

# Finite Automata

## Part Two

Recap from Last Time

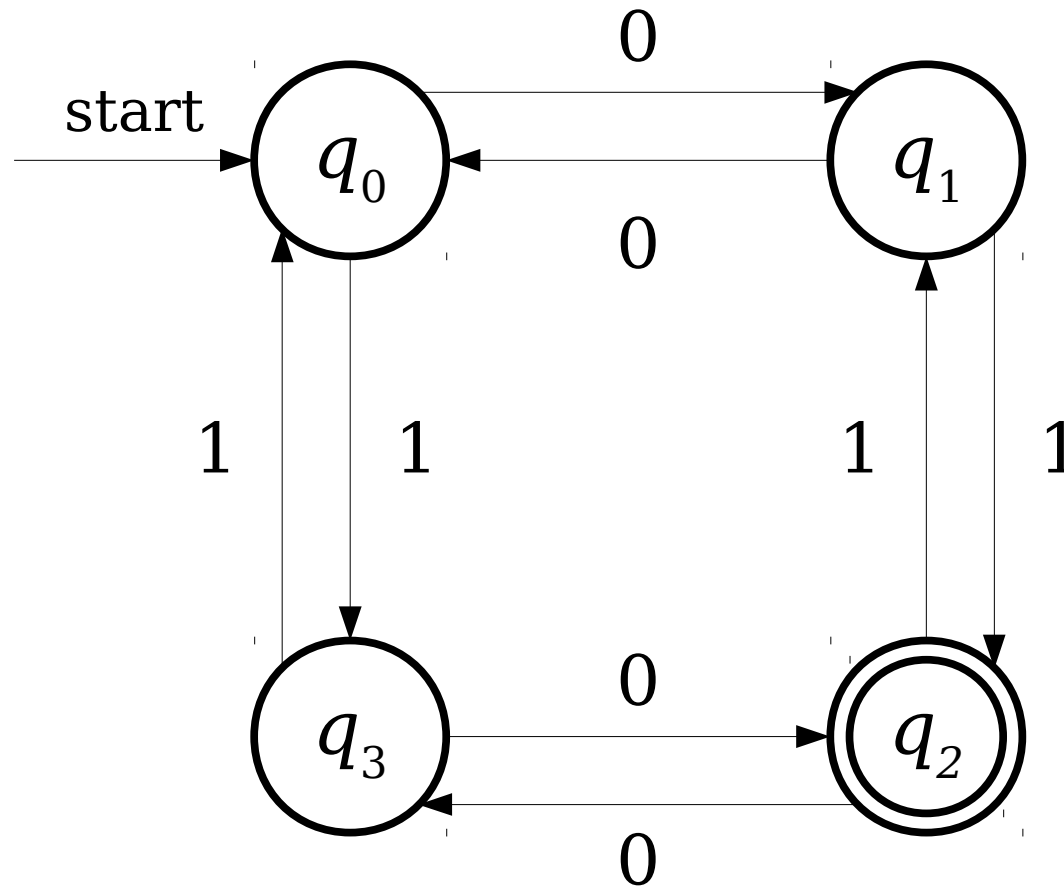
# Strings

- An **alphabet** is a finite, nonempty set of symbols called **characters**.
  - Typically, we use the symbol  $\Sigma$  to refer to an alphabet.
- A **string over an alphabet  $\Sigma$**  is a finite sequence of characters drawn from  $\Sigma$ .
- Example: If  $\Sigma = \{a, b\}$ , here are some valid strings over  $\Sigma$ :  
**a      aabaaabbabaaabaaaabbb      abbababba**
- The **empty string** has no characters and is denoted  $\epsilon$ .
- Calling attention to an earlier point: since all strings are finite sequences of characters from  $\Sigma$ , you cannot have a string of infinite length.

# Languages

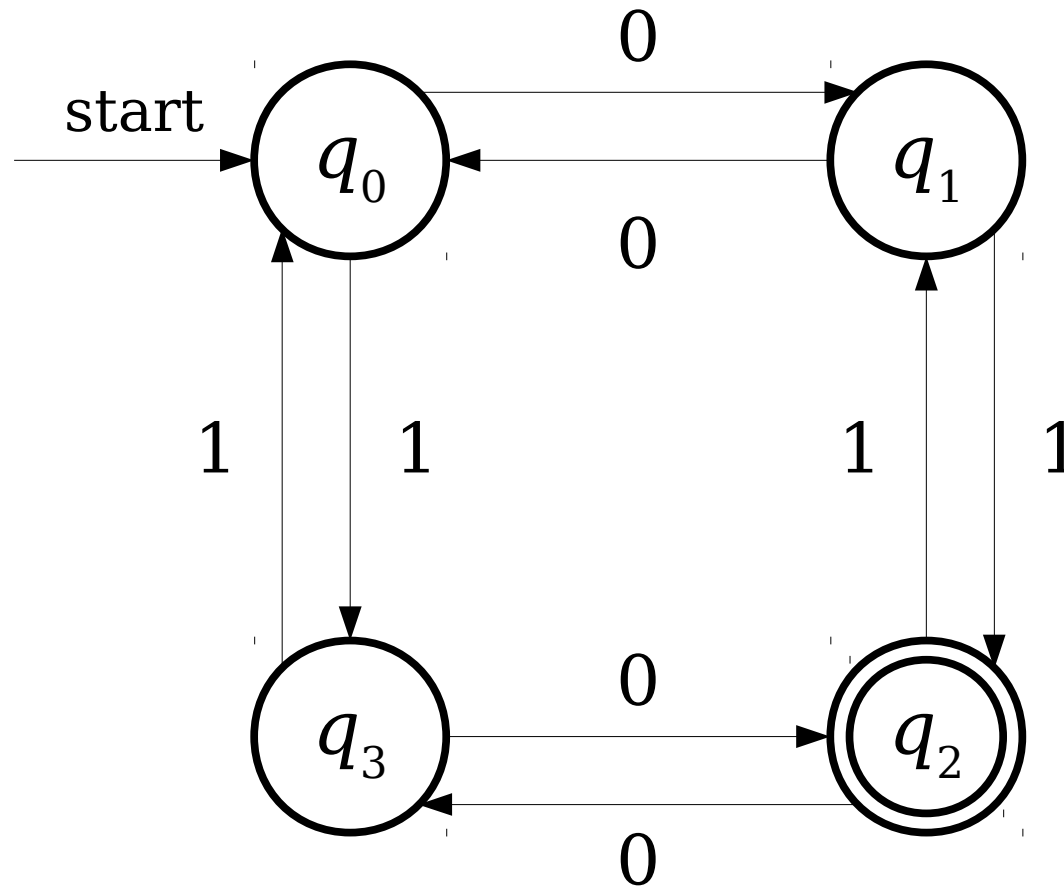
- A **formal language** is a set of strings.
- We say that  $L$  is a **language over  $\Sigma$**  if it is a set of strings over  $\Sigma$ .
- Example: The language of palindromes over  $\Sigma = \{a, b, c\}$  is the set
  - $\{\varepsilon, a, b, c, aa, bb, cc, aaa, aba, aca, bab, \dots\}$
- The set of all strings composed from letters in  $\Sigma$  is denoted  $\Sigma^*$ .
- Formally, we say that  $L$  is a language over  $\Sigma$  if  $L \subseteq \Sigma^*$ .

# A Simple Finite Automaton



**0 1 0 1 1 0**

# A Simple Finite Automaton



**1 0 1 0 0 0**

The *language of an automaton* is the set of strings that it accepts.

If  $D$  is an automaton, we denote the language of  $D$  as  $\mathcal{L}(D)$ .

$$\mathcal{L}(D) = \{ w \in \Sigma^* \mid D \text{ accepts } w \}$$

# DFAs

- A ***DFA*** is a
  - ***D***eterministic
  - ***F***inite
  - ***A***utomaton
- DFAs are the simplest type of automaton that we will see in this course.



# DFAs, Informally

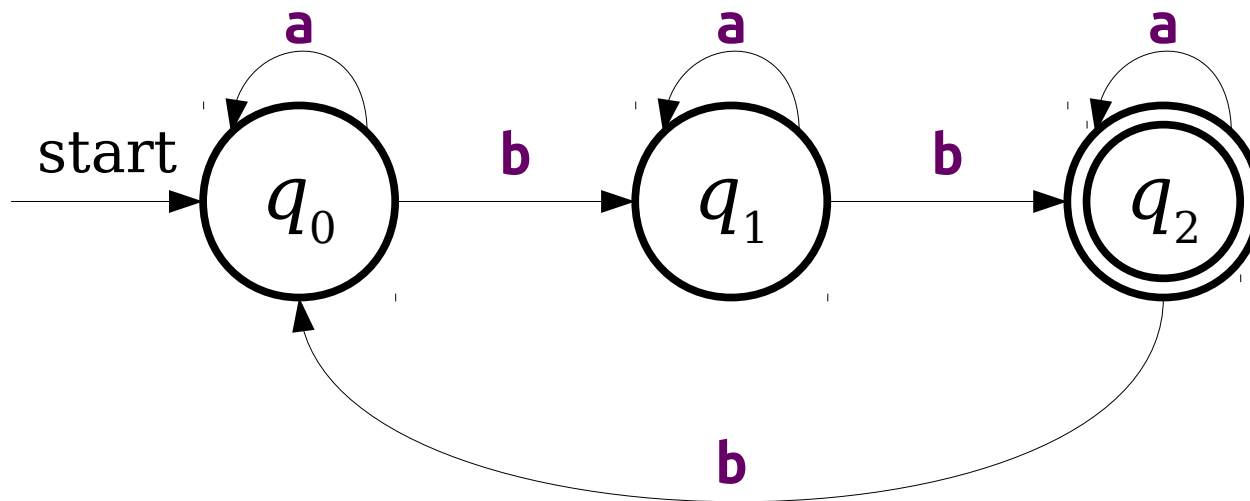
- A DFA is defined relative to some alphabet  $\Sigma$ .
- For each state in the DFA, there must be *exactly one* transition defined for each symbol in  $\Sigma$ .
  - This is the “deterministic” part of DFA.
- There is a unique start state.
- There are zero or more accepting states.

# Designing DFAs

- At each point in its execution, the DFA can only remember what state it is in.
- ***DFA Design Tip:*** Build each state to correspond to some piece of information you need to remember.
  - Each state acts as a “memento” of what you're supposed to do next.
  - Only finitely many different states  $\approx$  only finitely many different things the machine can remember.

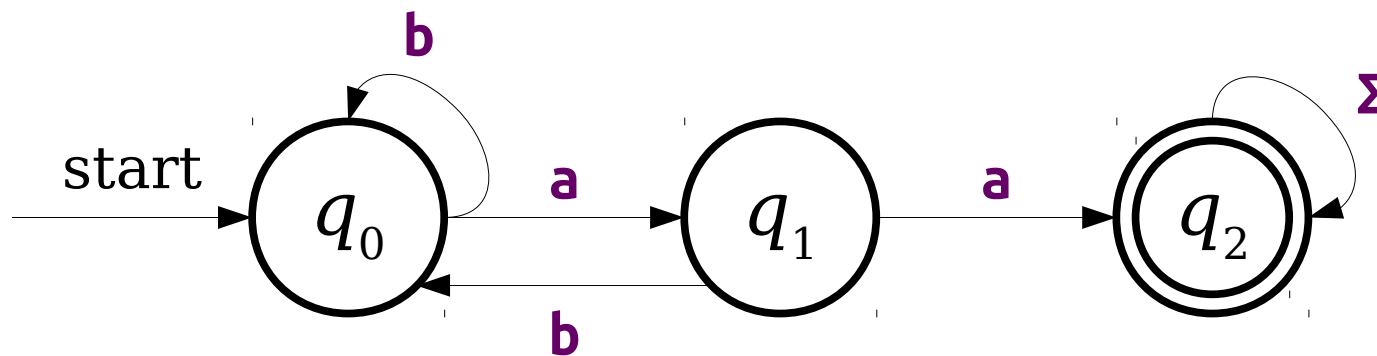
# Recognizing Languages with DFAs

$L = \{ w \in \{a, b\}^* \mid \text{the number of } b\text{'s in } w \text{ is congruent to two modulo three} \}$



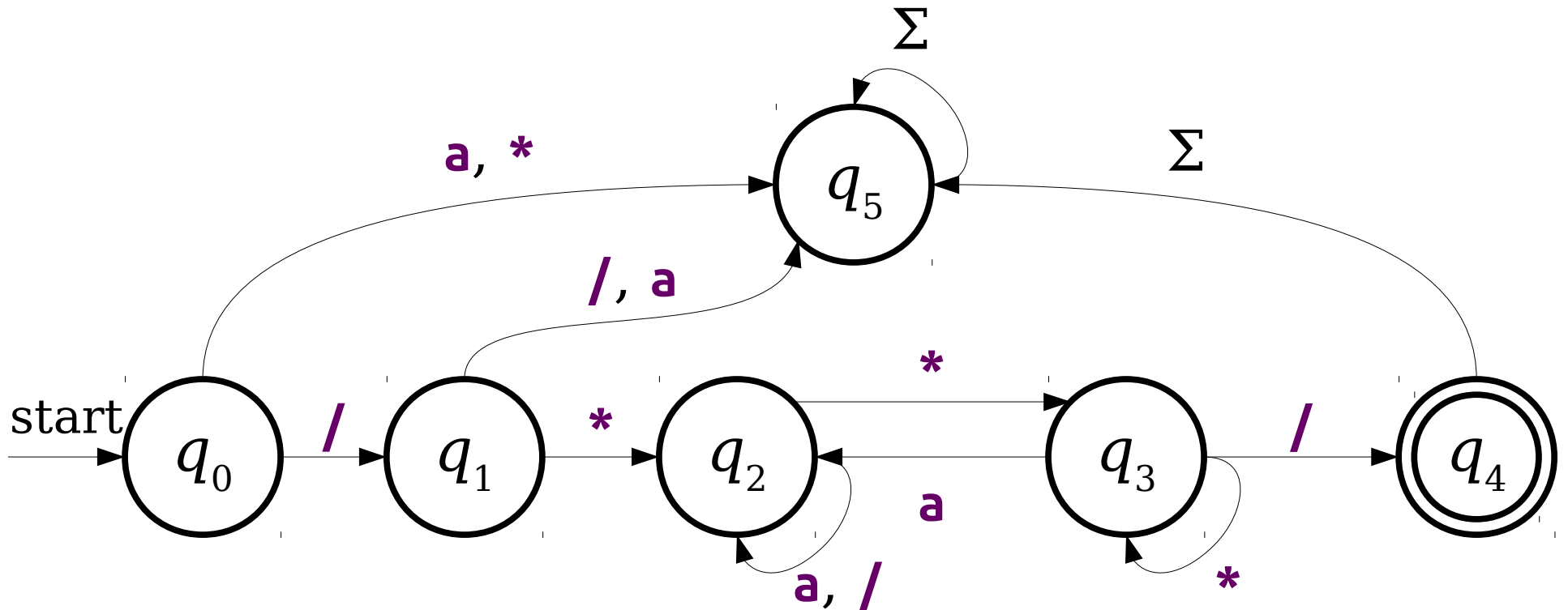
# Recognizing Languages with DFAs

$L = \{ w \in \{a, b\}^* \mid w \text{ contains } aa \text{ as a substring} \}$



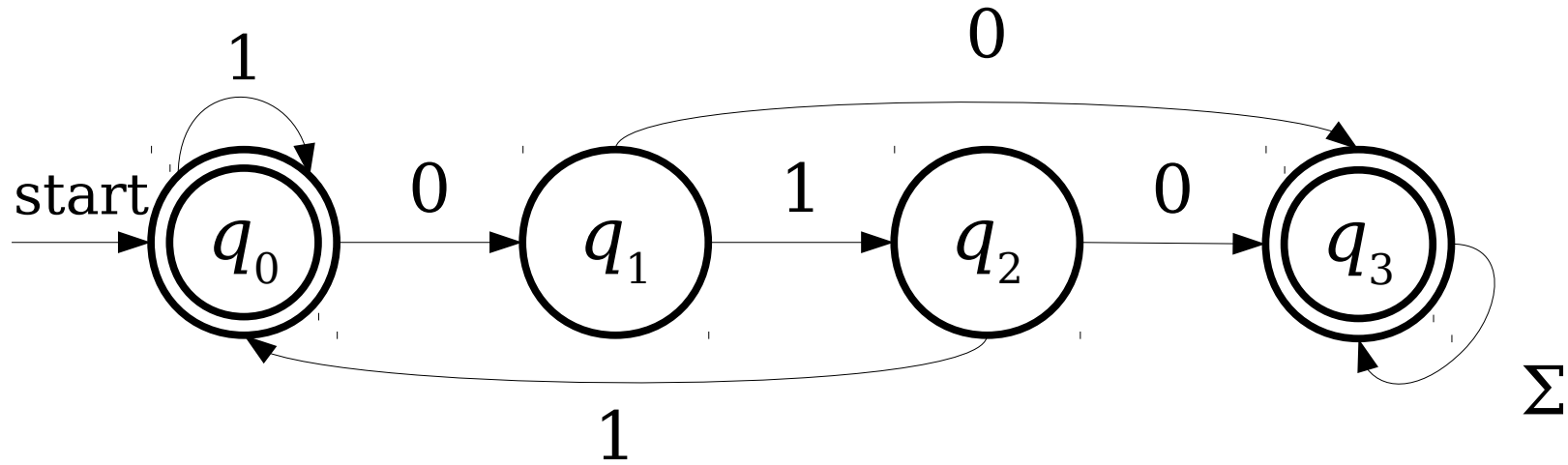
# Recognizing Languages with DFAs

$L = \{ w \in \{a, *, /\}^* \mid w \text{ represents a C-style comment} \}$

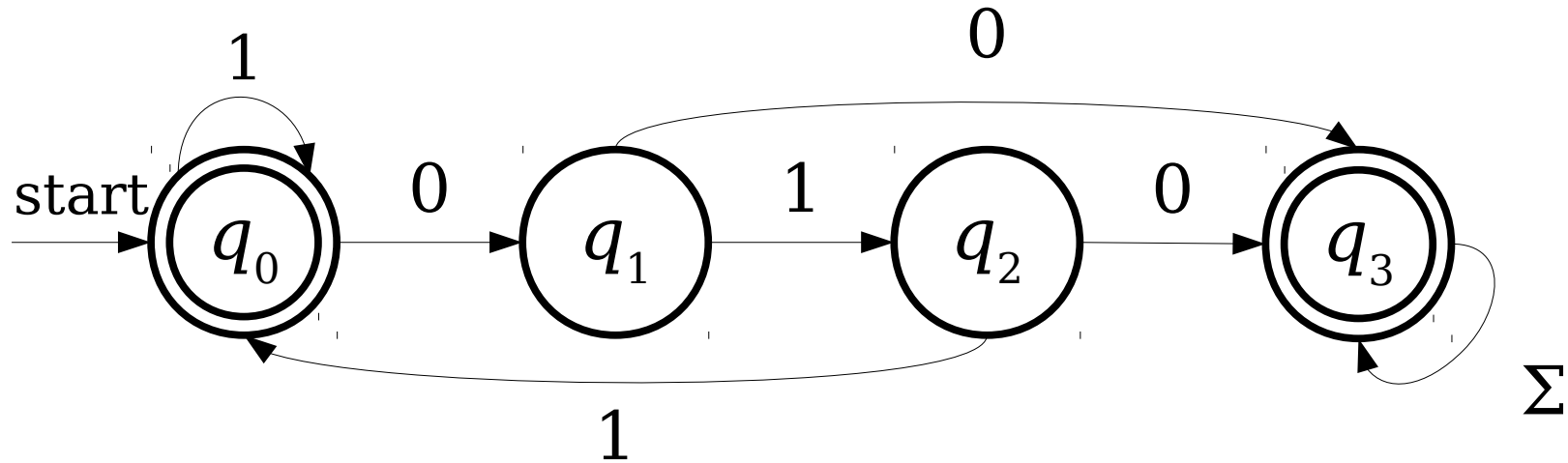


New Stuff!

# Tabular DFAs



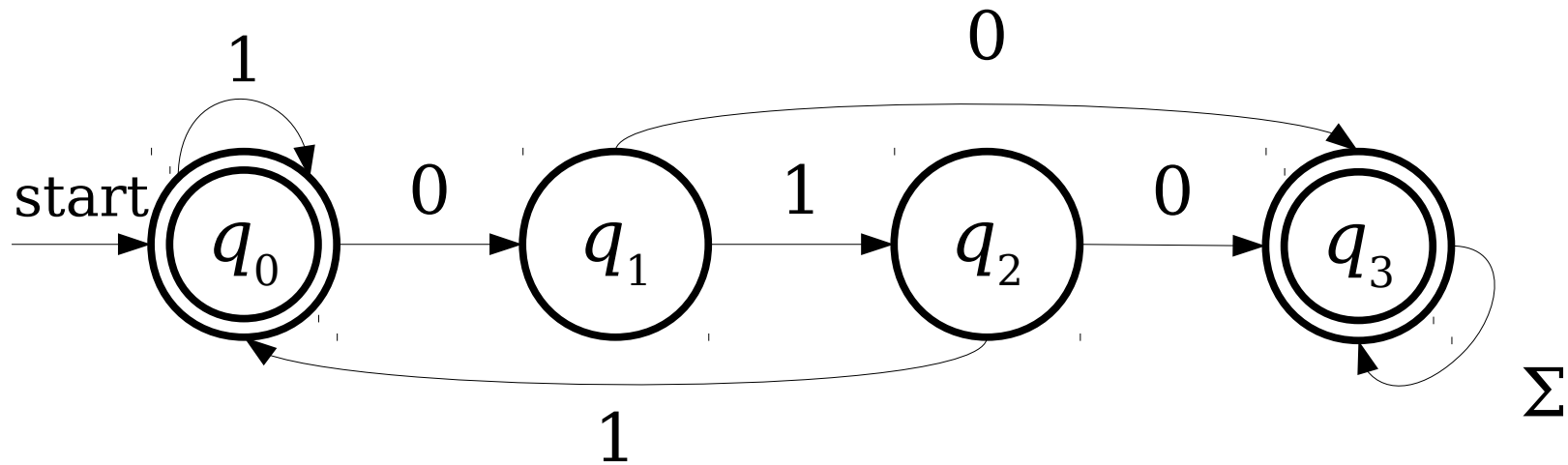
# Tabular DFAs



	0	1
$q_0$		
$q_1$		
$q_2$		
$q_3$		

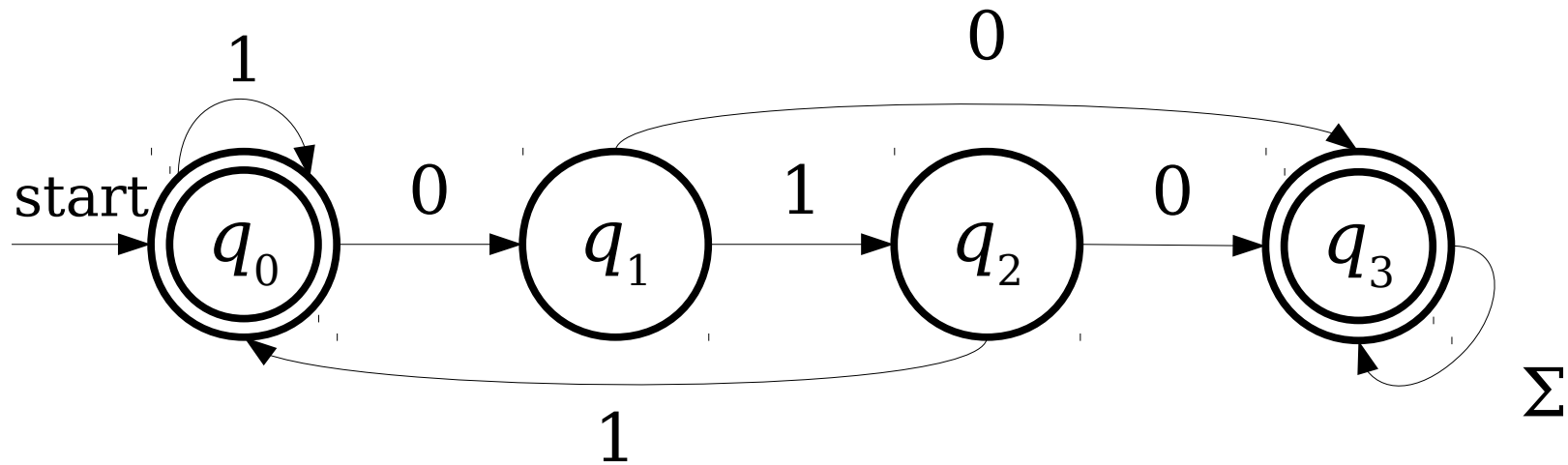


# Tabular DFAs



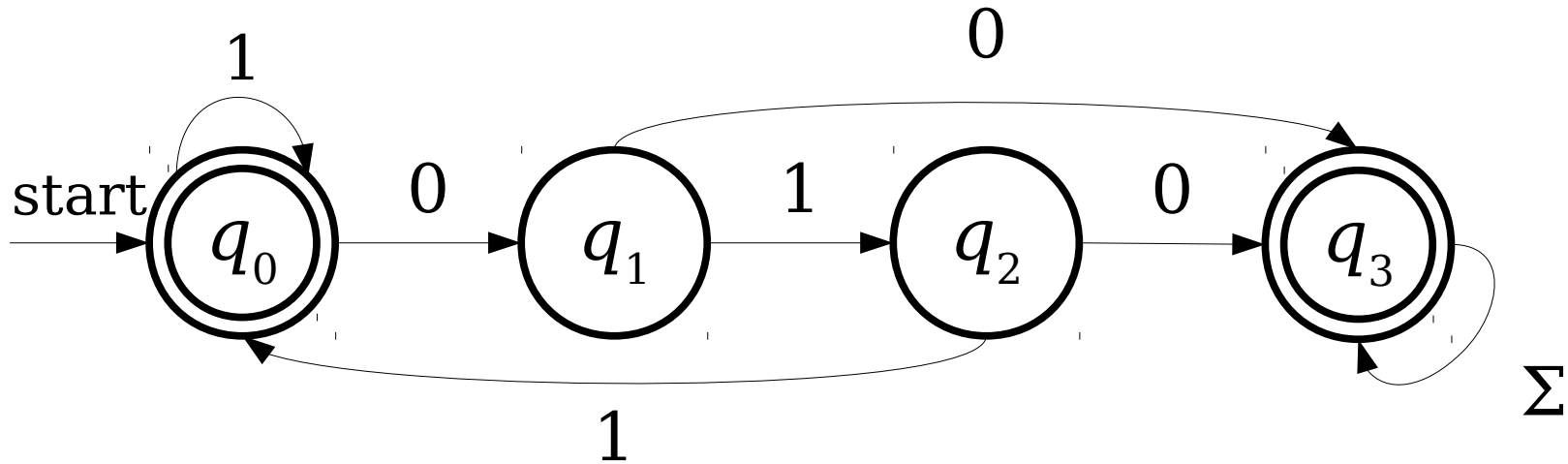
	0	1
$q_0$	$q_1$	$q_0$
$q_1$	$q_3$	$q_2$
$q_2$	$q_3$	$q_0$
$q_3$	$q_3$	$q_3$

# Tabular DFAs



	0	1
* $q_0$	$q_1$	$q_0$
$q_1$	$q_3$	$q_2$
$q_2$	$q_3$	$q_0$
* $q_3$	$q_3$	$q_3$

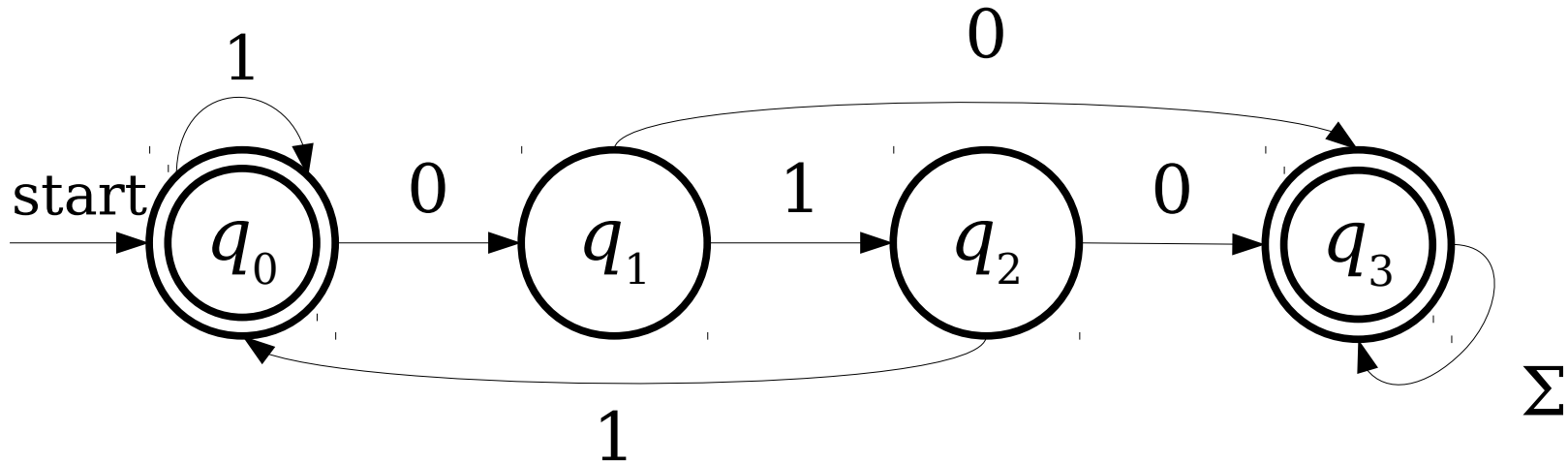
# Tabular DFAs



	0	1
* $q_0$	$q_1$	$q_0$
$q_1$	$q_3$	$q_2$
$q_2$	$q_3$	$q_0$
* $q_3$	$q_3$	$q_3$

These stars indicate accepting states.

# Tabular DFAs



Since this is the first row, it's the start state.

	0	1
* $q_0$	$q_1$	$q_0$
$q_1$	$q_3$	$q_2$
$q_2$	$q_3$	$q_0$
* $q_3$	$q_3$	$q_3$

# My Turn to Code Things Up!

```
int kTransitionTable[kNumStates][kNumSymbols] = {
    {0, 0, 1, 3, 7, 1, ...},
    ...
};
bool kAcceptTable[kNumStates] = {
    false,
    true,
    true,
    ...
};
bool SimulateDFA(string input) {
    int state = 0;
    for (char ch: input) {
        state = kTransitionTable[state][ch];
    }
    return kAcceptTable[state];
}
```

# The Regular Languages

A language  $L$  is called a ***regular language*** if there exists a DFA  $D$  such that  $\mathcal{L}(D) = L$ .

# The Complement of a Language

- Given a language  $L \subseteq \Sigma^*$ , the **complement** of that language (denoted  $\bar{L}$ ) is the language of all strings in  $\Sigma^*$  that aren't in  $L$ .
- Formally:

$$\bar{L} = \Sigma^* - L$$



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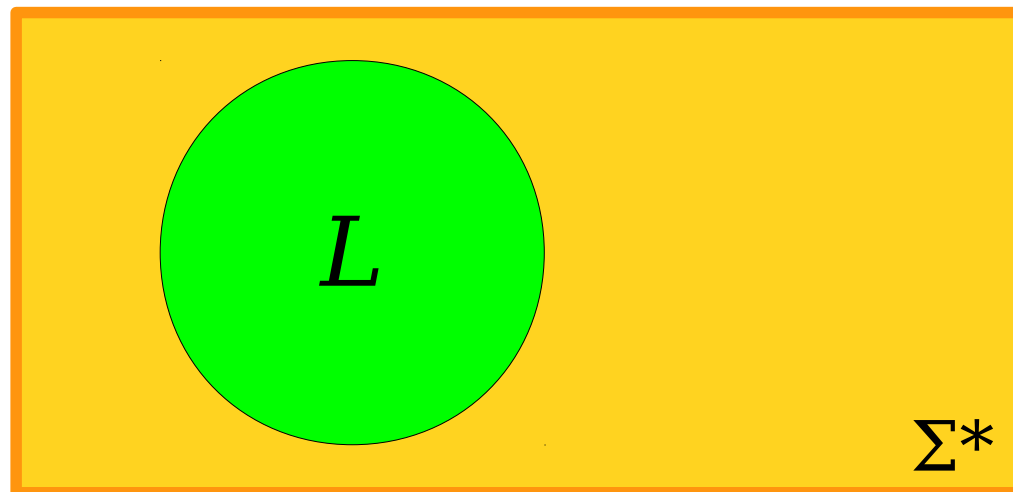
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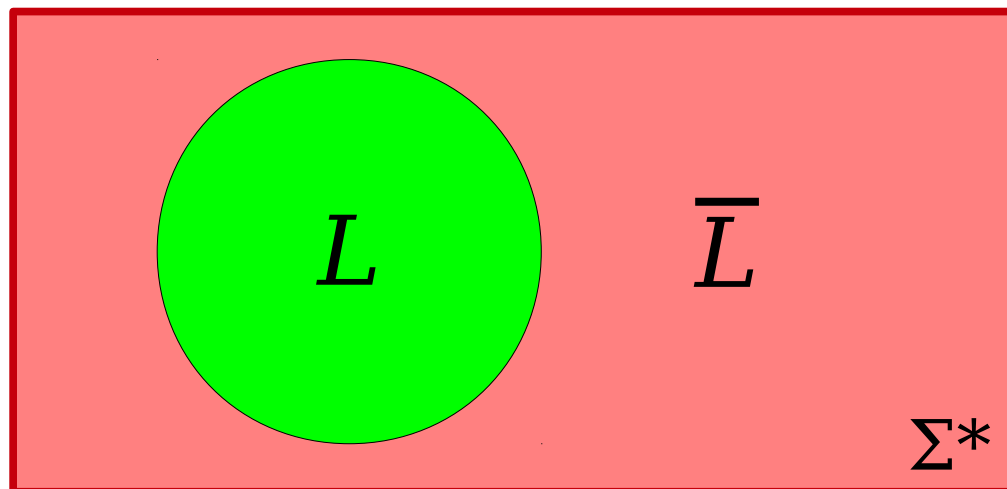
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# The Complement of a Language

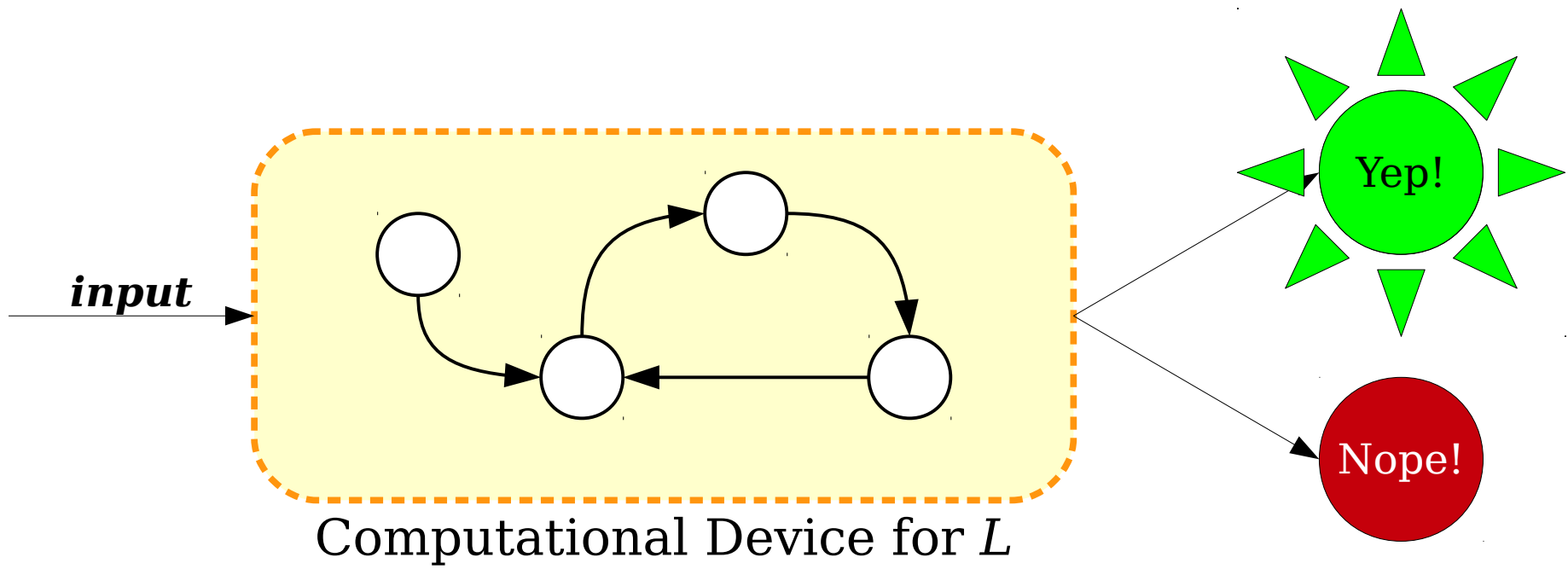
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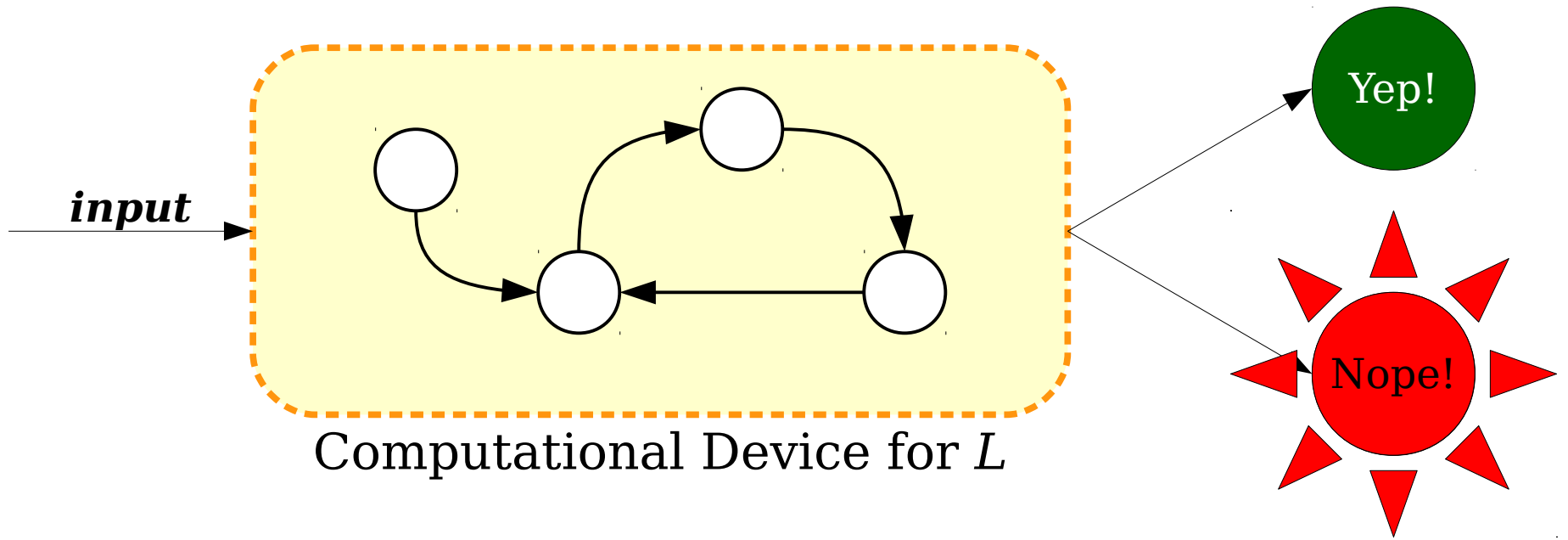
$$\bar{L} = \Sigma^* - L$$

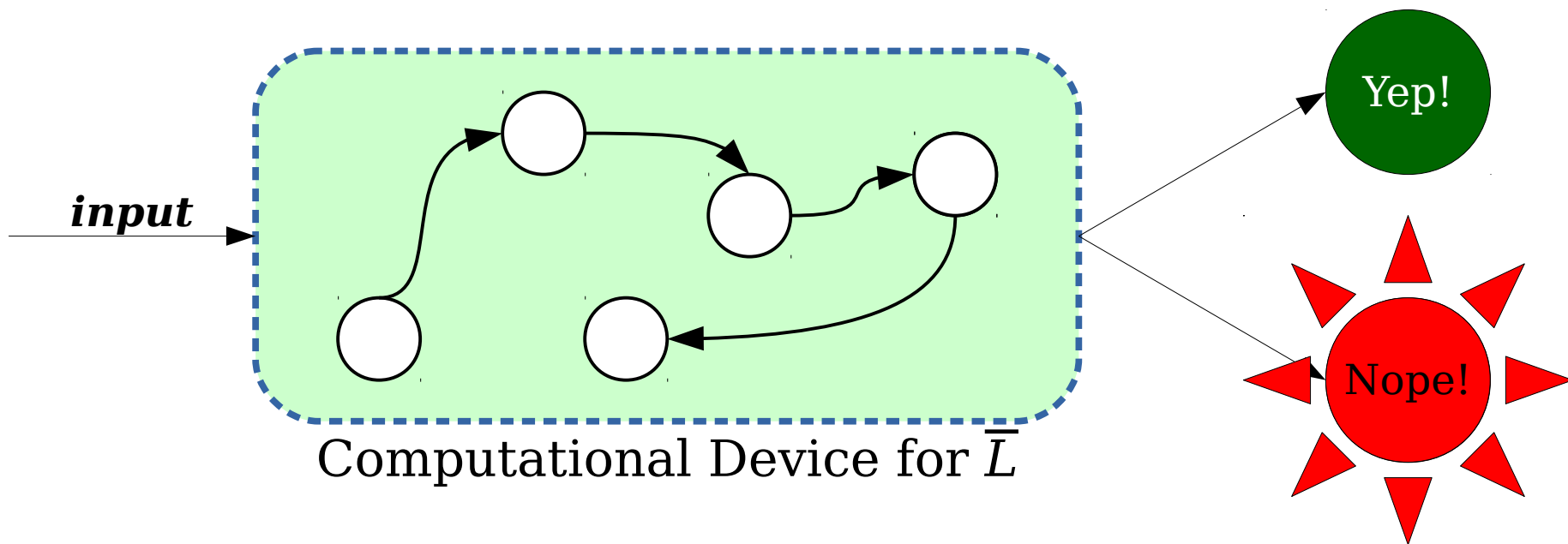
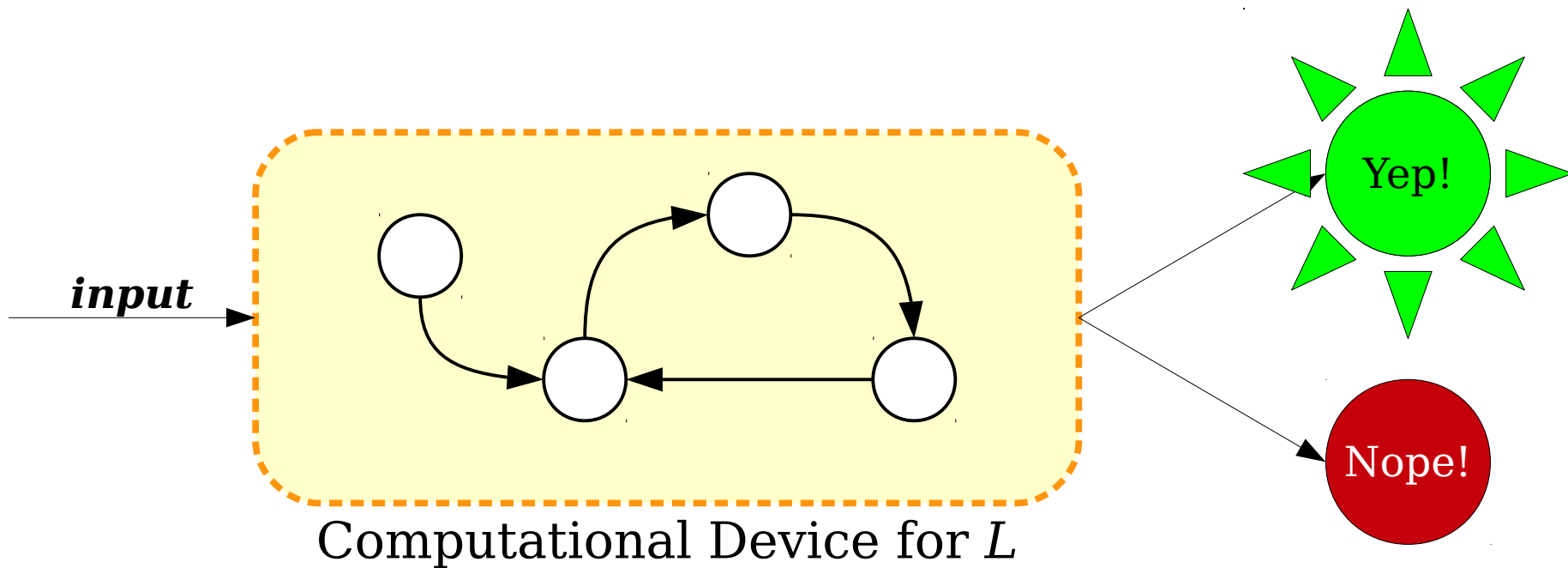


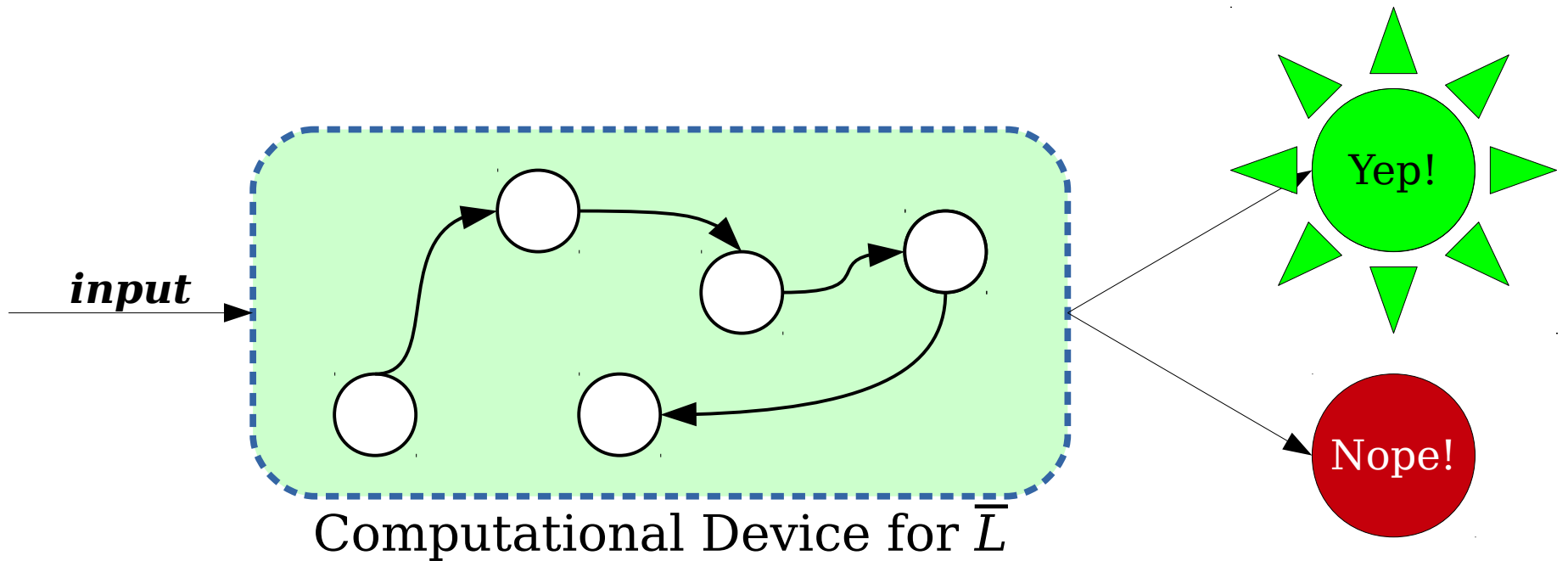
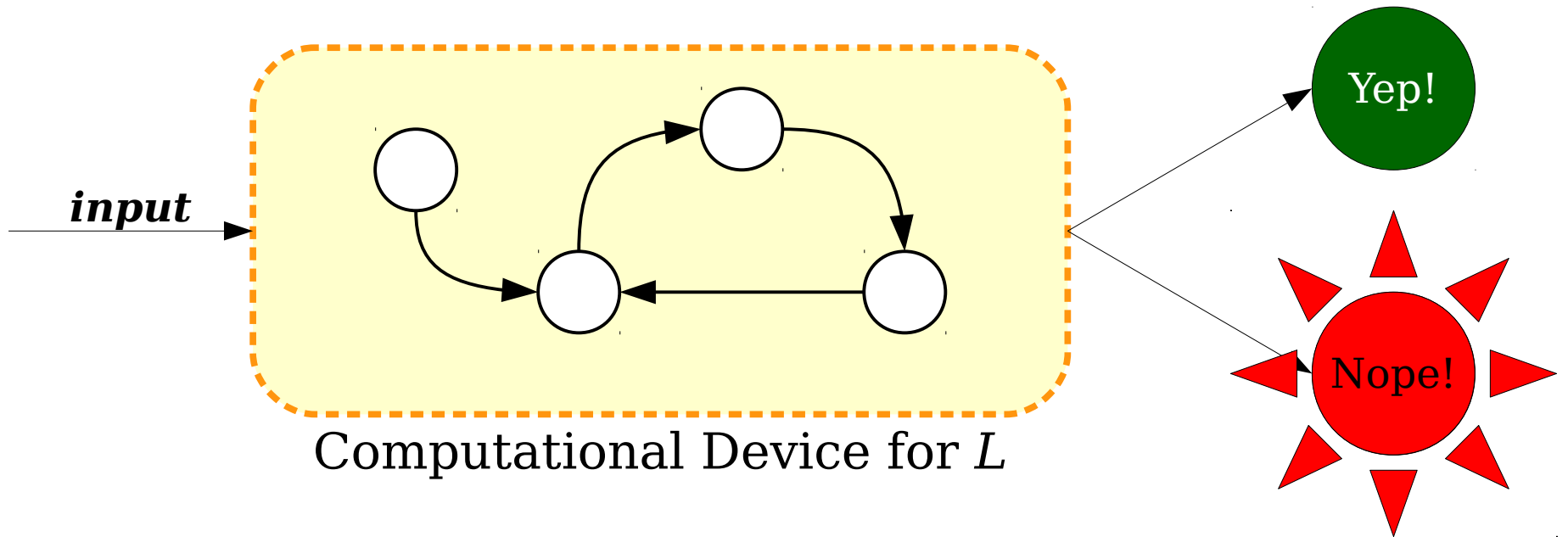
# Complements of Regular Languages

- As we saw a few minutes ago, a **regular language** is a language accepted by some DFA.
- **Question:** If  $L$  is a regular language, is  $\bar{L}$  necessarily a regular language?
- If the answer is “yes,” then if there is a way to construct a DFA for  $L$ , there must be some way to construct a DFA for  $\bar{L}$ .
- If the answer is “no,” then some language  $L$  can be accepted by some DFA, but  $\bar{L}$  cannot be accepted by any DFA.





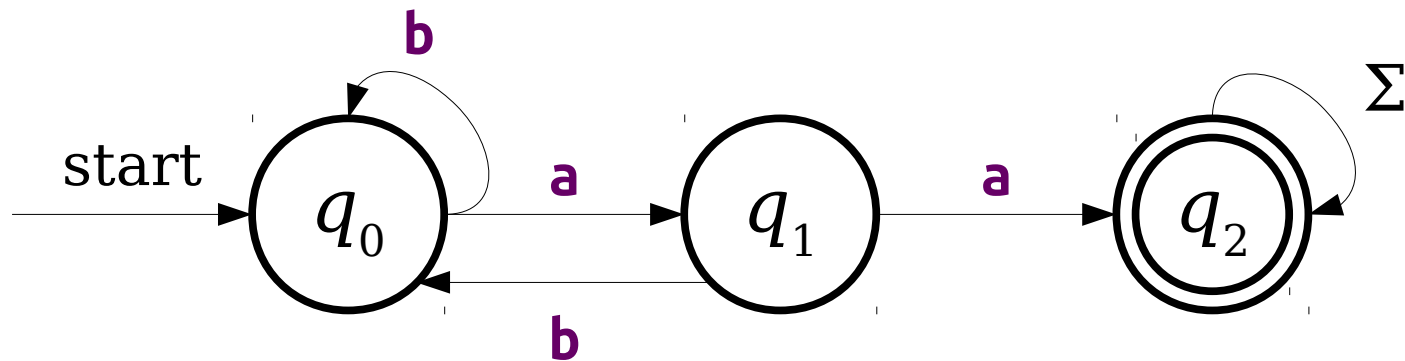




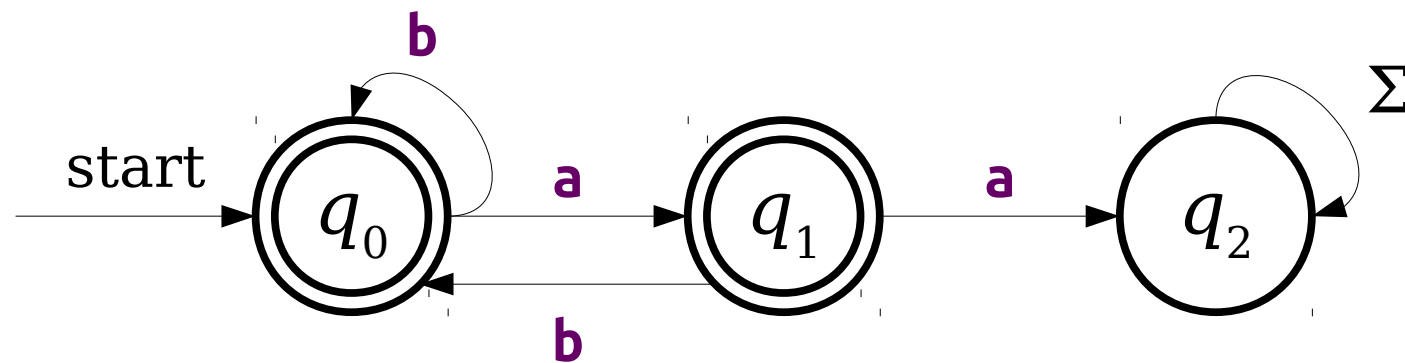


# Complementing Regular Languages

$$L = \{ w \in \{a, b\}^* \mid w \text{ contains } \mathbf{aa} \text{ as a substring} \}$$

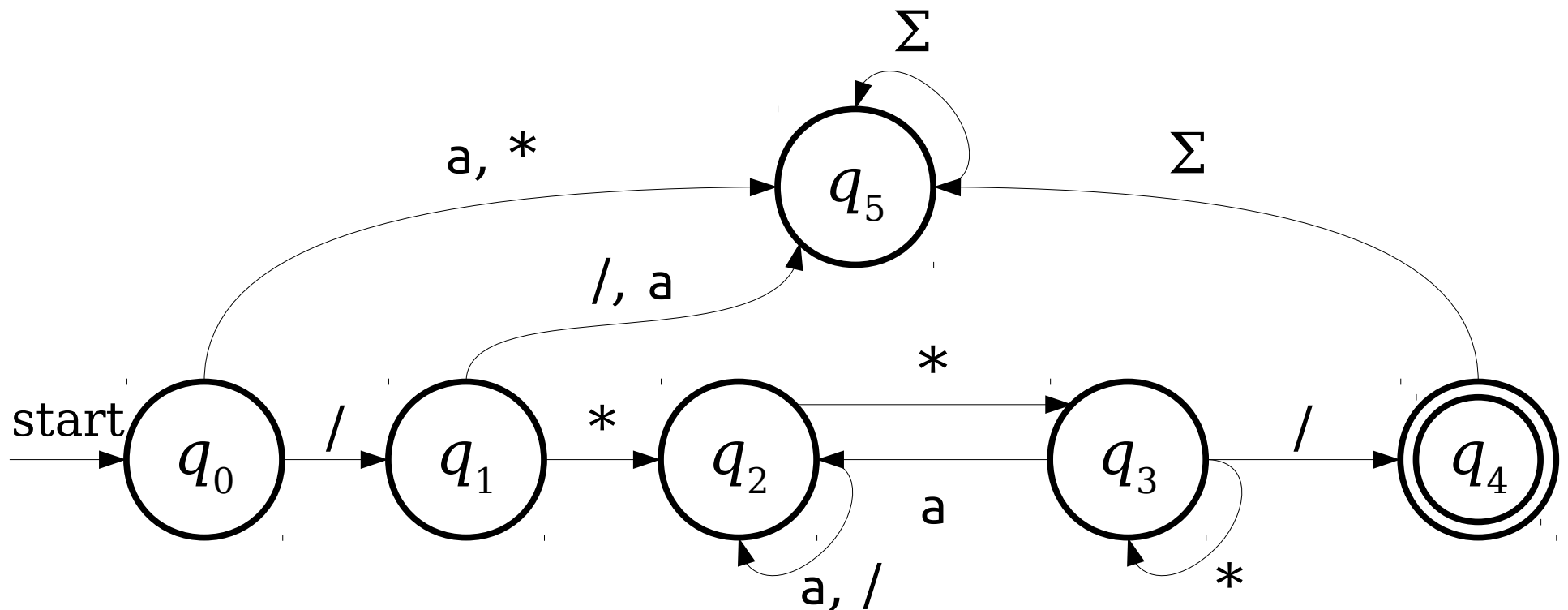


$$\bar{L} = \{ w \in \{a, b\}^* \mid w \text{ **does not** contain } \mathbf{aa} \text{ as a substring} \}$$



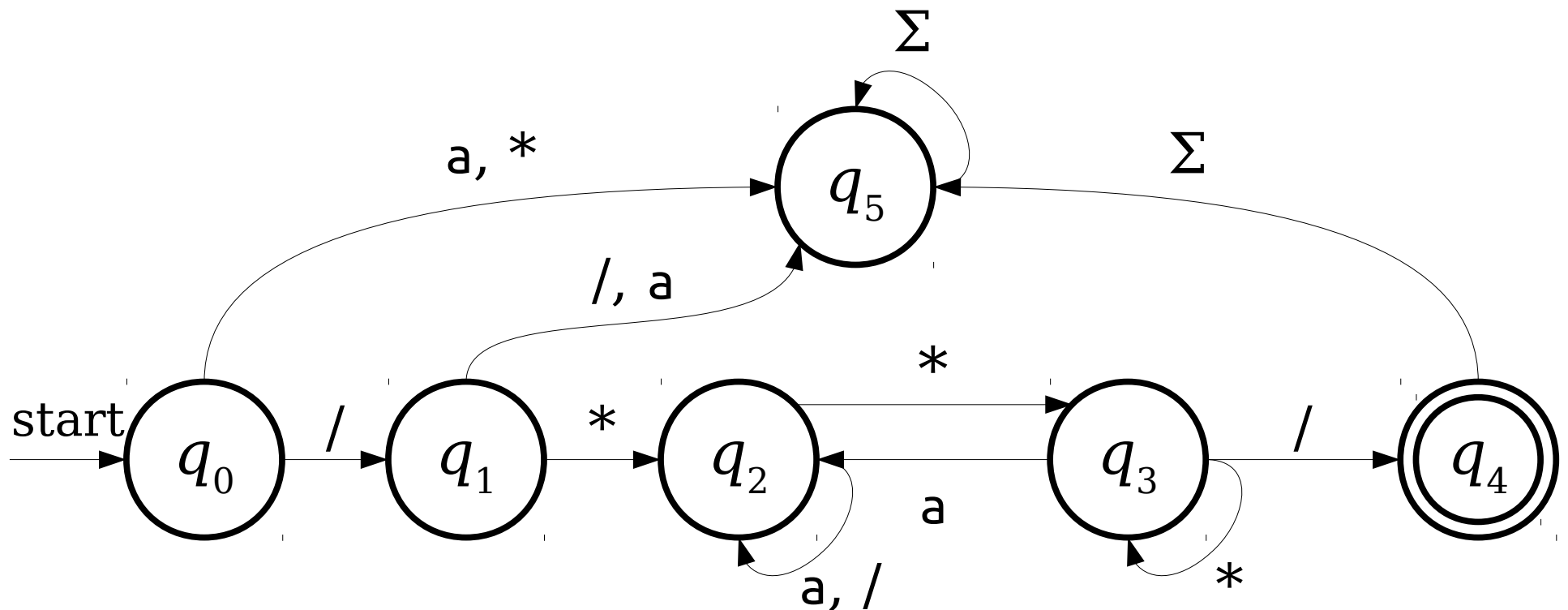
# More Elaborate DFAs

$L = \{ w \in \{a, *, /\}^* \mid w \text{ represents a C-style comment} \}$



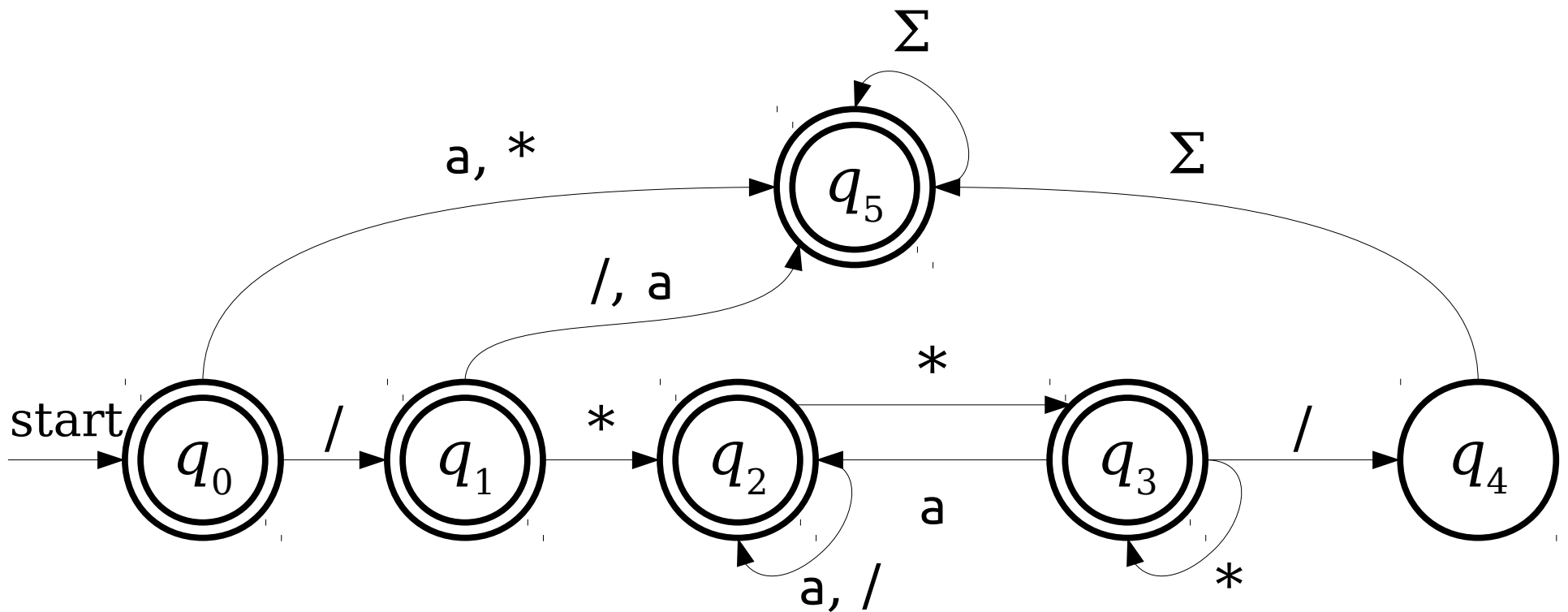
# More Elaborate DFAs

$\bar{L} = \{ w \in \{a, *, /\}^* \mid w \text{ doesn't represent a C-style comment} \}$



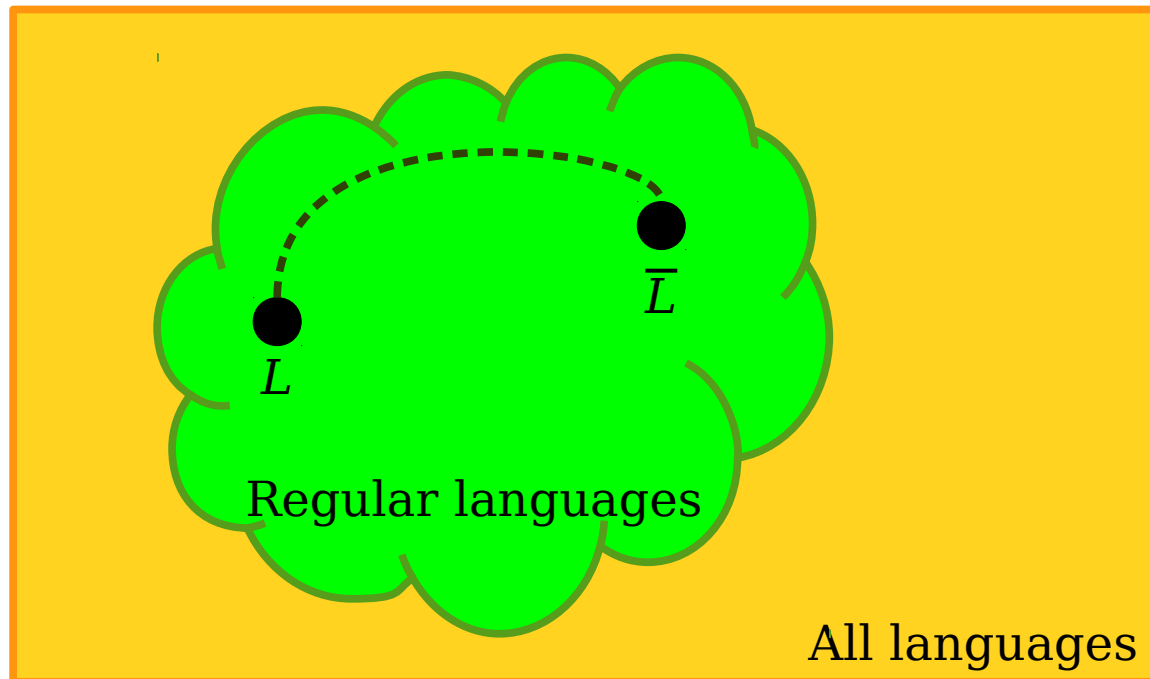
# More Elaborate DFAs

$\bar{L} = \{ w \in \{a, *, /\}^* \mid w \text{ doesn't represent a C-style comment} \}$



# Closure Properties

- **Theorem:** If  $L$  is a regular language, then  $\bar{L}$  is also a regular language.
- As a result, we say that the regular languages are **closed under complementation**.



**Time-Out For Announcements!**



# CODE2040 INFO SESSION

FRIDAY, NOV 3 | 7PM - 8PM

OLD UNION, ROOM 200



# Talk to Your Provost



- Provost Persis Drell will be holding office hours in Lathrop 143 next Monday, November 6, from 4PM - 6PM.
- Have any suggestions for the university? Want to change anything? Stop on by to chat!



# CS Career Panel

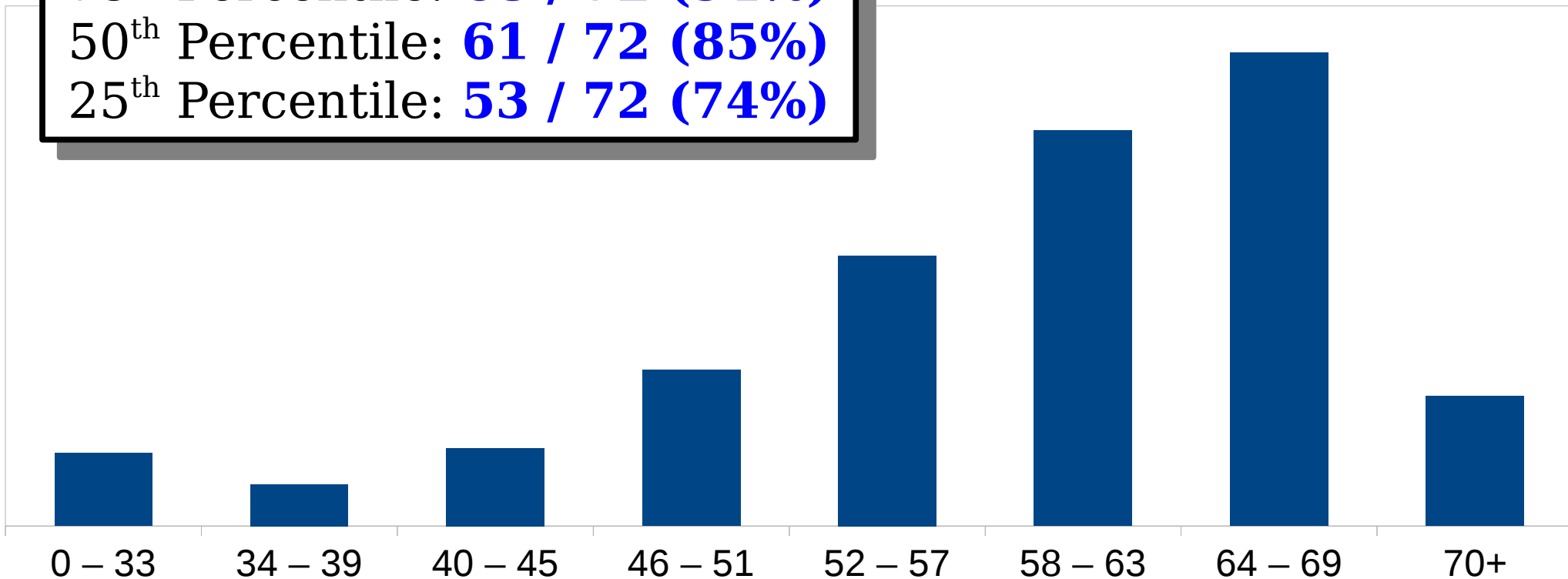
- Greg Ramel, our wonderful CS course advisor, is organizing a CS career panel.
- It's tomorrow (Thursday, November 2<sup>nd</sup>) in Gates 219 and runs from 5:45PM - 7:00PM.
- Please RSVP using [this link](#).
- There's a great mix of panelists. Highly recommended!

# Problem Set Four Graded

75<sup>th</sup> Percentile: **68 / 72 (94%)**

50<sup>th</sup> Percentile: **61 / 72 (85%)**

25<sup>th</sup> Percentile: **53 / 72 (74%)**



# Extra Practice Problems 2

- We've just released another set of extra practice problems to the course website.
- Need to review some concepts? Want more practice? Try these questions out! There's a ton of variety.
- Solutions will go out on Friday.

Your Questions

“I've struggled as a public speaker for as long as I can remember and watching your lectures, I'm amazed by how good you are. How do you do it?”

When I first started teaching I was so terrified of speaking to crowds that I literally memorized everything I was going to say and rehearsed for like ten hours each time. After I realized that most people are nice and won't eat you if you make a mistake, I started backing down from that and just worked out a general game plan for each lecture. Essentially, I just slowly stepped up the amount of improvisation I had in each lecture until I got the hang of it. Right now there's a balance between making things up as I go and falling back on things I know work well.

“Any movie / TV recommendations?”

My unsorted top movies: “Twelve Angry Men,” “Seven Samurai,” “The Lives of Others,” “Amelie,” “The Babadook,” “Brazil,” and “Lawrence of Arabia.”

Unsorted top TV shows: “The Wire.”

Back to CS103!

**NFAS**



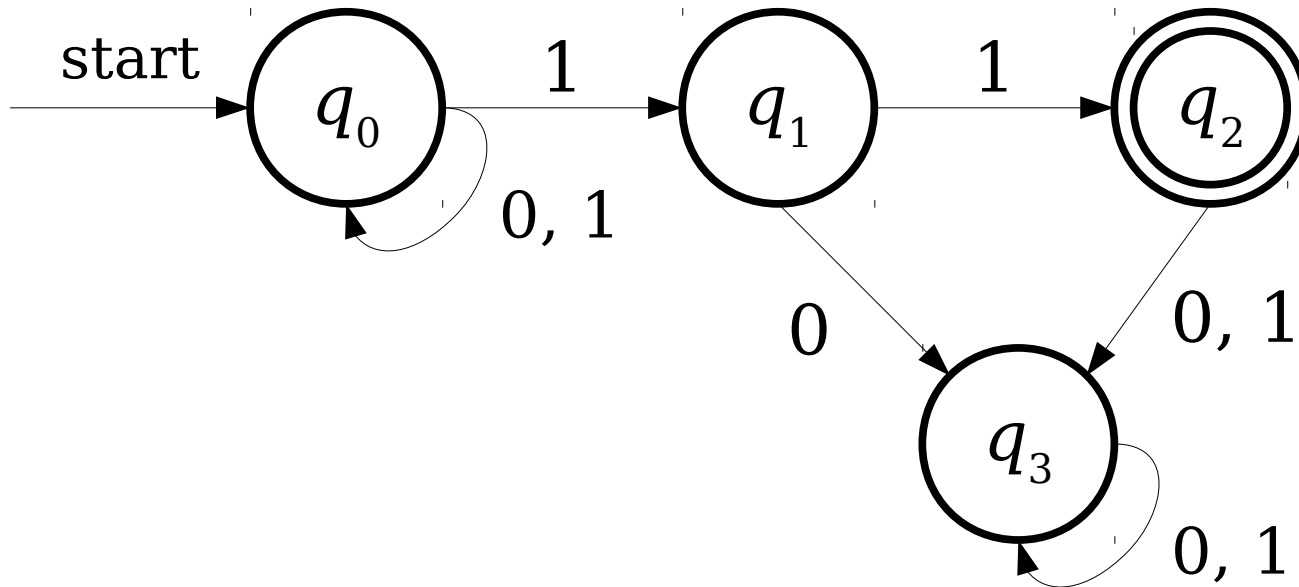
# NFAs

- An *NFA* is a
  - *N*ondeterministic
  - *F*inite
  - *A*utomaton
- Structurally similar to a DFA, but represents a fundamental shift in how we'll think about computation.

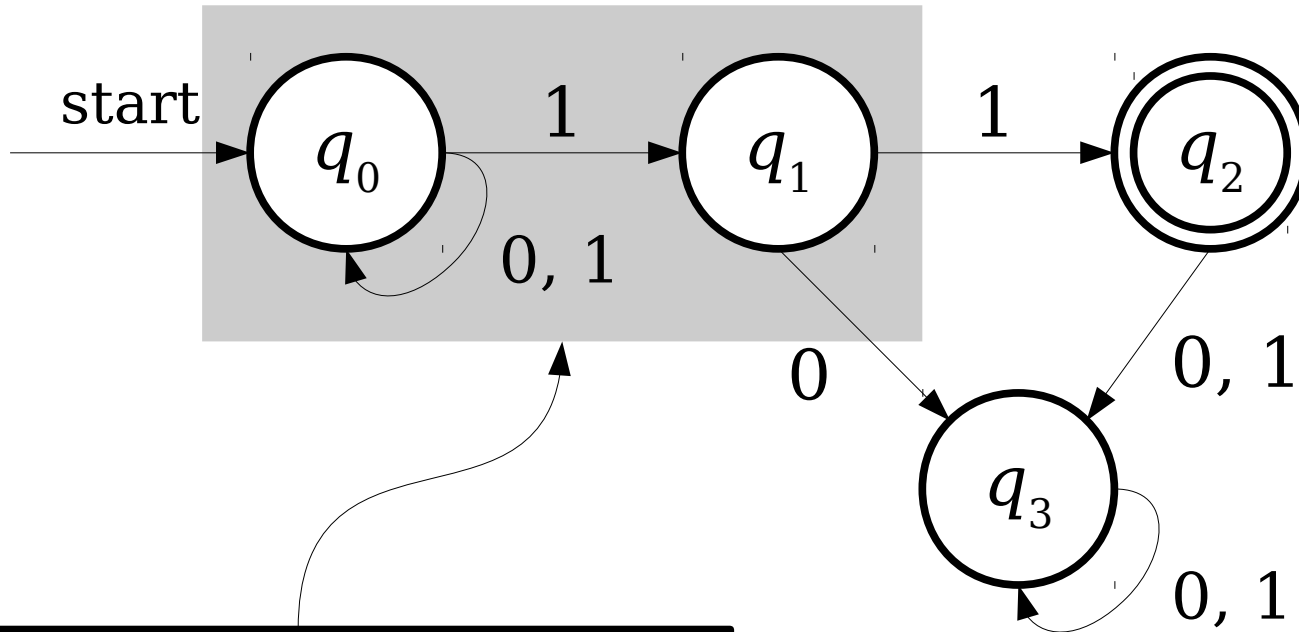
# (Non)determinism

- A model of computation is ***deterministic*** if at every point in the computation, there is exactly one choice that can be made.
- The machine accepts if that series of choices leads to an accepting state.
- A model of computation is ***nondeterministic*** if the computing machine may have multiple decisions that it can make at one point.
- The machine accepts if ***any*** series of choices leads to an accepting state.

# A Simple NFA

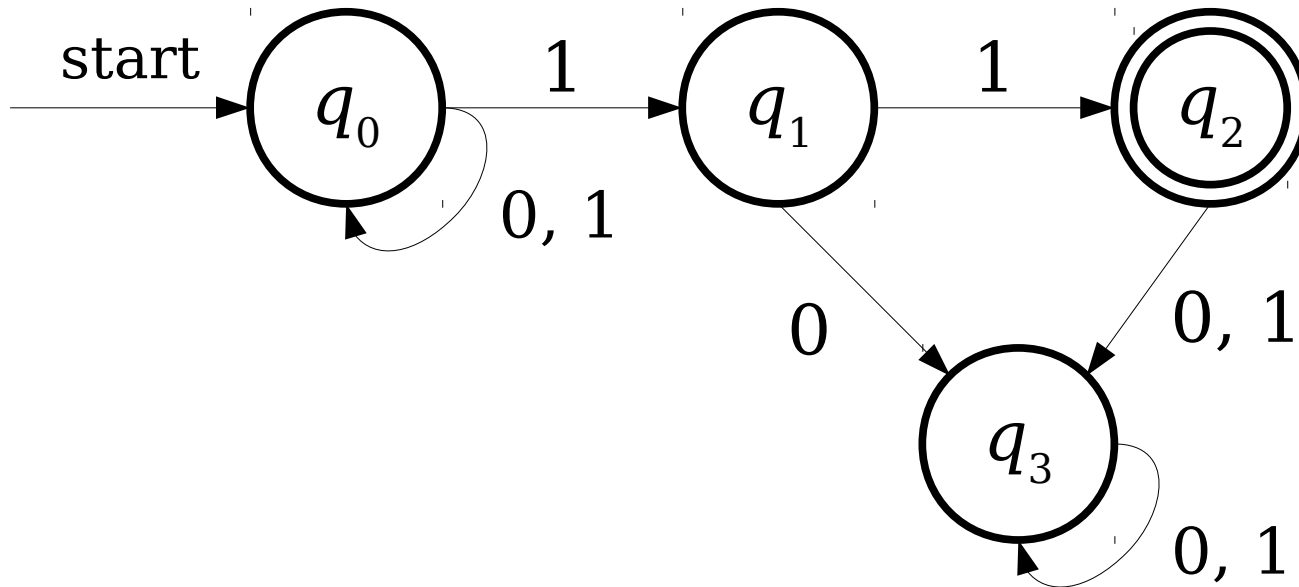


# A Simple NFA



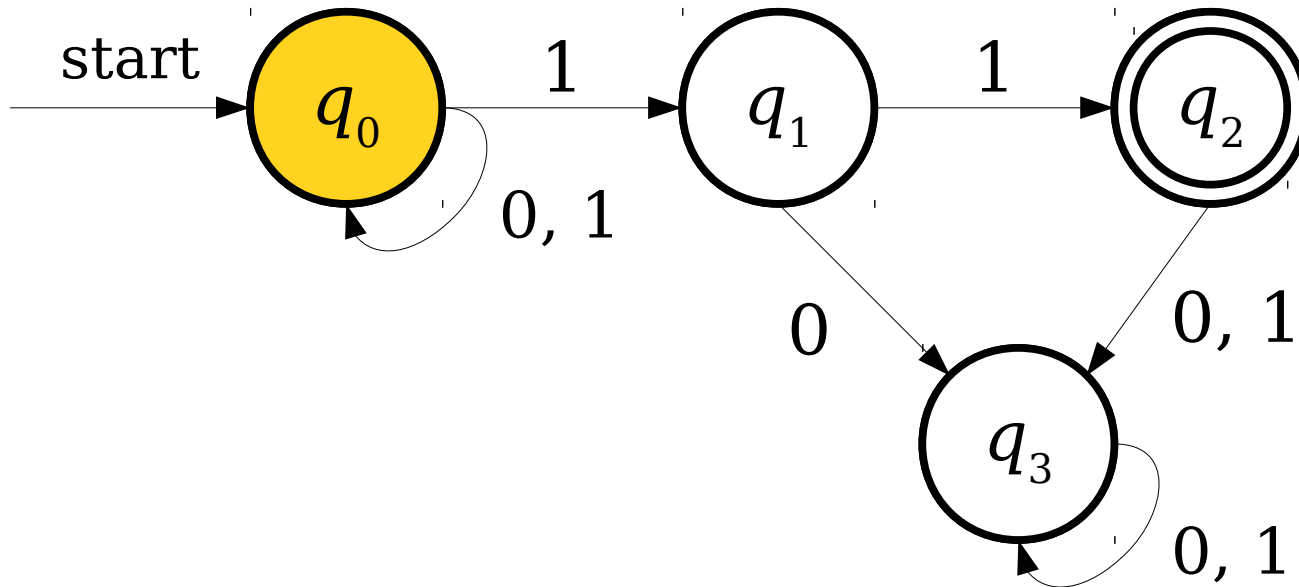
$q_0$  has two transitions defined on 1!

# A Simple NFA



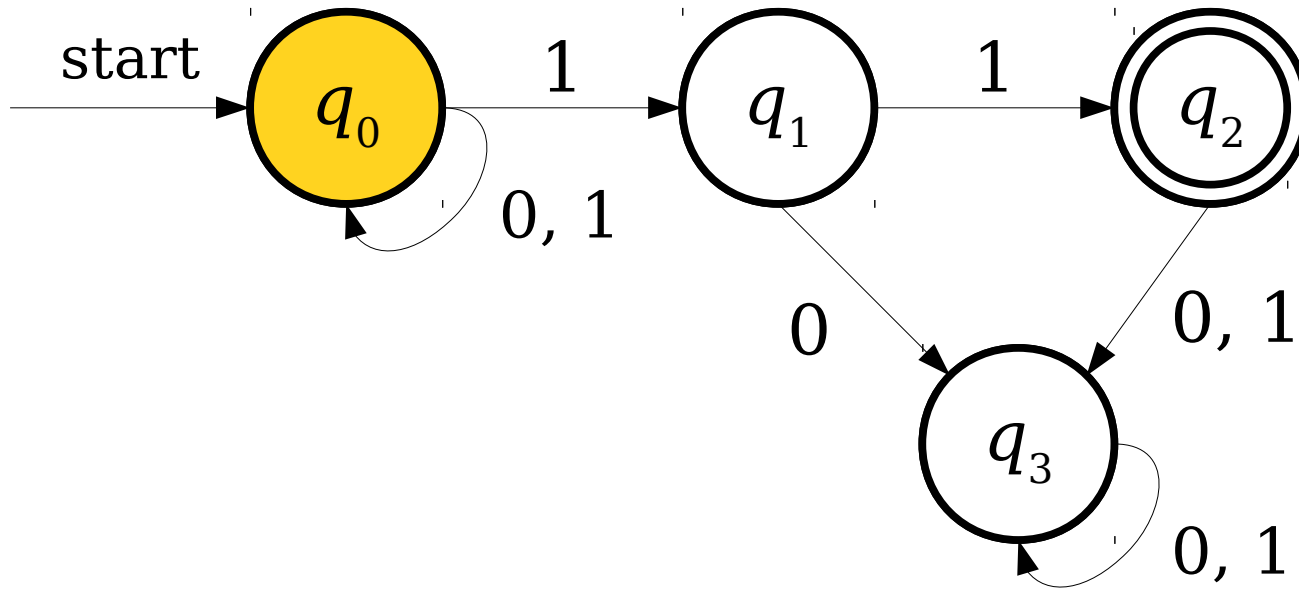
**0 1 0 1 1**

# A Simple NFA

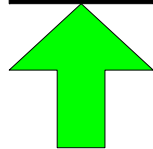


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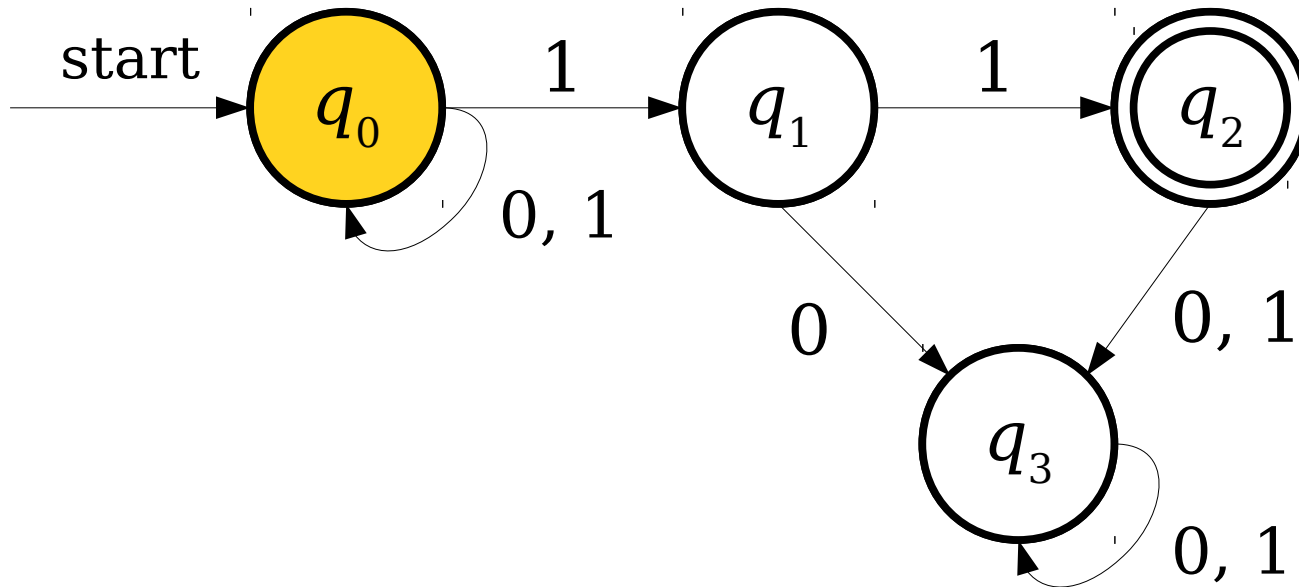
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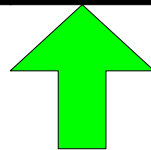
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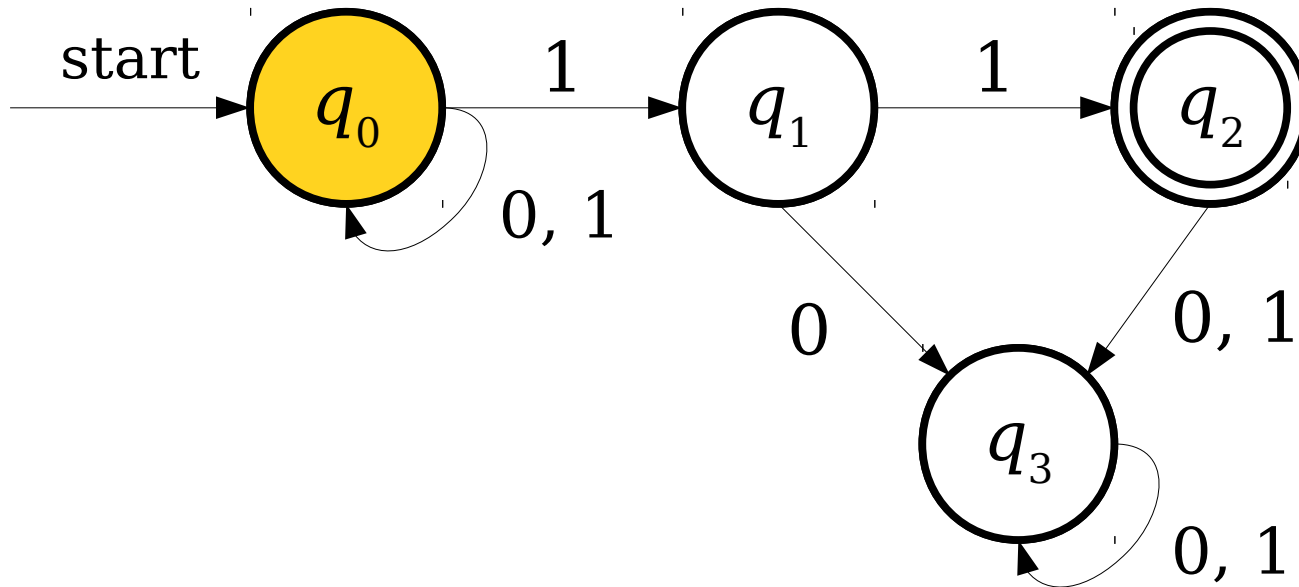


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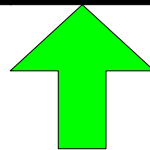




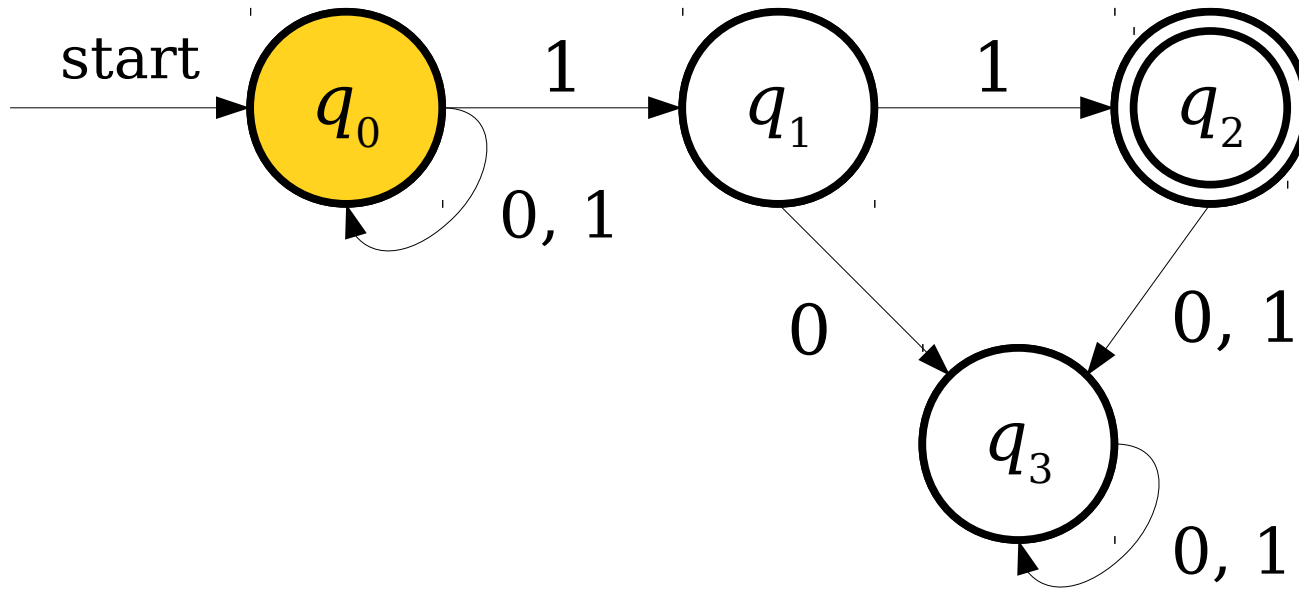
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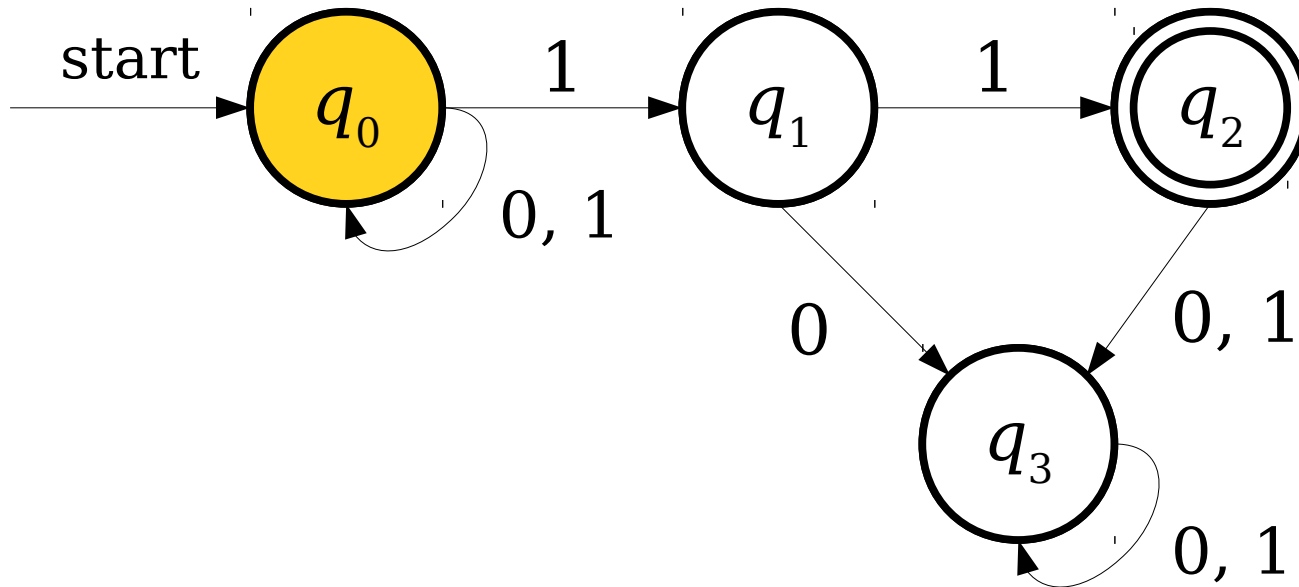
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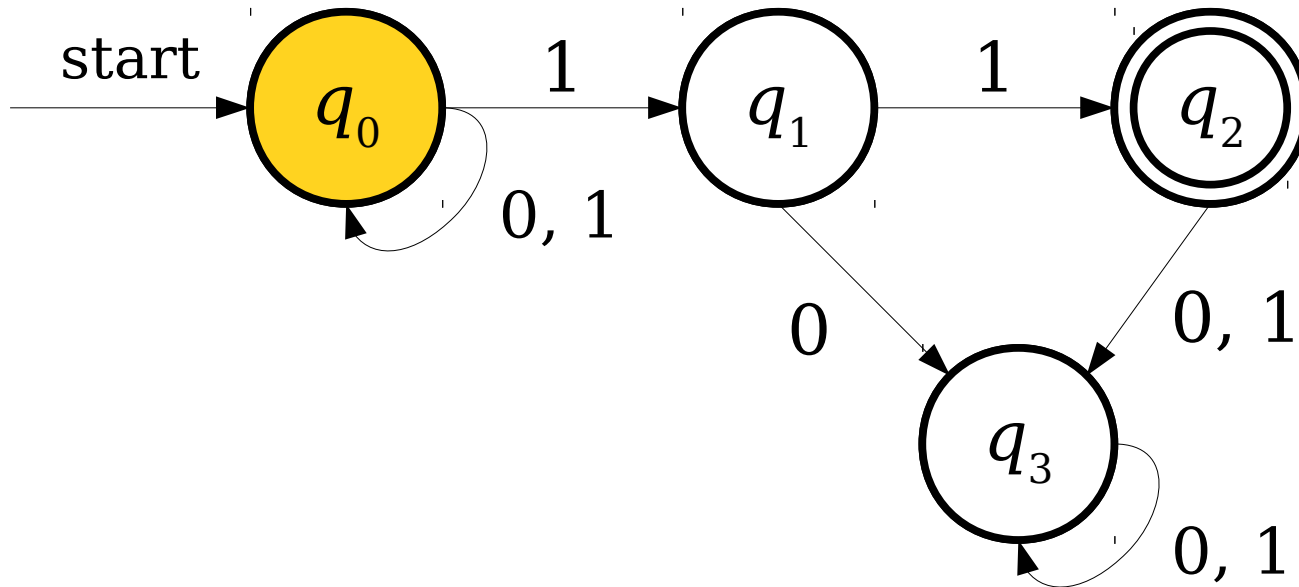
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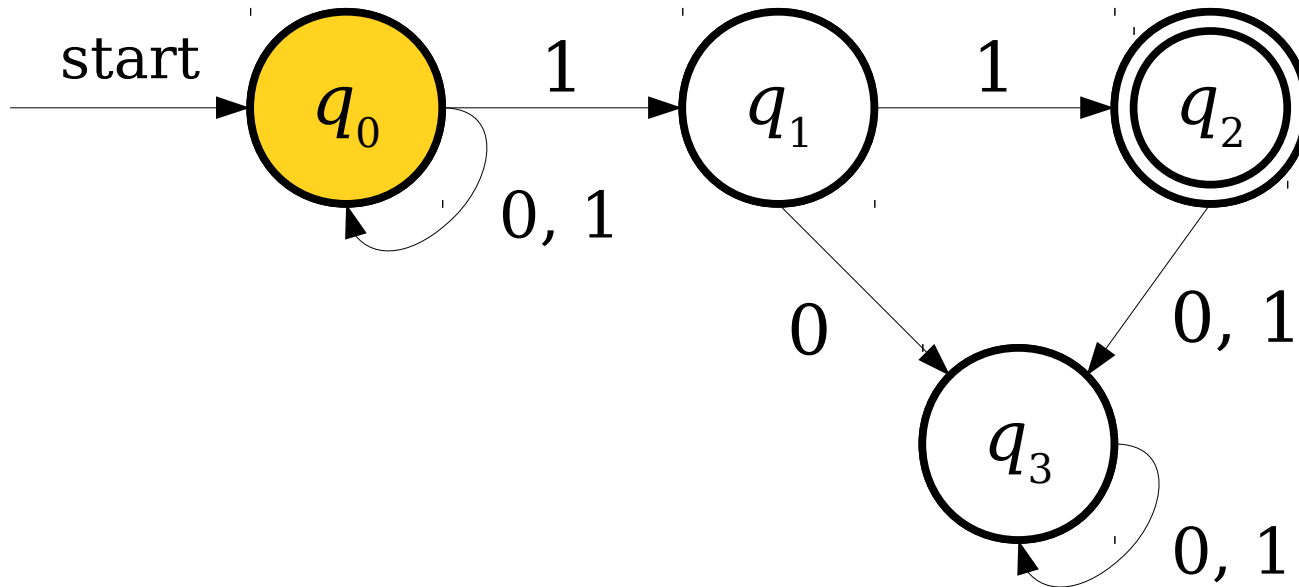


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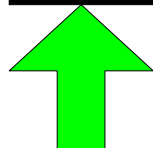


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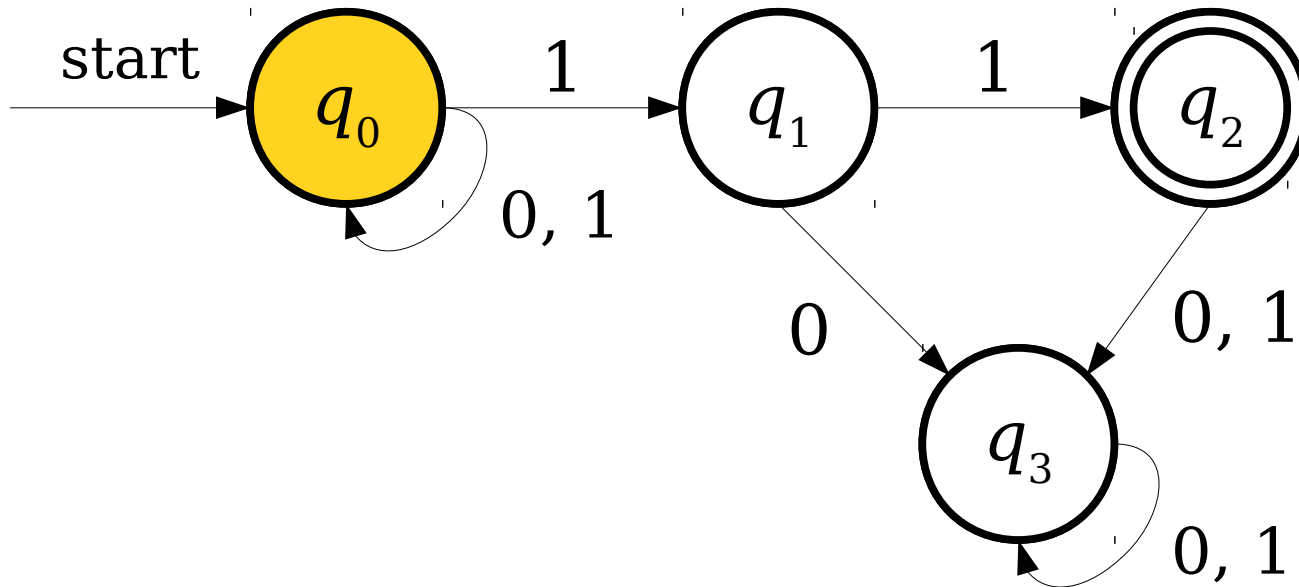
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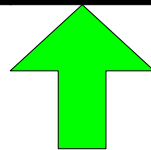
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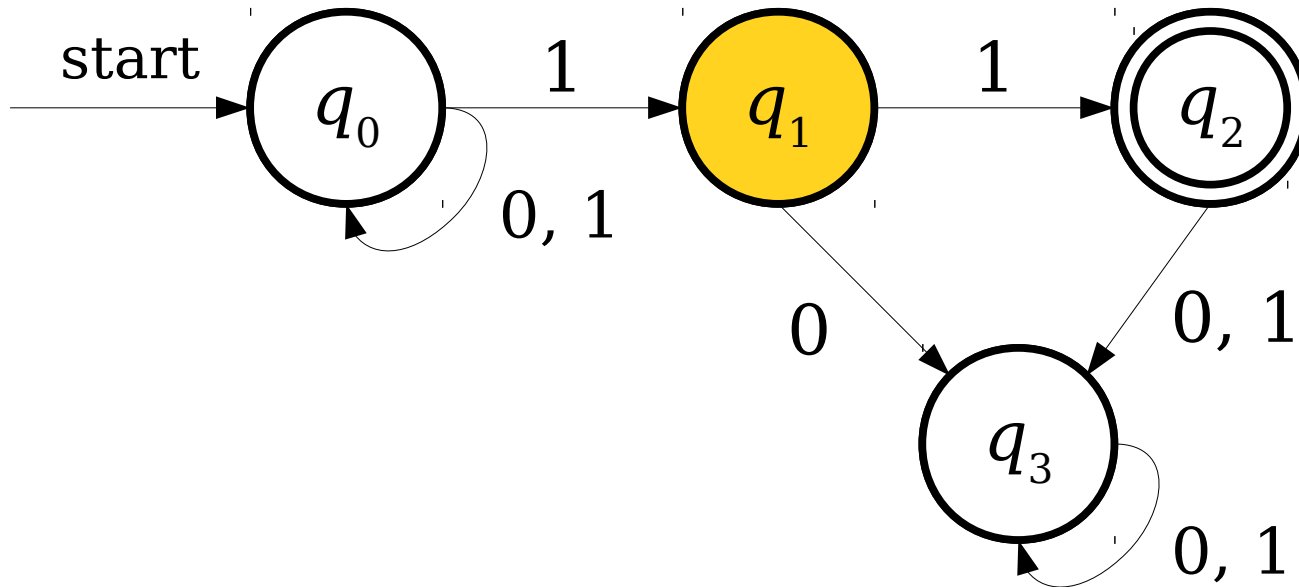
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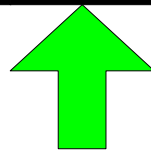
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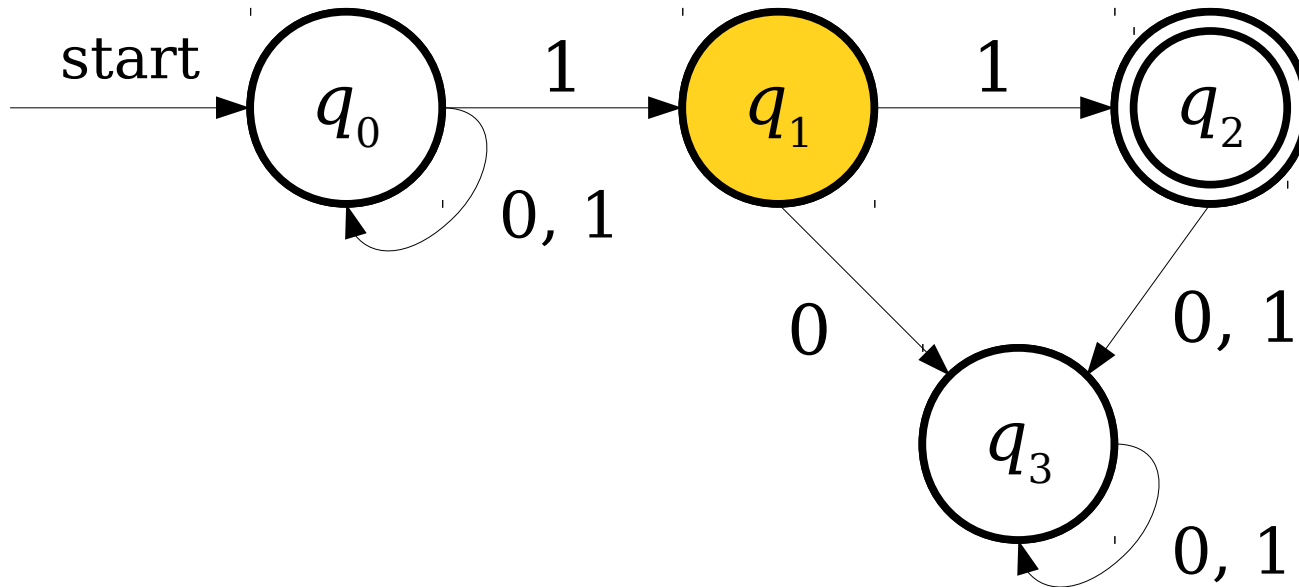
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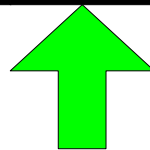
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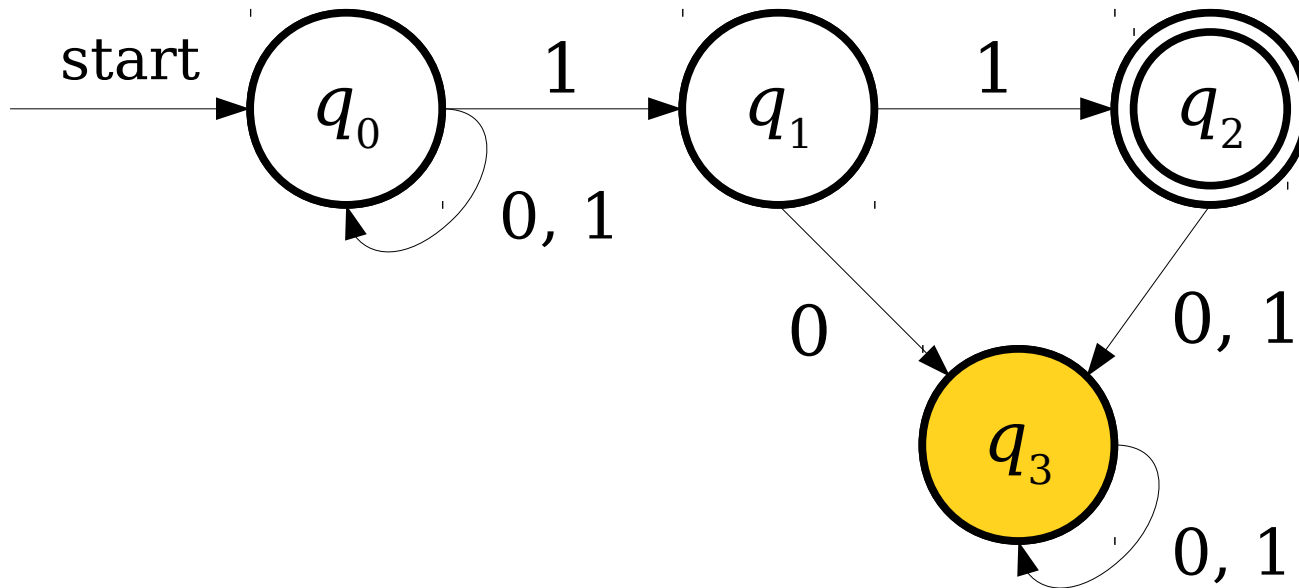


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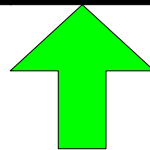




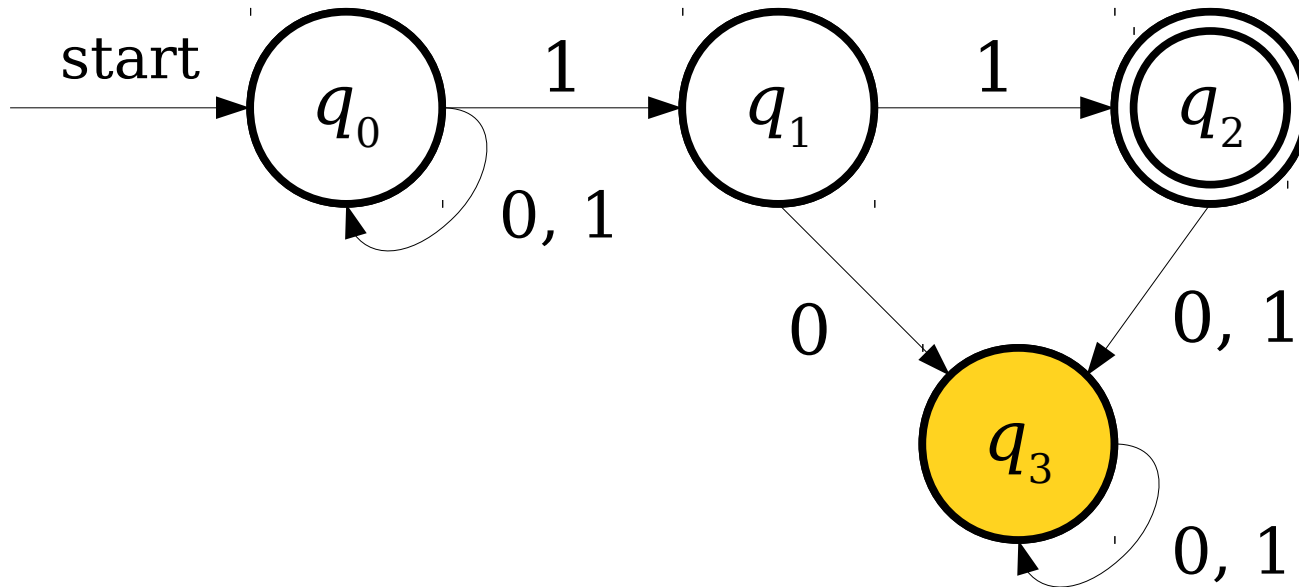
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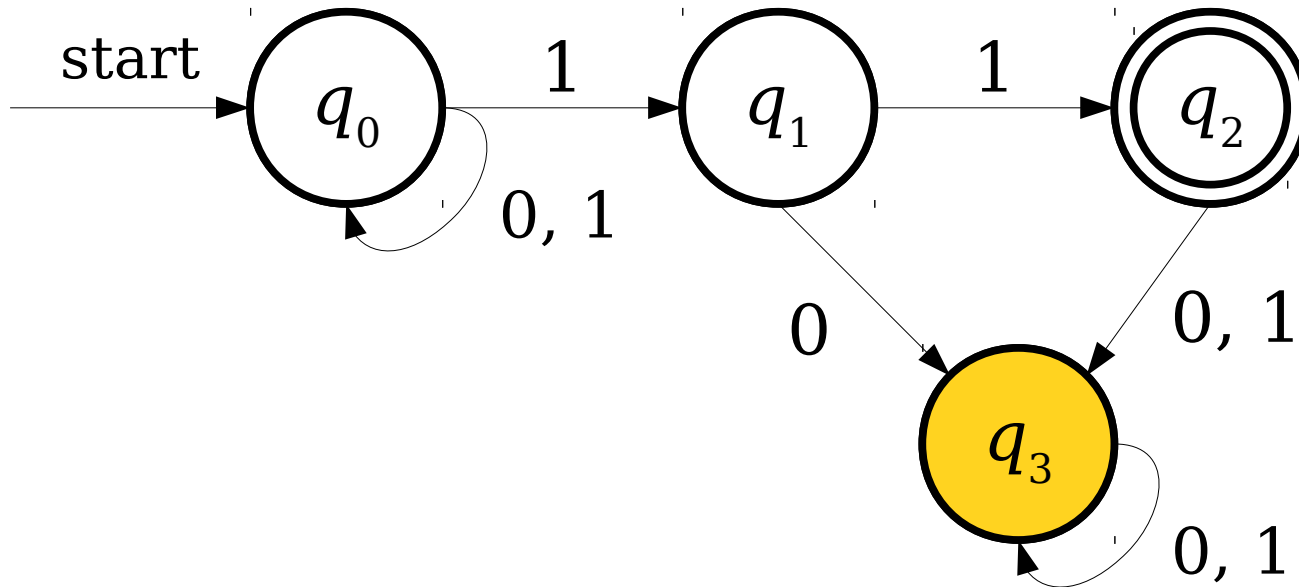
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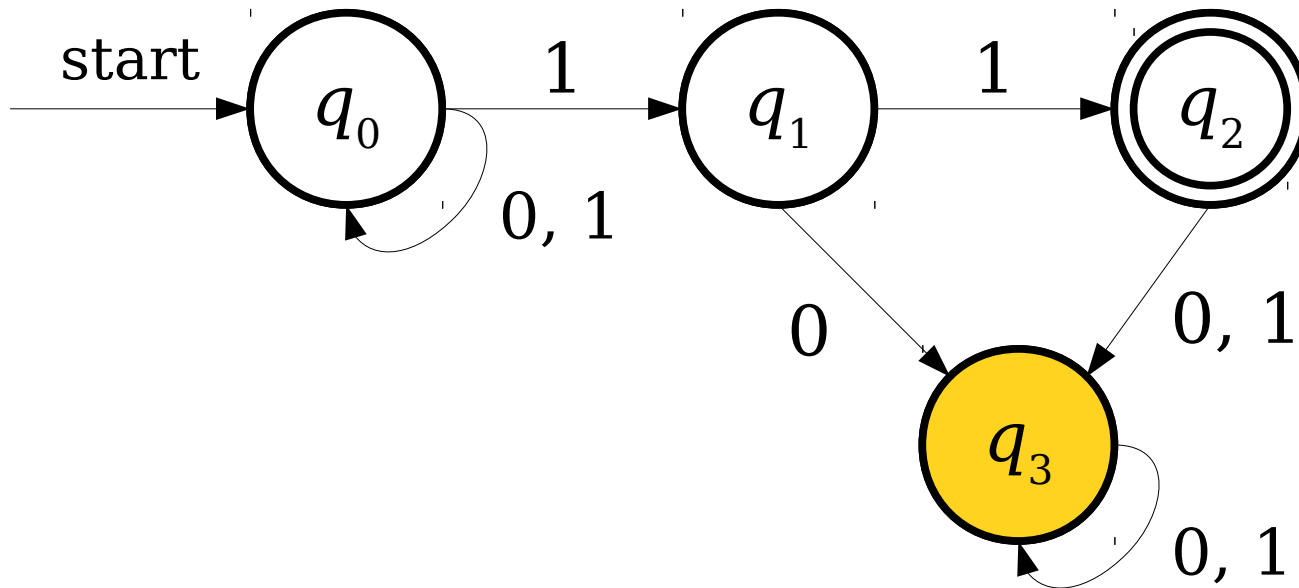
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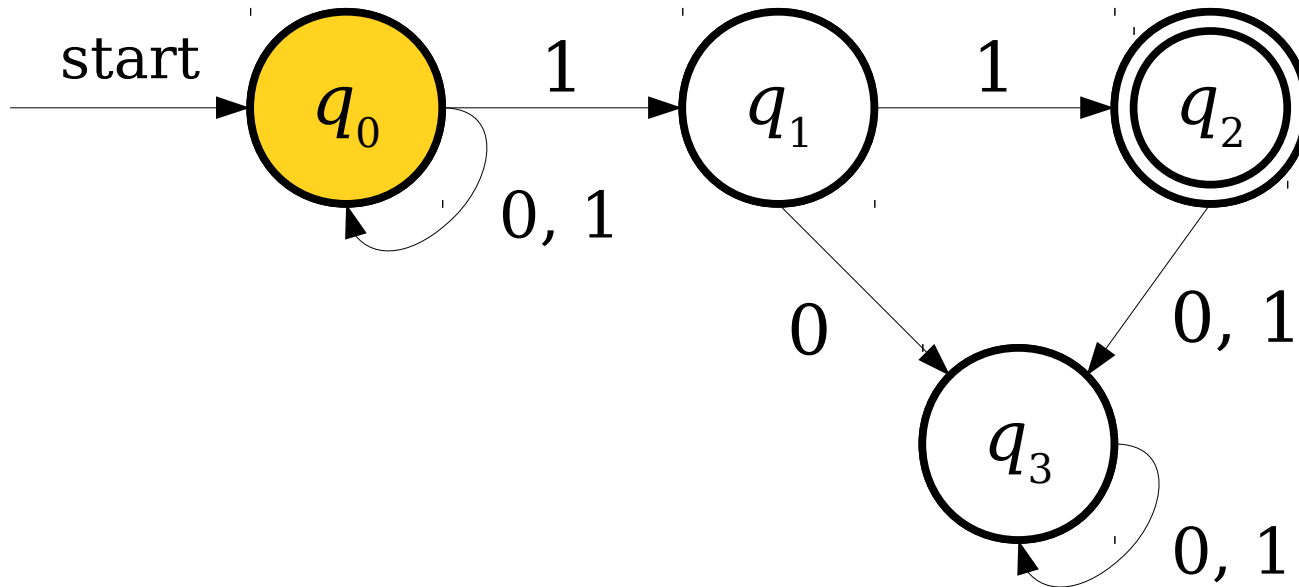


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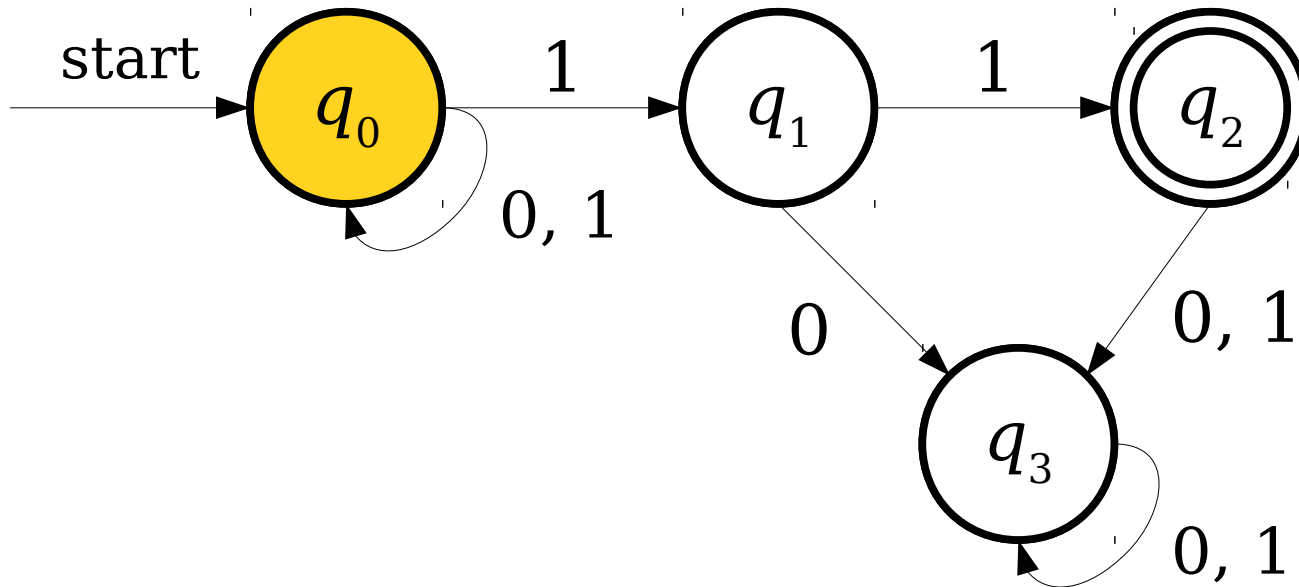
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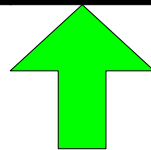
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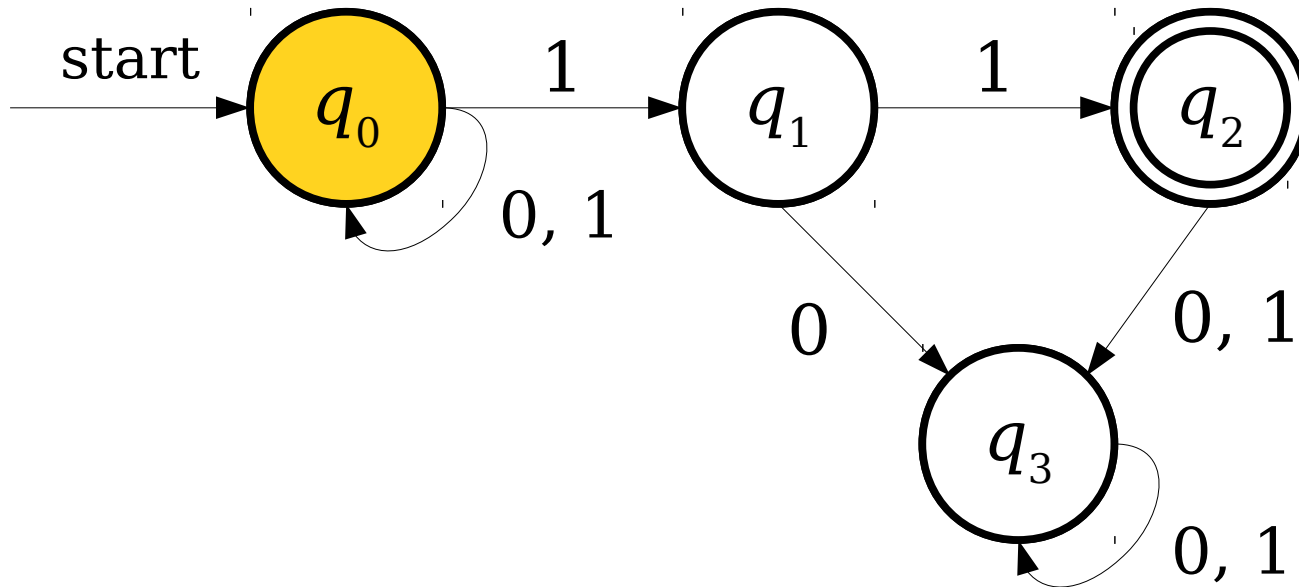
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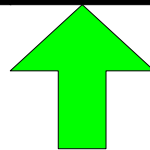
**0 1 0 1 1**



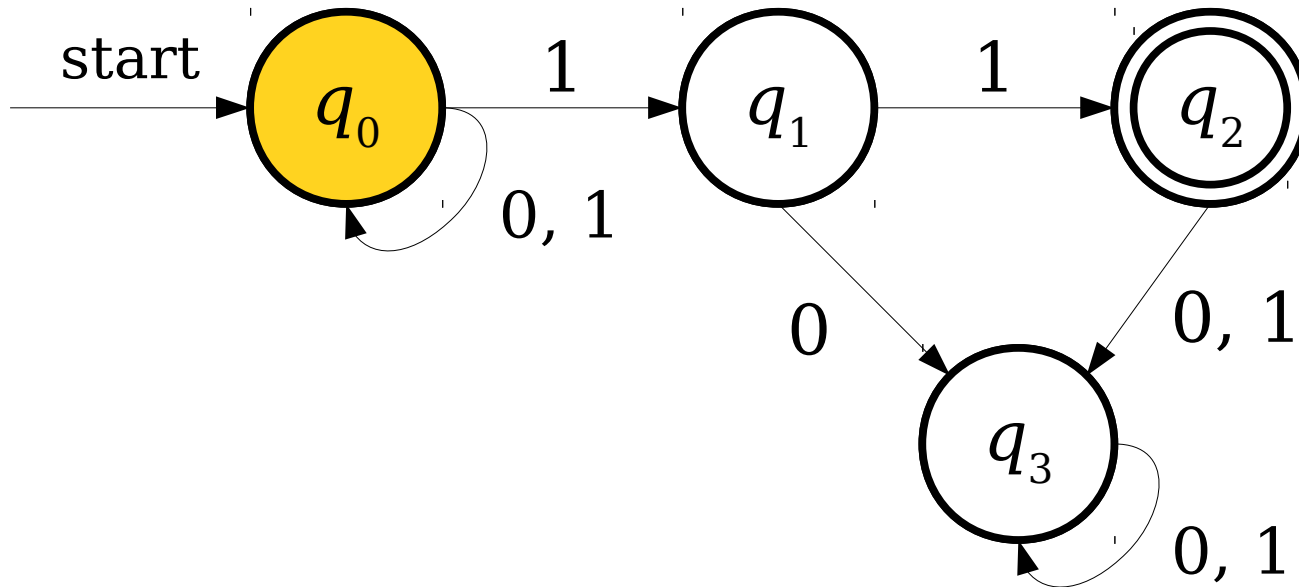
# A Simple NFA



**0 1 0 1 1**



# A Simple NFA

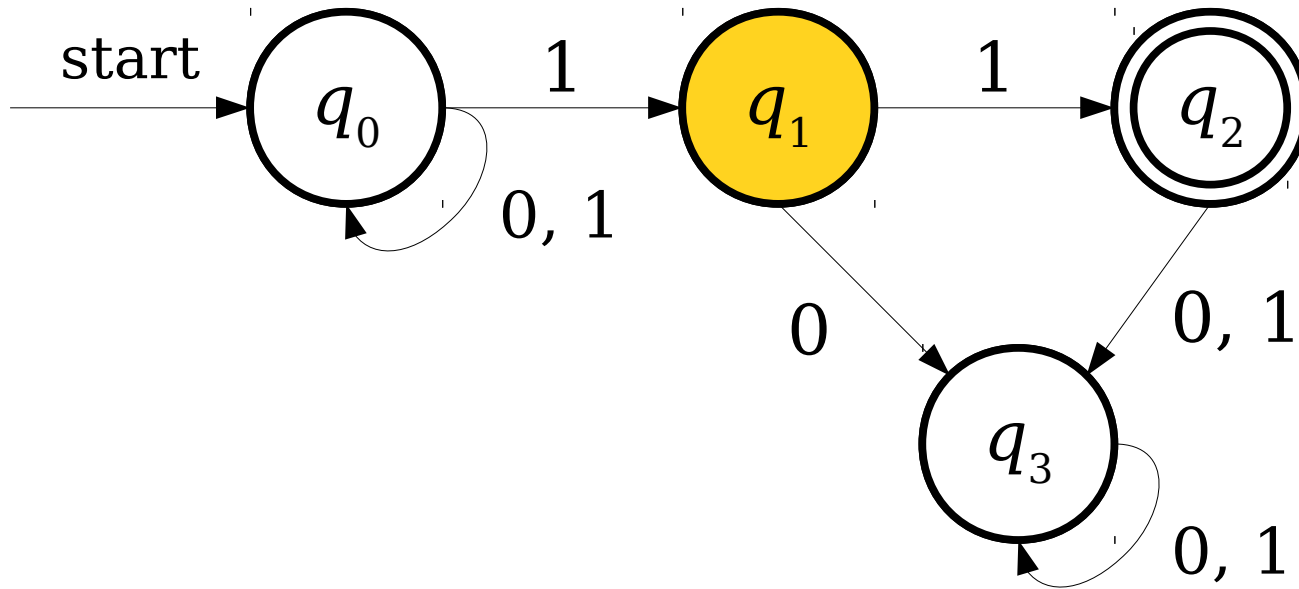


**0 1 0 1 1**





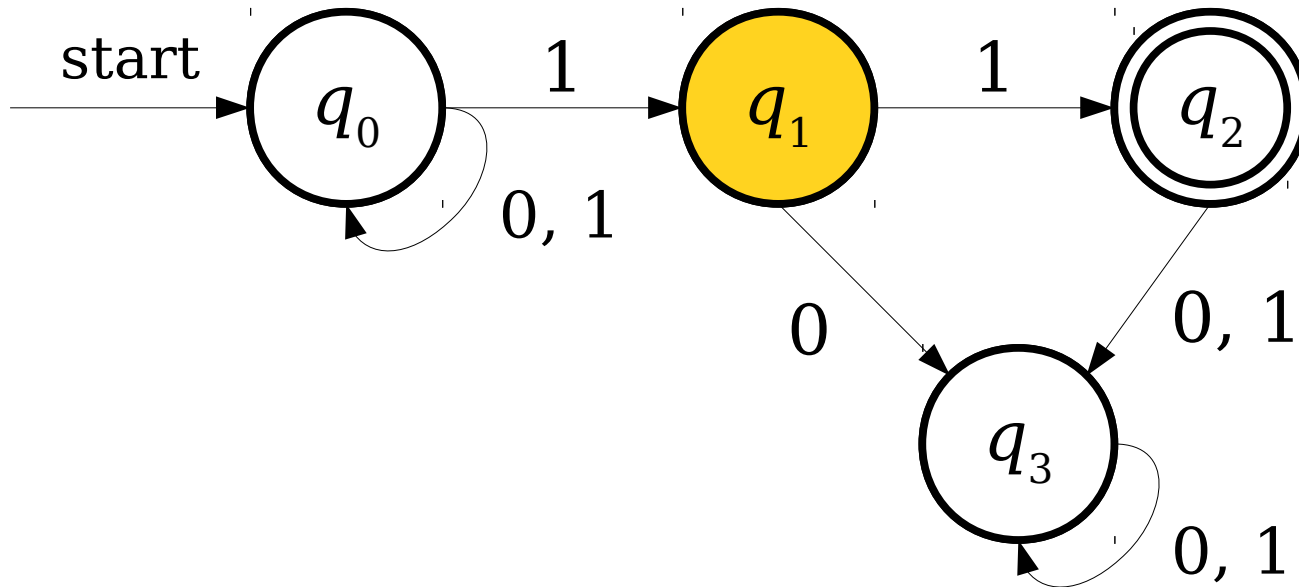
# A Simple NFA



**0 1 0 1 1**



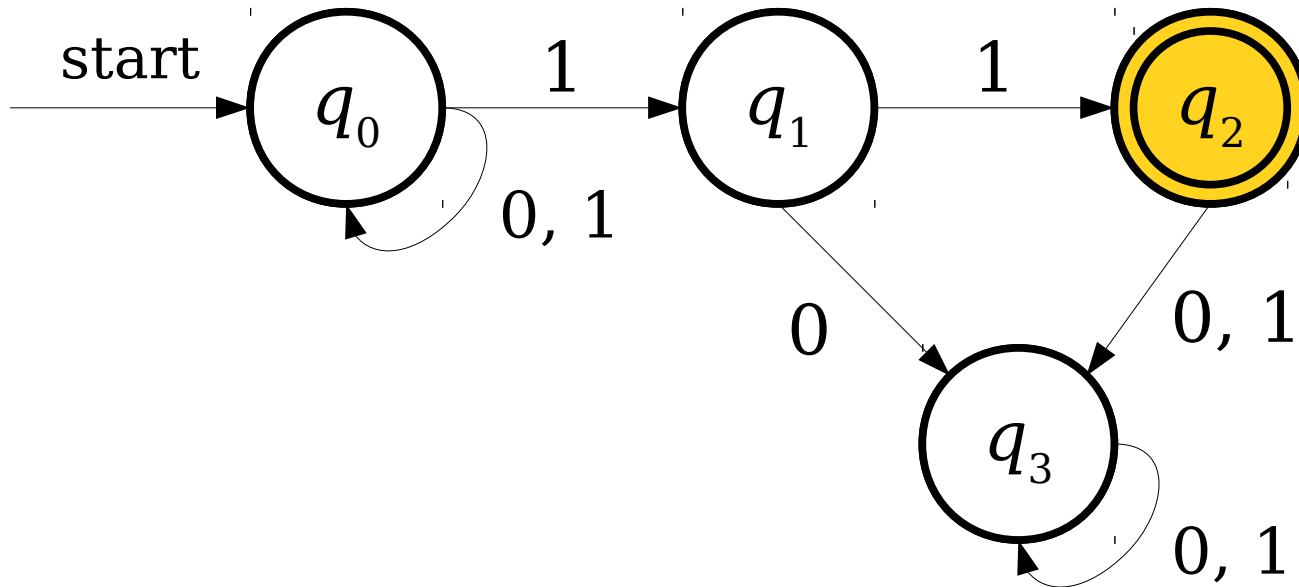
# A Simple NFA



**0 1 0 1 1**



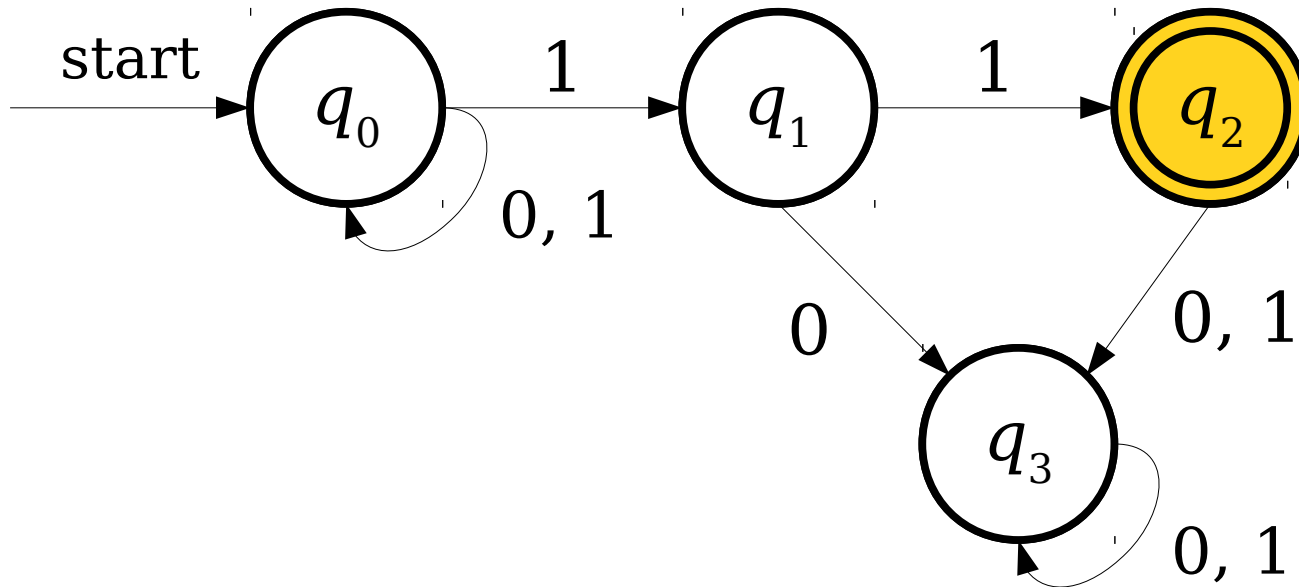
# A Simple NFA



**0 1 0 1 1**

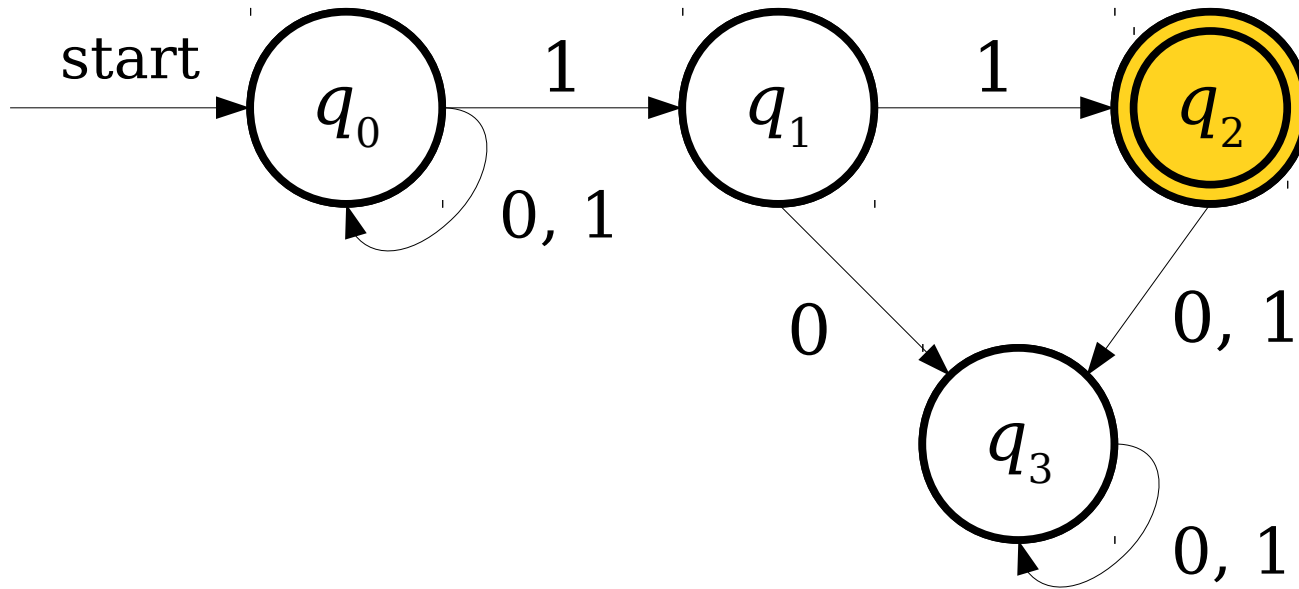


# A Simple NFA



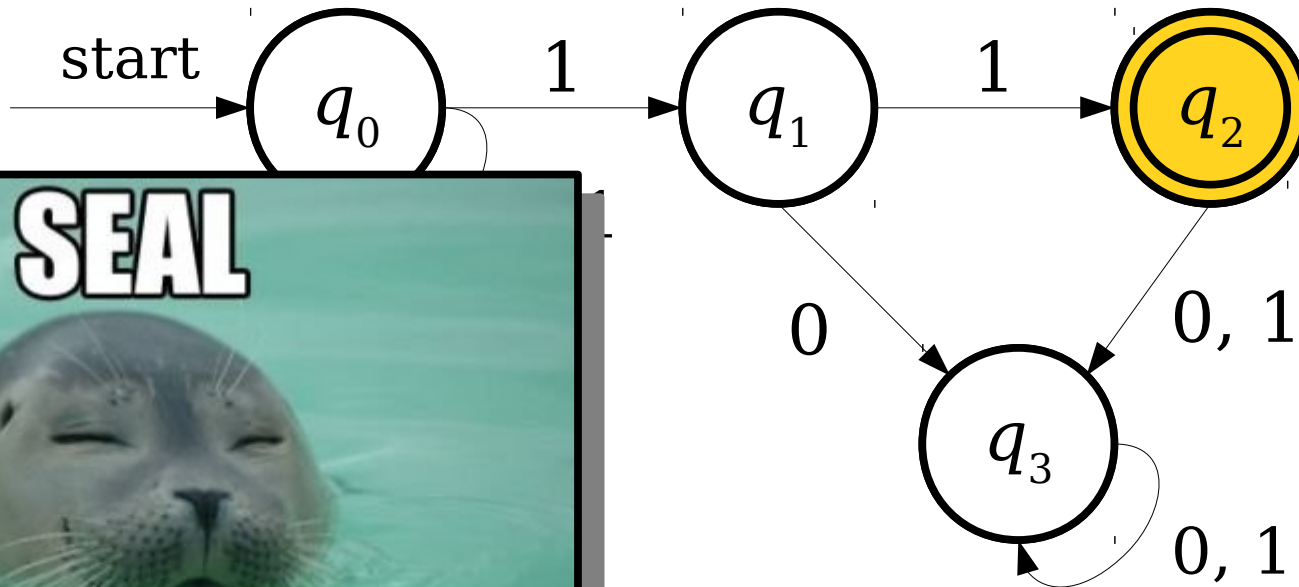
**0 1 0 1 1**

# A Simple NFA



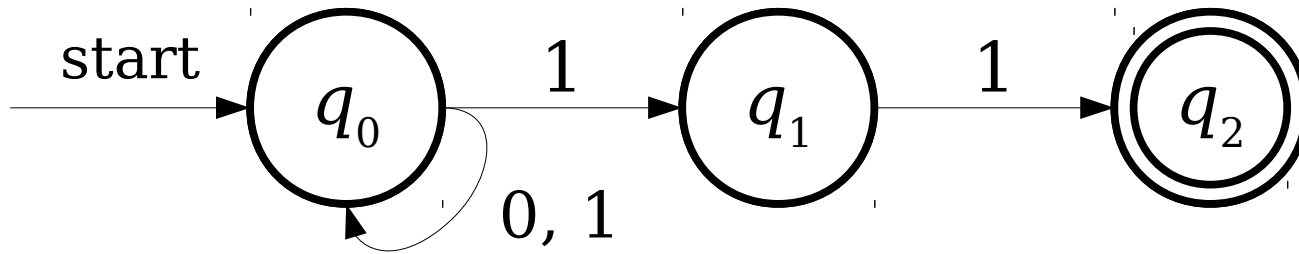
**0 1 0 1 1**

# A Simple NFA

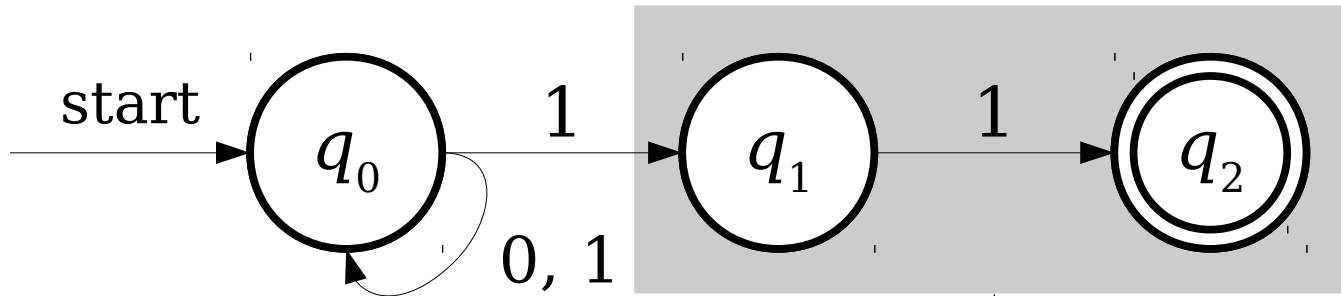


0 1 0 1 1

# A More Complex NFA



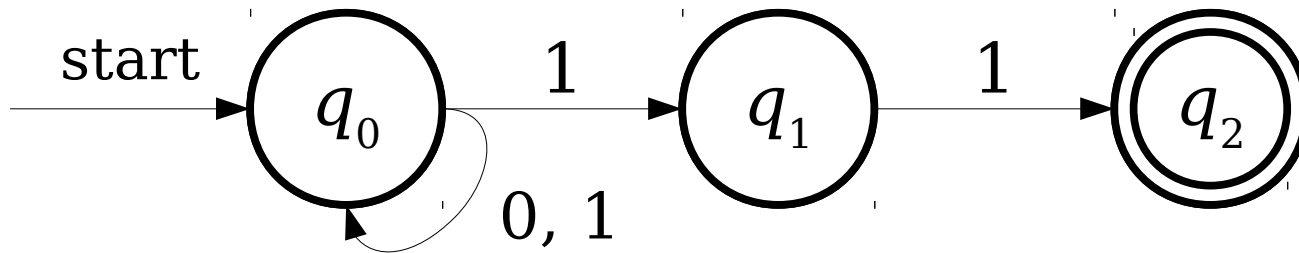
# A More Complex NFA



If a NFA needs to make a transition when no transition exists, the automaton **dies** and that particular path rejects.

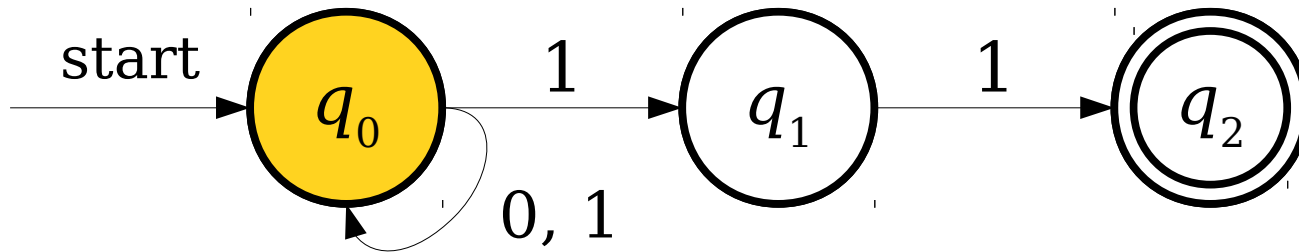


# A More Complex NFA



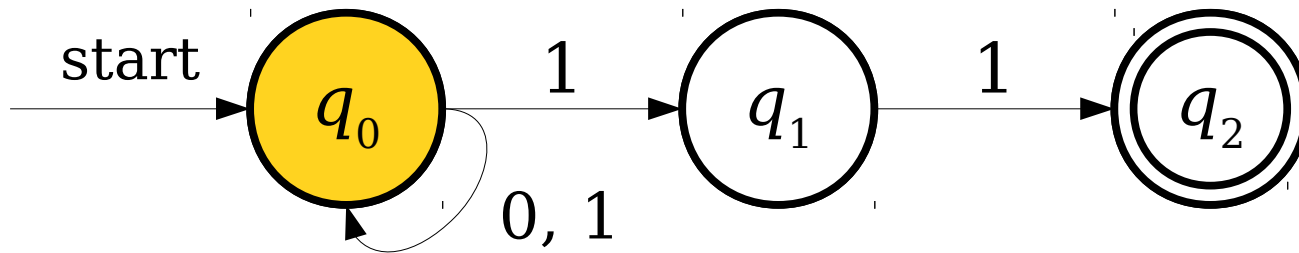
**0 1 0 1 1**

# A More Complex NFA



**0 1 0 1 1**

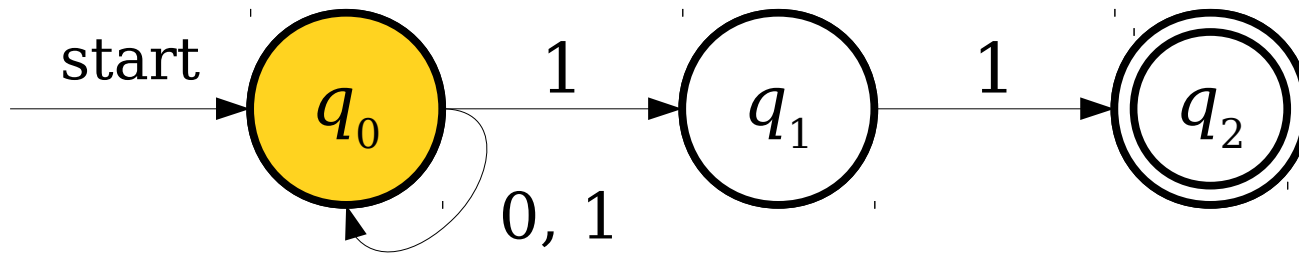
# A More Complex NFA



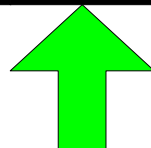
**0 1 0 1 1**

A green arrow points to the first character '0' of the string.

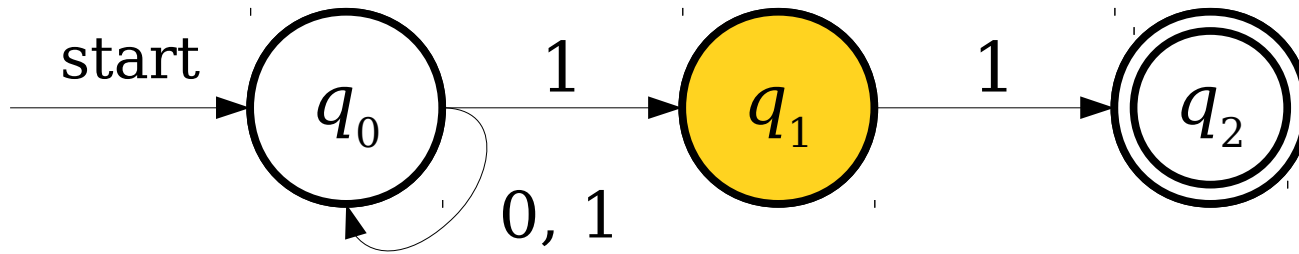
# A More Complex NFA



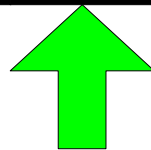
**0 1 0 1 1**



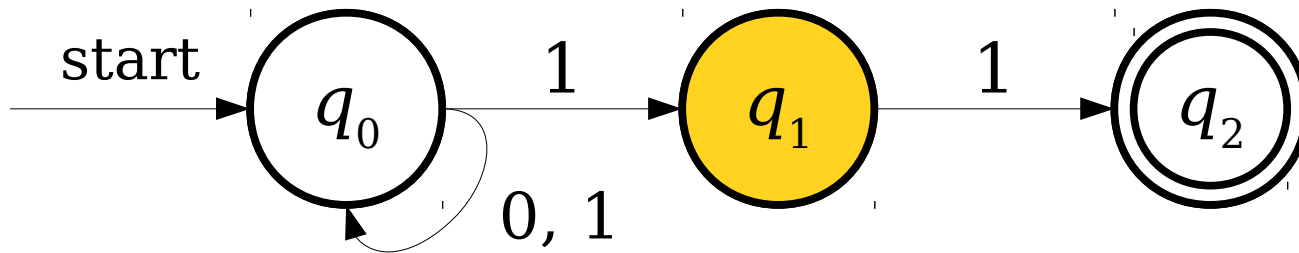
# A More Complex NFA



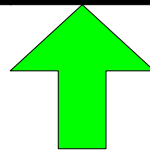
**0 1 0 1 1**



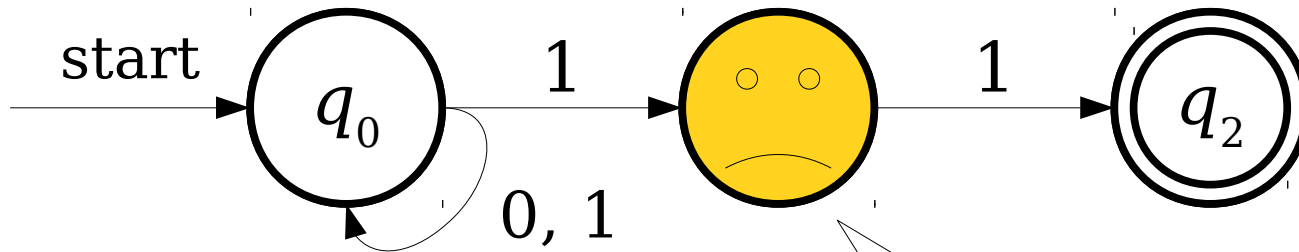
# A More Complex NFA



**0 1 0 1 1**

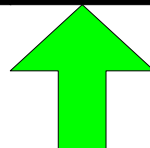


# A More Complex NFA

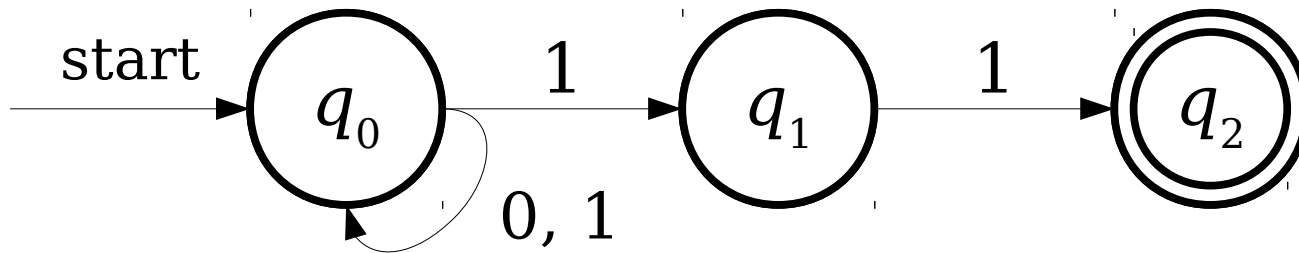


Oh no! There's no transition defined!

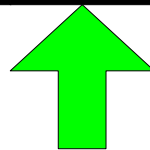
**0 1 0 1 1**



# A More Complex NFA

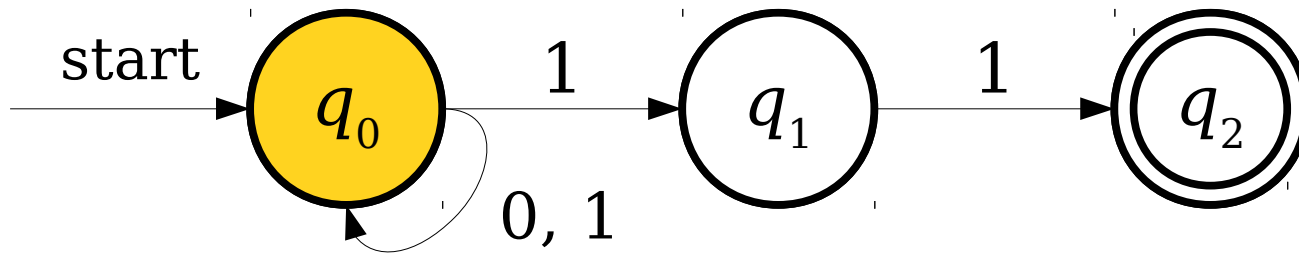


**0 1 0 1 1**





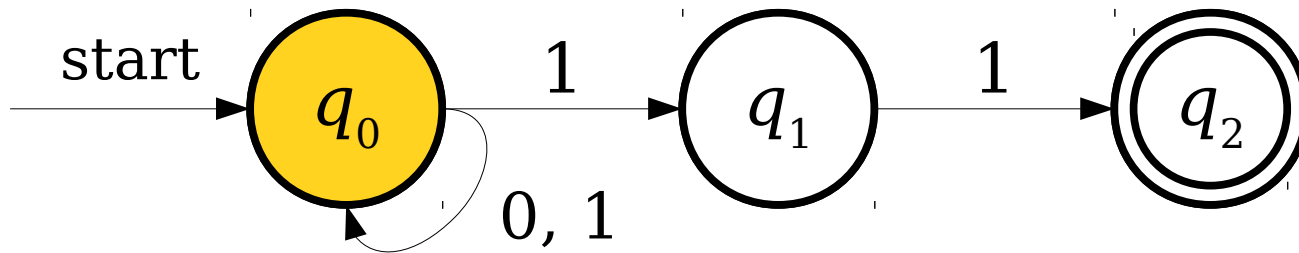
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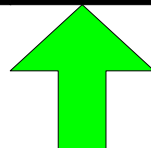
**0 1 0 1 1**

A green arrow points upwards to the first character '0' of the string '0 1 0 1 1'.

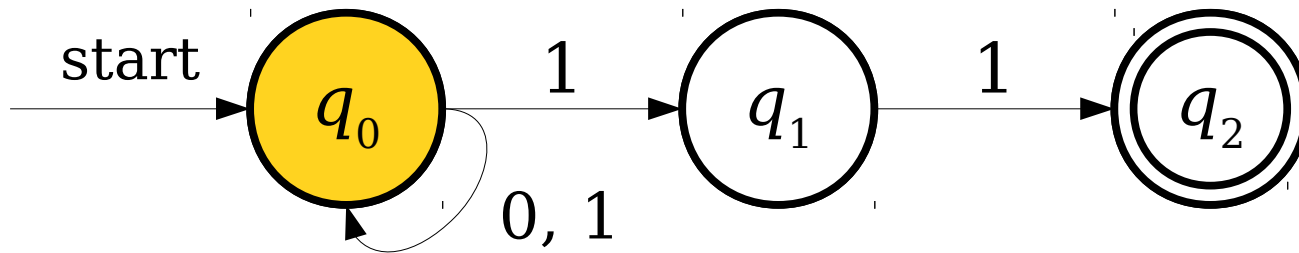
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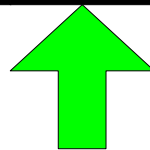
**0 1 0 1 1**



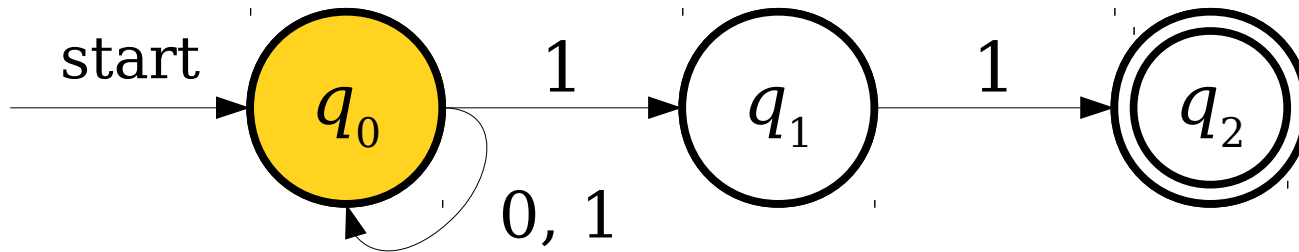
# A More Complex NFA



**0 1 0 1 1**



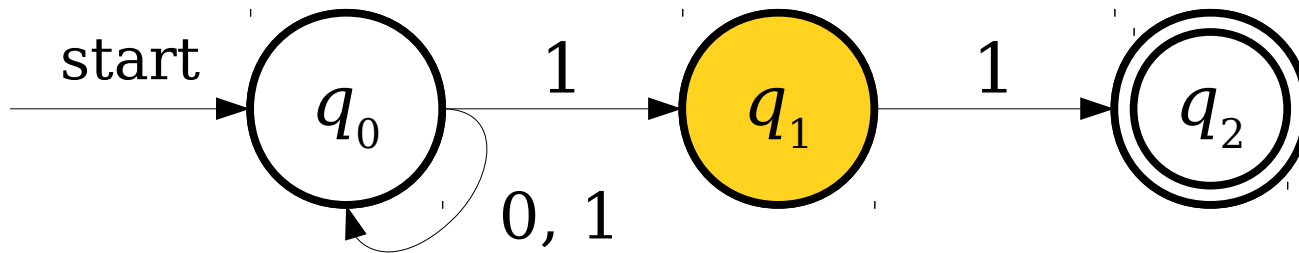
# A More Complex NFA



**0 1 0 1 1**



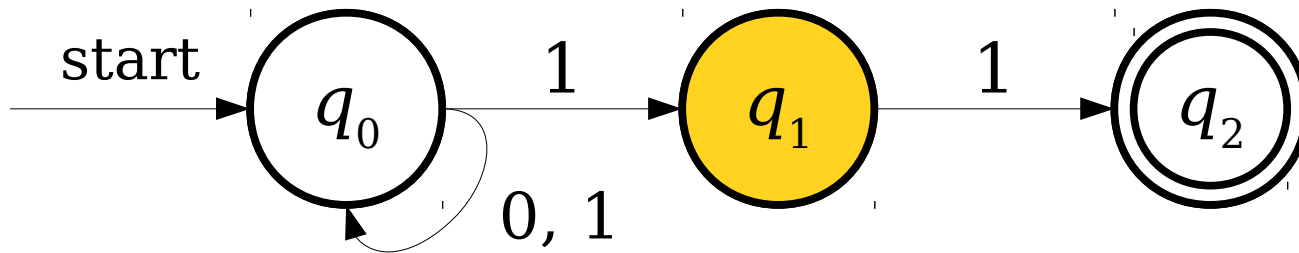
# A More Complex NFA



**0 1 0 1 1**



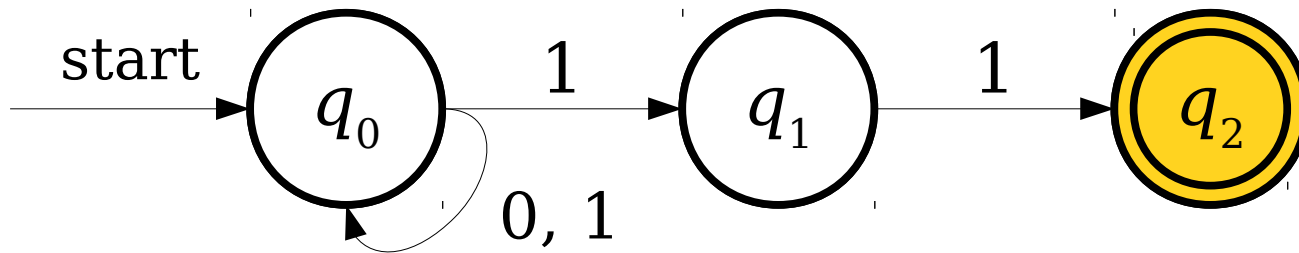
# A More Complex NFA



**0 1 0 1 1**

A green arrow points upwards to the final '1' in the string, indicating the current position in the input sequence.

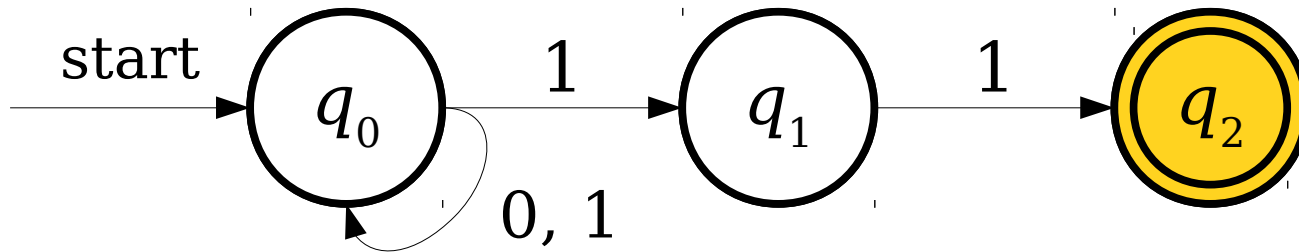
# A More Complex NFA



**0 1 0 1 1**



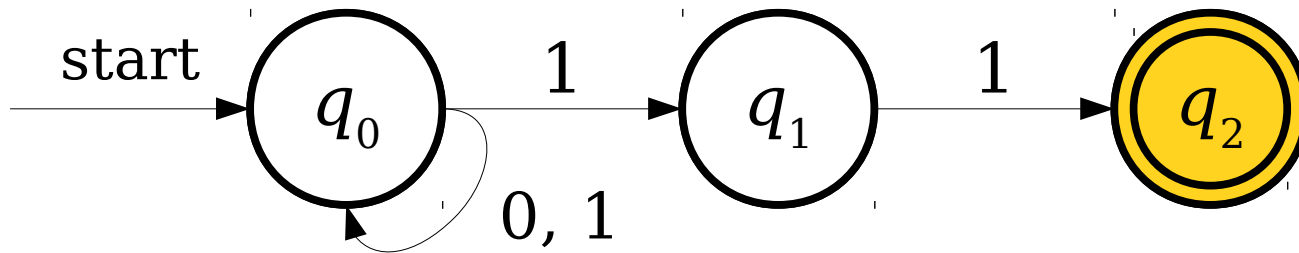
# A More Complex NFA



**0 1 0 1 1**

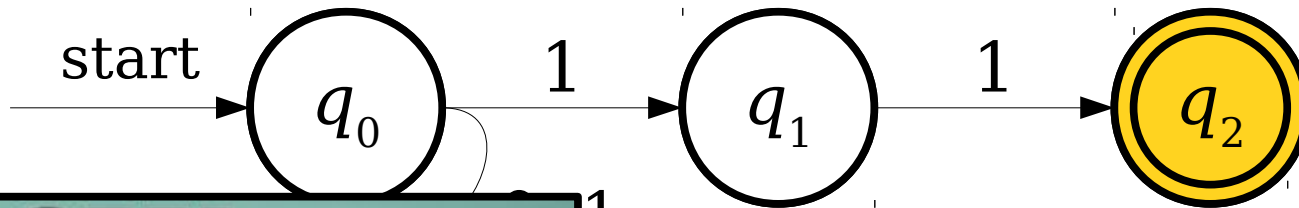


# A More Complex NFA



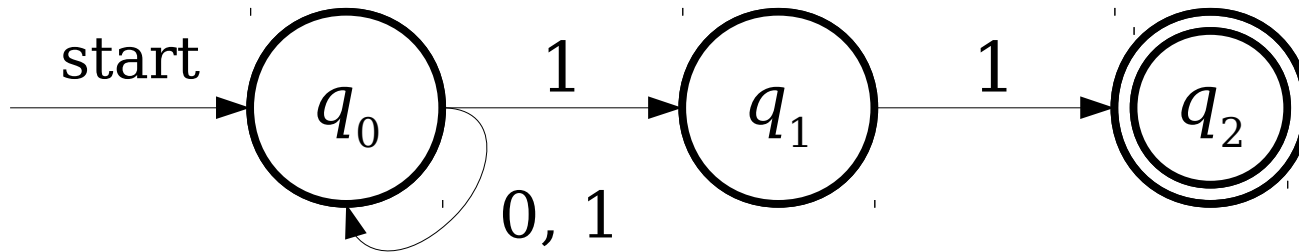
**0 1 0 1 1**

# A More Complex NFA



0 1 1

# A More Complex NFA



Question to ponder:  
What does this NFA  
accept?

# NFA Acceptance

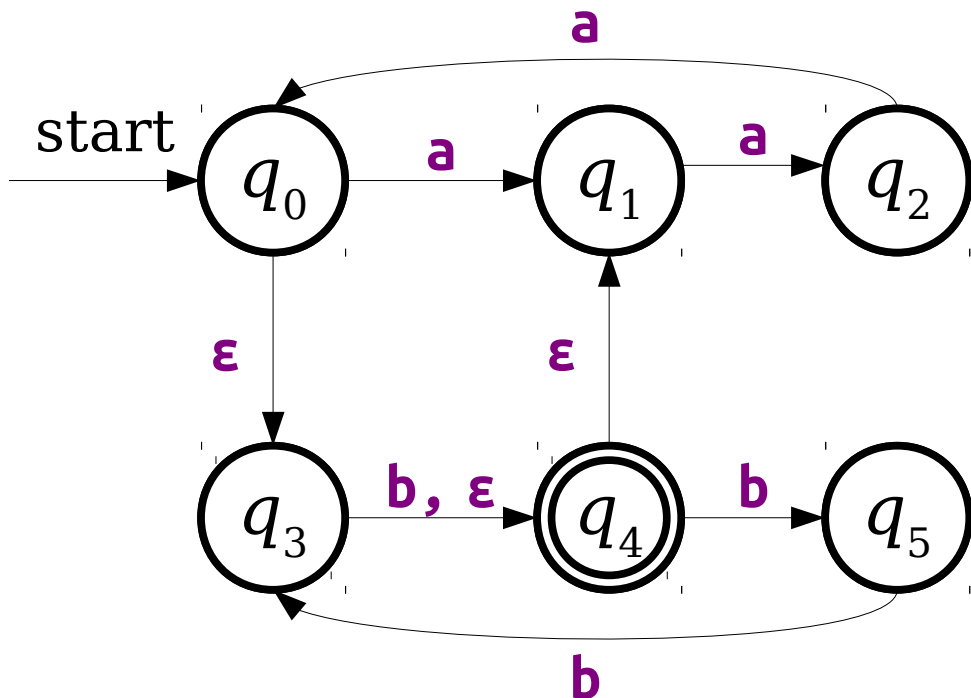
- An NFA  $N$  accepts a string  $w$  if there is some series of choices that lead to an accepting state.
- Consequently, an NFA  $N$  rejects a string  $w$  if *no possible* series of choices lead it into an accepting state.
- It's easier to show that an NFA does accept something than to show that it doesn't

# $\epsilon$ -Transitions

- NFAs have a special type of transition called the  **$\epsilon$ -transition**.
- An NFA may follow any number of  $\epsilon$ -transitions at any time without consuming any input.

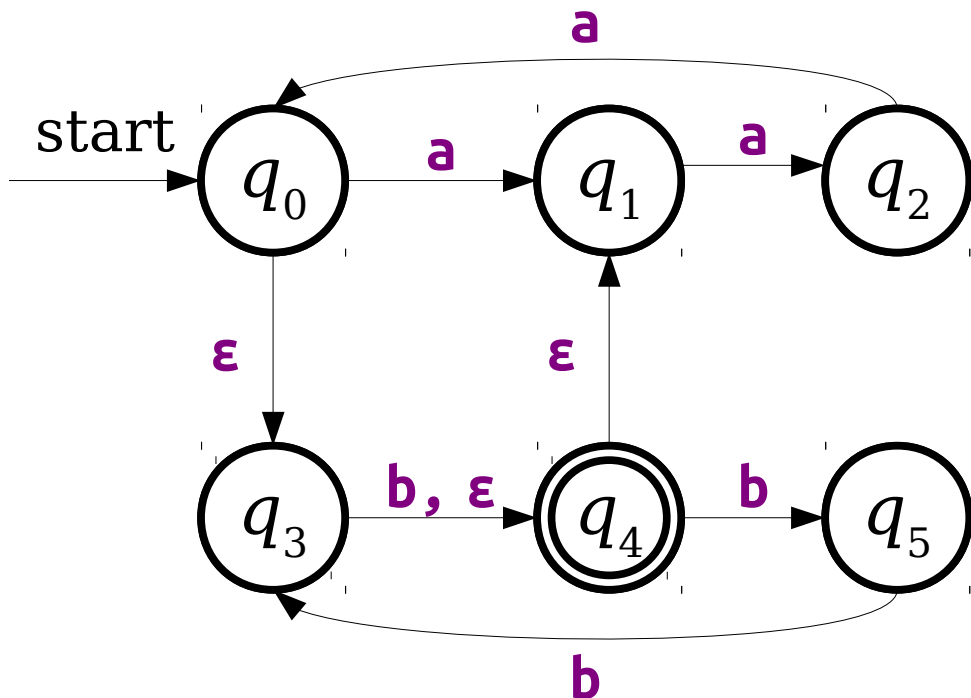
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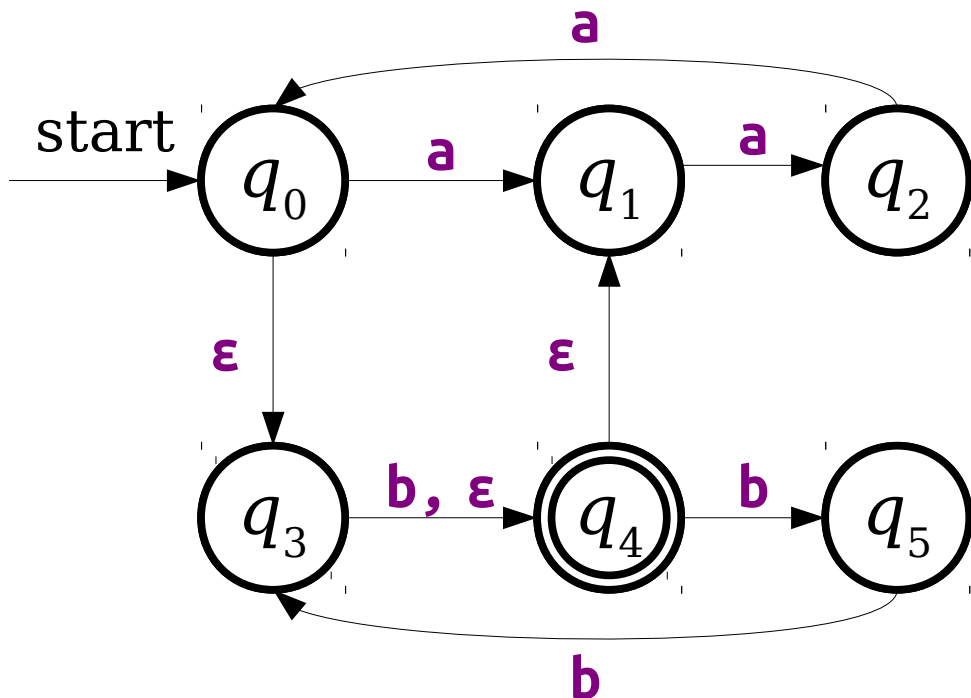
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**b a a b b**

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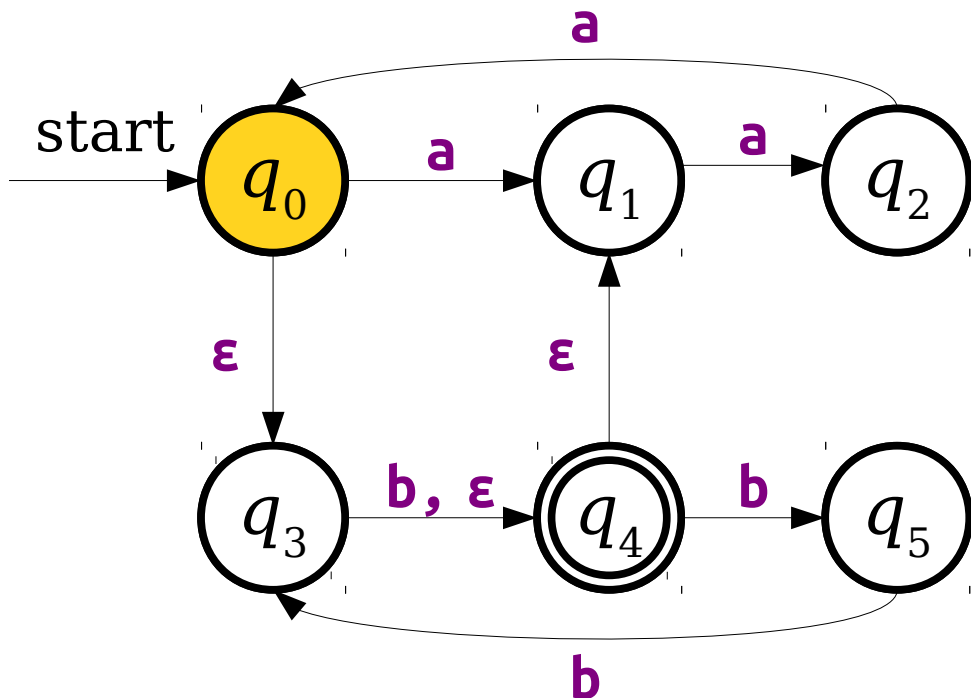
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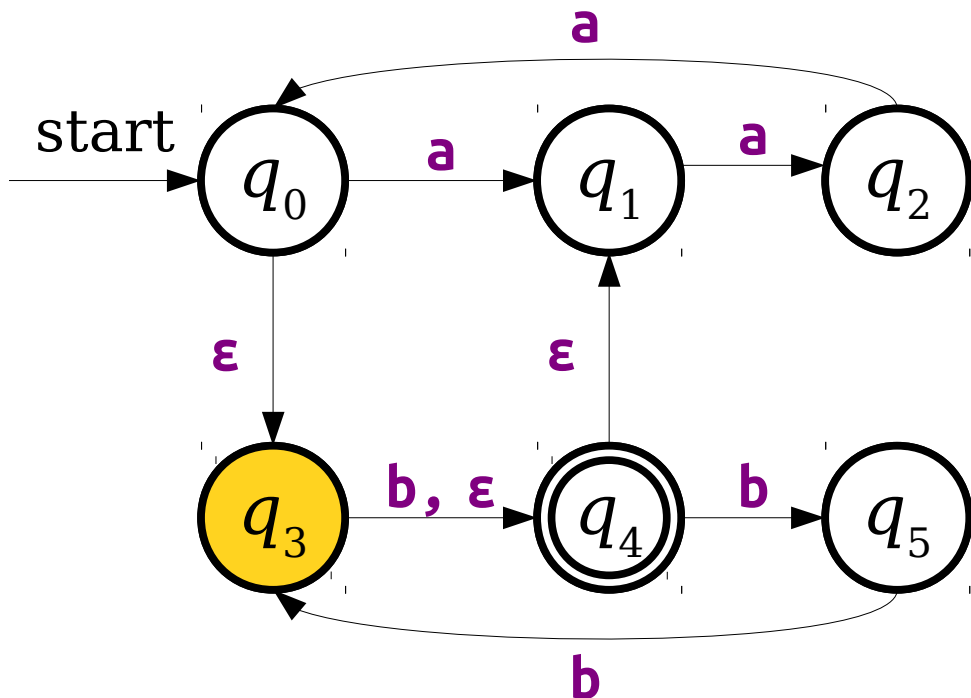
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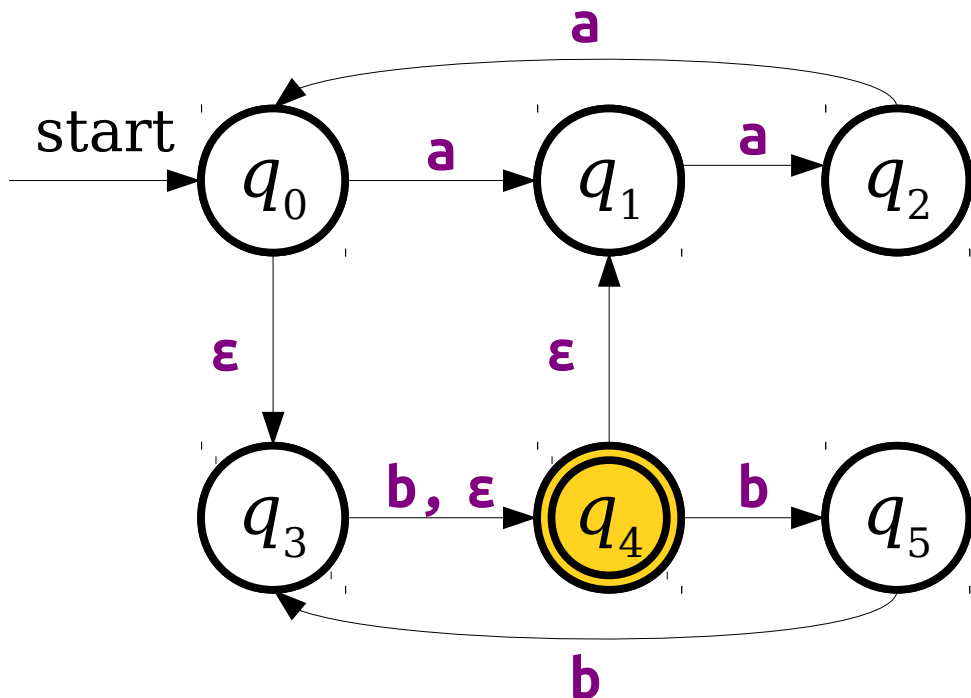
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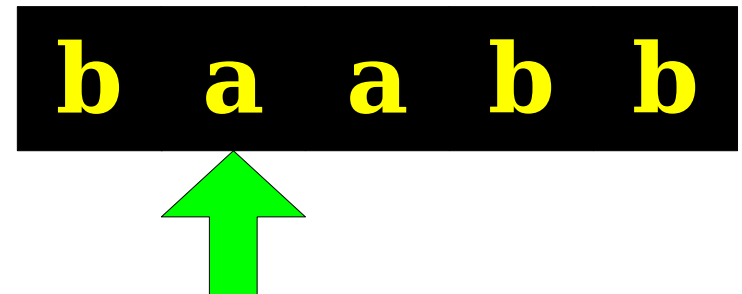
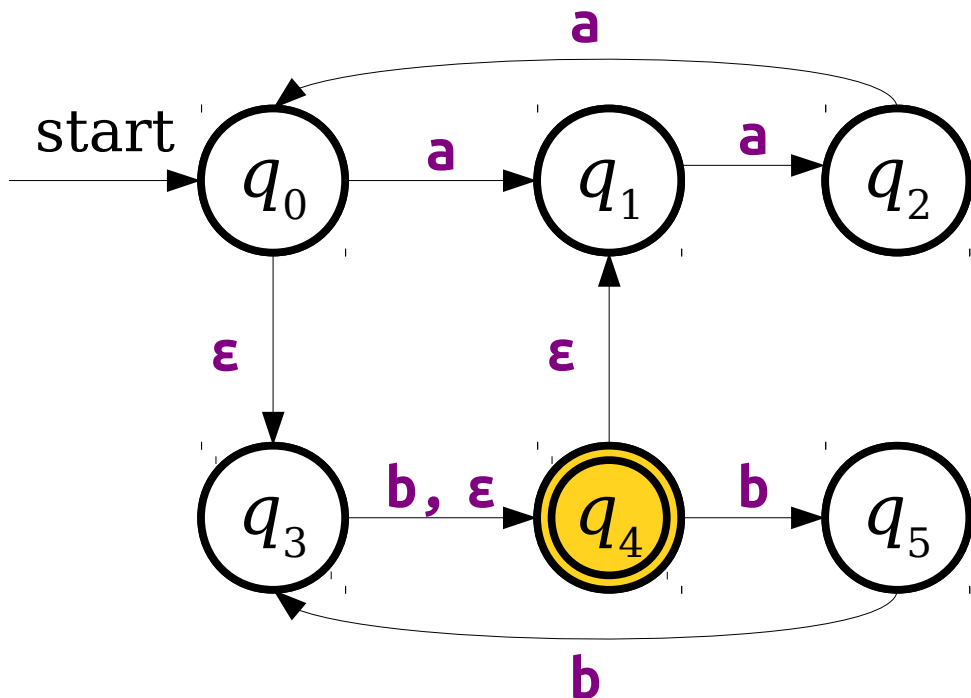
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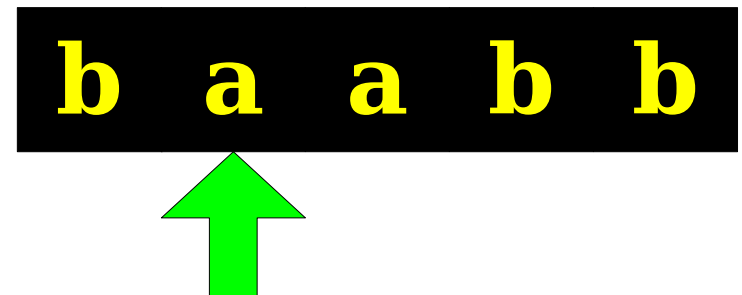
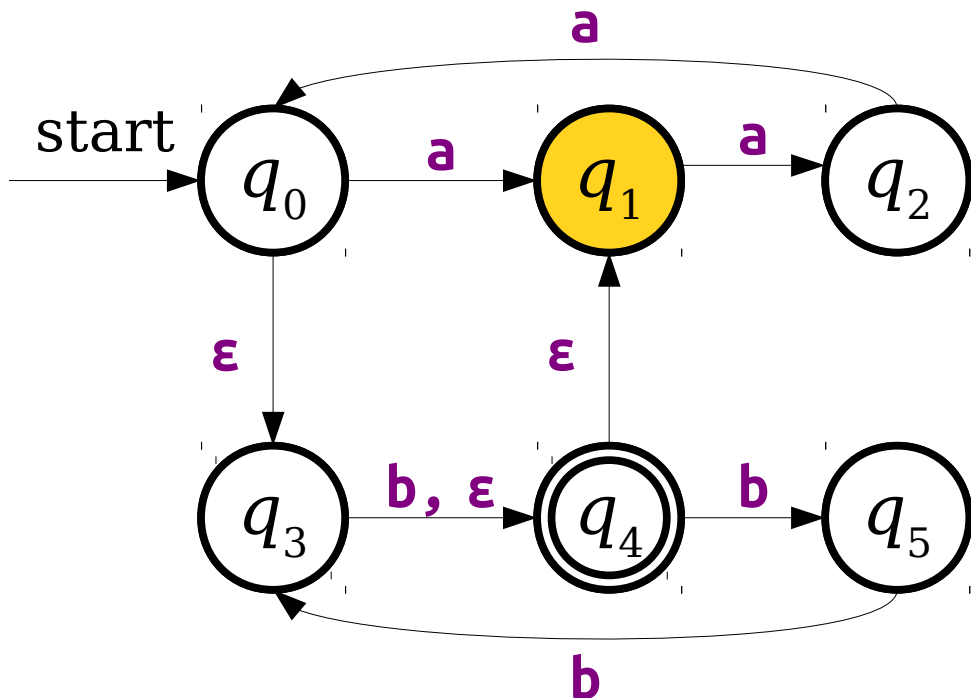
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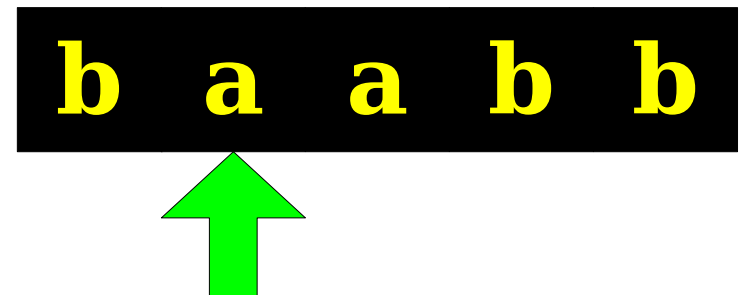
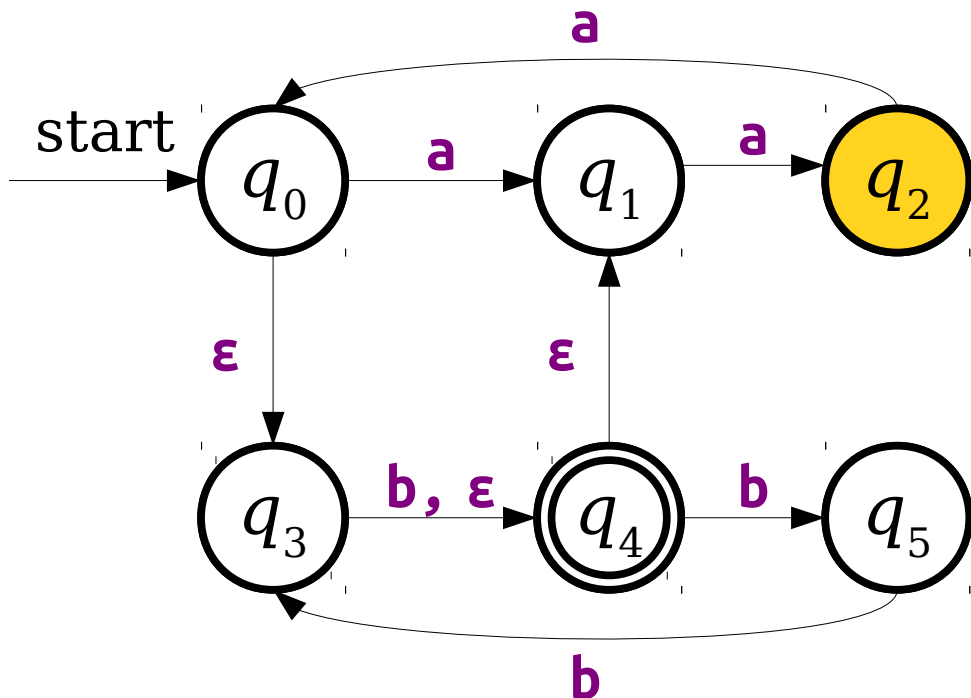
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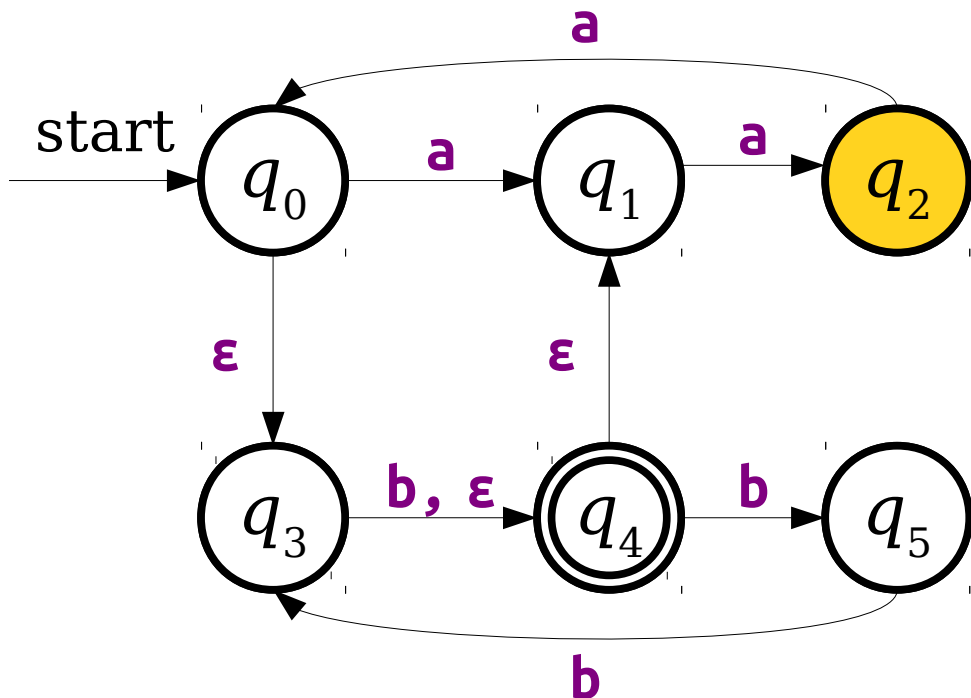
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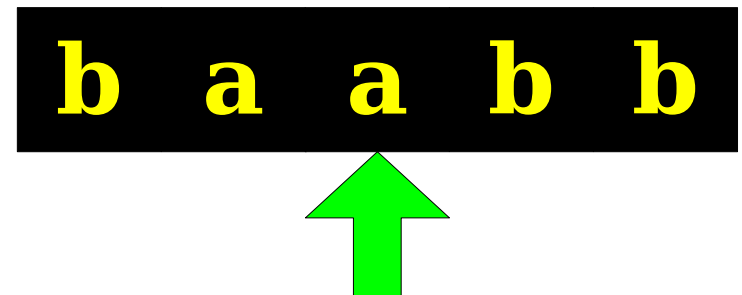
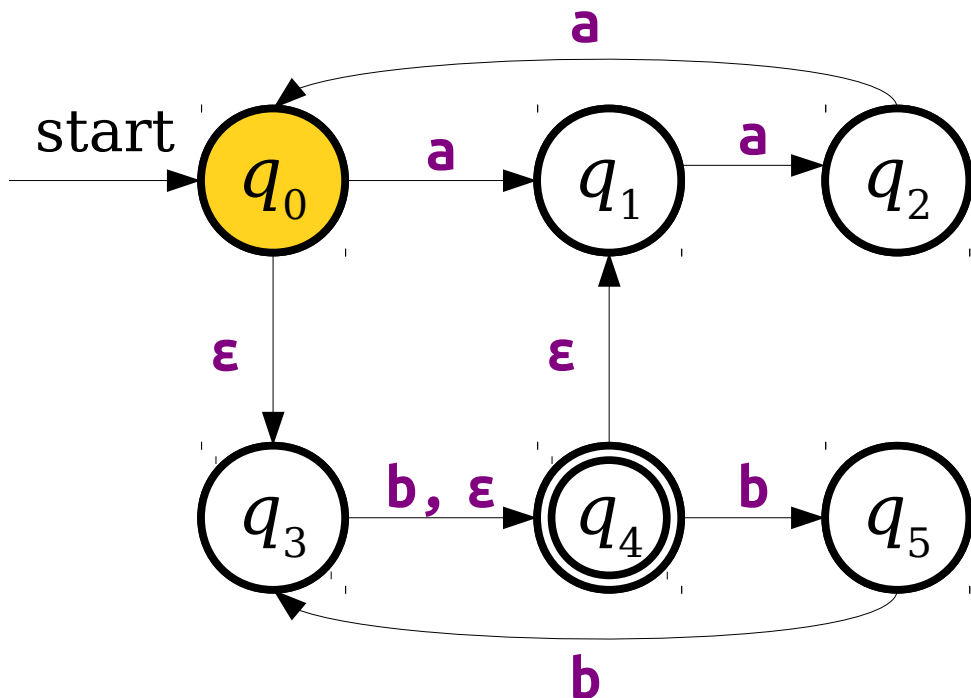
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**b a a b b**

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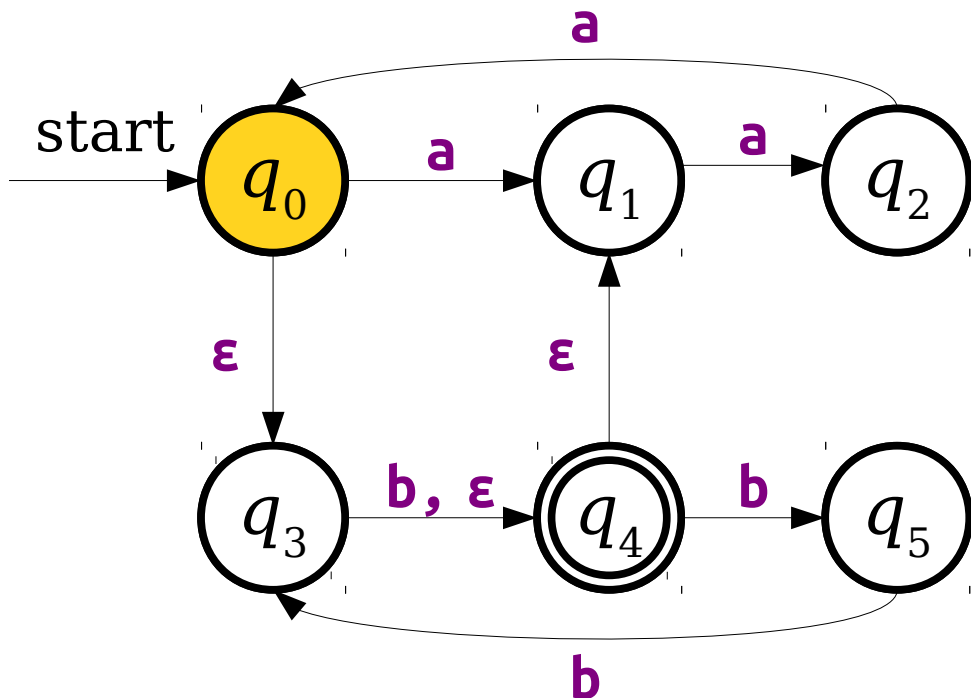
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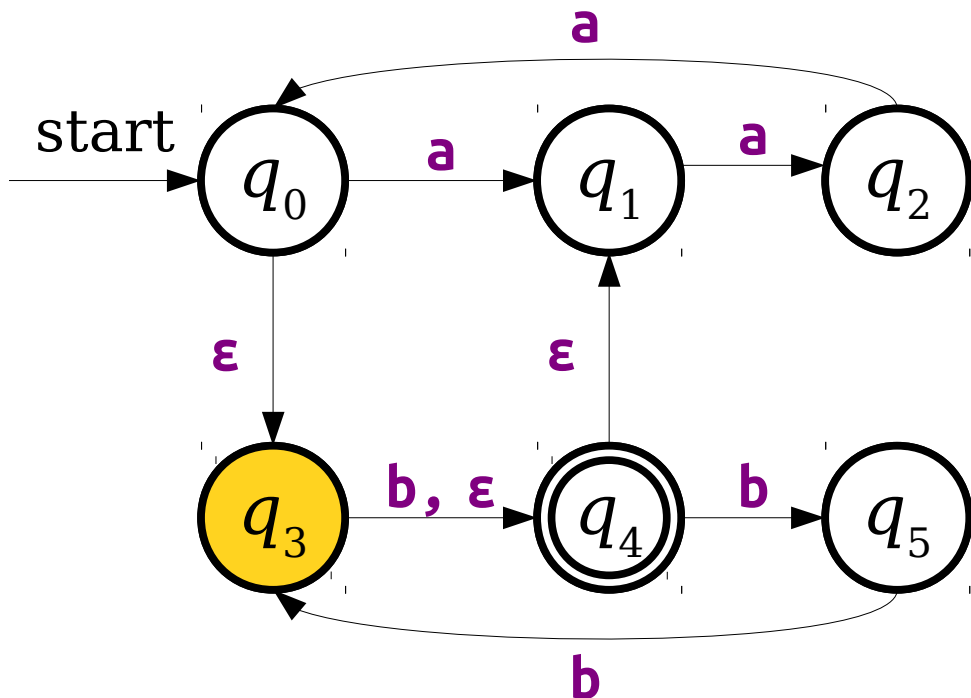
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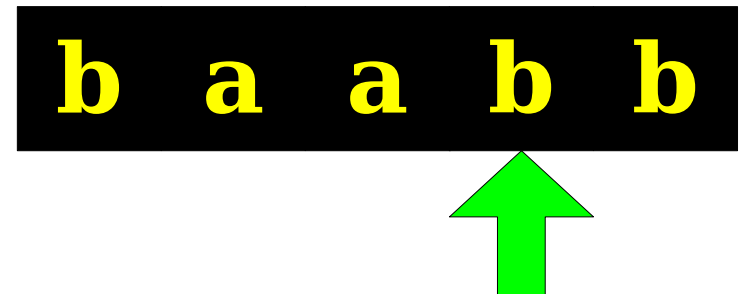
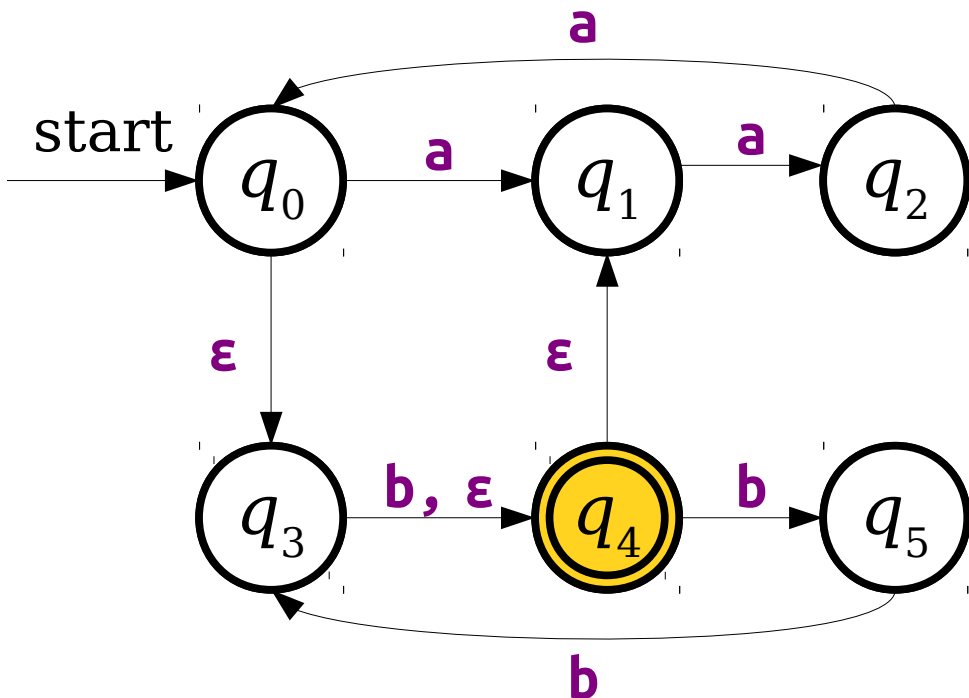
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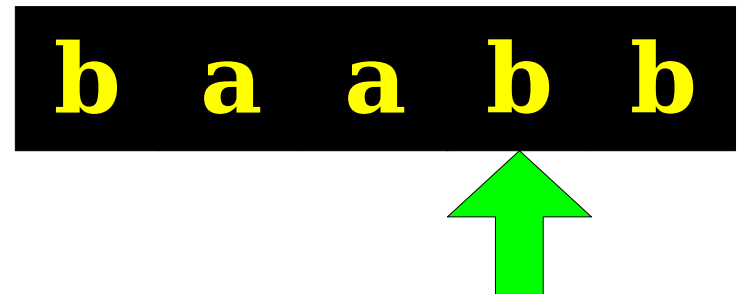
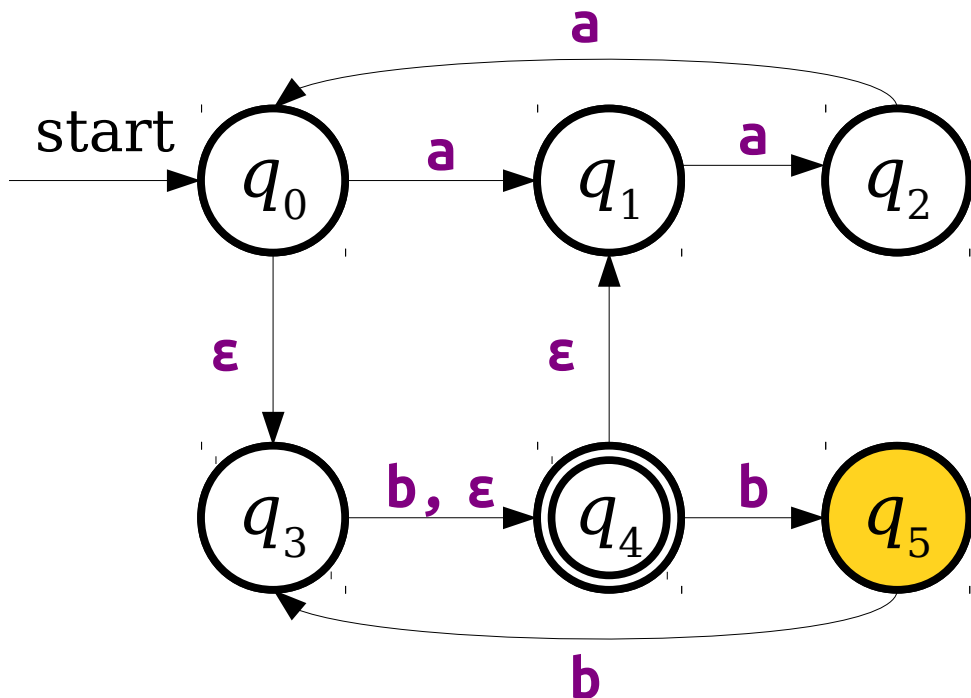
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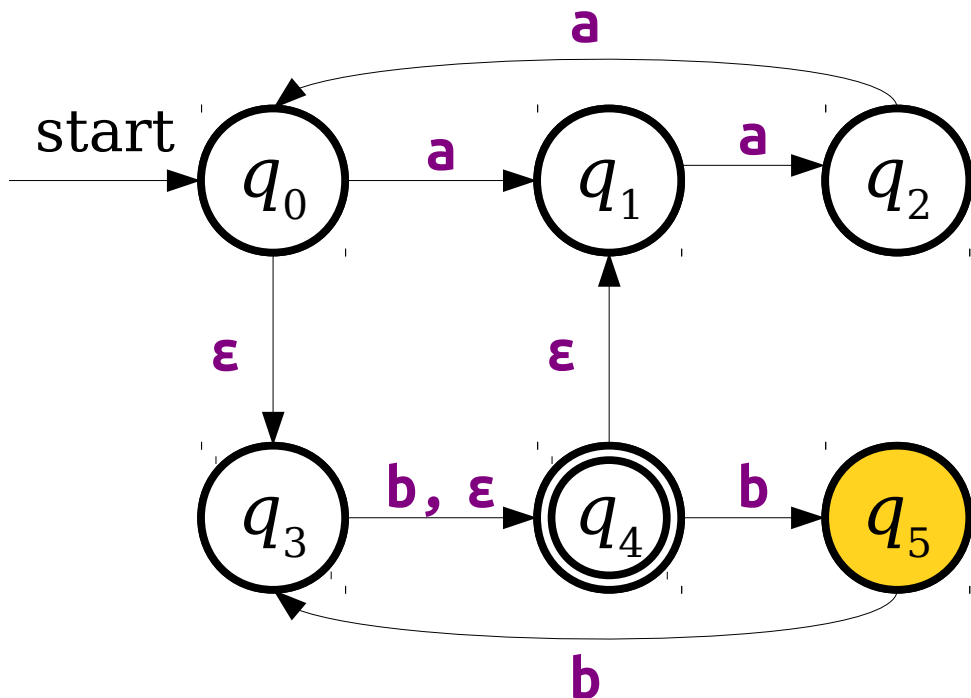
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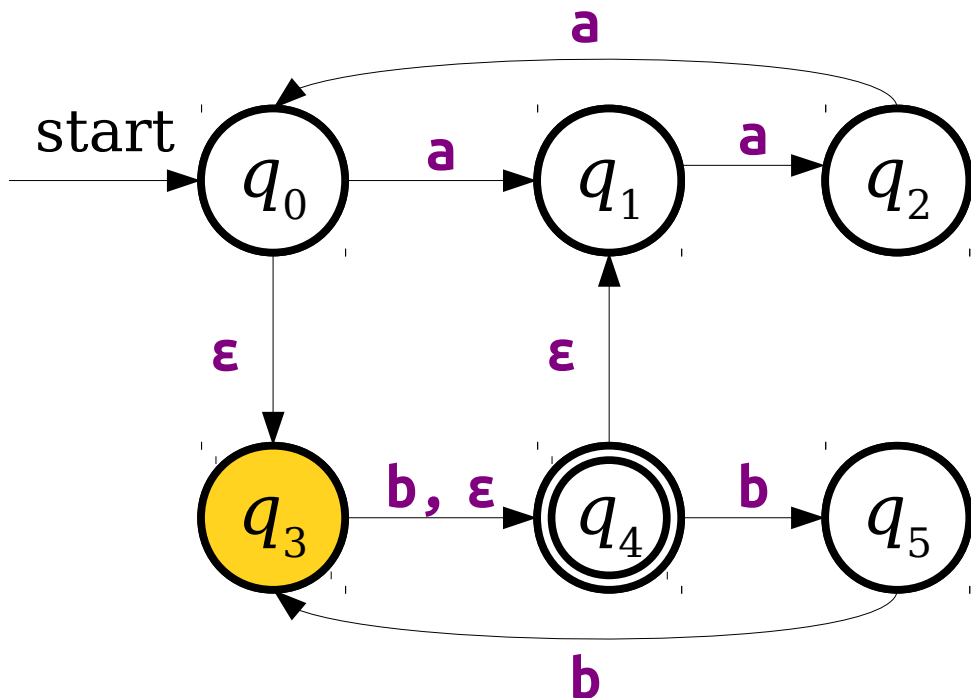


**b a a b b**

A green arrow points upwards to the final 'b' in the string "b a a b b".

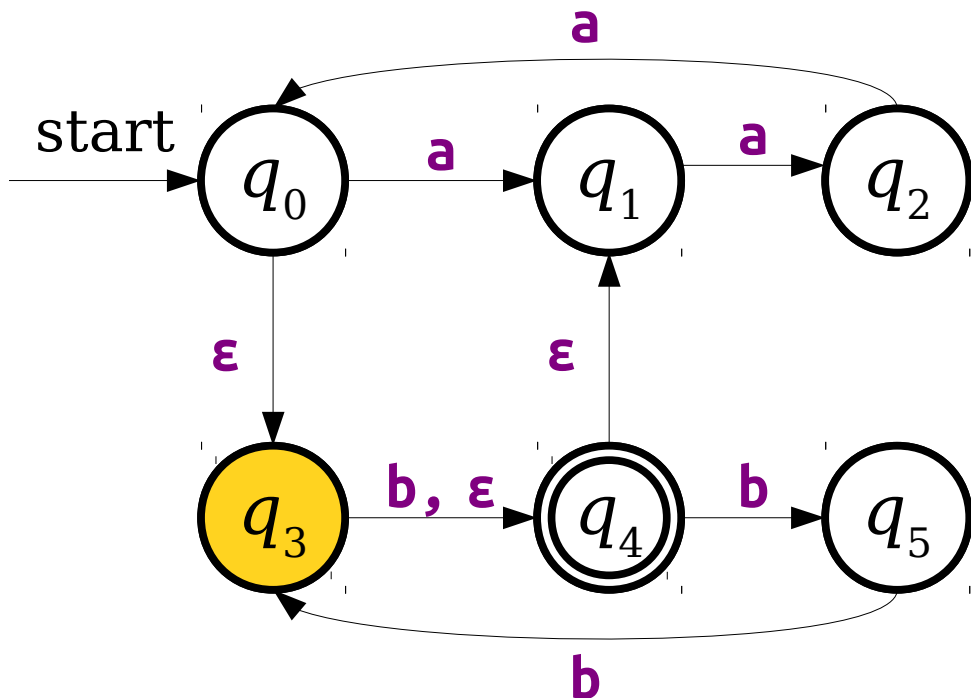
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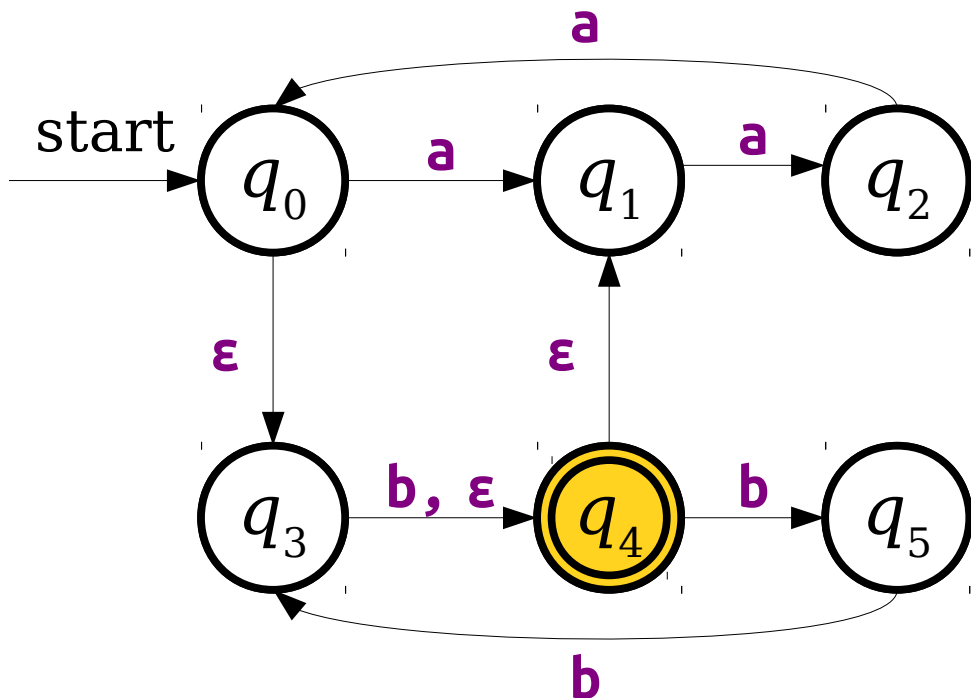
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**b a a b b**

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**b a a b b**



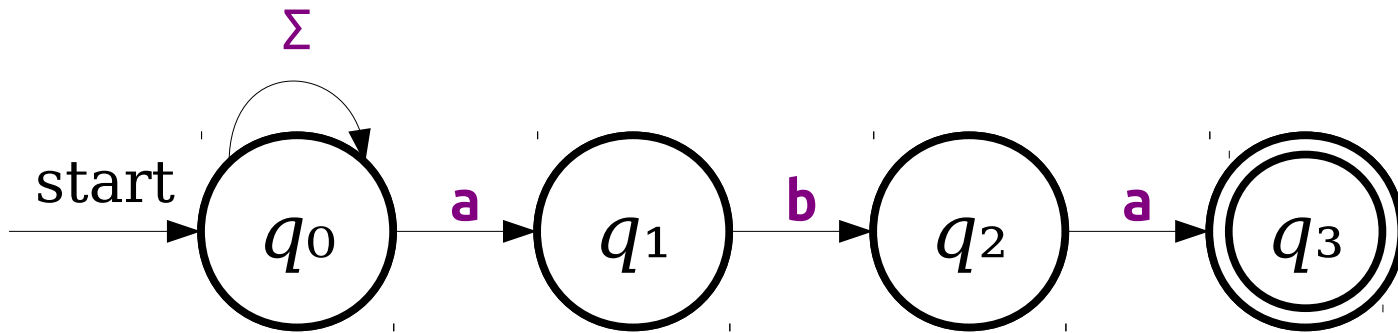
# $\epsilon$ -Transitions

- NFAs have a special type of transition called the  **$\epsilon$ -transition**.
- An NFA may follow any number of  $\epsilon$ -transitions at any time without consuming any input.
- NFAs are not *required* to follow  $\epsilon$ -transitions. It's simply another option at the machine's disposal.

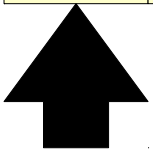
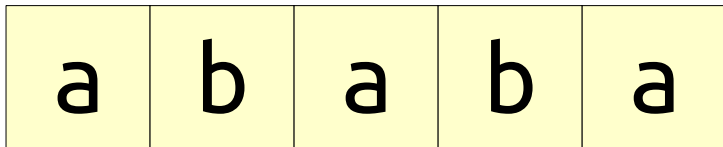
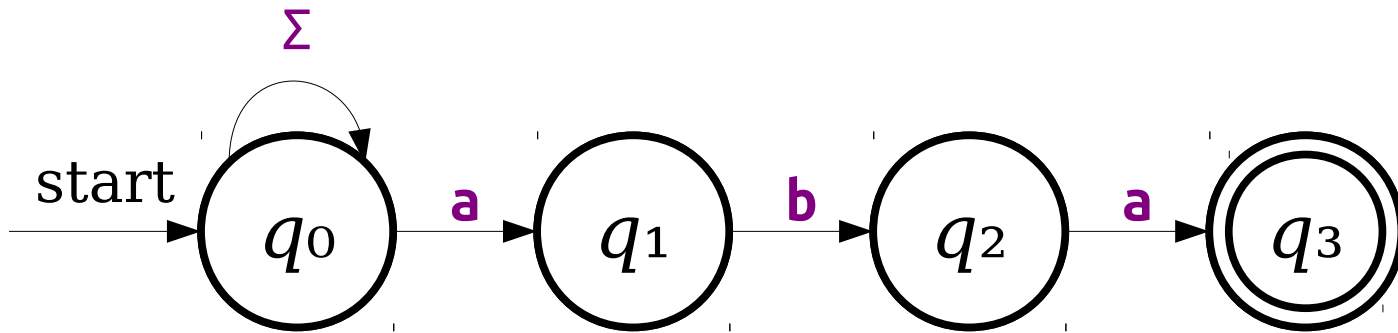
# Intuiting Nondeterminism

- Nondeterministic machines are a serious departure from physical computers. How can we build up an intuition for them?
- There are two particularly useful frameworks for interpreting nondeterminism:
  - *Perfect guessing*
  - *Massive parallelism*

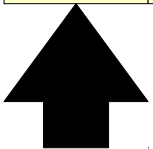
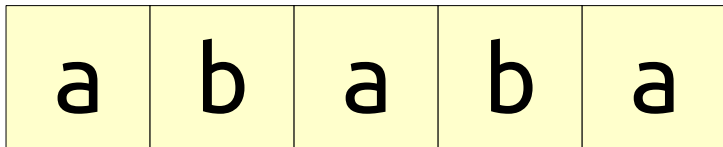
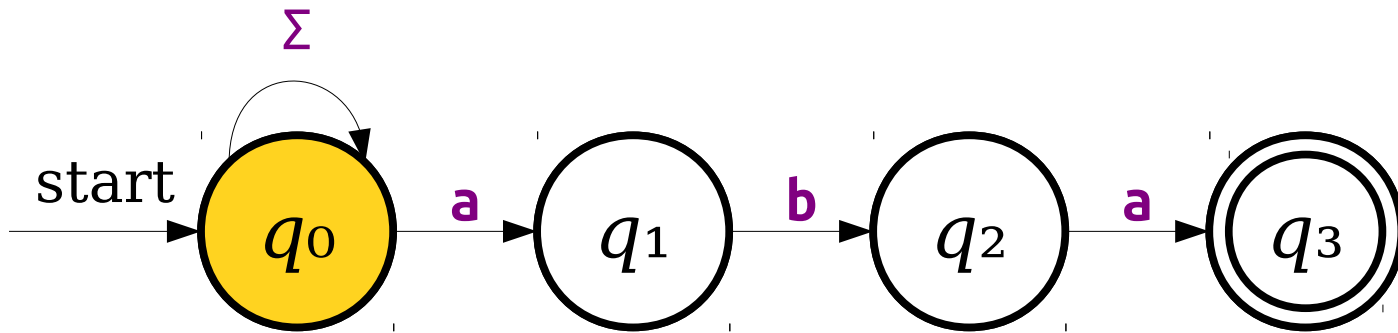
# Perfect Guessing



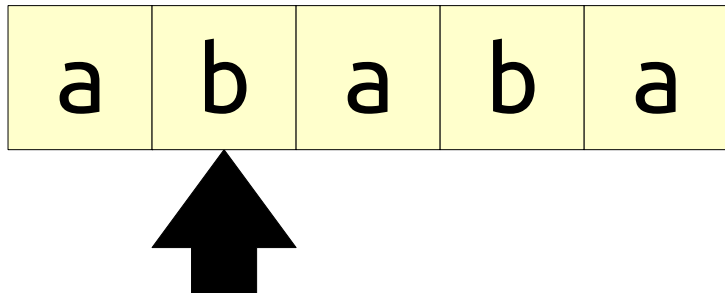
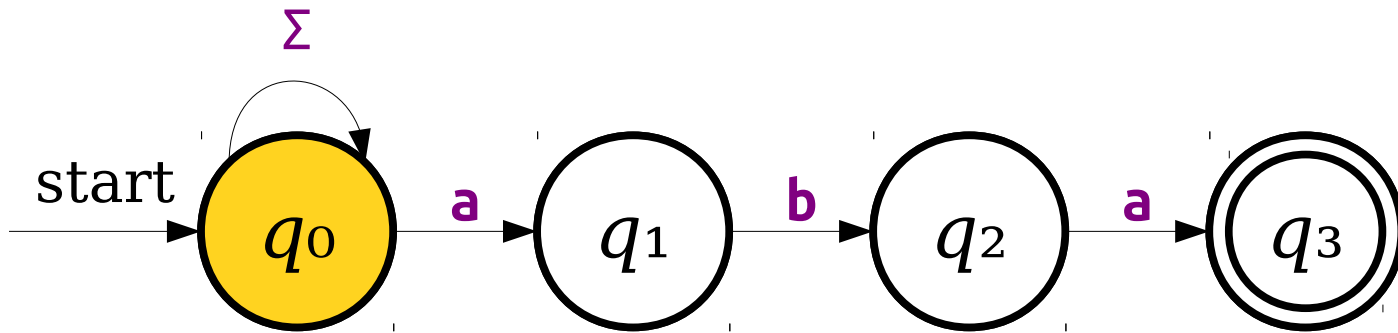
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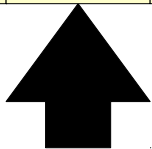
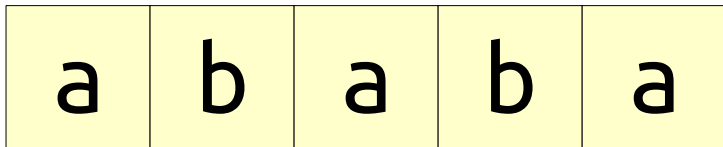
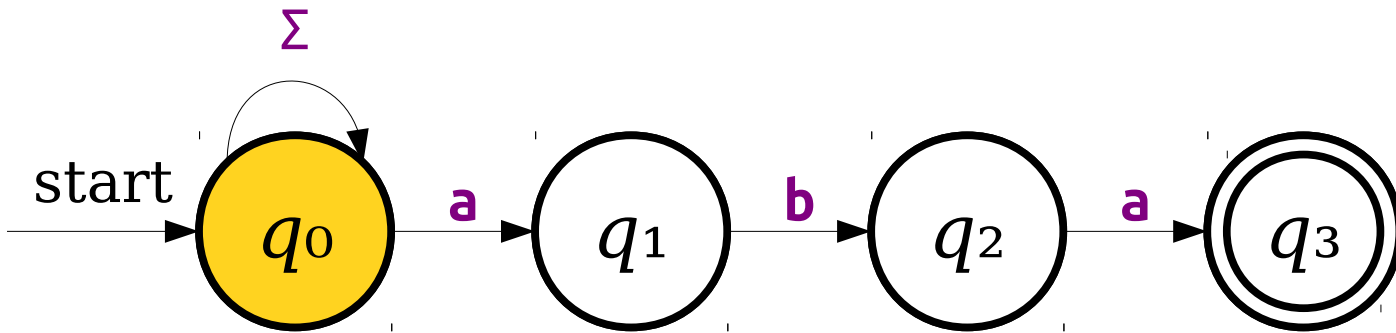
# Perfect Guessing



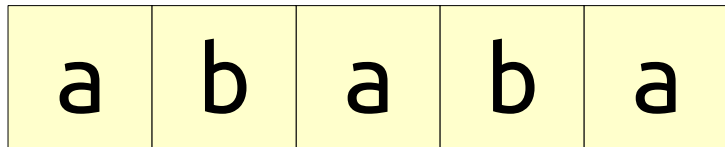
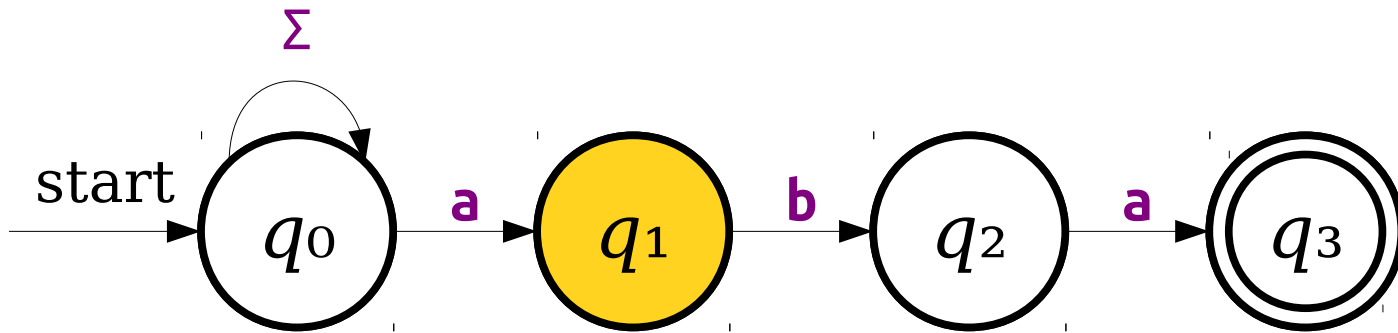
# Perfect Guessing



# Perfect Guessing

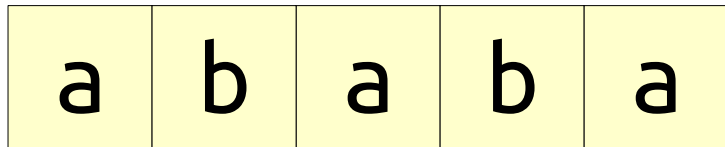
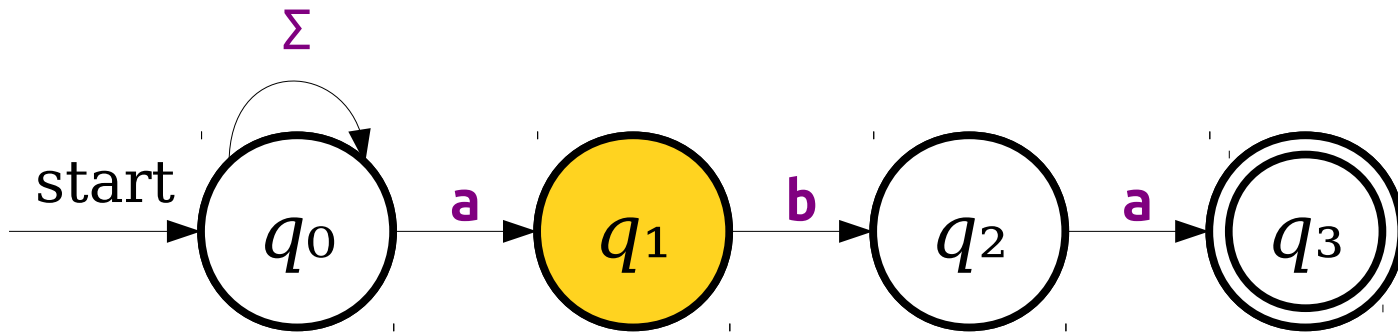


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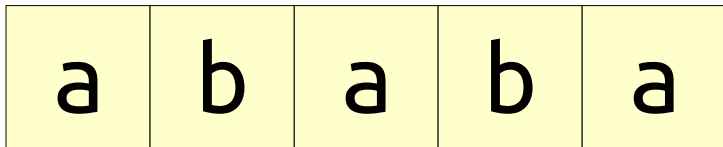
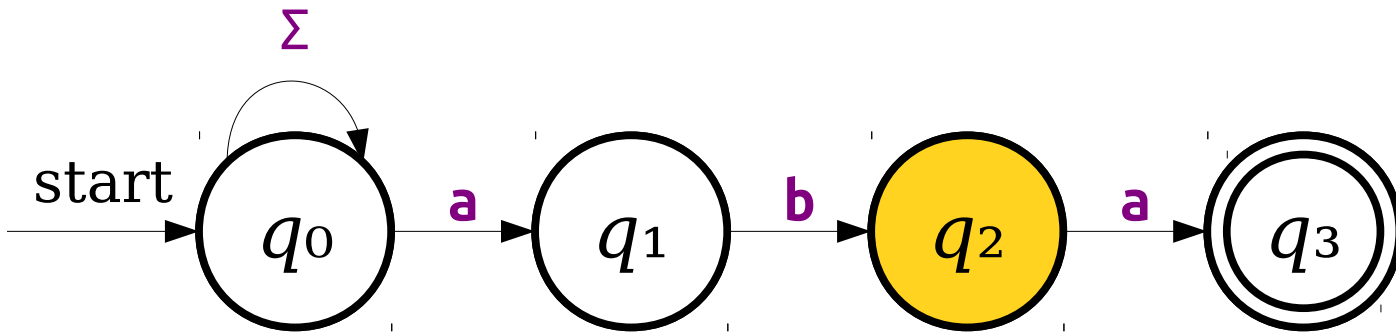




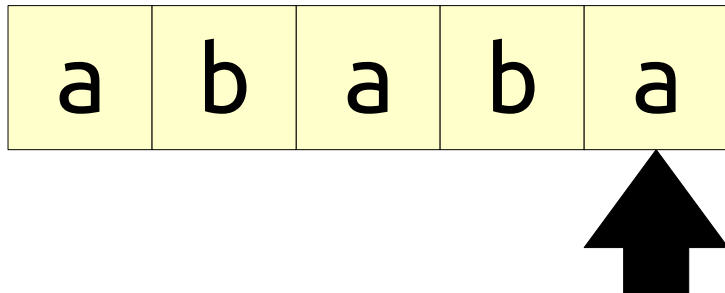
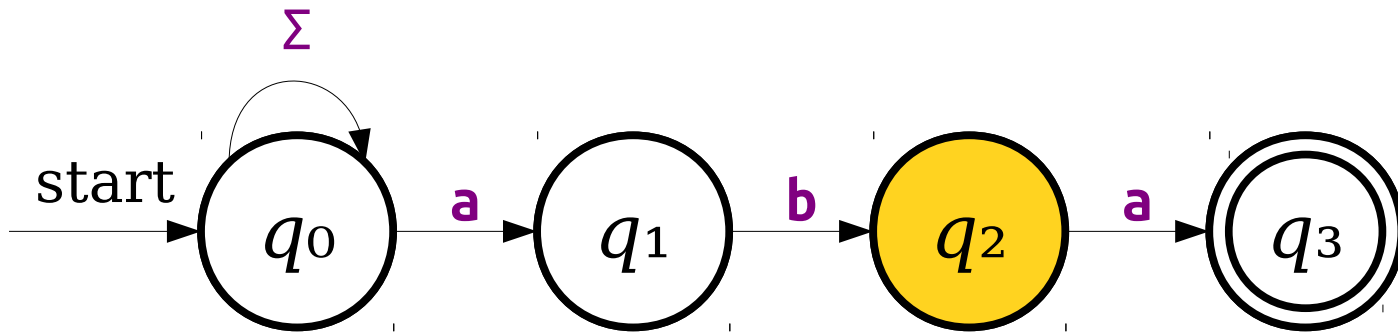
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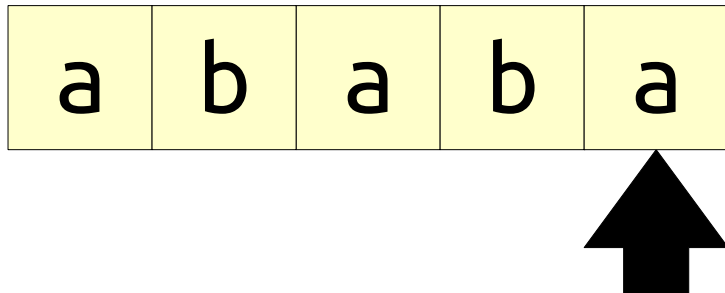
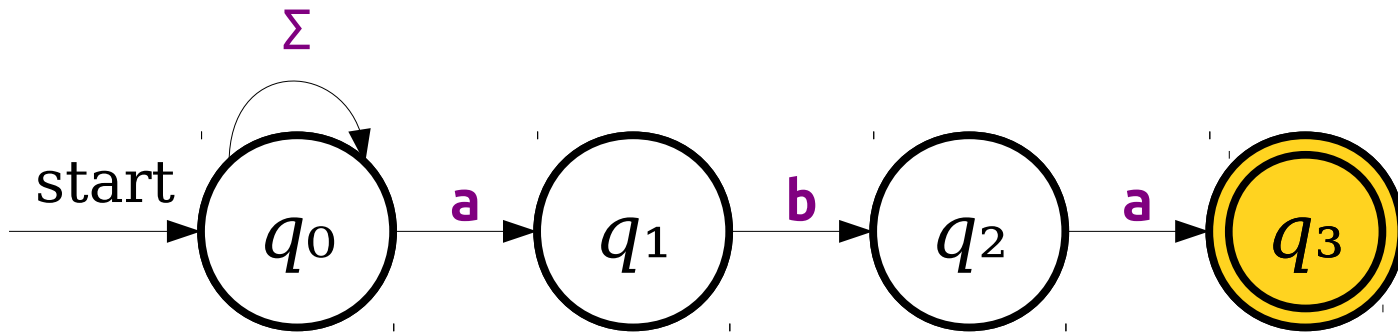
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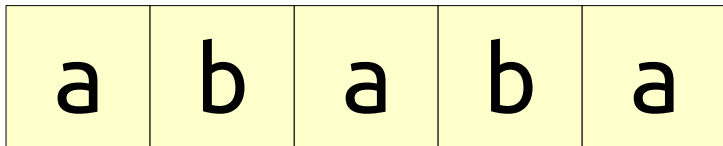
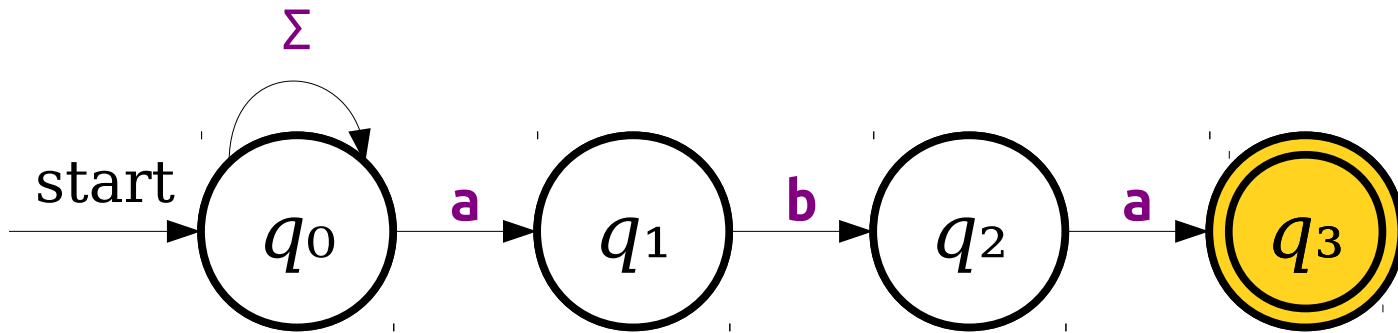
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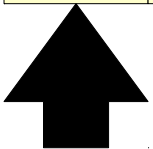
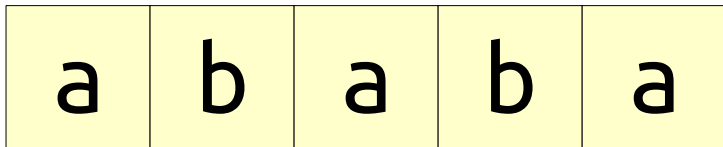
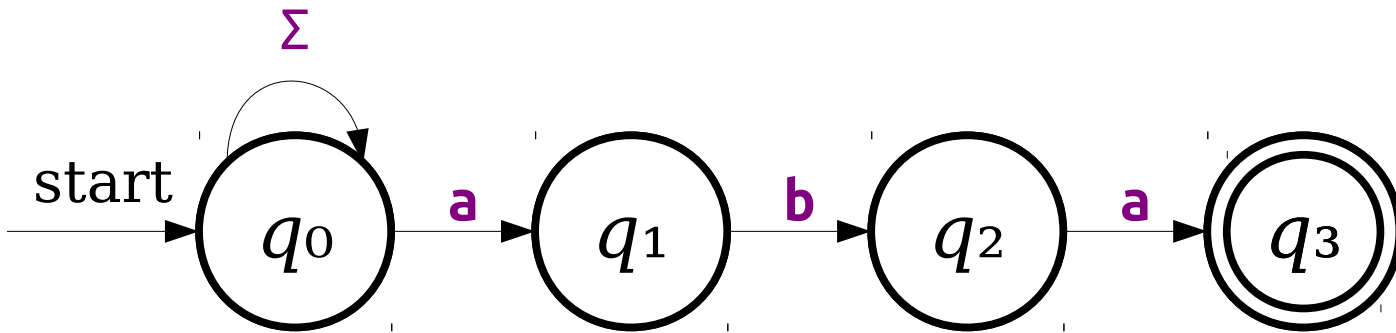
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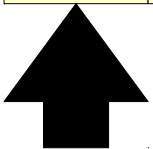
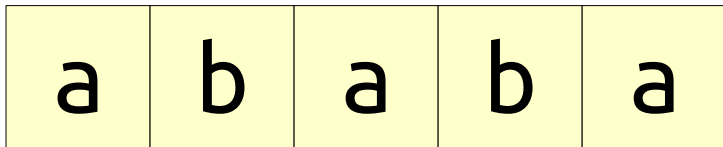
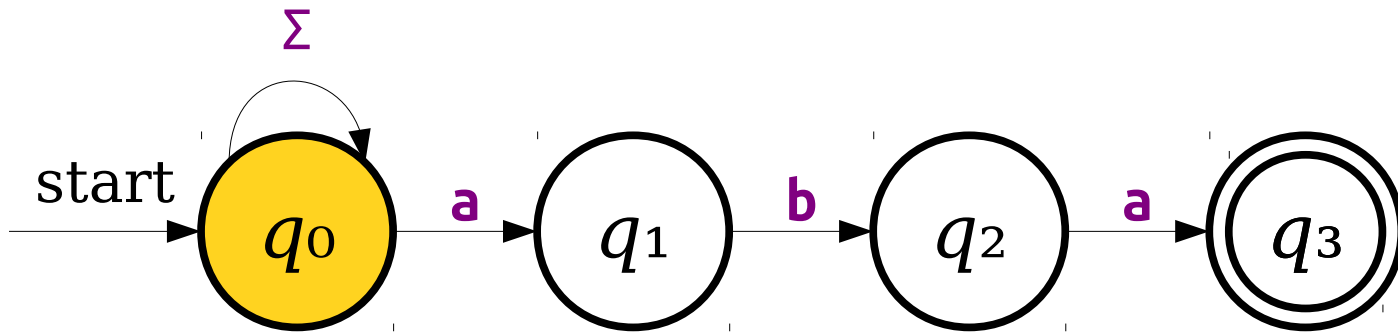
# Perfect Guessing

- We can view nondeterministic machines as having *Magic Superpowers* that enable them to guess choices that lead to an accepting state.
  - If there is at least one choice that leads to an accepting state, the machine will guess it.
  - If there are no choices, the machine guesses any one of the wrong guesses.
- No known physical analog for this style of computation – this is totally new!

# Massive Parallelism

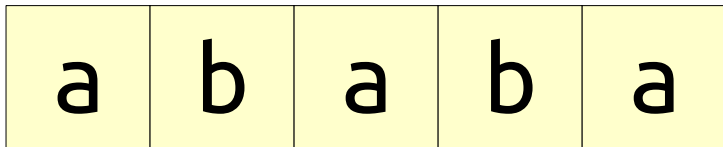
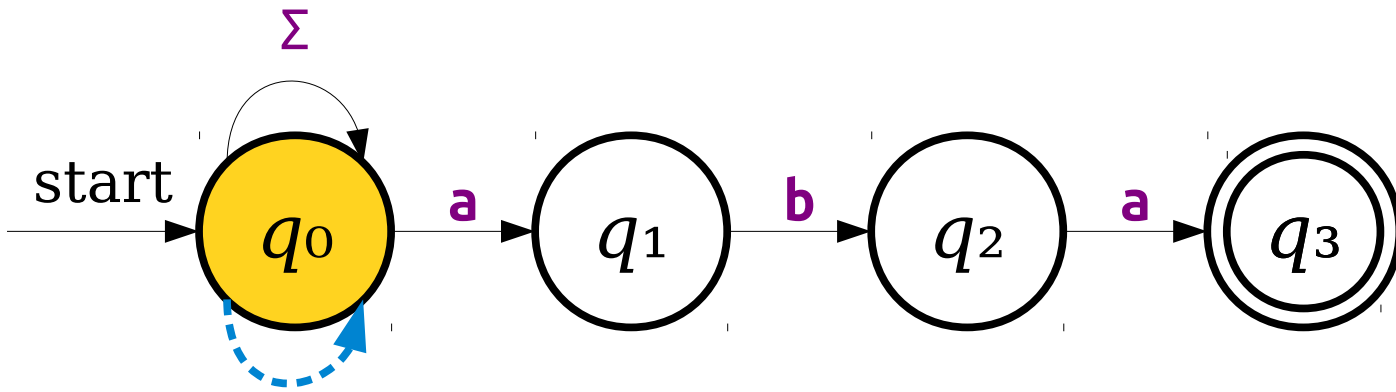


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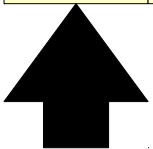
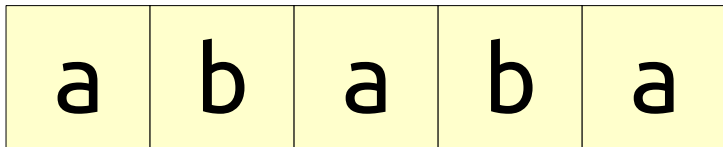
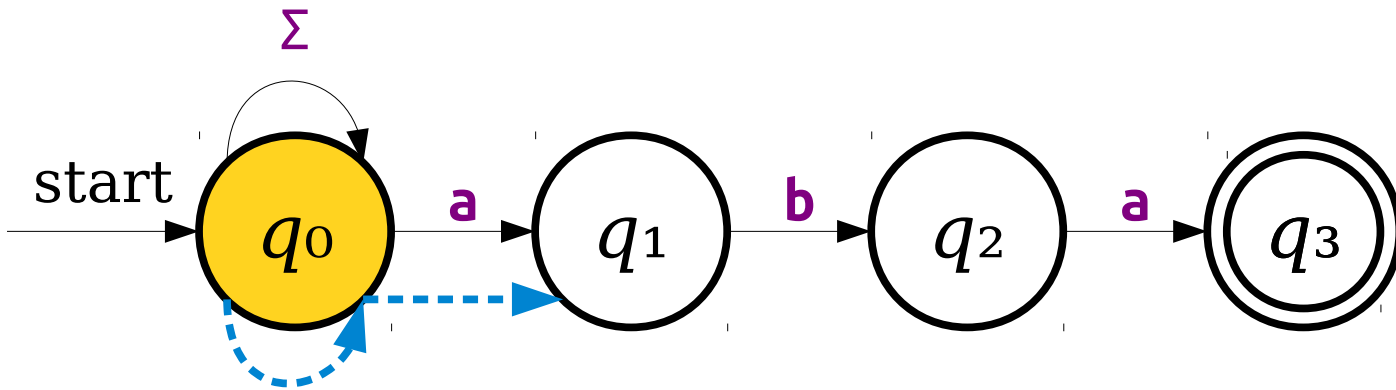




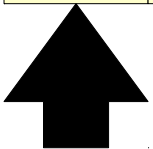
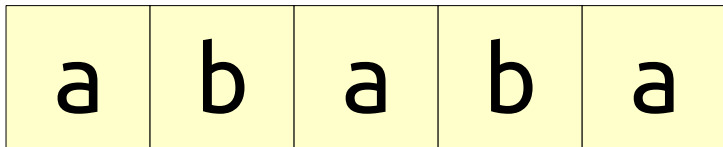
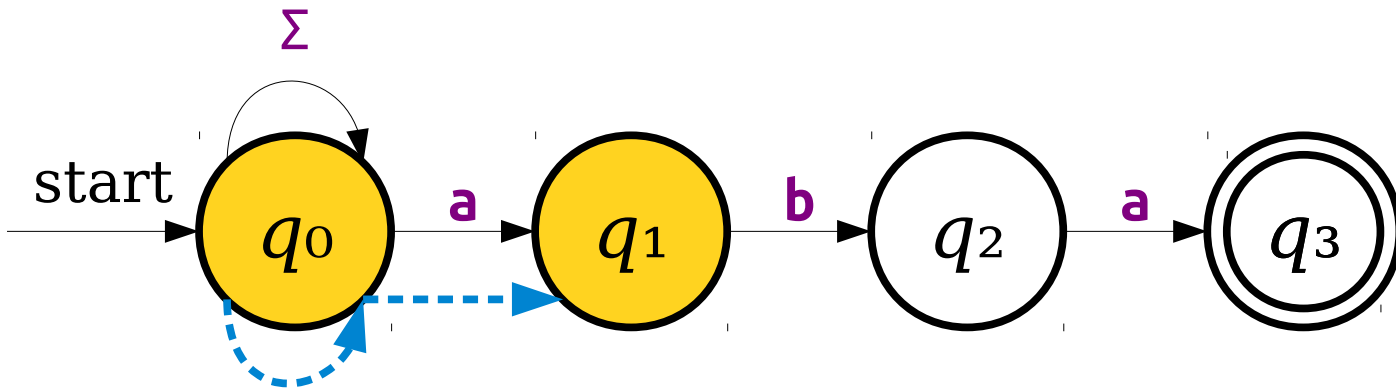
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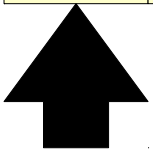
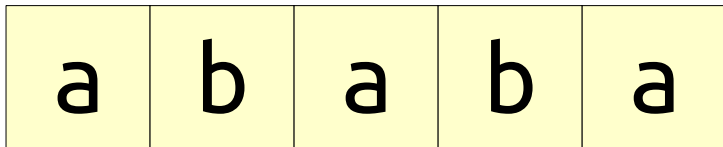
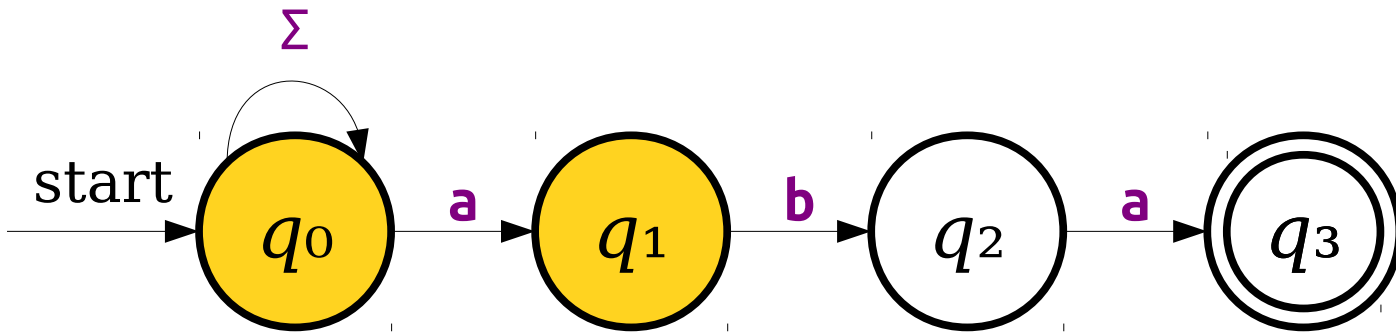
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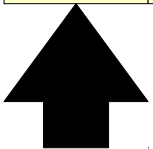
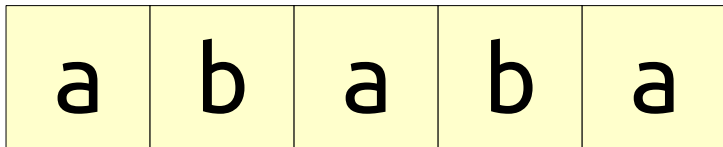
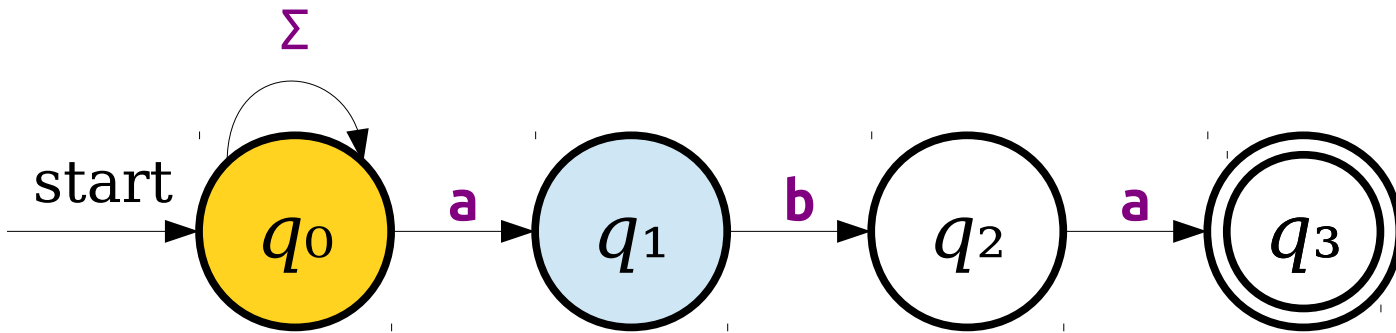
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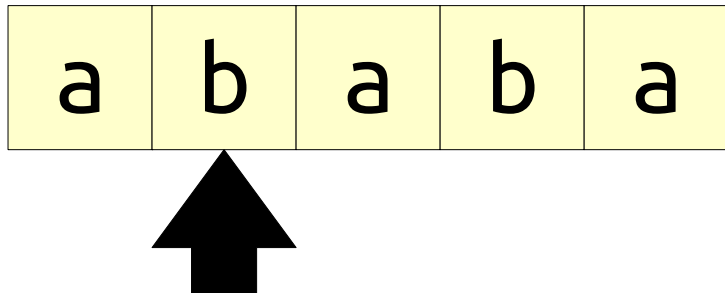
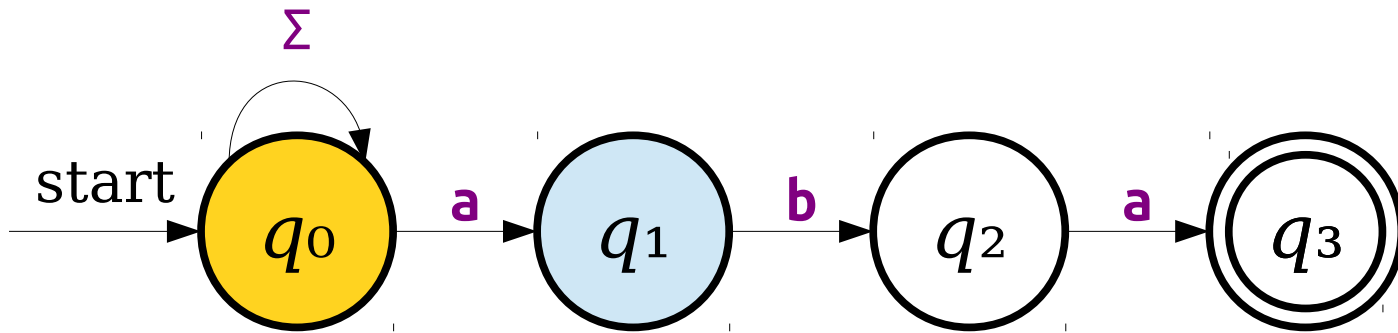
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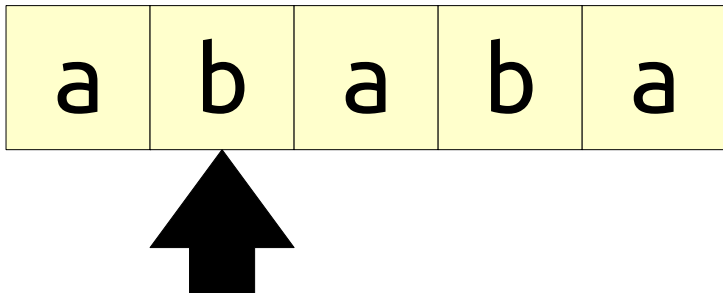
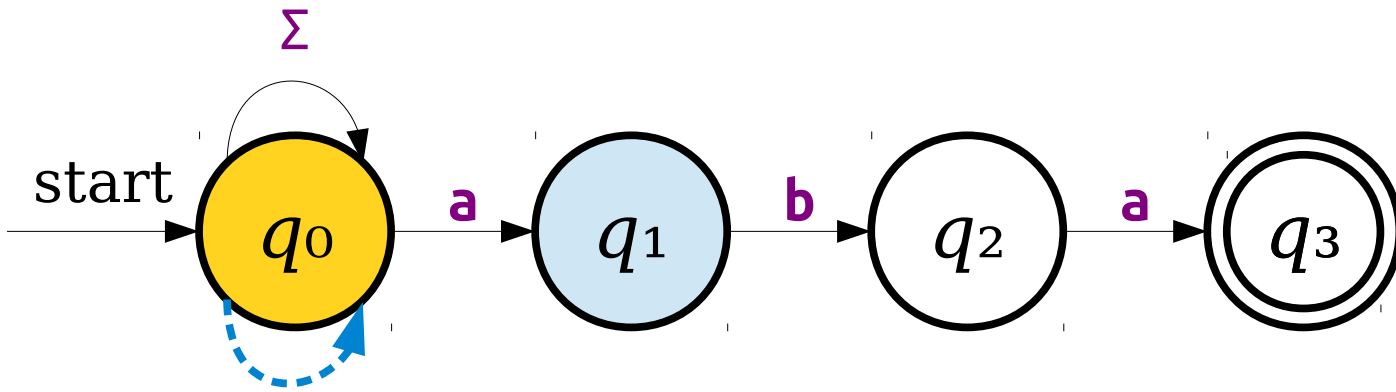
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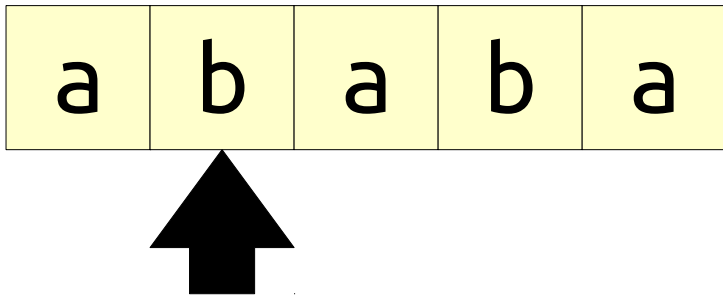
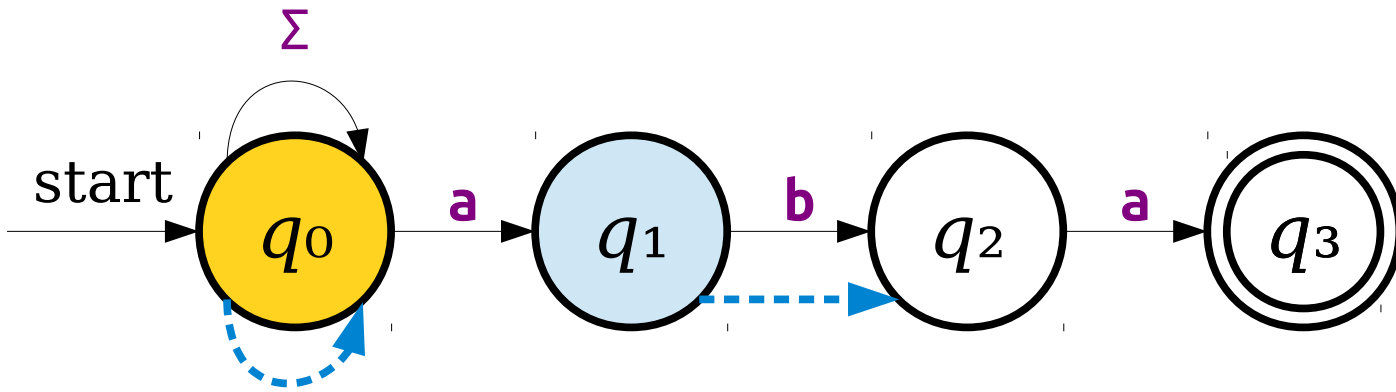
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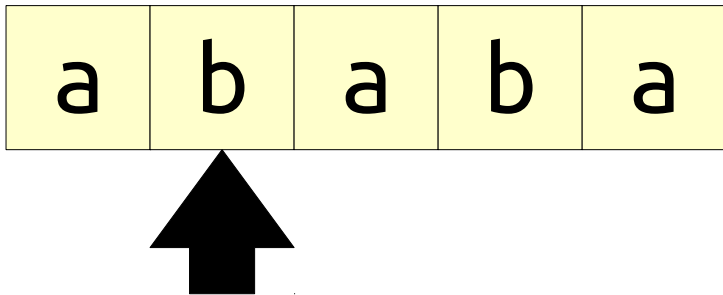
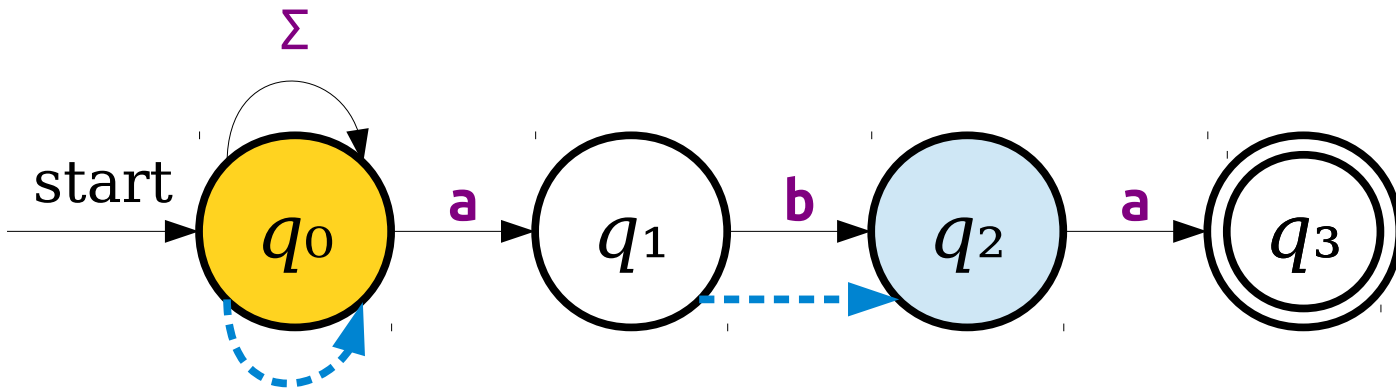


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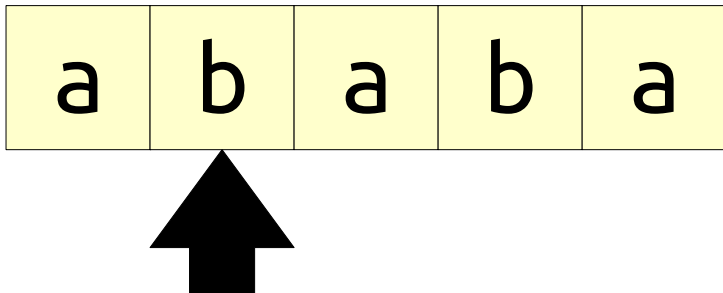
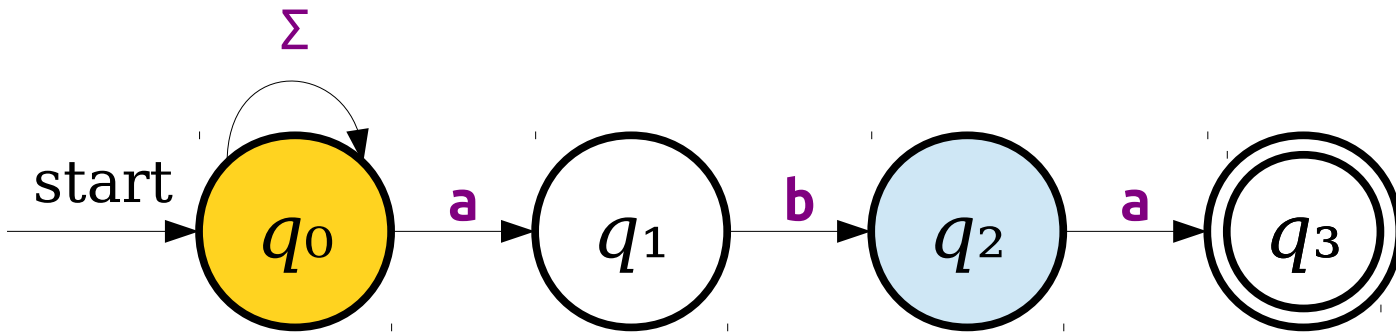




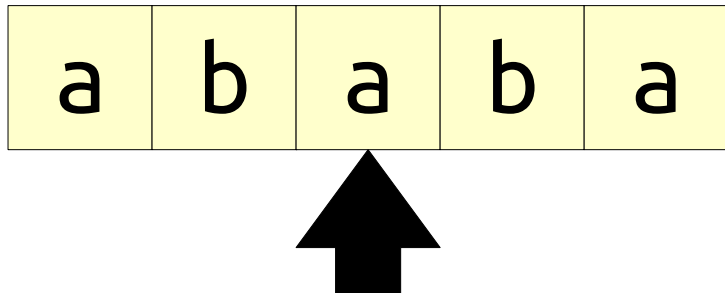
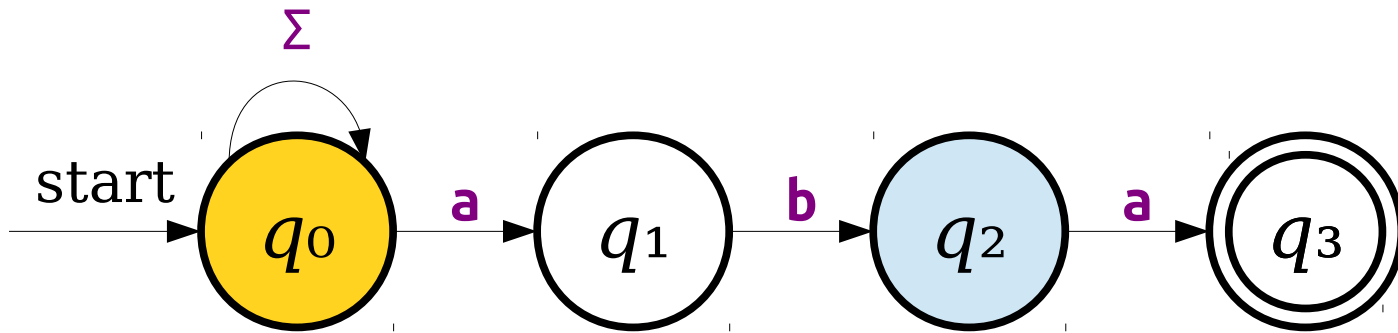
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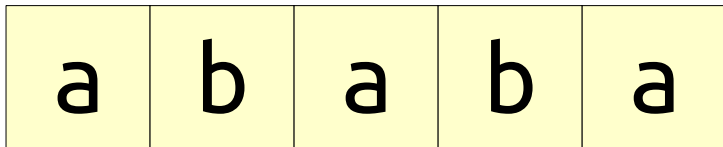
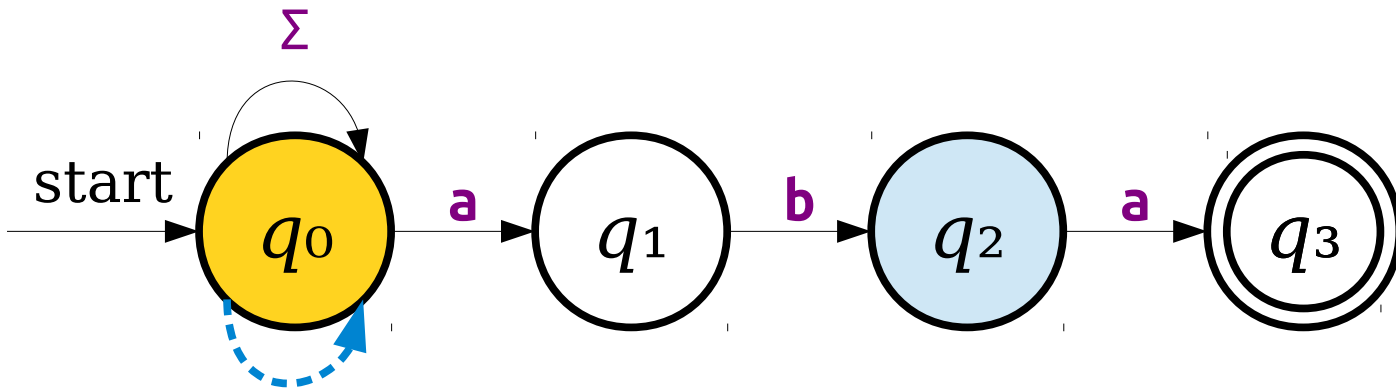
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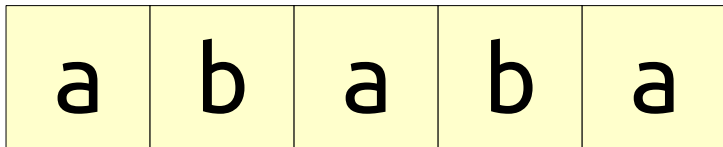
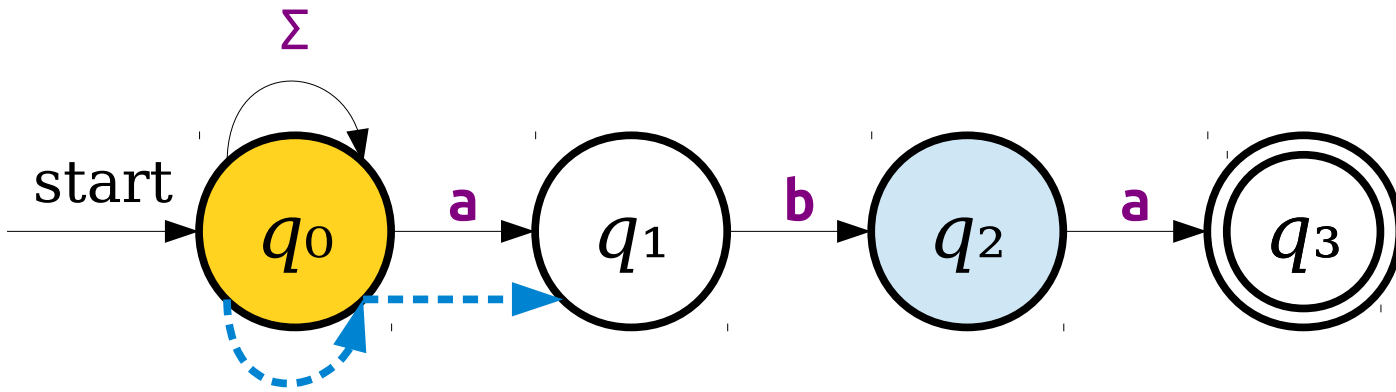
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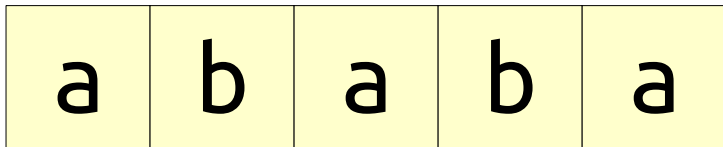
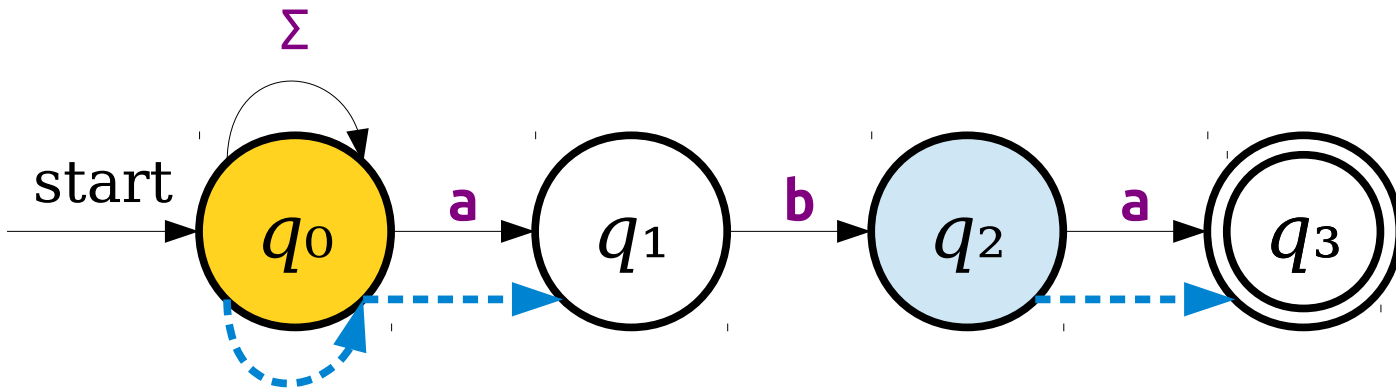
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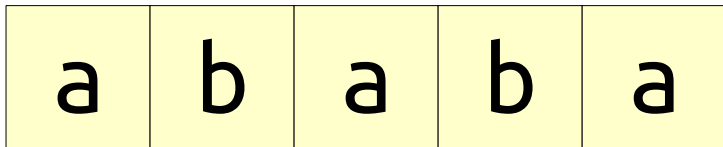
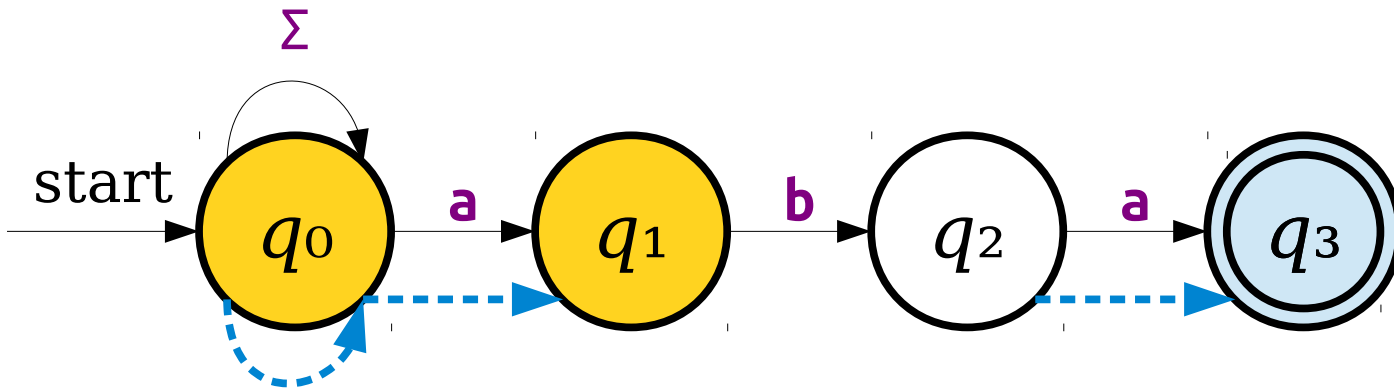
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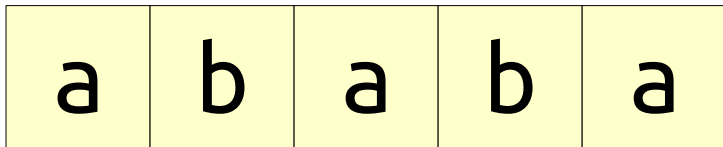
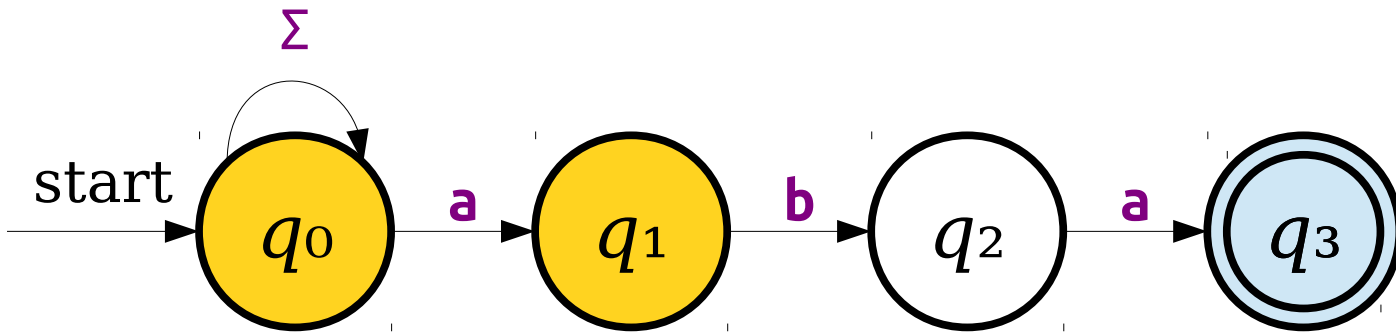
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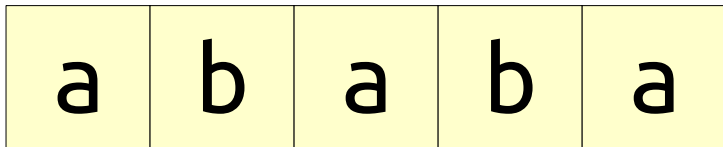
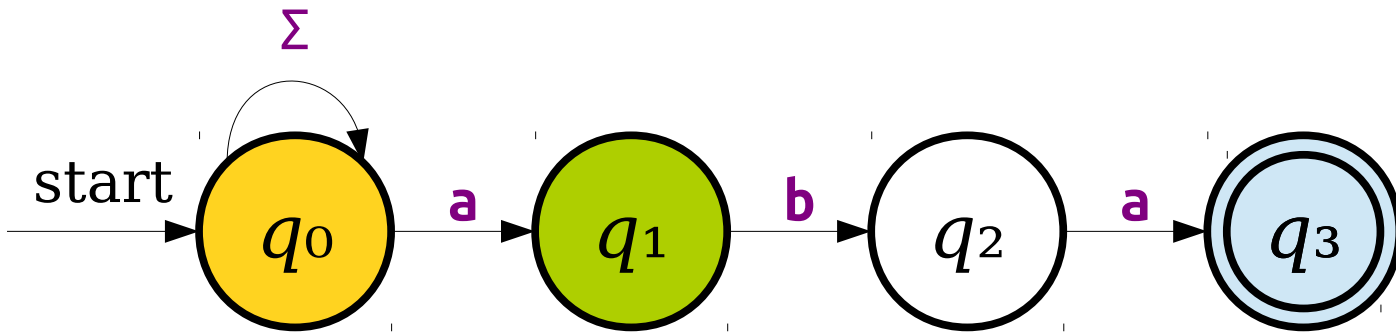


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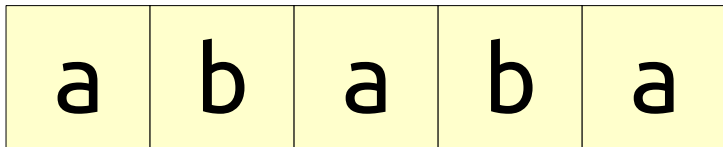
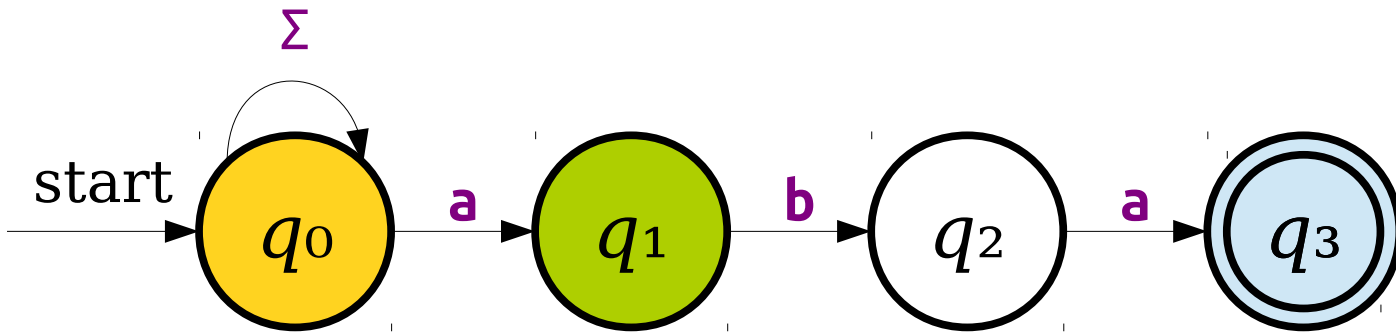




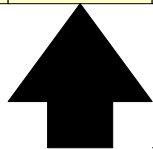
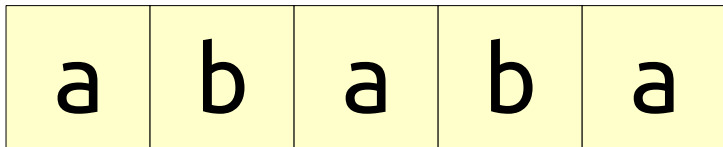
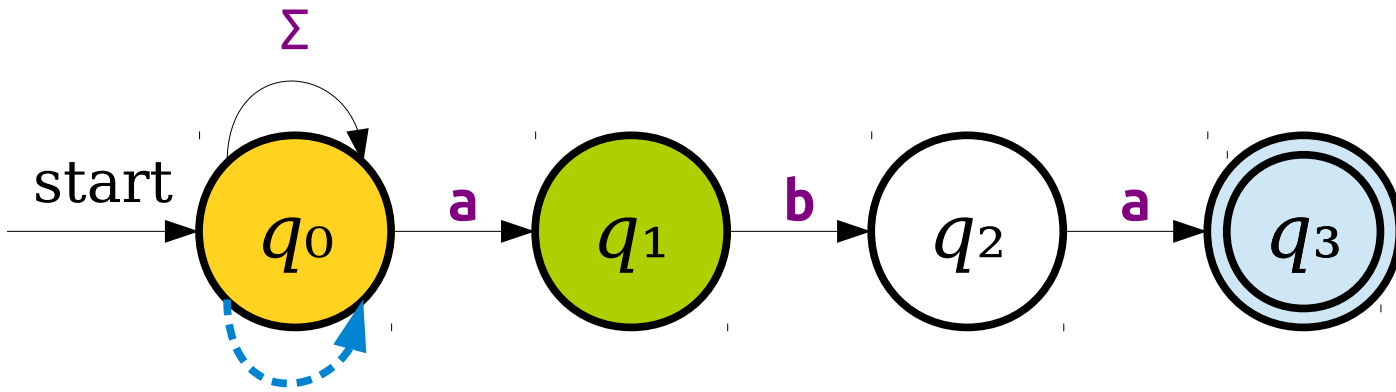
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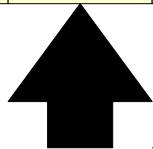
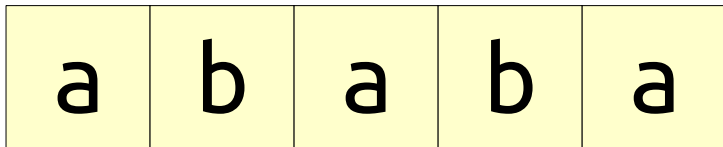
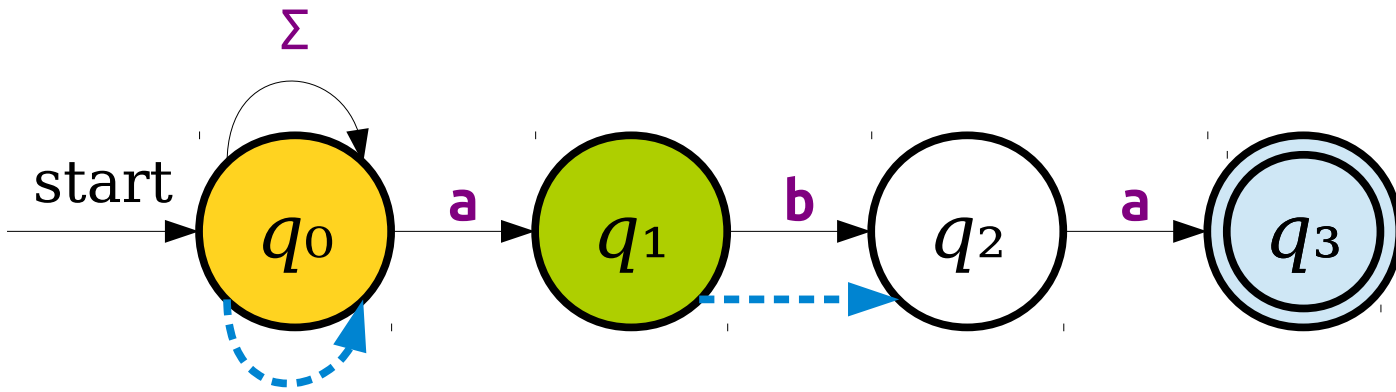
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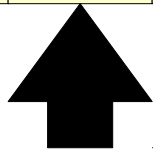
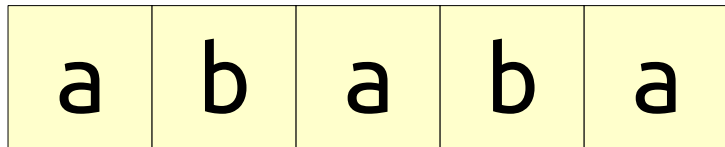
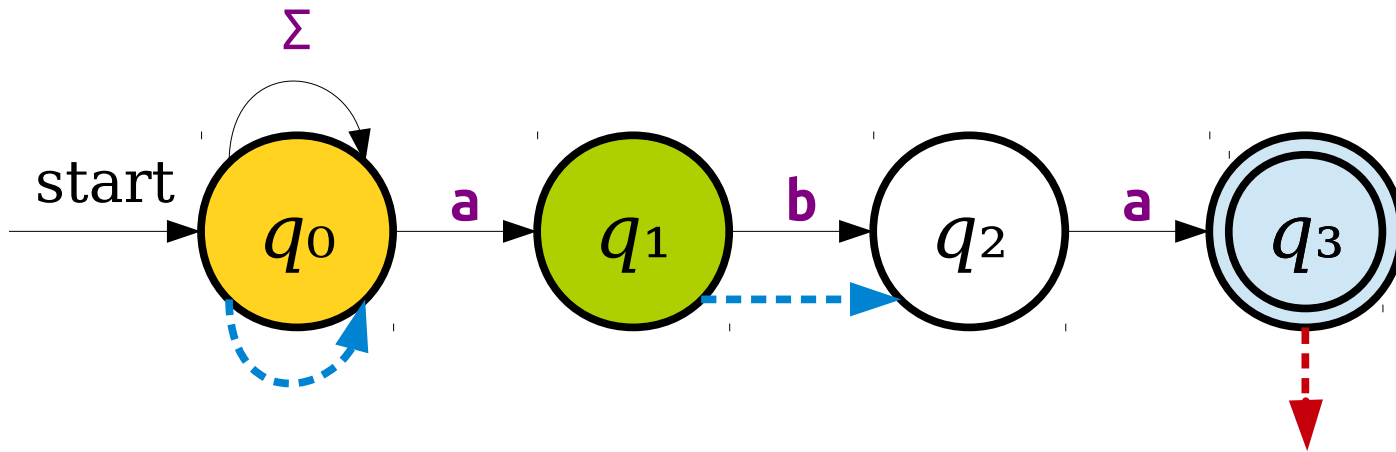
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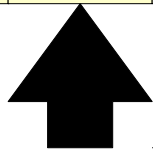
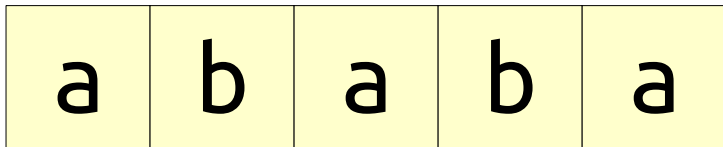
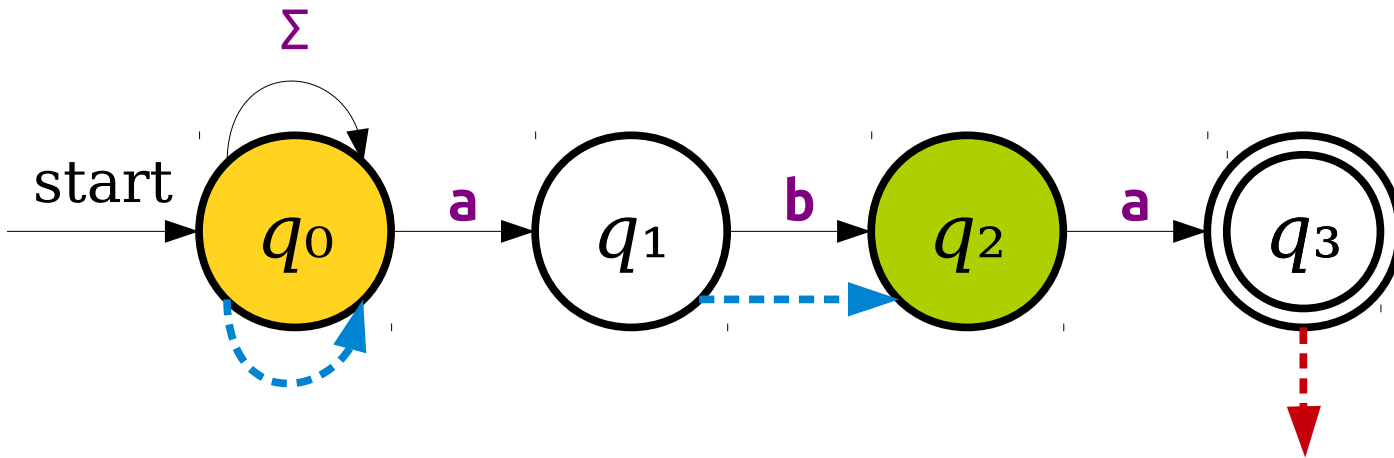
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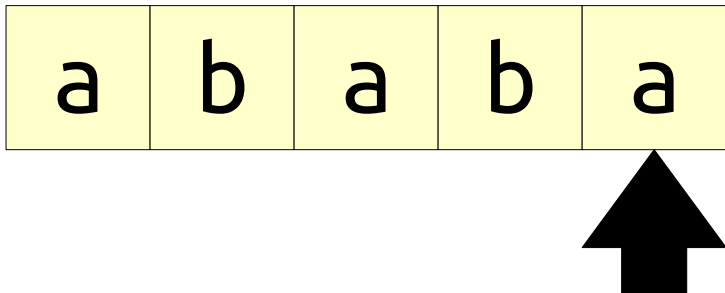
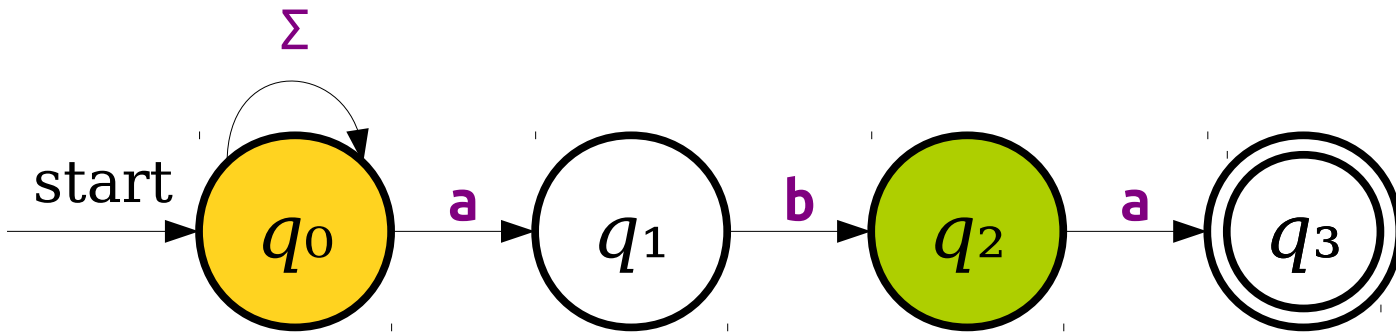
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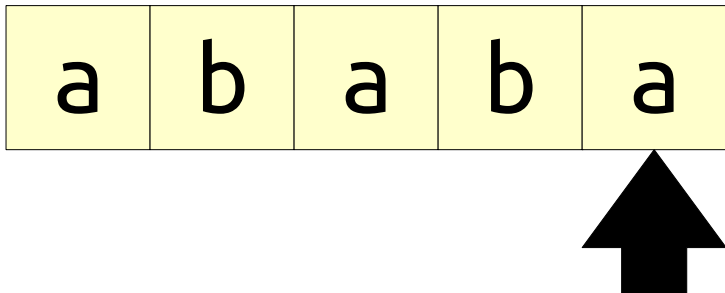
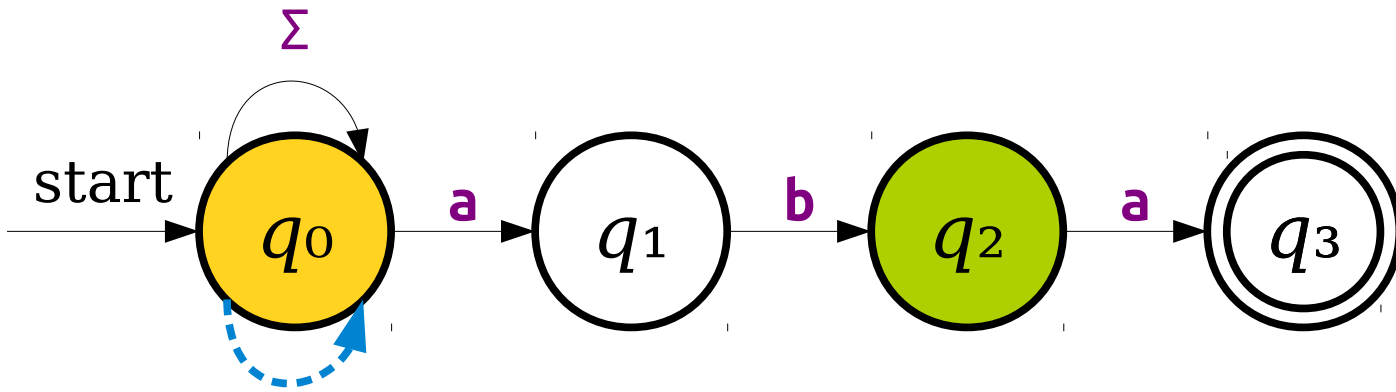
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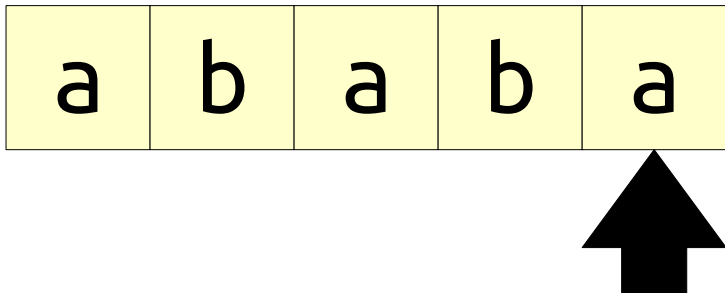
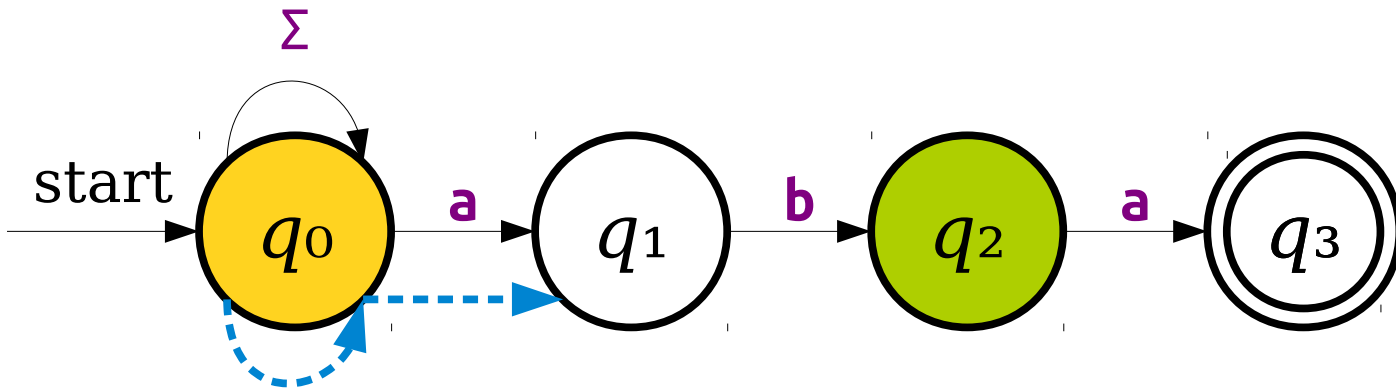


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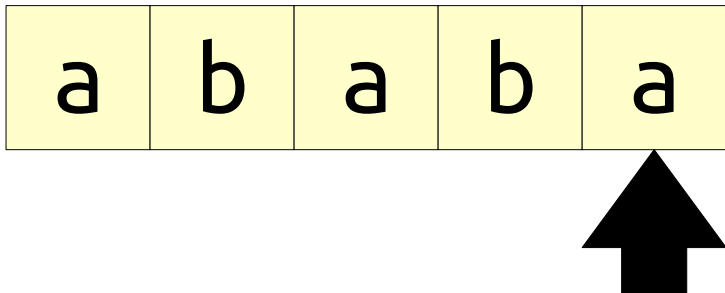
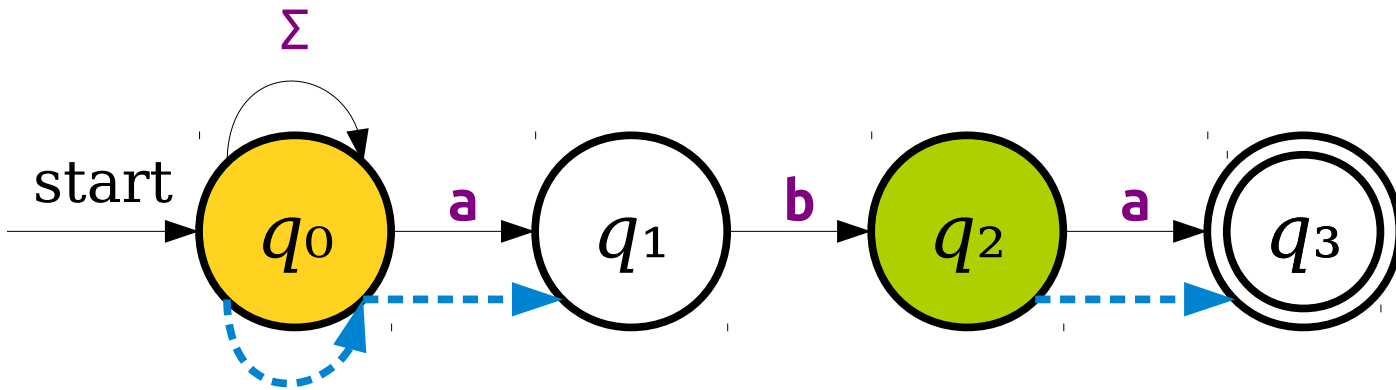




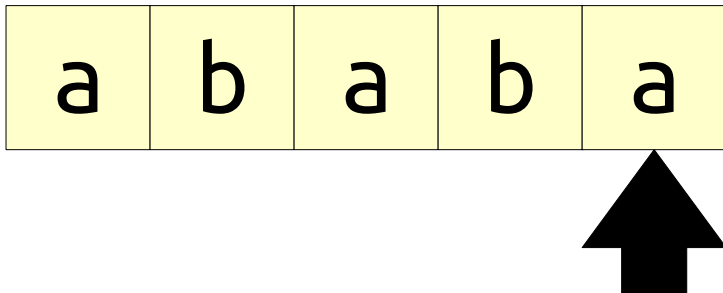
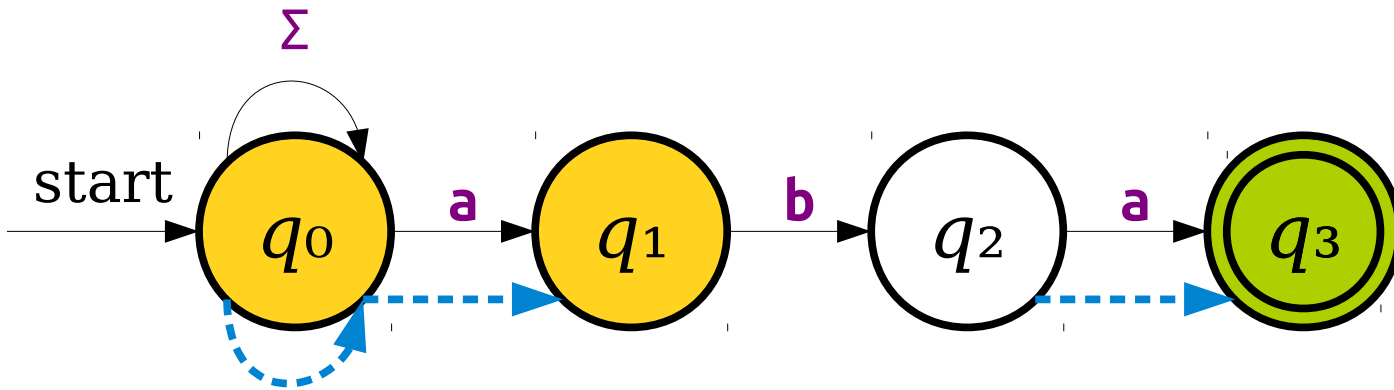
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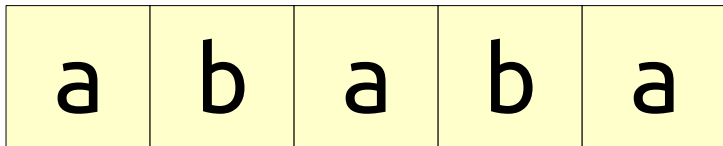
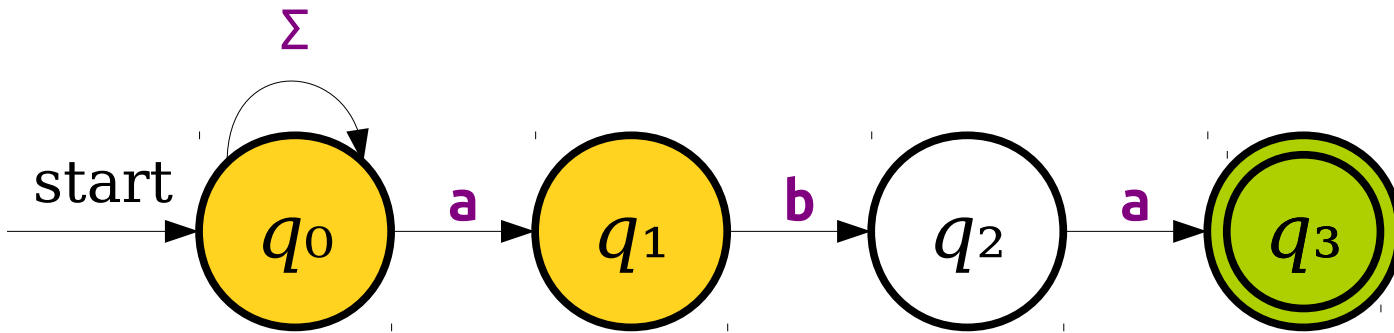
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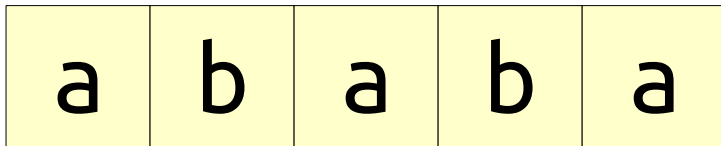
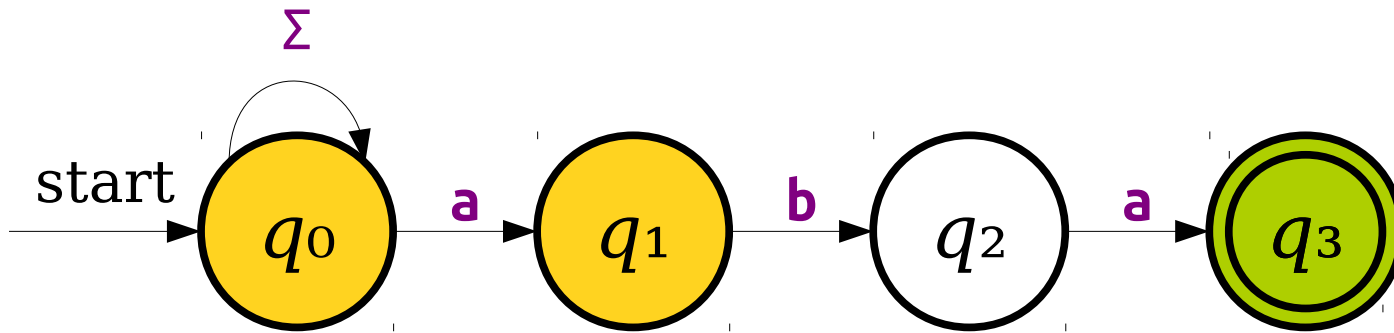
# Massive Parallelism



# Massive Parallelism



# Massive Parallelism



We're in at least one accepting state, so there's some path that gets us to an accepting state.

Therefore, we accept!

# Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- (Here's a rigorous explanation about how this works; read this on your own time).
  - Start off in the set of all states formed by taking the start state and including each state that can be reached by zero or more  $\epsilon$ -transitions.
  - When you read a symbol **a** in a set of states  $S$ :
    - Form the set  $S'$  of states that can be reached by following a single **a** transition from some state in  $S$ .
    - Your new set of states is the set of states in  $S'$ , plus the states reachable from  $S'$  by following zero or more  $\epsilon$ -transitions.

# So What?

- Each intuition of nondeterminism is useful in a different setting:
  - Perfect guessing is a great way to think about how to design a machine.
  - Massive parallelism is a great way to test machines – and has nice theoretical implications.
- Nondeterministic machines may not be feasible, but they give a great basis for interesting questions:
  - Can any problem that can be solved by a nondeterministic machine be solved by a deterministic machine?
  - Can any problem that can be solved by a nondeterministic machine be solved *efficiently* by a deterministic machine?
- The answers vary from automaton to automaton.

# Designing NFAs



# Designing NFAs

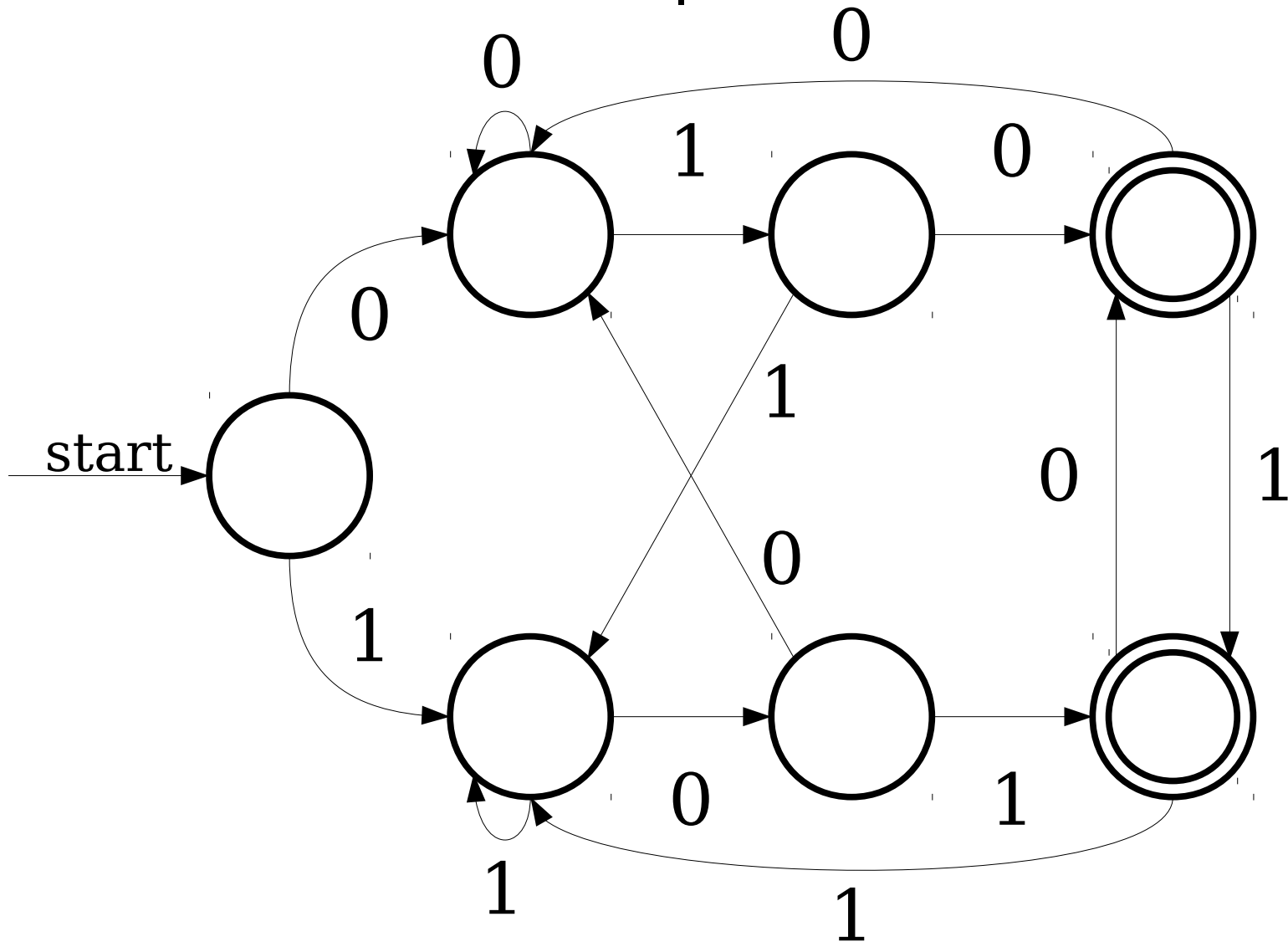
- When designing NFAs, *embrace the nondeterminism!*
- Good model: ***Guess-and-check***:
  - Is there some information that you'd really like to have? Have the machine *nondeterministically guess* that information.
  - Then, have the machine *deterministically check* that the choice was correct.
- The *guess* phase corresponds to trying lots of different options.
- The *check* phase corresponds to filtering out bad guesses or wrong options.

# Guess-and-Check

$$L = \{ w \in \{0, 1\}^* \mid w \text{ ends in } 010 \text{ or } 101 \}$$

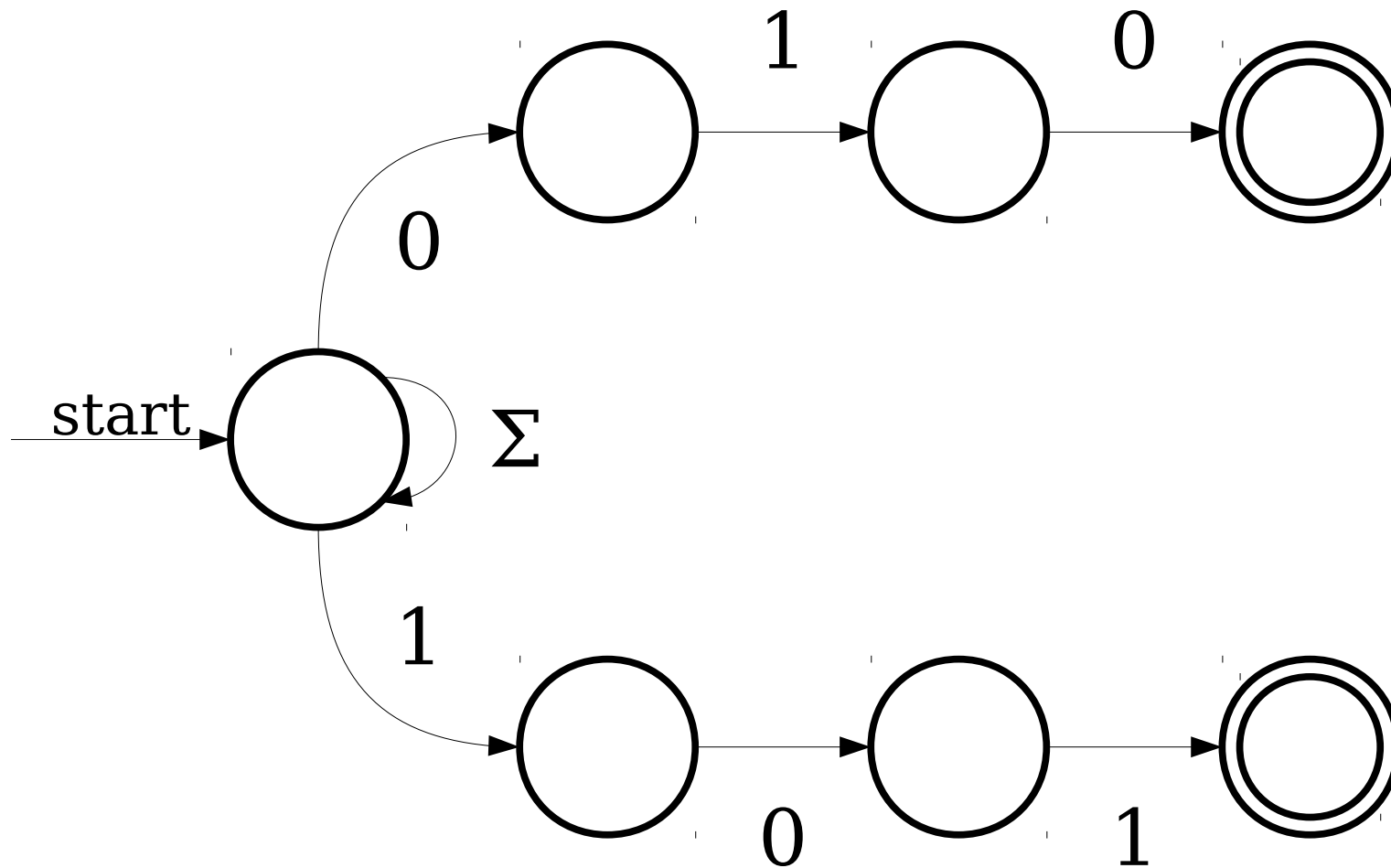
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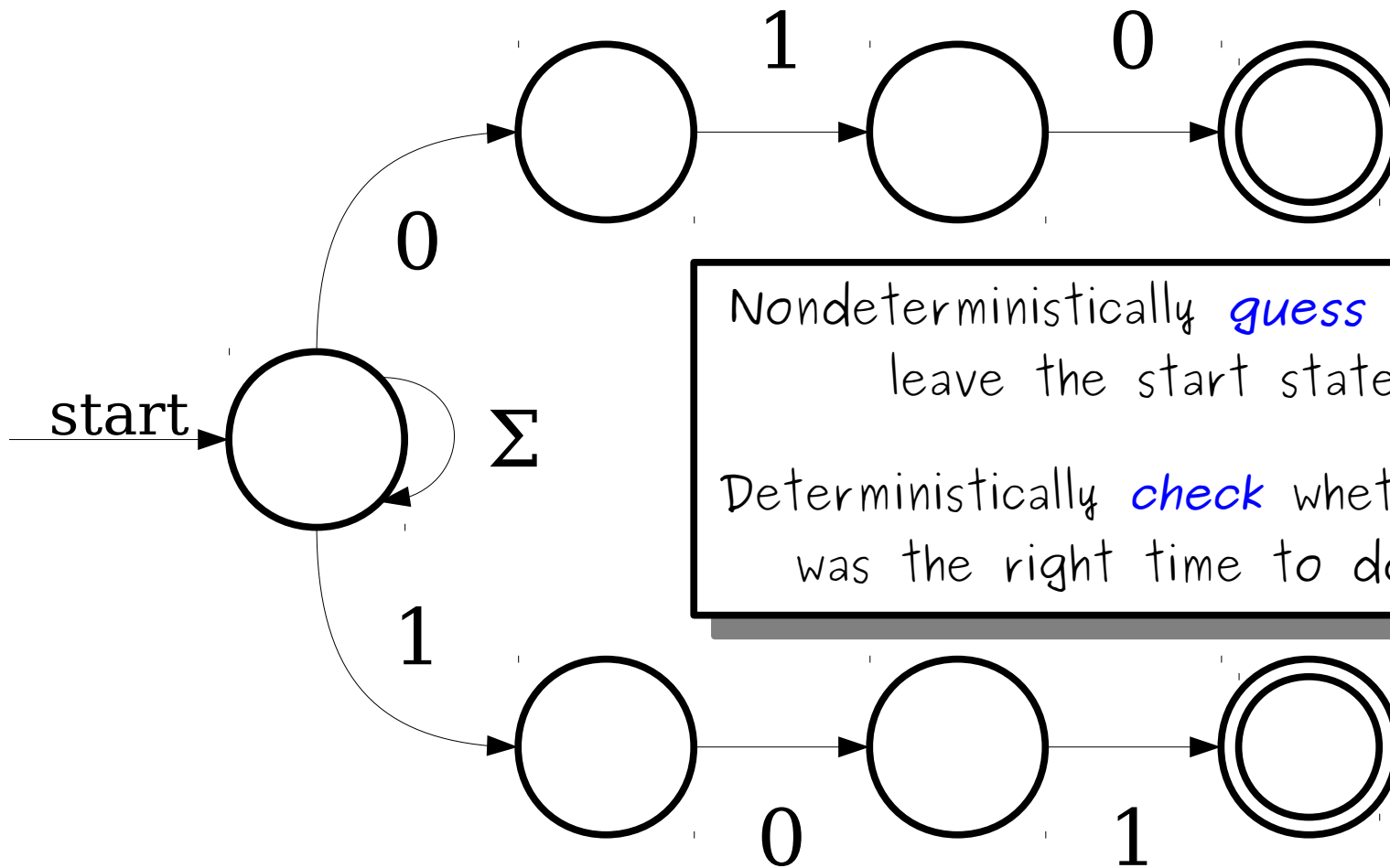
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Nondeterministically *guess* when to leave the start state.

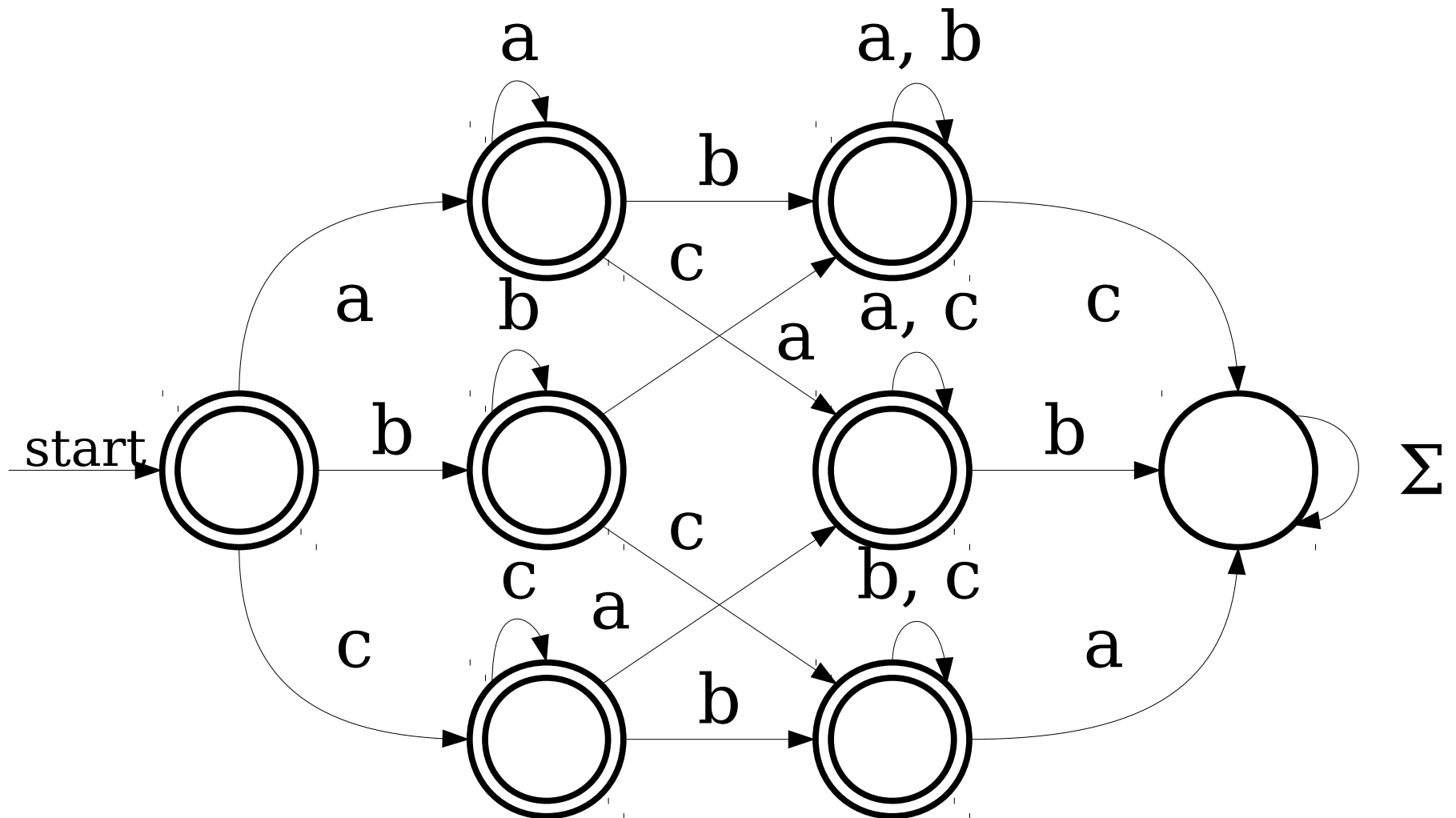
Deterministically *check* whether that was the right time to do so.

# Guess-and-Check

$L = \{ w \in \{a, b, c\}^* \mid \text{at least one of } a, b, \text{ or } c \text{ is not in } w \}$

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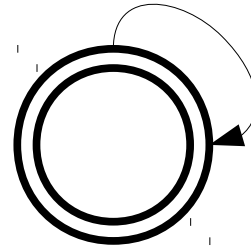
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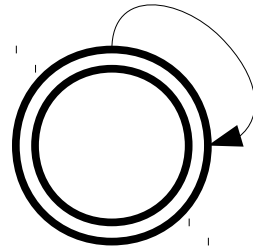
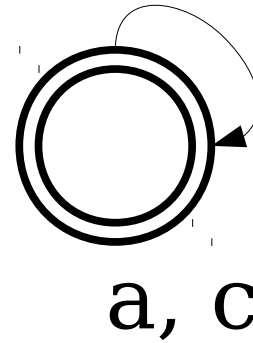
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a, b



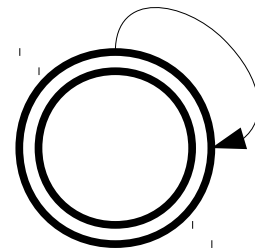
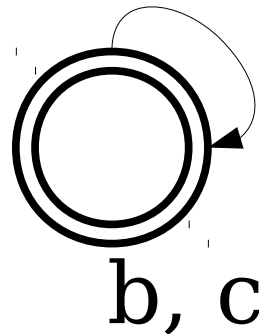
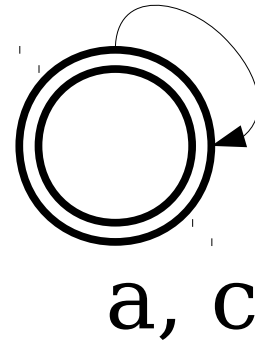
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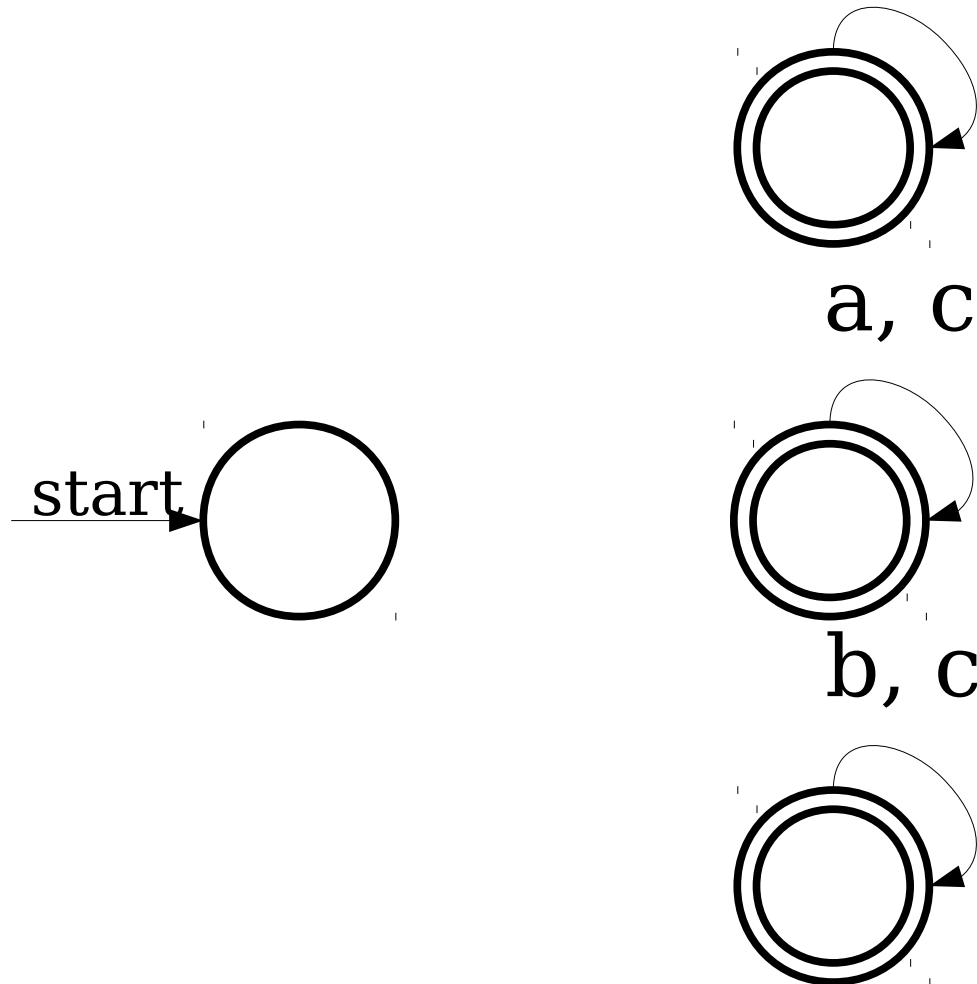
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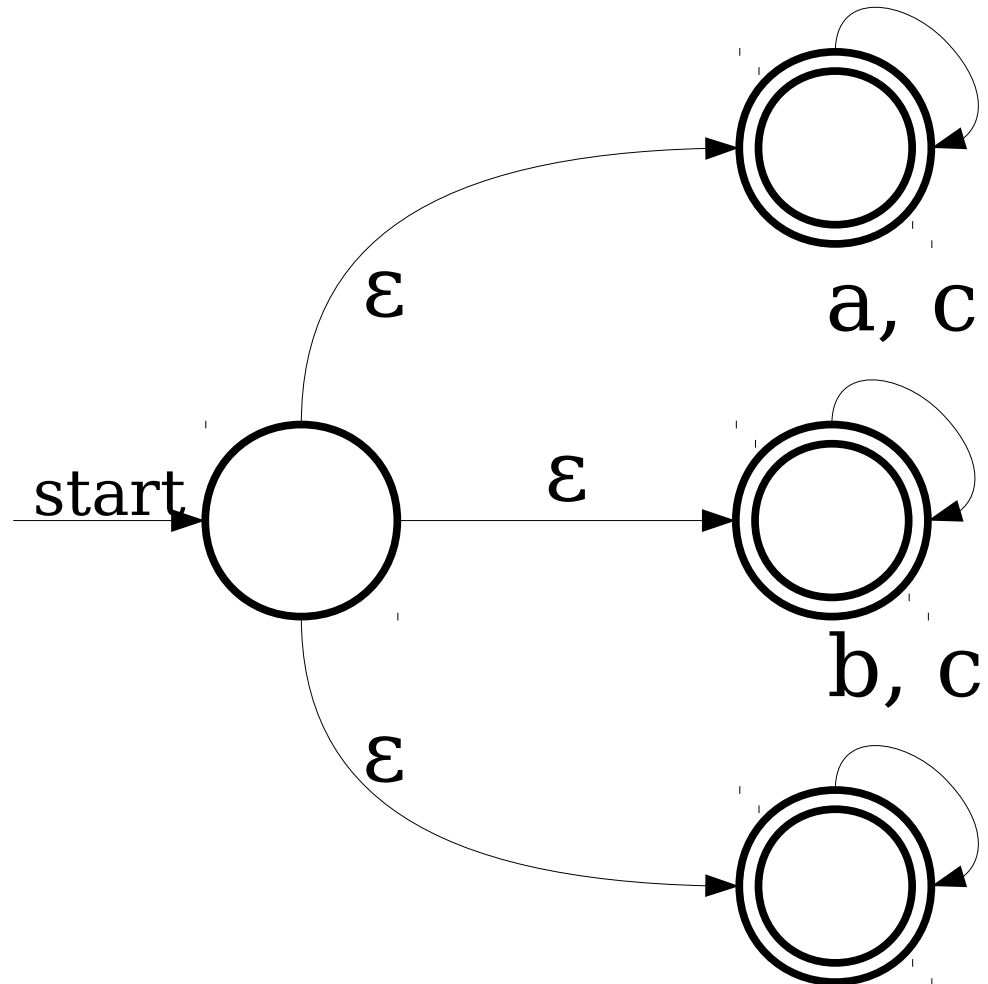
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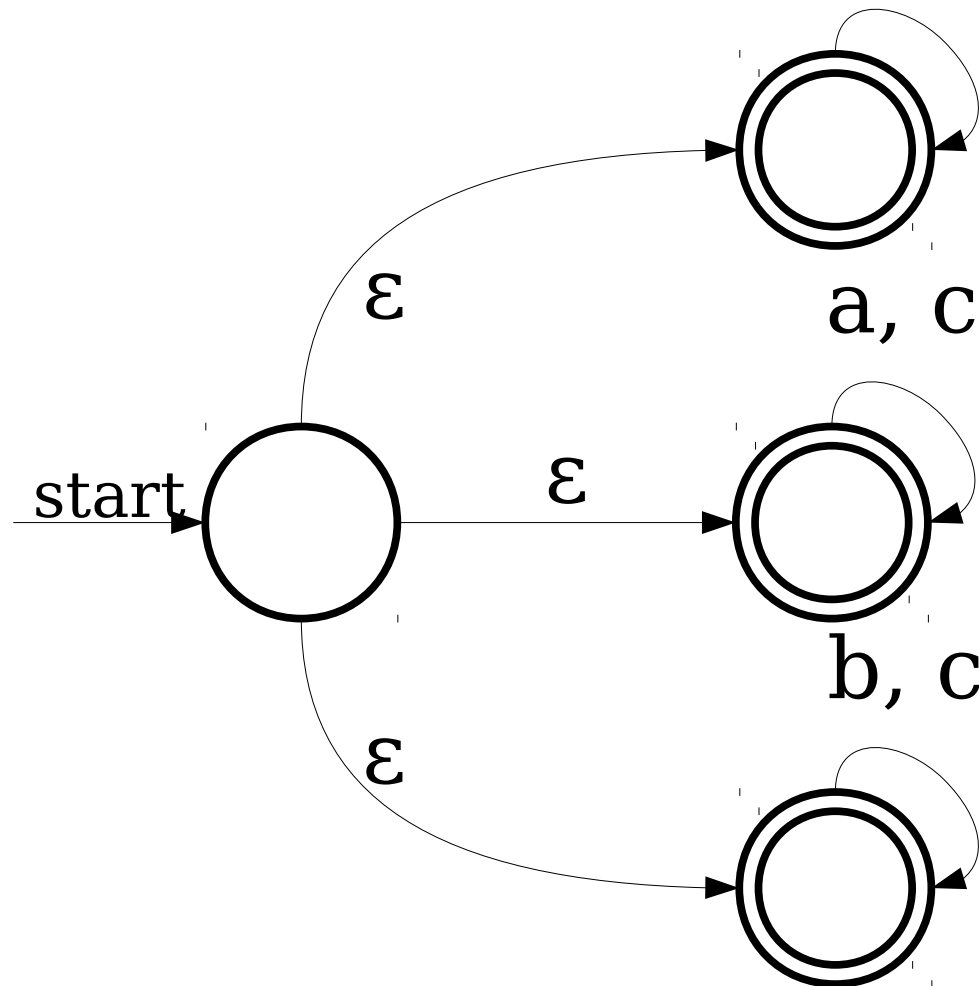
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Nondeterministically *guess* which character is missing.

Deterministically *check* whether that character is indeed missing.

Just how powerful are NFAs?