Finite Automata

Part Three

Recap from Last Time

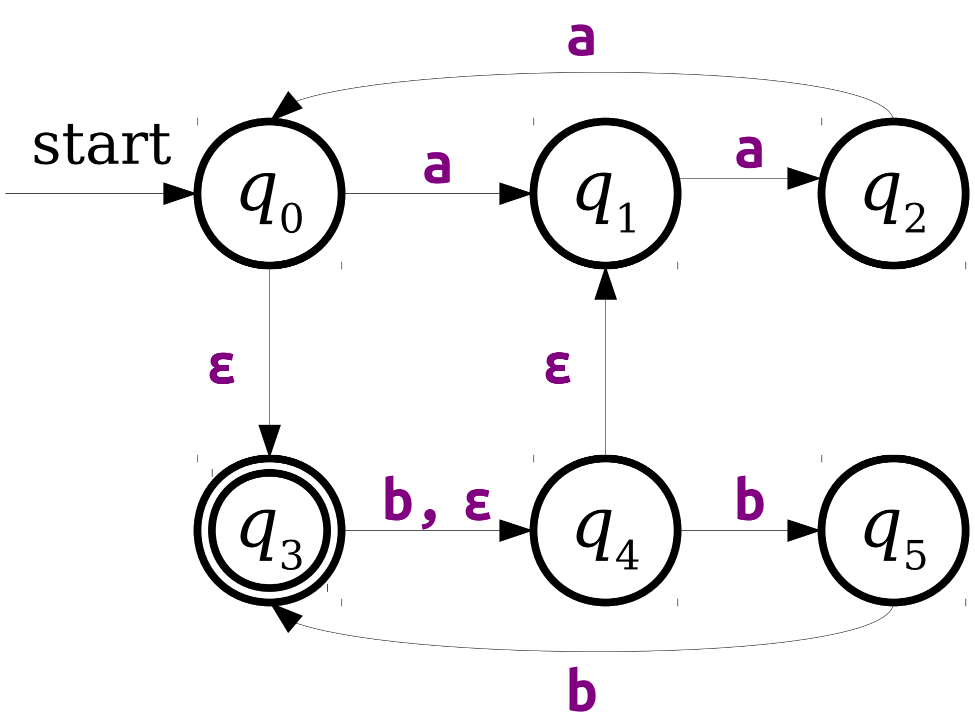
A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

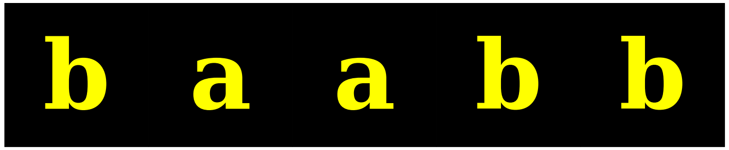
NFAs

- An **NFA** is a
  - **N**ondeterministic
  - **F**inite
  - **A**utomaton
- Can have missing transitions or multiple transitions defined on the same input symbol.
- Accepts if *any possible series of choices* leads to an accepting state.

ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.





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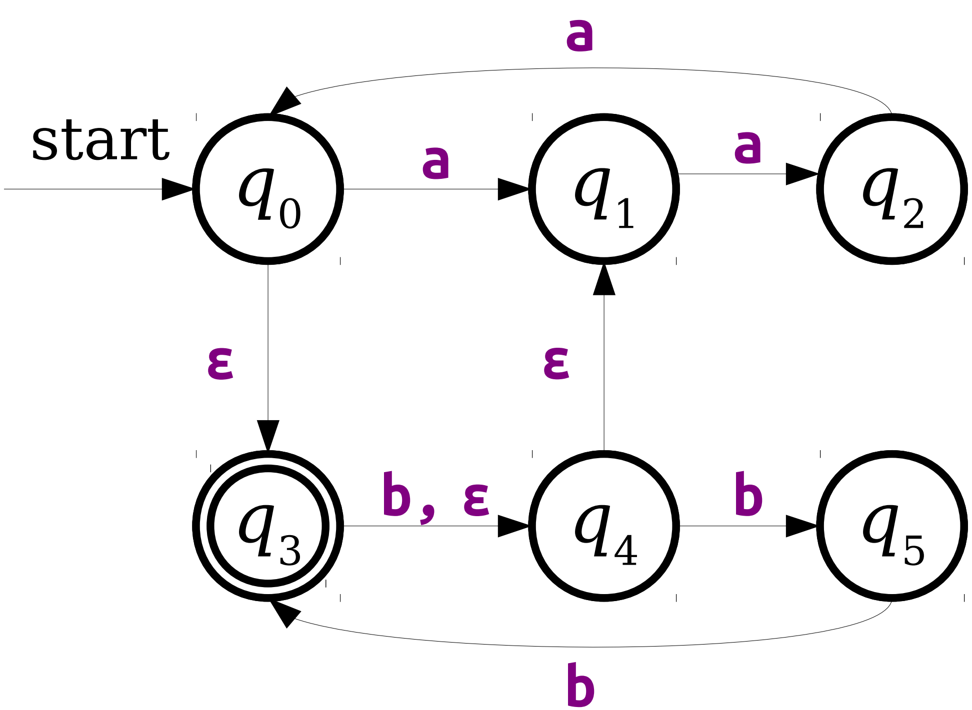

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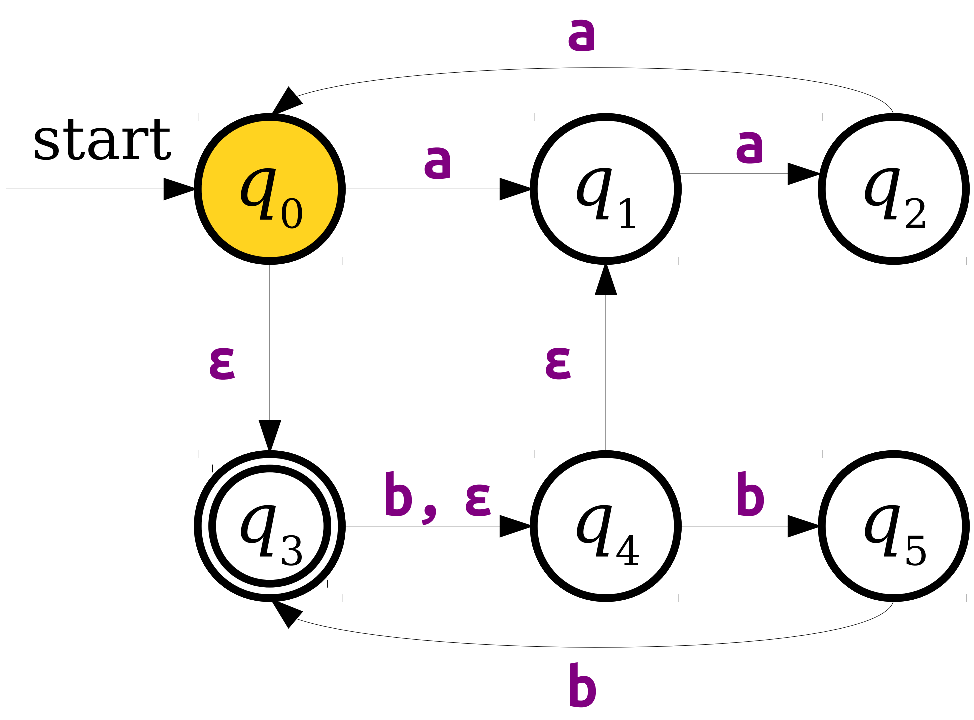

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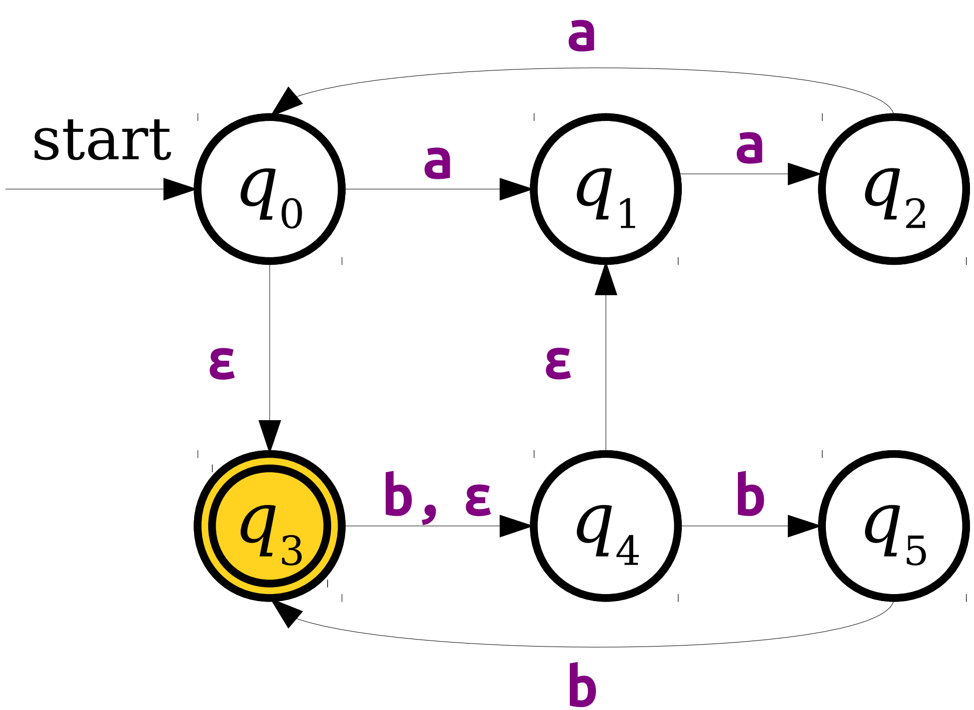

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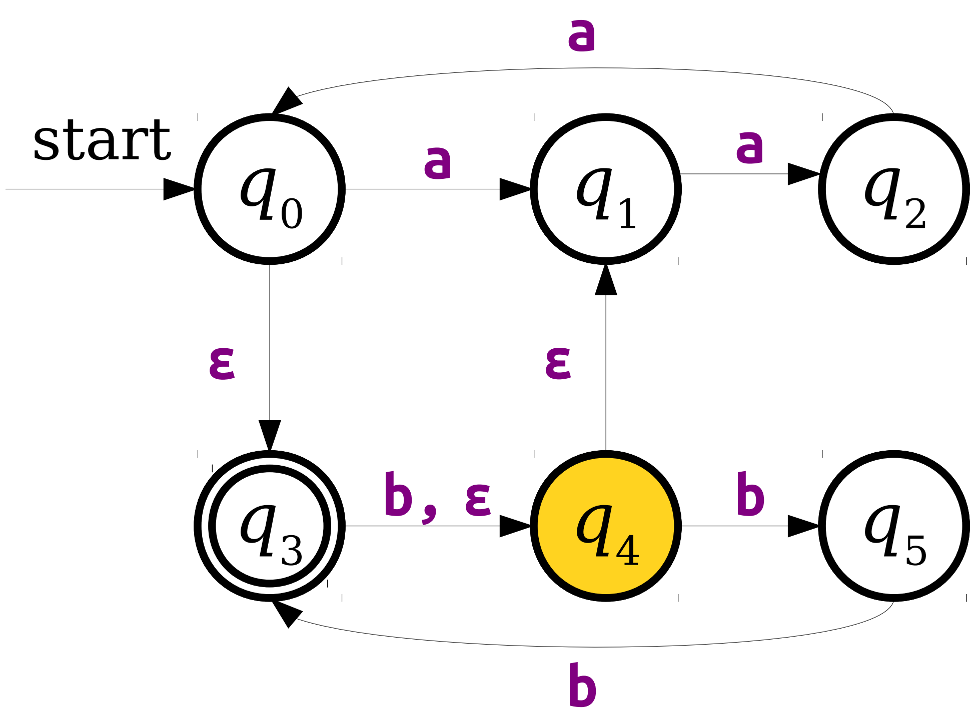

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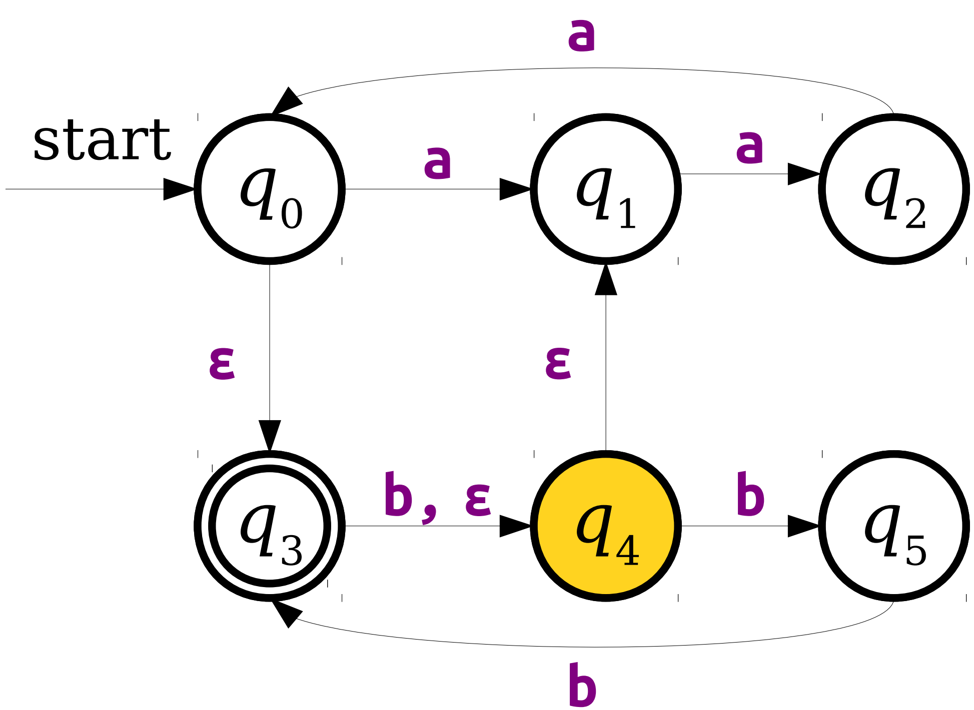

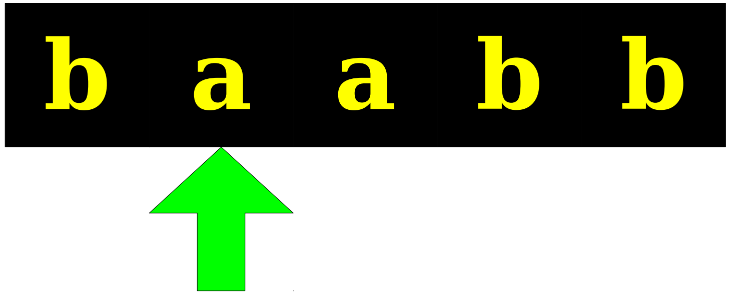

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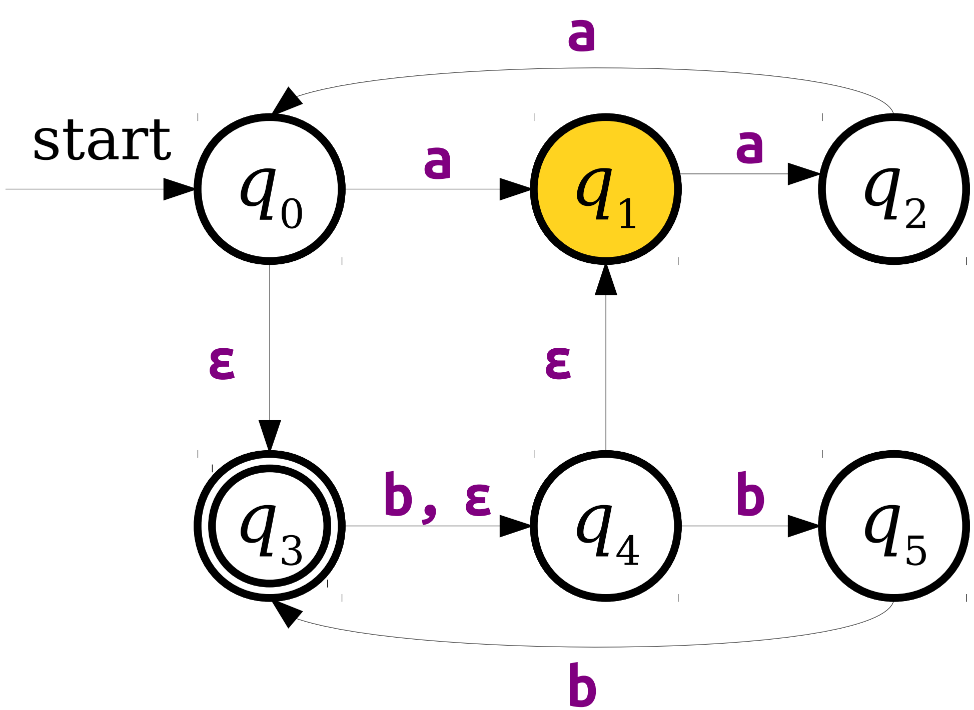

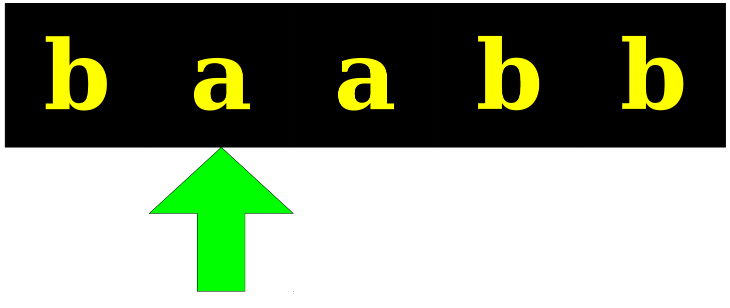

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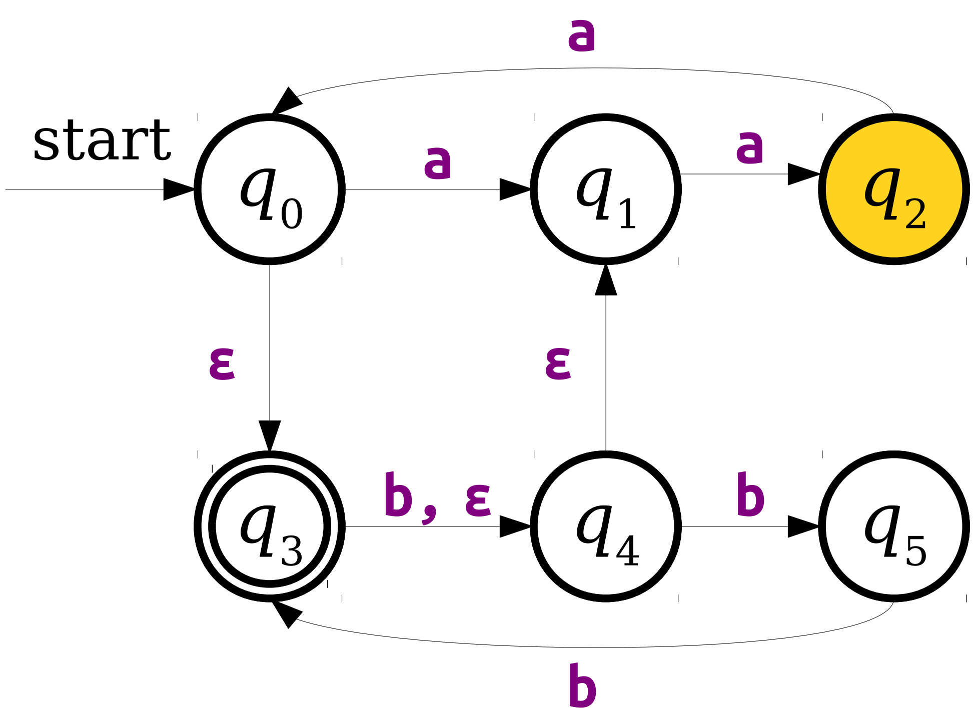

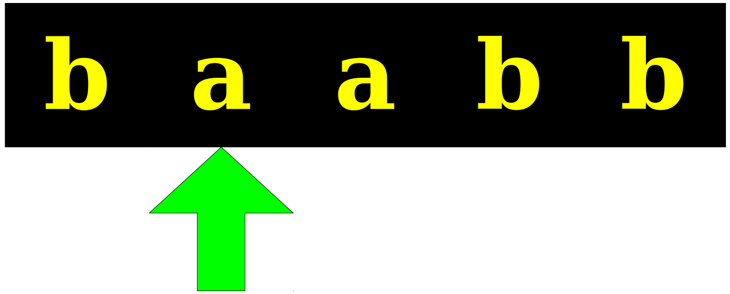

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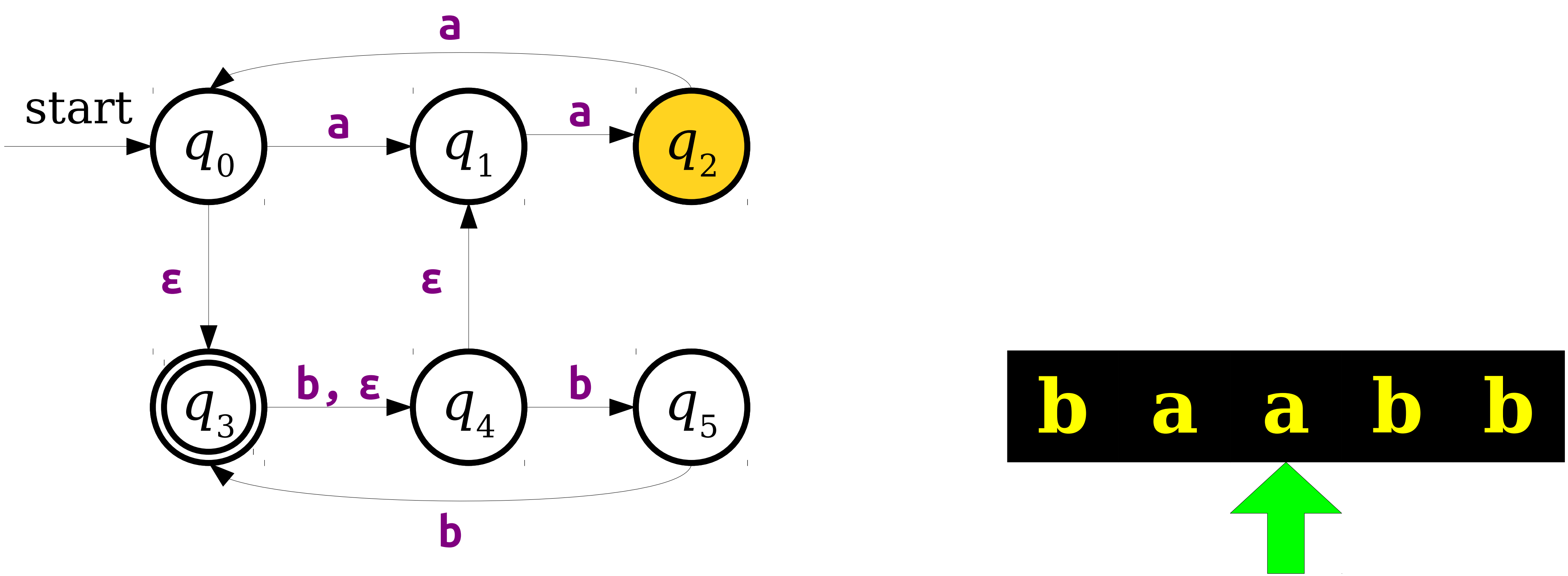

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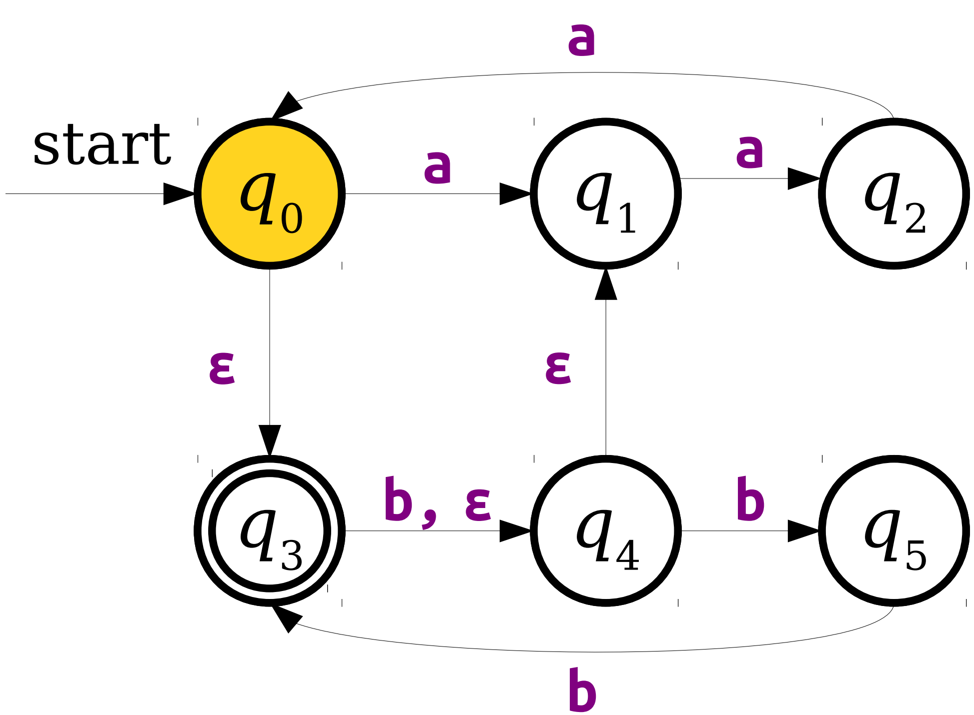

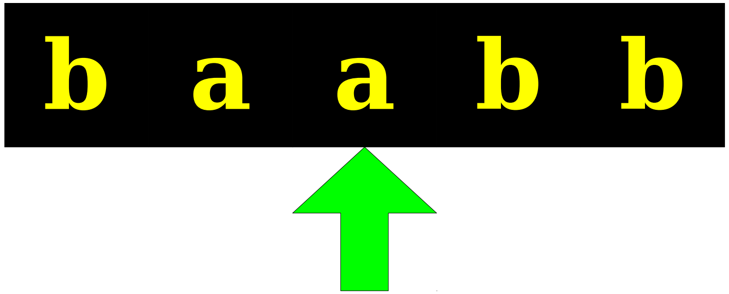

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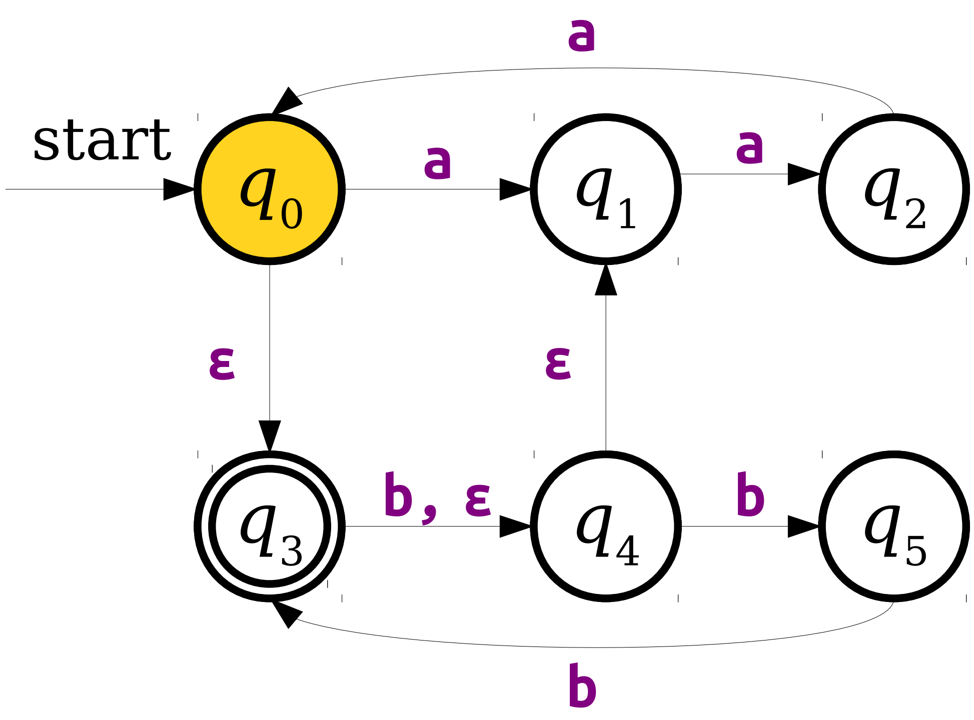

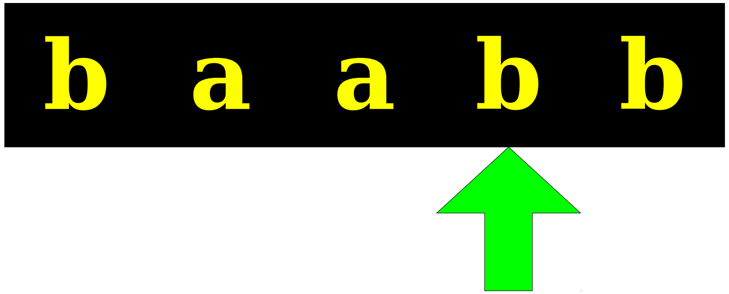

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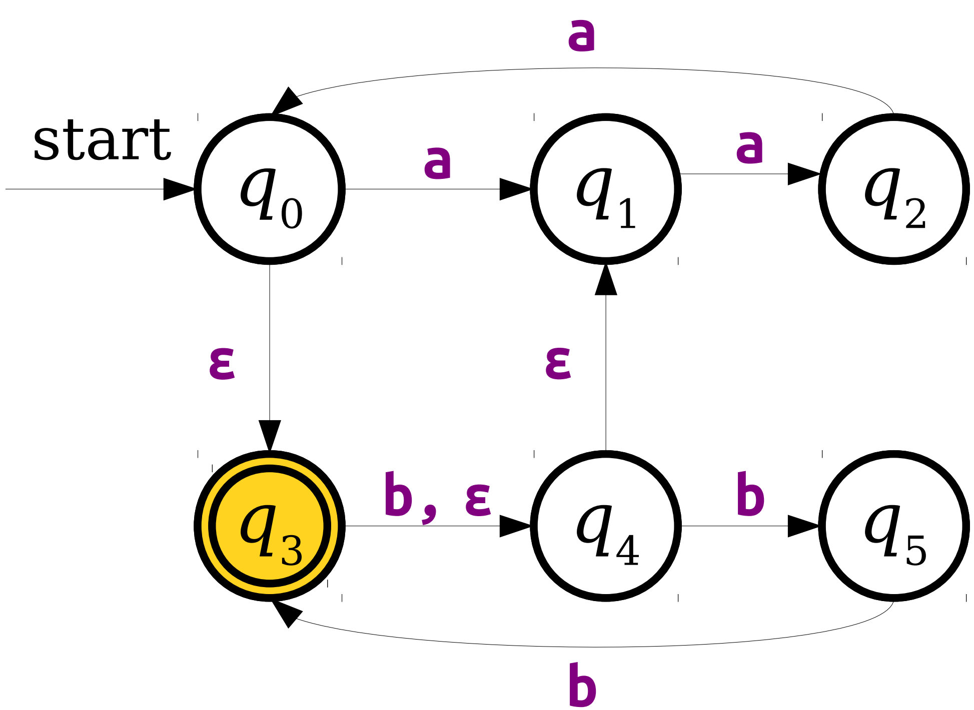

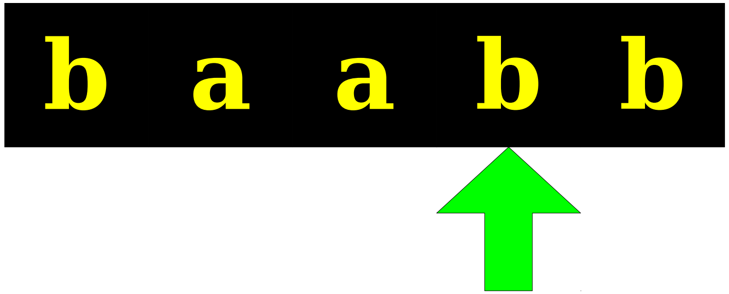

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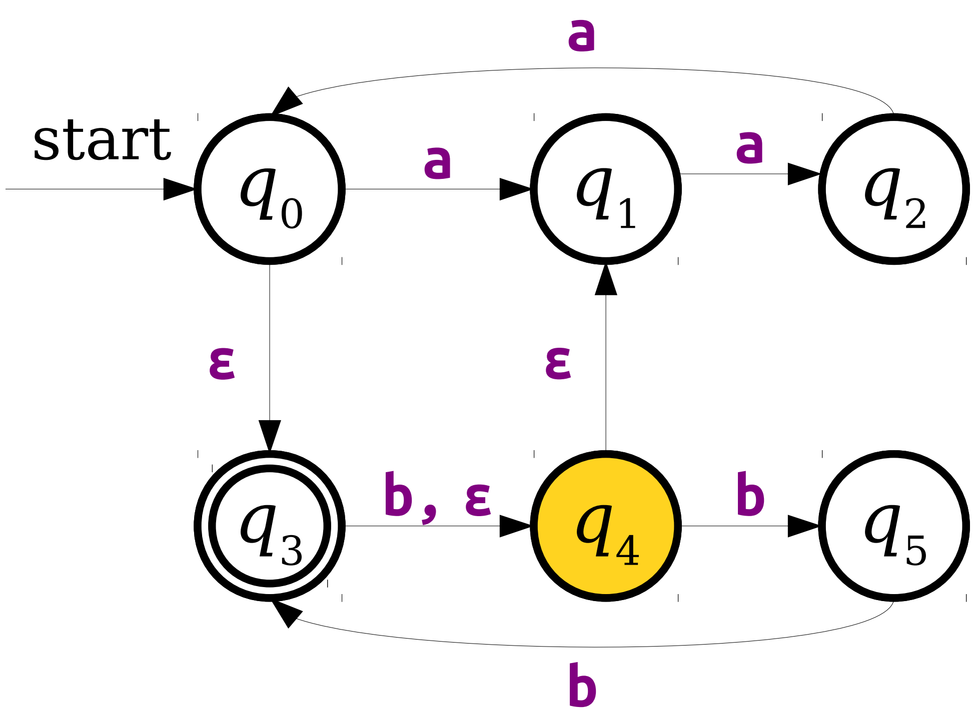

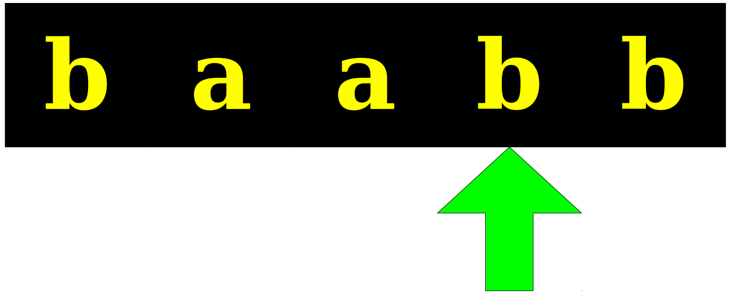

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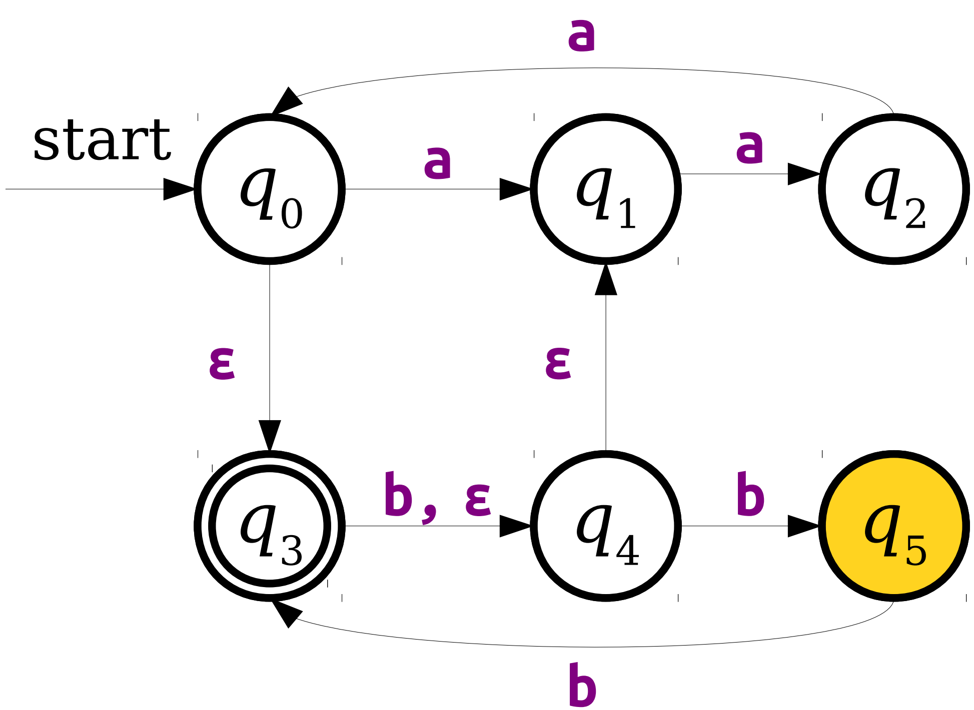

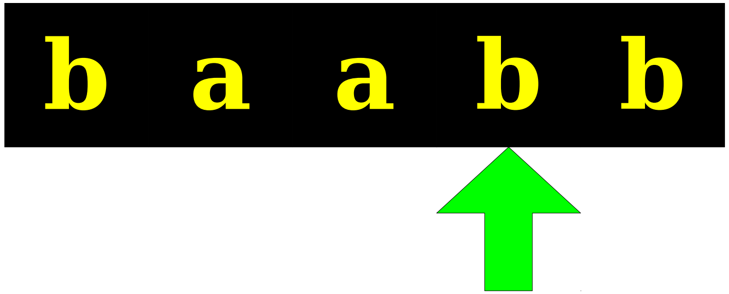

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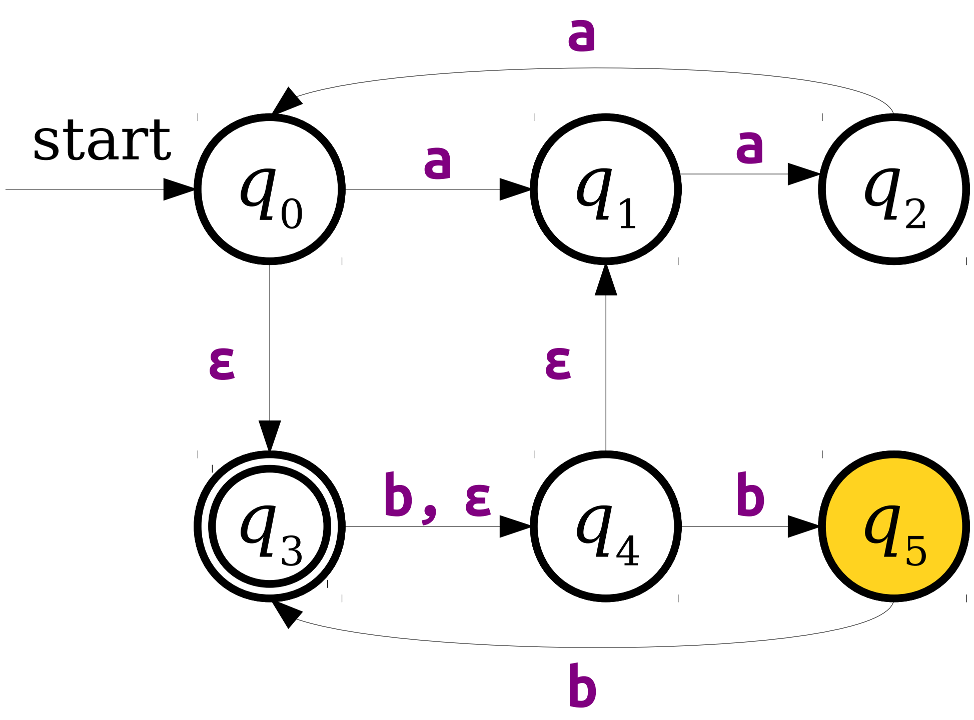

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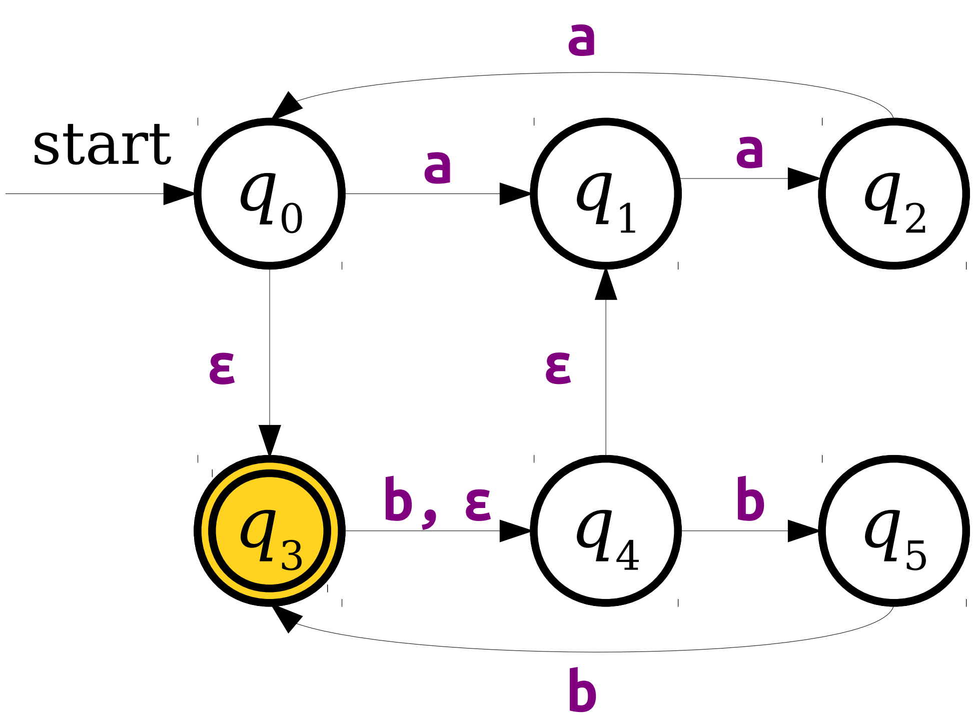

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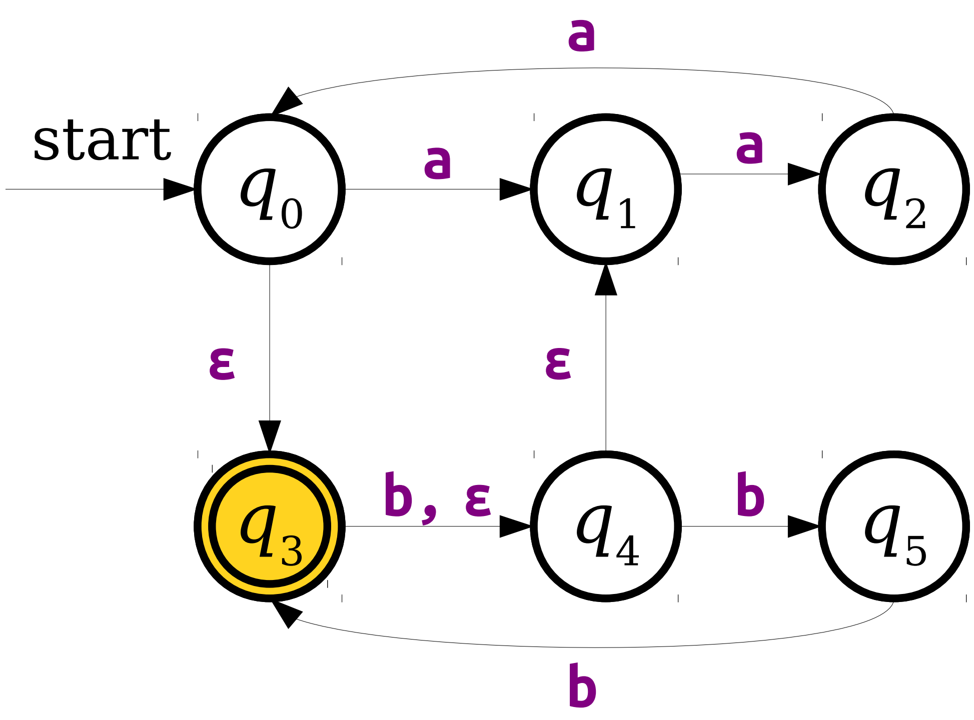
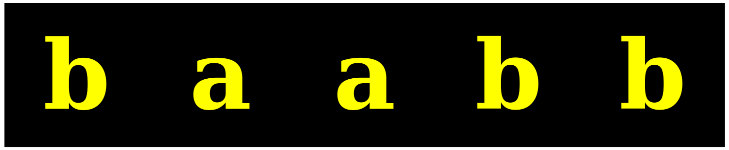




Perfect Guessing

- We can view nondeterministic machines as having ***Magic Superpowers*** that enable them to guess the correct choice of moves to make.
    - If there is at least one choice that leads to an accepting state, the machine will guess it.
    - If there are no choices, the machine guesses any one of the wrong guesses.
- No known physical analog for this style of computation – this is totally new!

Massive Parallelism

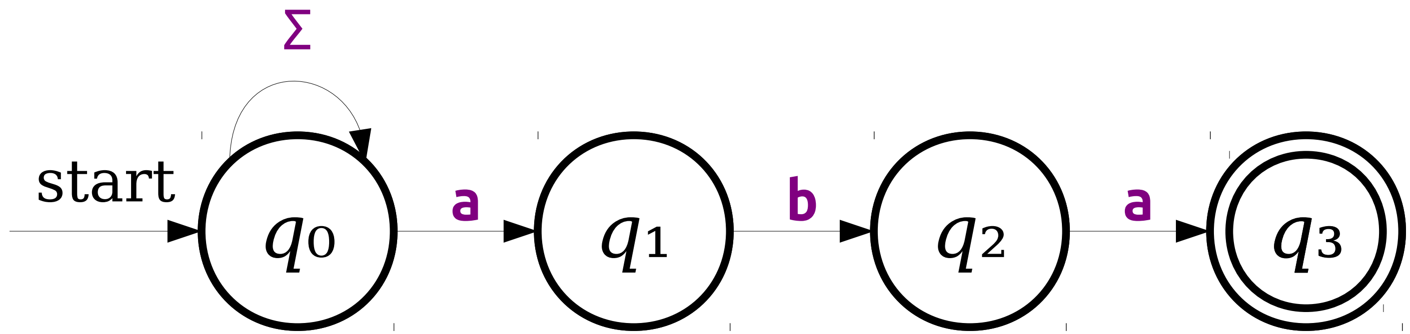
- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- The NFA accepts if *any* of the states that are active at the end are accepting states. It rejects otherwise.

Just how powerful *are* NFAs?

New Stuff!

NFAs and DFAs

- Any language that can be accepted by a DFA can be accepted by an NFA.
- Why?
  - Every DFA essentially already *is* an NFA!
- ***Question:*** Can any language accepted by an NFA also be accepted by a DFA?
- Surprisingly, the answer is ***yes***!



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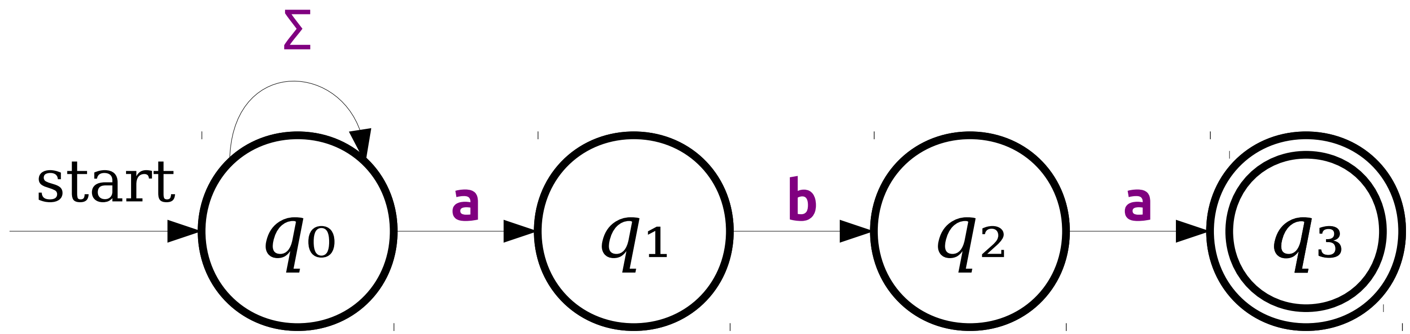
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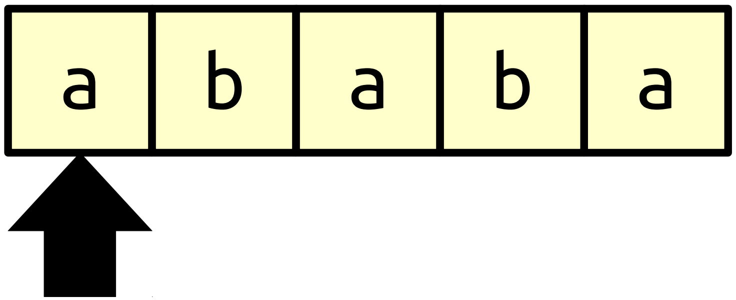
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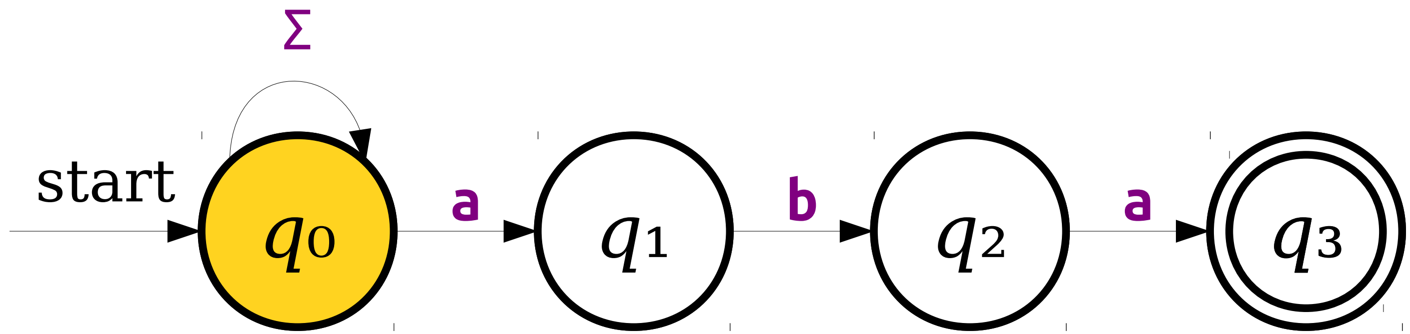
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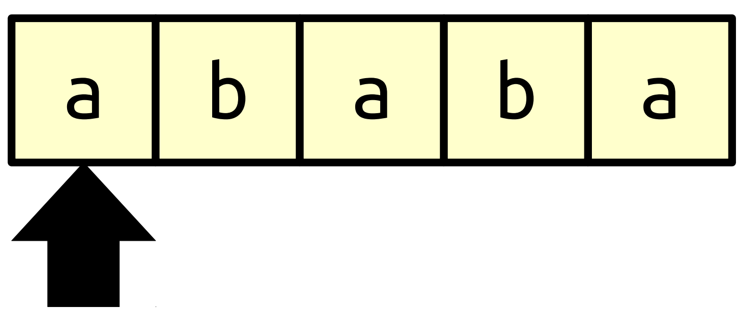
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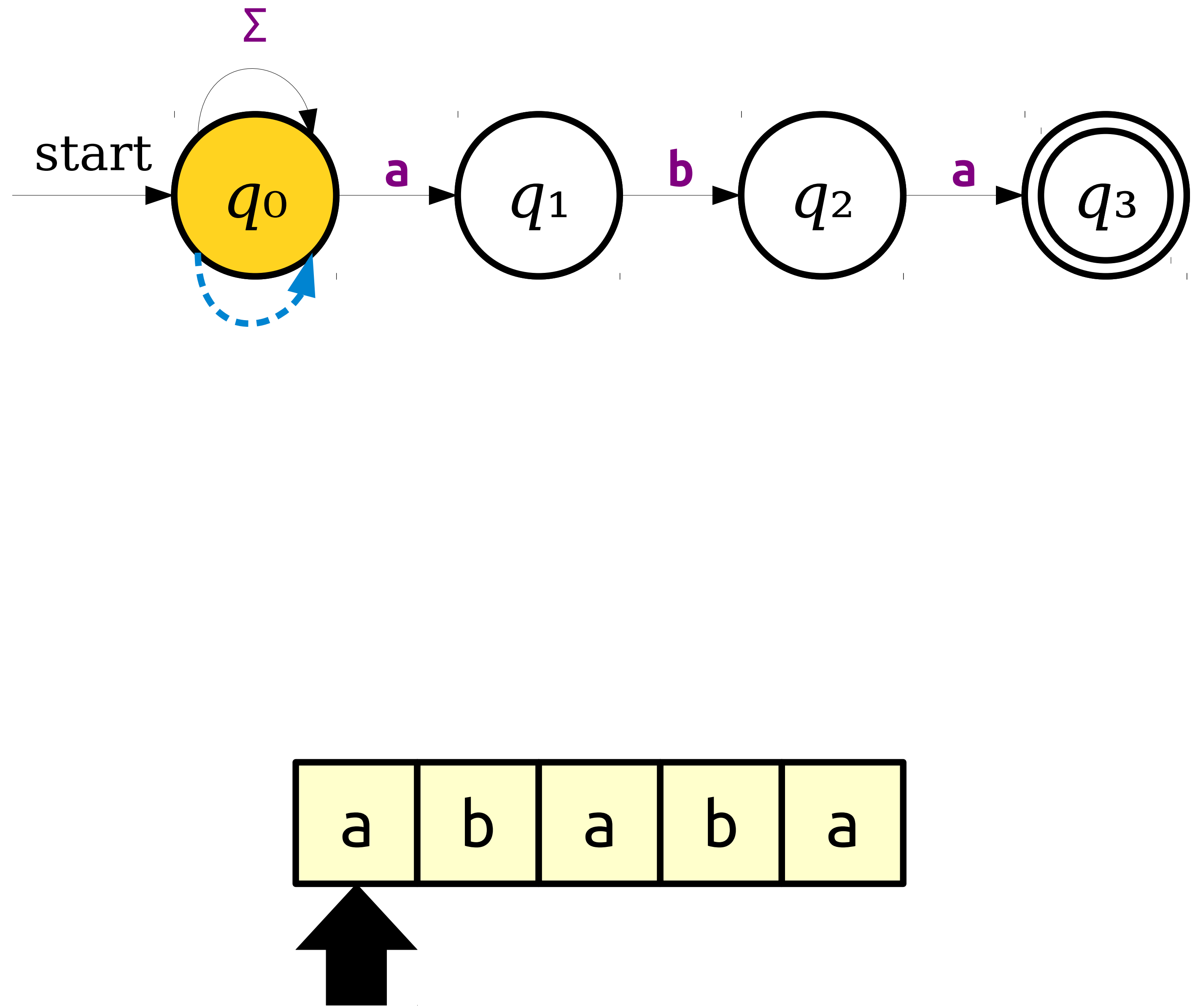

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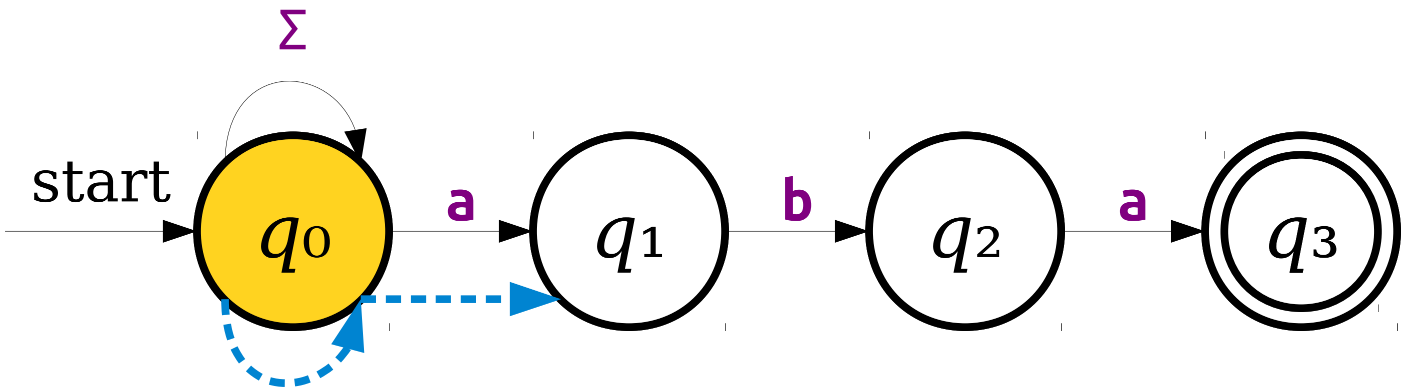
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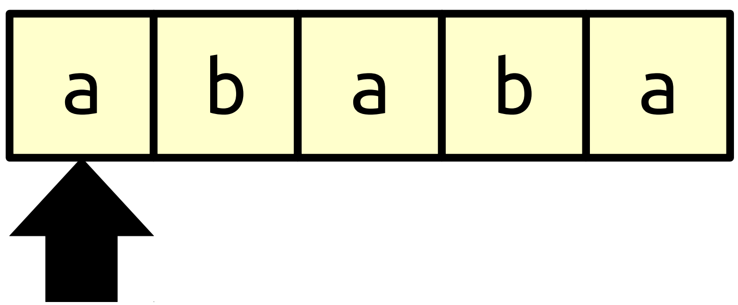
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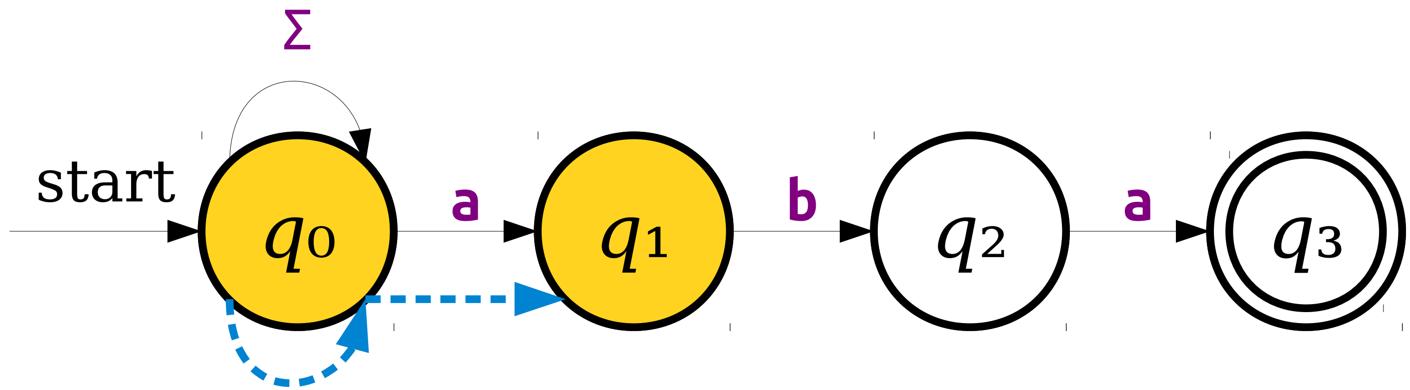
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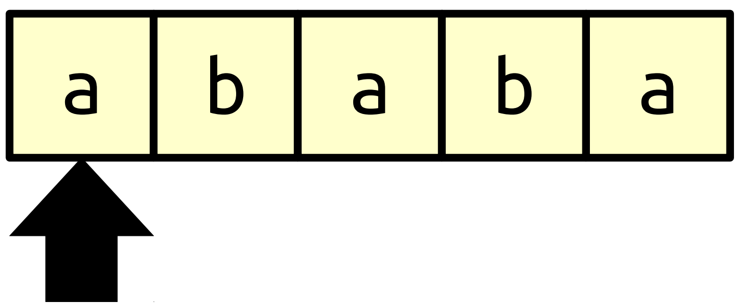
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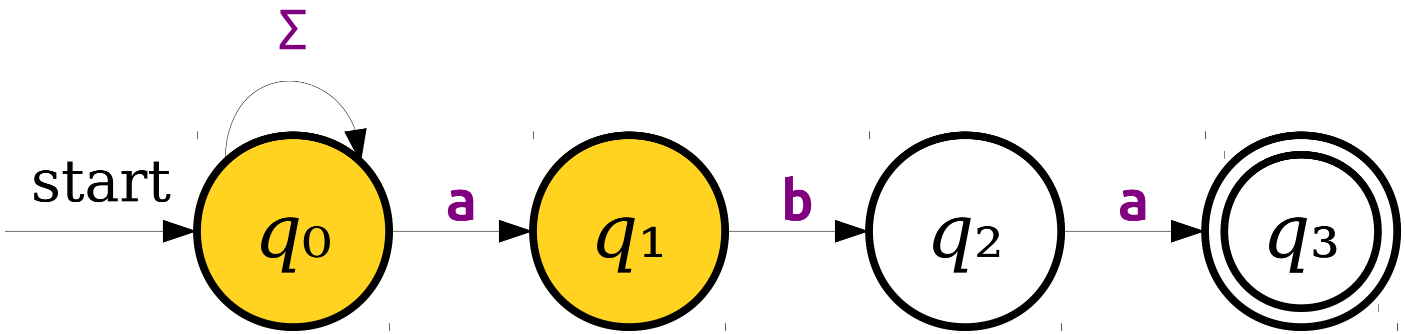
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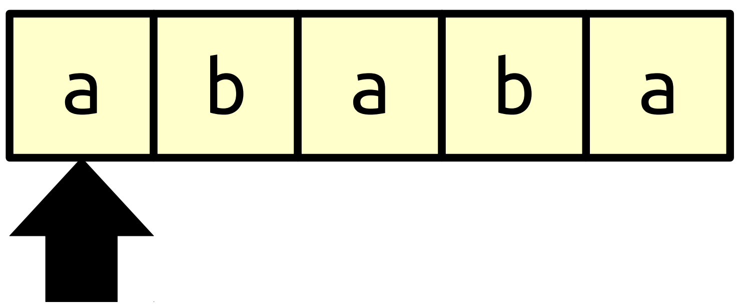
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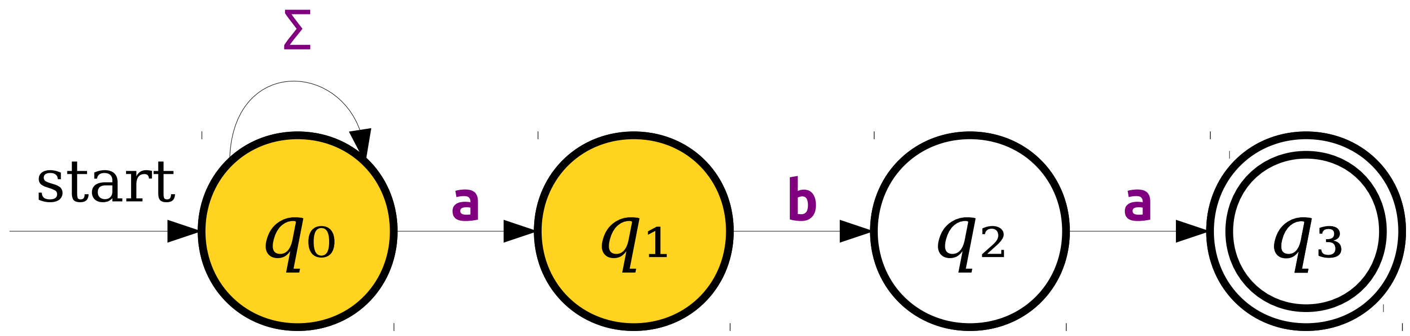
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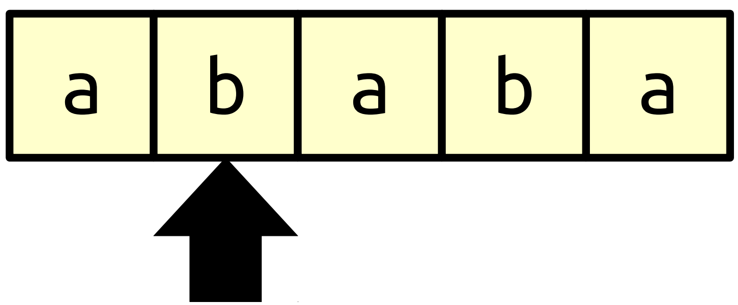
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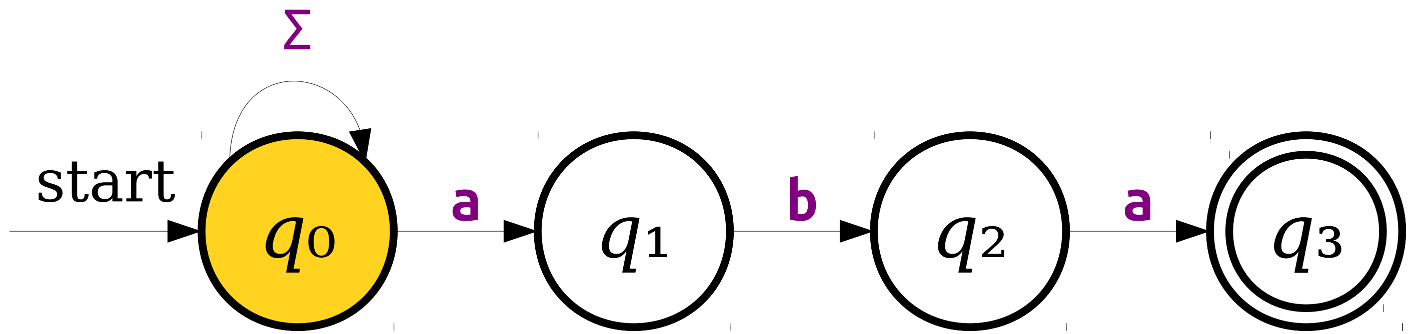
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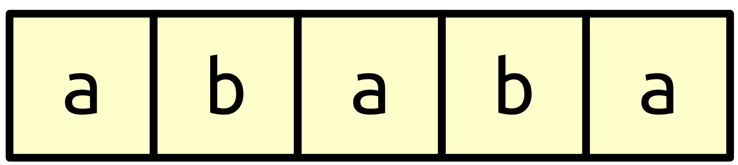
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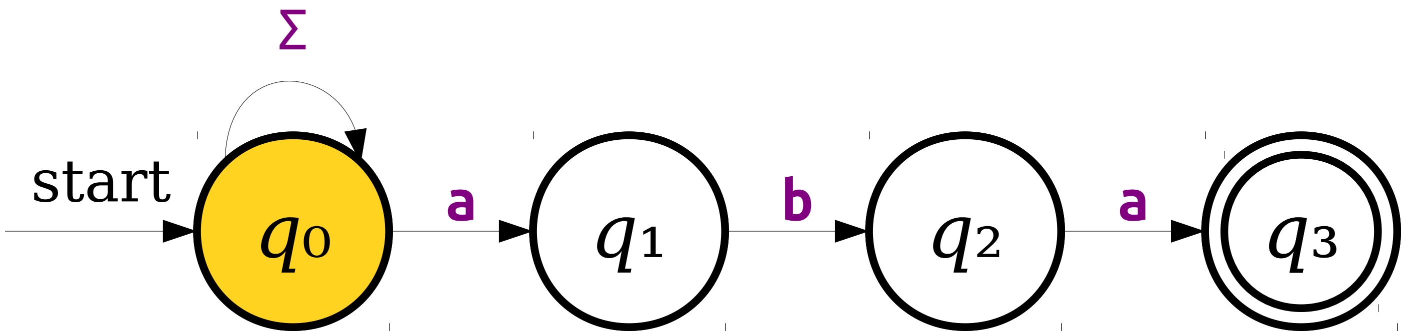
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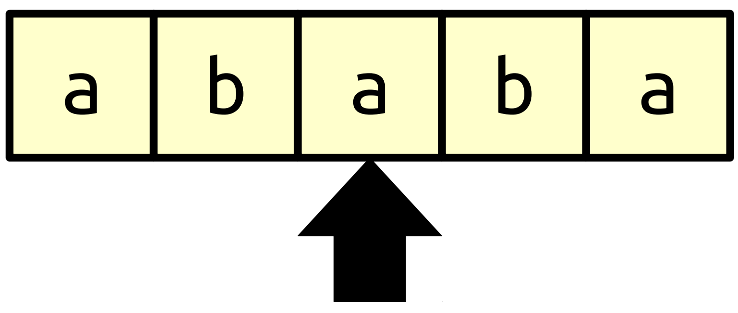
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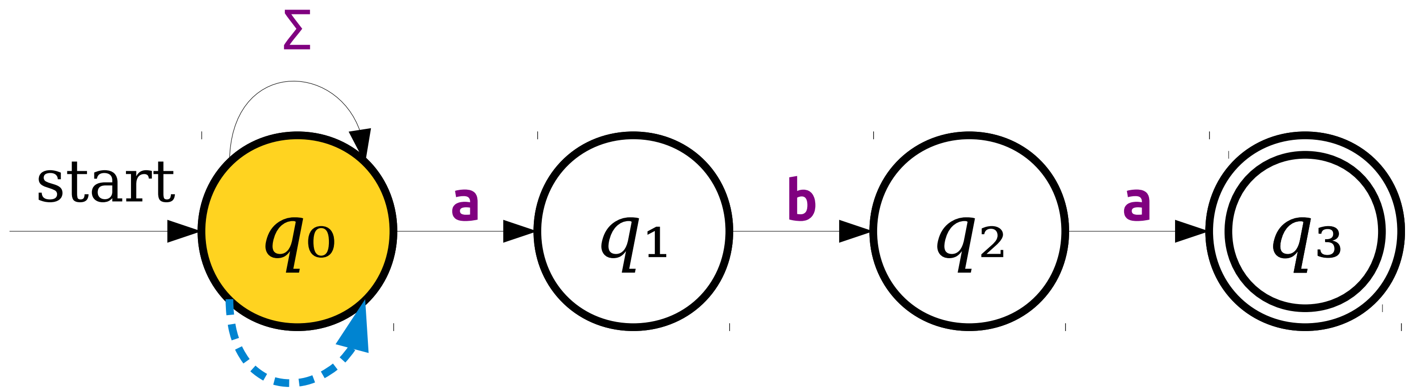
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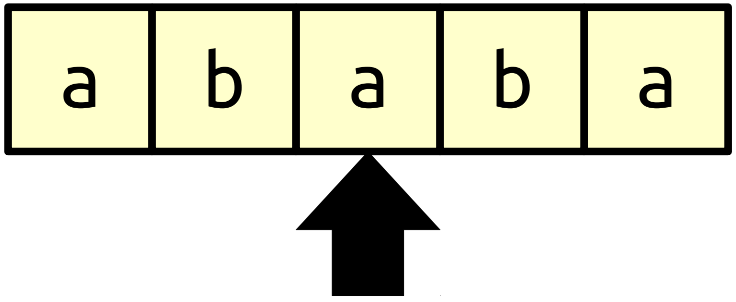
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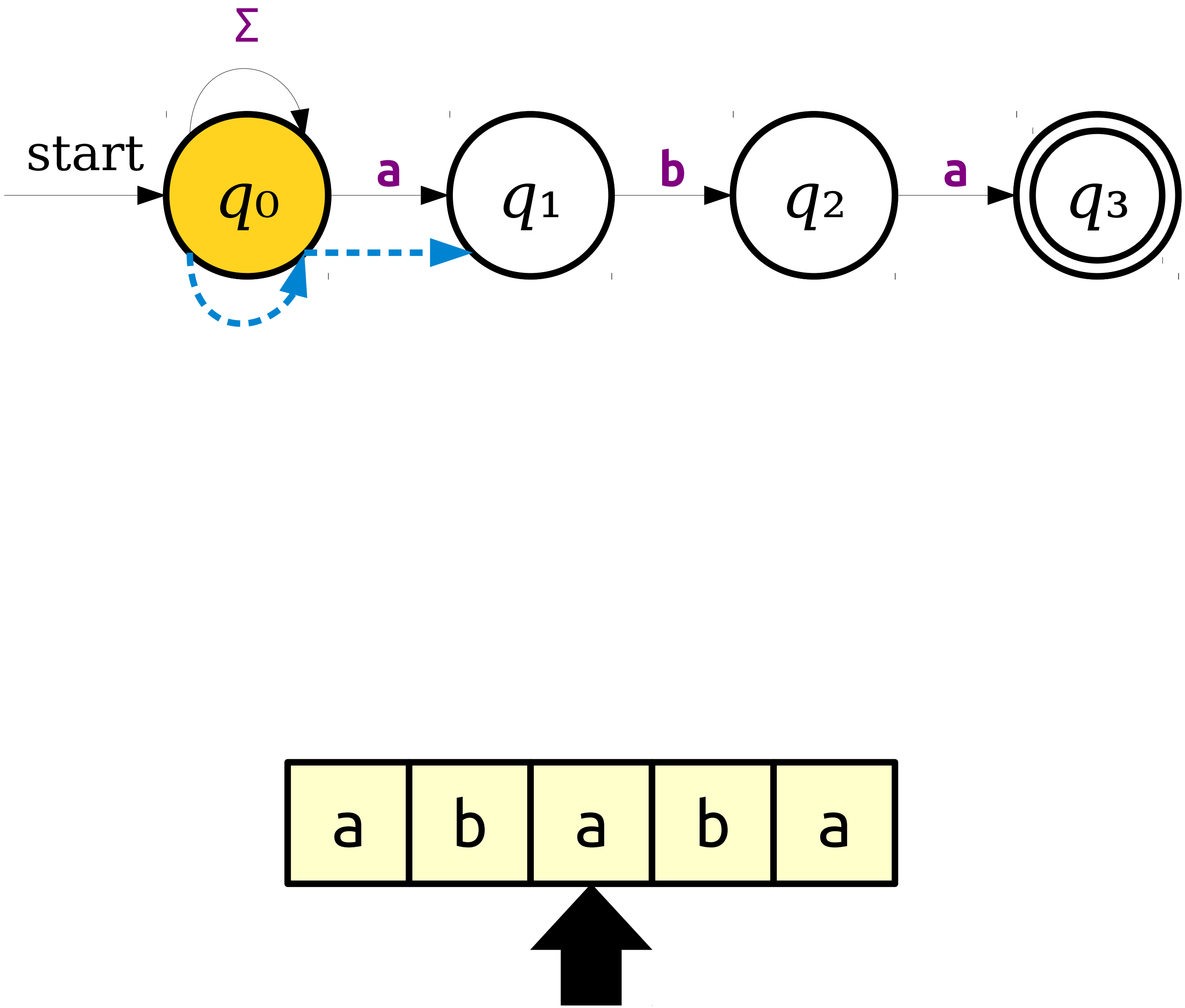
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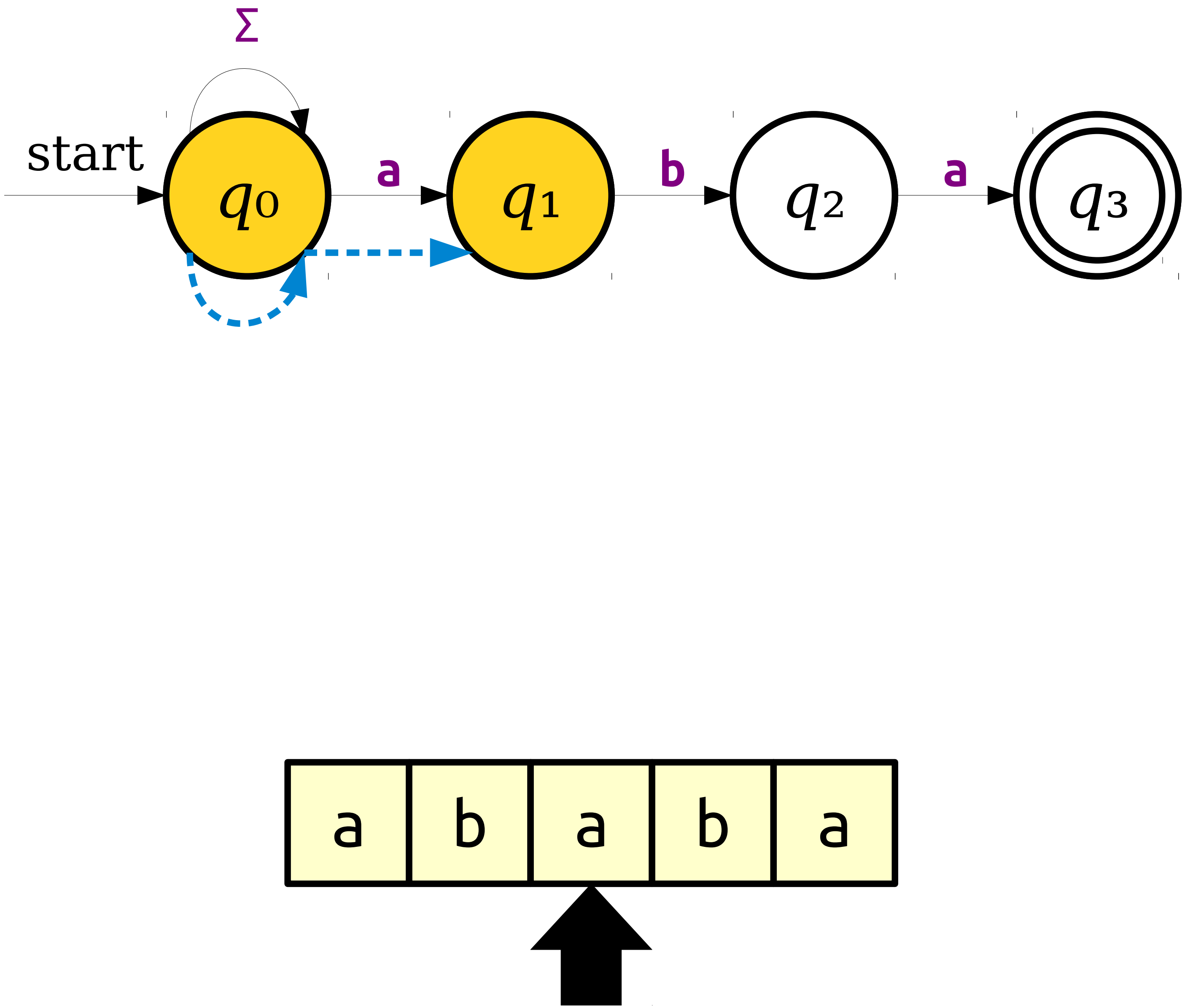
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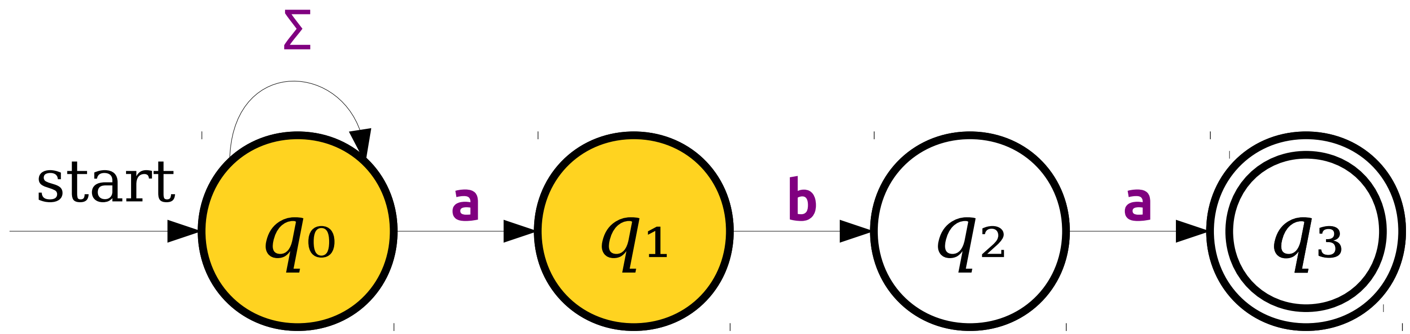
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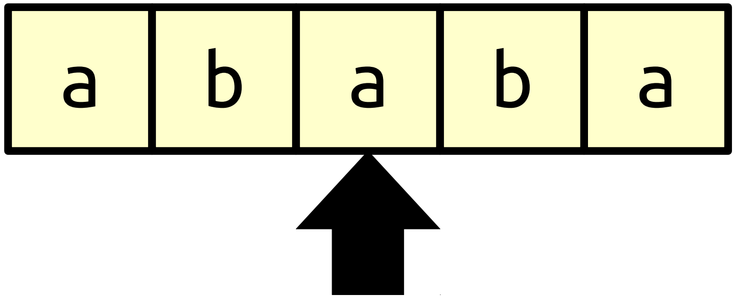
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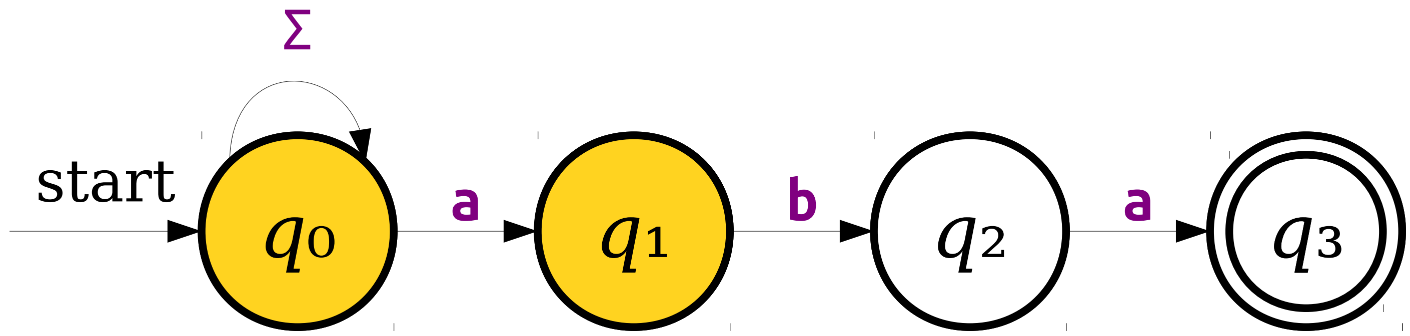
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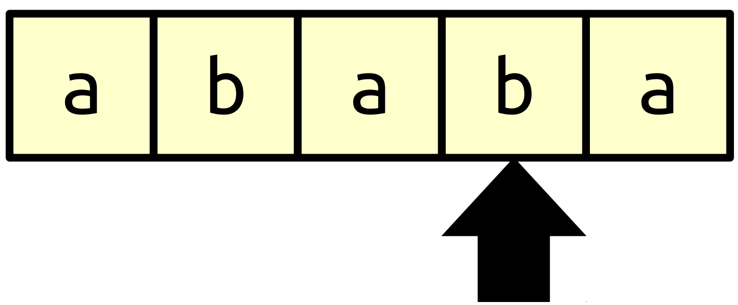
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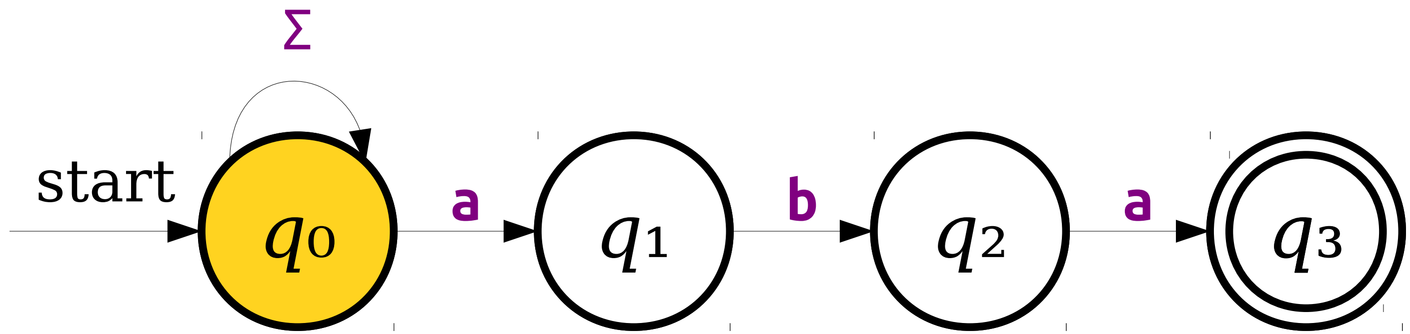
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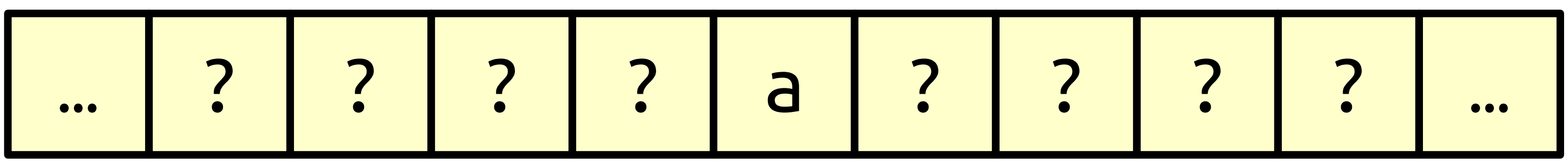
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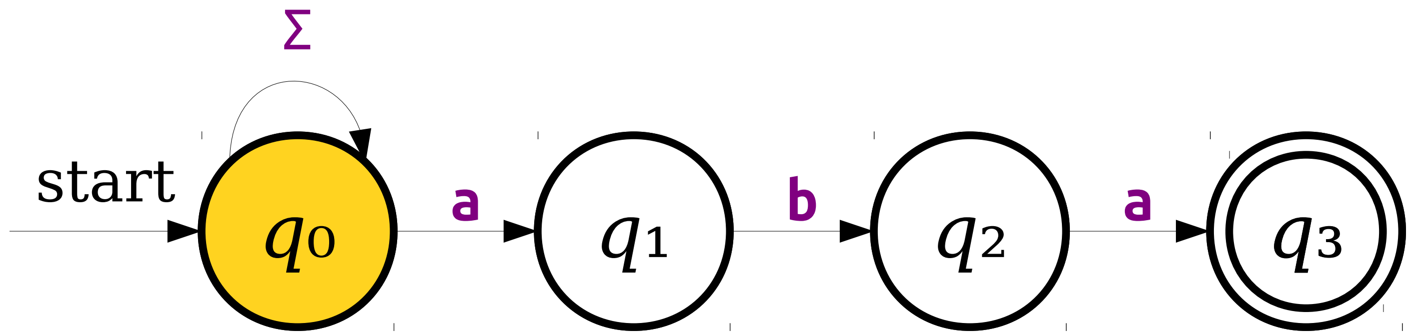
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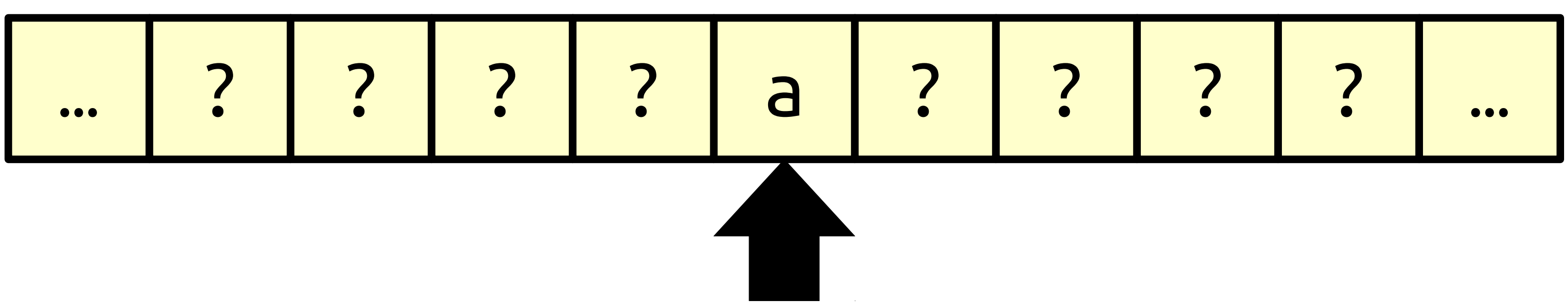
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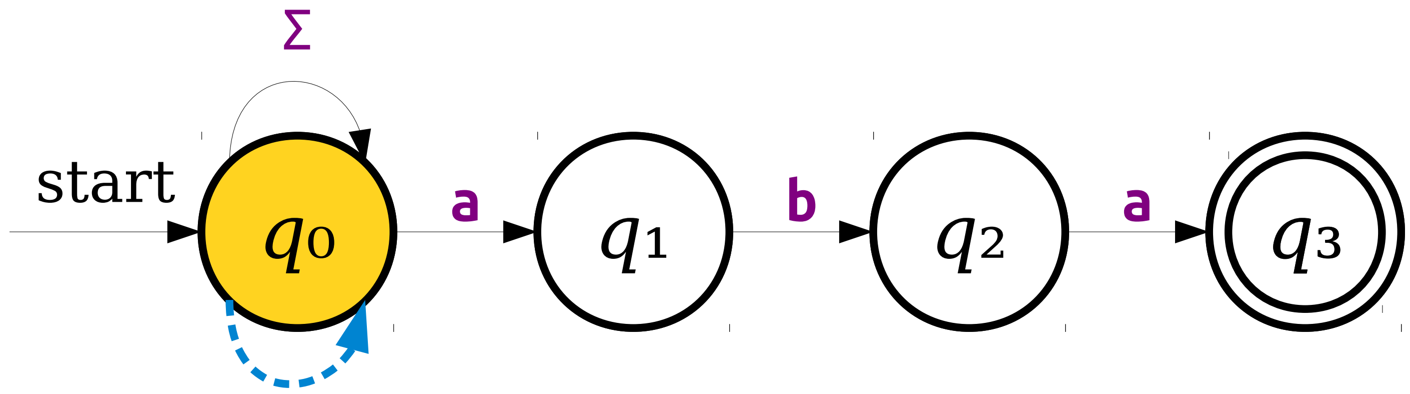
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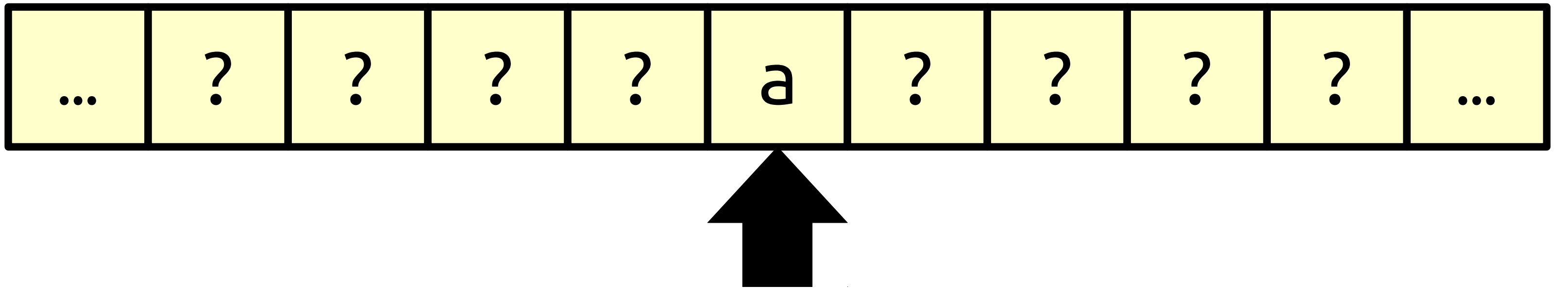
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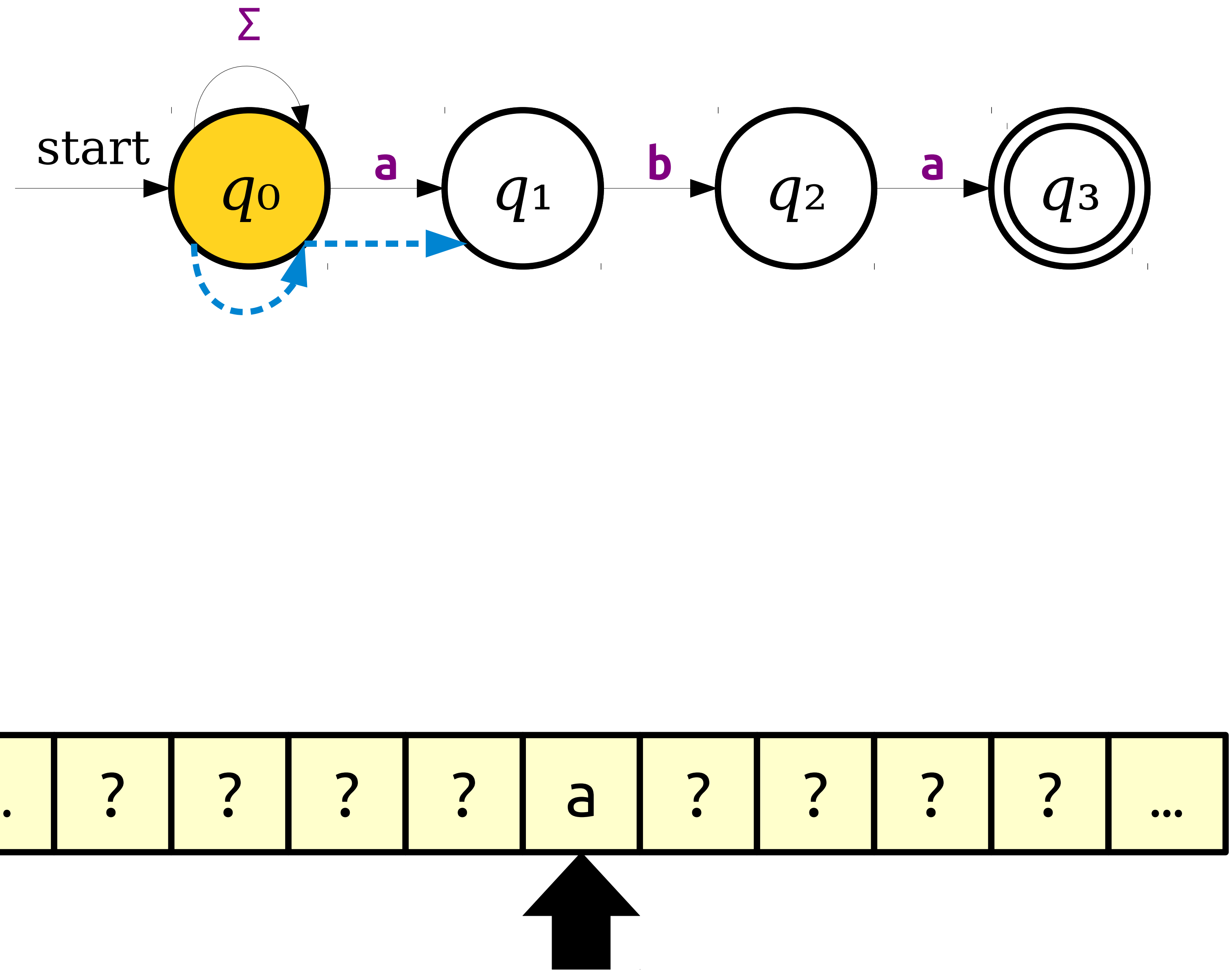
Σ
start
q_0
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q_1
b
q_2
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q_3
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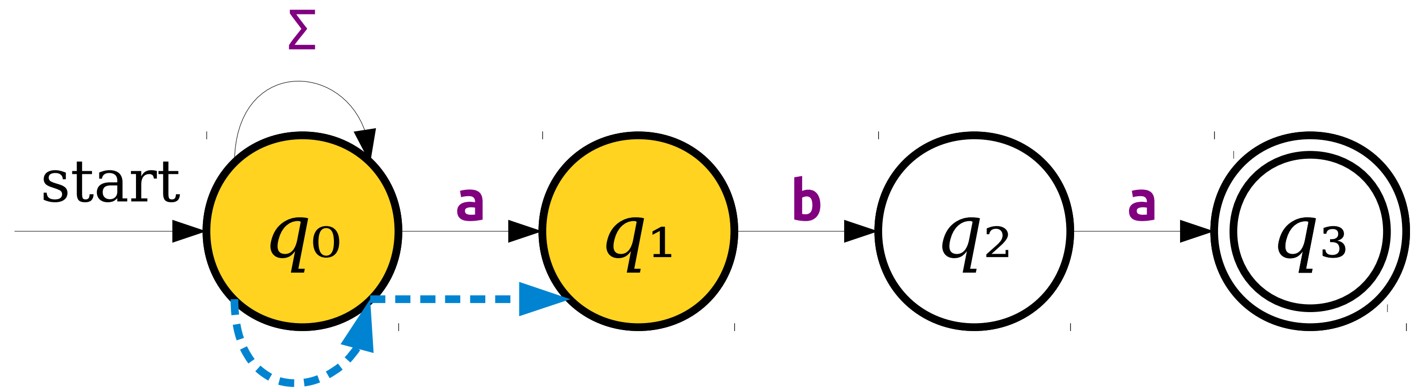
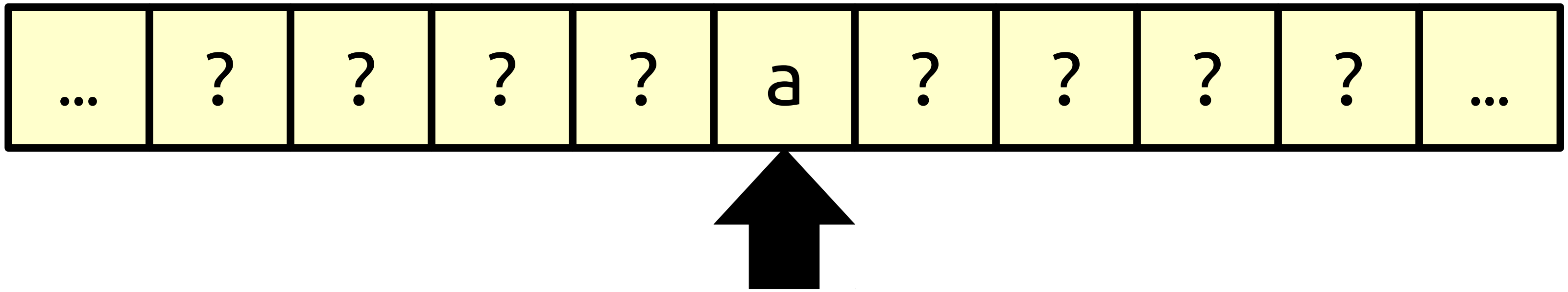


Σ
start
q_0
a
q_1
b
q_2
a
q_3



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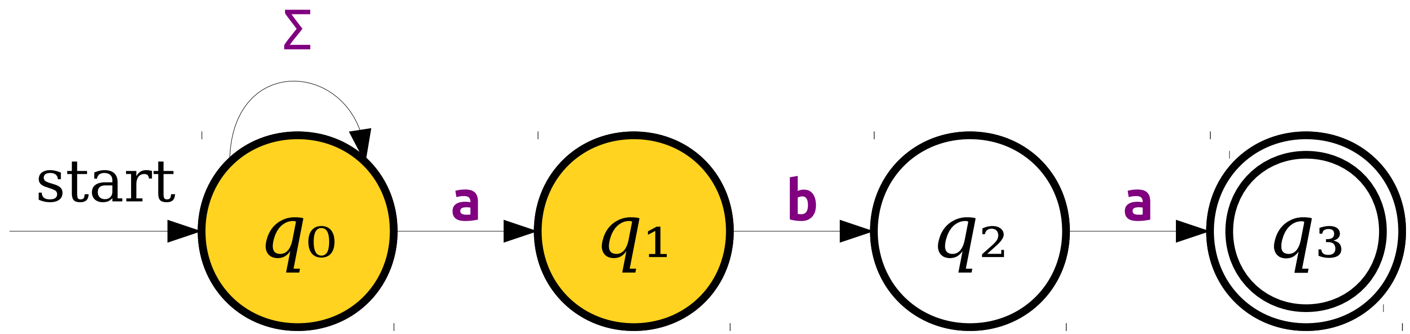
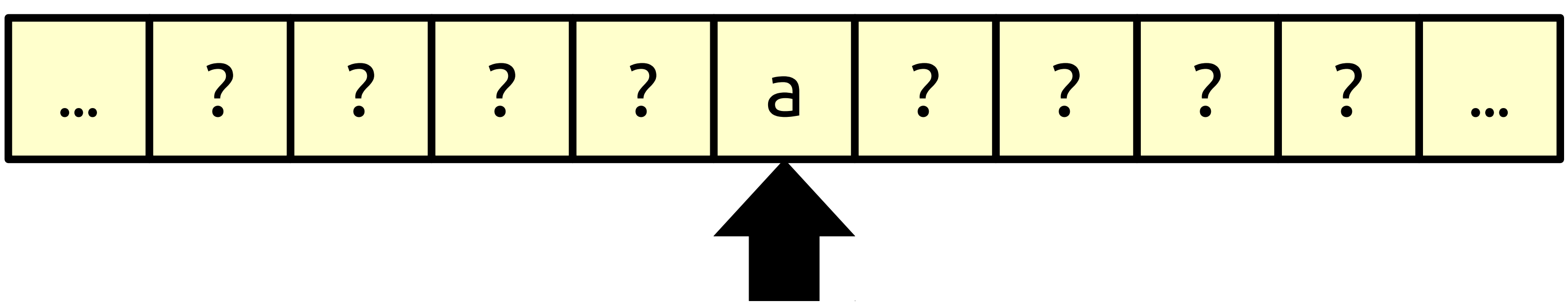

Σ
start
q_0
a
q_1
b
q_2
a
q_3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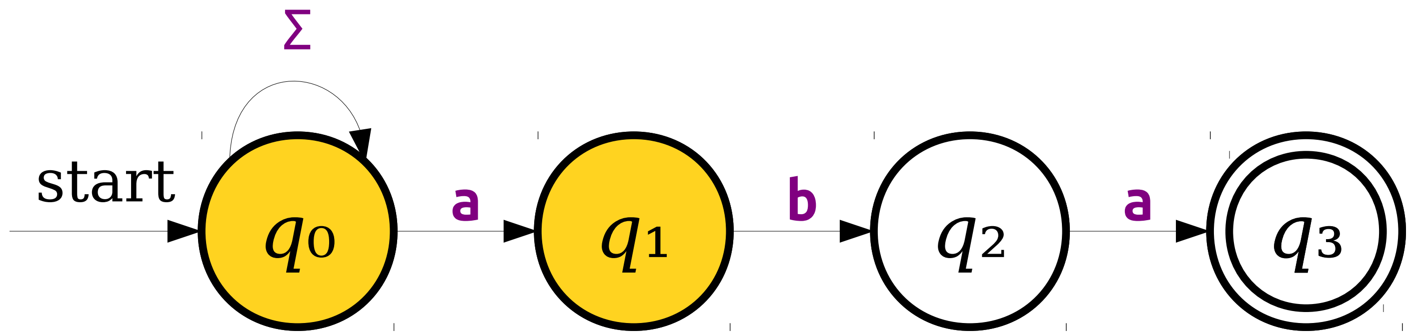
Σ
start
q0
a
q1
b
q2
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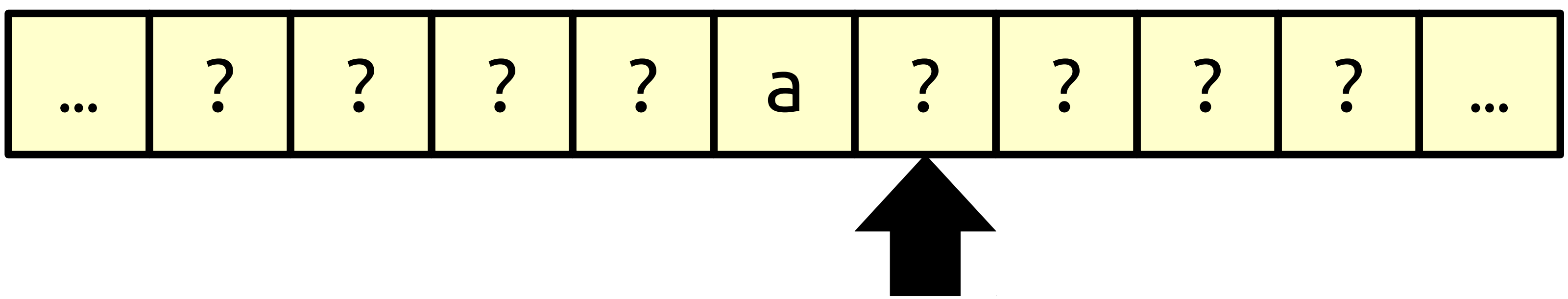
q3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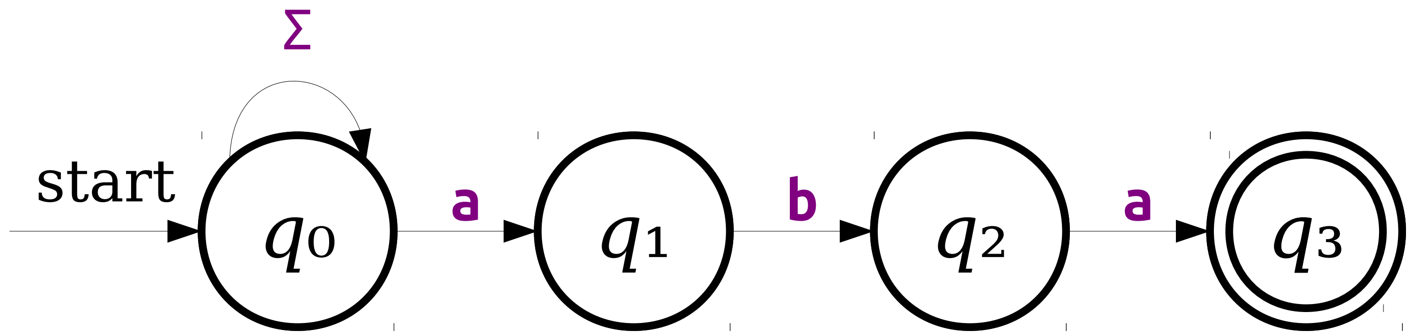
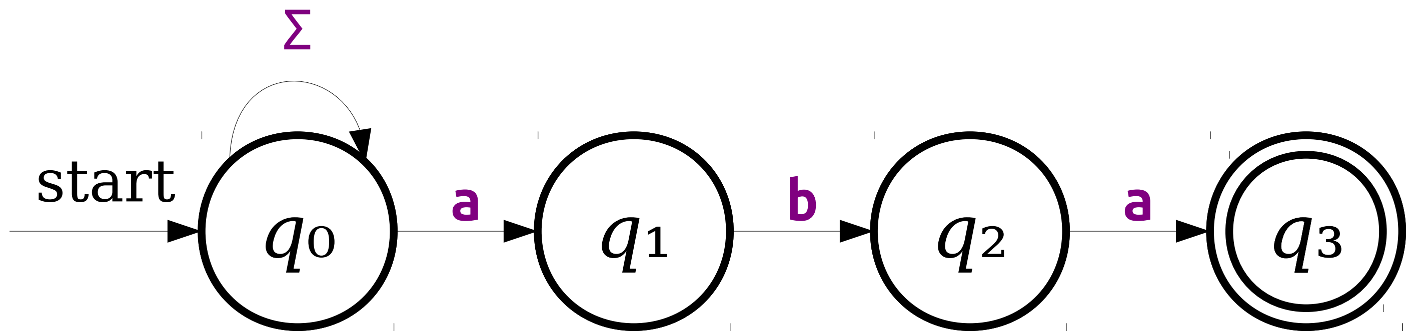
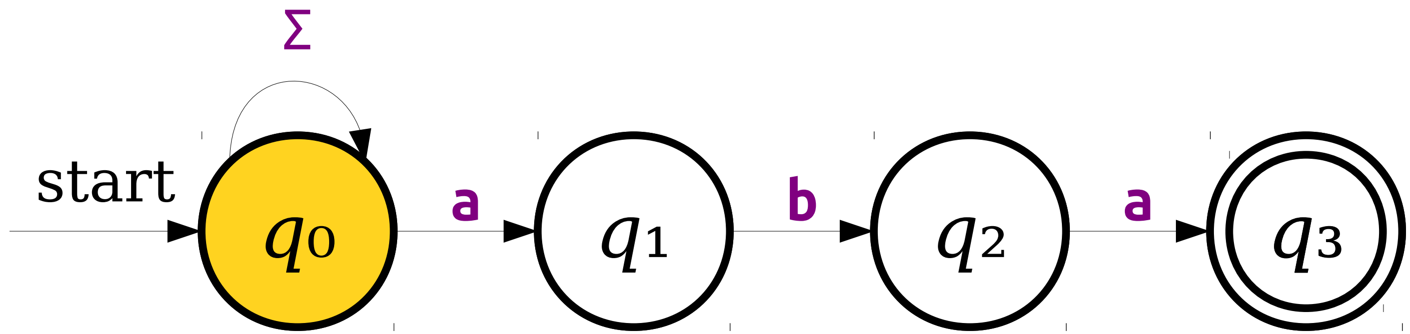


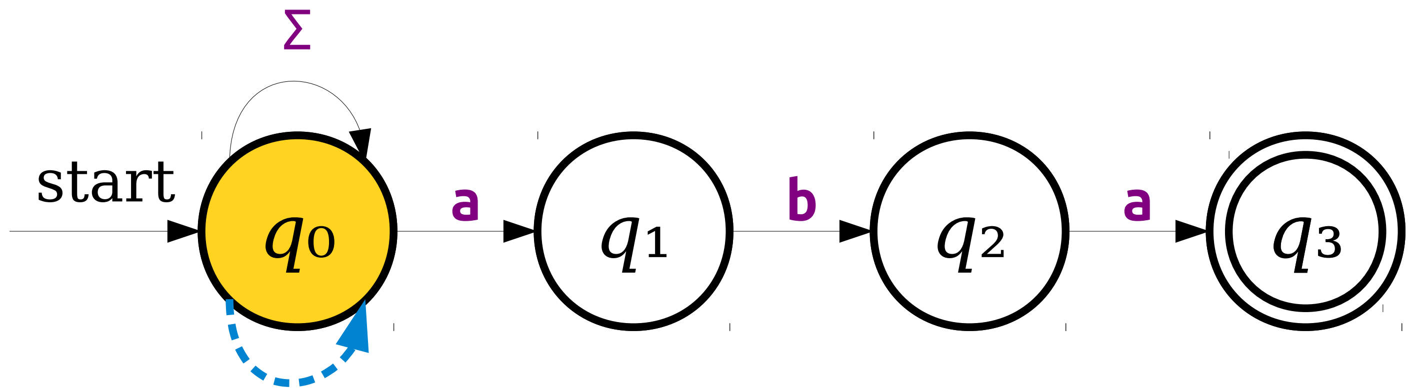
	a
$\{q_0\}$	$\{q_0, q_1\}$



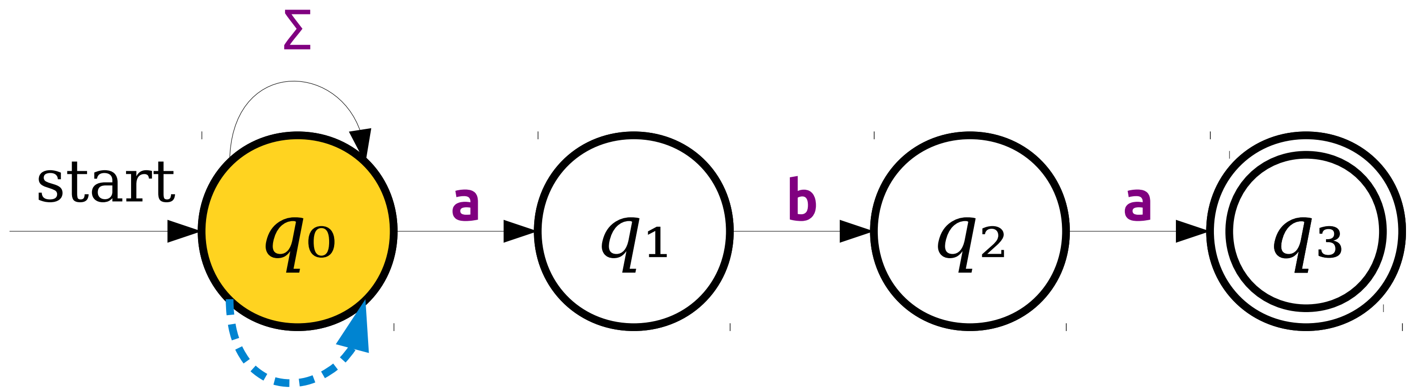
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



	a	b
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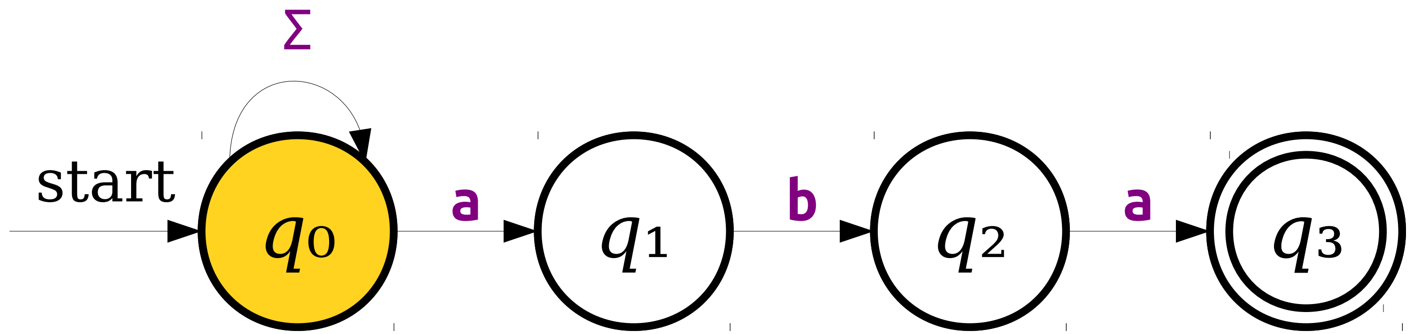
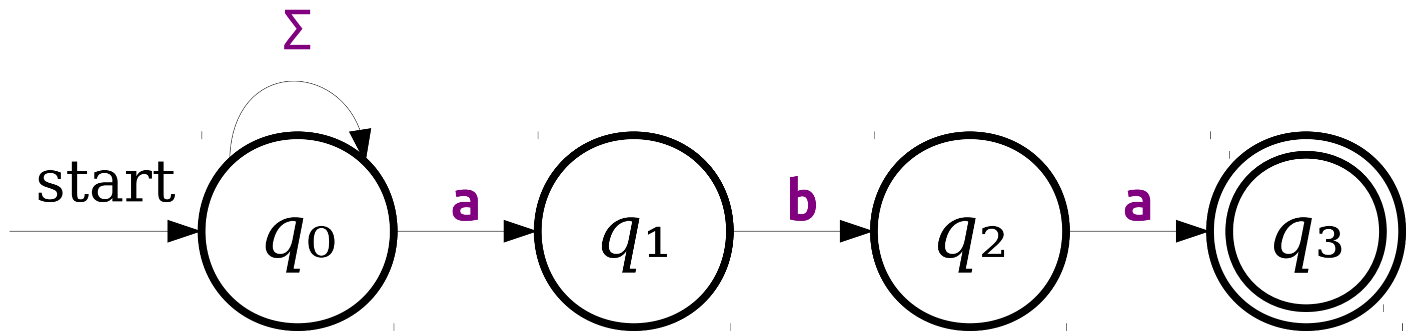


	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



	a	b
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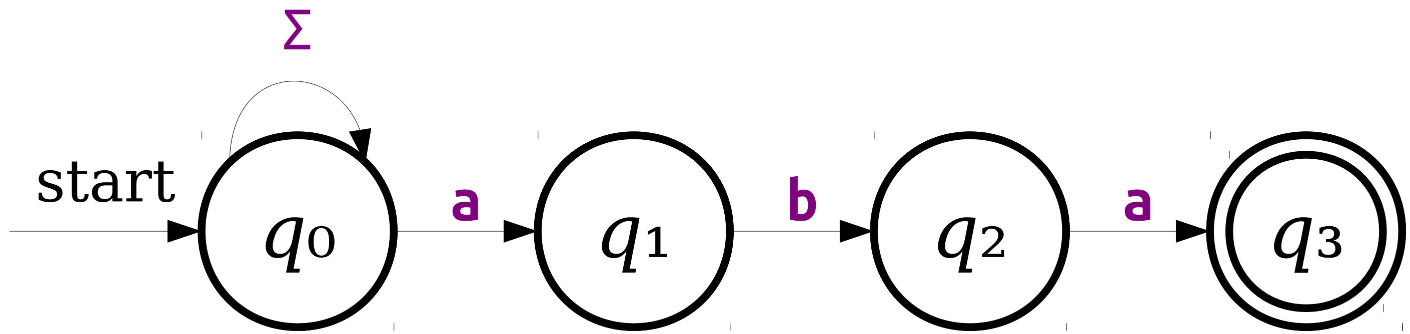
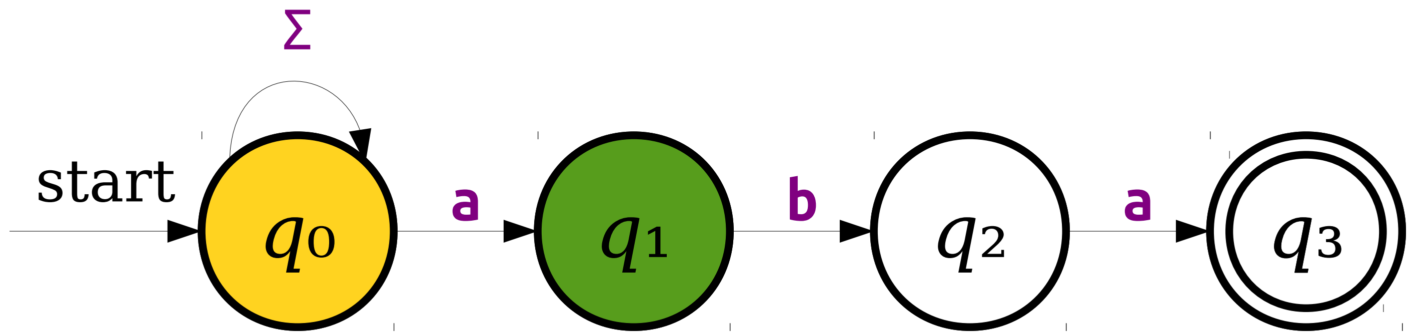
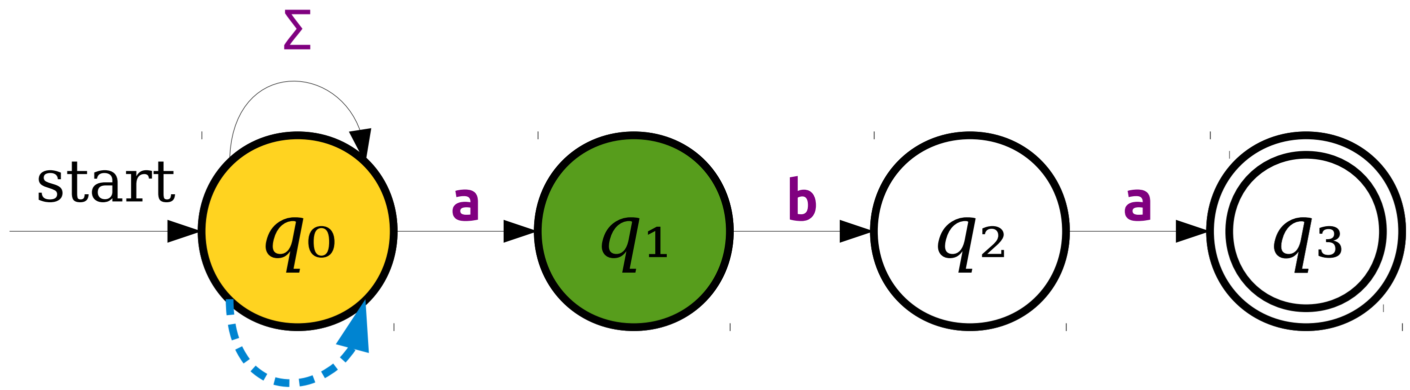


	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



	a	b
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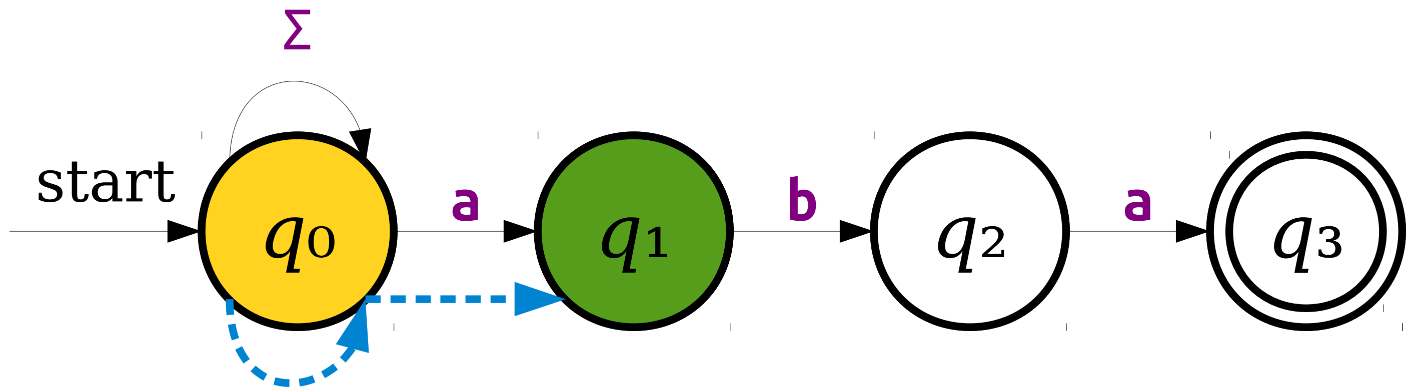
$\{q_0, q_1\}$		



	a	b
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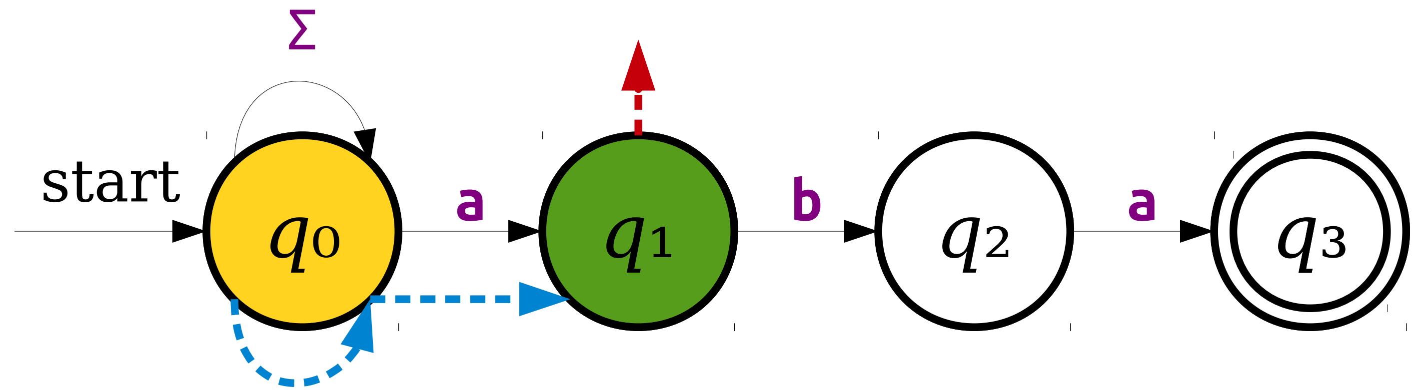
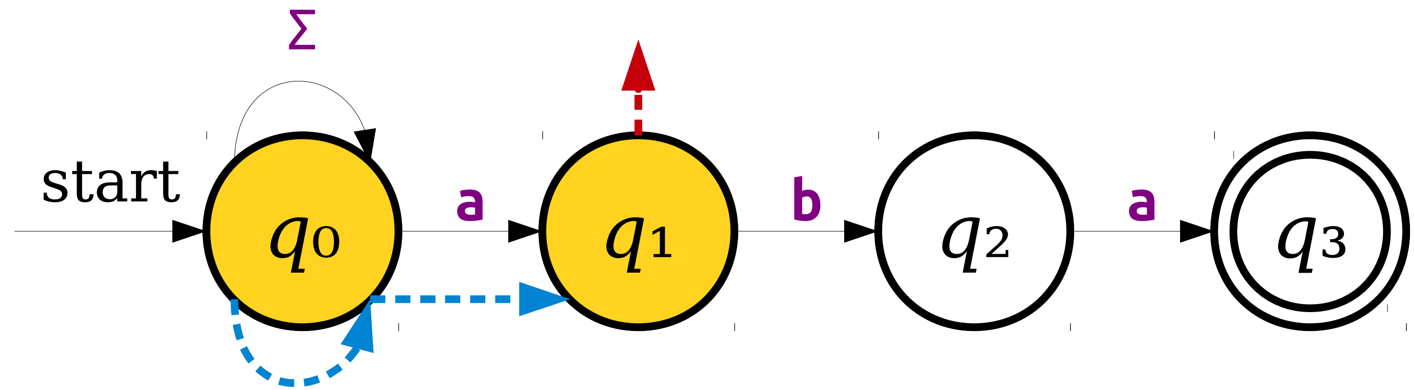
$\{q_0, q_1\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



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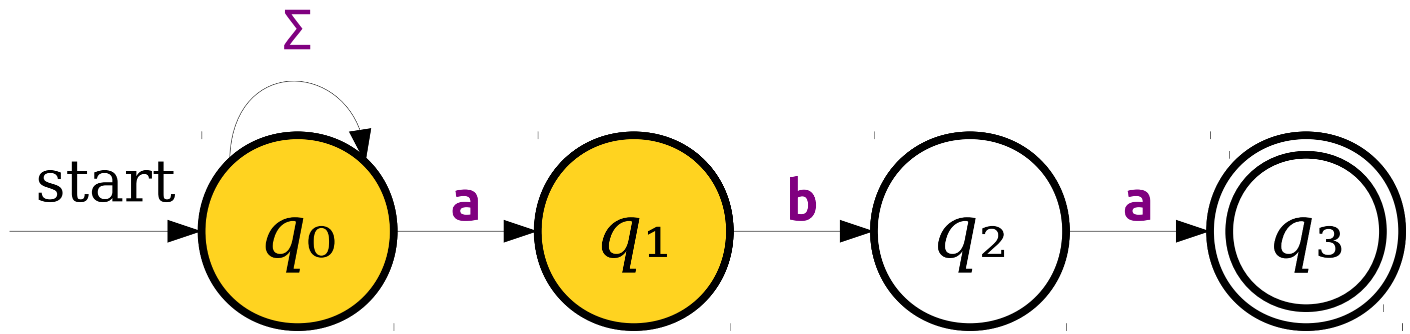
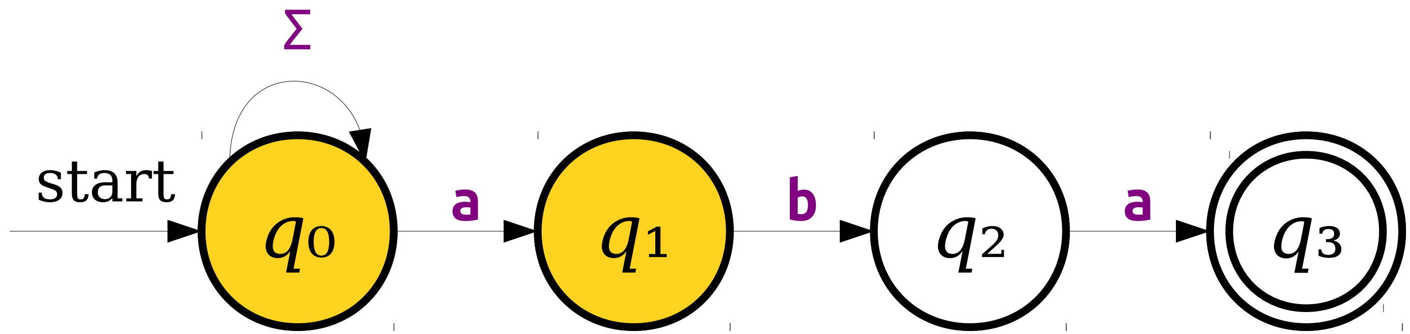
$\{q_0, q_1\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



	a	b
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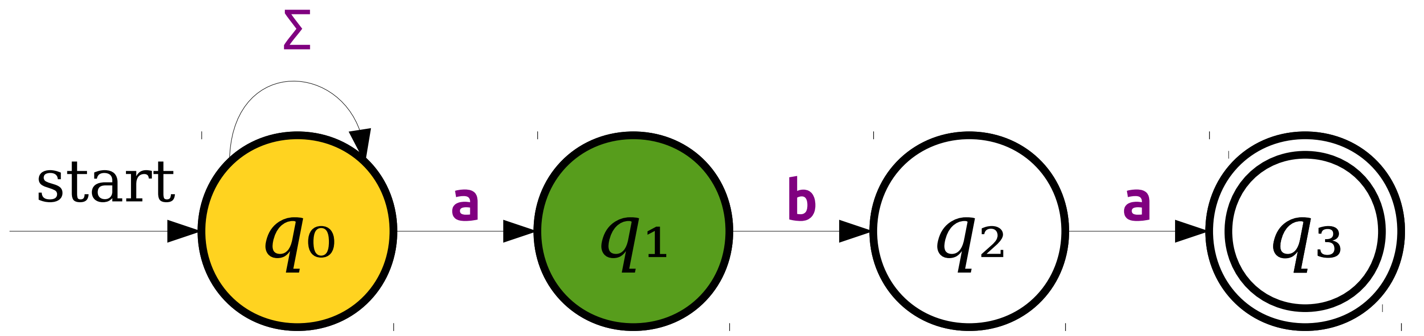
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



	a	b
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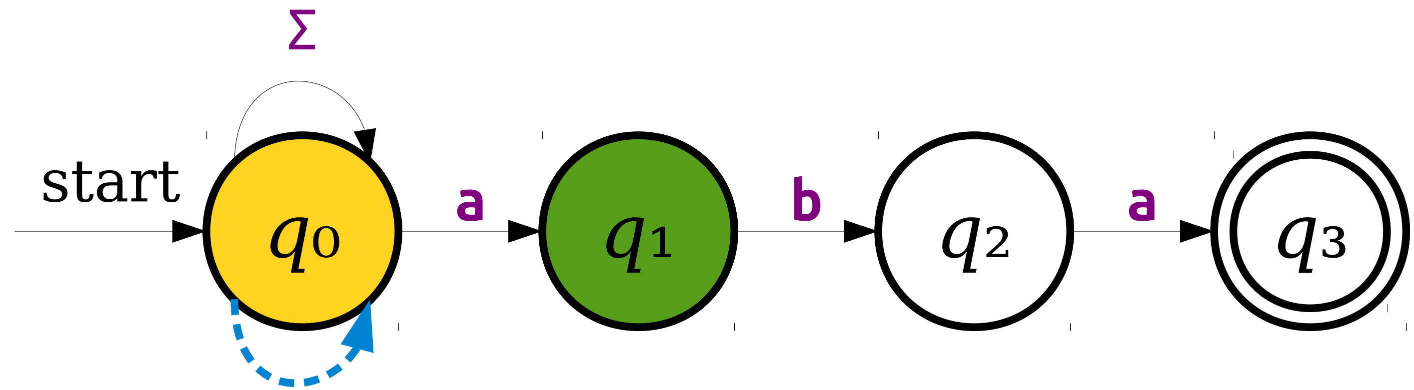
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



	a	b
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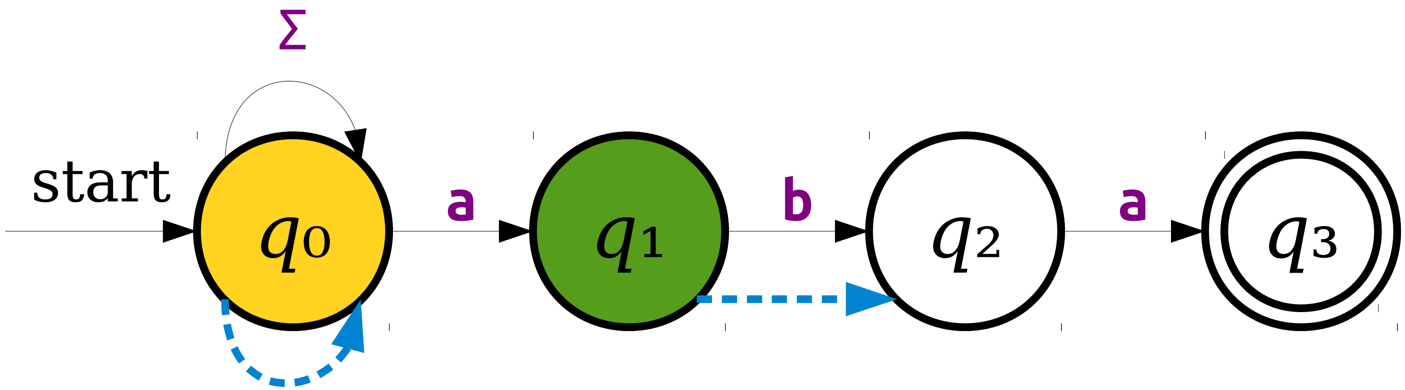

	a	b
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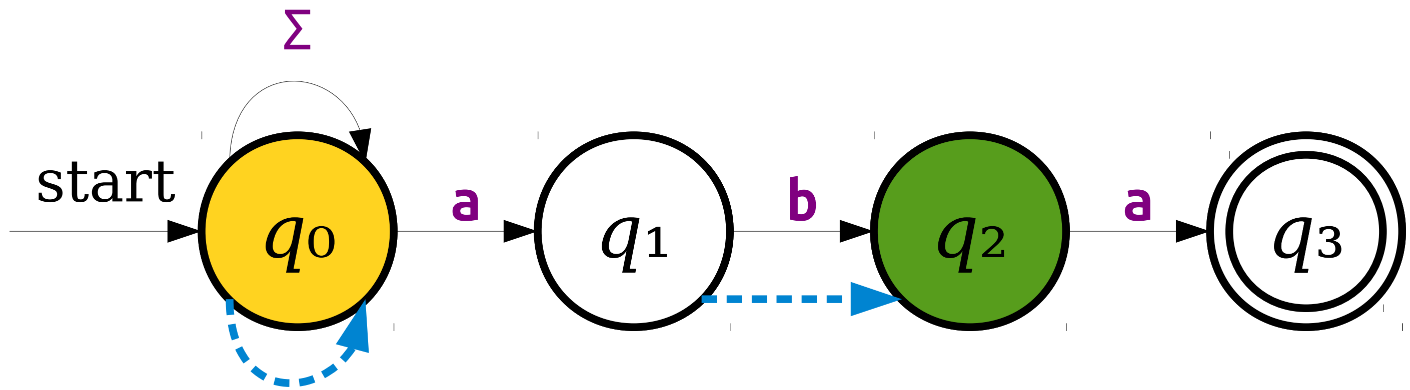

	a	b
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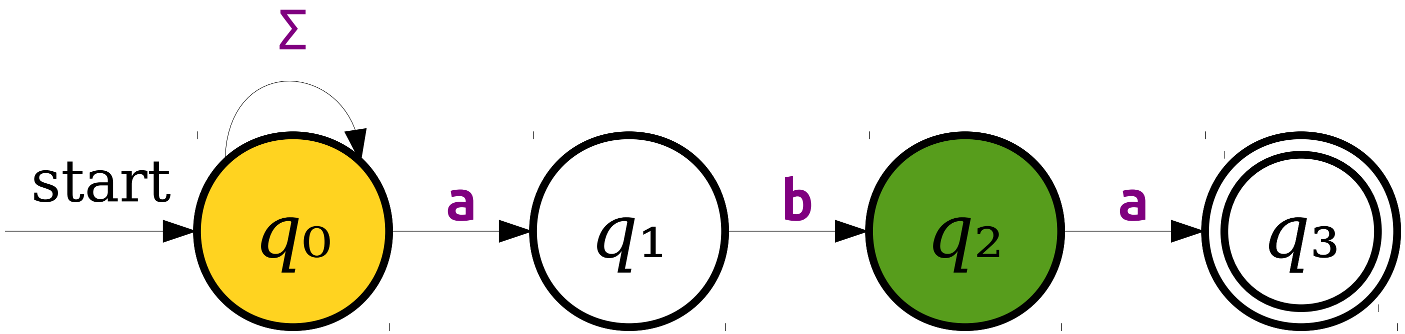

	a	b
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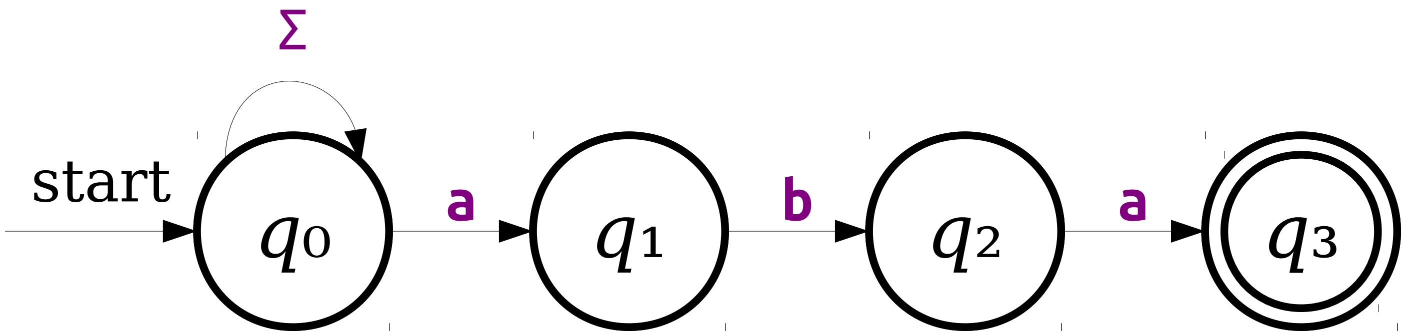

	a	b
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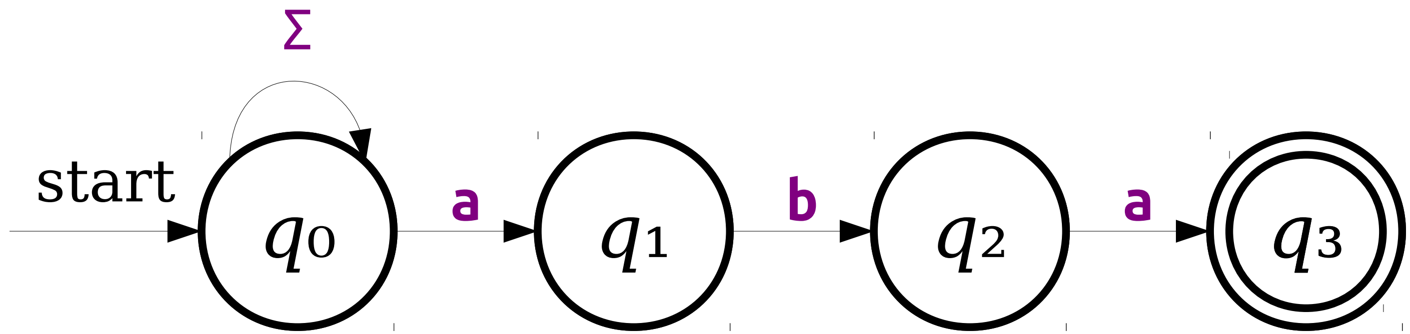

	a	b
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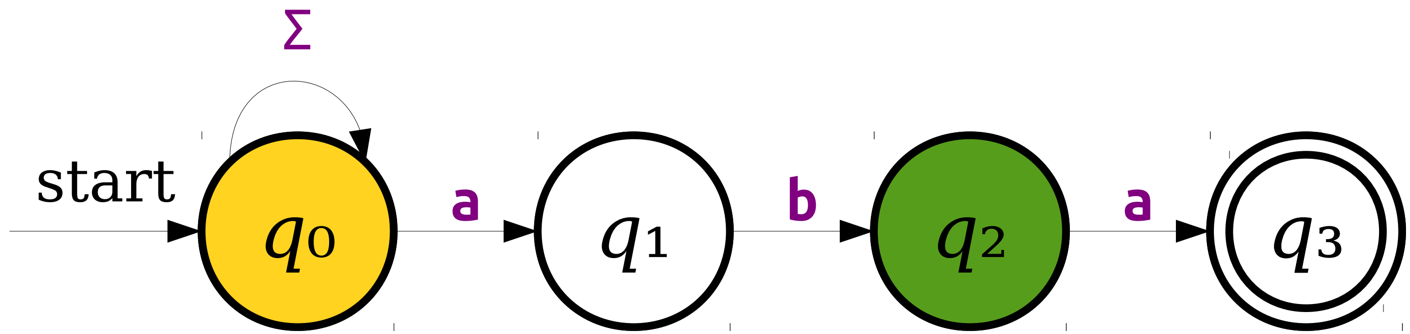

	a	b
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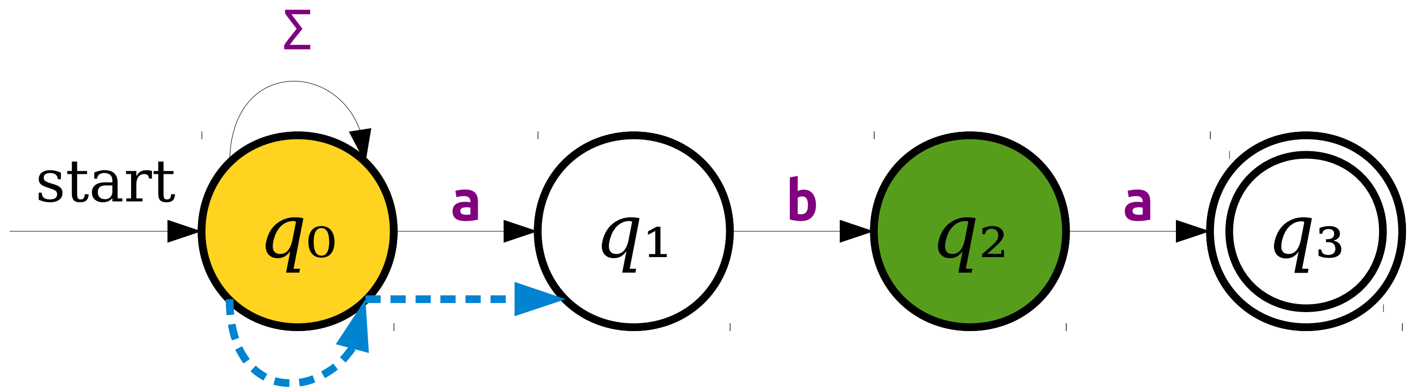

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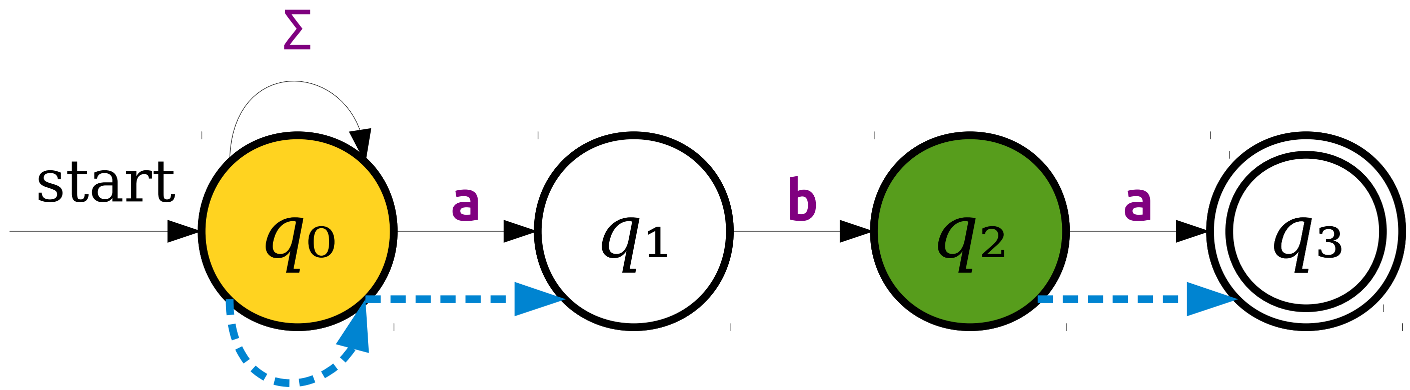

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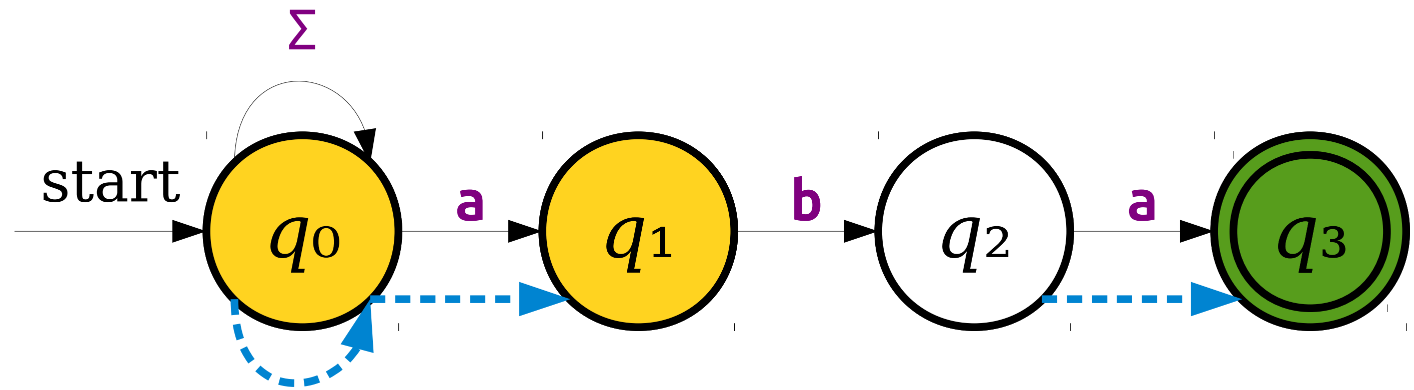

	a	b
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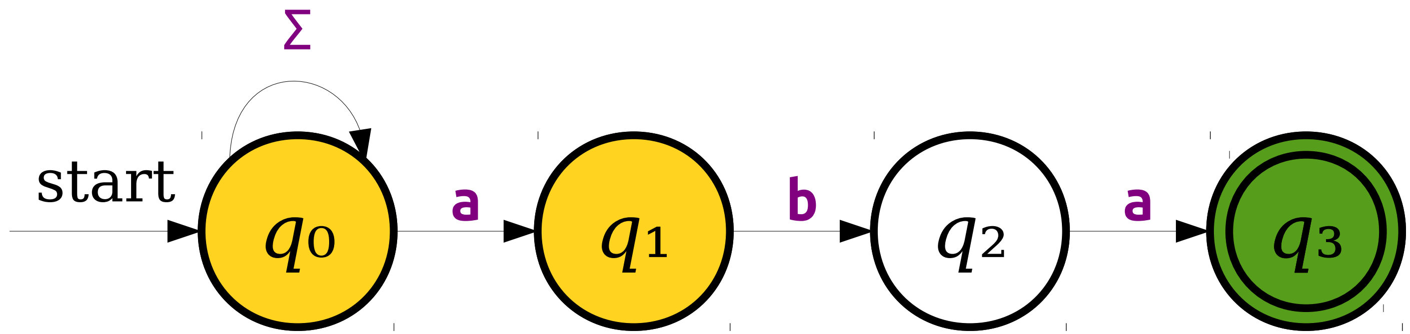

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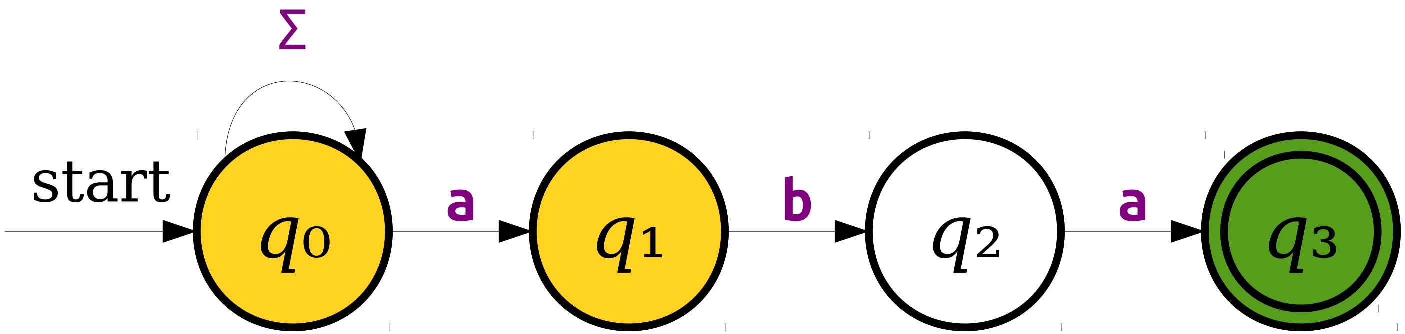

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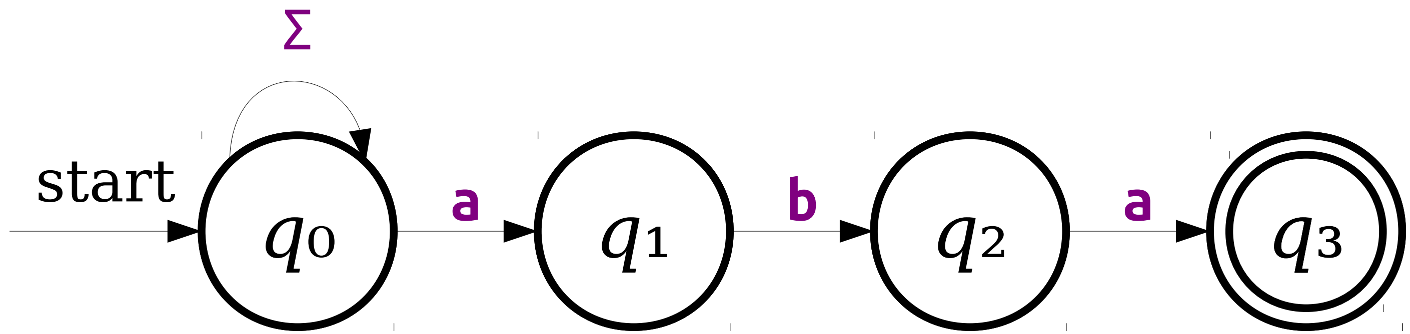

	a	b
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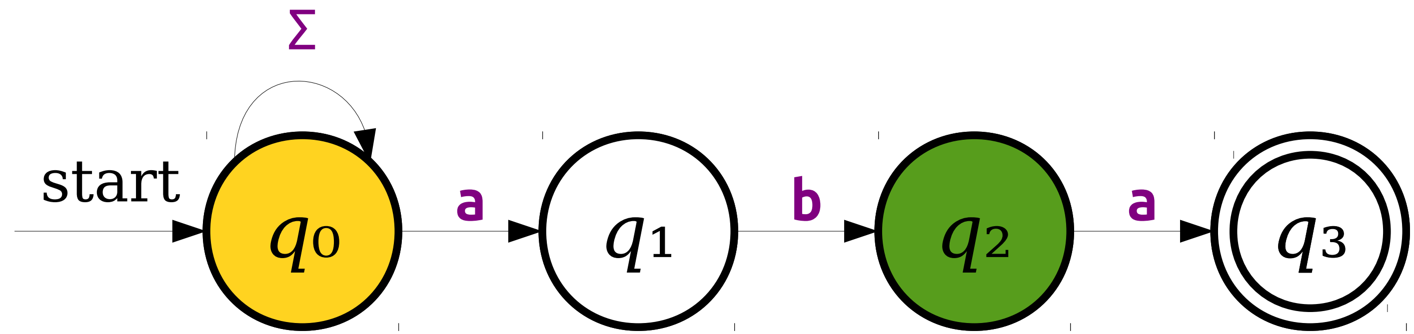

	a	b
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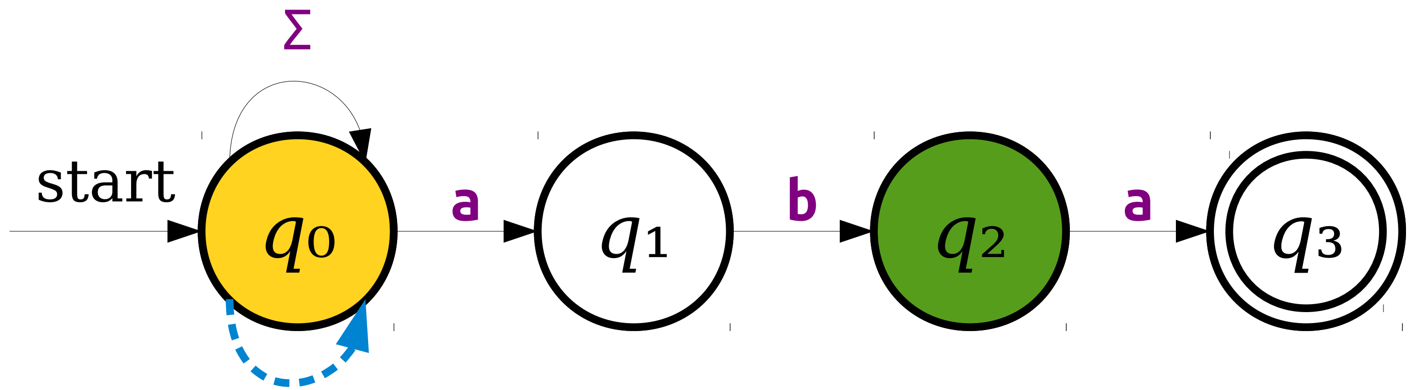

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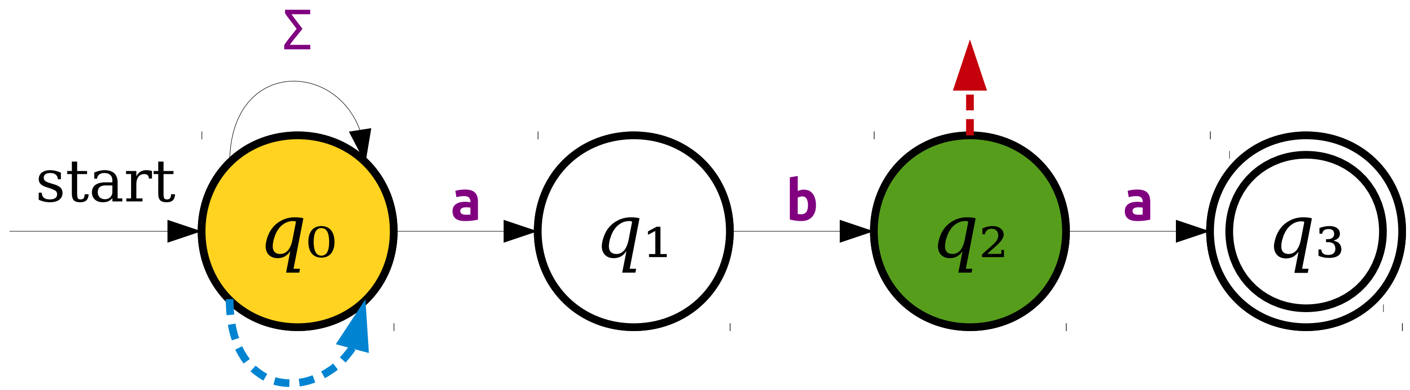

	a	b
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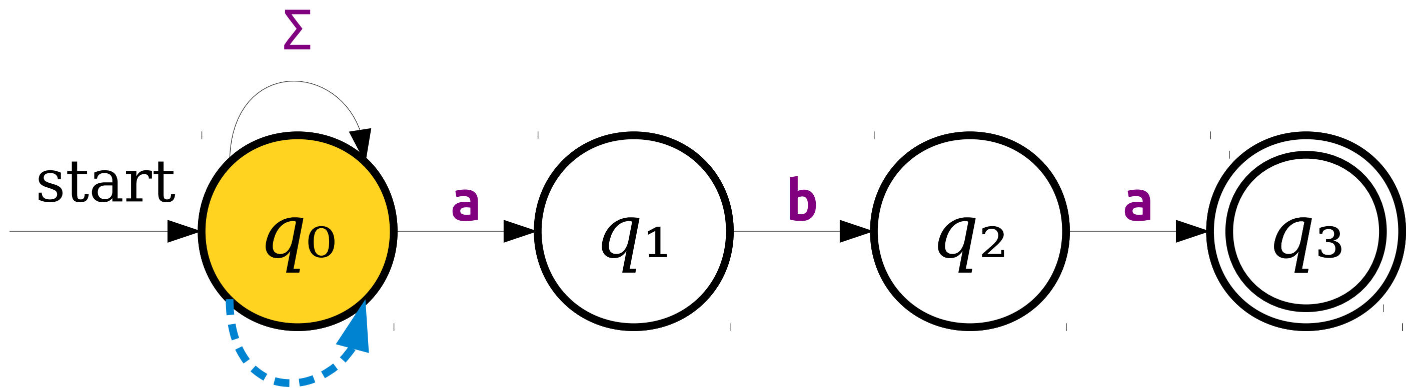

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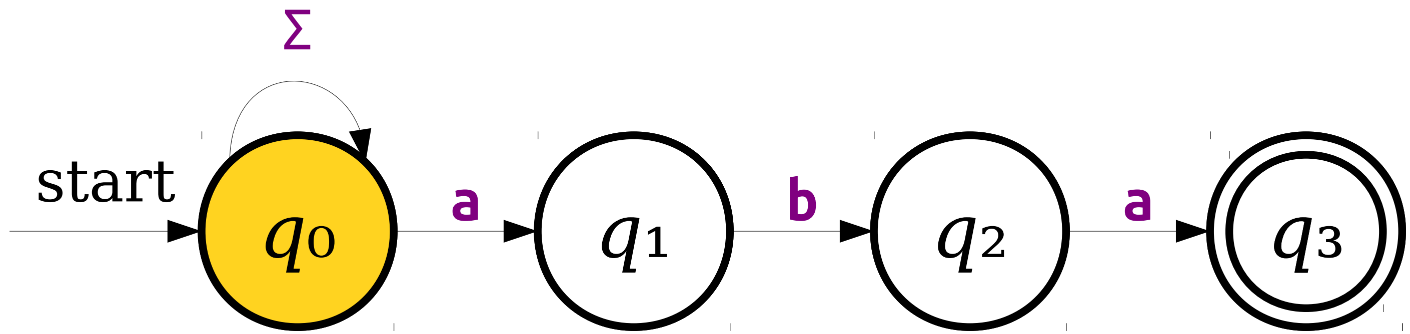

	a	b
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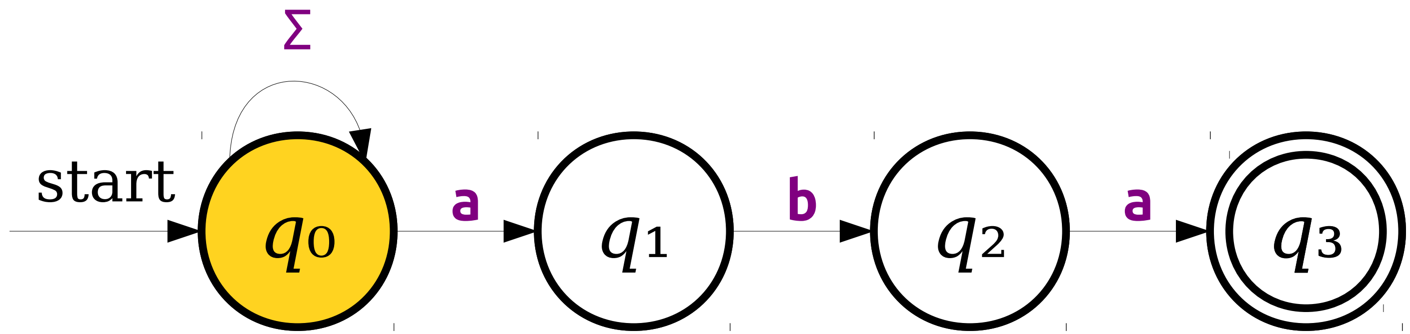

	a	b
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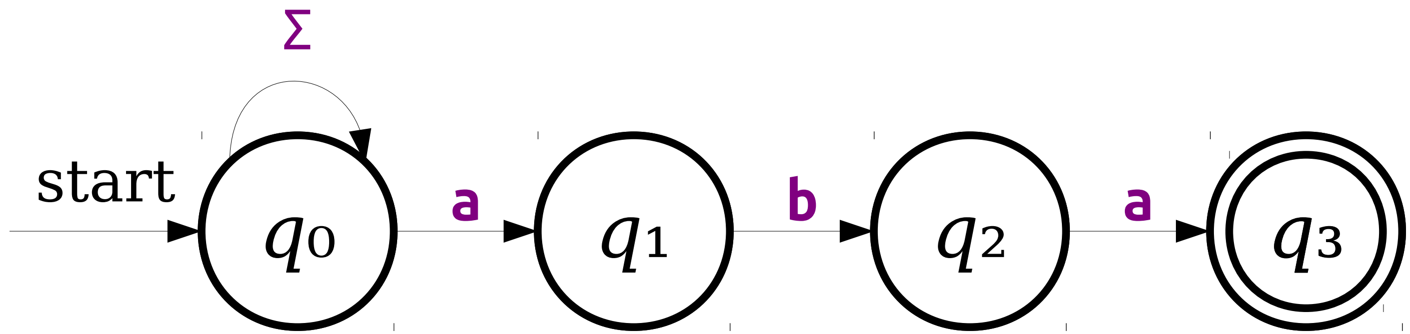

	a	b
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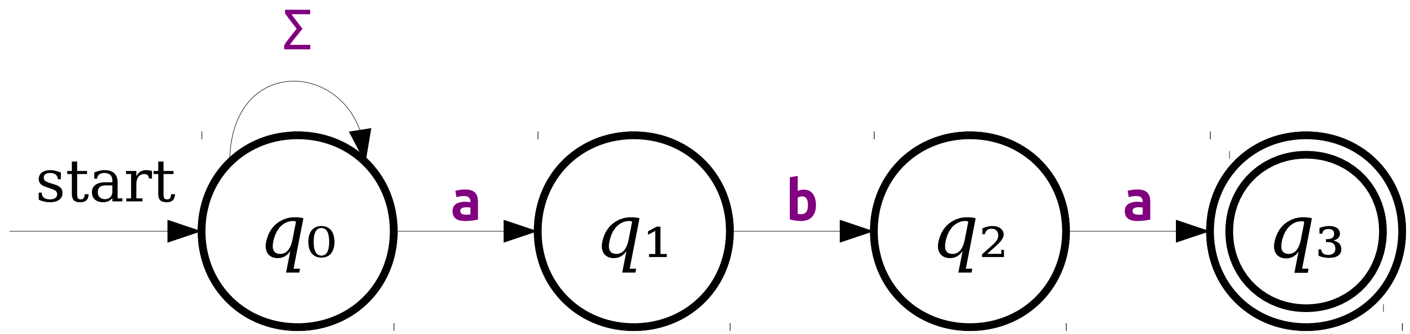

	a	b
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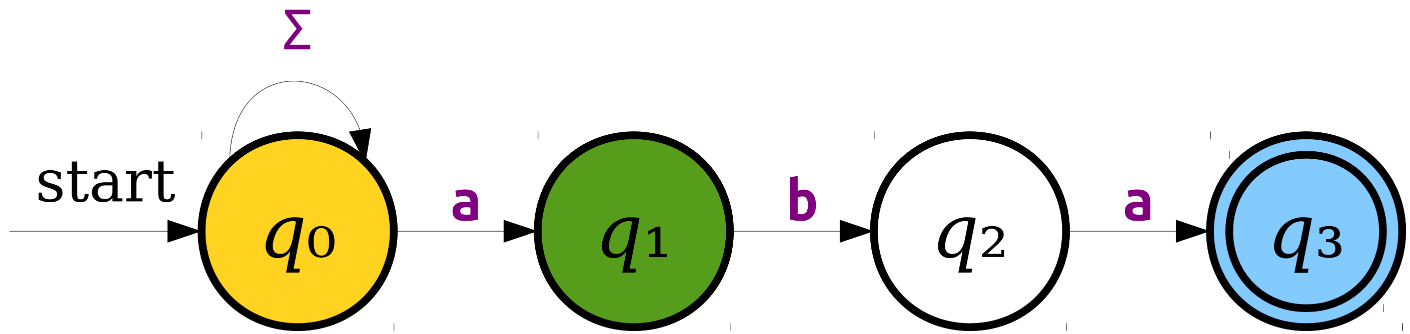

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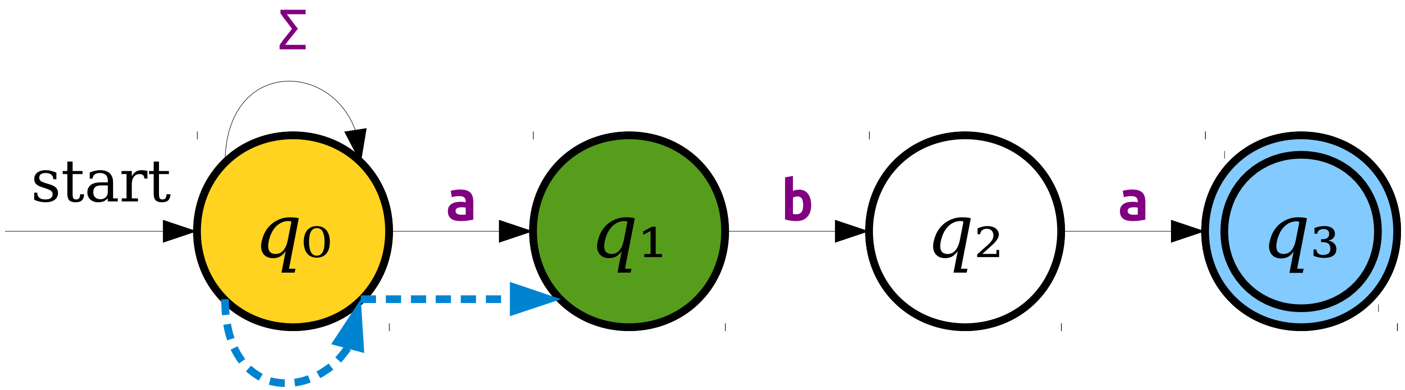

	a	b
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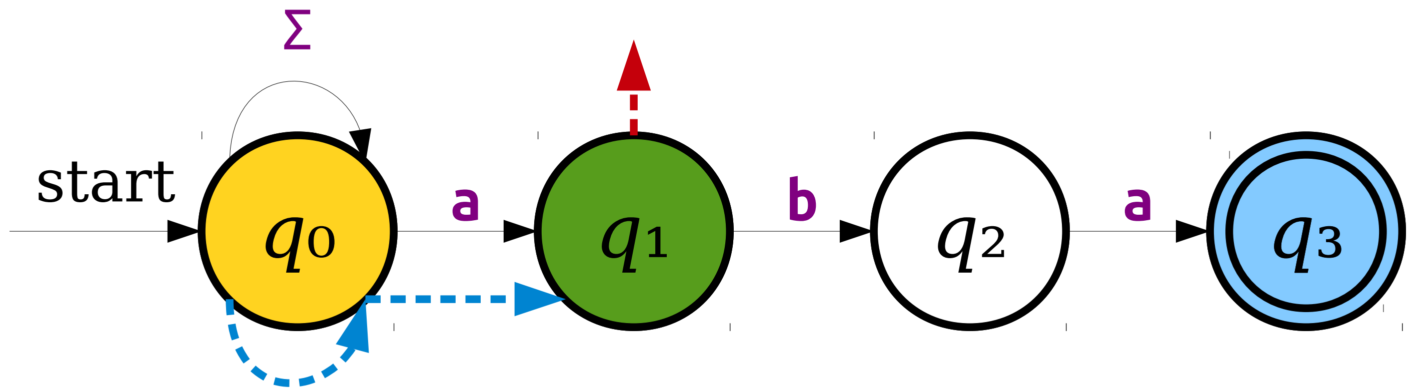
$\{q_0, q_1, q_3\}$		



	a	b
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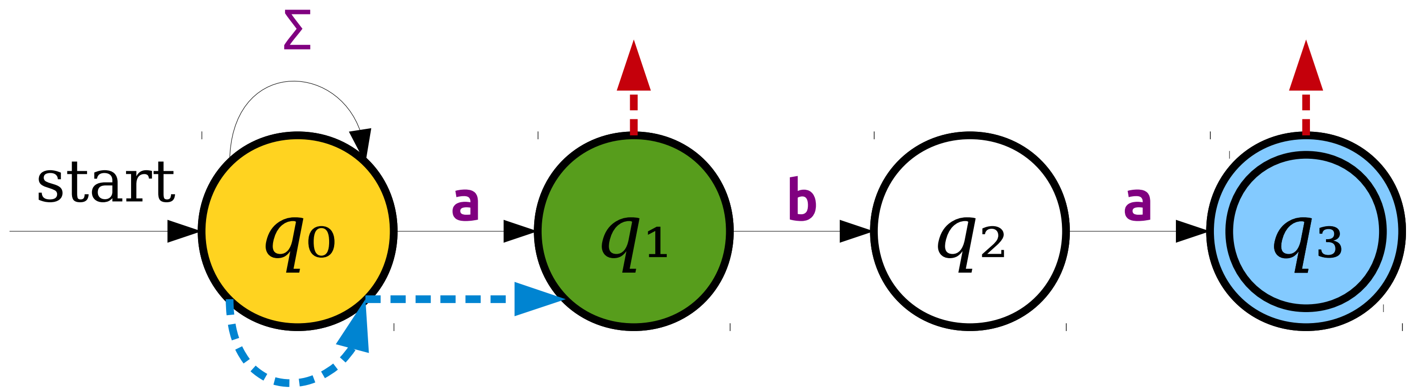
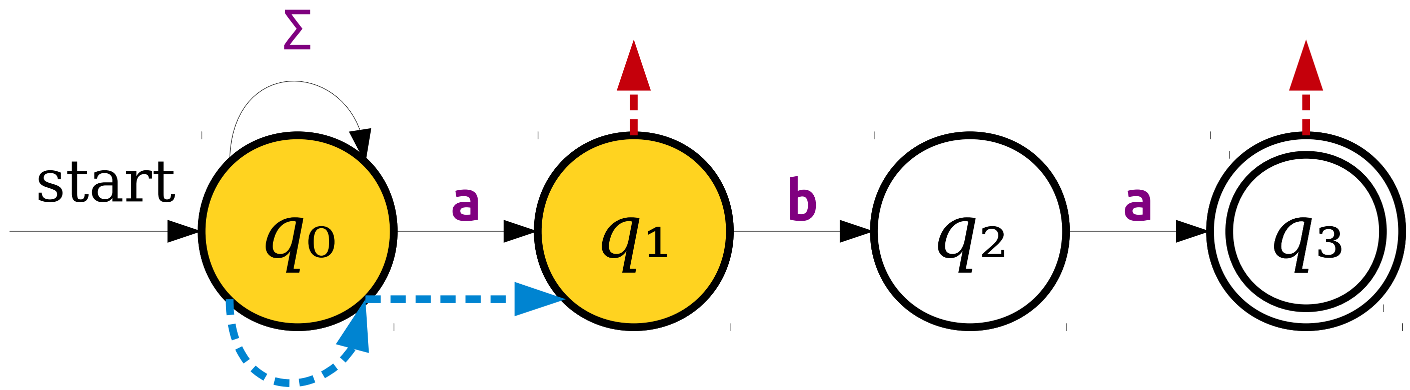
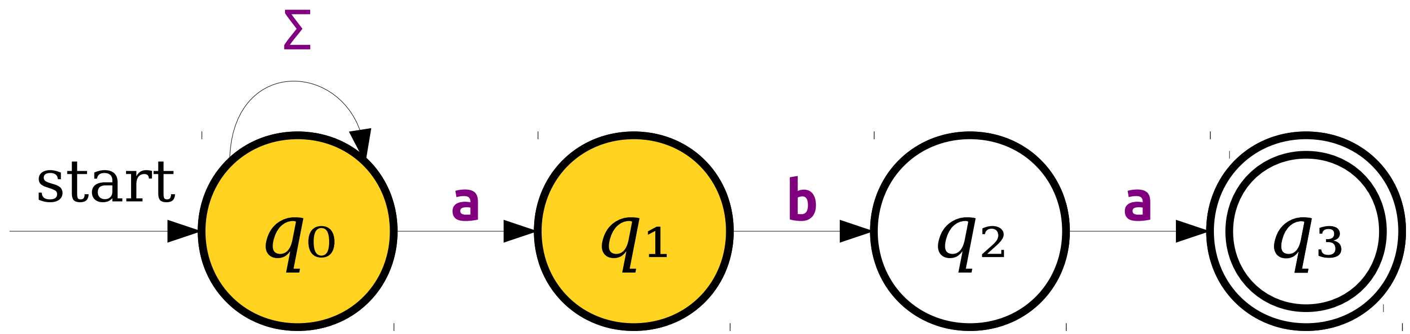


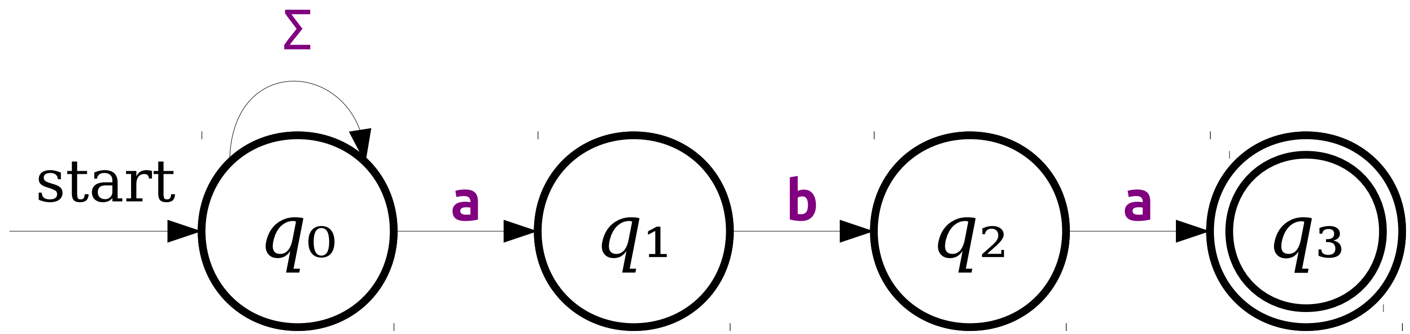
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



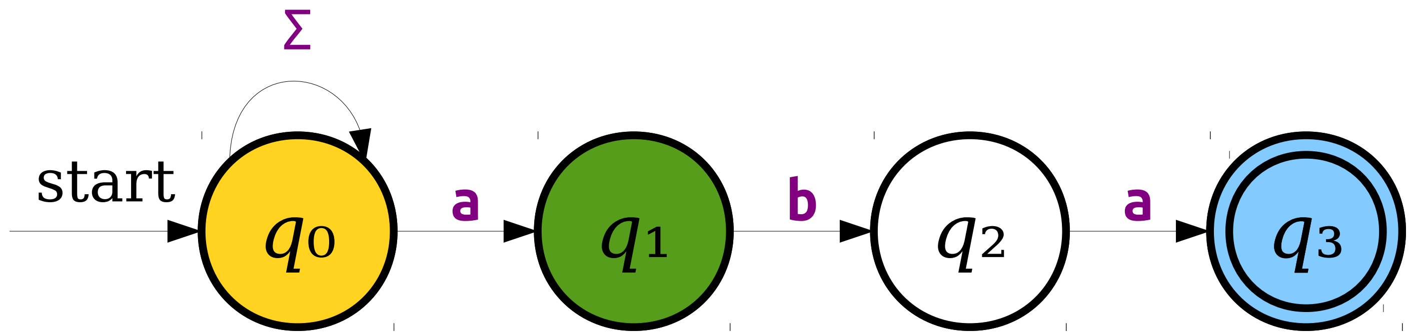
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



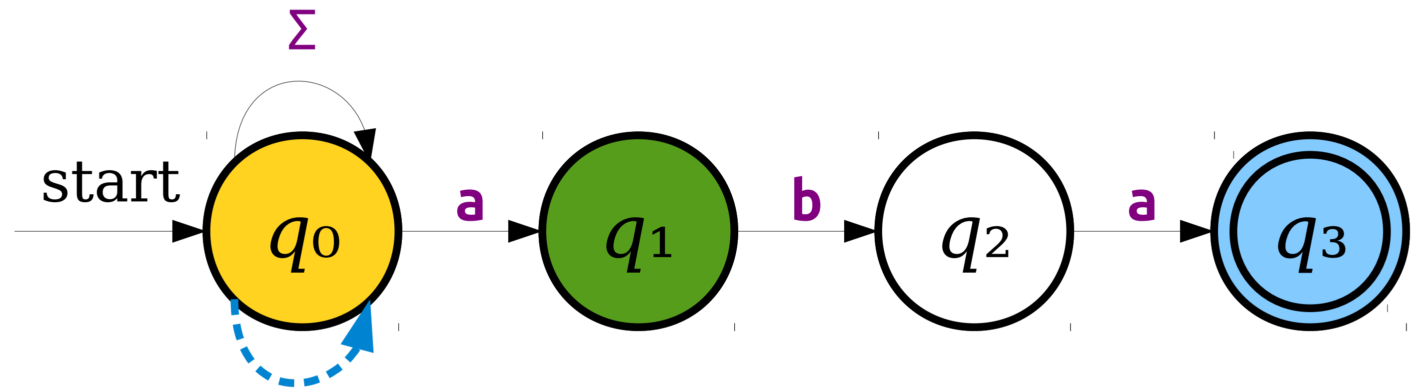
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



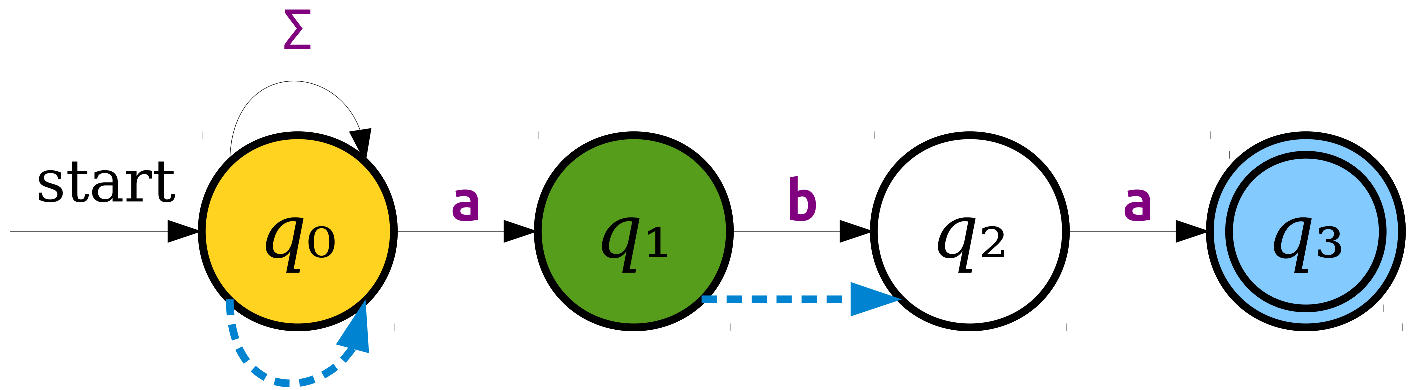
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



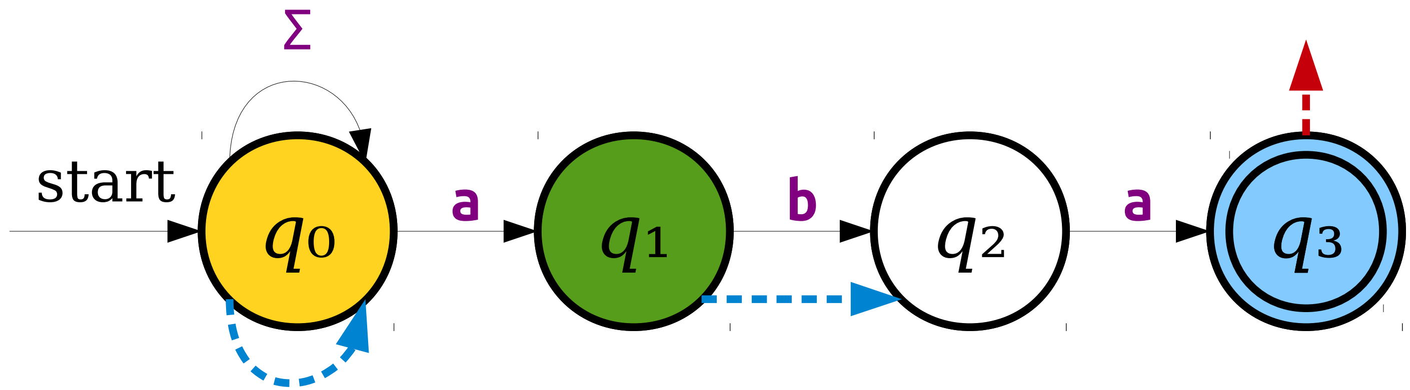
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



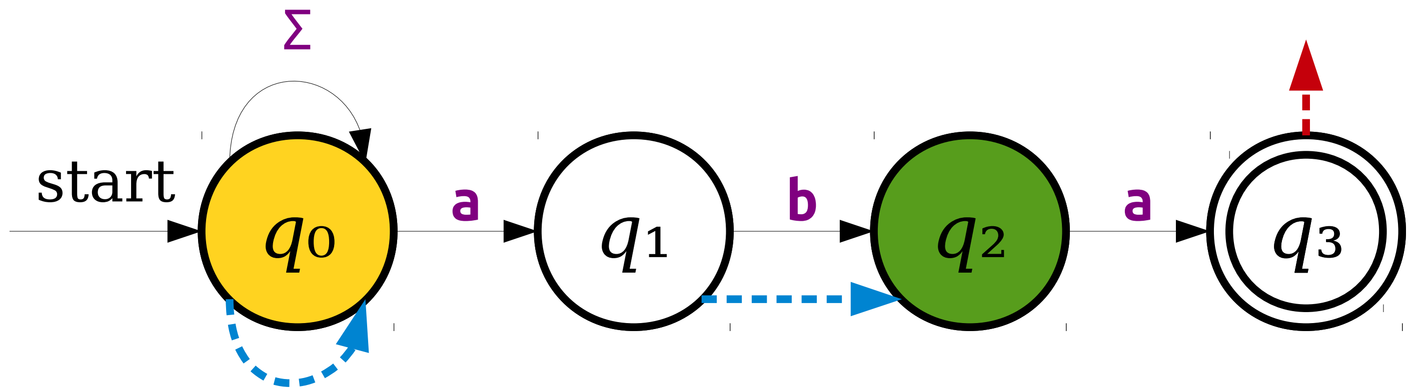
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



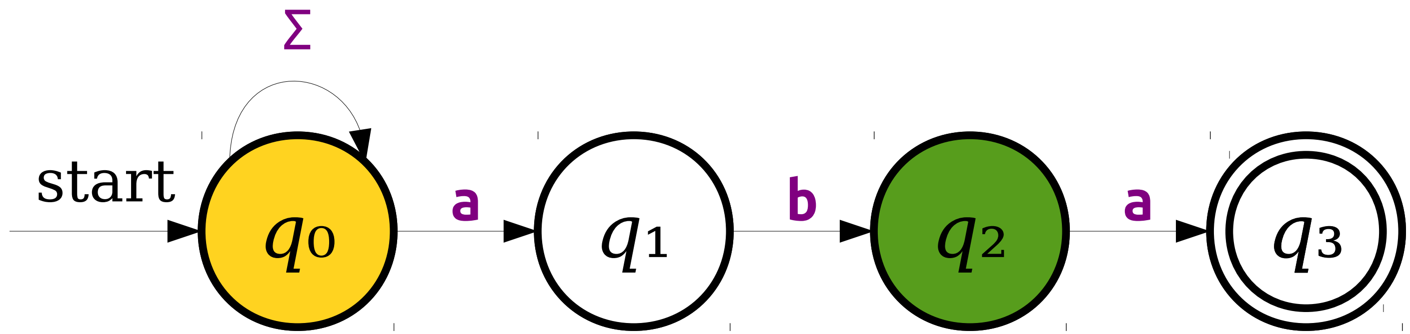
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



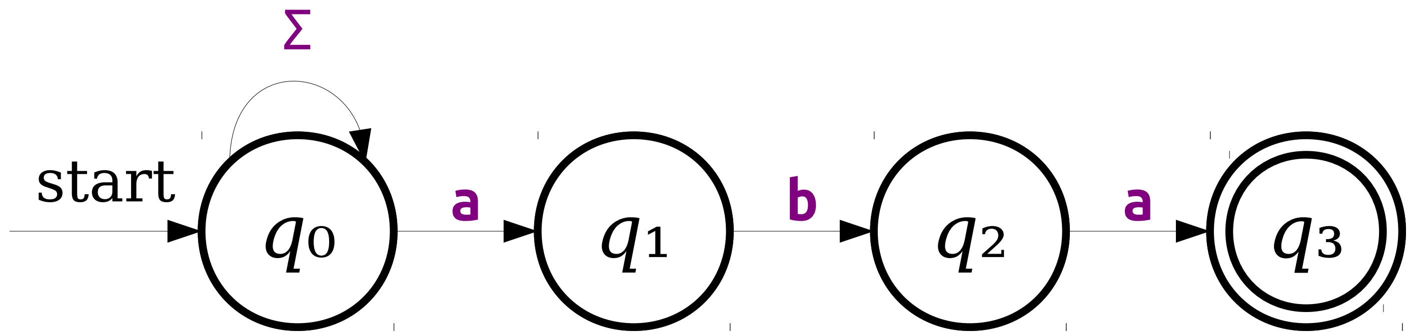
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



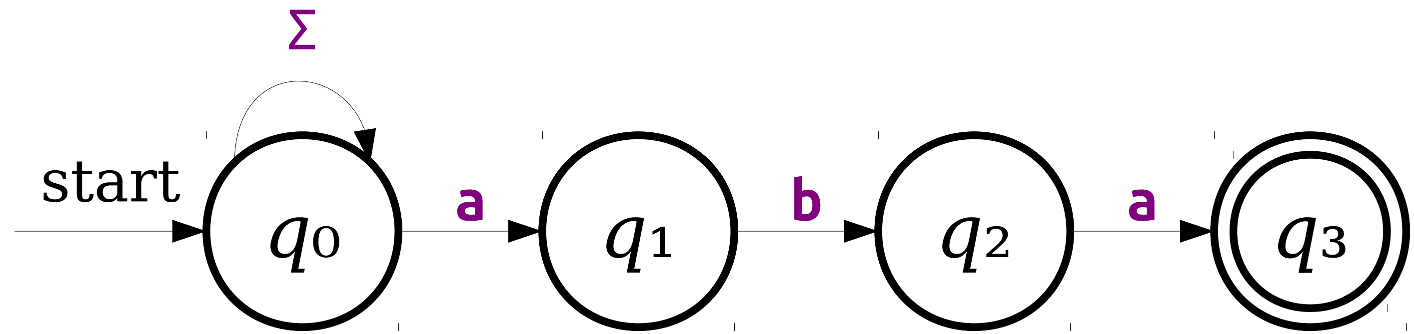
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



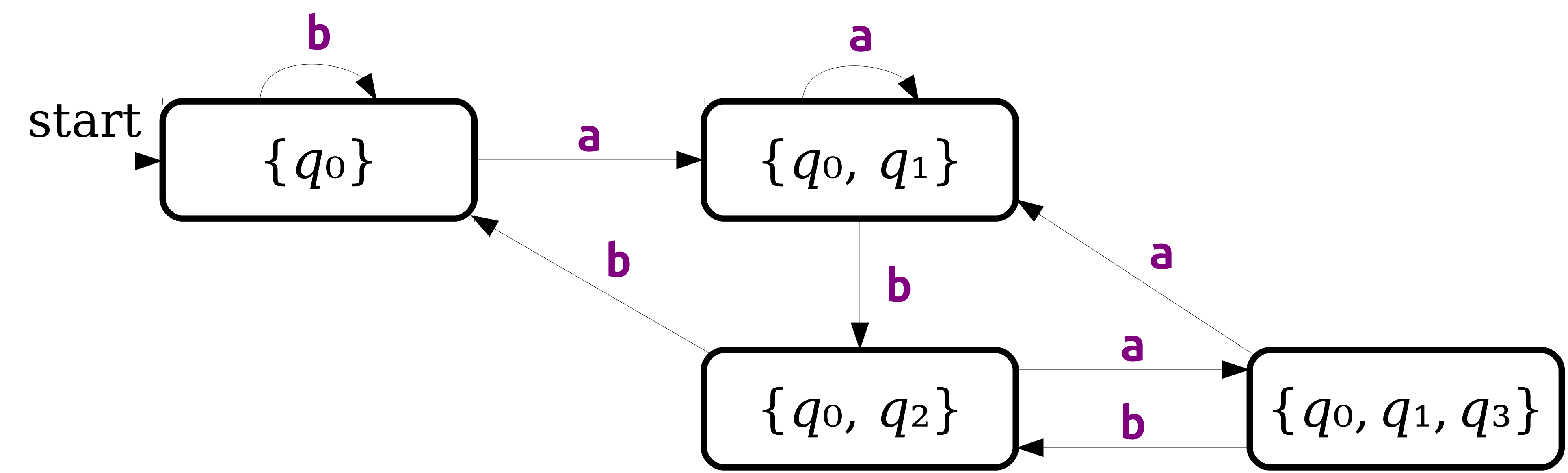
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

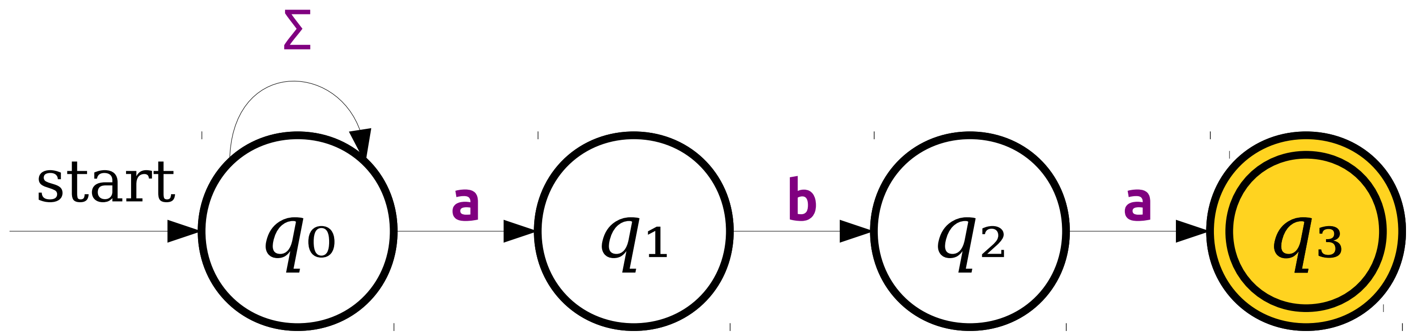


	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

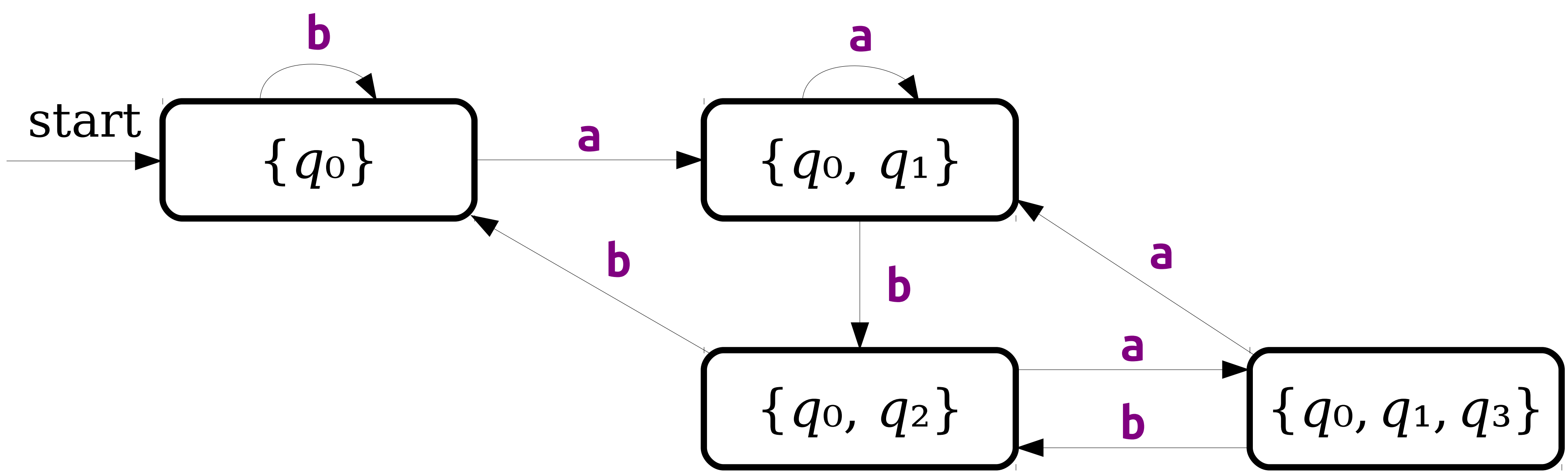


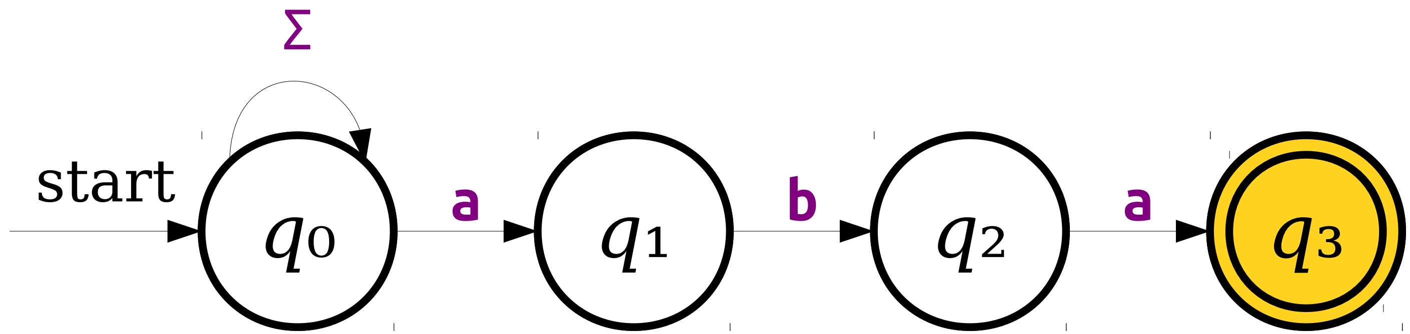
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



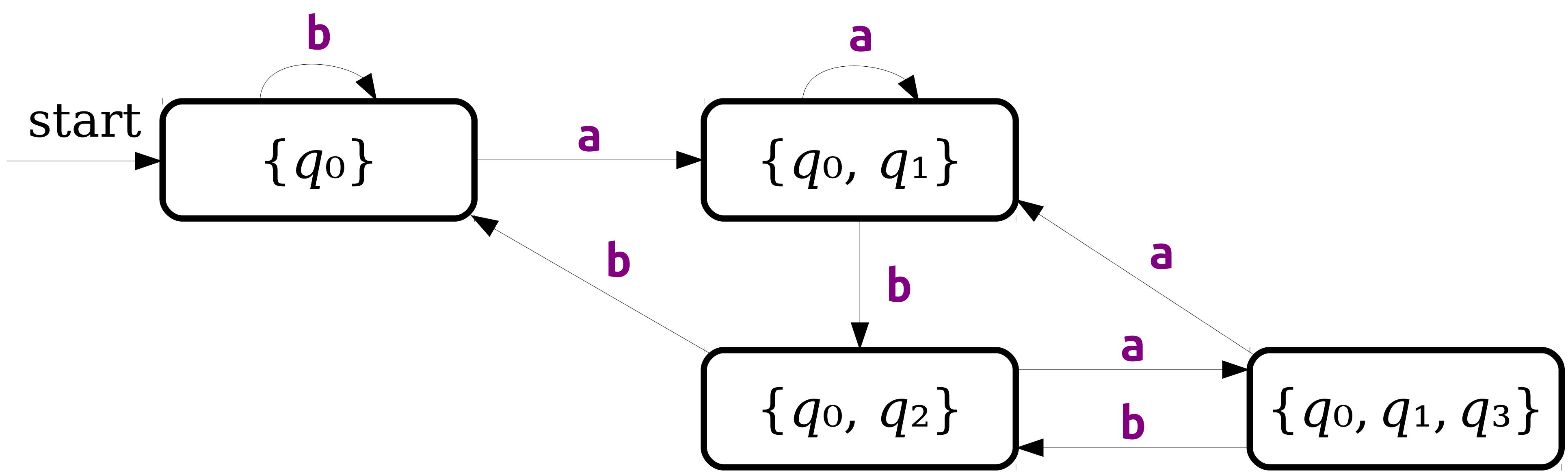


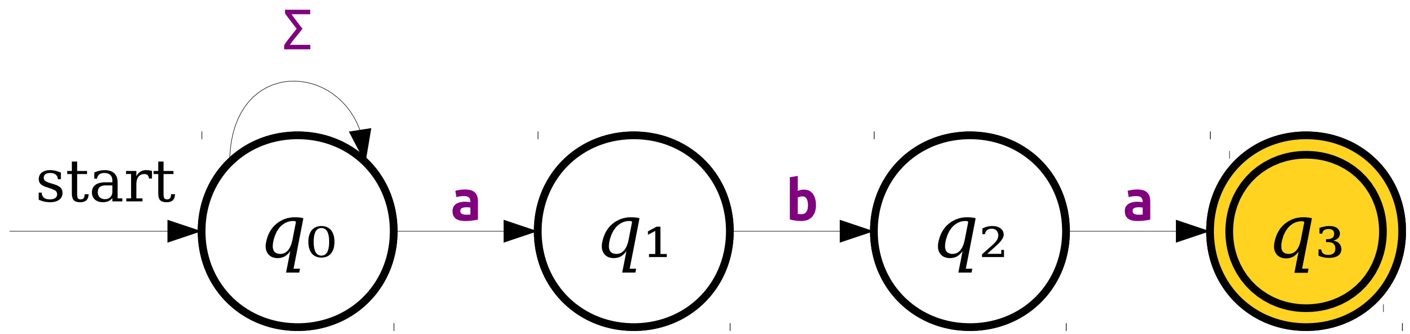
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



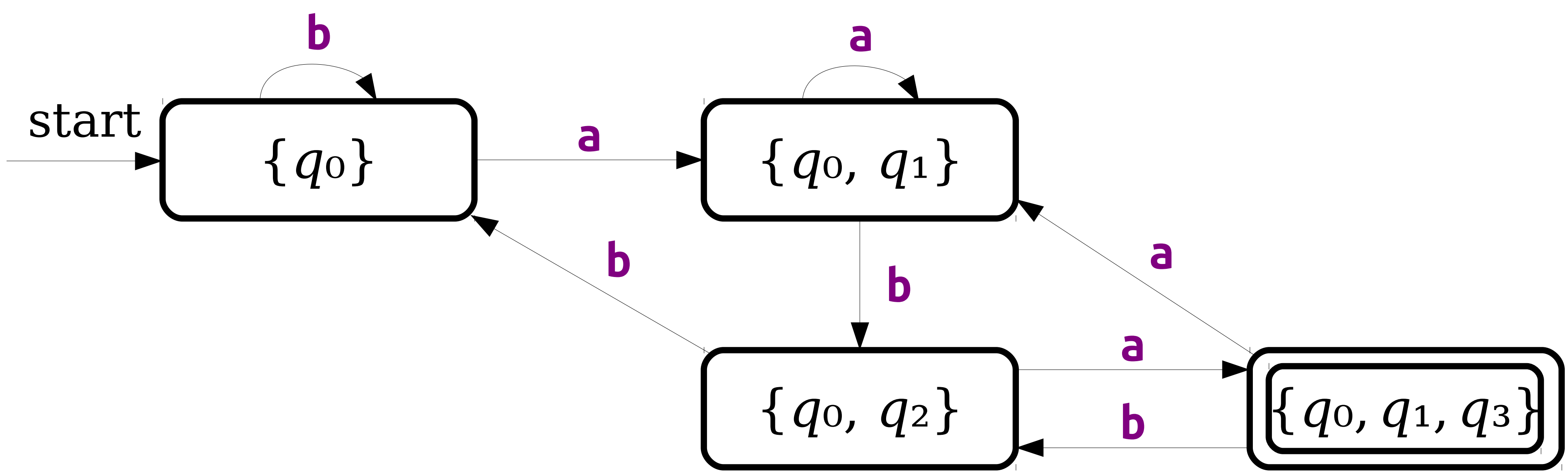


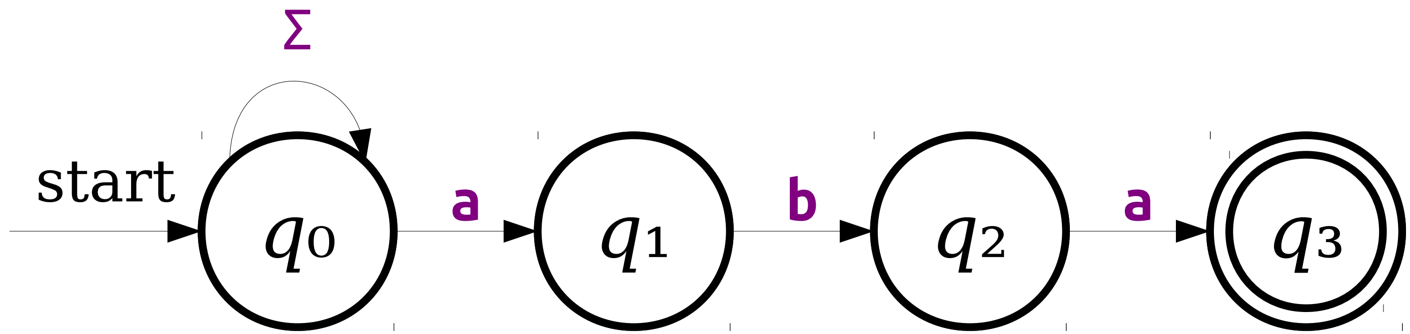
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



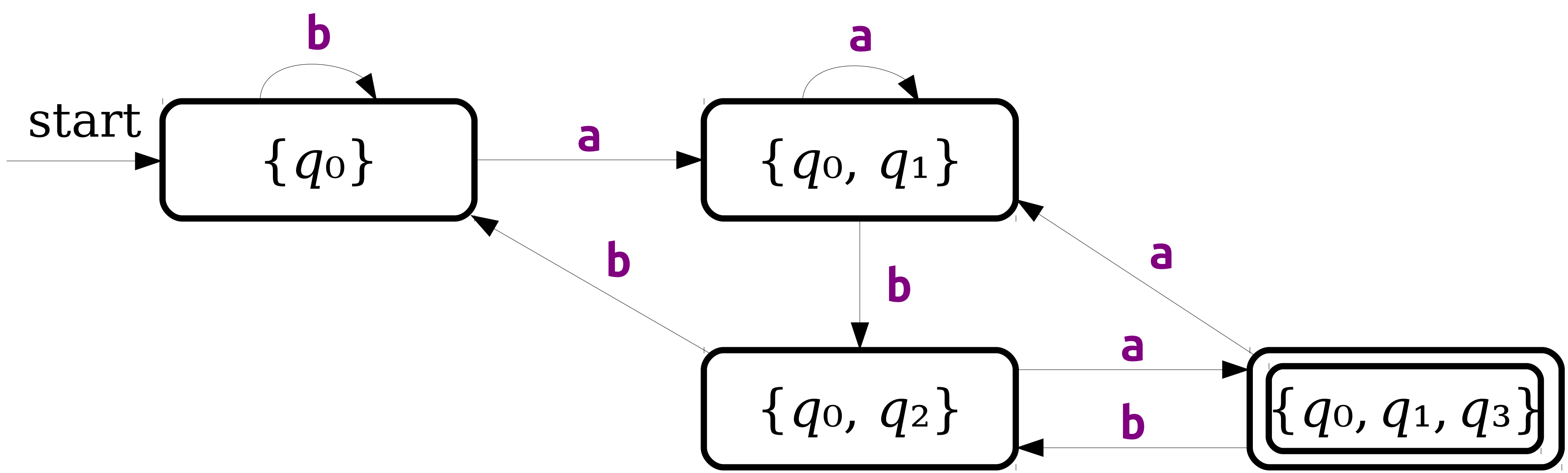


	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

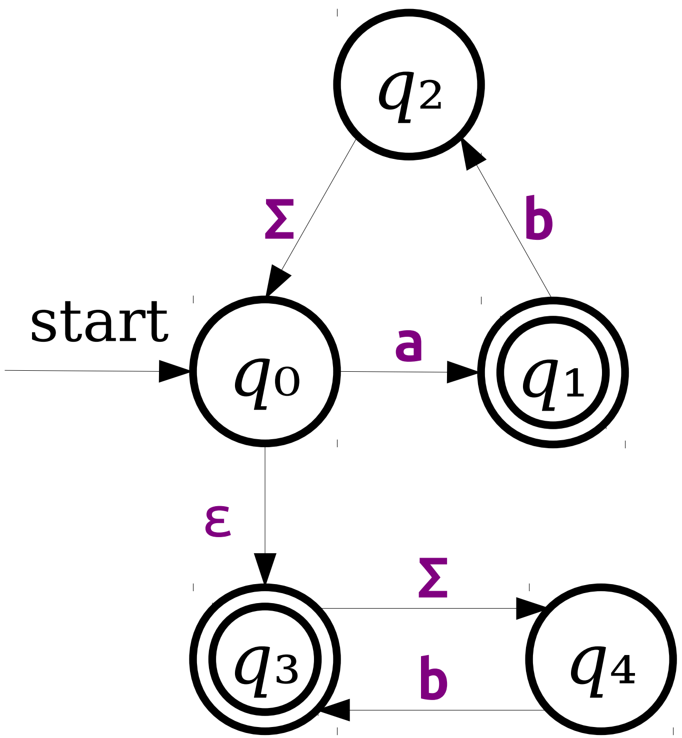




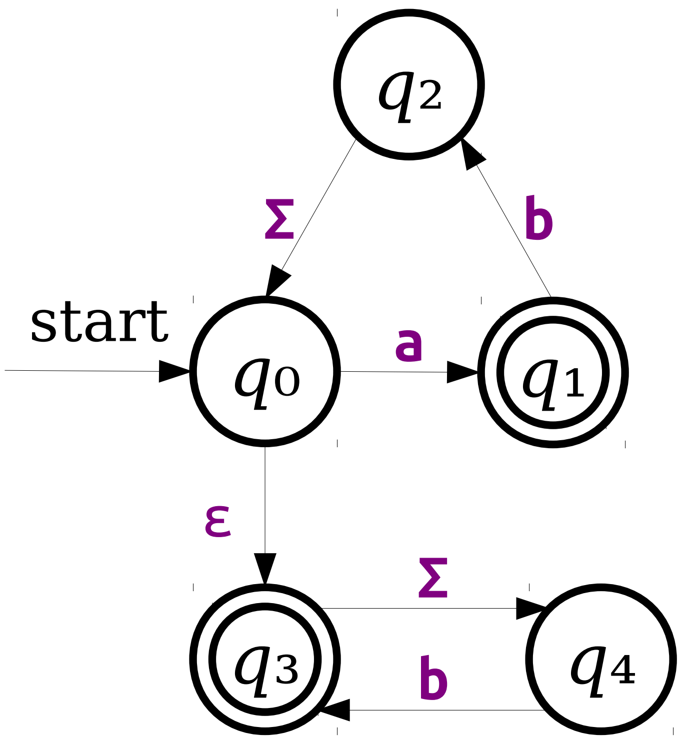
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



Once More, With Epsilons!

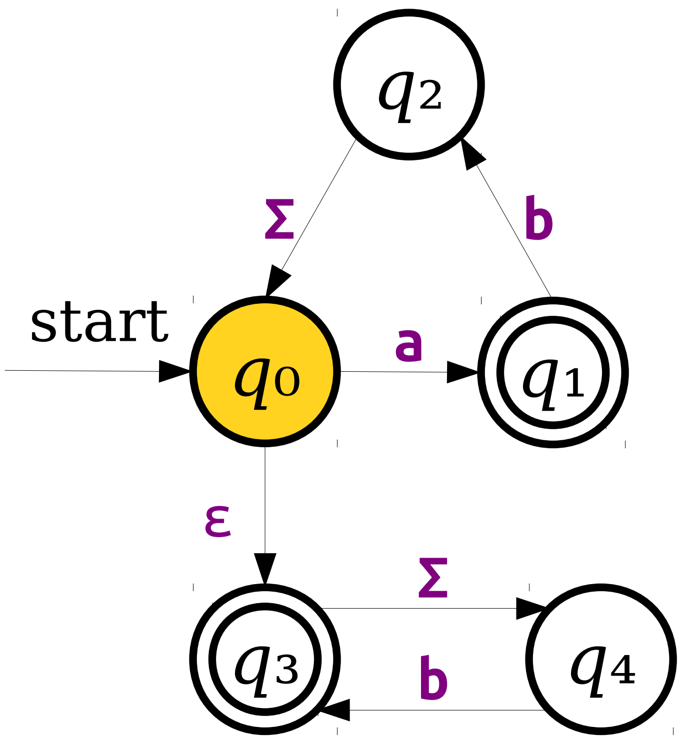


Once More, With Epsilons!



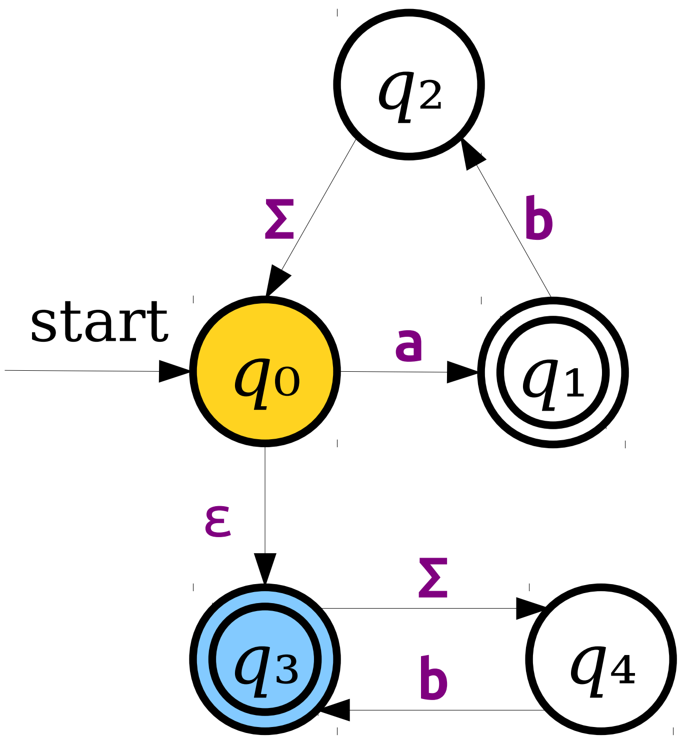
	a	b

Once More, With Epsilons!



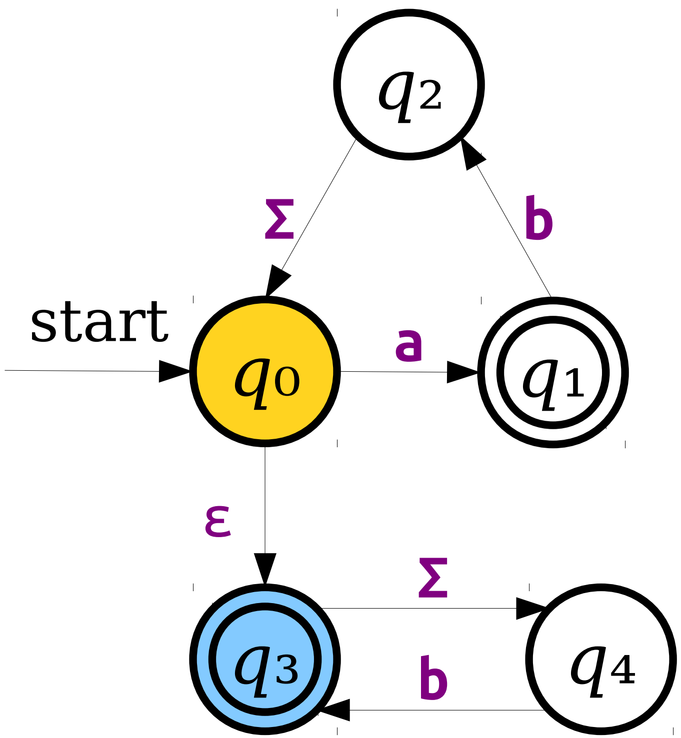
	a	b

Once More, With Epsilons!



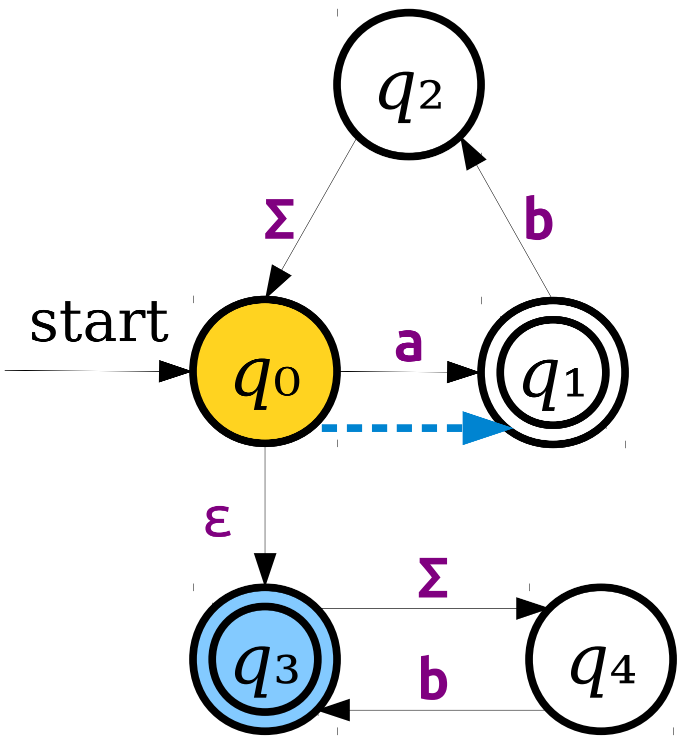
	a	b

Once More, With Epsilons!



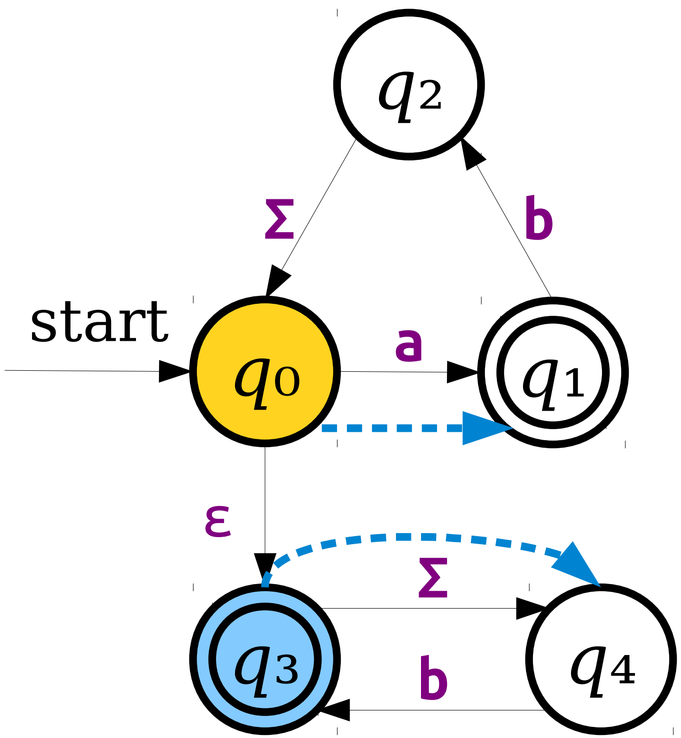
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



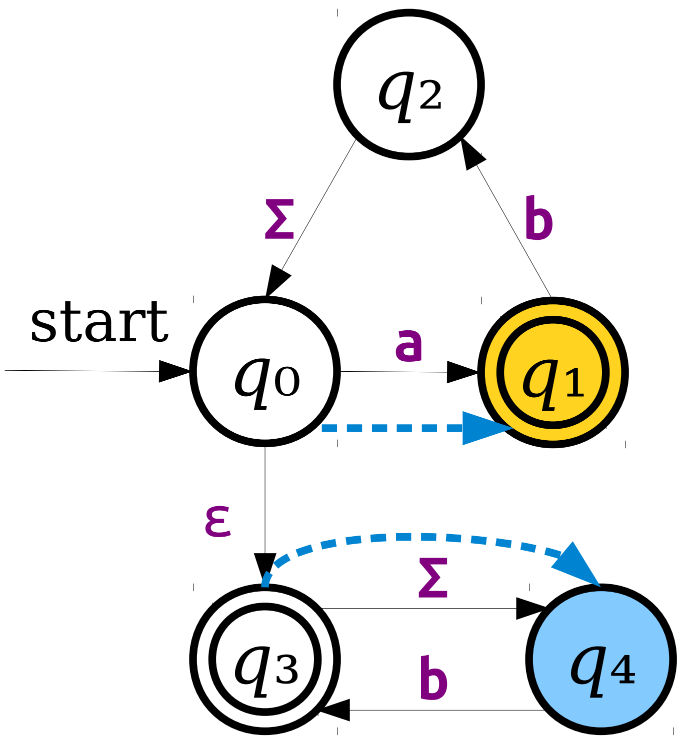
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



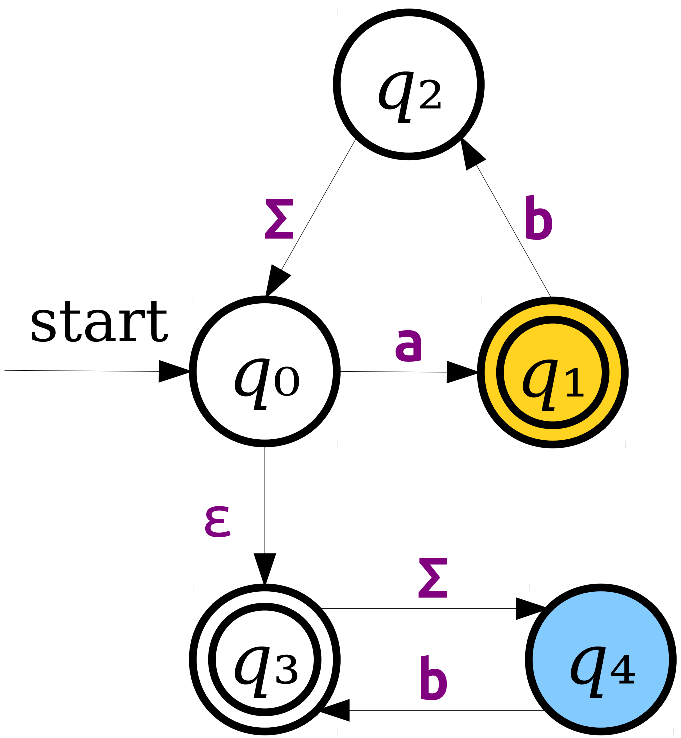
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



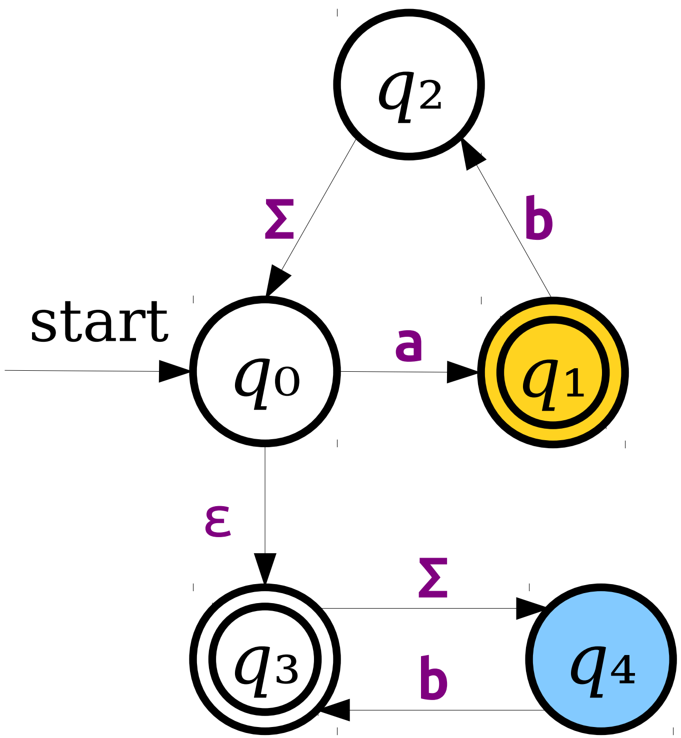
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



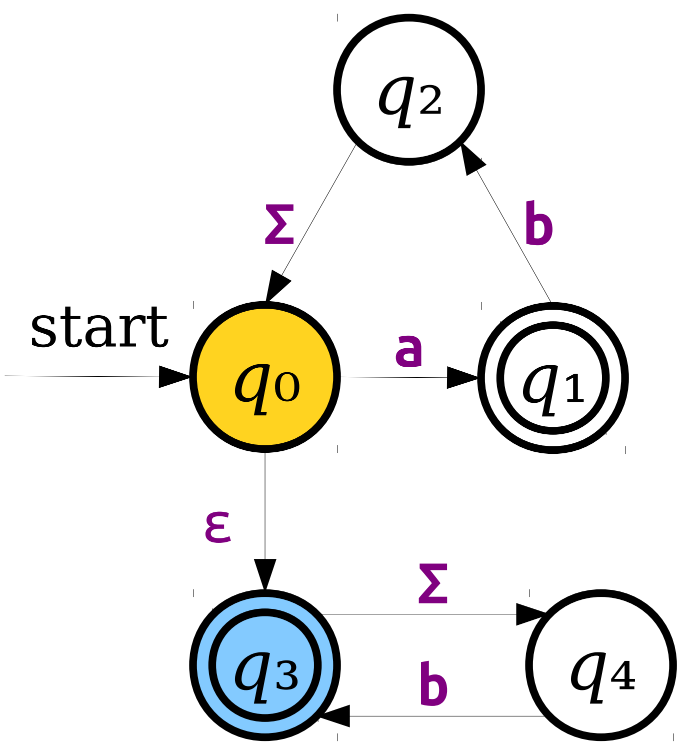
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



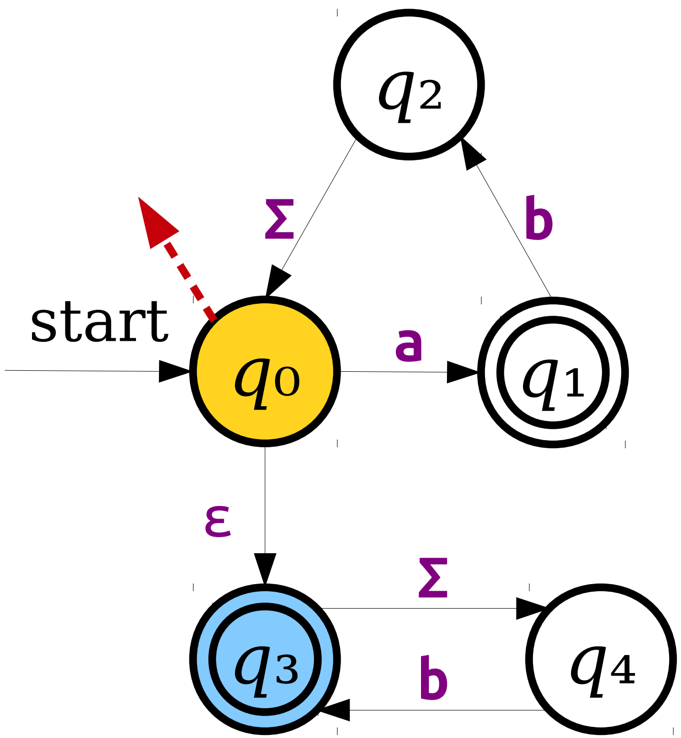
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



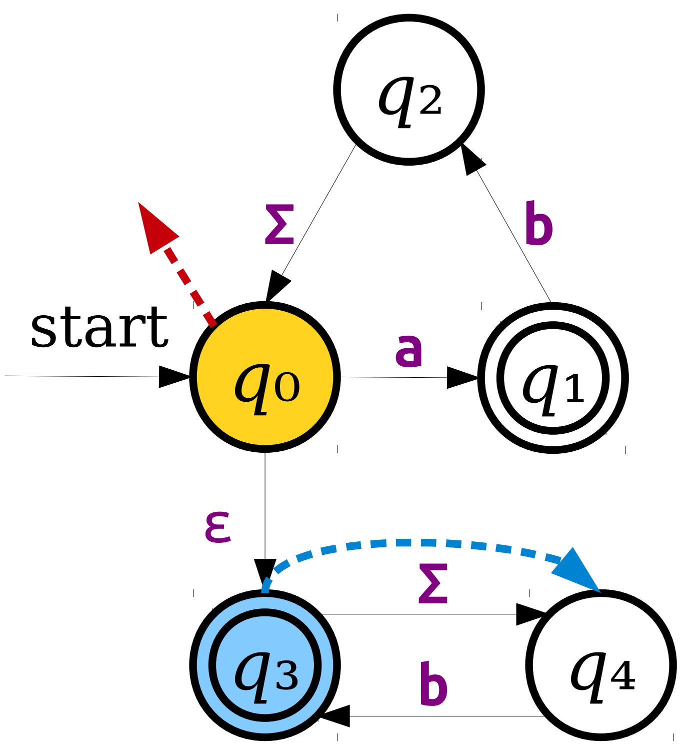
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



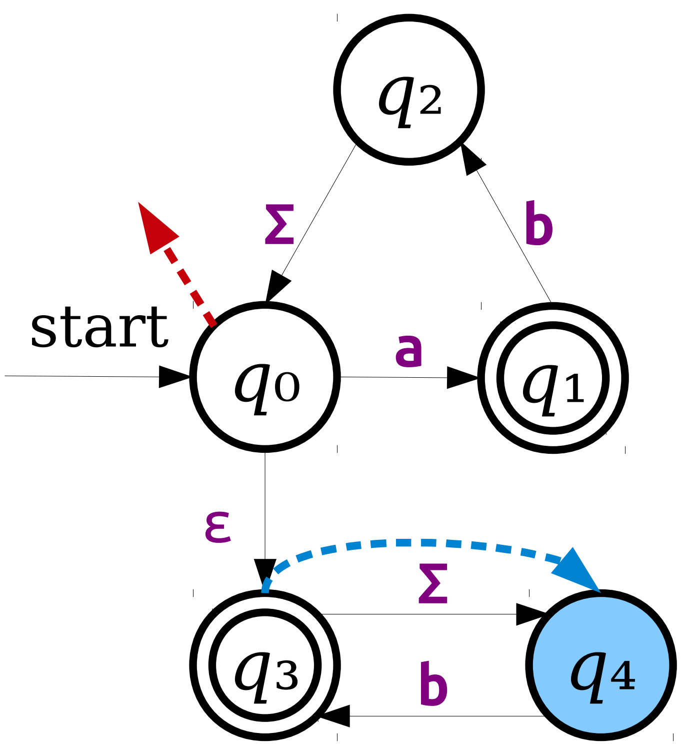
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



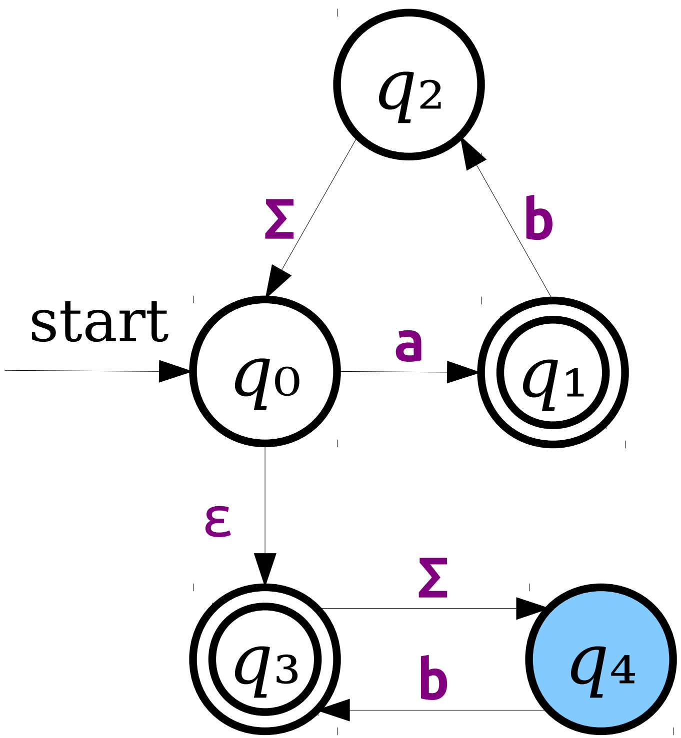
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



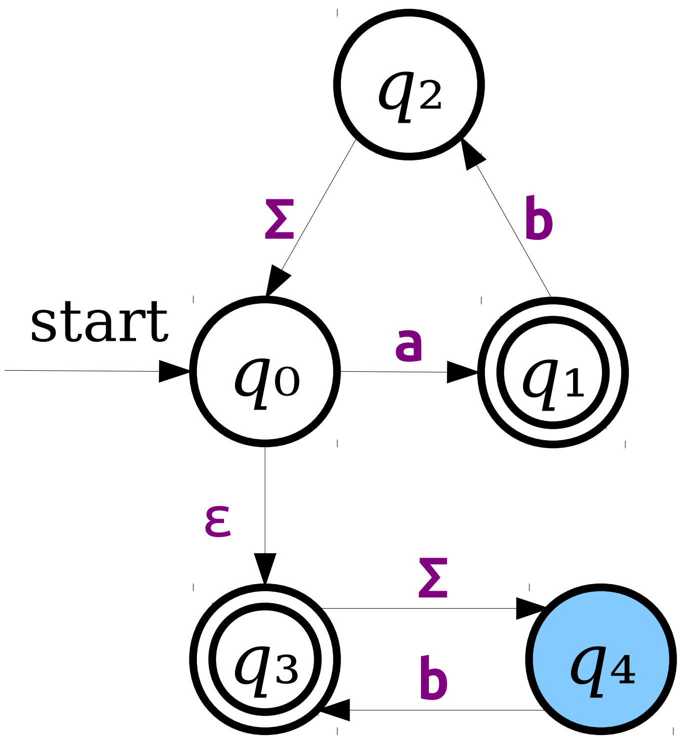
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



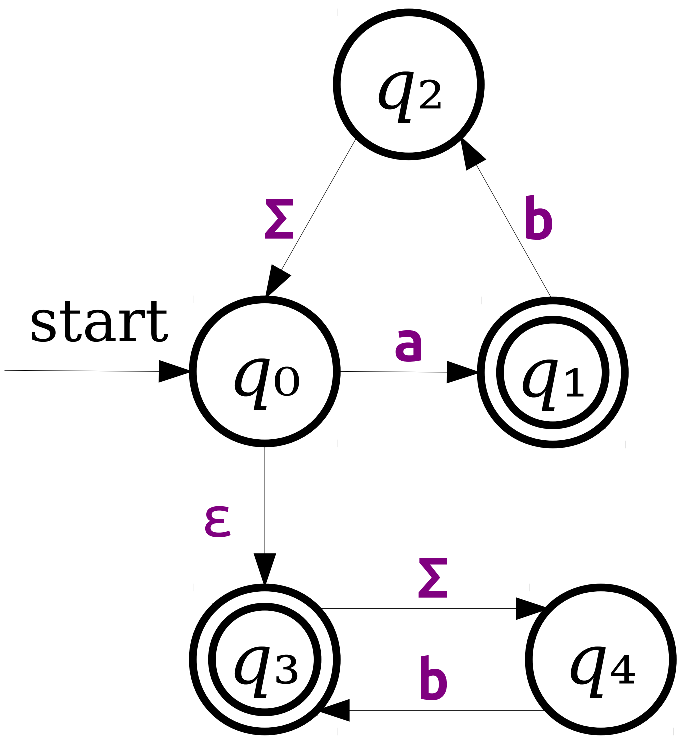
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



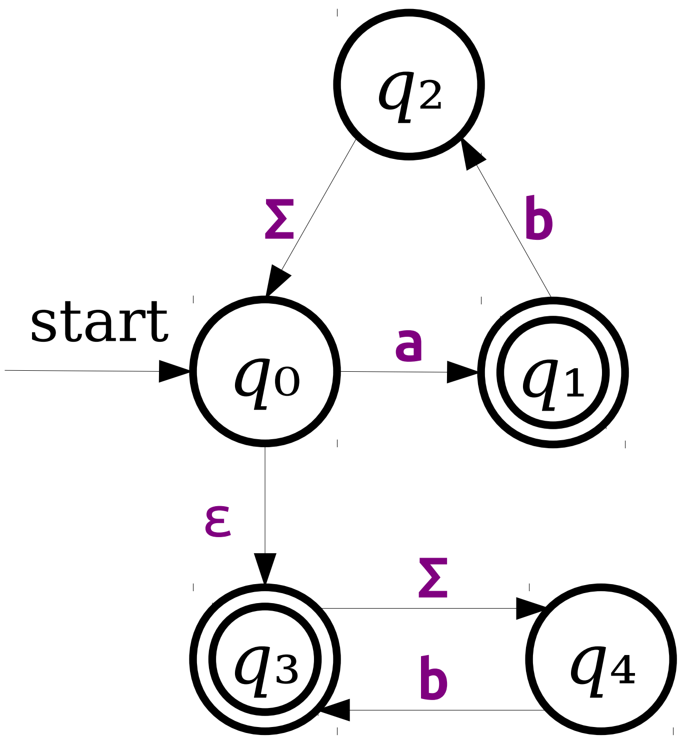
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$

Once More, With Epsilons!



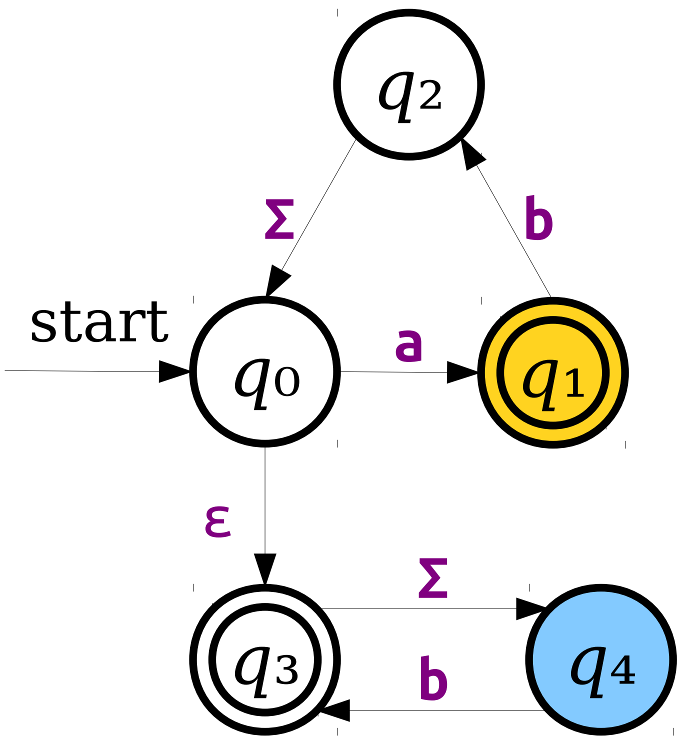
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$

Once More, With Epsilons!



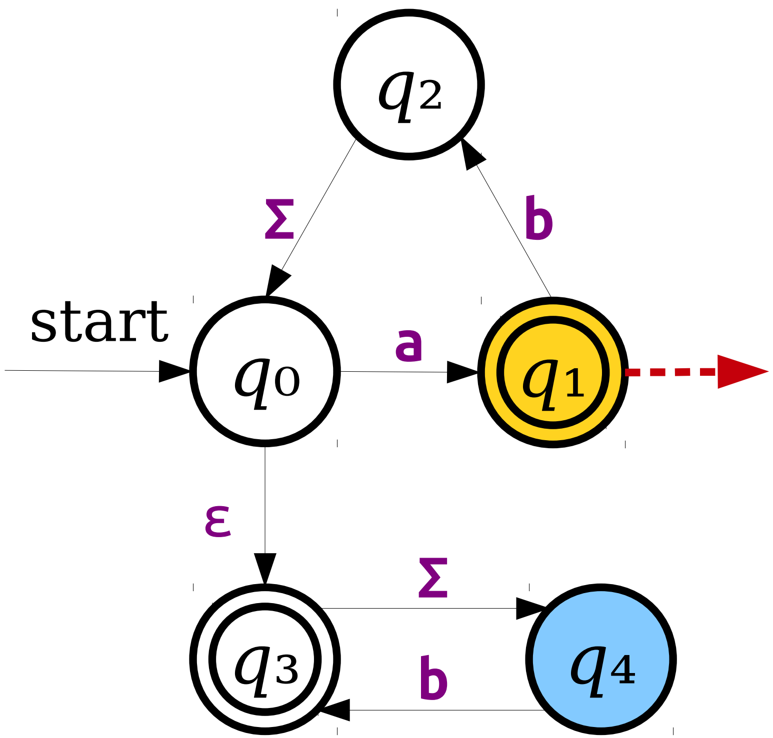
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



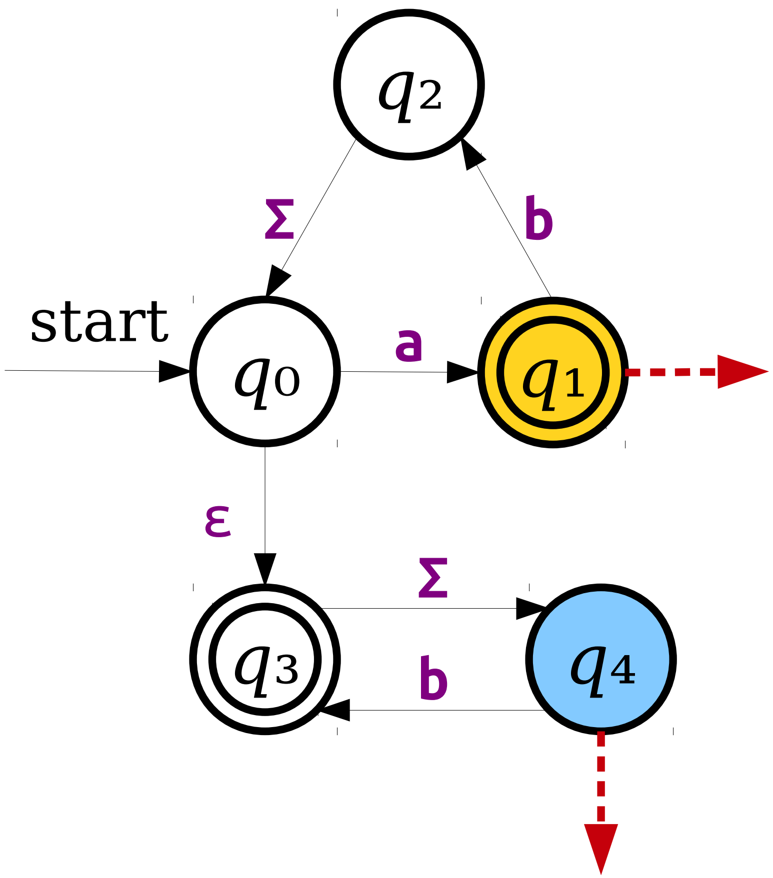
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



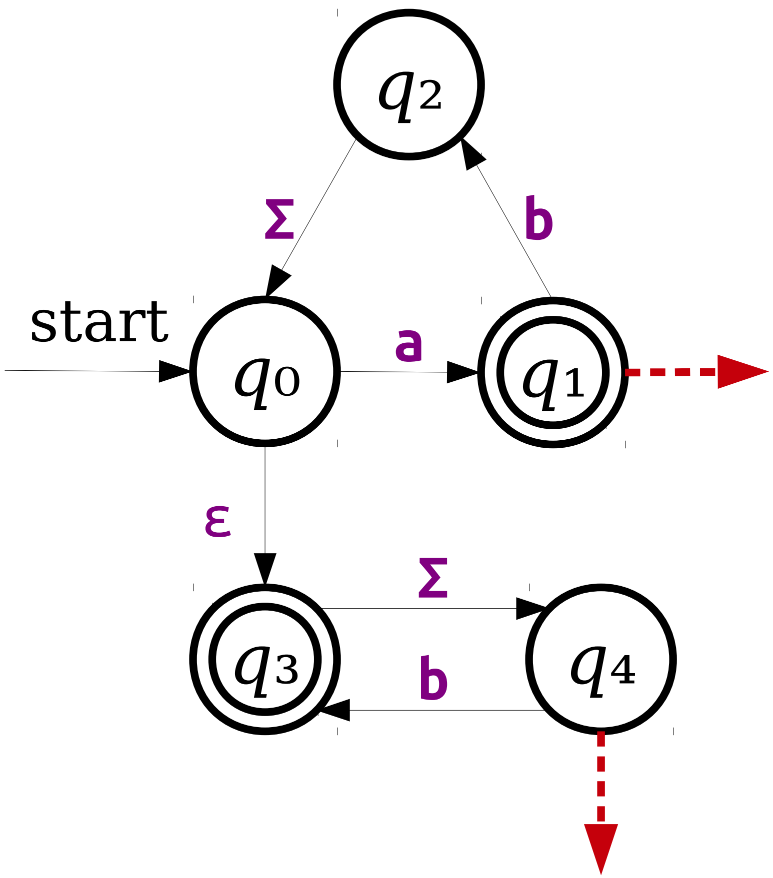
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



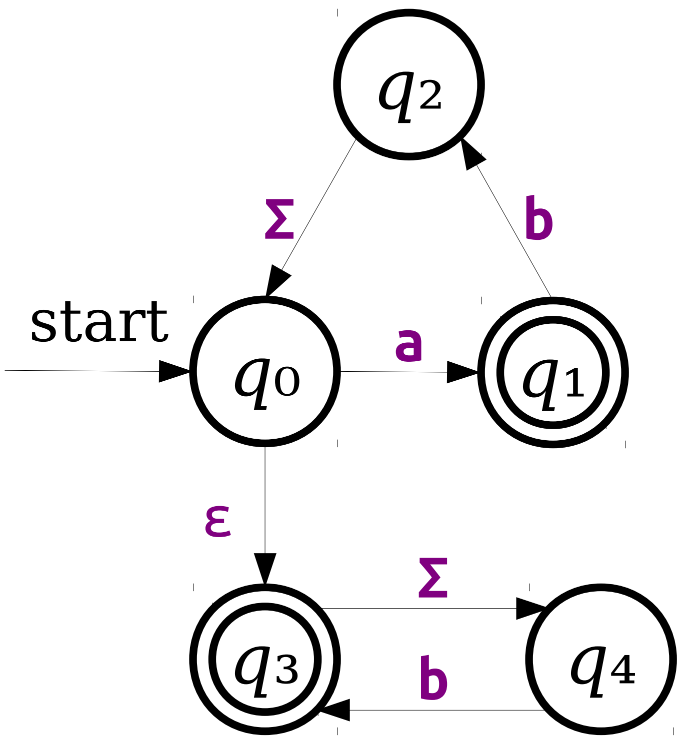
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



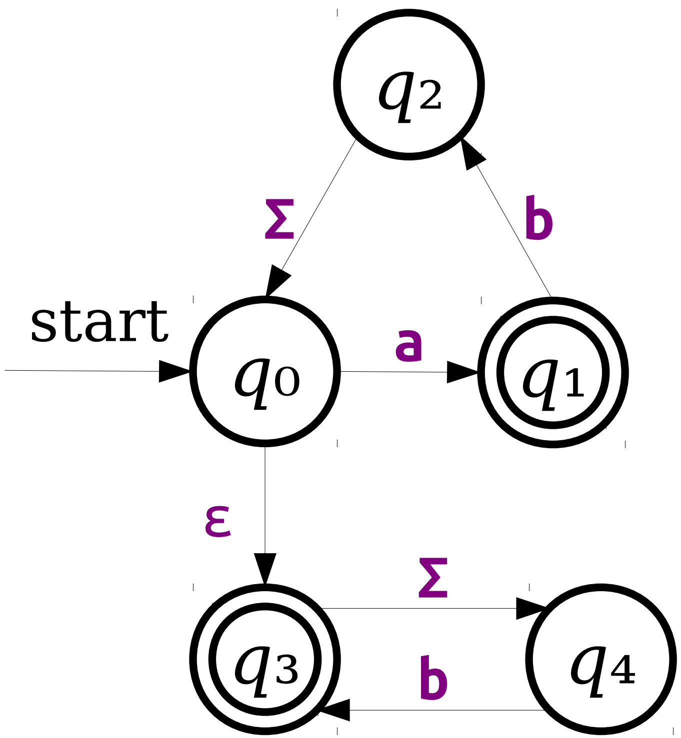
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



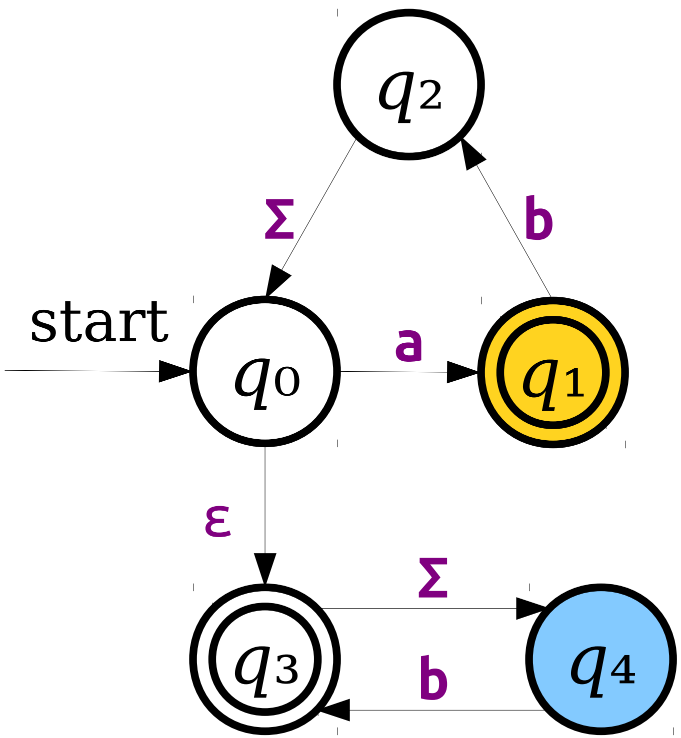
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



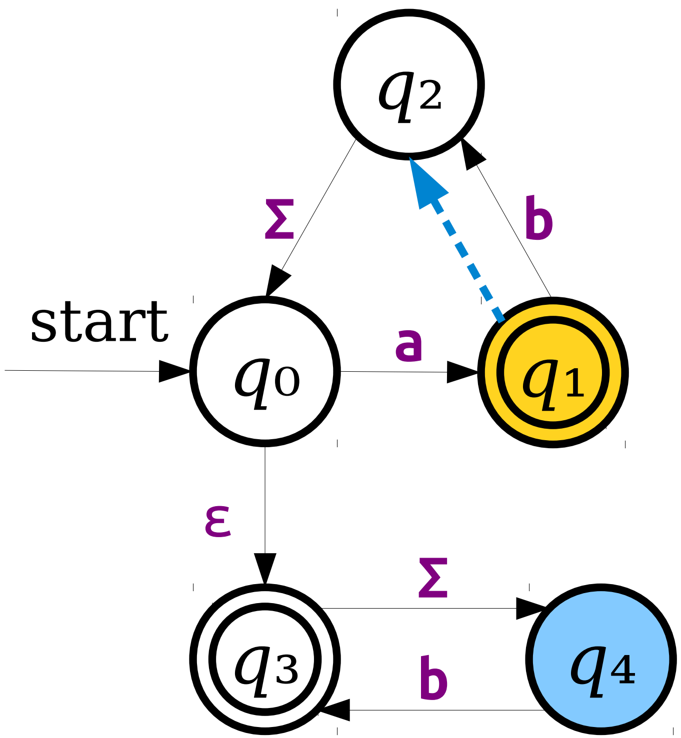
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



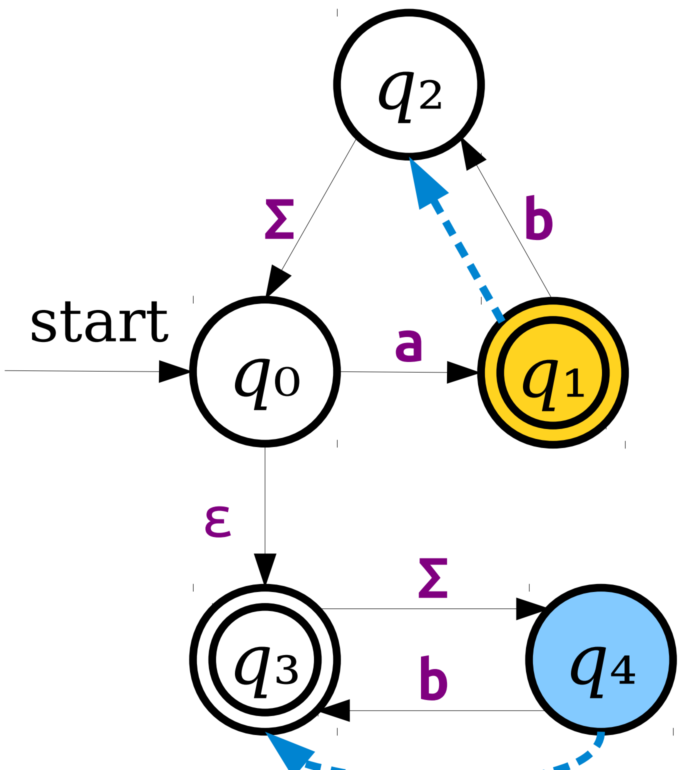
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



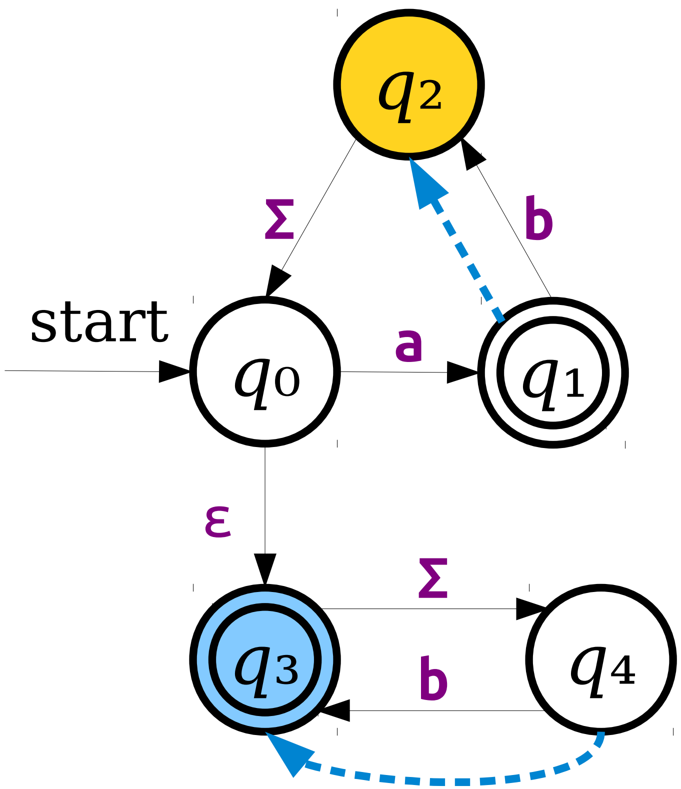
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



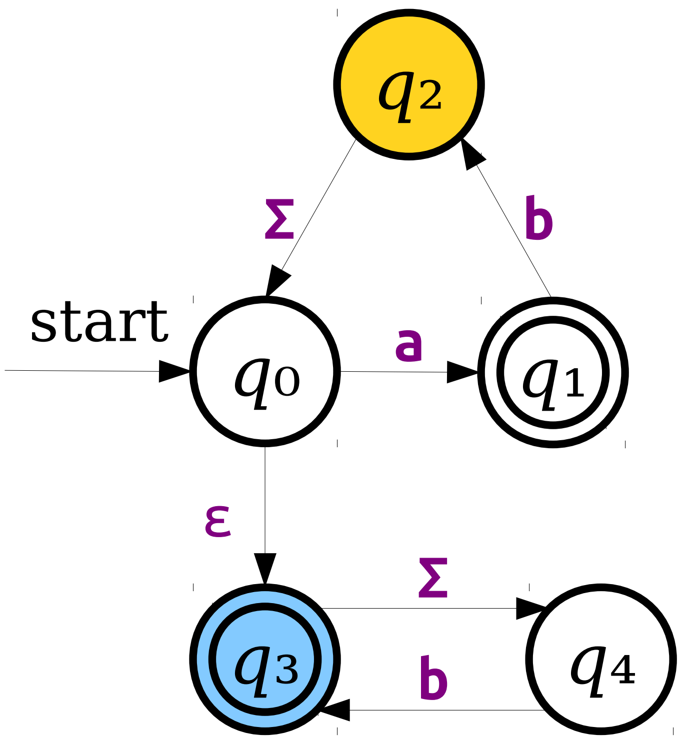
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



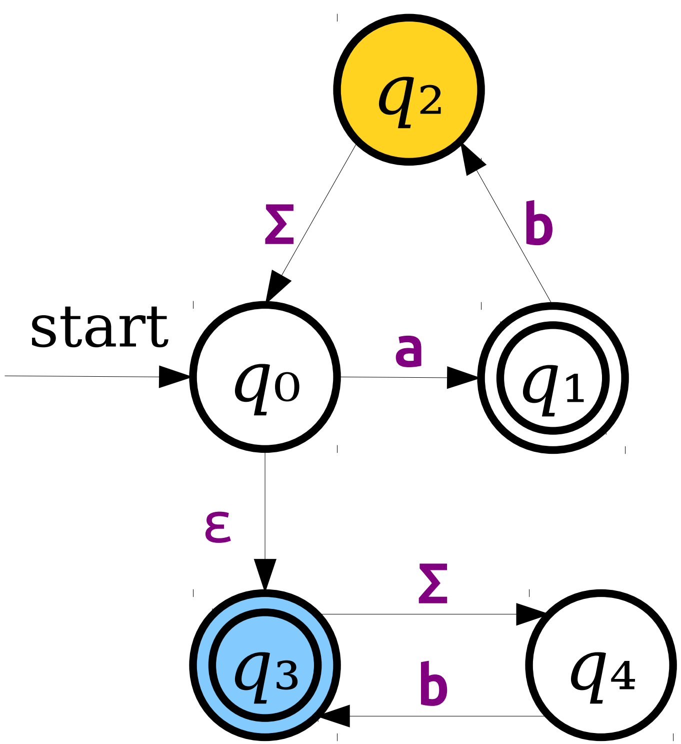
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



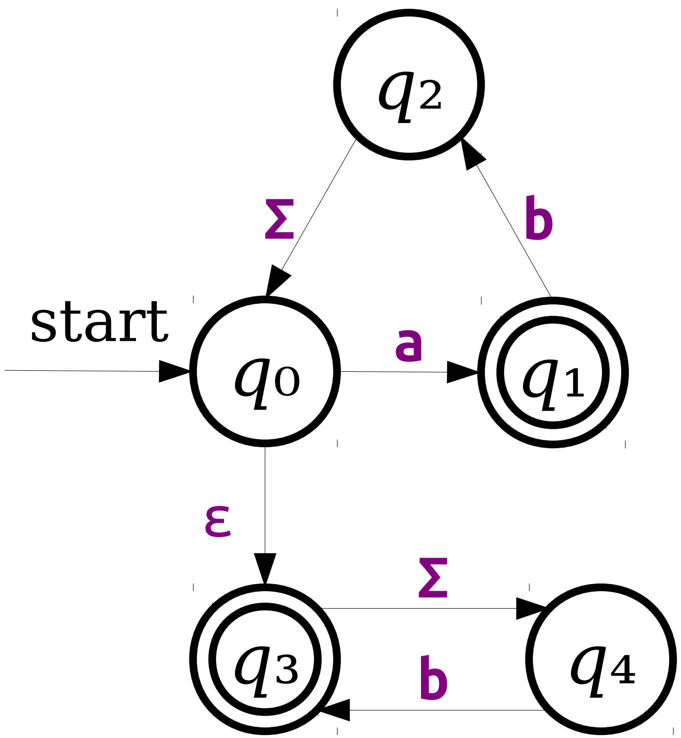
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



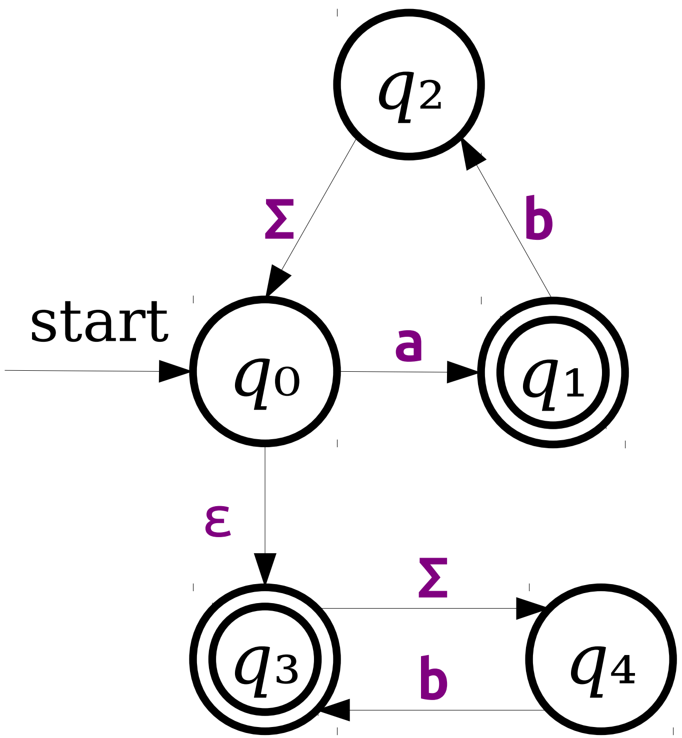
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$

Once More, With Epsilons!



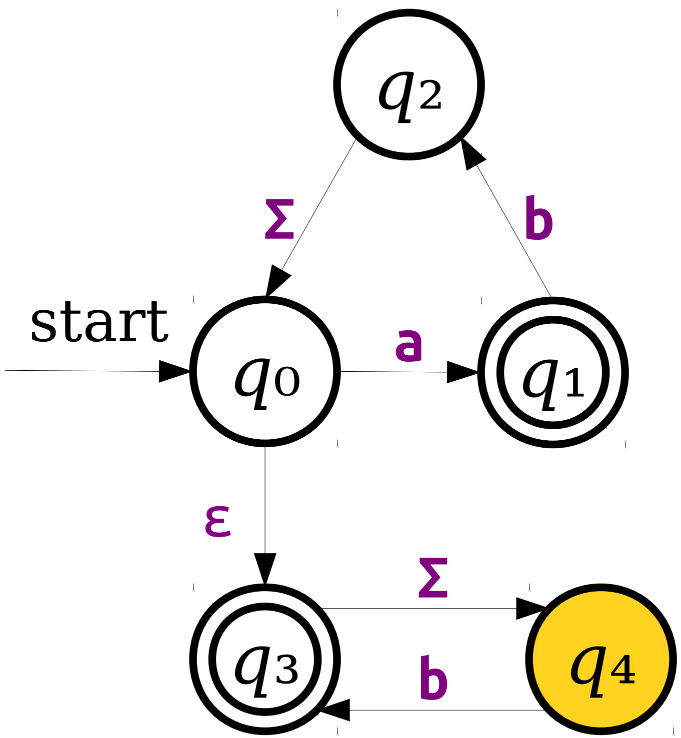
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$

Once More, With Epsilons!



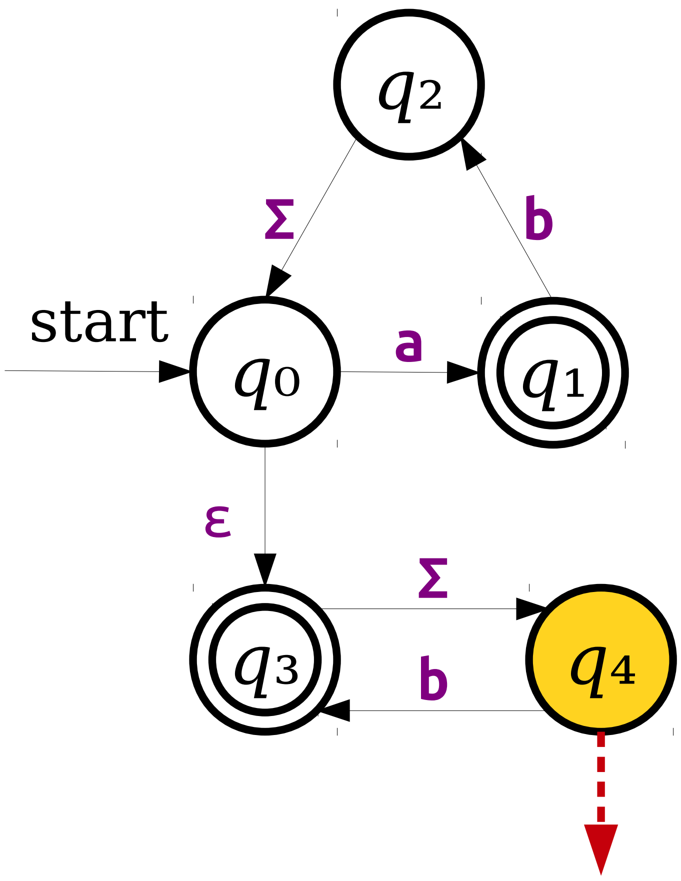
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



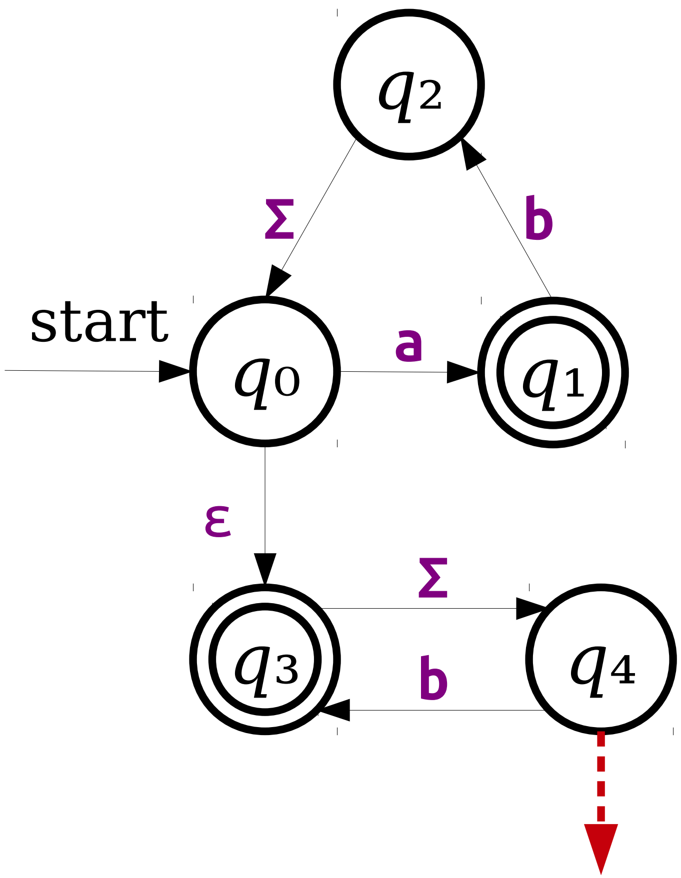
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



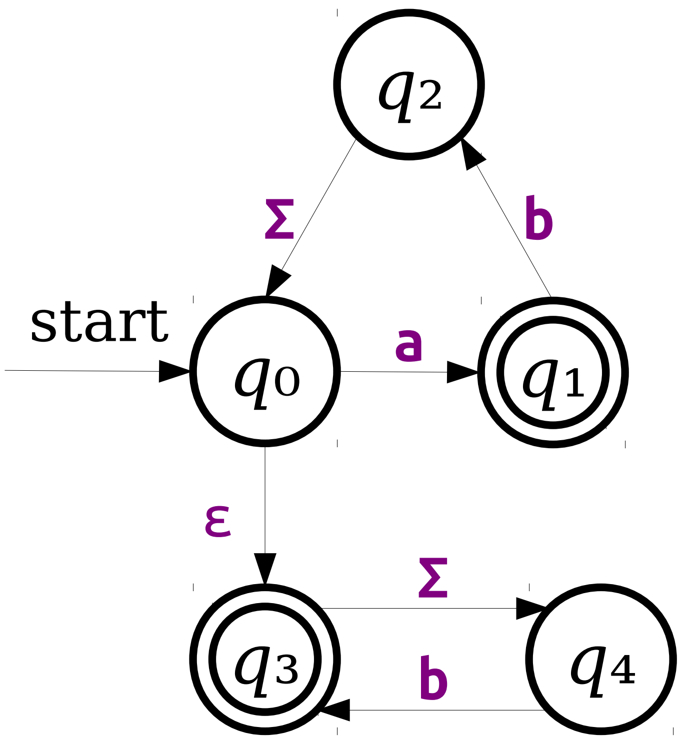
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



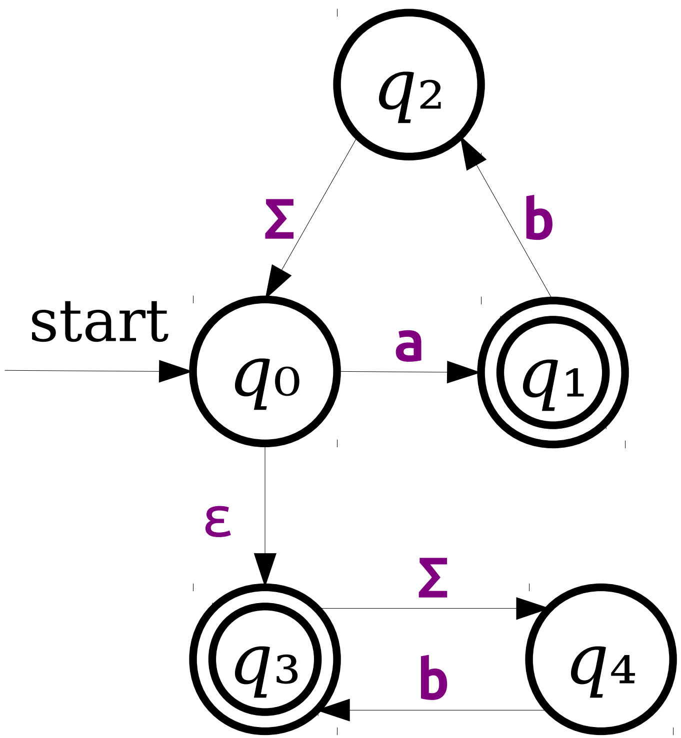
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



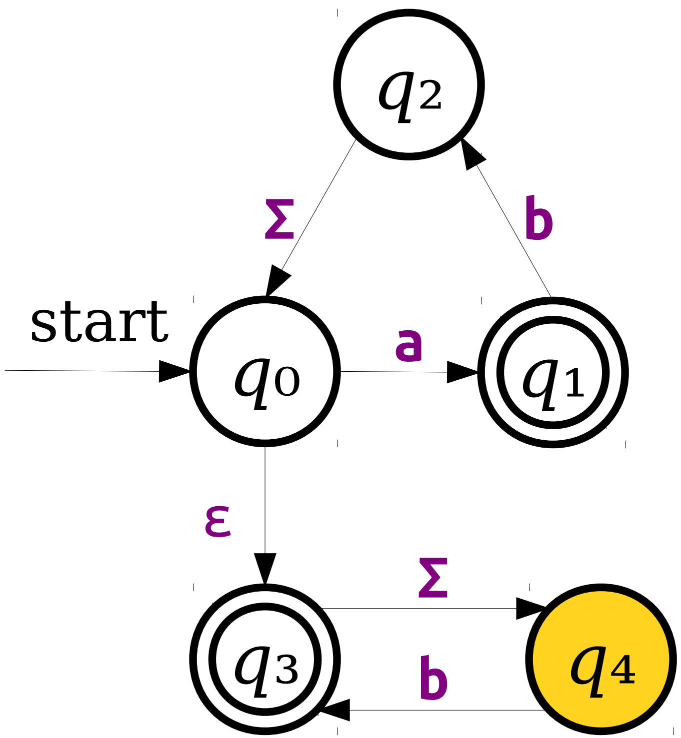
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



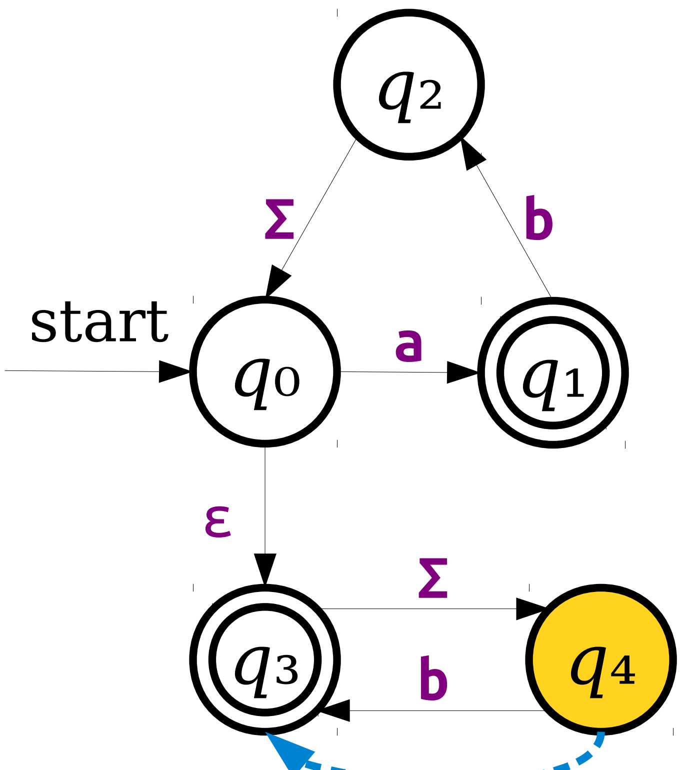
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

Once More, With Epsilons!



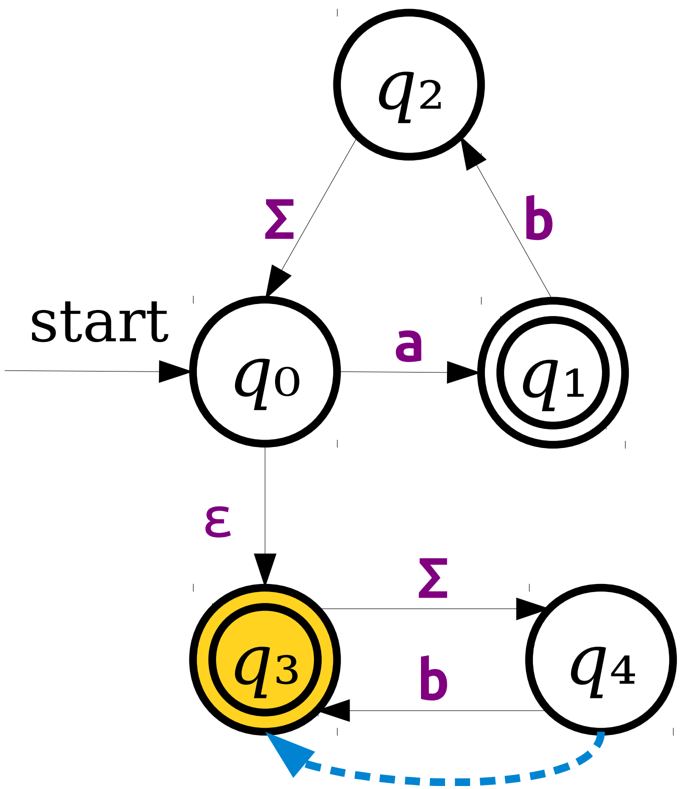
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	

Once More, With Epsilons!



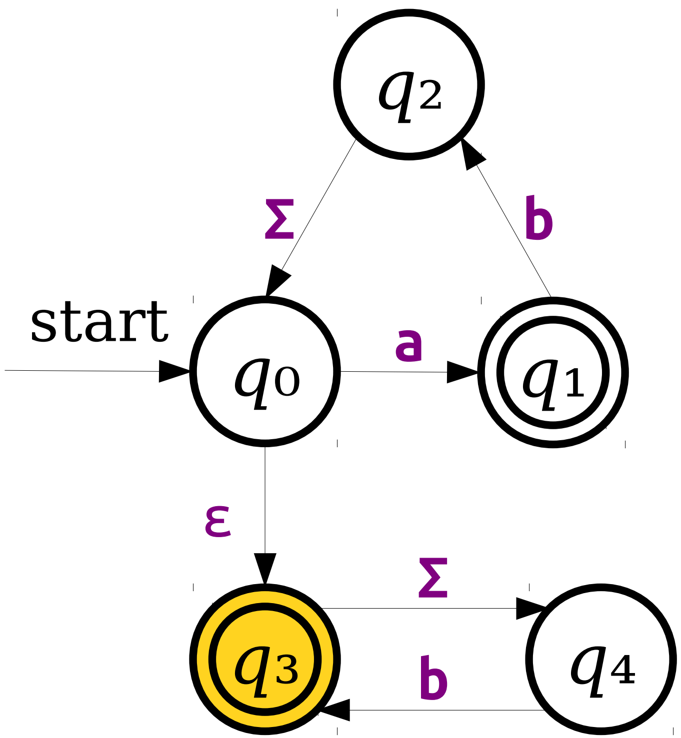
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

Once More, With Epsilons!



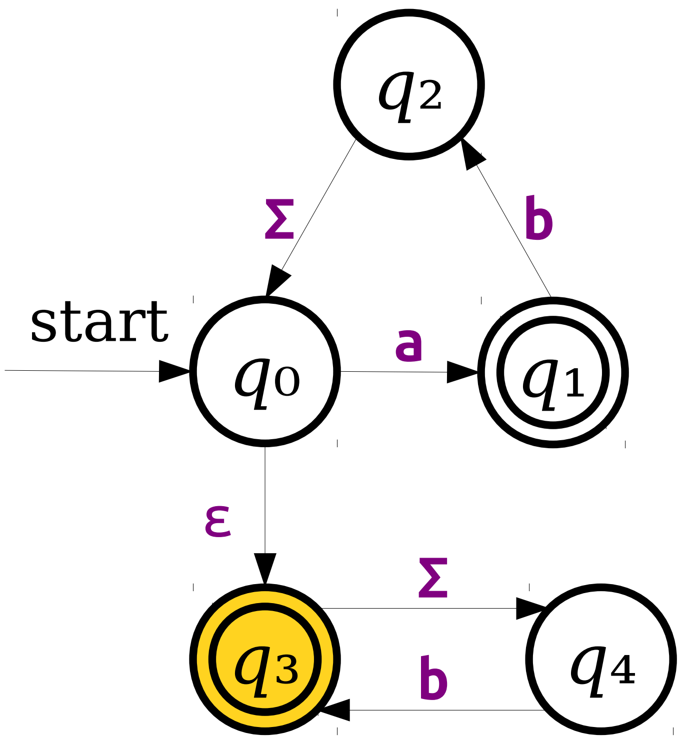
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

Once More, With Epsilons!



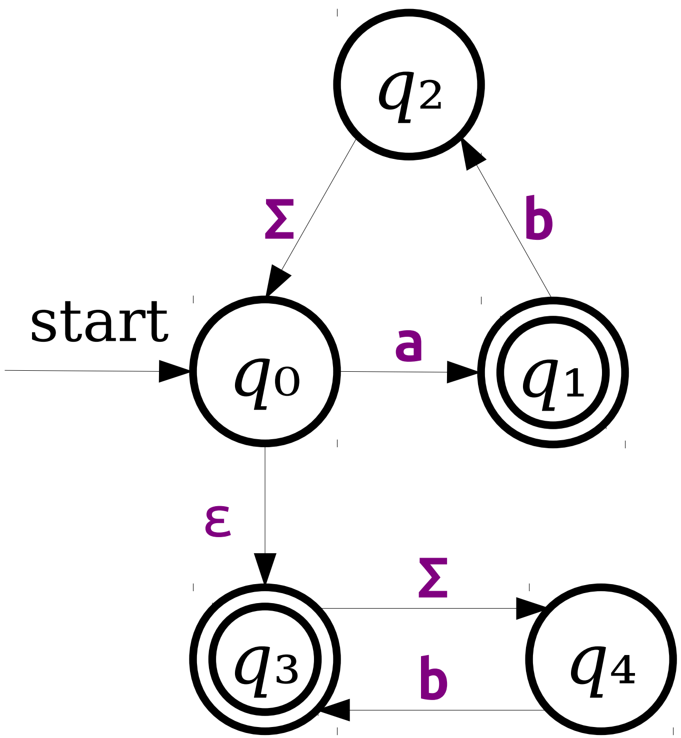
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

Once More, With Epsilons!



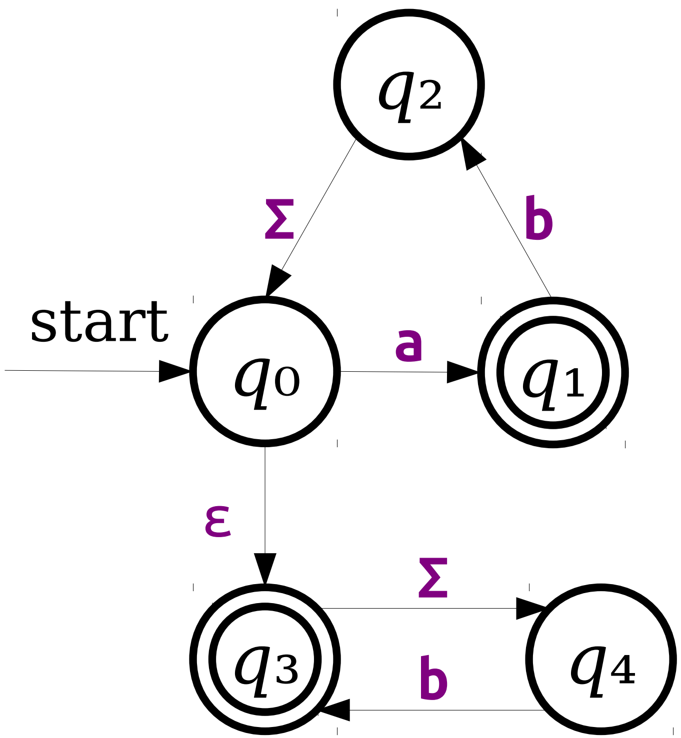
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$

Once More, With Epsilons!



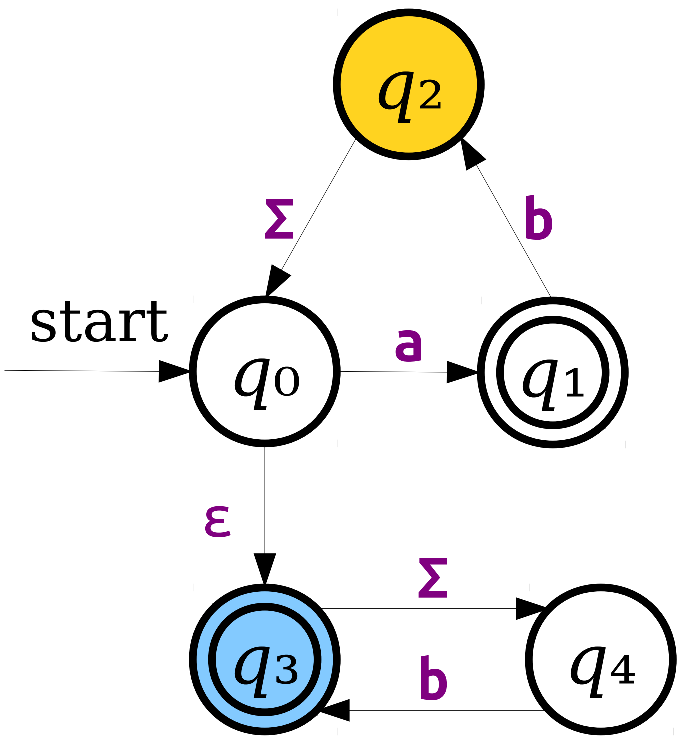
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$

Once More, With Epsilons!



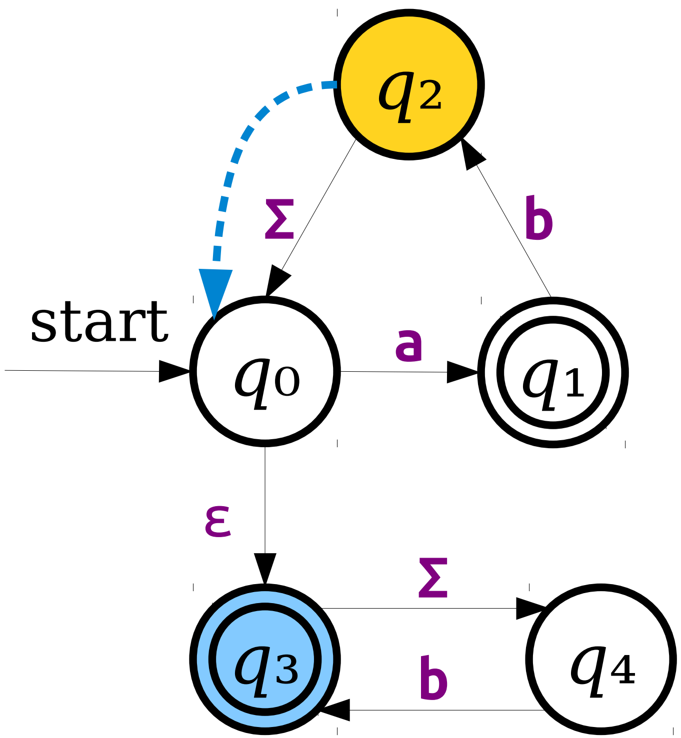
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



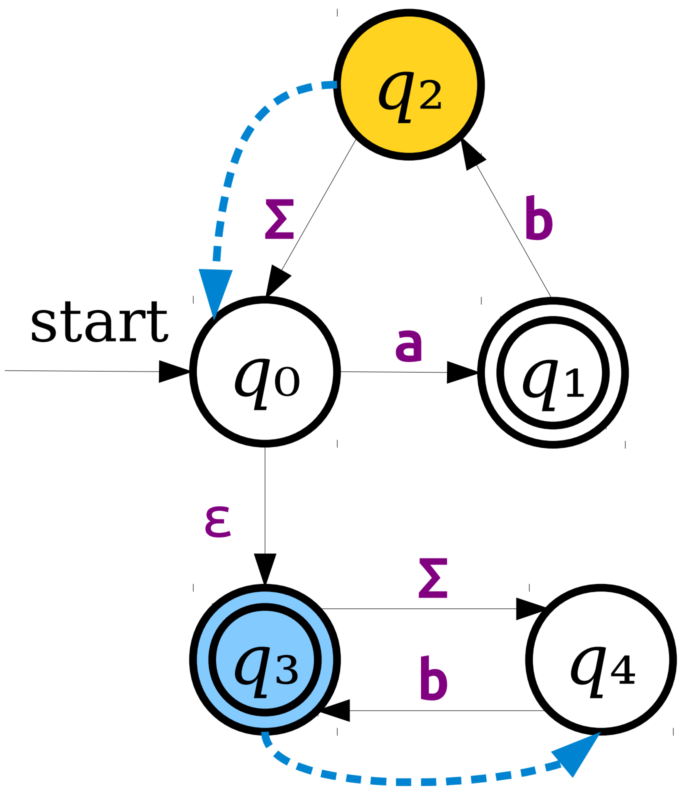
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



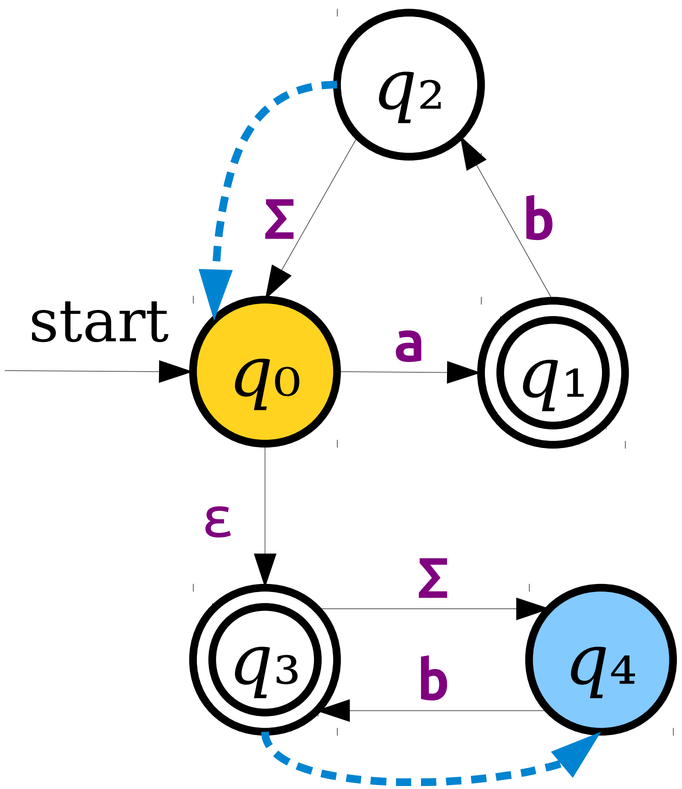
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



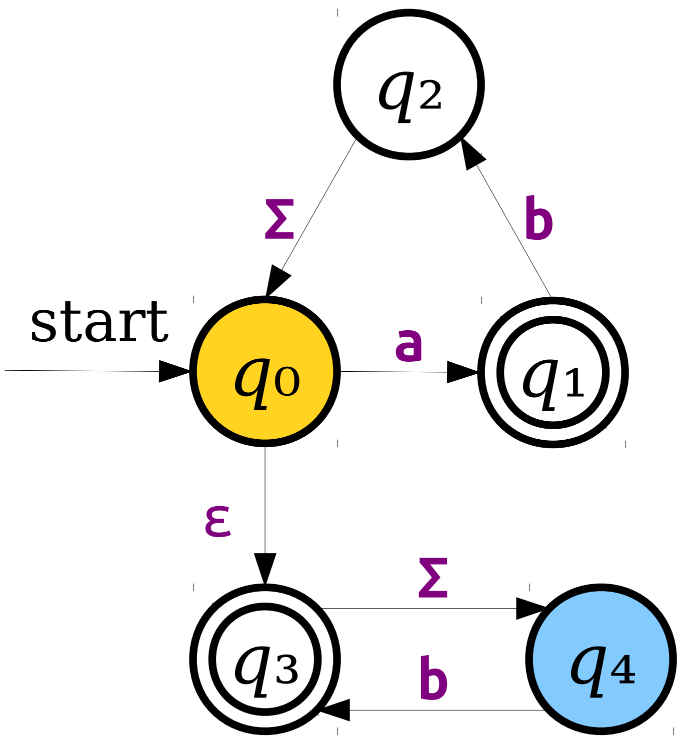
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



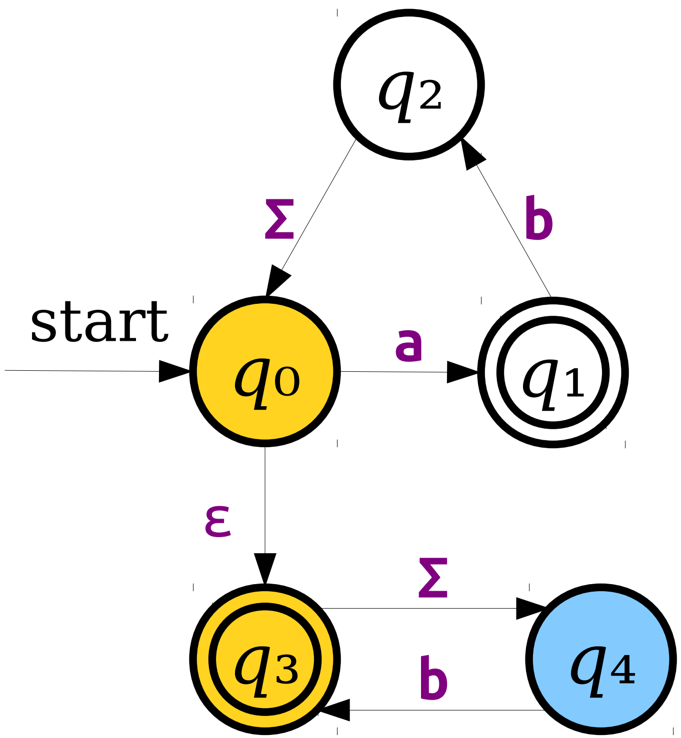
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



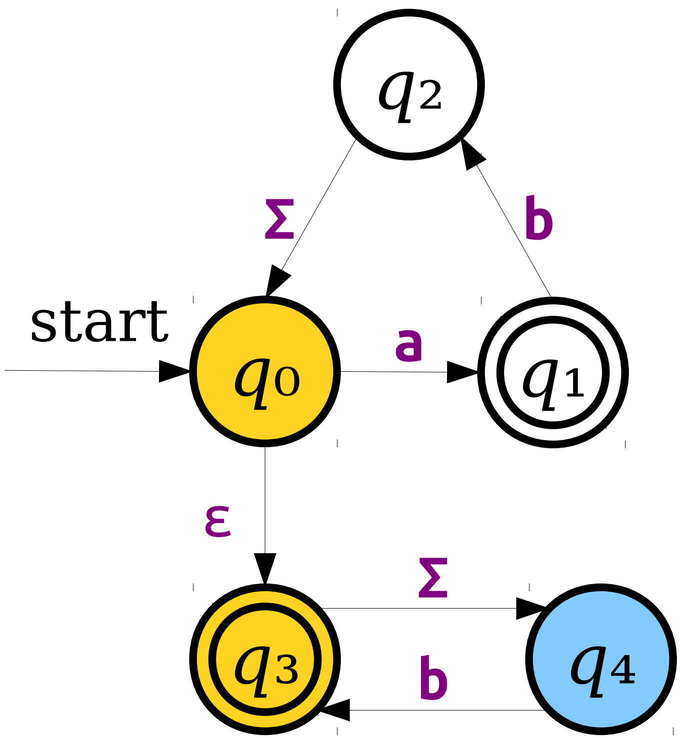
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



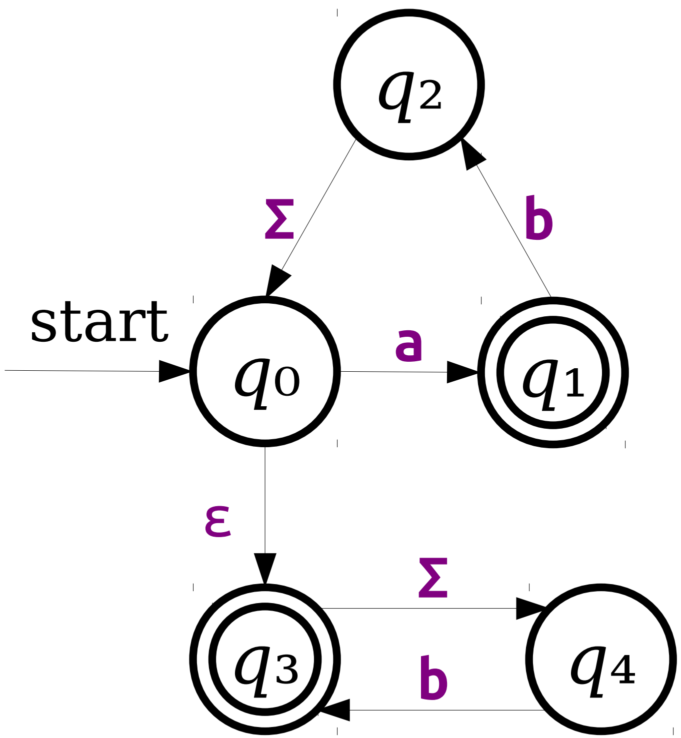
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



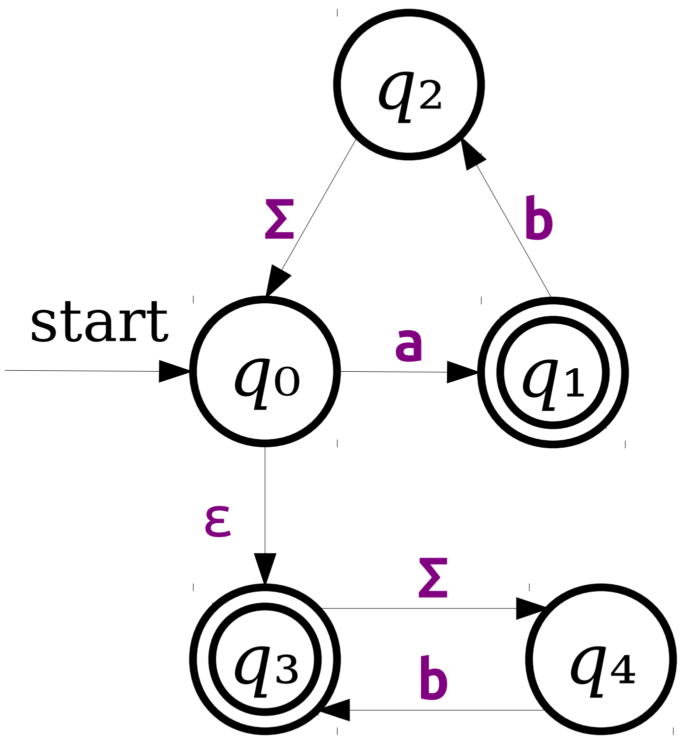
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$

Once More, With Epsilons!



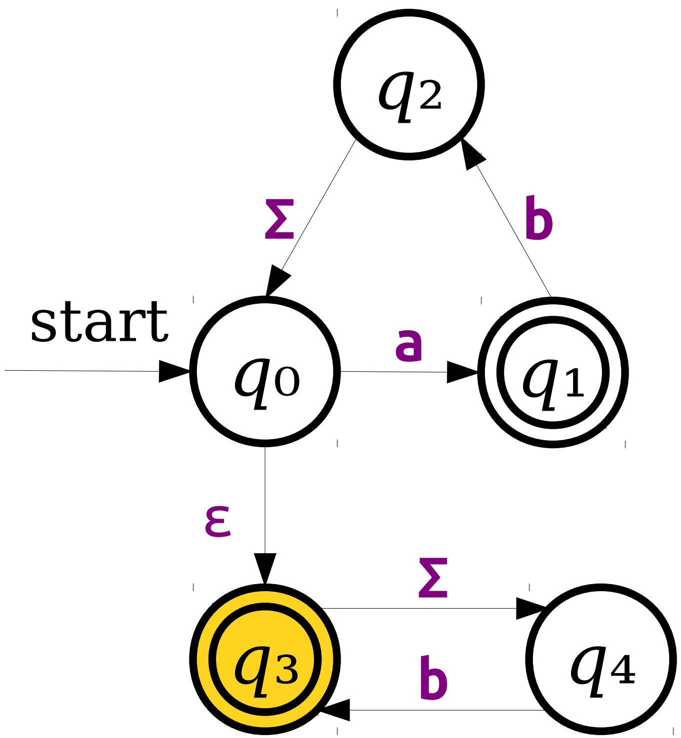
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



	a	b
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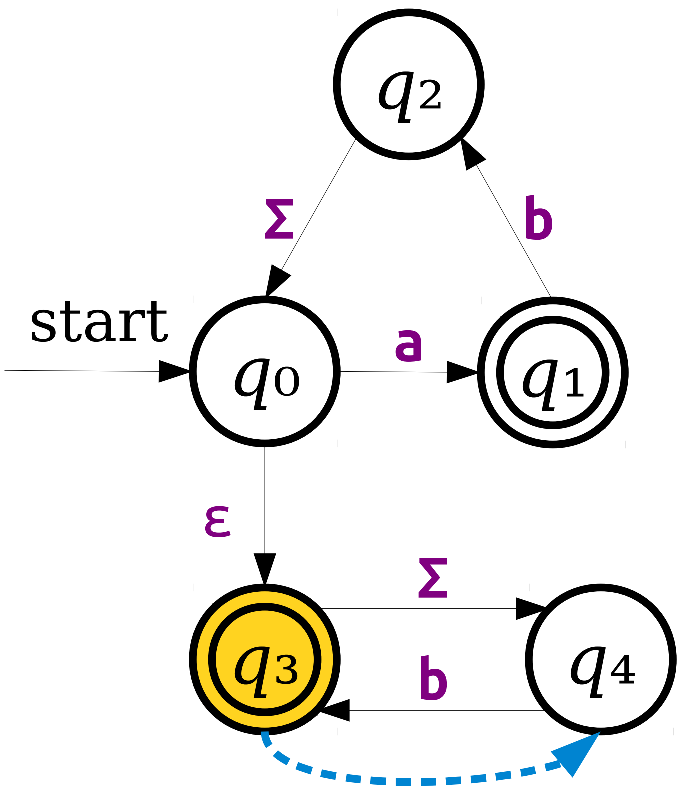
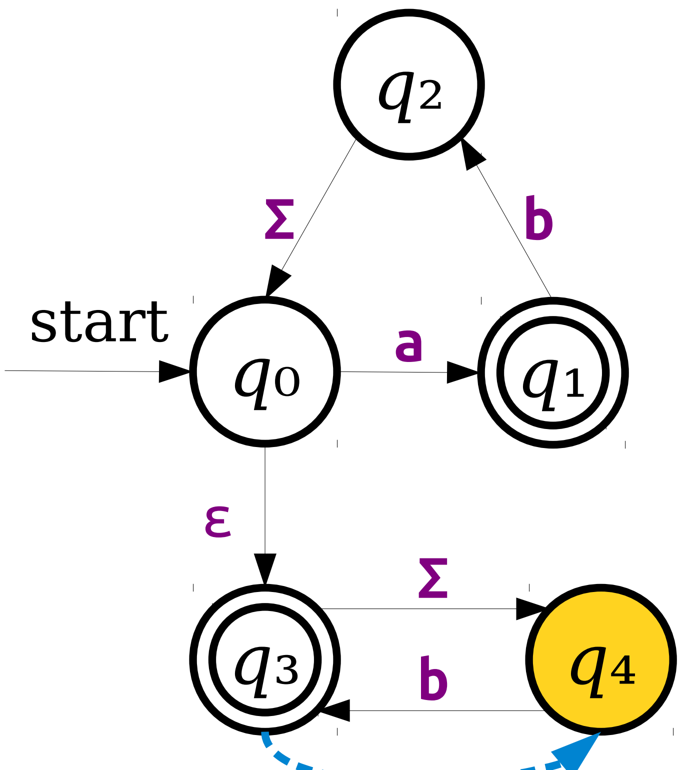
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



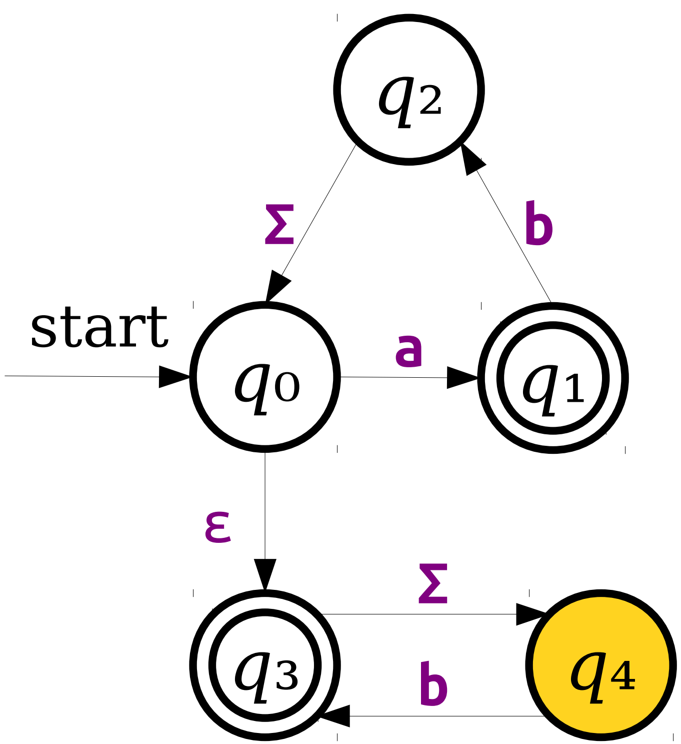
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



	a	b
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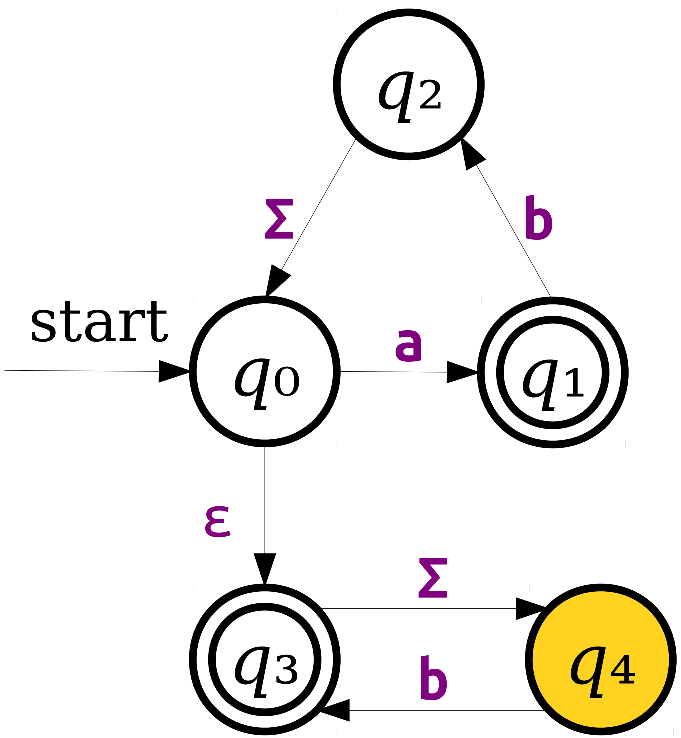
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



	a	b
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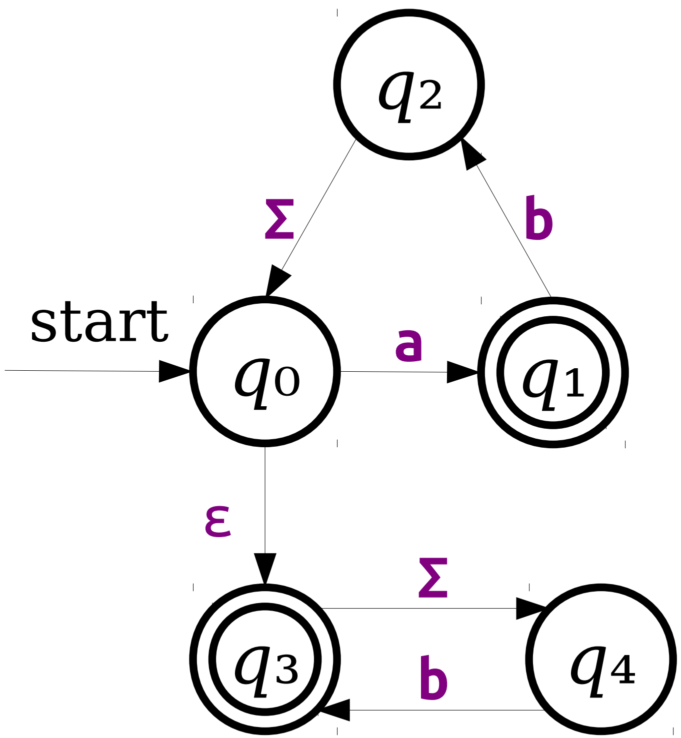
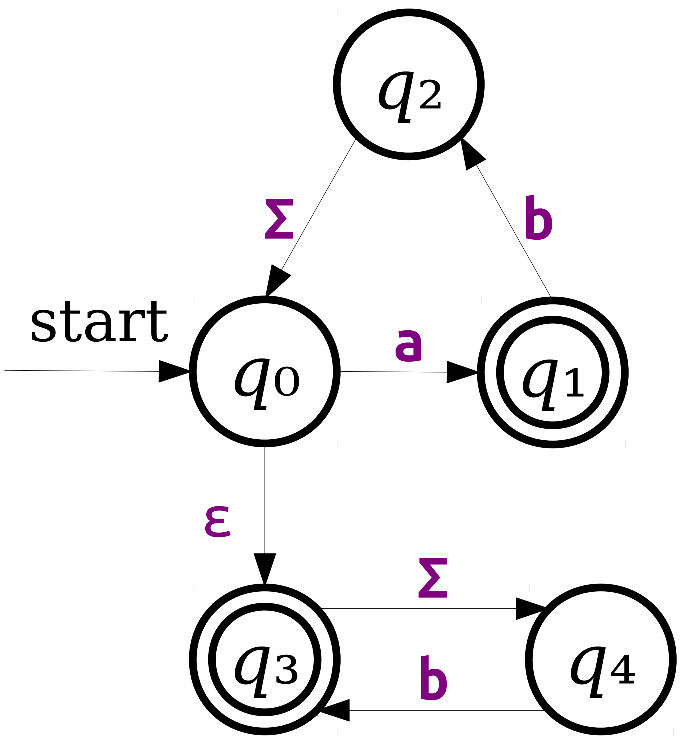
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$

Once More, With Epsilons!



	a	b
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$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

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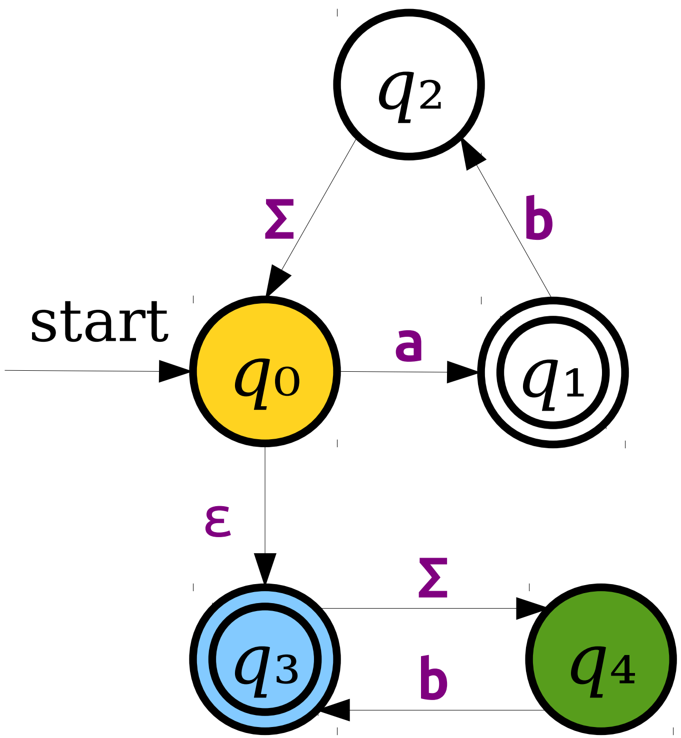
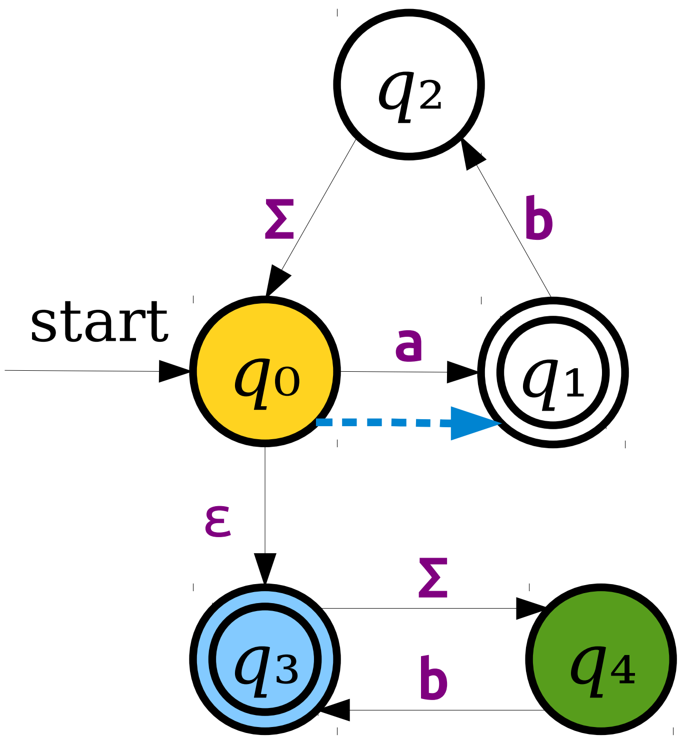


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



	a	b
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$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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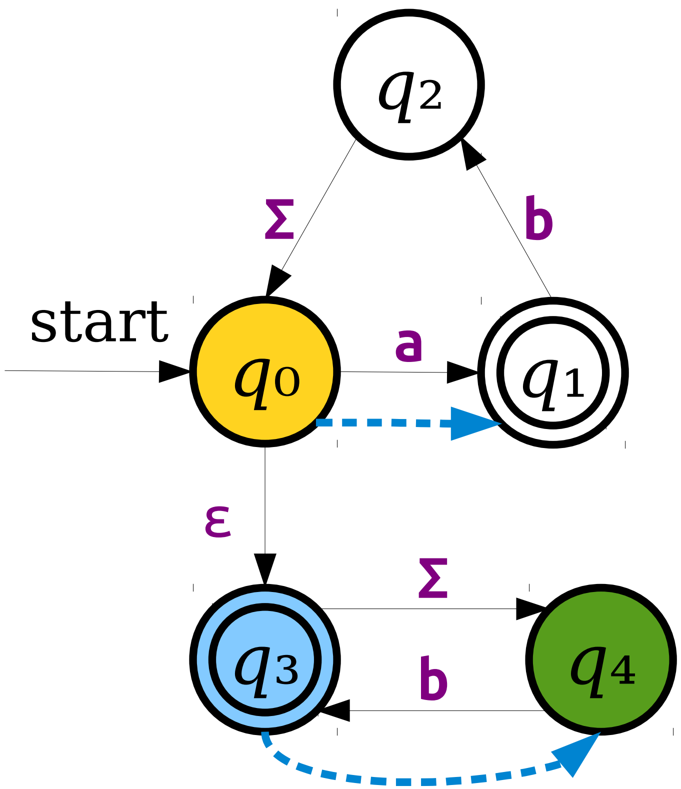

	a	b
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$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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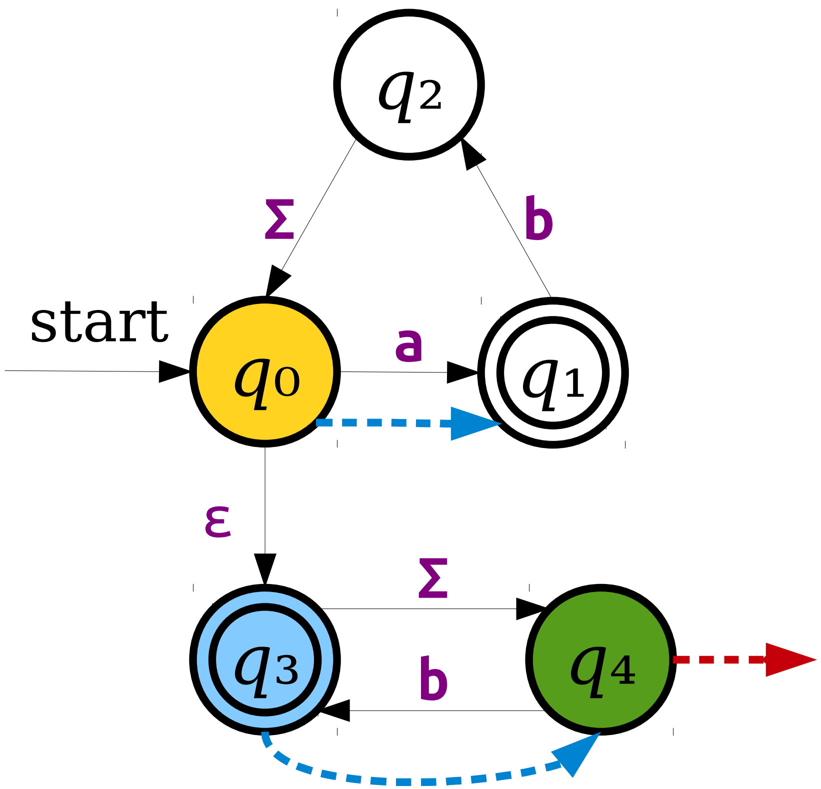

	a	b
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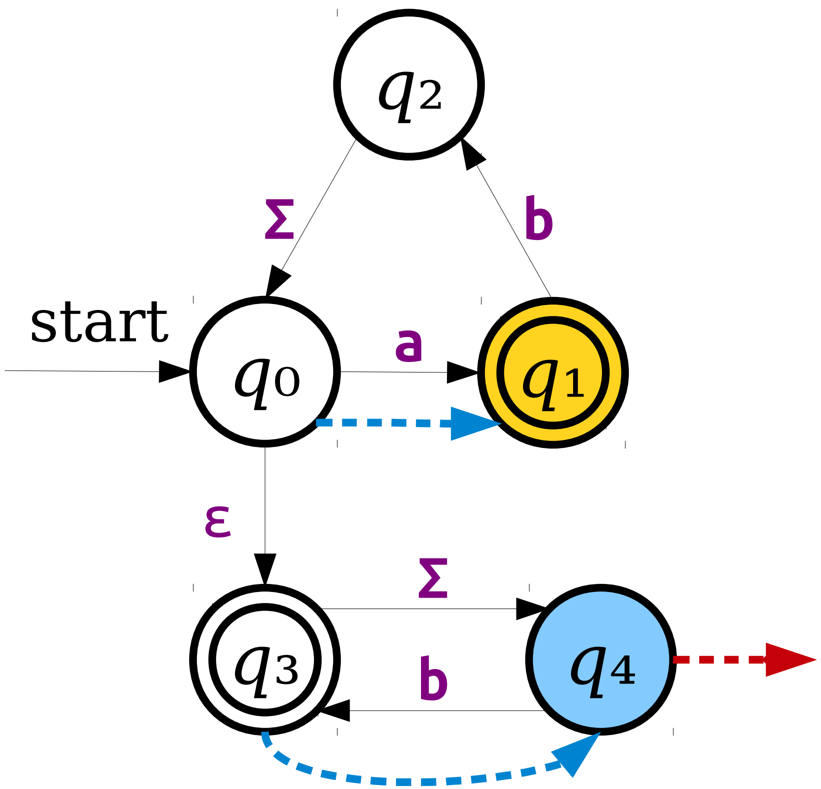
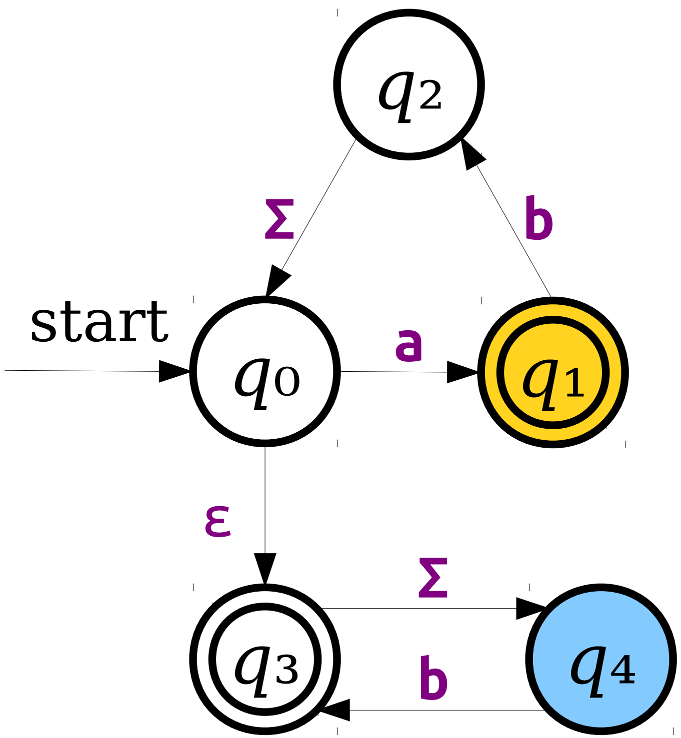
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



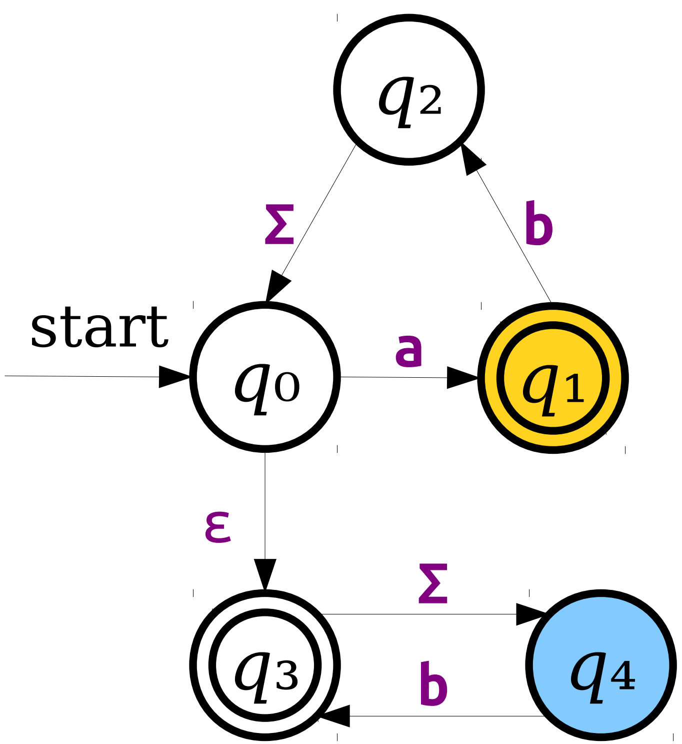
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



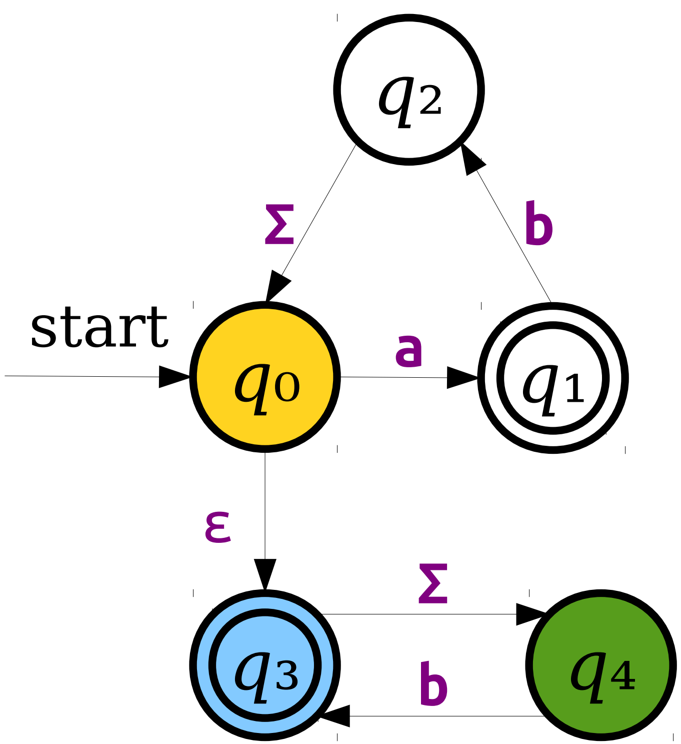
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



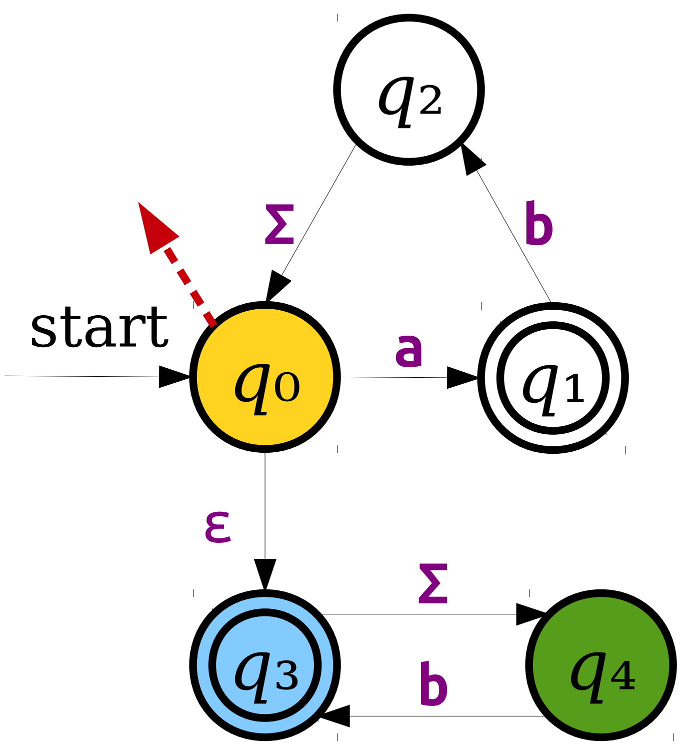
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



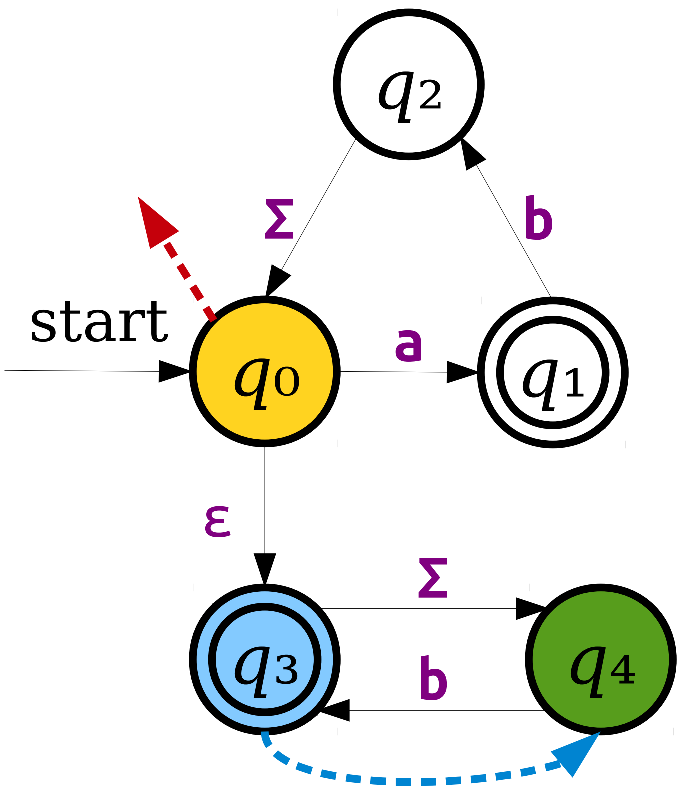
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



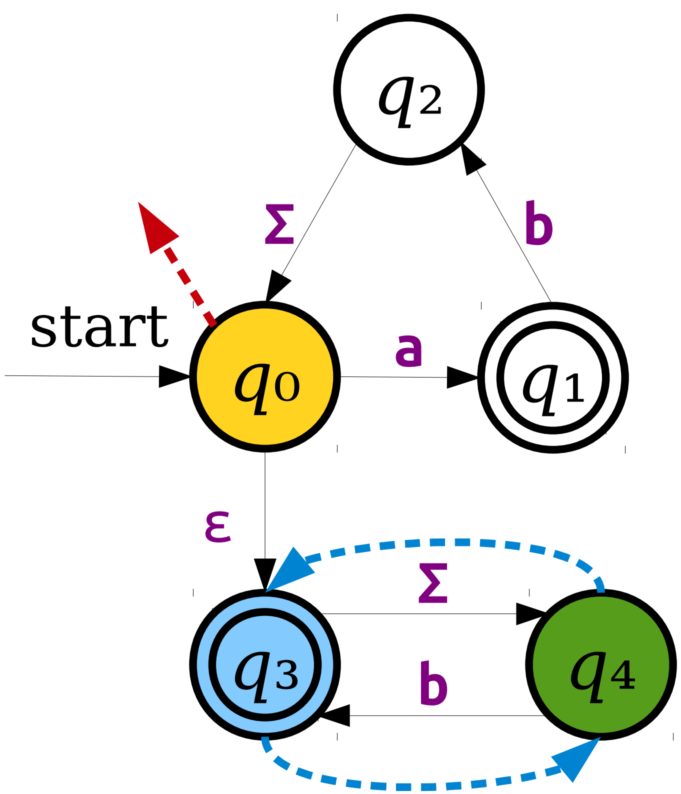
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



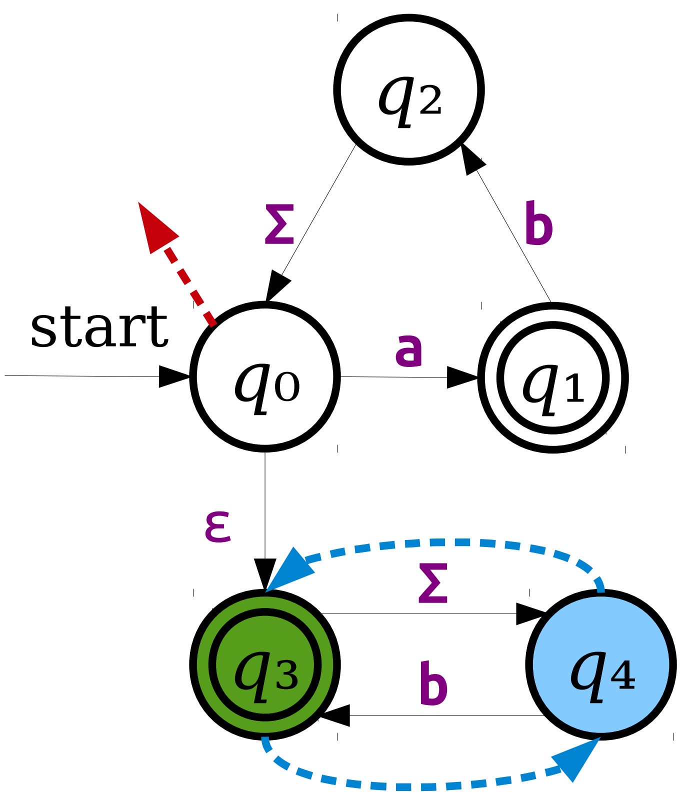
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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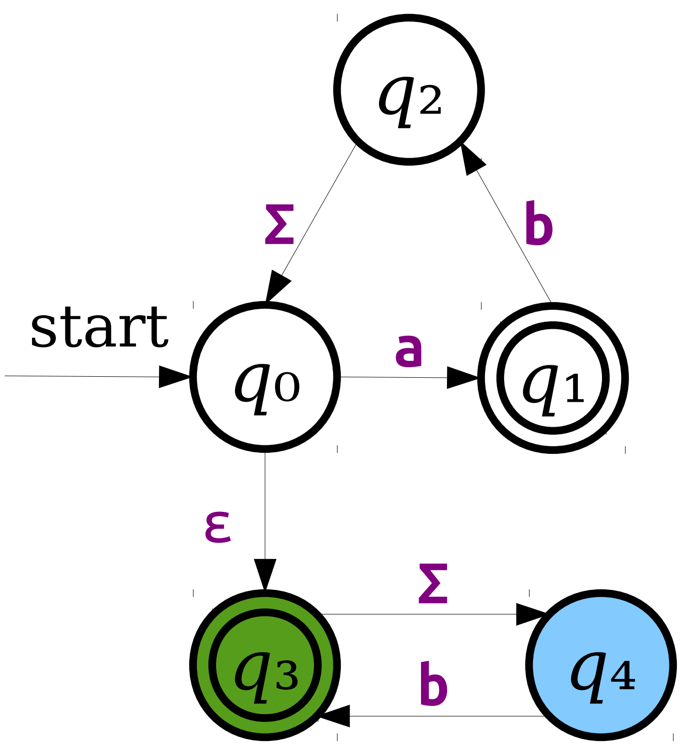
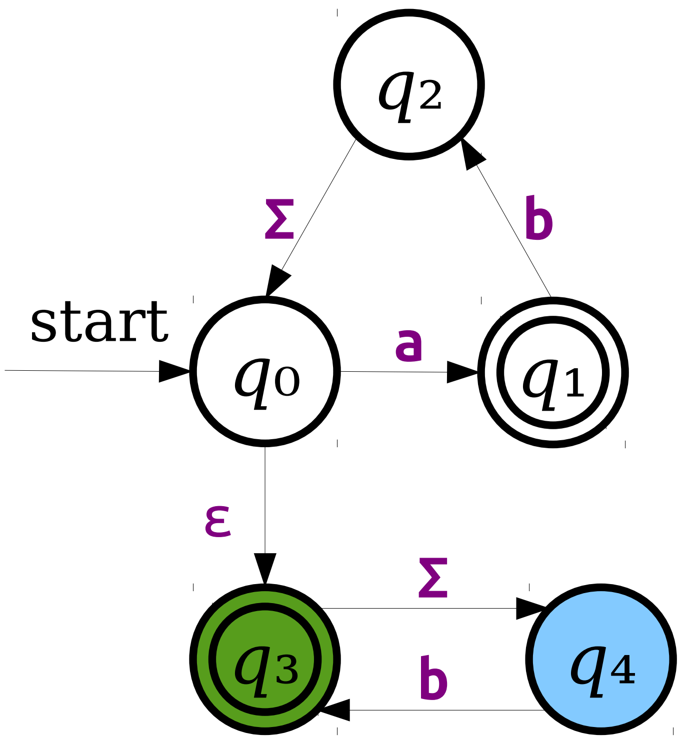
Once More, With Epsilons!



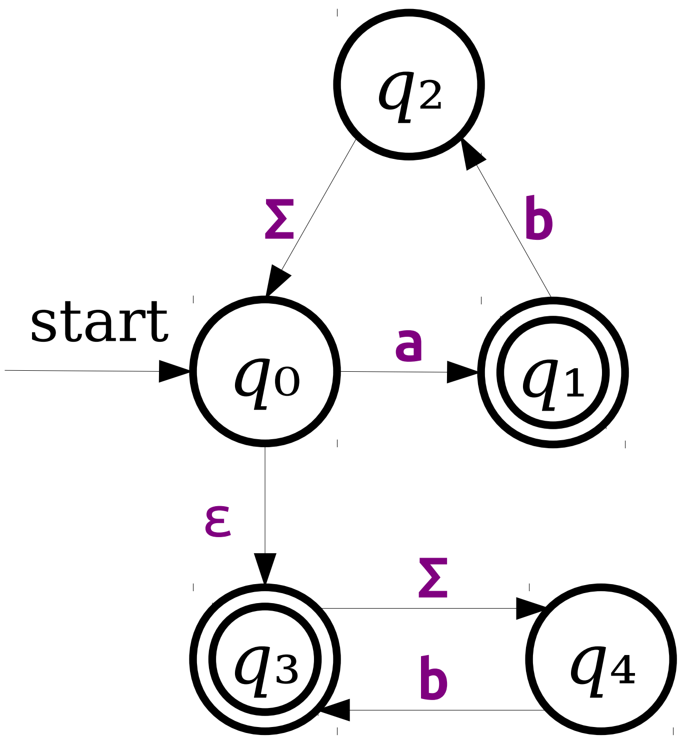
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



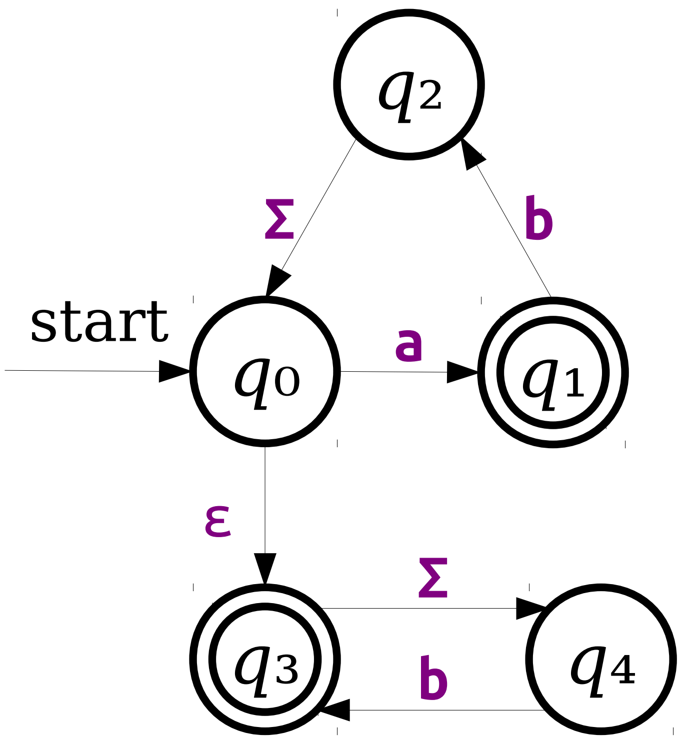
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



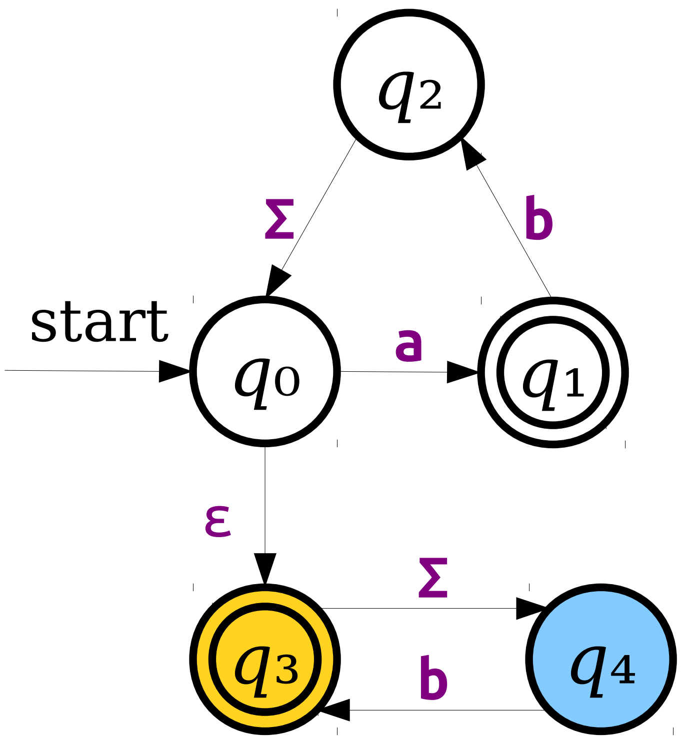
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



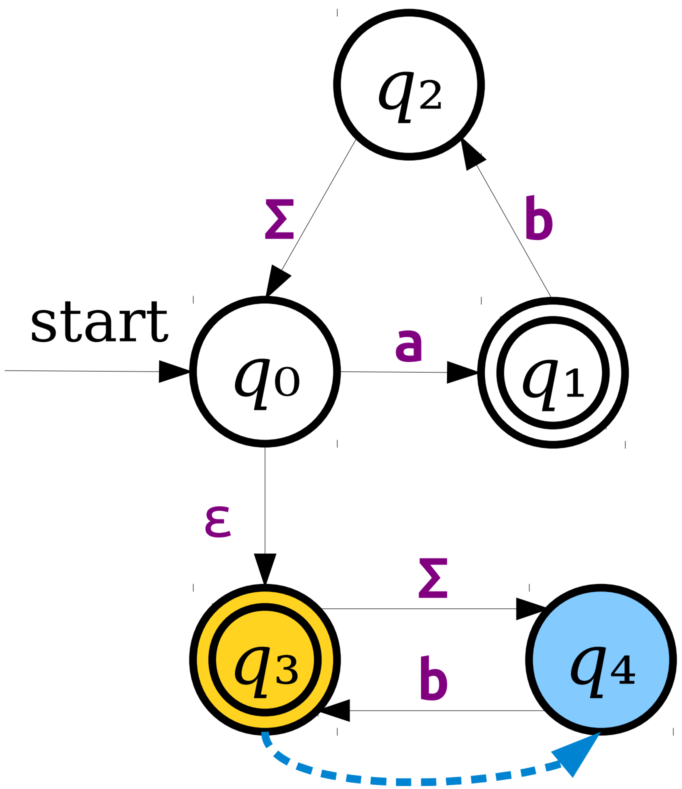
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



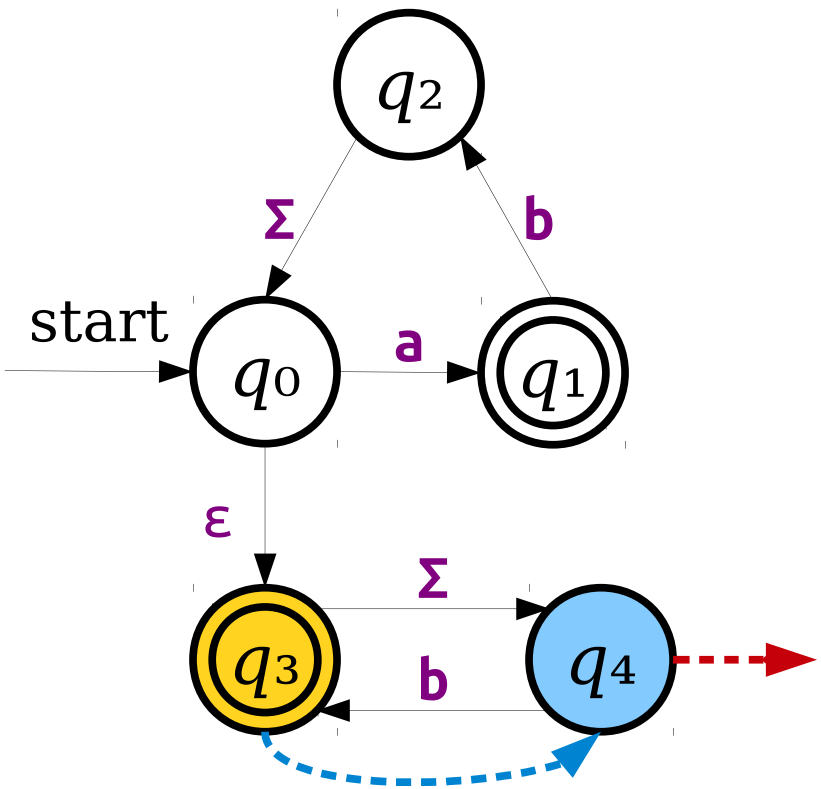
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
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$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

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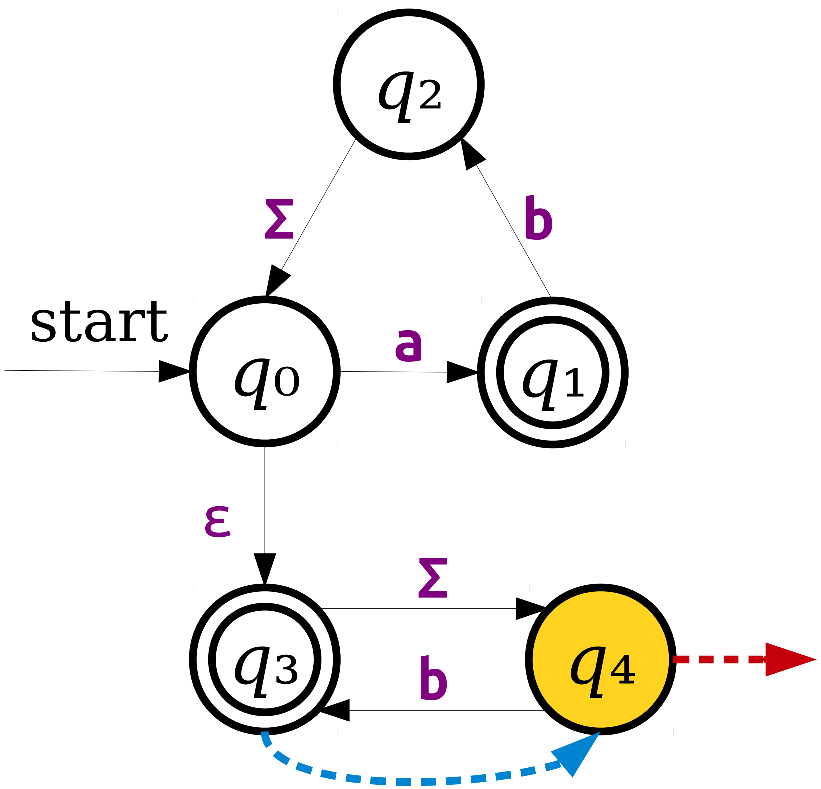
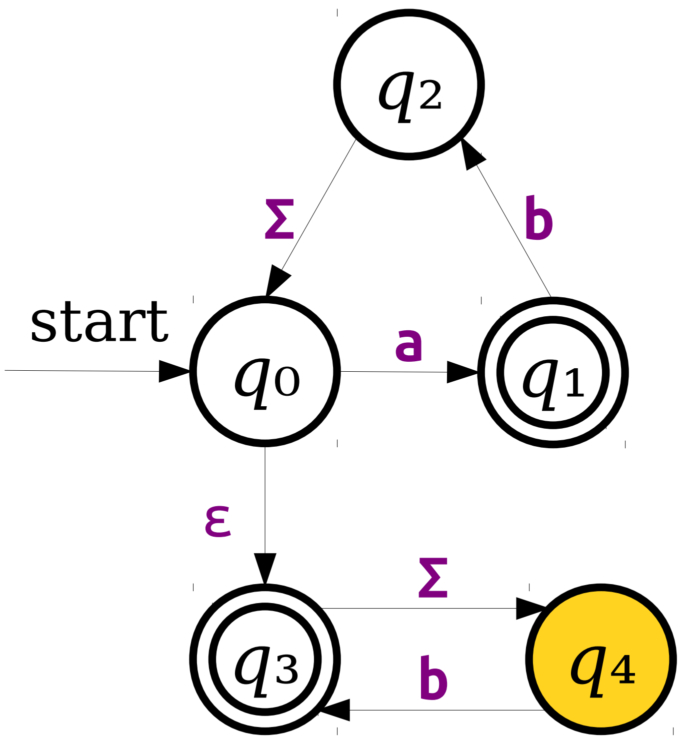


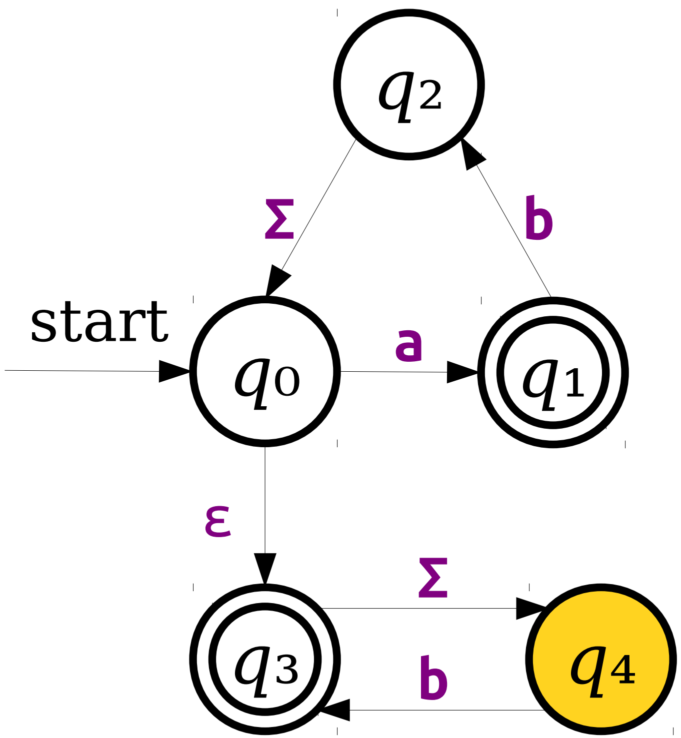
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



	a	b
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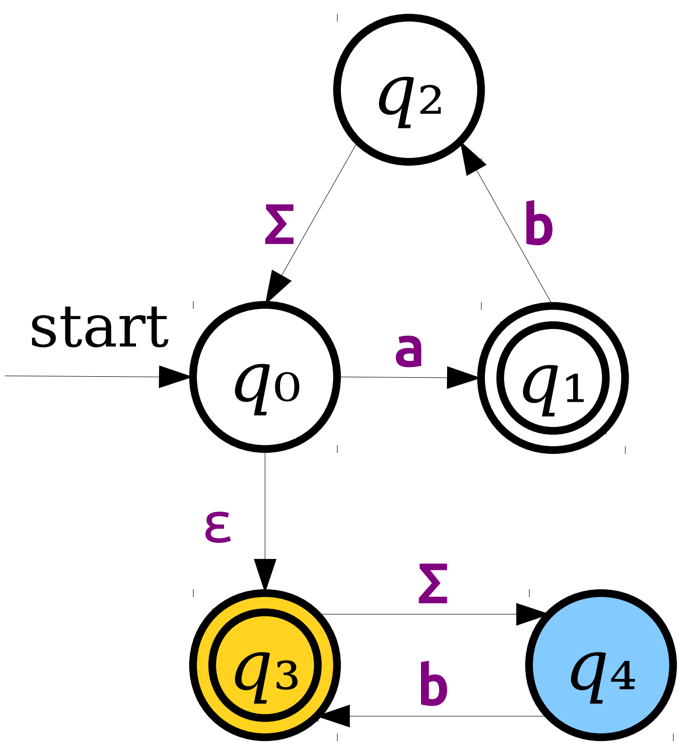
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



	a	b
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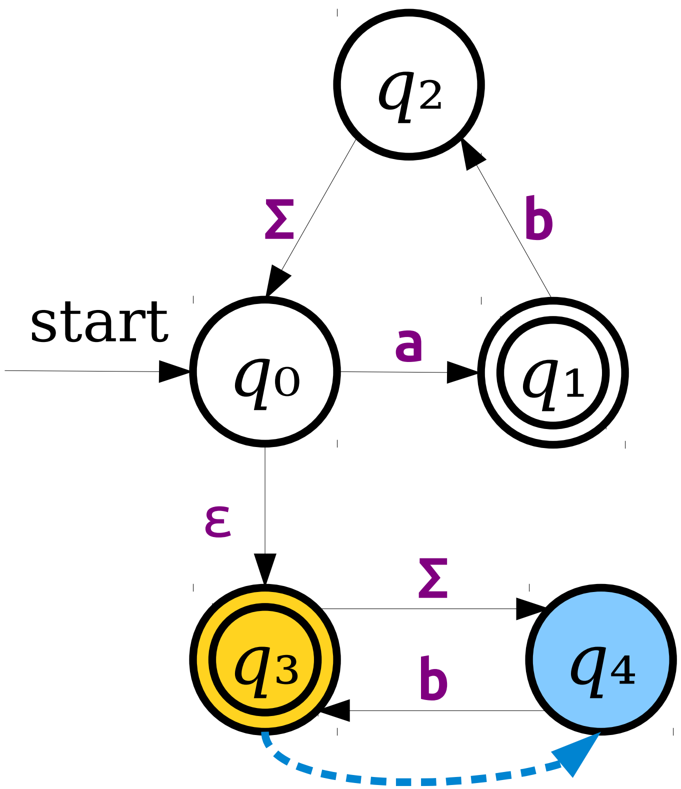
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



	a	b
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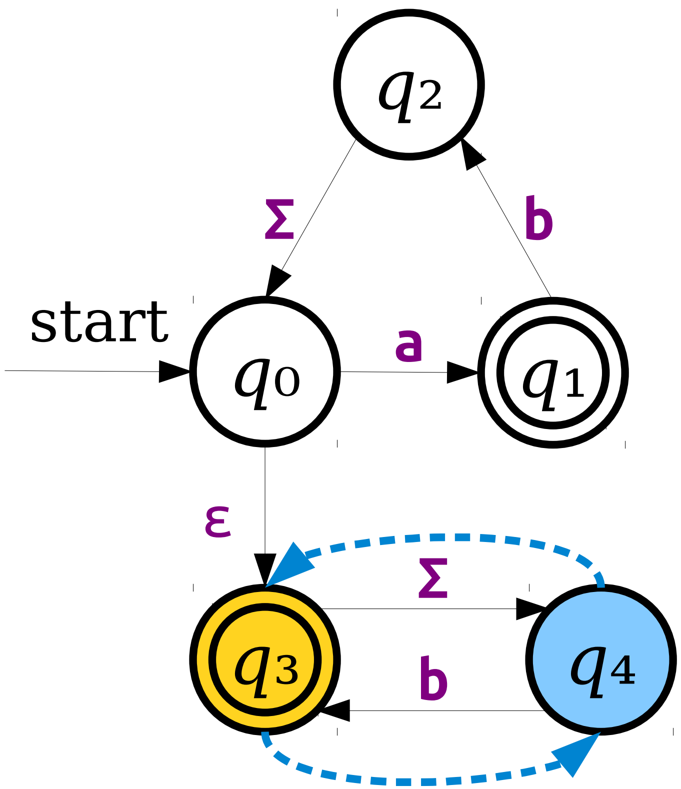
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



	a	b
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$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
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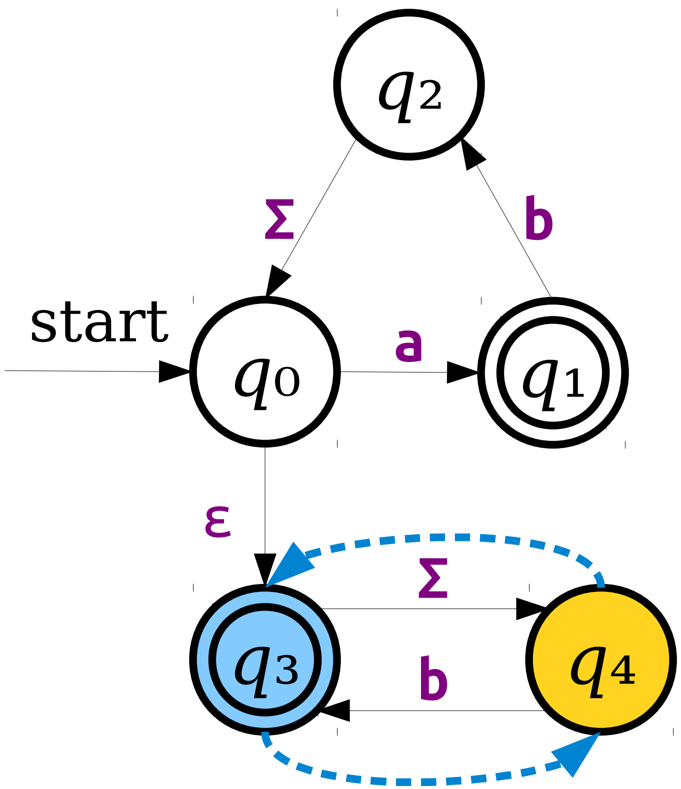
Once More, With Epsilons!



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$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
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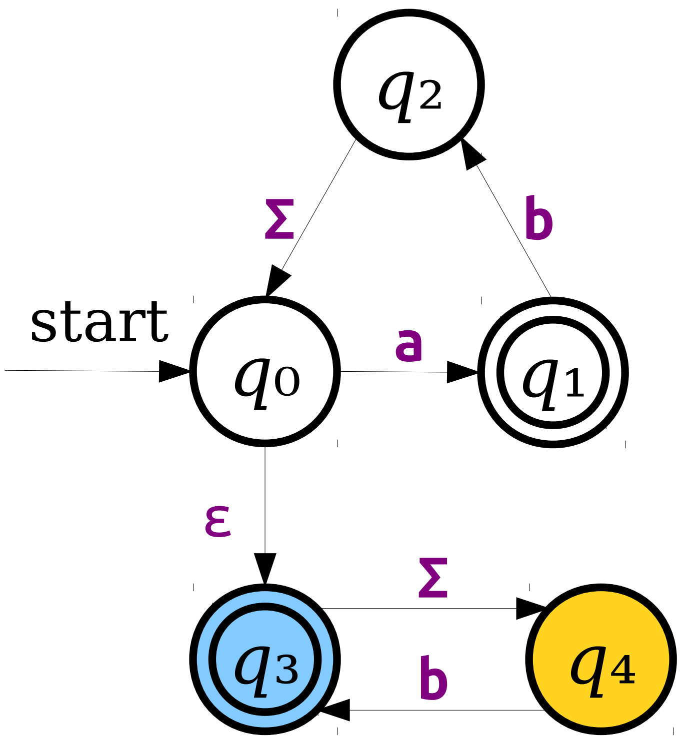
Once More, With Epsilons!



	a	b
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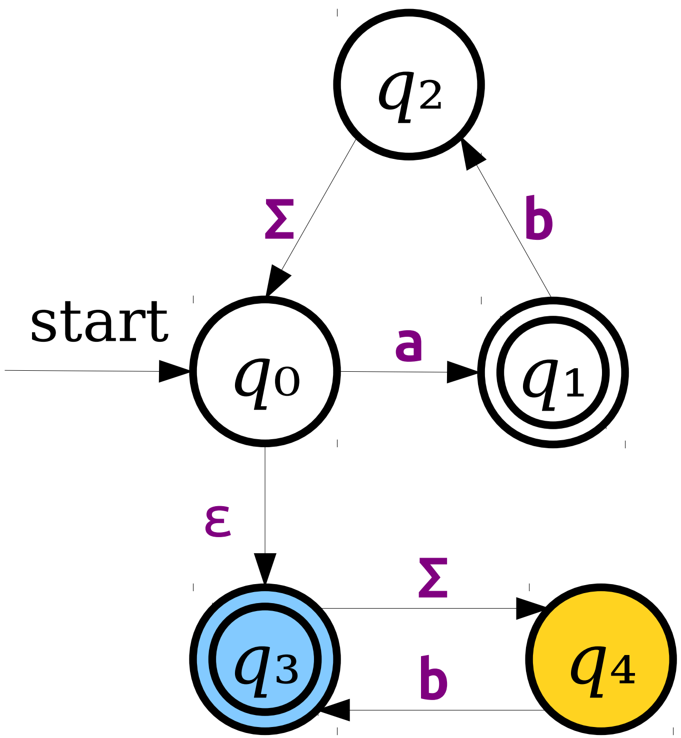
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



	a	b
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$\{q_4\}$	$\emptyset$	$\{q_3\}$
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$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
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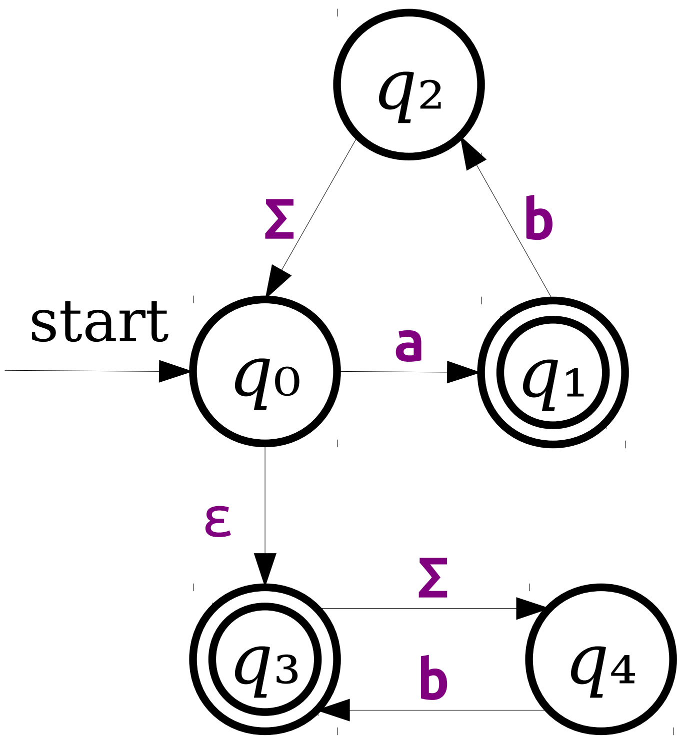
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
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$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
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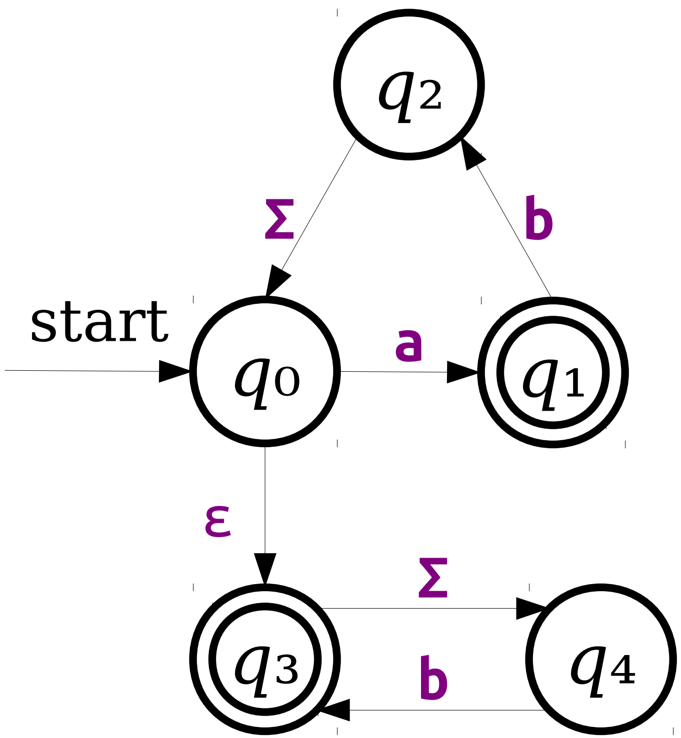
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
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$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
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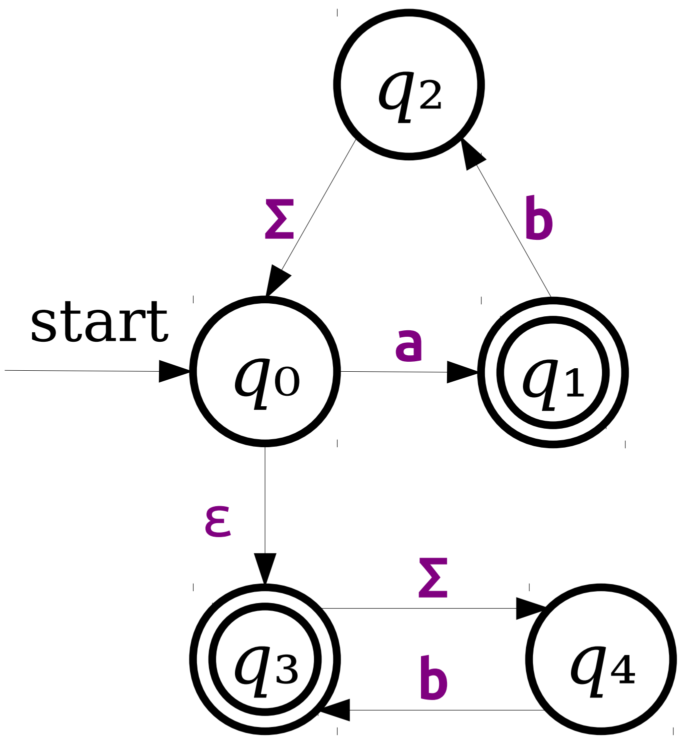
Once More, With Epsilons!



	a	b
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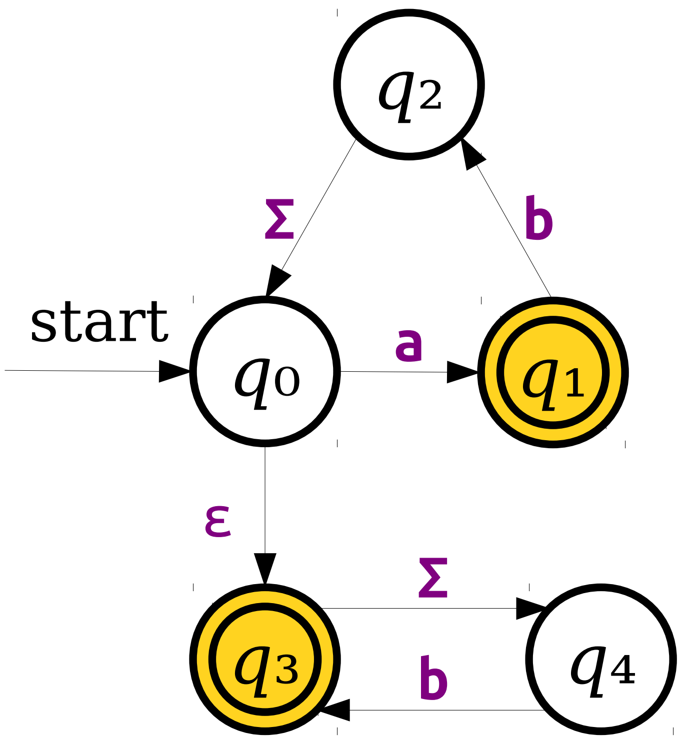
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\emptyset$	$\emptyset$	$\emptyset$

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\varnothing$	$\varnothing$	$\varnothing$

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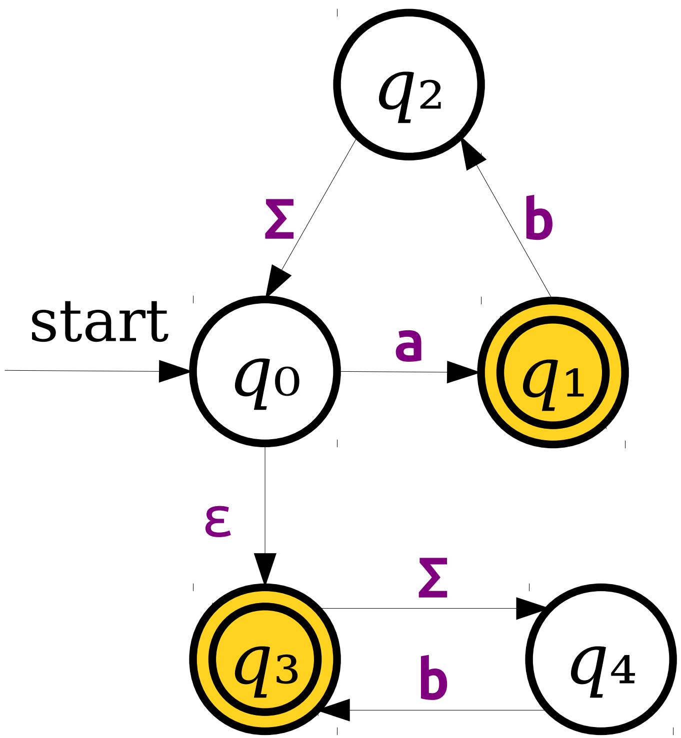

	a	b
$*\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$*\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$*\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$*\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$*\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$*\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\varnothing$	$\varnothing$	$\varnothing$

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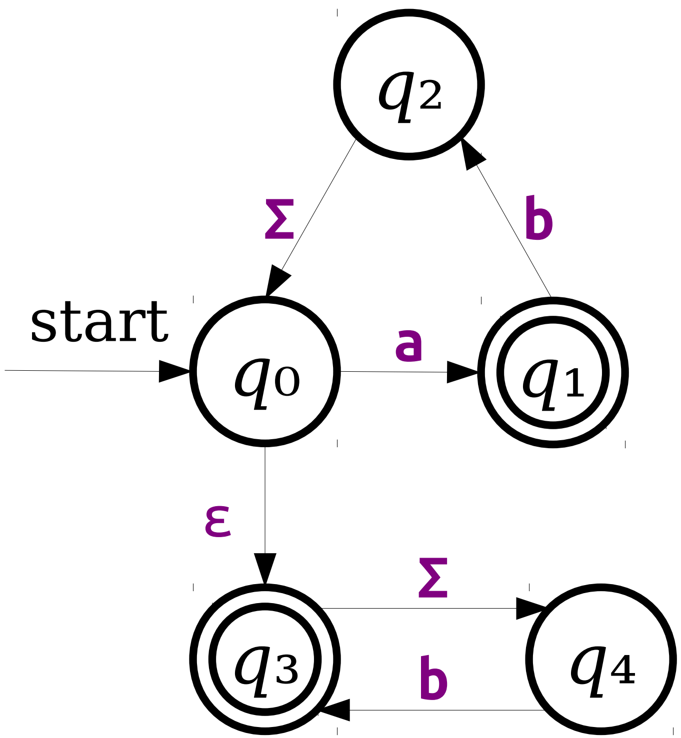

	a	b
$*\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$*\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$*\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$*\{q_3\}$	$\{q_4\}$	$\{q_4\}$
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$*\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\emptyset$	$\emptyset$	$\emptyset$

The Subset Construction

- This construction for transforming an NFA into a DFA is called the ***subset construction*** (or sometimes the ***powerset construction***).
  - Each state in the DFA is associated with a set of states in the NFA.
  - The start state in the DFA corresponds to the start state of the NFA, plus all states reachable via ε-transitions.
  - If a state q in the DFA corresponds to a set of states S in the NFA, then the transition from state q on a character `a` is found as follows:
    - Let S' be the set of states in the NFA that can be reached by following a transition labeled `a` from any of the states in S. *(This set may be empty.)*
    - Let S'' be the set of states in the NFA reachable from some state in S' by following zero or more epsilon transitions.
    - The state q in the DFA transitions on `a` to a DFA state corresponding to the set of states S''.
- ***Read Sipser for a formal account.***

The Subset Construction

- In converting an NFA to a DFA, the DFA's states correspond to sets of NFA states.
- ***Useful fact:*** $|\wp(S)| = 2^{|S|}$ for any finite set S.
- In the worst-case, the construction can result in a DFA that is *exponentially larger* than the original NFA.
- Interesting challenge: Find a language for which this worst-case behavior occurs (there are infinitely many of them!)

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

An Important Result

Theorem: A language L is regular iff there is some NFA N such that $\mathscr{L}(N) = L$.

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Why This Matters

- We now have two perspectives on regular languages:
  - Regular languages are languages accepted by DFAs.
  - Regular languages are languages accepted by NFAs.
- We can now reason about the regular languages in two different ways.

Time-Out for Announcements!

Midterm Grading

- You're done with the midterm! Woohoo!
- We'll be grading the exams over the weekend. We'll try to get them back to you as soon as possible given that we have to work around the Monday holiday.

Problem Set Five

- Problem Set Five is currently out. It's due on Friday of this week.
  - Want to use late days? Turn it in by Monday at 3:00PM.
- As always, feel free to ask questions on Piazza or to stop by office hours with questions.

Your Questions

"Why do you care about diversity in CS?"

For a lot of different reasons. Here's a few:

1. Engineering is about solving problems, and people only tend to work on problems that matter to them. Increasing diversity in CS means that more problems – and, likely, more meaningful problems – get addressed.

2. A lot of people choose not to go into CS because they don't feel welcome there, regardless of their actual interest or talent. If people are leaving the field because the culture is toxic, it's important that we fix.

3. CS provides an amazing avenue of upward economic mobility and can totally transform peoples' lives – and their family's lives. It's critical that these opportunities are available to everyone.

4. Quoting Sam Altman: "anyone who claims that CS is a meritocracy has a lot of explaining to do."

Back to CS103!

Properties of Regular Languages

The Union of Two Languages

- If L_1 and L_2 are languages over the alphabet Σ, the language $L_1 \cup L_2$ is the language of all strings in at least one of the two languages.
- If L_1 and L_2 are regular languages, is $L_1 \cup L_2$?

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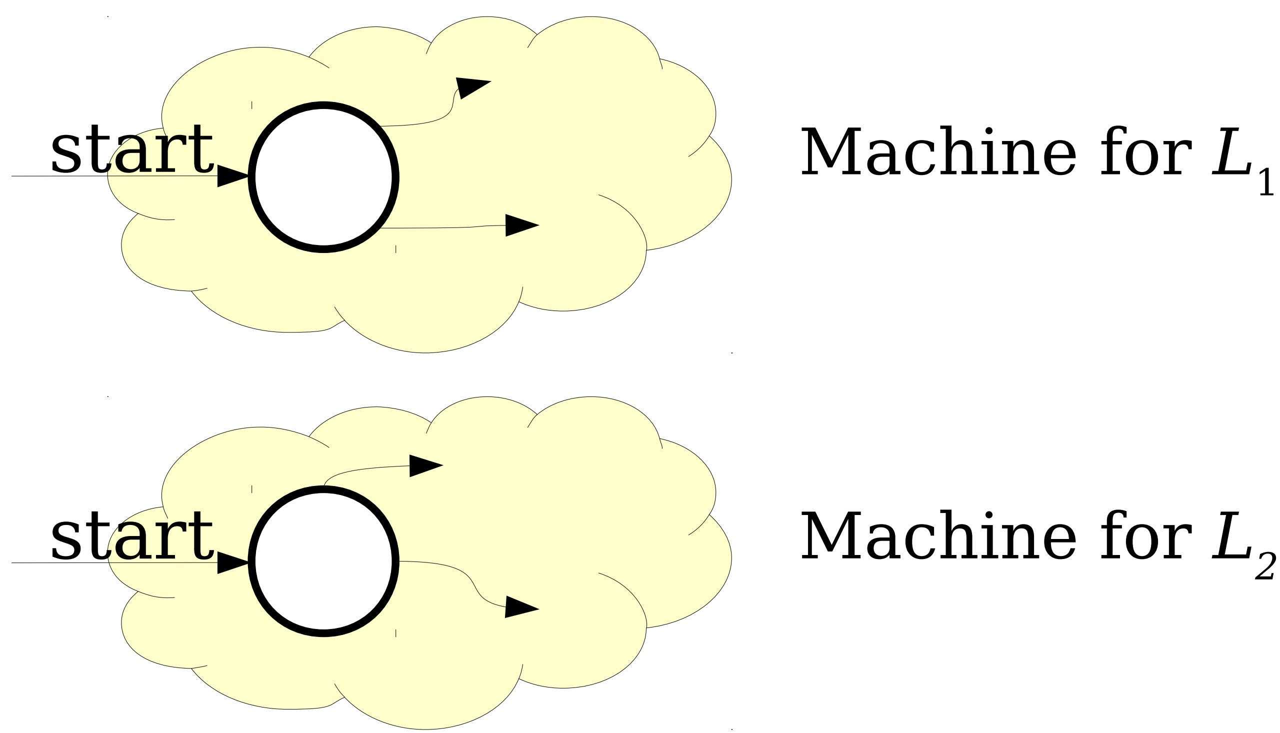

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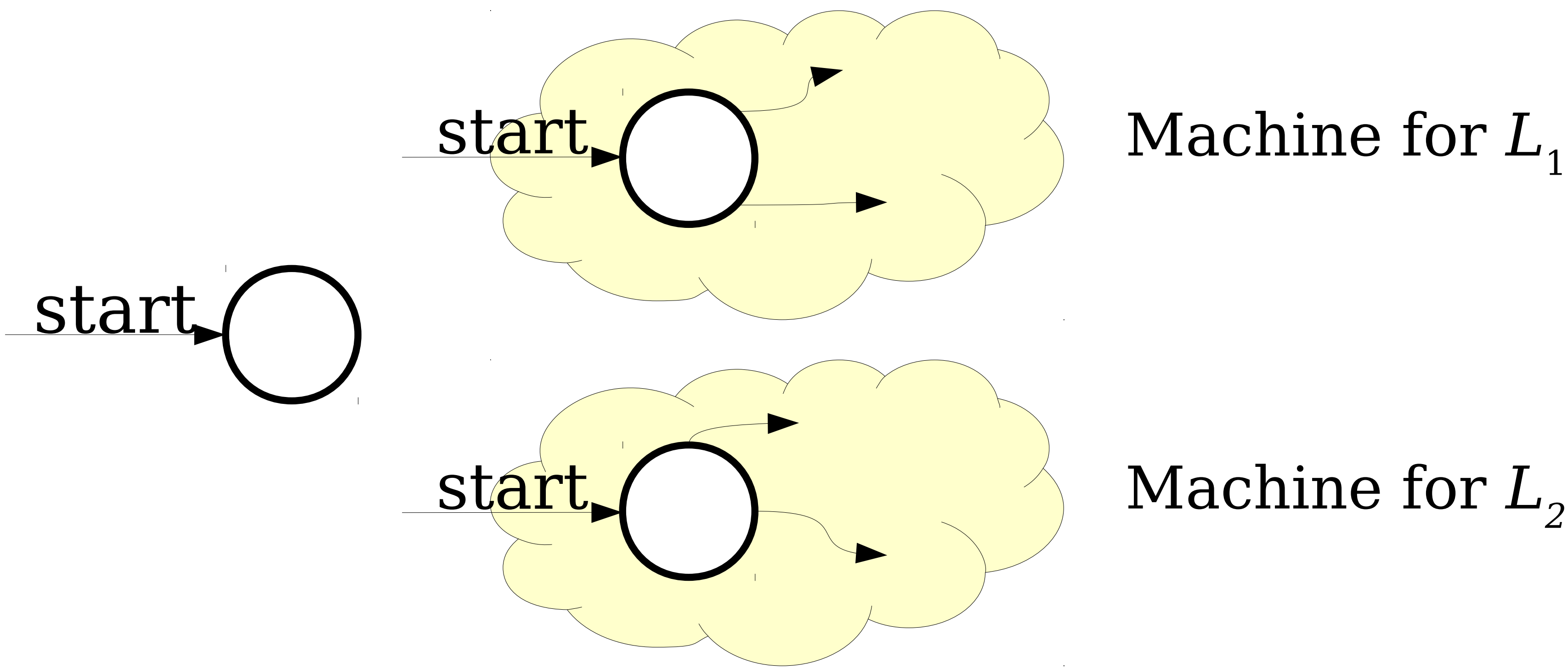

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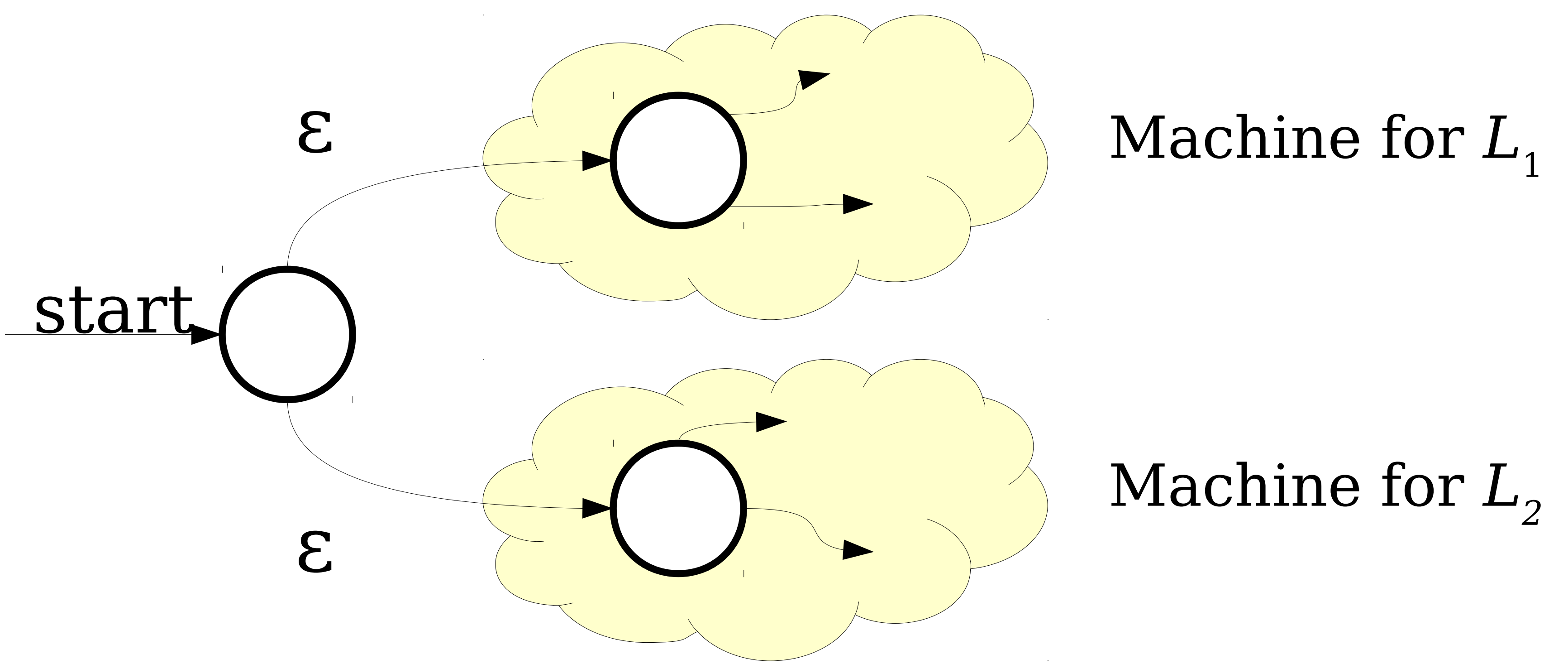

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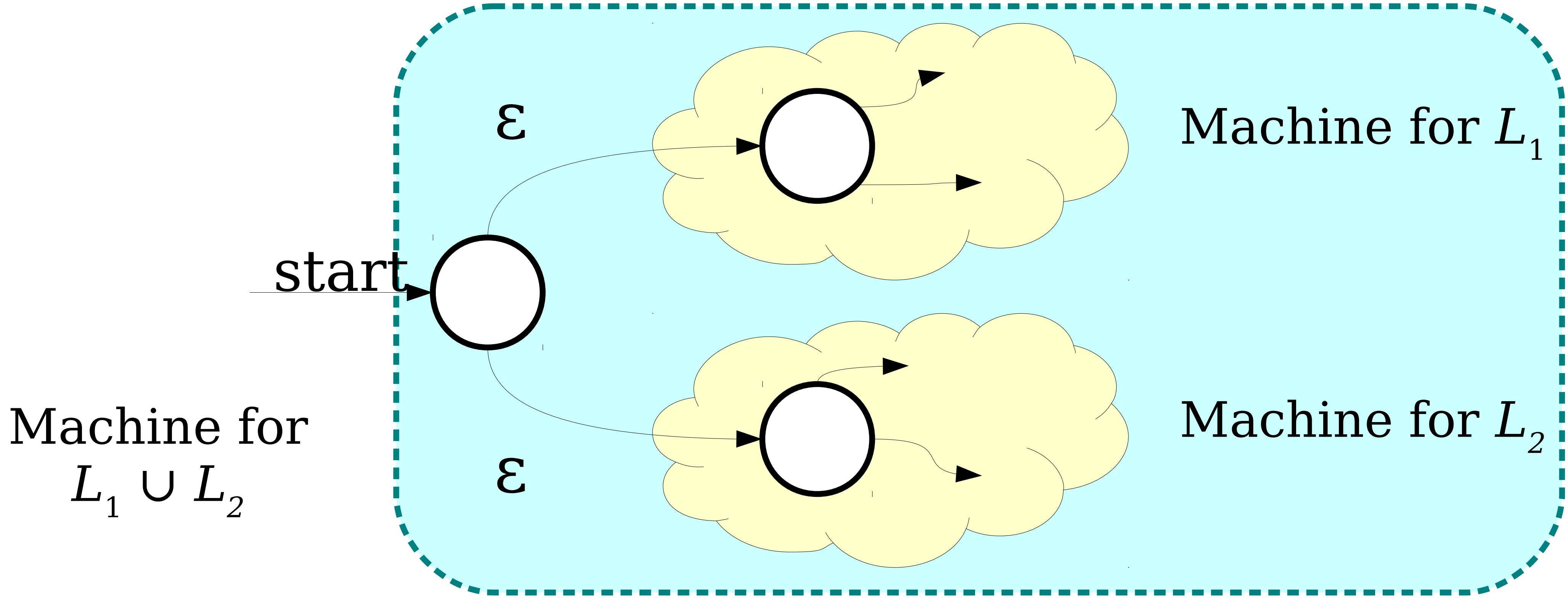

The Intersection of Two Languages

- If L_1 and L_2 are languages over Σ, then $L_1 \cap L_2$ is the language of strings in both L_1 and L_2.
- Question: If L_1 and L_2 are regular, is $L_1 \cap L_2$ regular as well?

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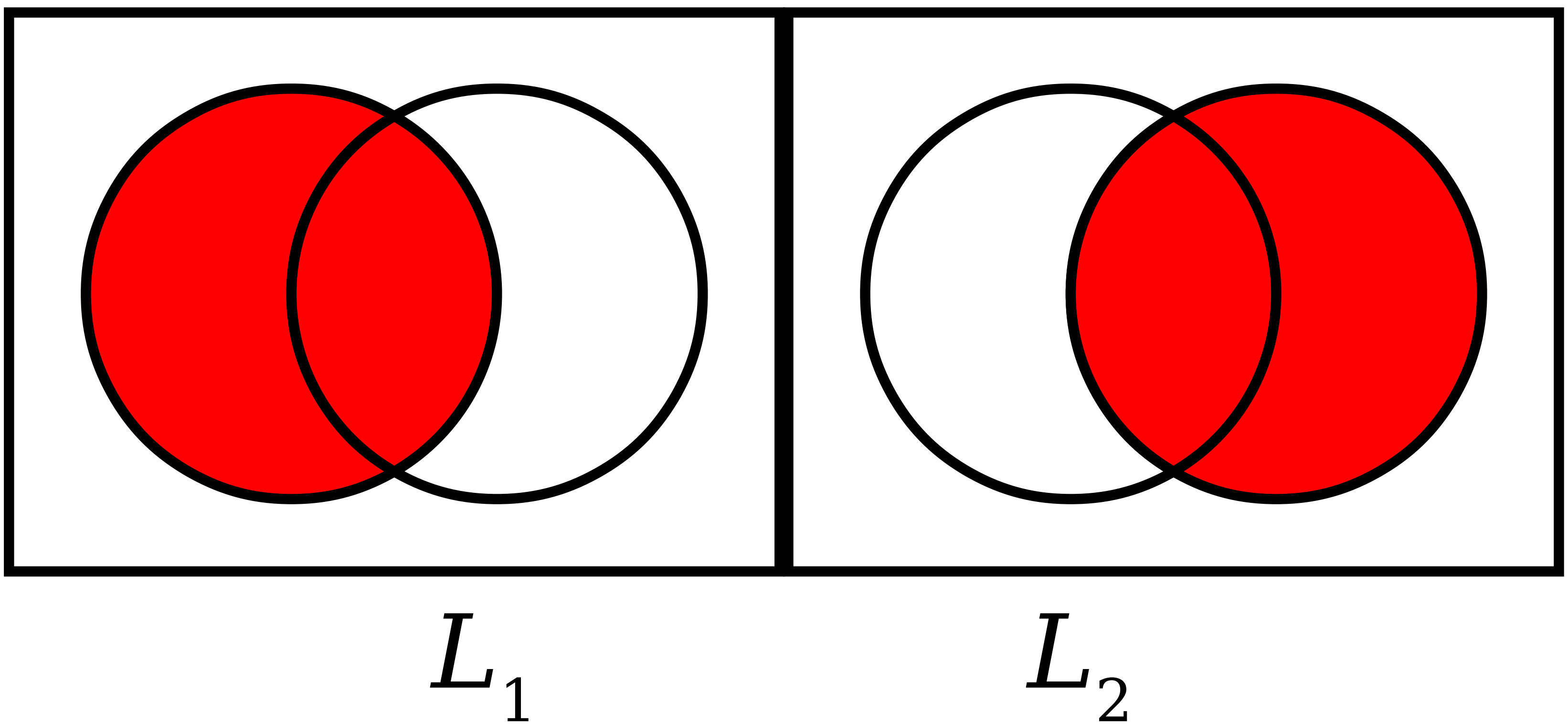

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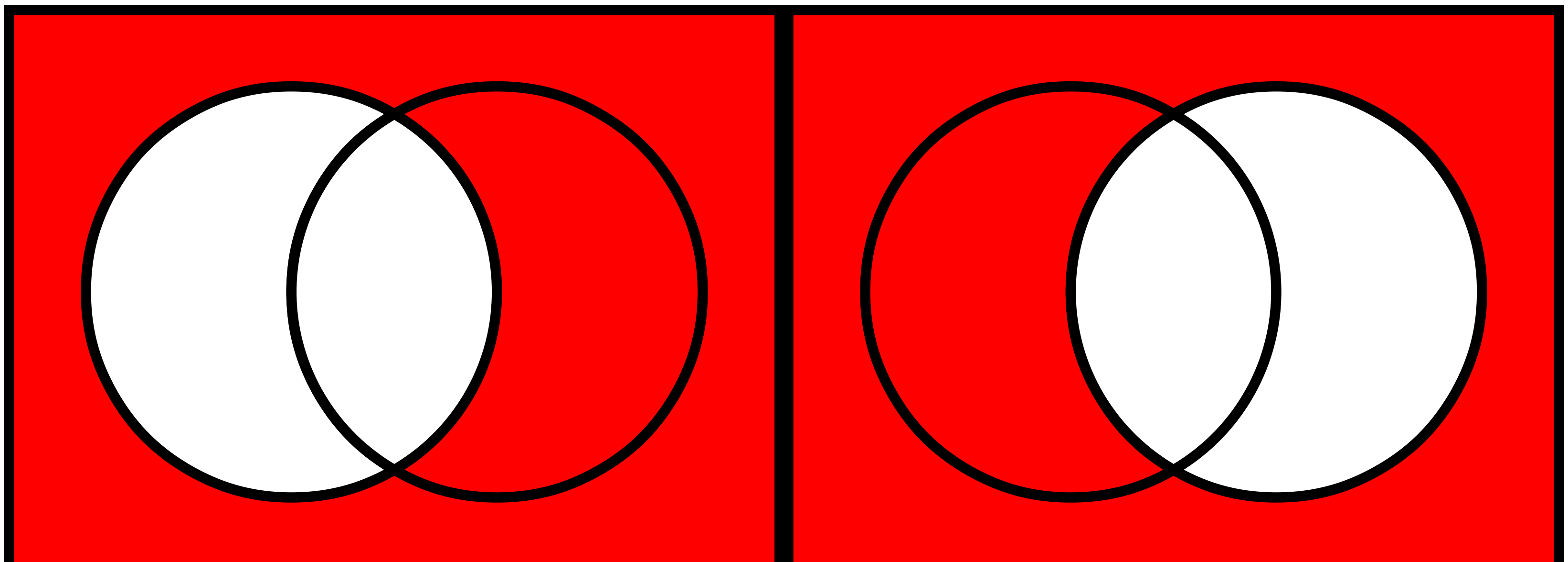

$\overline{L_1}$ $\overline{L_2}$

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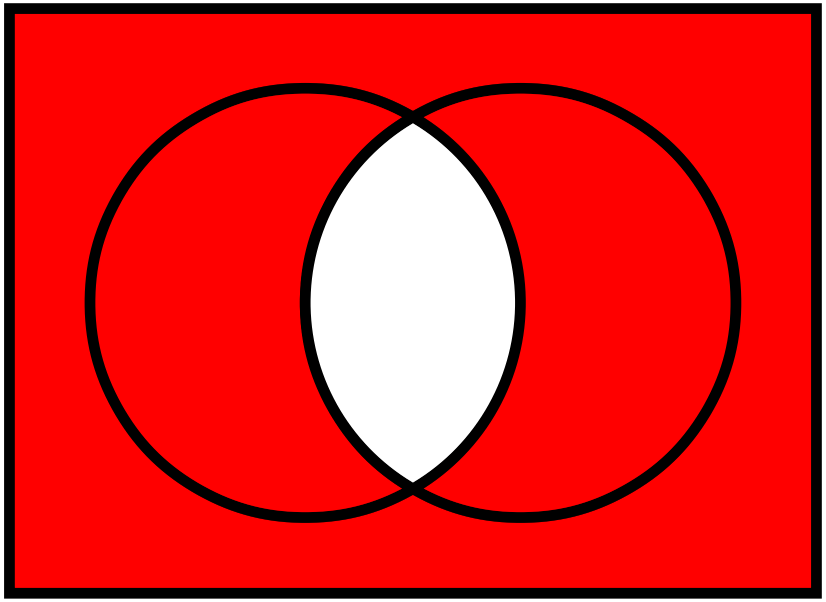

$$\overline{L}_1 \cup \overline{L}_2$$

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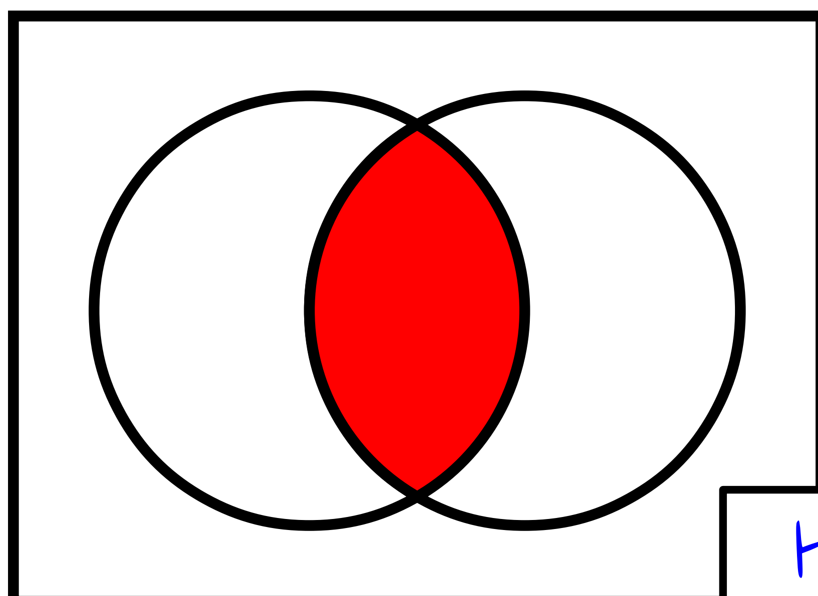

$$\overline{\overline{L_1} \cup \overline{L_2}}$$

Hey, it's De Morgan's laws!

Concatenation

String Concatenation

- If $w \in \Sigma^*$ and $x \in \Sigma^*$, the ***concatenation*** of w and x, denoted wx, is the string formed by tacking all the characters of x onto the end of w.
- Example: if w = **quo** and x = **kka**, the concatenation wx = **quokka**.
- Analogous to the + operator for strings in many programming languages.

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$\boldsymbol{L_1 L_2 = \{\, wx \in \Sigma^* \mid w \in L_1 \land x \in L_2 \,\}}$$

Concatenation Example

- Let Σ = { **`a`**, **`b`**, …, **`z`**, **`A`**, **`B`**, …, **`Z`** } and consider these languages over Σ:
  - ***Noun*** = { **`Puppy`**, **`Rainbow`**, **`Whale`**, … }
  - ***Verb*** = { **`Hugs`**, **`Juggles`**, **`Loves`**, … }
  - ***The*** = { **`The`** }
- The language ***TheNounVerbTheNoun*** is

  { **`ThePuppyHugsTheWhale`**,
  `TheWhaleLovesTheRainbow`,
  `TheRainbowJugglesTheRainbow`, … }

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$L_1L_2 = \{ wx \in \Sigma^* \mid w \in L_1 \land x \in L_2 \}$$

- Two views of L_1L_2:
  - The set of all strings that can be made by concatenating a string in L_1 with a string in L_2.
  - The set of strings that can be split into two pieces: a piece from L_1 and a piece from L_2.
- Conceptually similar to the Cartesian product of two sets, only with strings.

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?

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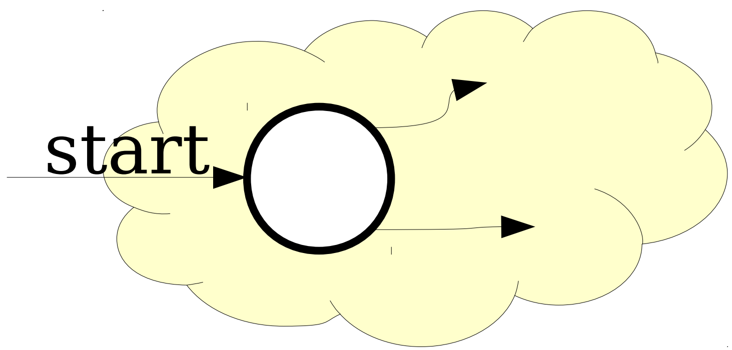
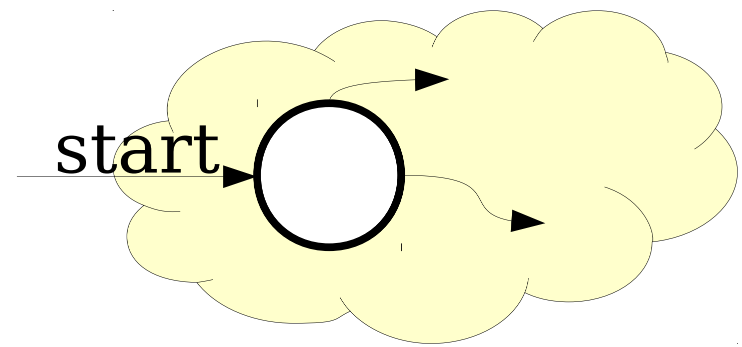


Machine for L_1



Machine for L_2

Concatenating Regular Languages

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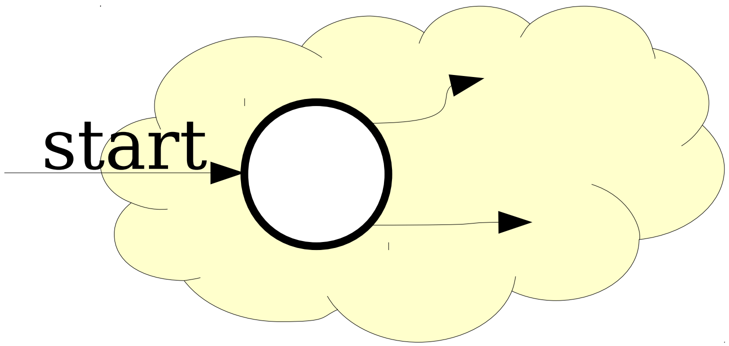

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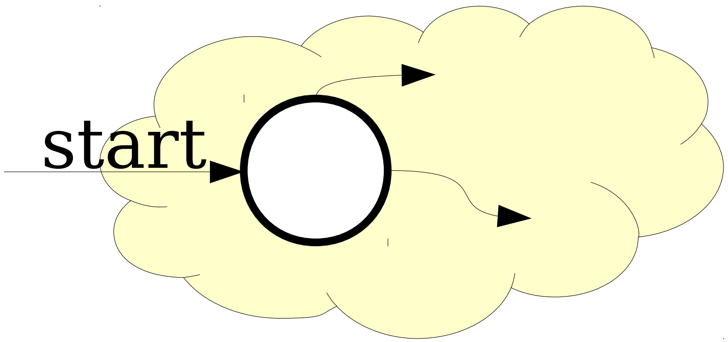

Machine for L_2

b	**o**	**o**	**k**	**k**	**e**	**e**	**p**	**e**	**r**

Concatenating Regular Languages

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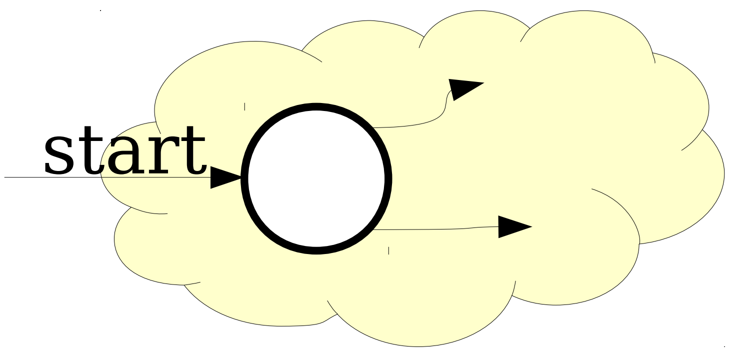

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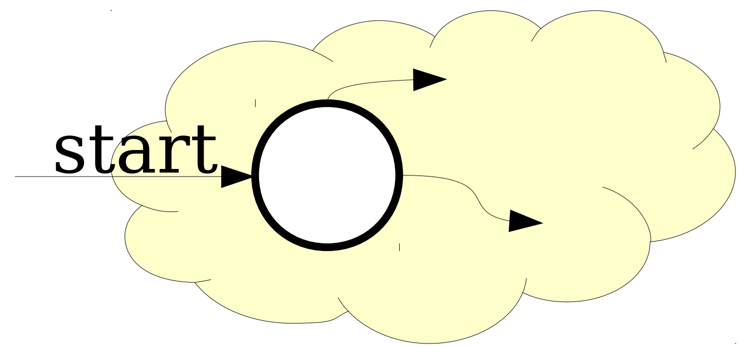

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b	**o**	**o**	**k**	**k**	**e**	**e**	**p**	**e**	**r**

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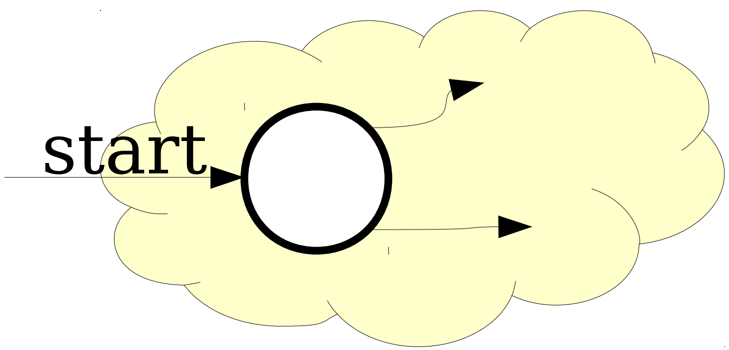

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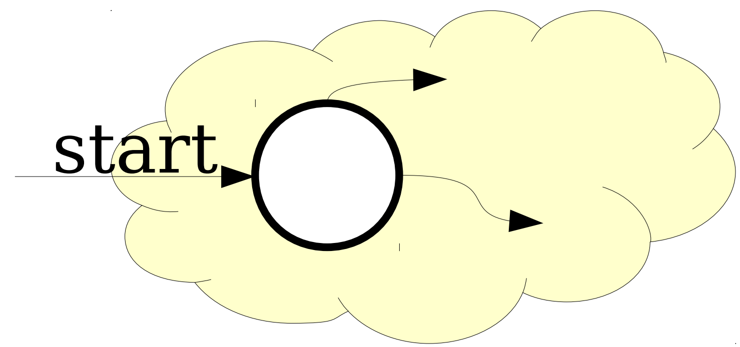


Machine for L_2

b	**o**	**o**	**k**

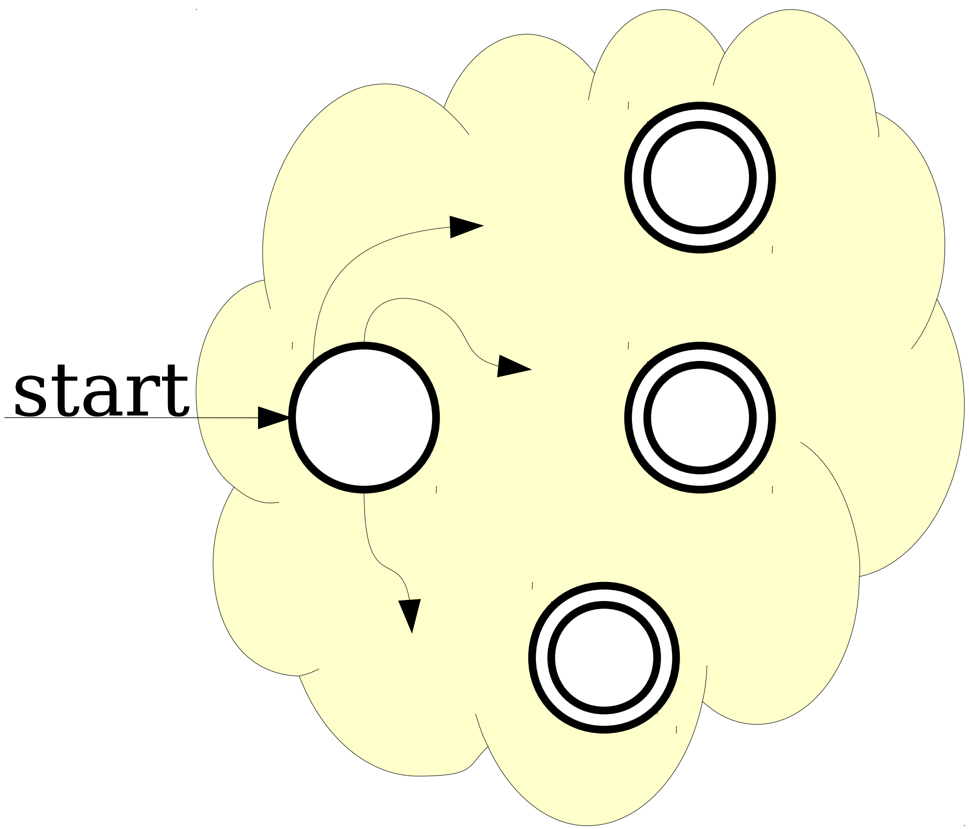
k	**e**	**e**	**p**	**e**	**r**

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?
- **Idea**: Run the automaton for L_1 on w, and whenever L_1 reaches an accepting state, optionally hand the rest off w to L_2.
  - If L_2 accepts the remainder, then L_1 accepted the first part and the string is in L_1L_2.
  - If L_2 rejects the remainder, then the split was incorrect.

Concatenating Regular Languages

Concatenating Regular Languages



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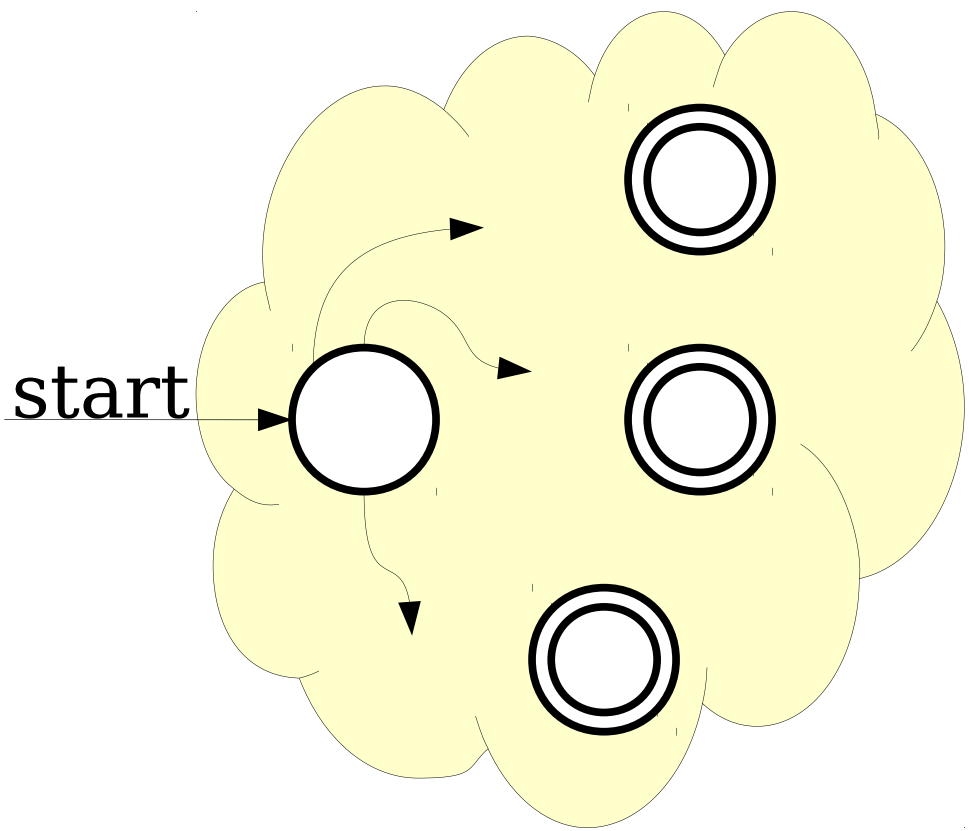
Concatenating Regular Languages



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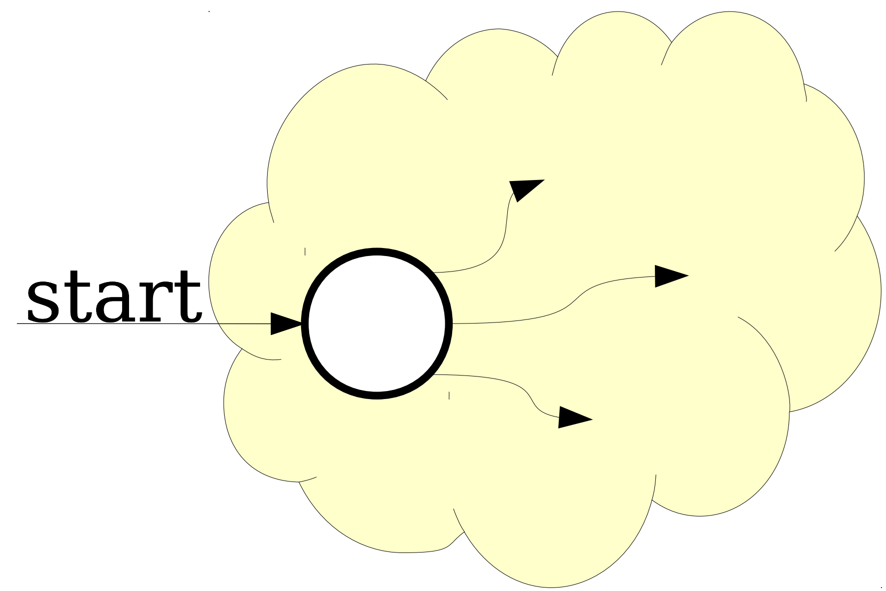

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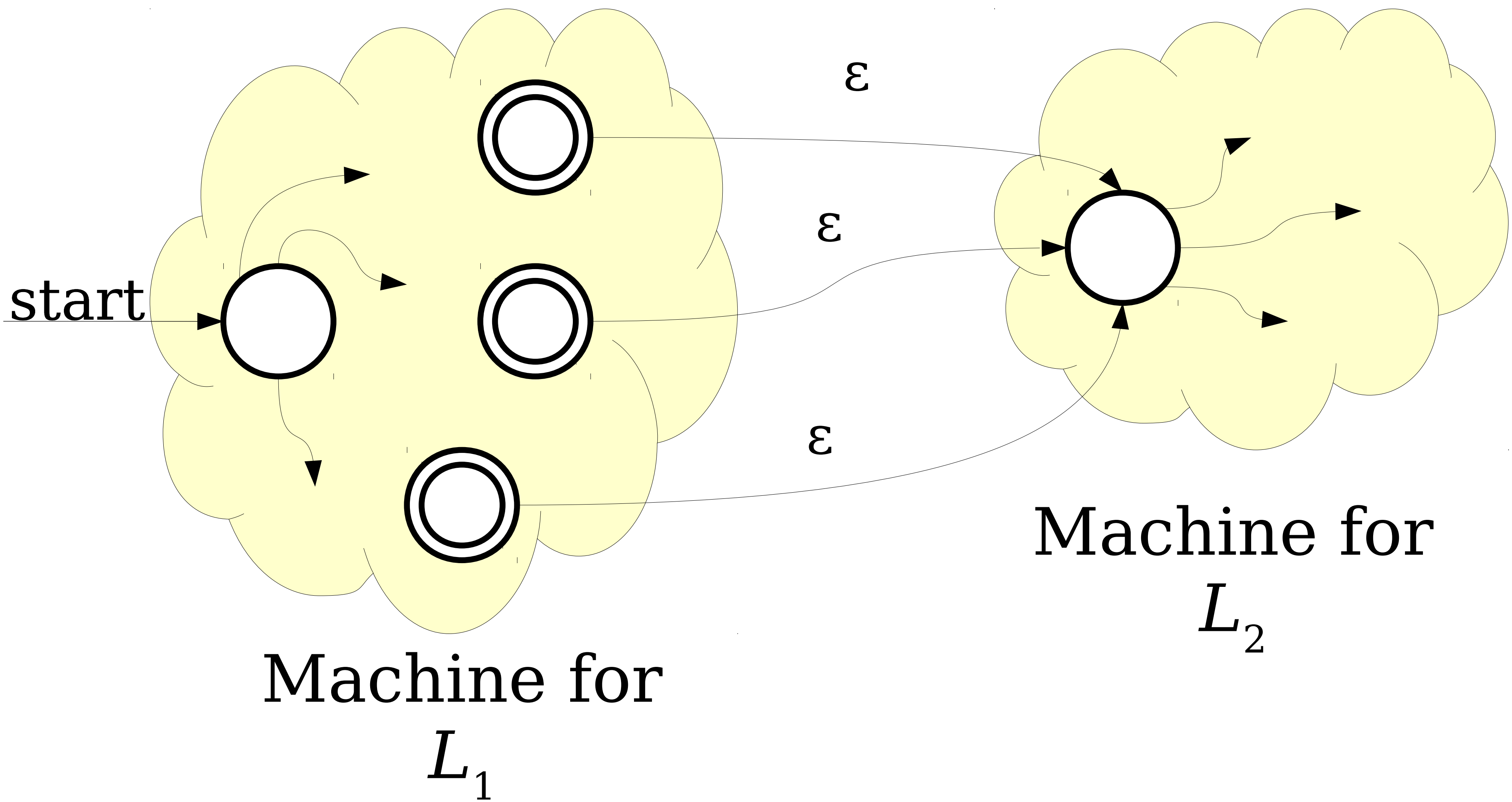

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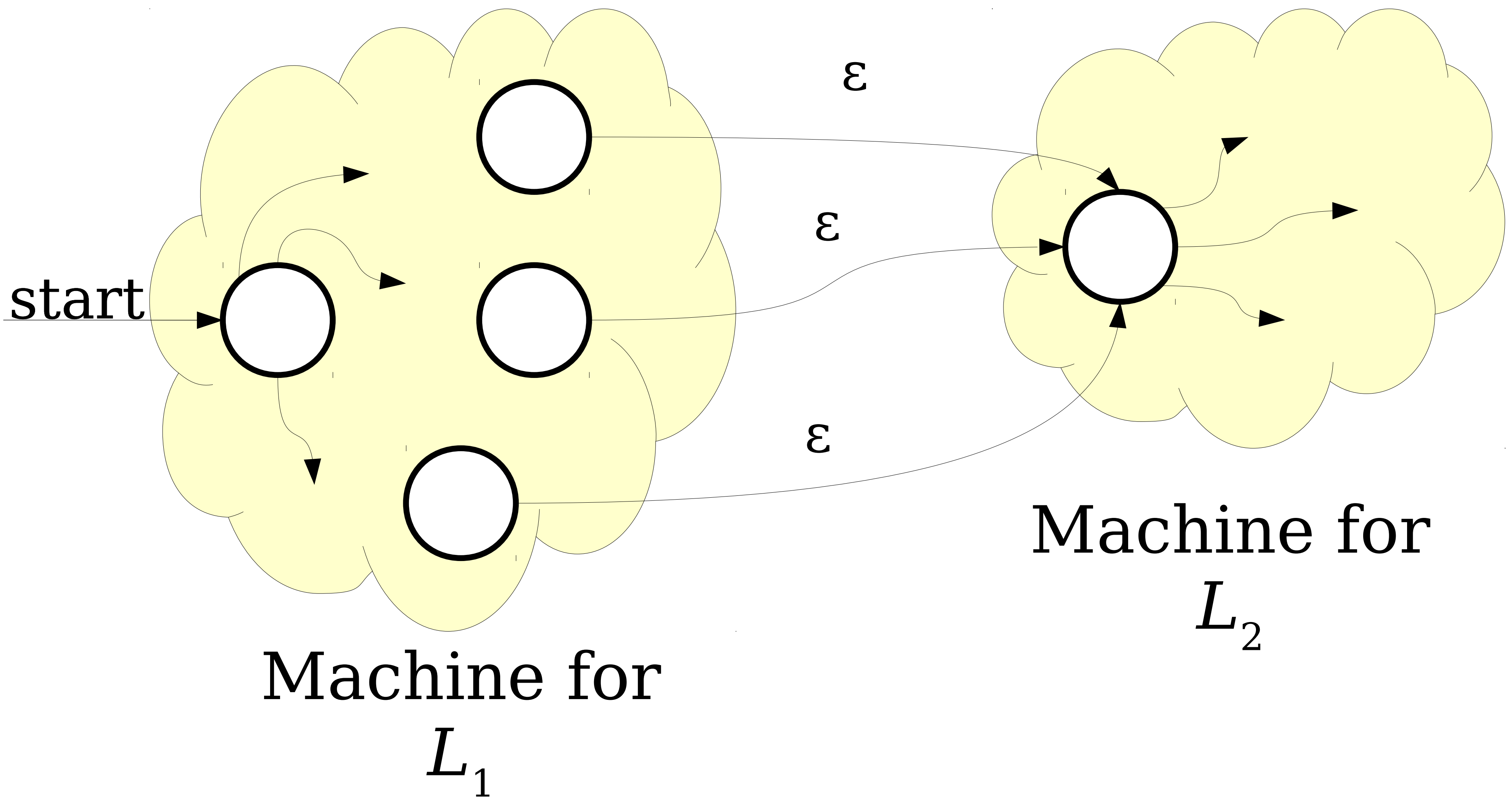

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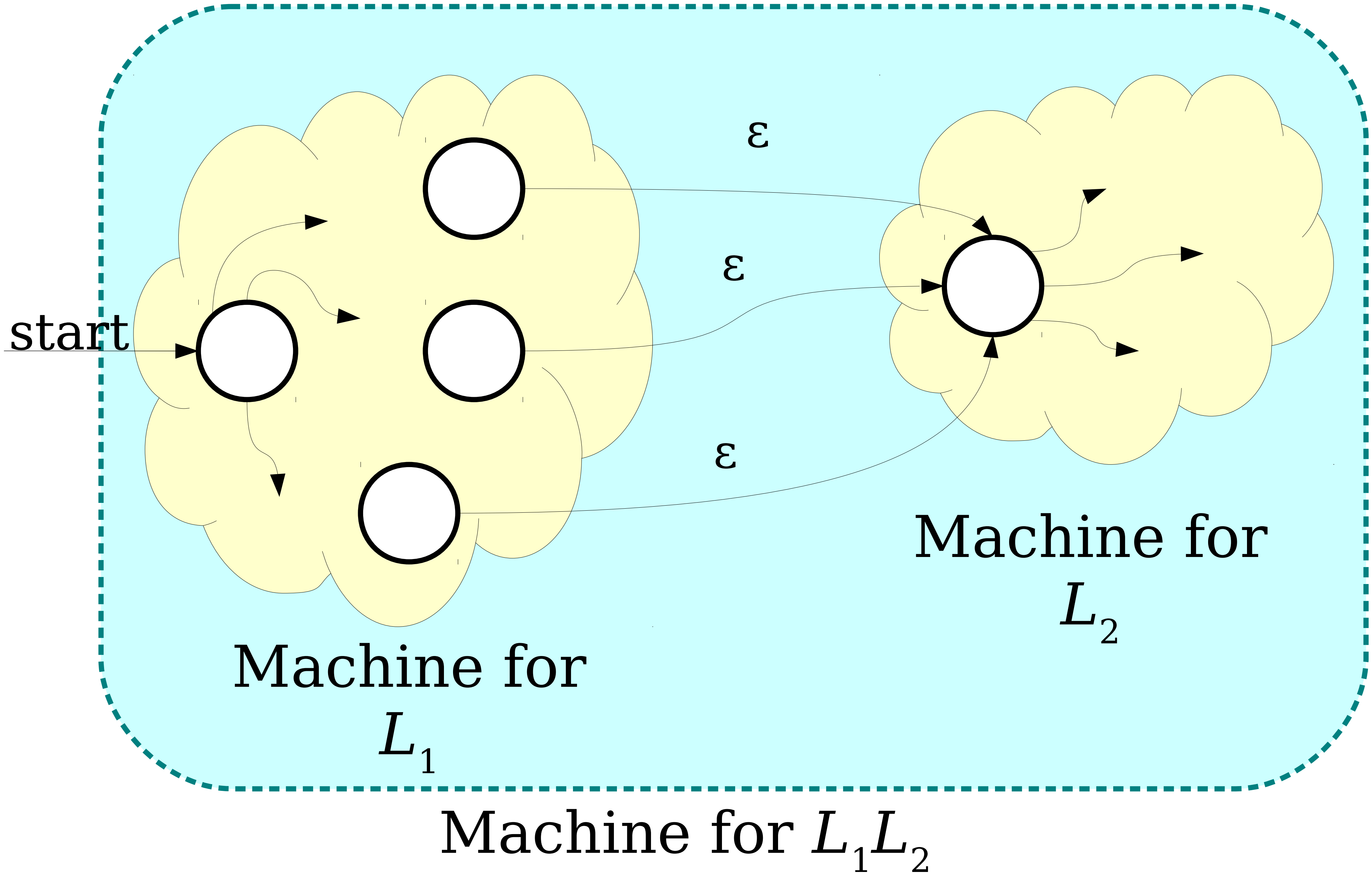


Lots and Lots of Concatenation

- Consider the language L = { **aa**, **b** }
- LL is the set of strings formed by concatenating pairs of strings in L.

{ **aaaa**, **aab**, **baa**, **bb** }

- LLL is the set of strings formed by concatenating triples of strings in L.

{ **aaaaaa**, **aaaab**, **aabaa**, **aabb**, **baaaa**, **baab**, **bbaa**, **bbb**}

- $LLLL$ is the set of strings formed by concatenating quadruples of strings in L.

{ **aaaaaaaa**, **aaaaaab**, **aaaabaa**, **aaaabb**, **aabaaaa**, **aabaab**, **aabbaa**, **aabbb**, **baaaaaa**, **baaaab**, **baabaa**, **baabb**, **bbaaaa**, **bbaab**, **bbbaa**, **bbbb**}

Language Exponentiation

- We can define what it means to "exponentiate" a language as follows:
- $L^0 = \{\varepsilon\}$
  - The set containing just the empty string.
  - Idea: Any string formed by concatenating zero strings together is the empty string.
- $L^{n+1} = LL^n$
  - Idea: Concatenating (n+1) strings together works by concatenating n strings, then concatenating one more.
- ***Question:*** Why define $L^0 = \{\varepsilon\}$?

The Kleene Closure

- An important operation on languages is the ***Kleene Closure***, which is defined as

$$L^* = \bigcup_{i=0}^{\infty} L^i$$

- Mathematically:

$$\boldsymbol{w \in L^*} \quad \textbf{iff} \quad \boldsymbol{\exists n \in \mathbb{N}.\ w \in L^n}$$

- Intuitively, all possible ways of concatenating any number of copies of strings in L together.

The Kleene Closure

If L = { **a**, **bb** }, then L^* = {

ε,

a, **bb**,

aa, **abb**, **bba**, **bbbb**,

aaa, **aabb**, **abba**, **abbbb**, **bbaa**, **bbabb**, **bbbba**, **bbbbbb**,

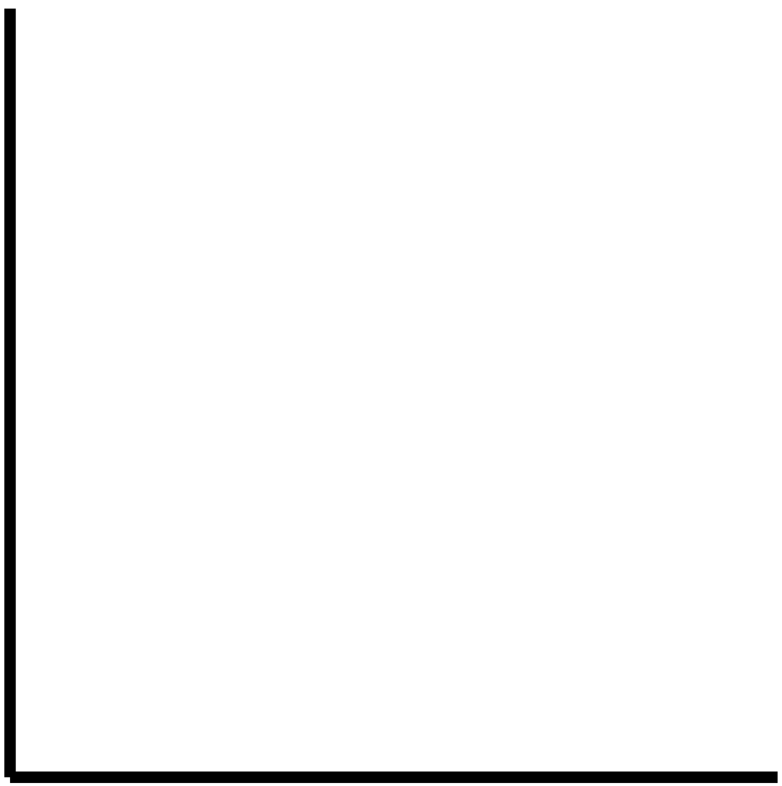
…

}

Reasoning about Infinity

- If L is regular, is L^* necessarily regular?
- **A Bad Line of Reasoning**:
  - $L^0 = \{ \varepsilon \}$ is regular.
  - $L^1 = L$ is regular.
  - $L^2 = LL$ is regular
  - $L^3 = L(LL)$ is regular
  - …
  - Regular languages are closed under union.
  - So the union of all these languages is regular.

Reasoning about Infinity



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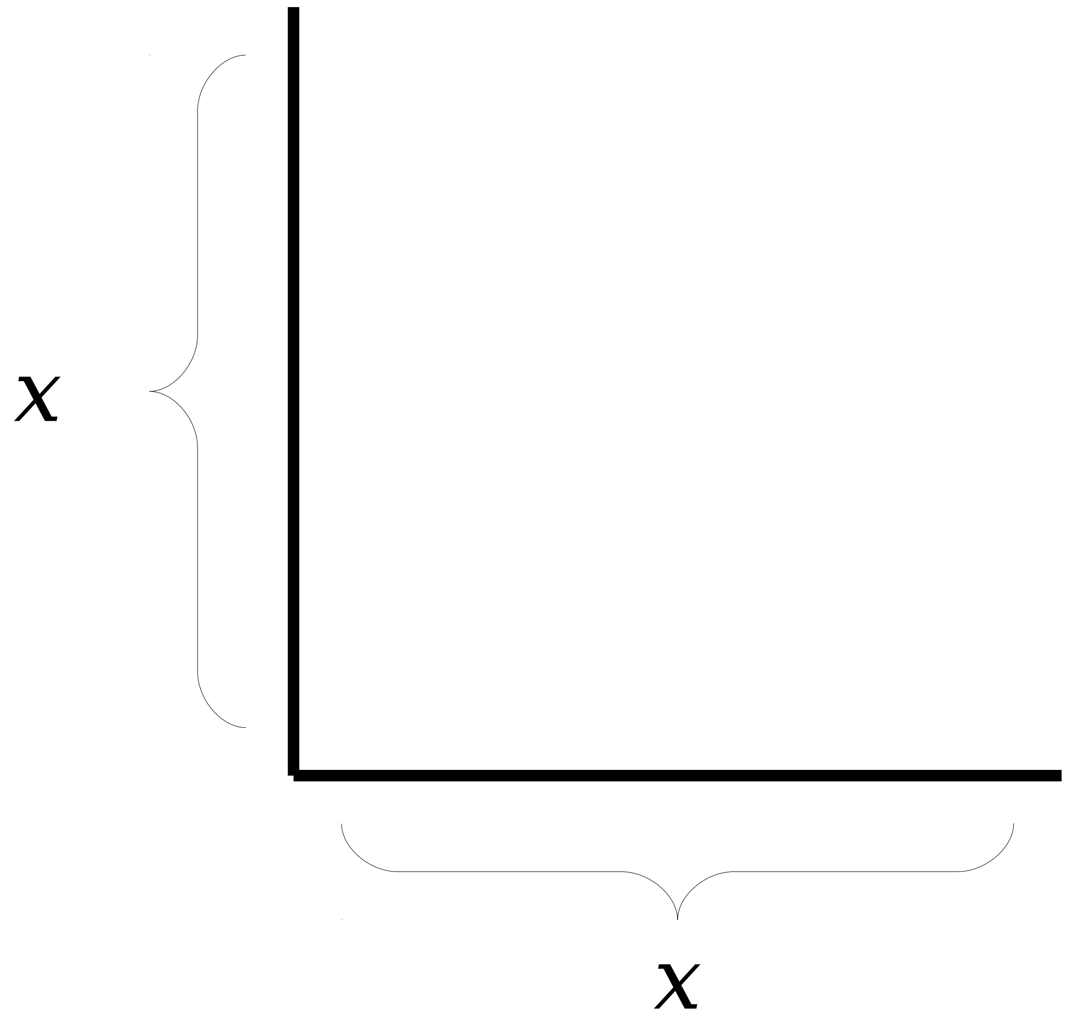

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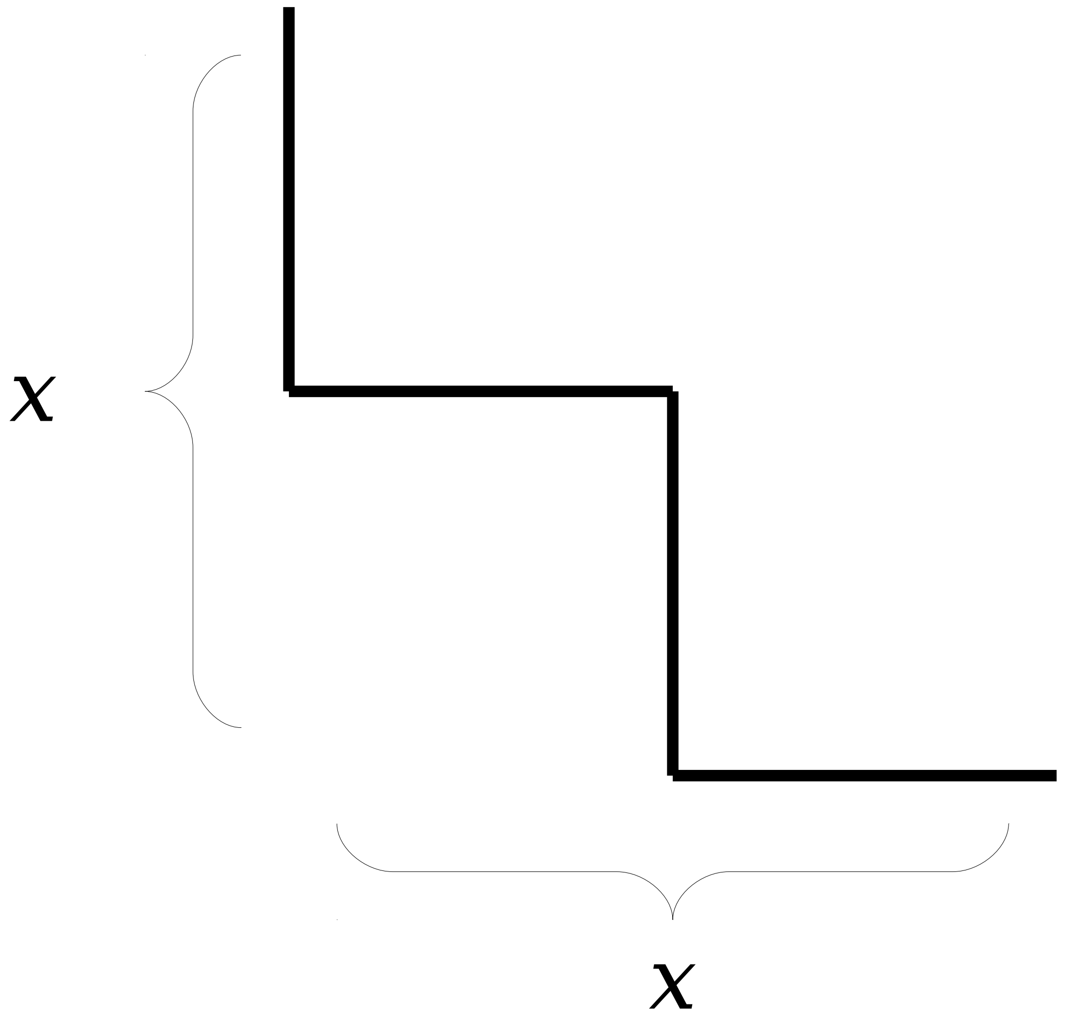

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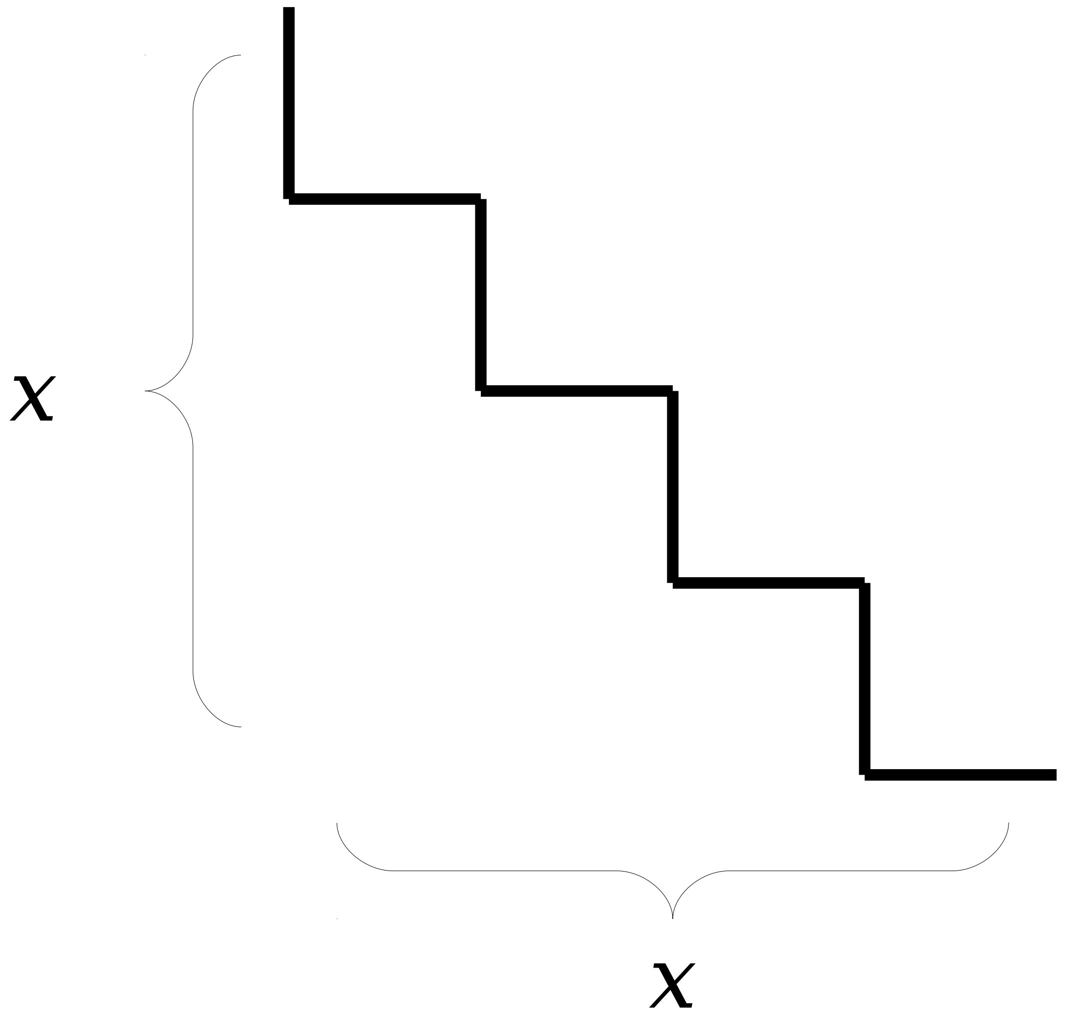

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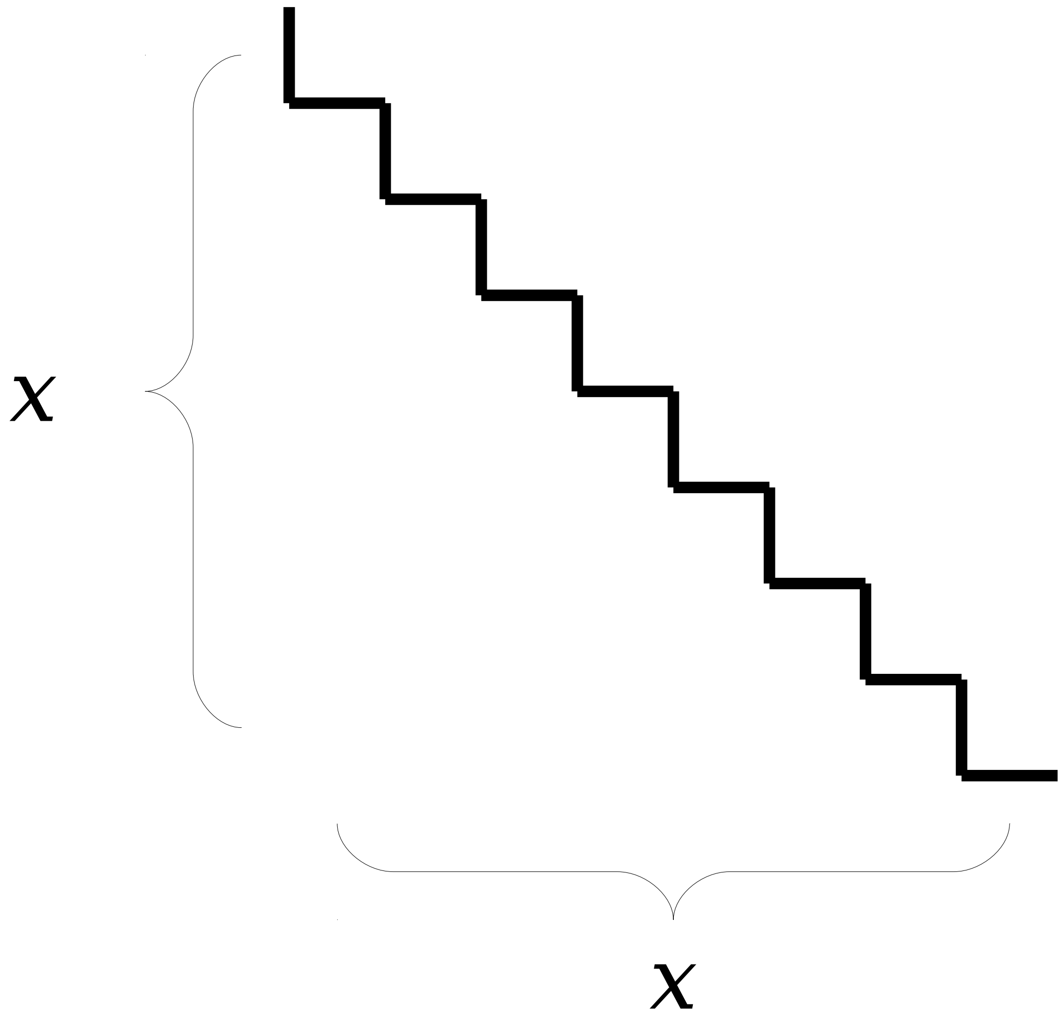

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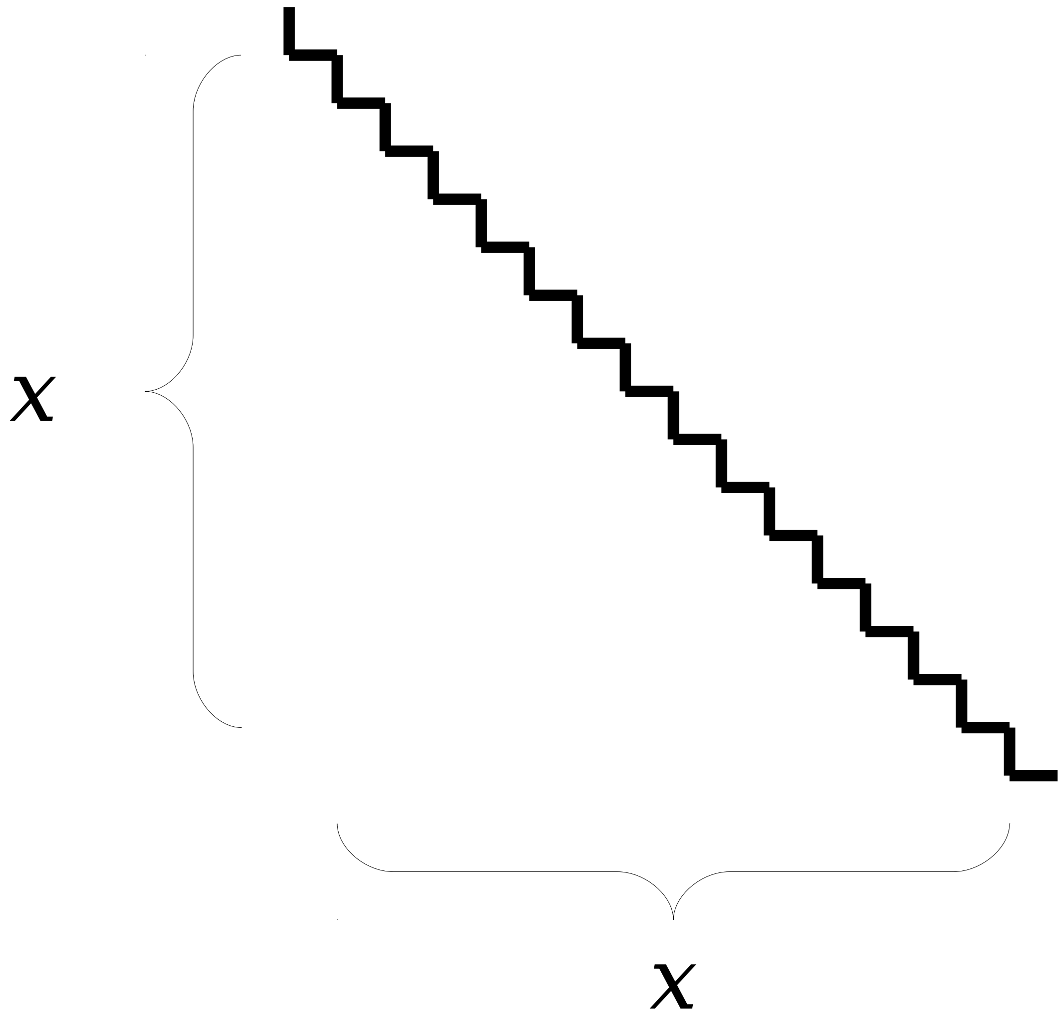
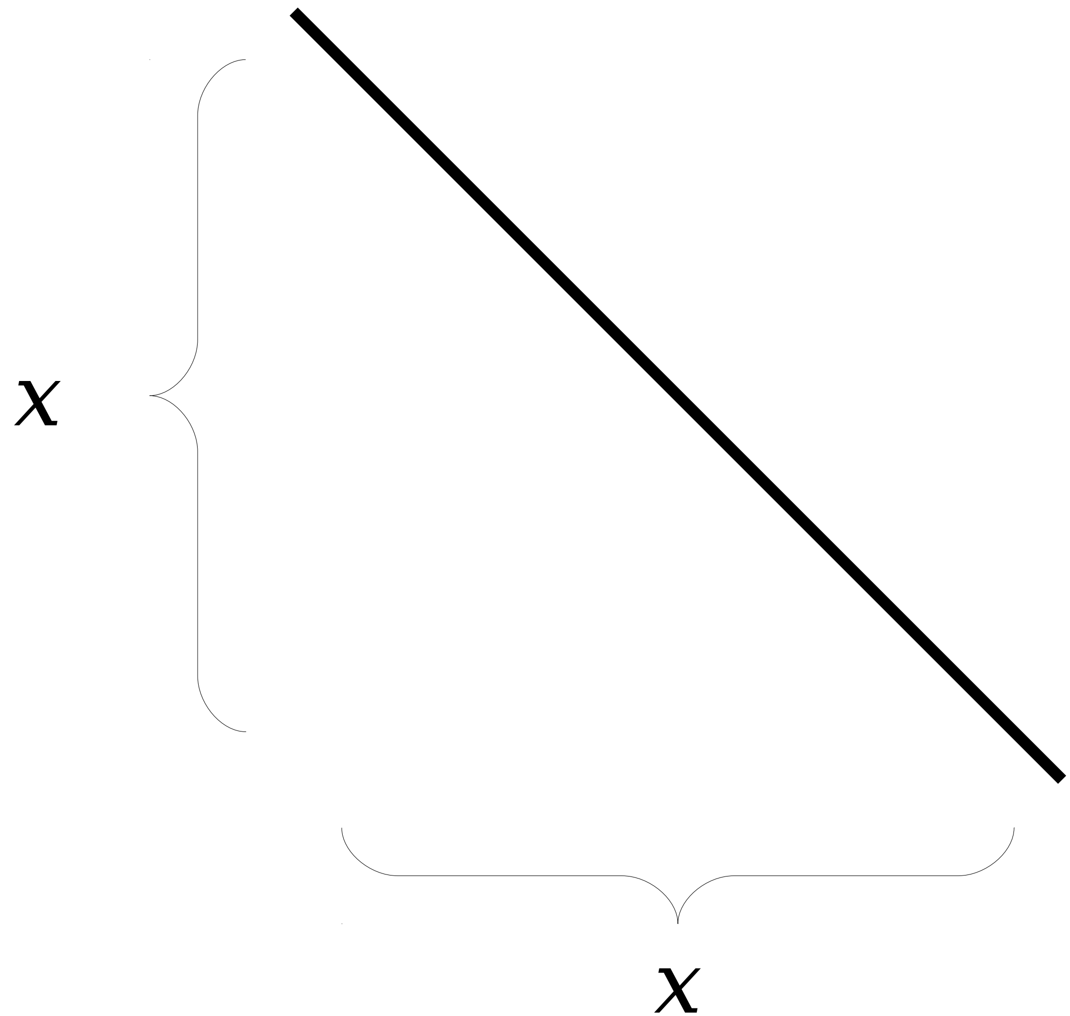
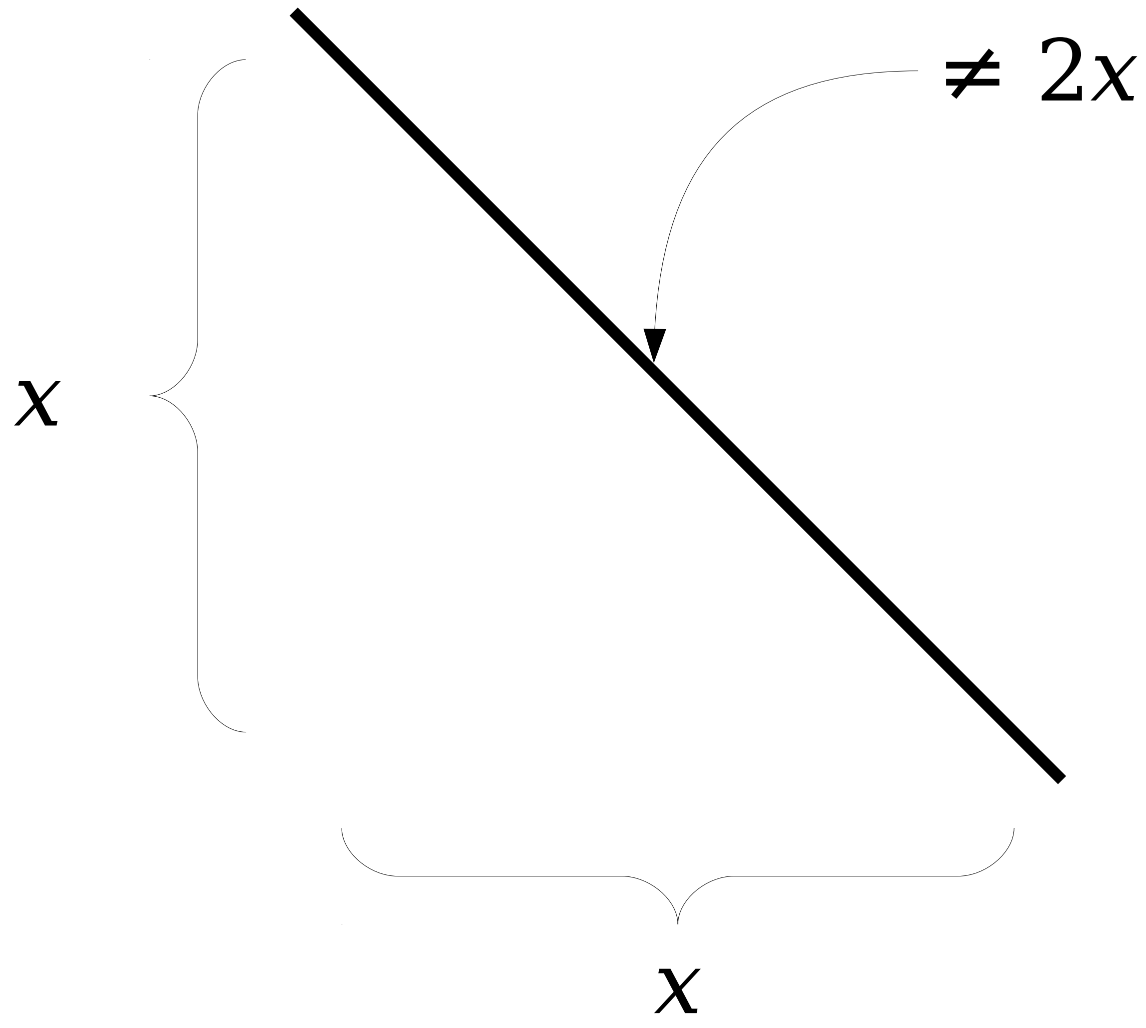


Reasoning about Infinity



Reasoning about Infinity



Reasoning about Infinity

$$0.9 < 1$$

Reasoning about Infinity

$$0.99 < 1$$

Reasoning about Infinity

$$0.999 < 1$$

Reasoning about Infinity

$$0.9999 < 1$$

Reasoning about Infinity

$$0.9999\overline{9} < 1$$

Reasoning about Infinity

$$0.9999\overline{9} \not< 1$$

Reasoning about Infinity

0 is finite

Reasoning about Infinity

1 is finite

Reasoning about Infinity

2 is finite

Reasoning about Infinity

3 is finite

Reasoning about Infinity

4 is finite

Reasoning about Infinity

∞ is finite

Reasoning about Infinity

∞ is finite

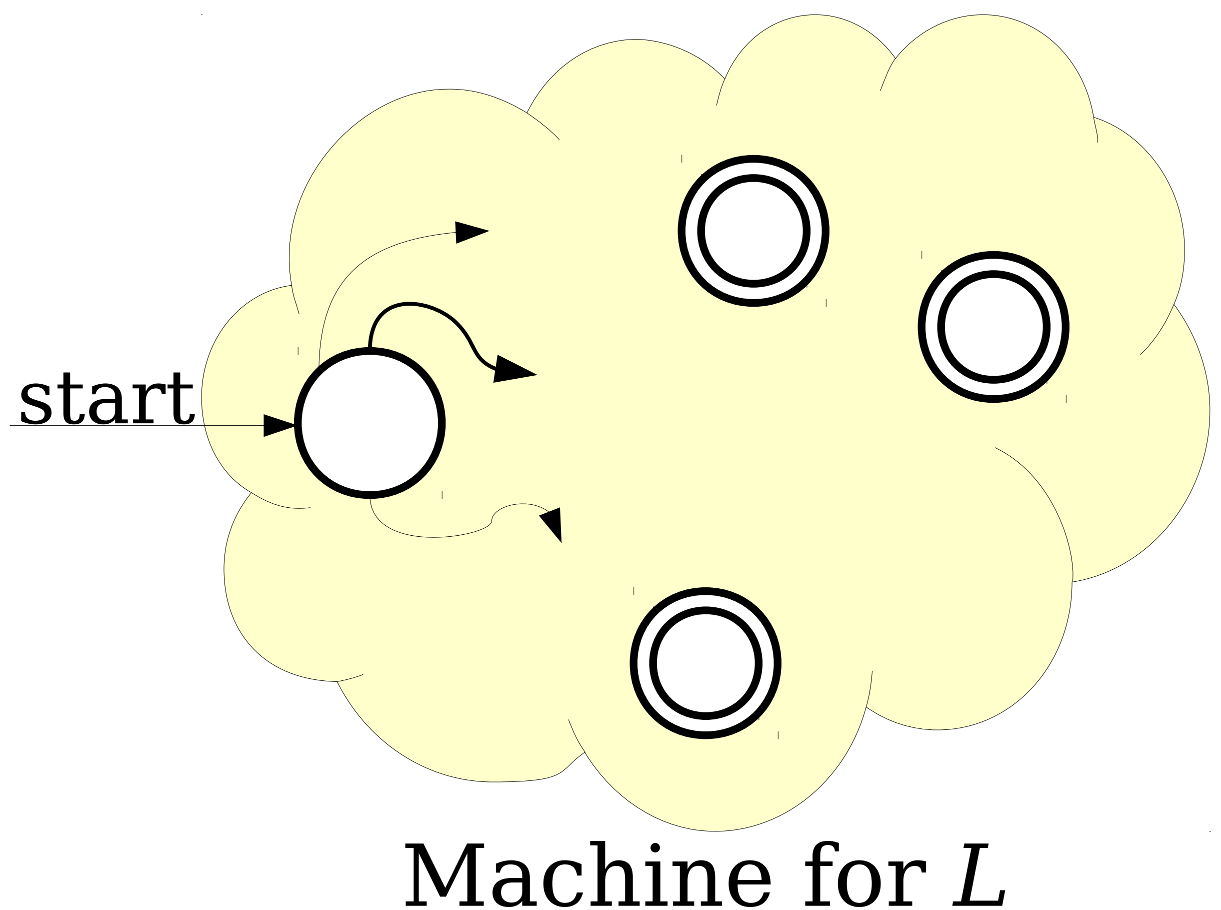
^ not

Reasoning About the Infinite

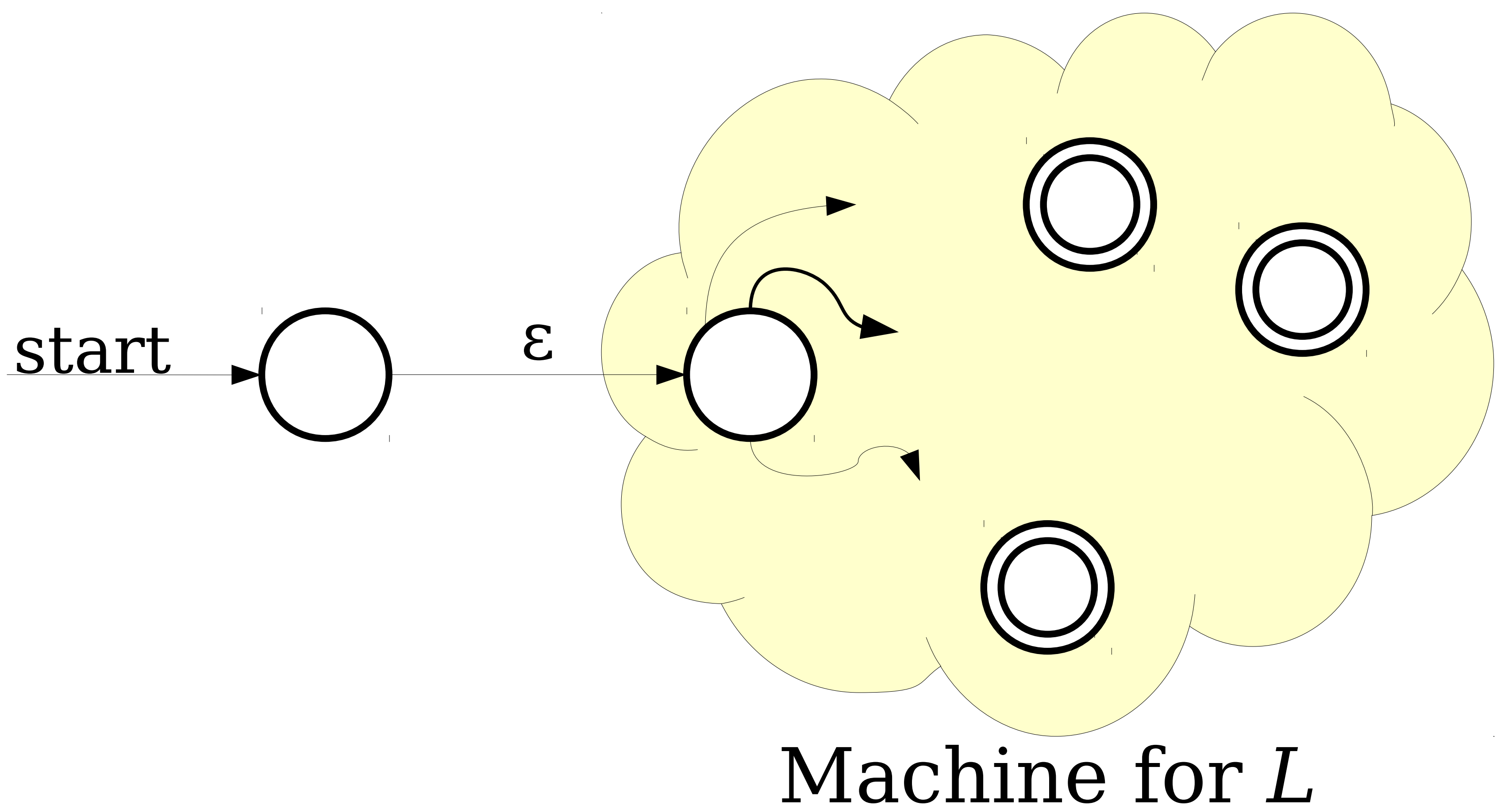
- If a series of finite objects all have some property, the “limit” of that process *does not* necessarily have that property.
- In general, it is not safe to conclude that some property that always holds in the finite case must hold in the infinite case.
  - (This is why calculus is interesting).

Idea: Can we directly convert an NFA for language L to an NFA for language L^*?

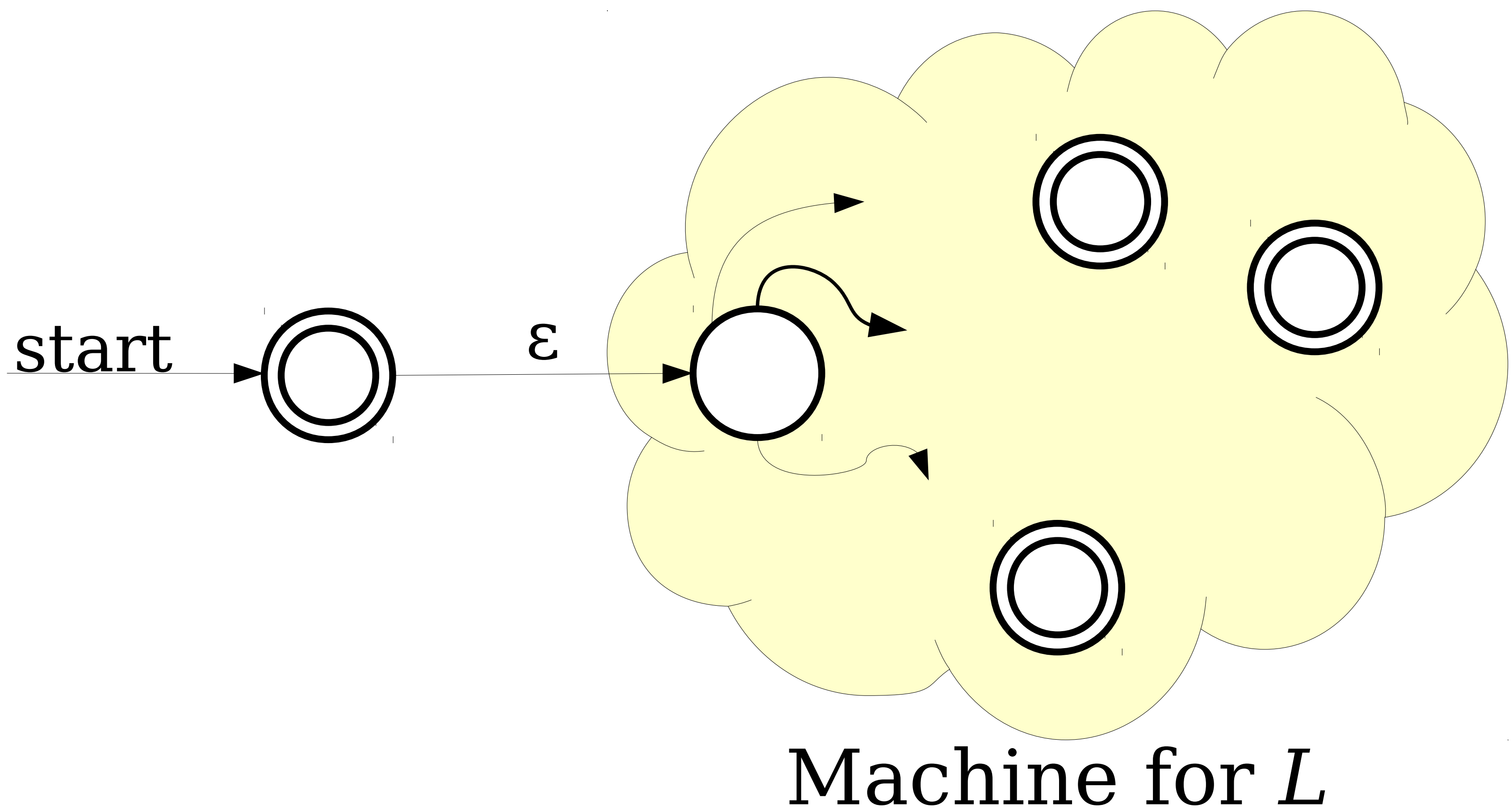
The Kleene Star



The Kleene Star



The Kleene Star



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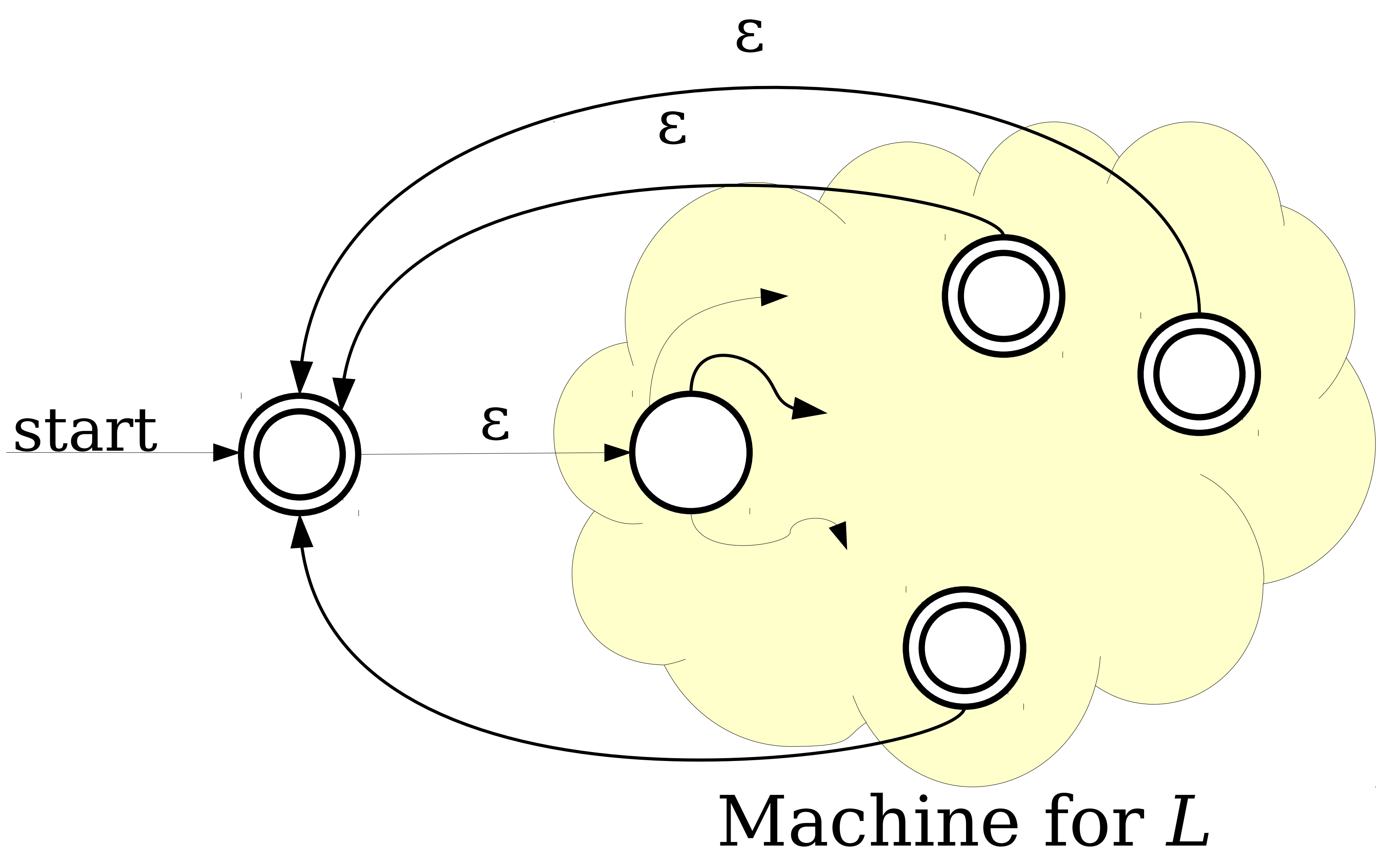

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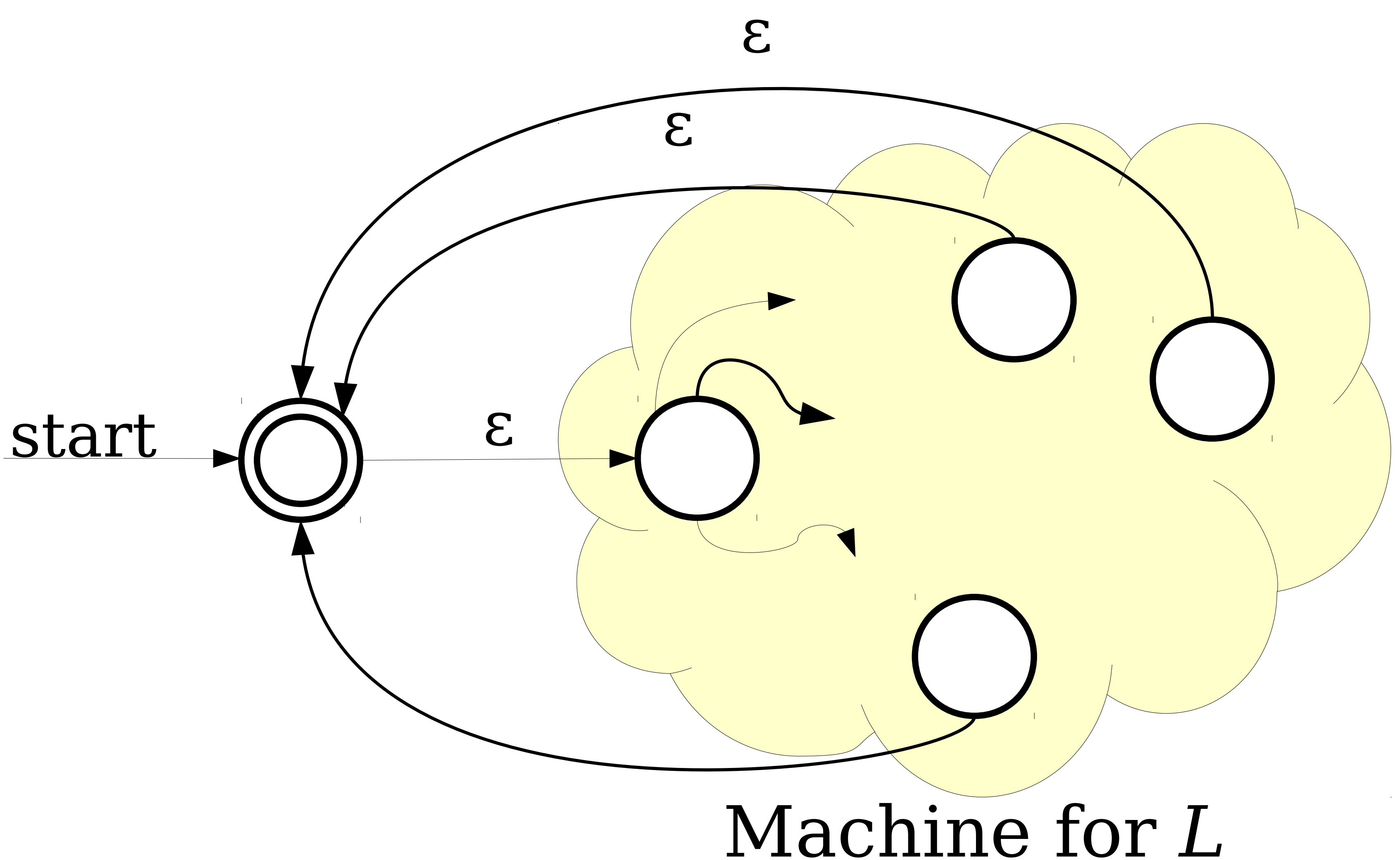
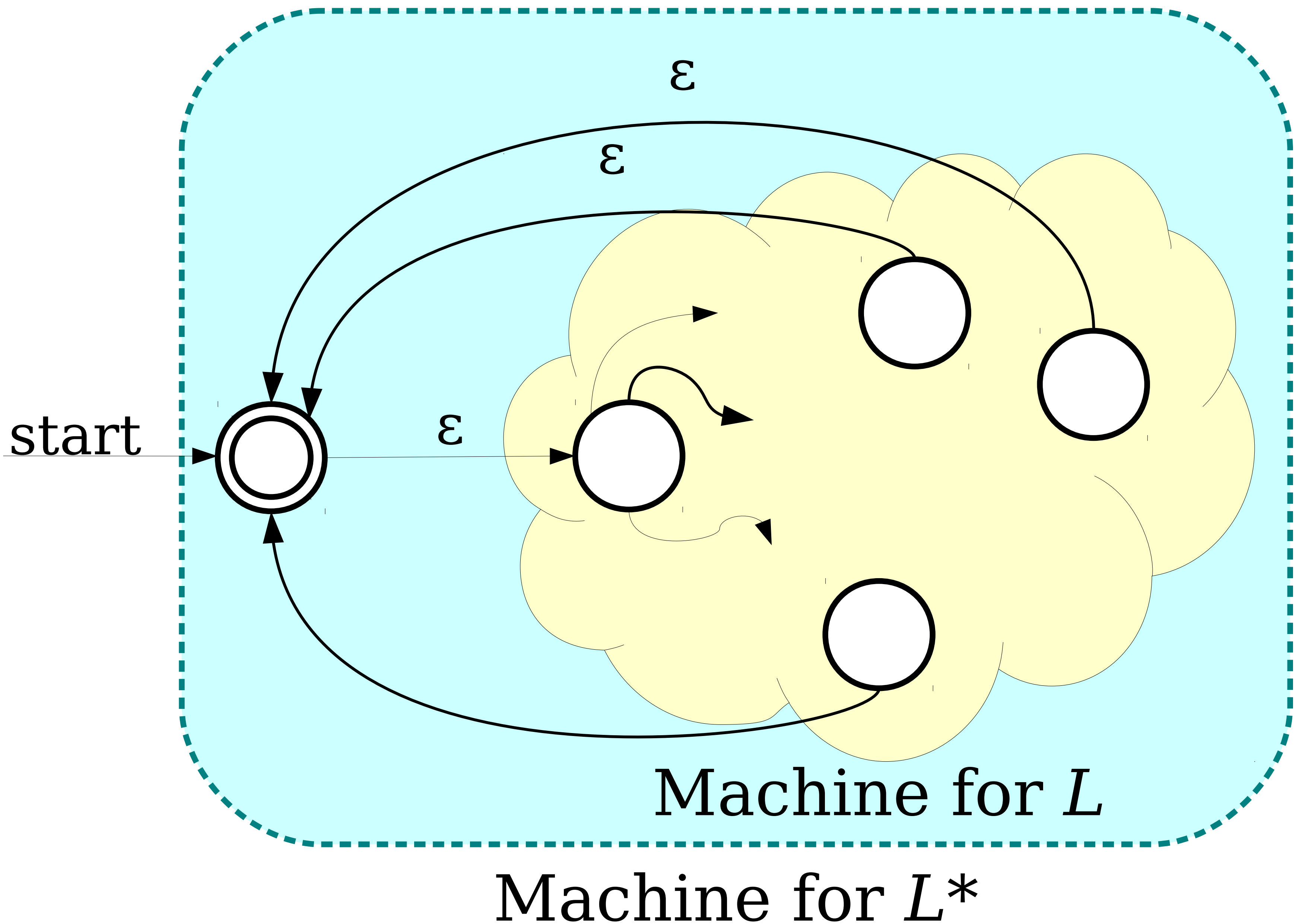
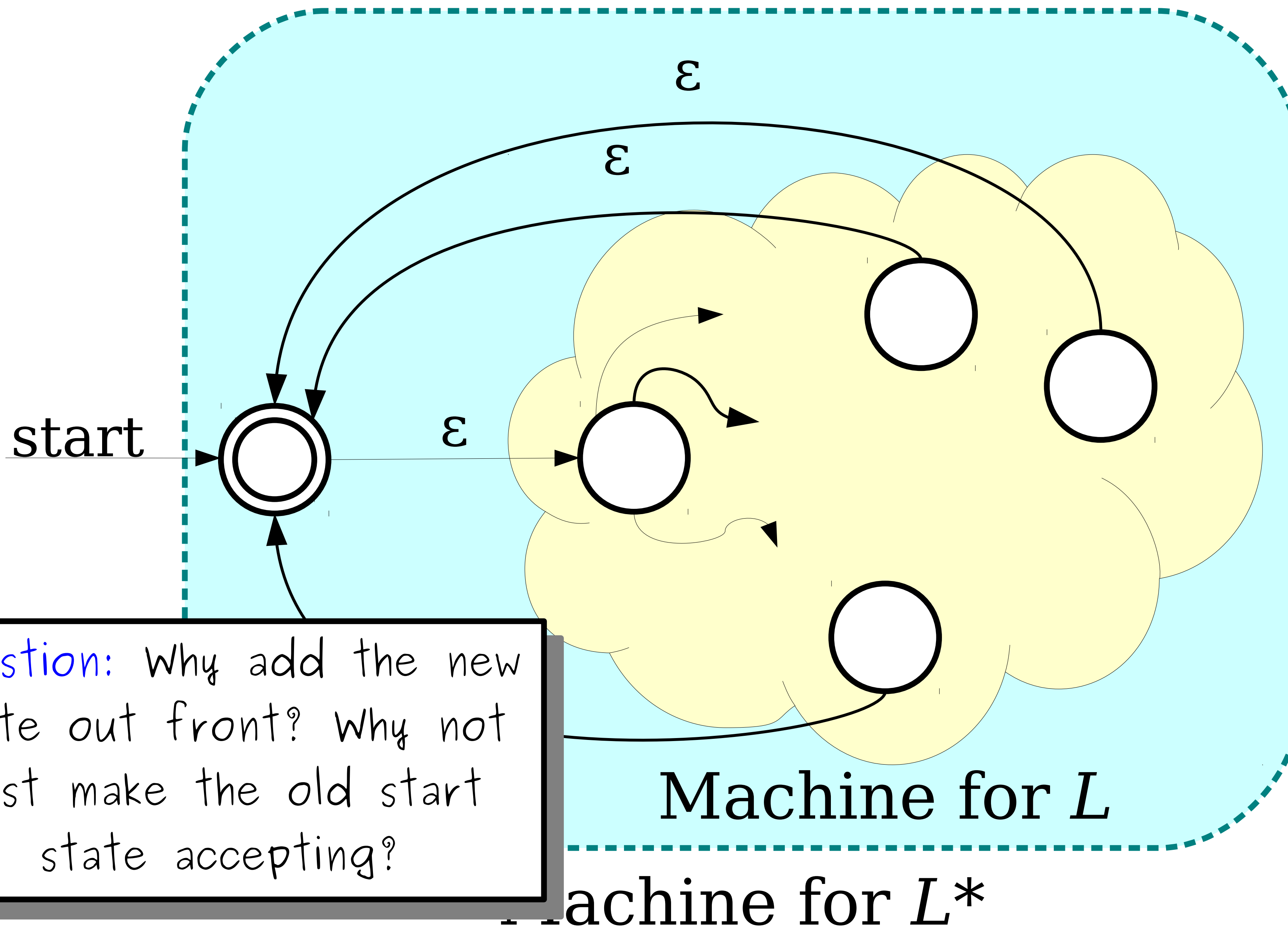


The Kleene Star



The Kleene Star



Question: Why add the new state out front? Why not just make the old start state accepting?

Summary

- NFAs are a powerful type of automaton that allows for ***nondeterministic*** choices.
- NFAs can also have ***ε-transitions*** that move from state to state without consuming any input.
- The ***subset construction*** shows that NFAs are not more powerful than DFAs, because any NFA can be converted into a DFA that accepts the same language.
- The union, intersection, complement, concatenation, and Kleene closure of regular languages are all regular languages.