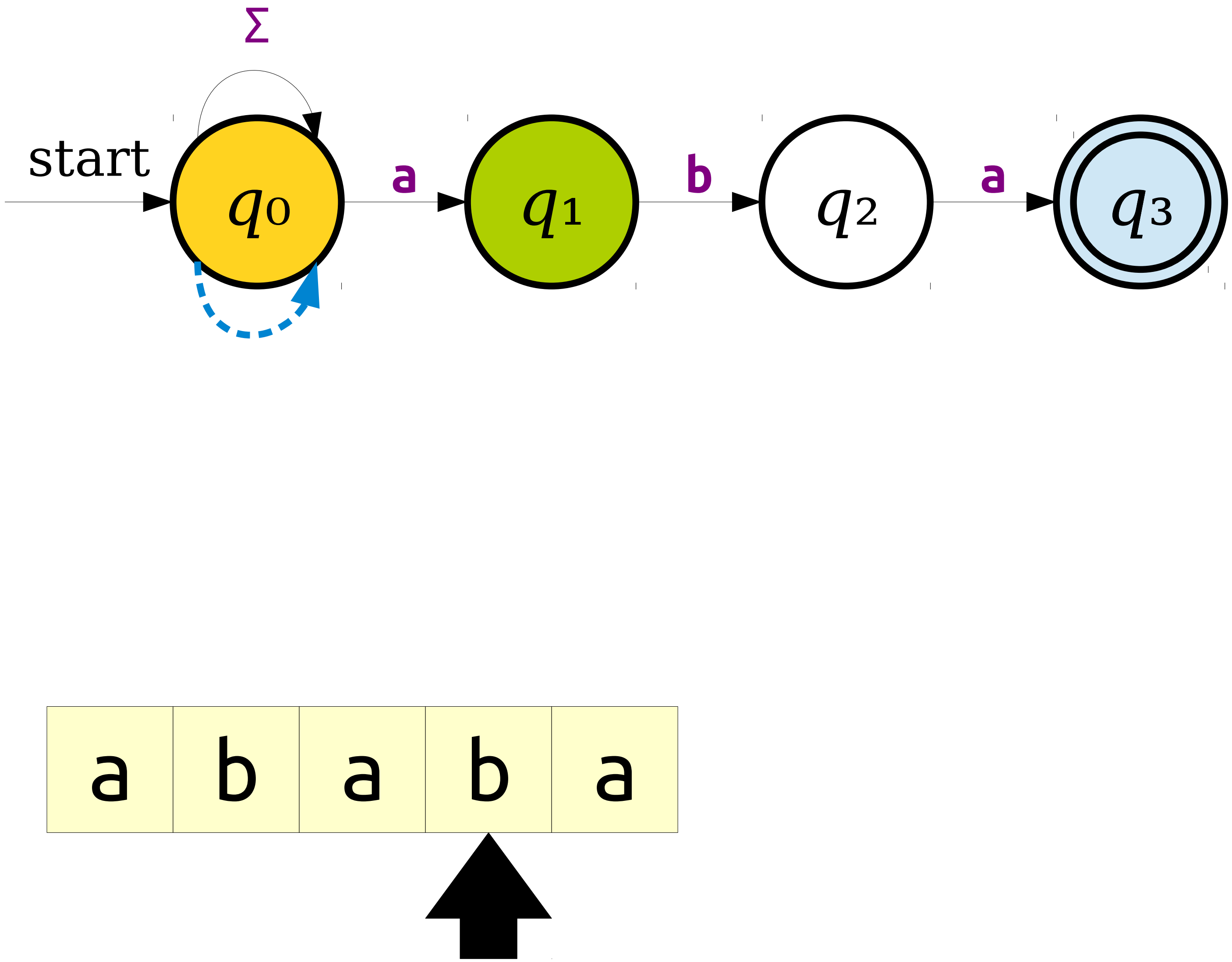

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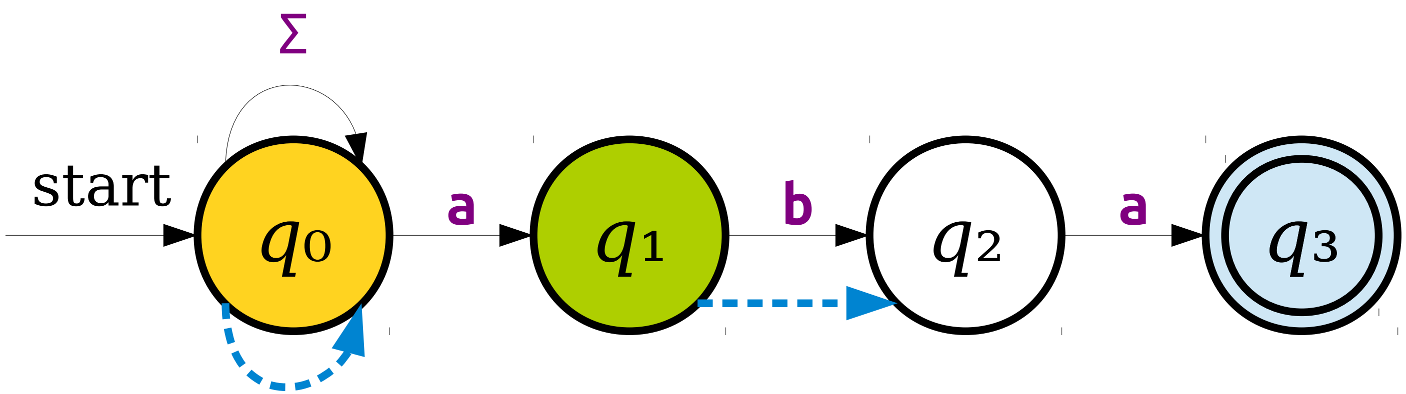

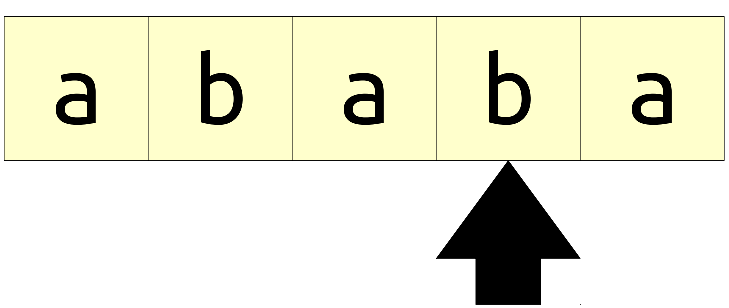

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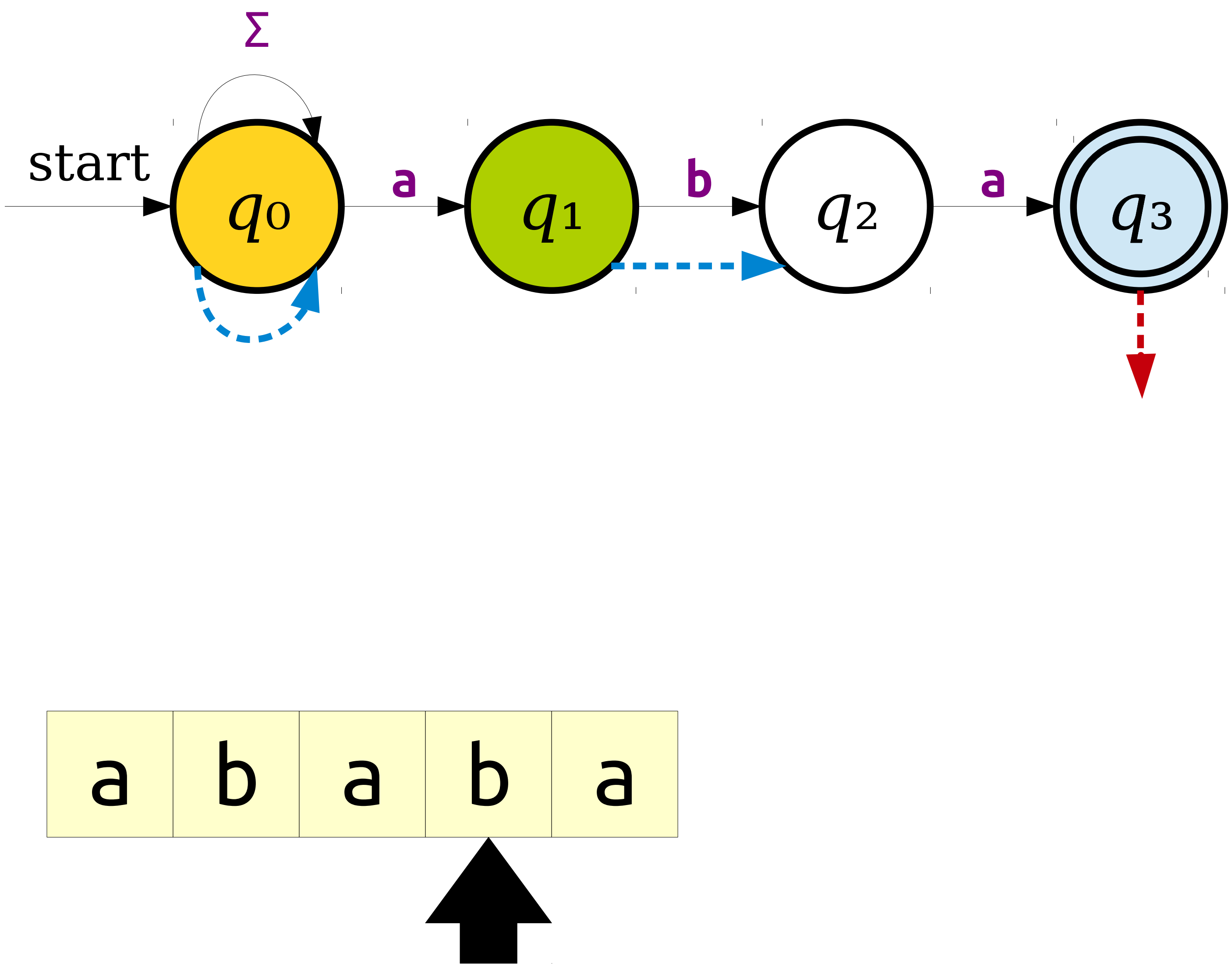

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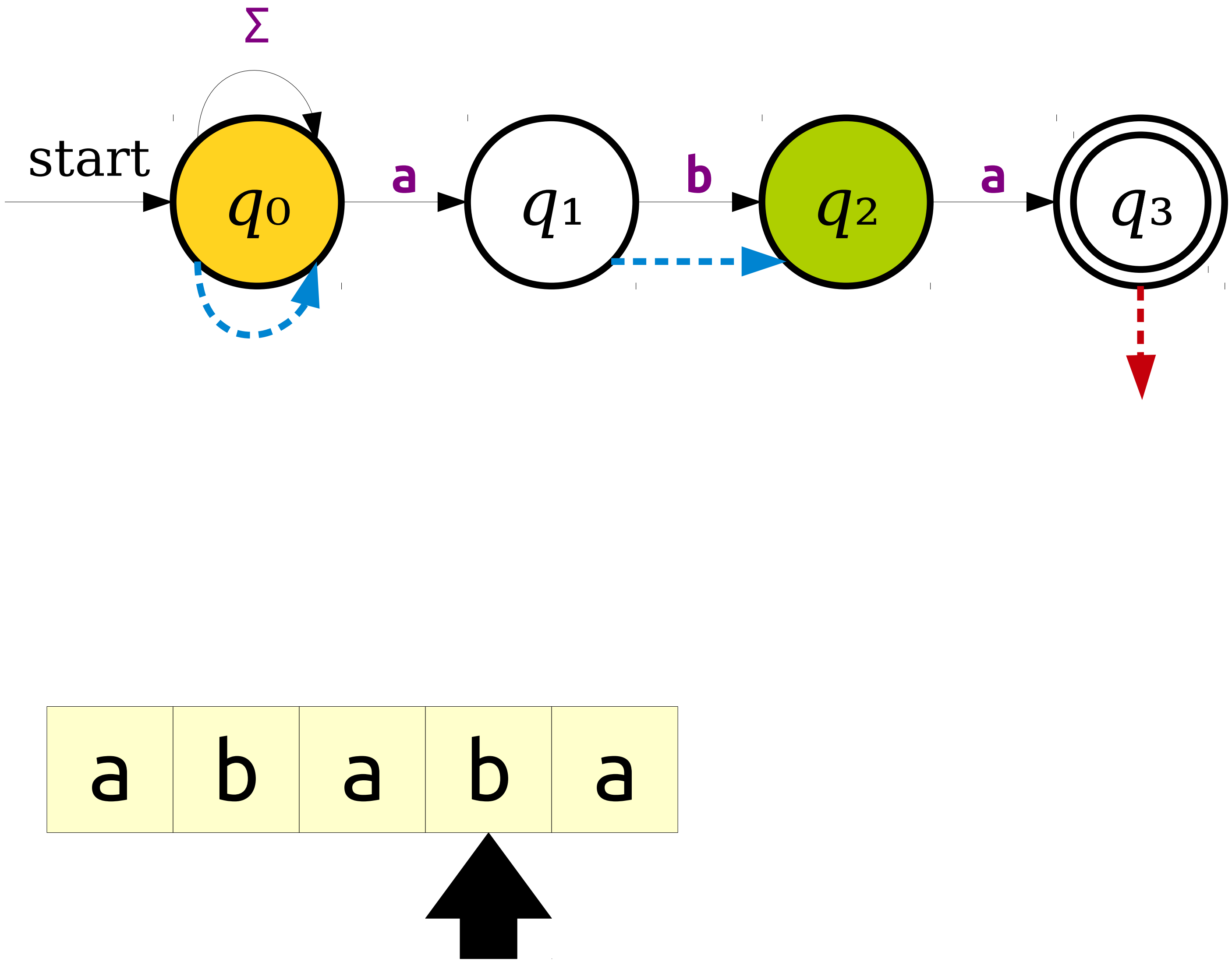

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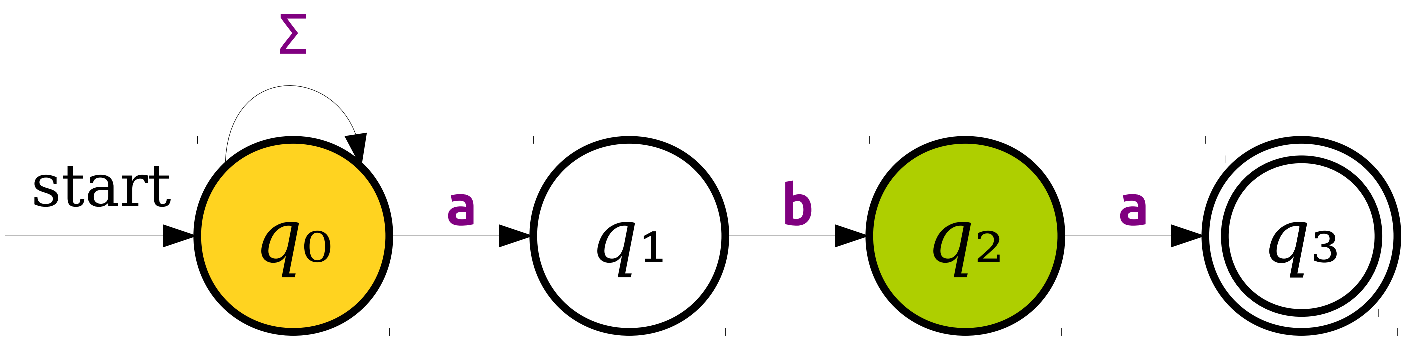

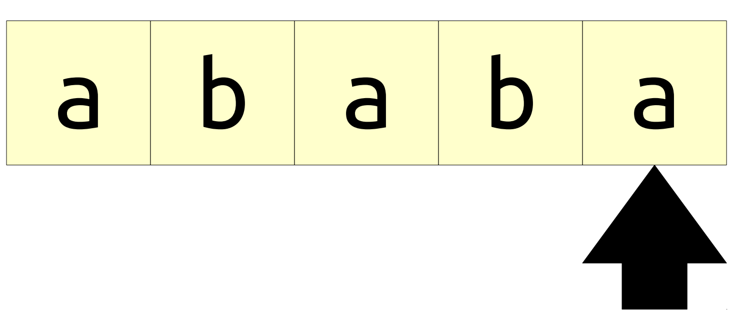

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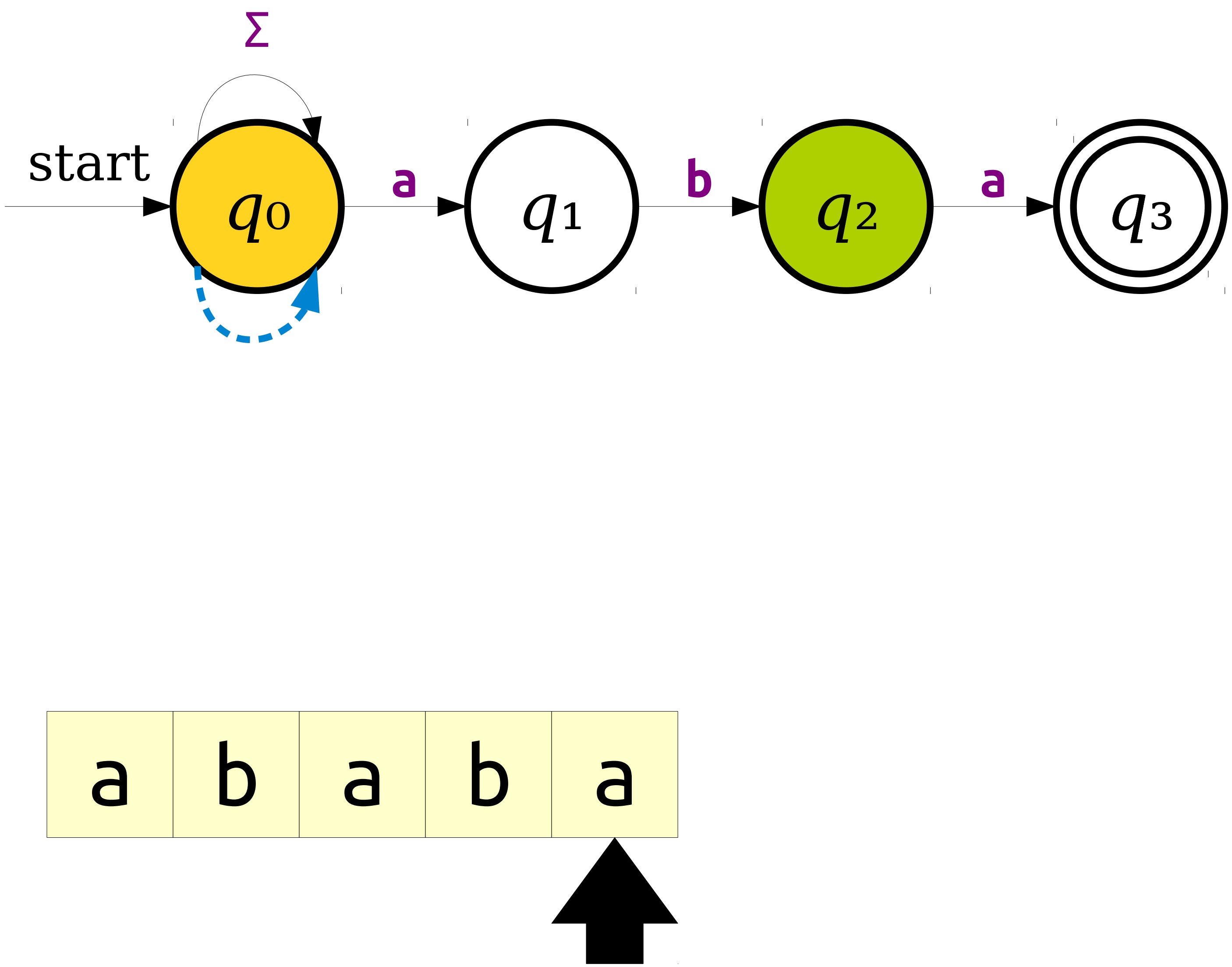

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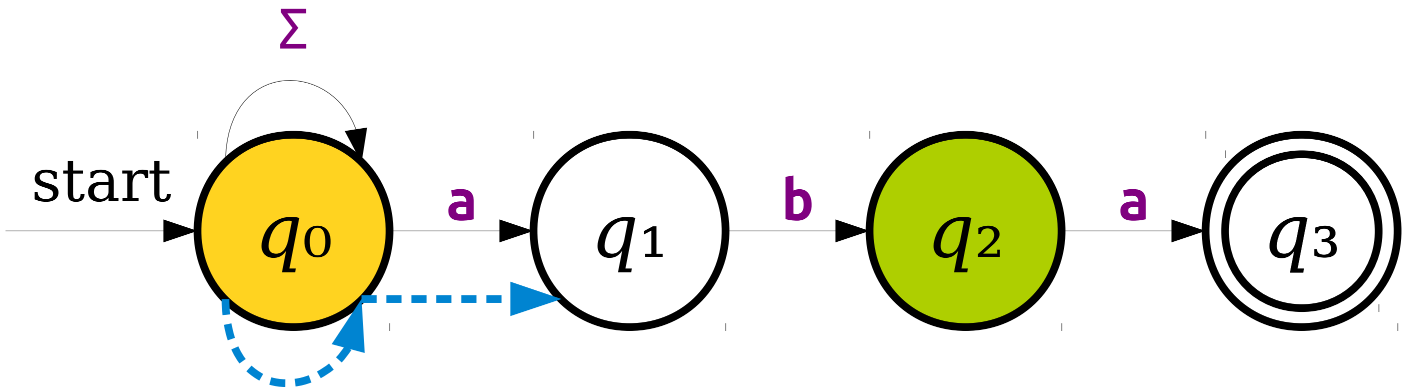

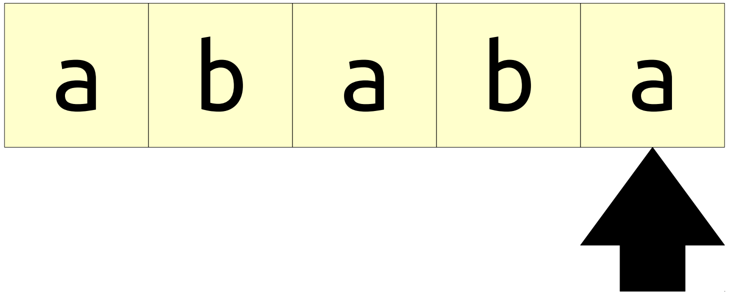

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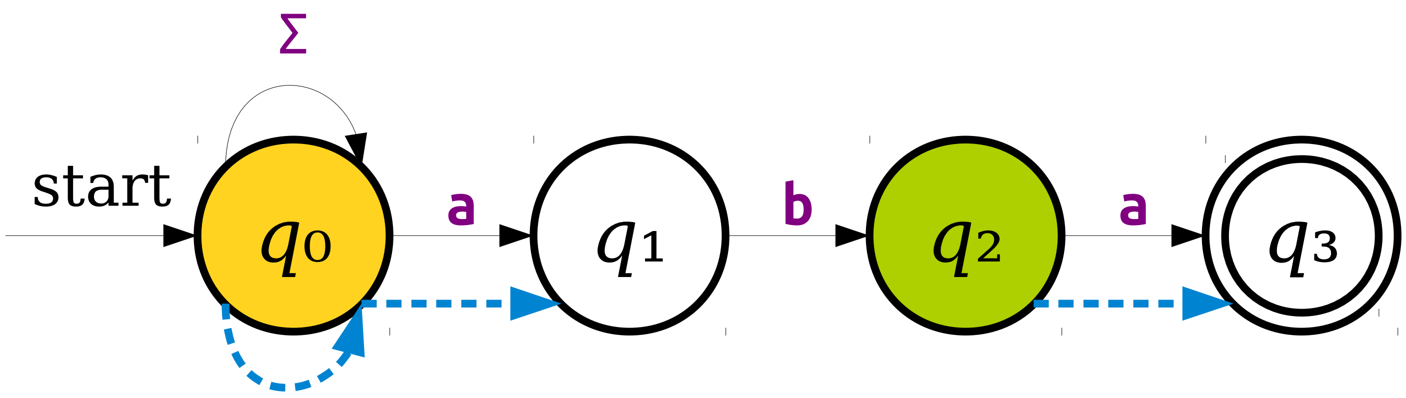

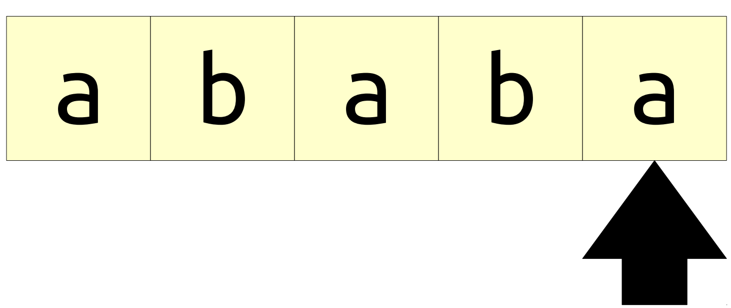

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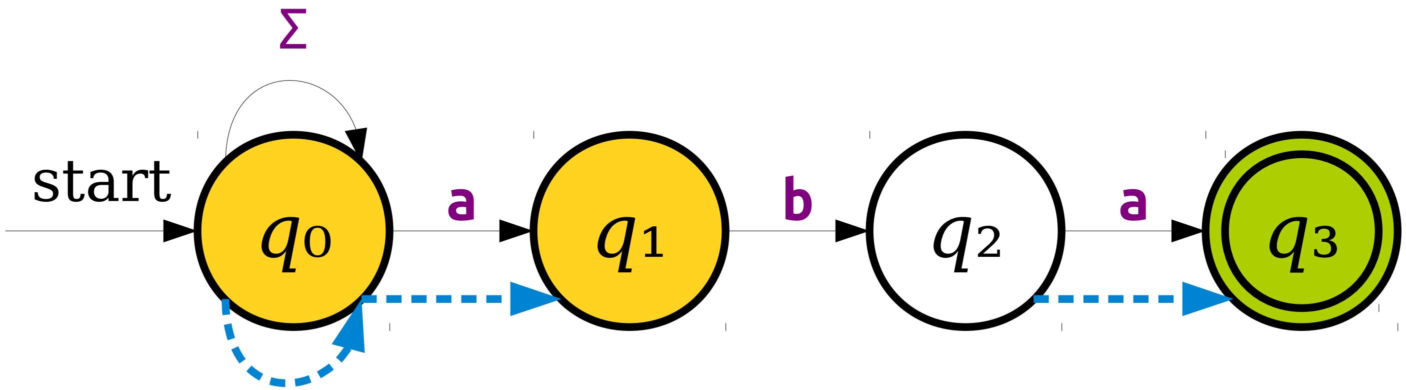

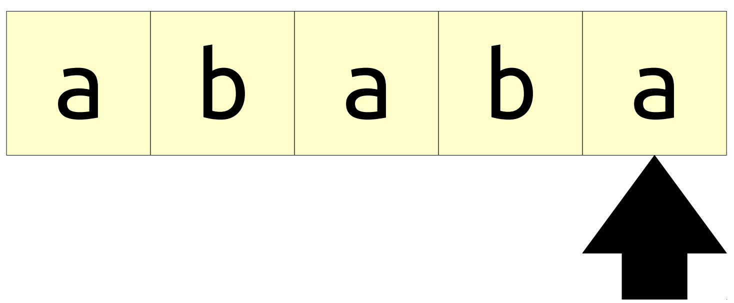

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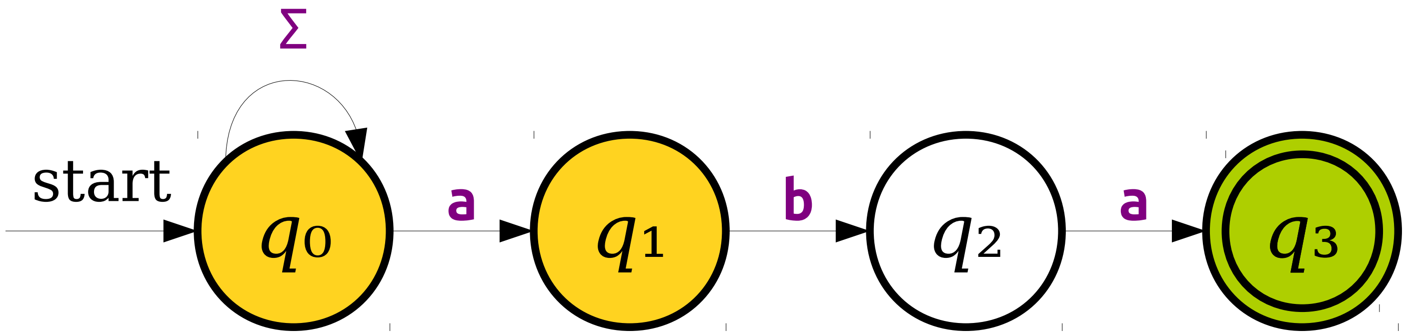

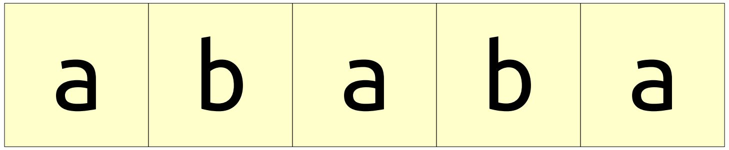

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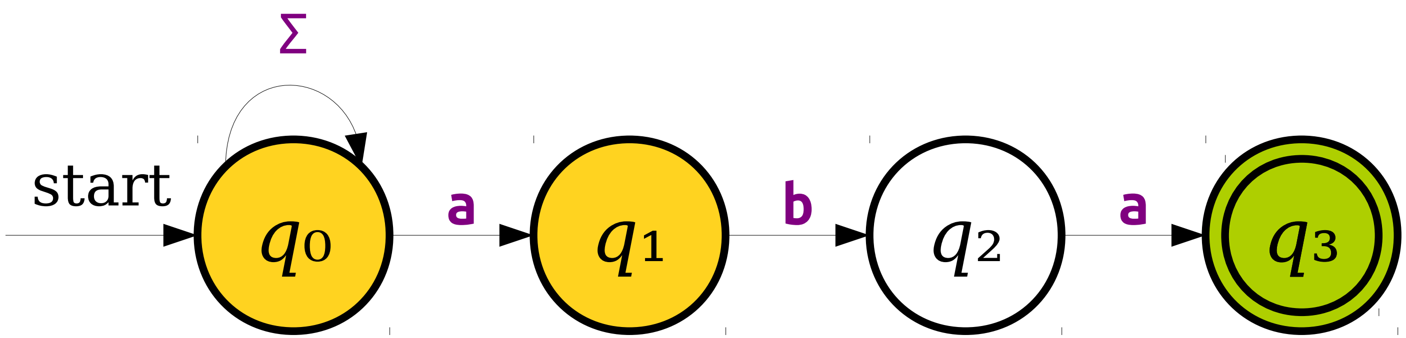

a	b	a	b	a

We're in at least one accepting state, so there's some path that gets us to an accepting state.

Therefore, we accept!

Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- (Here's a rigorous explanation about how this works; read this on your own time). If you're in an initial set of states S, to determine the next set of states S', do the following:
  - For each active state, find all the transitions leaving the state on the current symbol.
  - Follow all those transitions and add the endpoints to S'.
  - Follow all ε-transitions out of S' and add them to S'.
  - Your new set of states is the resulting set S'.

So What?

- Each intuition of nondeterminism is useful in a different setting:
  - Perfect guessing is a great way to think about how to design a machine.
  - Massive parallelism is a great way to test machines – and has nice theoretical implications.
- Nondeterministic machines may not be feasible, but they give a great basis for interesting questions:
  - Can any problem that can be solved by a nondeterministic machine be solved by a deterministic machine?
  - Can any problem that can be solved by a nondeterministic machine be solved ***efficiently*** by a deterministic machine?
- The answers vary from automaton to automaton.

Designing NFAs

Designing NFAs

- When designing NFAs, *embrace the nondeterminism!*
- Good model: ***Guess-and-check***:
  - Is there some information that you'd really like to have? Have the machine *nondeterministically guess* that information.
  - Then, have the machine *deterministically check* that the choice was correct.
- The *guess* phase corresponds to trying lots of different options.
- The *check* phase corresponds to filtering out bad guesses or wrong options.

Guess-and-Check

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ ends in } \texttt{010} \text{ or } \texttt{101} \}$

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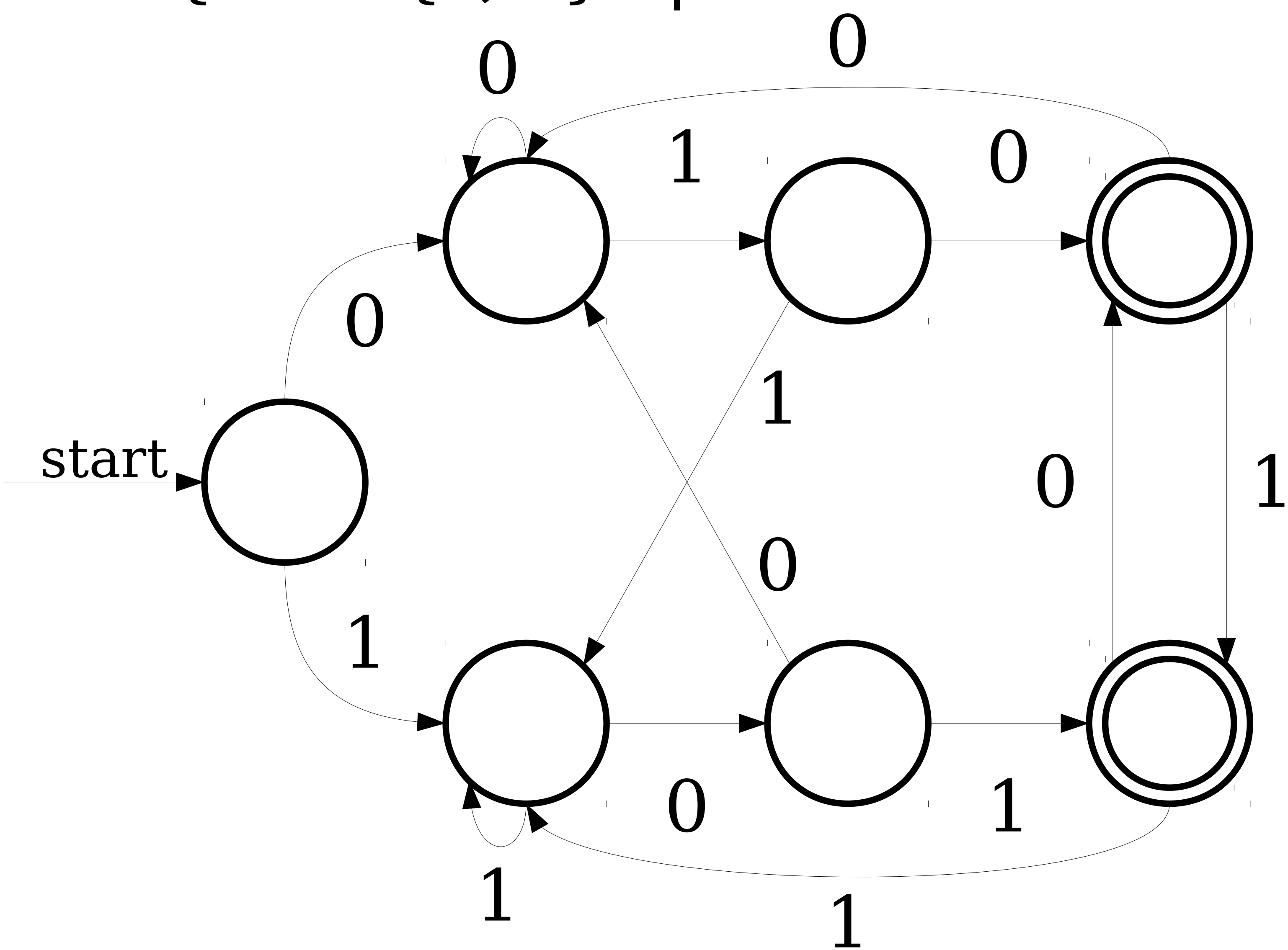

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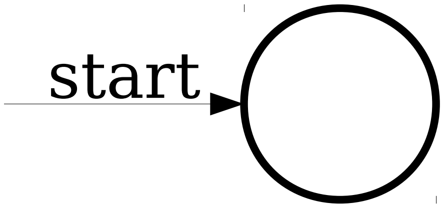

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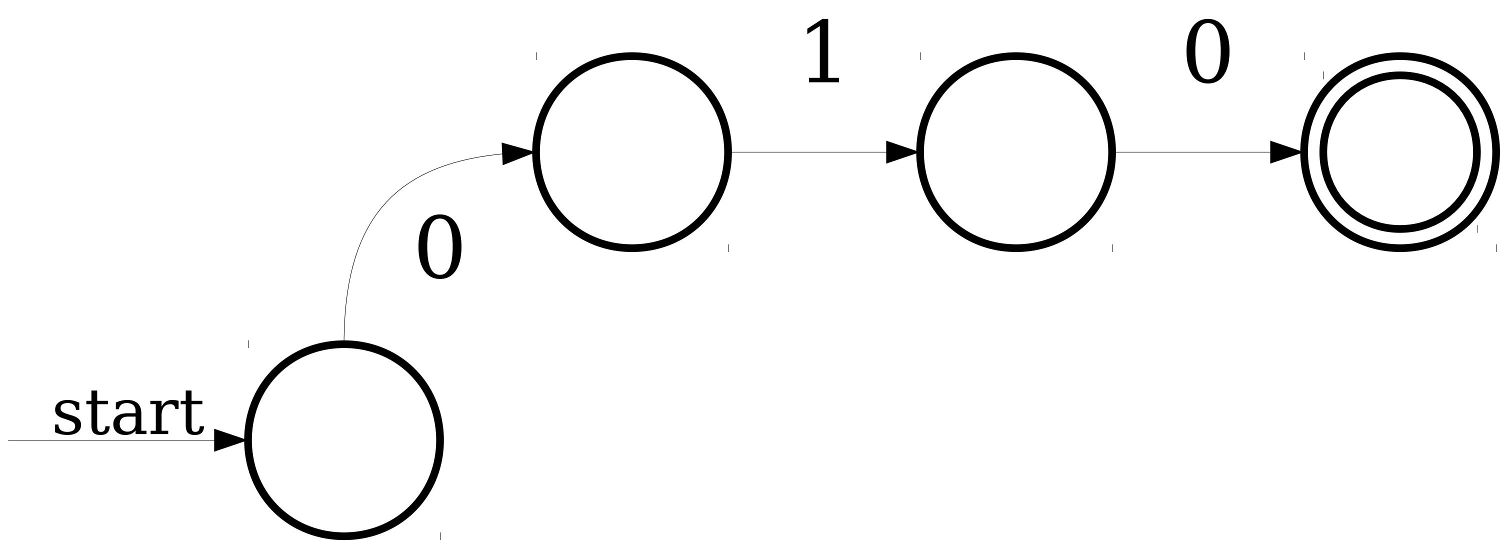

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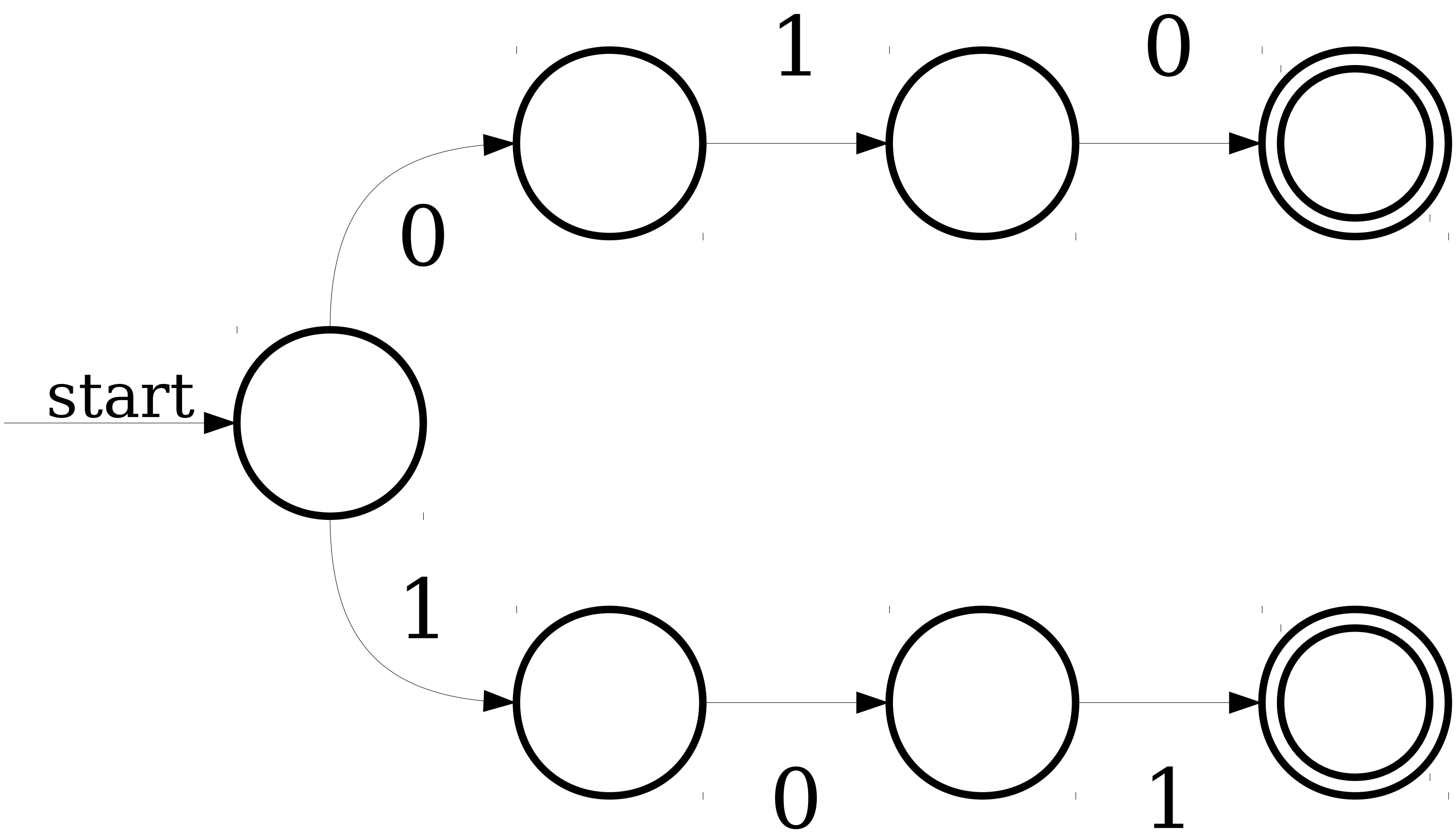

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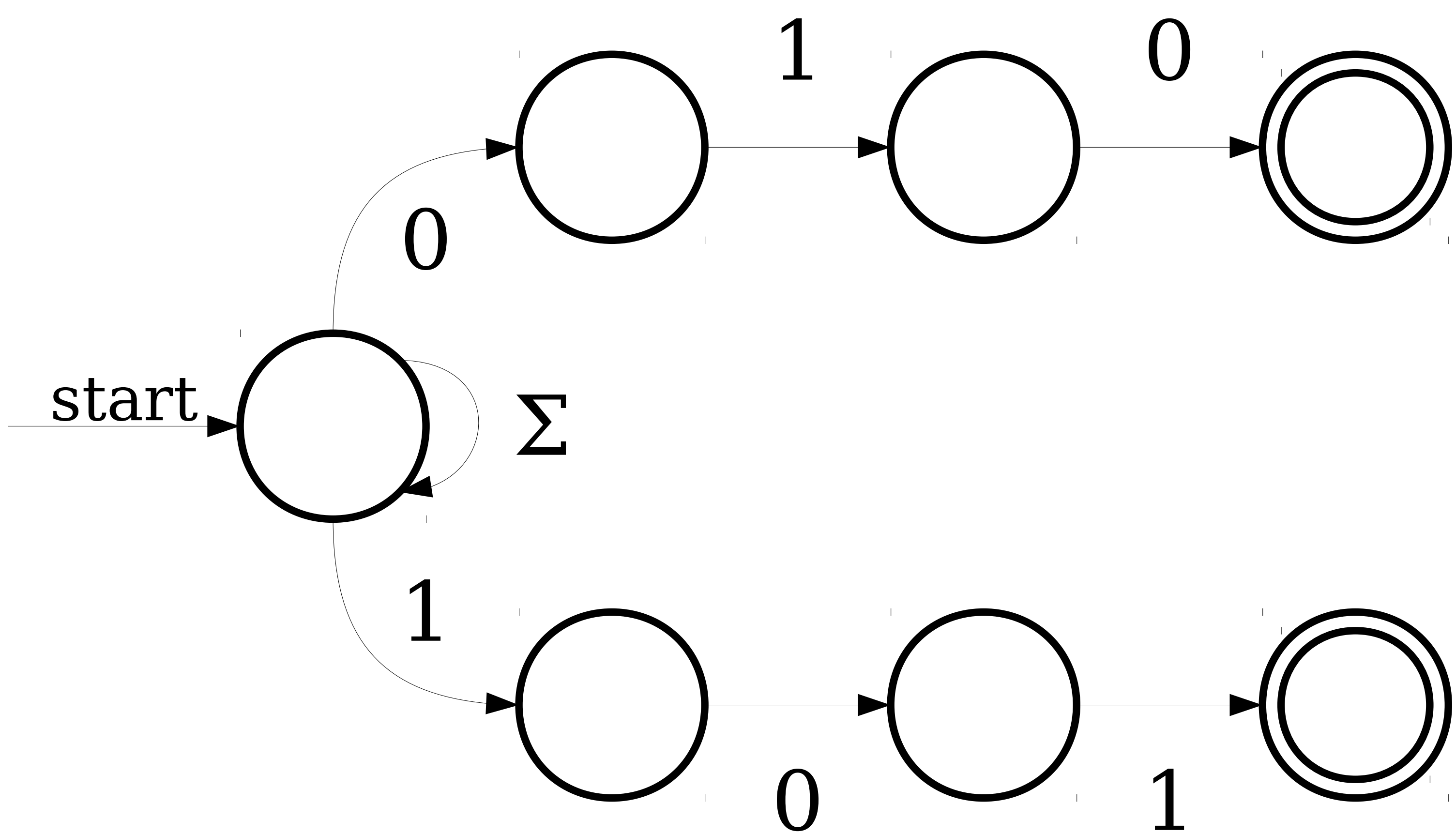

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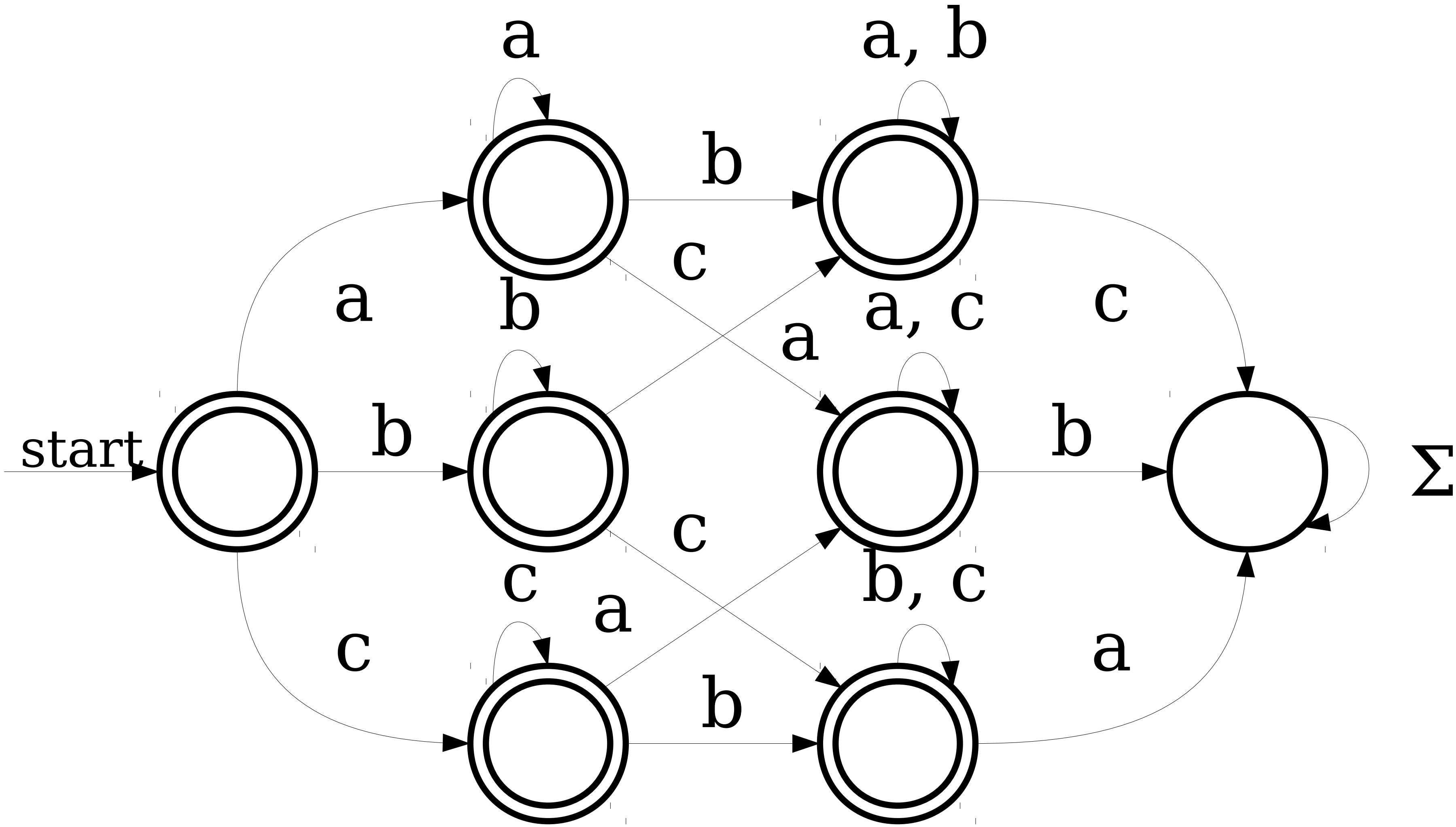

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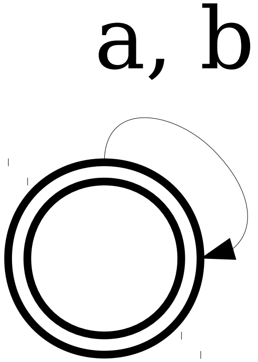

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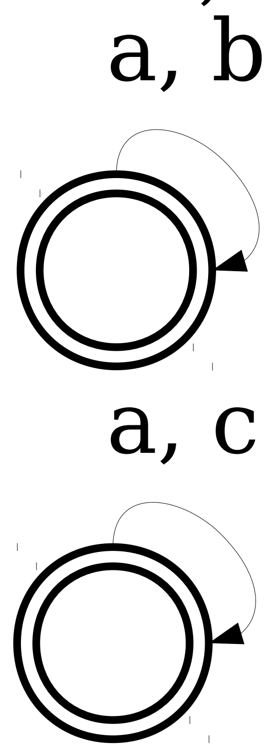

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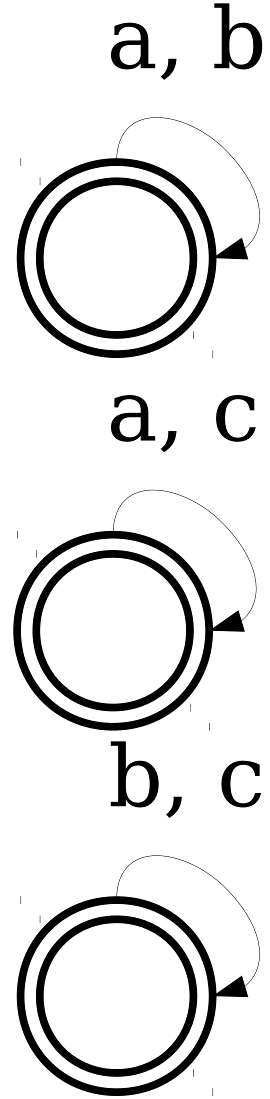

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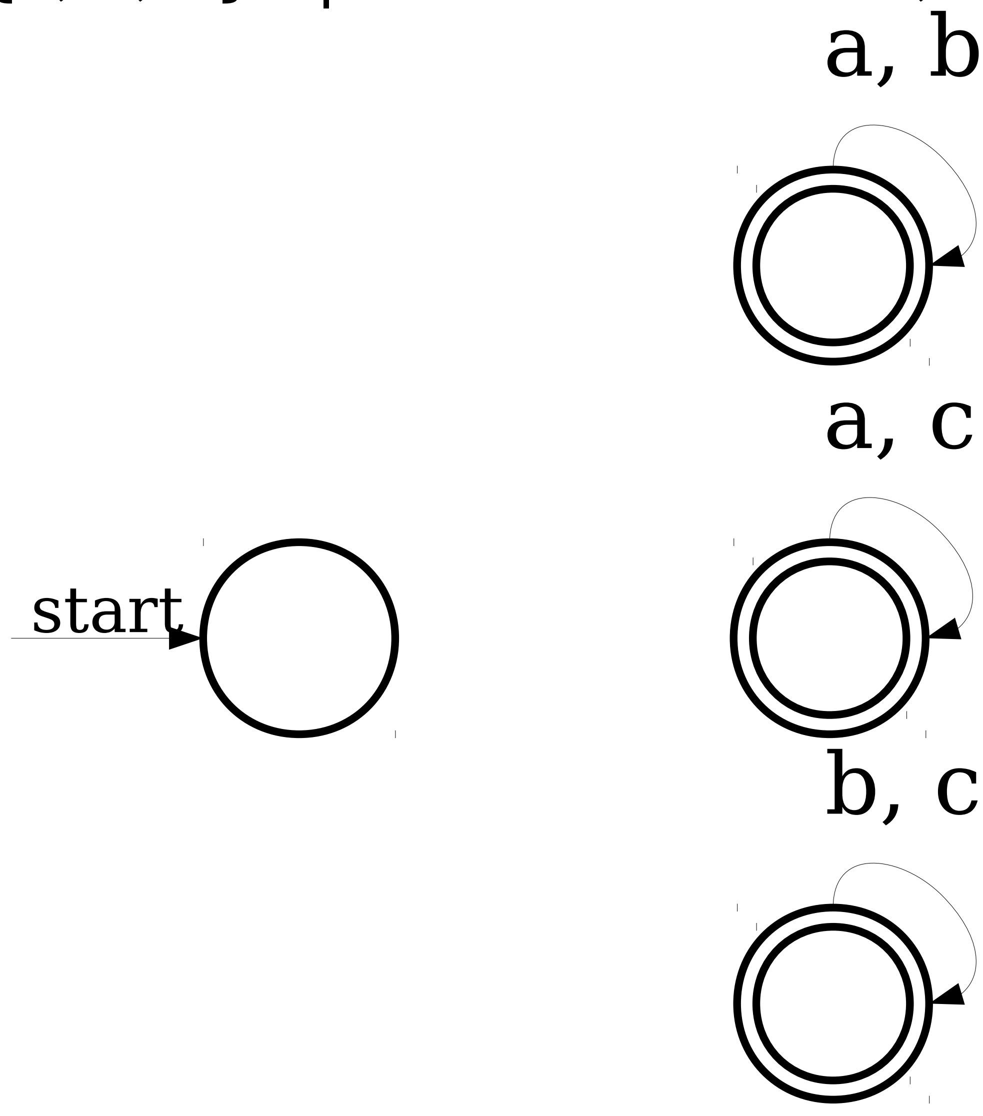

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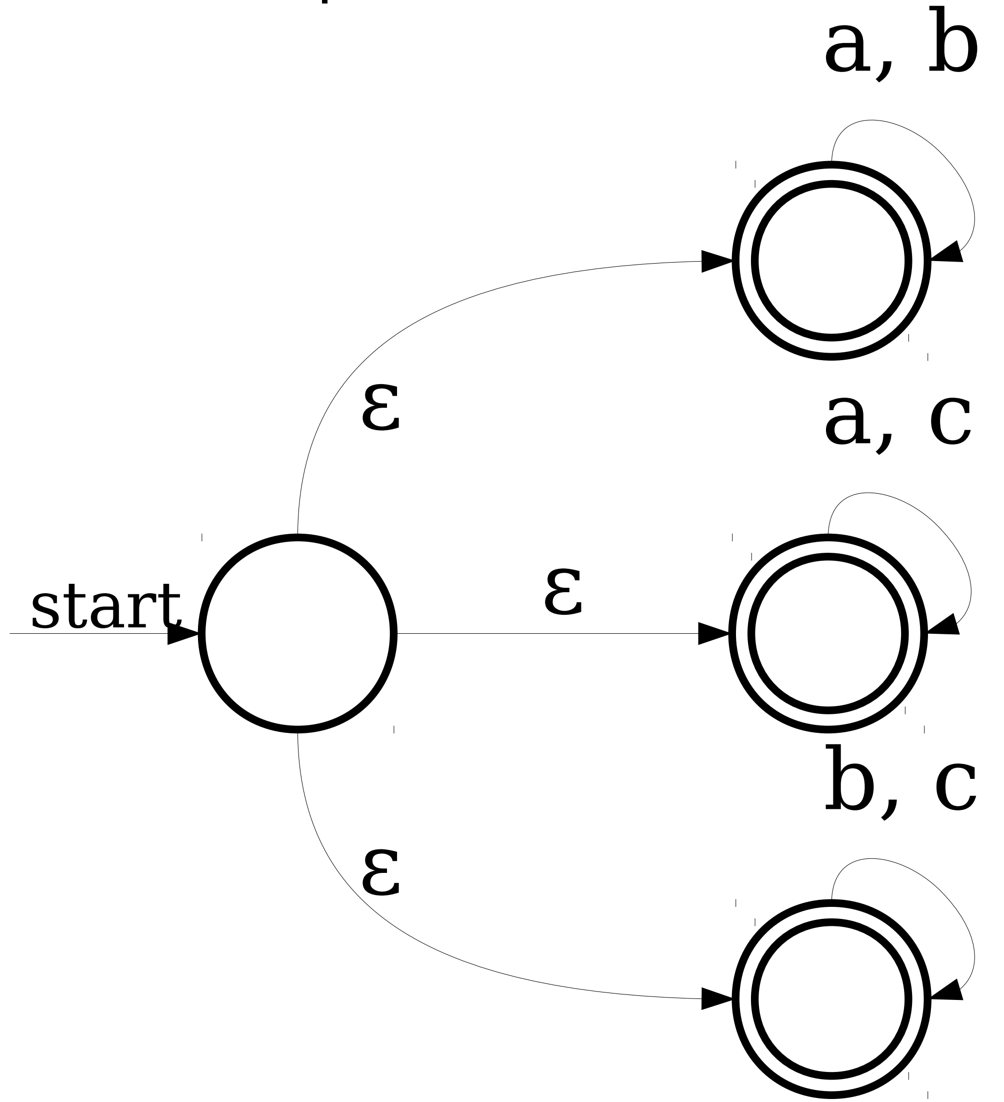

Just how powerful are NFAs?

Some Words of Encouragement