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CS106B

Handout #22A
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Solutions to Section Handout #3

Problem 1. Weights and balances

```
/*
 * Function: isMeasurable
 * Usage: if (isMeasurable(target, weights) . . .
 * ---------------------------------------------
 * Determines whether it is possible to measure the specified target
 * weight using some combination of the weights stored in the vector
 * weights.  To do so, it recursively attacks the problem by considering
 * only the first weight in the array, which gives rise to the following
 * possibilities:
 *
 * 1. The first weight is unused.  In this case, it is possible
 *    to measure the target weight only if it is possible to do
 *    so using the remaining weights.
 *
 * 2. The first weight goes on the opposite side of the balance
 *    from the sample.  In this case, the target weight is
 *    effectively decreased by first, which means it can be
 *    measured only if it is possible to measure target - first
 *    ounces using the other weights.
 *
 * 3. The first weight goes on the same side of the balance
 *    from the sample.  In this case, the target weight is
 *    effectively increased by first, which means it can be
 *    measured only if it is possible to measure target + first
 *    ounces using the other weights.
 *
 * The simple case occurs when there are no weights at all, in
 * which case the target weight is measurable only if it is 0.
 */

bool isMeasurable(int target, Vector<int> & weights) {
   if (weights.isEmpty()) {
      return target == 0;
   } else {
      int first = weights[0];
      Vector<int> rest = weights;
      rest.remove(0);
      return isMeasurable(target, rest)
          || isMeasurable(target - first, rest)
          || isMeasurable(target + first, rest);
   }
}
```

Problem 2. Shortest path

```
/*
 * Function: shortestPathLength
 * Usage: int len = shortestPathLength(maze, start);
 * -------------------------------------------------
 * Finds the shortest exit path in the maze beginning at the specified
 * starting square.  If no solution exists, shortestPathLength returns
 * the sentinel value -1.
 */

int shortestPathLength(Maze & maze, Point pt) {
   if (maze.isOutside(pt)) return 0;
   if (maze.isMarked(pt)) return -1;
   maze.markSquare(pt);
   int shortest = -1;
   for (Direction dir = NORTH; dir <= WEST; dir++) {
      if (!maze.wallExists(pt, dir)) {
         int len = shortestPathLength(maze, adjacentPoint(pt, dir)) + 1;
         if (len > 0) {
            if (shortest == -1 || len < shortest) {
               shortest = len;
            }
         }
      }
   }
   maze.unmarkSquare(pt);
   return shortest;
}
```

Problem 3. Filling a region

```
/*
 * Function: fillRegion
 * Usage: fillRegion(grid, row, col);
 * ----------------------------------
 * This function paints black pixels everywhere inside the
 * region at the specified row and column.
 */

void fillRegion(Grid<bool> & pixels, int row, int col) {
   if (pixels.inBounds(row, col) && !pixels[row][col]) {
      pixels[row][col] = true;
      fillRegion(pixels, row + 1, col);
      fillRegion(pixels, row - 1, col);
      fillRegion(pixels, row, col + 1);
      fillRegion(pixels, row, col - 1);
   }
}
```

Problem 4. Generating multiword anagrams

```
/*
 * Function: findAnagram
 * Usage: bool found = findAnagram(letters, english, words);
 * --------------------------------------------------------
 * Finds a multiword anagram for the specified set of letters.
 * using only English words from the dictionary in english in
 * which each word must be at least MIN_WORD characters long.
 * If the program finds any anagrams, it stores the list of words
 * in the vector words and returns true.  If no anagrams exist,
 * the function returns false.
 */

bool findAnagram(string letters, Lexicon & english, Vector<string> & words) {
   return findAnagramWithFixedPrefix("", letters, english, words);
}

/*
 * Function: findAnagram
 * Usage: bool found = findAnagram(prefix, letters, english, words);
 * ----------------------------------------------------------------
 * Finds a multiword anagram for the specified set of letters, where
 * the current word must begin with the specified prefix.
 */

bool findAnagramWithFixedPrefix(string prefix, string rest,
                                Lexicon & english,
                                Vector<string> & words) {
   if (!english.containsPrefix(prefix)) return false;
   if (english.contains(prefix) && prefix.length() >= MIN_WORD) {
      if (rest == "" || findAnagram(rest, english, words)) {
         words.add(prefix);
         return true;
      }
   }
   for (int i = 0; i < rest.length(); i++) {
      string otherLetters = rest.substr(0, i) + rest.substr(i + 1);
      if (findAnagramWithFixedPrefix(prefix + rest[i], otherLetters,
                                     english, words)) return true;
   }
   return false;
}
```

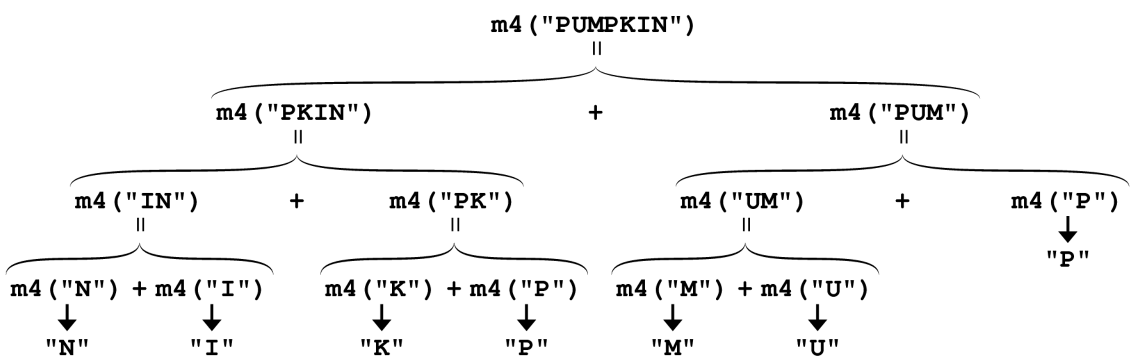
Problem 5. Recursion and Big-O

(a) $O(N^2)$. The outer loop will run `n` times. Each time through the outer loop, the inner loop will run `i` times, where `i` runs from 0 to `n-1`. A constant amount of work is done in the body of the inner loop. This leads to the series: 0 + 1 + 2 + . . . + `n-1` which, as you know from the selection sort analysis, is `n`(`n-1`)/2. Keeping only the highest order term and throwing away any constant factors gives the $O(N^2)$ computational complexity.

(b) $O(1)$. The outer loop executes 10 times, the inner loop `i` times where `i` runs from 0 to 9, so there are ~100 multiply/add operations, but it does that same amount of work for any value of `n`. Thus the computational complexity is constant with respect to `n`. The constant 1 in $O(1)$ signifies this. Constant time doesn't necessarily mean that a function computes its result instantly, but the function always does the same amount of work.

(c) $O(\log N)$. On each recursive call, the argument `n` is divided by 2. The complexity of this operation is therefore the number of times you can divide `n` by 2 until you reach 1. From the discussion of either binary search or merge sort, you know that this in $O(\log N)$.

(d) Calling `mystery4("PUMPKIN")` returns `"NIKPMUP"`, which consists of the letters in reverse order. The calls to concatenation and `mystery4` (which is abbreviated here as `m4` to save space) look like this:



In problems such as this one, the structure of the tree is often sufficient to compute the complexity order. The key points to observe are:

- The work done at each level is $O(N)$.
- There are $O(\log N)$ levels, because the string is always divided in half.

The computational complexity is therefore the same as that for the merge sort algorithm, which is $O(N \log N)$.