

# Efficiency

# Algorithms Revisited

- An **algorithm** is a procedure for effecting some result.
- There can be many different algorithms for solving the same problem.
- How can we compare algorithms against one another?
- What's the best algorithm for solving a given problem?

# Two Famous Problems

- **Searching**
  - Given an array of values, determine whether some value is contained in that array.
  - Very important: finding medical records, seeing if a genome sequence is in a database, etc.
- **Sorting**
  - Given an array of values, rearrange those values to put them in sorted order.
  - Enormously important: shows up in iTunes, Google, Facebook, etc.

# Searching

Can I get some volunteers?

# Linear Search

# Linear Search

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private int linearSearch(int[] arr, int key) {  
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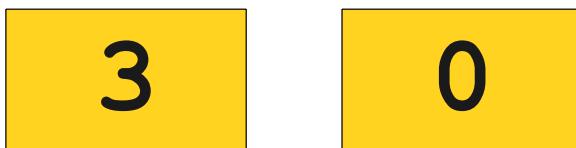
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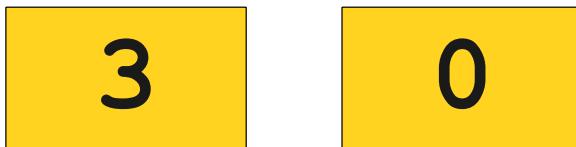
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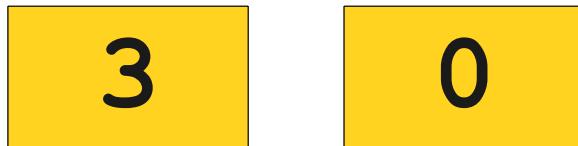
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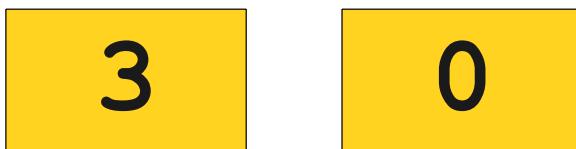
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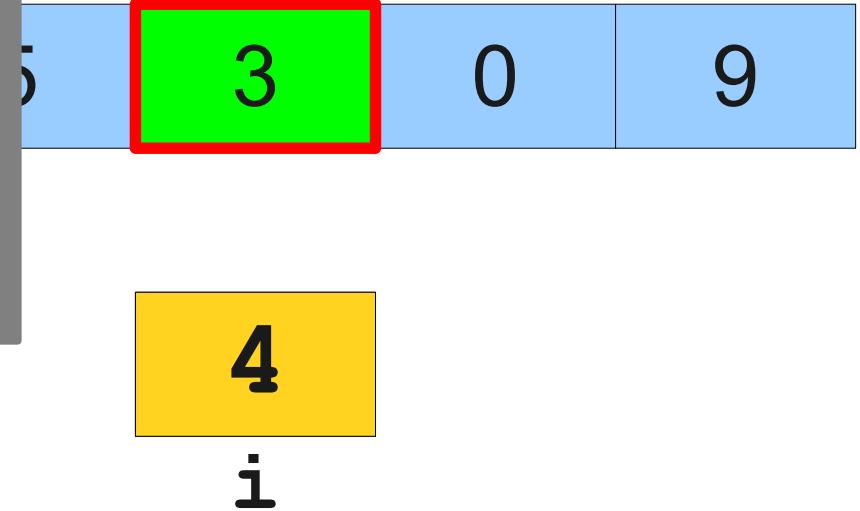
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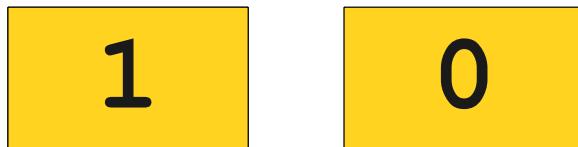
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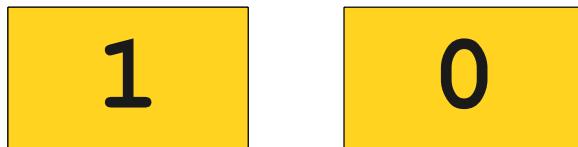
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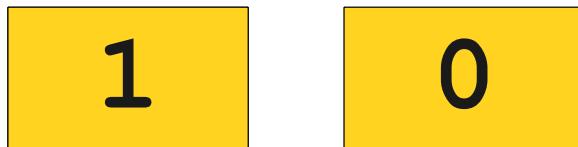
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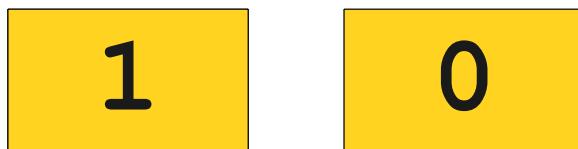
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# Searching II

# Binary Search

2	3	5	7	11	13	17	19	23	29	31	37	41
---	---	---	---	----	----	----	----	----	----	----	----	----

43	47	53	59	61	67	71	73	79	83	89	97	101
----	----	----	----	----	----	----	----	----	----	----	----	-----

103	107	109	113	127	131	137	139	149	151	157	163	167
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

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# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
    int rhs = arr.length - 1;  
  
    while (lhs <= rhs) {  
        int mid = (lhs + rhs) / 2;  
  
        if (arr[mid] == key)  
            return mid;  
        else if (arr[mid] < key)  
            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
    int rhs = arr.length - 1;  
  
    while (lhs <= rhs) {  
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            return mid;  
        else if (arr[mid] < key)  
            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

6

arr

1

2

3

5

6

8

9

# Binary Search

```
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key

6

arr

1

2

3

5

6

8

9

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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

**key**

6

**lhs**

0

**rhs**

6

**arr**



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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        if (arr[mid] == key)  
            return mid;  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

**key**

6

**lhs**

0

**rhs**

6

**arr**



1

2

3

5

6

8

9

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

key

6

lhs

0

mid

3

rhs

6

arr

1

2

3

5

6

8

9

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
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            rhs = mid - 1;  
    }  
    return -1;  
}
```

key

6

lhs

0

mid

3

rhs

6

arr

1

2

3

5

6

8

9

# Binary Search

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private int binarySearch(int[] arr, int key) {  
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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

key

6

lhs

0

mid

3

rhs

6

arr

1

2

3

5

6

8

9



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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    }  
    return -1;  
}
```

key

6

lhs

0

mid

3

rhs

6

arr

1

2

3

5

6

8

9



# Binary Search

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private int binarySearch(int[] arr, int key) {  
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            rhs = mid - 1;  
    }  
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}
```

**key**

6

**lhs**

0

**mid**

3

**rhs**

6

**arr**

1

2

3

5

6

8

9



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

key

6

mid    lhs

3

4

rhs

6

arr



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
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        int mid = (lhs + rhs) / 2;  
  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

key  
6

mid	lhs	rhs
3	4	6

arr → 1 2 3 5 6 8 9

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
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    while (lhs <= rhs) {  
        int mid = (lhs + rhs) / 2;  
  
        if (arr[mid] == key)  
            return mid;  
        else if (arr[mid] < key)  
            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

**key**

6

**arr**



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

**key**

6

**lhs**

4

**rhs**

6

**arr**

1

2

3

5

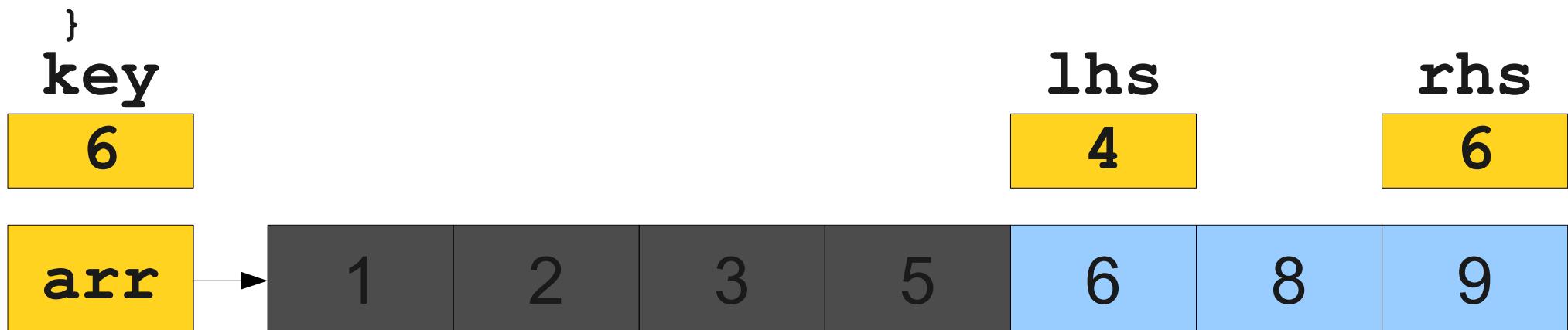
6

8

9

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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    }  
    return -1;  
}
```



# Binary Search

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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

6

arr

lhs	mid	rhs
4	5	6
1	2	3

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

6

arr

lhs	mid	rhs
4	5	6
1	2	3



# Binary Search

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private int binarySearch(int[] arr, int key) {  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

6

arr

lhs	mid	rhs
4	5	6
1	2	3



The array arr is shown with indices 0 to 6. The element at index 6 (value 8) is highlighted with a red border.

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
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    while (lhs <= rhs) {  
        int mid = (lhs + rhs) / 2;  
  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

6

arr



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

6

arr

lhs mid rhs

4 5 6

1 2 3 5 6 8 9

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

lhs	mid	rhs
-----	-----	-----

4	5	6
---	---	---



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

lhs    rhs

4	4
---	---

arr



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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    return -1;  
}  
key
```

lhs	rhs
4	4



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

6

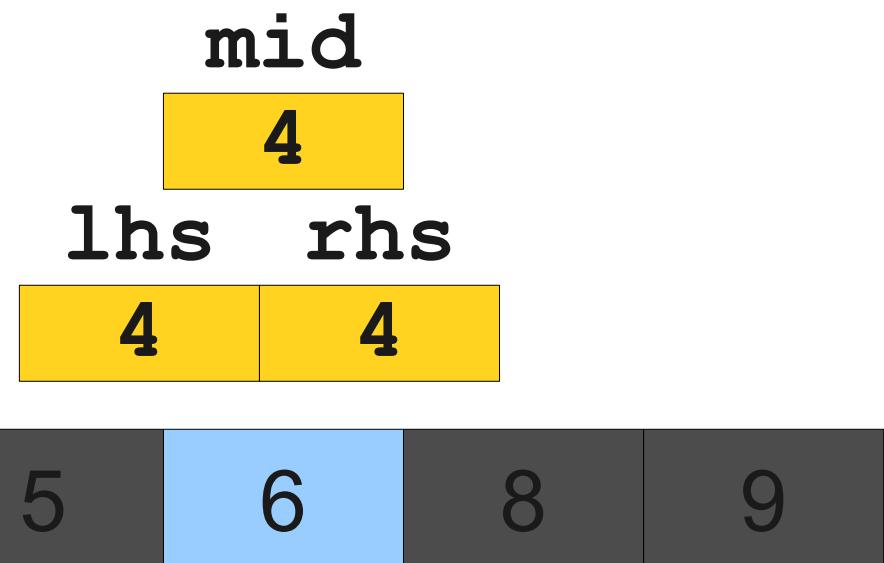
lhs	rhs
4	4

arr

arr → [1 | 2 | 3 | 5 | 6 | 8 | 9]

# Binary Search

```
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}  
key
```



# Binary Search

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}  
key
```

6

arr



# Binary Search

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key
```

6

arr



mid

4

lhs      rhs

4      4

# Binary Search

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}  
key
```

6

arr

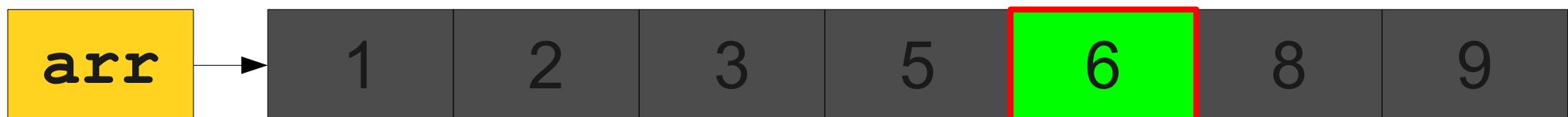


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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

6

arr



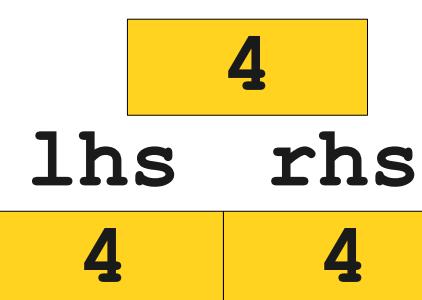
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}  
key
```

6

arr →

1 2 3 5 6 8 9



# Binary Search

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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

7

arr

1

2

3

5

6

8

9

# Binary Search

```
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        else  
            rhs = mid - 1;  
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}  
key
```

7

arr

1

2

3

5

6

8

9

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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

**key**

7

**lhs**

0

**rhs**

6

**arr**

1	2	3	5	6	8	9
---	---	---	---	---	---	---

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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}
```

**key**

7

**lhs**

0

**rhs**

6

**arr**



# Binary Search

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}
```

**key**

7

**lhs**

0

**mid**

3

**rhs**

6

**arr**

1

2

3

5

6

8

9

# Binary Search

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private int binarySearch(int[] arr, int key) {  
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}
```

**key**

7

**lhs**

0

**mid**

3

**rhs**

6

**arr**

1

2

3

5

6

8

9

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}
```

key

7

lhs

0

mid

3

rhs

6

arr

1

2

3

5

6

8

9



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```

**key**

7

**lhs**

0

**mid**

3

**rhs**

6

**arr**

1

2

3

5

6

8

9



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

**key**

**lhs**

**mid**

**rhs**

7

0

3

6

**arr**

1

2

3

5

6

8

9



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
    int rhs = arr.length - 1;  
  
    while (lhs <= rhs) {  
        int mid = (lhs + rhs) / 2;  
  
        if (arr[mid] == key)  
            return mid;  
        else if (arr[mid] < key)  
            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

key

7

mid    lhs

3

4

rhs

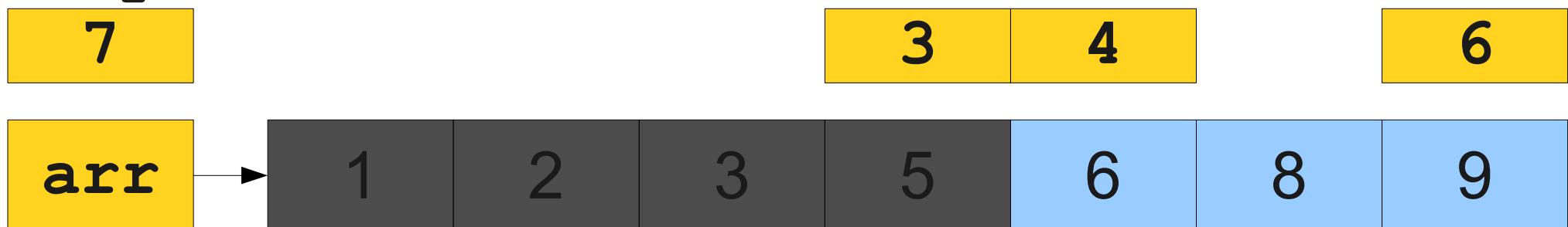
6

arr



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    }  
    return -1;  
}
```

**key**

7

**arr**

arr	1	2	3	4	5	6	7	8	9
lhs	4	5	6	7	8	9	10	11	12
rhs	6	7	8	9	10	11	12	13	14

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
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            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

7

lhs  
4

rhs  
6

arr →

1

2

3

5

6

8

9

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**key**

7

**arr**



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key
```

7

arr

lhs mid rhs

4 5 6

1 2 3 5 6 8 9

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key
```

7

arr

lhs	mid	rhs
4	5	6
1	2	3

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key
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7

arr



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key
```

7

arr

lhs mid rhs

4 5 6

1 2 3 5 6 8 9

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key
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7

arr

lhs mid rhs

4 5 6

1 2 3 5 6 8 9

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key
```

7

arr

lhs mid rhs

4 5 6

1 2 3 5 6 8 9

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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

lhs    rhs

4	4
---	---

arr

The diagram shows an array `arr` with elements 1, 2, 3, 5, 6, 8, 9. The element 6 is highlighted in light blue. To the left of the array, there is a yellow box labeled `arr`. Above the array, there are two yellow boxes labeled `lhs` and `rhs`, both containing the value 4. An arrow points from the `arr` label to the first element of the array.

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
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    while (lhs <= rhs) {  
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            rhs = mid - 1;  
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}  
key
```

lhs	rhs
4	4



# Binary Search

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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

7

lhs	rhs
4	4

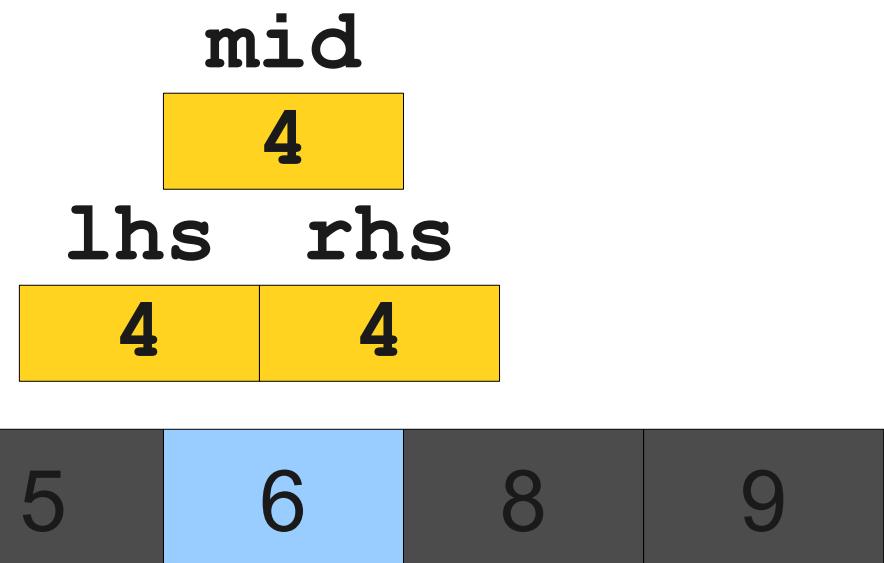
arr



arr → [1, 2, 3, 5, 6, 8, 9]

# Binary Search

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private int binarySearch(int[] arr, int key) {  
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key
```



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            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

7

arr

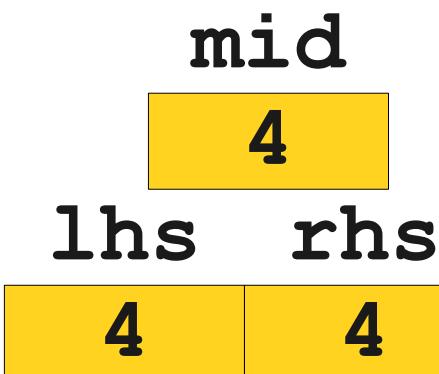


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private int binarySearch(int[] arr, int key) {  
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    }  
    return -1;  
}  
key
```

7

arr

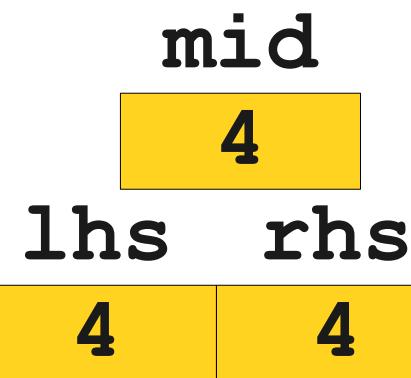


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}  
key
```

7

arr



# Binary Search

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private int binarySearch(int[] arr, int key) {  
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            lhs = mid + 1;  
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            rhs = mid - 1;  
    }  
    return -1;  
}
```

key  
7

arr →

mid  
4

lhs      rhs

4	4
---	---

1	2	3	5	6	8	9
---	---	---	---	---	---	---

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
    int rhs = arr.length - 1;  
  
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            lhs = mid + 1;  
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    }  
    return -1;  
}  
key
```

rhs    lhs

4	5
---	---



# Binary Search

```
private int binarySearch(int[] arr, int key) {  
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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```

rhs    lhs

4    5

arr

1    2    3    5    6    8    9

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
    int rhs = arr.length - 1;  
  
    while (lhs <= rhs) {  
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        else  
            rhs = mid - 1;  
    }  
    return -1;  
}
```

key

7

rhs    lhs

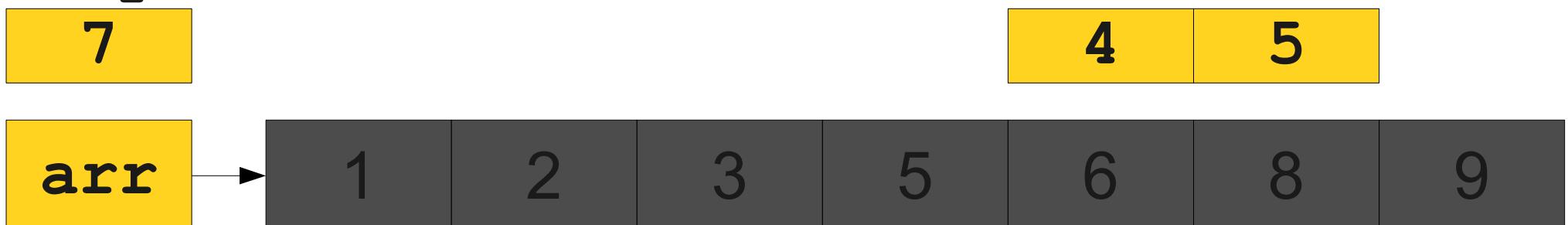
4	5
---	---

arr

The diagram shows an array `arr` with elements 1, 2, 3, 5, 6, 8, 9. The element 4 is highlighted in yellow, representing the key being searched for. The search range is indicated by indices: `rhs` is at index 4 (containing 4), and `lhs` is at index 5 (containing 5). The array elements are shown in dark grey boxes, while the key and indices are in yellow boxes.

# Binary Search

```
private int binarySearch(int[] arr, int key) {  
    int lhs = 0;  
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    while (lhs <= rhs) {  
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            return mid;  
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            lhs = mid + 1;  
        else  
            rhs = mid - 1;  
    }  
    return -1;  
}  
key
```



# Analyzing the Algorithms

# For Comparison

```
private int linearSearch(int[] arr,
                        int key) {
    for (int i = 0; i < arr.length; i++) {
        if (arr[i] == key)
            return i;
    }
    return -1;
}
```

```
private int binarySearch(int[] arr,
                        int key) {
    int lhs = 0;
    int rhs = arr.length - 1;

    while (lhs <= rhs) {
        int mid = (lhs + rhs) / 2;

        if (arr[mid] == key)
            return mid;
        else if (arr[mid] < key)
            lhs = mid + 1;
        else
            rhs = mid - 1;
    }
    return -1;
}
```

# Some Quick Facts

- Elementary operations (arithmetic, choosing an if/else branch, deciding whether to loop again, etc.) all take roughly the same amount of time to complete.
- Array accesses take roughly the same time to complete regardless of the index.

# Analyzing Linear Search

- How many elements of the array do we have to look at to do a linear search?
- Let's suppose that there are  $N$  elements in the array.
- We may have to look at each of them once.
- Number of lookups:  $N$ .

# Analyzing Binary Search

- How many elements of the array do we have to look at to do a binary search?
- Let's suppose that there are  $N$  elements in the array.
- Each lookup cuts the size of the array in half.
- How many times can we cut the array in half before we run out of elements?

# Slicing and Dicing

- After zero lookups:  $N$
- After one lookup:  $N / 2$
- After two lookups:  $N / 4$
- After three lookups:  $N / 8$
- ...
- After  $k$  lookups:  $N / 2^k$

# Cutting in Half

- After doing  $k$  lookups, there are  $N / 2^k$  elements left.
- The algorithm stops when there is just one element left.
- Solving for the number of iterations:

$$N / 2^k = 1$$

$$N = 2^k$$

$$\log_2 N = k$$

- So binary search stops after  $\log_2 N$  lookups.

# For Comparison

$N$	$\log_2 N$
10	3
100	7
1000	10
1,000,000	20
1,000,000,000	30

Binary search can check whether a value exists in an array of **one billion elements** in just 30 array accesses!

# A Feel for $\log_2 N$

- It is conjectured that the number of atoms in the universe is  $10^{100}$ .
- $\log_2 10^{100} \approx 300$ .
- If you (somehow) listed all the atoms in the universe in sorted order, you would need to look at 300 before you found the one you were looking for.

# Why Efficiency Matters

# An Interesting Lecture Excerpt

Admiral Grace Hopper on nanoseconds:

**<http://www.youtube.com/watch?v=JEpsKnWZrJ8>**

# Sorting

# Bubble Sort

- Until the array is sorted:
  - Look at each adjacent pair of elements.
  - If they are out of order, swap them.

Should we use bubble sort?

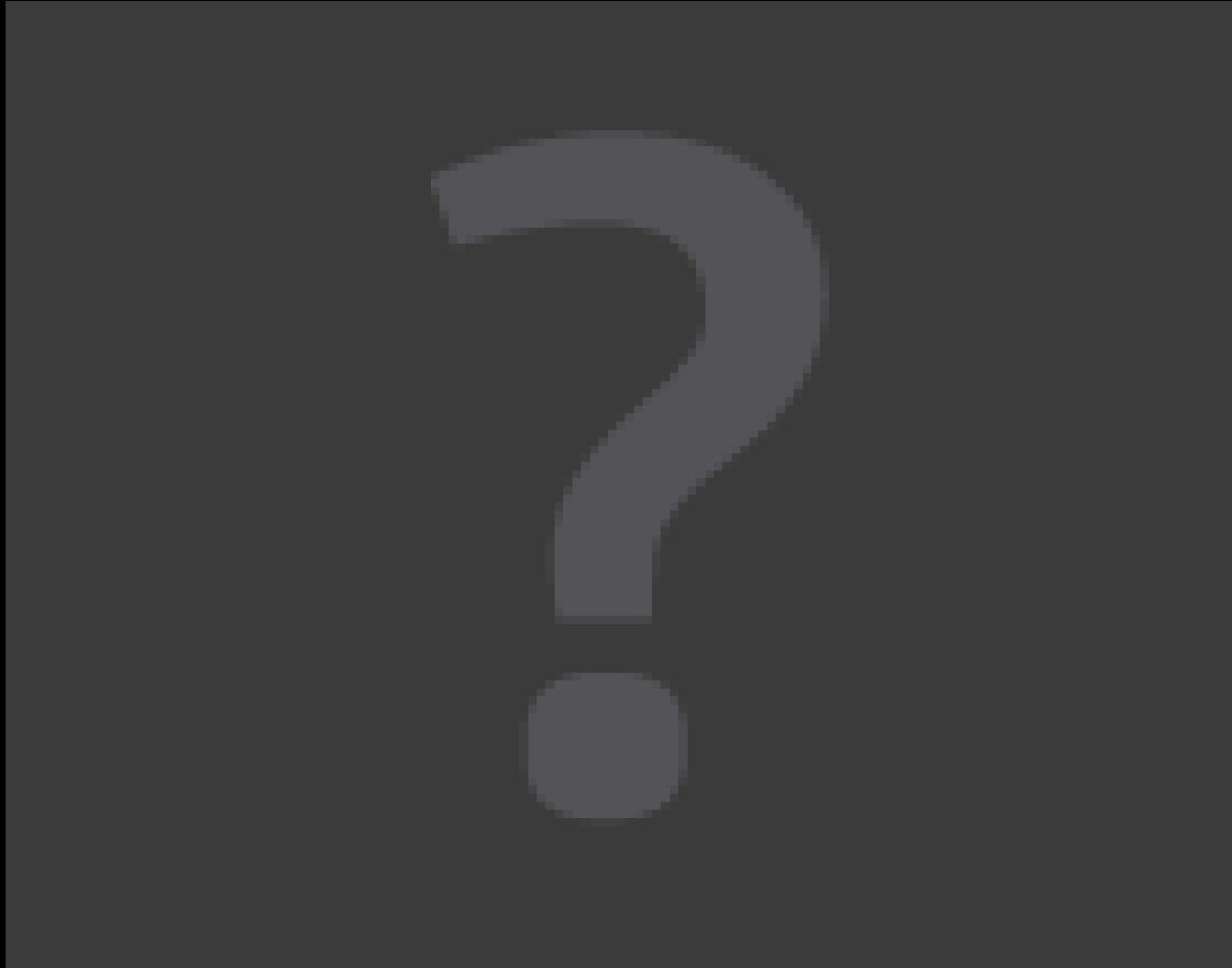
Should we use bubble sort?

Let's ask the President of the United States!

Barack  
Obama, in  
2008.



Eric Schmidt,  
then CEO of  
Google.



# A Presidential Decree



“The bubble sort  
would be the  
wrong way to go.”

- Barack Obama

What other  
algorithms could  
we use instead?

# Quicksort

# Yes, This Exists

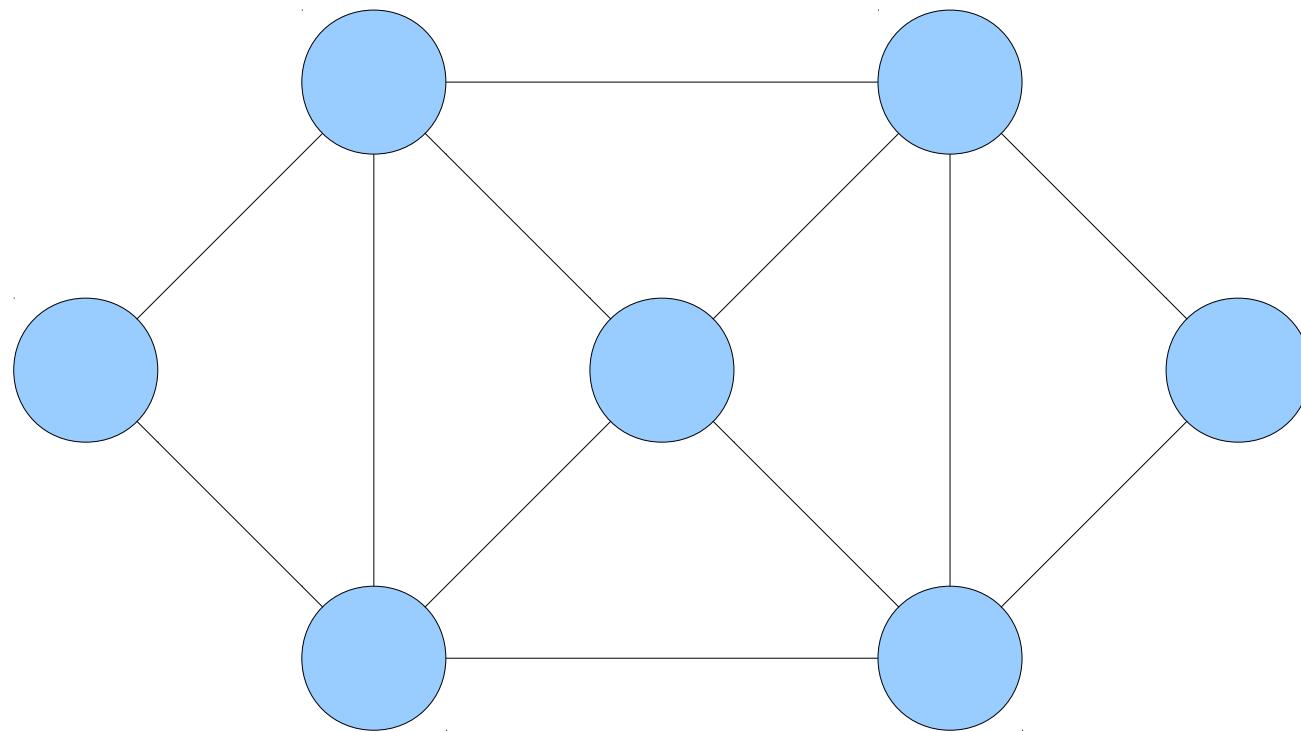
Sorting algorithms done as  
Hungarian folk dances:

<http://www.youtube.com/user/AlgoRhythmics>

# Theoretical Limits of Efficiency

		7		6		1		
					3		5	2
3			1		5	9		7
6		5		3		8		9
	1						2	
8		2		1		5		4
1		3	2		7			8
5	7		4					
		4		8		7		

2	5	7	9	6	4	1	8	3
4	9	1	8	7	3	6	5	2
3	8	6	1	2	5	9	4	7
6	4	5	7	3	2	8	1	9
7	1	9	5	4	8	3	2	6
8	3	2	6	1	9	5	7	4
1	6	3	2	5	7	4	9	8
5	7	8	4	9	6	2	3	1
9	2	4	3	8	1	7	6	5



If you can check a solution to a problem efficiently, can you necessarily solve that problem efficiently?

This is called the **P versus NP problem** and is the biggest open problem in theoretical computer science.

There is a \$1,000,000 prize for the answer.