Implementing Abstractions

Part Two

Announcements

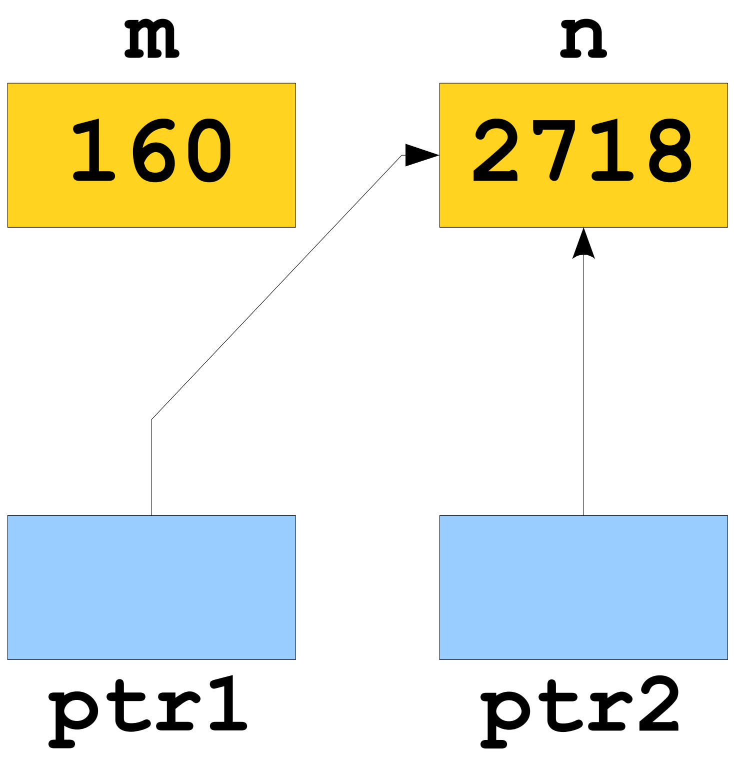
- Aubrey's Office hours cancelled today because he is sleepy

Exams

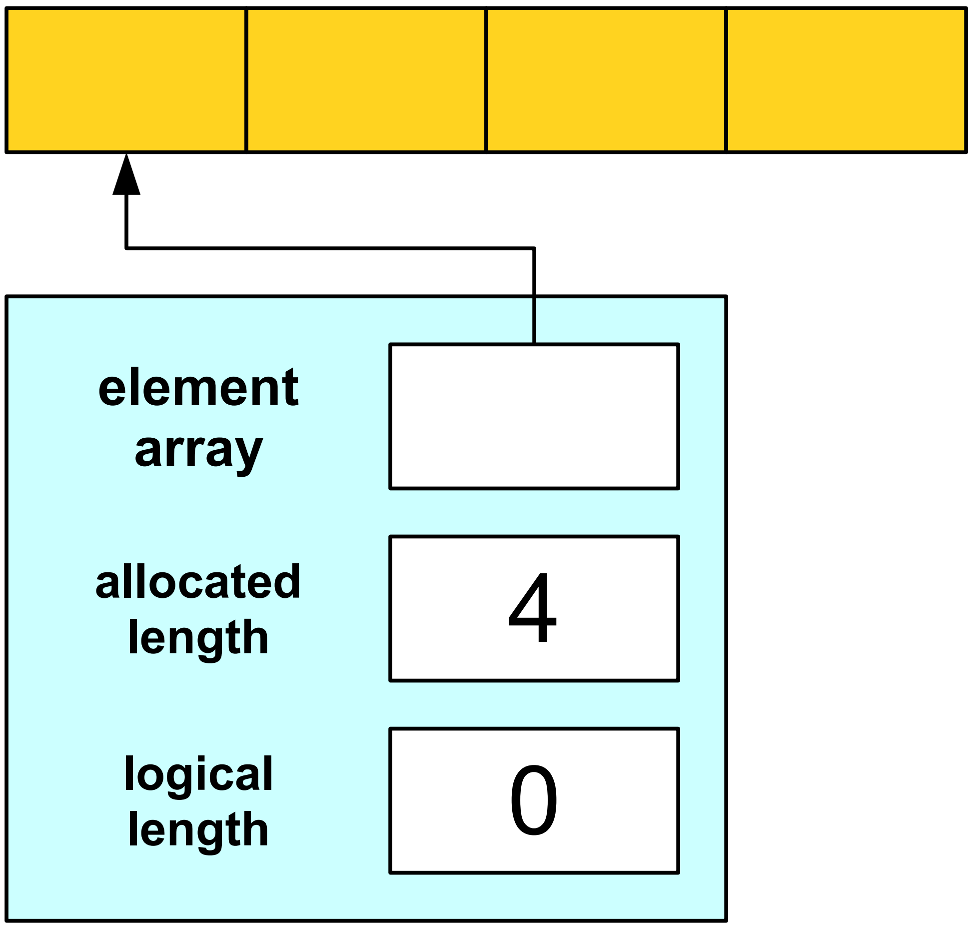
- Can pick the up after class today or when Michael or I are in Gates 160
- Criteria will be posted online later today
  - Please take a look, especially if you think there's a grading error
  - Regrade requests need to happen in the next week
    - We reserve the right to fix any grading mistakes we see (not just ones in your favor) so please look at the entire exam!
- Exam statistics at the end of lecture

Pointers, Visually

```
int m = 137;
int n = 42;

int* ptr1 = &m;
int* ptr2 = &n;

*ptr1 = 2718;
*ptr2 = *ptr1;
m = 160;

ptr1 = ptr2;
```



An Initial Idea



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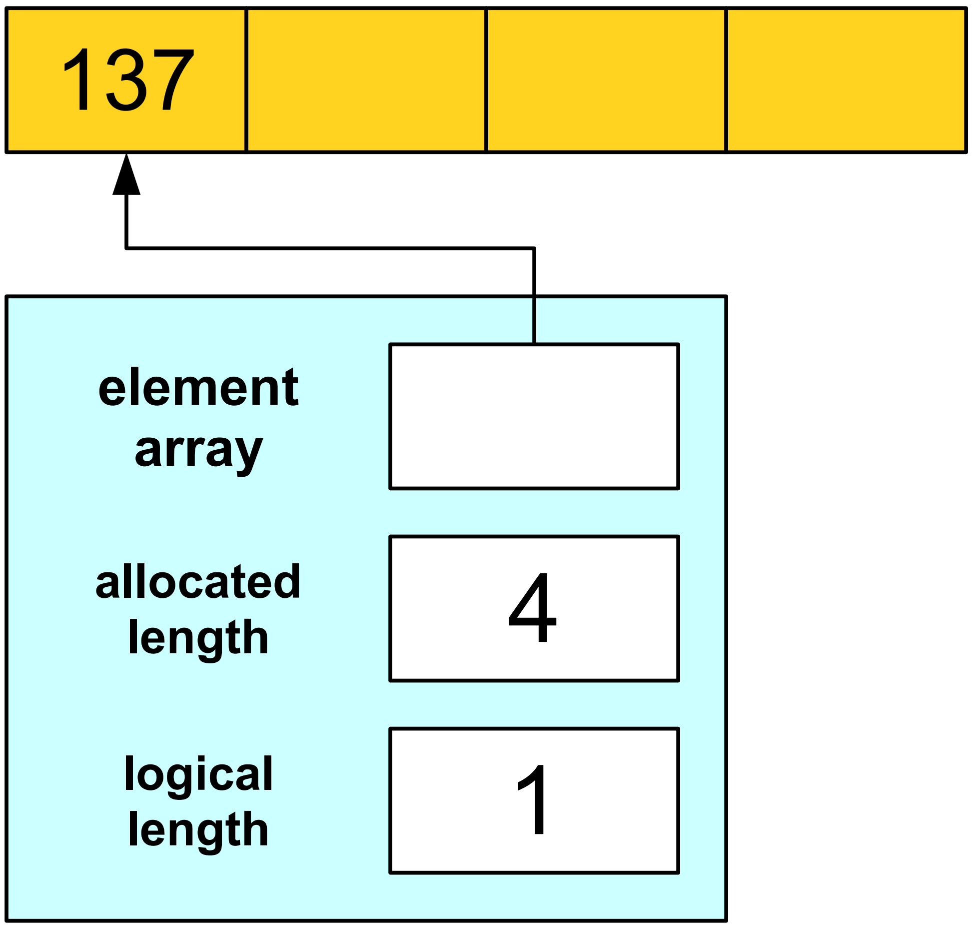

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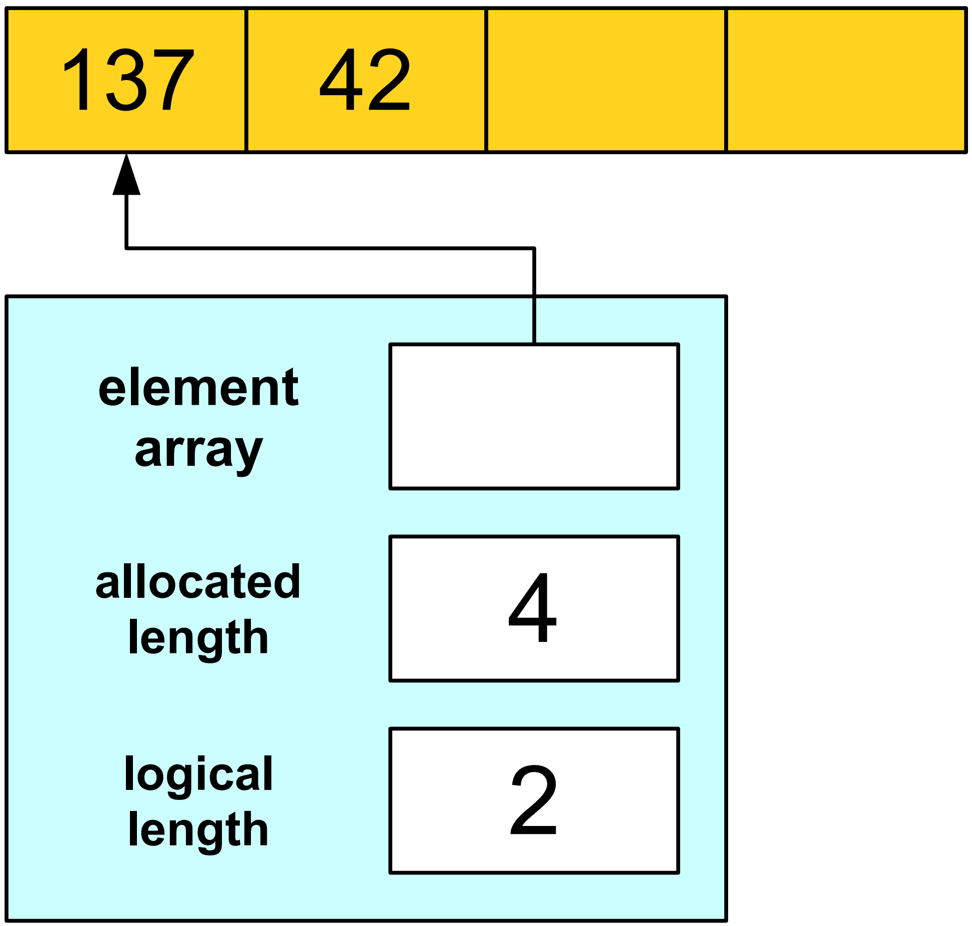

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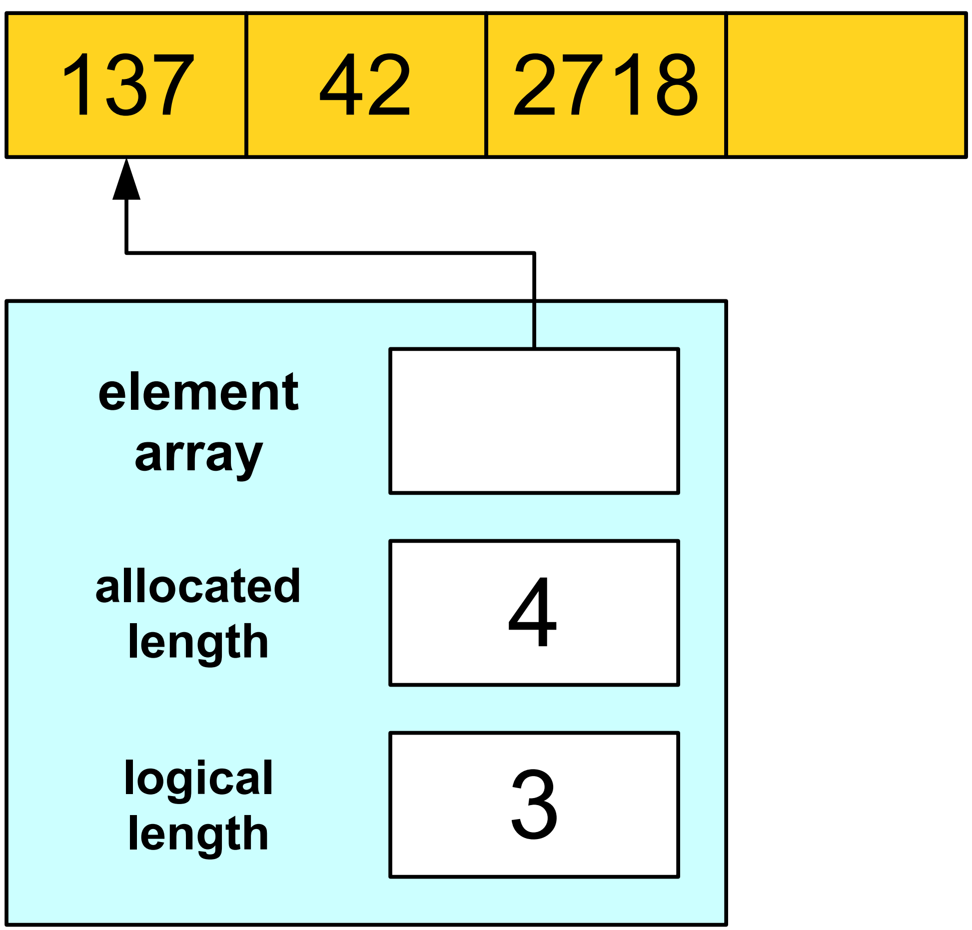

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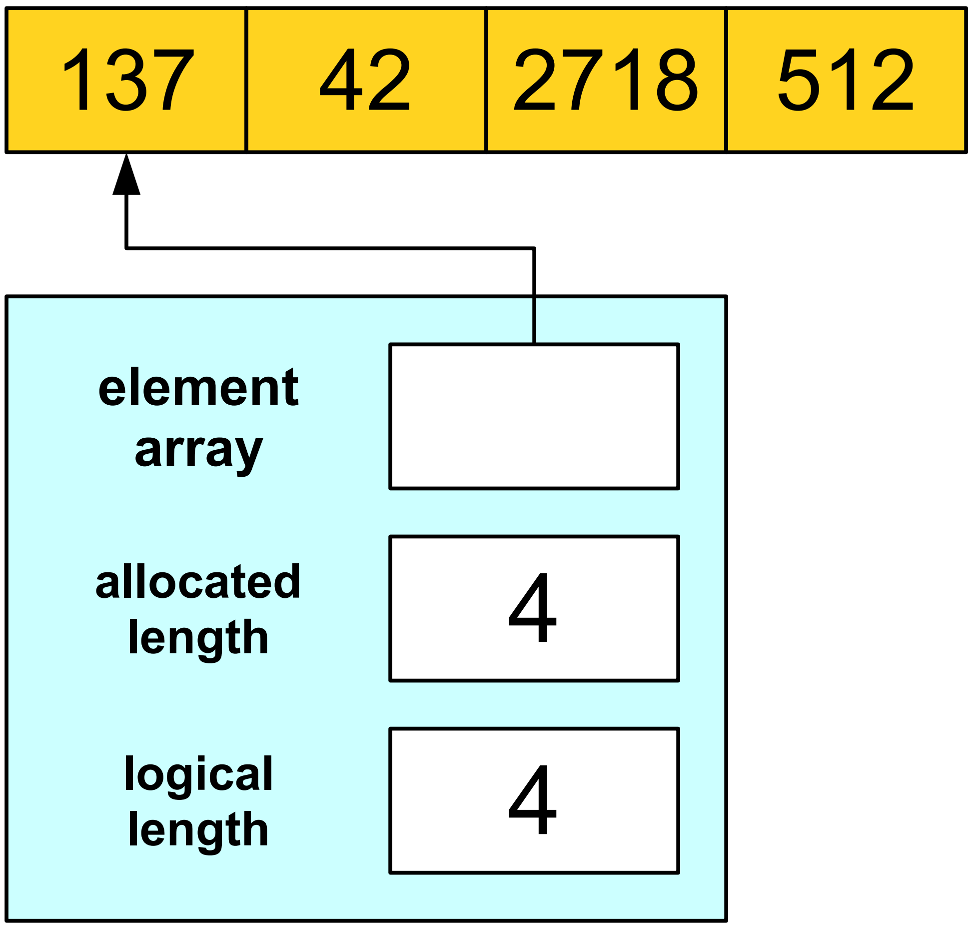

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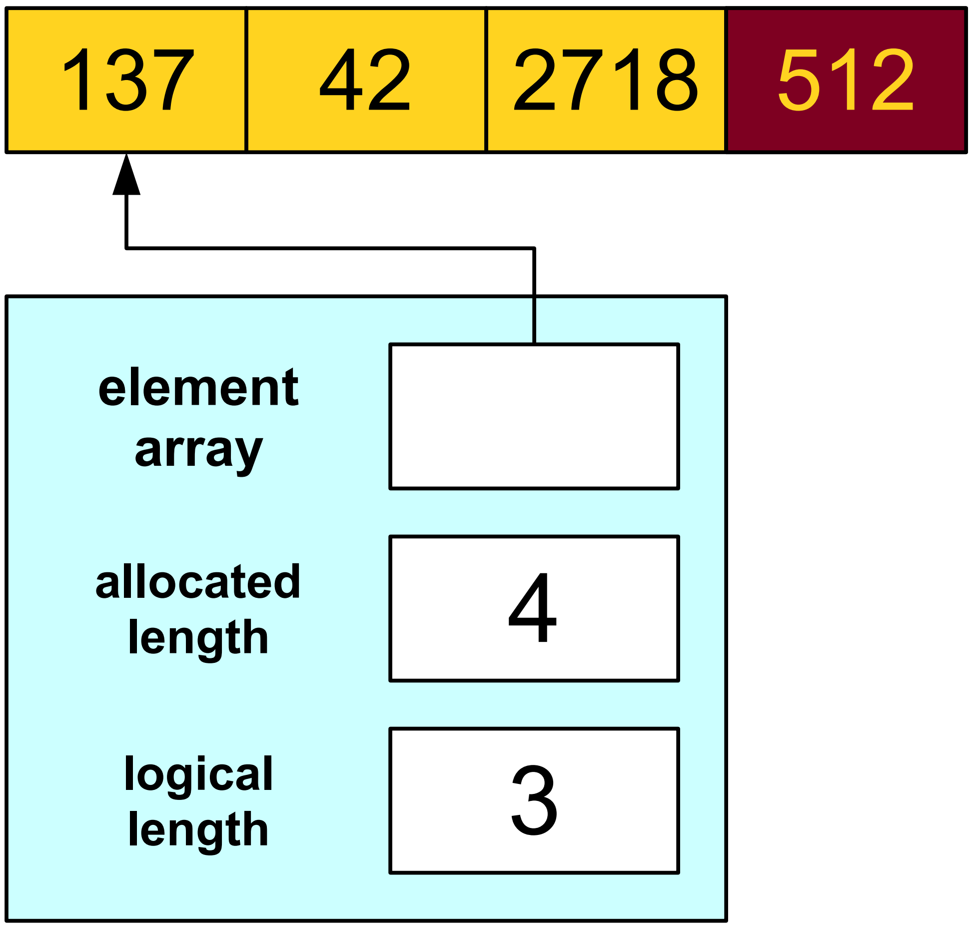

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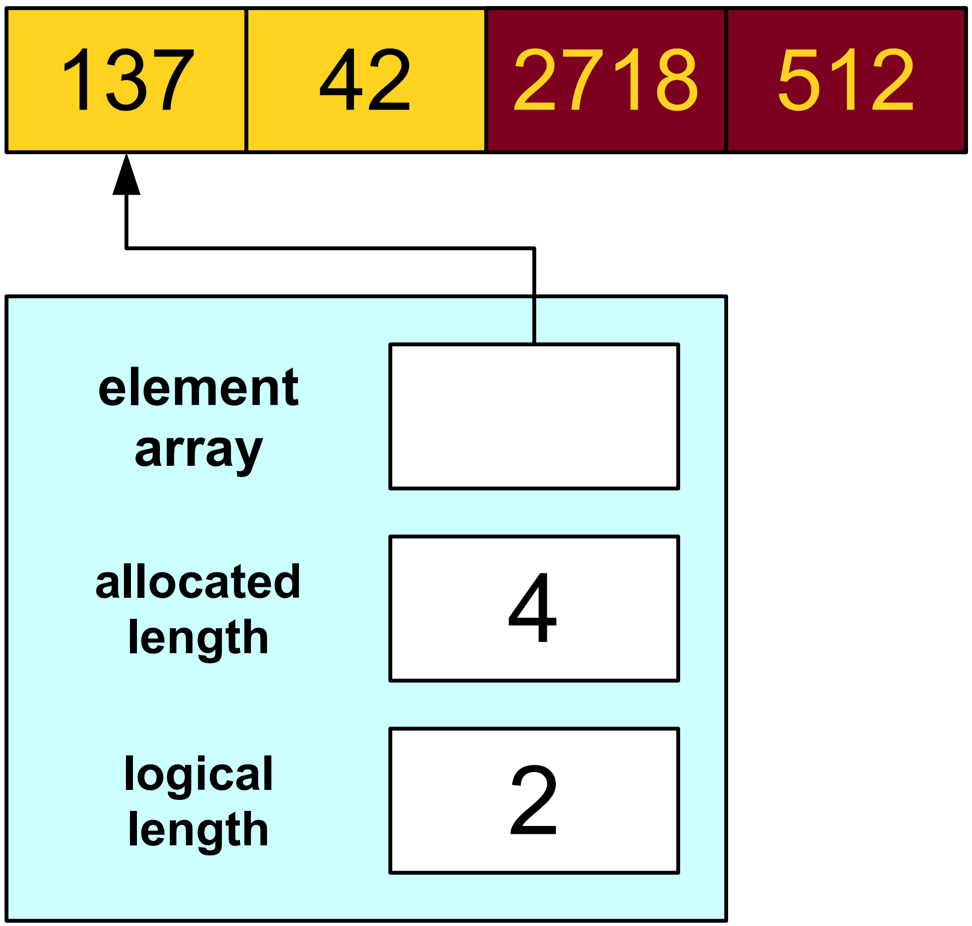

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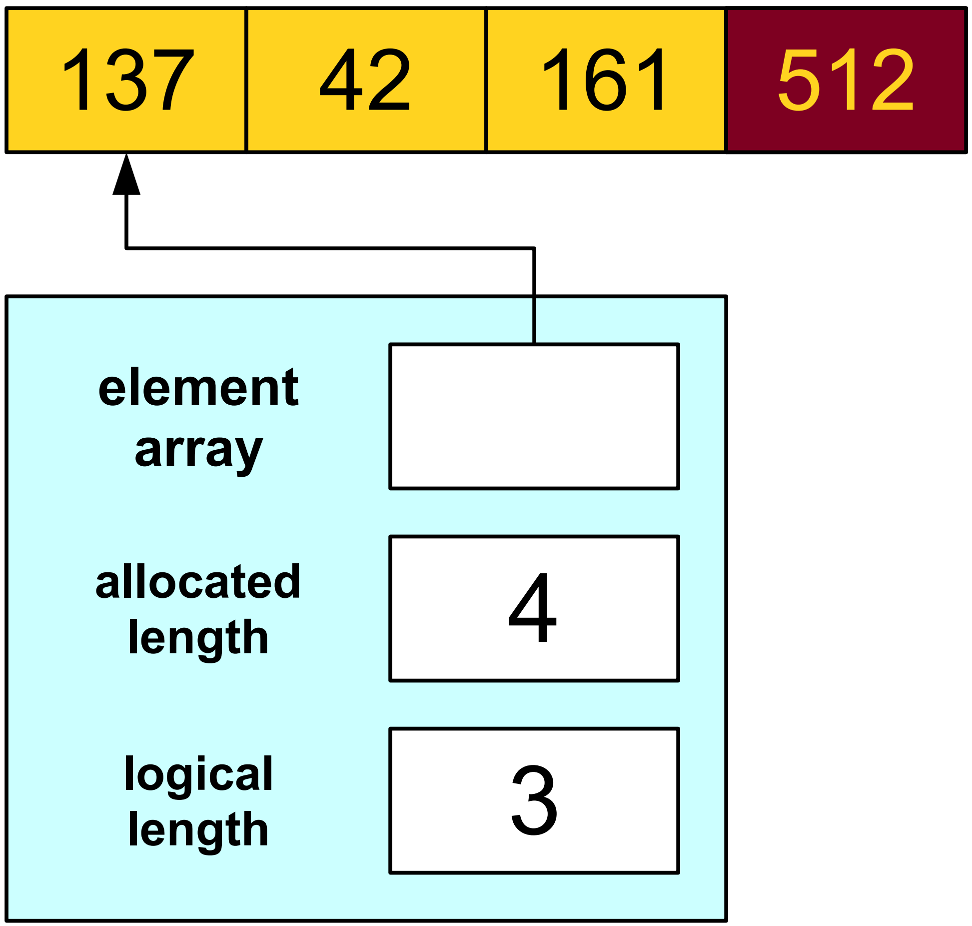

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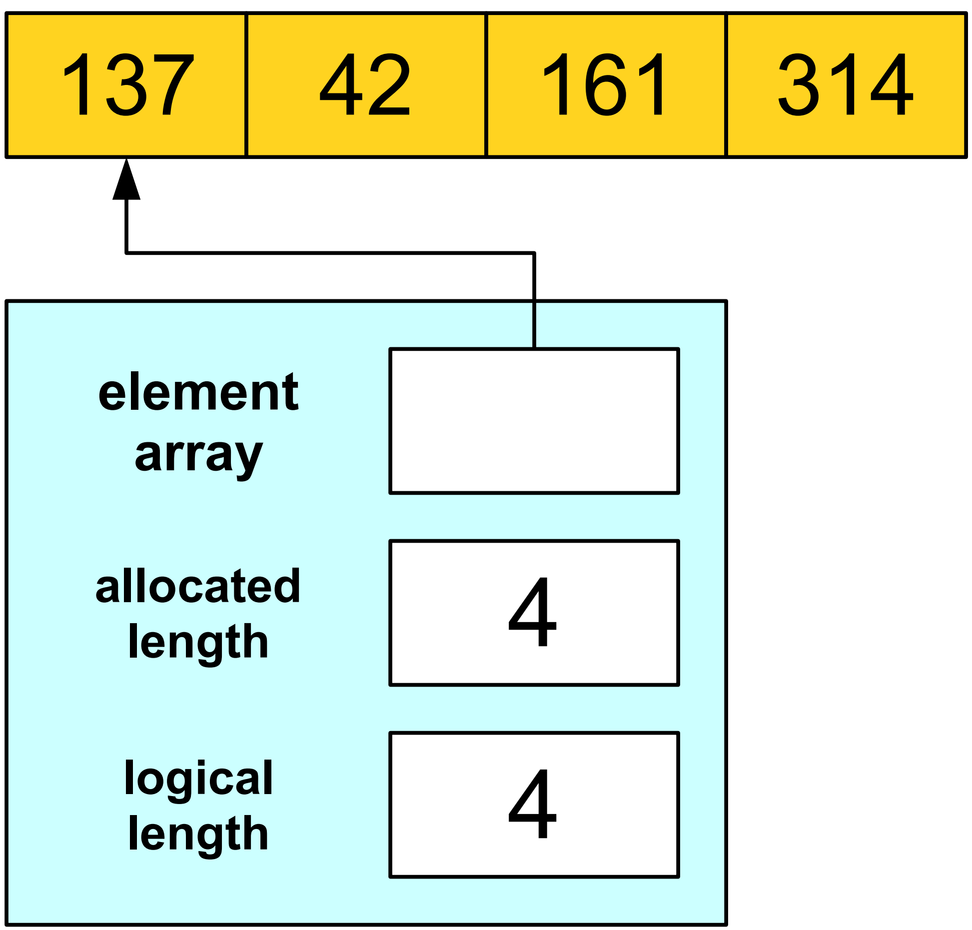
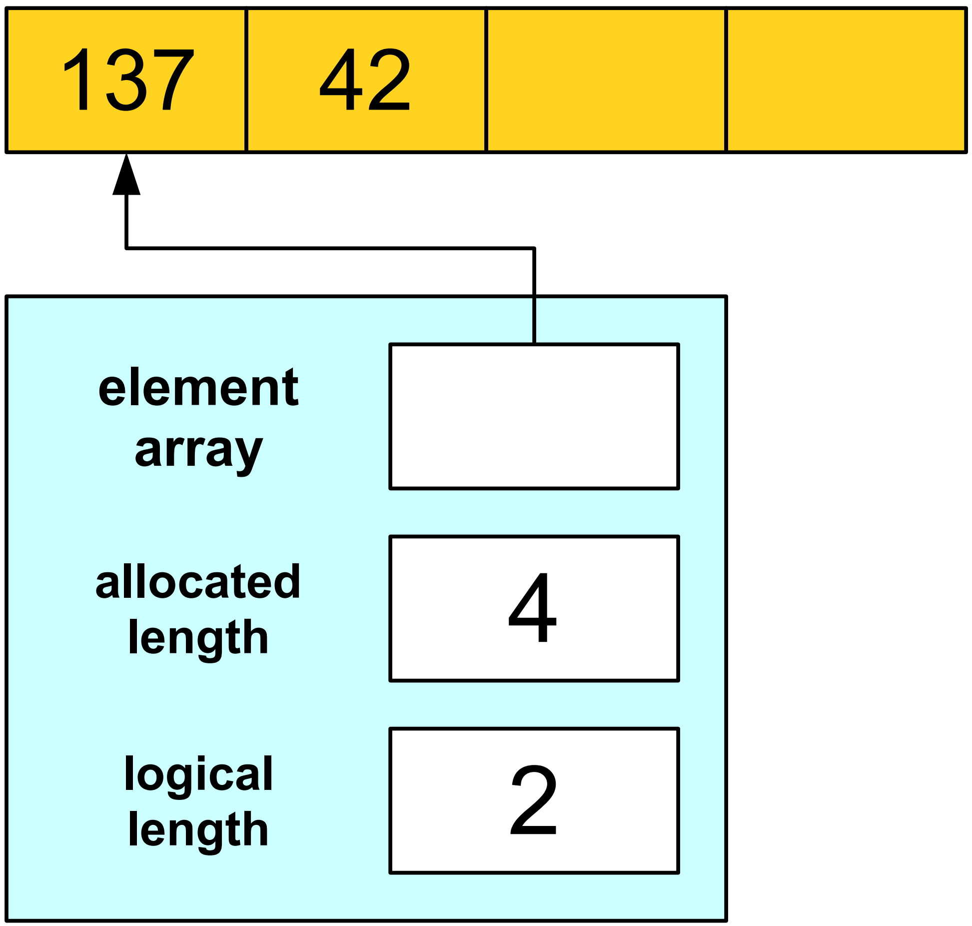


Finish Bounded Stack...

Running out of Space

- Our current implementation very quickly runs out of space to store elements.
- What should we do when this happens?

An Initial Idea



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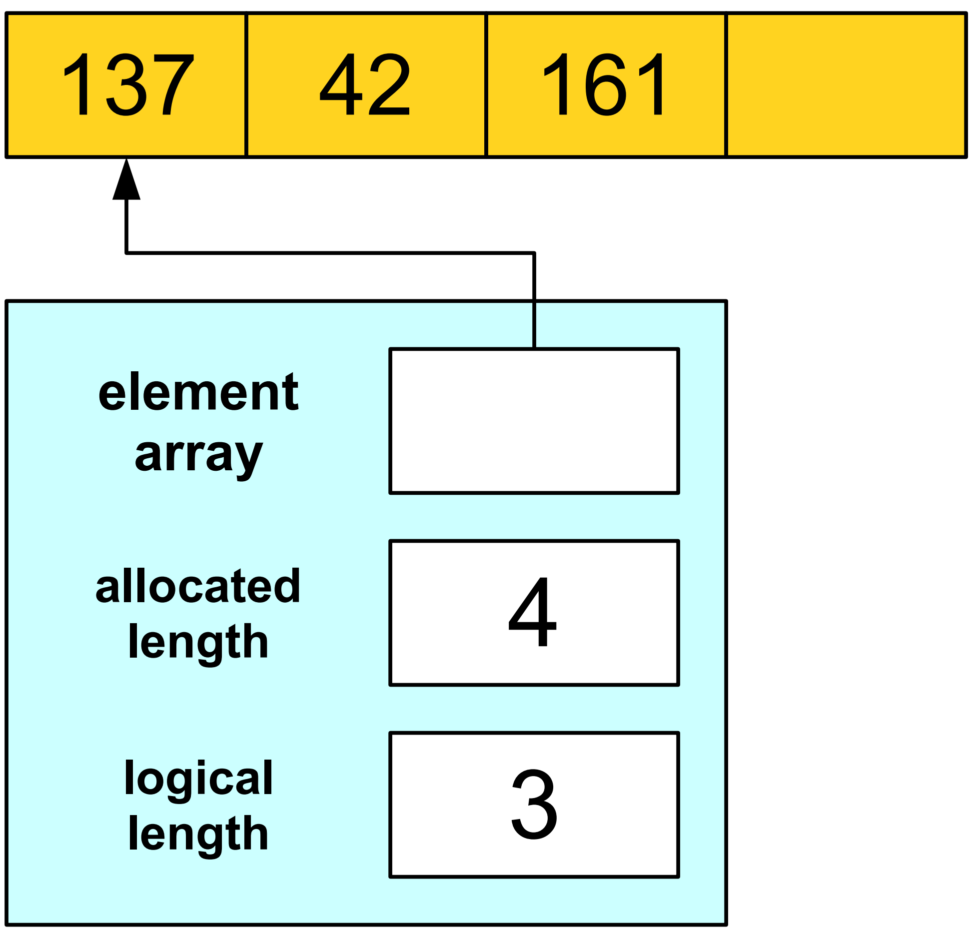

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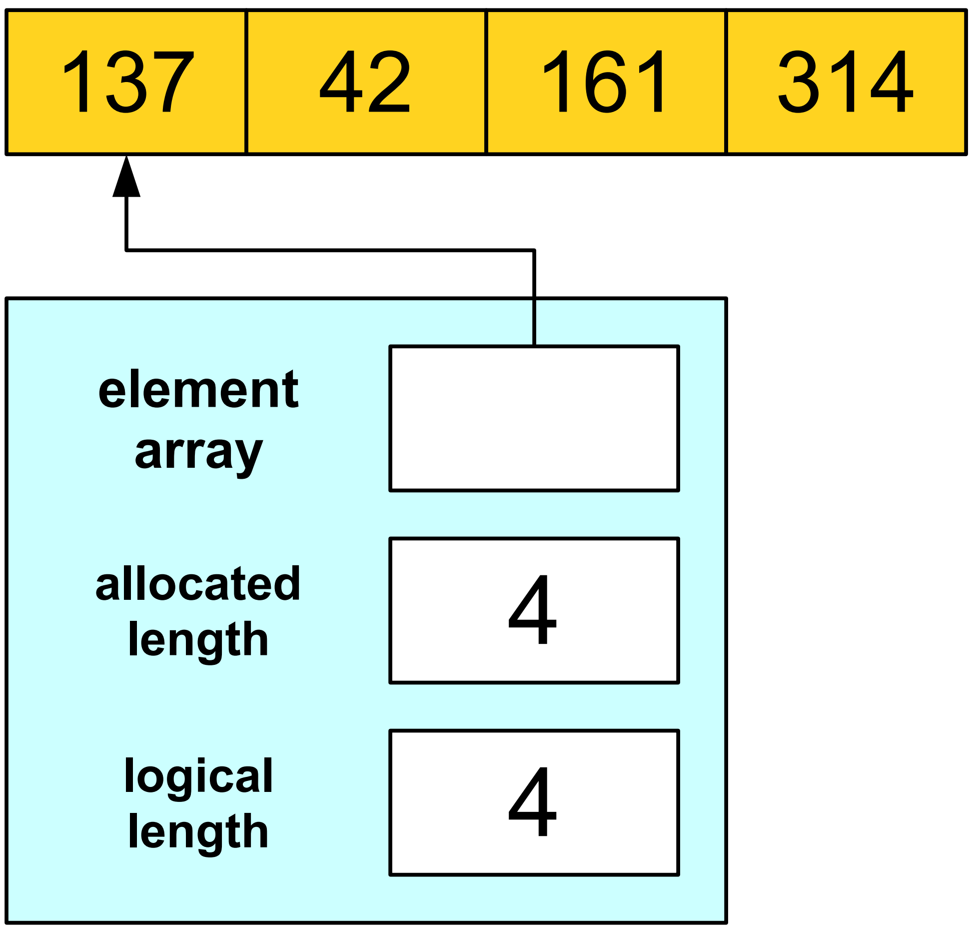

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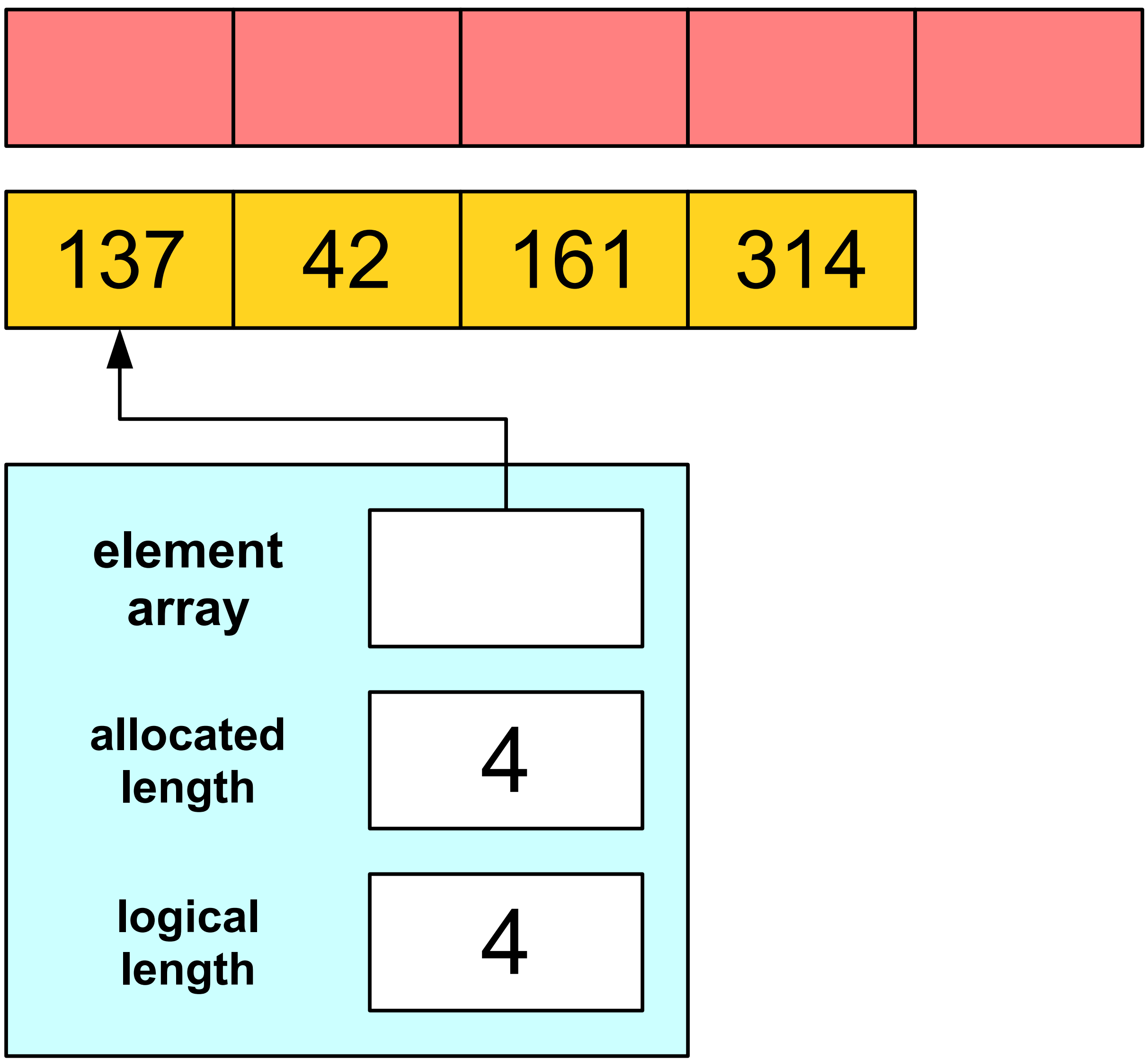

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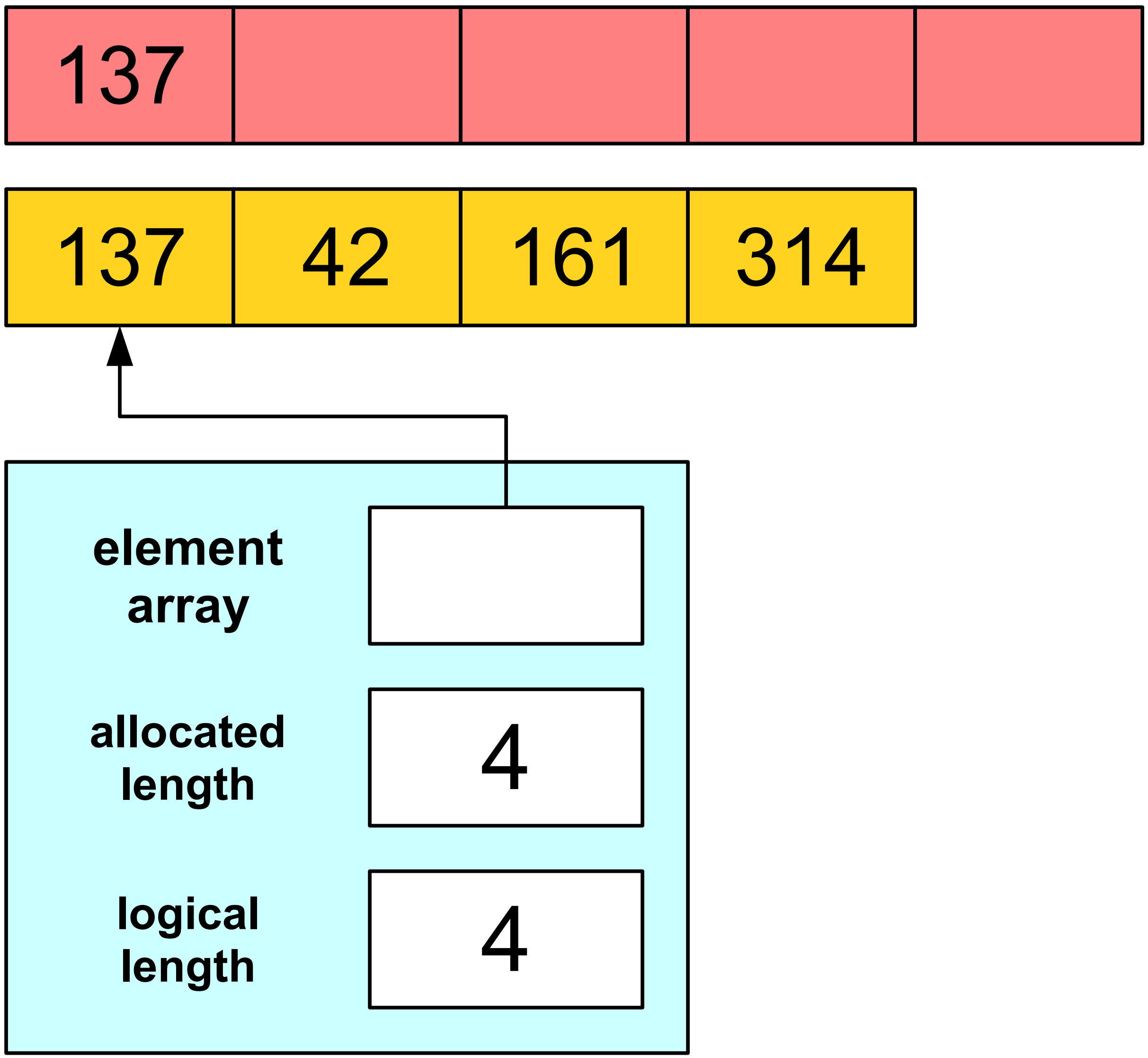

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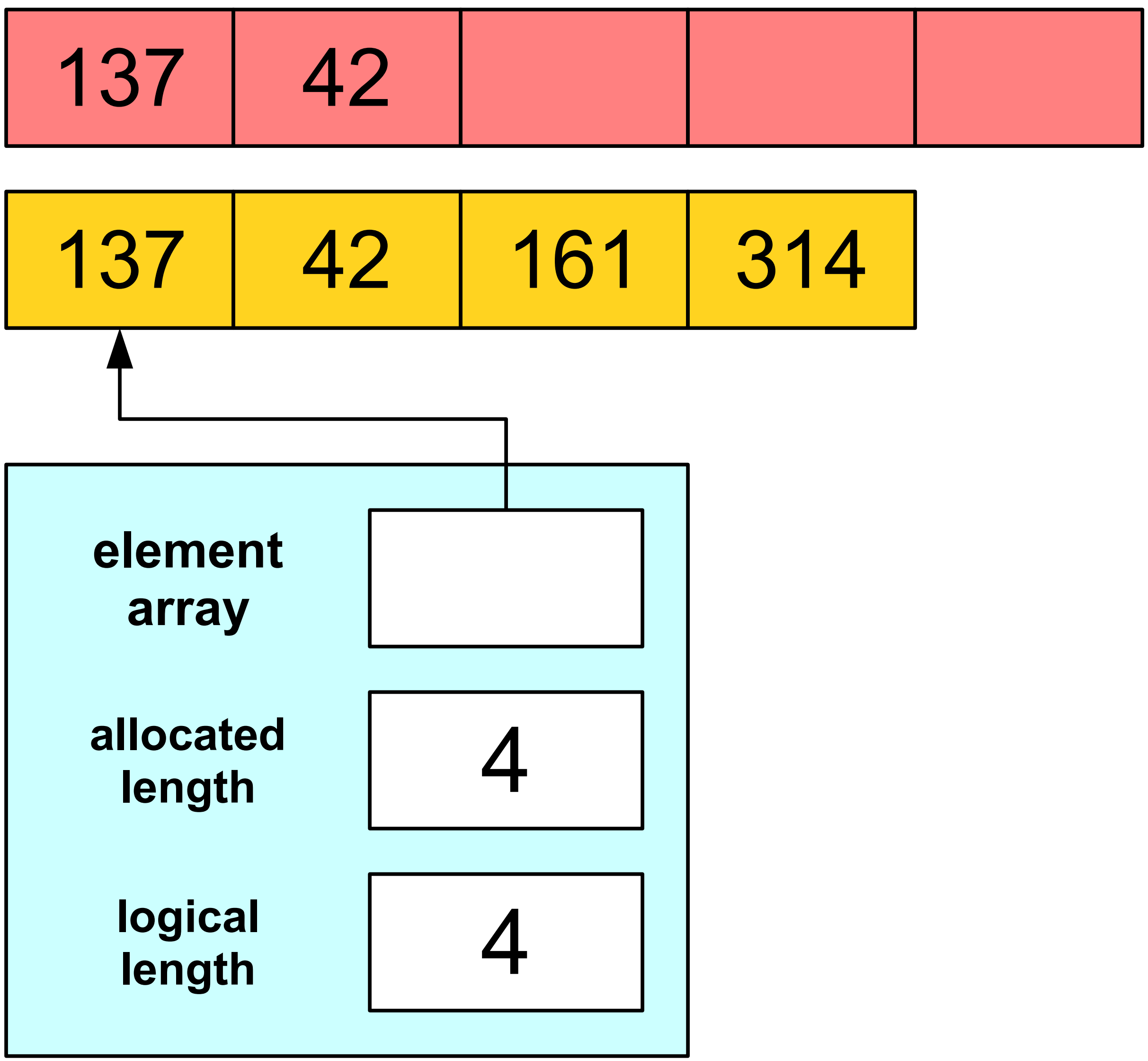

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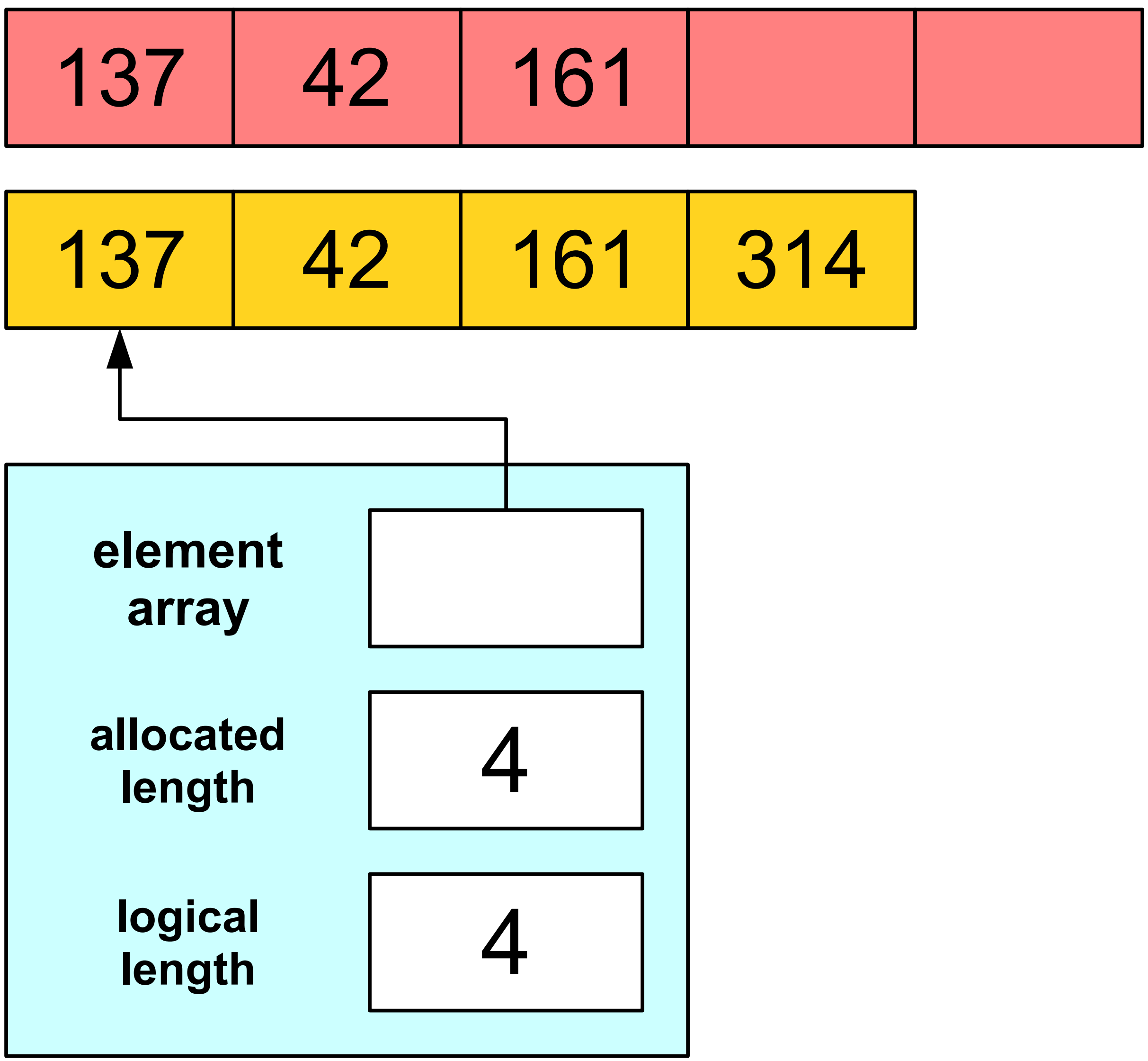

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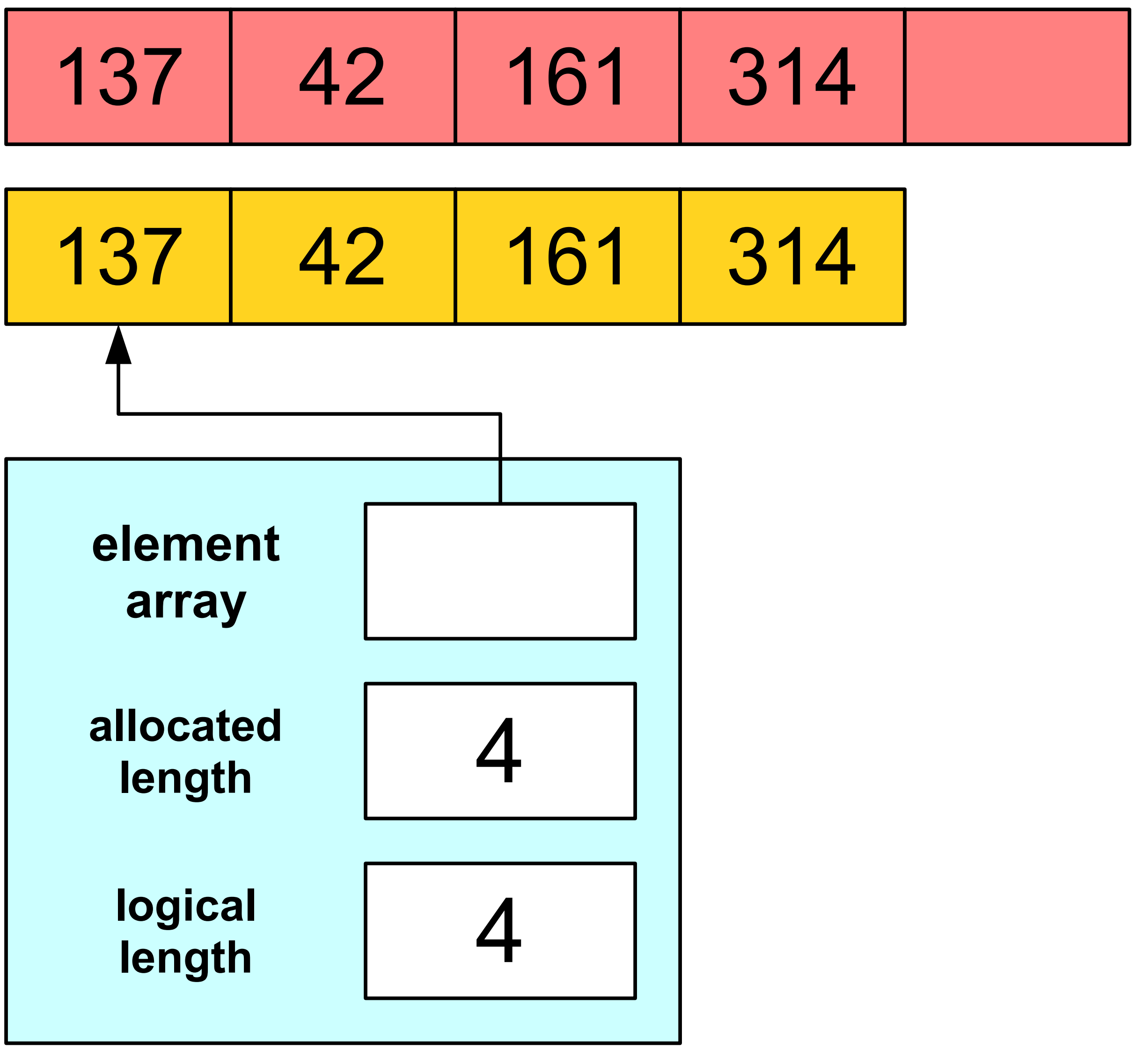

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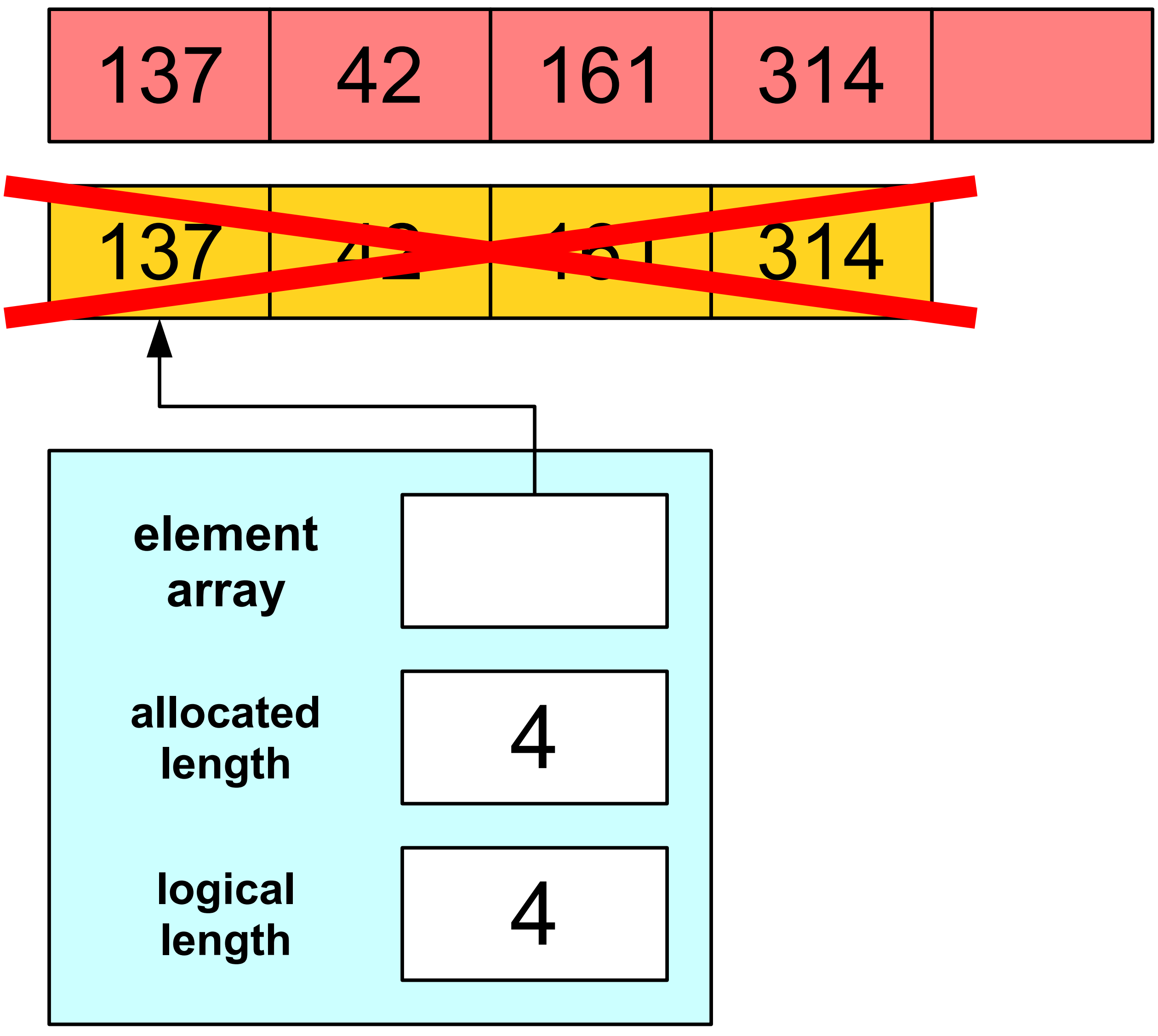

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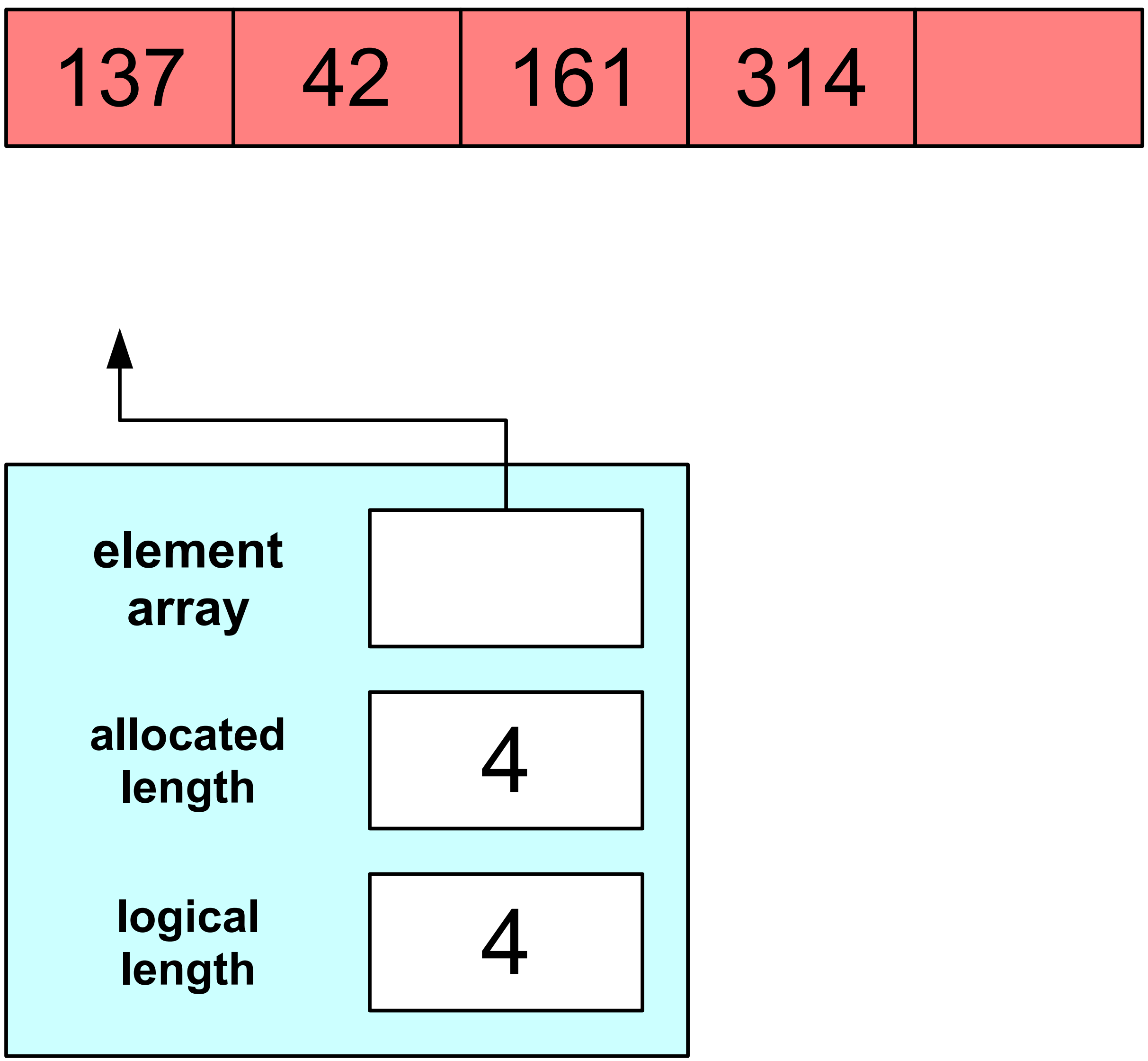

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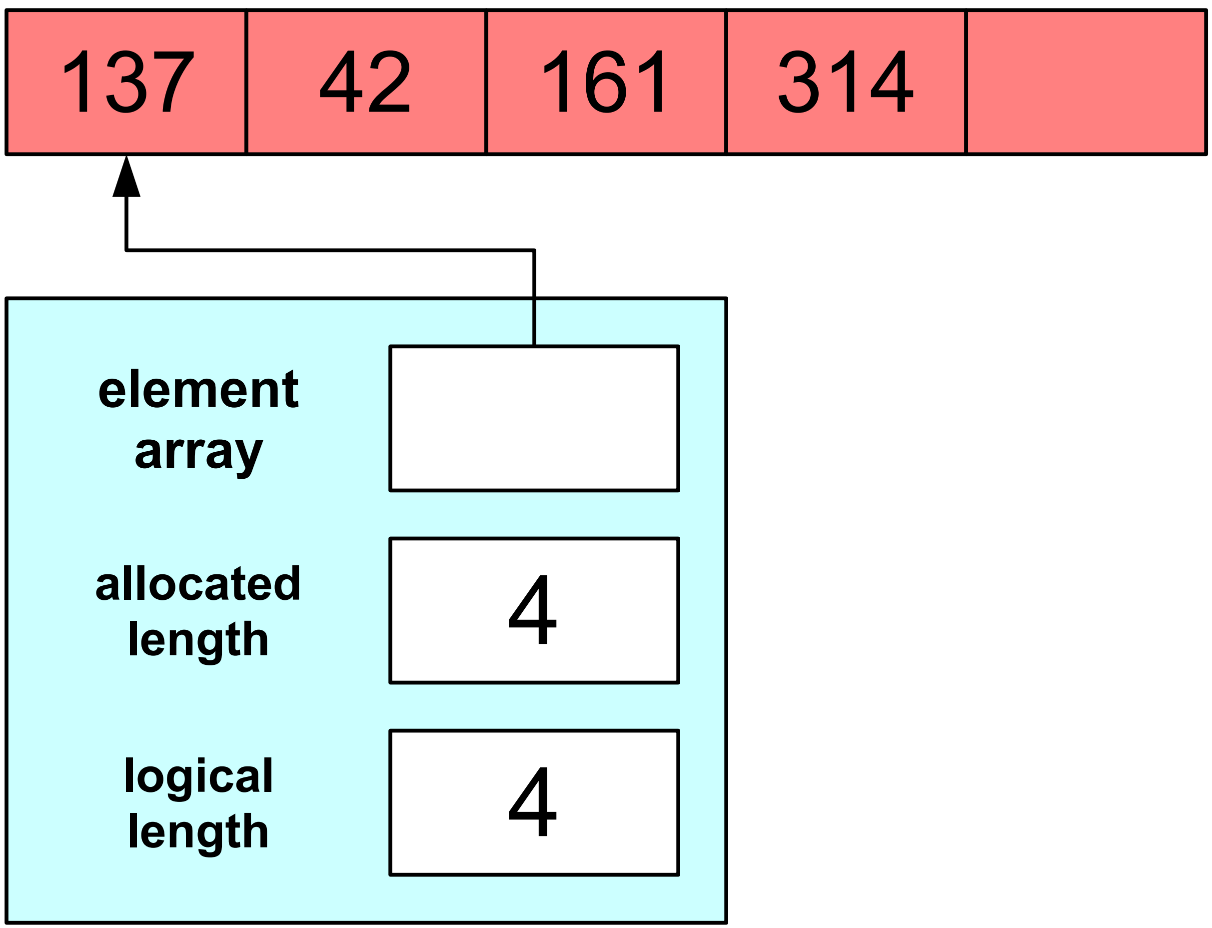

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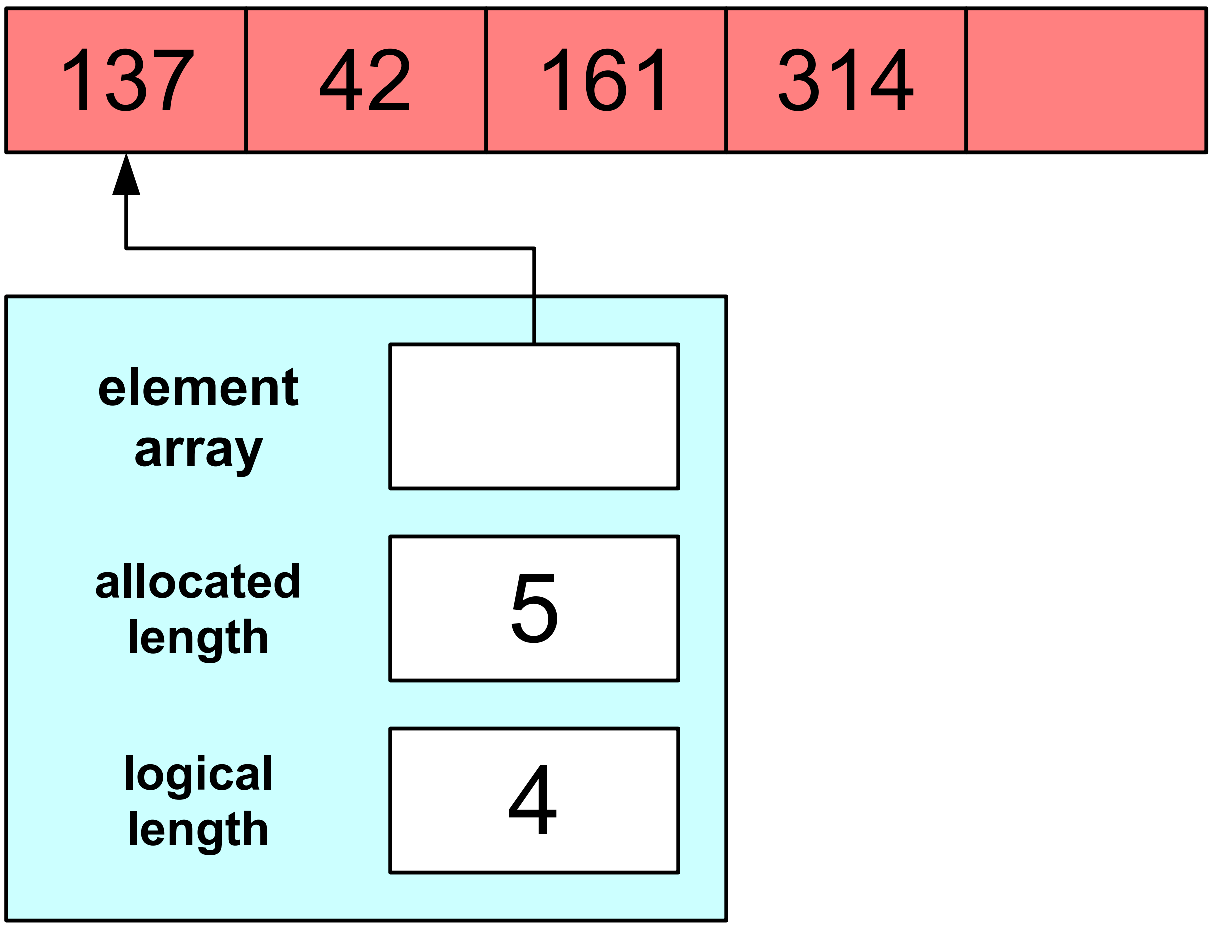

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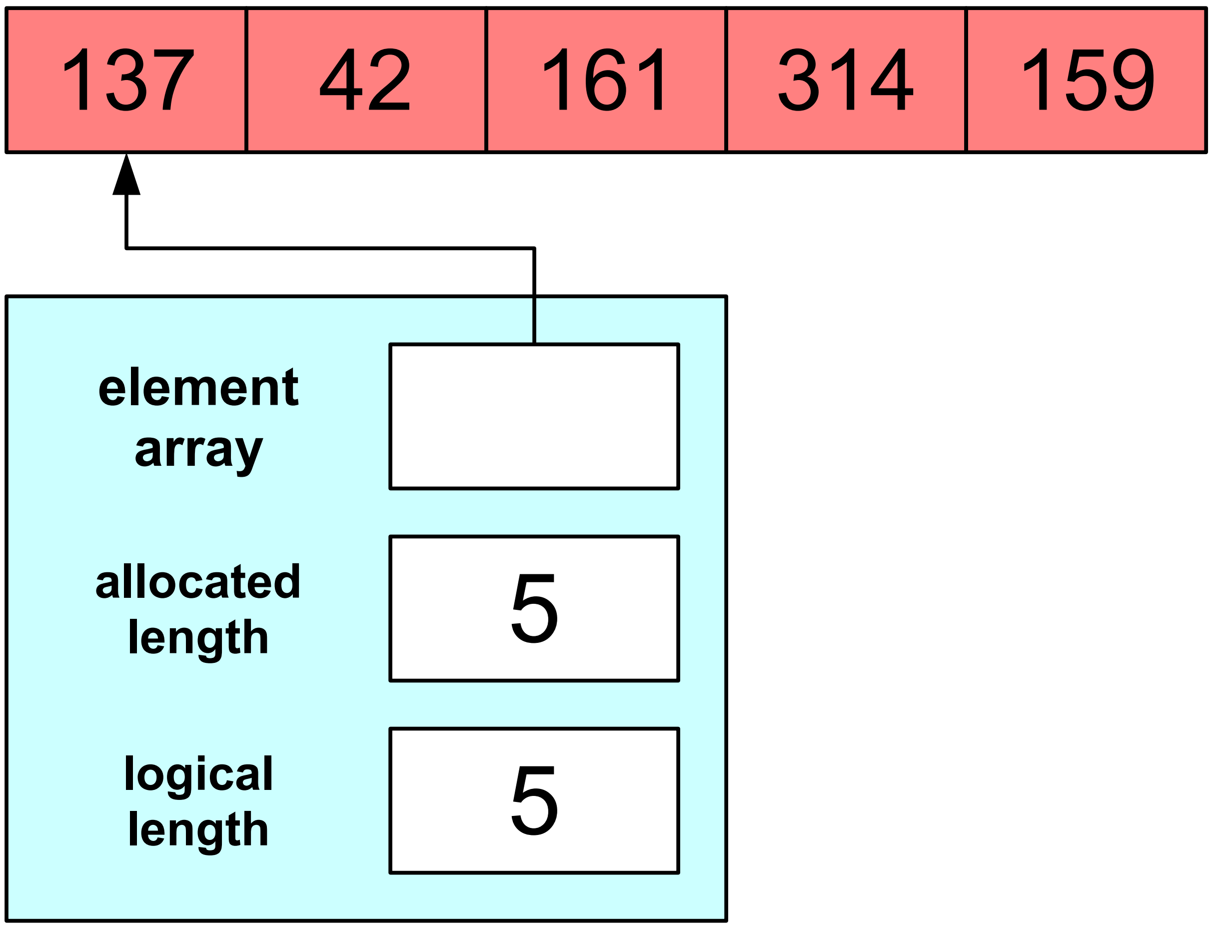

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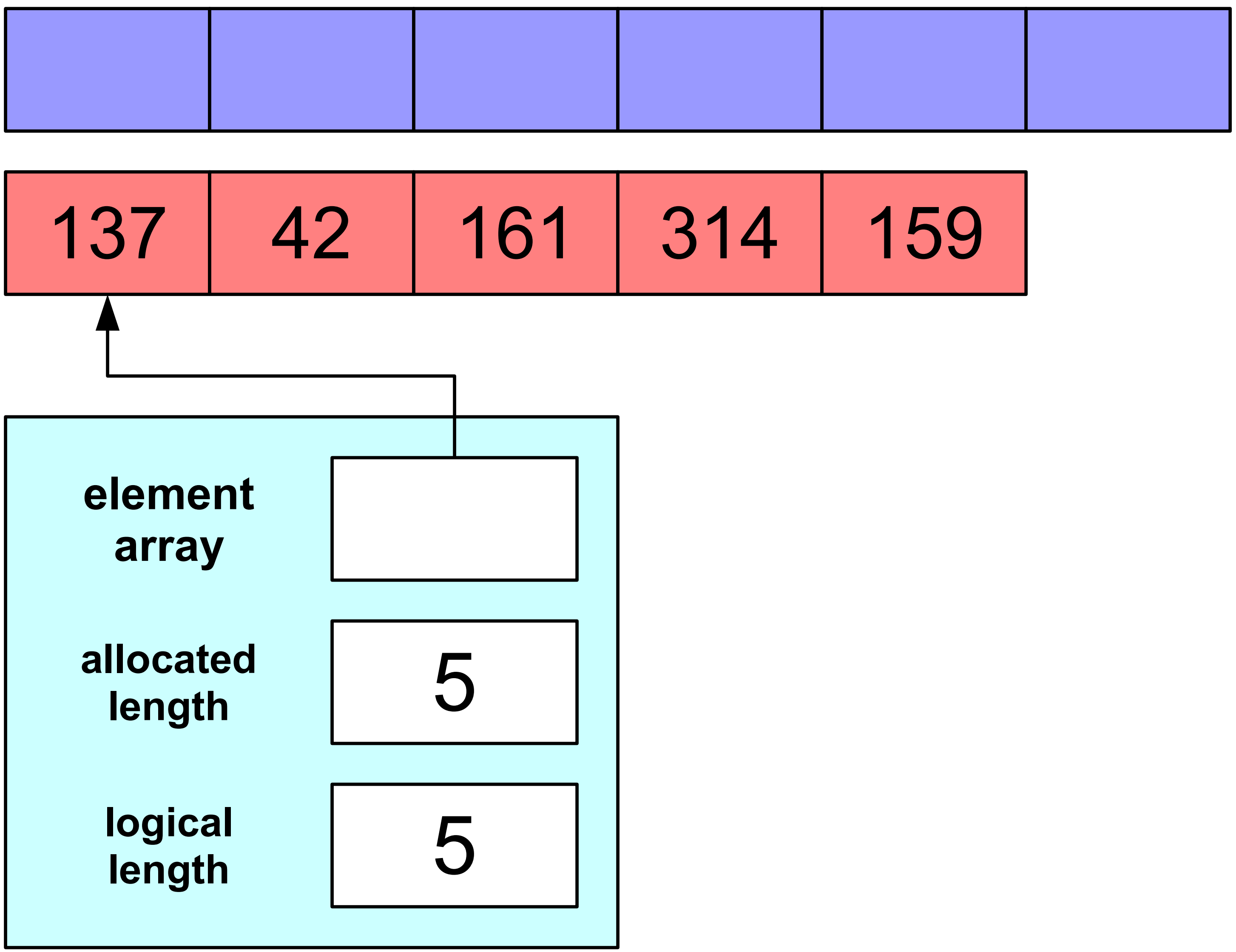

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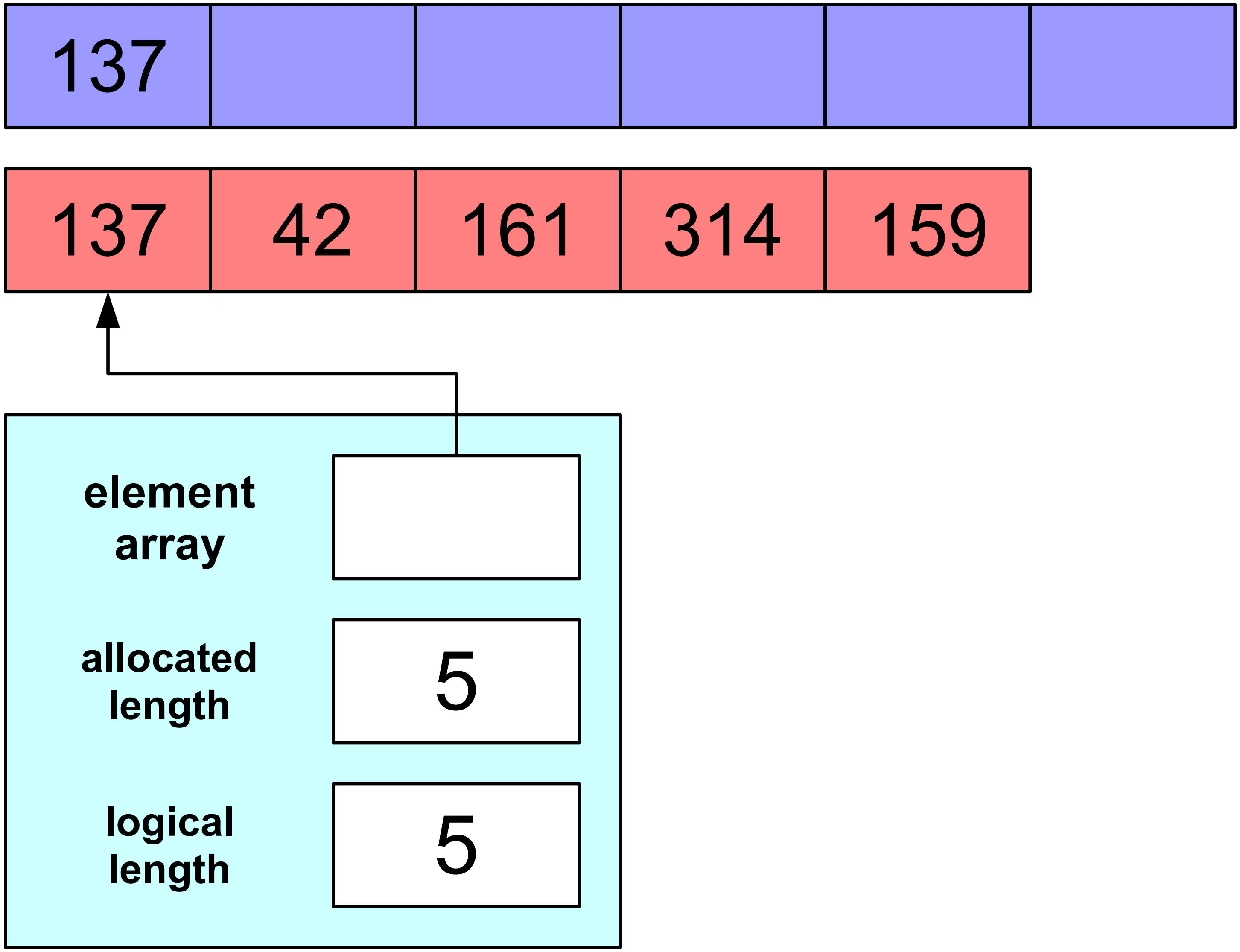

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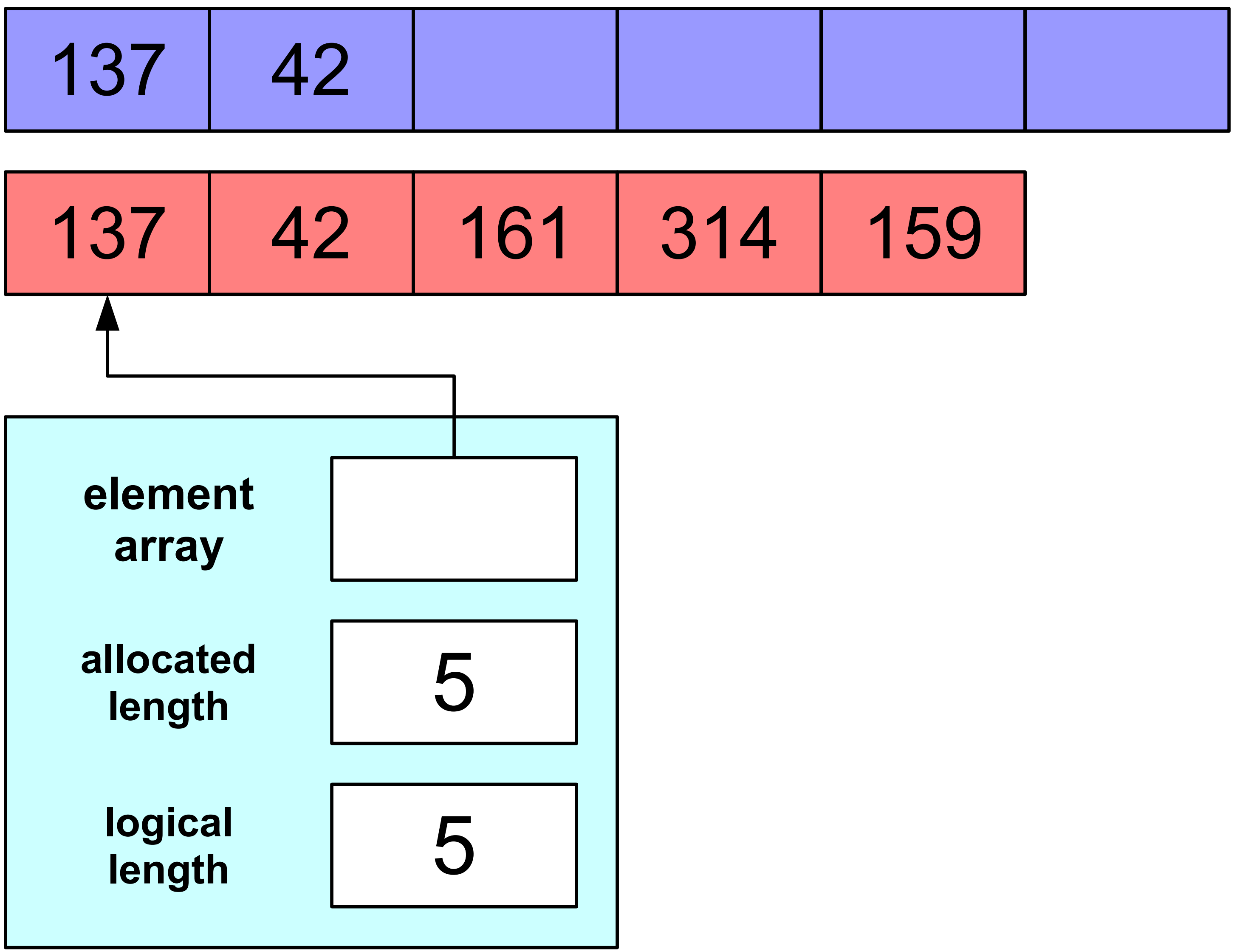

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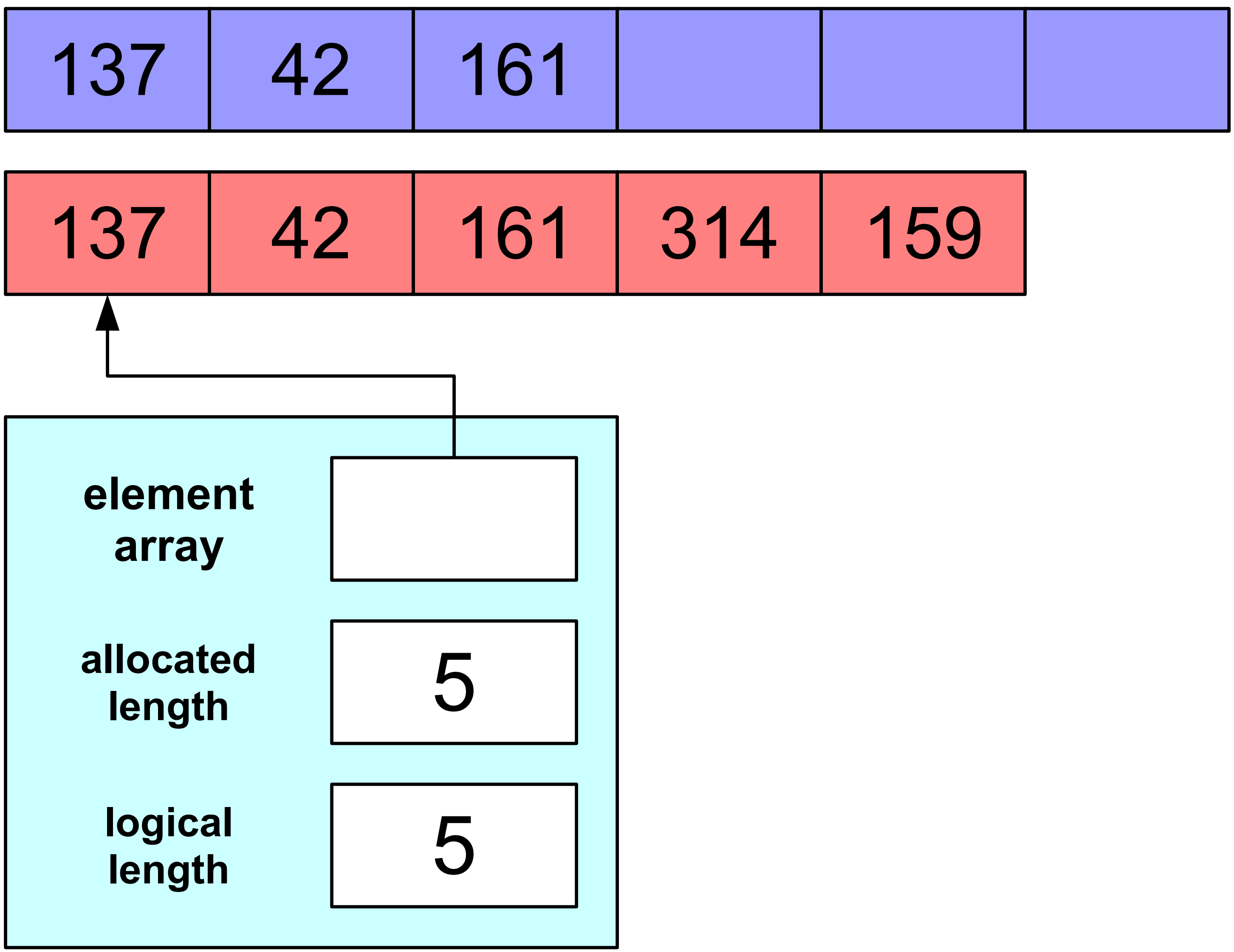

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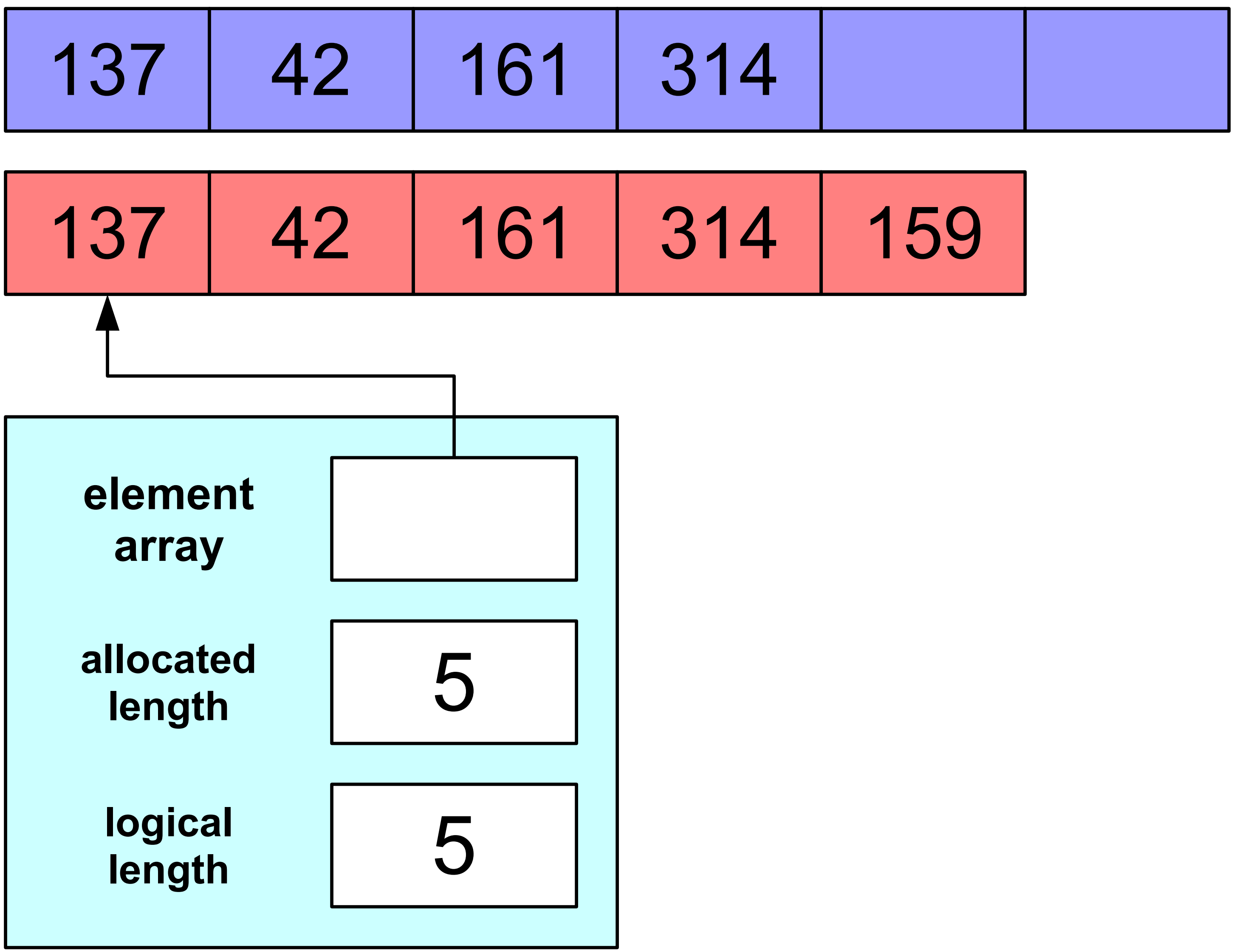

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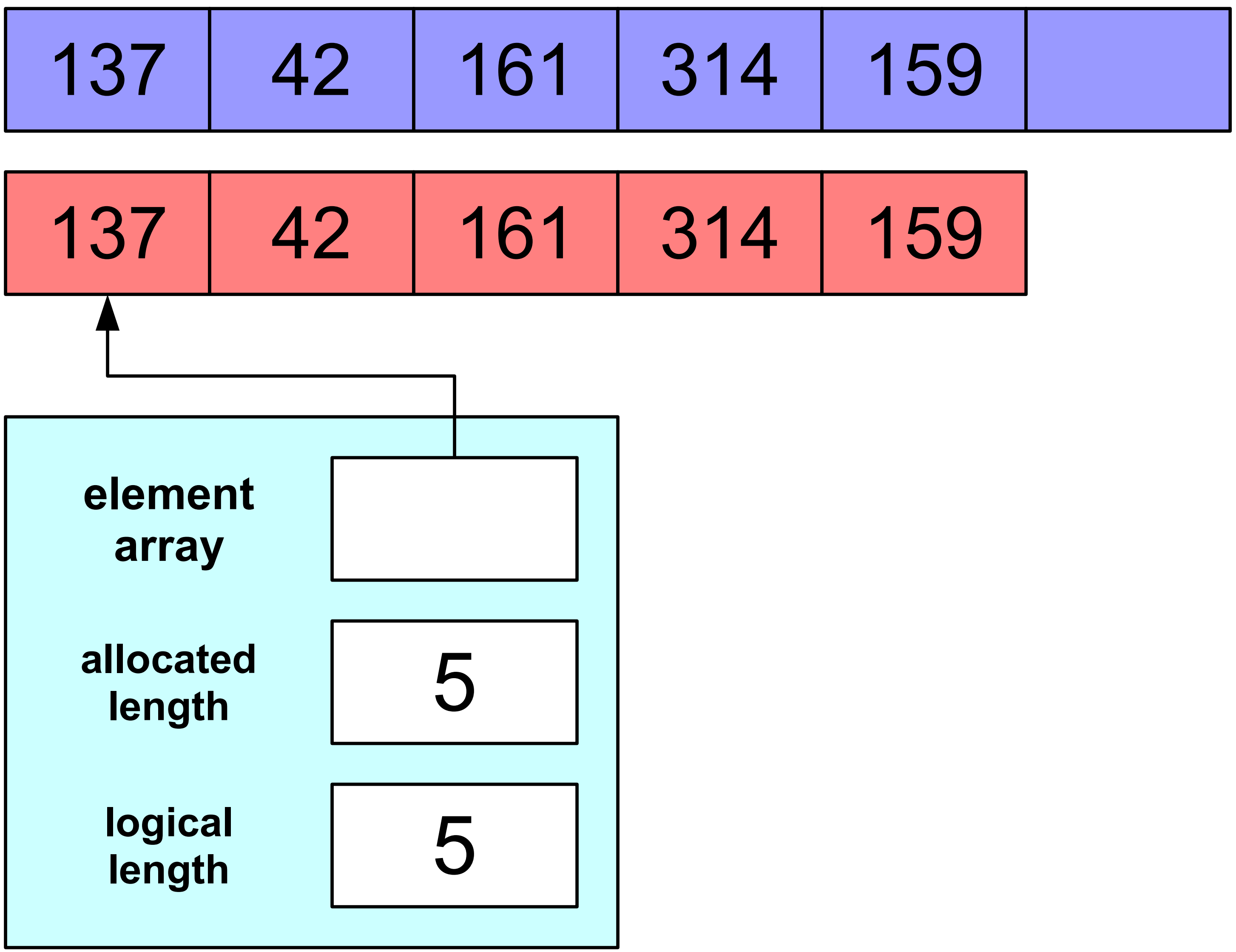

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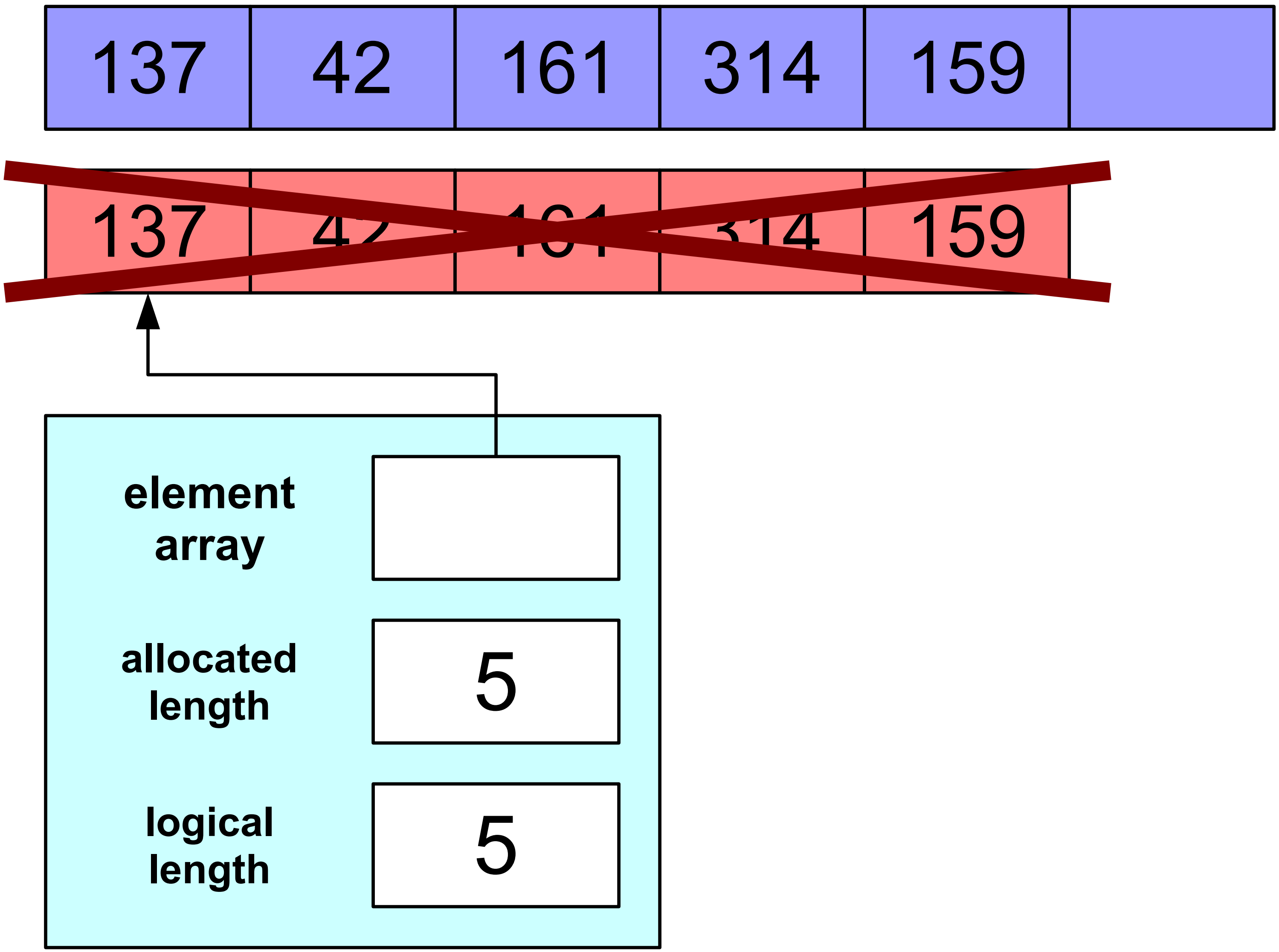

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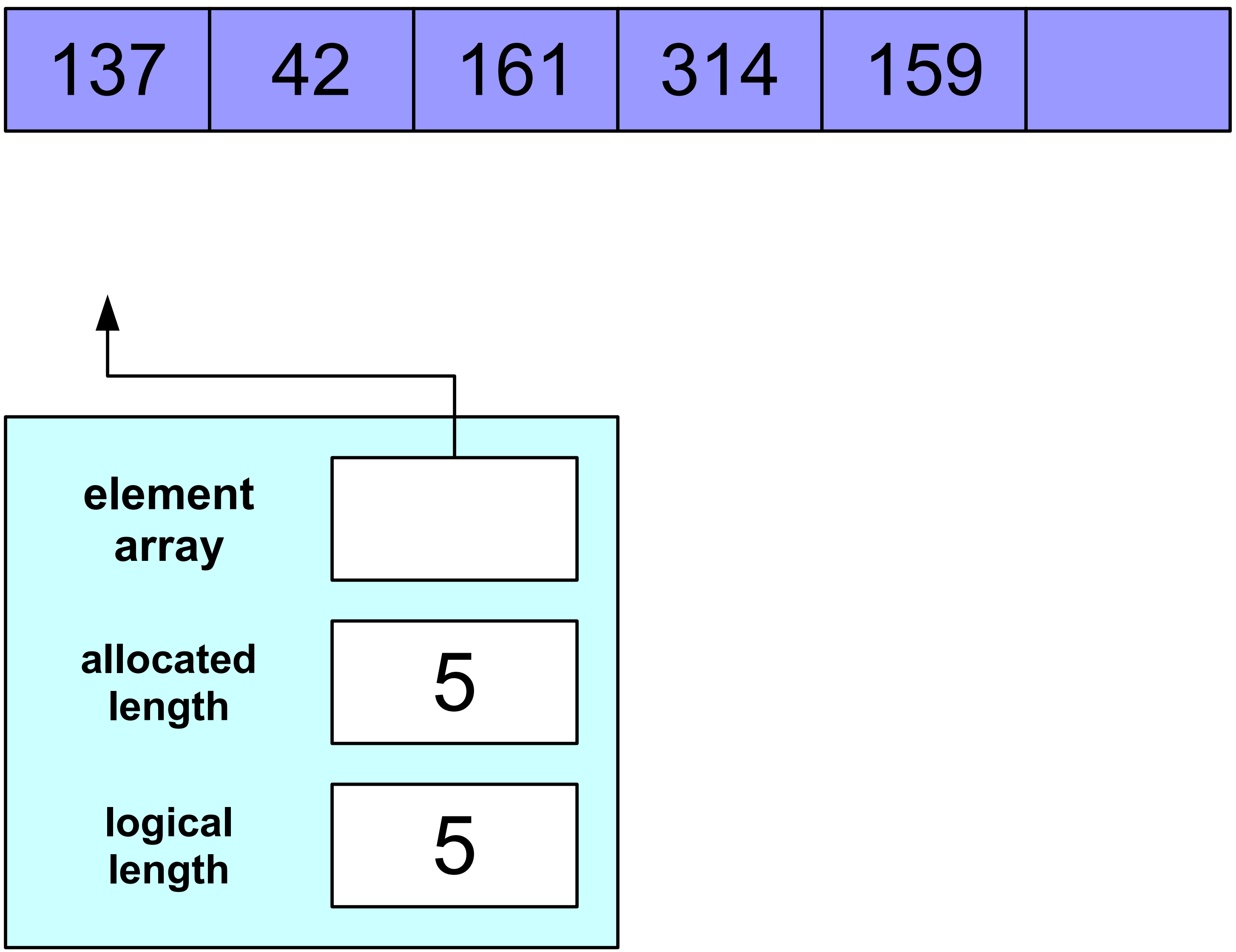

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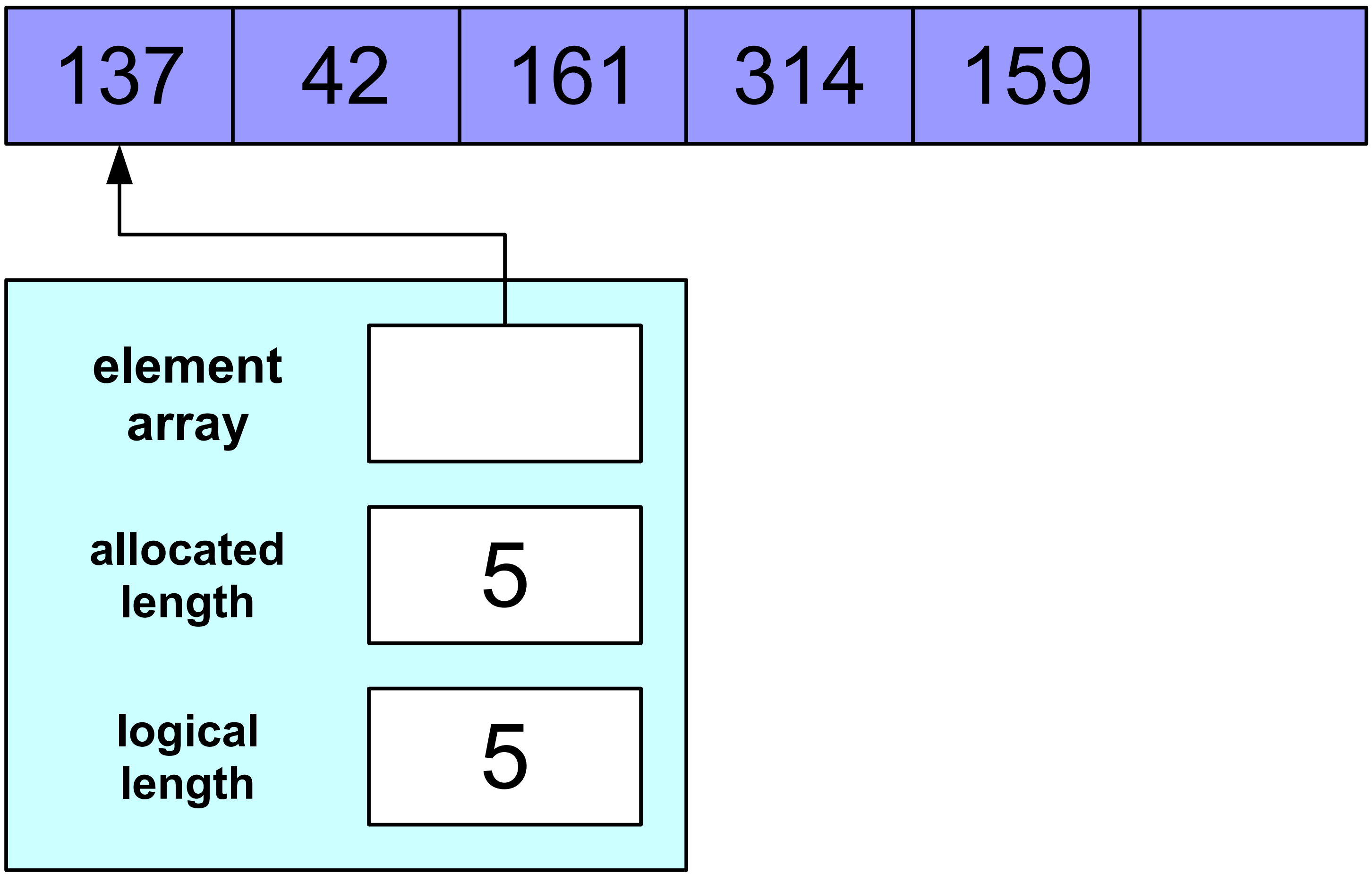

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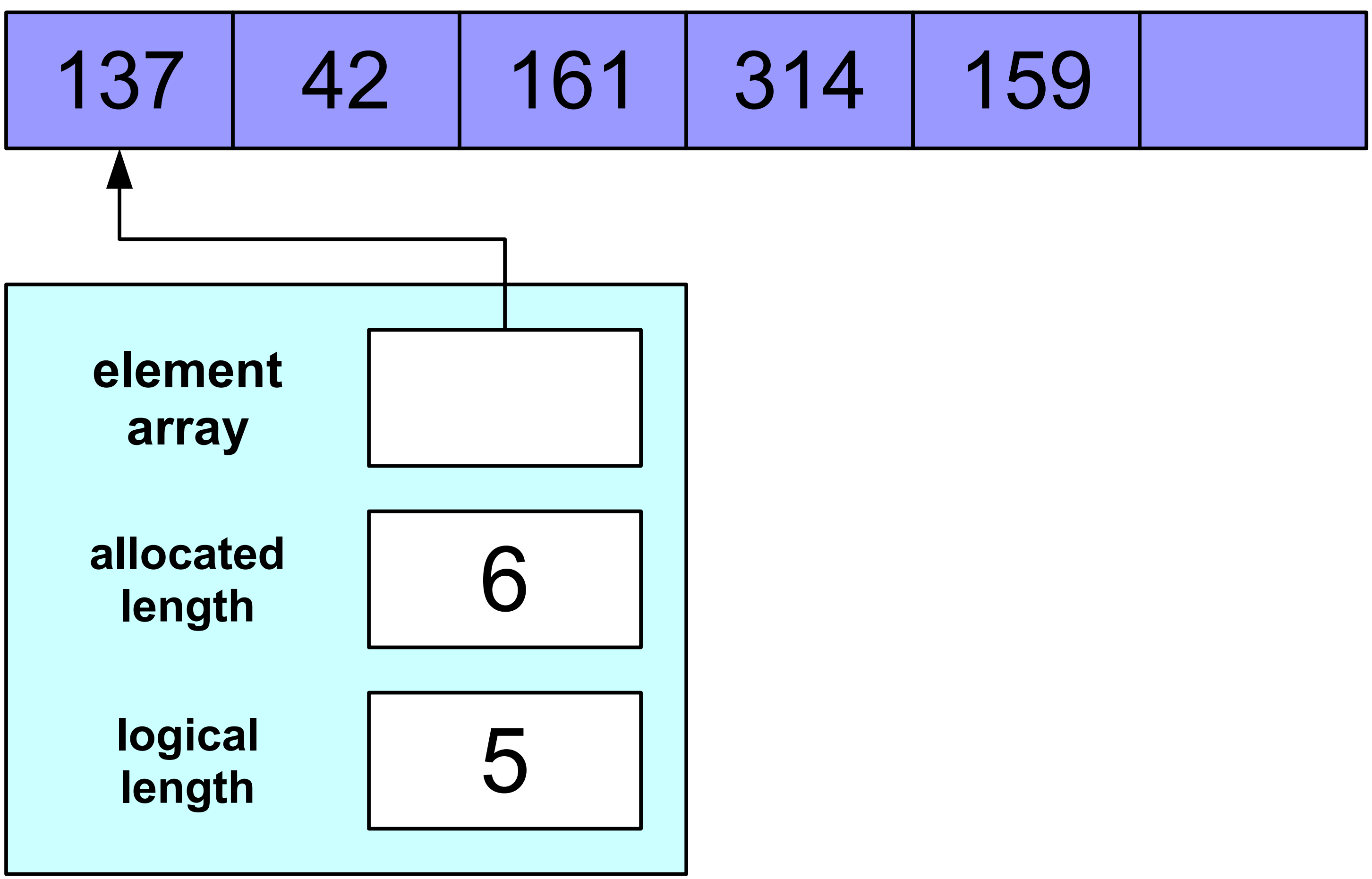

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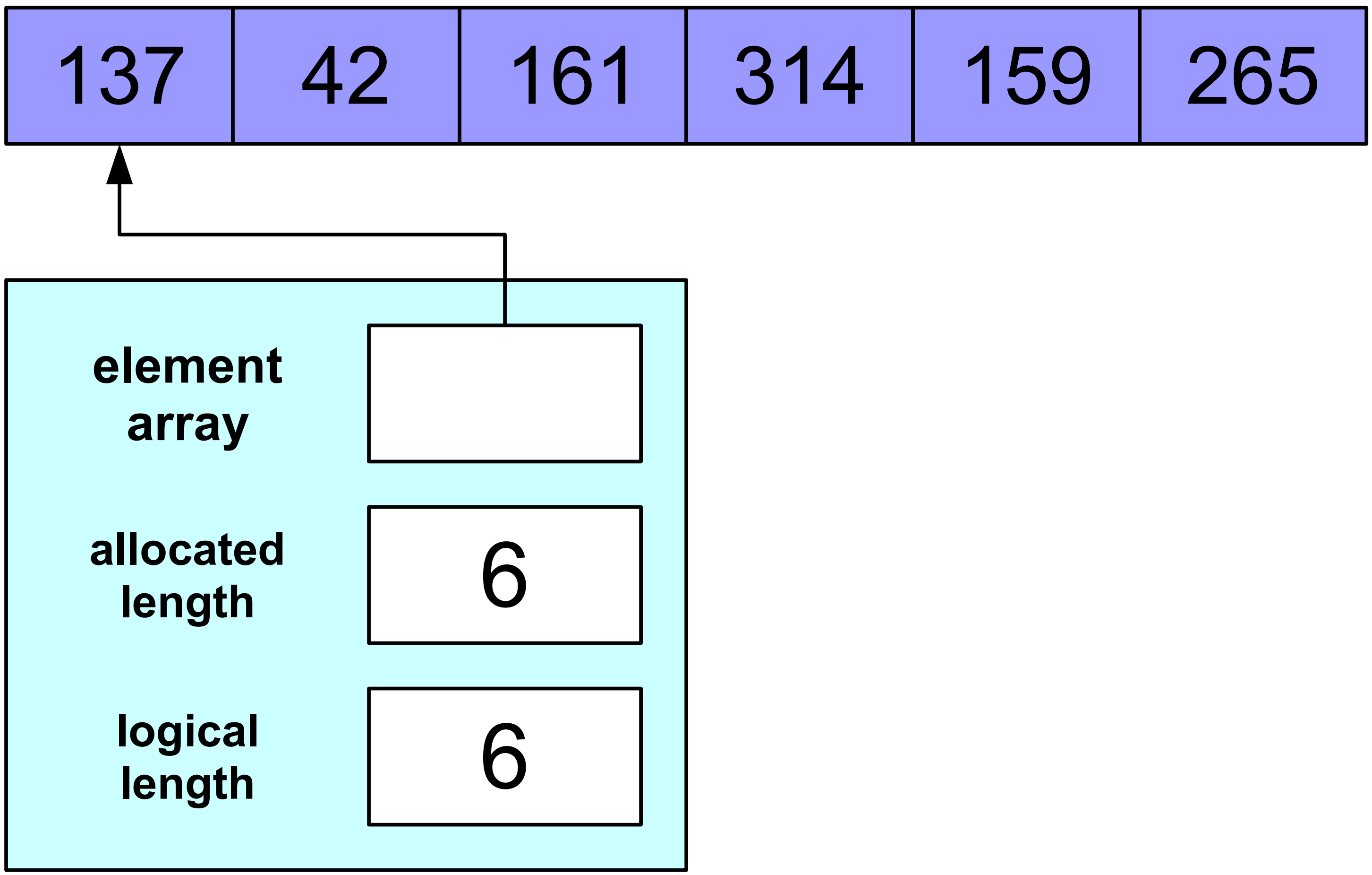

OurStack::grow()

Analyzing Our Approach

- We now have a working solution, but is it an *efficient* solution?
- Let's analyze the big-O complexity of the five operations.
  - **`size`**:
  - **`isEmpty`**:
  - **`push`**:
  - **`pop`**:
  - **`top`**:

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  - **`size`**: **O(1)**
  - **`isEmpty`**: **O(1)**
  - **`push`**: **O(*n*)**
  - **`pop`**: **O(1)**
  - **`top`**: **O(1)**

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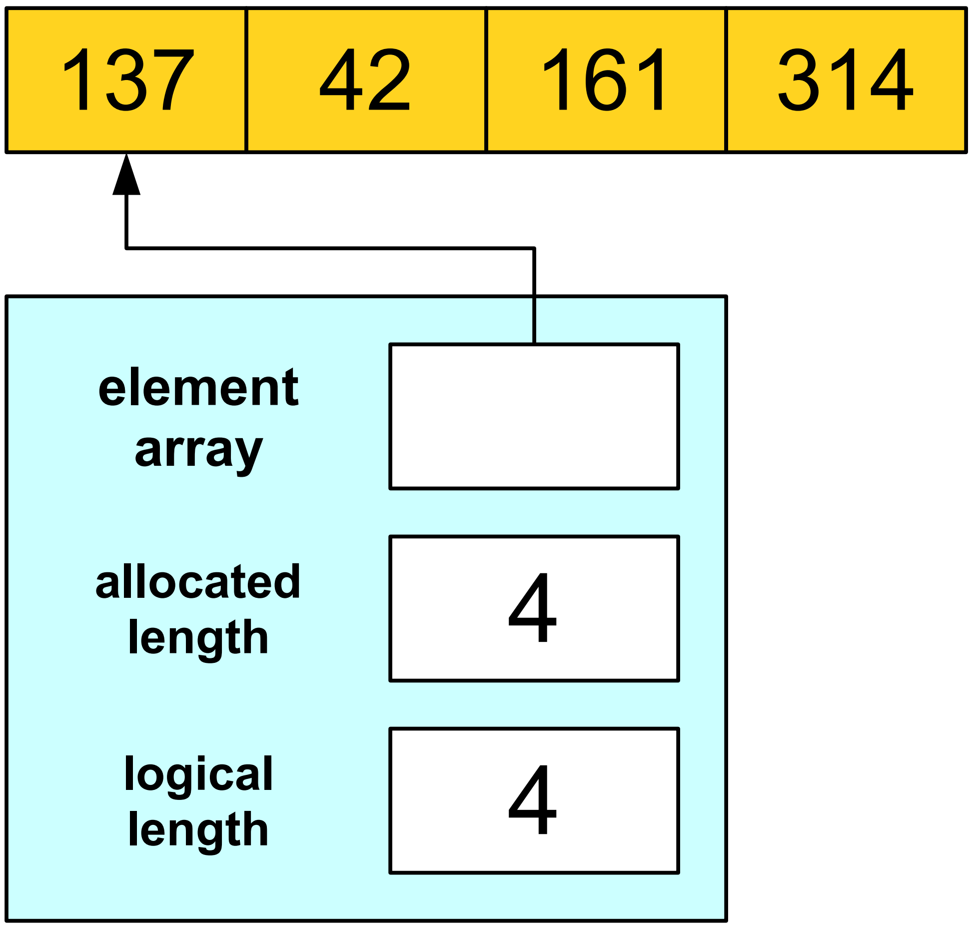
- Total cost: **$O(n^2)$**

Validating Our Model

Speeding up the Stack

A Better Idea



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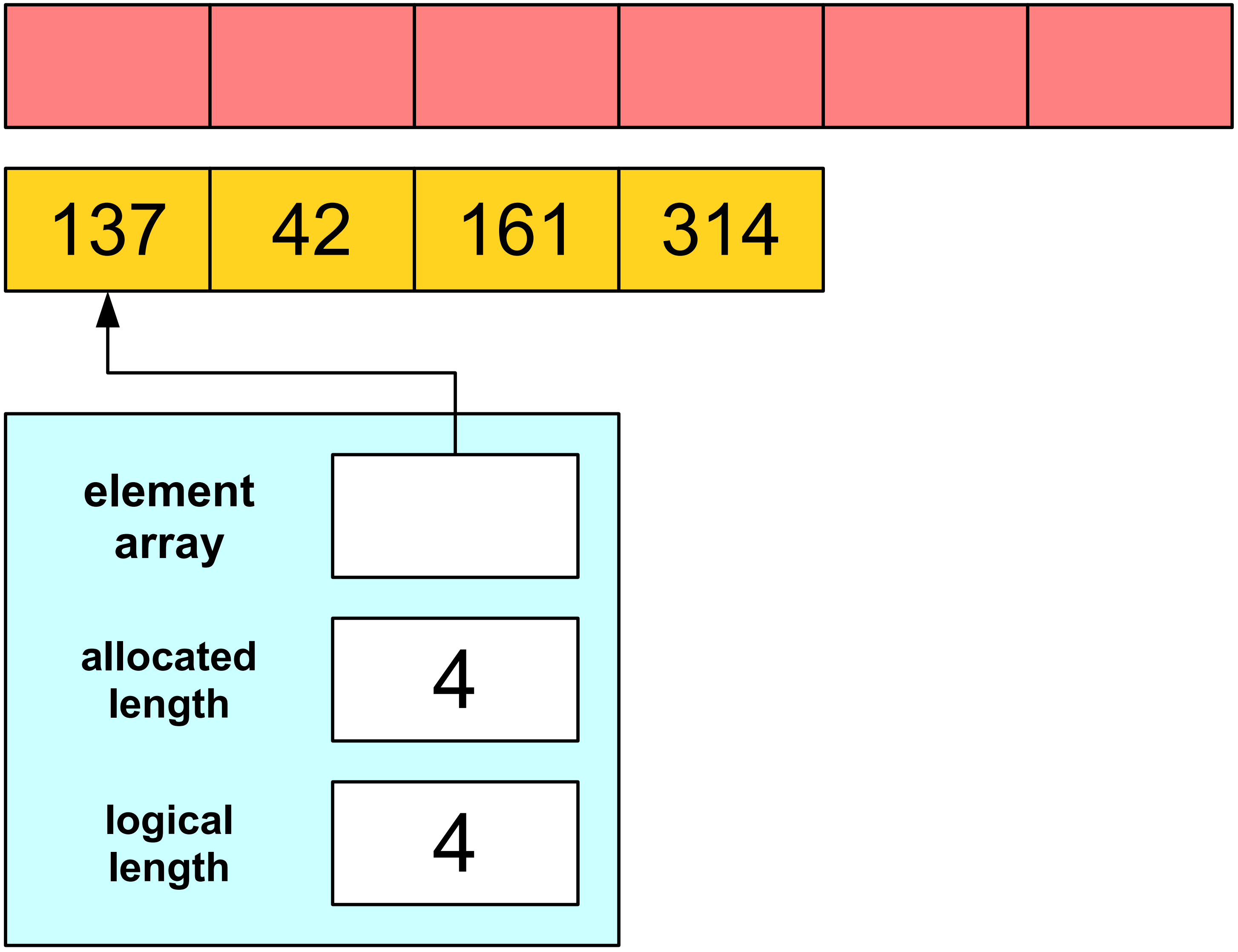

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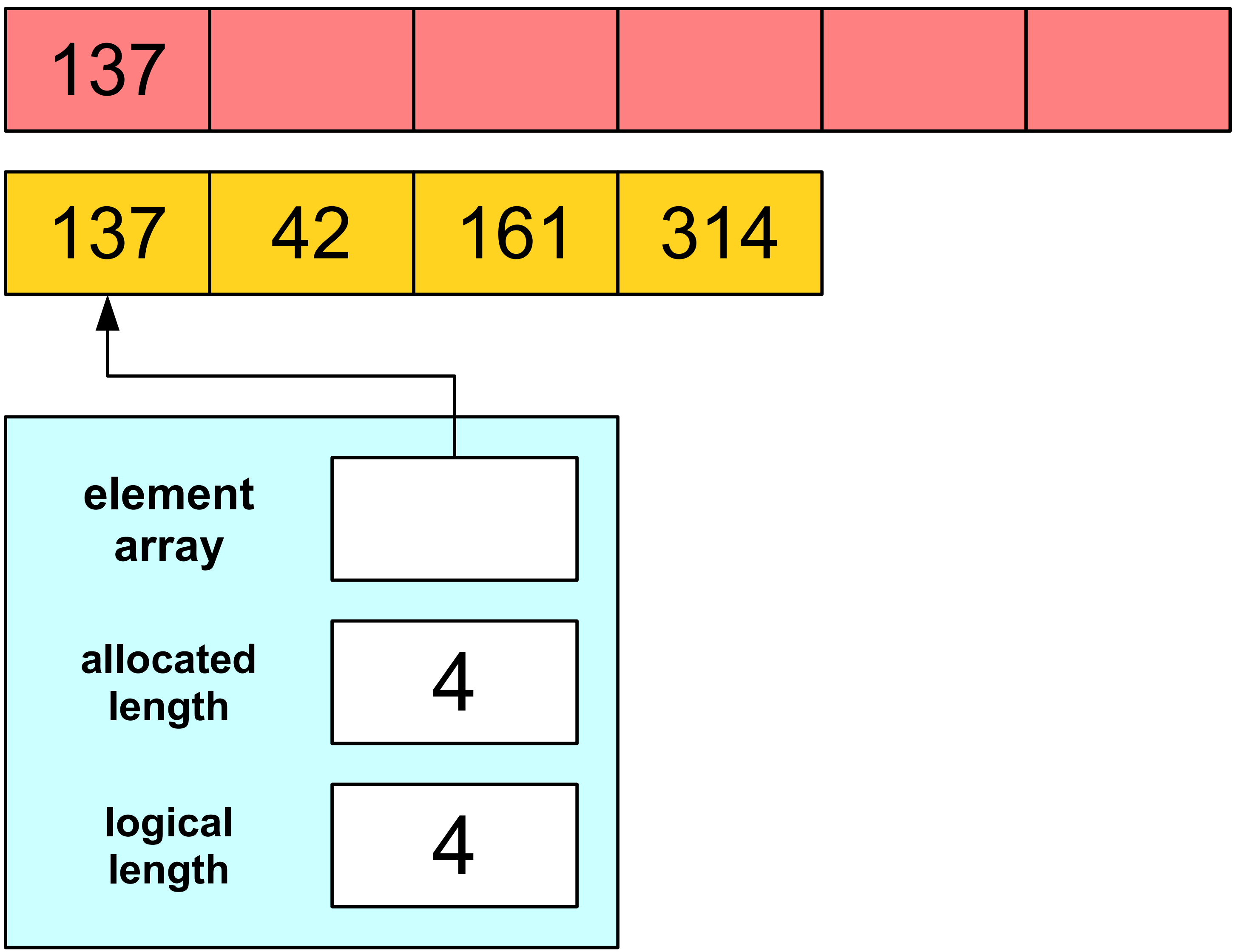

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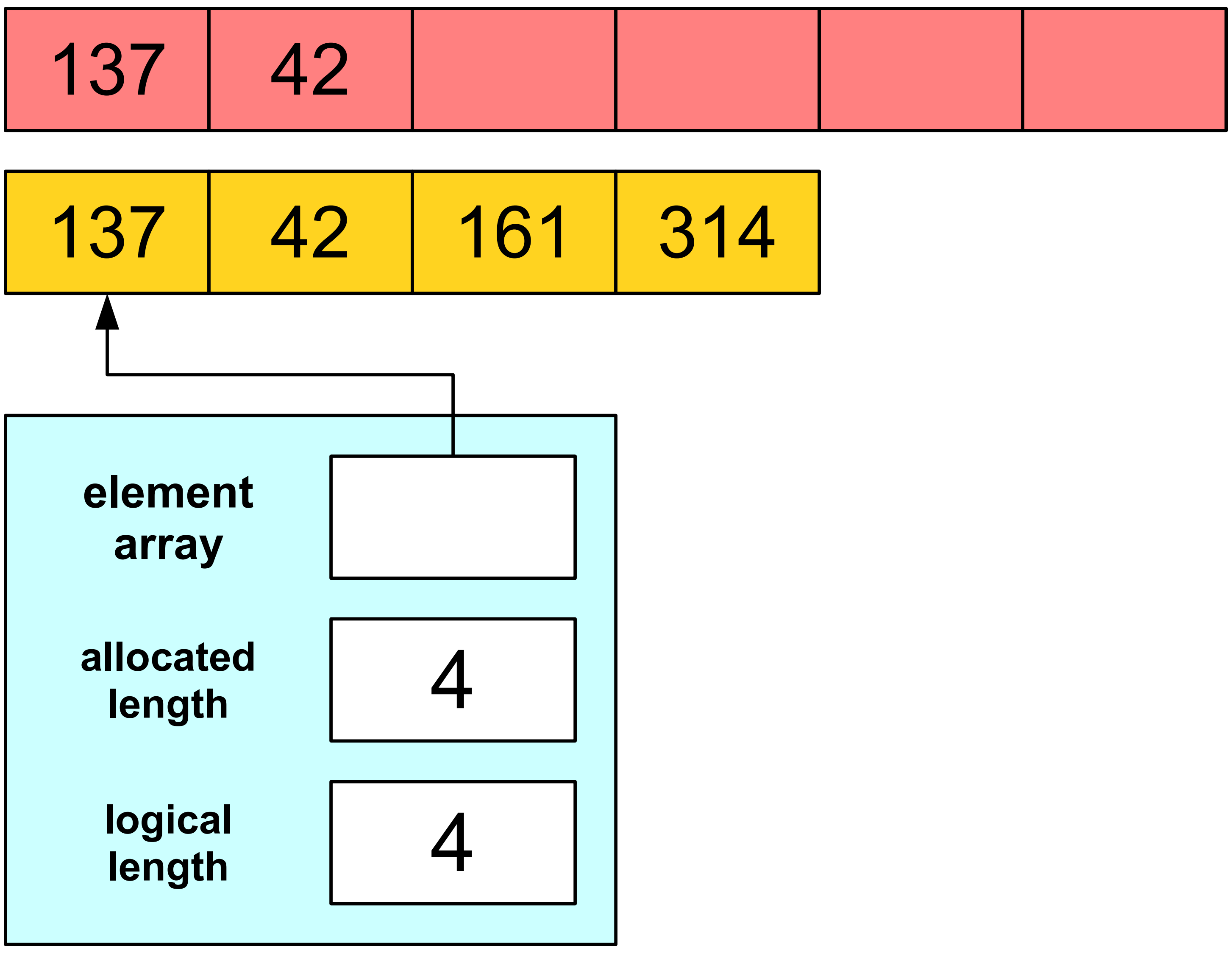

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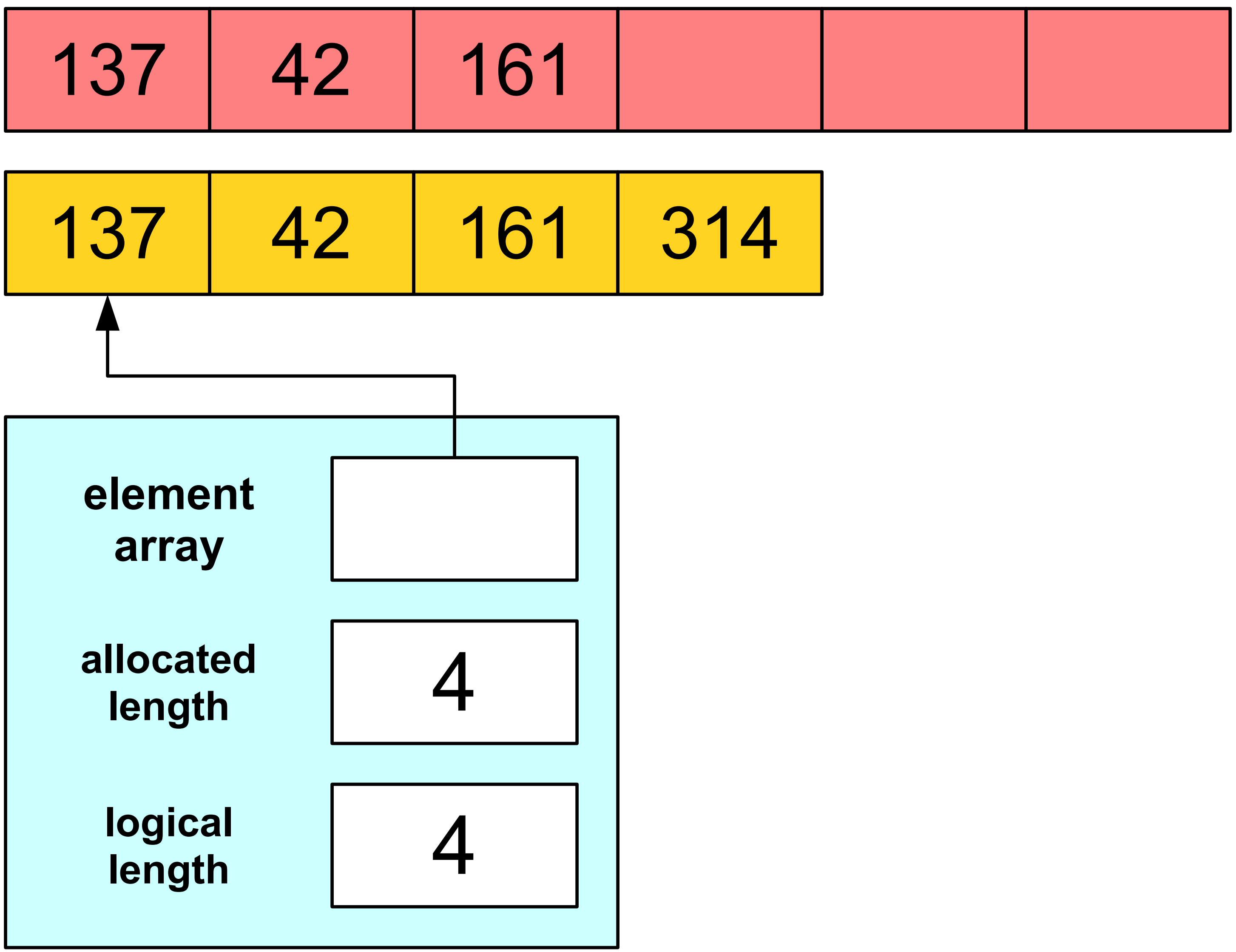

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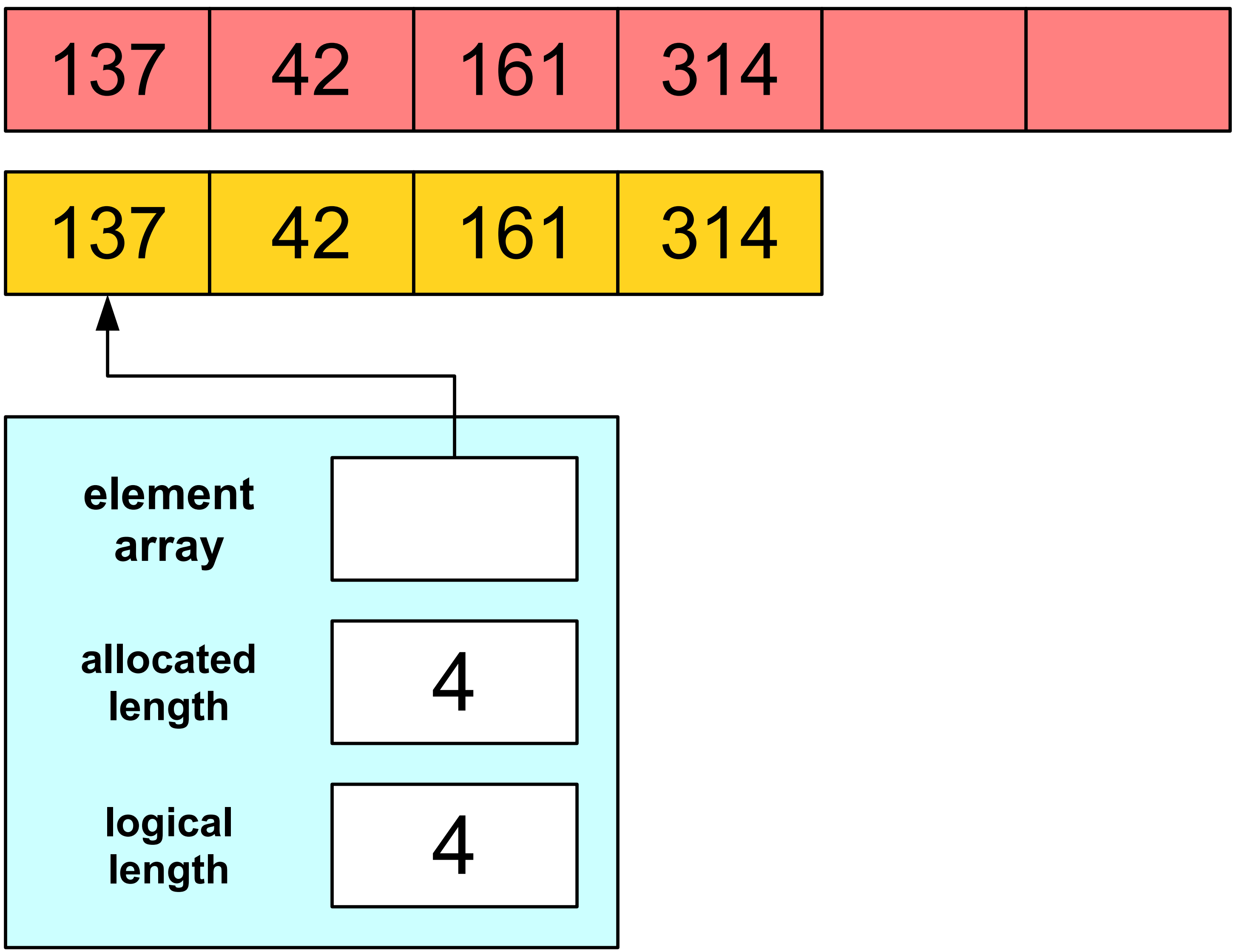

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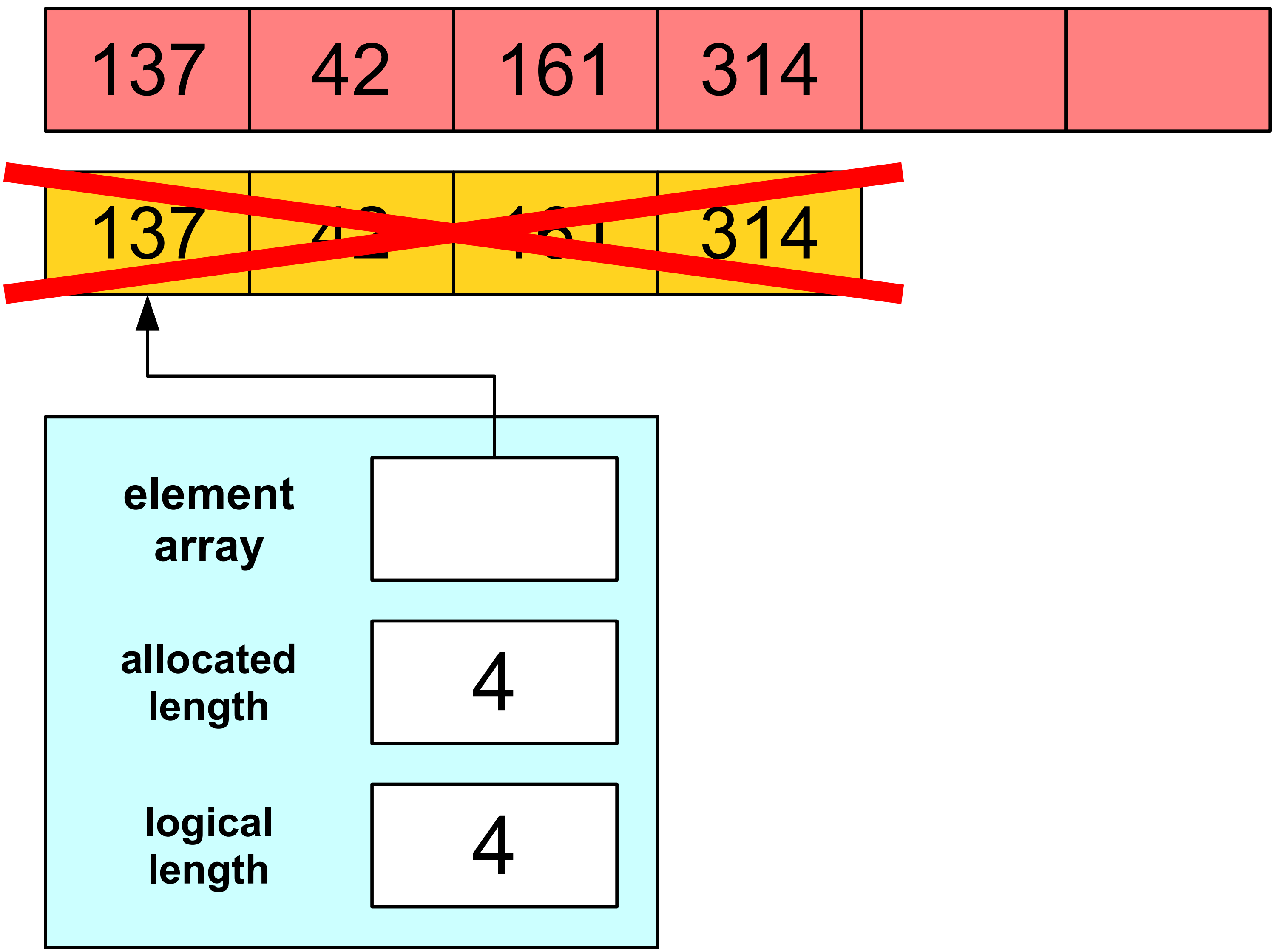

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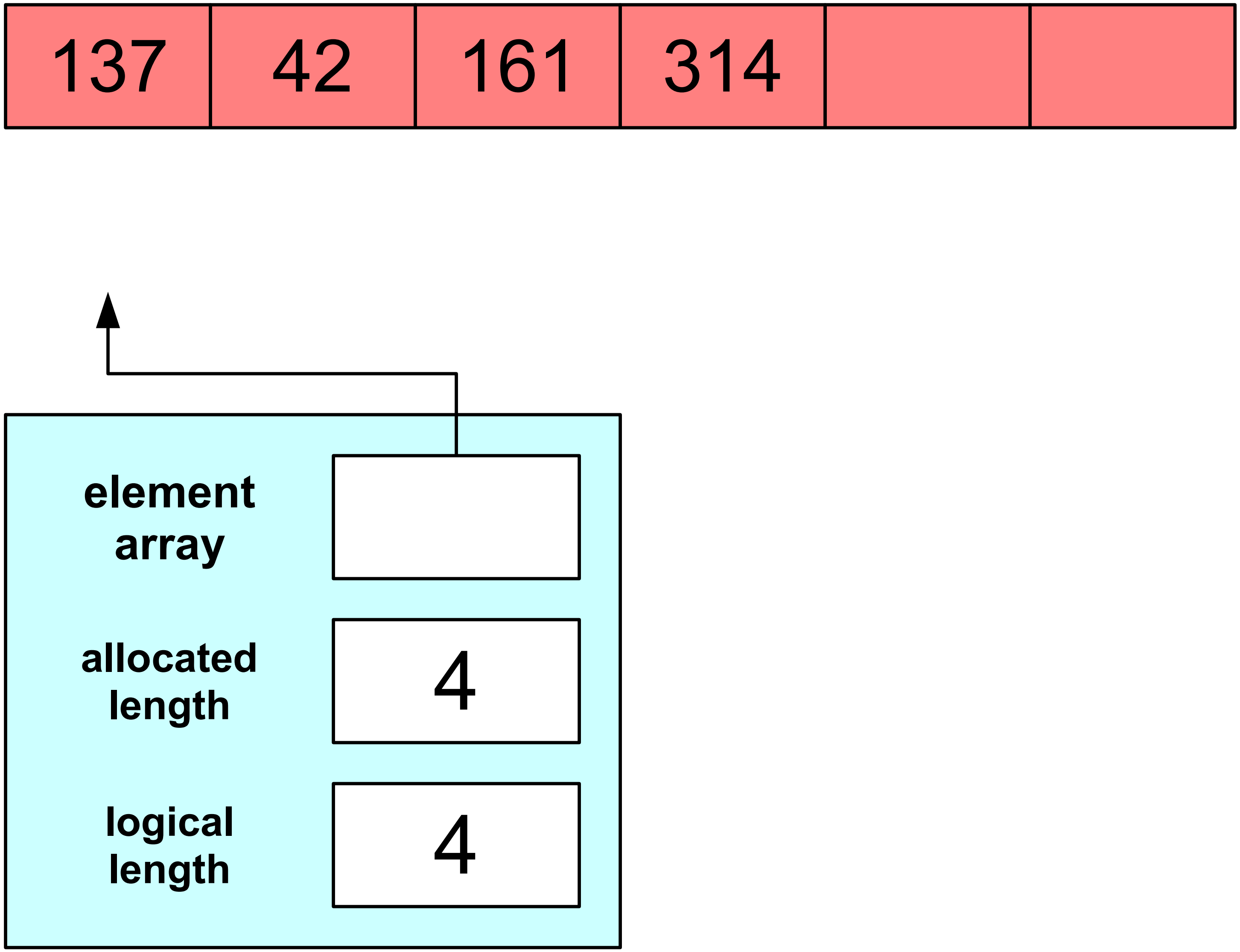

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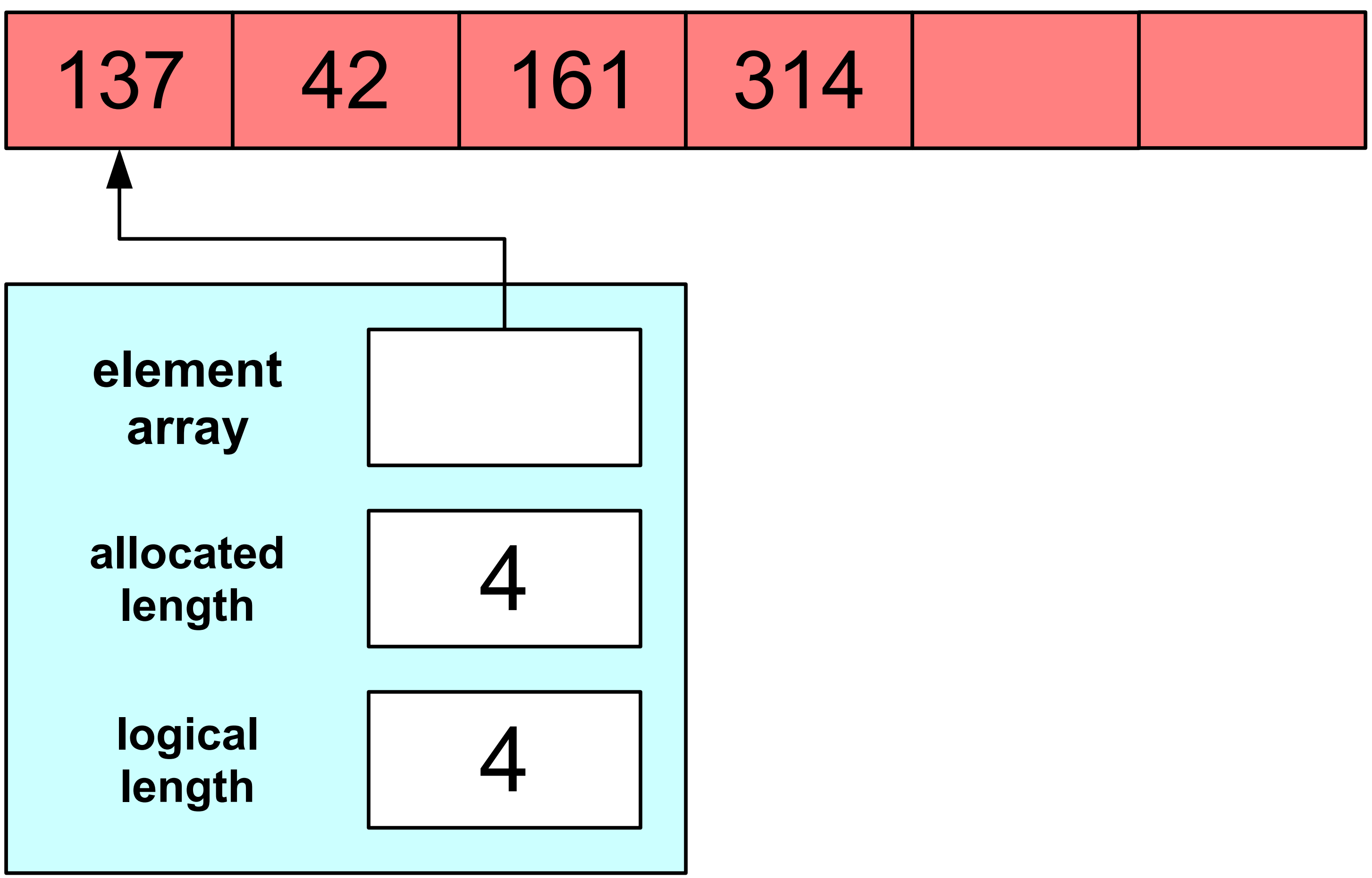

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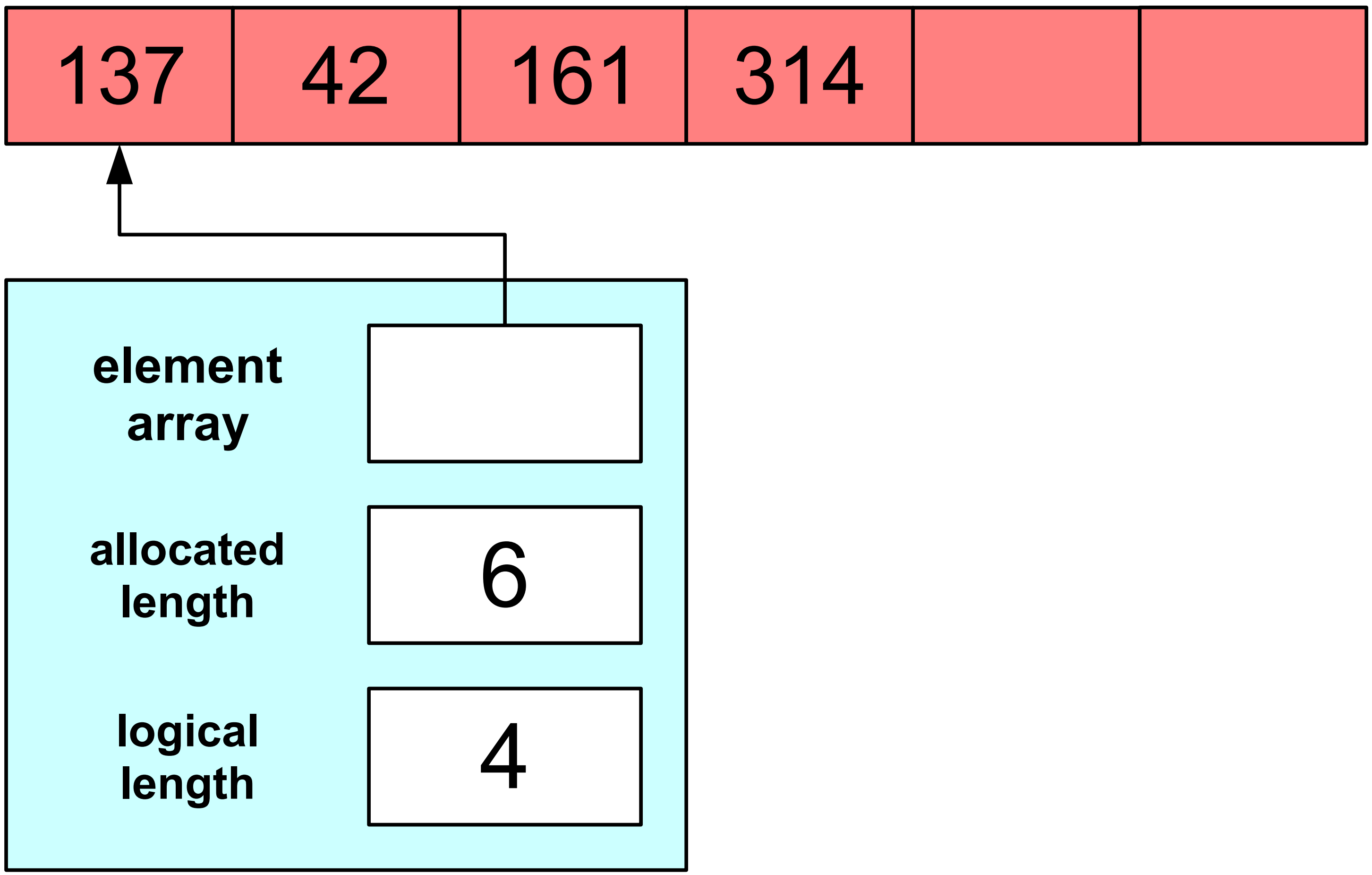

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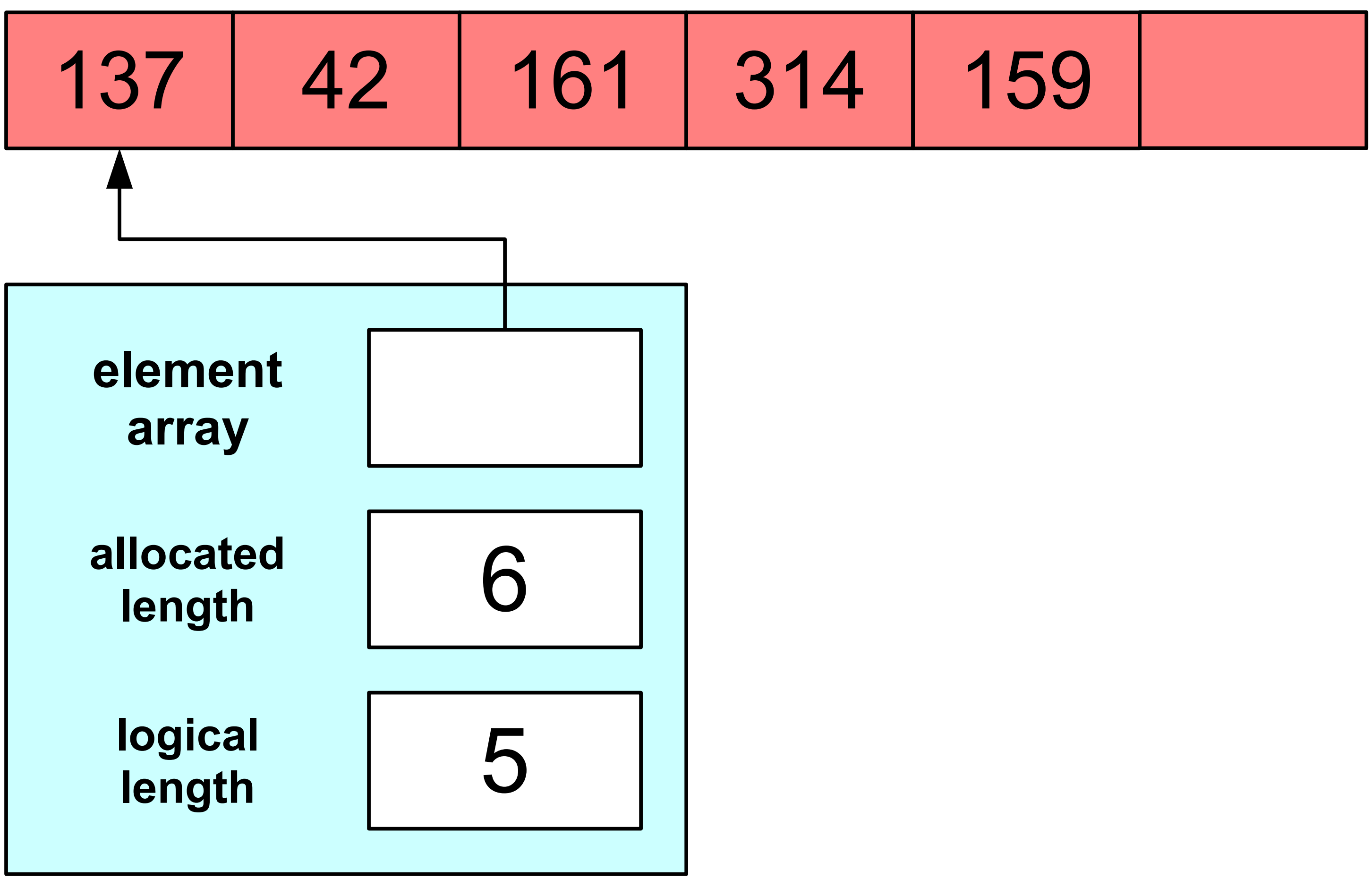

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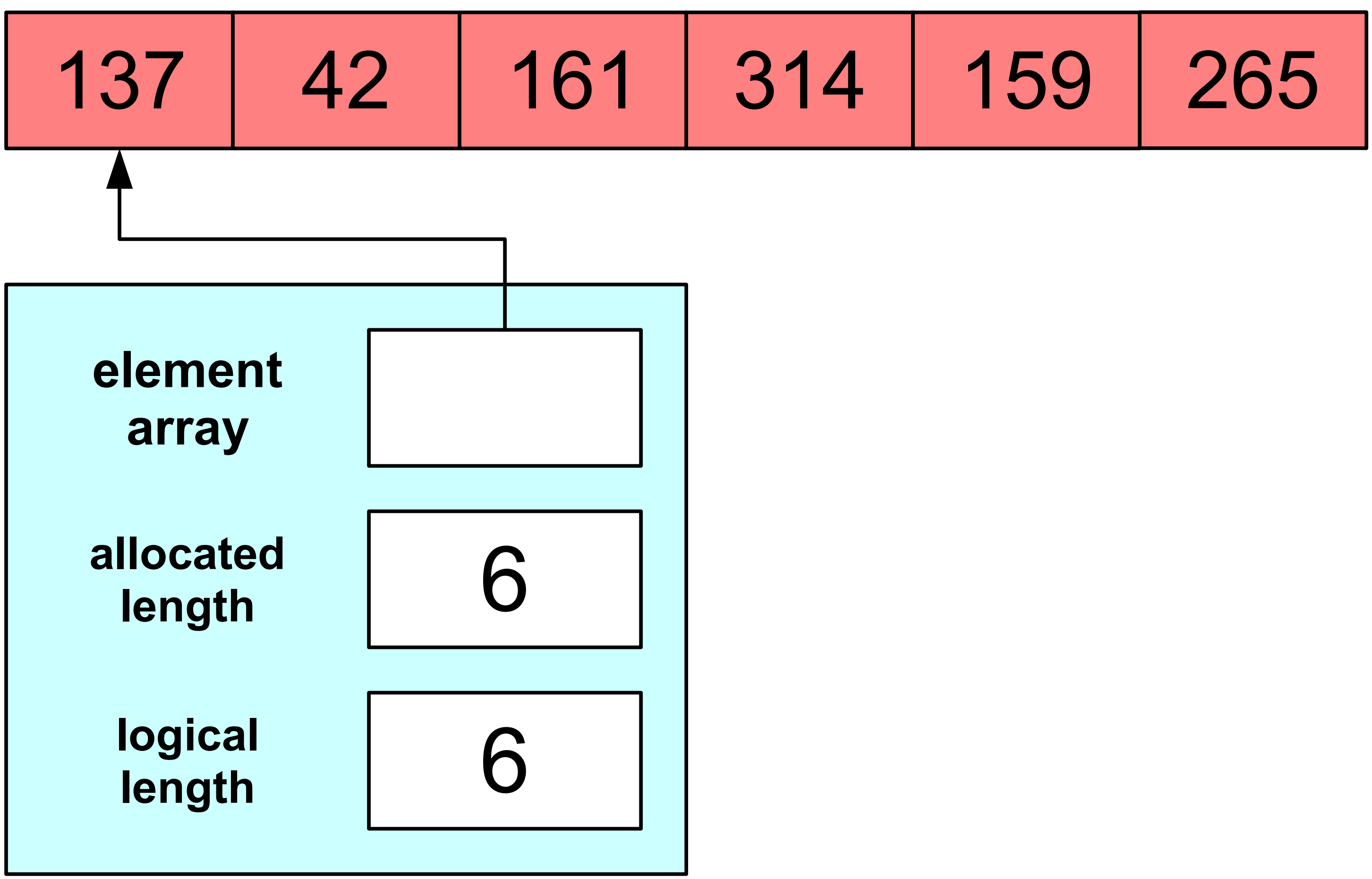
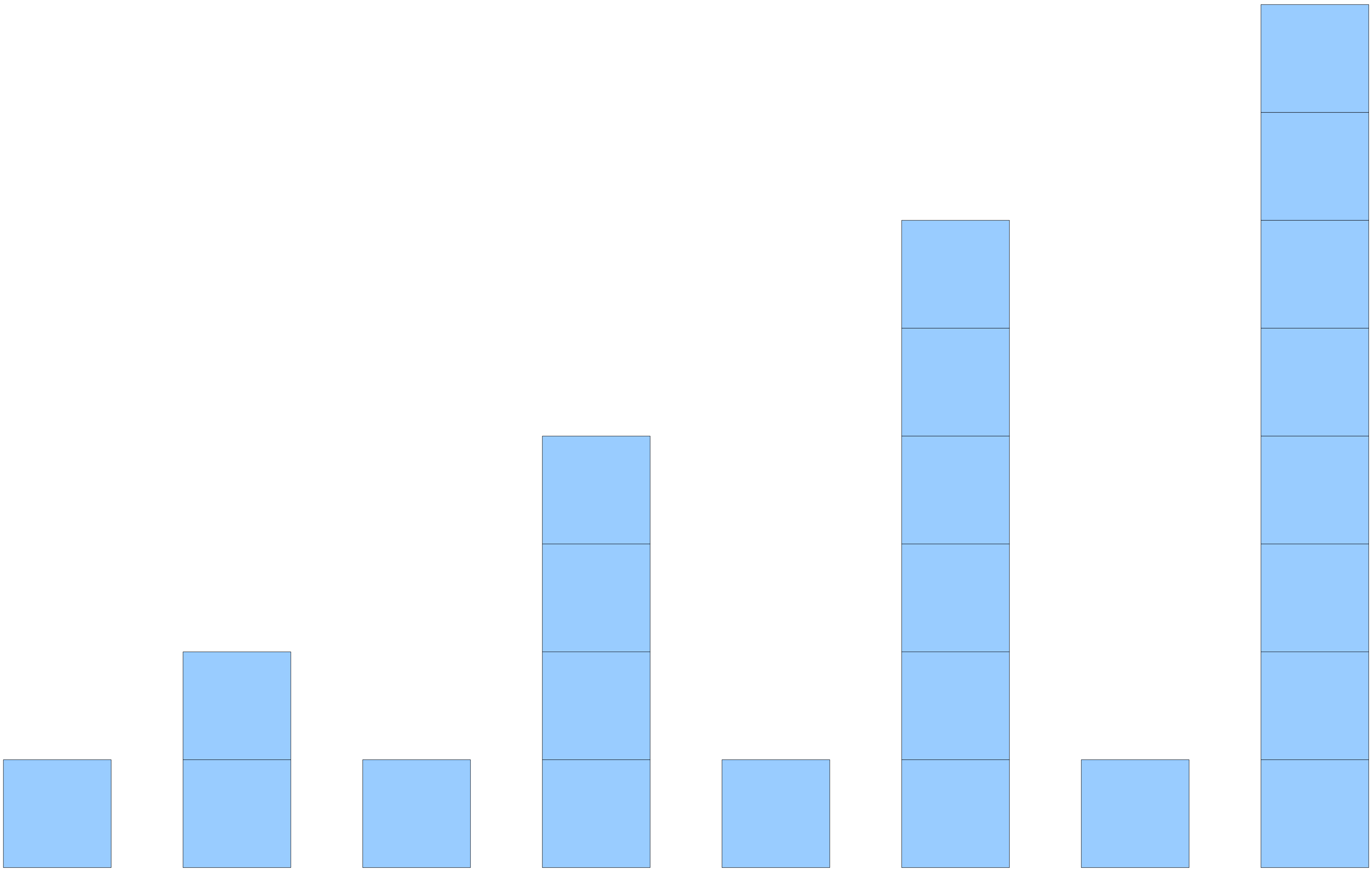


What Just Happened?

- Half of our pushes are now “easy” pushes, and half of our pushes are now “hard” pushes.
- Hard pushes still take time O(n).
- Easy pushes only take time O(1).
- Worst-case is still O(n).
- What about the average case?

Analyzing the Work



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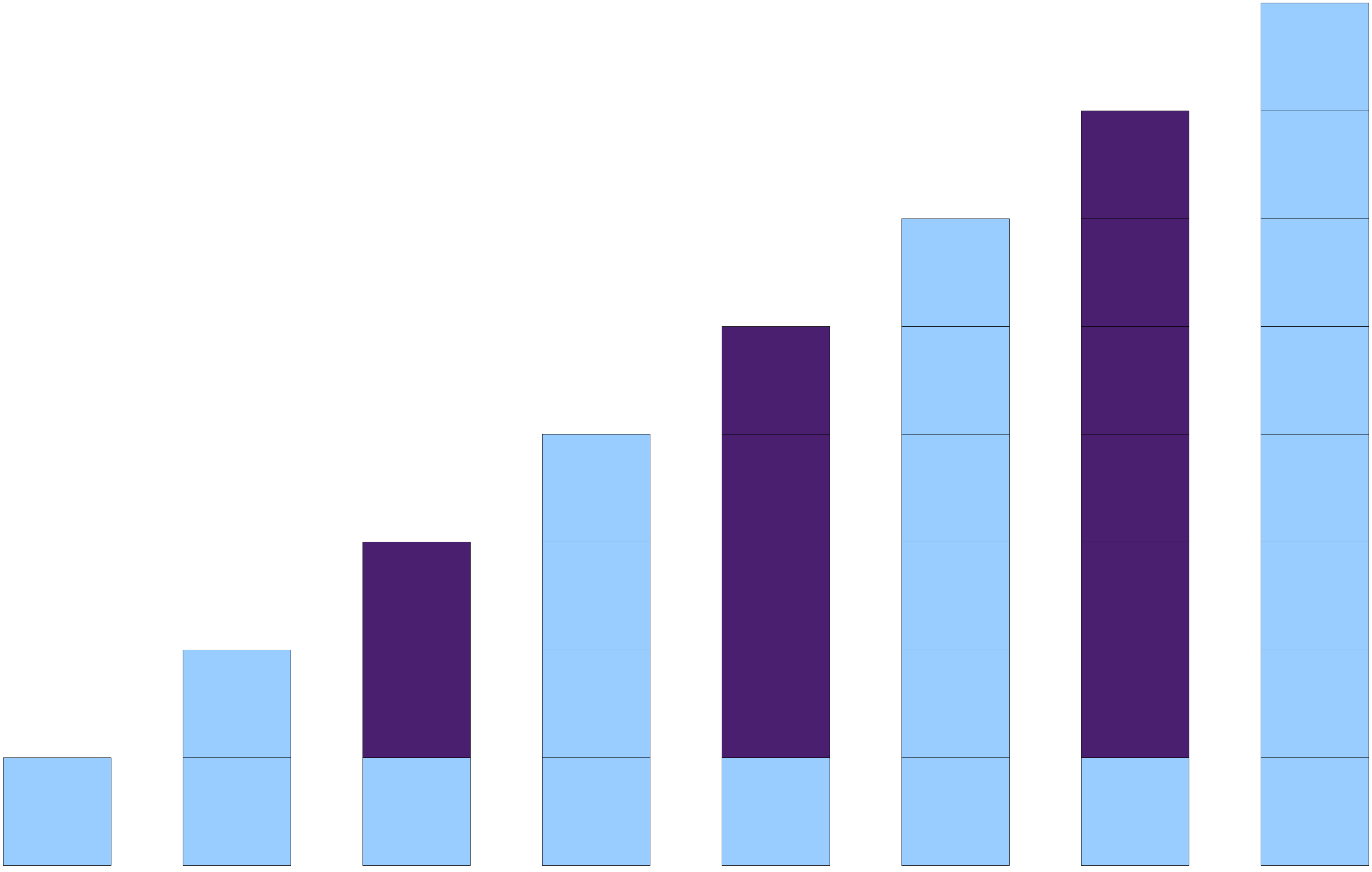

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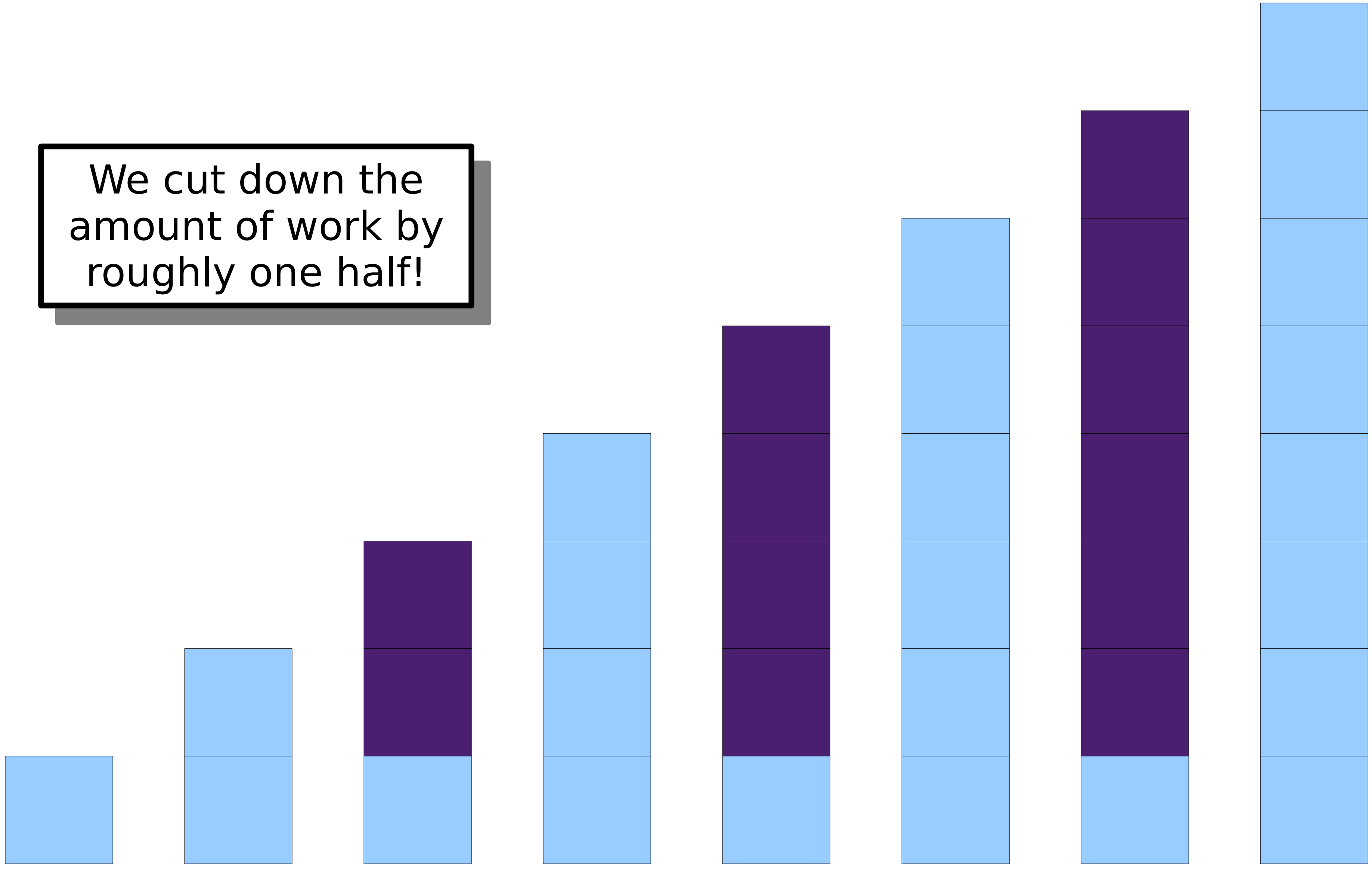

A Different Analysis

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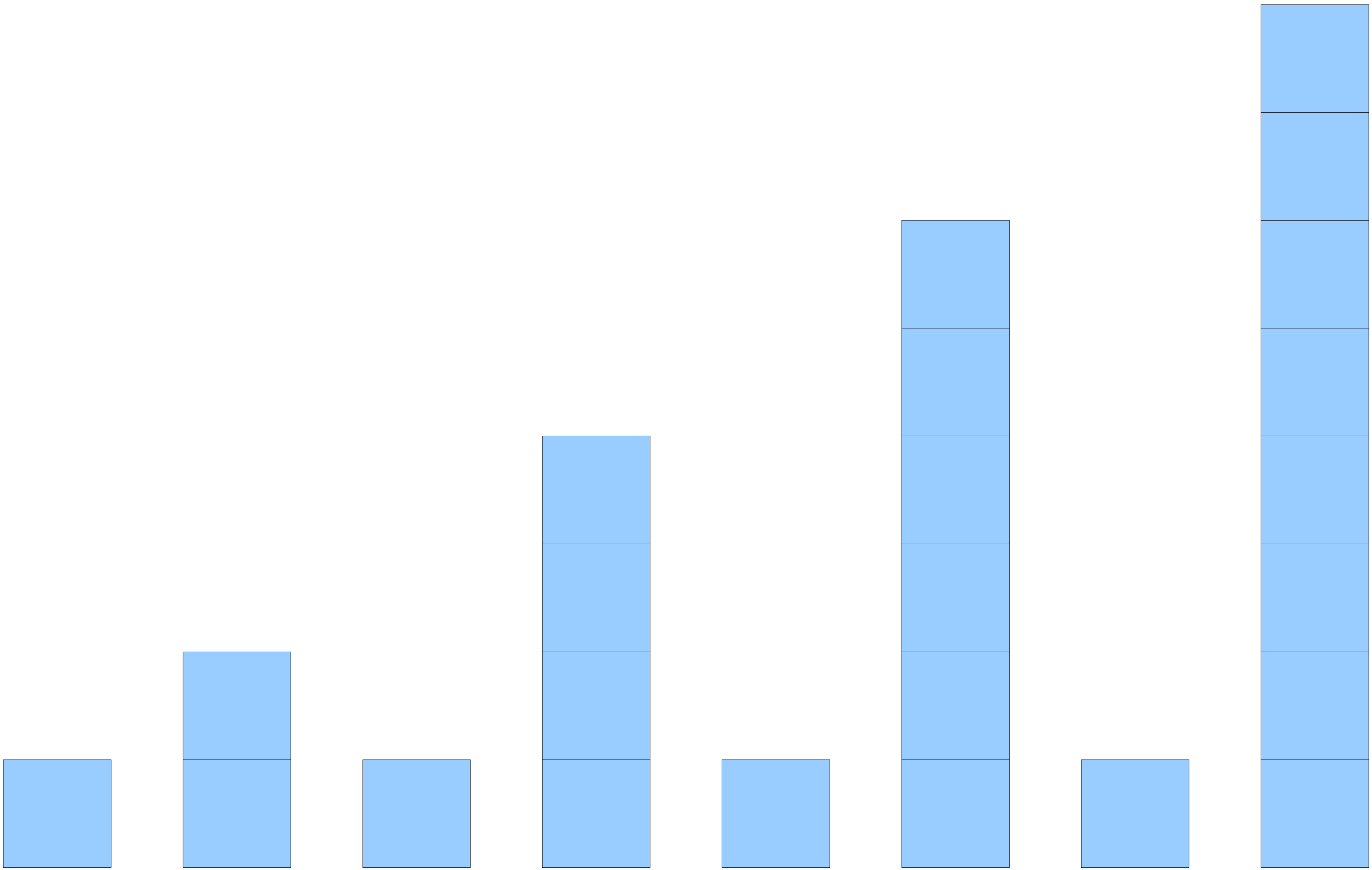

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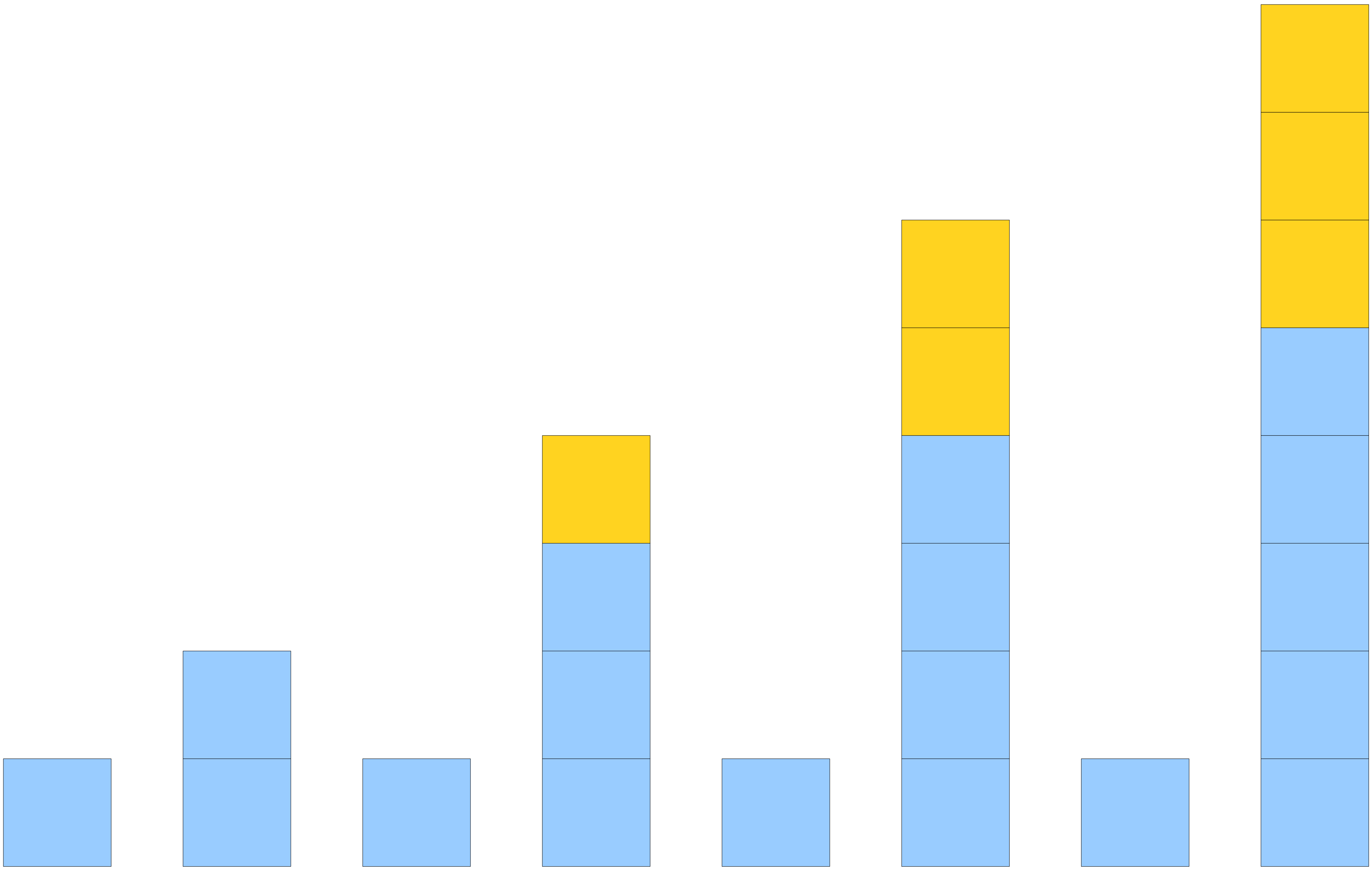

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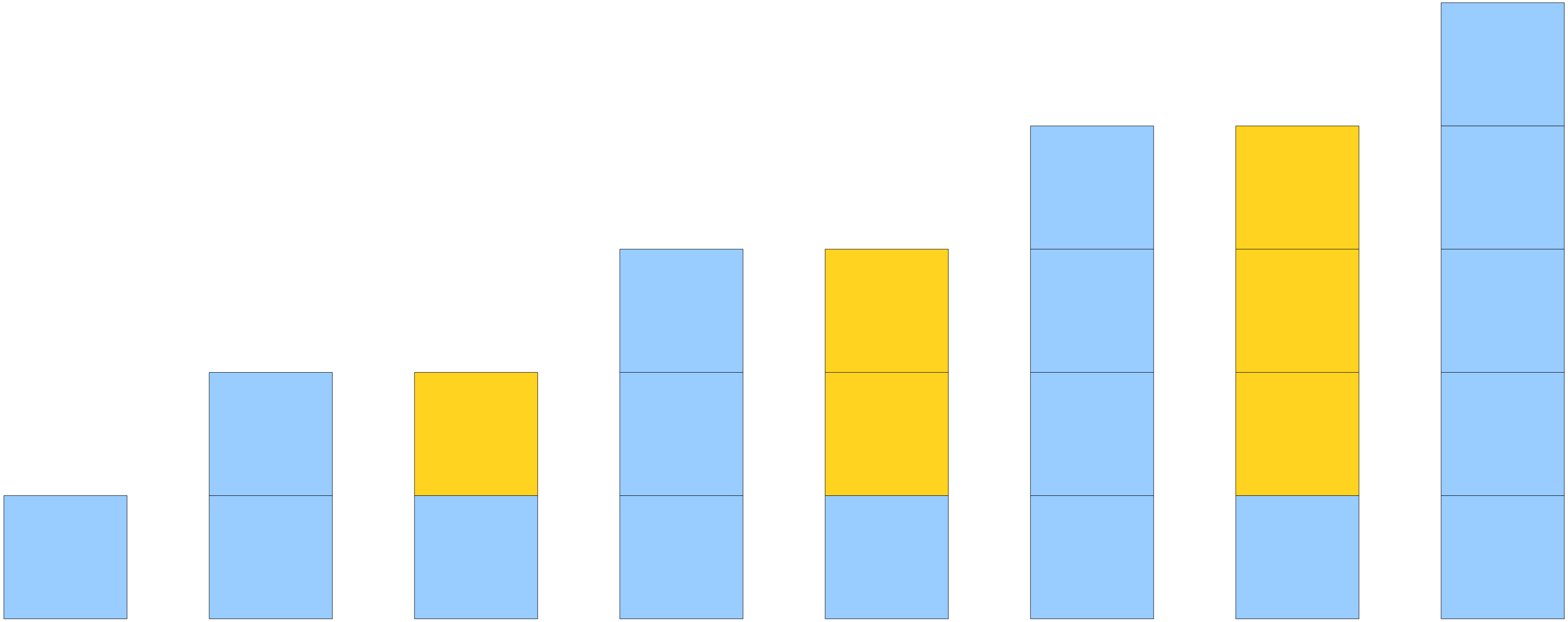

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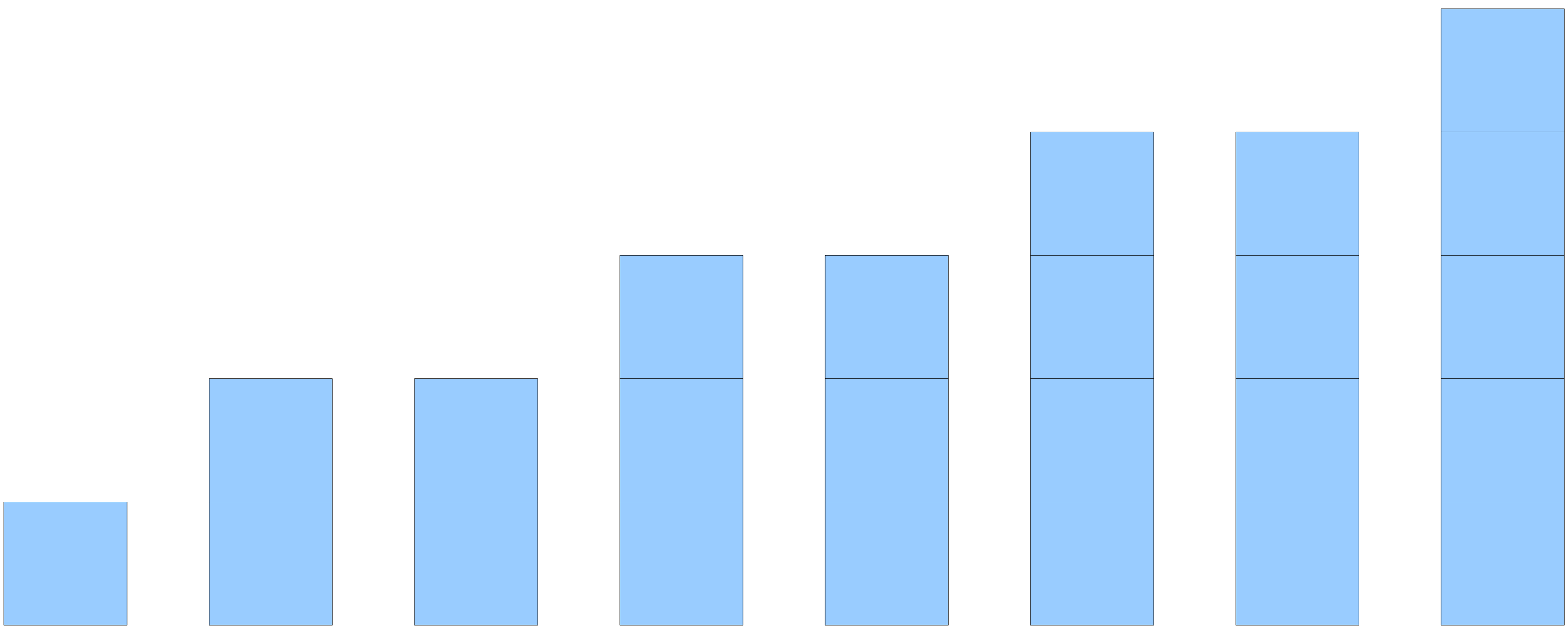

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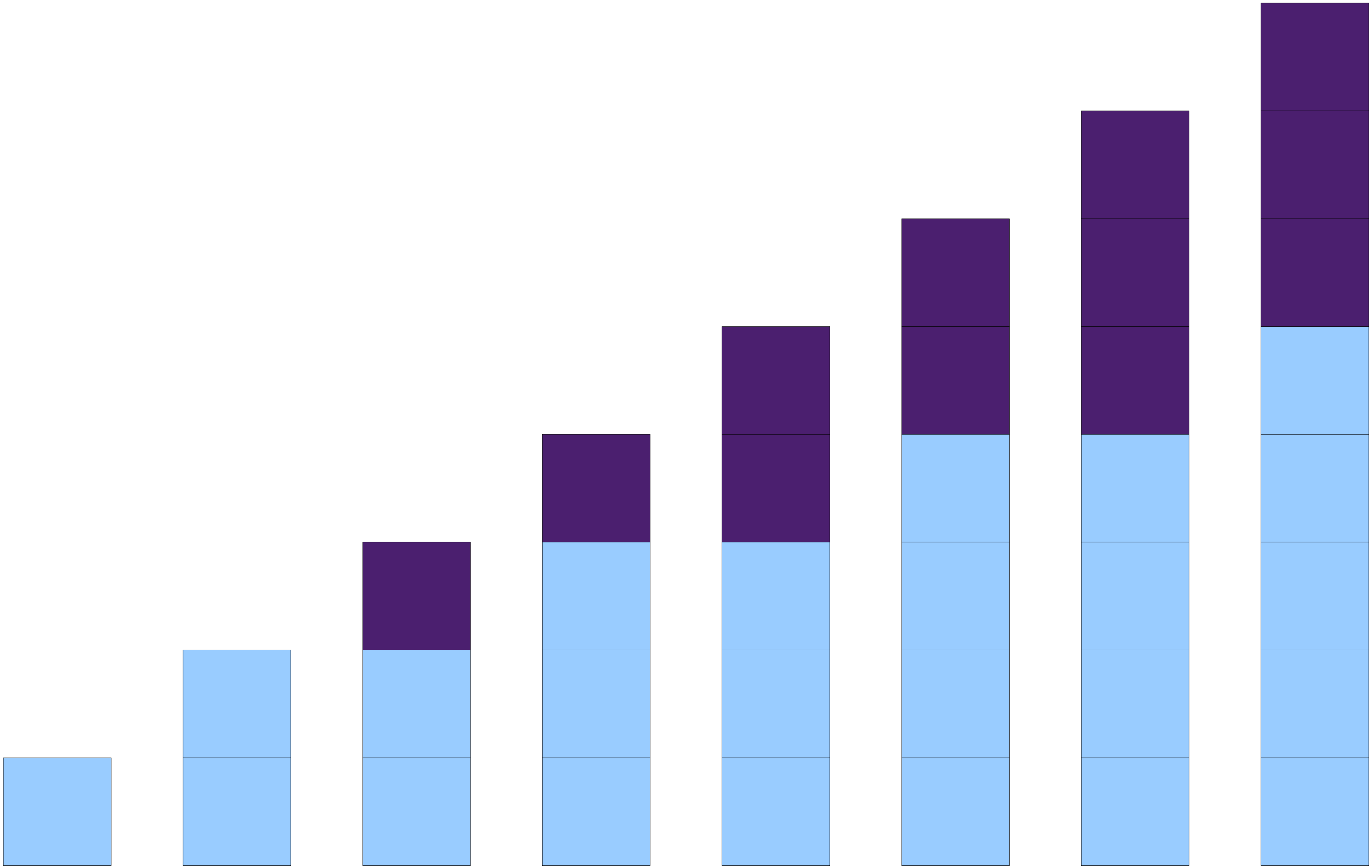

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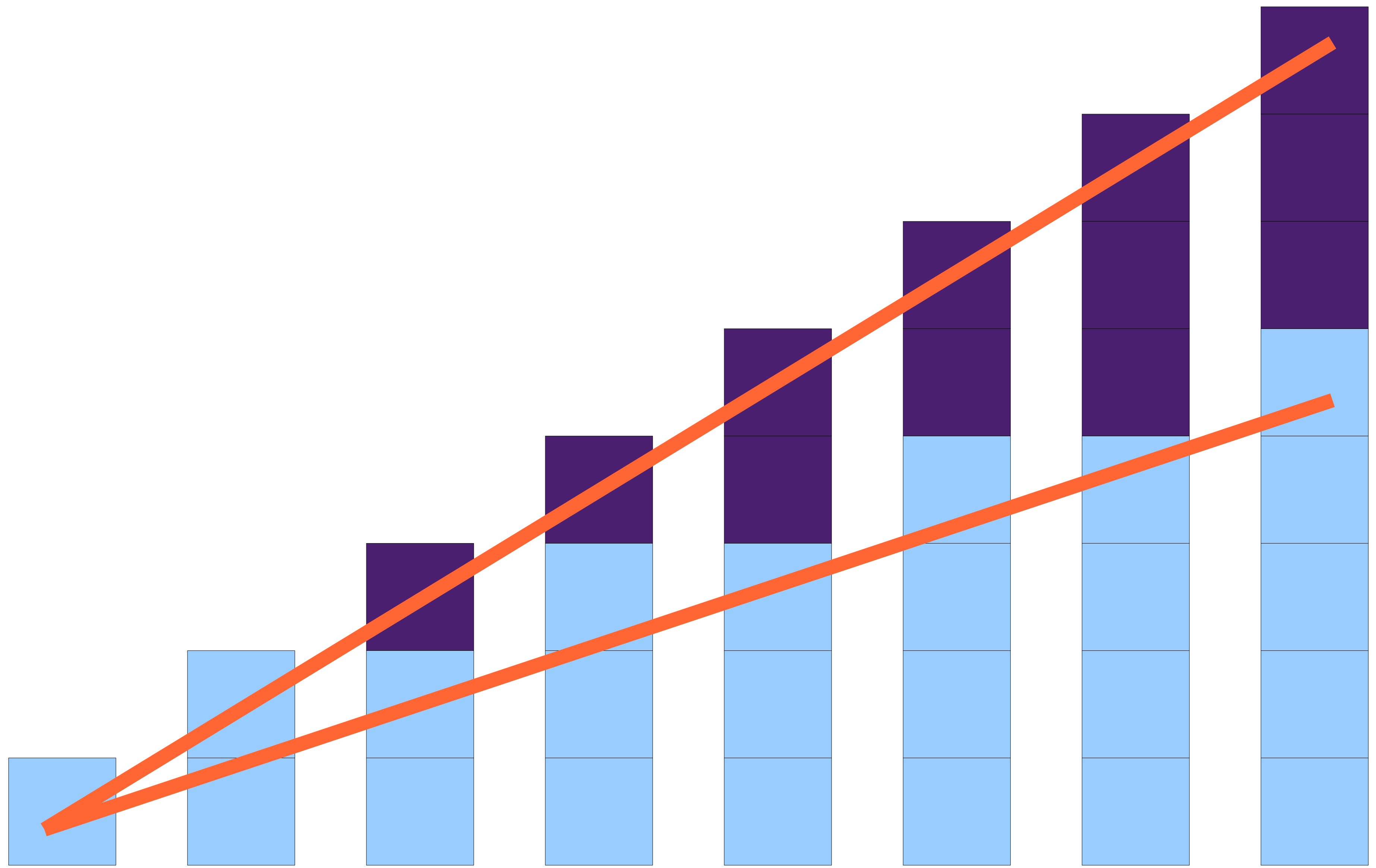

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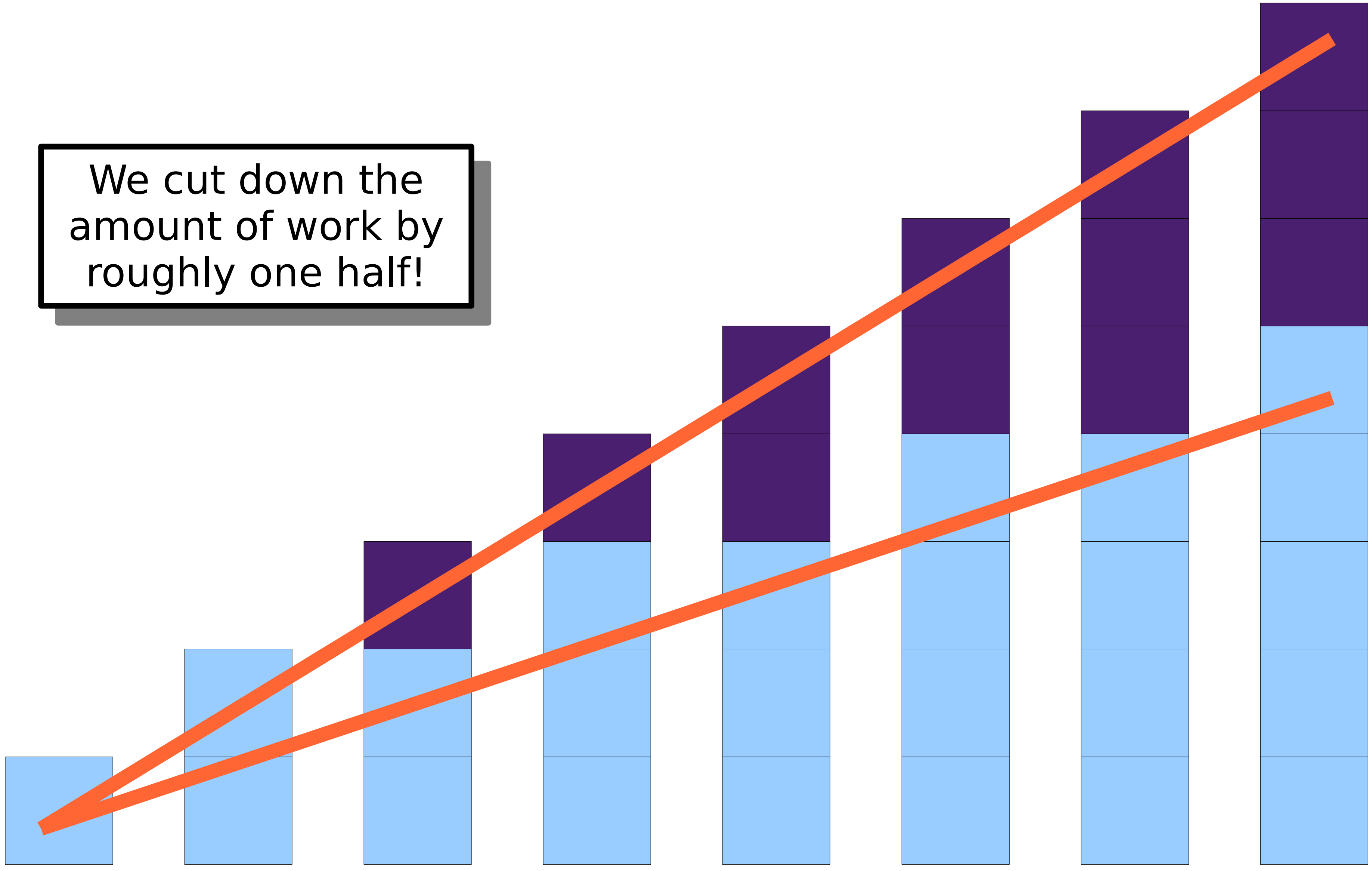
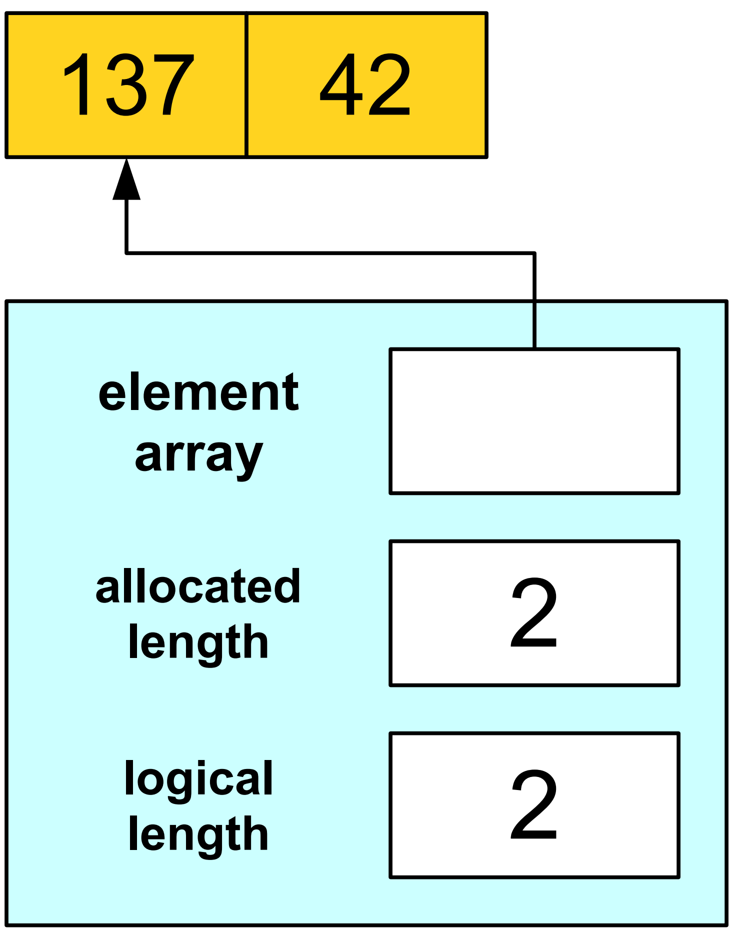


How does it stack up?

A Much Better Idea



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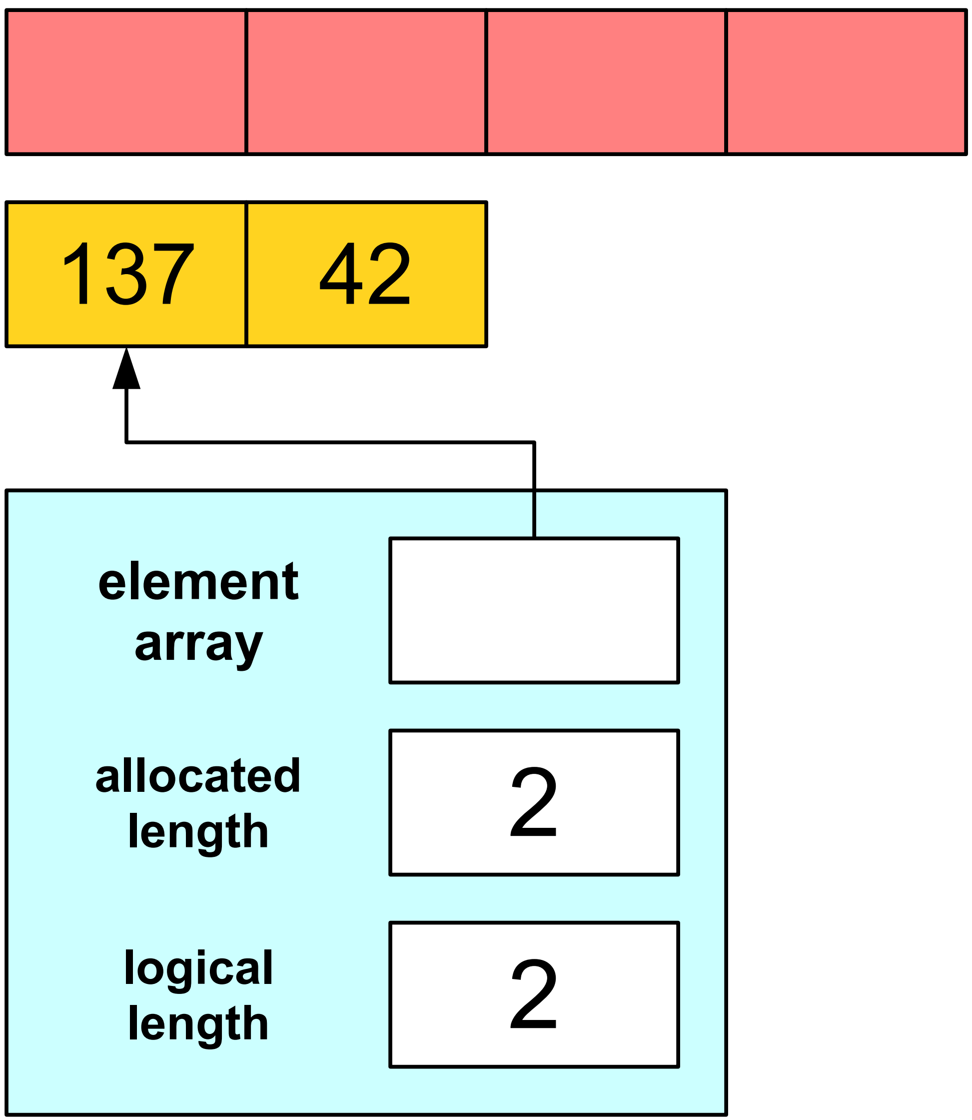

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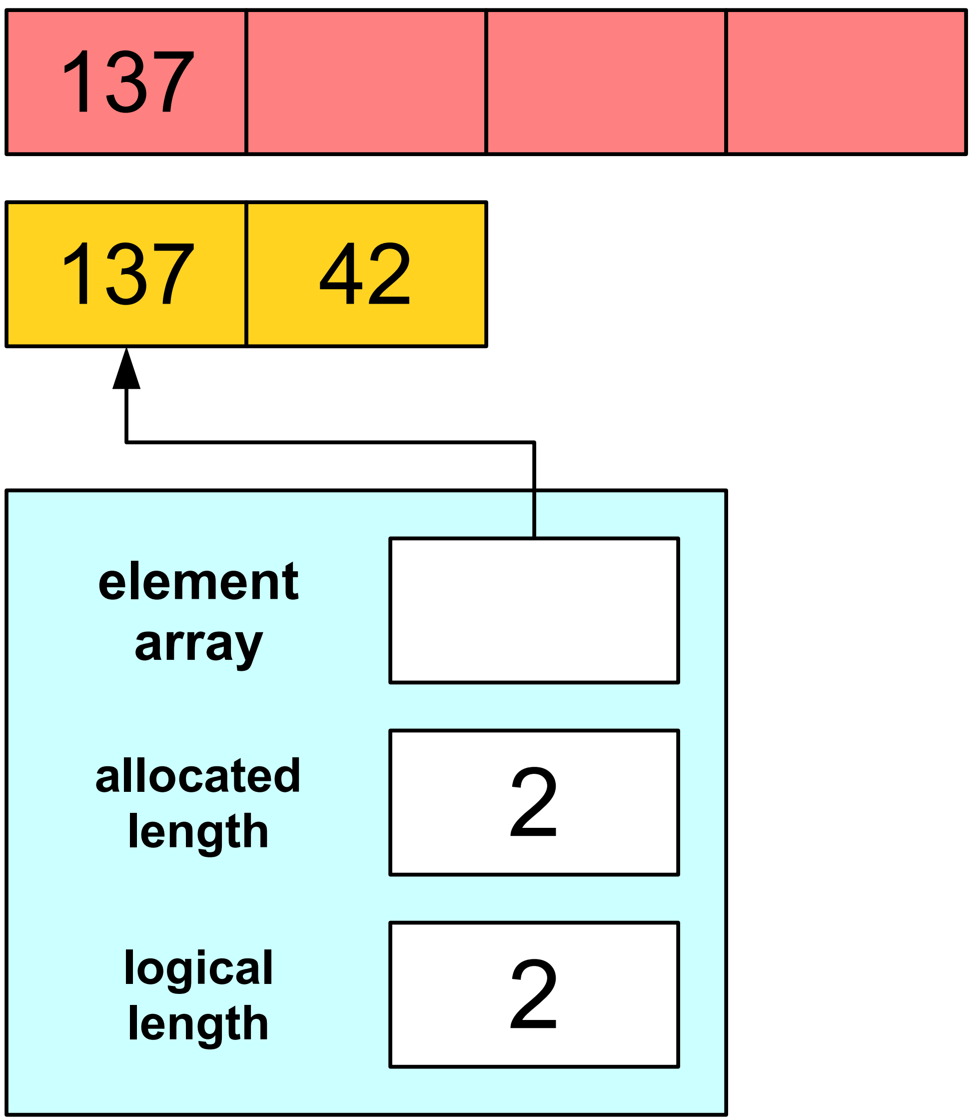

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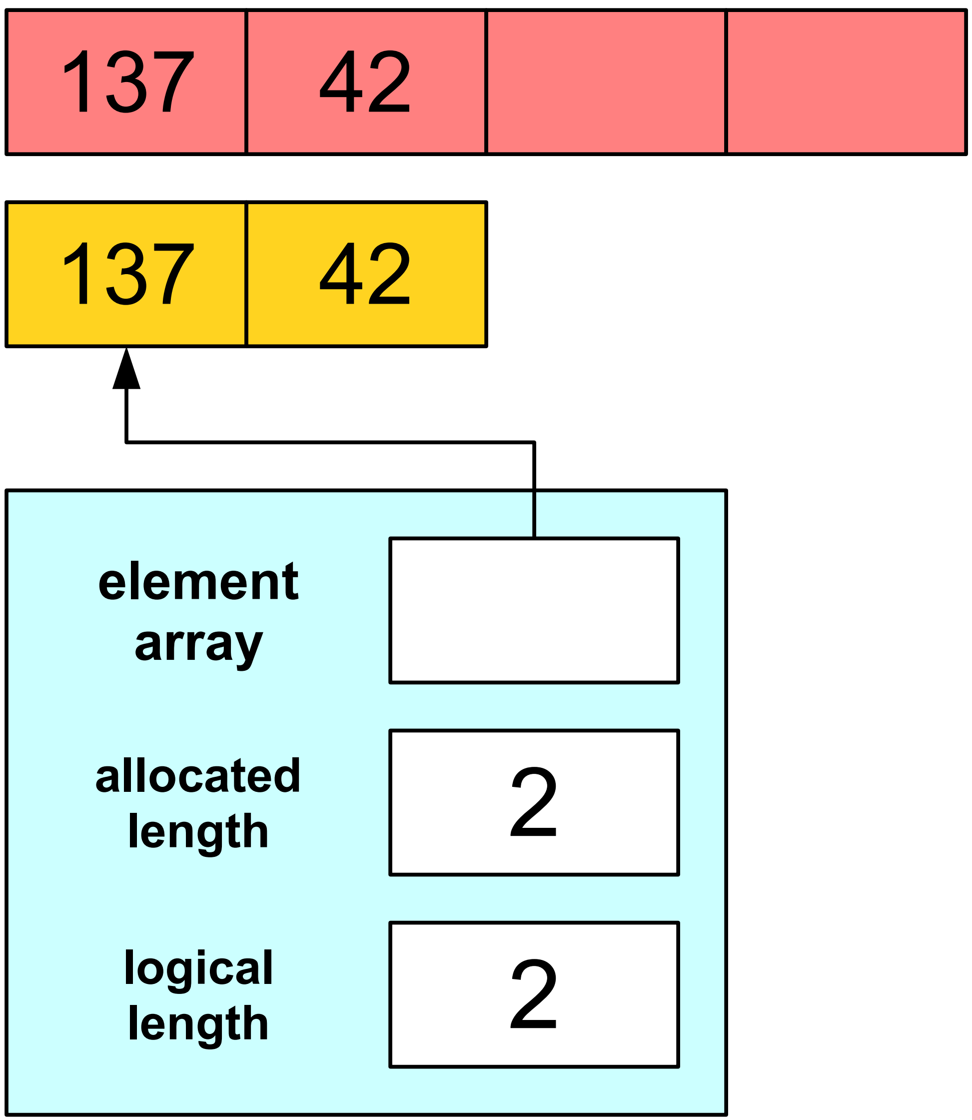

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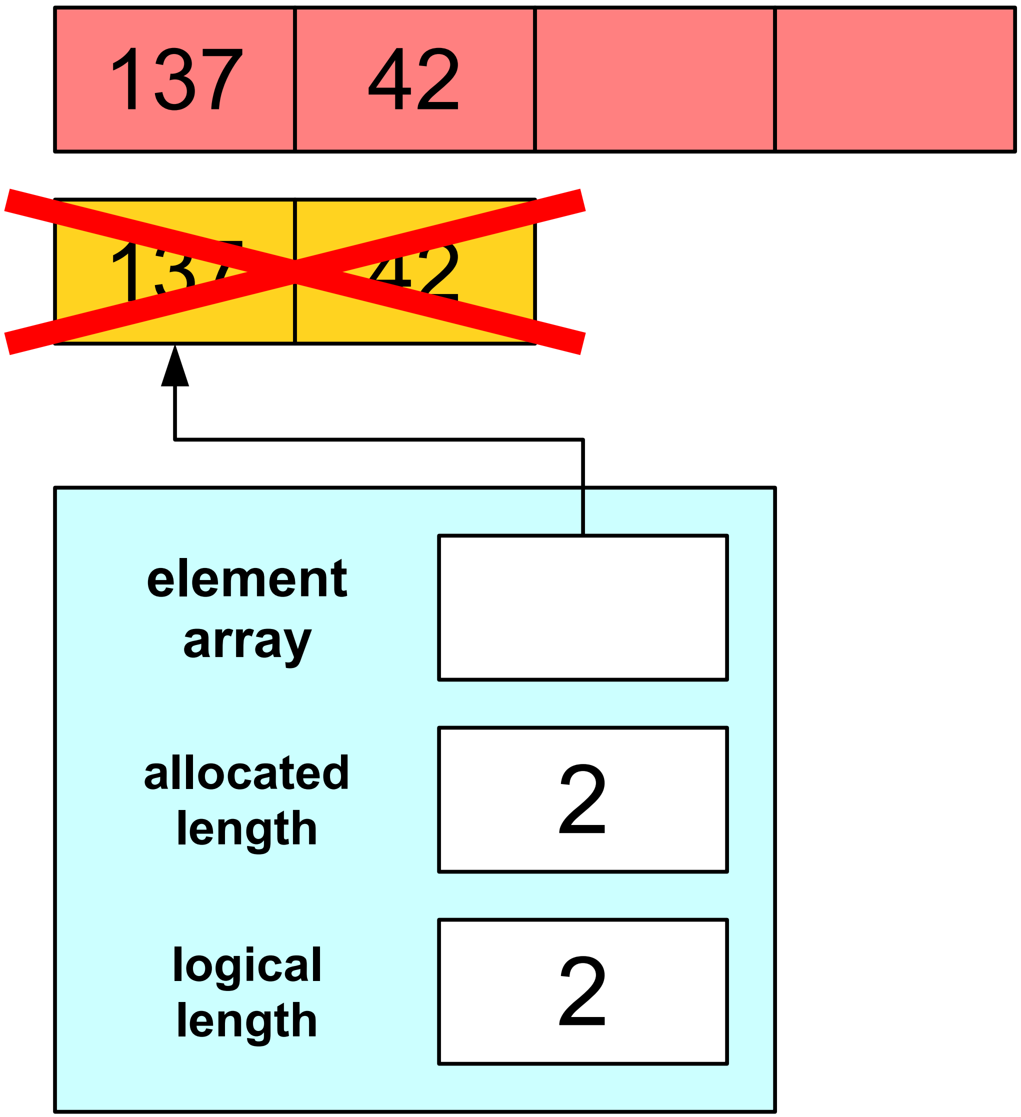

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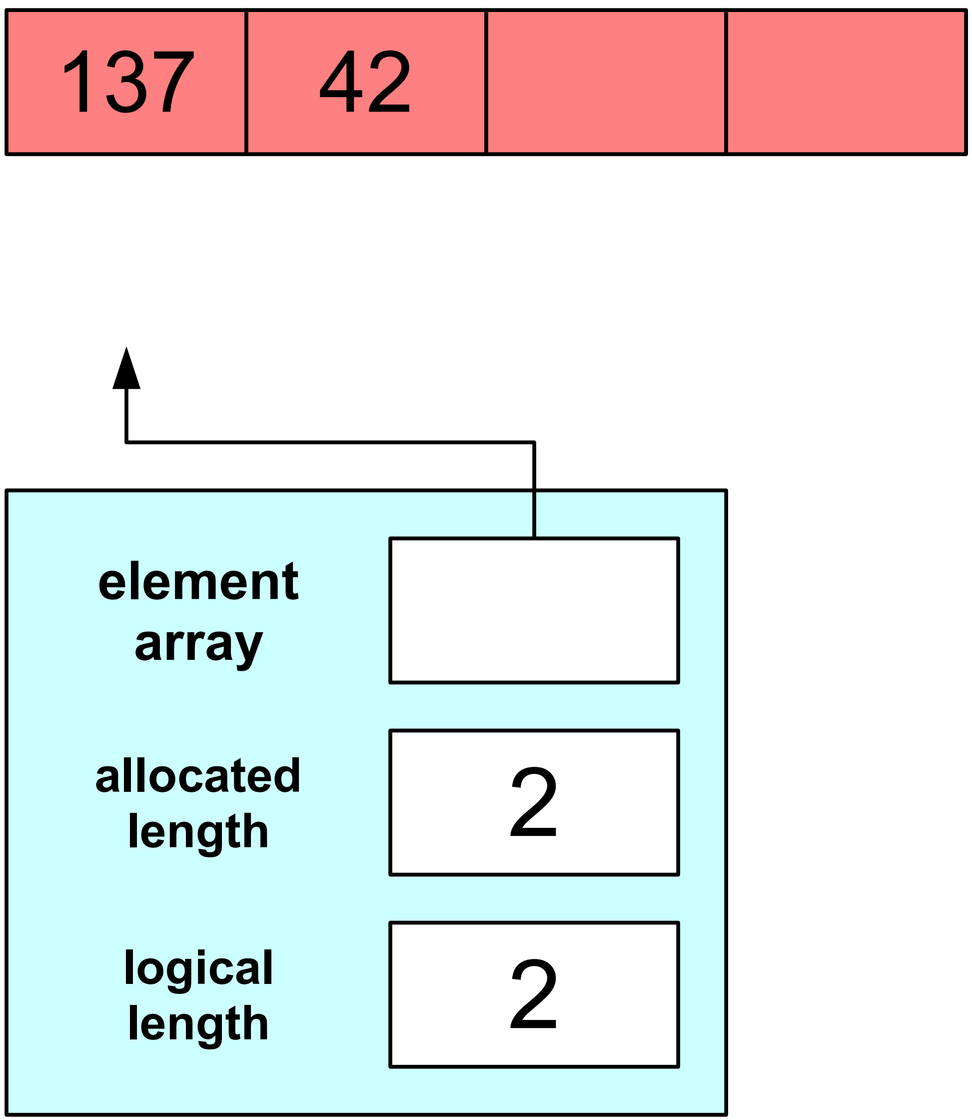

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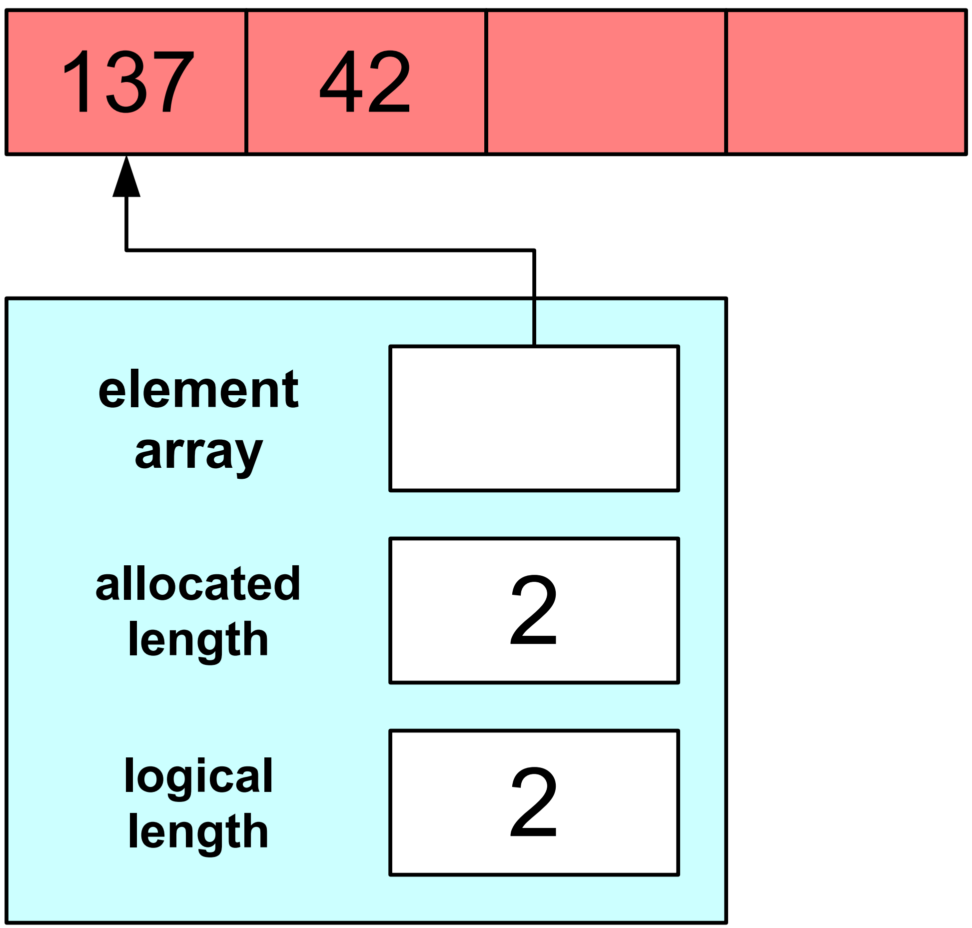

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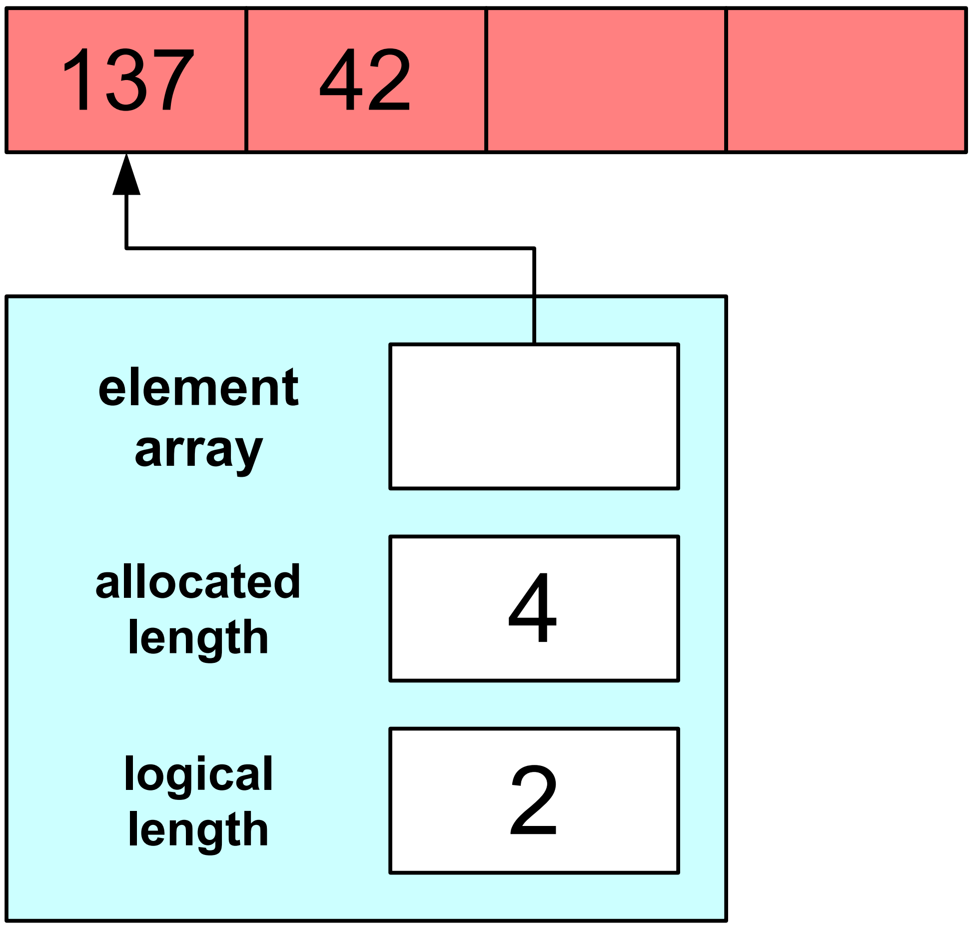

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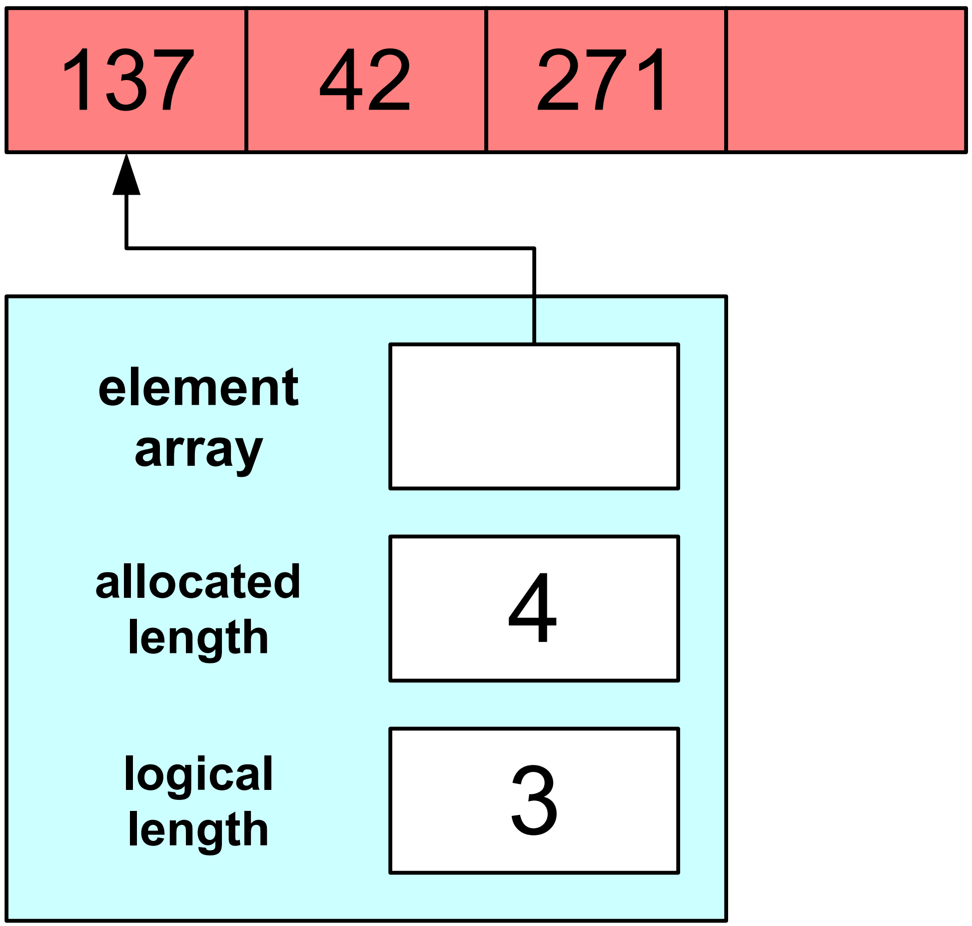

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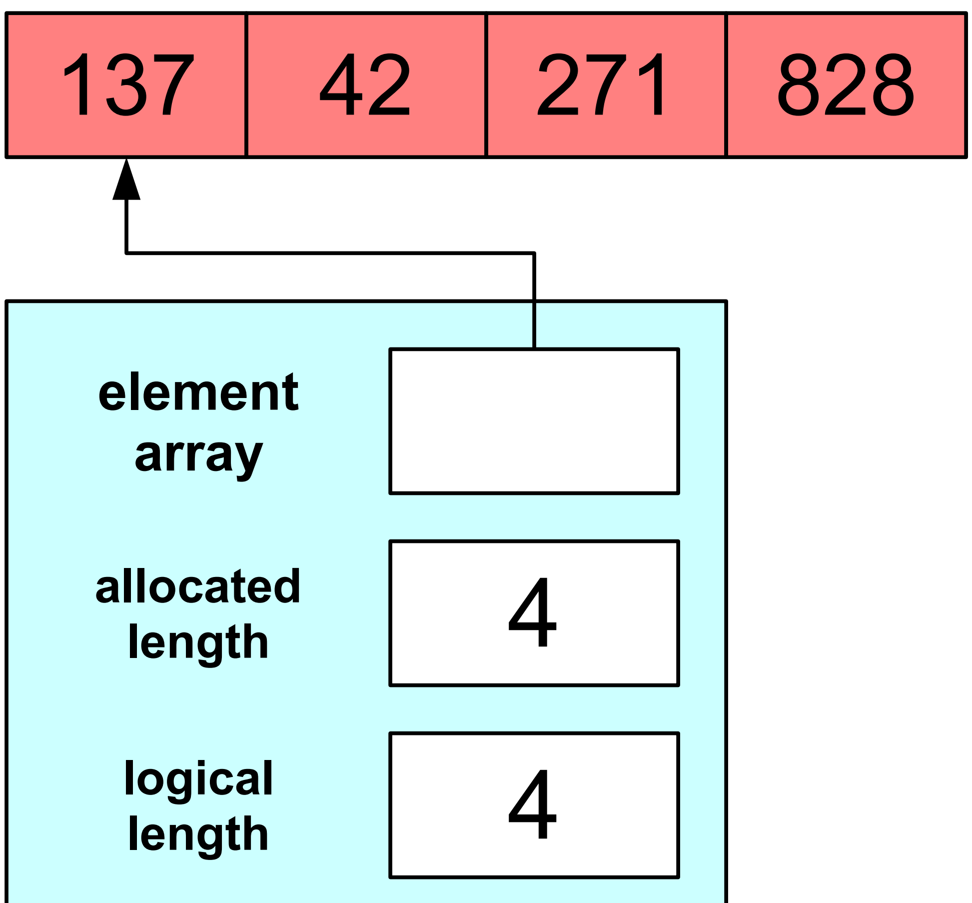

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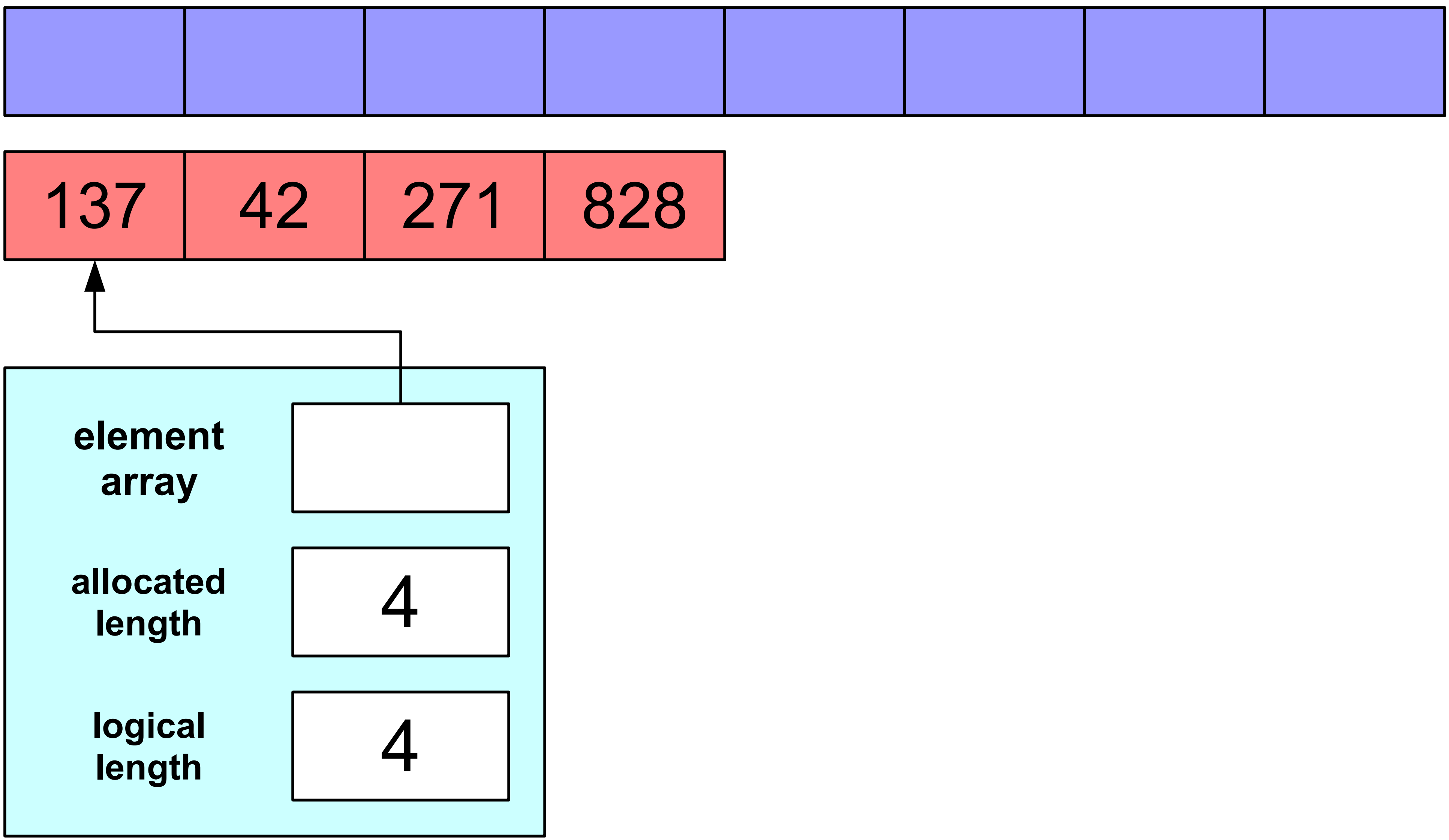

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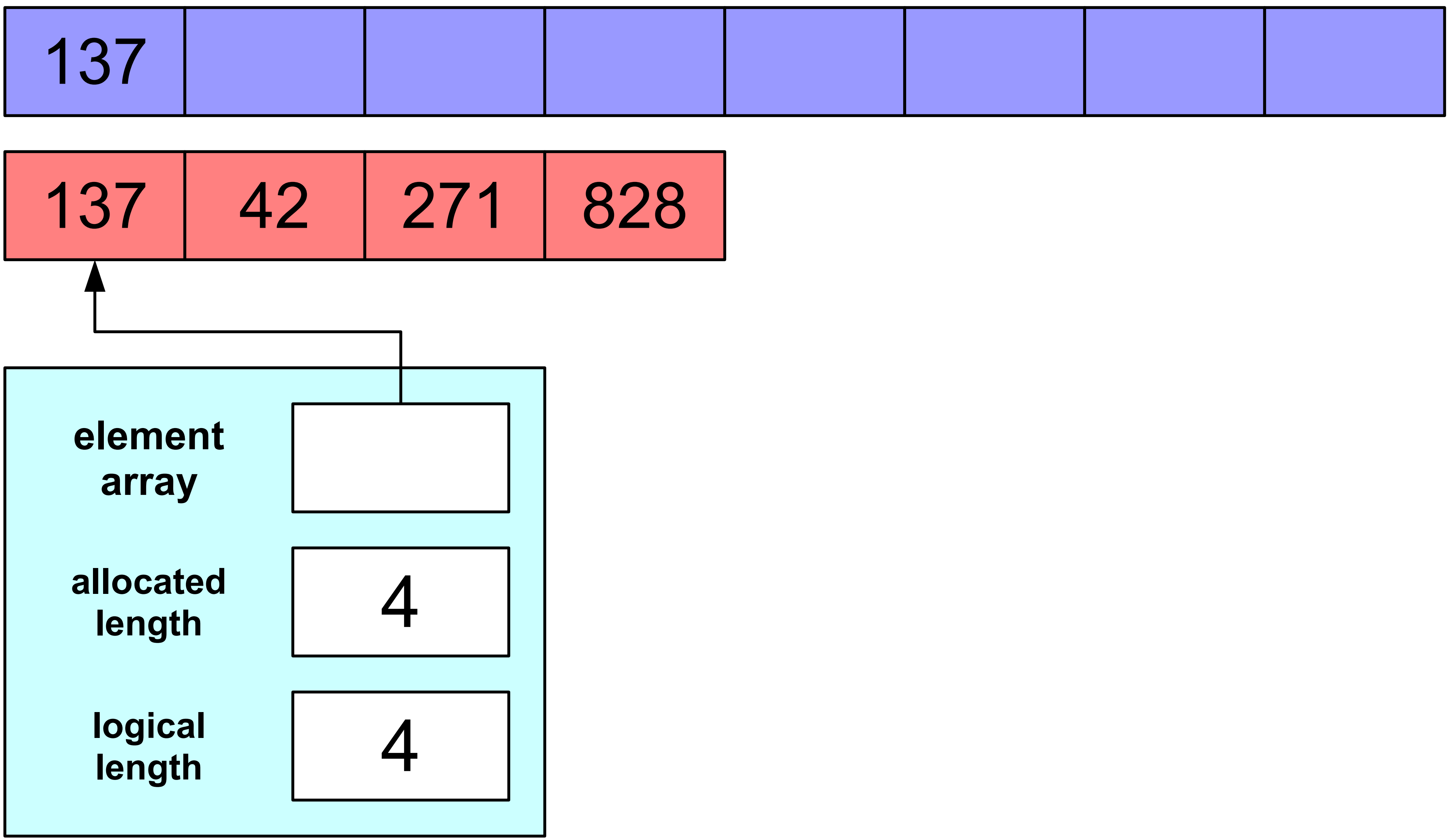

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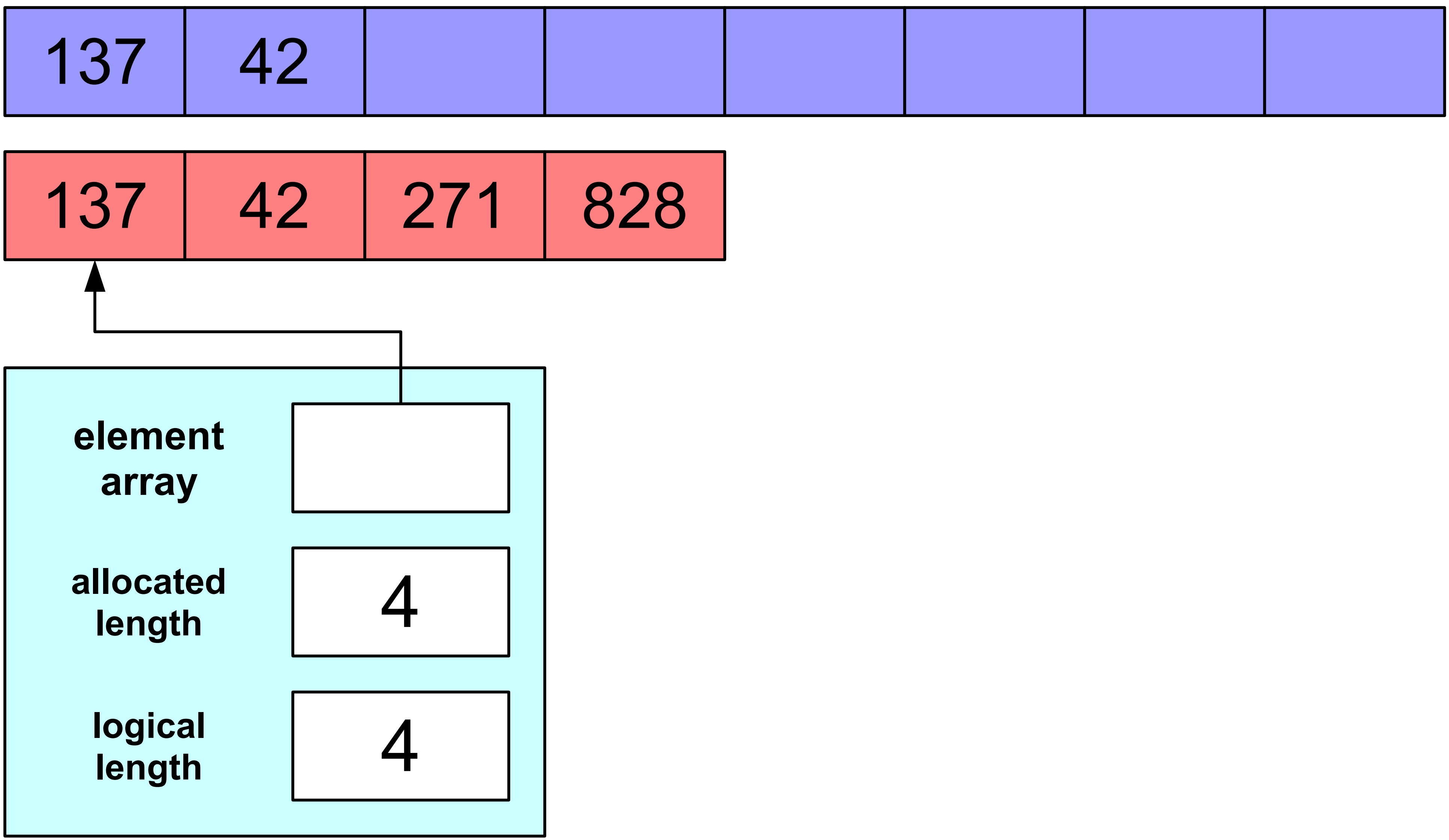

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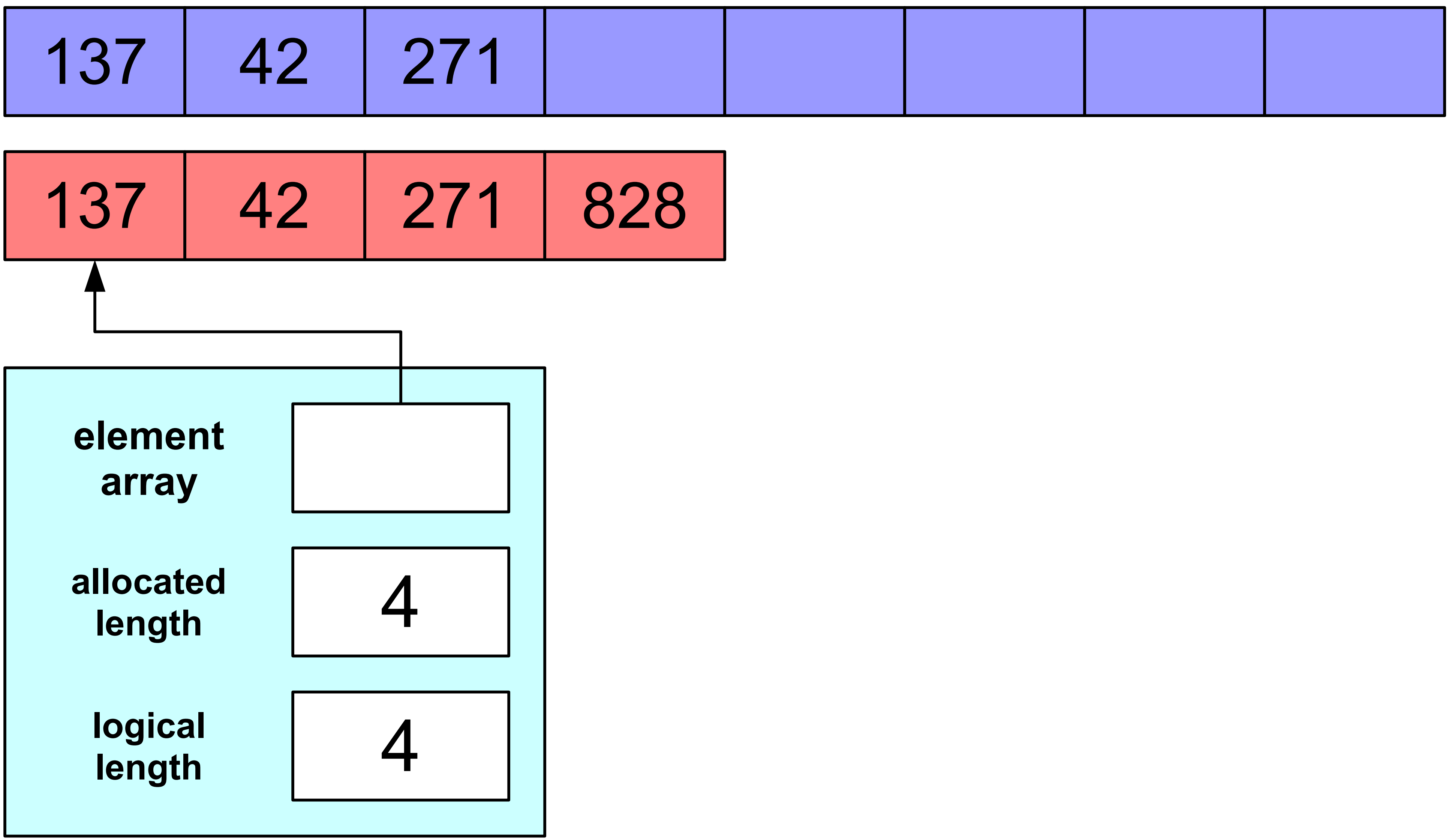

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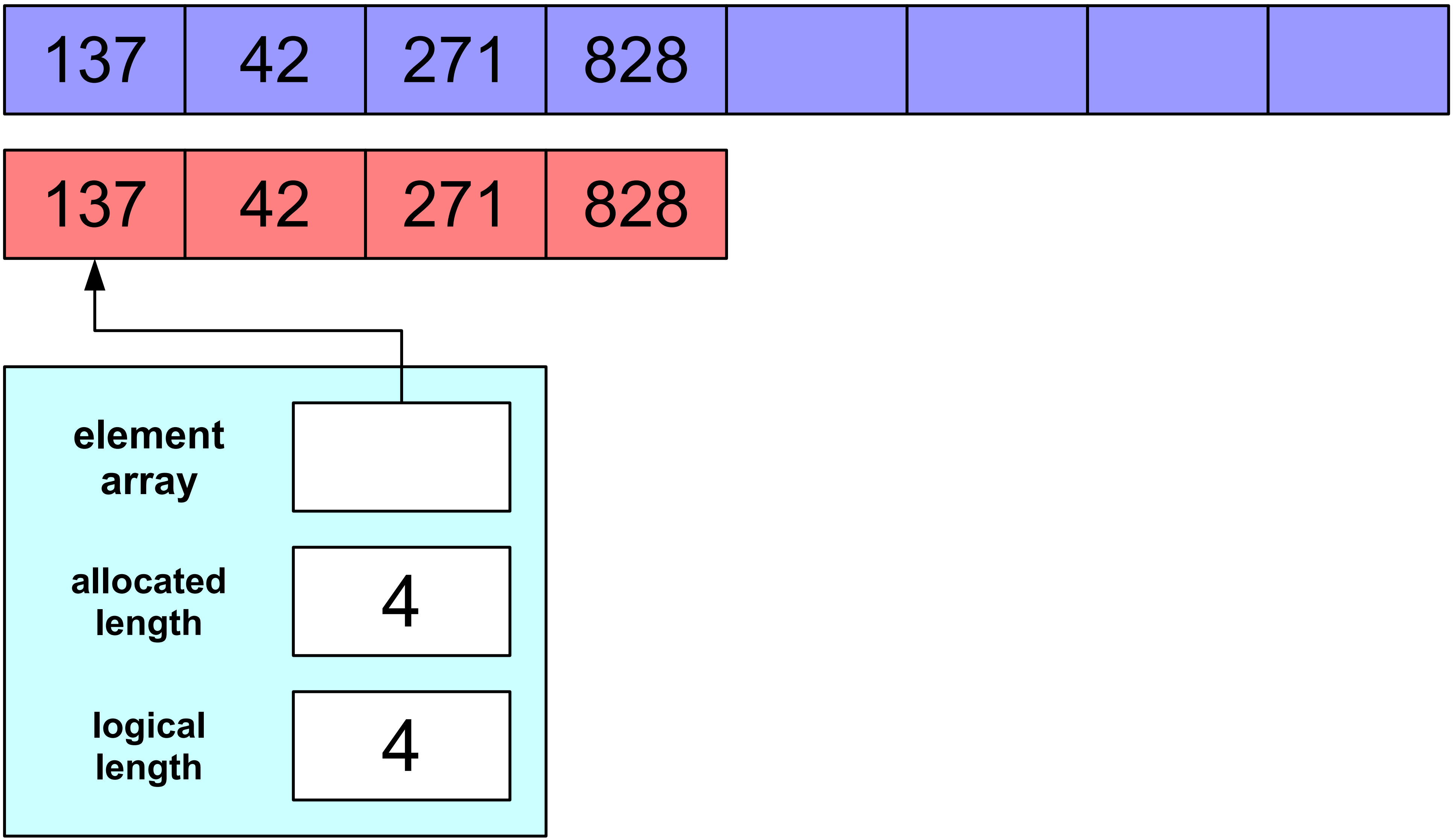

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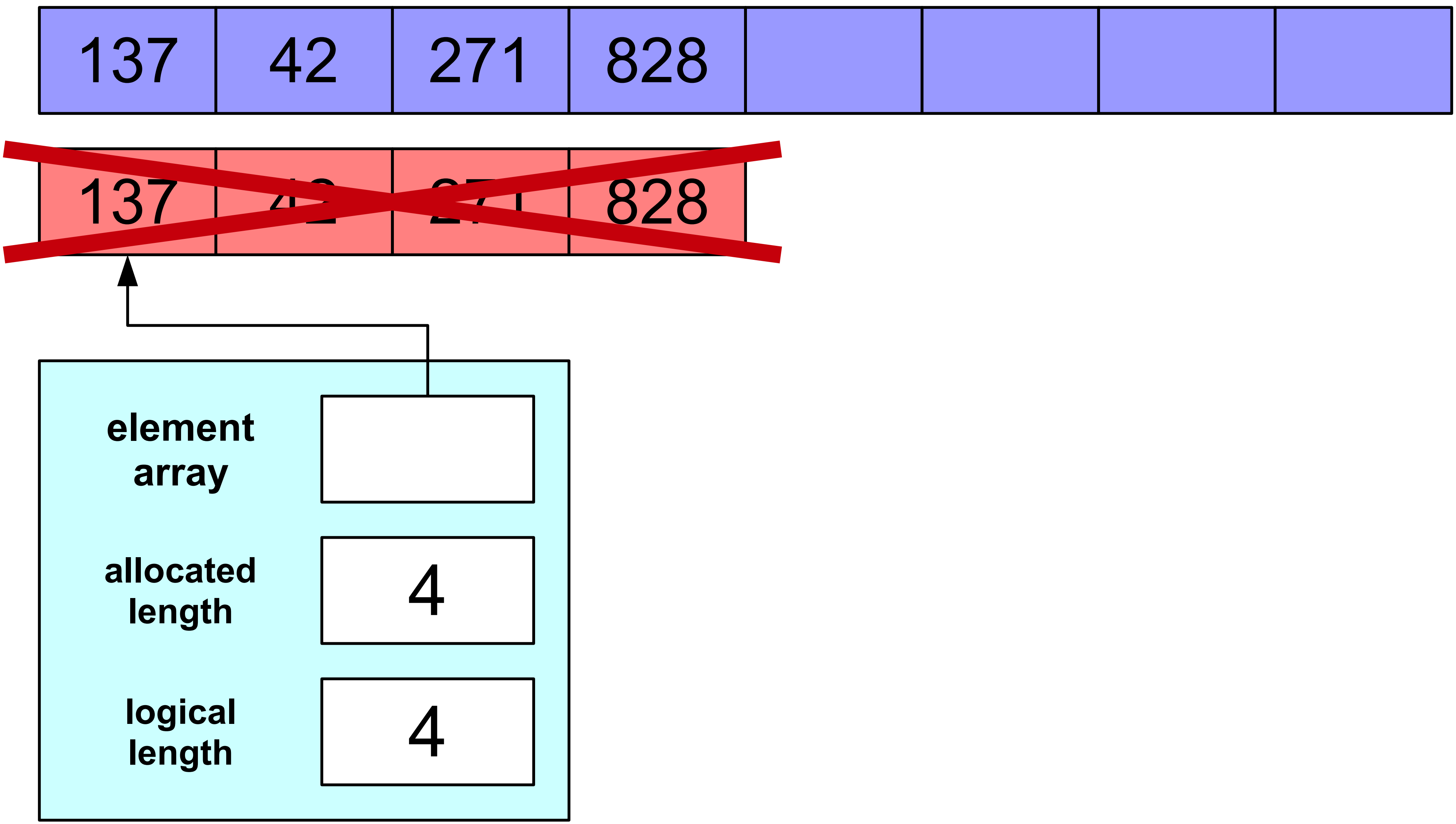

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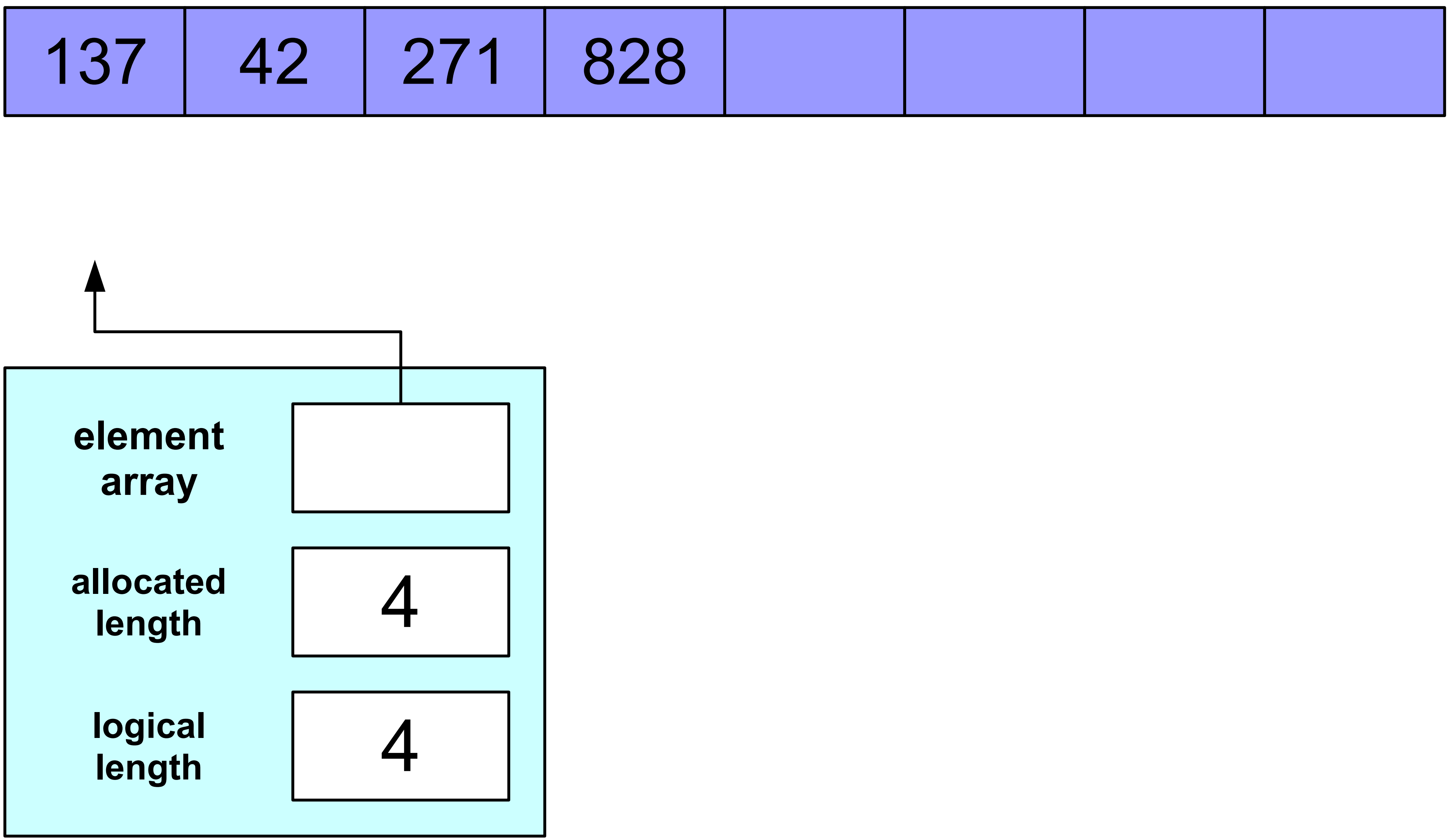

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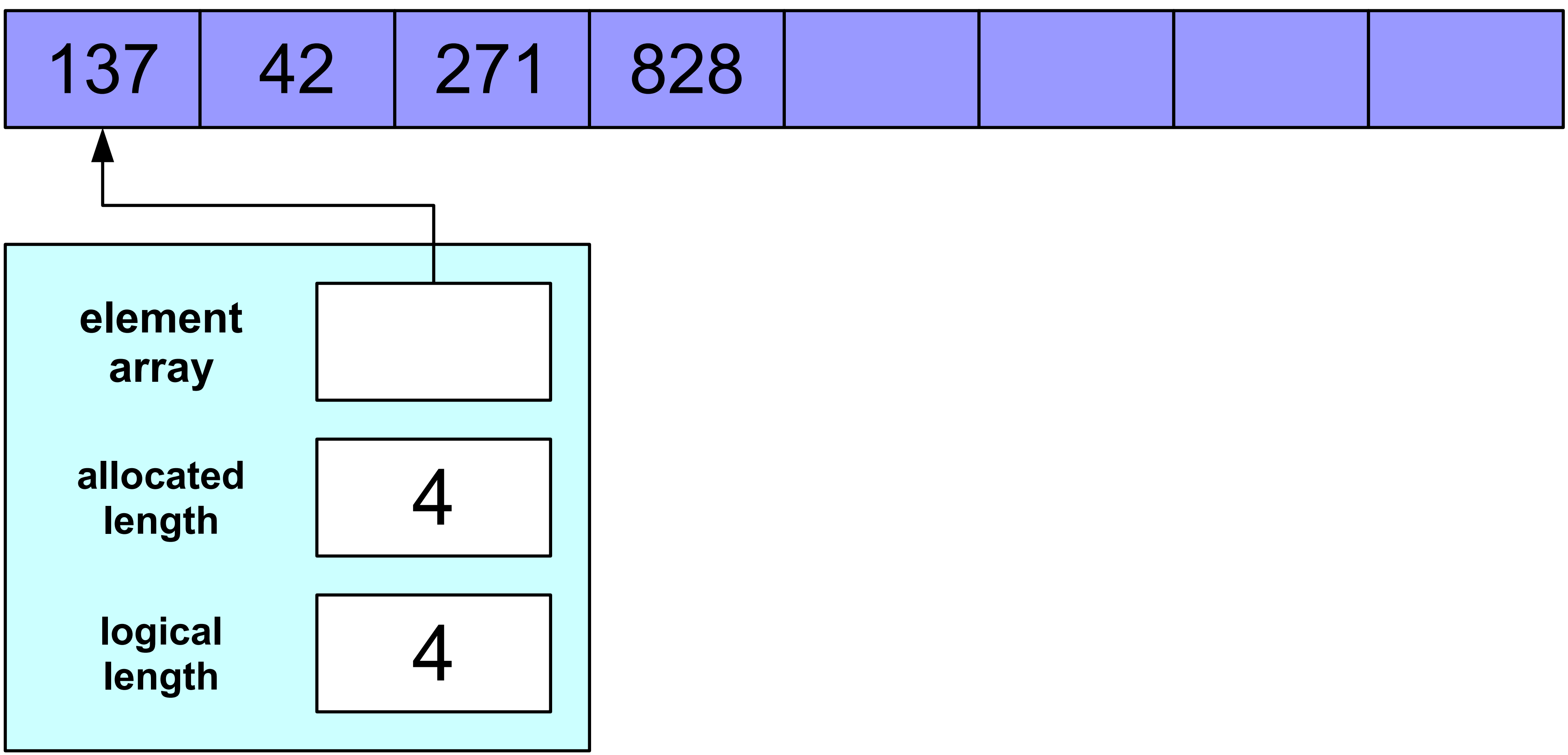

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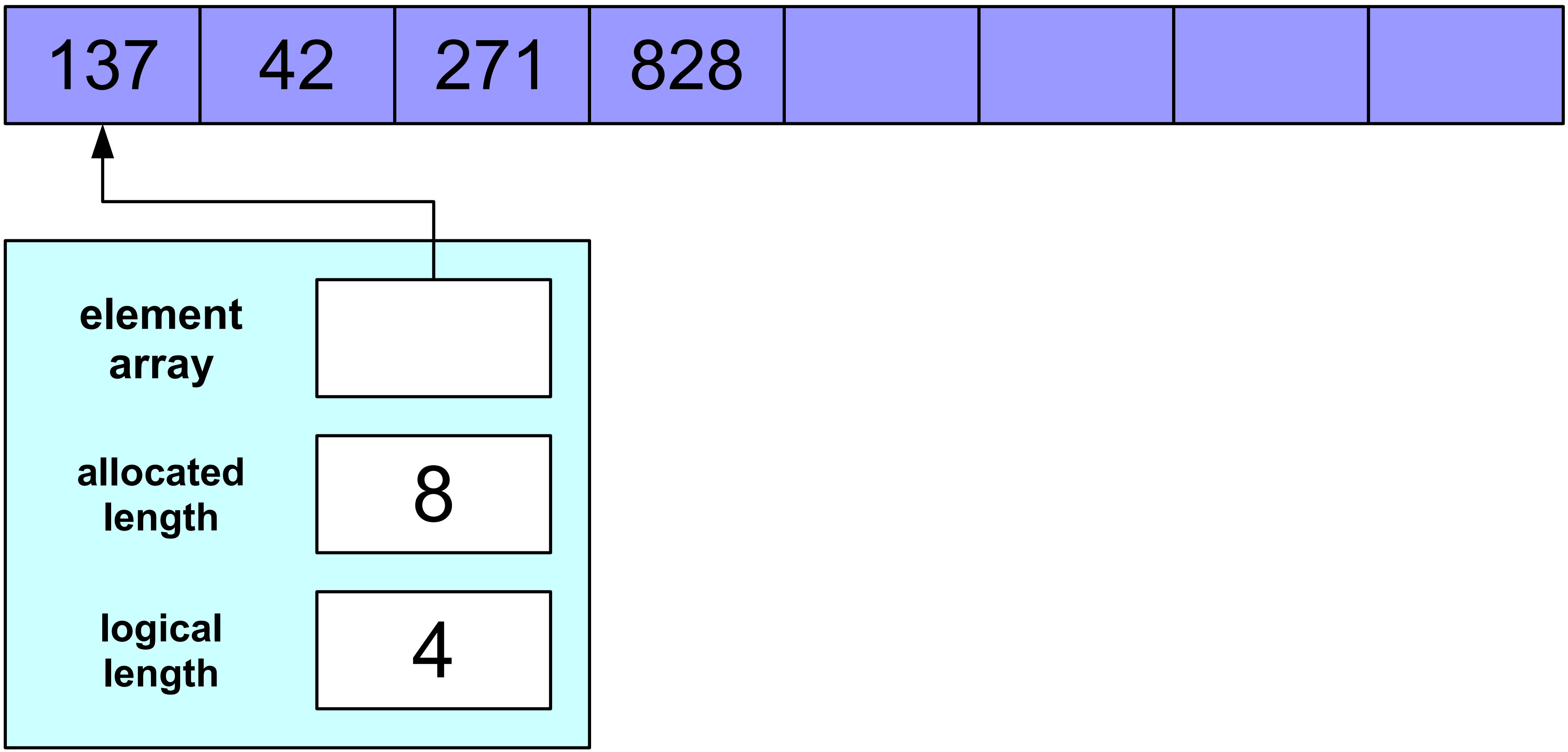

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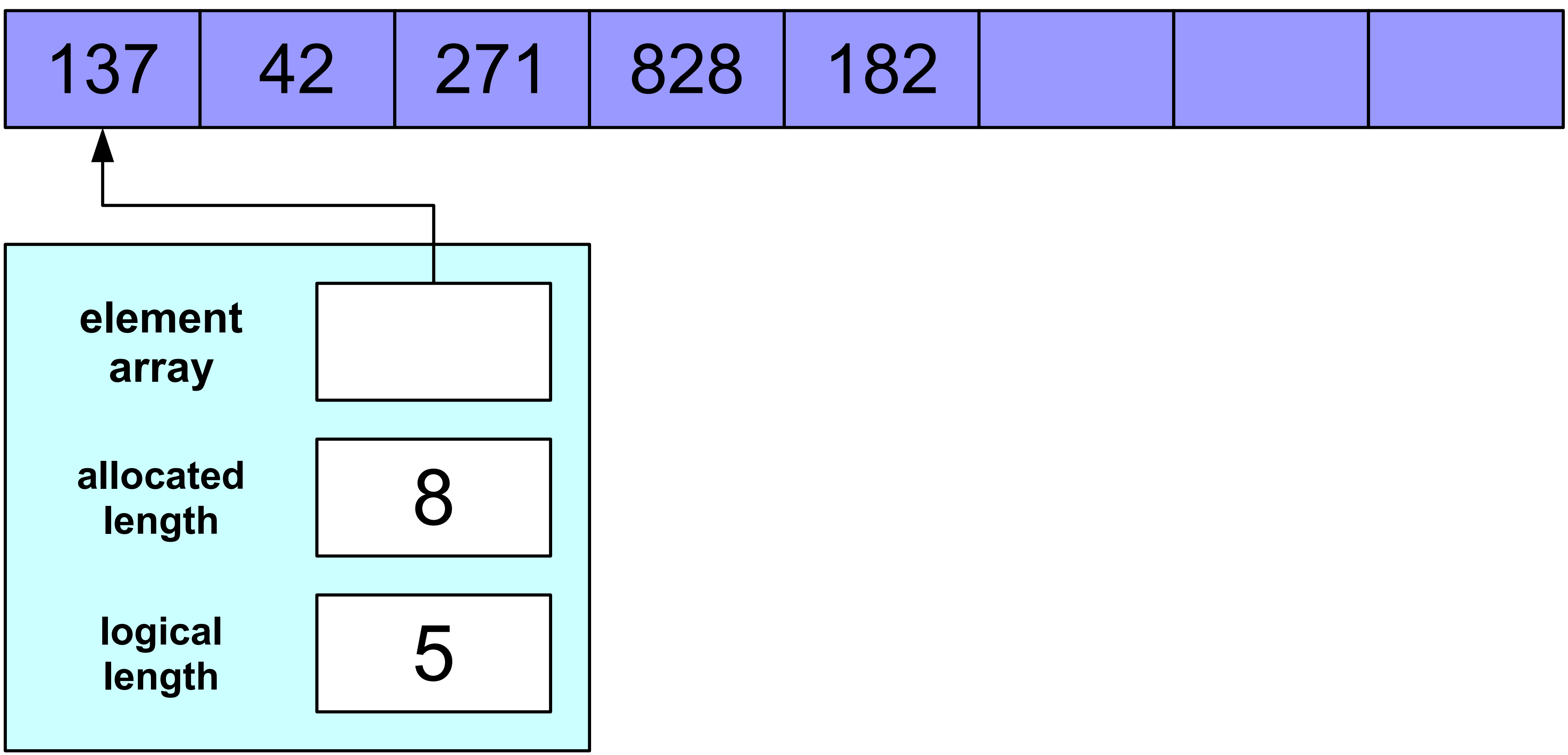

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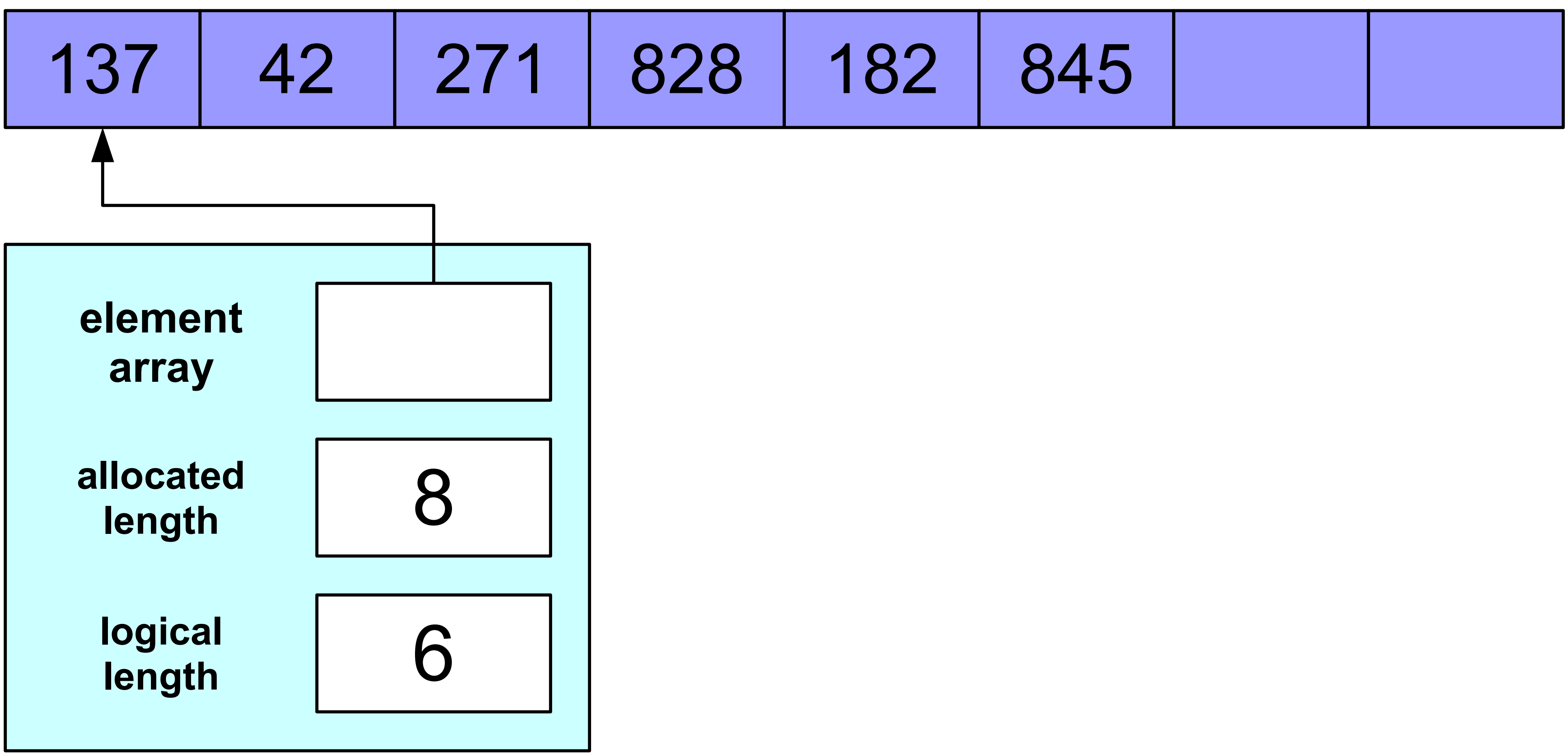

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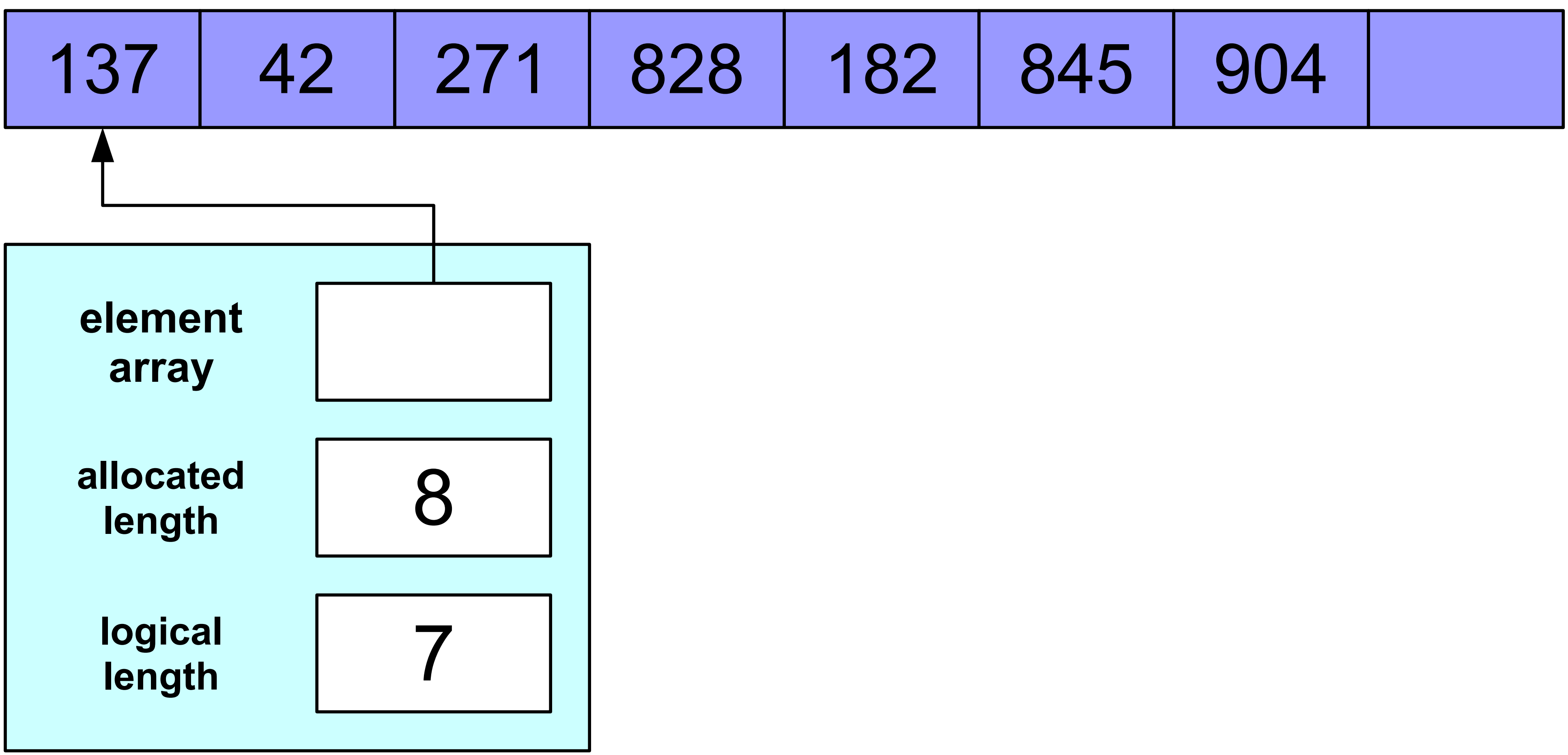

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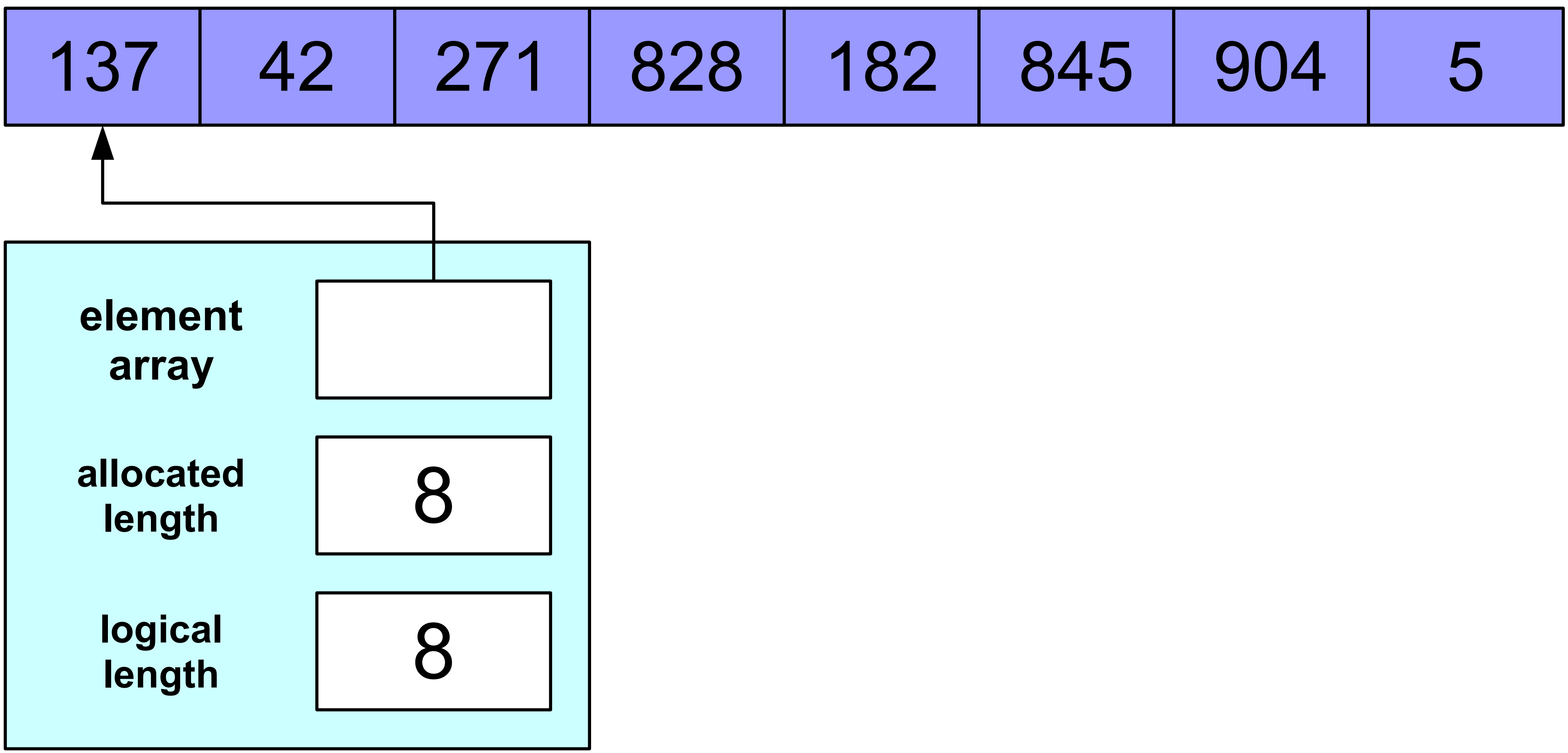
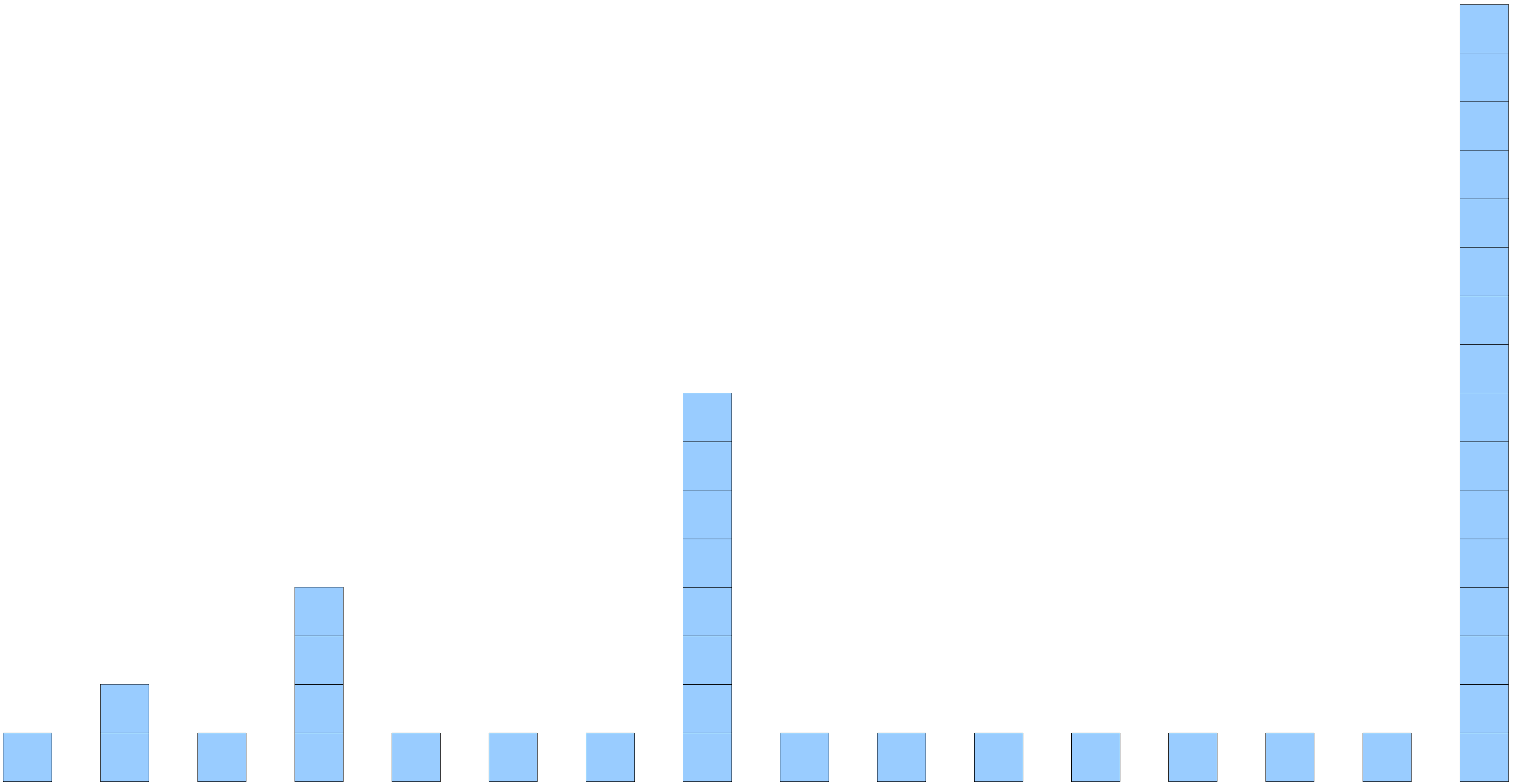


Let's Give it a Try!

How do we analyze this?

Spreading the Work



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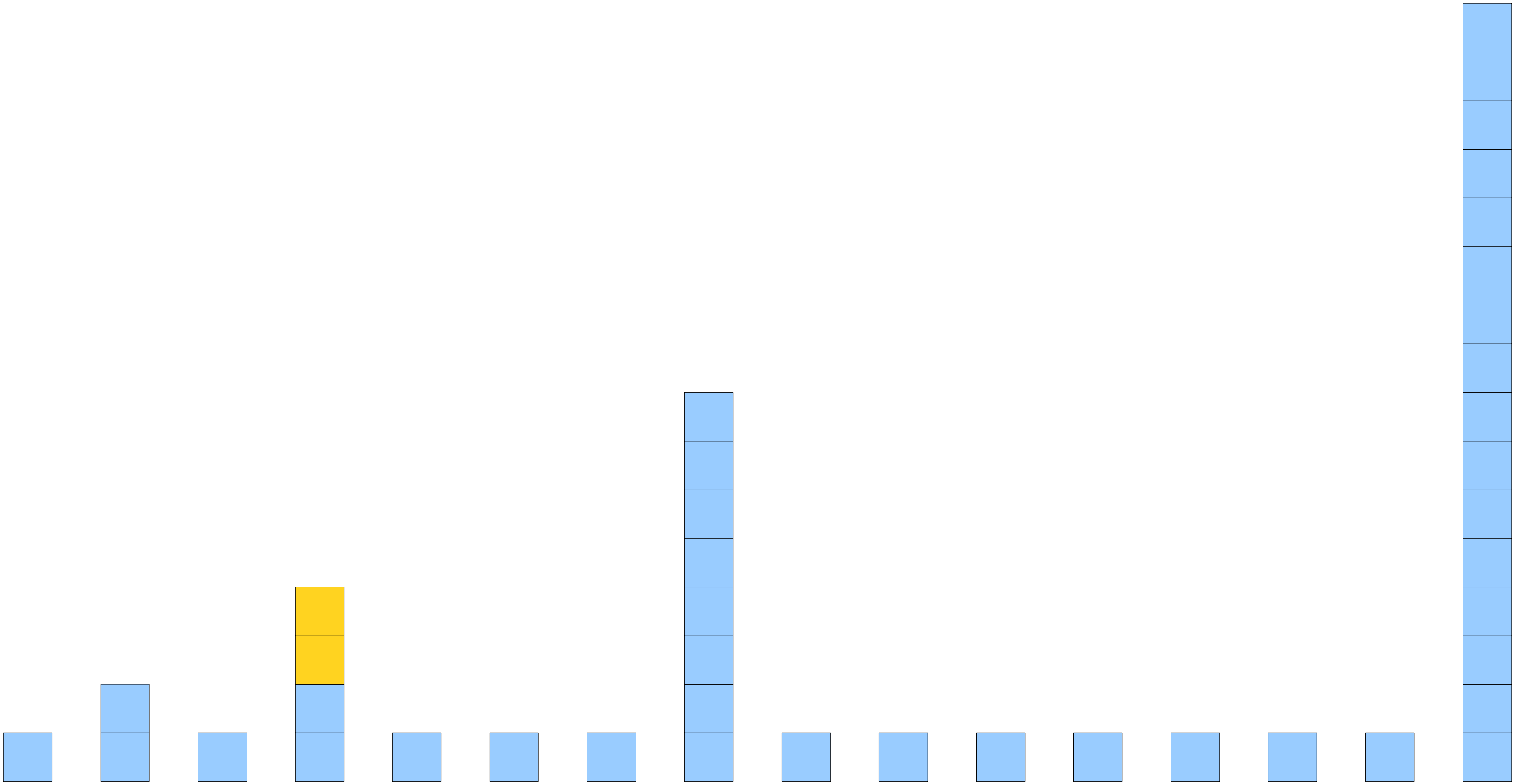

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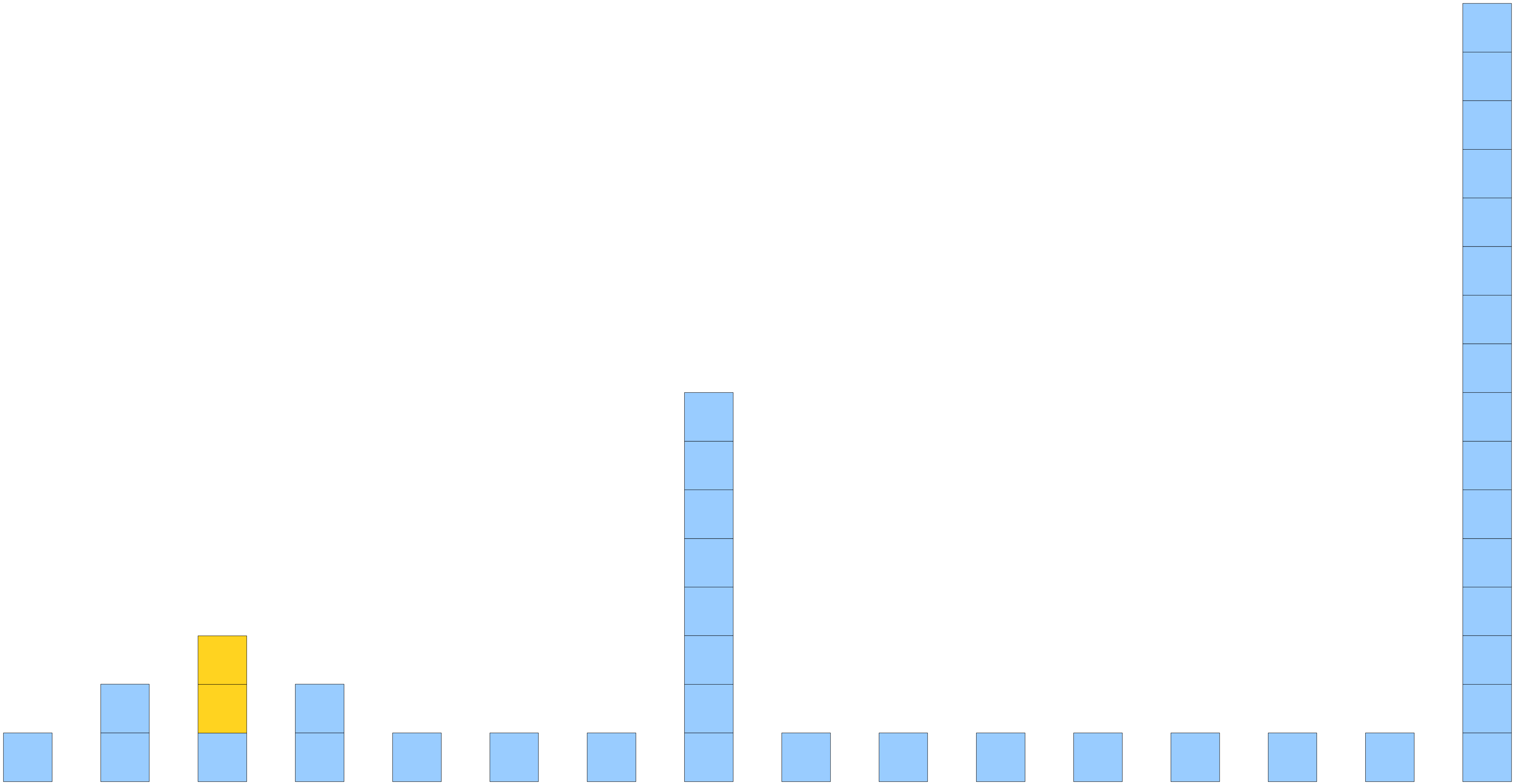

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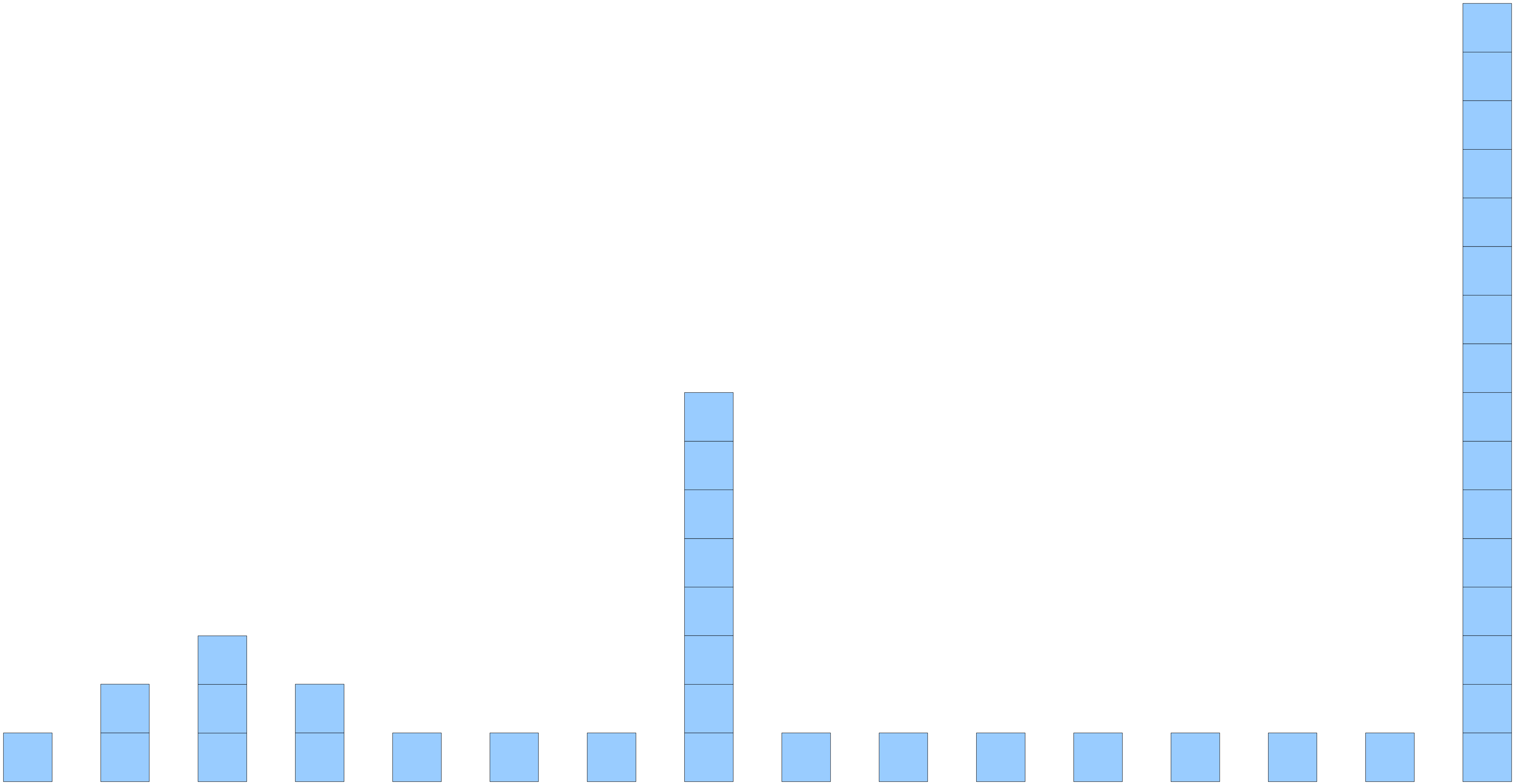

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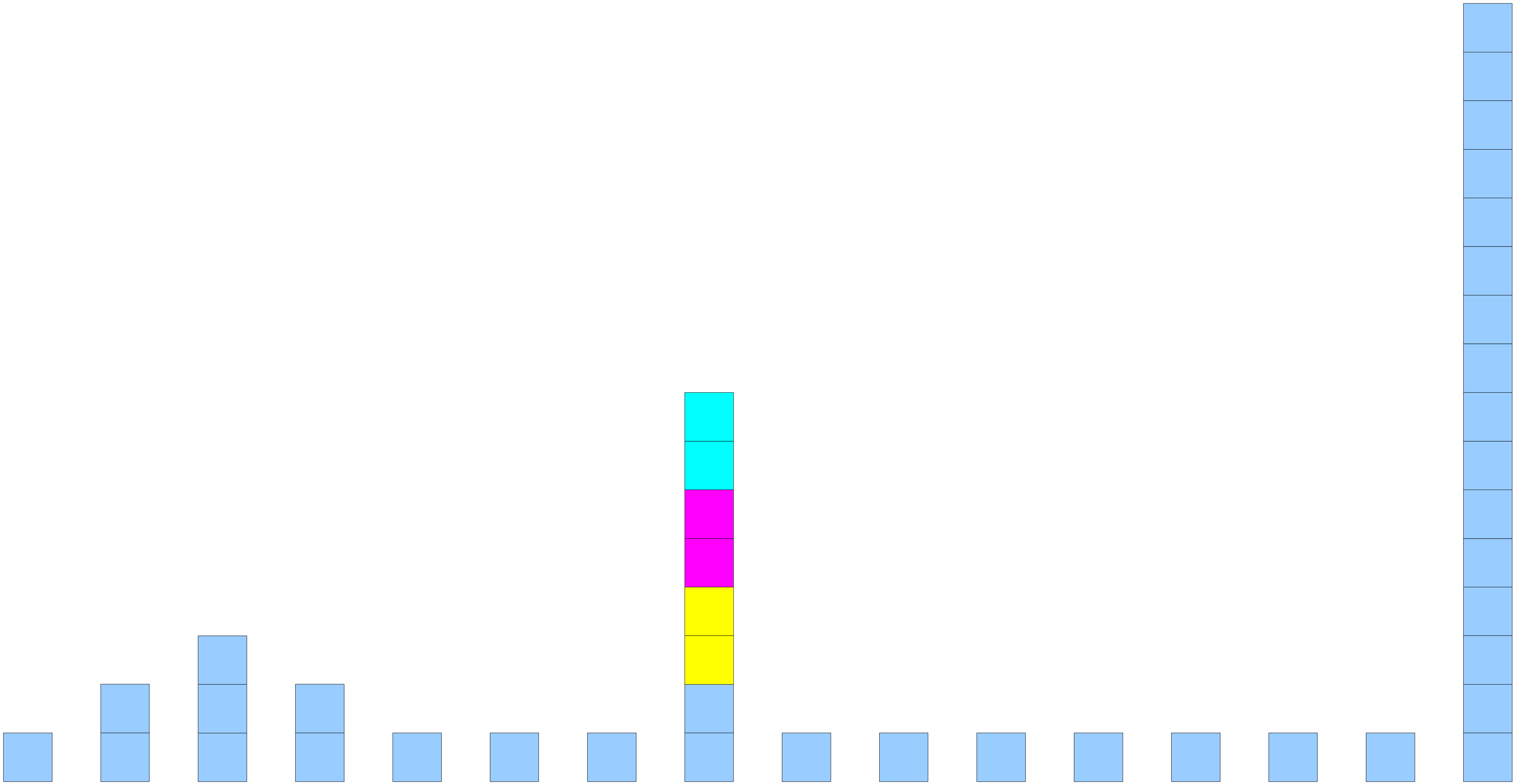

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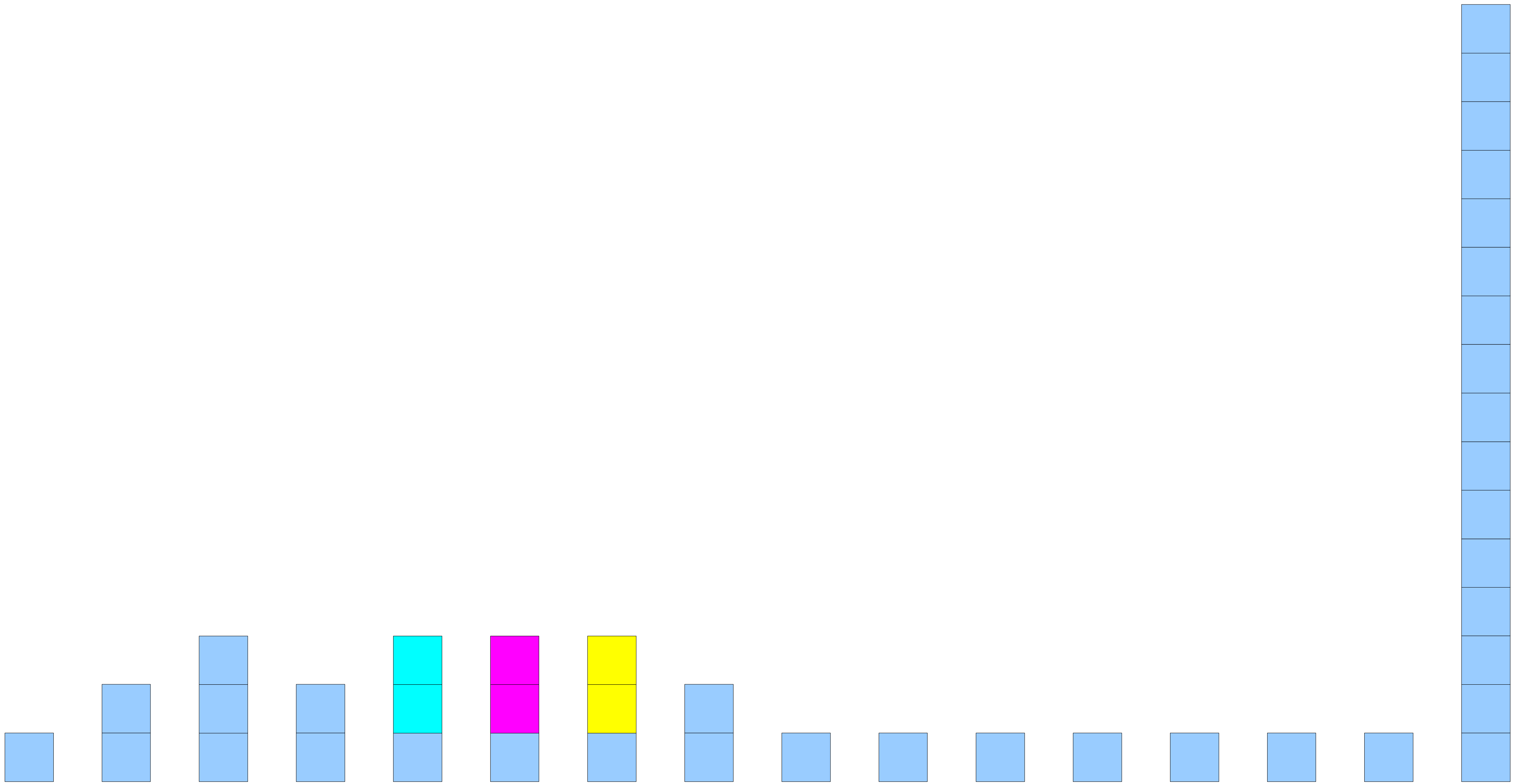

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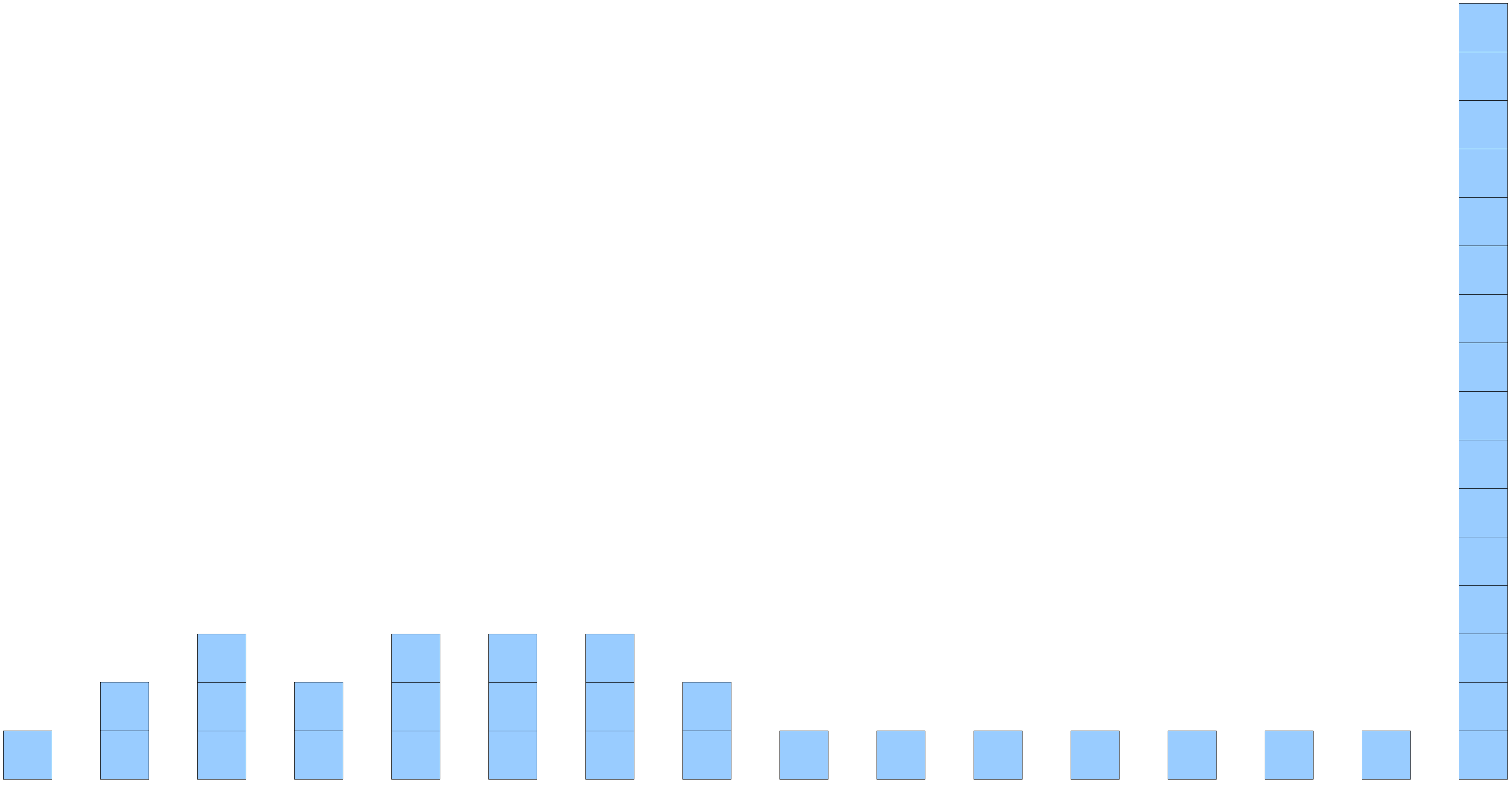

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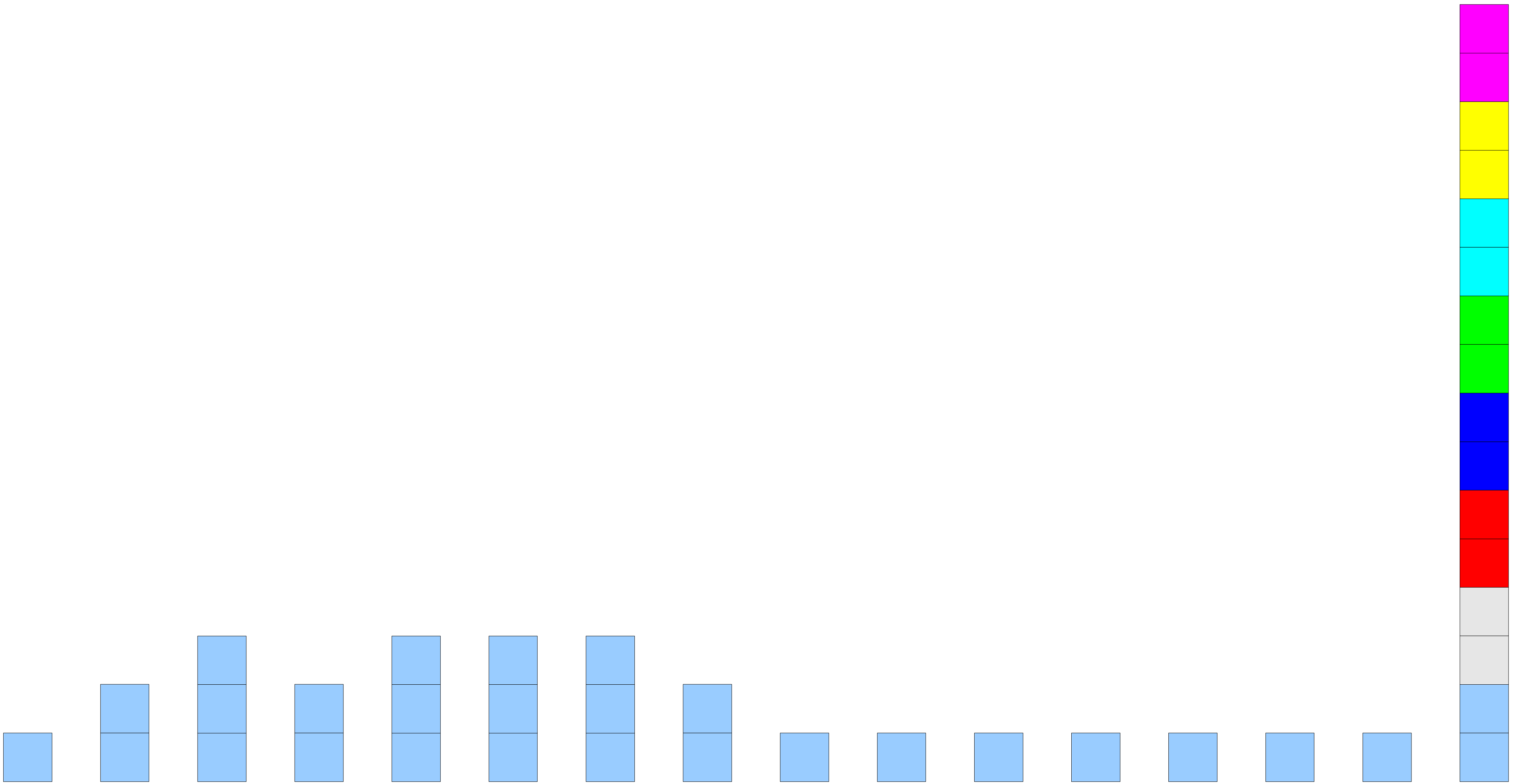

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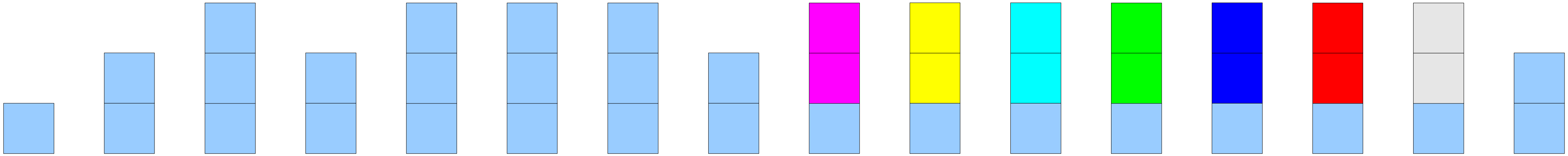

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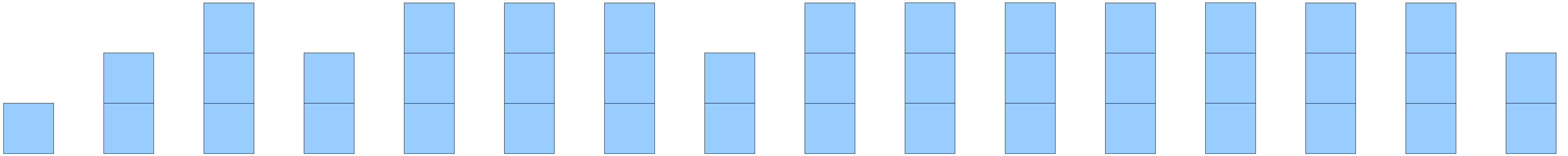
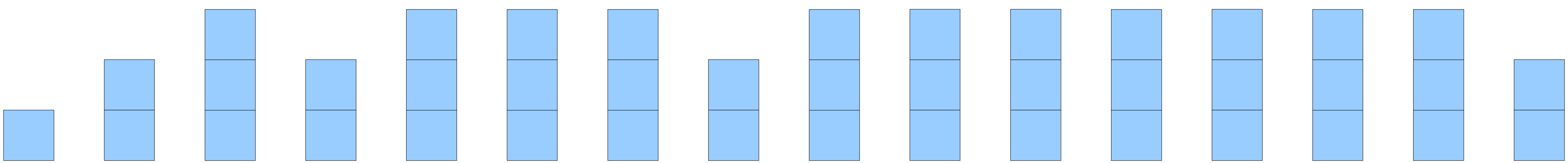


Spreading the Work

On average, we do just 3 units of work!

This is O(1) work on average!



Sharing the Burden

- We still have "heavy" pushes taking time O(n) and "light" pushes taking time O(1).
- Worst-case time for a push is O(n).
- Heavy pushes become so rare that the **average** time for a push is O(1).
- Can we confirm this?

Amortized Analysis

- The analysis we have just done is called an **amortized analysis**.
- Reason about the total amount of work done, not the work done per operation.
- In an amortized sense, our implementation of the stack is extremely fast!
- This is one of the most common approaches to implementing **`Stack`**.

Amortized Analysis

- People are amortizing ***all the time!***
  - E.g. Buying Groceries
    - Driving to Store takes 30 minutes roundtrip.
    - Actually shopping takes 5 minutes per days worth of food you buy
    - If we only buy 1 days worth of food per visit then we'll spend 35 minutes a day shopping
    - If we buy 1 weeks worth of food per visit, then we'll spend only 65/7 ~ 9 minutes *on average* per day.

Implementing `Queue`

Implementing `Queue`

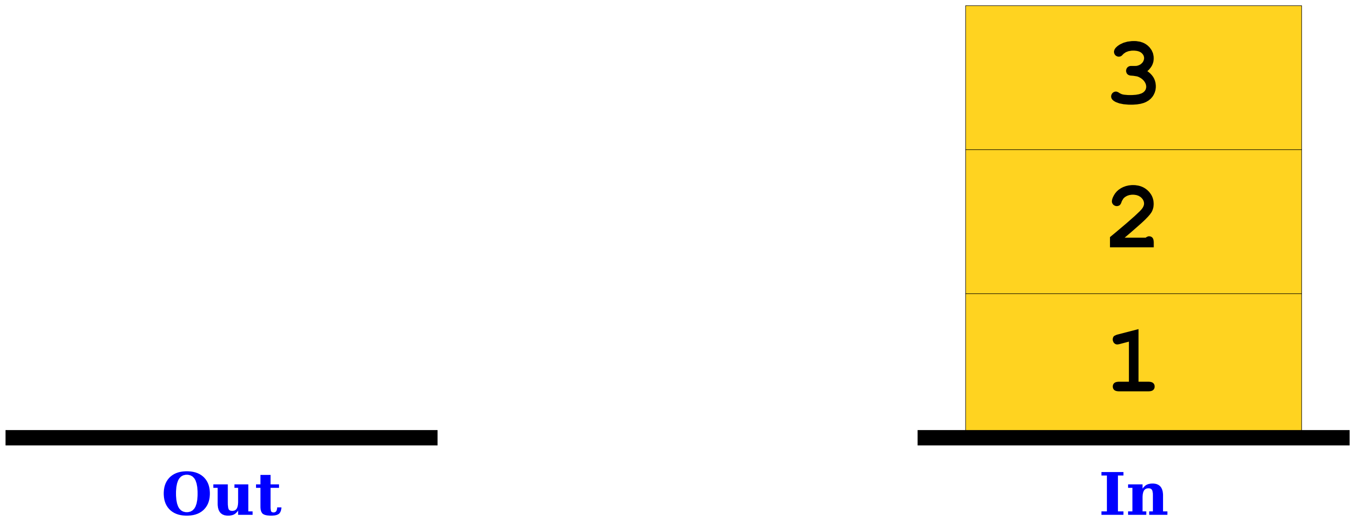
- We've just used dynamic arrays to implement a stack. Could we use them to implement a queue?
- Yes, but here's a better idea: **could we use our stack to implement a queue?**

The Two-Stack Queue

The Two-Stack Queue

The Two-Stack Queue

The Two-Stack Queue



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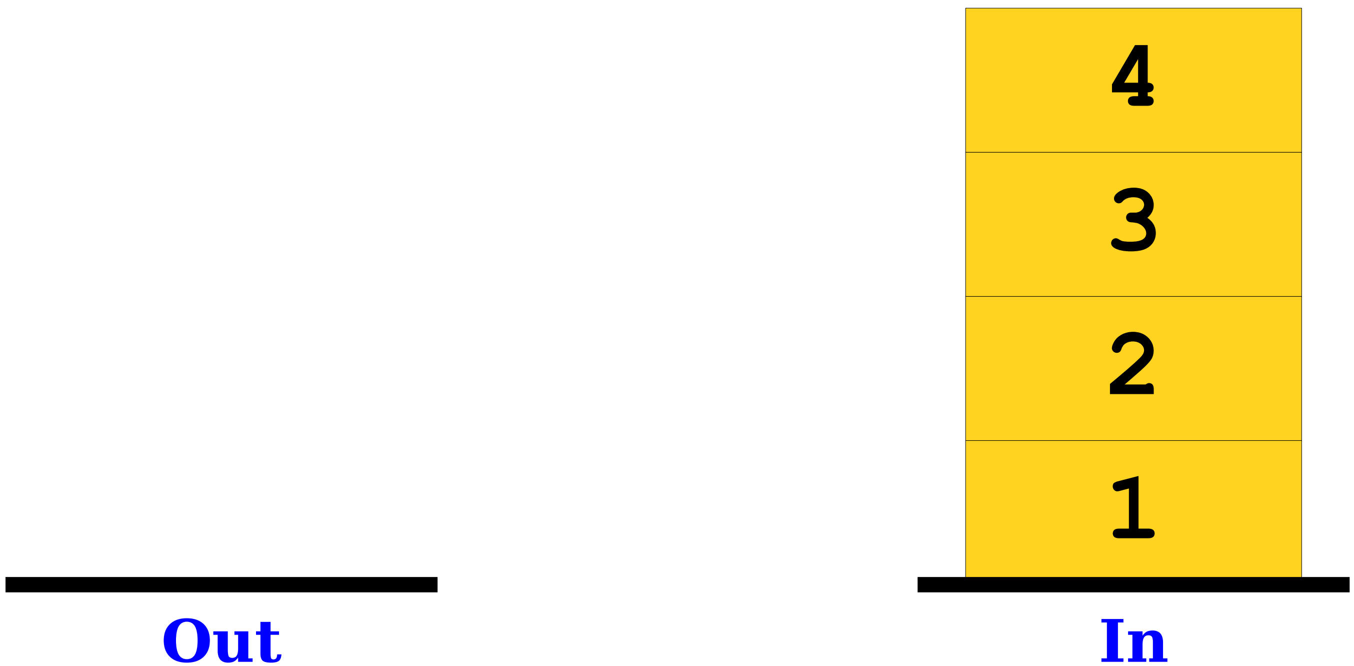

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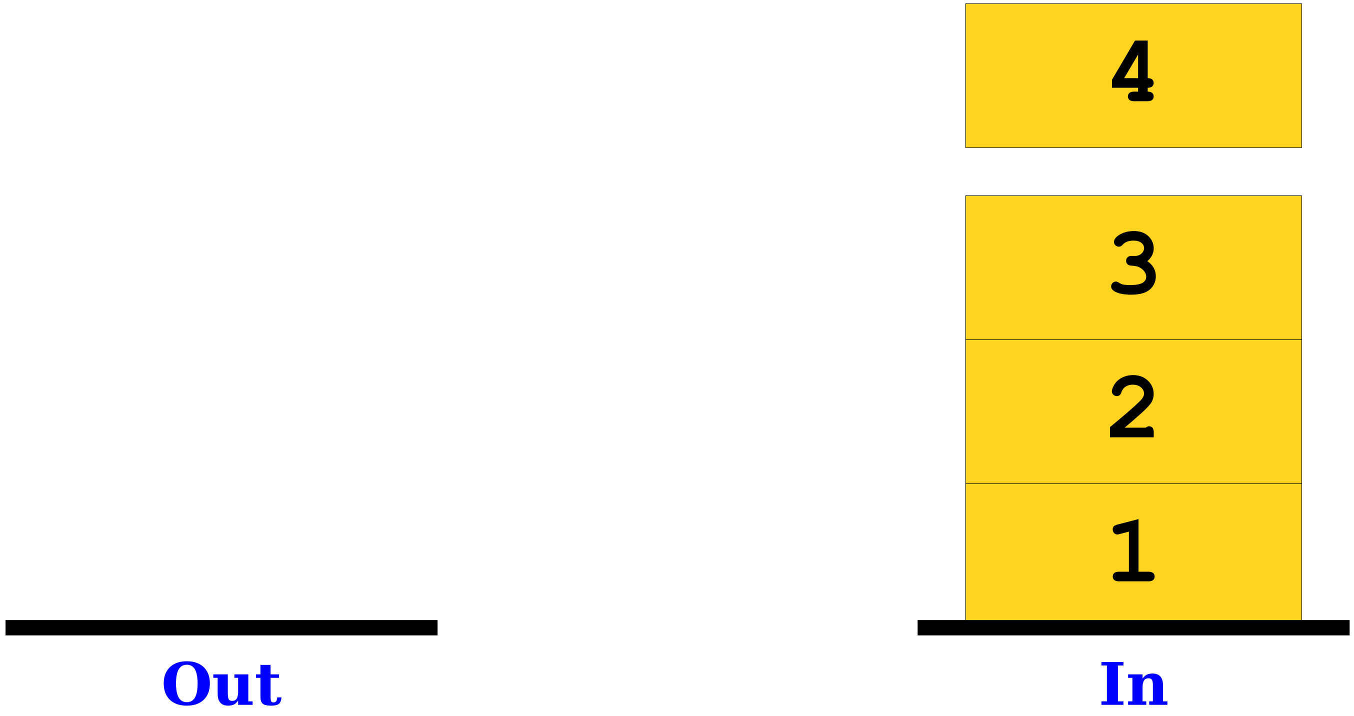

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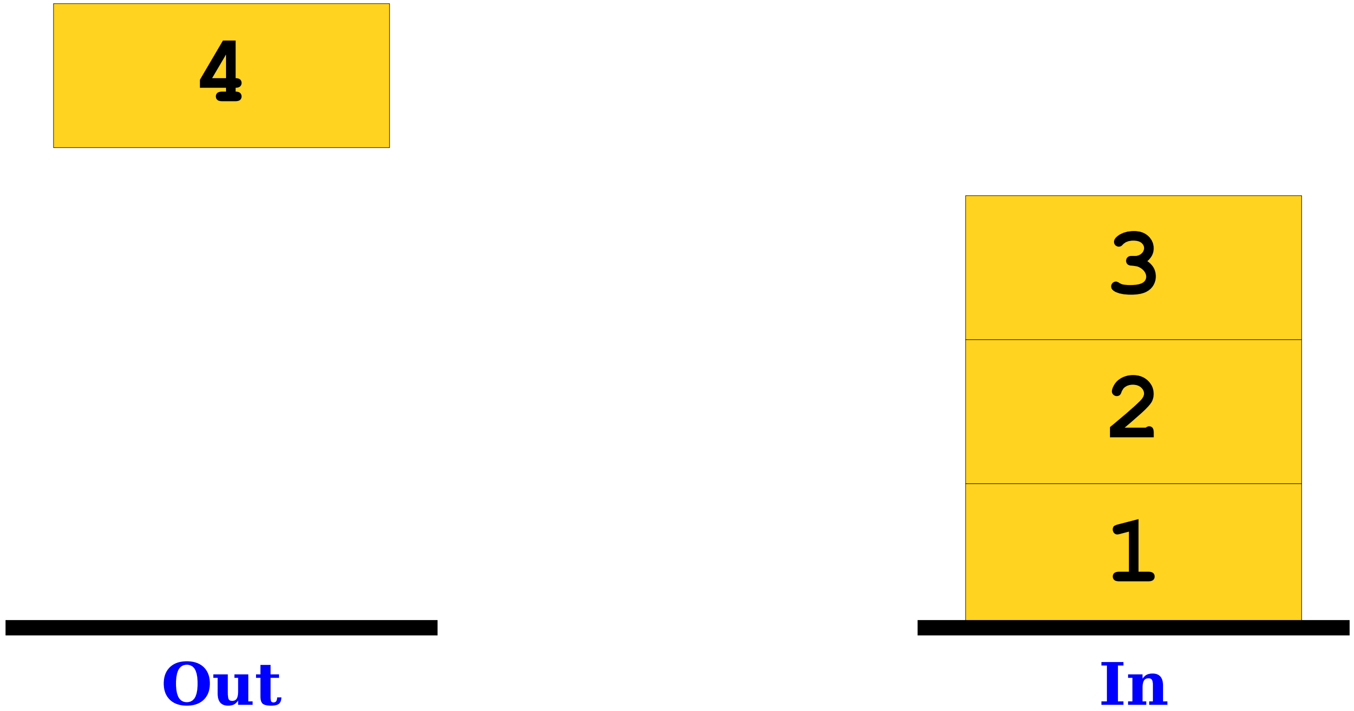

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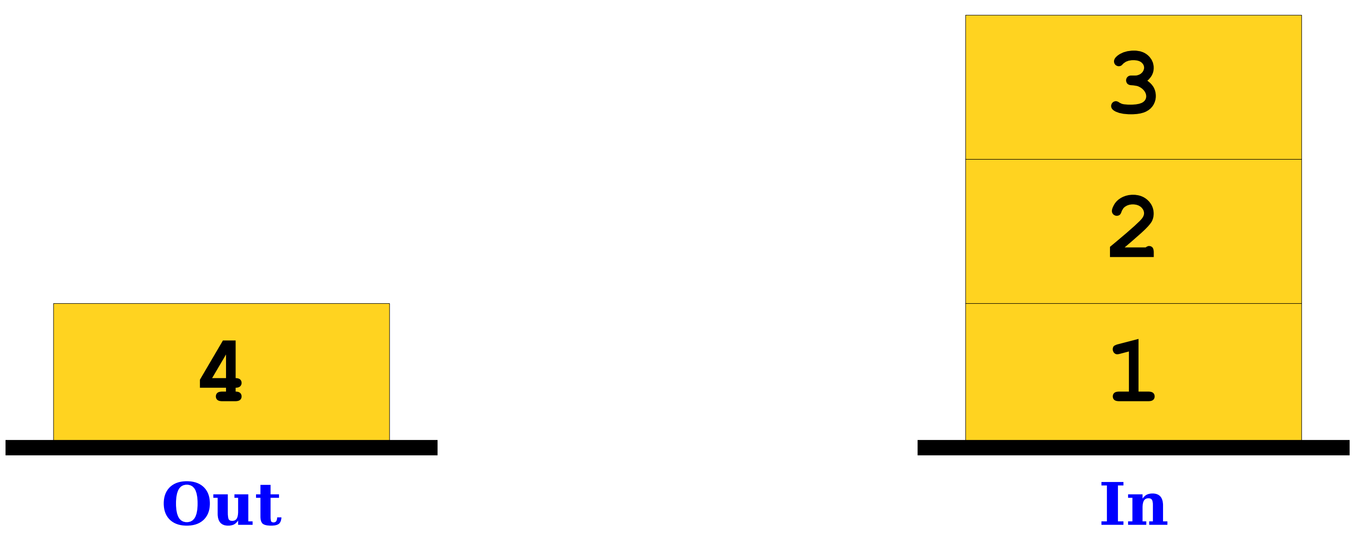

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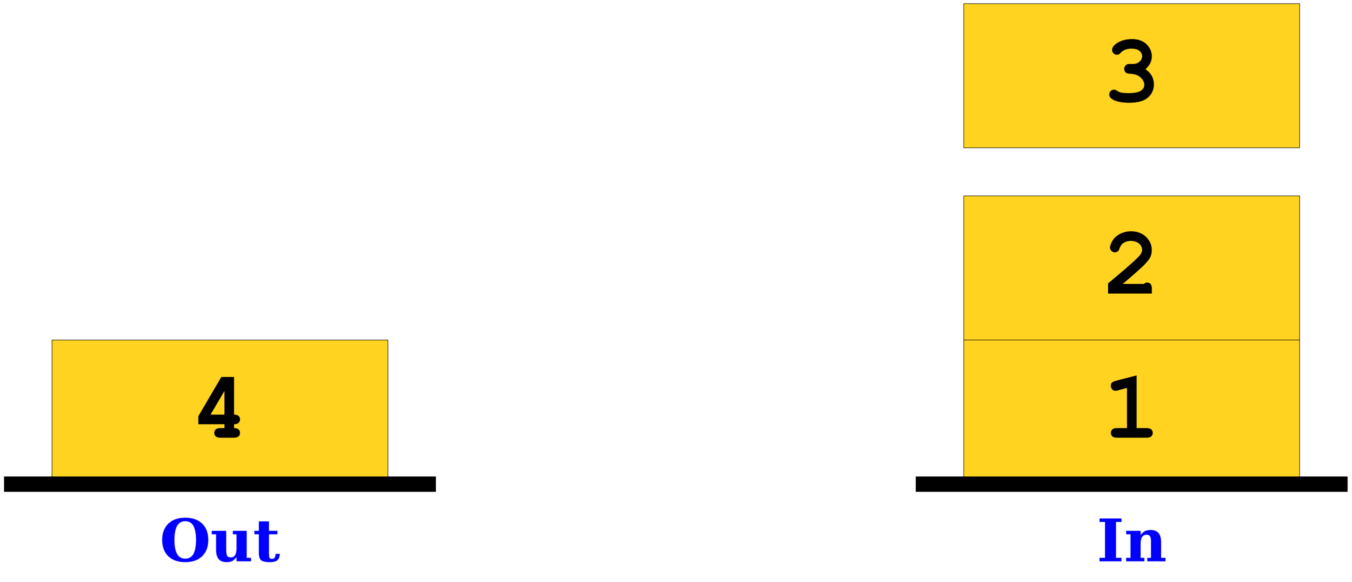

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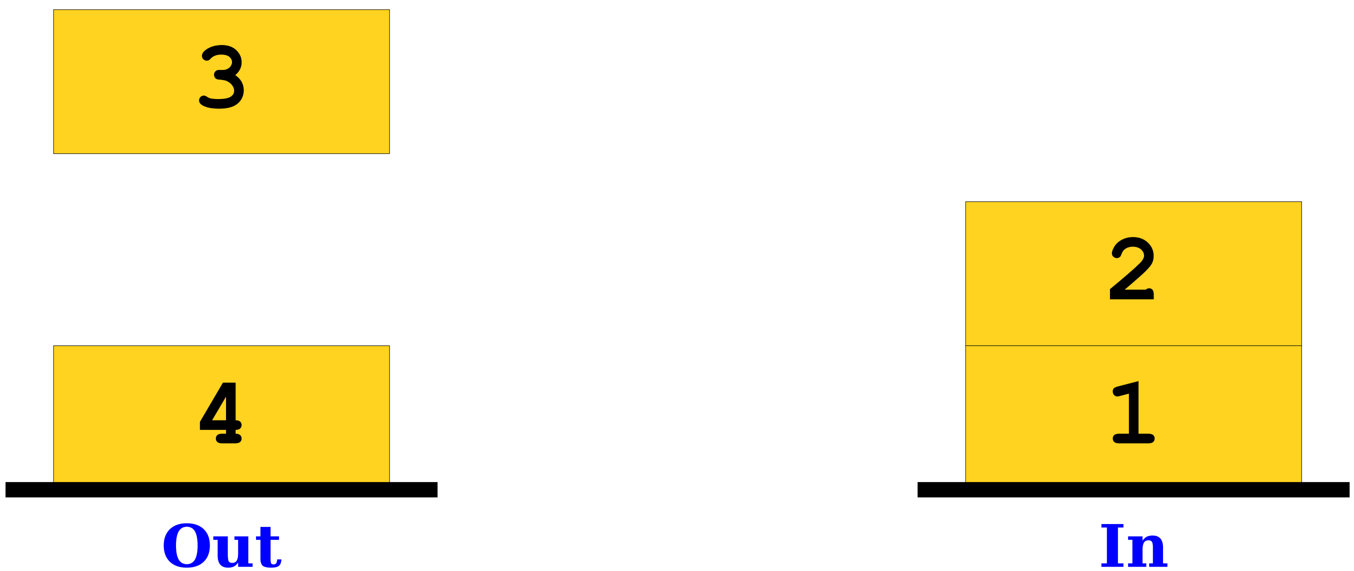

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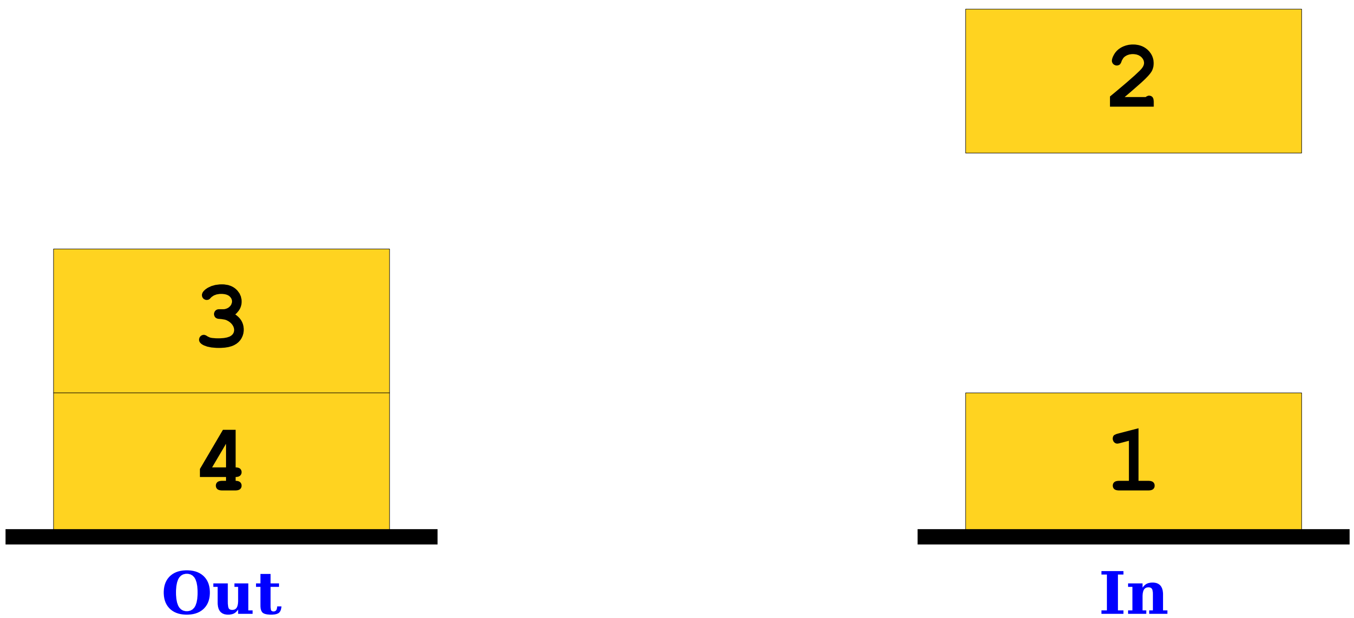

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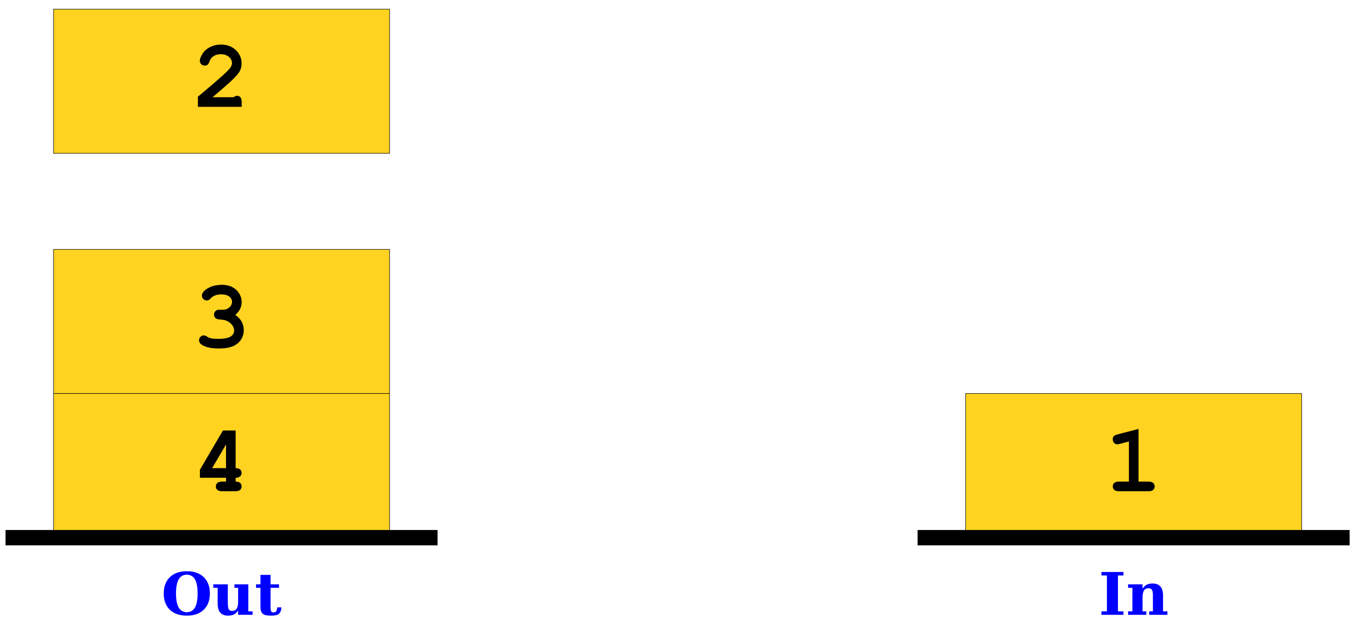

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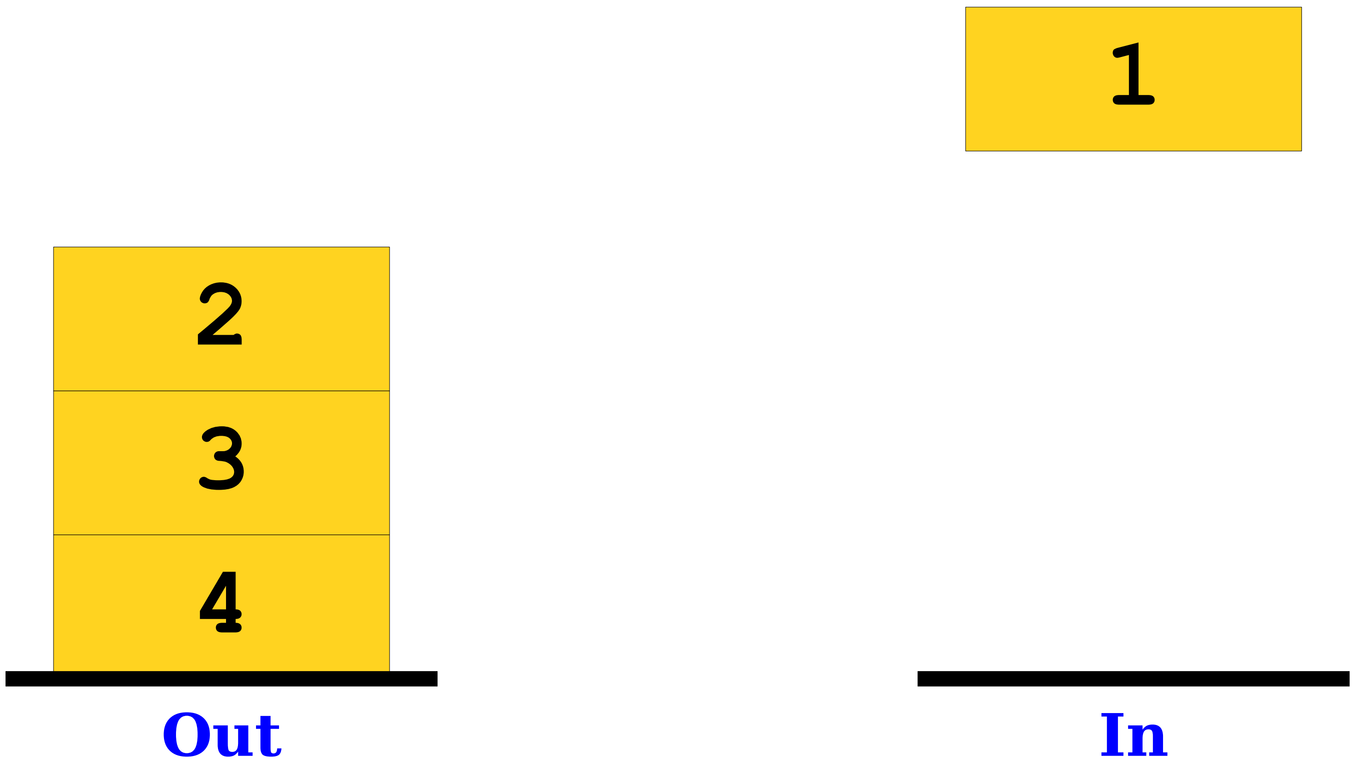

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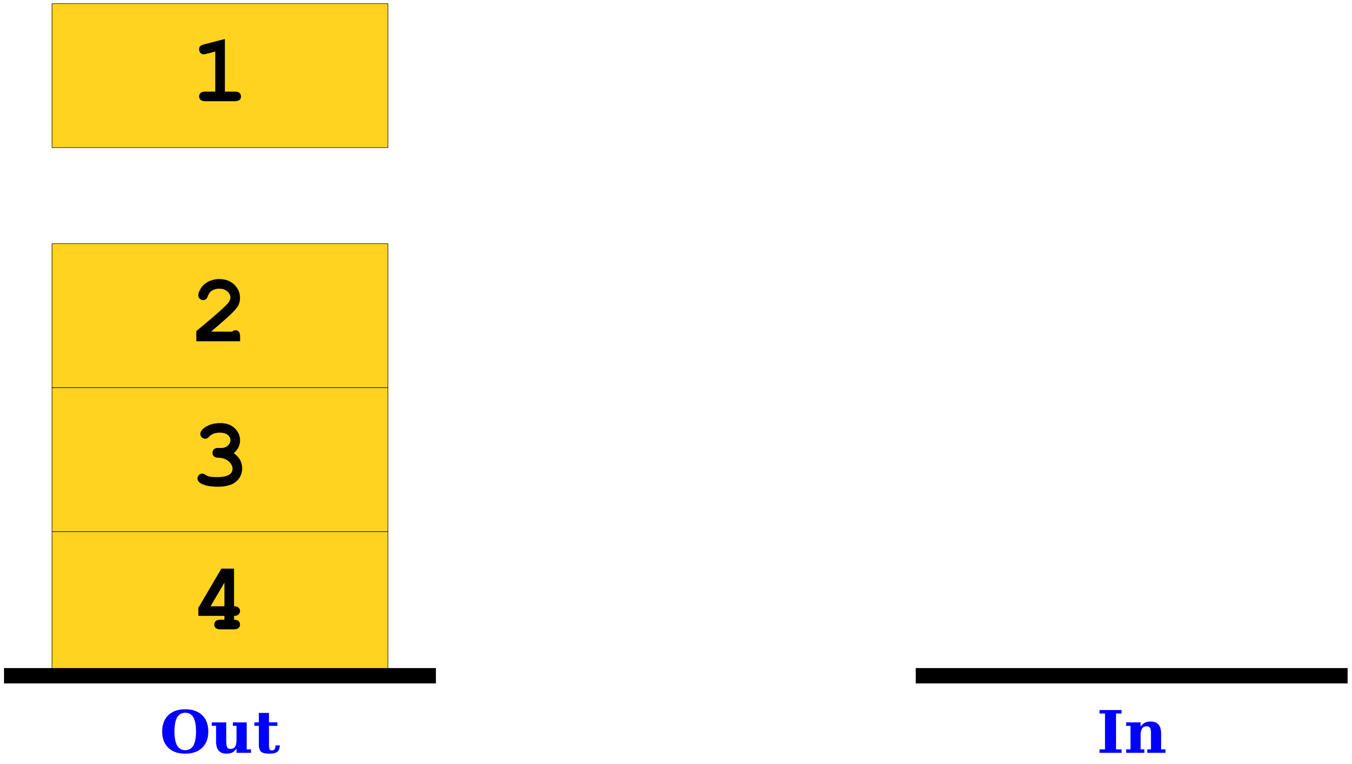

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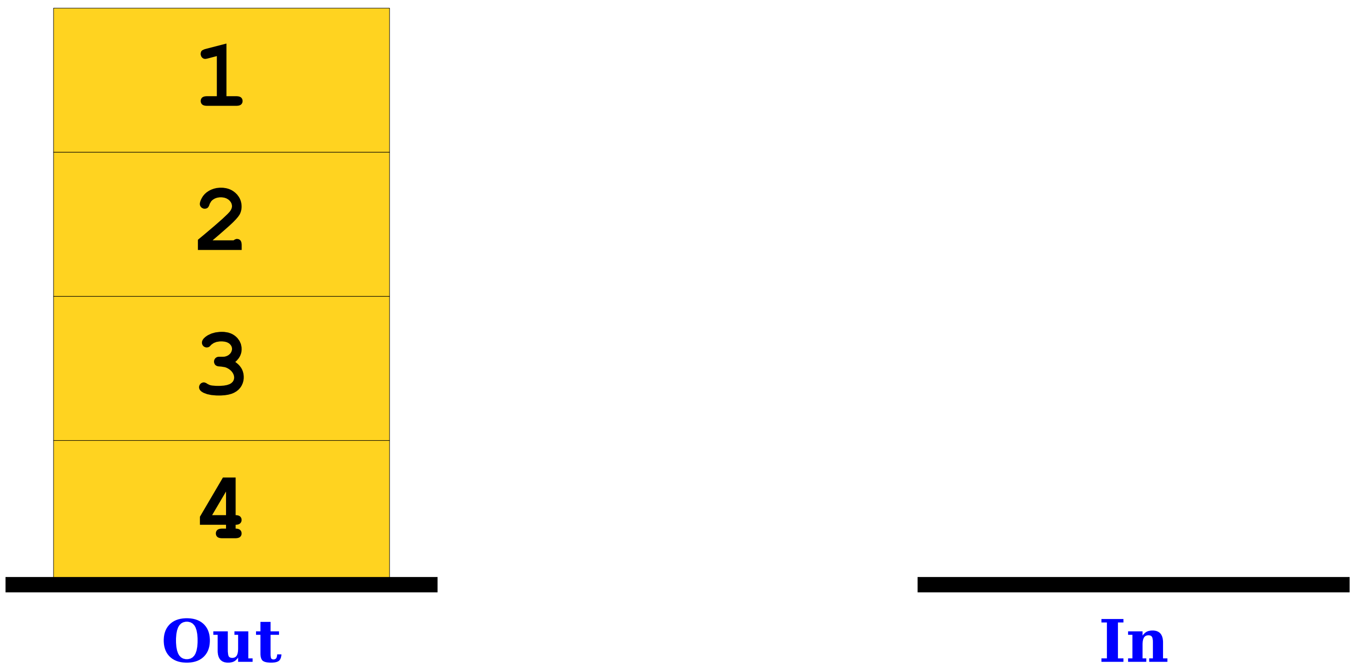

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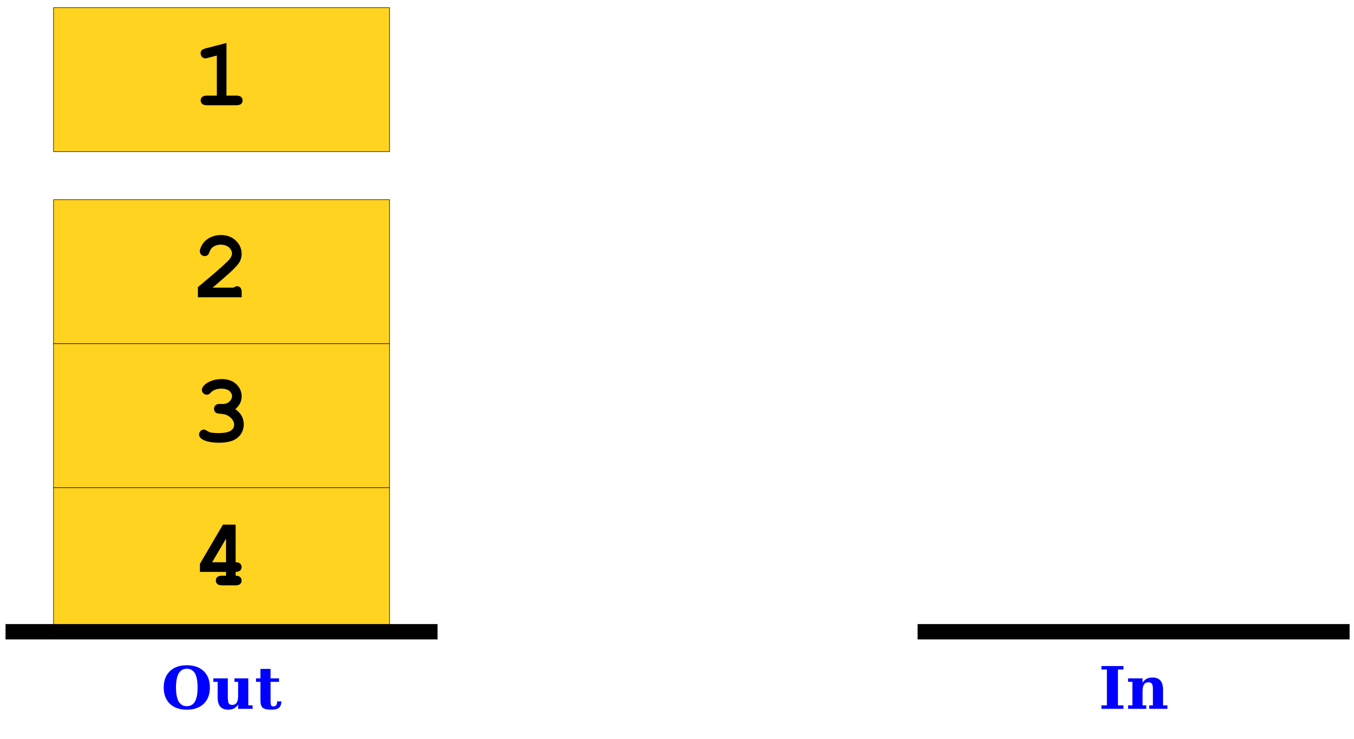

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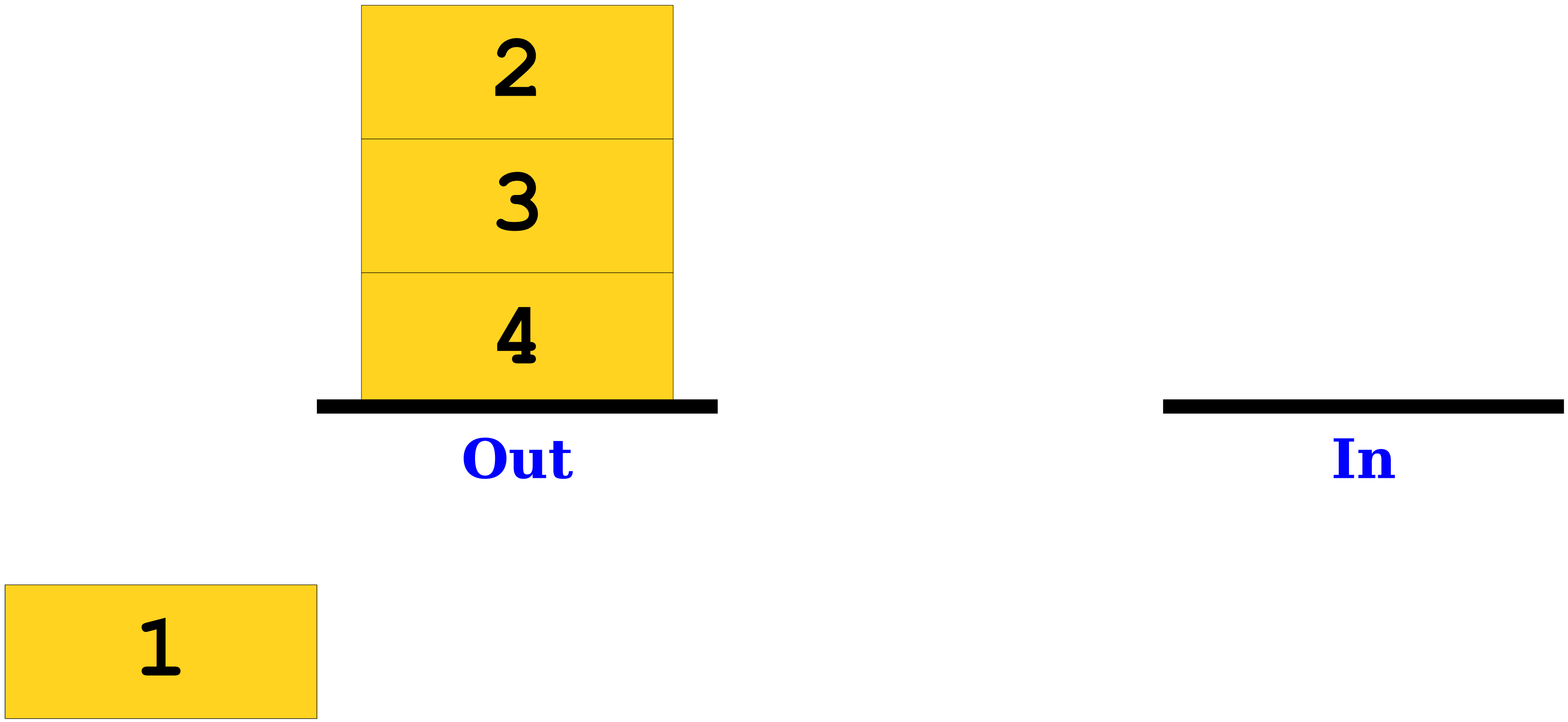

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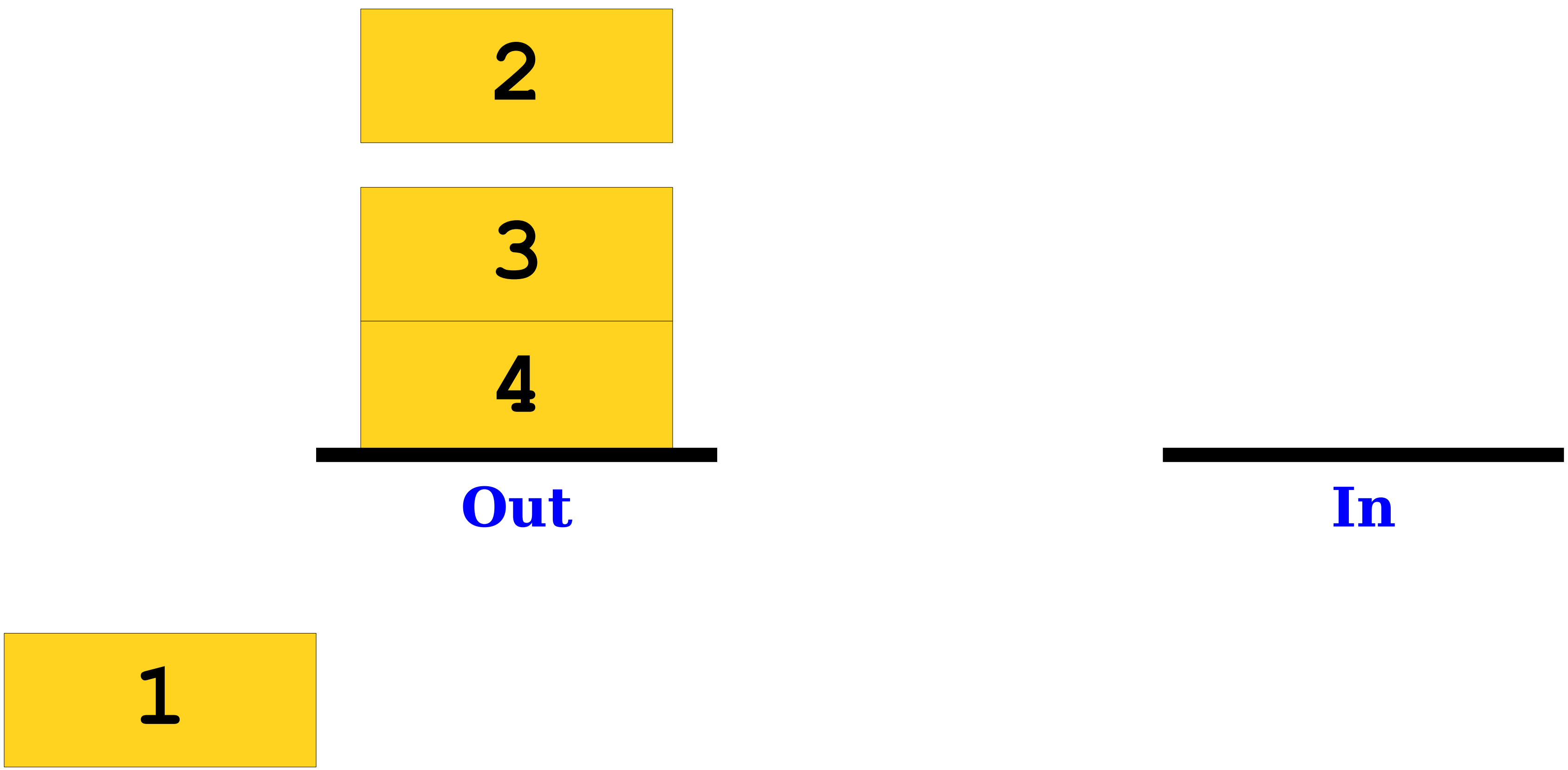

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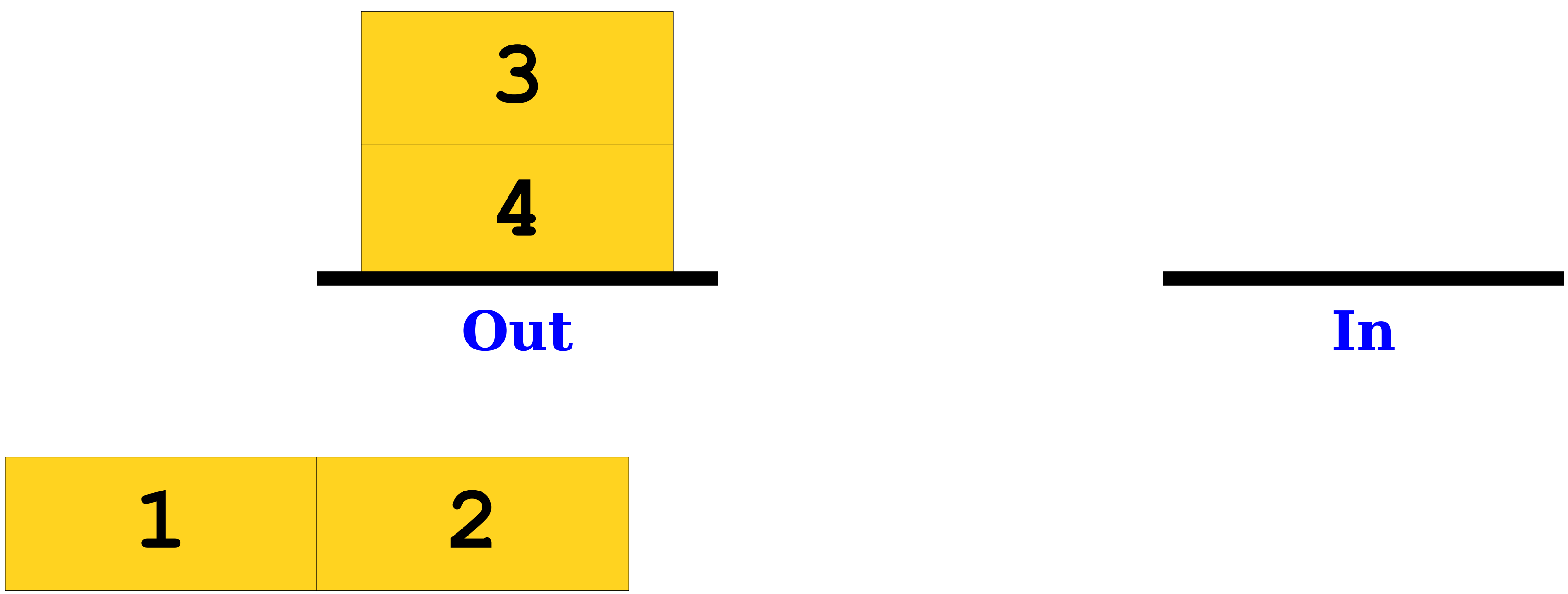

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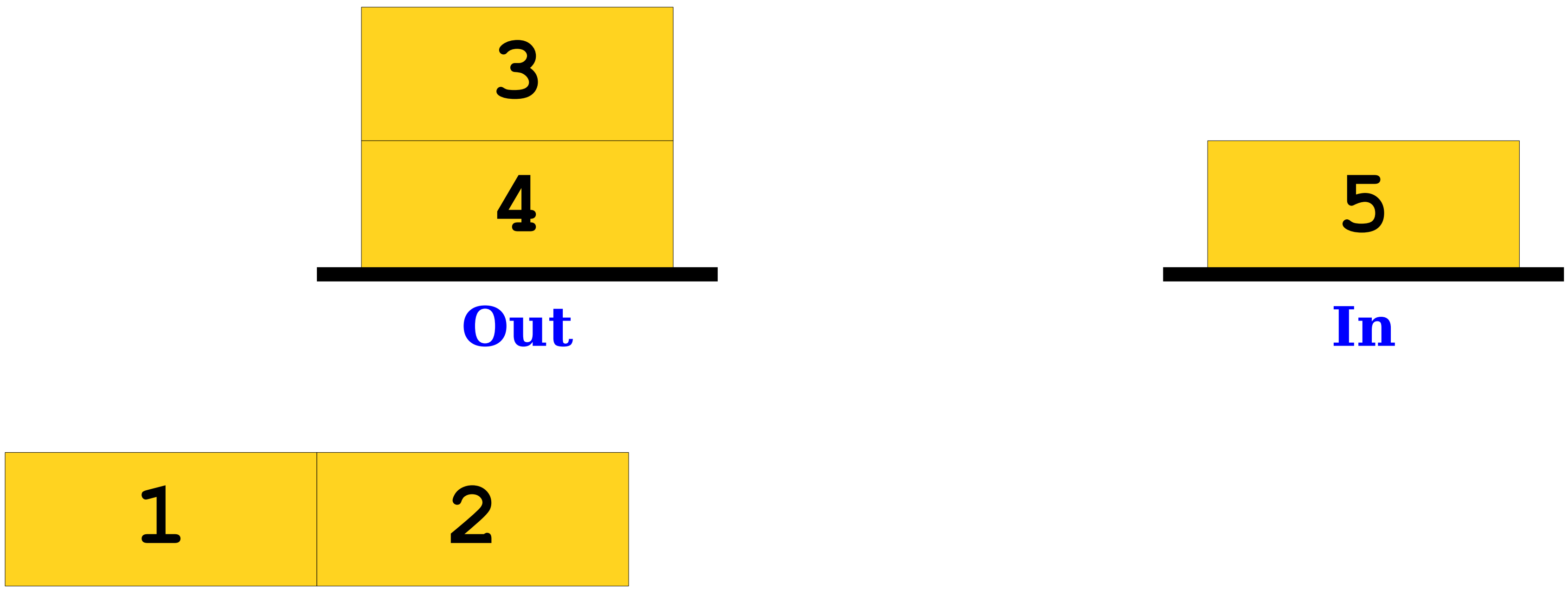

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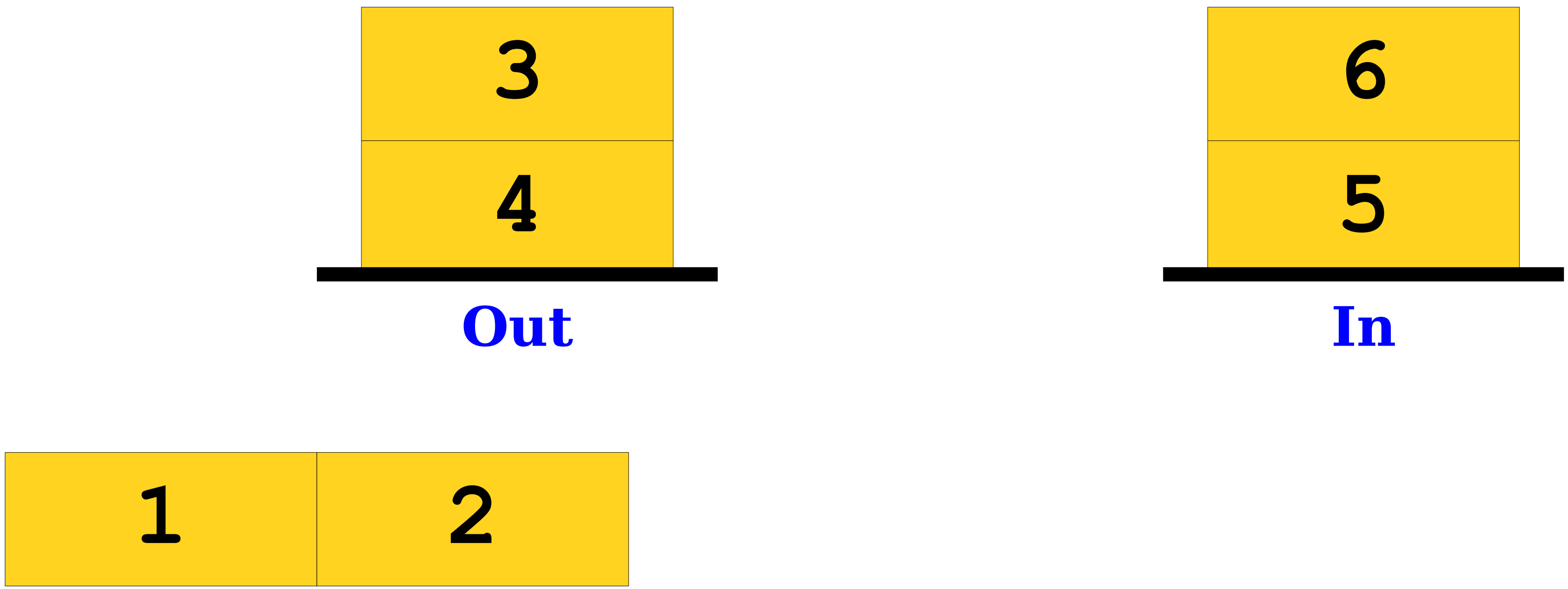

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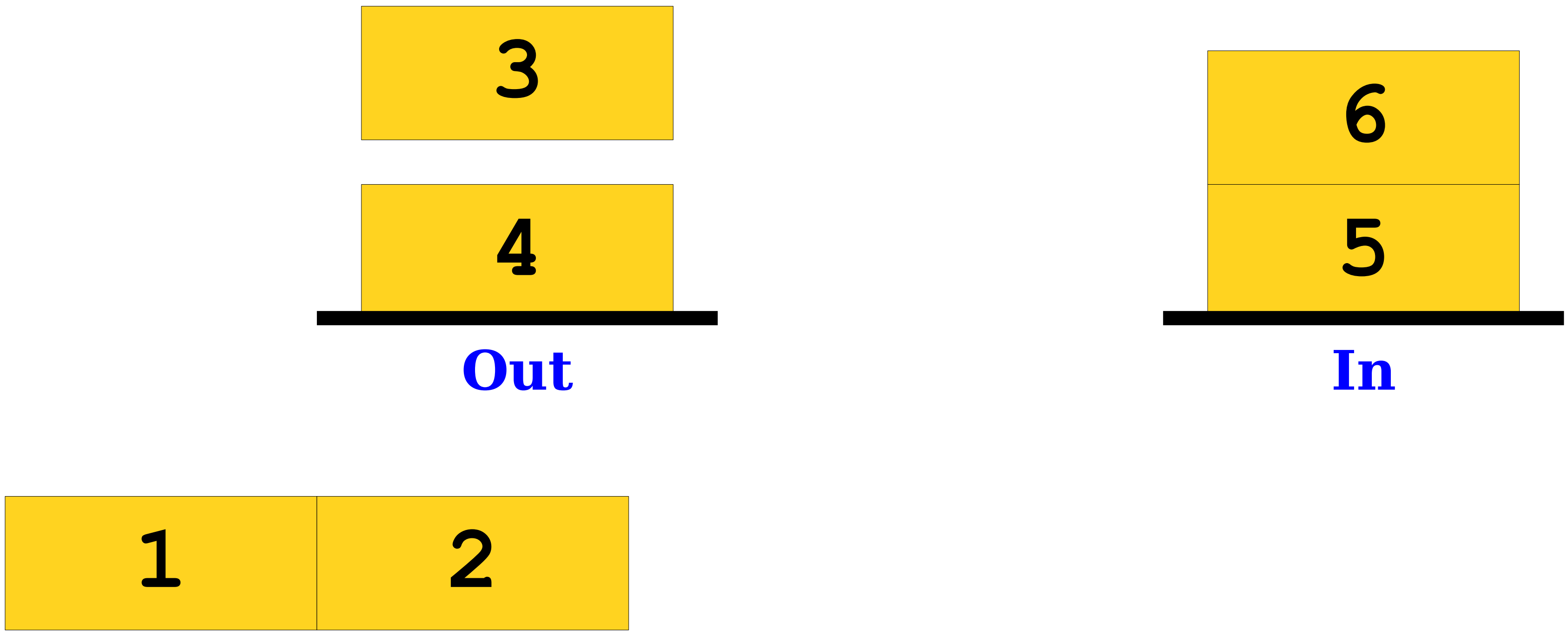

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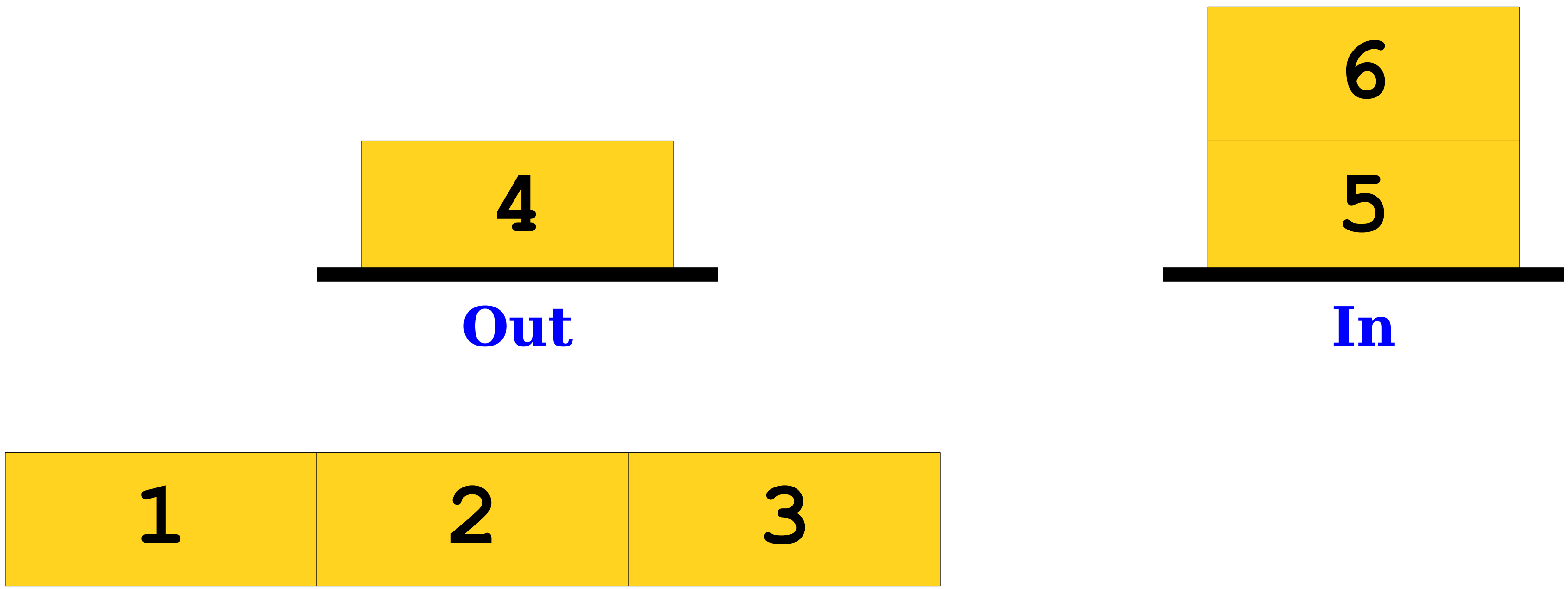

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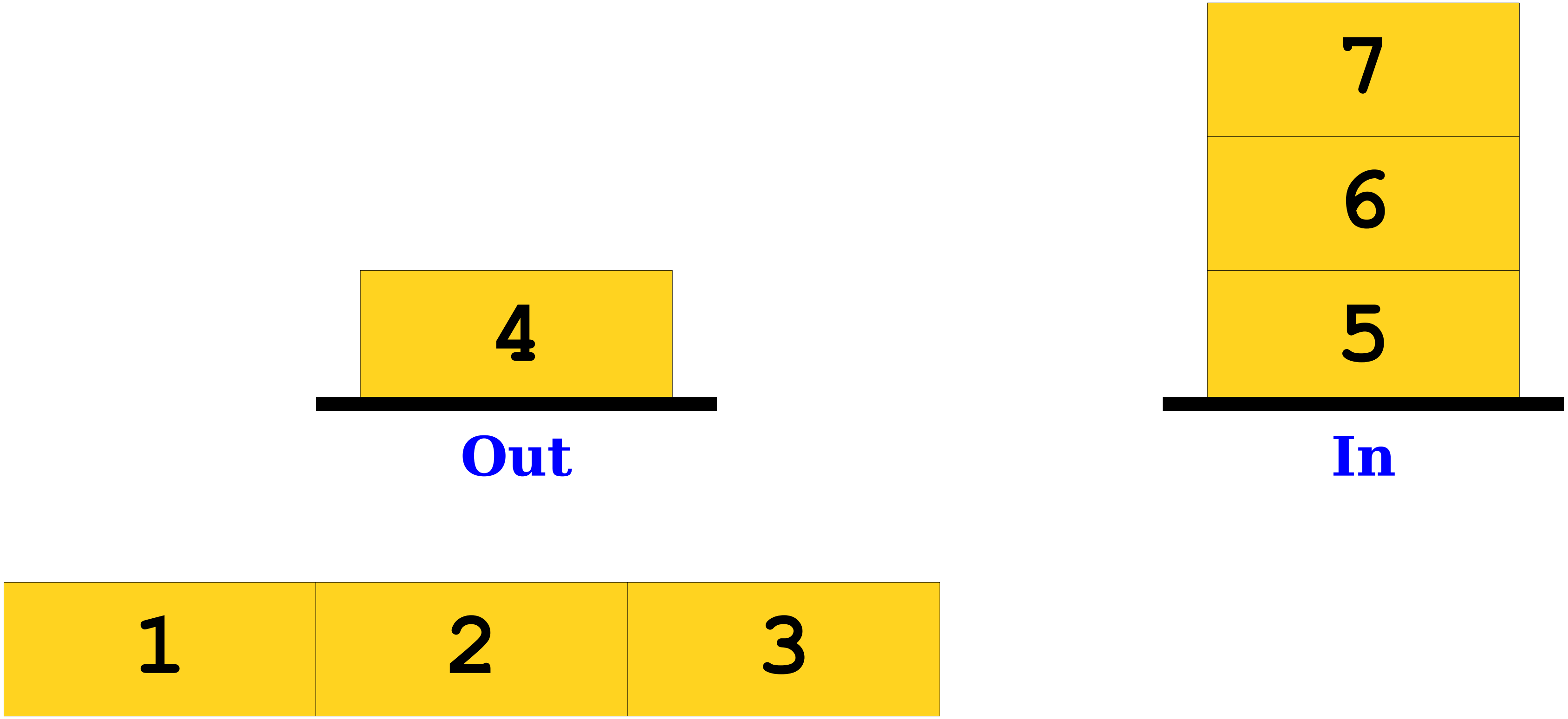

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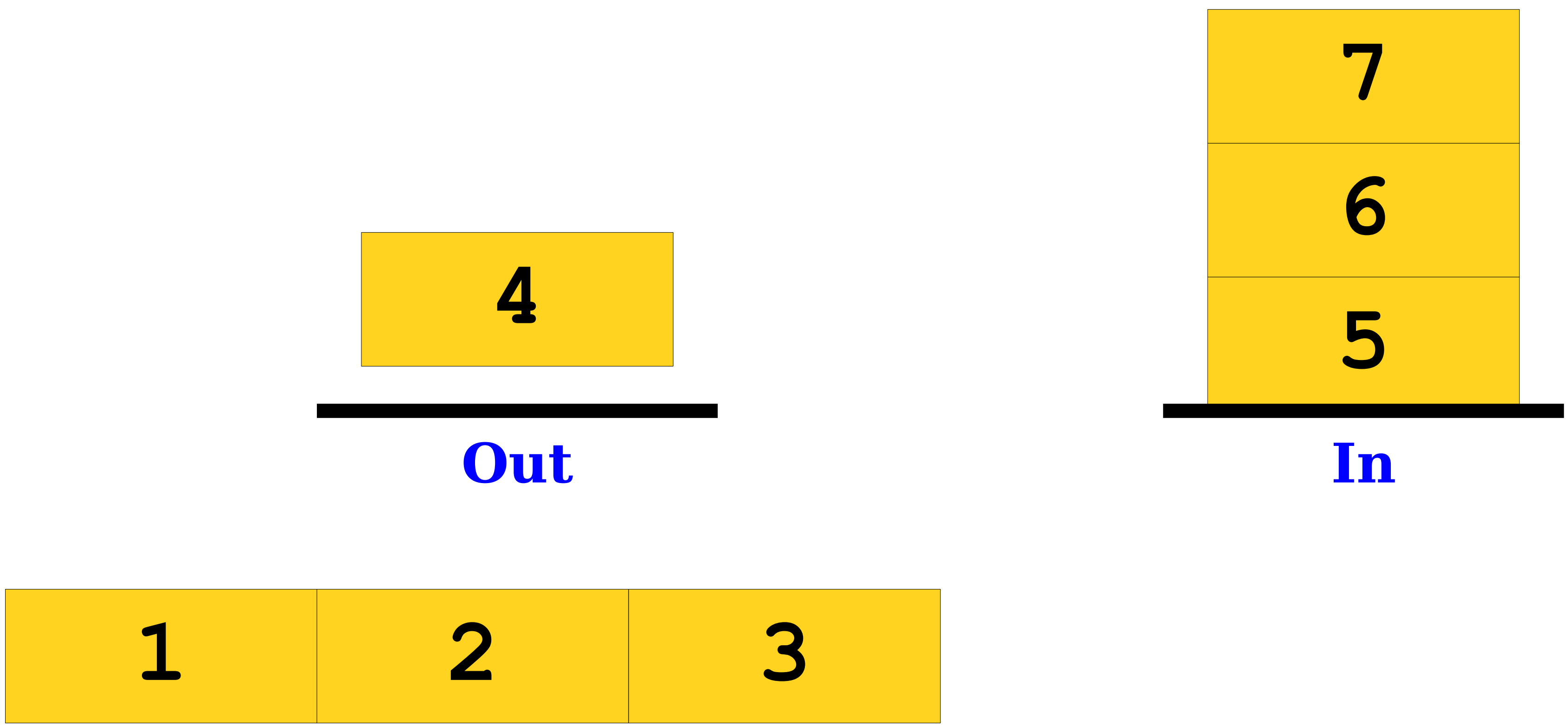

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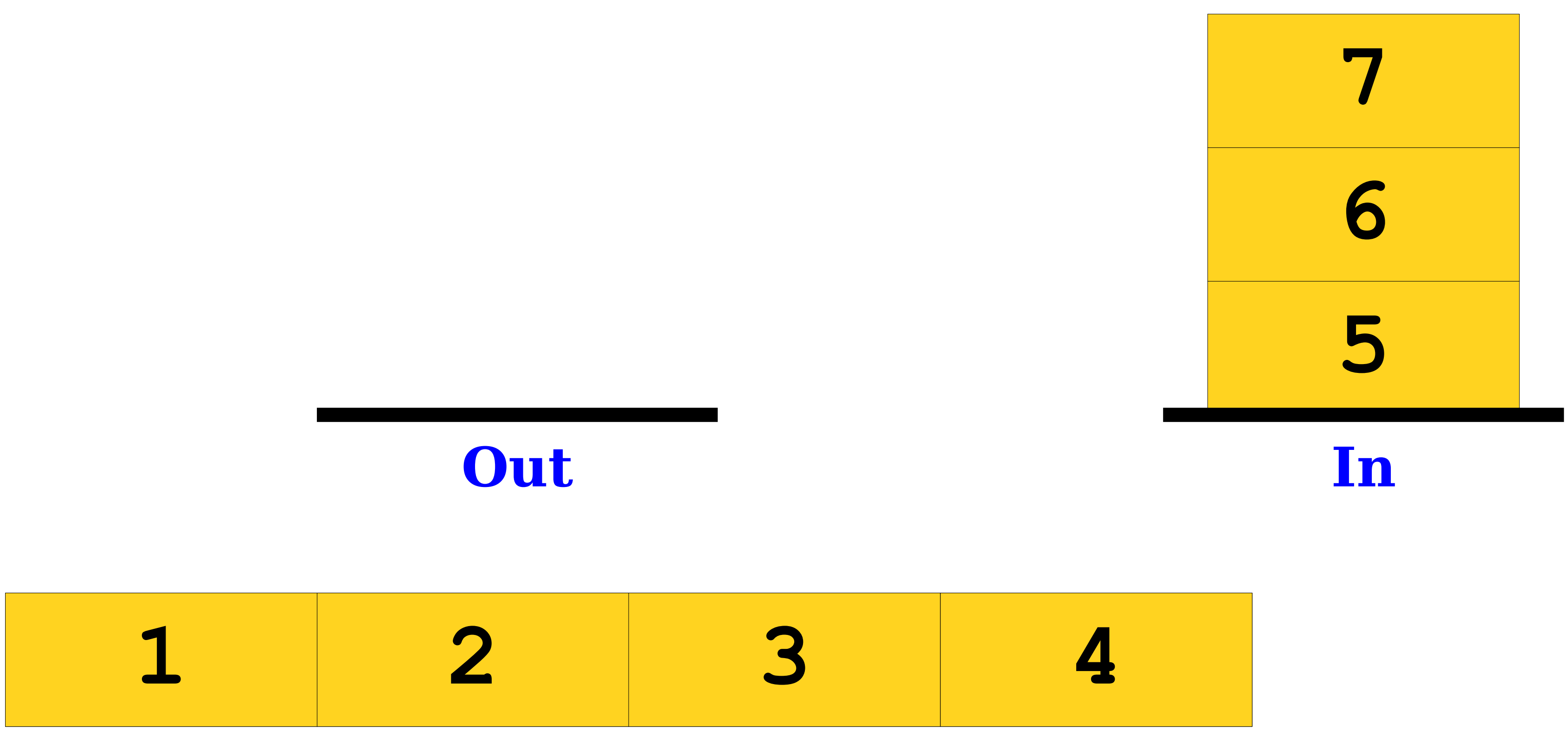

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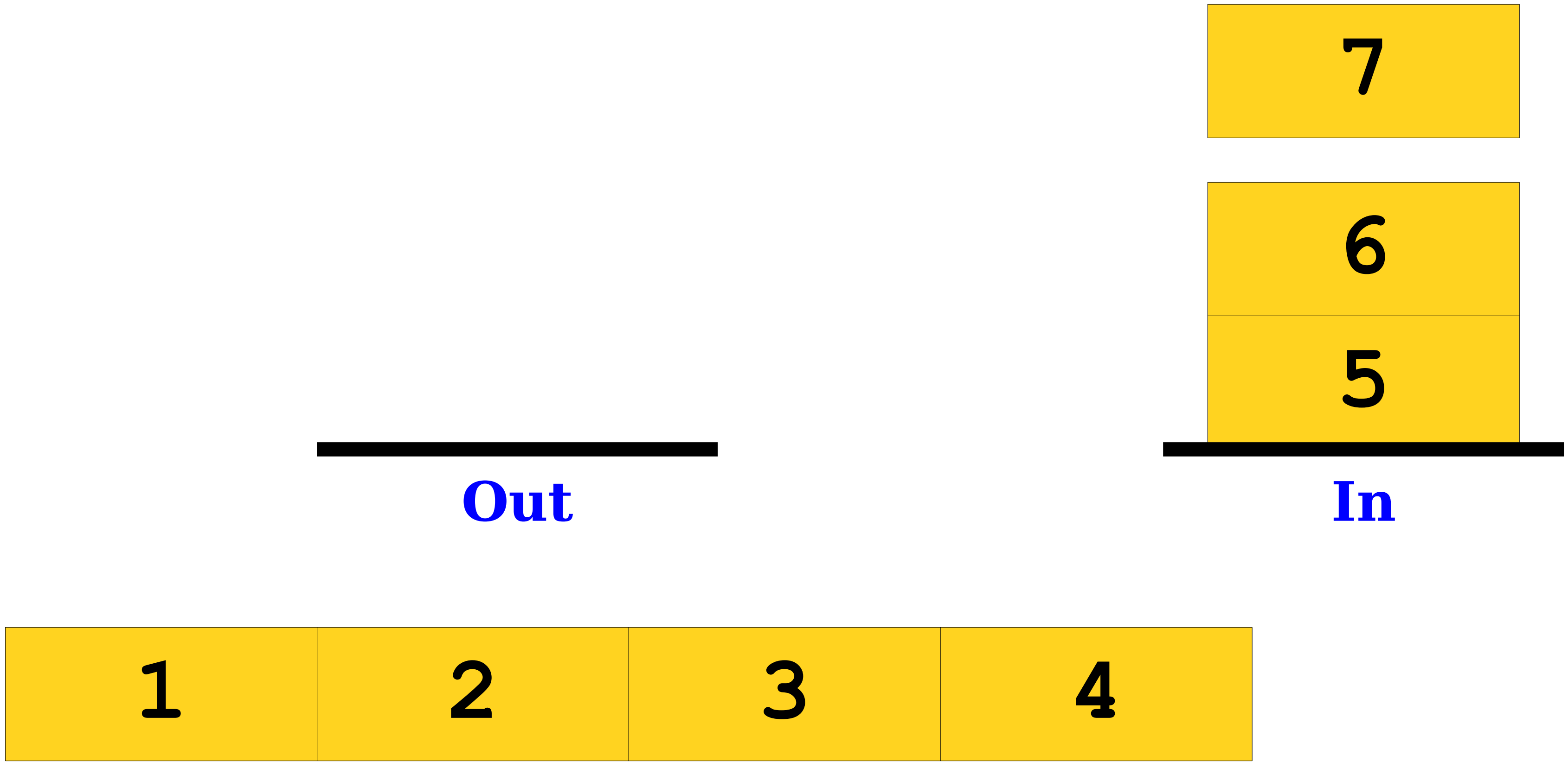

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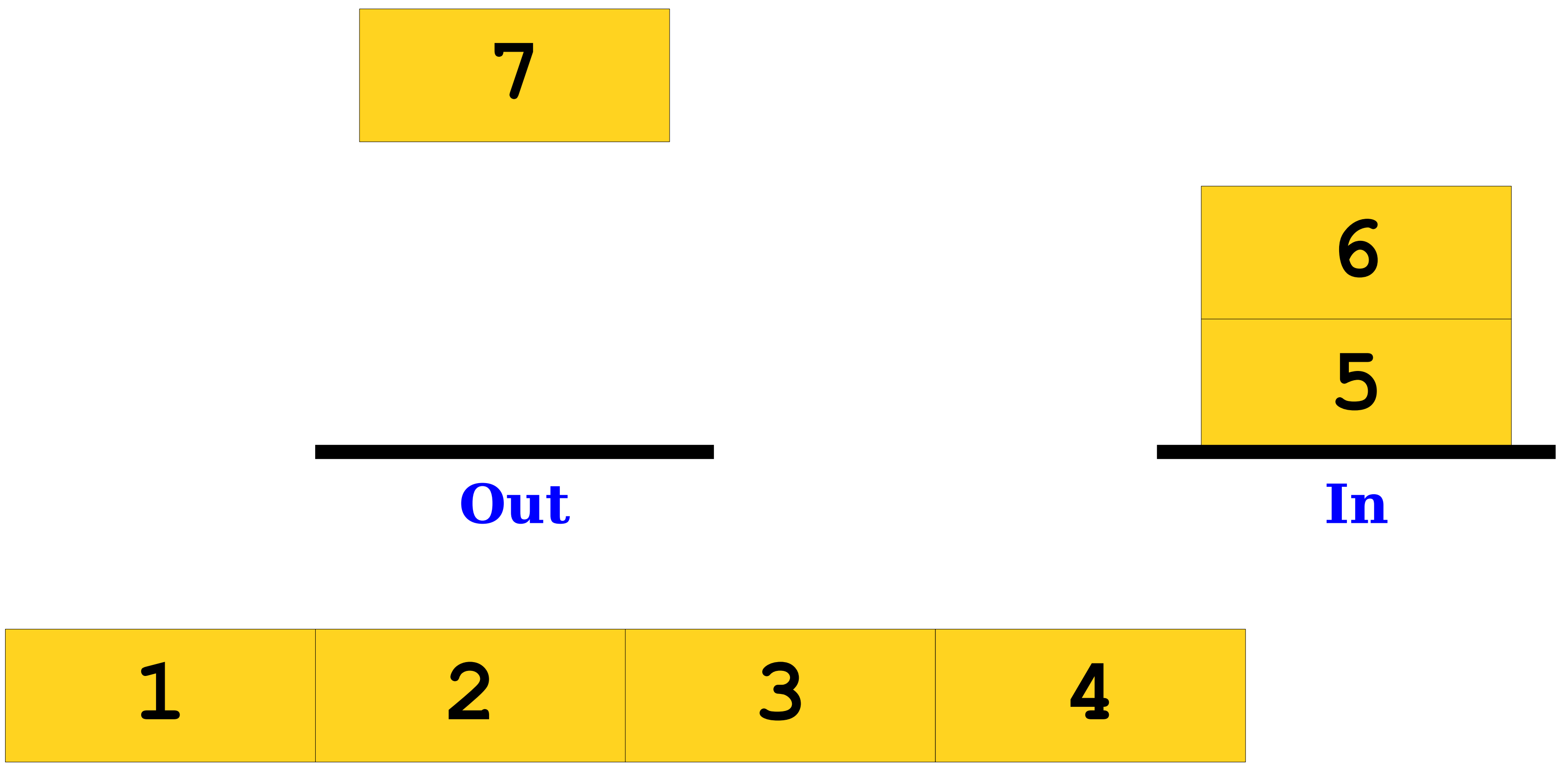

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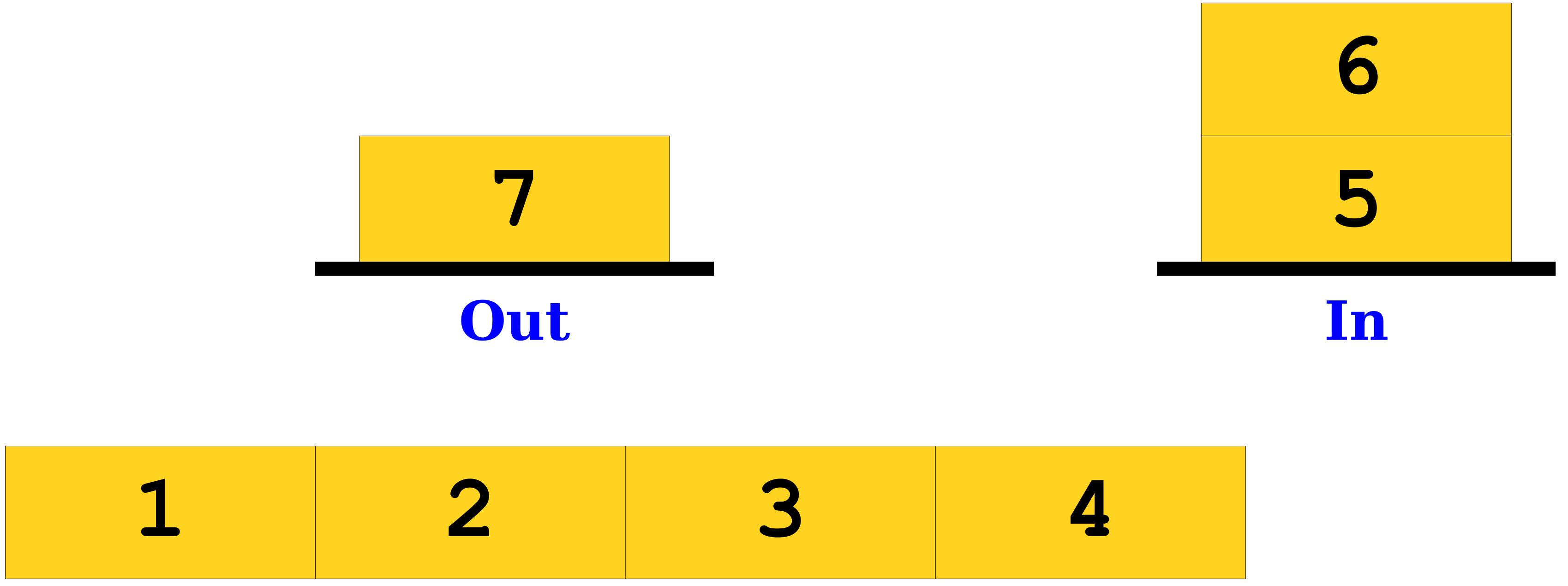

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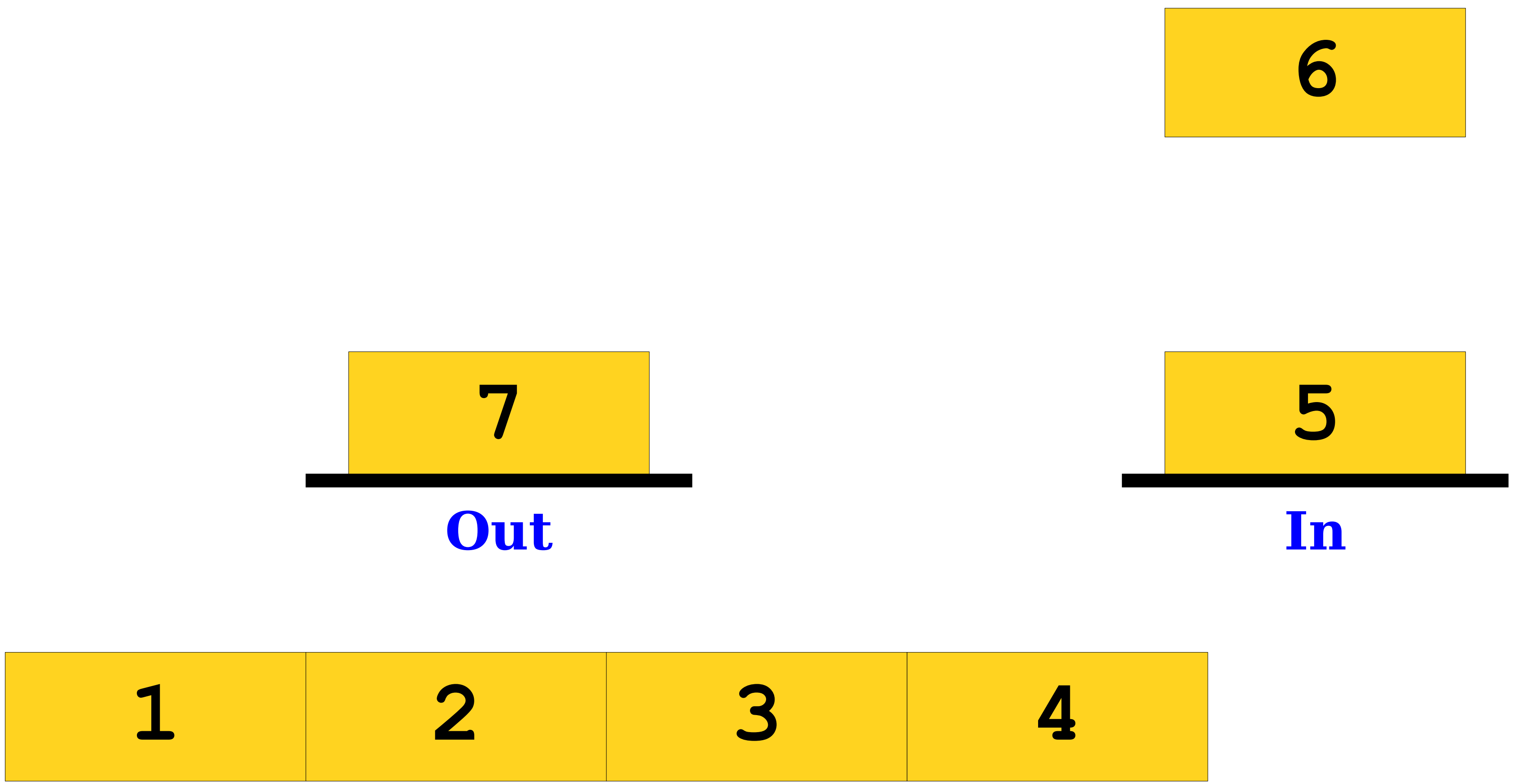

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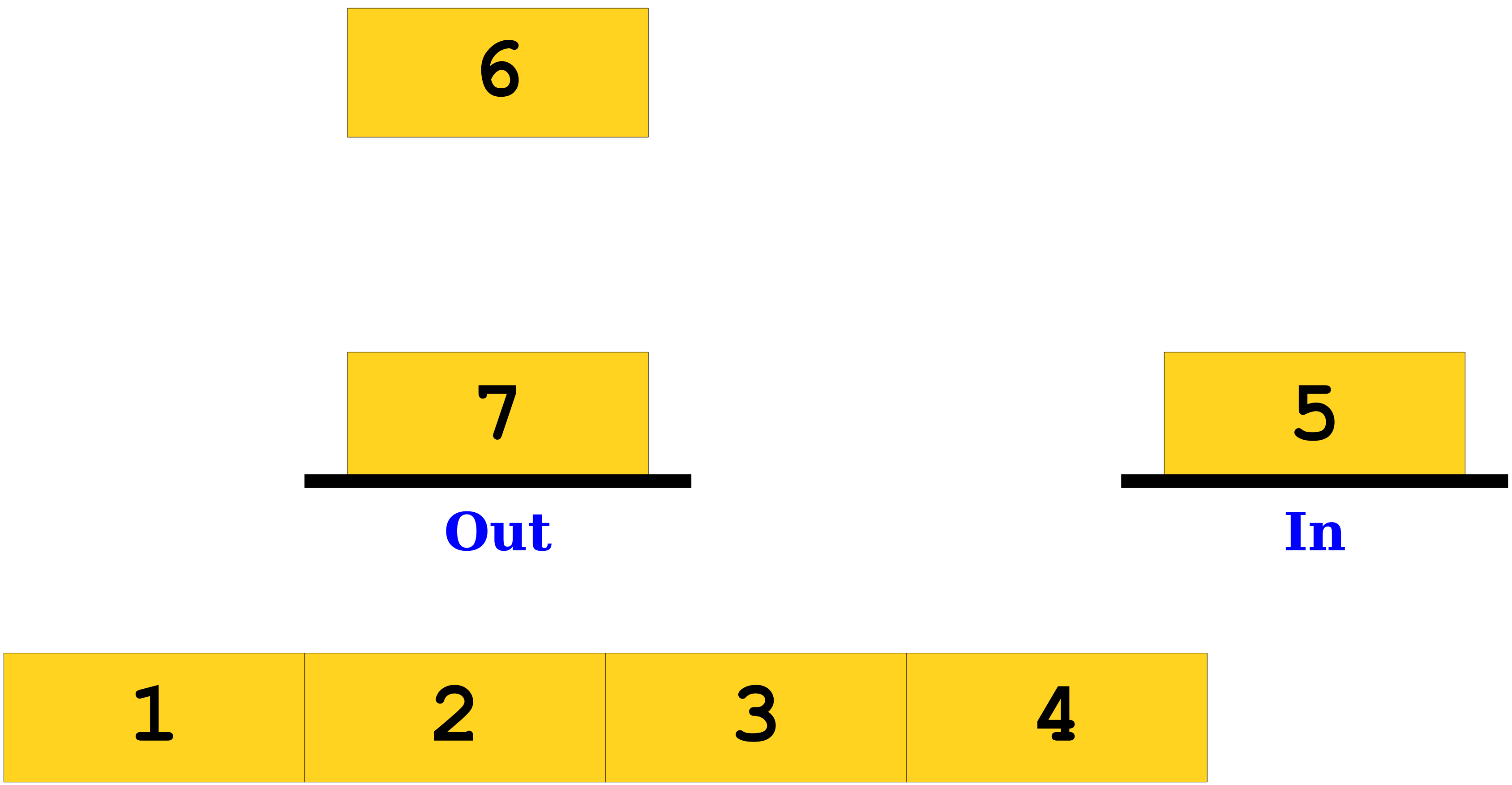

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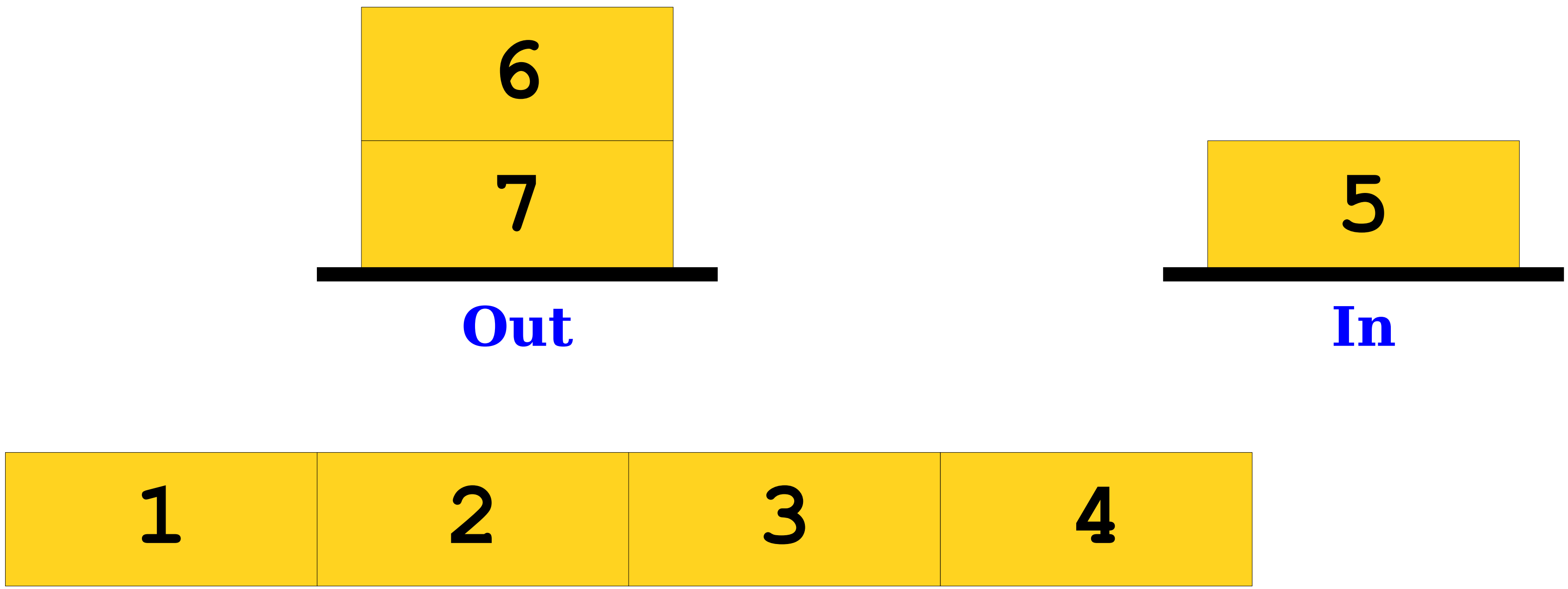

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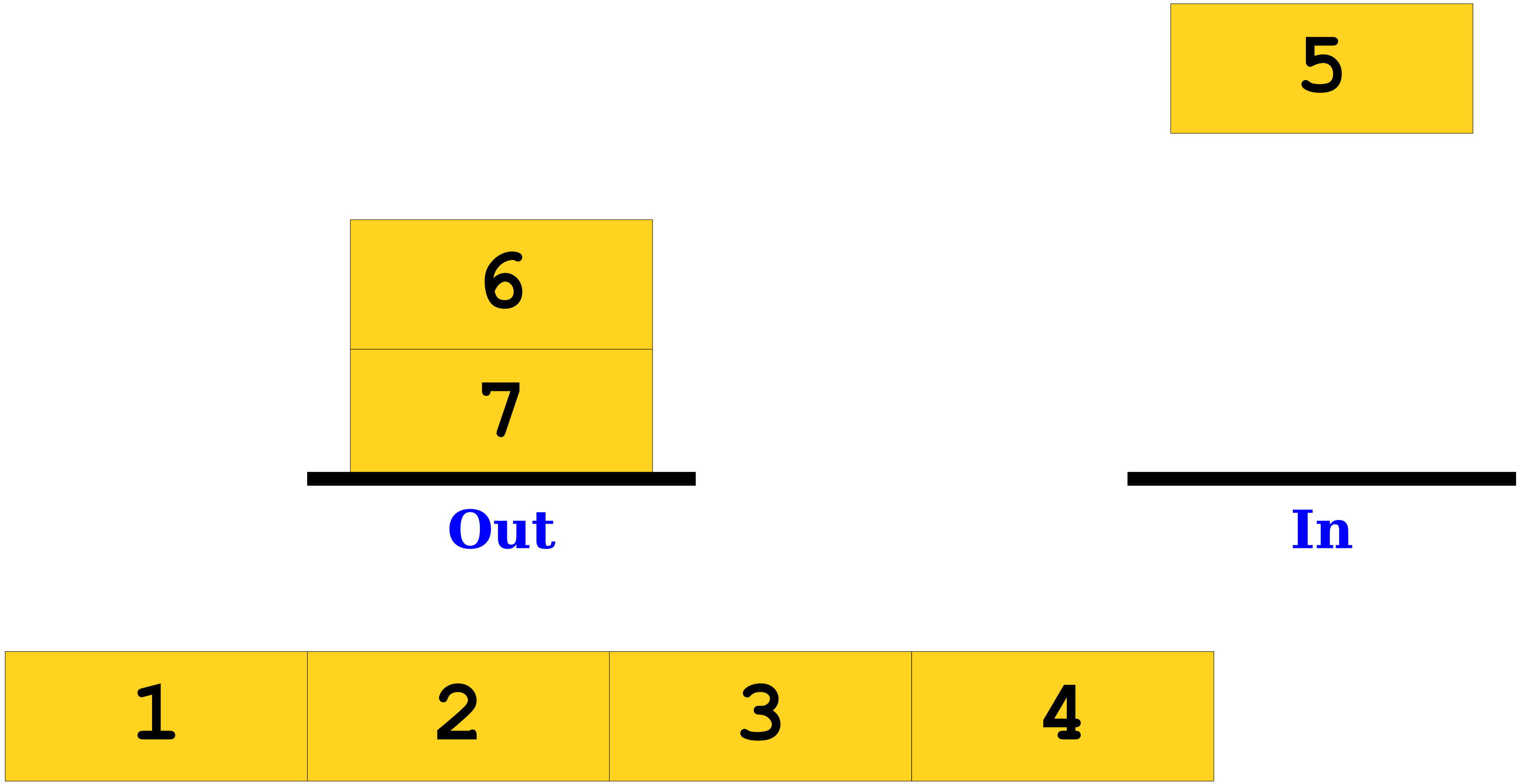

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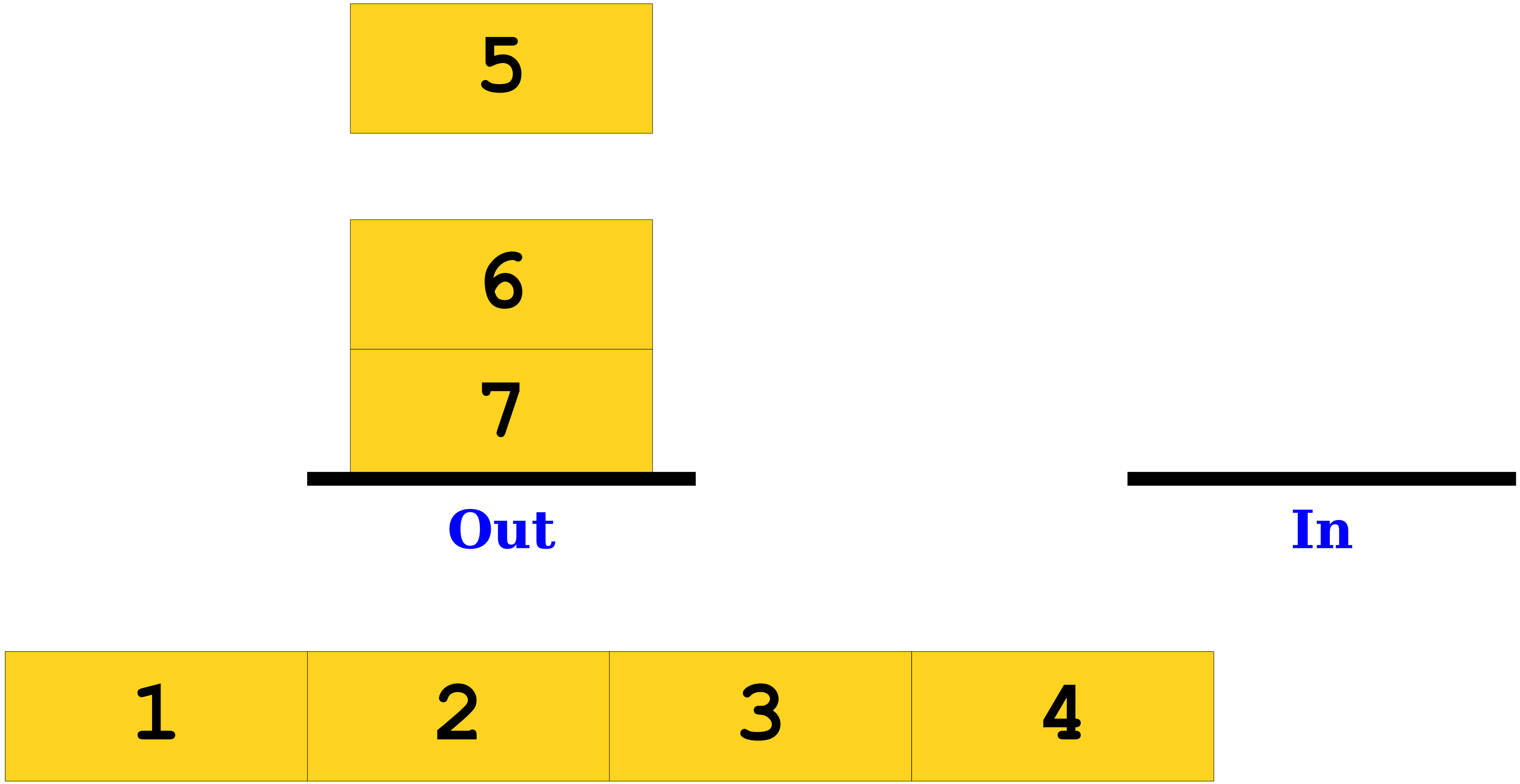

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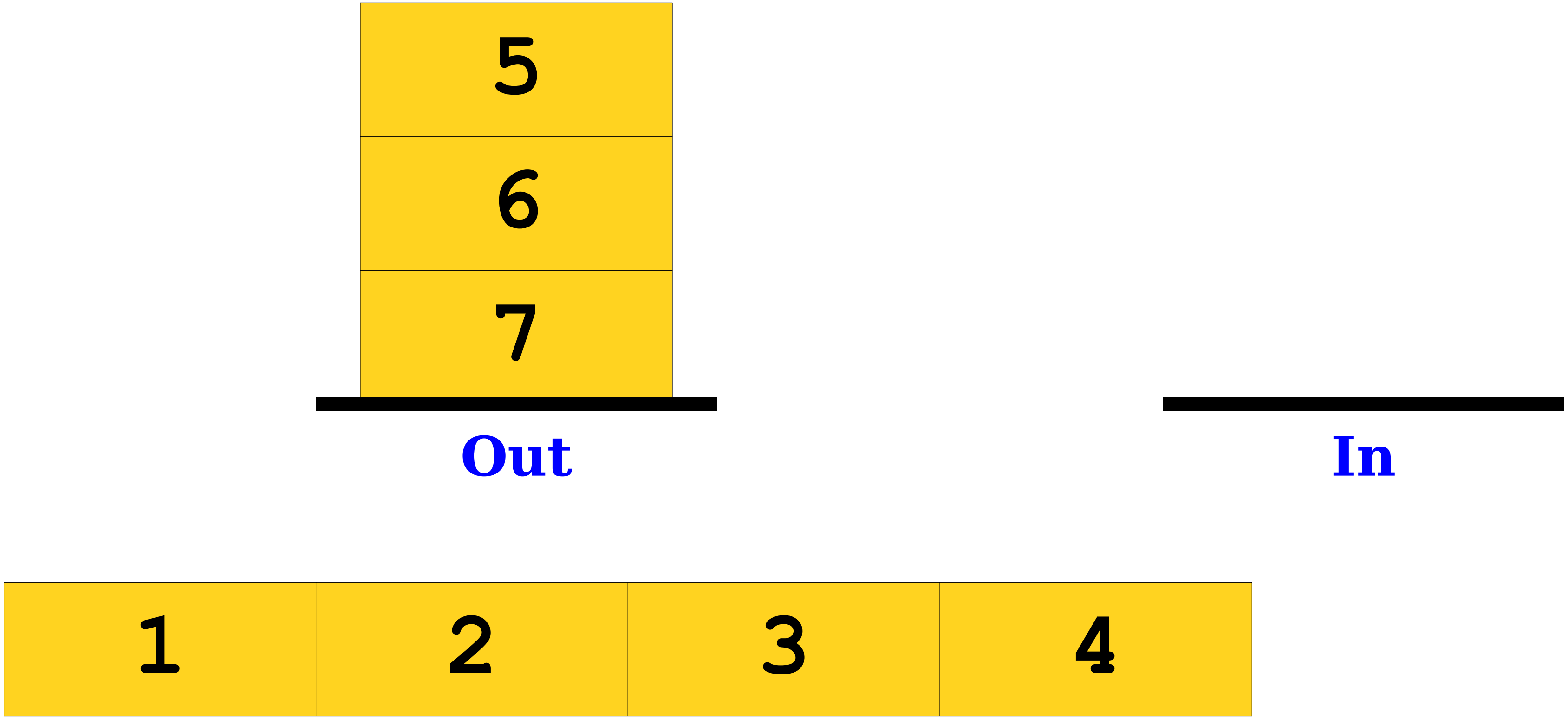

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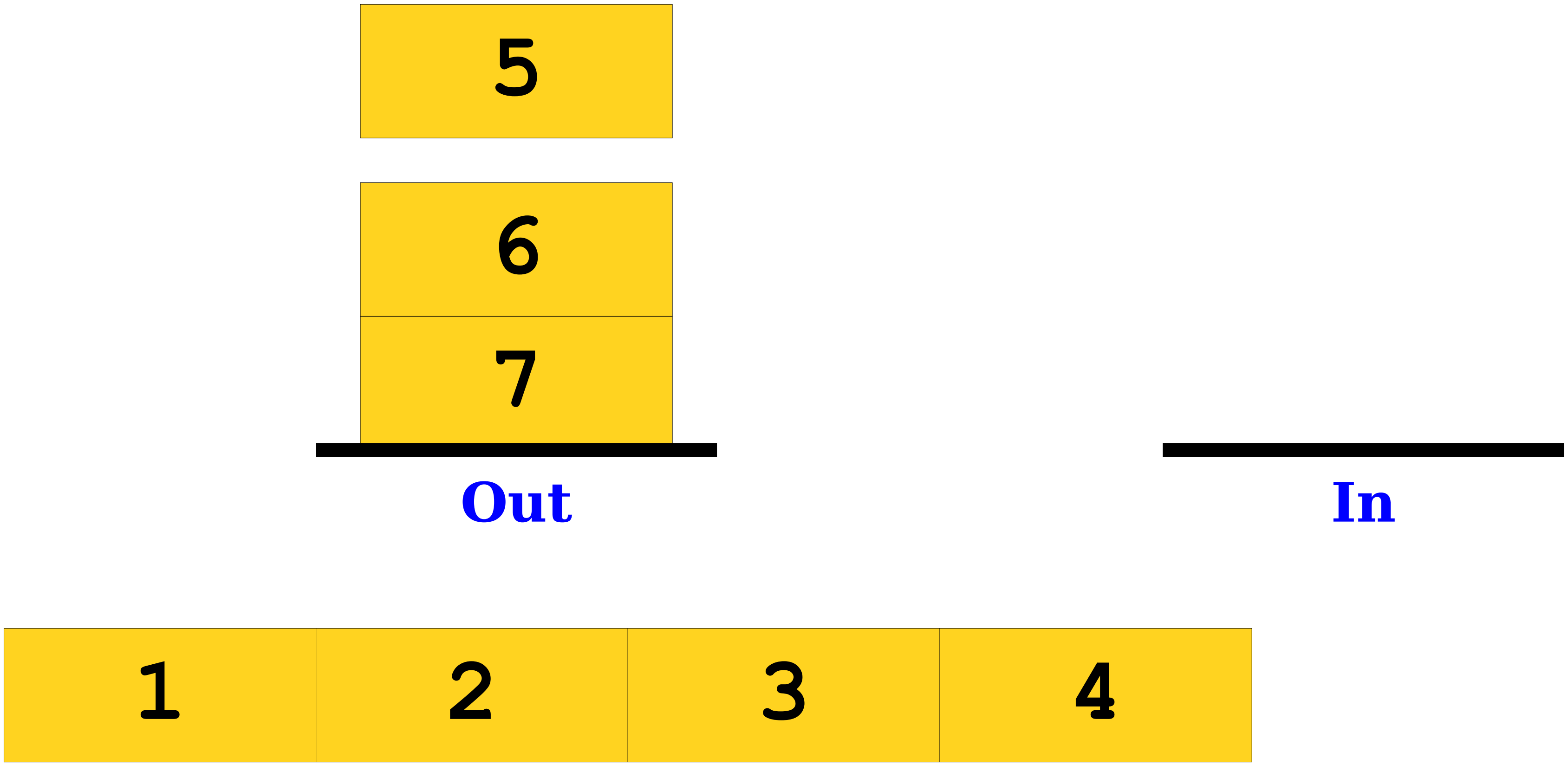

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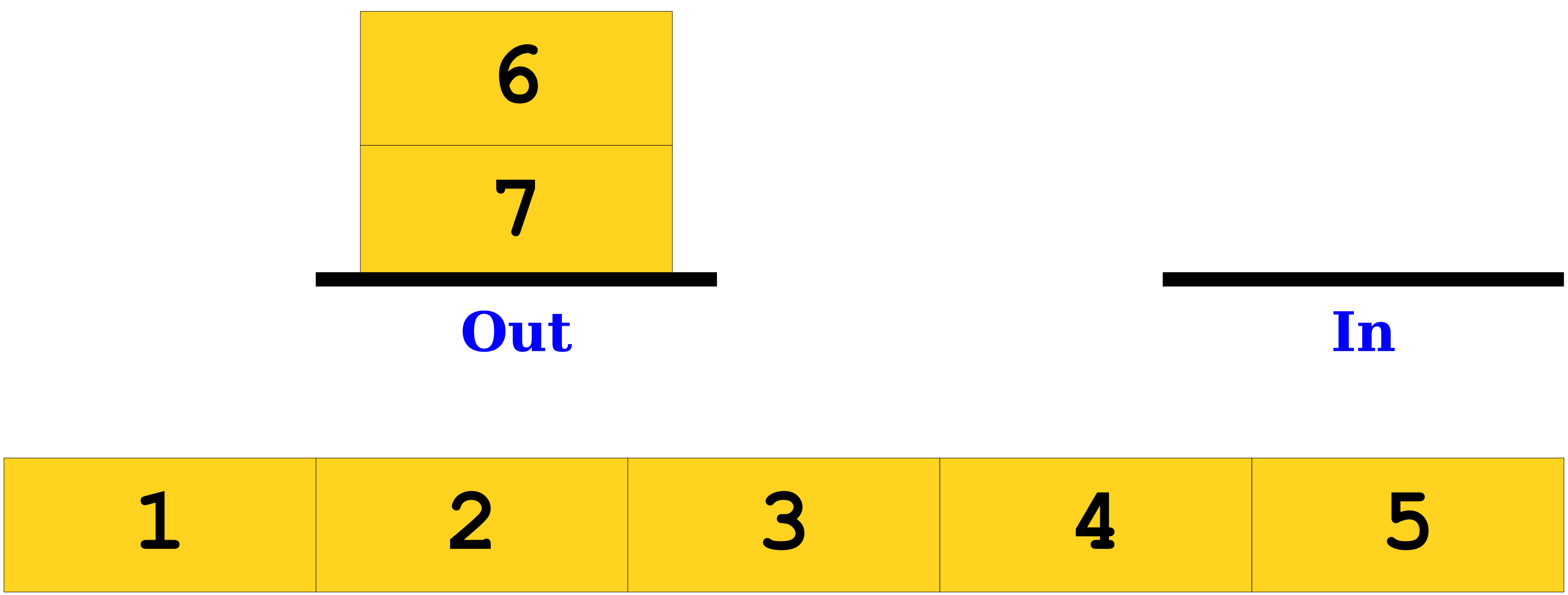

The Two-Stack Queue

- Maintain two stacks, an **In** stack and an **Out** stack.
- To enqueue an element, push it onto the **In** stack.
- To dequeue an element:
  - If the **Out** stack is empty, pop everything off the **In** stack and push it onto the **Out** stack.
  - Pop the **Out** stack and return its value.

Let's code it up...

Analyzing Efficiency

- How efficient is our two-stack queue?
- All enqueues just do one push.
- A dequeue might do a lot of pushes *and* a lot of pops.
- However, let's do an amortized analysis:
  - Each element is pushed at most twice and popped at most twice.
  - n enqueues and n dequeues thus do at most $4n$ pushes and pops.
  - Any $4n$ pushes / pops takes $O(n)$ amortized time.
  - Amortized cost: **O(1)** per operation.

Exam Notes

- Out of 71 Points
  - Mean: 56
  - Standard Deviation: 13
- If you have any concerns with your grade then come chat with me at some point.

Next Time

- **Linked Lists**
  - A different way to represent sequences of elements.
- **Dynamic Allocation Revisited**
  - What else can we allocate?