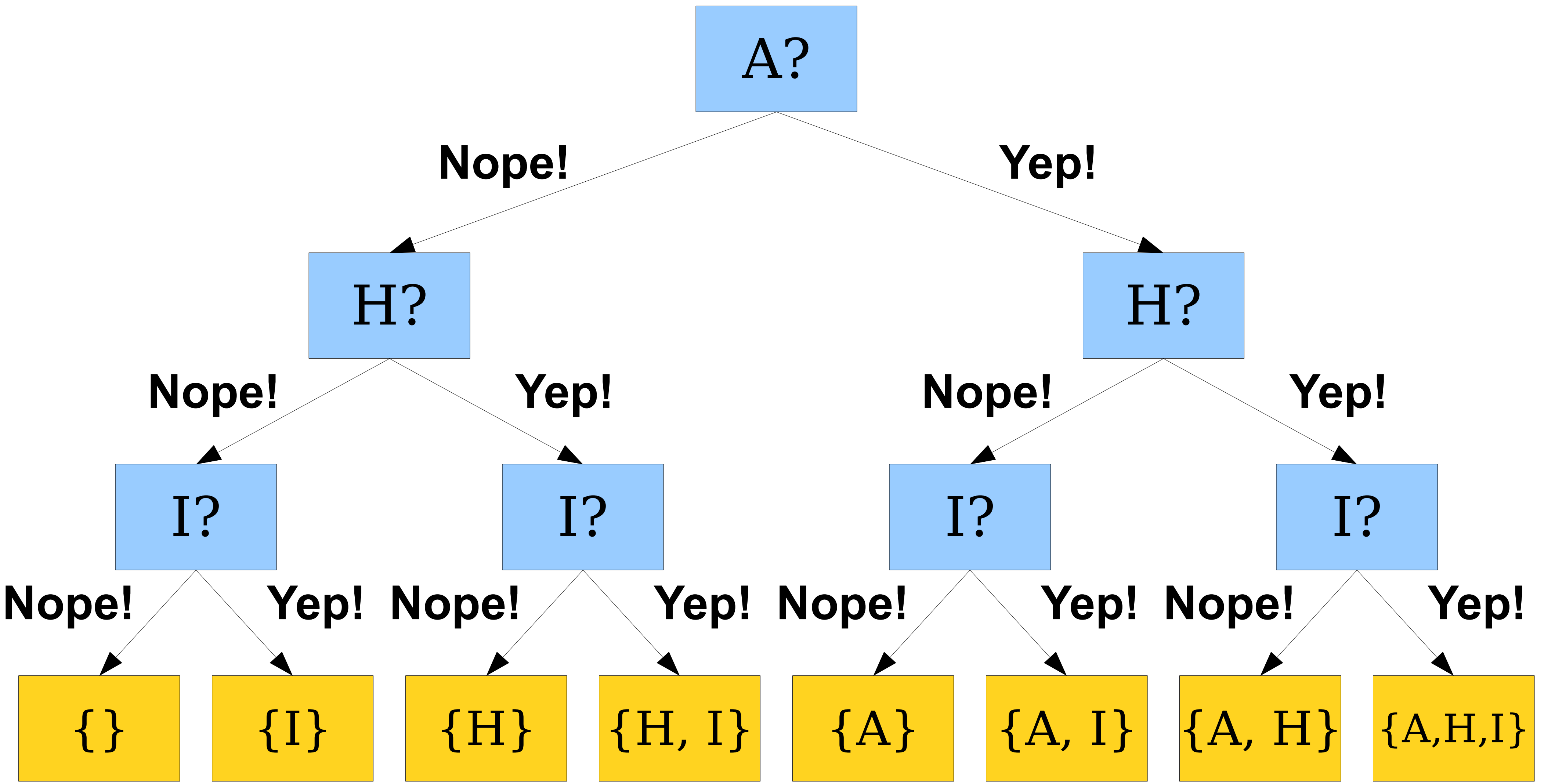
# Thinking Recursively

## Part IV

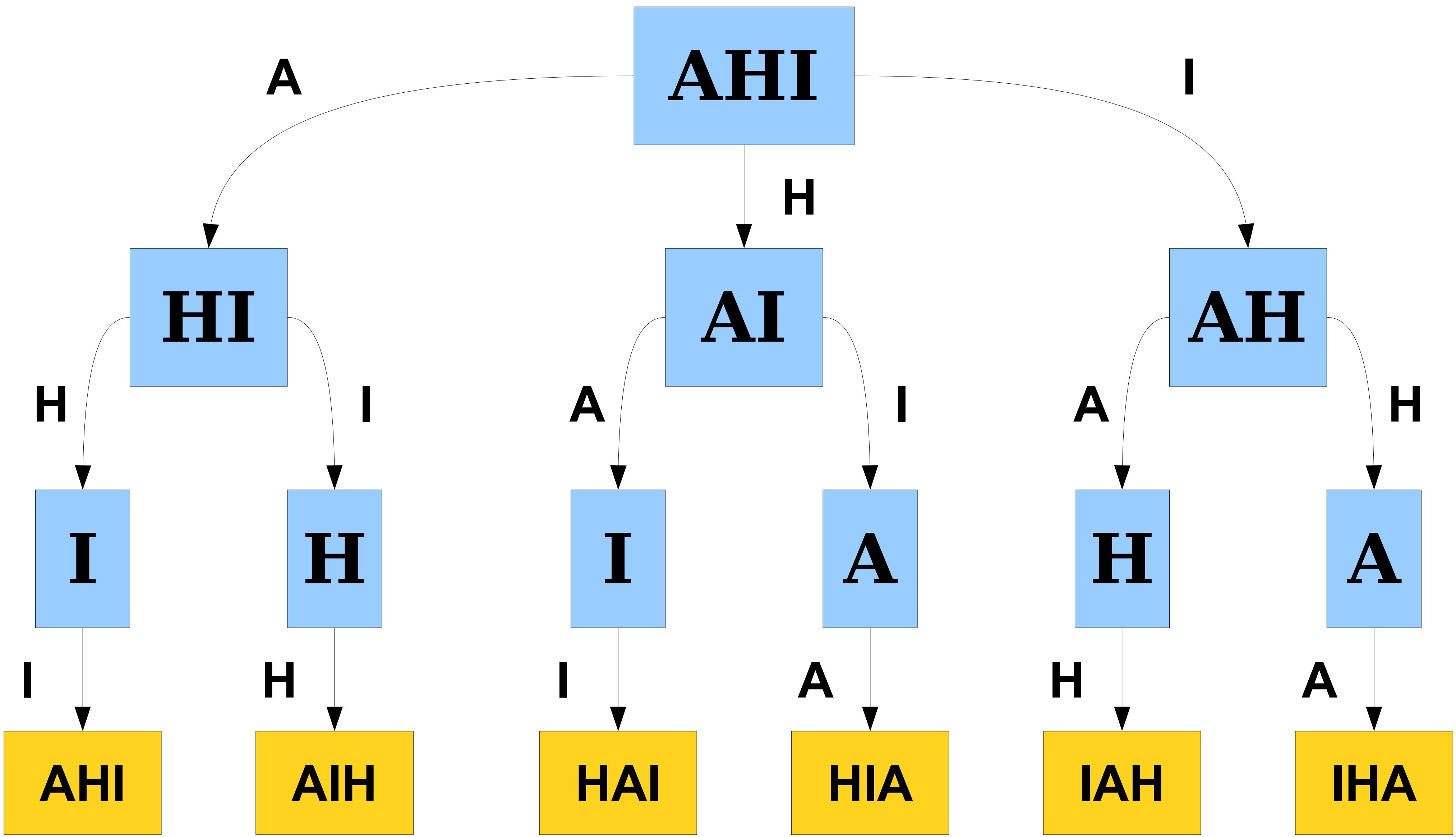
# From Last Time…

Most of the “interesting” exhaustive recursive programs can be reduced to either generating subsets or permutations

# Subset Decision Tree



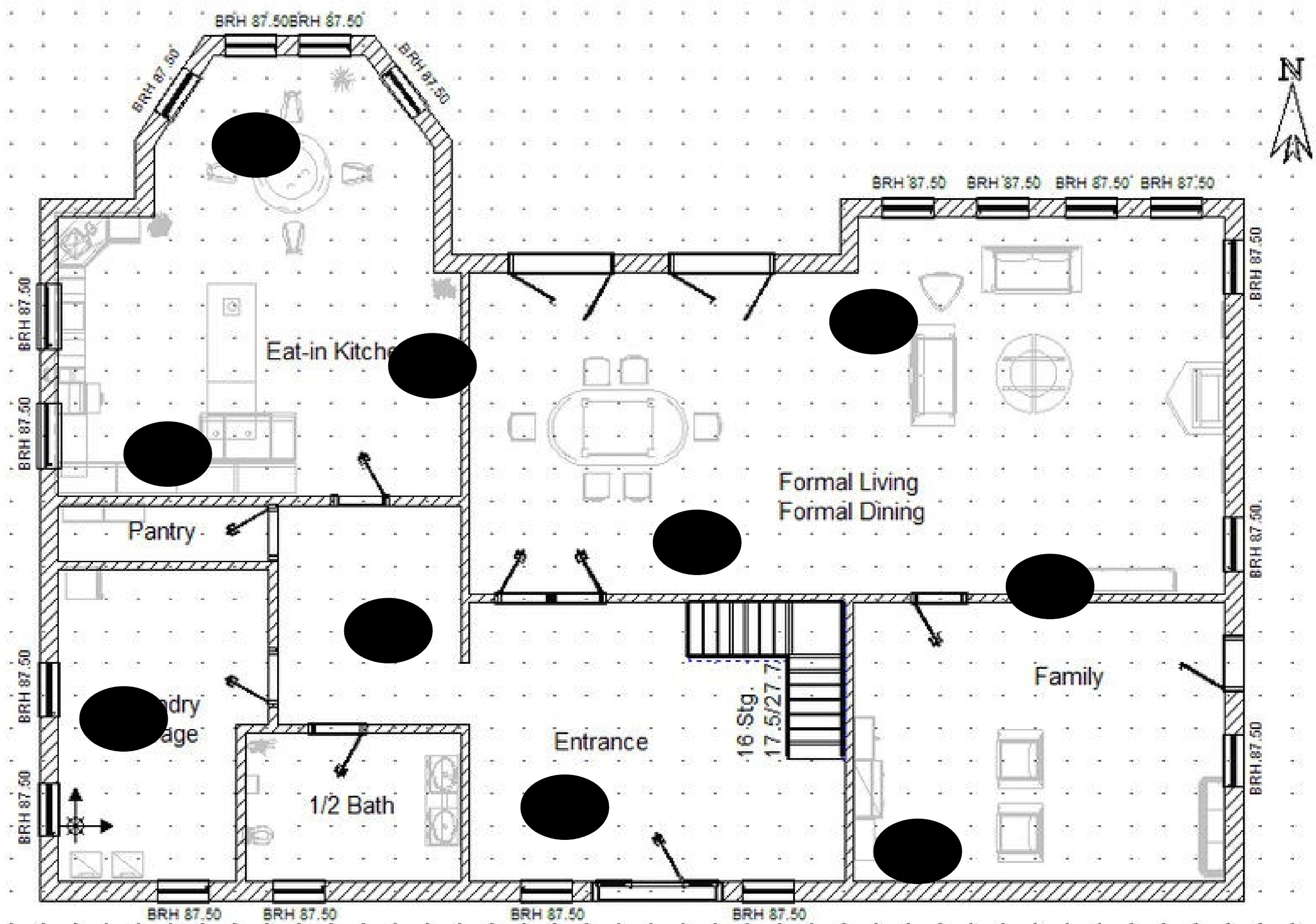
# Permutations Decision Tree



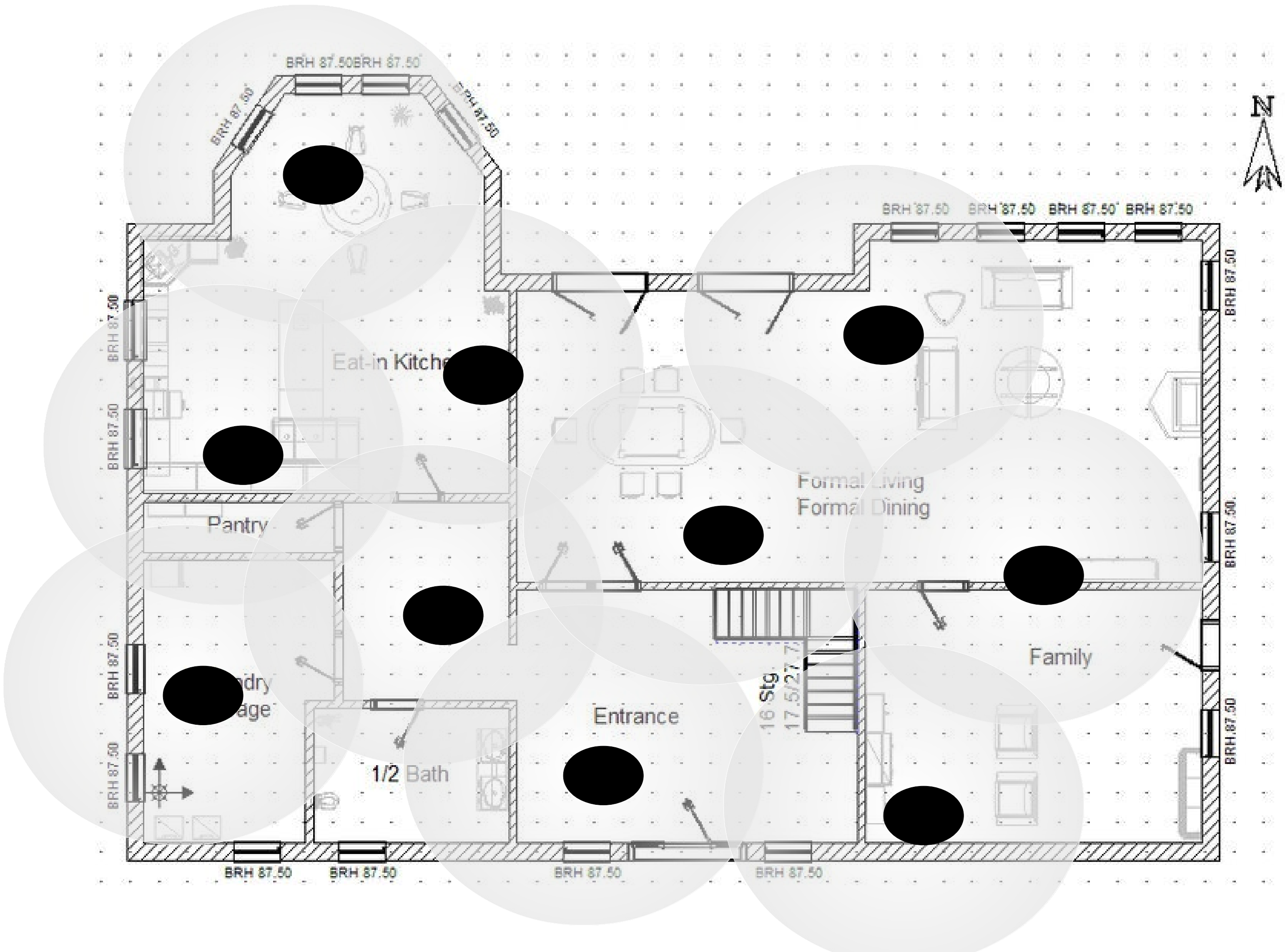
A good first step to solving an exhaustive recursive problem is first determine if it's related to generating subsets or permutations.

# New Stuff…

# Sensor Placement



# Sensor Placement



# Sensor Placement

- Goal is still to maximize covered area
  - New Constraint: Can only pick **k** sensors
- Similar to subset example, **no known** efficient algorithms for solving this problem *perfectly* for arbitrary **k**
- How can we generate all possible choices?

# Generating Combinations

- Suppose that we want to find every way to choose exactly **one** element from a set.
- We could do something like this:

```
foreach (int x in mySet) {
    cout << x << endl;
}
```

# Generating Combinations

- Suppose that we want to find every way to choose exactly **two** elements from a set.
- We could do something like this:

```
foreach (int x in mySet) {
  foreach (int y in mySet) {
    if (x != y) {
      cout << x << ", " << y << endl;
    }
  }
}
```

# Generating Combinations

- Suppose that we want to find every way to choose exactly **three** elements from a set.
- We could do something like this:

```
foreach (int x in mySet) {
  foreach (int y in mySet) {
    foreach (int z in mySet) {
      if (x != y && x != z && y != z) {
         cout << x << ", " << y << ", " << z << endl;
      }
    }
  }
}
```

# Generating Combinations

- If we know how many elements we want in advance, we can always just nest a whole bunch of loops.
- But what if we don't know in advance?

# Pascal's Triangle Revisited

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

# Pascal's Triangle Revisited

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| | | | 1 | | 2 | | 1 | | | |
| | | 1 | | 3 | | 3 | | 1 | | |
| | 1 | | 4 | | 6 | | 4 | | 1 | |
| 1 | | 5 | | 10 | | 10 | | 5 | | 1 |

# Pascal's Triangle Revisited

1

1 1

1 2 1

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# Pascal's Triangle Revisited

(0, 0)

(0, 1) (1, 1)

(0, 2) (1, 2) (2, 2)

(0, 3) (1, 3) (2, 3) (3, 3)

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(0, 5) (1, 5) (2, 5) (3, 5) (4, 5) (5, 5)

# Pascal's Triangle Revisited

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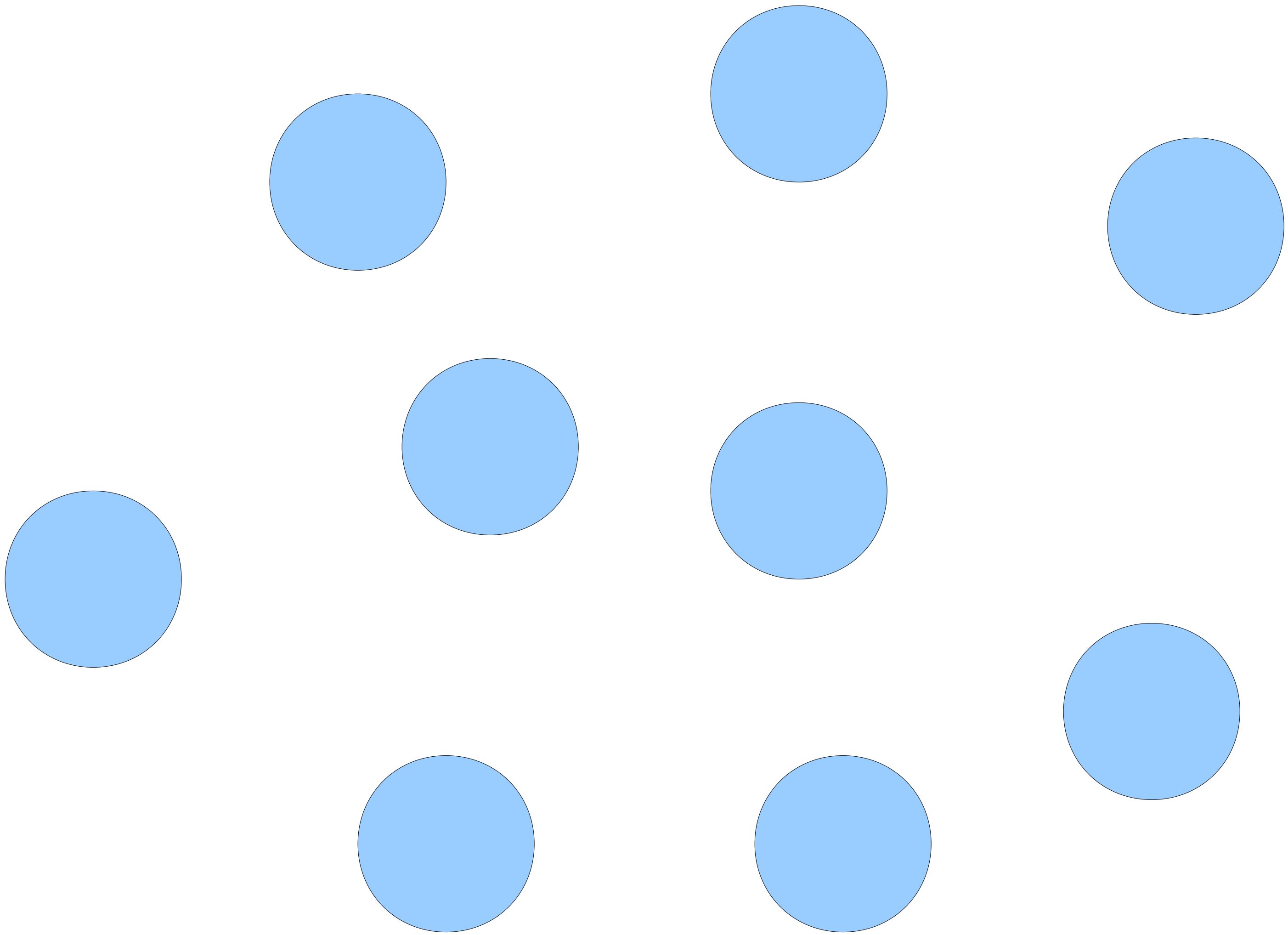
(0, 3) (1, 3) (2, 3) (3, 3)

(0, 4) (1, 4) (2, 4) (3, 4) (4, 4)

(0, 5) (1, 5) (2, 5) (3, 5) (4, 5) (5, 5)

What's up with that?

# Generating Combinations



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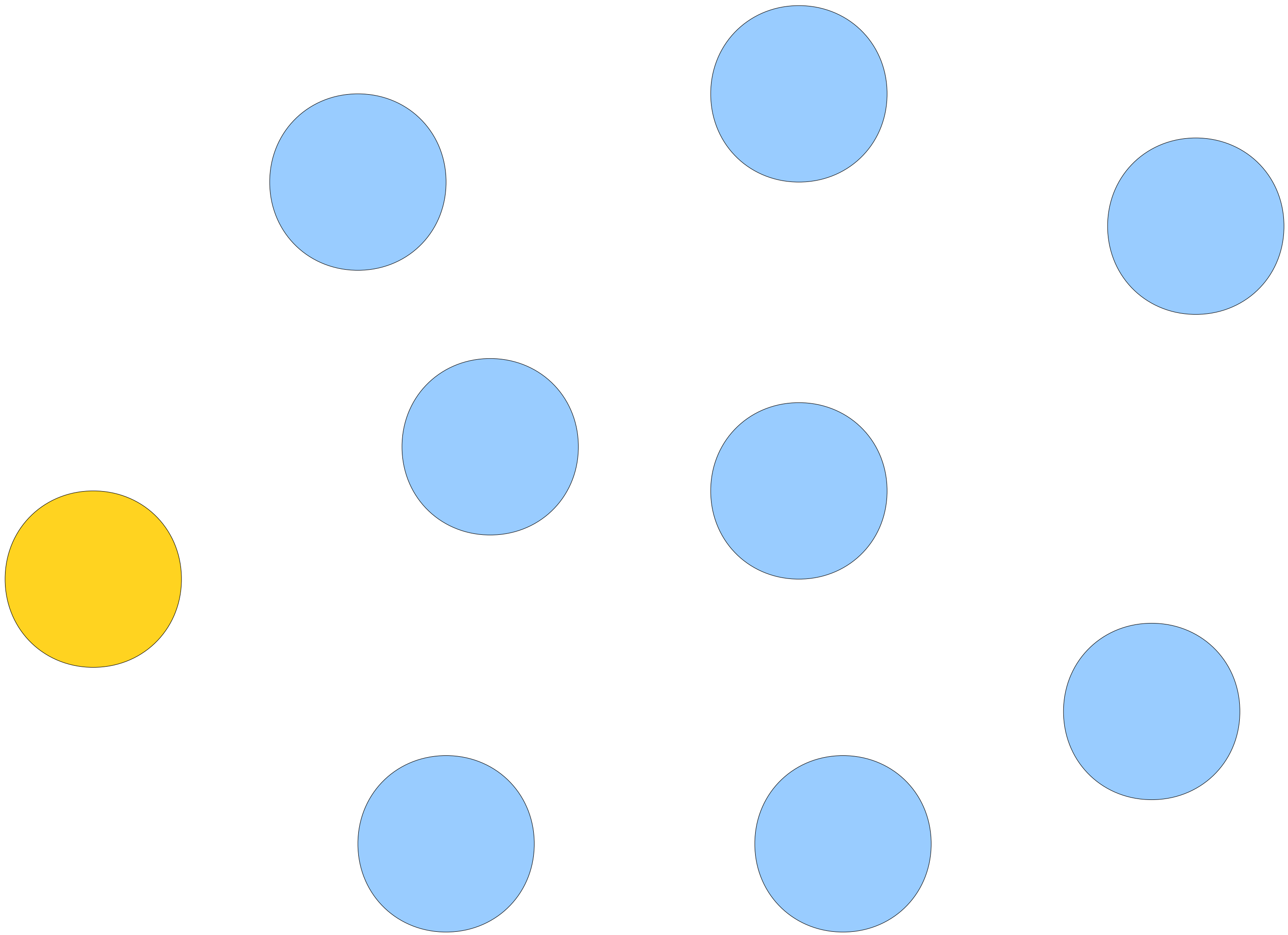
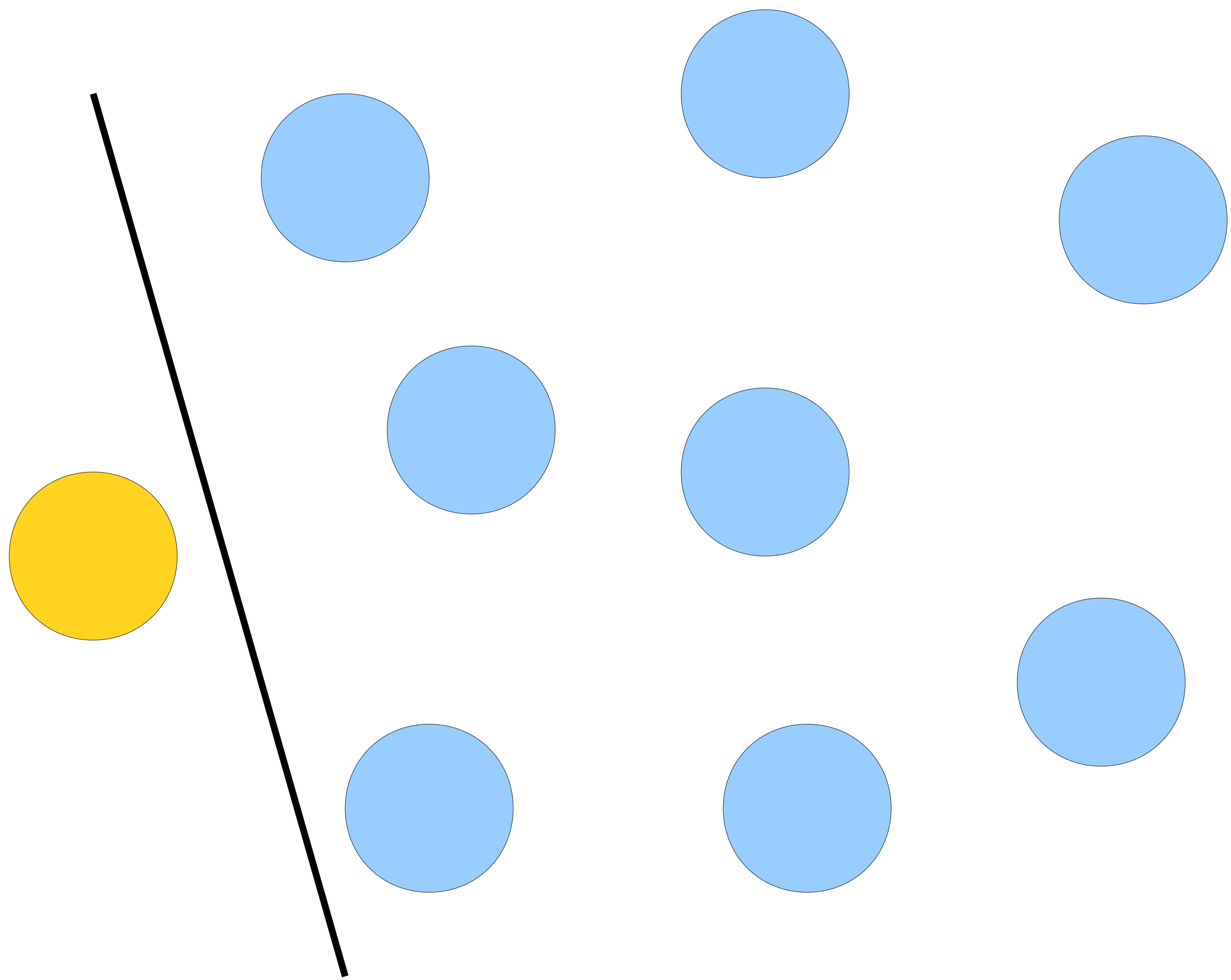


# Generating Combinations



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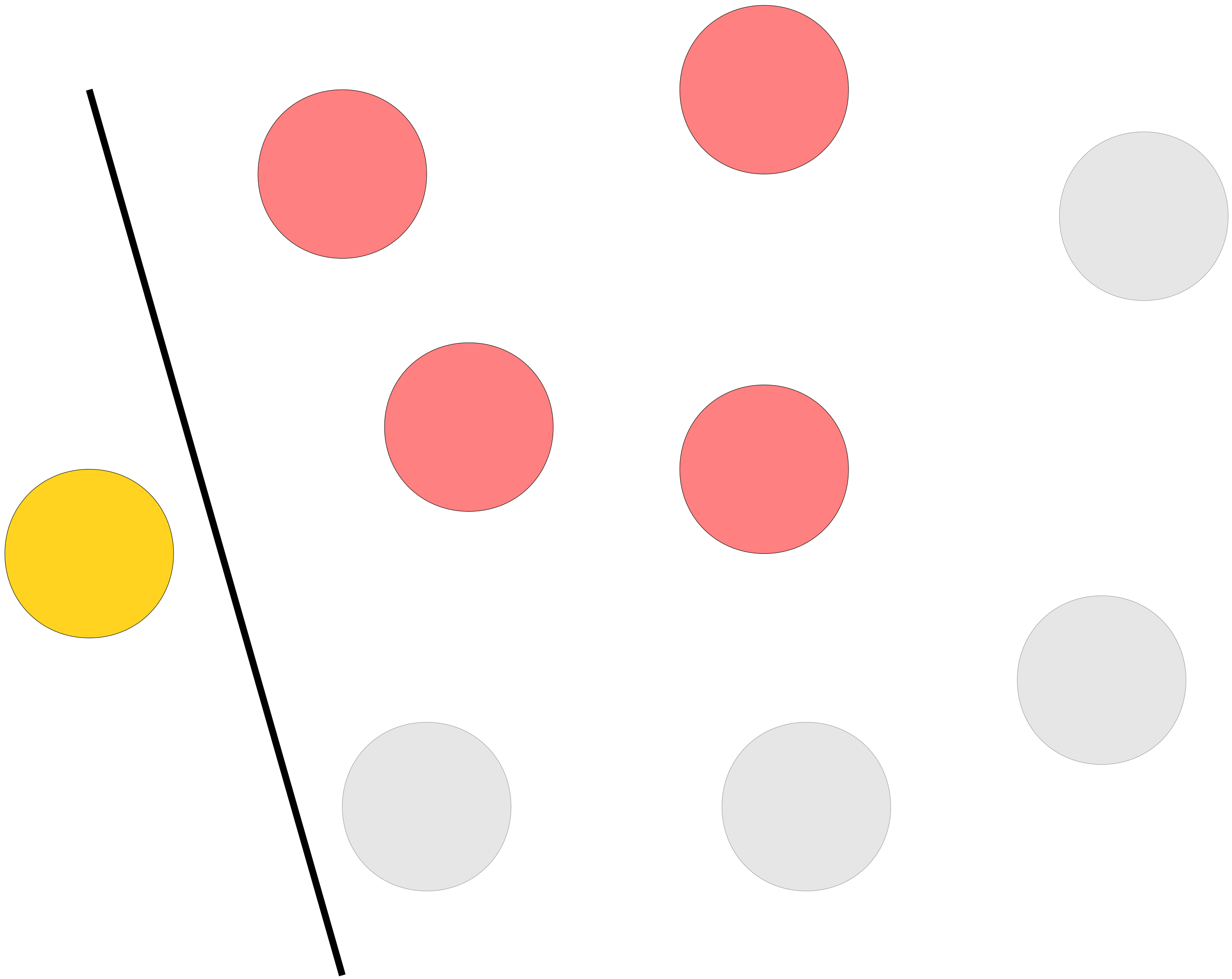

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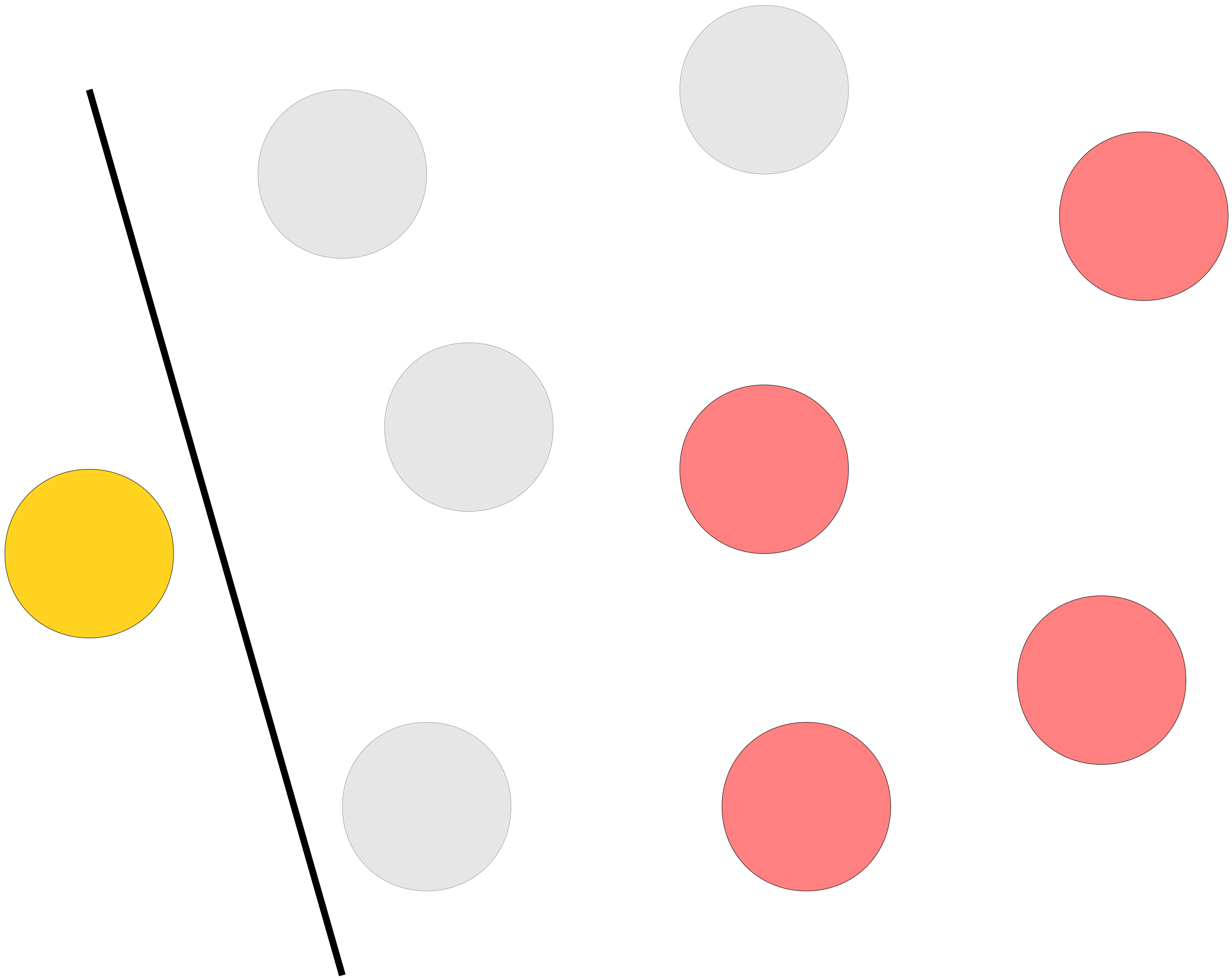

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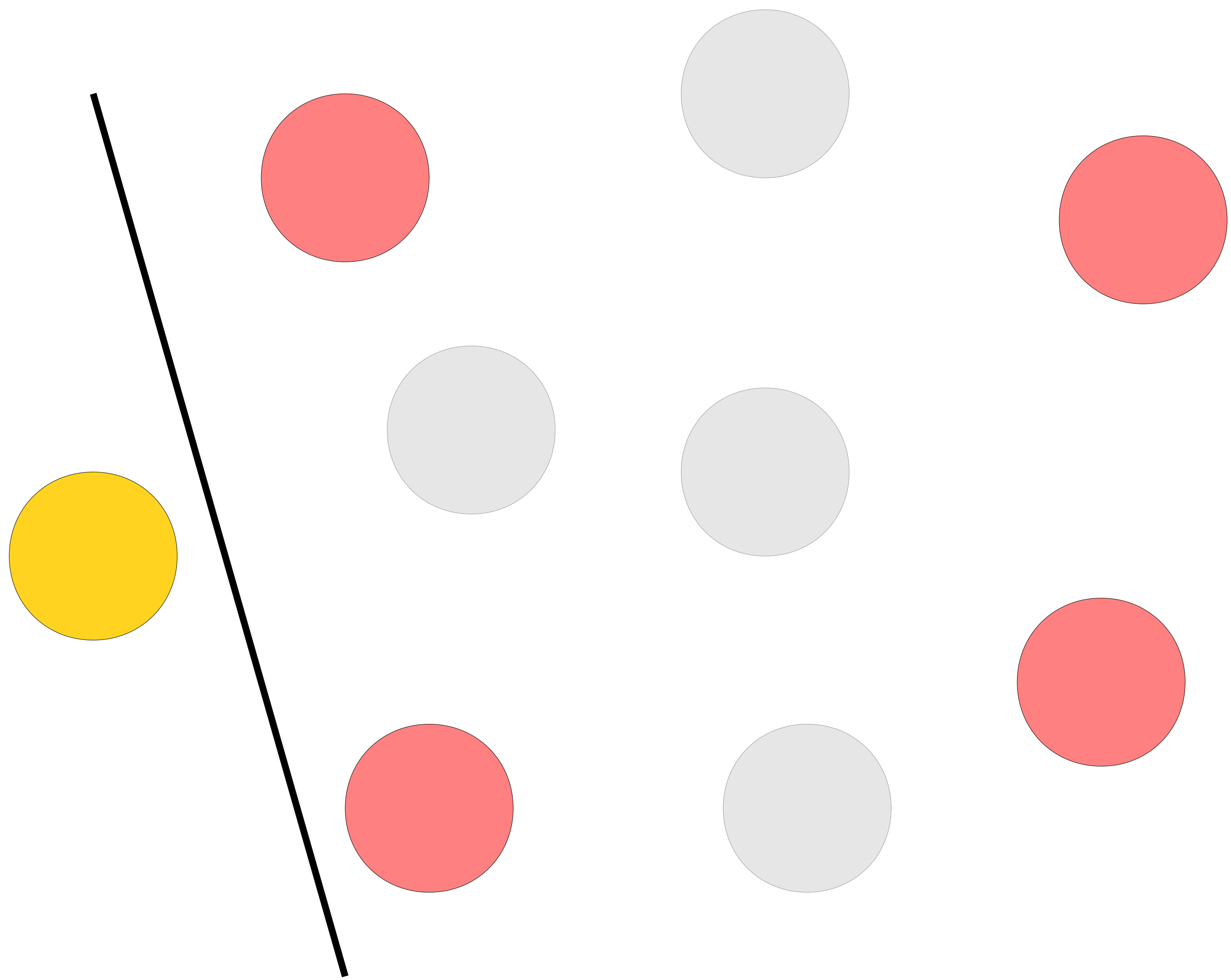

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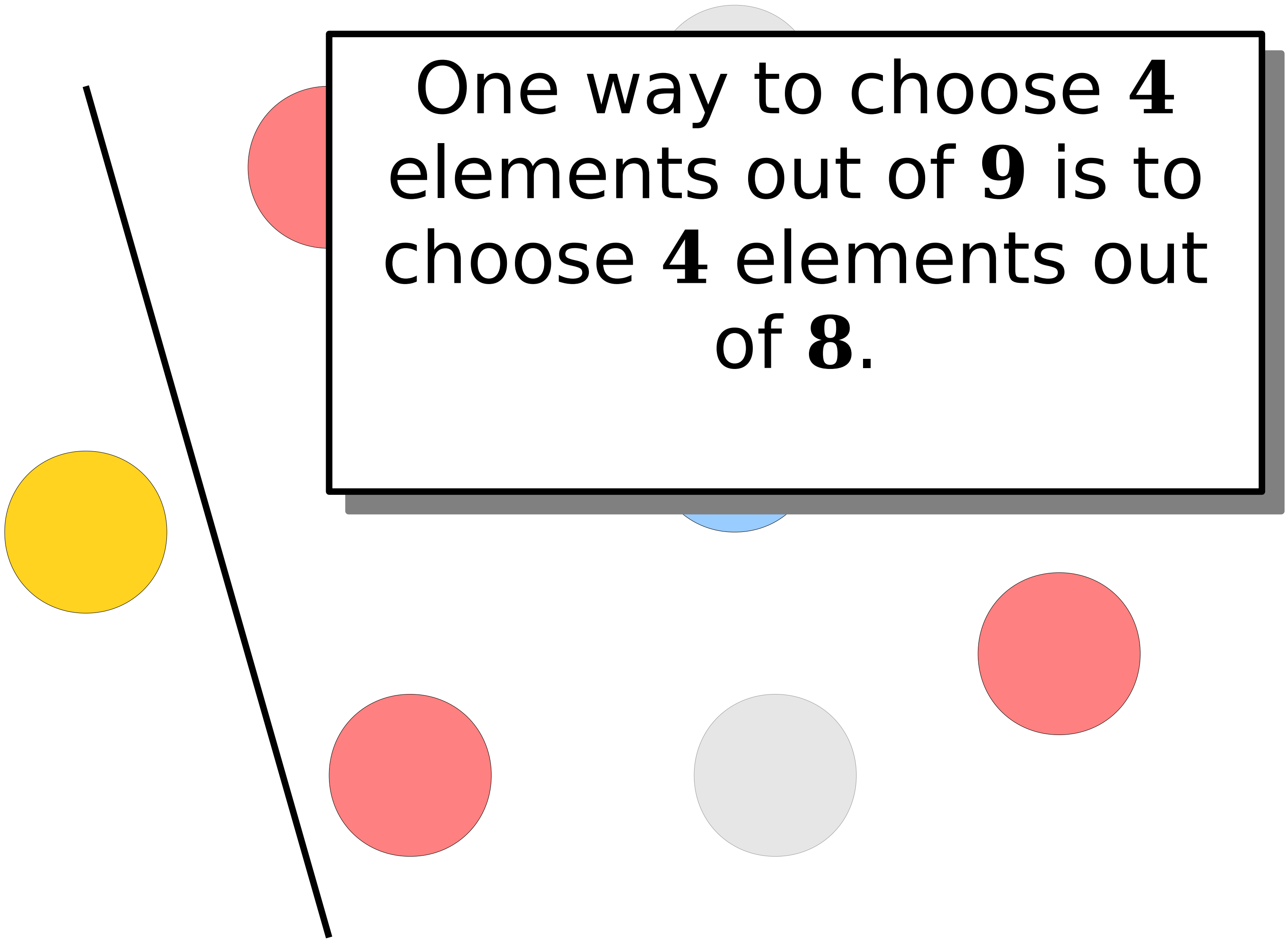

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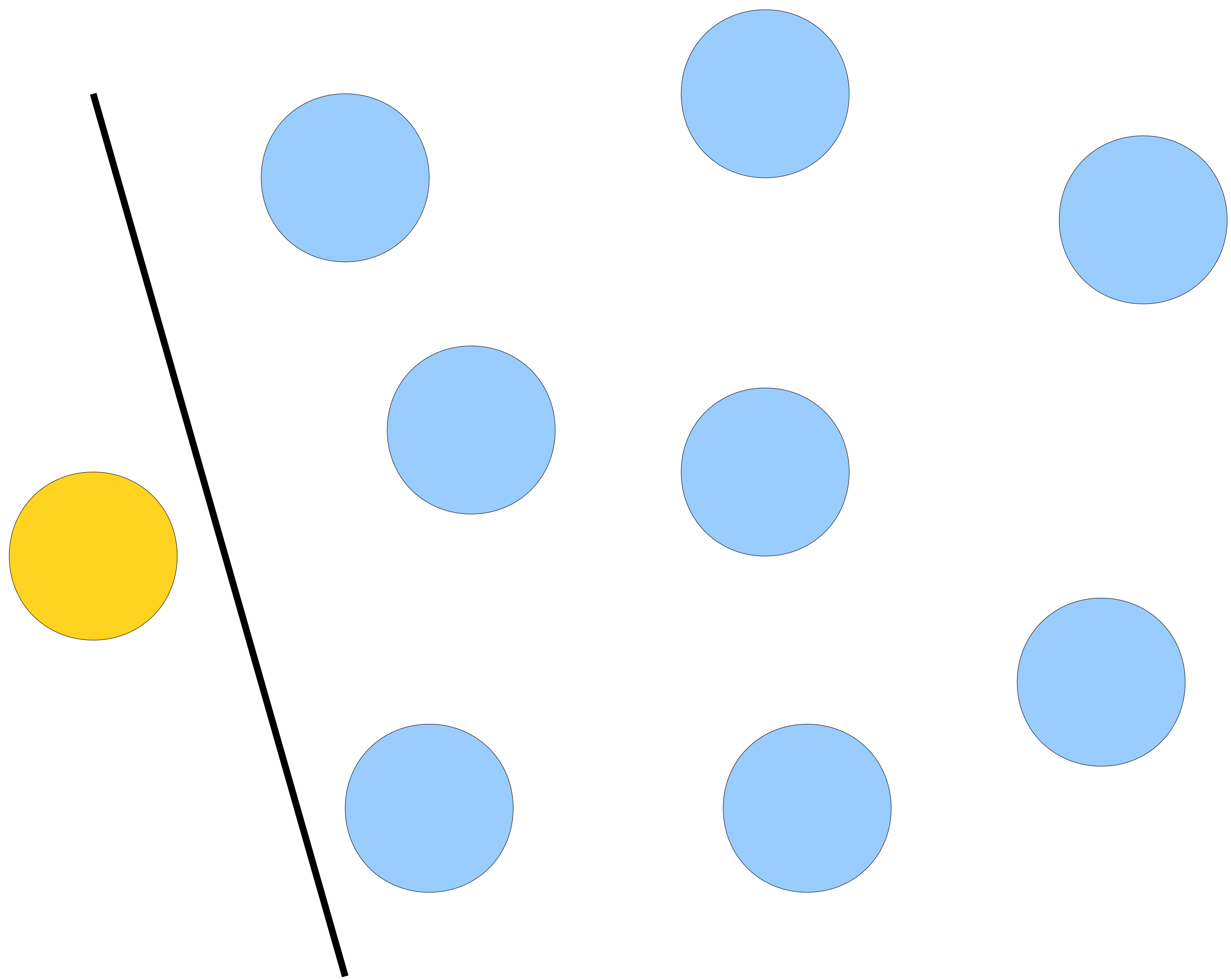

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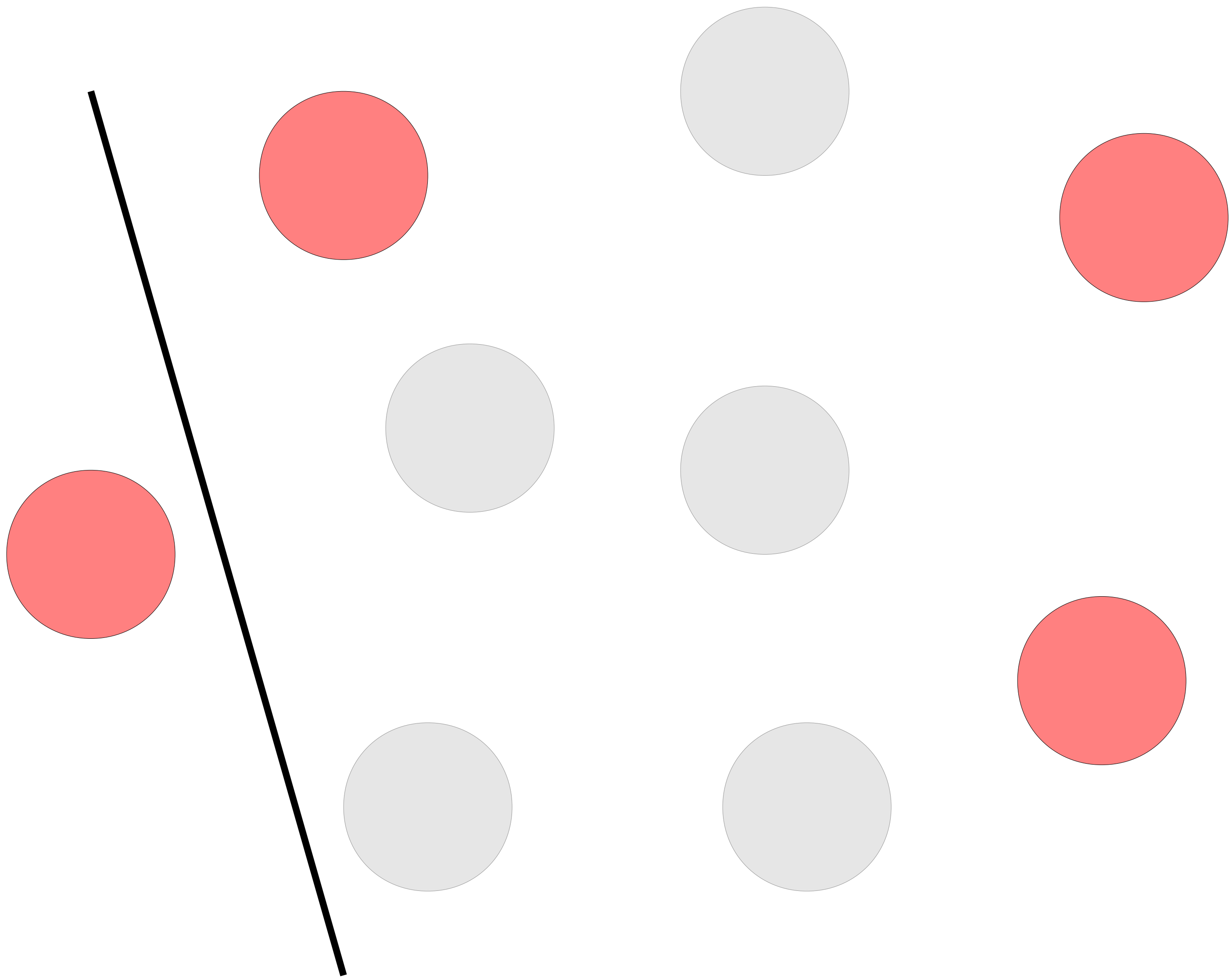

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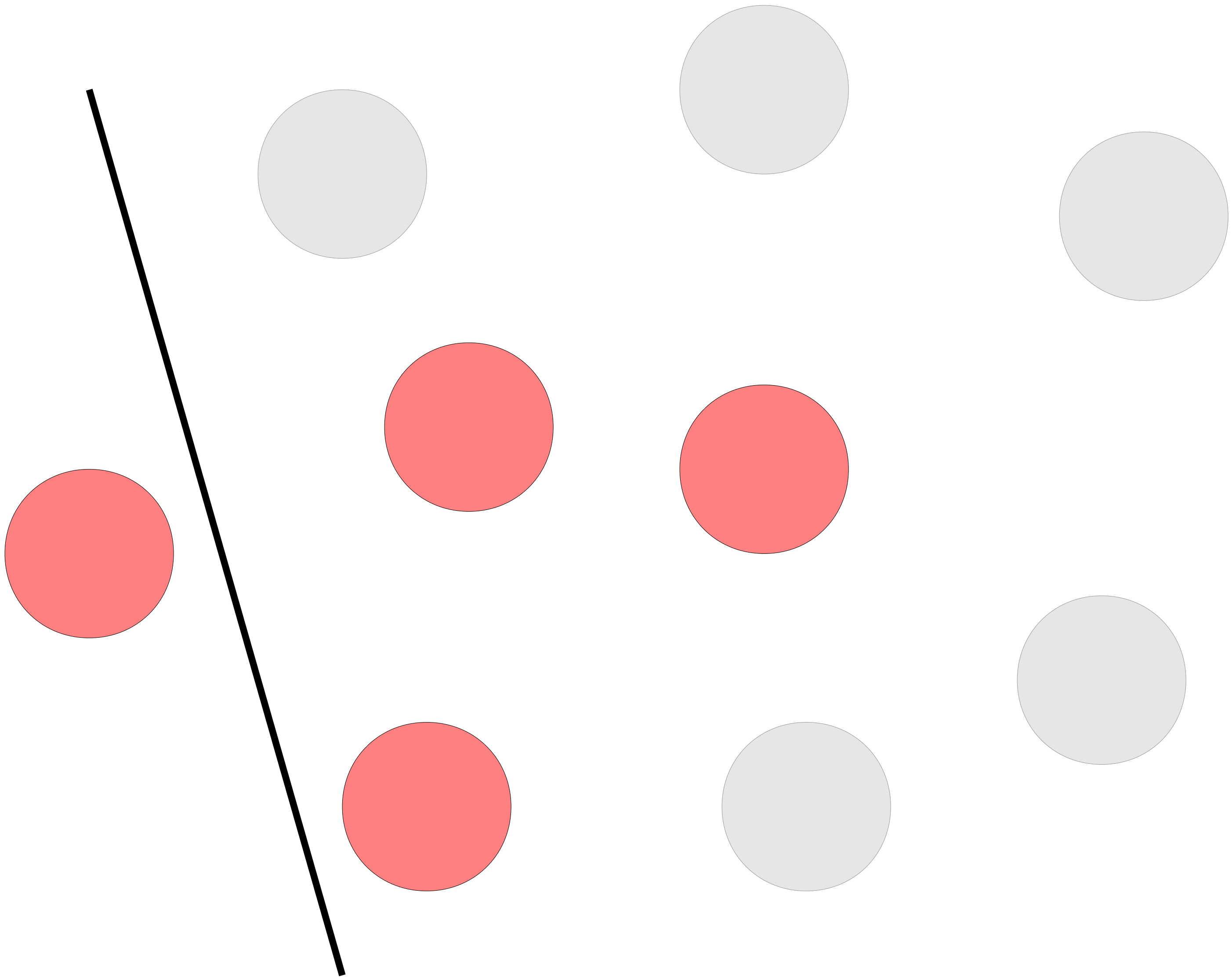

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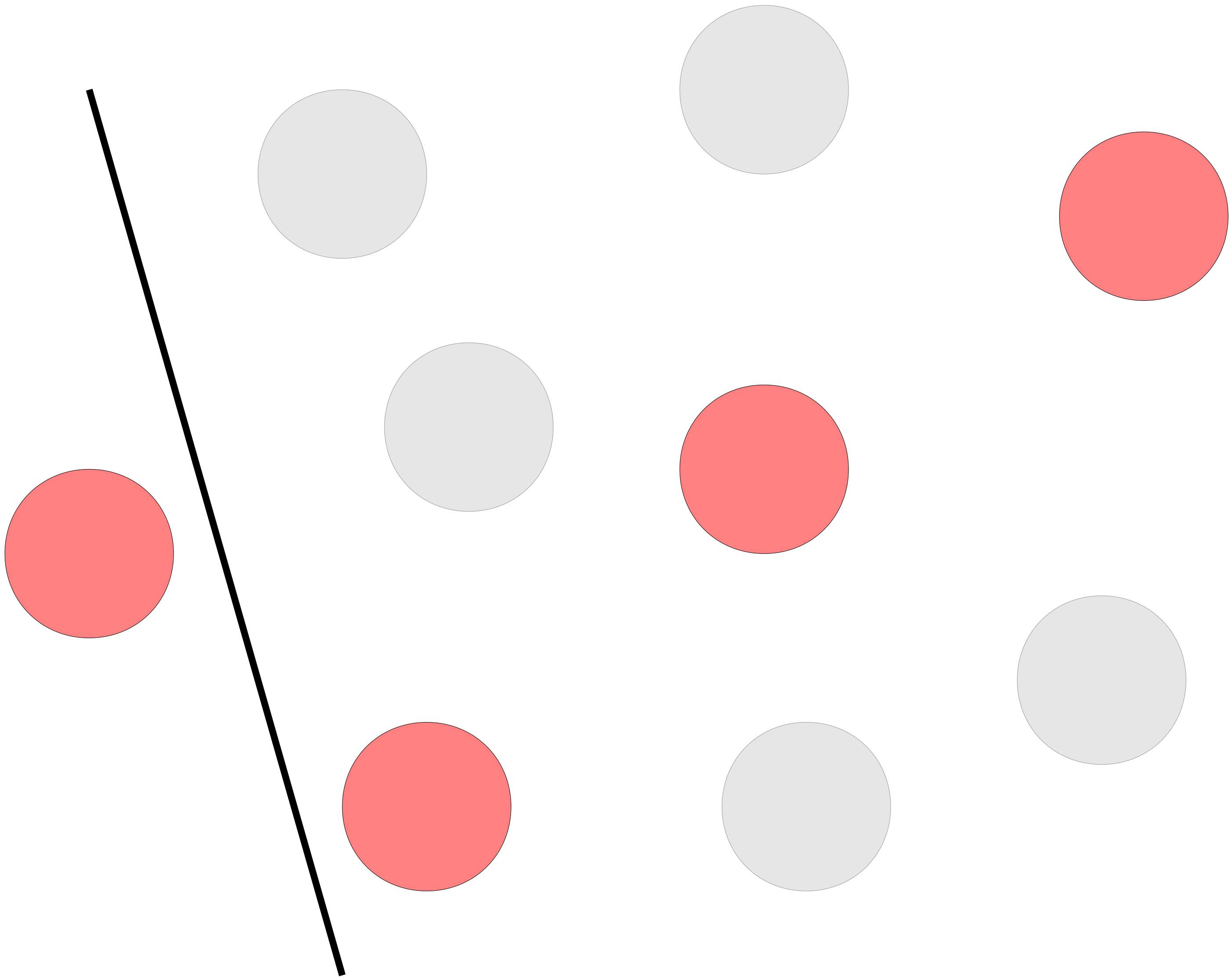

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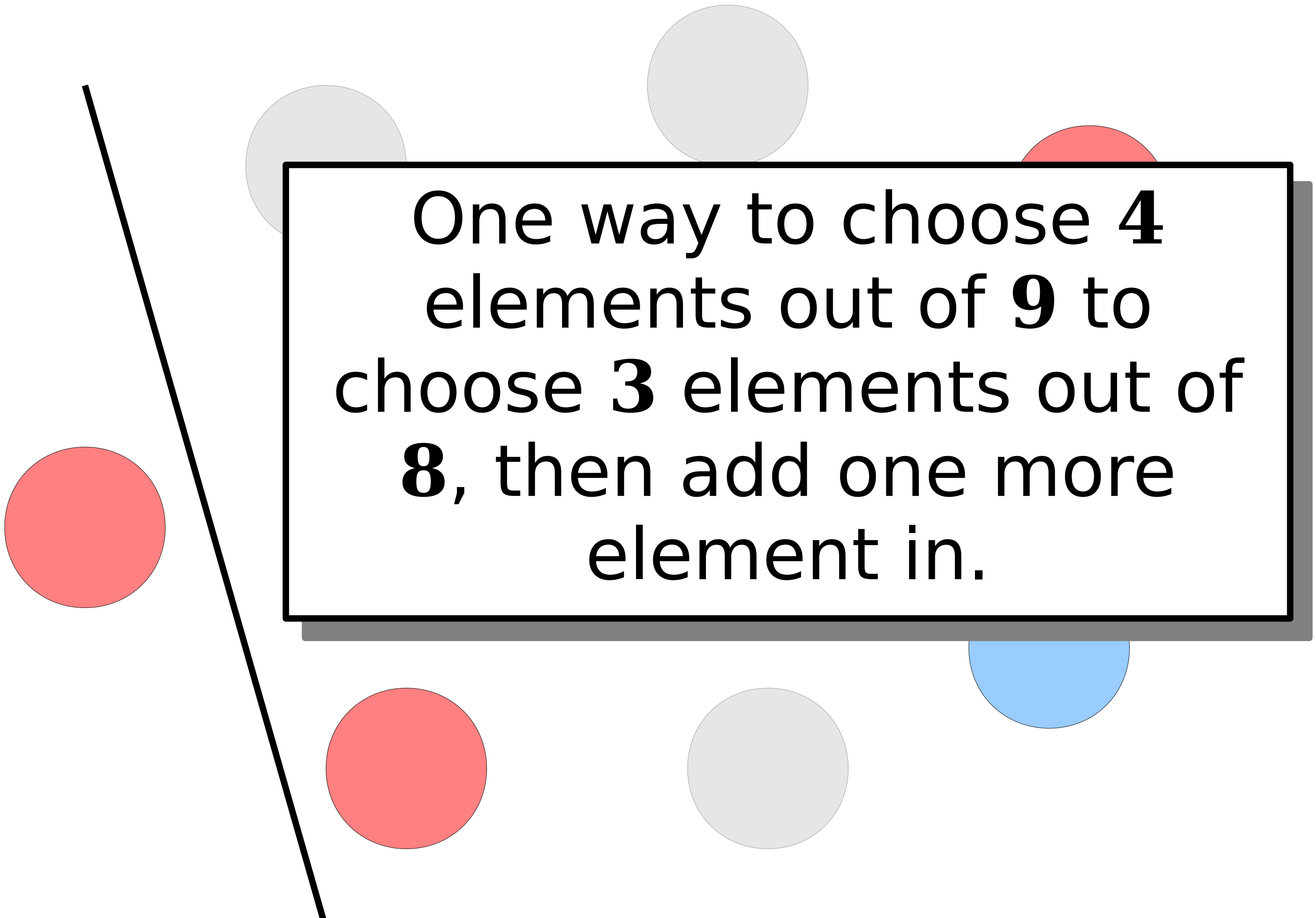

One way to choose **4** elements out of **9** to choose **3** elements out of **8**, then add one more element in.

# Pascal's Triangle Revisited

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| | | | 1 | | 2 | | 1 | | | |
| | | 1 | | 3 | | 3 | | 1 | | |
| | 1 | | 4 | | 6 | | 4 | | 1 | |
| 1 | | 5 | | 10 | | 10 | | 5 | | 1 |

# Pascal's Triangle Revisited

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1 | | | | | |
| | | | | 1 | | 1 | | | | |
| | | | 1 | | 2 | | 1 | | | |
| | | 1 | | 3 | | 3 | | 1 | | |
| | 1 | | 4 | | 6 | | 4 | | 1 | |
| 1 | | 5 | | 10 | | 10 | | 5 | | 1 |

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(0, 0)

(0, 1) (1, 1)

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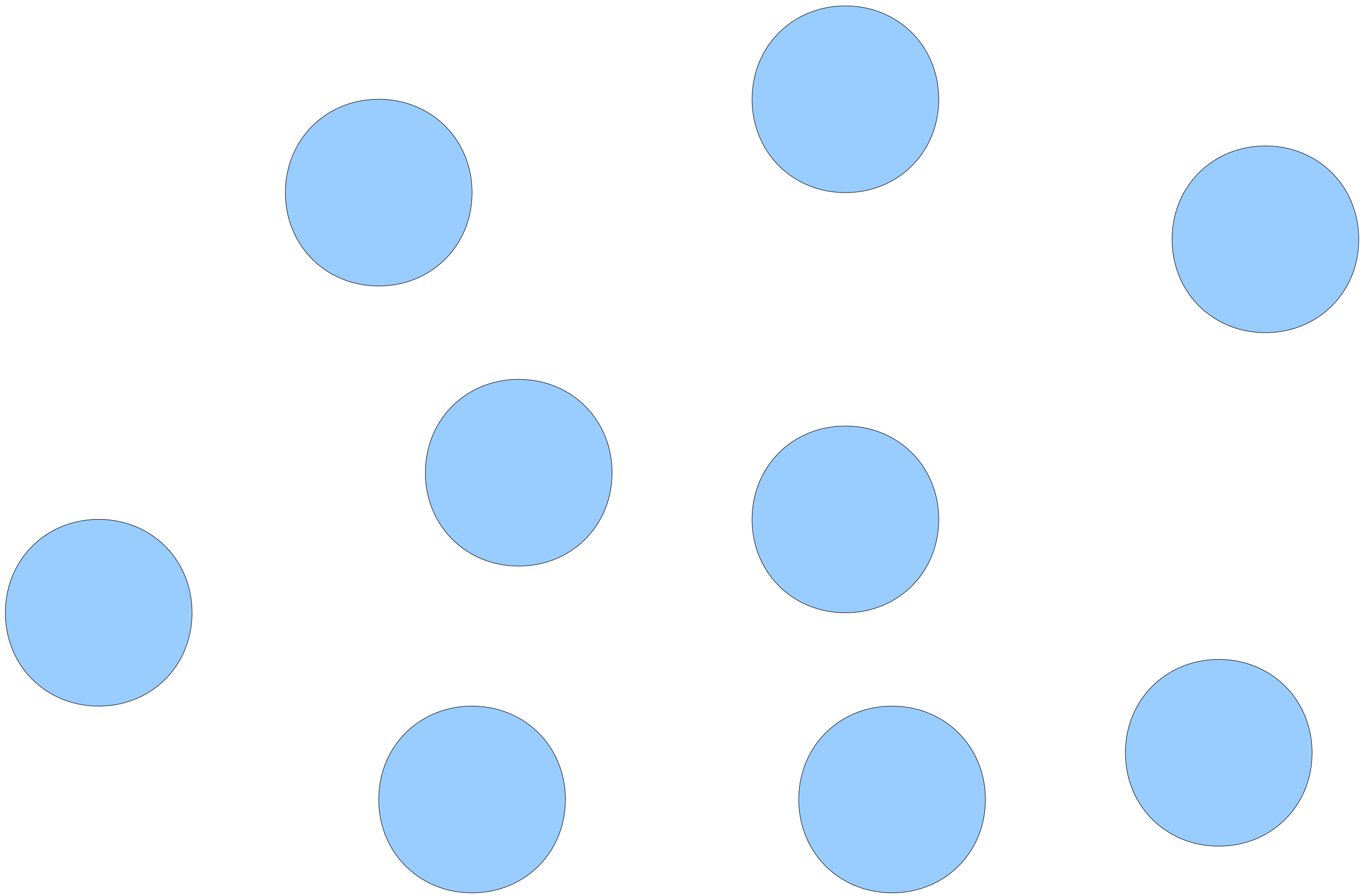
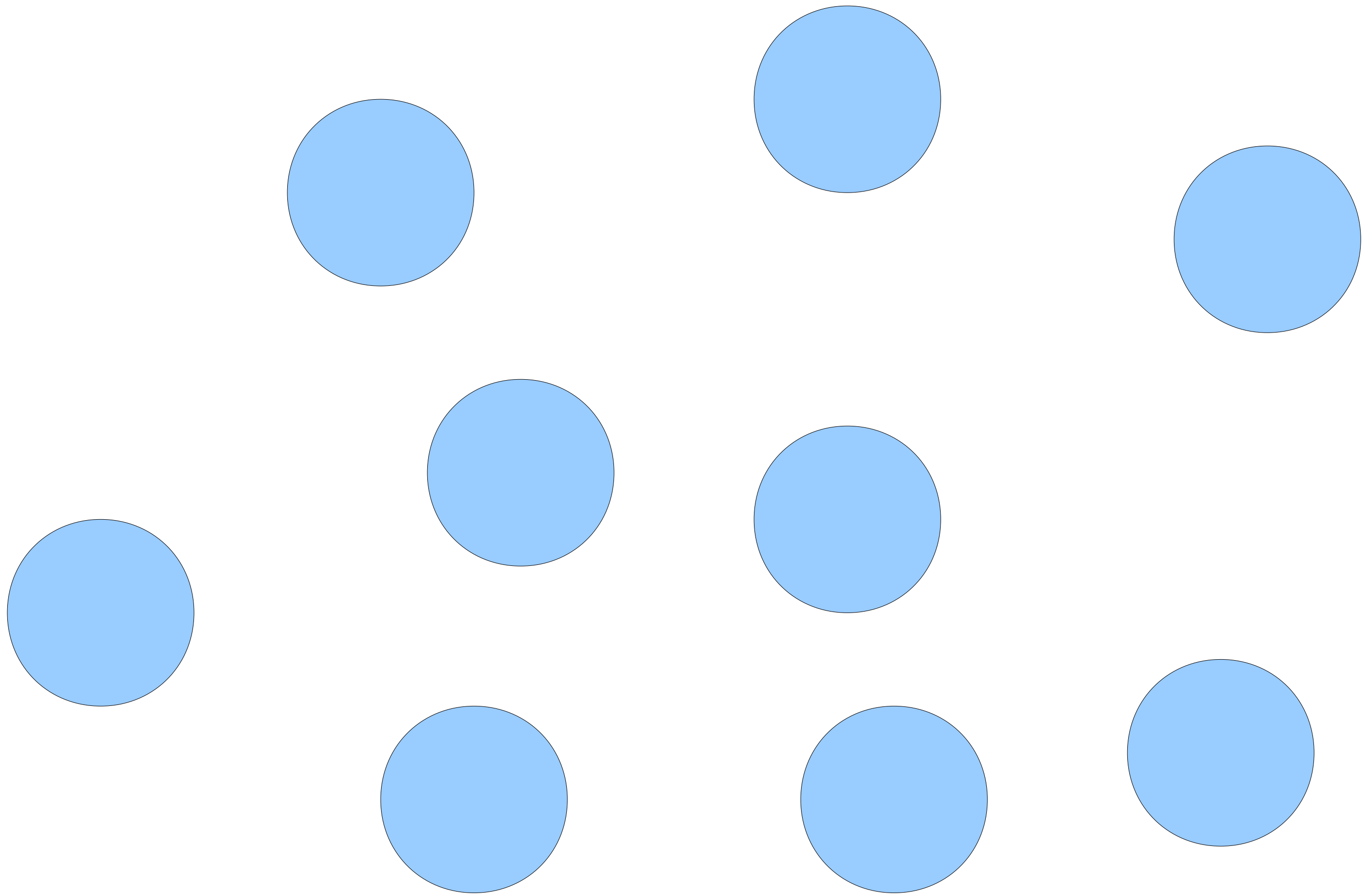
(0, 4) (1, 4) (2, 4) (3, 4) (4, 4)

(0, 5) (1, 5) (2, 5) (3, 5) (4, 5) (5, 5)

# Generating Combinations

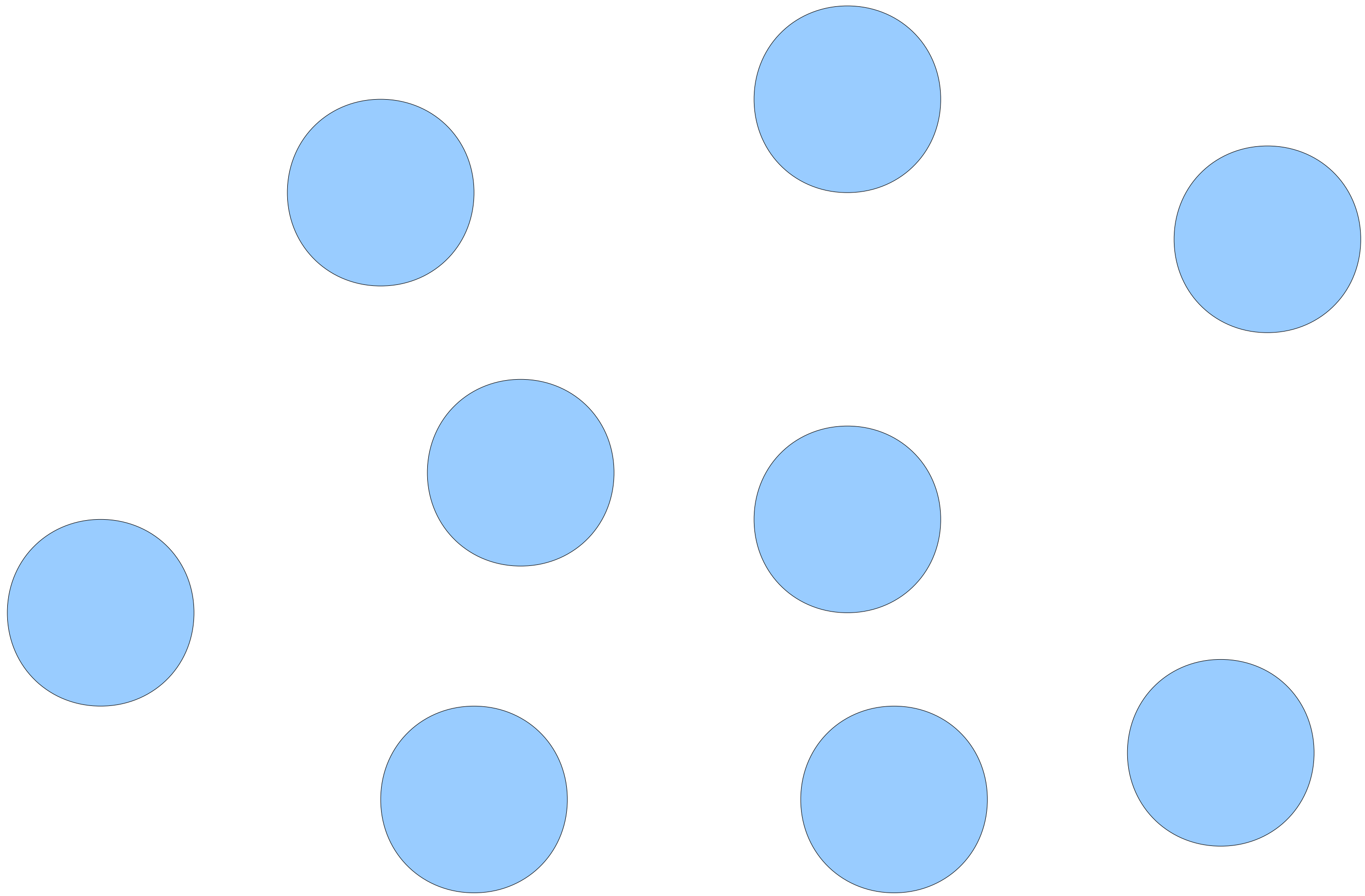


# Generating Combinations



How many ways are there to pick **0** things from this set?

# Generating Combinations



How many ways are there to pick **100** things from this set?

# `combinations` (Pseudocode)

# Combinations, Recursively

- How to pick $k$ elements from a set?
- **Base Cases:**
  - If $k$ is 0, the only option is to pick the empty set.
  - Otherwise, if $k$ is greater than the number of elements of the set, there are no options.
- **Recursive Step:**
  - Pick some element $x$ from the set.
  - Find all ways of picking $k$ elements of what remains.
  - Find all ways of picking $k$ – 1 elements of what remains, then add $x$ back in.

`combinations.cpp`

(Computer)

# Combinations

- Even though a combination is a different mathematical structure, generating combinations is nearly identical to generating subsets
  - All we needed to add was an extra parameter and an extra base case.

# A Little Word Puzzle

“What nine-letter word can be reduced to a single-letter word one letter at a time by removing letters, leaving it a legal word at each step?”

# The Startling Truth

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# The Startling Truth

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# The Startling Truth

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# The Startling Truth

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# The Startling Truth

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# The Startling Truth

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# The Startling Truth

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# The Startling Truth

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# The Startling Truth

I

Is there **really** just one nine-letter word with this property?

# Shrinkable Words

- Let's define a **shrinkable word** as a word that can be reduced down to one letter by removing one character at a time, leaving a word at each step.
- **Base Cases:**
  - Any string that is not a word cannot be a shrinkable word.
  - Any single-letter word is shrinkable.
    - A, I, O
- **Recursive Step:**
  - Any multi-letter word is shrinkable if you can remove a letter to form a shrinkable word.

# `shrinkable-words.cpp`
# (Pseudocode)

# `shrinkable-words.cpp`
# (Computer)

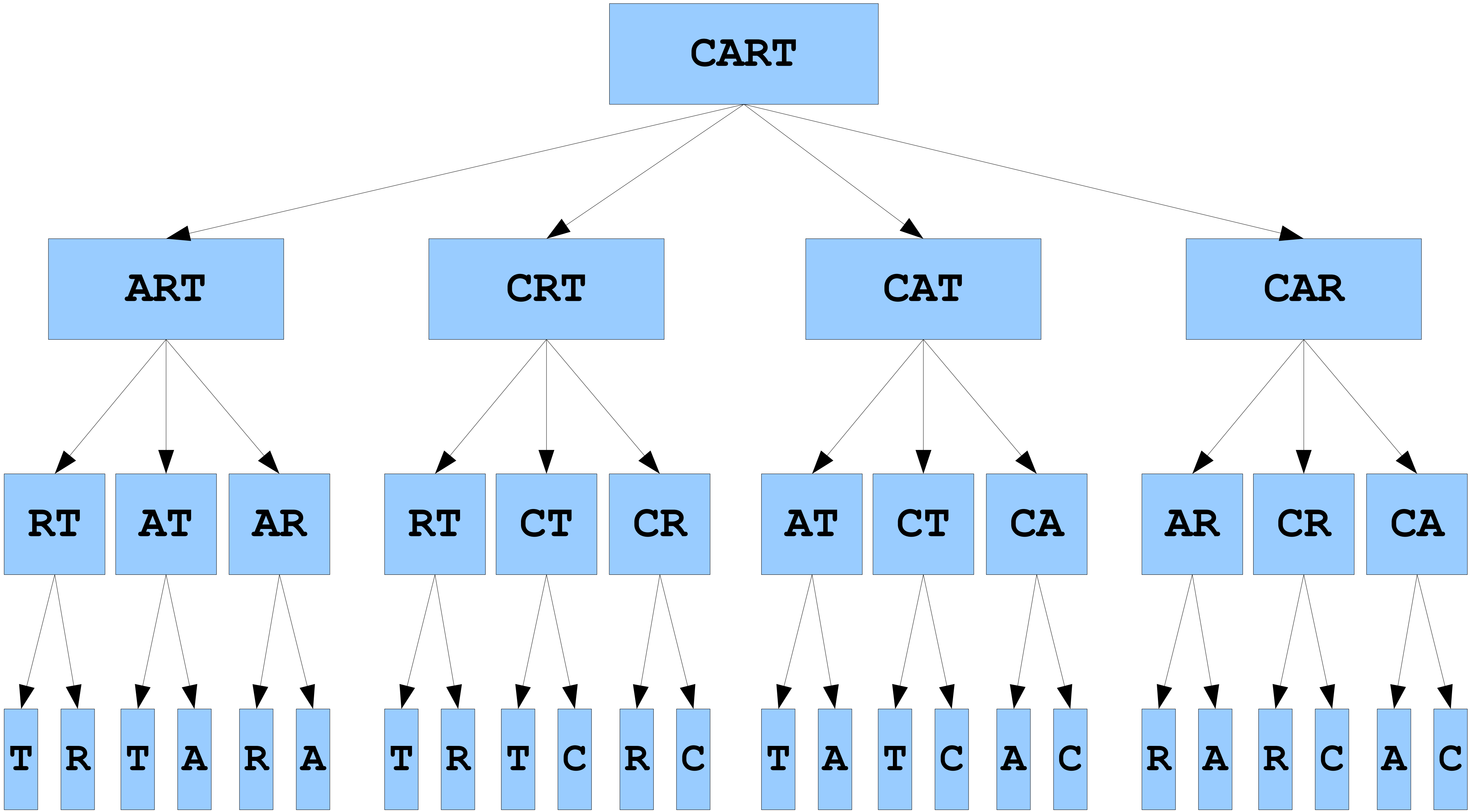
# Recursive Backtracking

- The function we have just written is an example of **recursive backtracking**.
- At each step, we try one of many possible options.
- If *any* option succeeds, that's great! We're done.
- If *none* of the options succeed, then this particular problem can't be solved.
- In recursive backtracking we care about finding “one thing” instead of “generating all things”

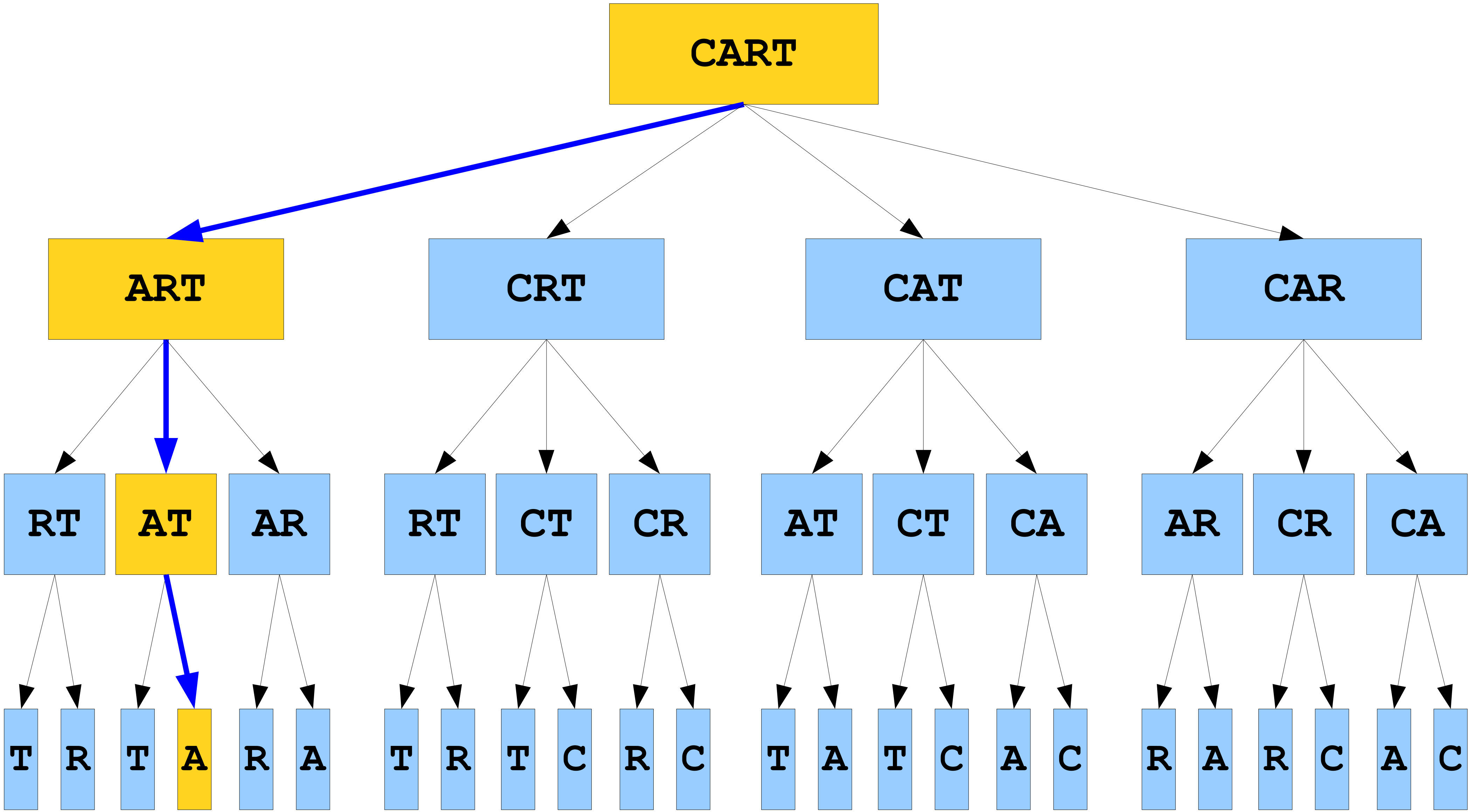
# Recursive Backtracking

- I claimed that most exhaustive recursive problems can be reduced to generating permutations or subsets.
- Is shrinkable words a subsets or permutations problem?
  - Like permutations, we are computing an ordering: the order in which we remove characters.
  - Instead of *adding* characters to a string we are *removing* characters from a string.

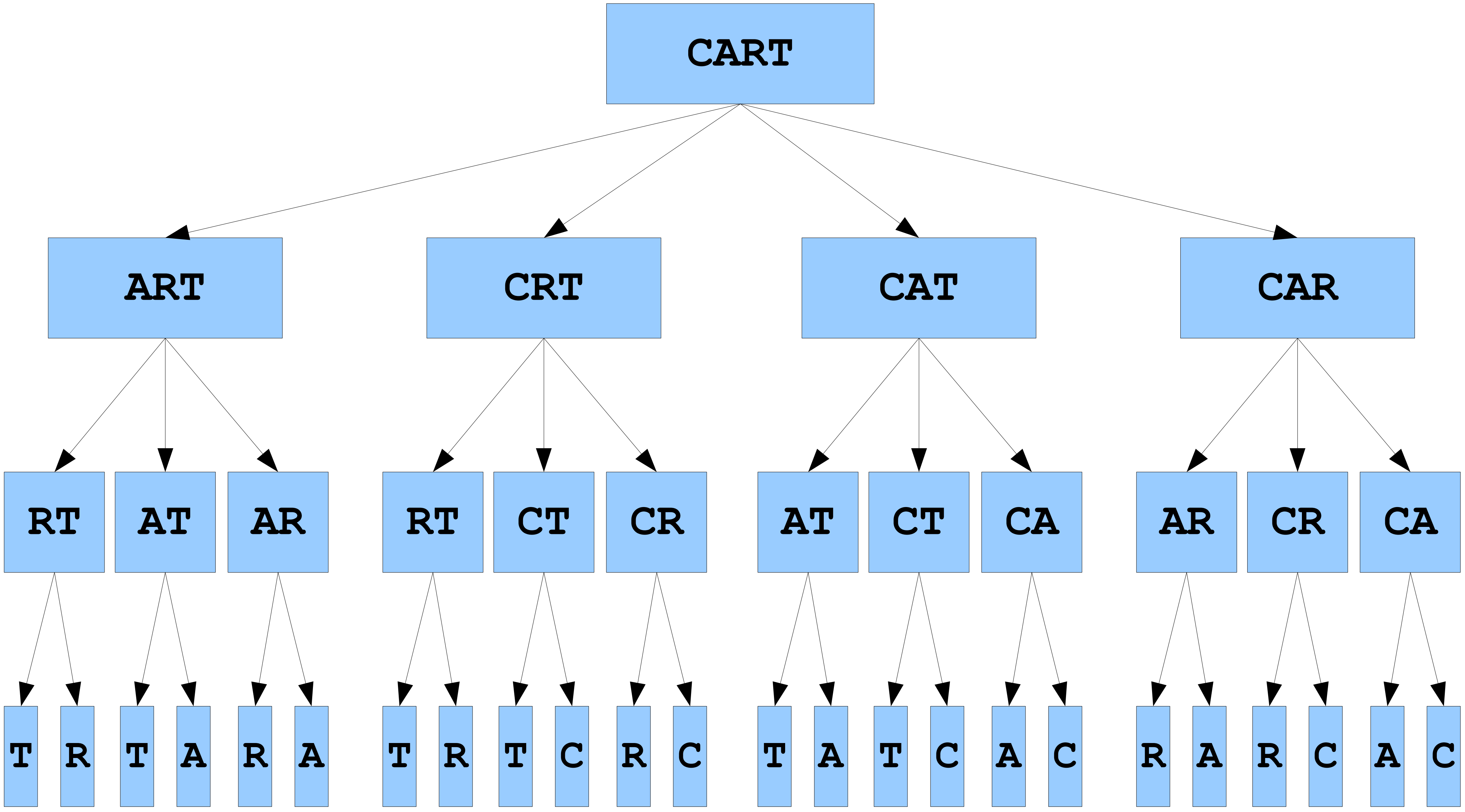
# Decision Tree



# Decision Tree



# Recursive Backtracking



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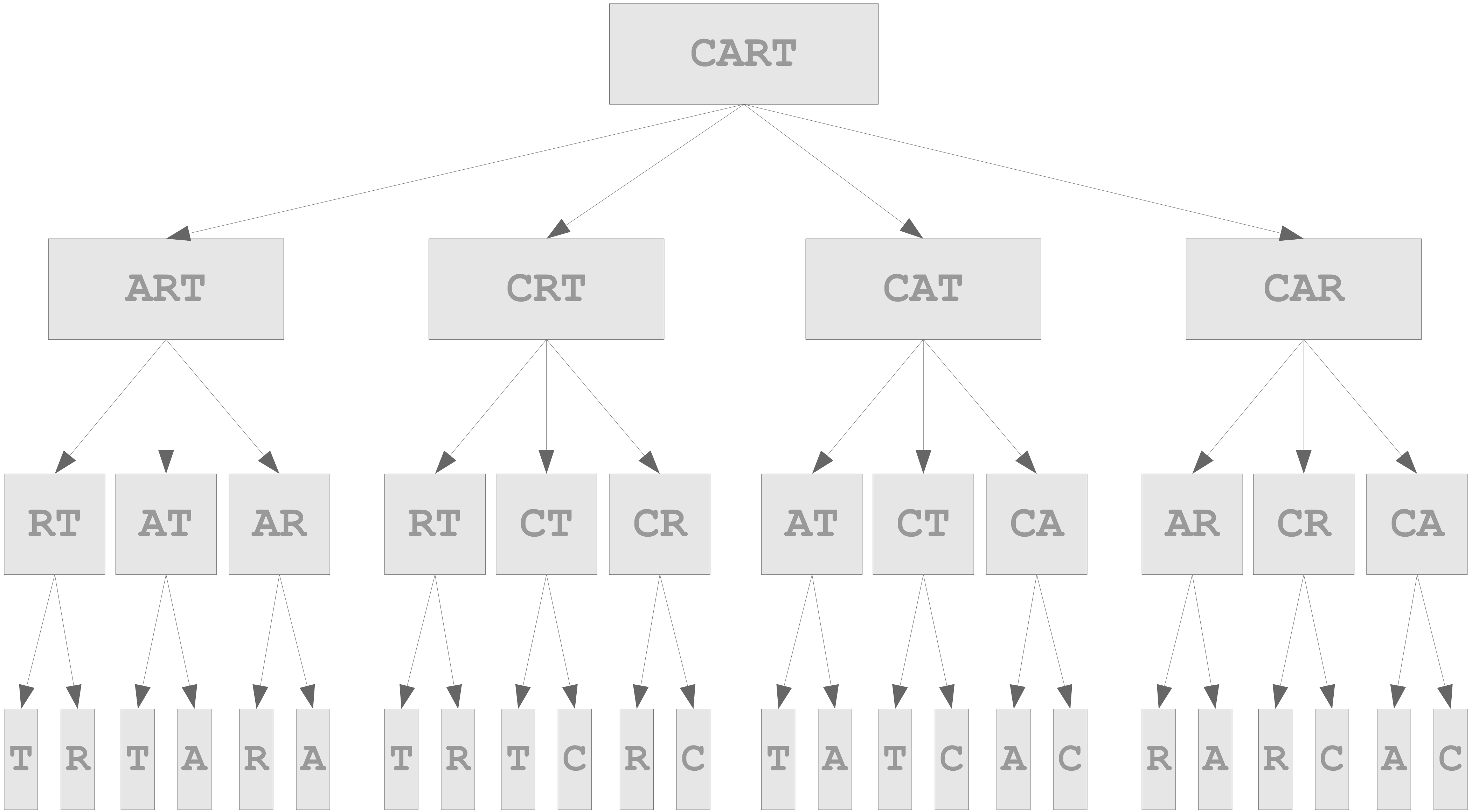

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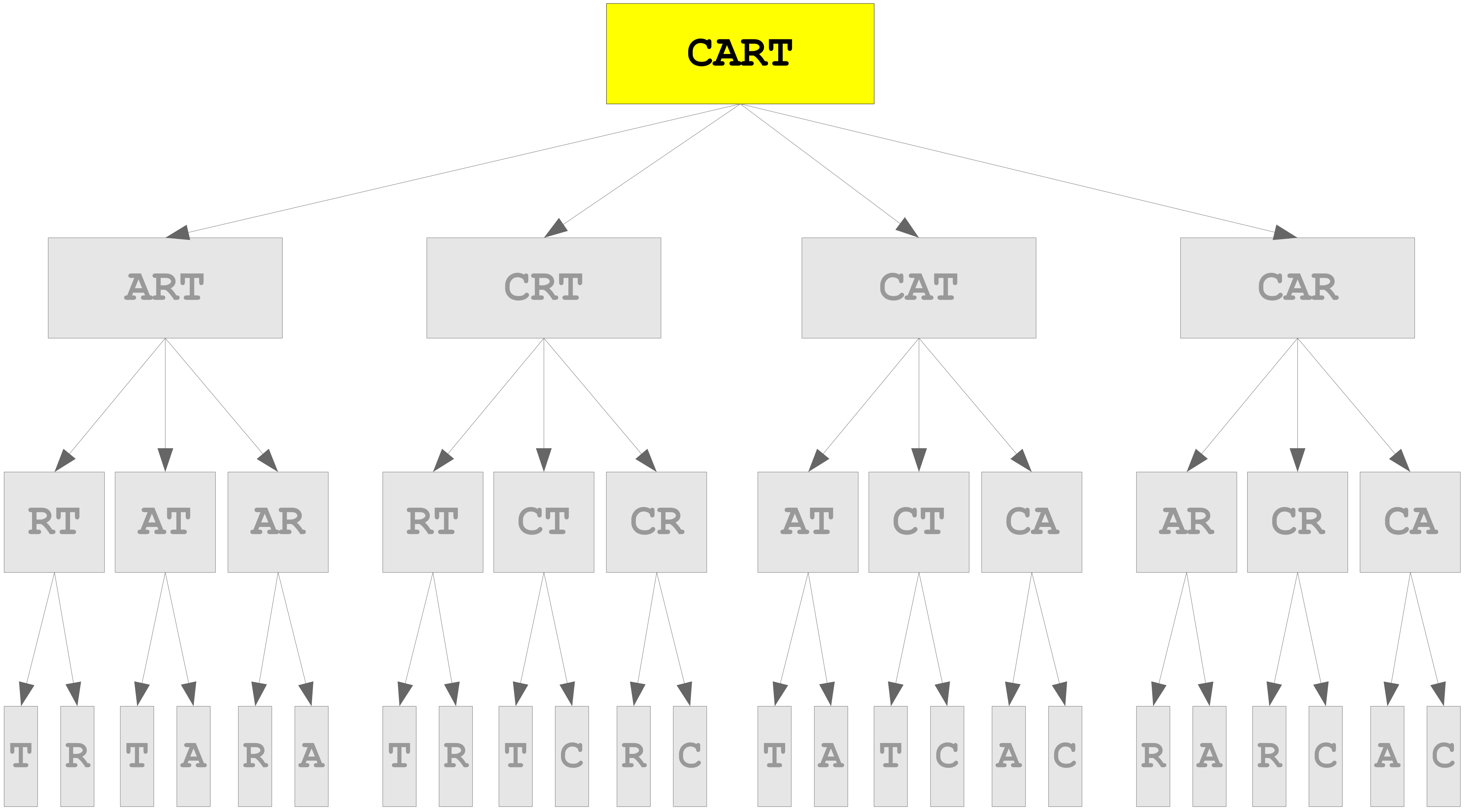

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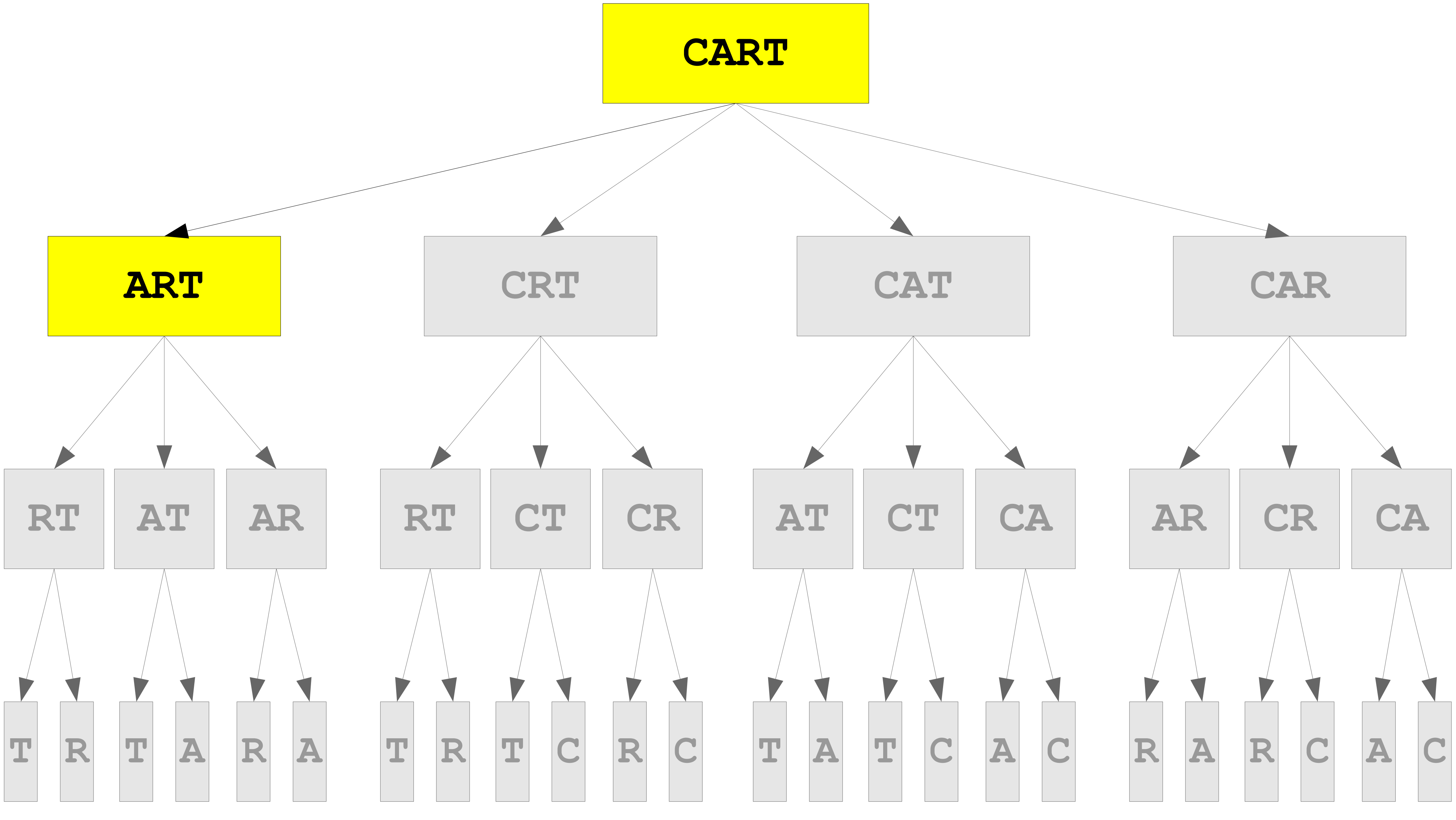

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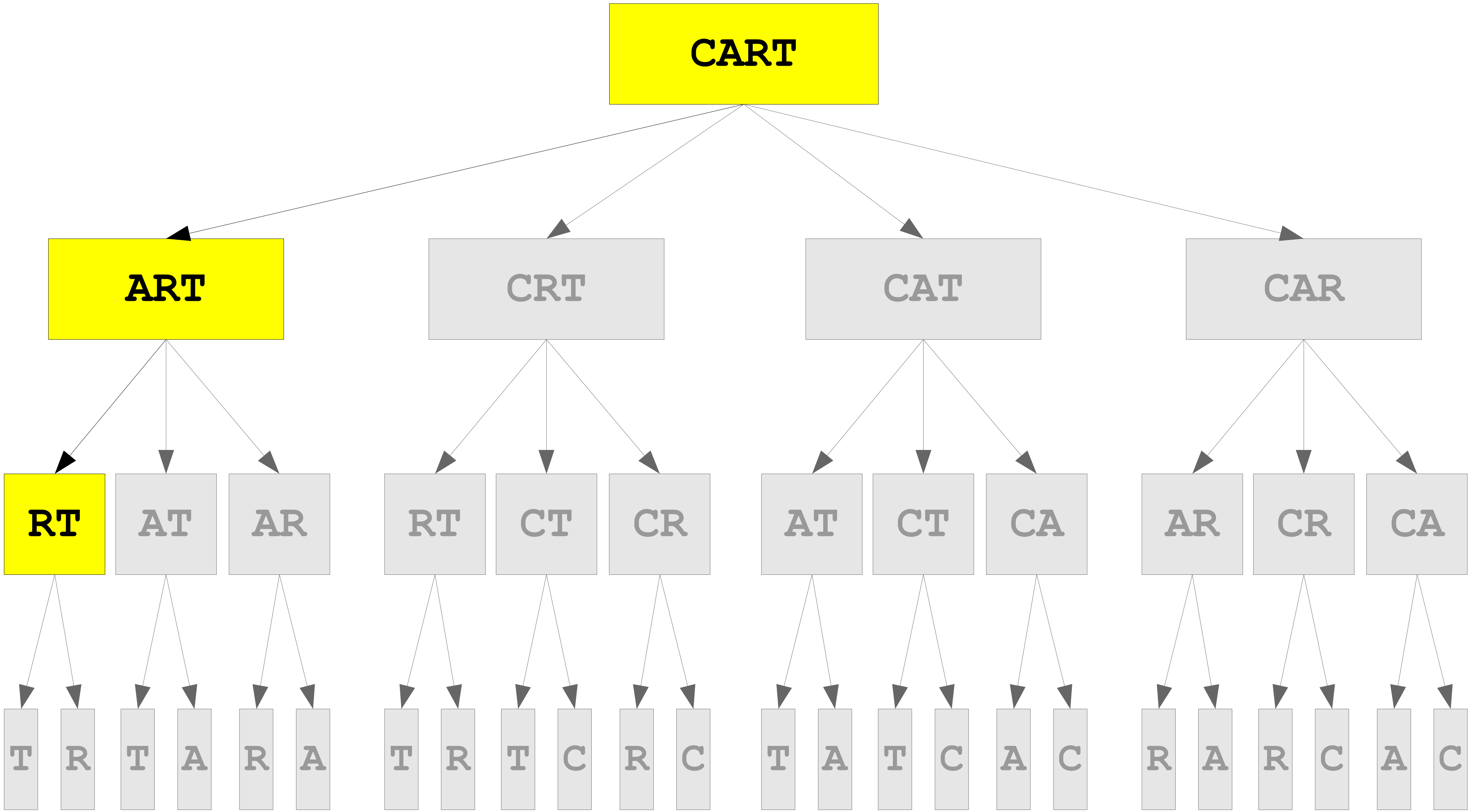

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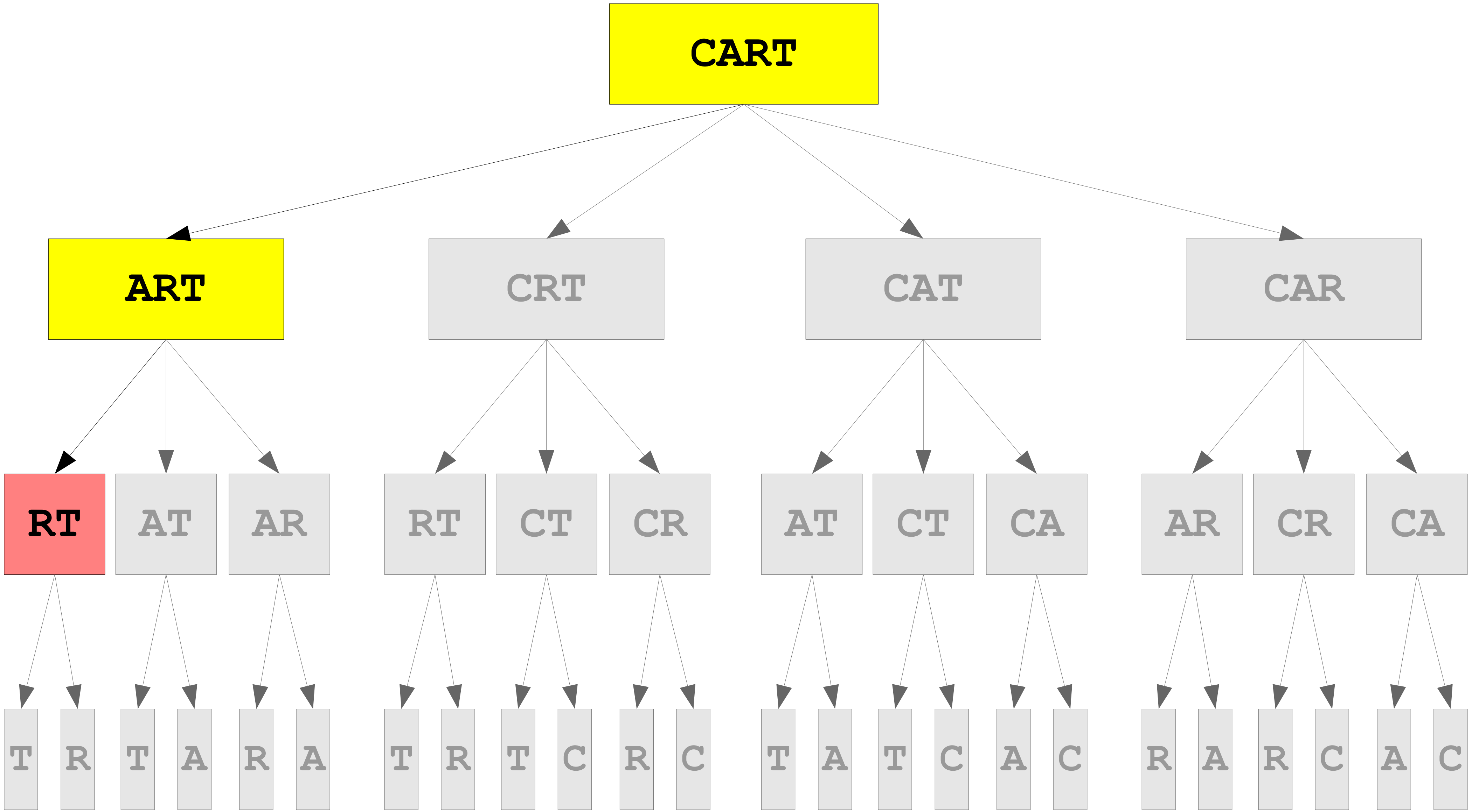
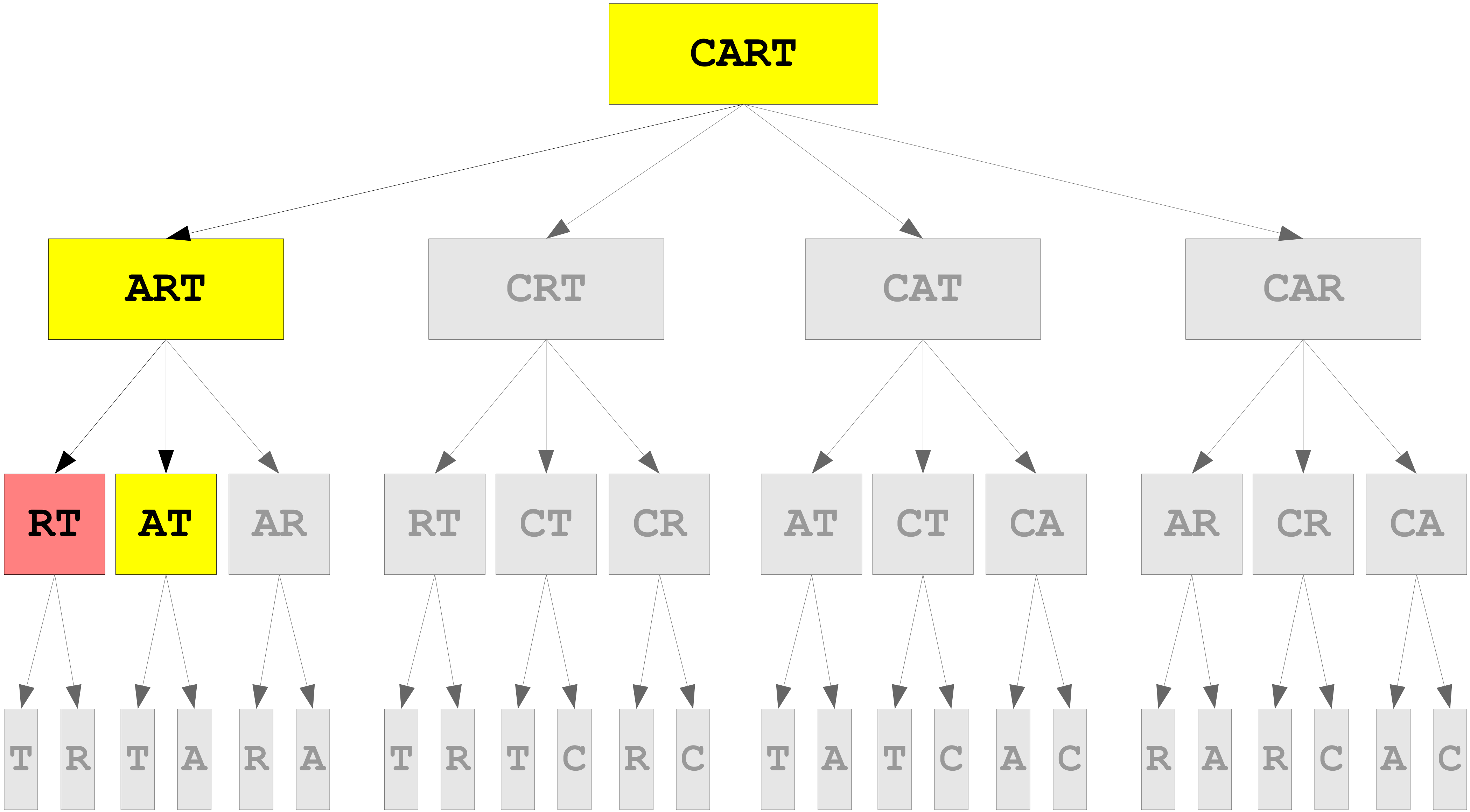


# Recursive Backtracking



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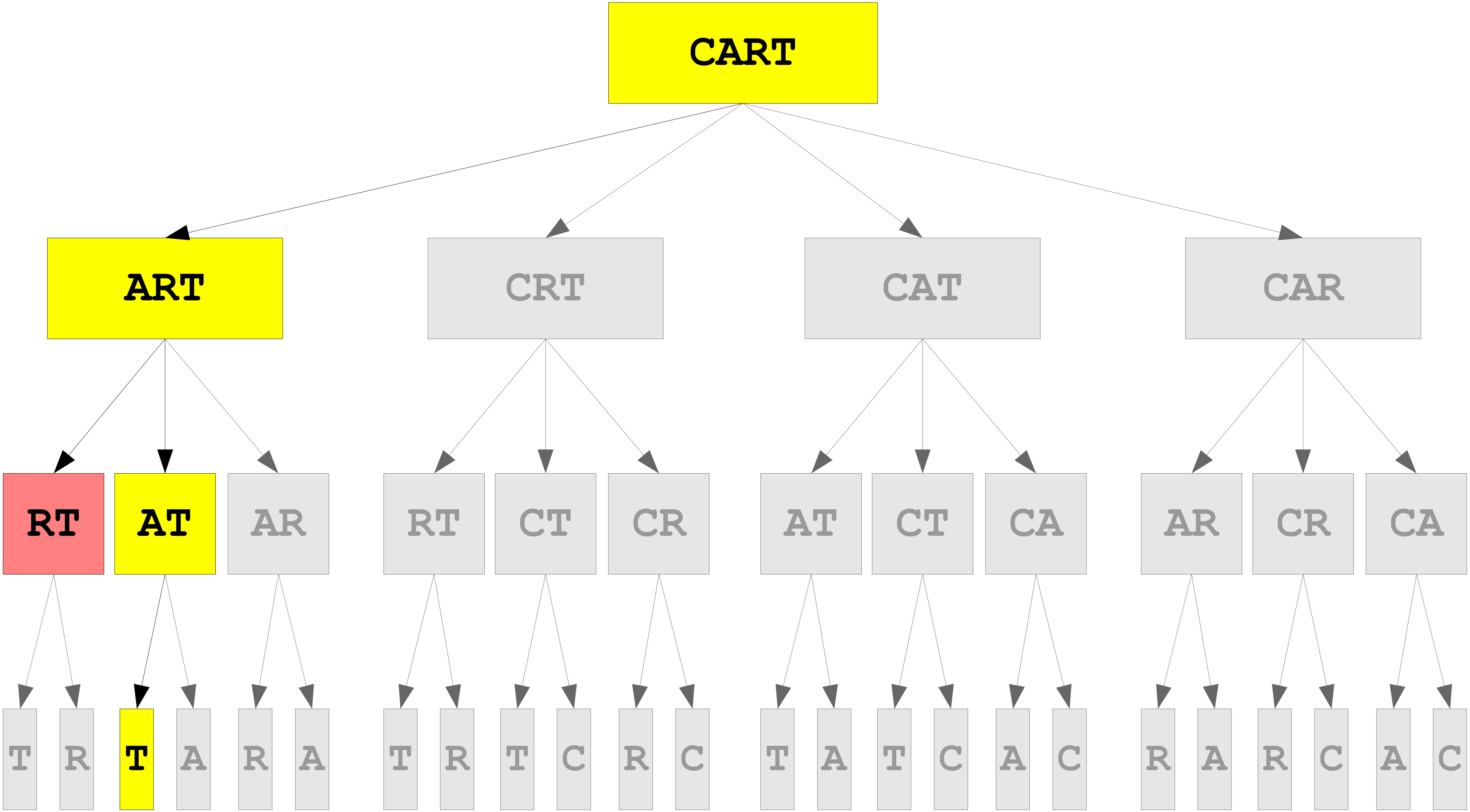

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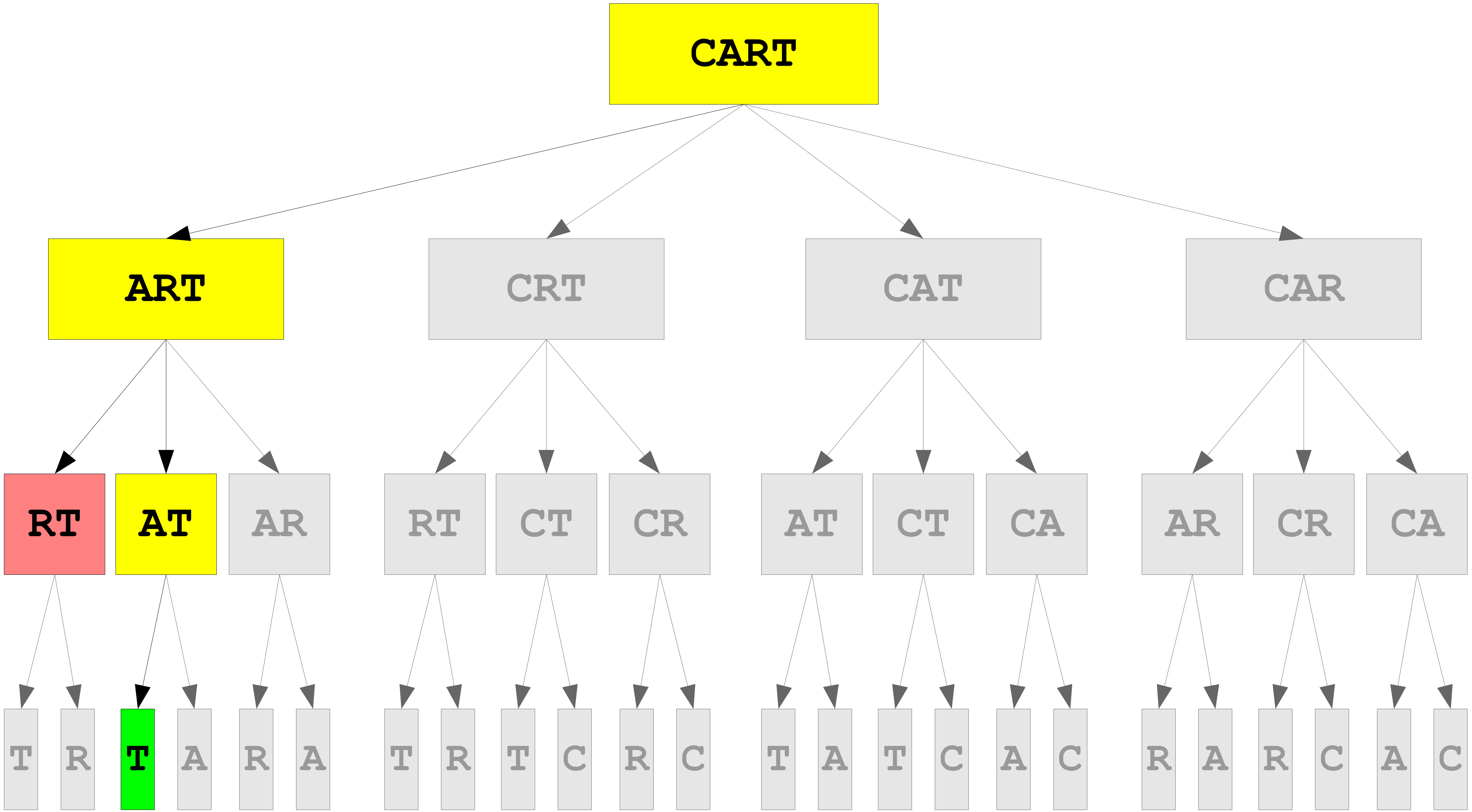

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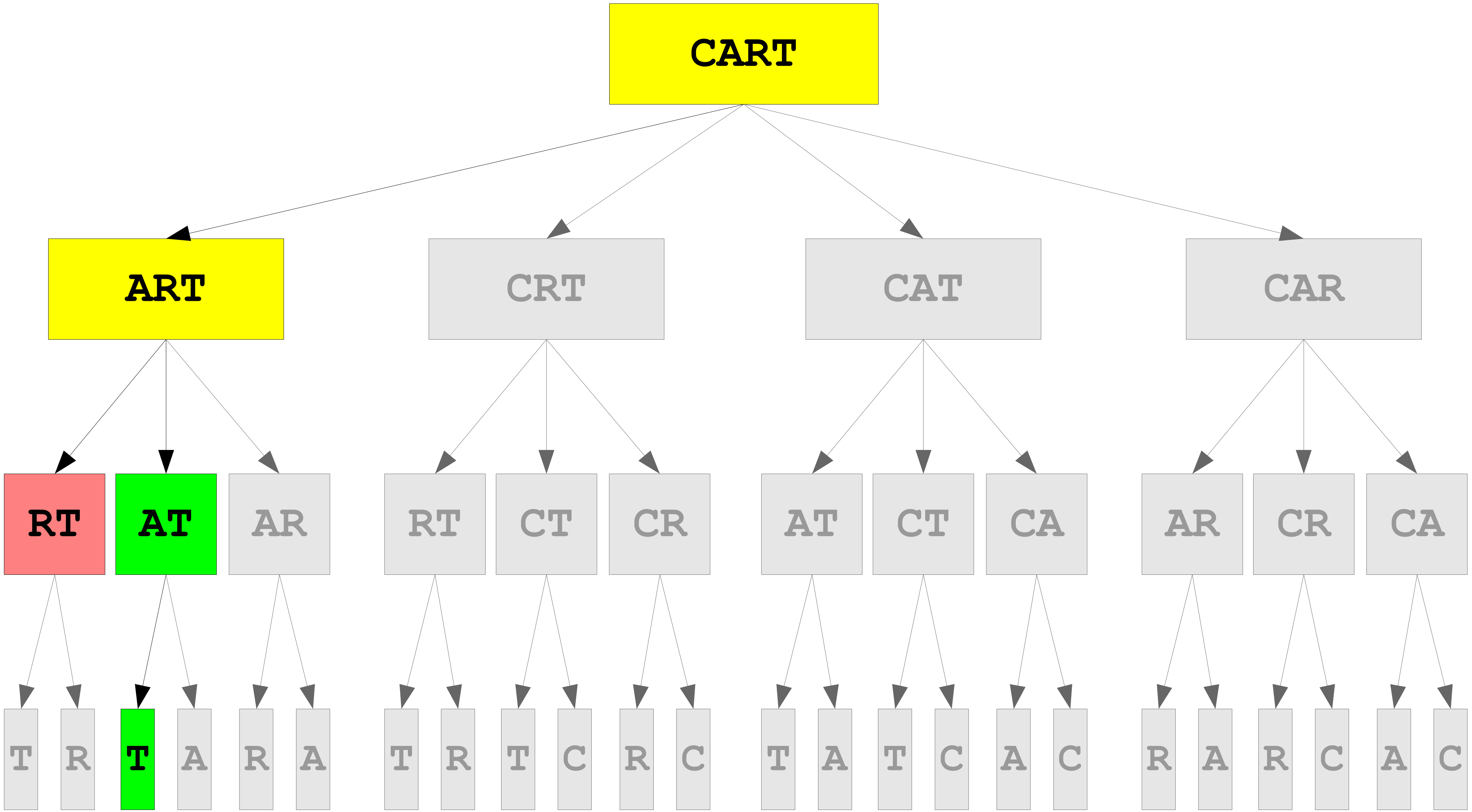

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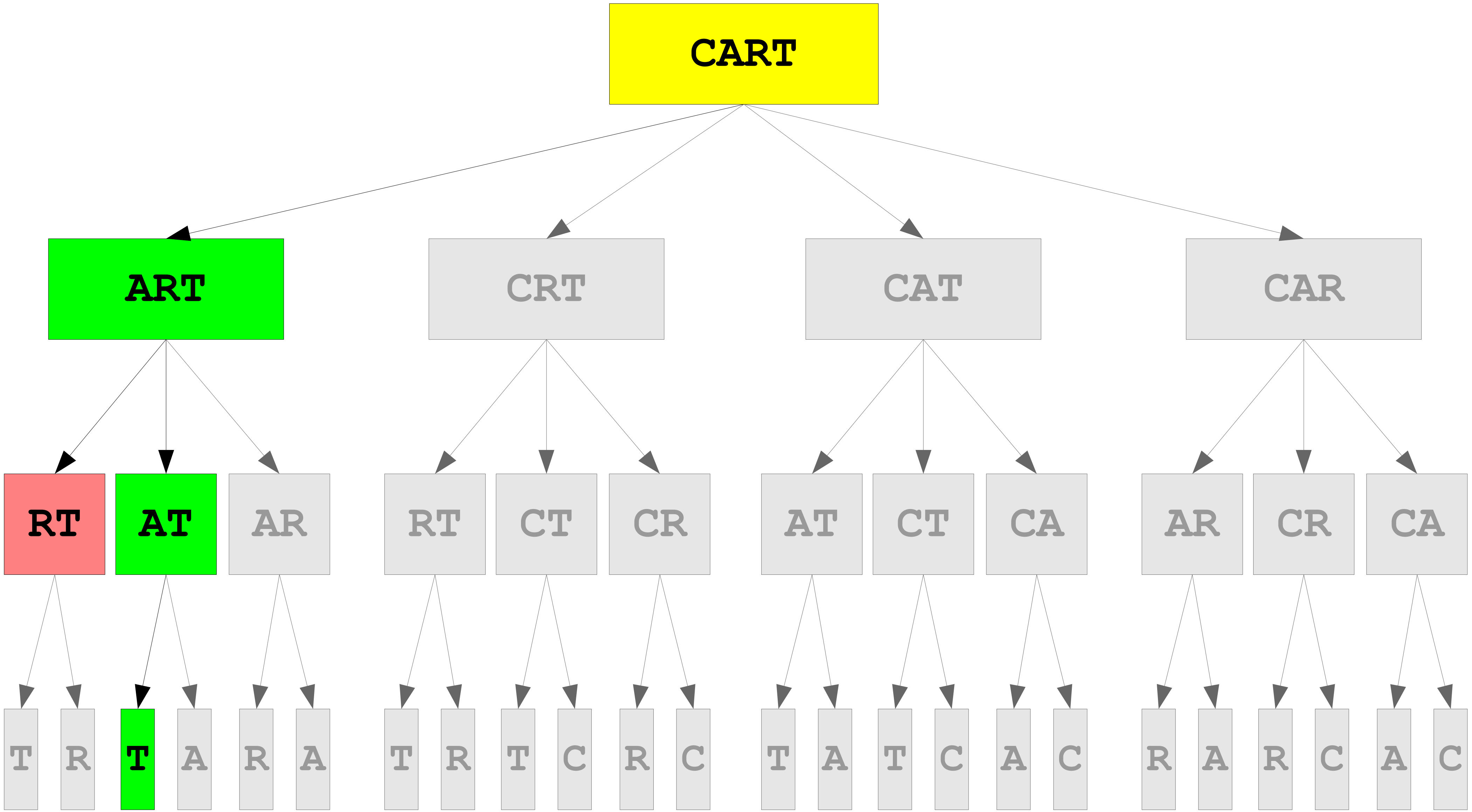

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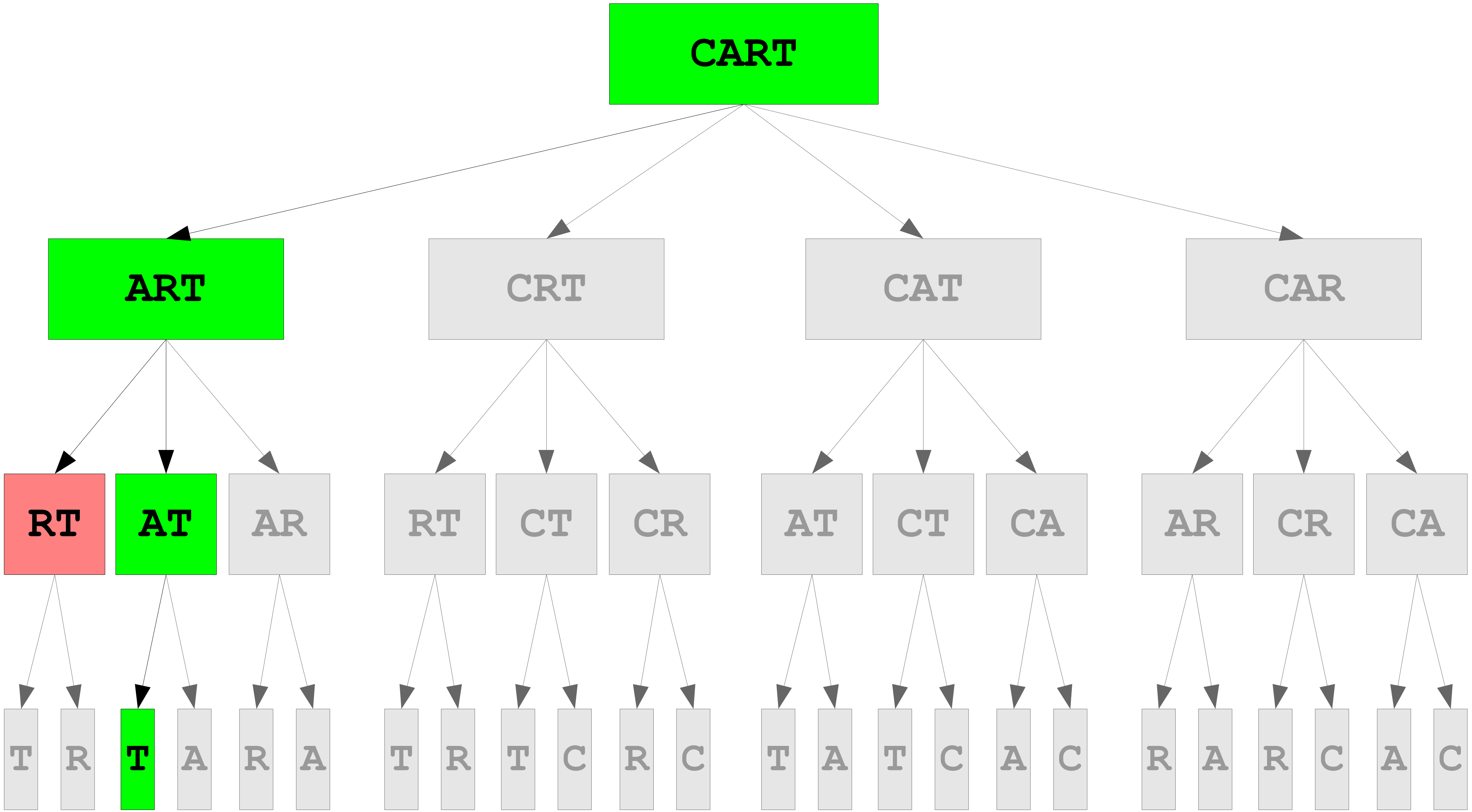

# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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# Failure in Backtracking

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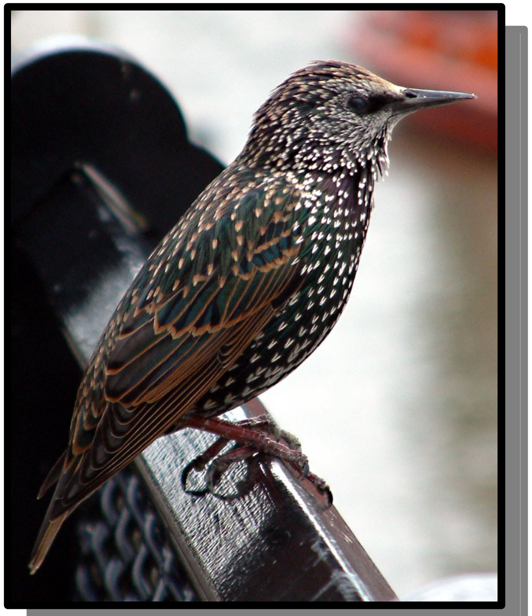

Sources: http://www.rubyinside.com/wp-content/uploads/2008/07/starling.jpg an
http://2.bp.blogspot.com/-hCzVof1xqWo/TvaFXmpJAmI/AAAAAAAAIdk/QzUg1sK mmons.jpg

# Failure in Backtracking

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# Failure in Backtracking

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| S | A | R | L | I | N | G |

# Recursive Backtracking

```
if (problem is sufficiently simple) {
    return whether or not the problem is solvable
} else {
    for (each choice) {
        try out that choice.
        if (that choice leads to success) {
            return success
        }
    }
    return failure
}
```

# Recursive Backtracking

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if (problem is sufficiently simple) {
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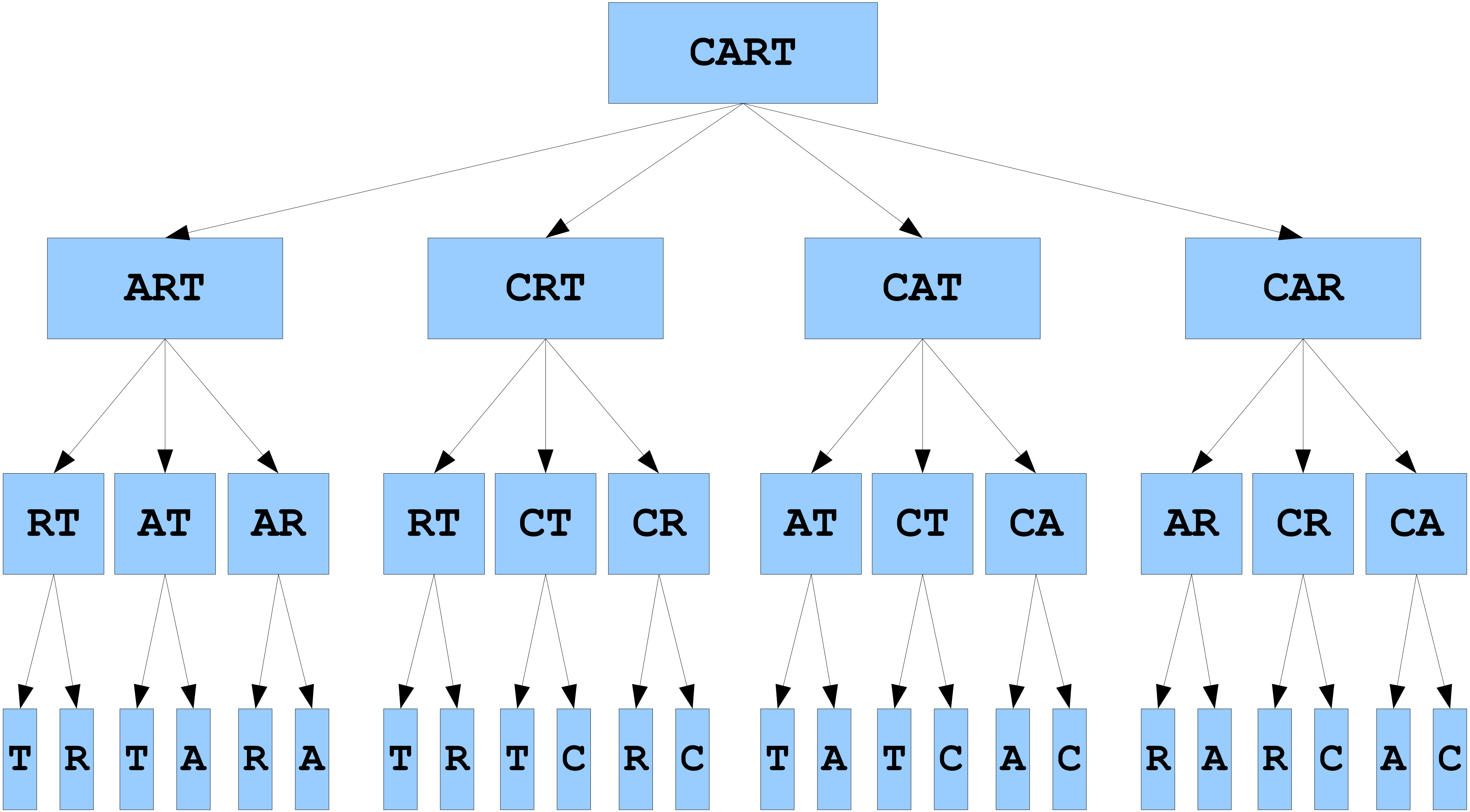
        if (that choice leads to success) {
            return success;
        }
    }
    return failure
}
```

Note that ***if*** it succeeds, ***then*** we return success. If it doesn't succeed, that doesn't mean we've failed – it just means we need to try out the next option.

# Failure in Backtracking

- Returning false in recursive backtracking does ***not*** mean that the entire problem is unsolvable!
- Instead, it just means that the current subproblem is unsolvable.
- Whoever made the call to this function can then try other options.
- Only when all options are exhausted can we know that the problem is unsolvable.

# Ur Doin It Rong!



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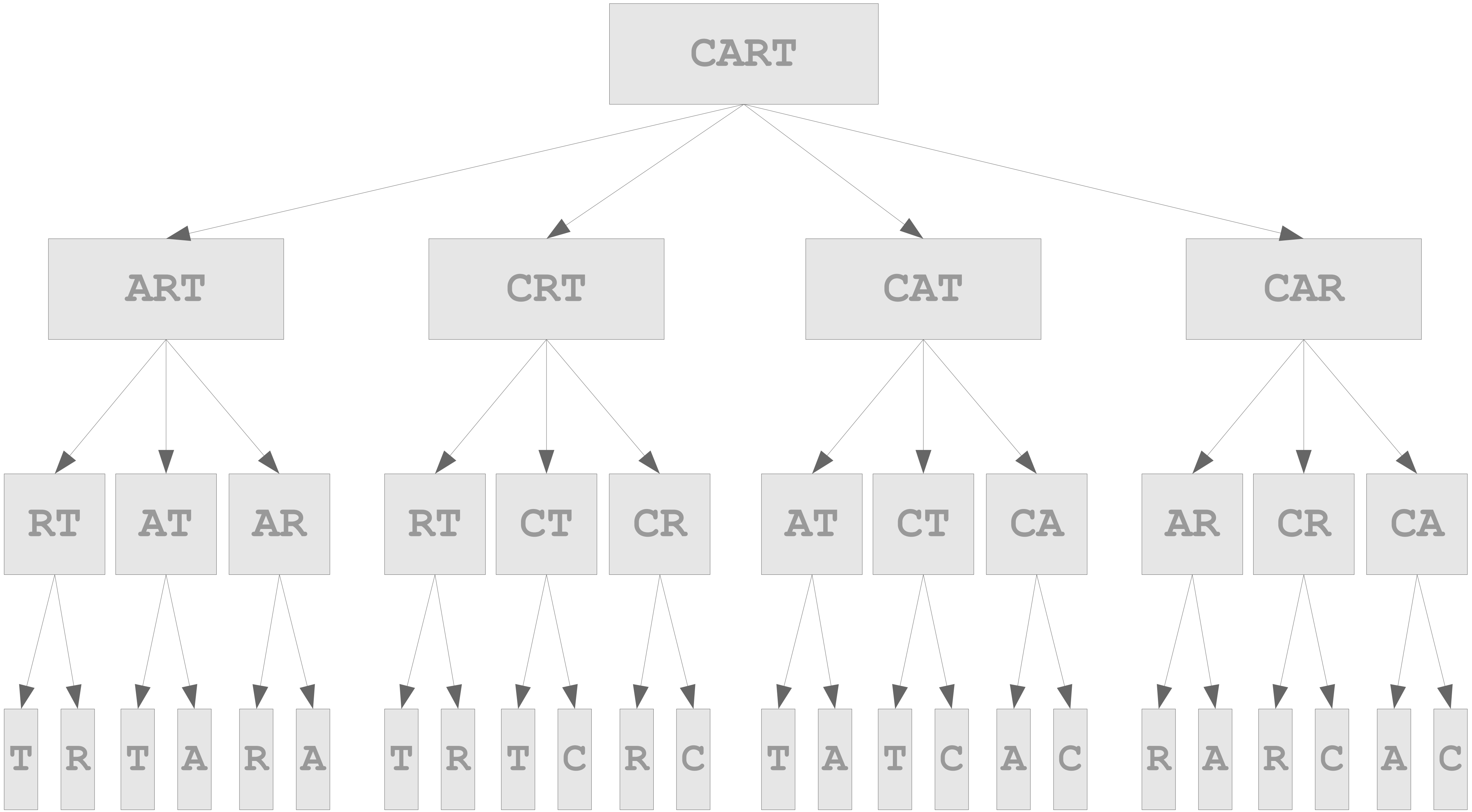

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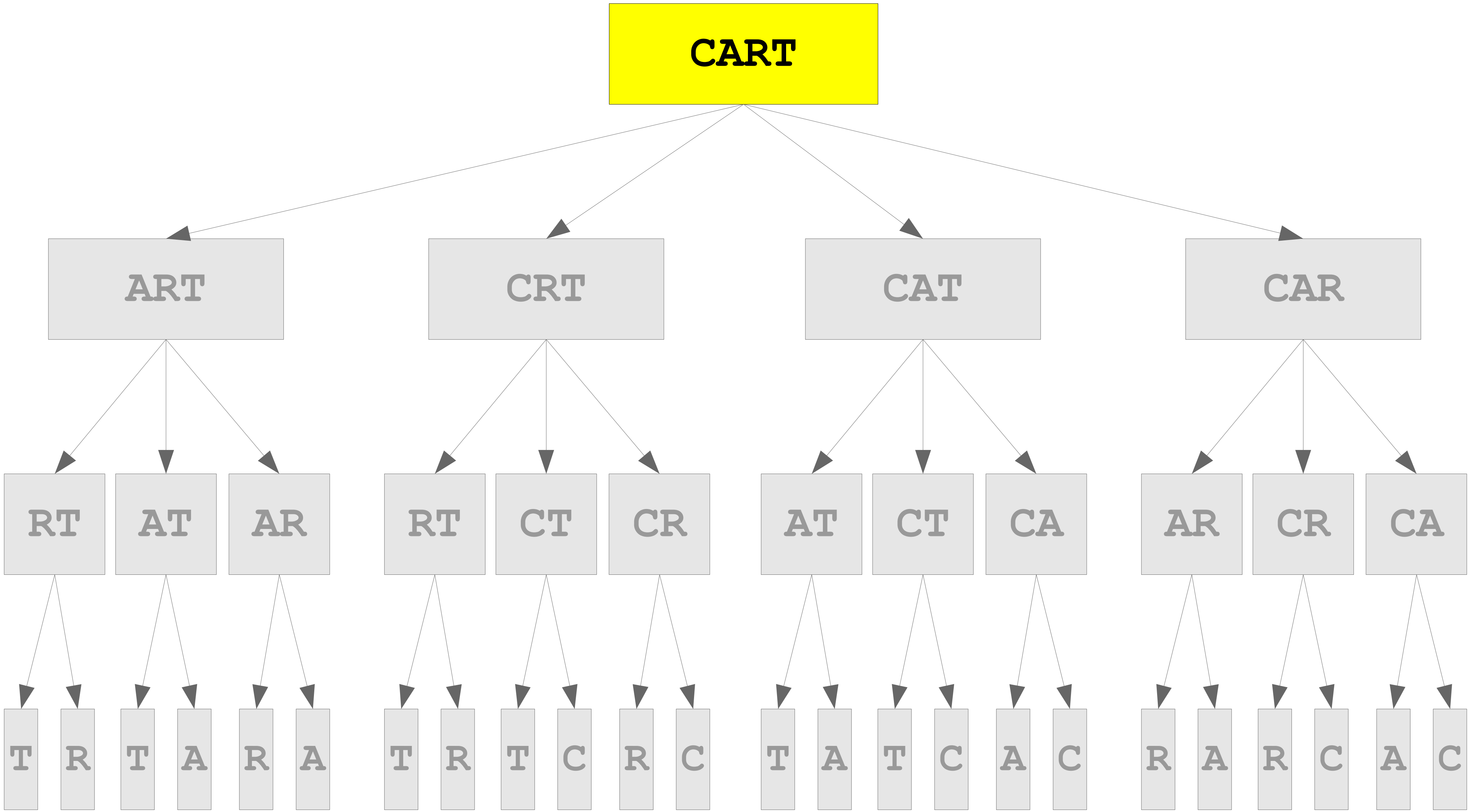

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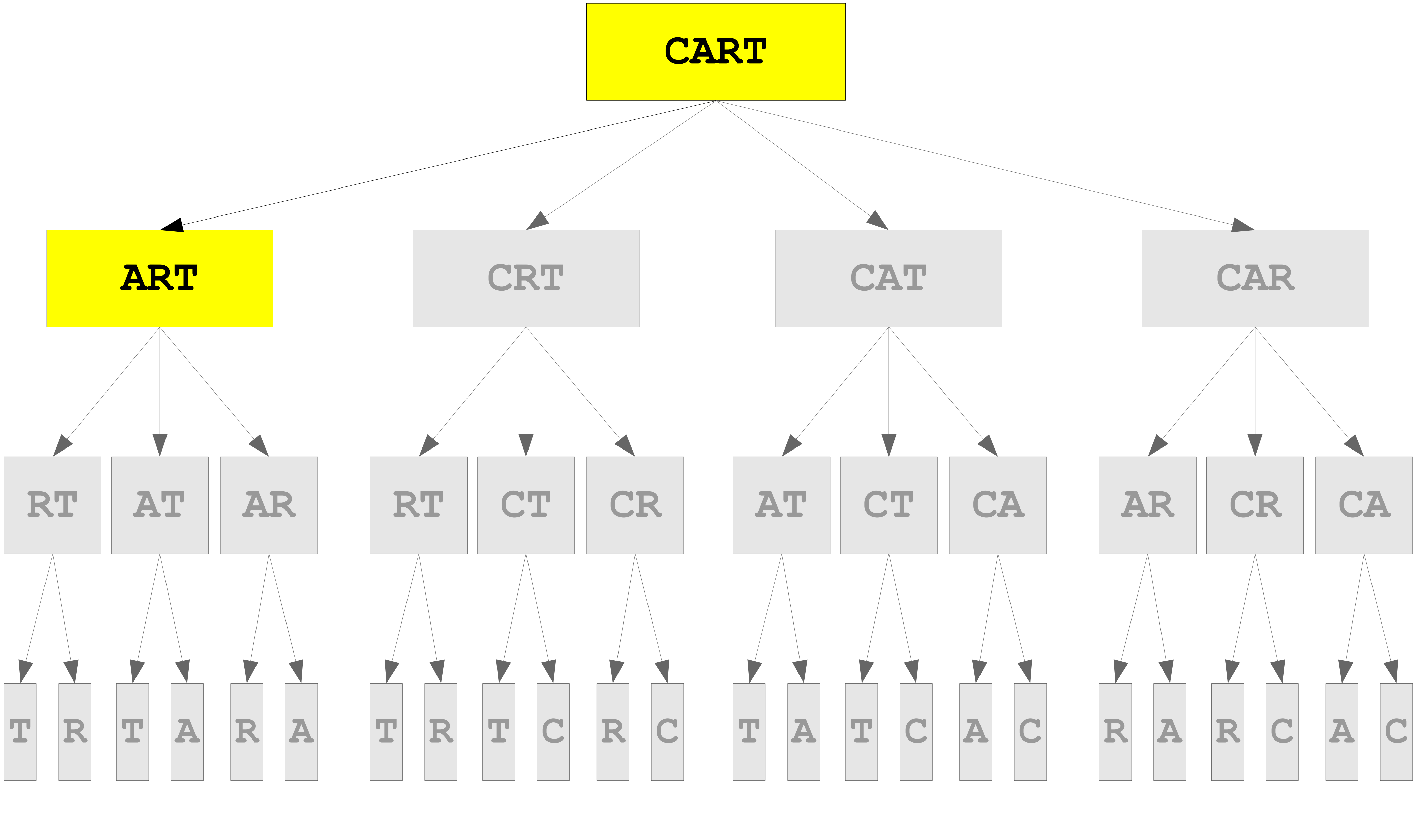

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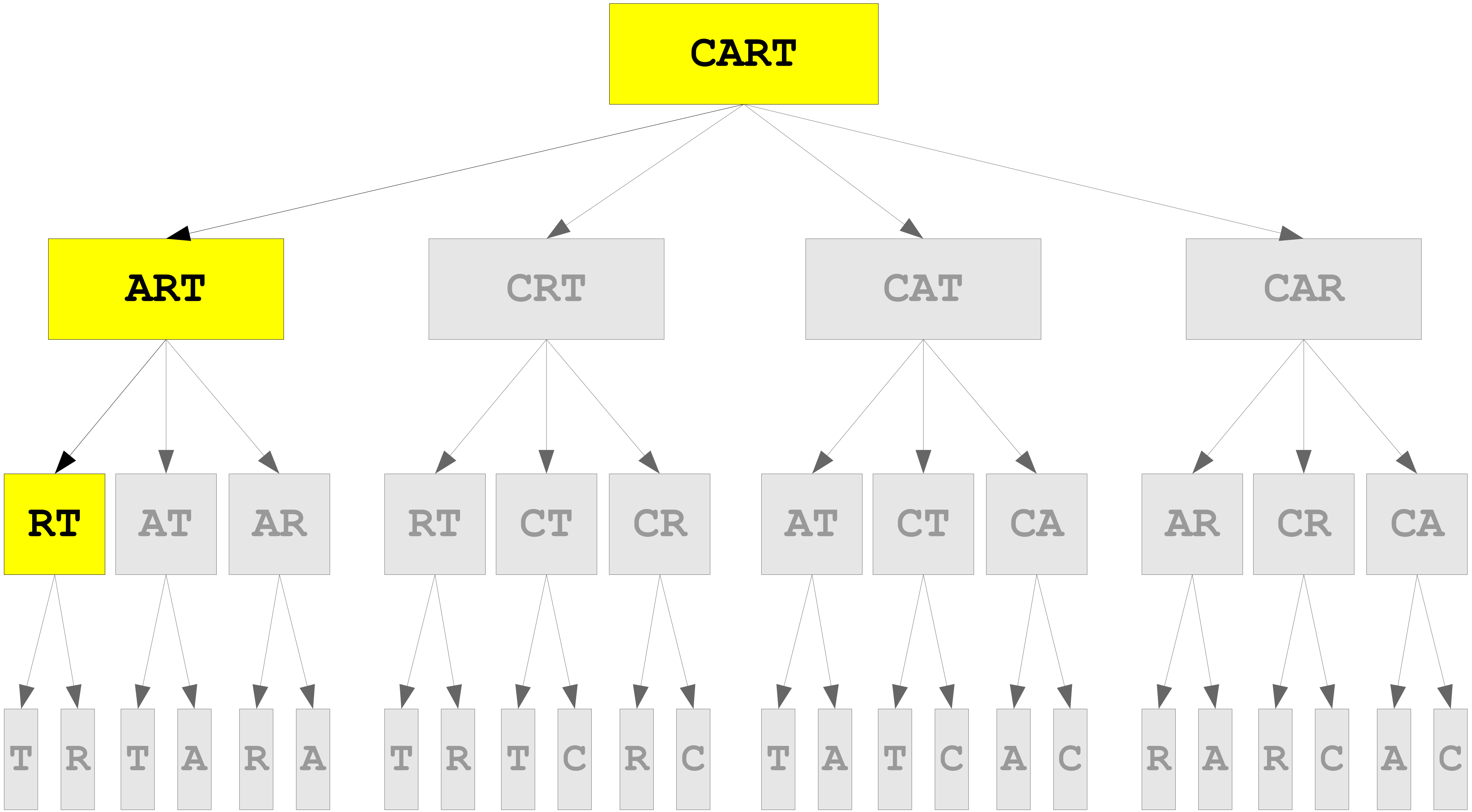

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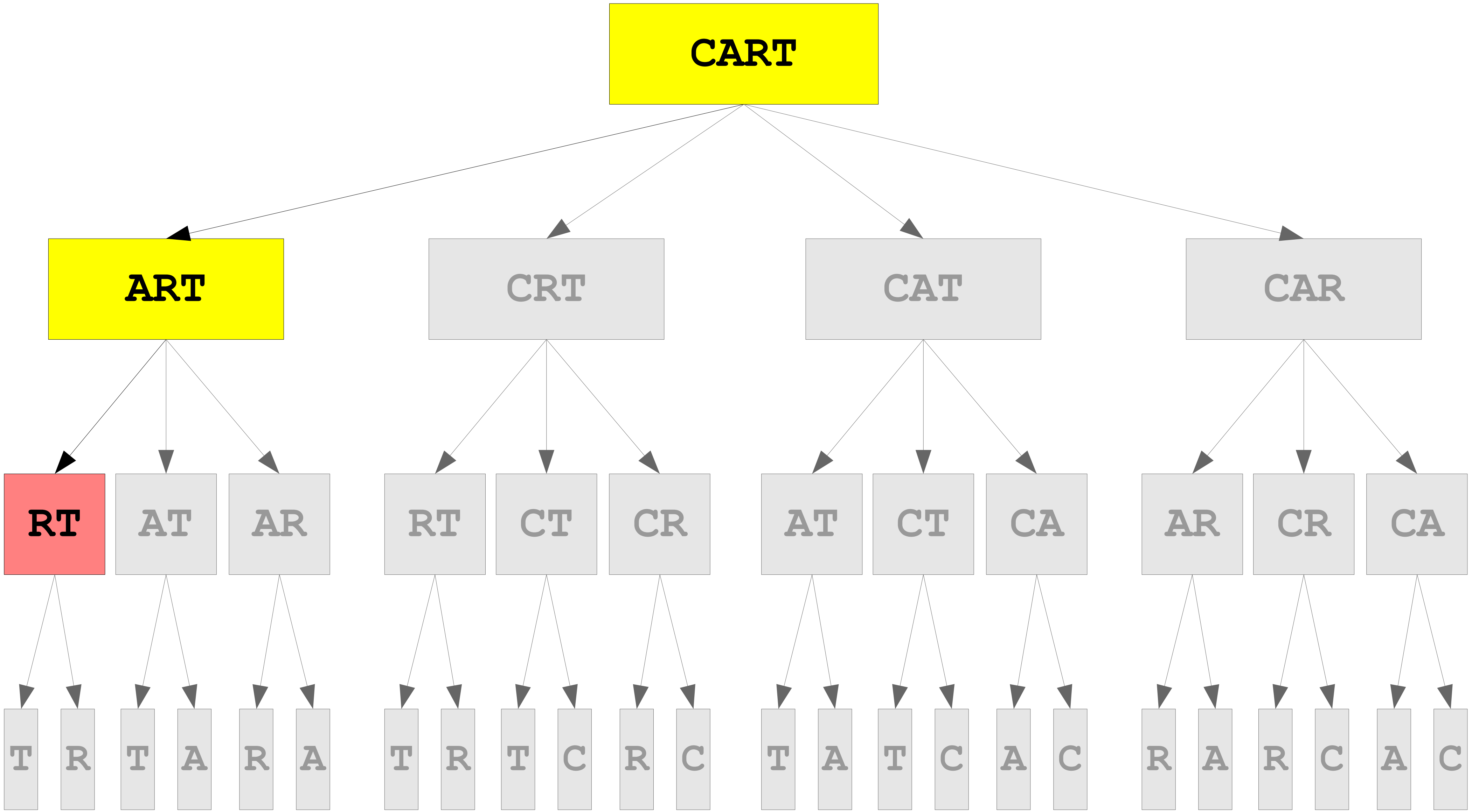

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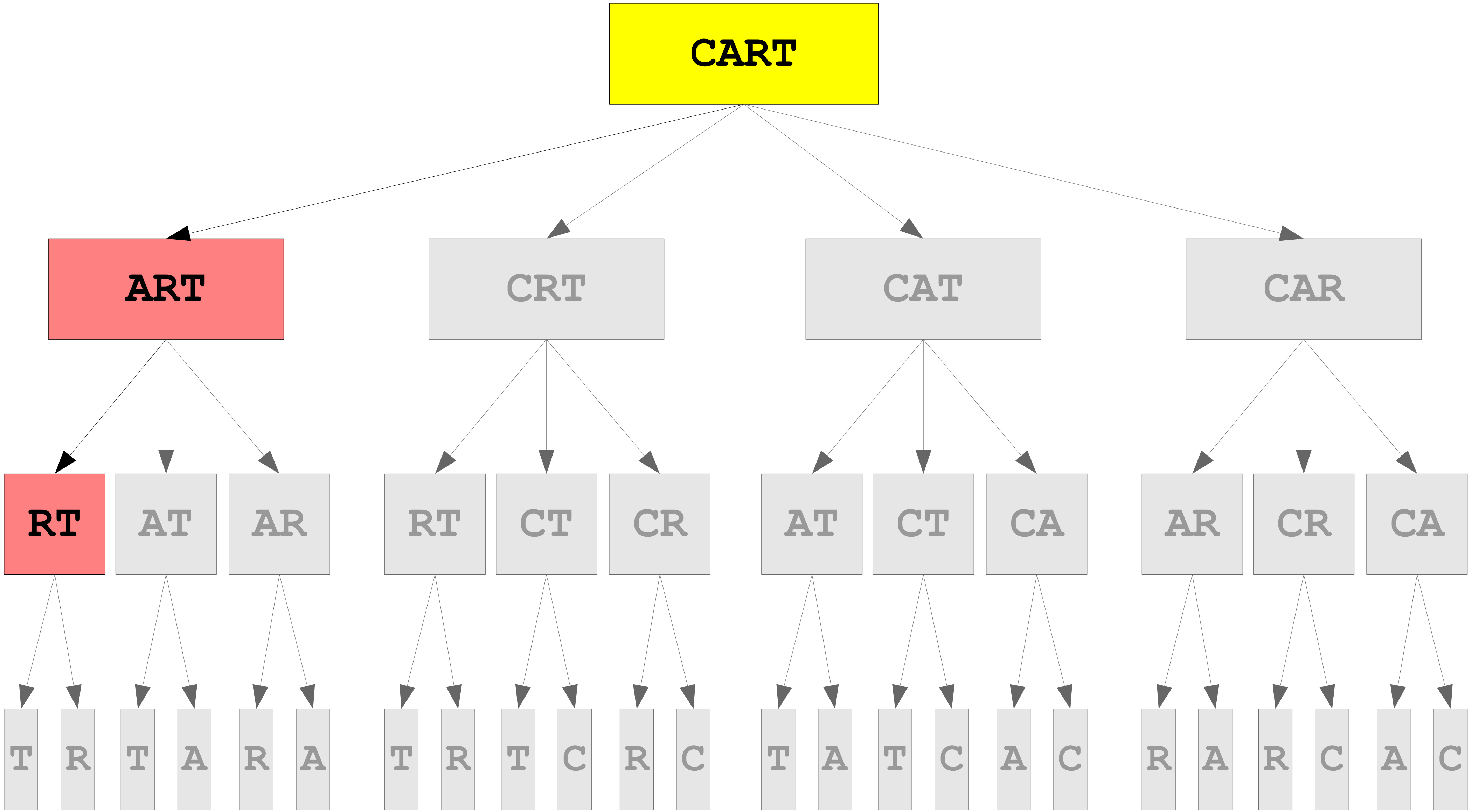

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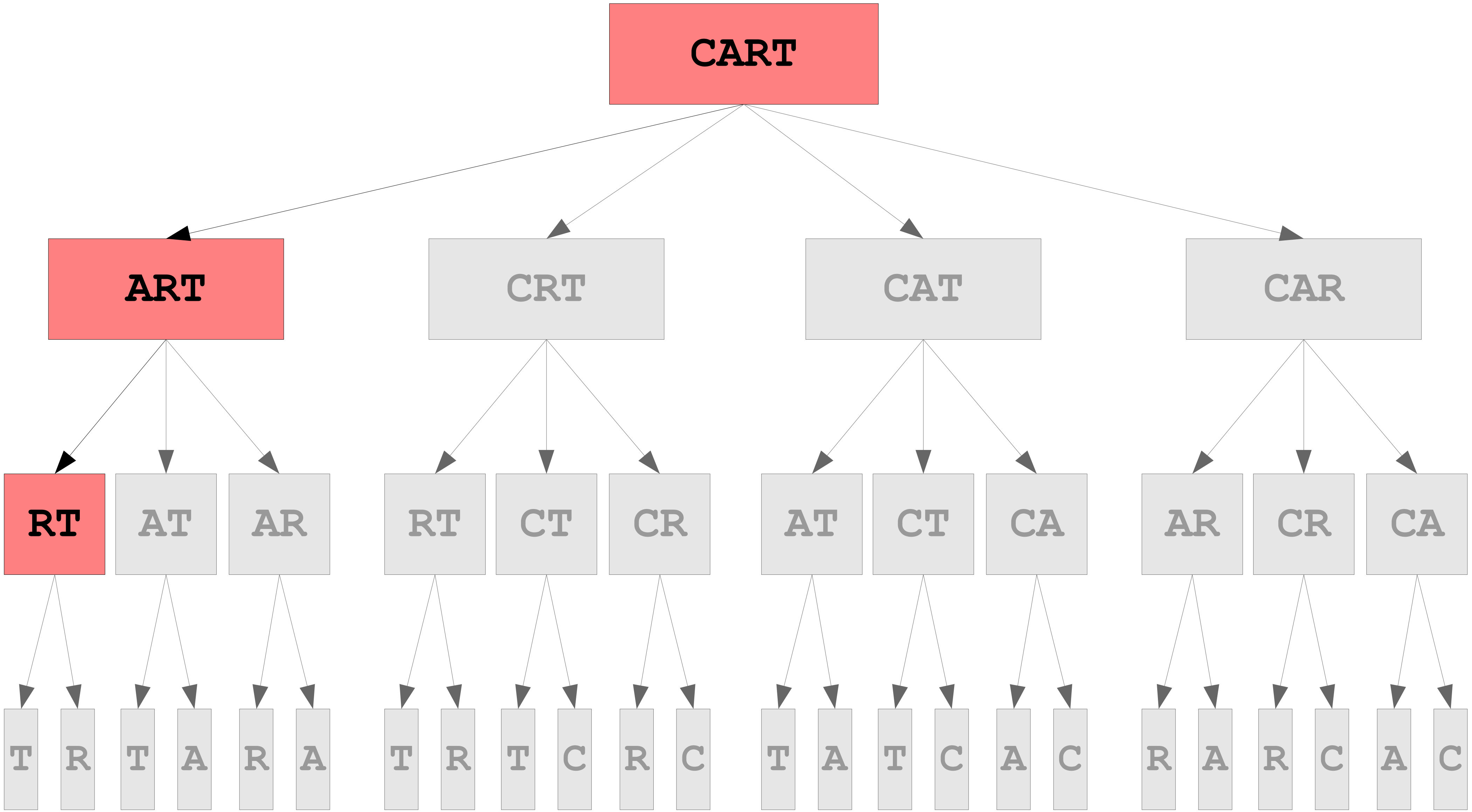

# Next Week

- **Algorithmic Efficiency**
  - How can we compare the speed of two different algorithms?
- **Sorting Algorithms**
- **Implementing Collections Classes**