Problem 1. Weights and balances

```cpp
/*
 * Function: isMeasurable
 * Usage: if (isMeasurable(target, weights))
 * Determines whether it is possible to measure the specified target
 * weight using some combination of the weights stored in the vector
 * weights. To do so, it recursively attacks the problem by considering
 * only the first weight in the array, which gives rise to the following
 * possibilities:
 *
 * 1. The first weight is unused. In this case, it is possible
 * to measure the target weight only if it is possible to do
 * so using the remaining weights.
 *
 * 2. The first weight goes on the opposite side of the balance
 * from the sample. In this case, the target weight is
 * effectively decreased by first, which means it can be
 * measured only if it is possible to measure target - first
 * ounces using the other weights.
 *
 * 3. The first weight goes on the same side of the balance
 * from the sample. In this case, the target weight is
 * effectively increased by first, which means it can be
 * measured only if it is possible to measure target + first
 * ounces using the other weights.
 * The simple case occurs when there are no weights at all, in
 * which case the target weight is measurable only if it is 0.
 */

bool isMeasurable(int target, Vector<int> & weights) {
    if (weights.isEmpty()) {
        return target == 0;
    } else {
        int first = weights[0];
        Vector<int> rest = weights;
        rest.remove(0);
        return isMeasurable(target, rest)
            || isMeasurable(target - first, rest)
            || isMeasurable(target + first, rest);
    }
}
```
Problem 2. Shortest path

```java
/*
 * Function: shortestPathLength
 * Usage: int len = shortestPathLength(maze, start);
 * --------------------------------------------------------------
 * Finds the shortest exit path in the maze beginning at the specified
 * starting square. If no solution exists, shortestPathLength returns
 * the sentinel value -1.
 */

int shortestPathLength(Maze & maze, Point pt) {
    if (maze.isOutside(pt)) return 0;
    if (maze.isMarked(pt)) return -1;
    maze.markSquare(pt);
    int shortest = -1;
    for (Direction dir = NORTH; dir <= WEST; dir++) {
        if (!maze.wallExists(pt, dir)) {
            int len = shortestPathLength(maze, adjacentPoint(pt, dir)) + 1;
            if (len > 0) {
                if (shortest == -1 || len < shortest) {
                    shortest = len;
                }
            }
        }
    }
    maze.unmarkSquare(pt);
    return shortest;
}
```

Problem 3. Filling a region

```java
/*
 * Function: fillRegion
 * Usage: fillRegion(grid, row, col);
 * -----------------------------
 * This function paints black pixels everywhere inside the
 * region at the specified row and column.
 */

void fillRegion(Grid<bool> & pixels, int row, int col) {
    if (pixels.inBounds(row, col) && !pixels[row][col]) {
        pixels[row][col] = true;
        fillRegion(pixels, row + 1, col);
        fillRegion(pixels, row - 1, col);
        fillRegion(pixels, row, col + 1);
        fillRegion(pixels, row, col - 1);
    }
}
```
Problem 4. Generating multiword anagrams

```cpp
/*
 * Function: findAnagram
 * Usage: bool found = findAnagram(letters, english, words);
 * -------------------------------------------------------------
 * Finds a multiword anagram for the specified set of letters.
 * using only English words from the dictionary in english in
 * which each word must be at least MIN_WORD characters long.
 * If the program finds any anagrams, it stores the list of words
 * in the vector words and returns true. If no anagrams exist,
 * the function returns false.
 */
bool findAnagram(string letters, Lexicon & english, Vector<string> & words) {
    return findAnagramWithFixedPrefix("", letters, english, words);
}

/*
 * Function: findAnagram
 * Usage: bool found = findAnagram(prefix, letters, english, words);
 * -------------------------------------------------------------
 * Finds a multiword anagram for the specified set of letters, where
 * the current word must begin with the specified prefix.
 */
bool findAnagramWithFixedPrefix(string prefix, string rest,
                                 Lexicon & english,
                                 Vector<string> & words) {
    if (!english.containsPrefix(prefix)) return false;
    if (english.contains(prefix) && prefix.length() >= MIN_WORD) {
        if (rest == "" || findAnagram(rest, english, words)) {
            words.add(prefix);
            return true;
        }
    }
    for (int i = 0; i < rest.length(); i++) {
        string otherLetters = rest.substr(0, i) + rest.substr(i + 1);
        if (findAnagramWithFixedPrefix(prefix + rest[i], otherLetters,
                                        english, words)) return true;
    }
    return false;
}
```
Problem 5. Recursion and Big-O

(a) $O(N^2)$. The outer loop will run $n$ times. Each time through the outer loop, the inner loop will run $i$ times, where $i$ runs from 0 to $n-1$. A constant amount of work is done in the body of the inner loop. This leads to the series: $0 + 1 + 2 + \ldots + n-1$ which, as you know from the selection sort analysis, is $n(n-1)/2$. Keeping only the highest order term and throwing away any constant factors gives the $O(N^2)$ computational complexity.

(b) $O(1)$. The outer loop executes 10 times, the inner loop $i$ times where $i$ runs from 0 to 9, so there are ~100 multiply/add operations, but it does that same amount of work for any value of $n$. Thus the computational complexity is constant with respect to $n$. The constant 1 in $O(1)$ signifies this. Constant time doesn’t necessarily mean that a function computes its result instantly, but the function always does the same amount of work.

(c) $O(\log N)$. On each recursive call, the argument $n$ is divided by 2. The complexity of this operation is therefore the number of times you can divide $n$ by 2 until you reach 1. From the discussion of either binary search or merge sort, you know that this in $O(\log N)$.

(d) Calling mystery4("PUMPKIN") returns "NIKPMUP", which consists of the letters in reverse order. The calls to concatenation and mystery4 (which is abbreviated here as m4 to save space) look like this:

```
In problems such as this one, the structure of the tree is often sufficient to compute the complexity order. The key points to observe are:

• The work done at each level is $O(N)$.
• There are $O(\log N)$ levels, because the string is always divided in half.

The computational complexity is therefore the same as that for the merge sort algorithm, which is $O(N \log N)$.