# Tries

# Implementing **`Lexicon`**

- The **`Lexicon`** represents a set of English words.
- Main operations:
  - Add word.
  - Remove word.
  - Is word contained?
  - Is prefix contained?
- How can we efficiently implement the **`Lexicon`**?

# Sorted Array Implementation

- We could implement the `Lexicon` as an array of all the words it contains.
- To add a word: **O(*n*)**
  - Check if the word already exists.
  - If not, insert it in sorted order.
- To remove a word: **O(*n*)**
  - Find and remove it from the array.
- To see if a word exists: **O(log *n*)**
  - Perform binary search of the array for the word.
- To see if a prefix exists: **O(log *n*)**
  - Perform binary search of the array to see if the word is a prefix of some word in the array.

# A Better Implementation

- Use a Binary Search Tree.
- Adding, removing, checking for a word AND **`containsPrefix`** now run very quickly (**O(log *n*)** comparisons needed).
- Can we do any better?

# A (kinda) Better Implementation

- Use a hash table.
- Adding, removing, and checking for a word now run even faster (**O(1)** comparisons needed).
- How would you implement **`containsPrefix`**?
  - Would have to check all words in all buckets.
    - Linear search!

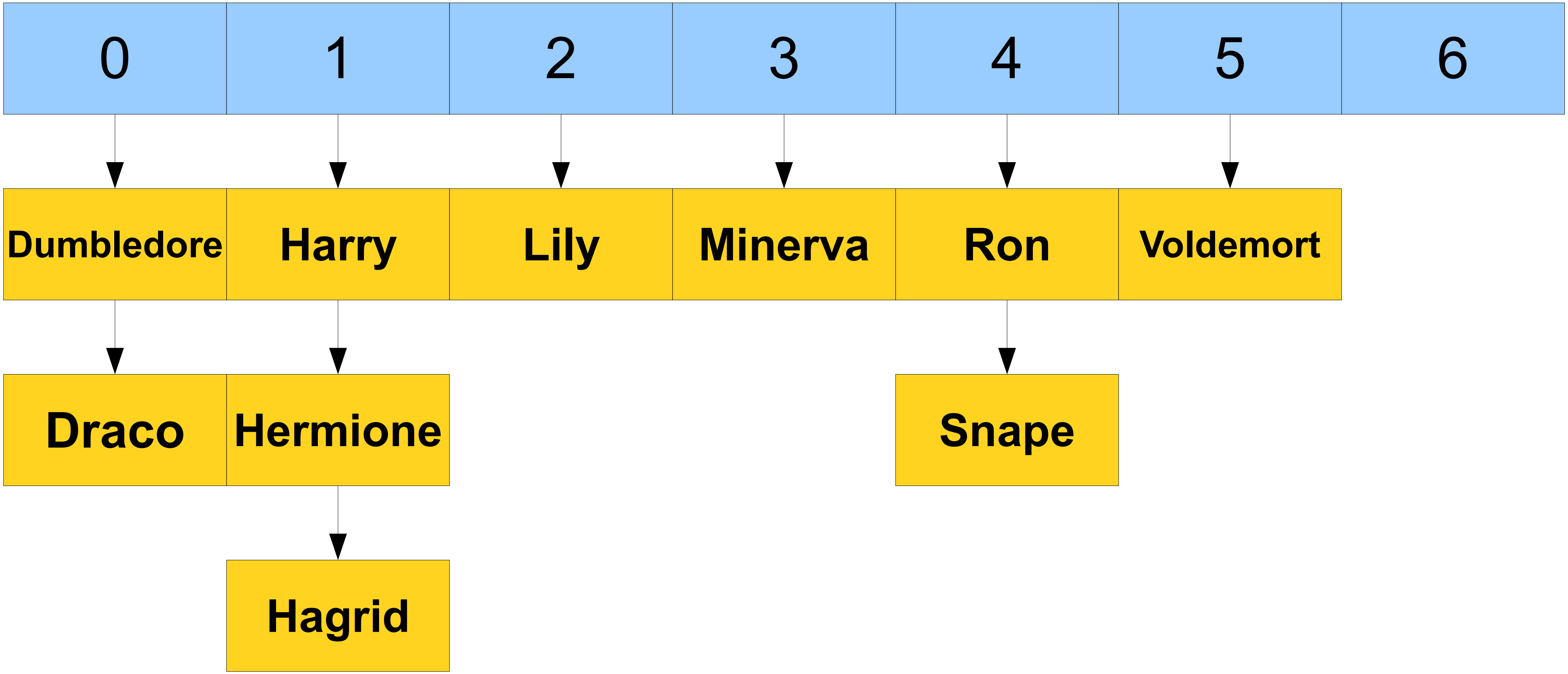
# What We Want

- What we want is a data structure that allows us to lookup, insert, remove AND check for a prefix in **O(1)**
- It isn't possible to do this with any of the structures we've covered so far.
- We need a new data structure!

# Rethinking Hashing

- Our motivation behind hashing was to put values into places where we would know to look for them.
- When storing strings as our keys, one initial idea was to break strings apart by their first letter.
- Let's look at that idea again.

# An Initial Hashing Idea



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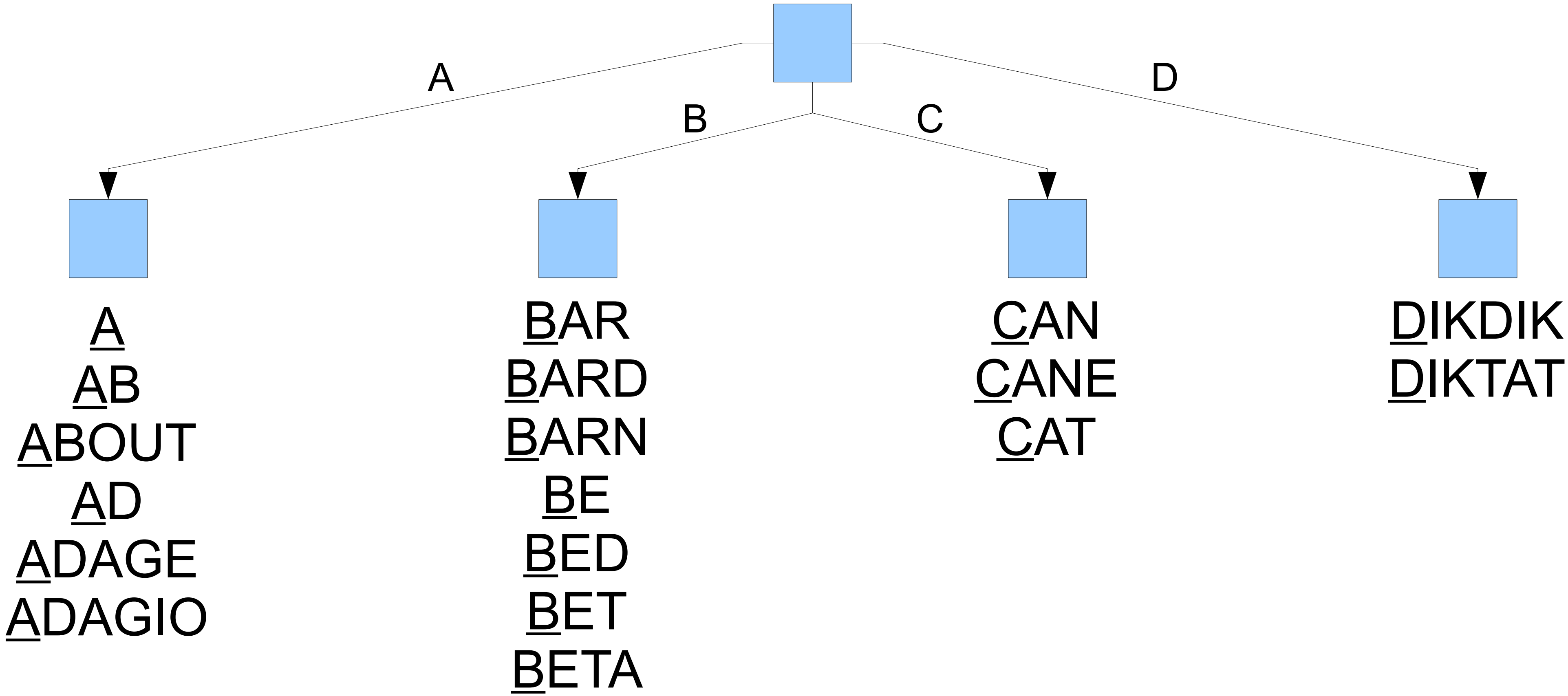
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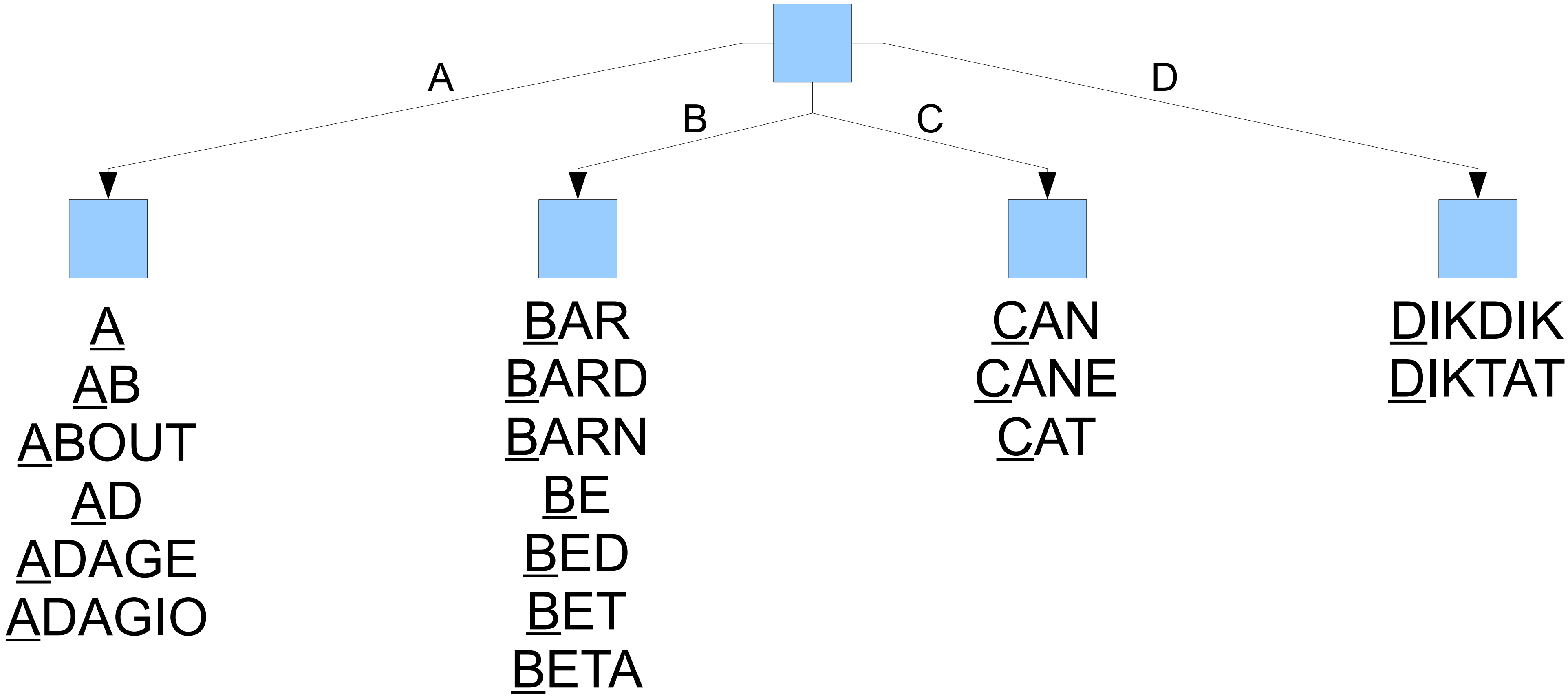
# How does this affect performance?

- If we assume that roughly the same number of words start with each letter, then we've sped up **`containsPrefix`** by a factor of 1/26
  - Totally not a fair assumption to make. But it still gives us a good constant factor speedup
- **`containsPrefix`** still runs in **O(*n*)** (unsorted) or **O(log(*n*))** (sorted)

What happens if we *split again*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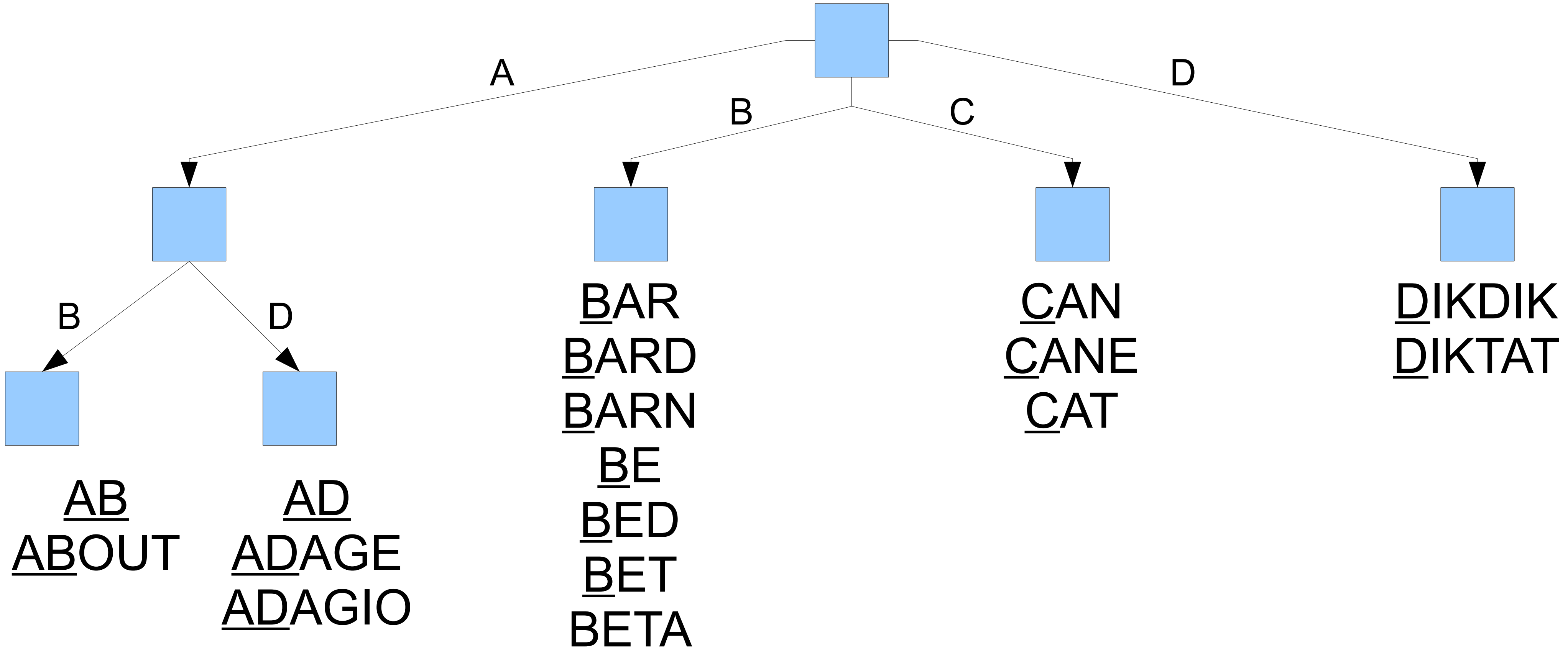
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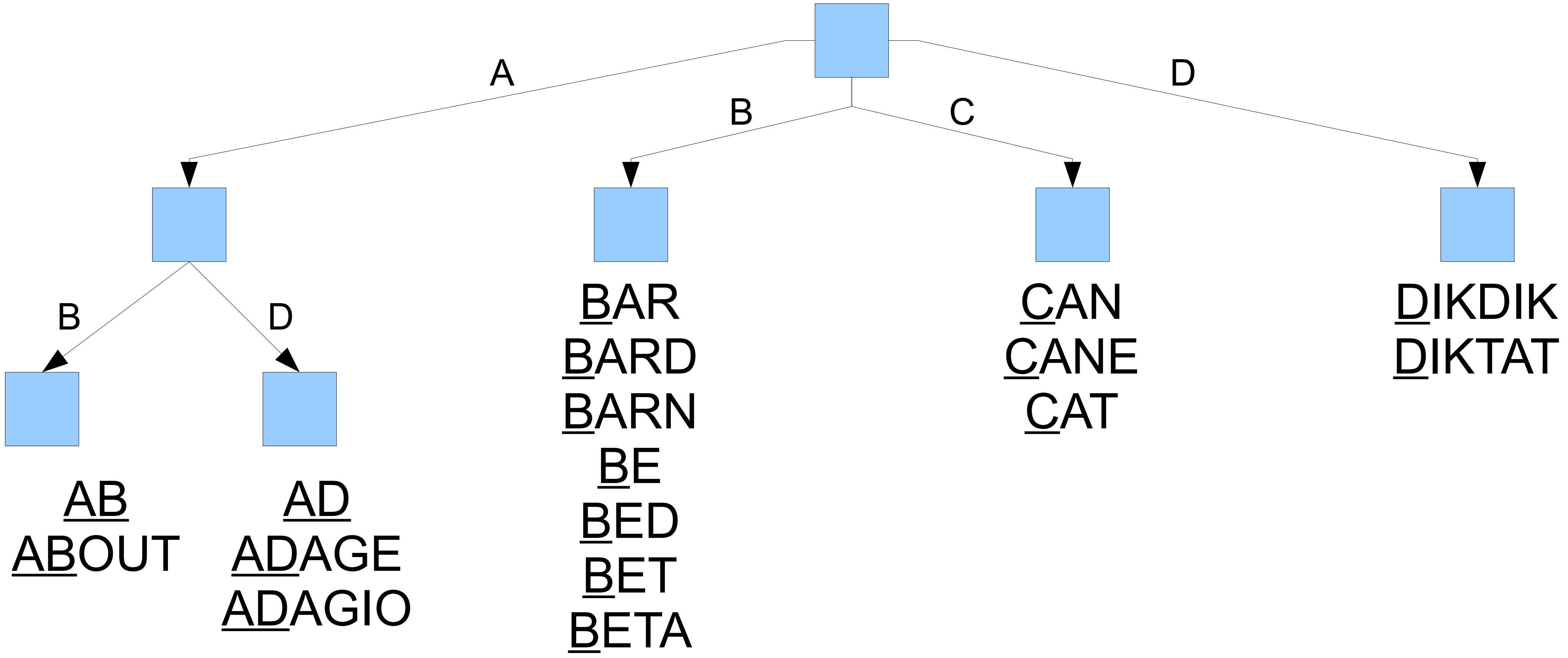
# How does this affect performance?

- This gives us another constant factor speedup on words that start with “AB” and “AD”
- What happens if we continue this process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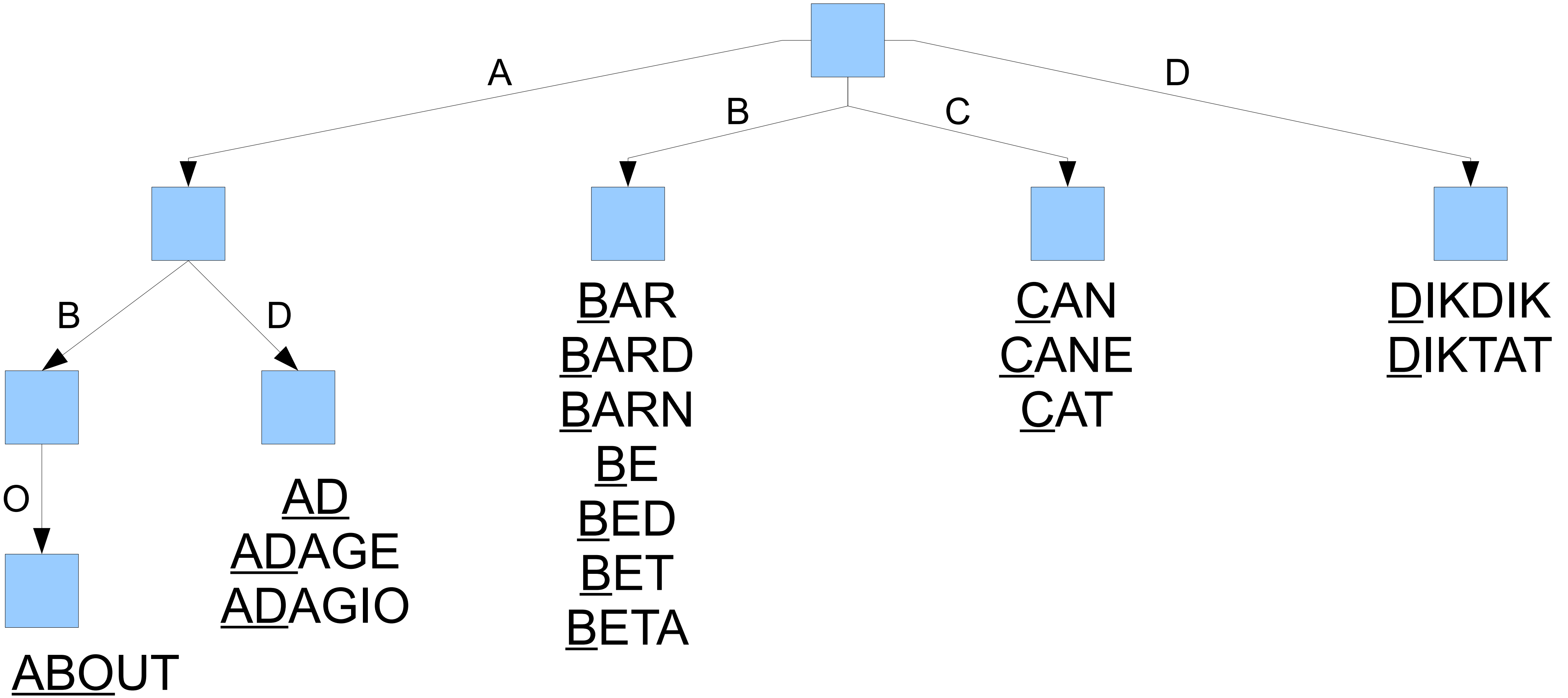
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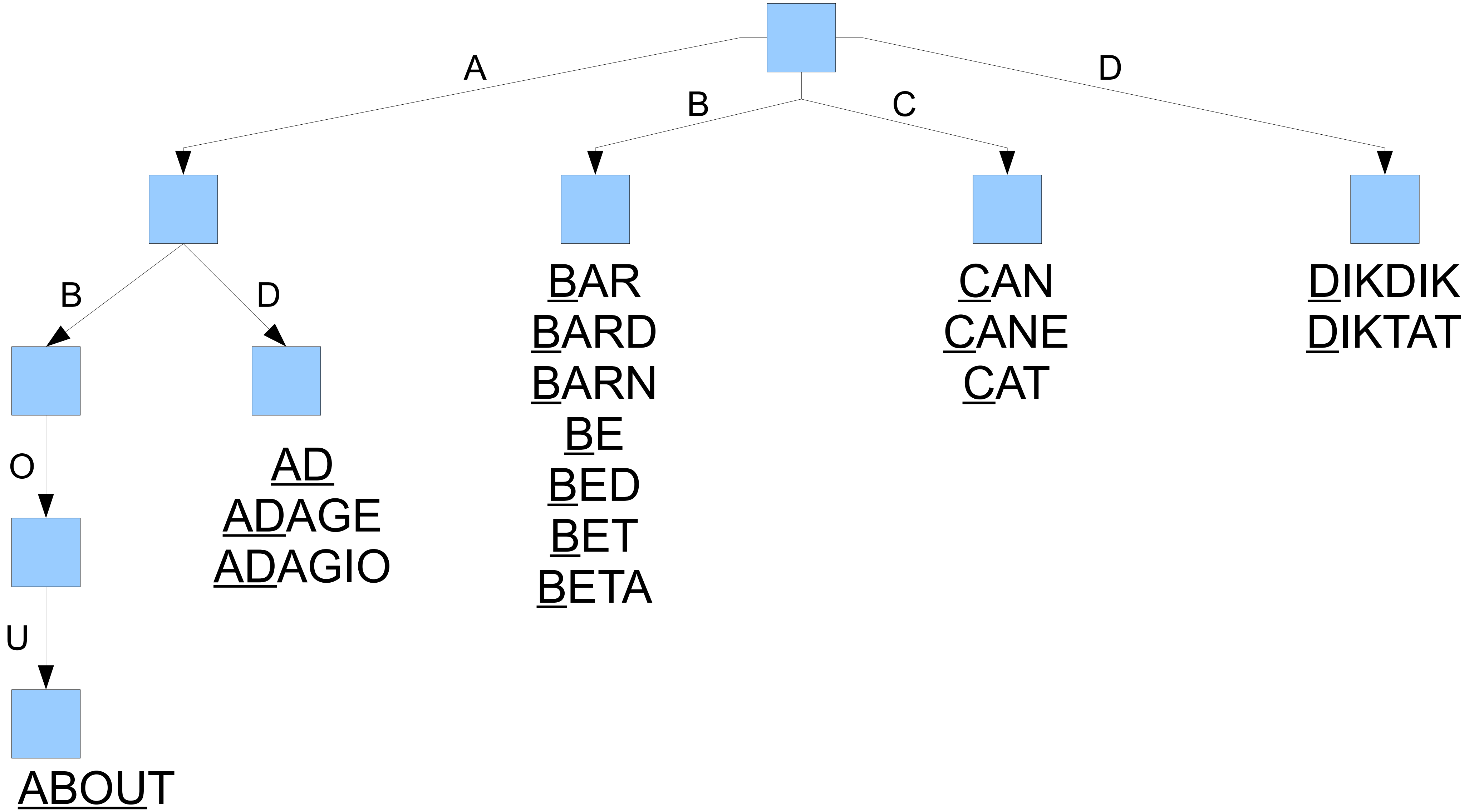
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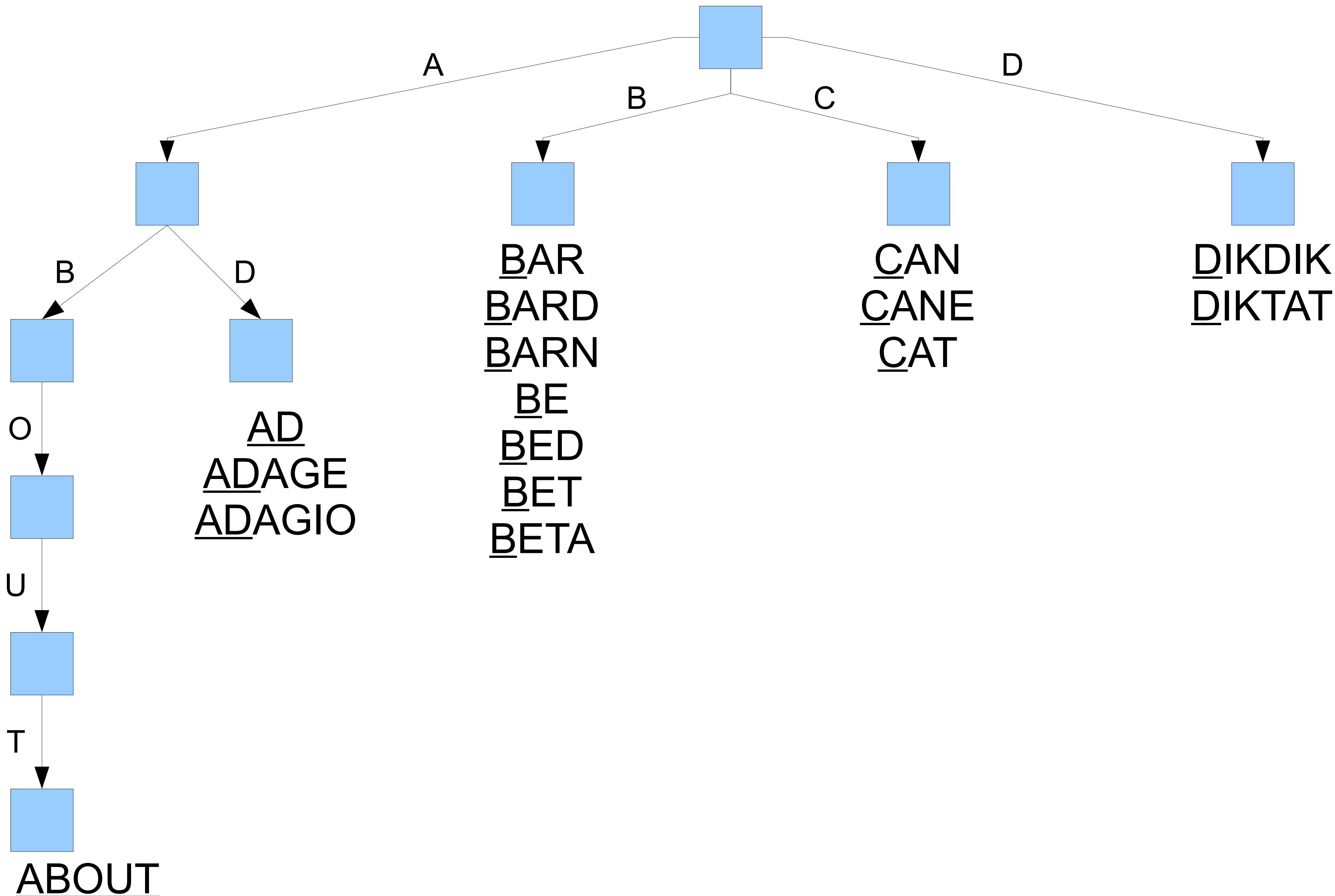

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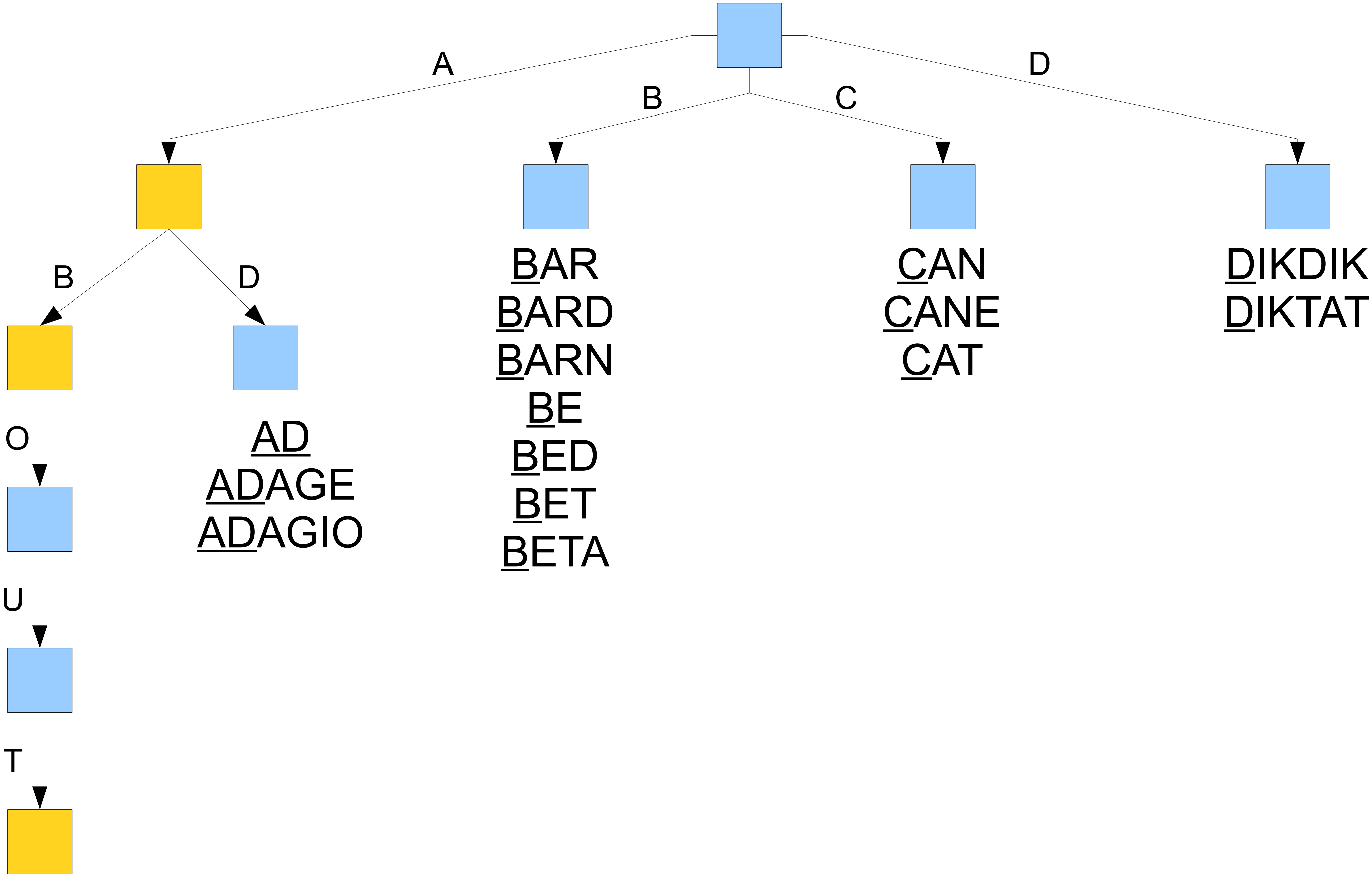
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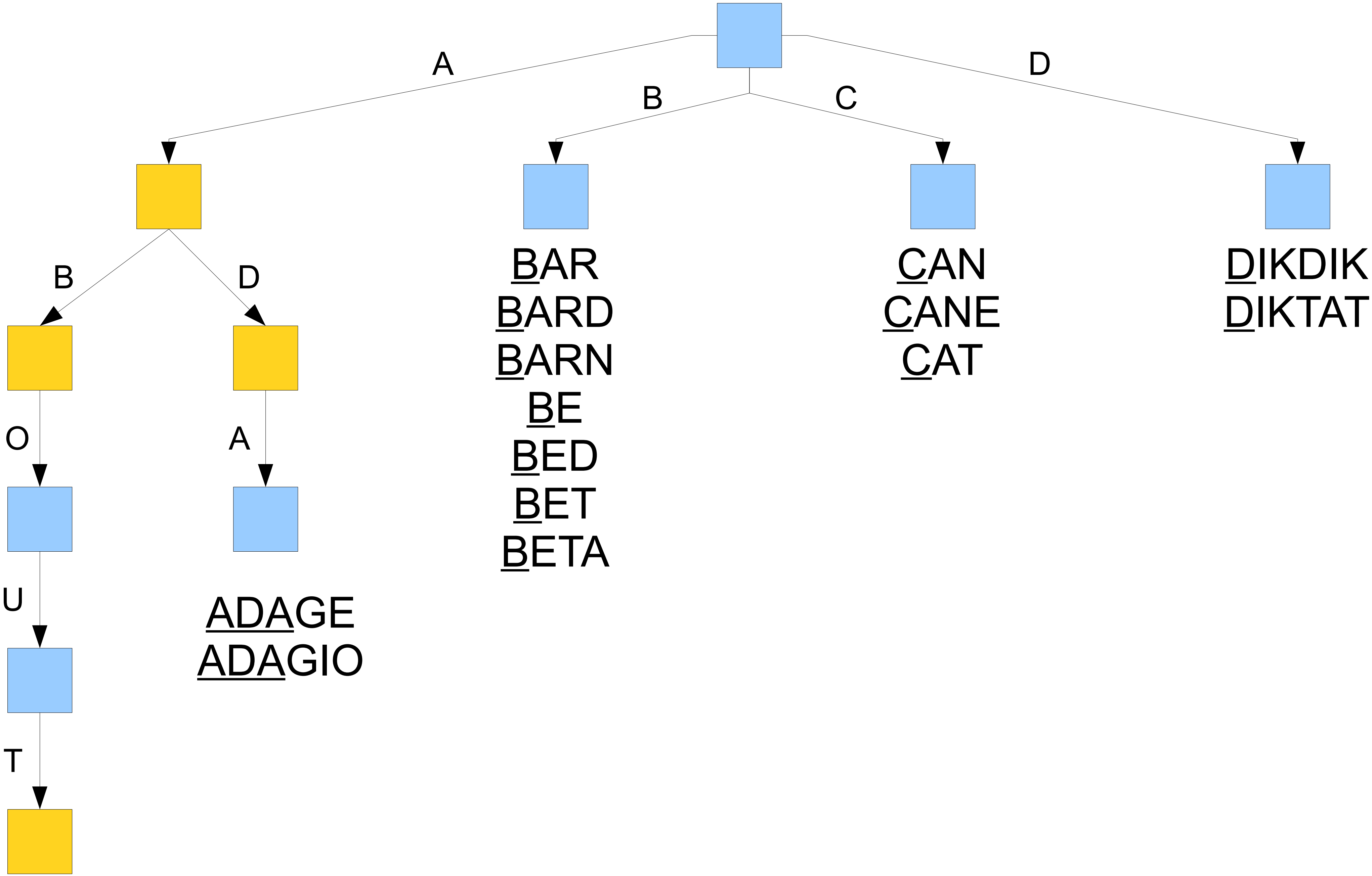
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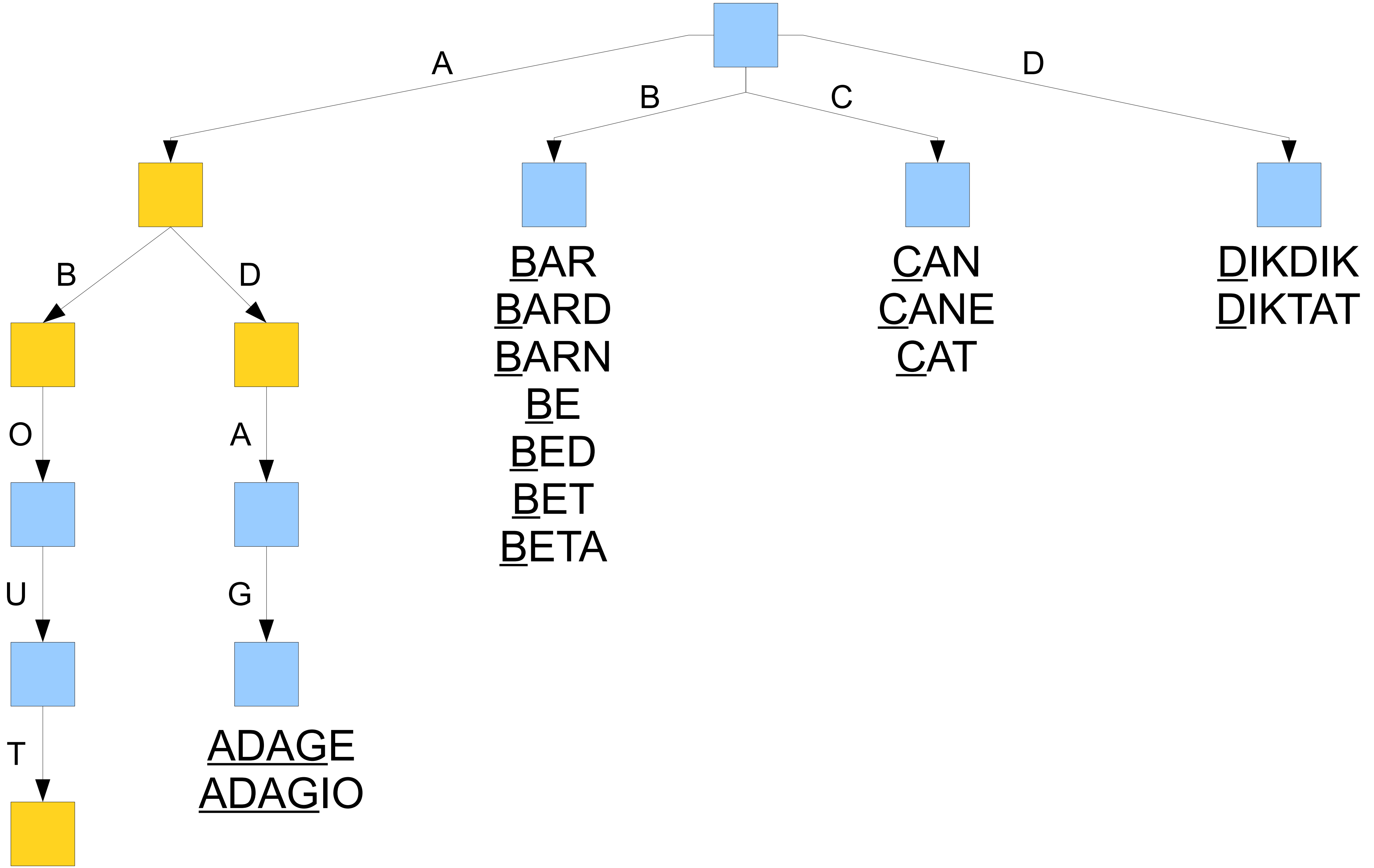
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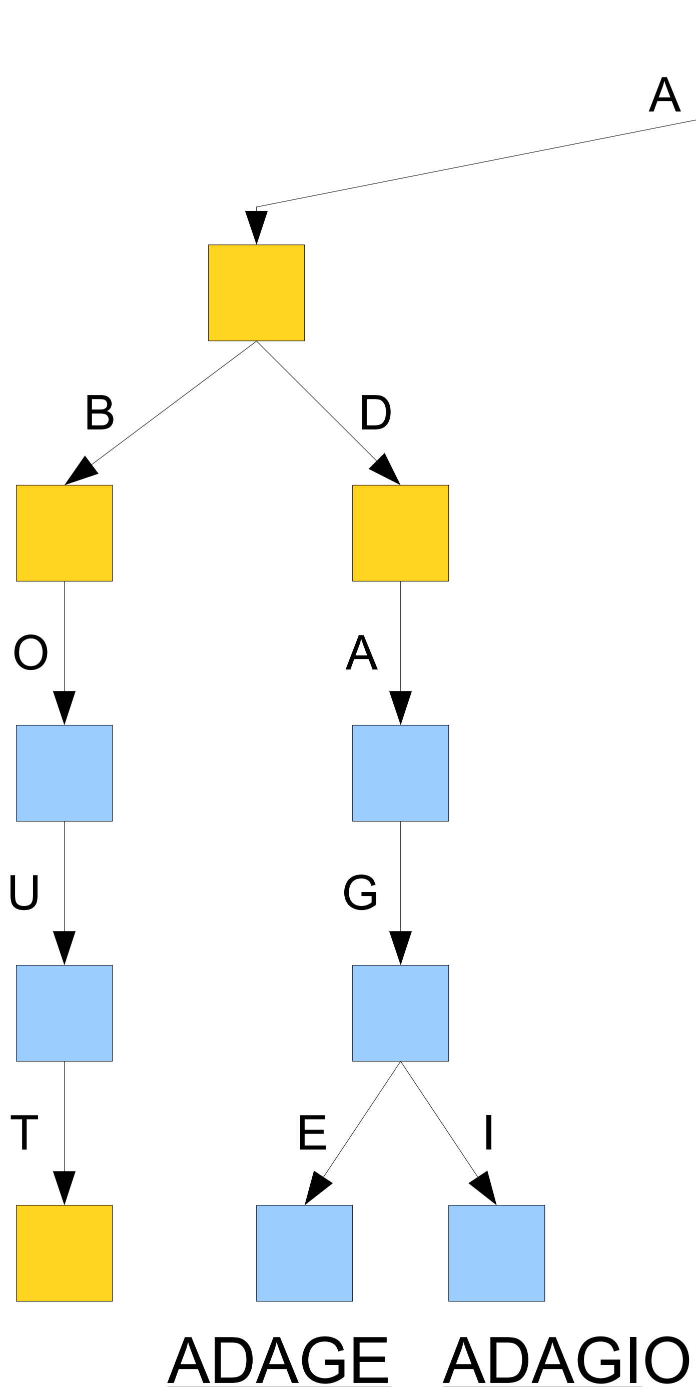

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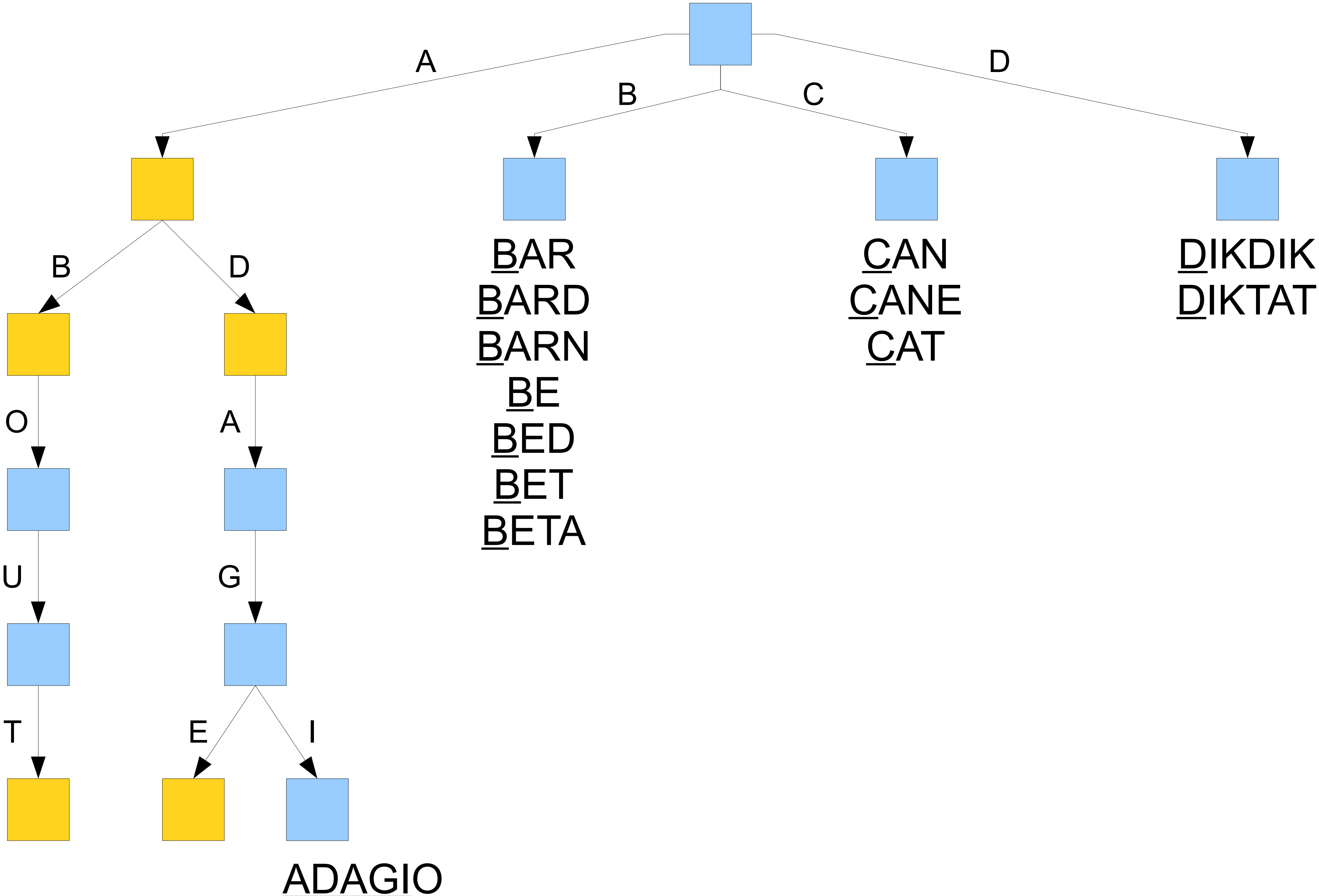
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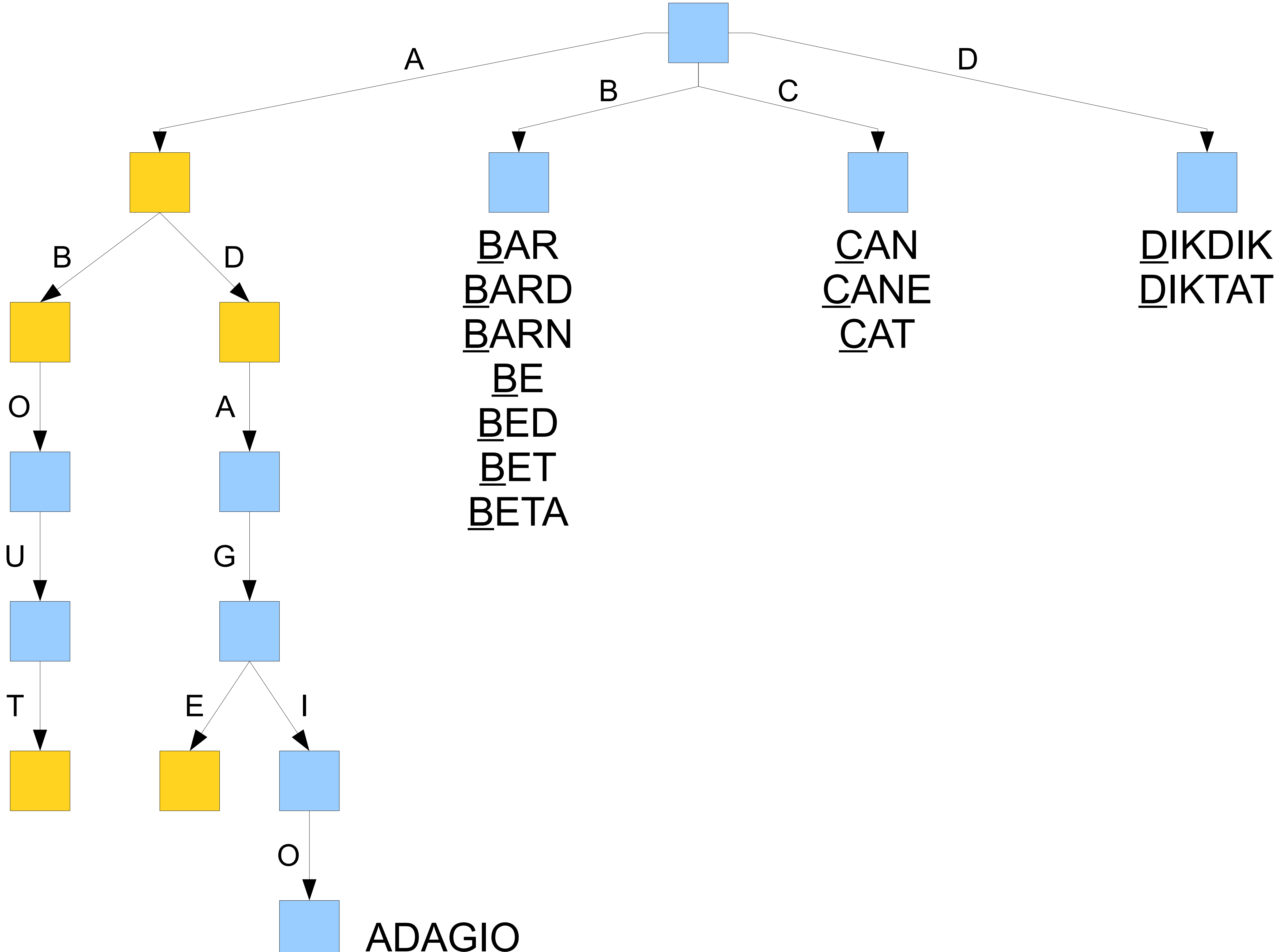
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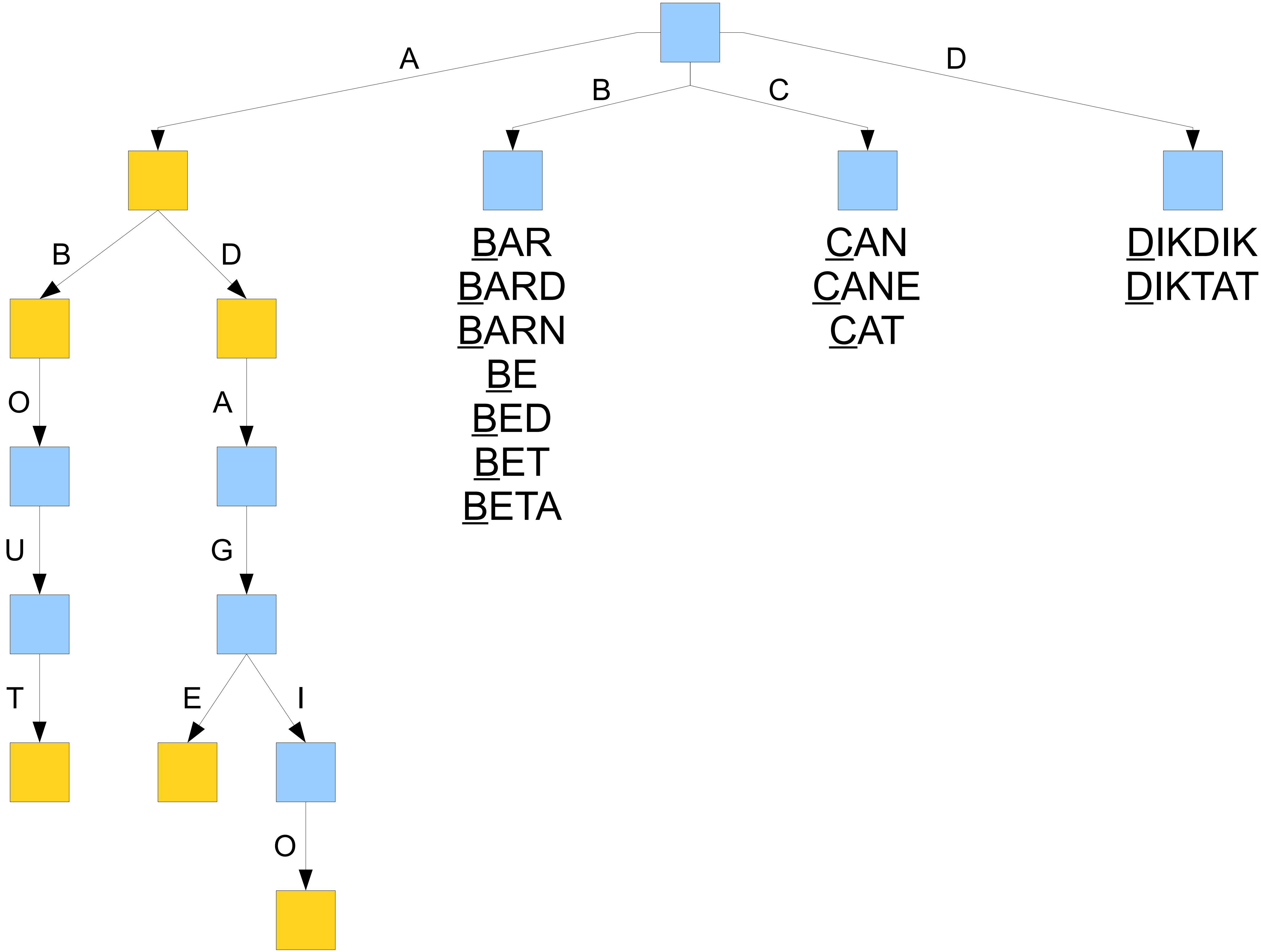
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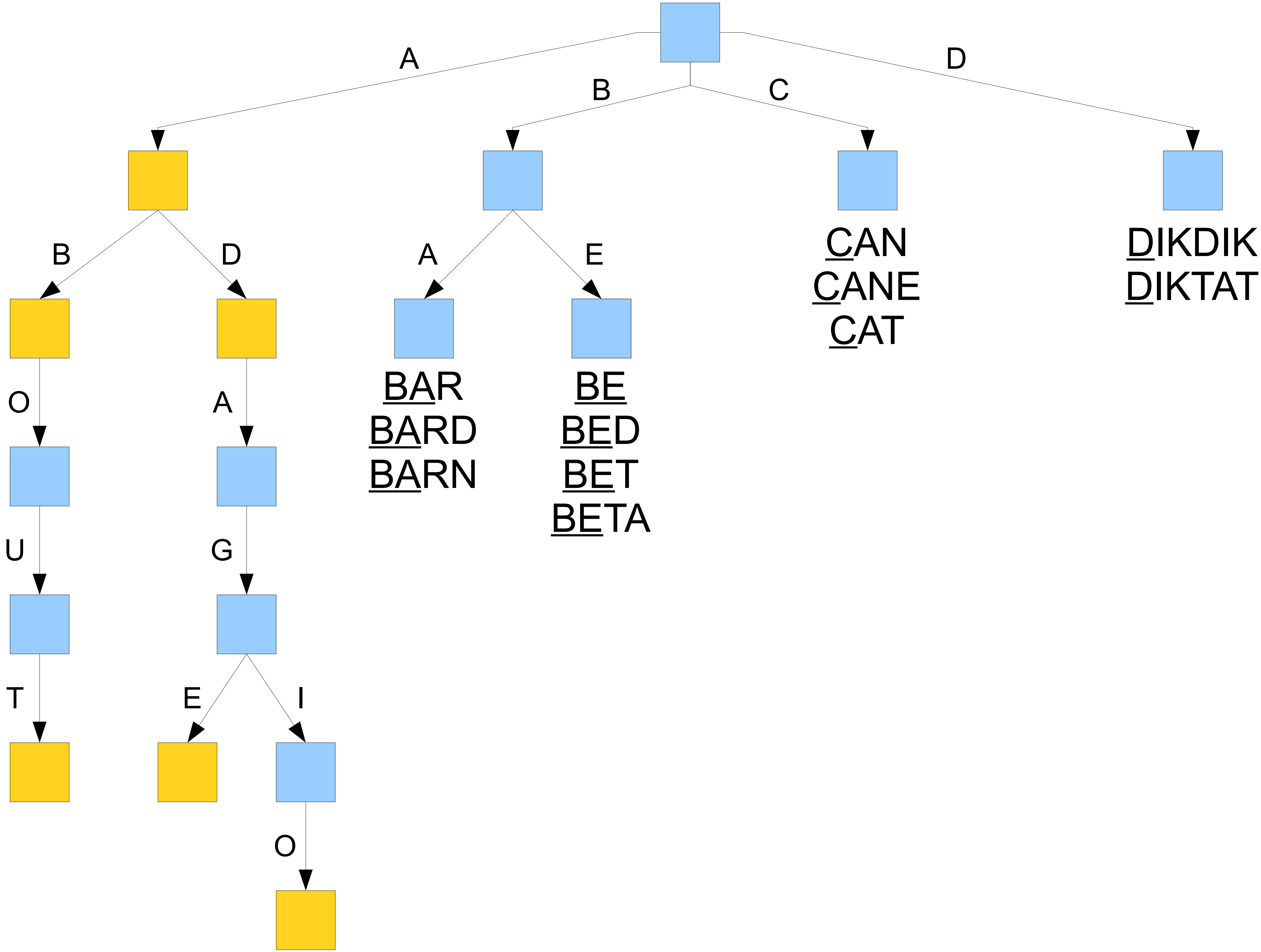
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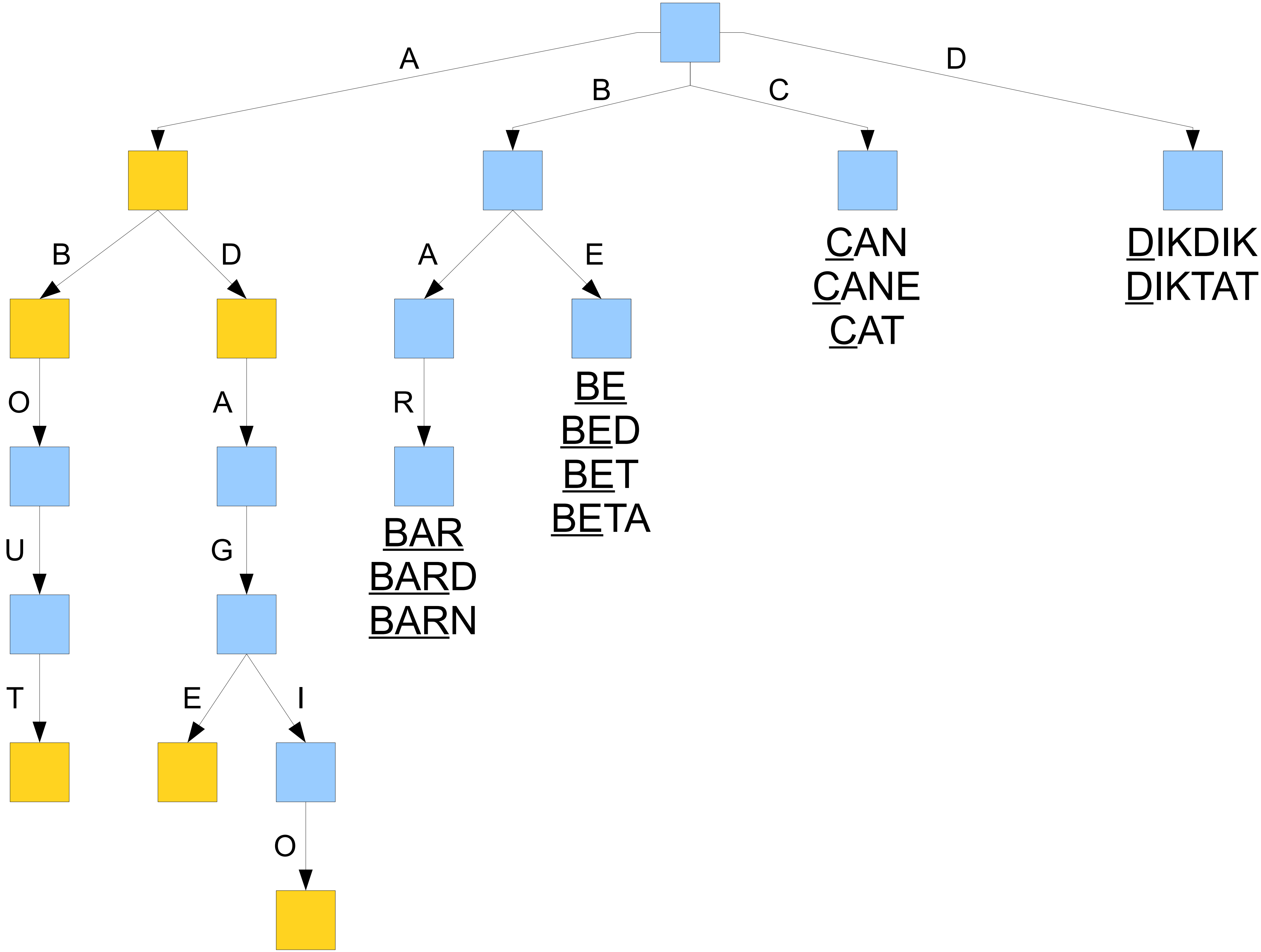
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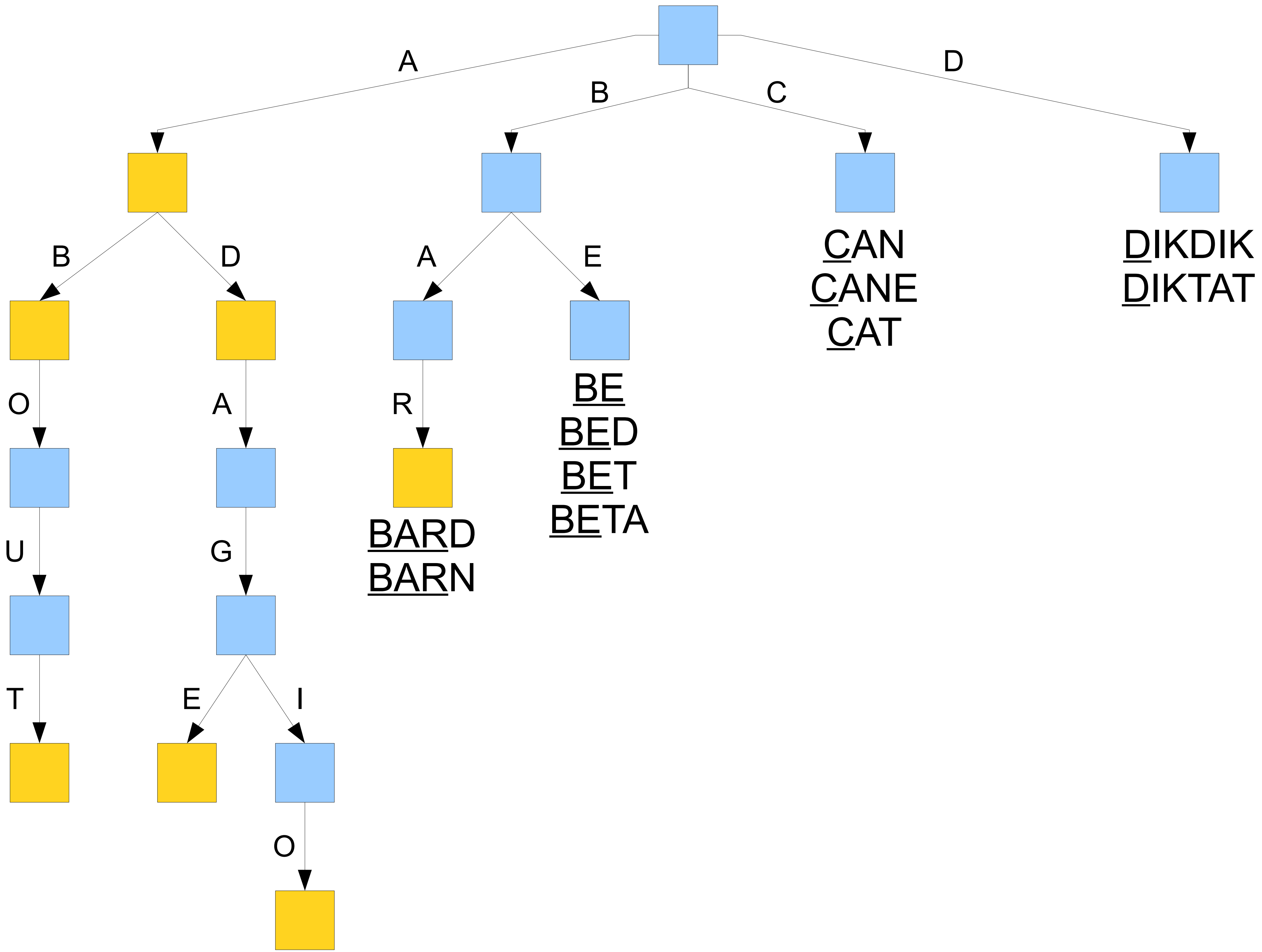
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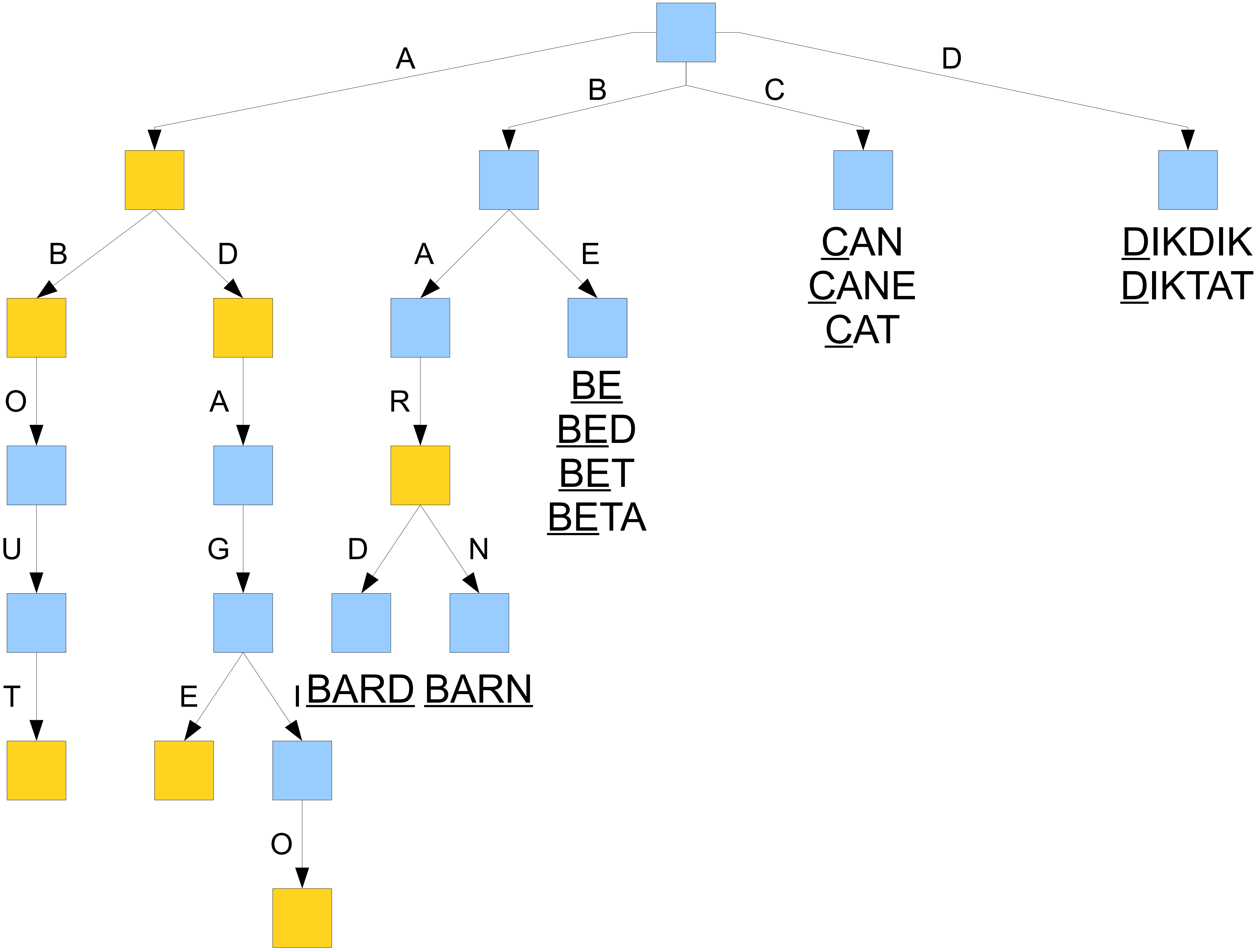
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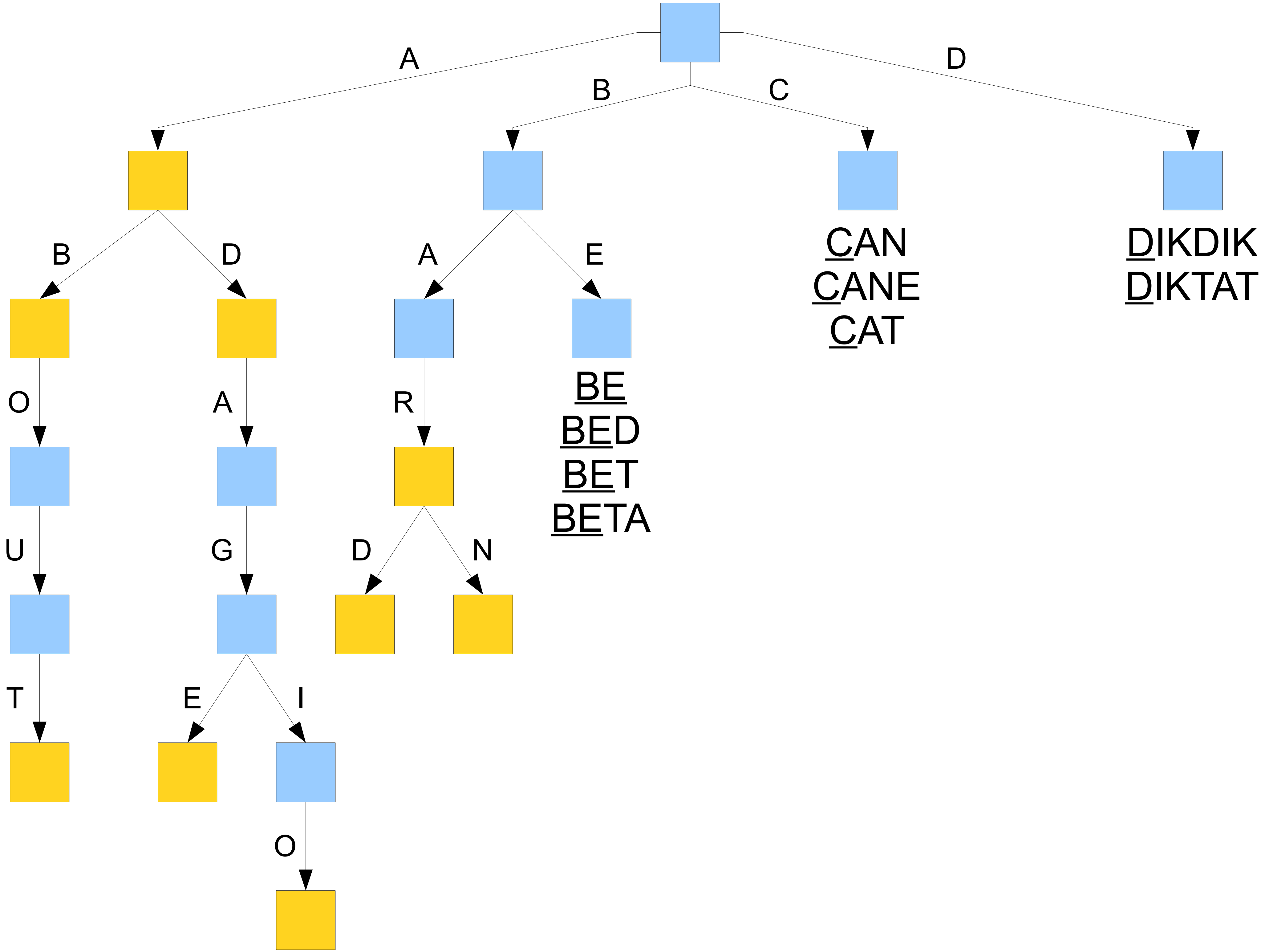
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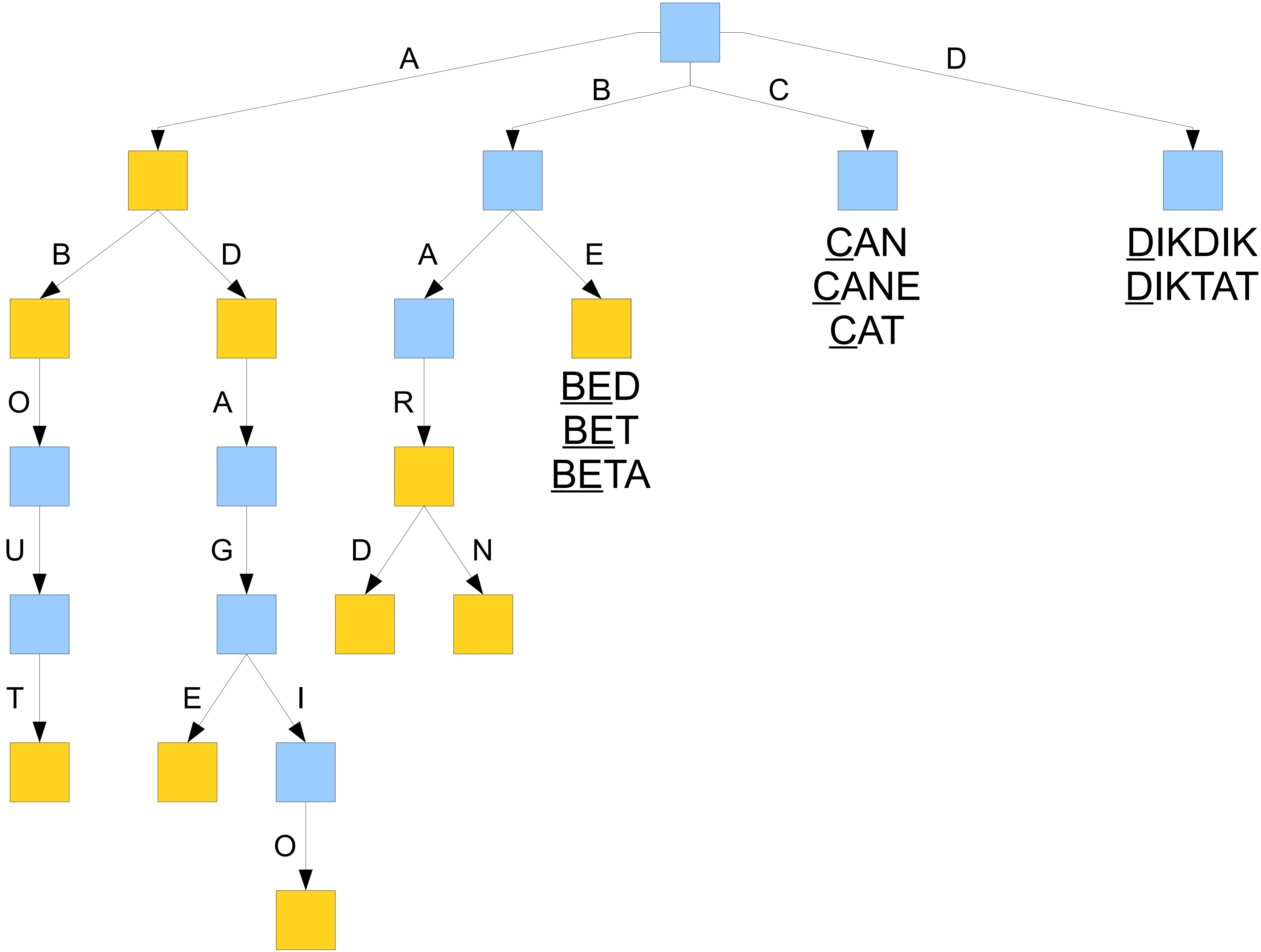
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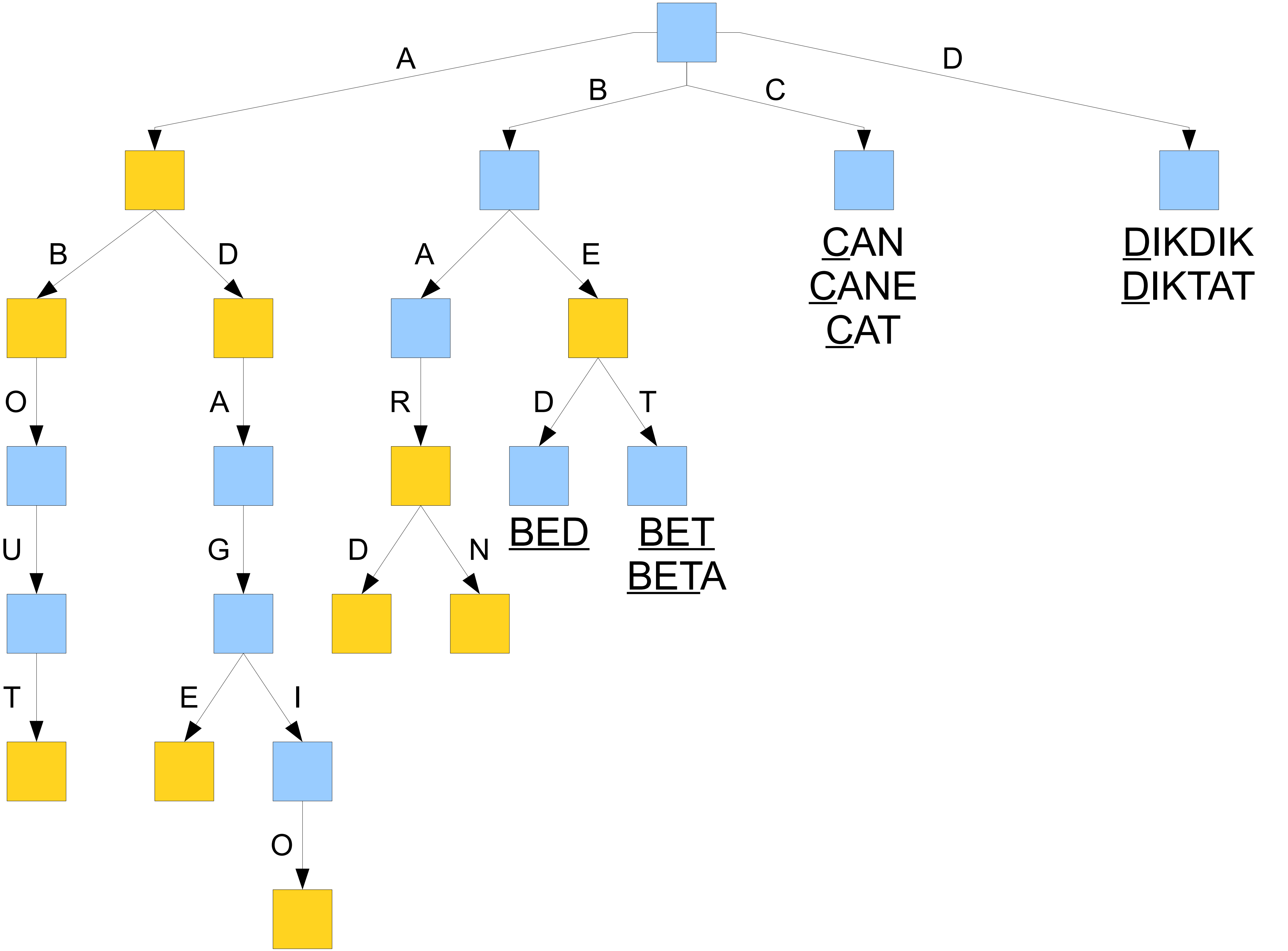
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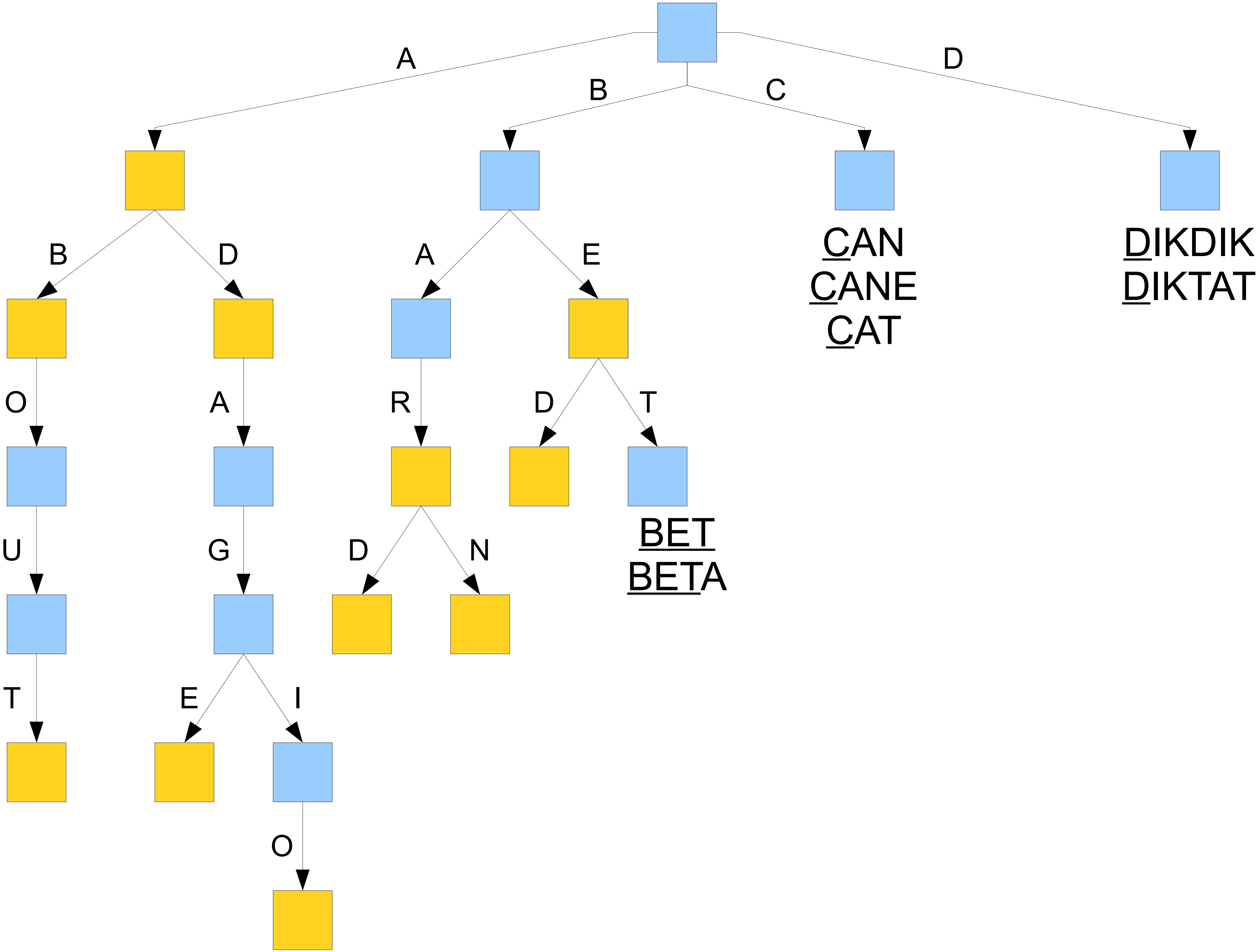
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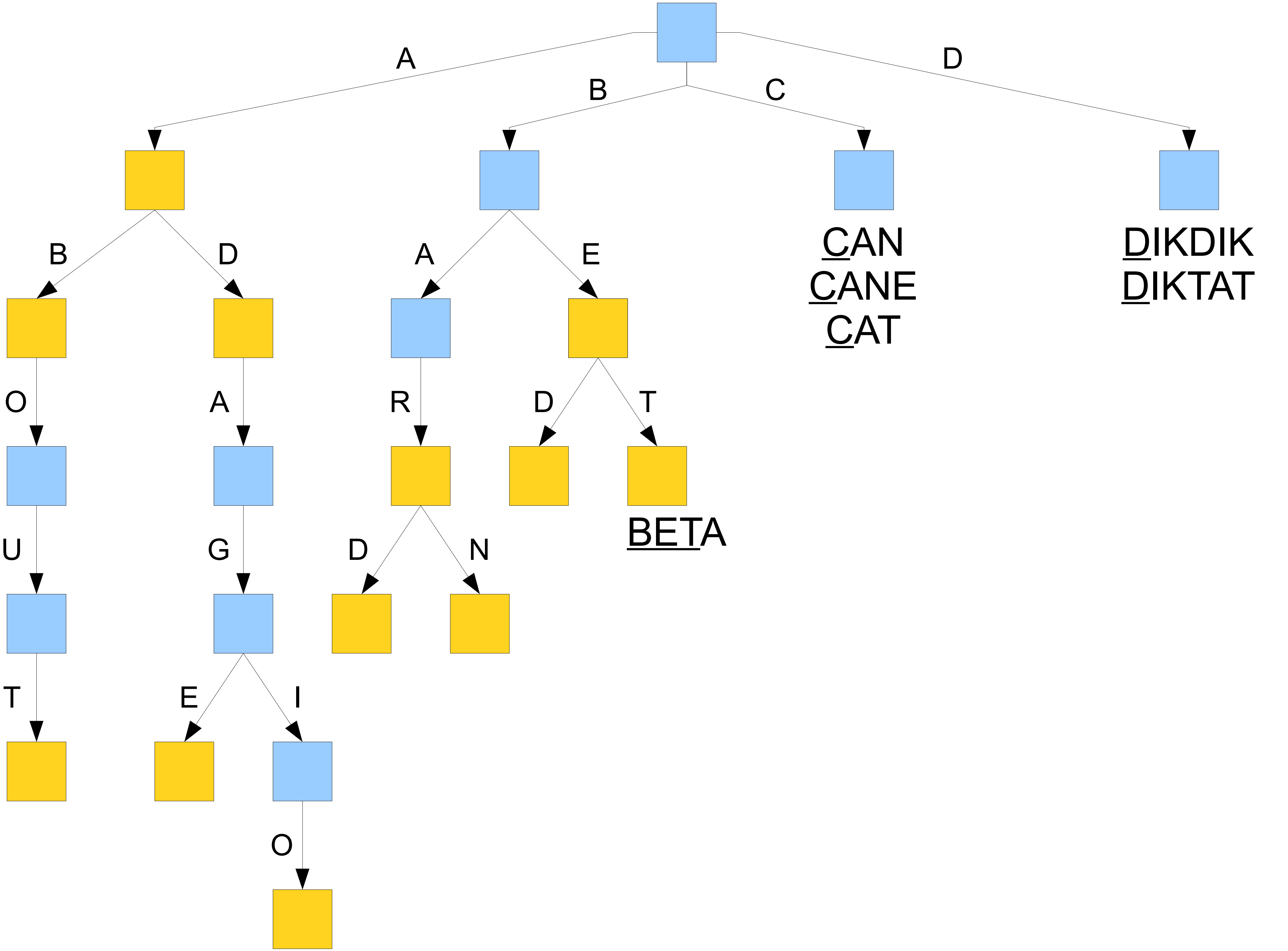
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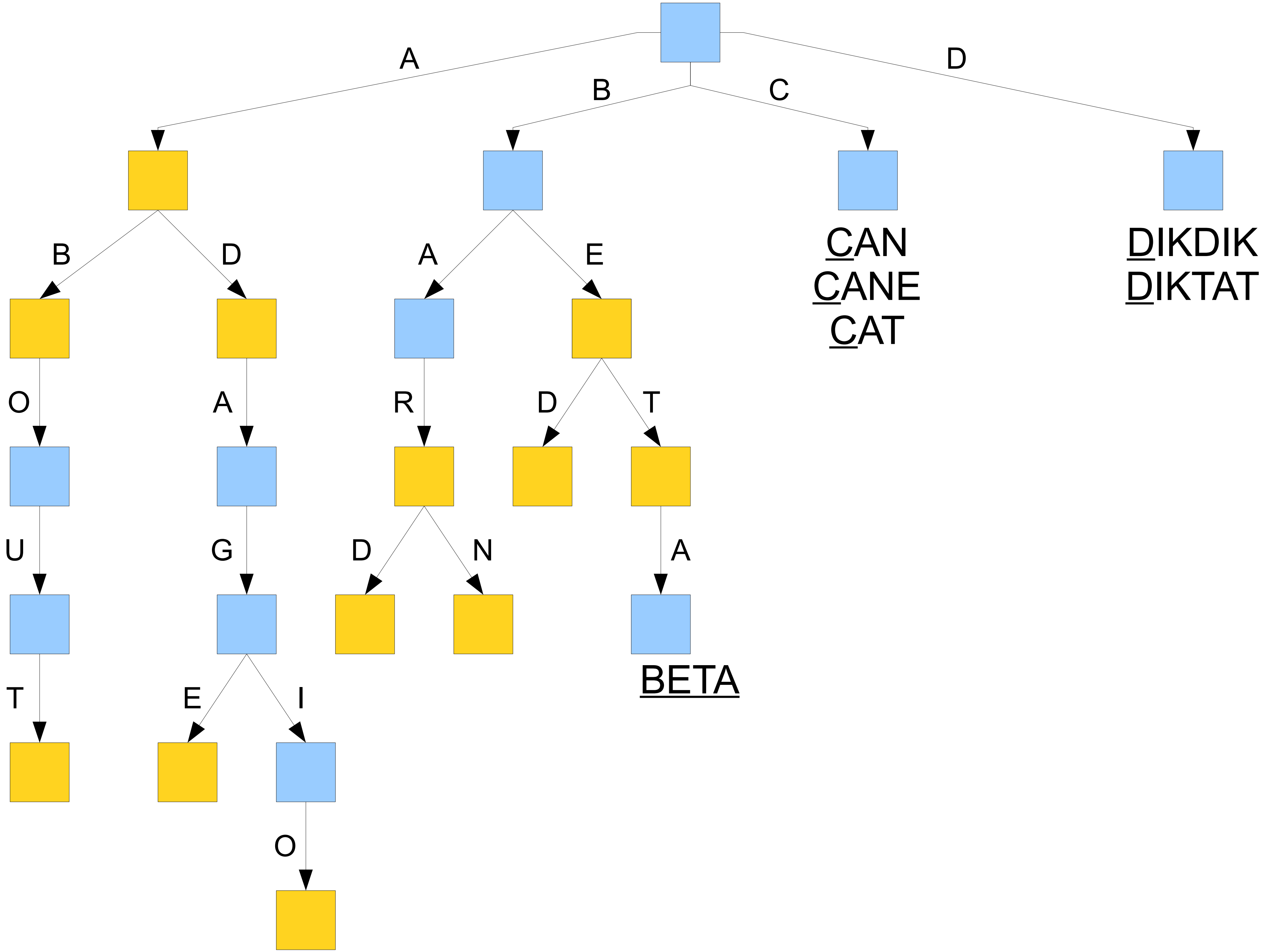
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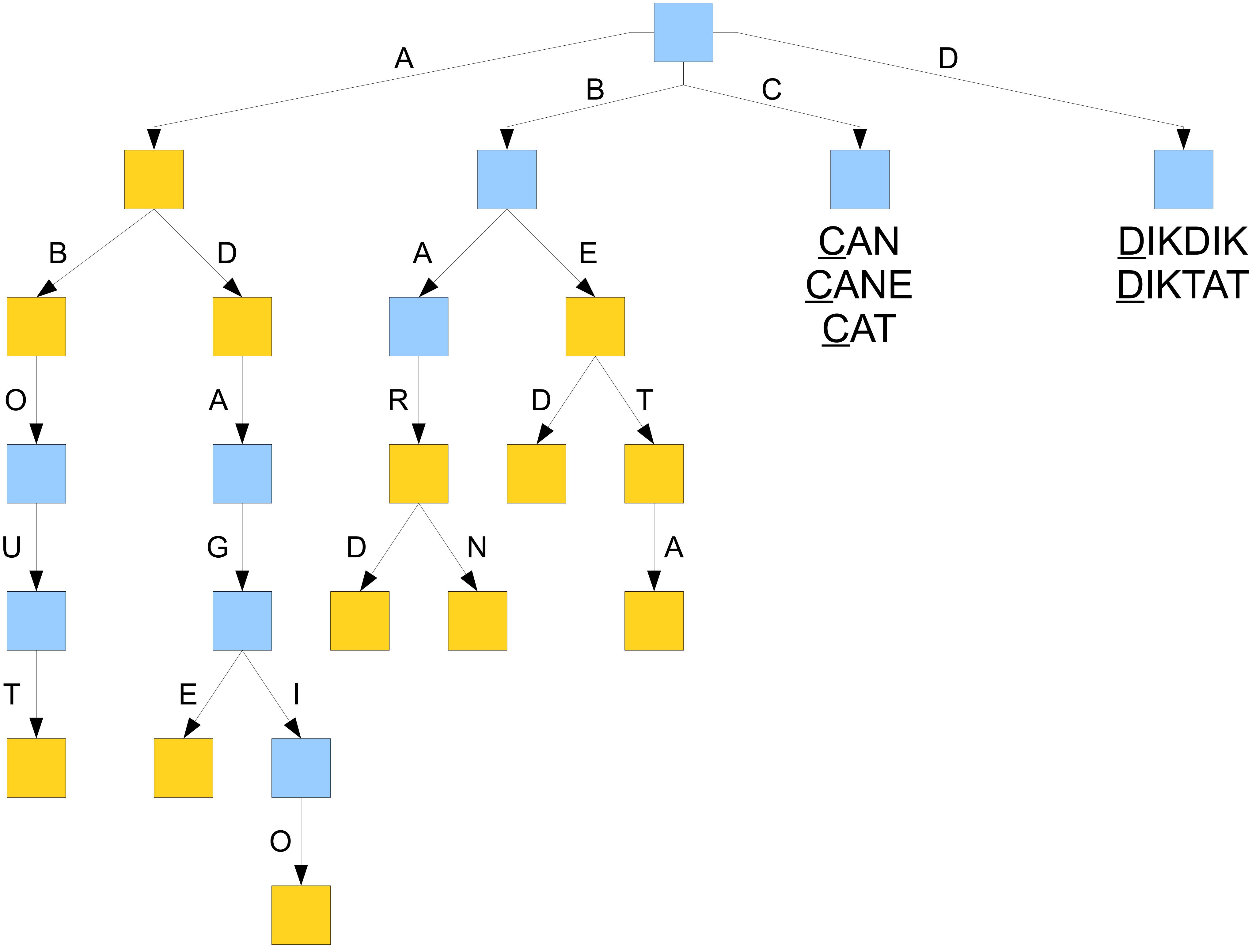
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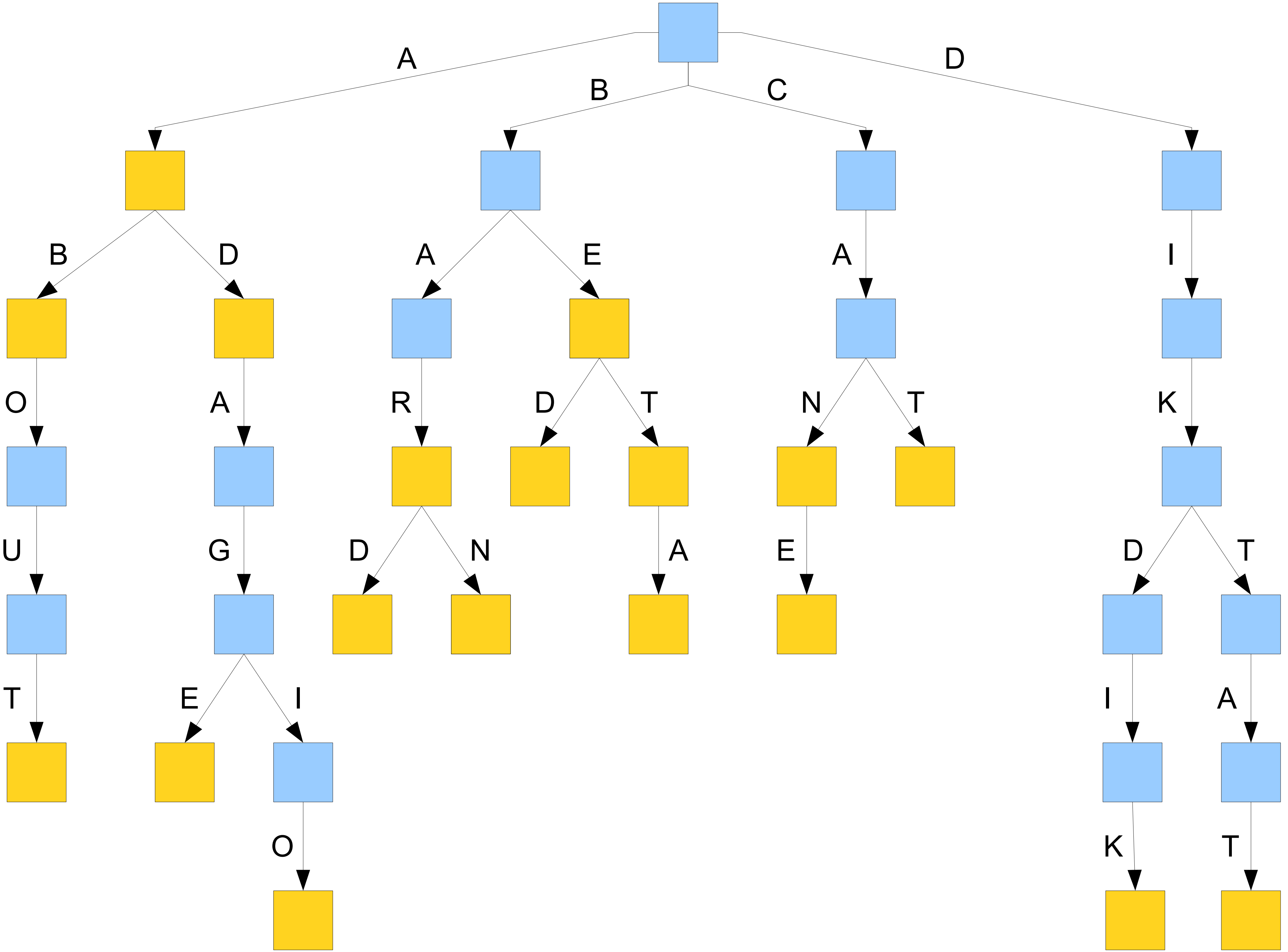
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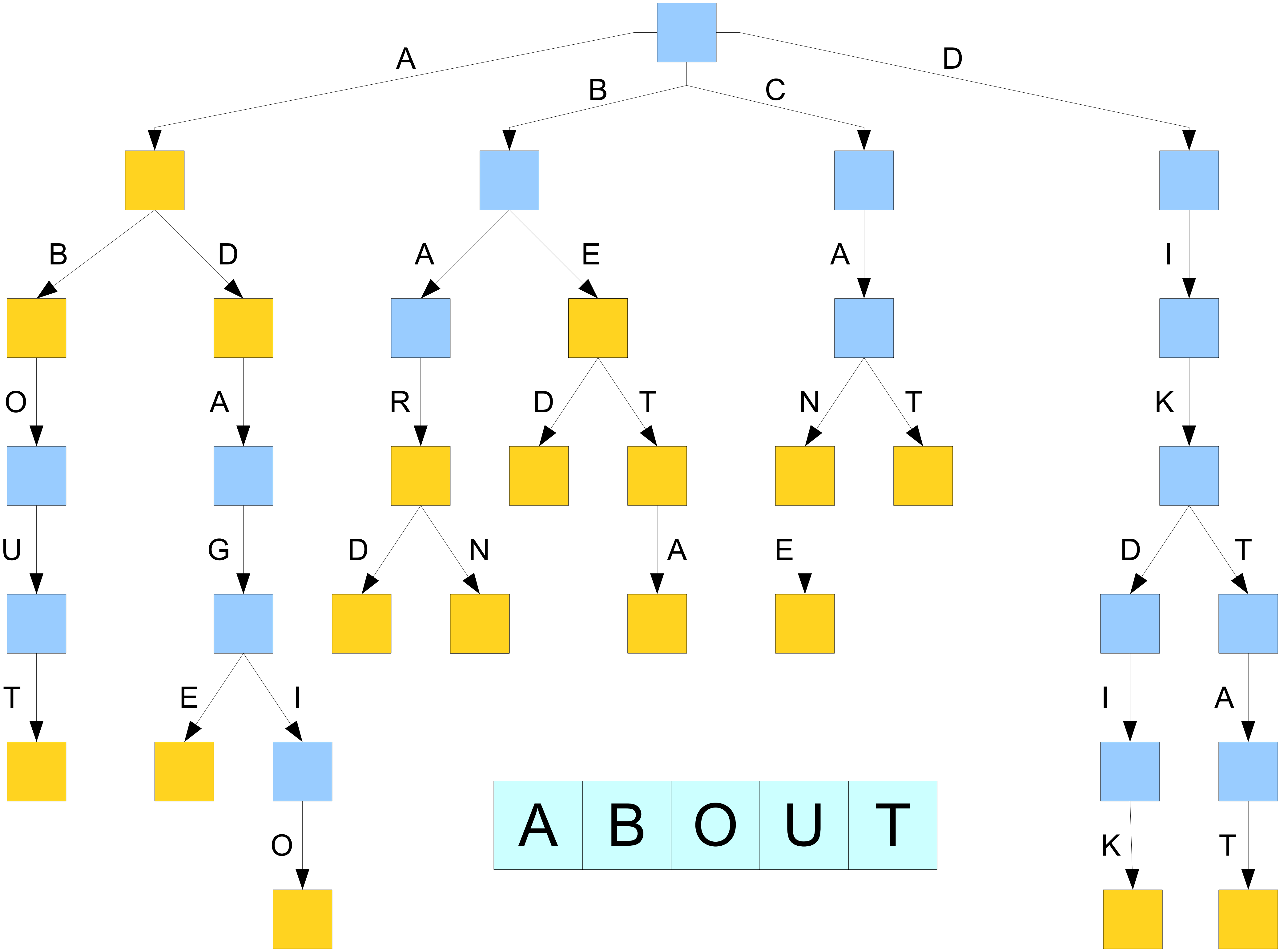
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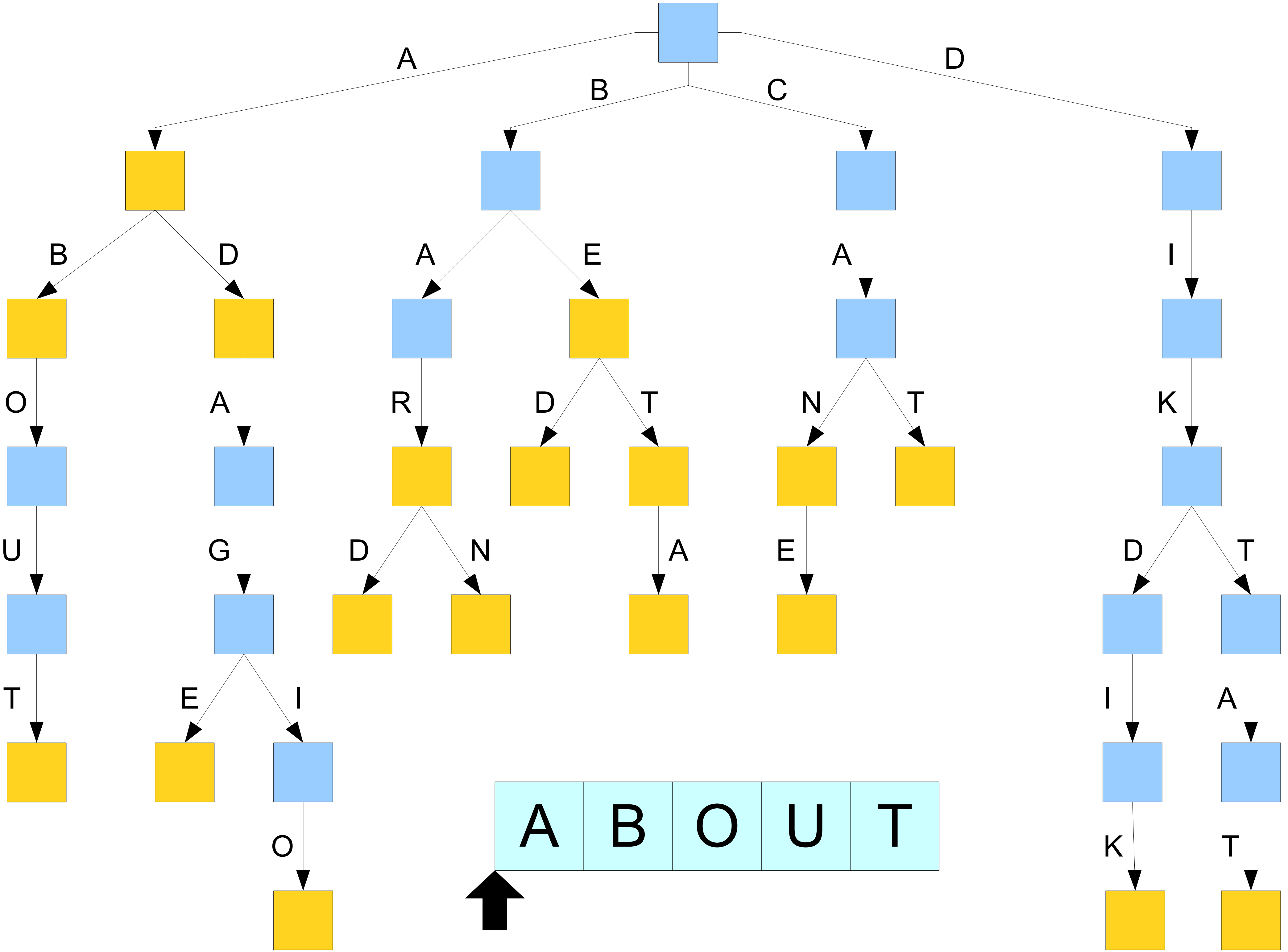
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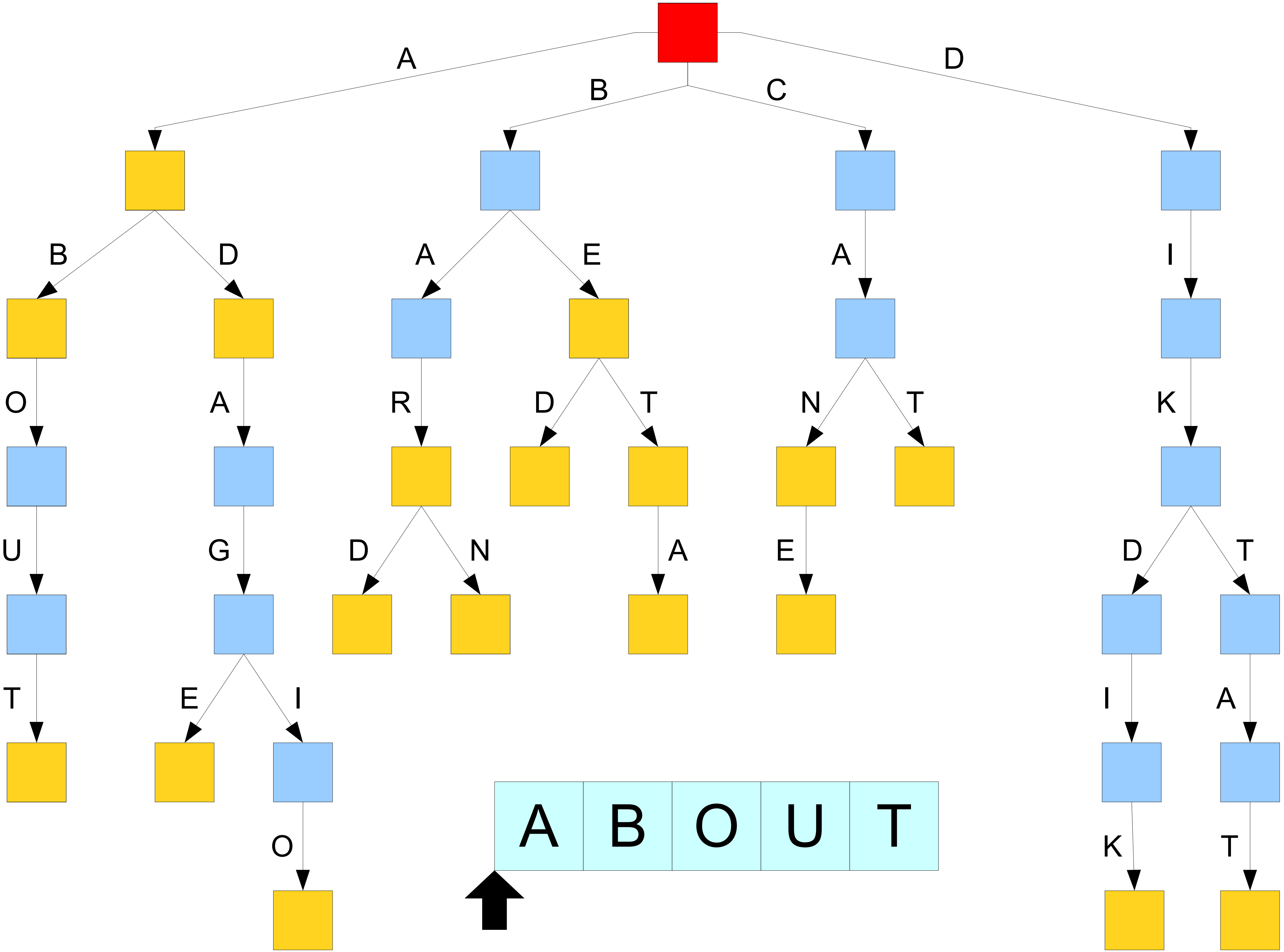
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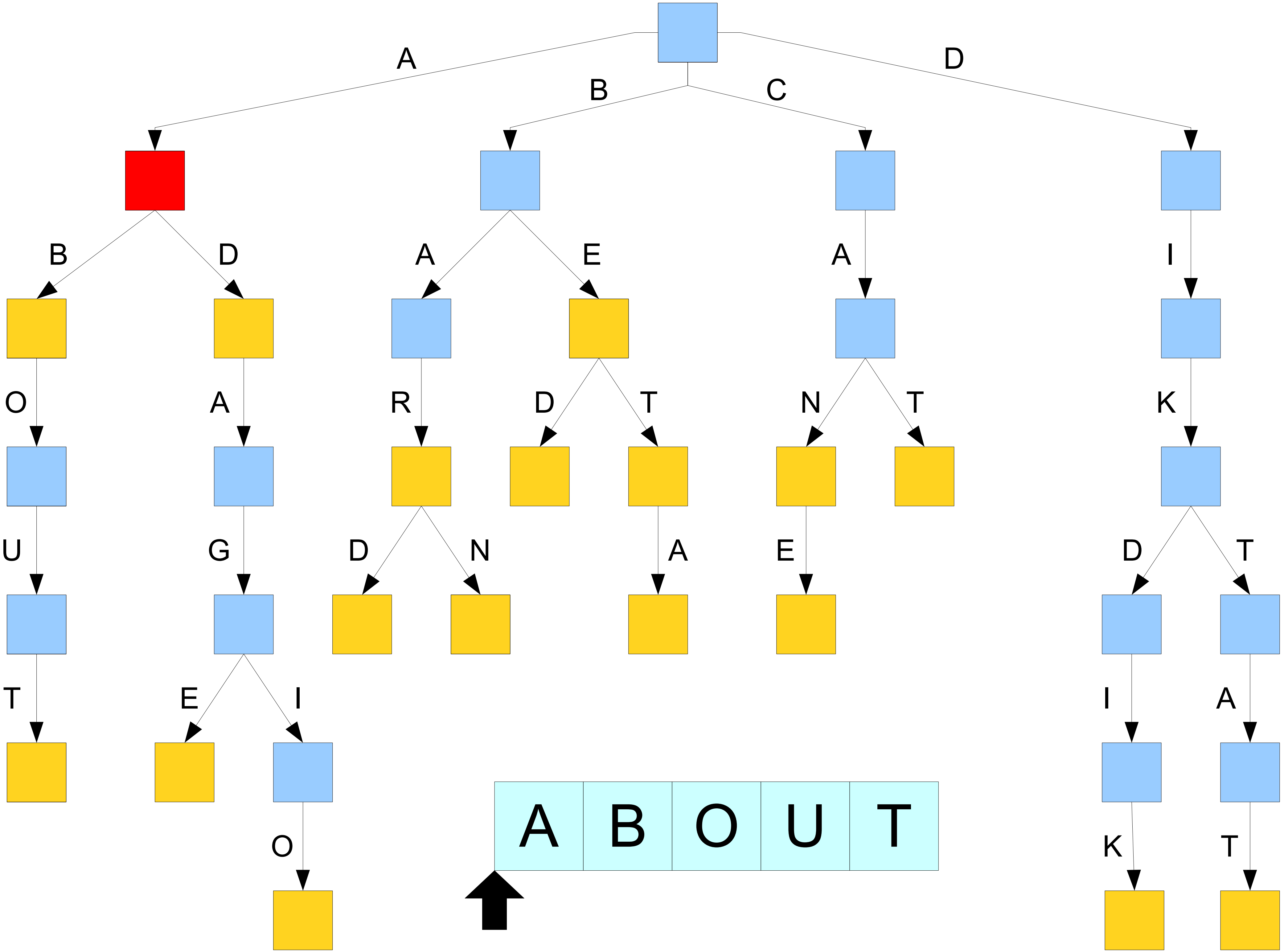

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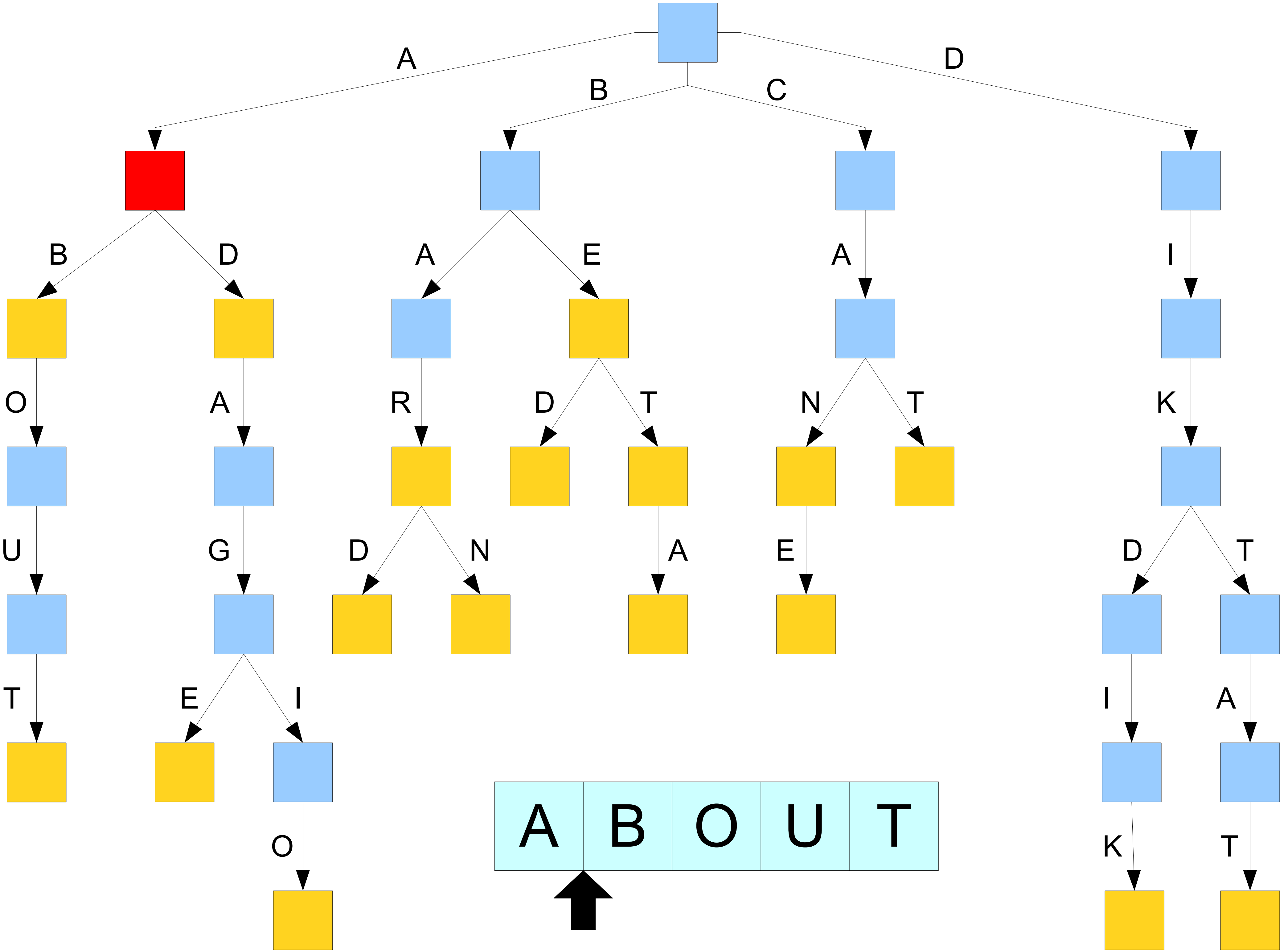
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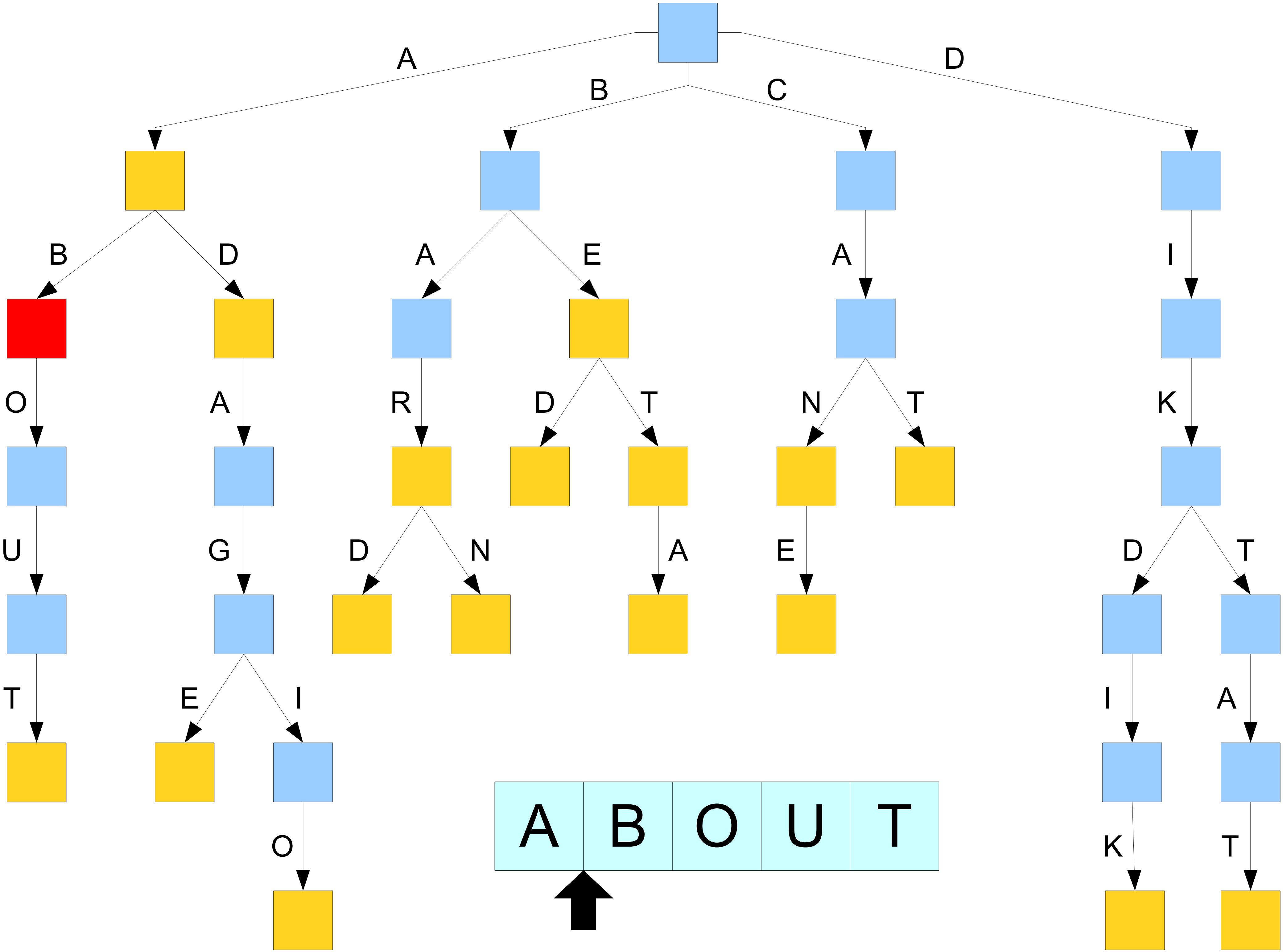

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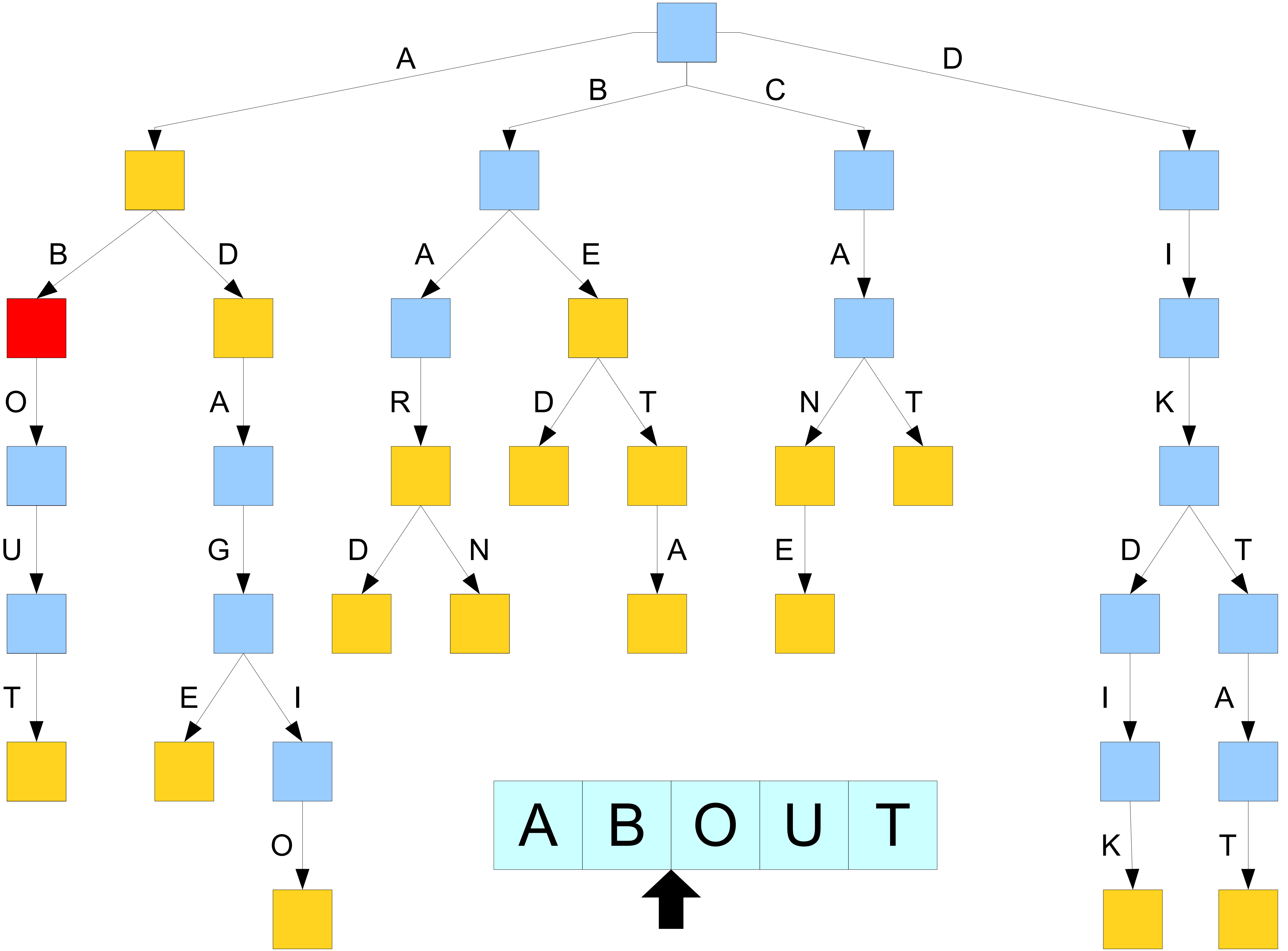
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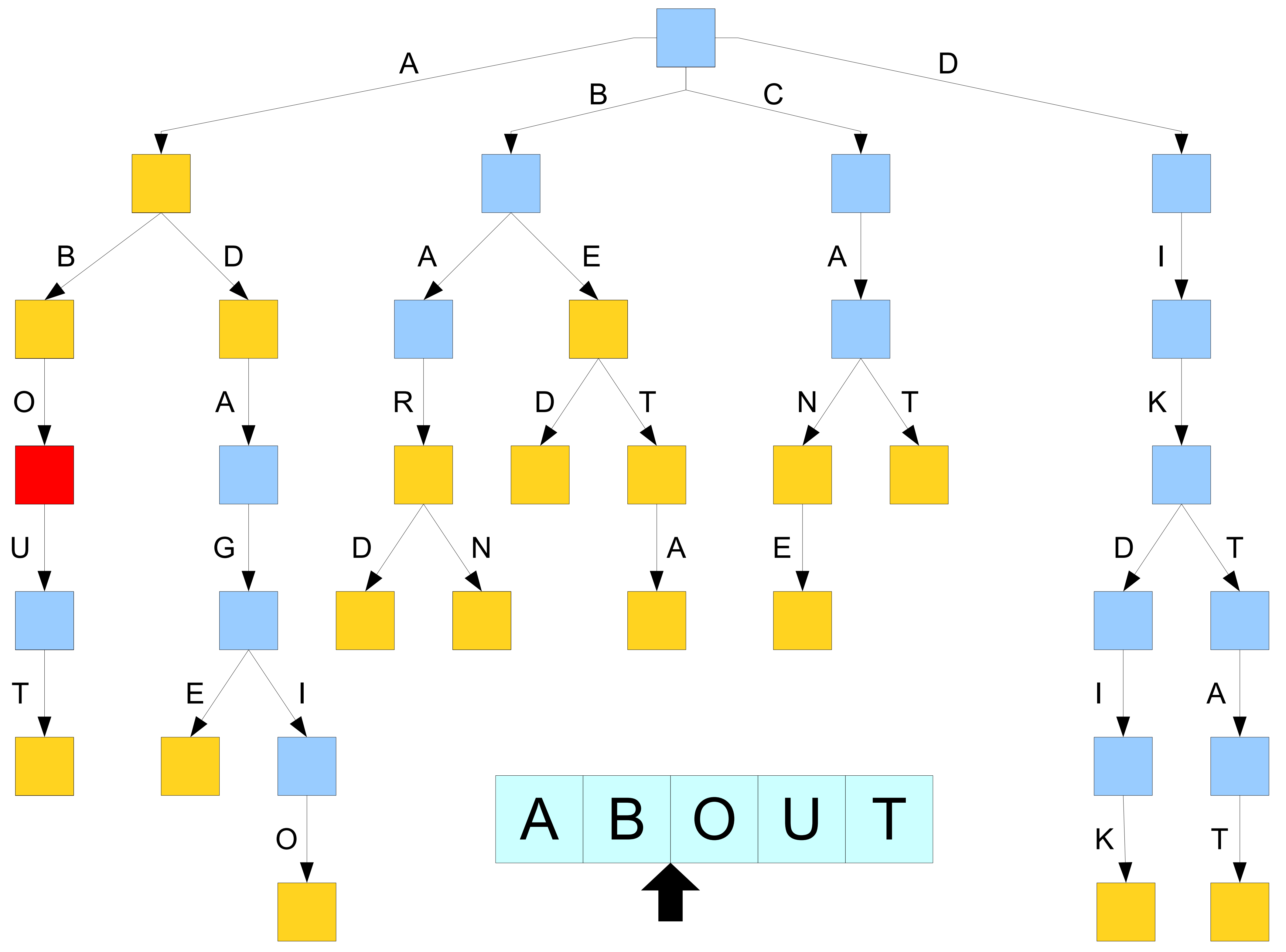
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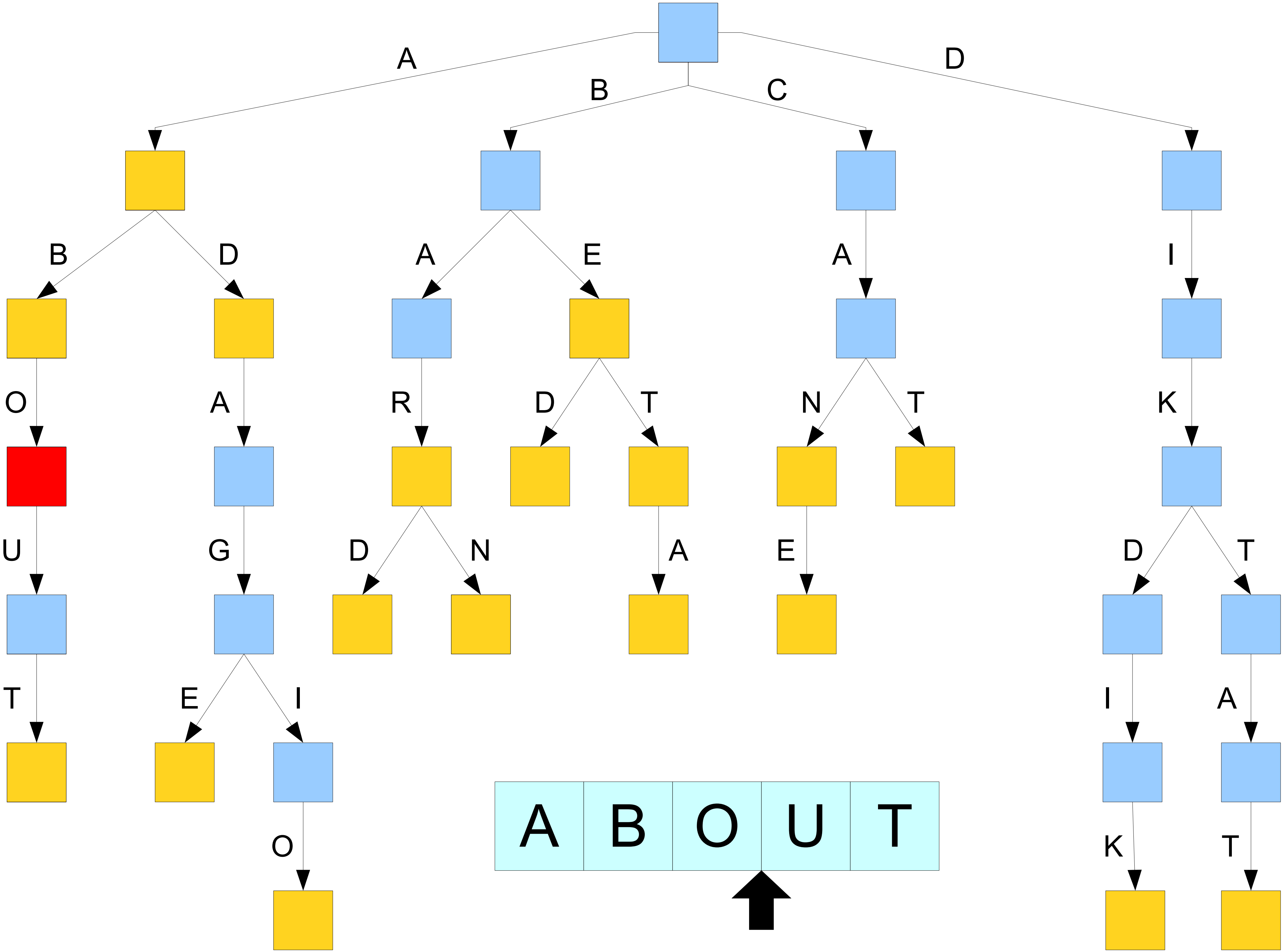

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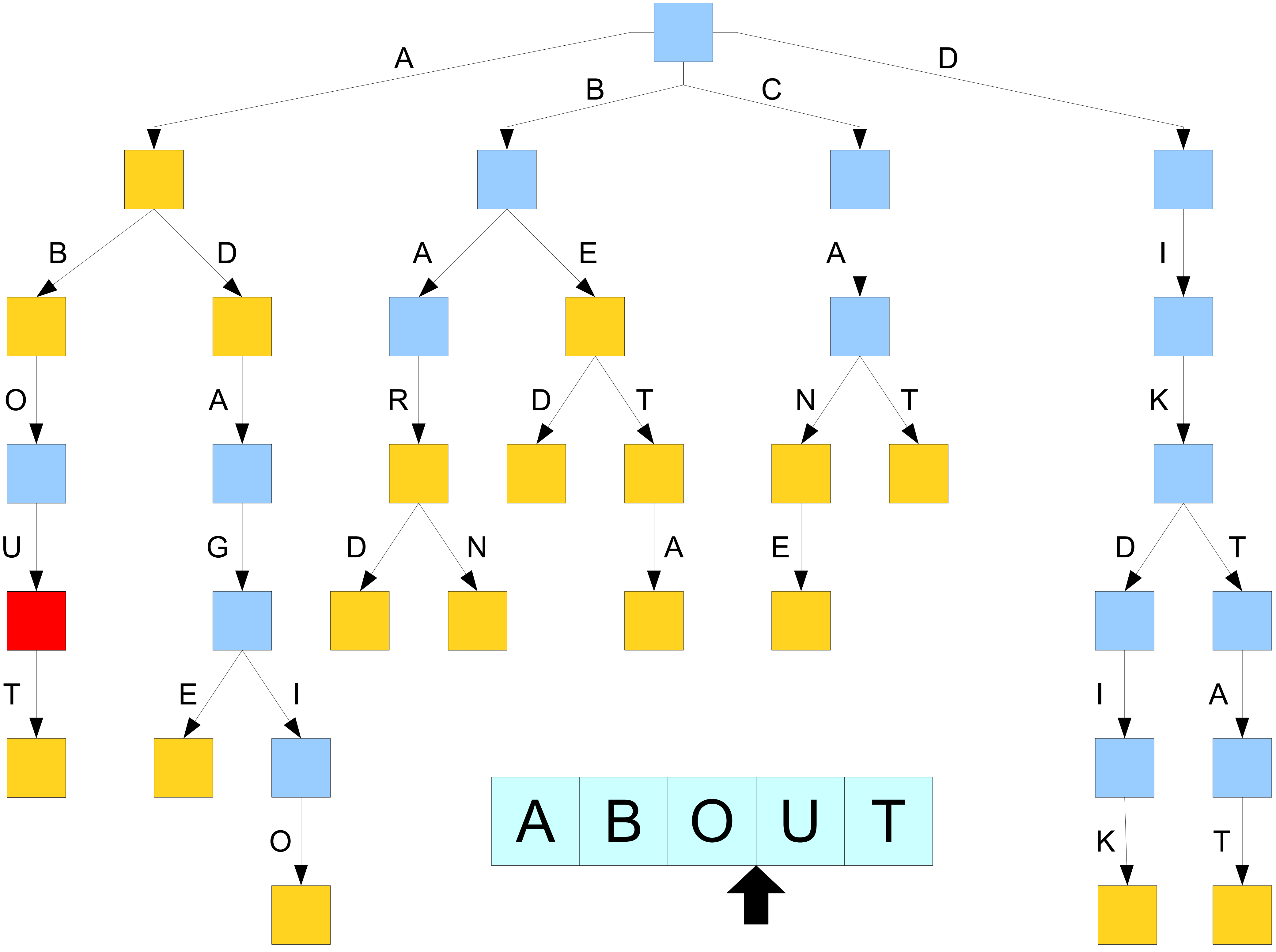
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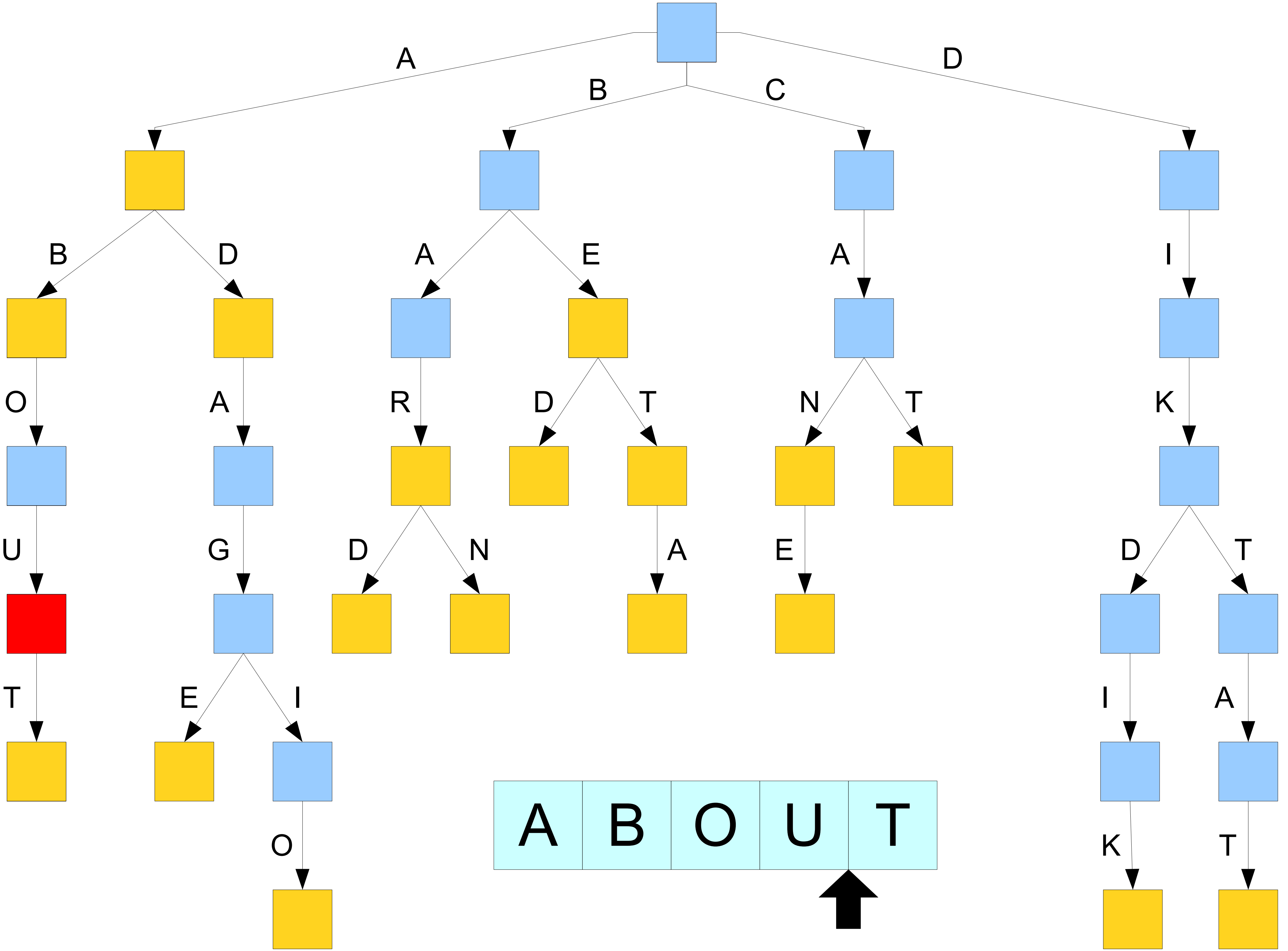
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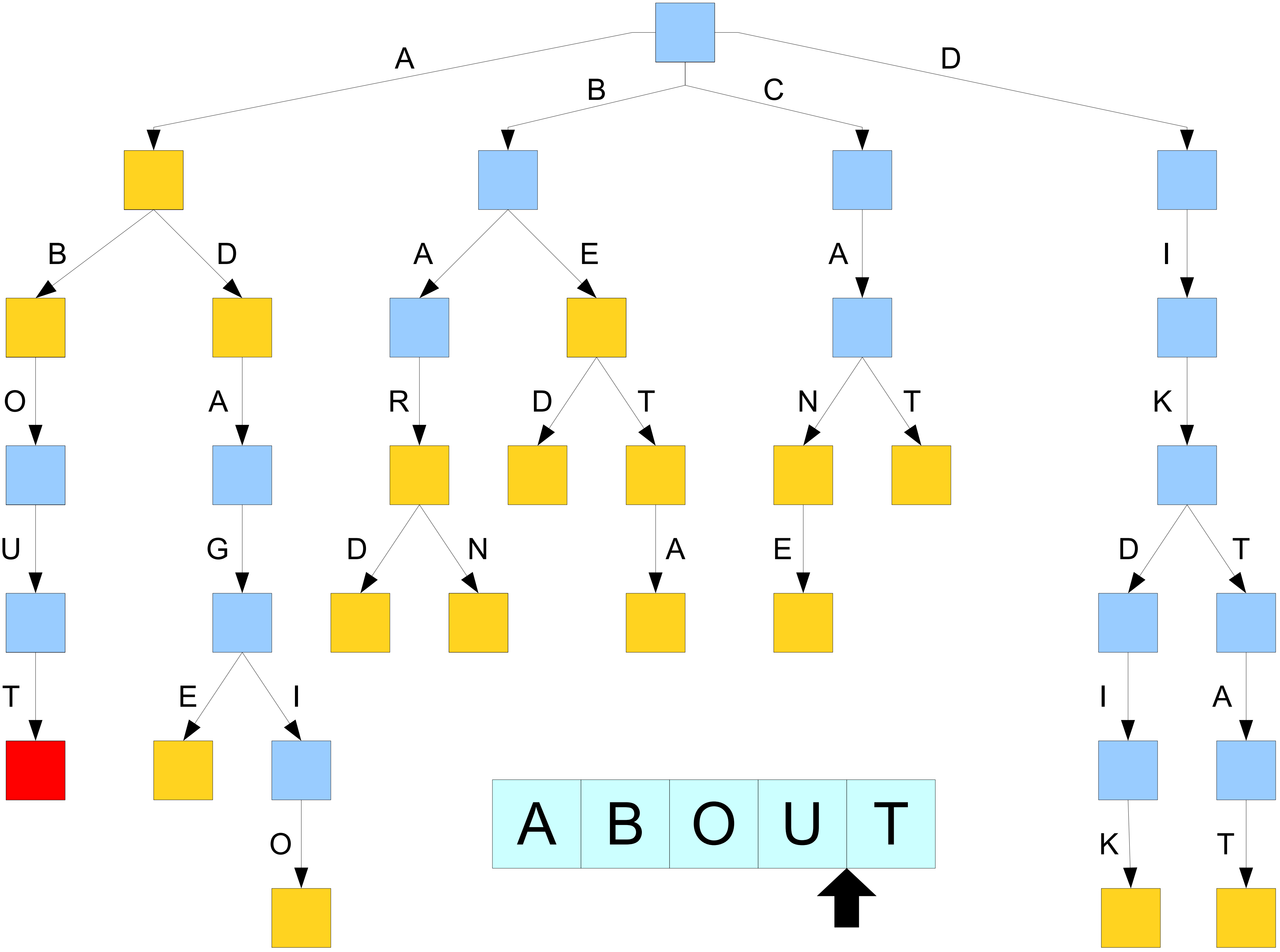
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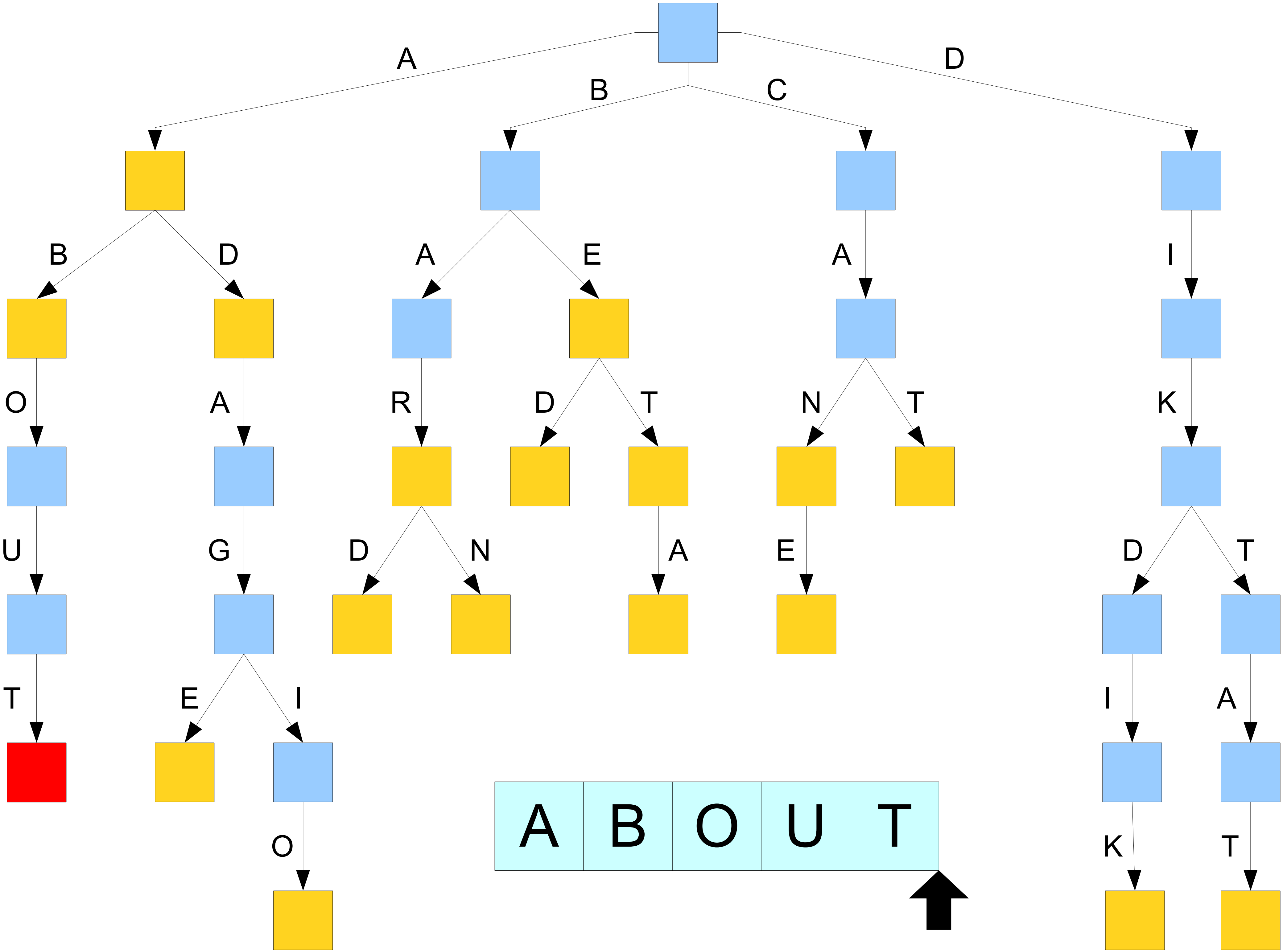

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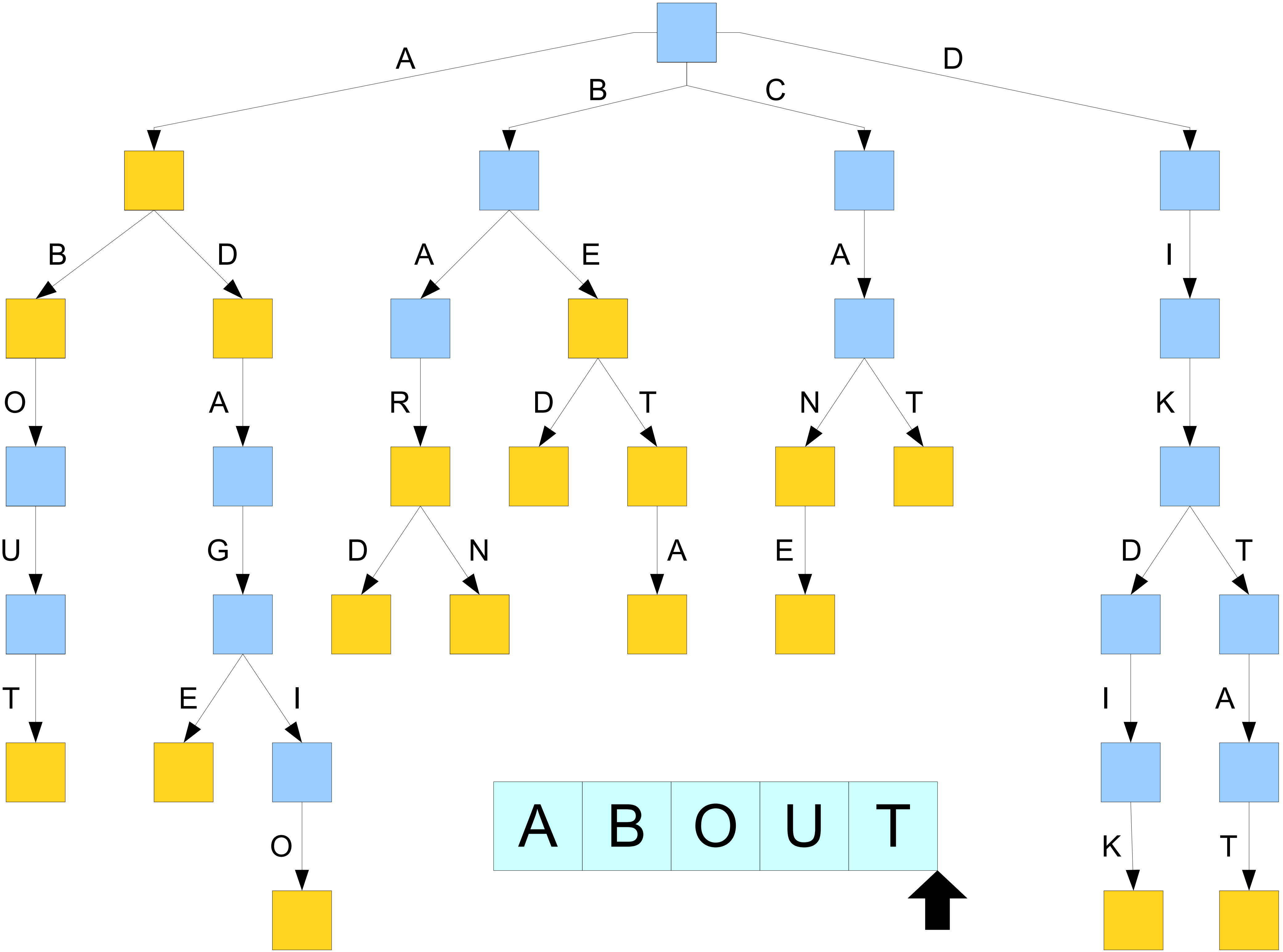
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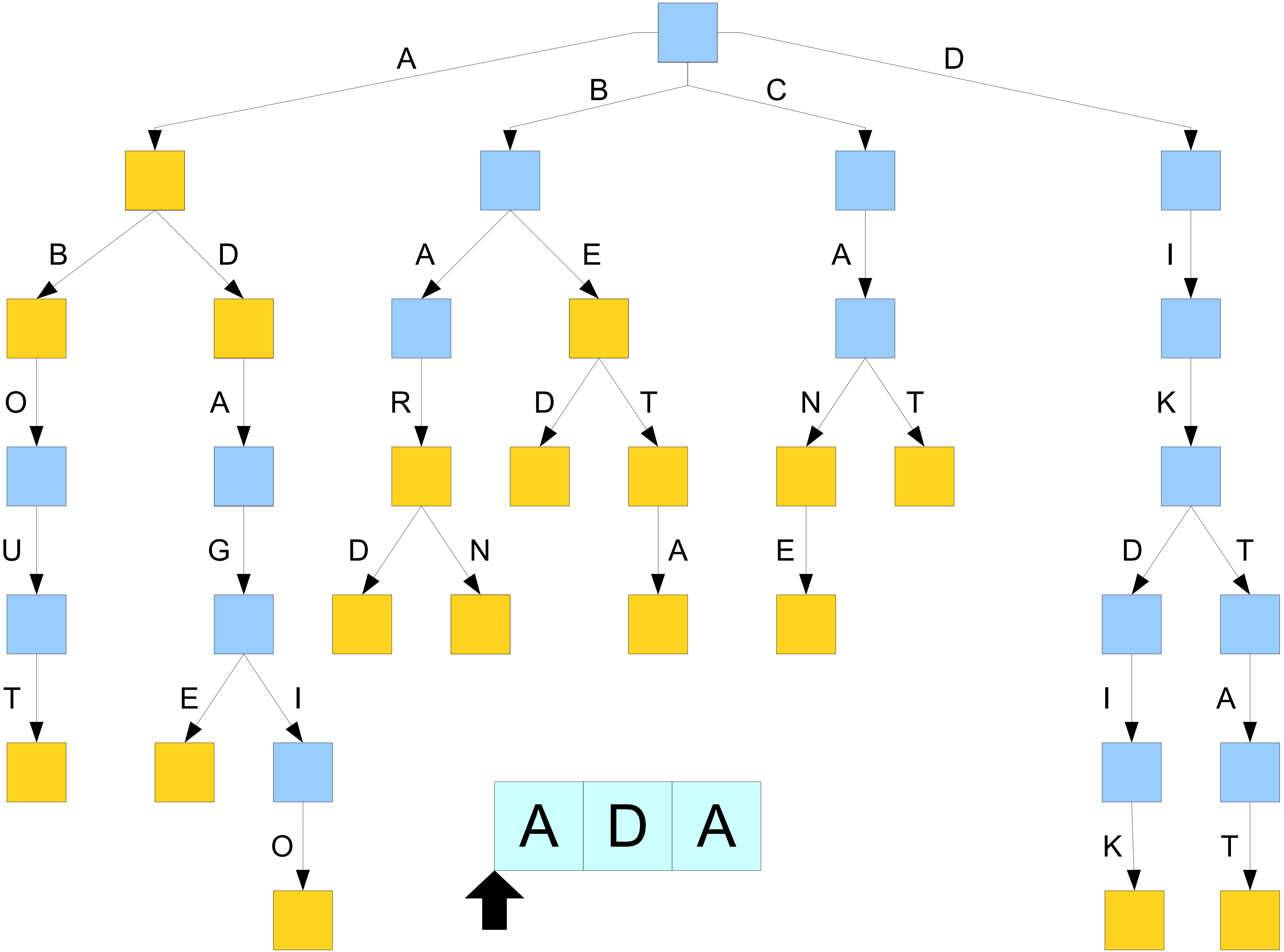
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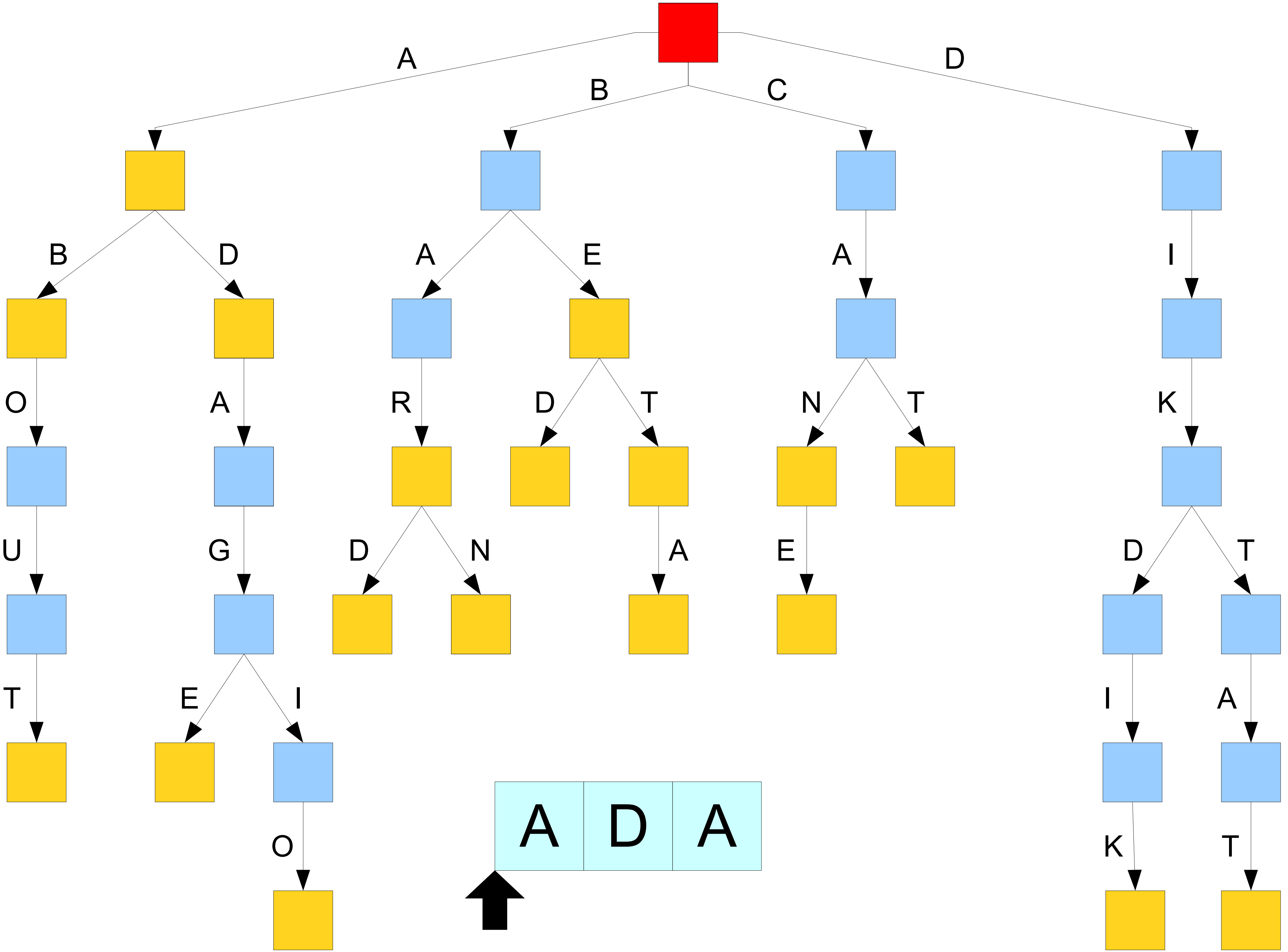

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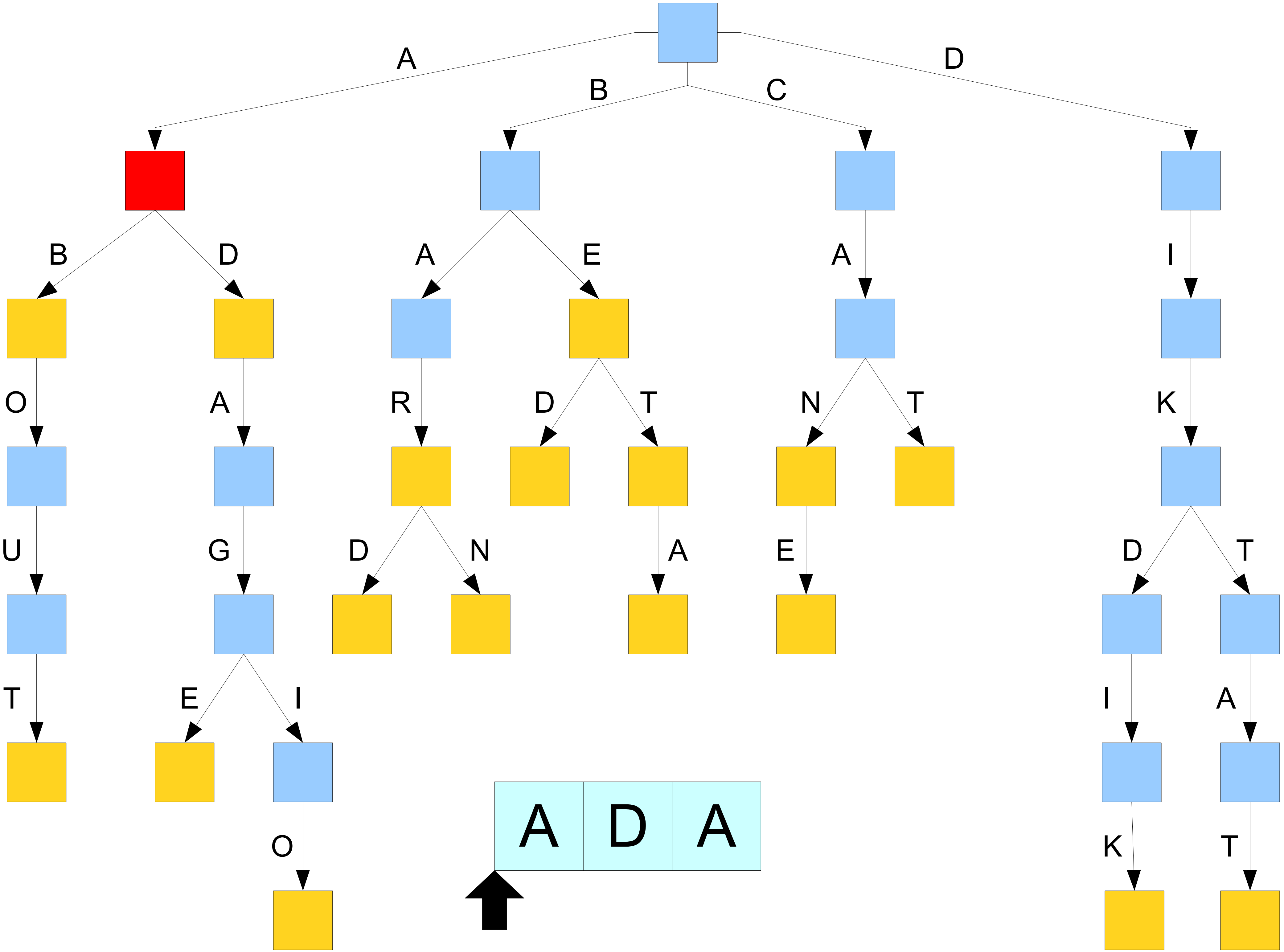
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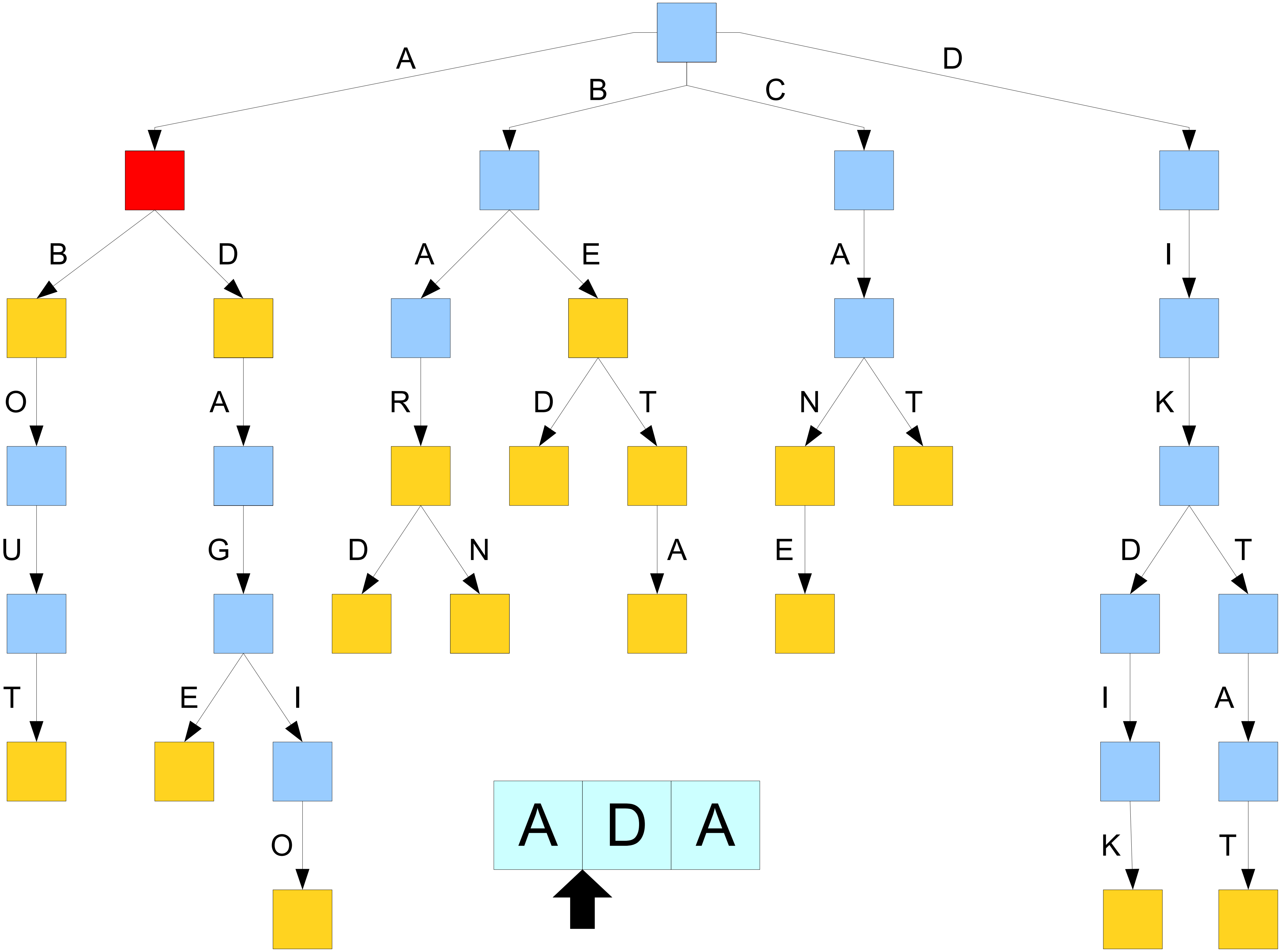
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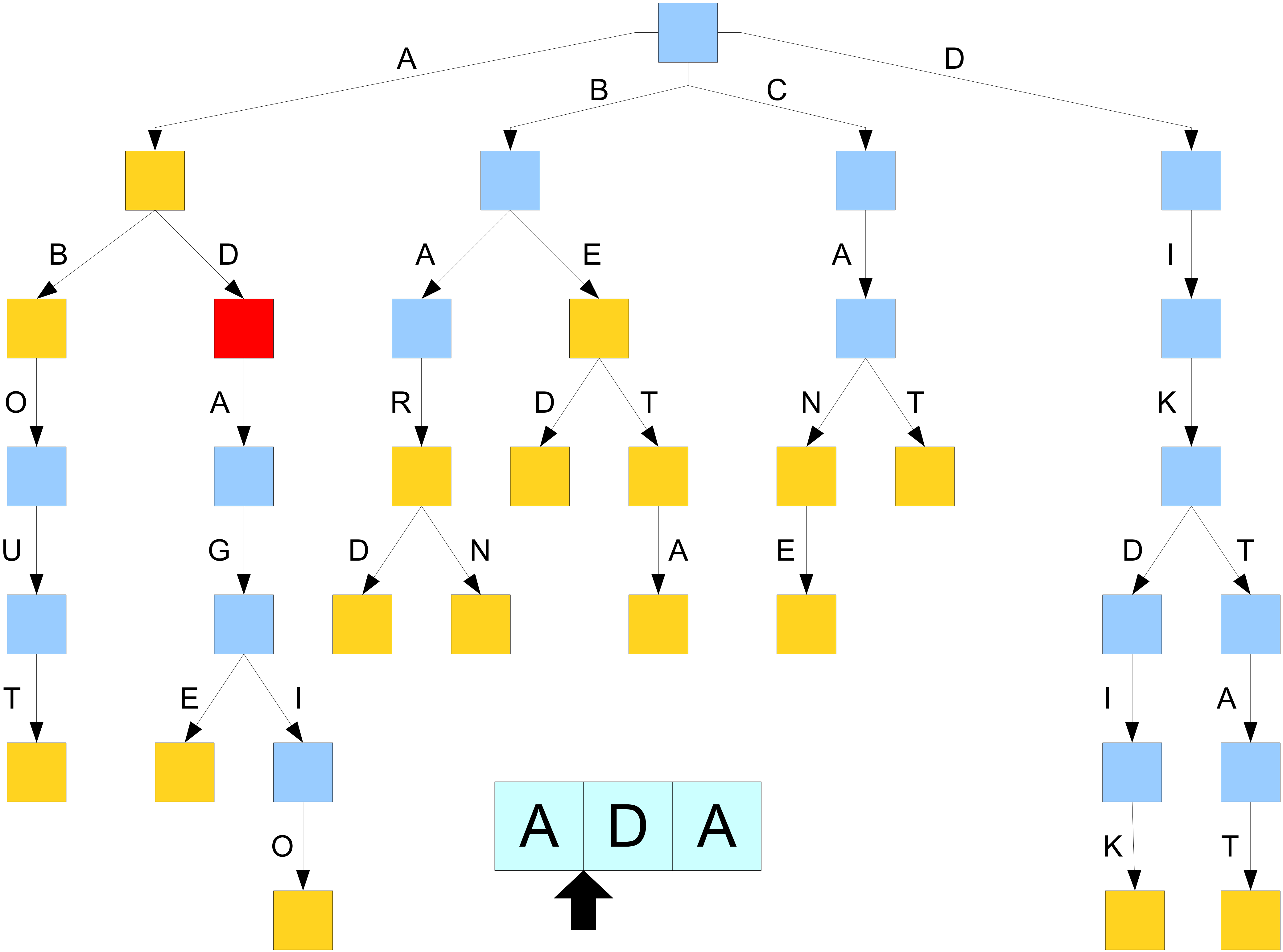
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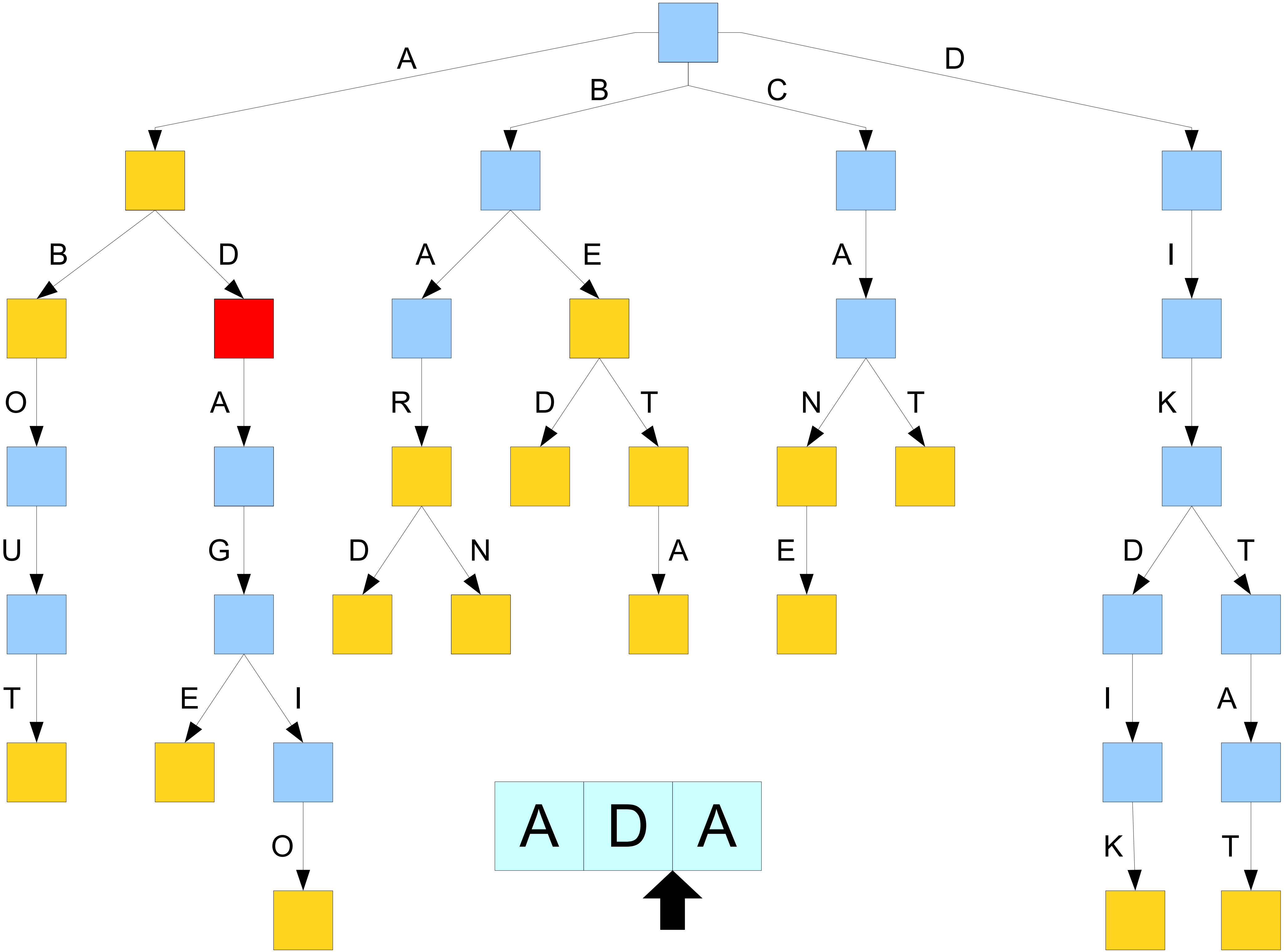

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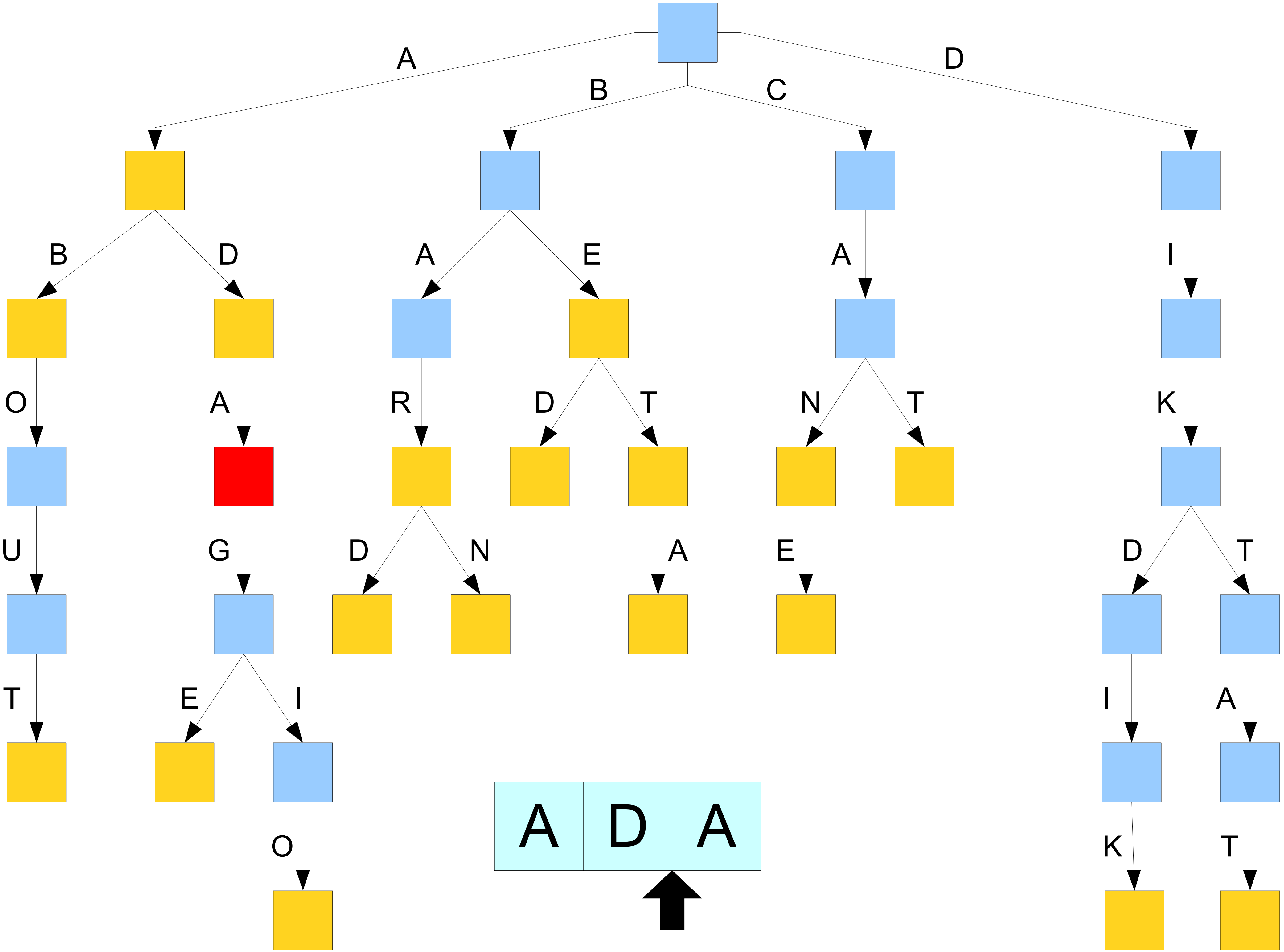
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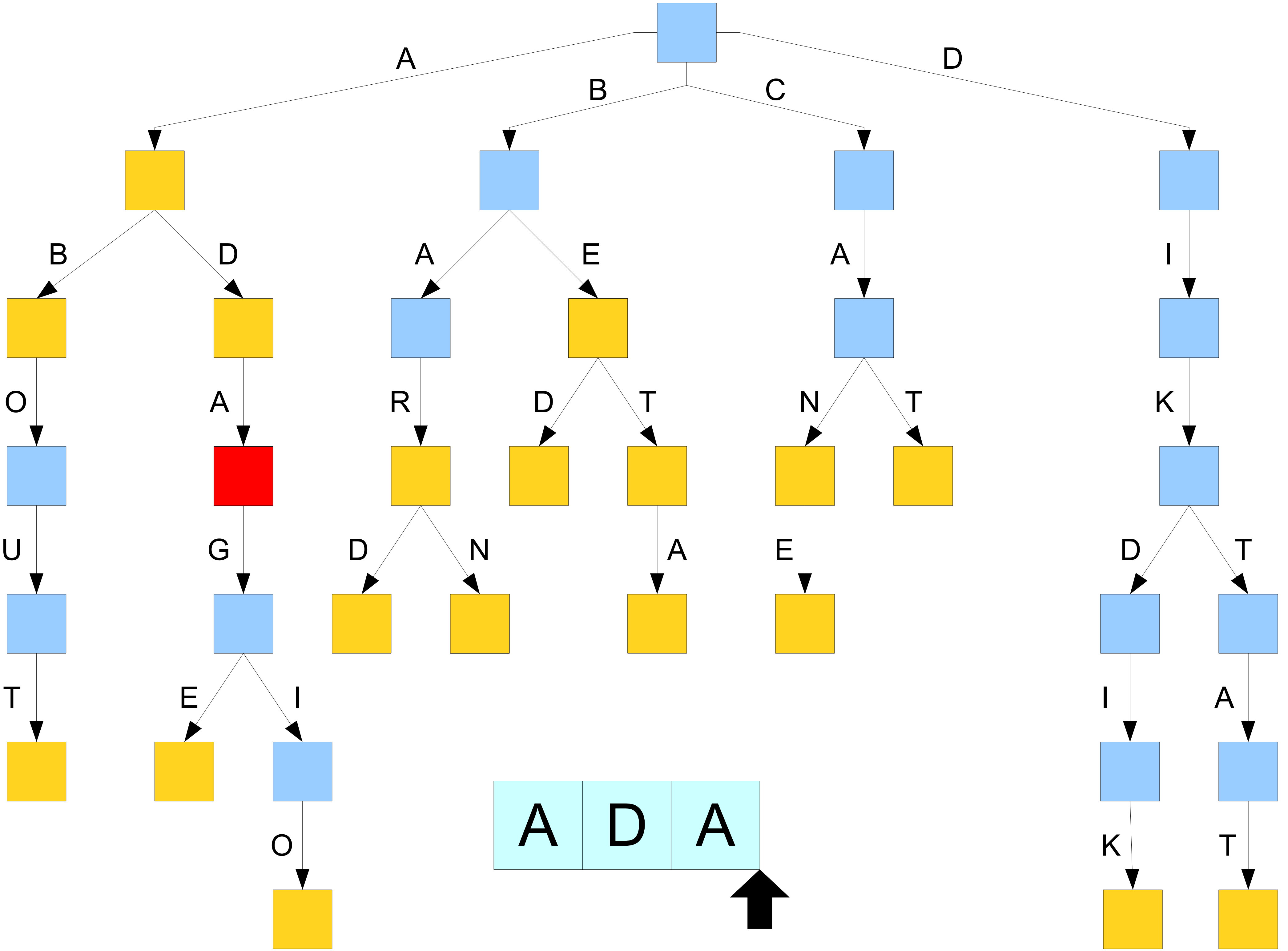

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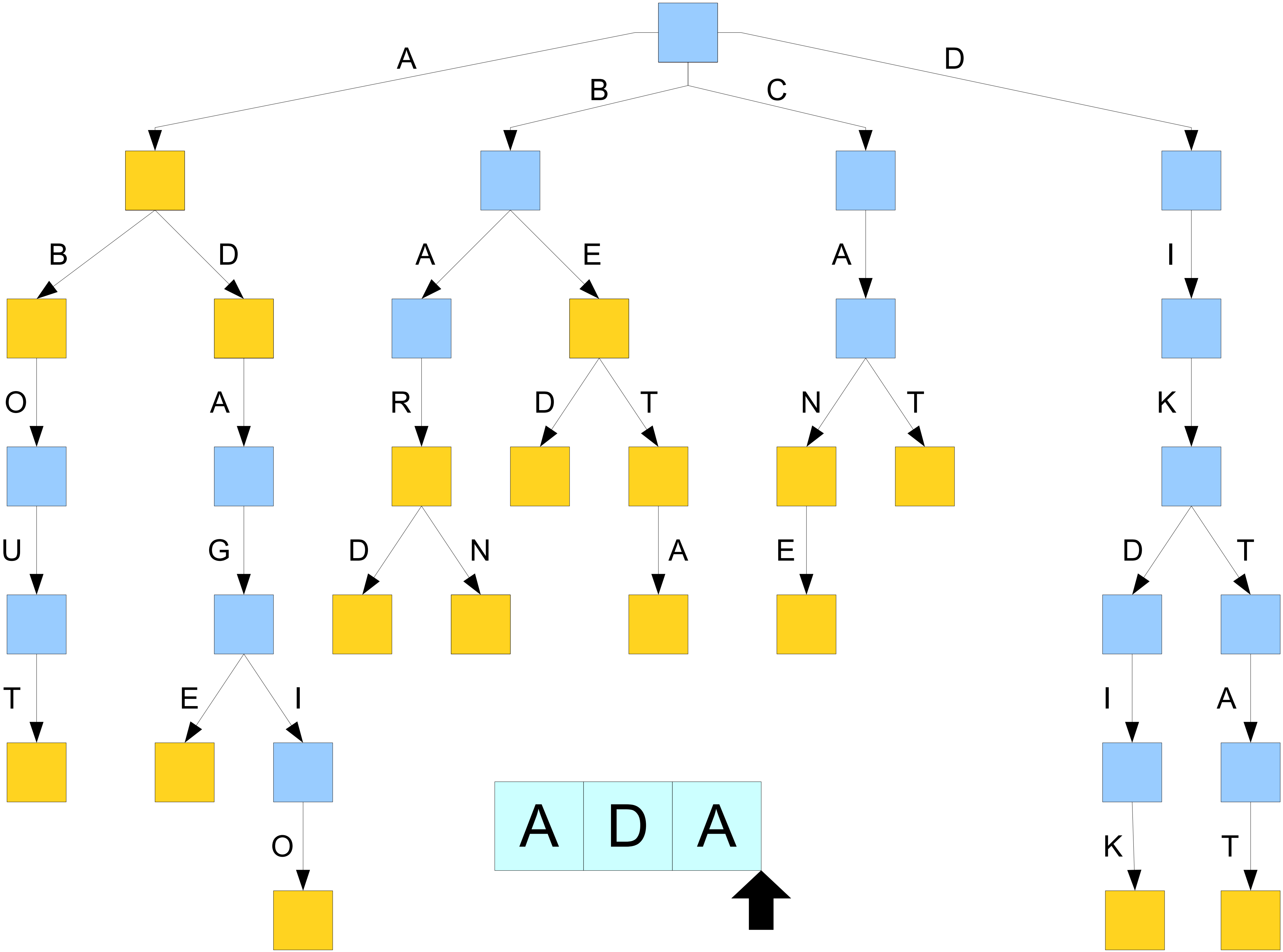
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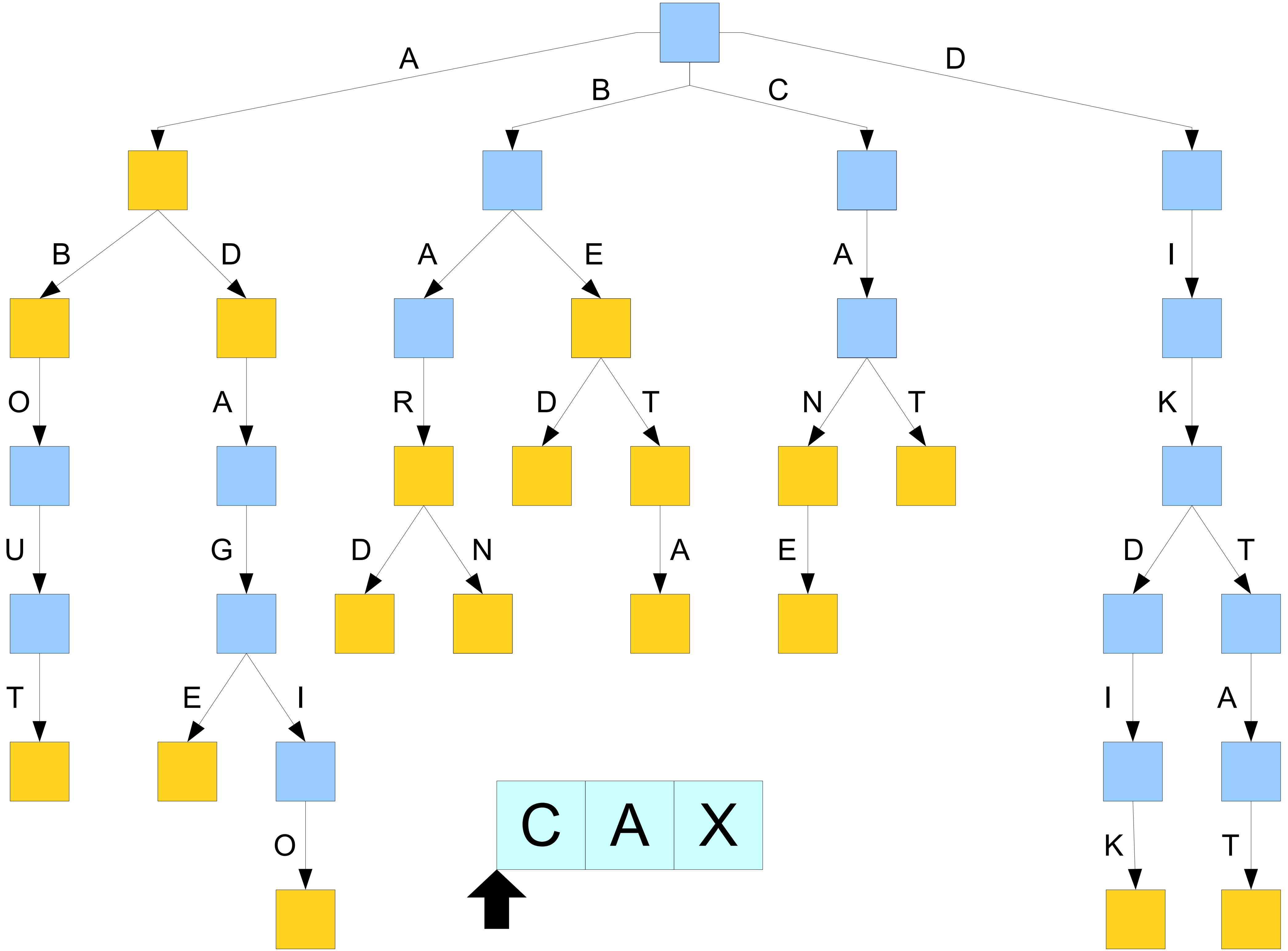

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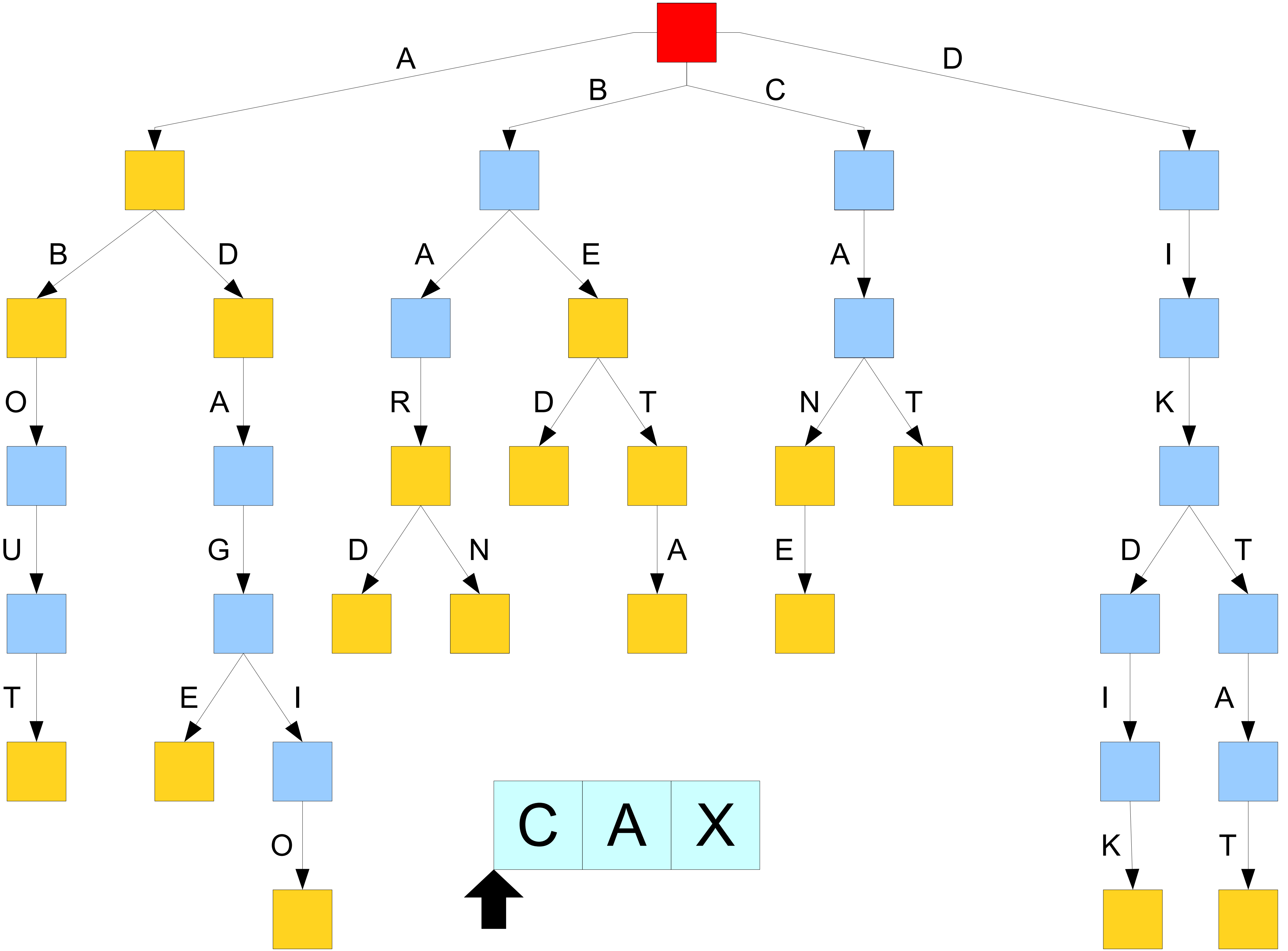
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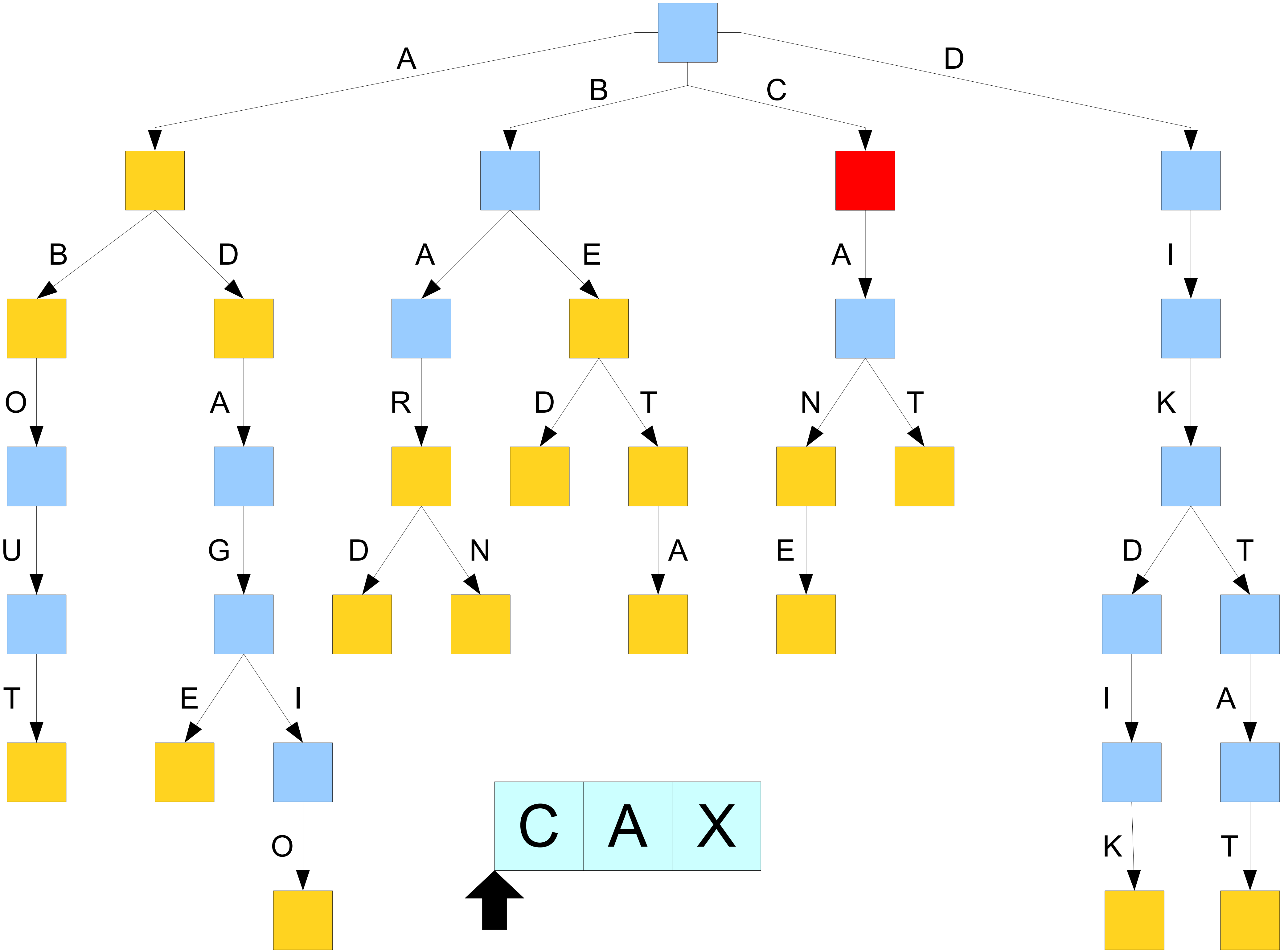
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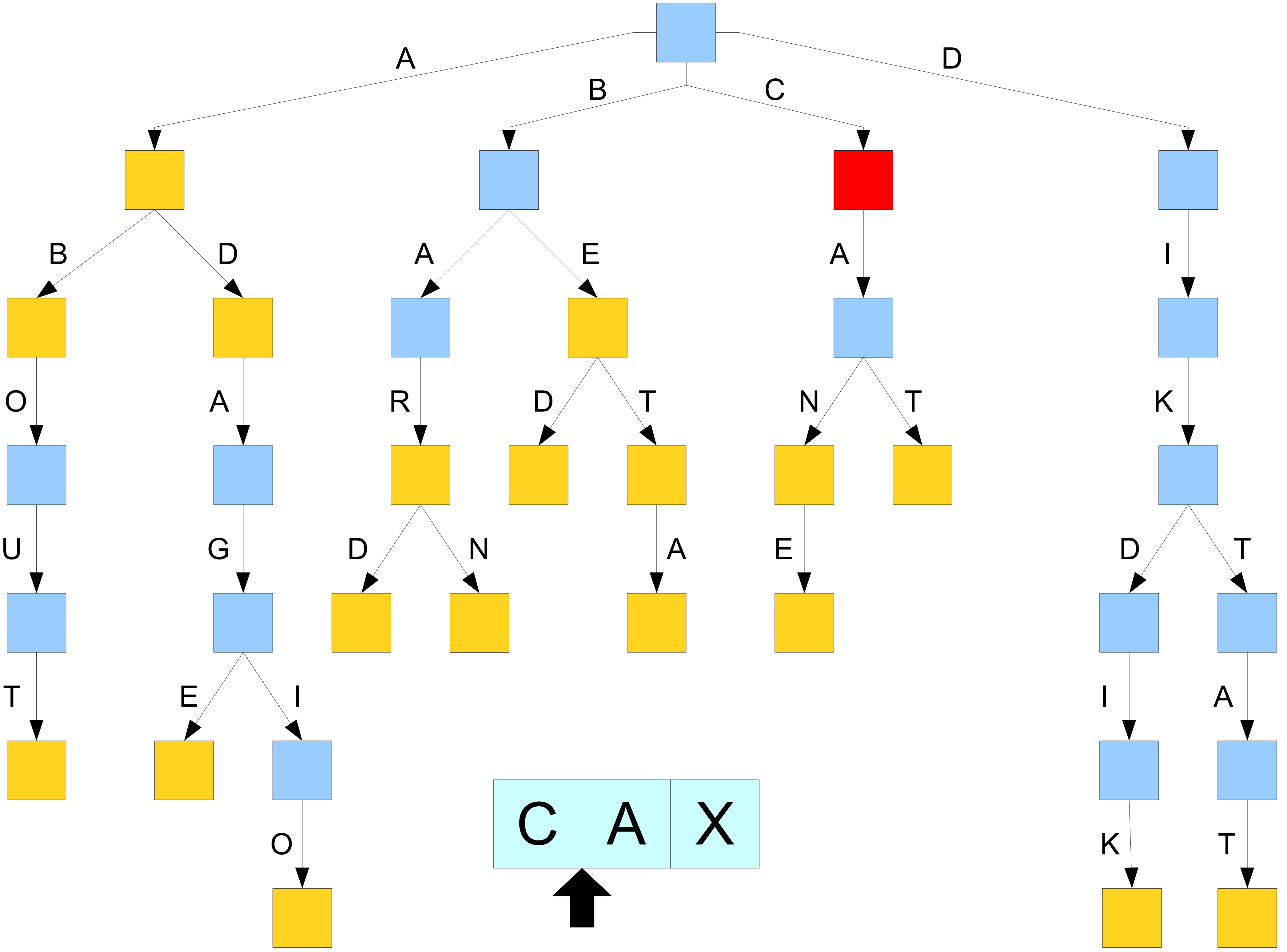
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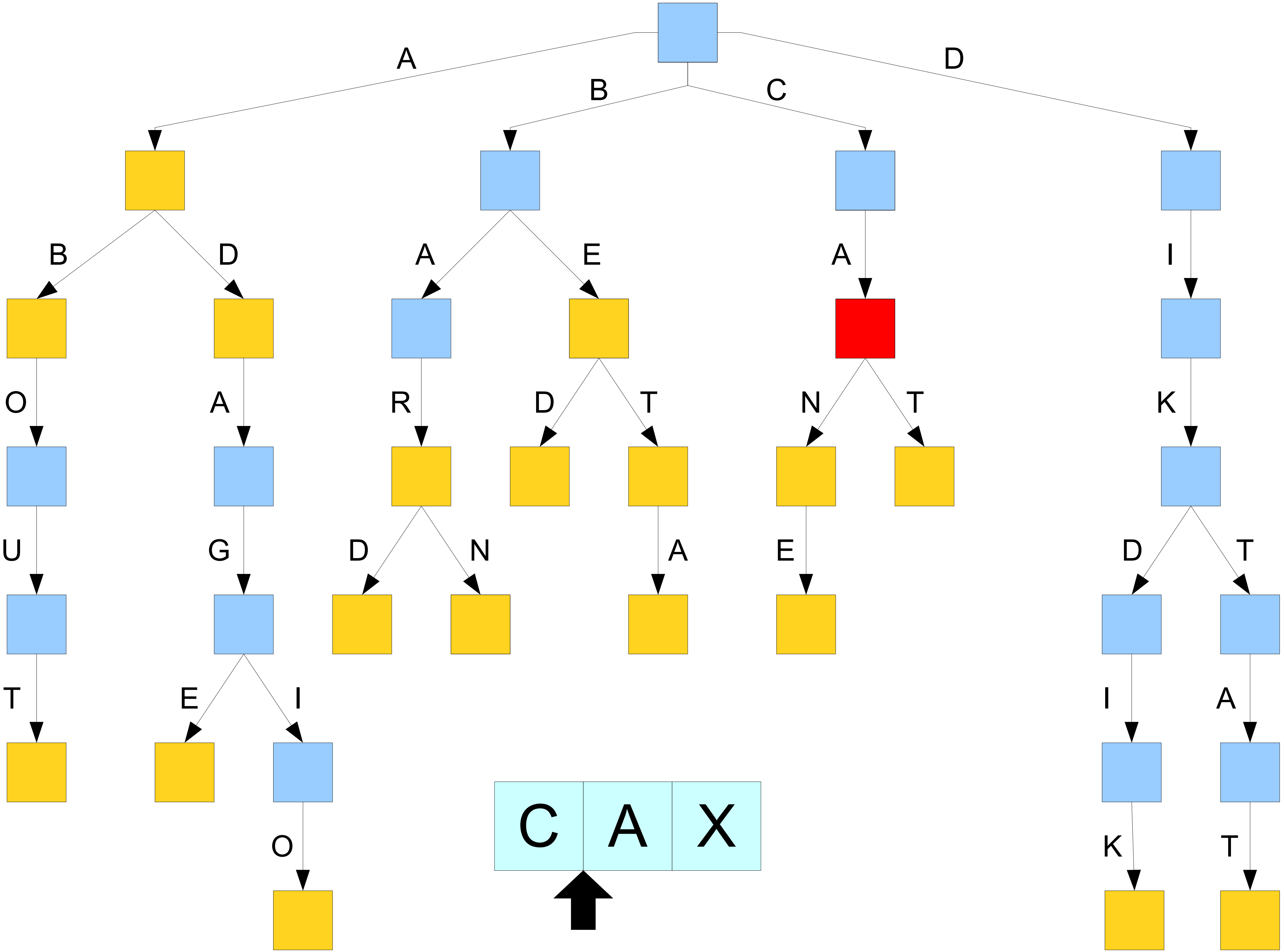
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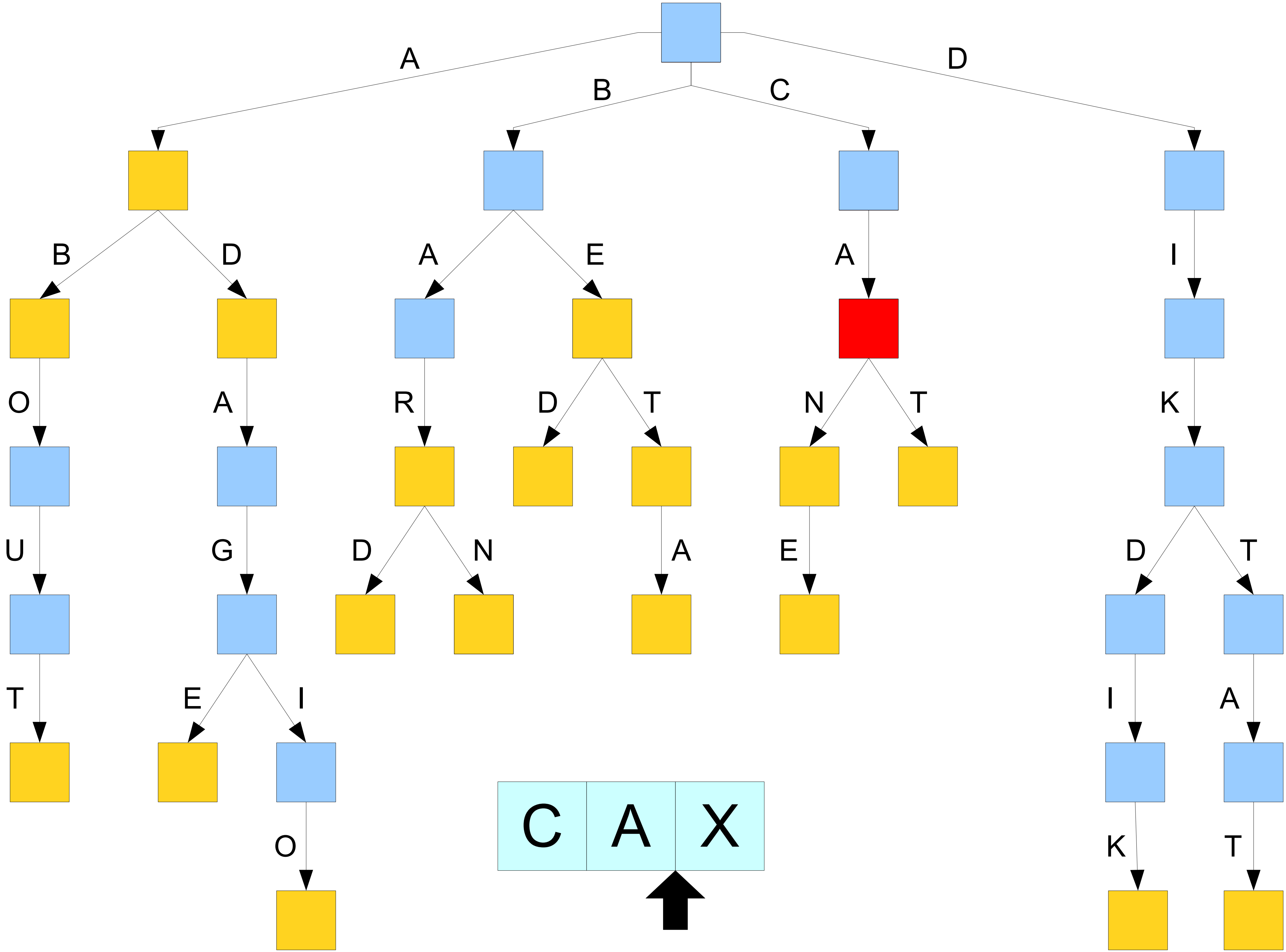
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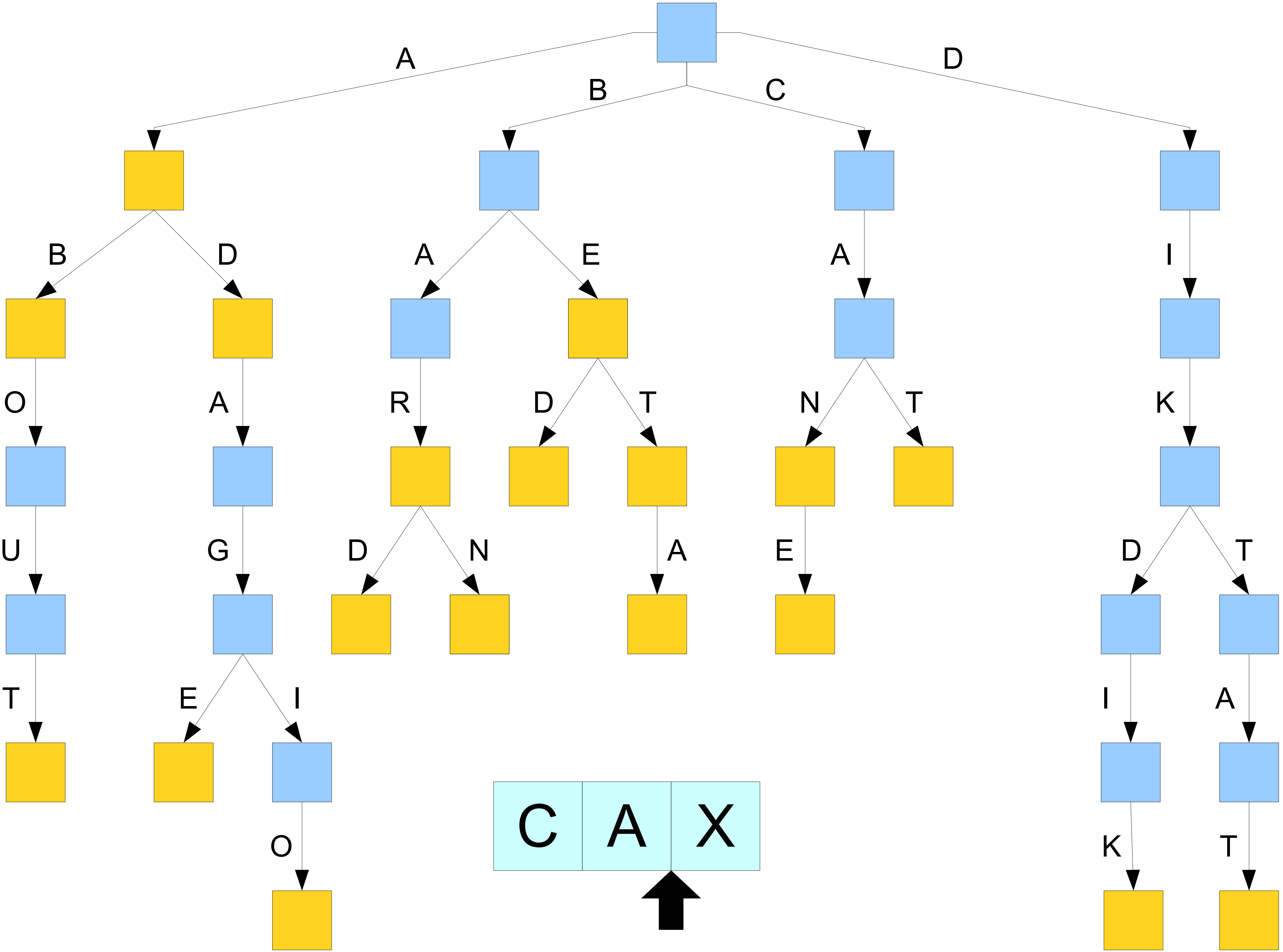
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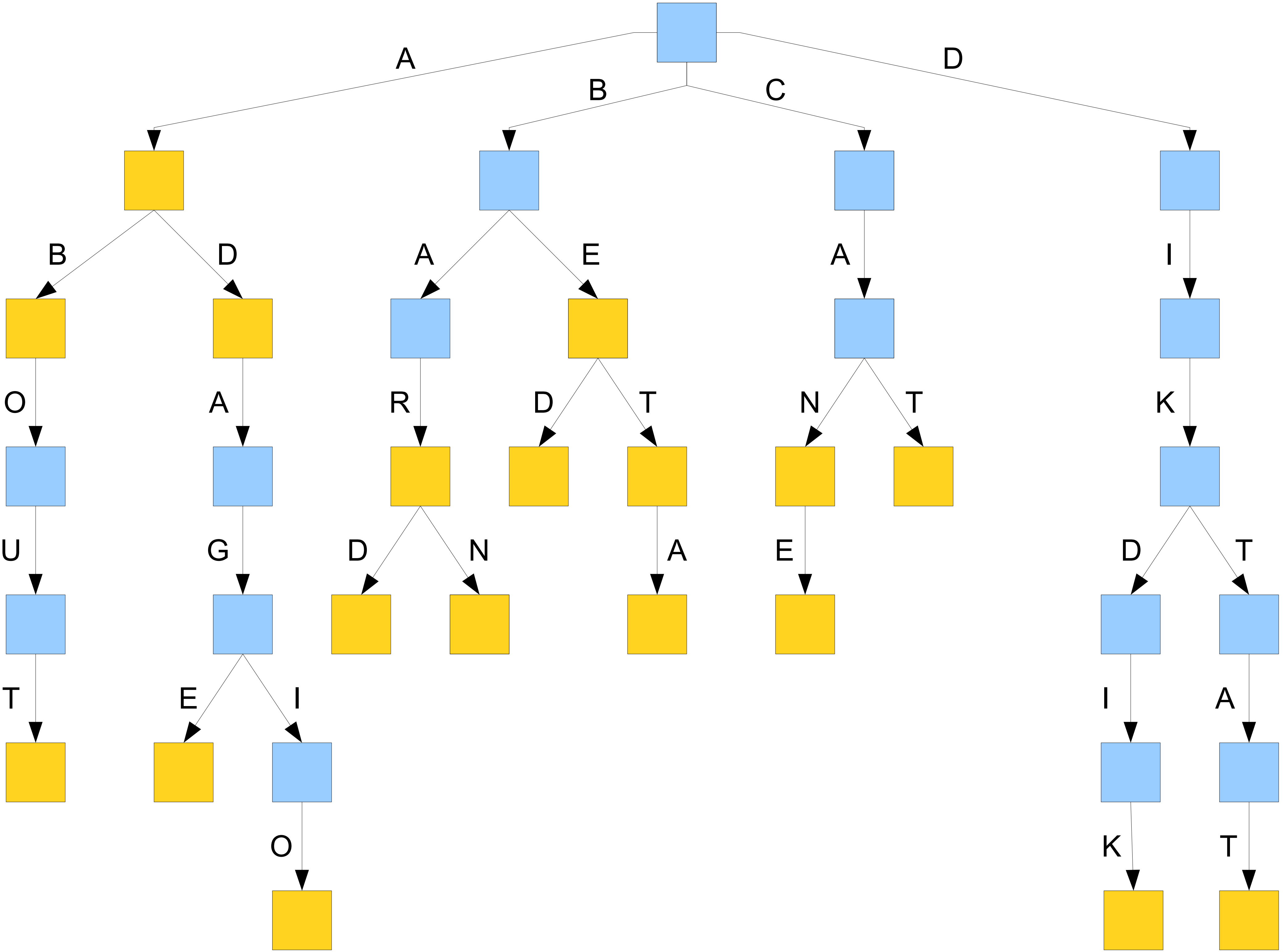
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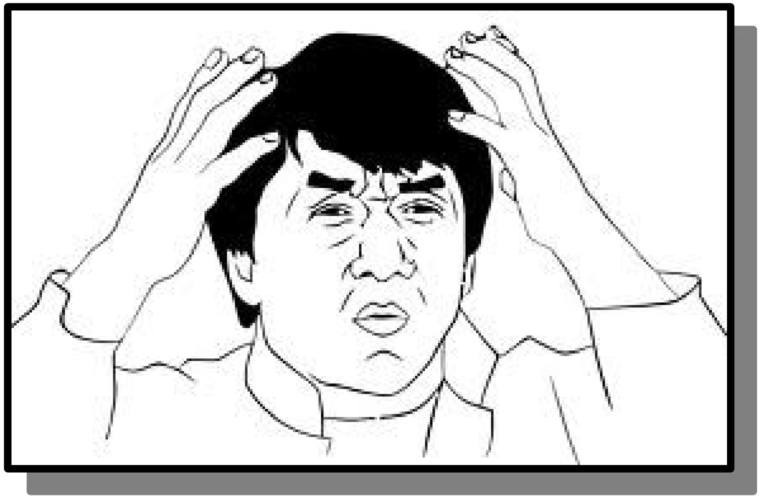
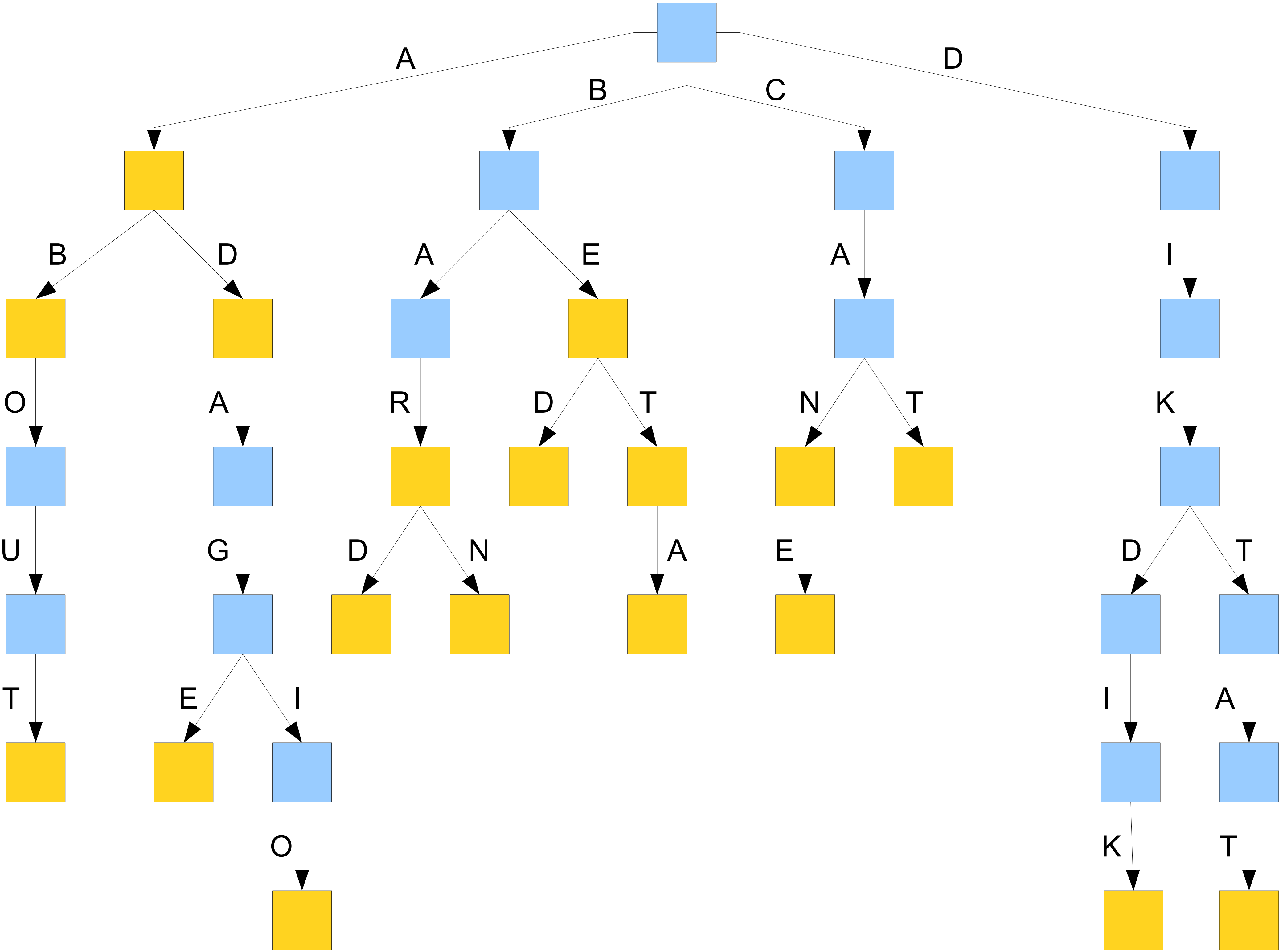
# Tries

- The data structure we have just seen is called a **trie**.
- Comes from the word re**trie**val.
- Pronounced “try,” not “tree.”



# Trie Nodes

- The pieces of the trie are called **nodes**.
- Each node stores two pieces of information:
    - Whether, at this point in the trie, you have arrived at a word, and
    - Pointers to child nodes in the trie.
- The node at the top of the trie is called the **root node**.



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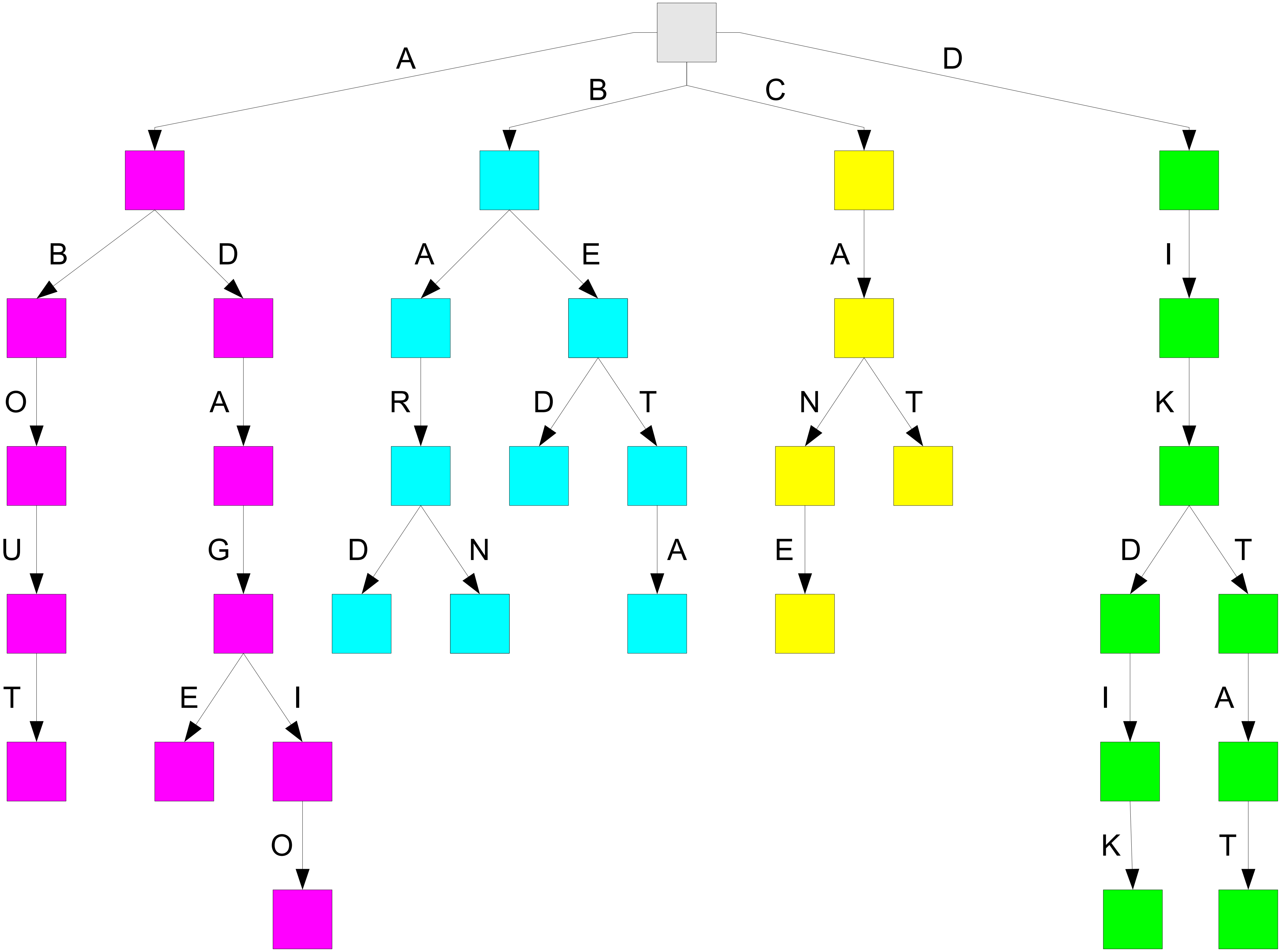
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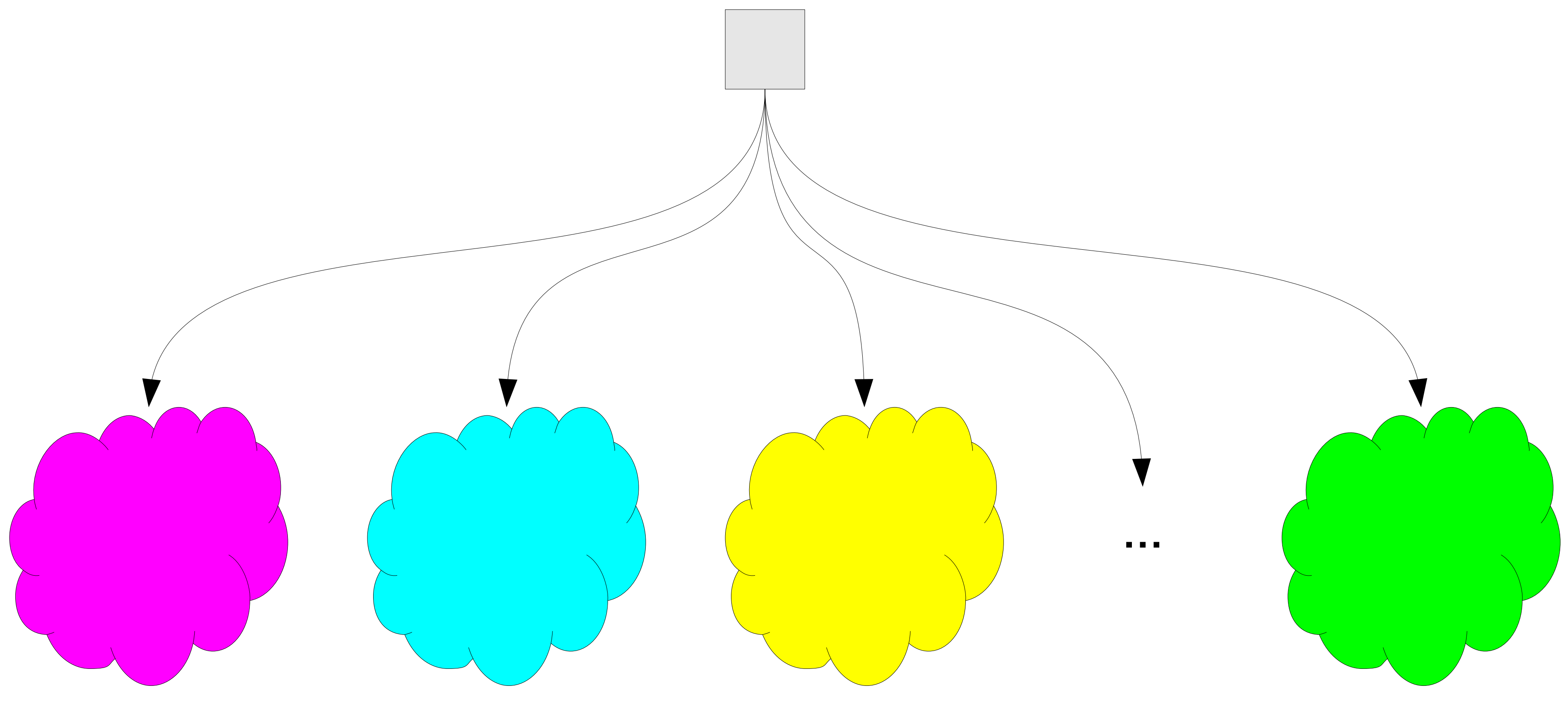
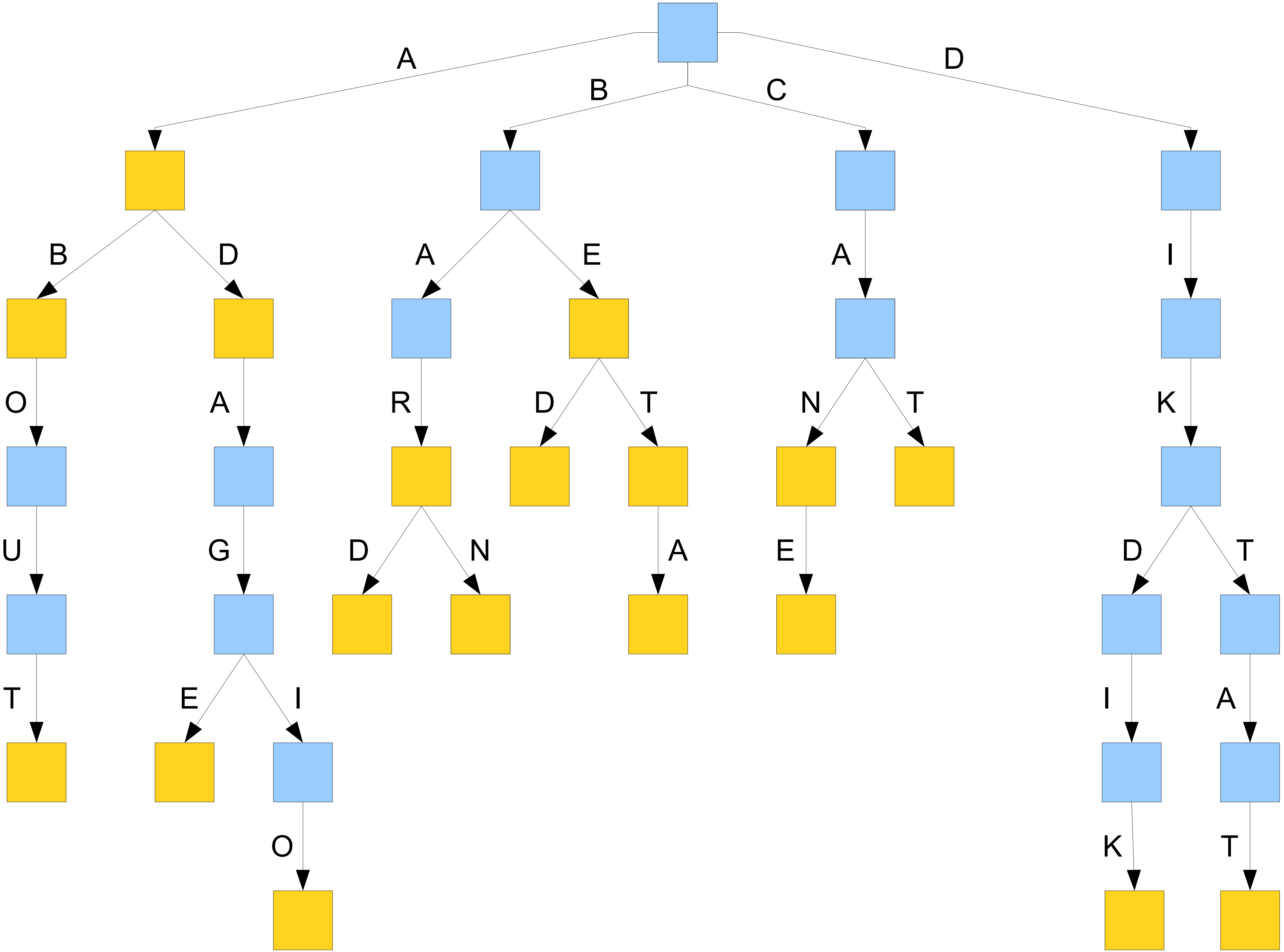


Let's trie coding up **`Lexicon`**!
(Constructor,
Destructor
Variables,
**`size`**
**`contains`**,
**`containsPrefix`**)
(OurLexicon.cpp/h)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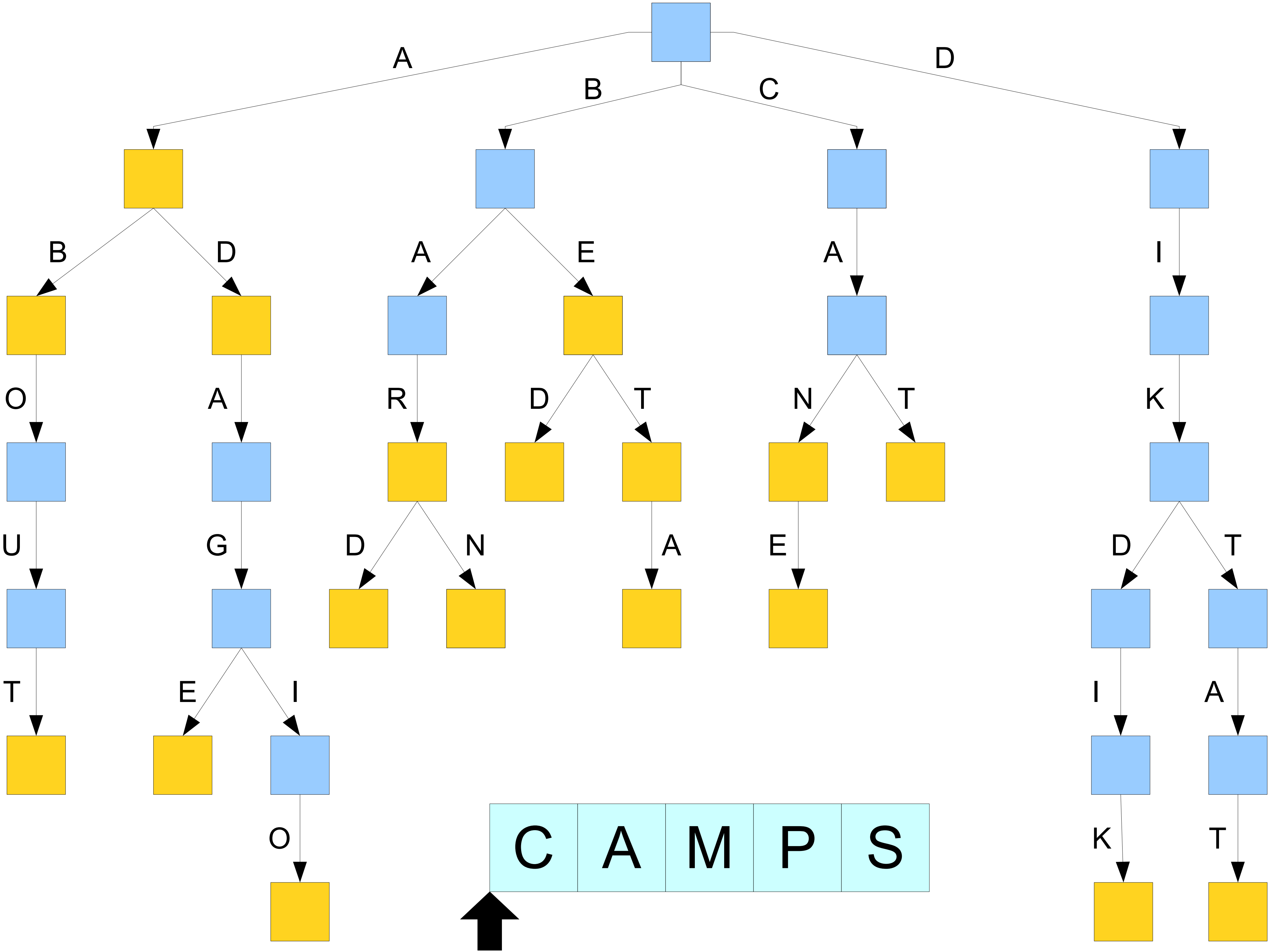
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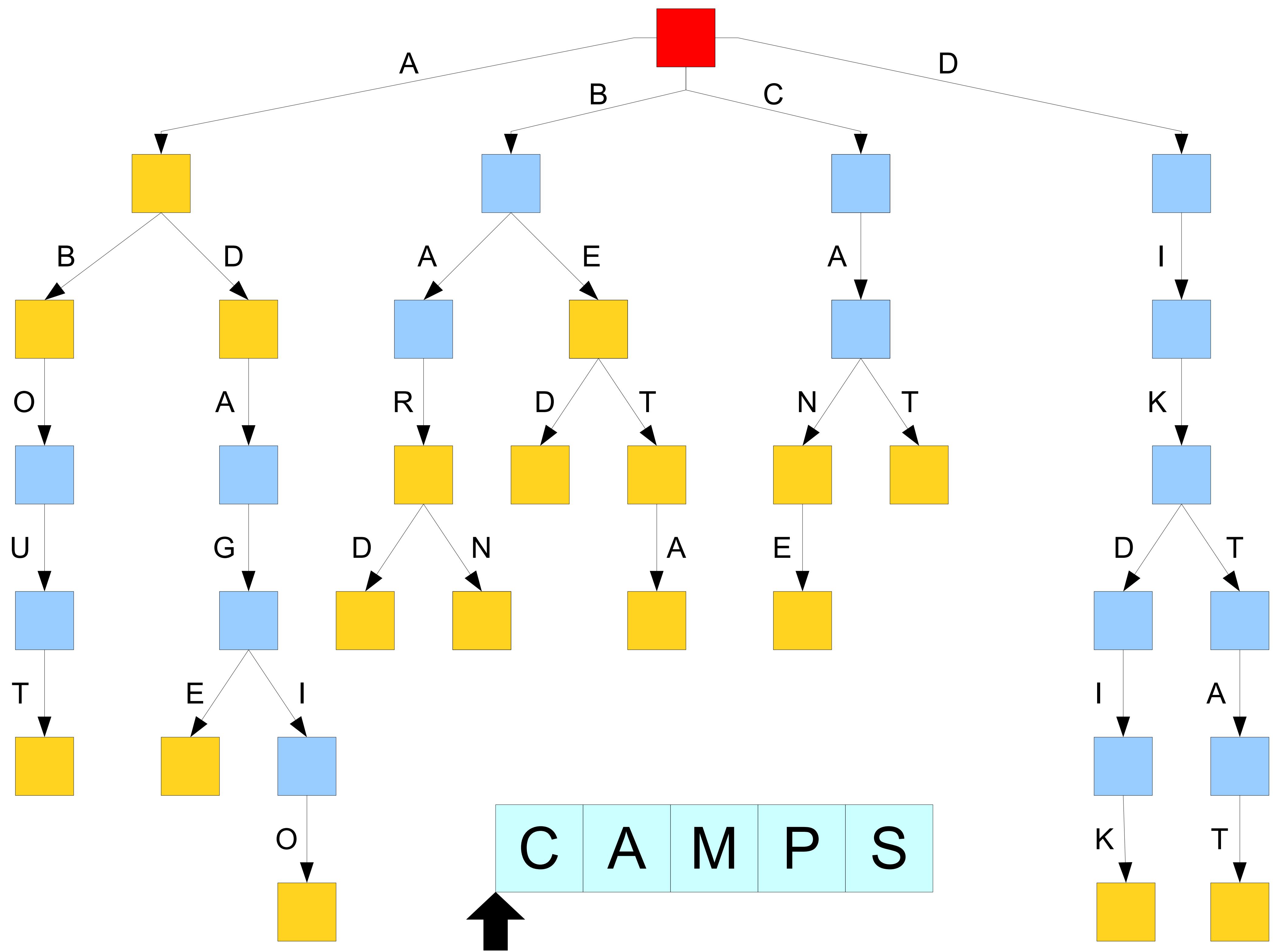
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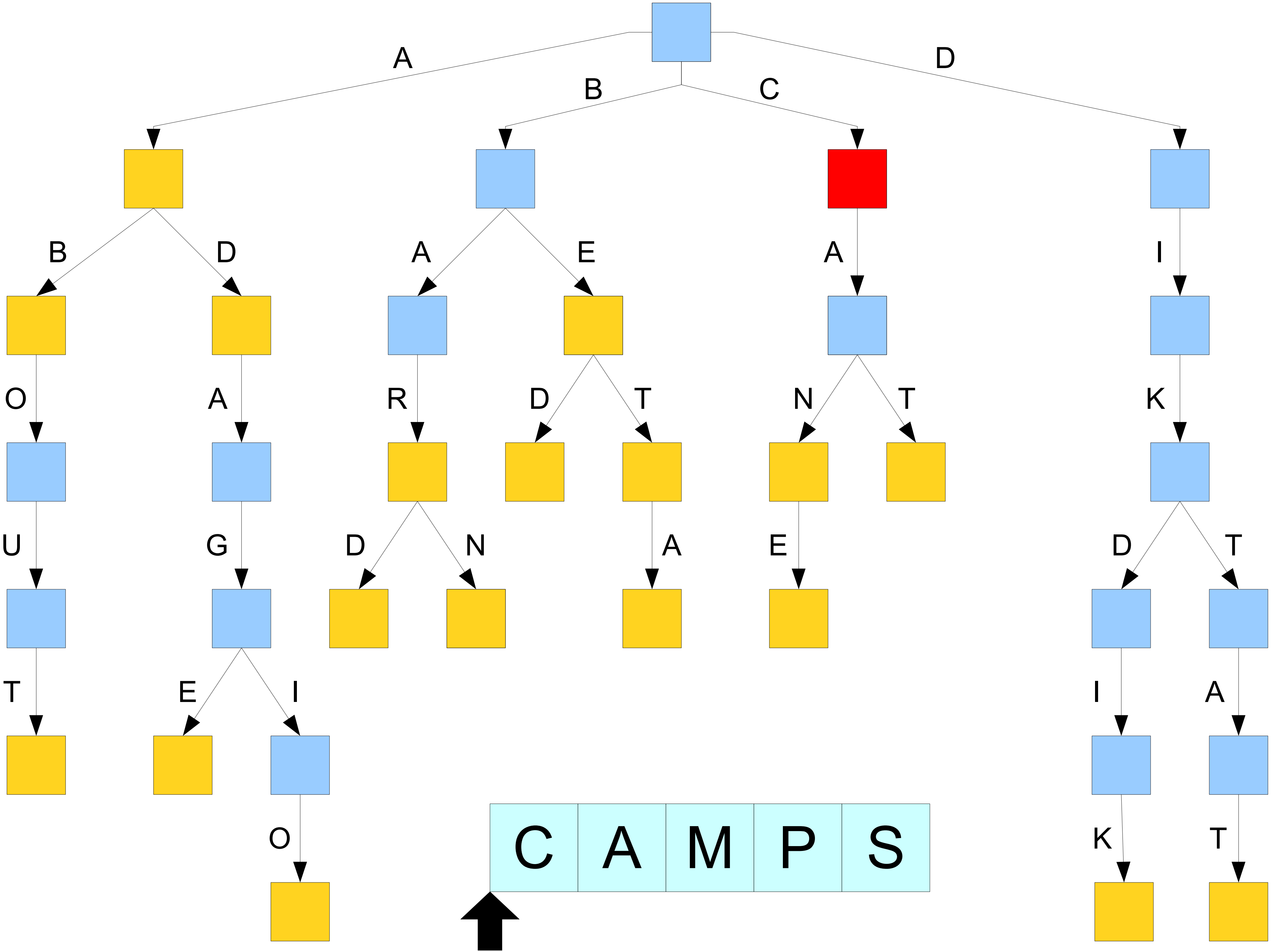
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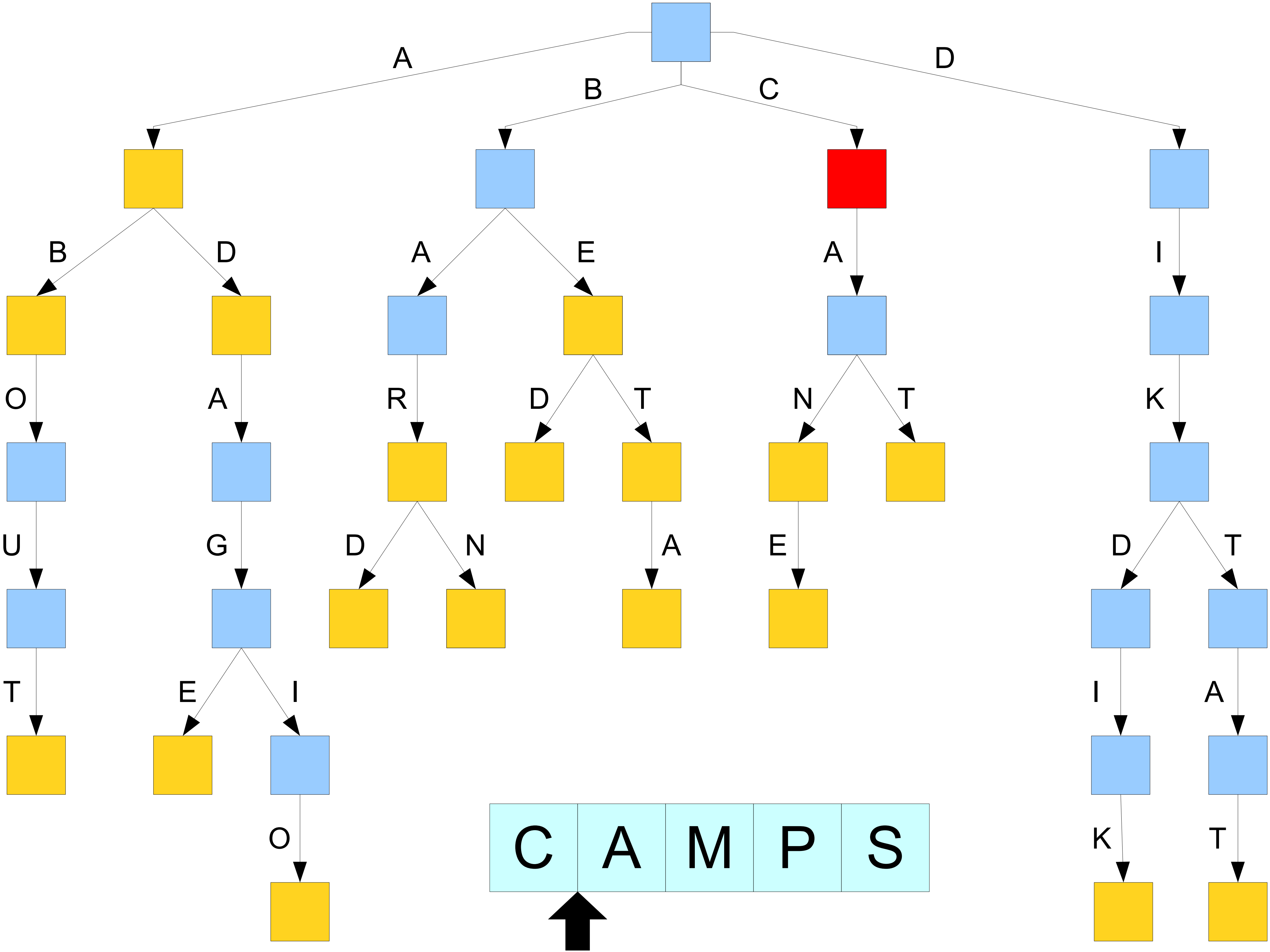
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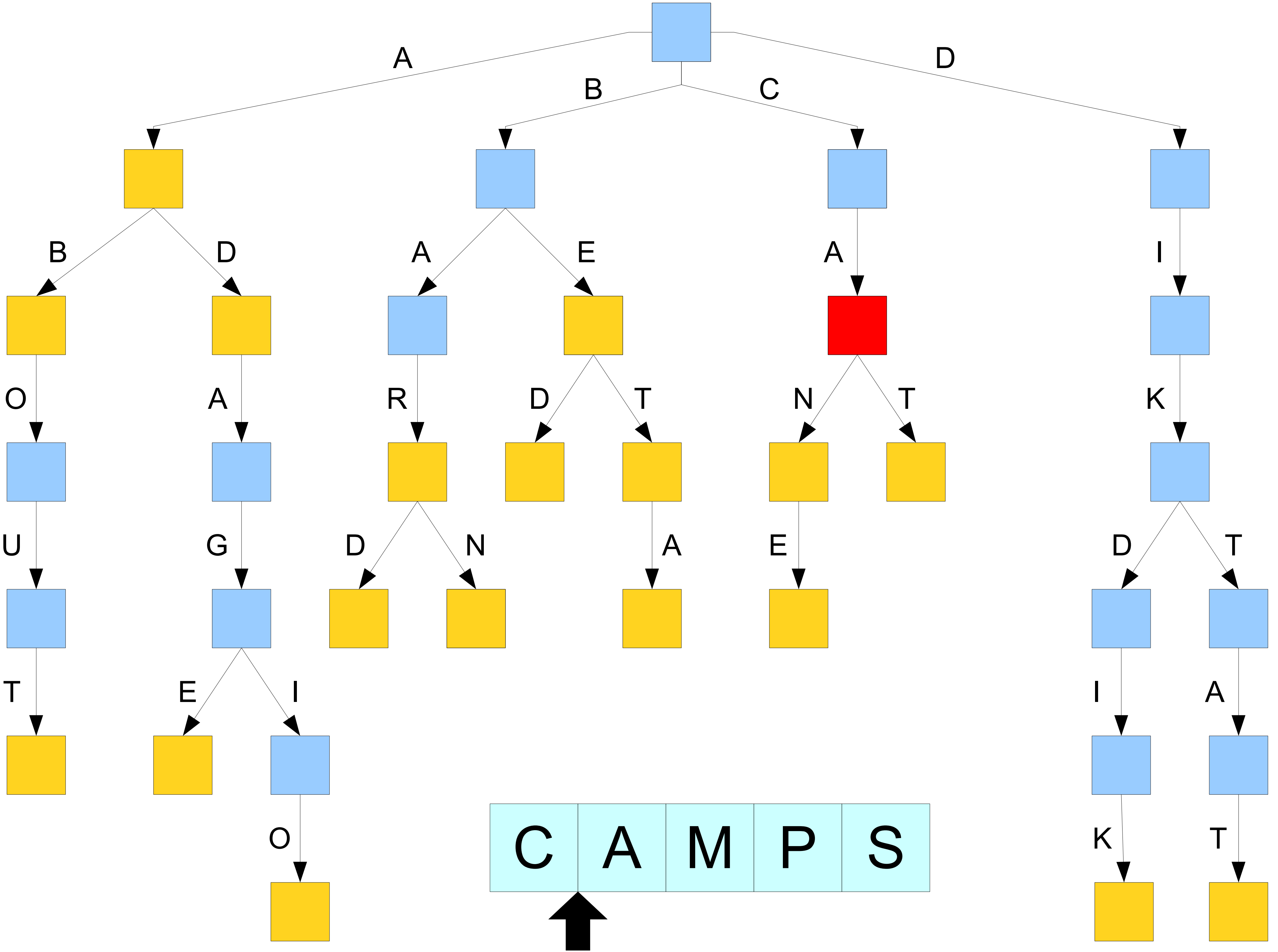
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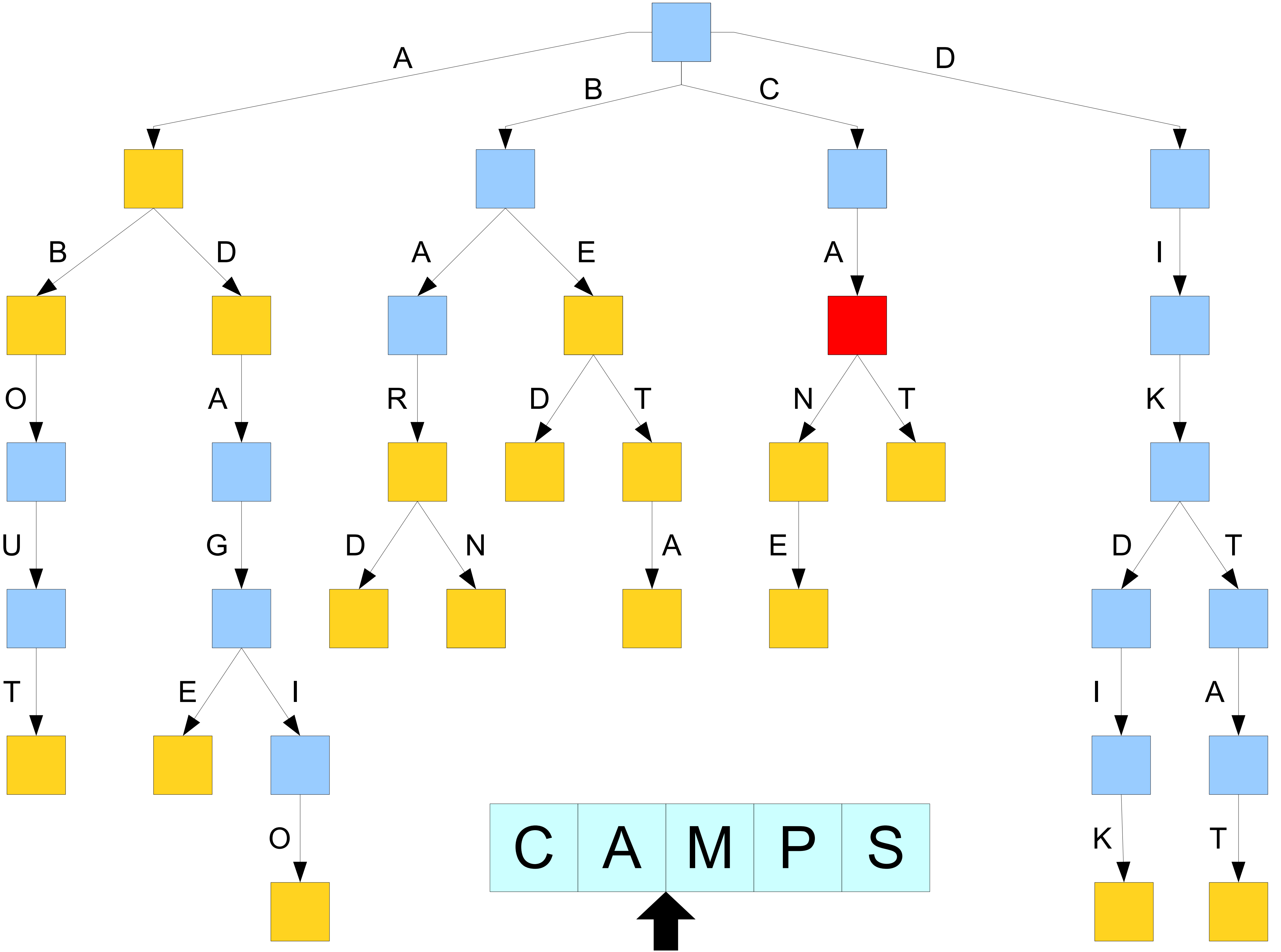
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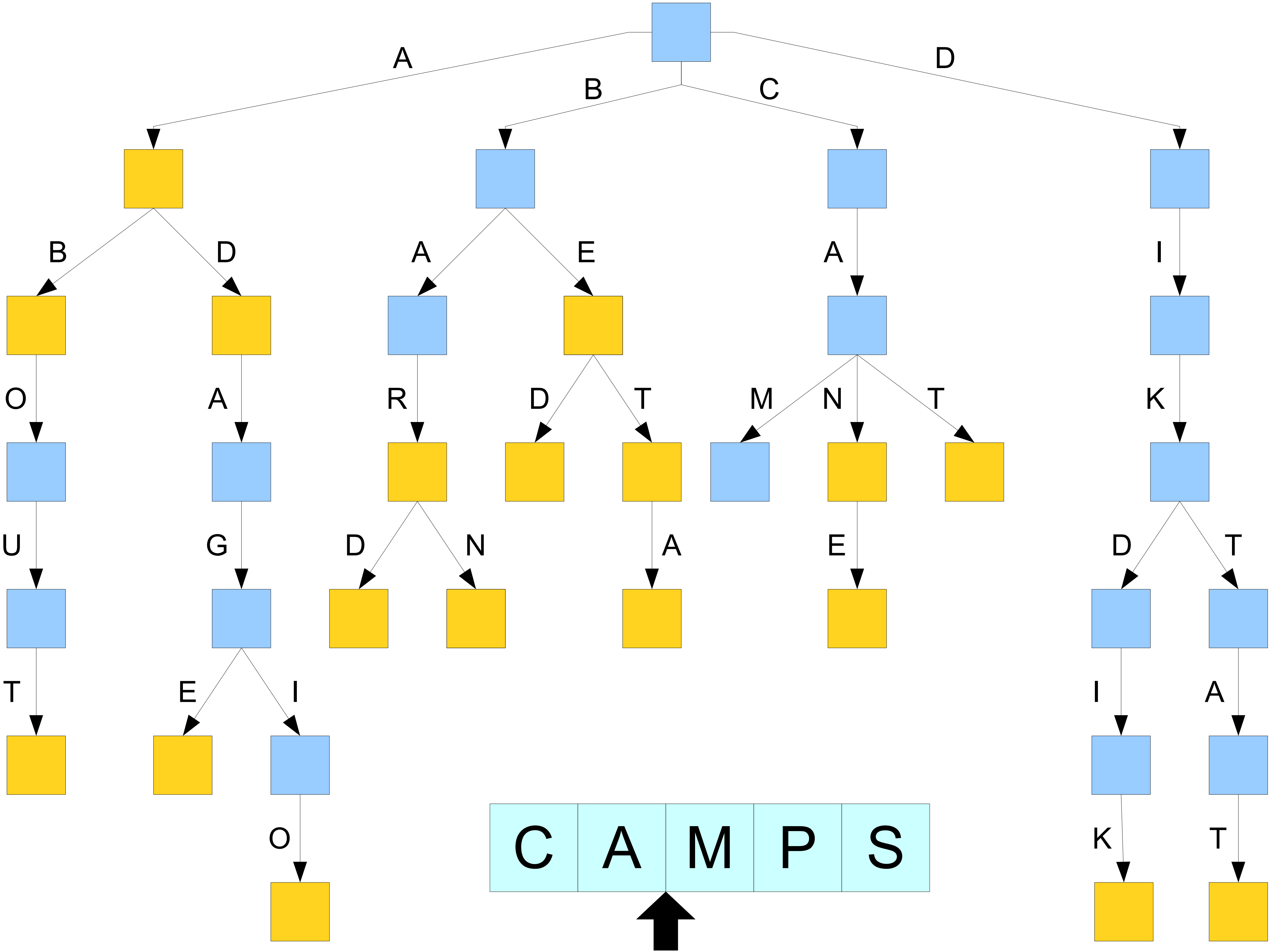
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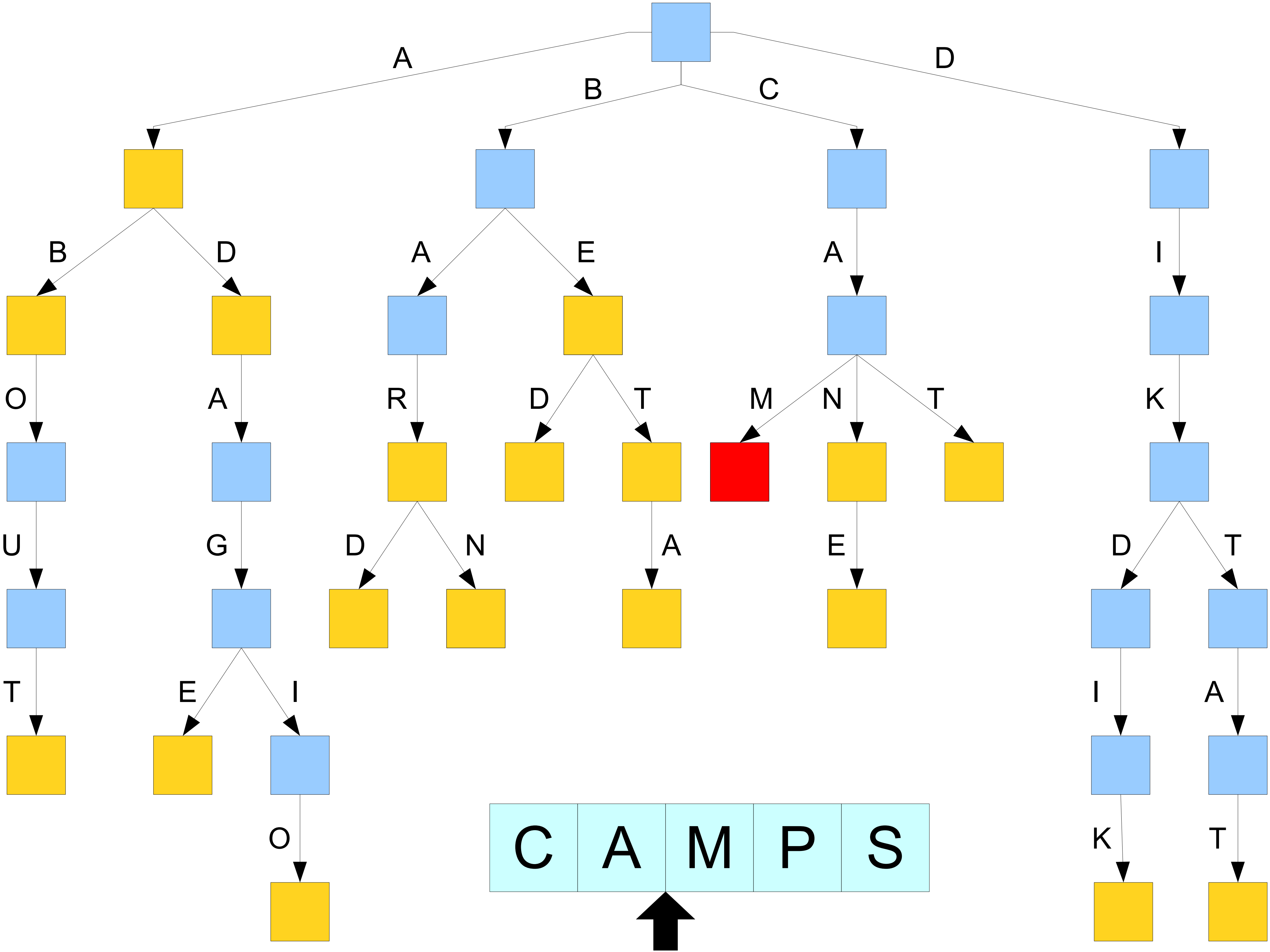
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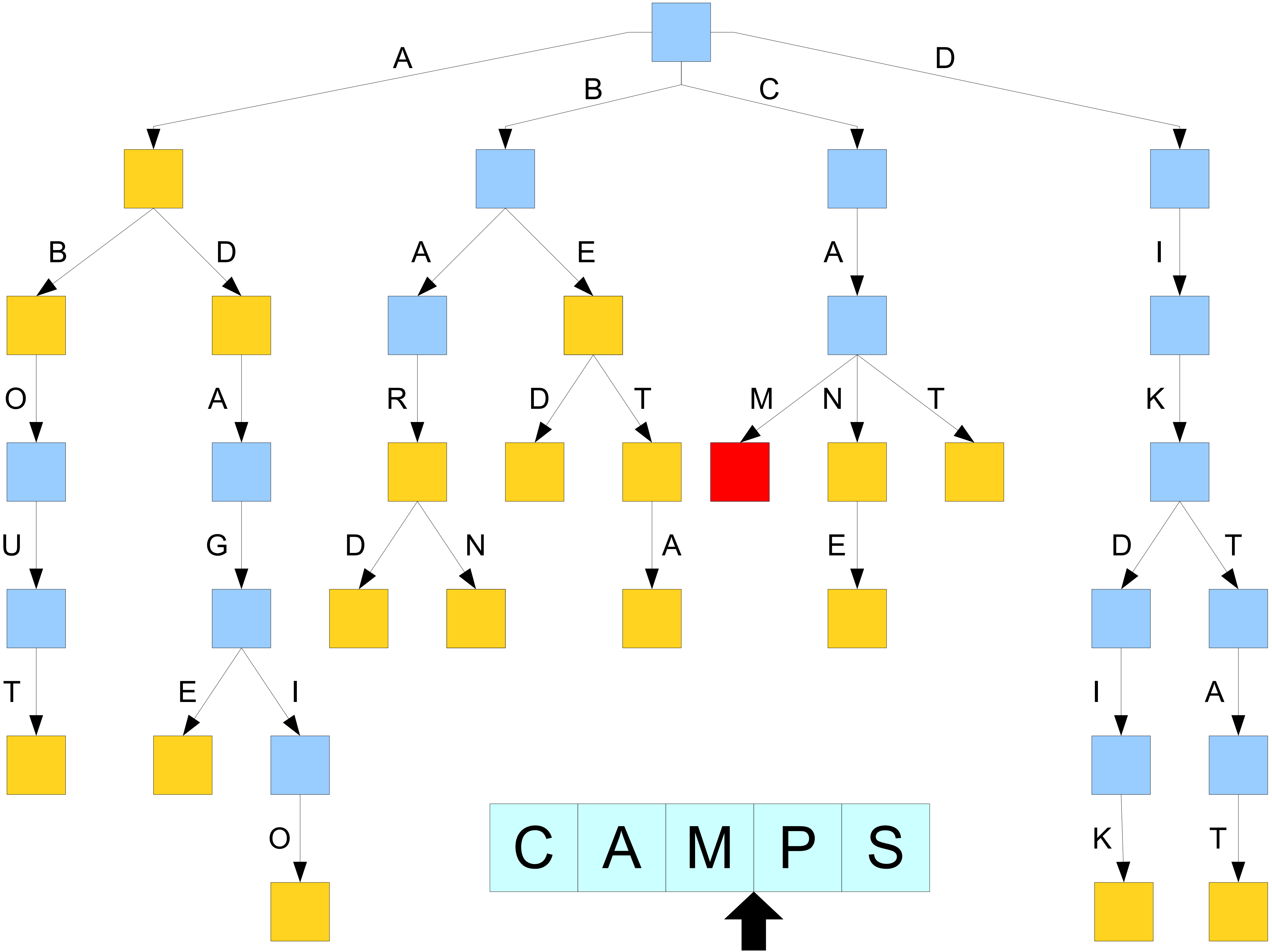
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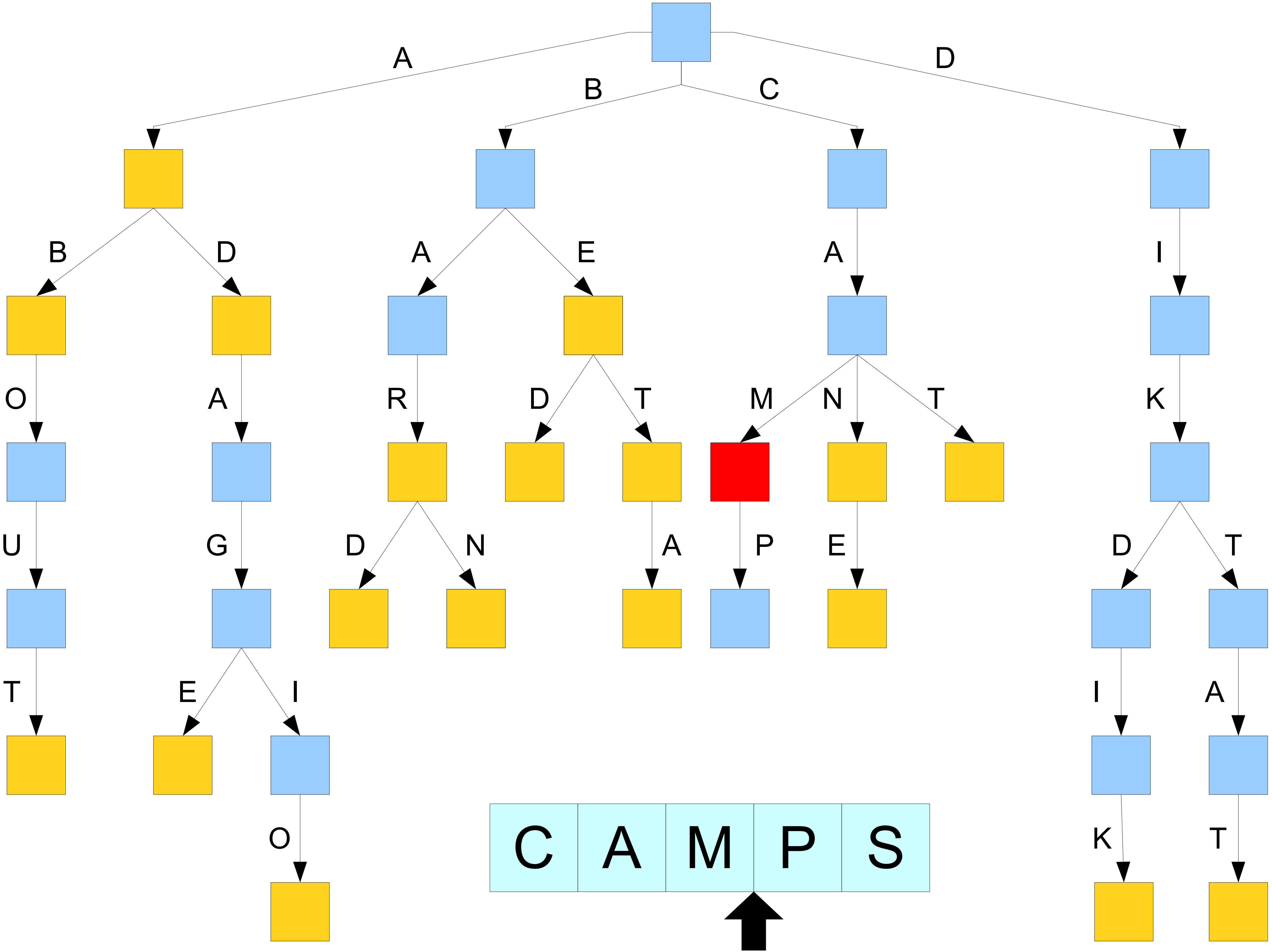
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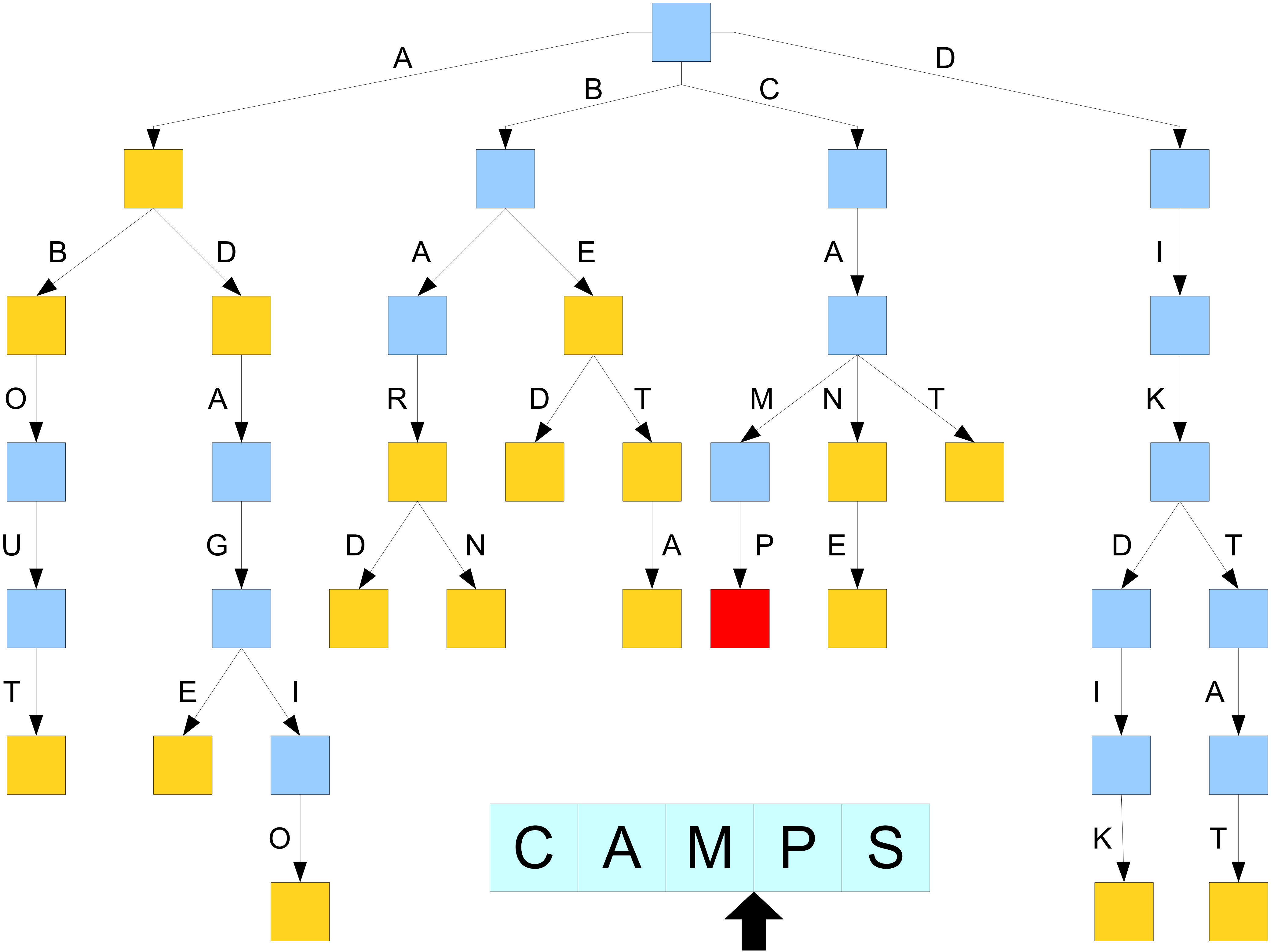

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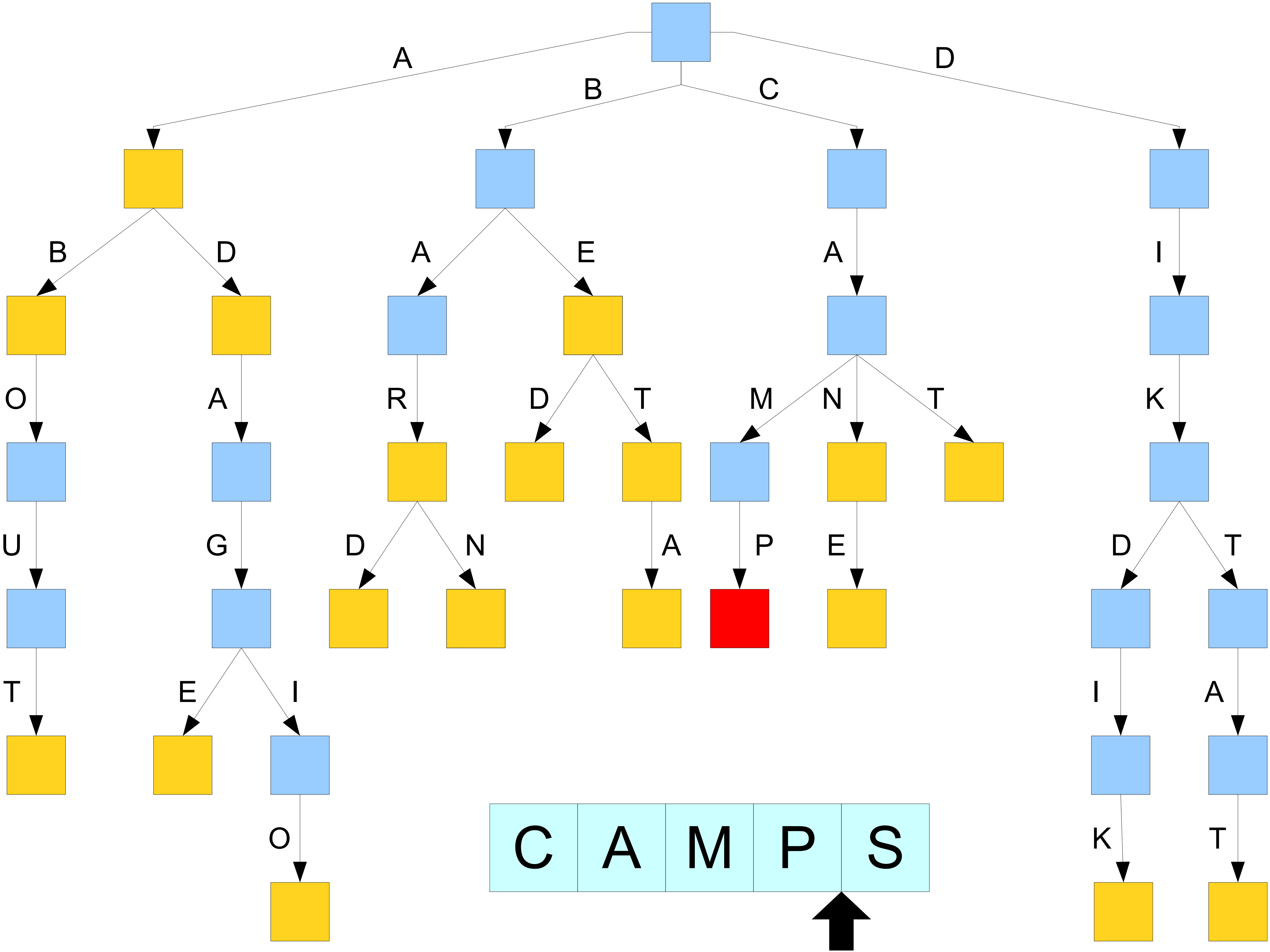

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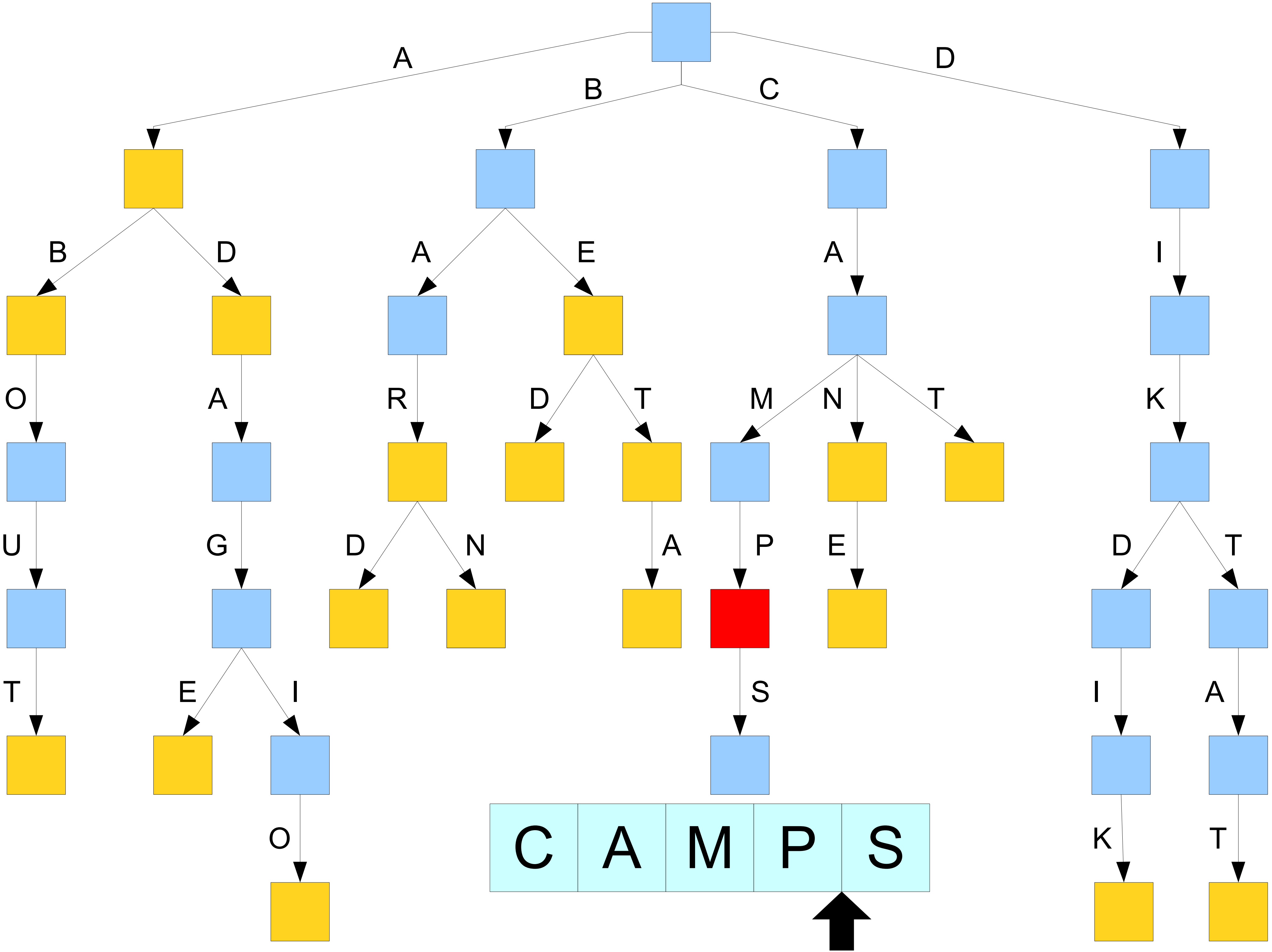
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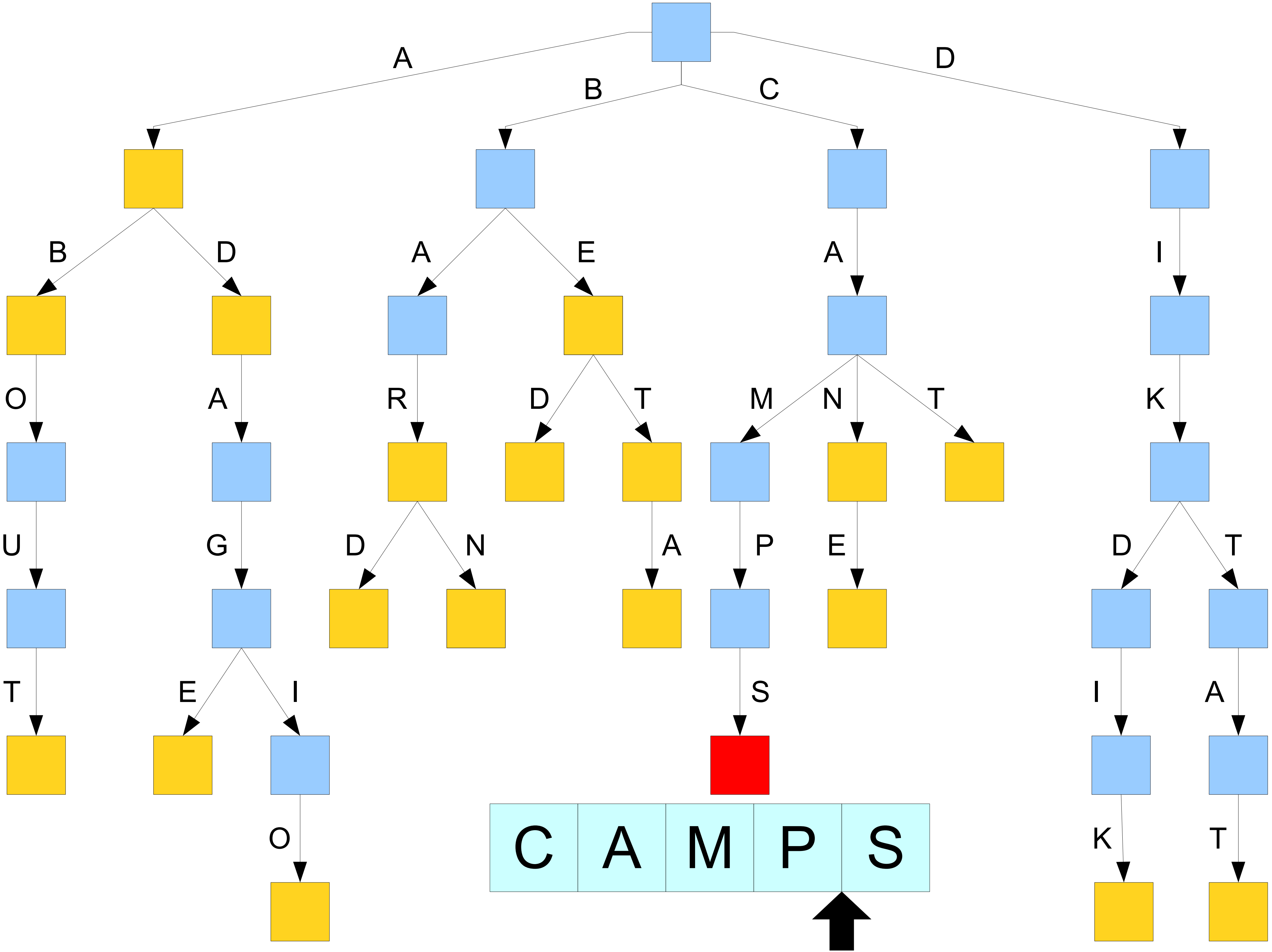
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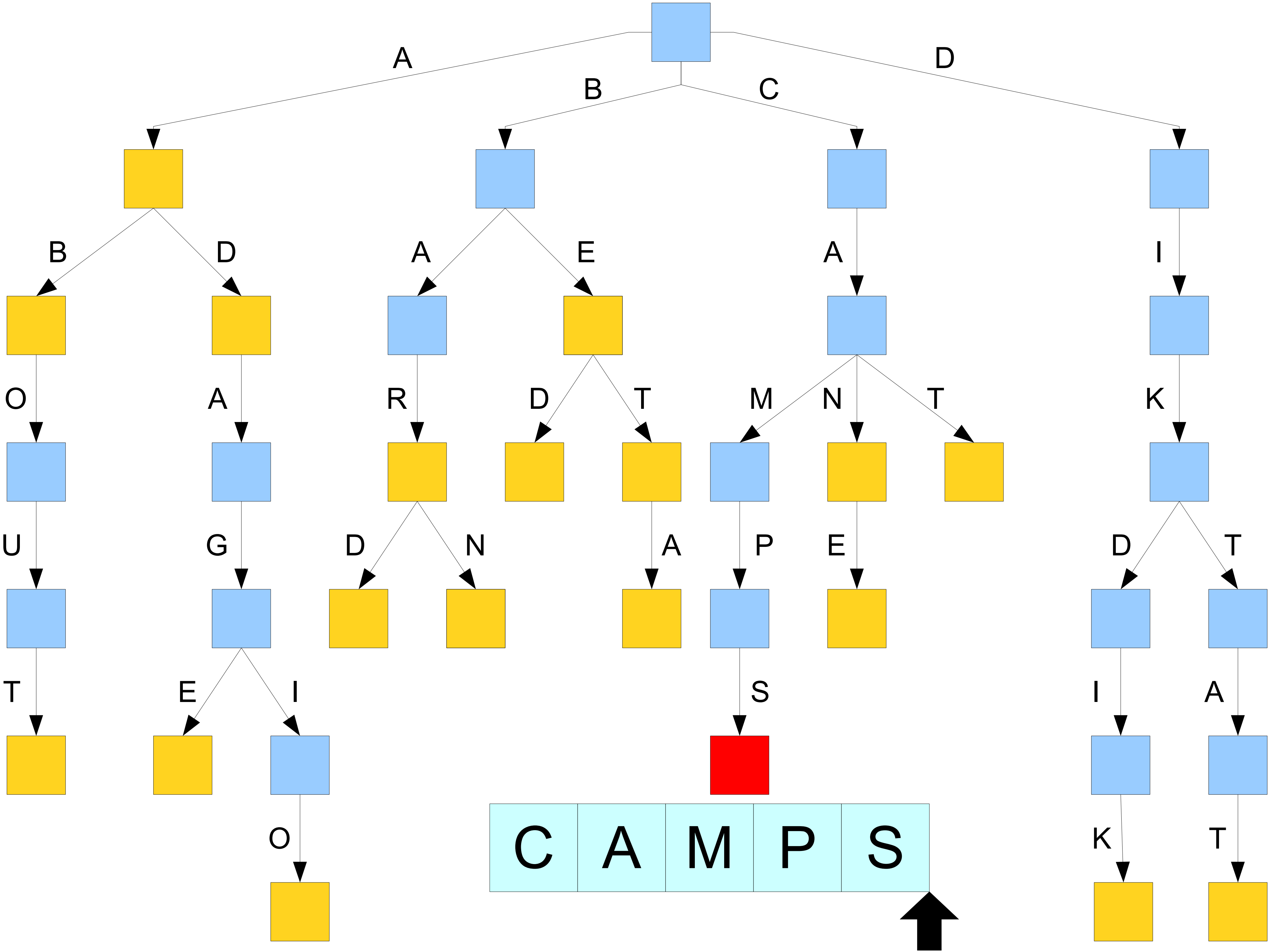
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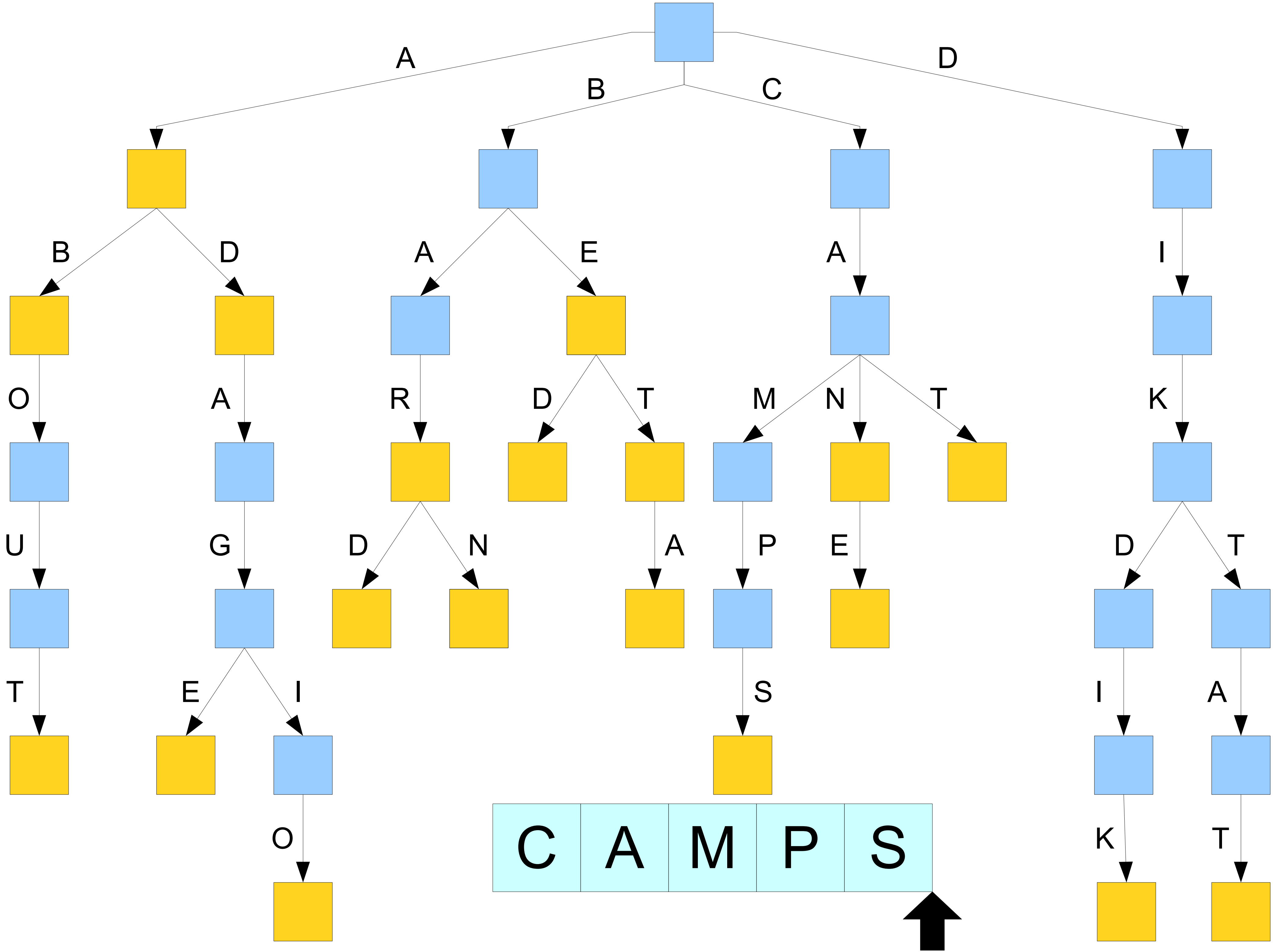
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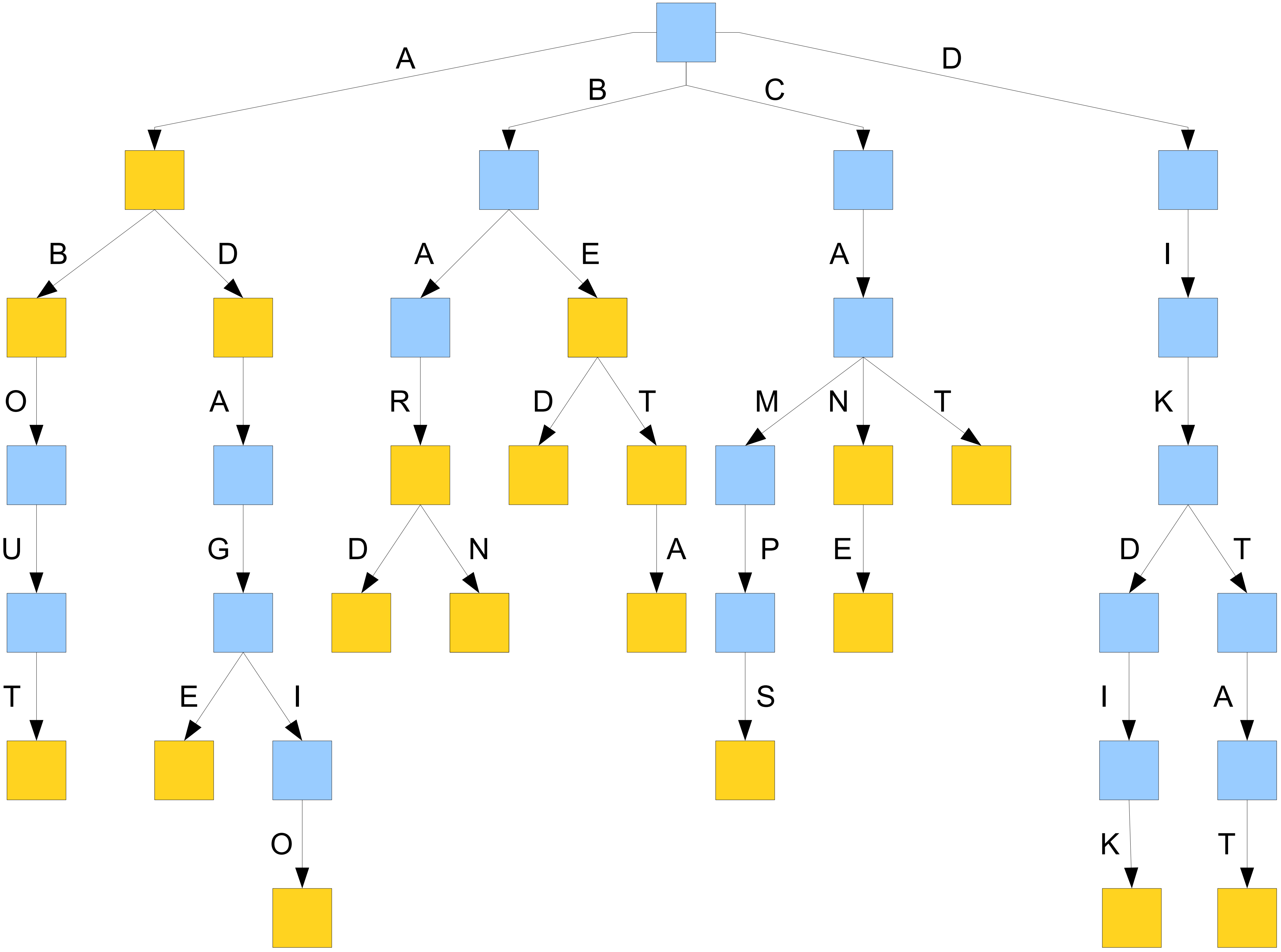
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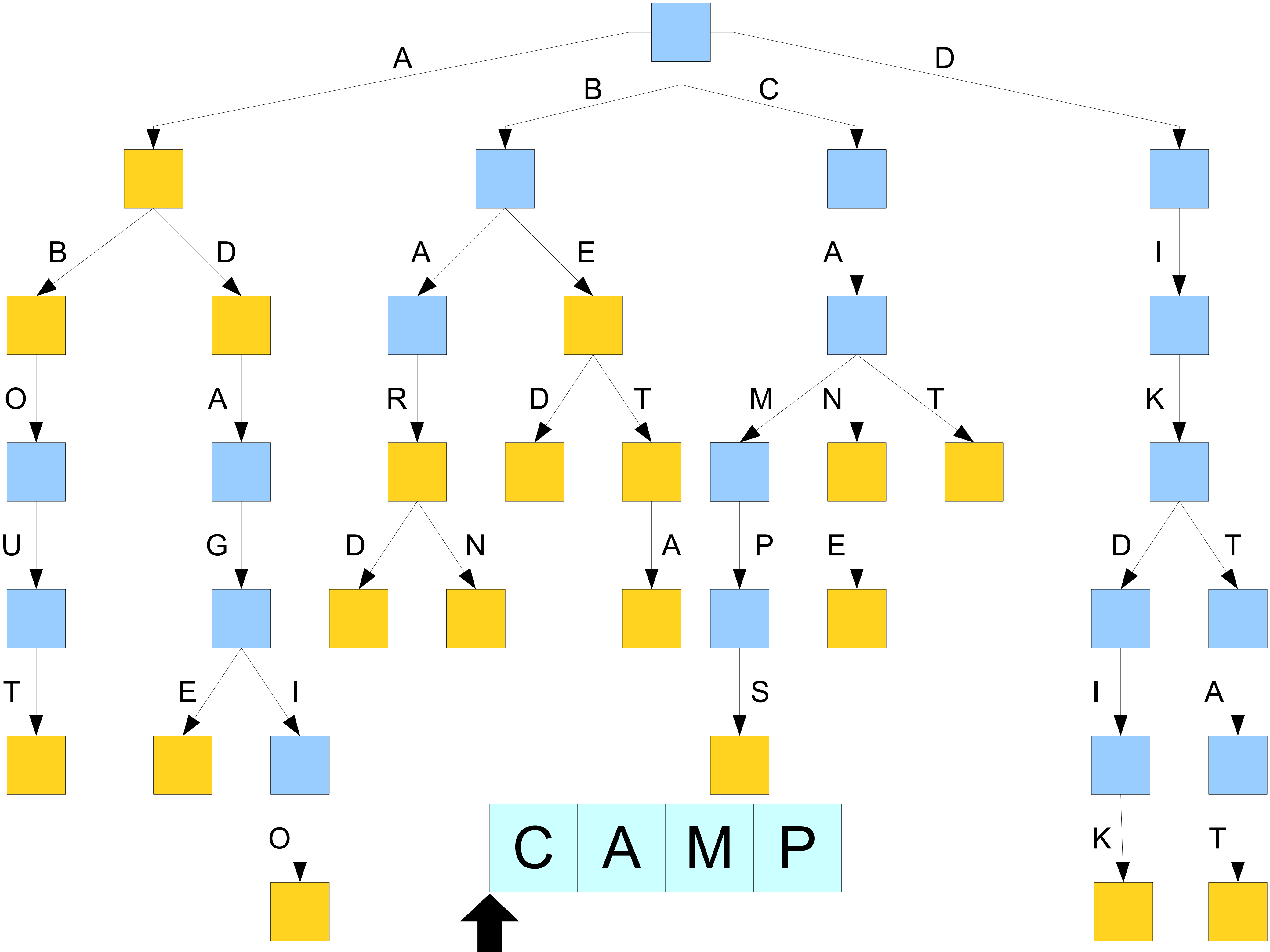
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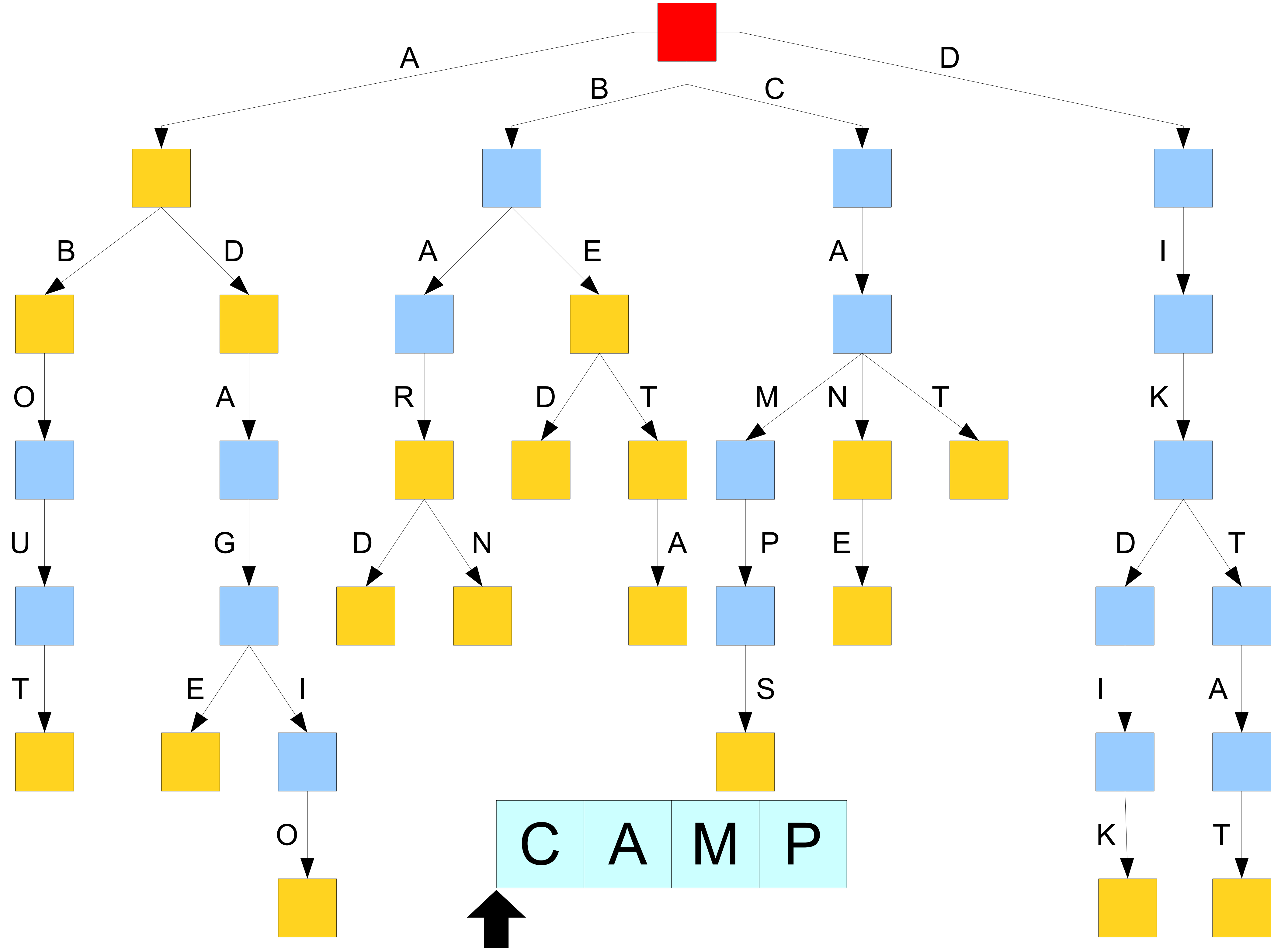
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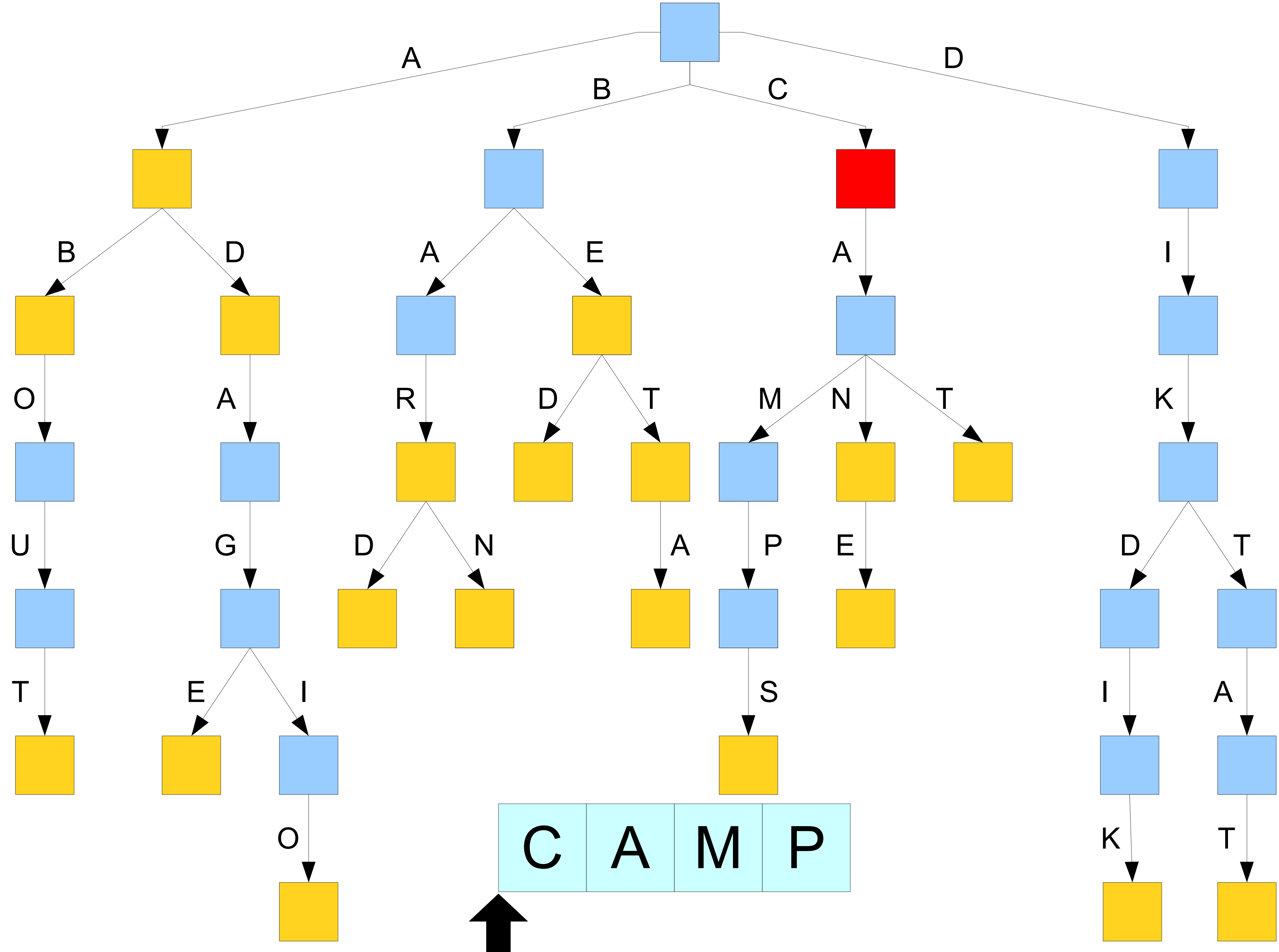
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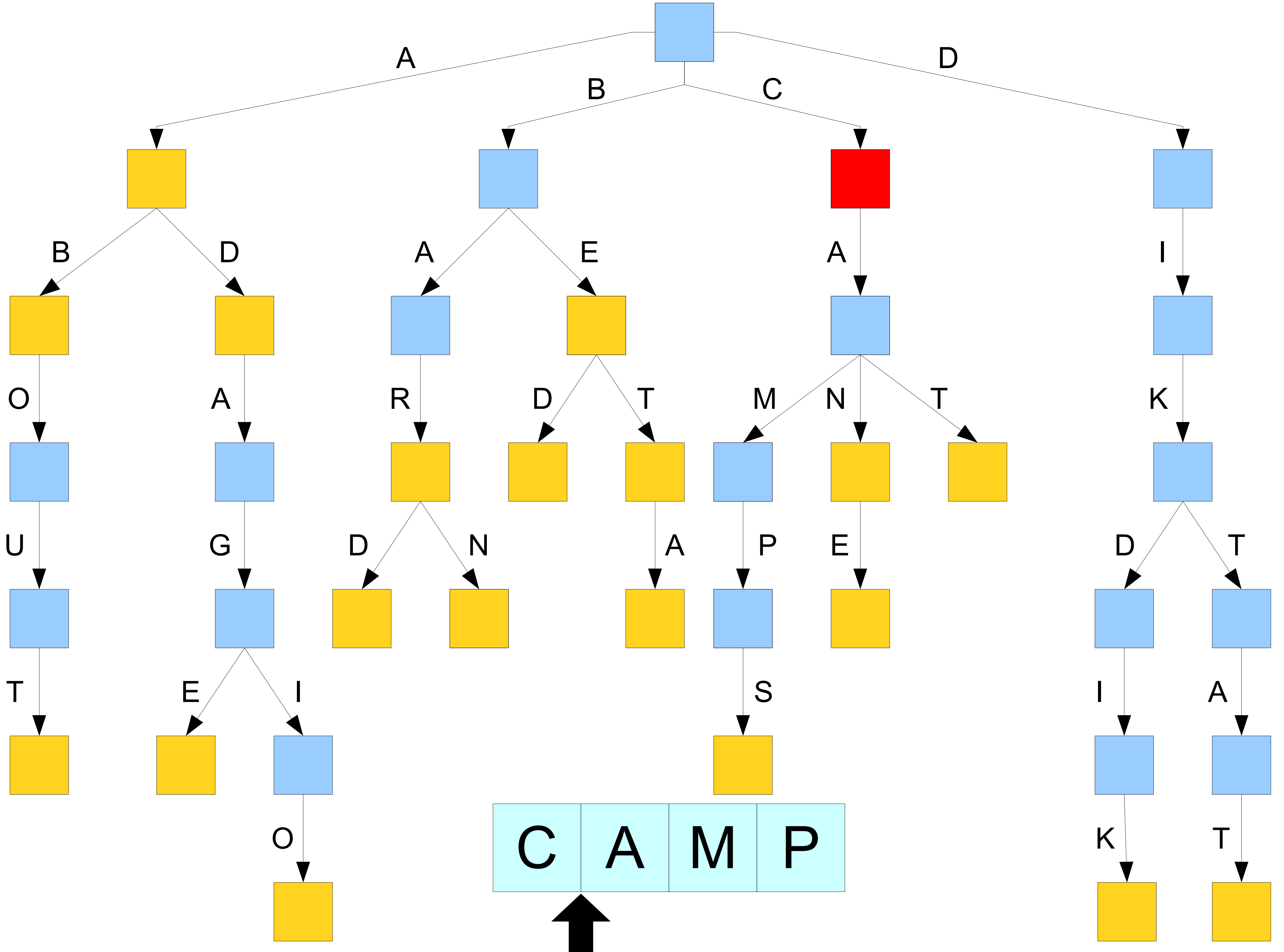

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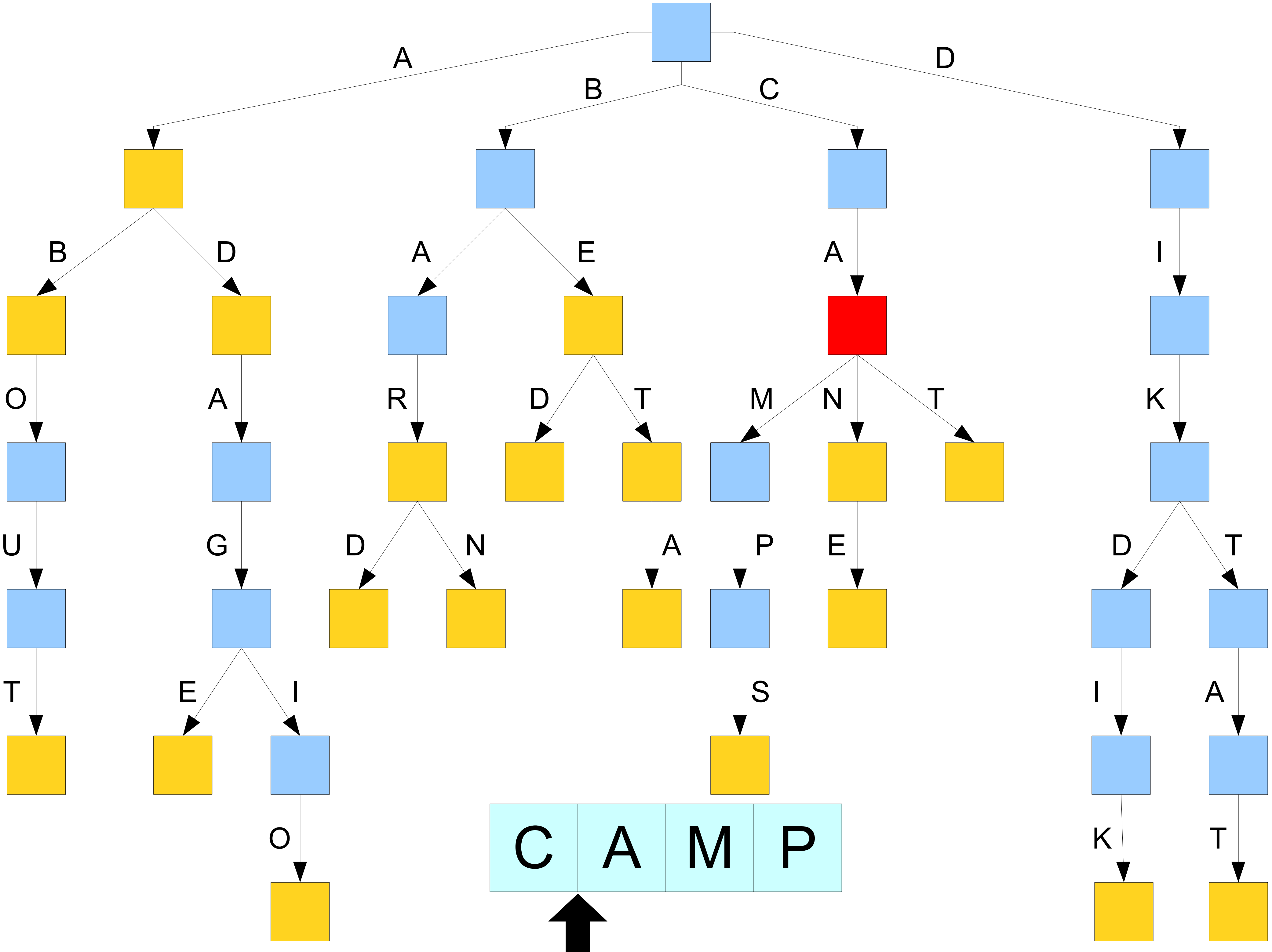
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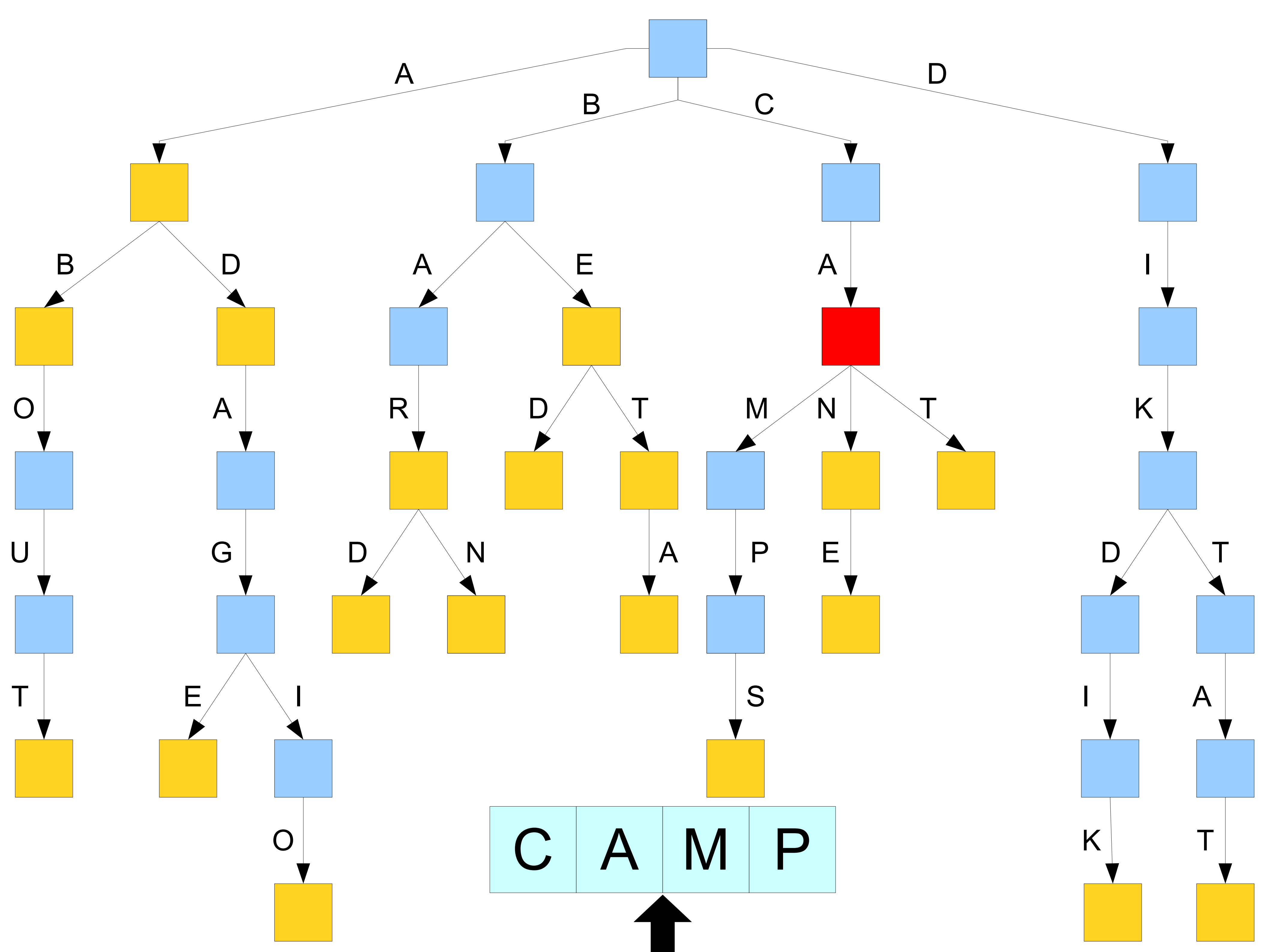
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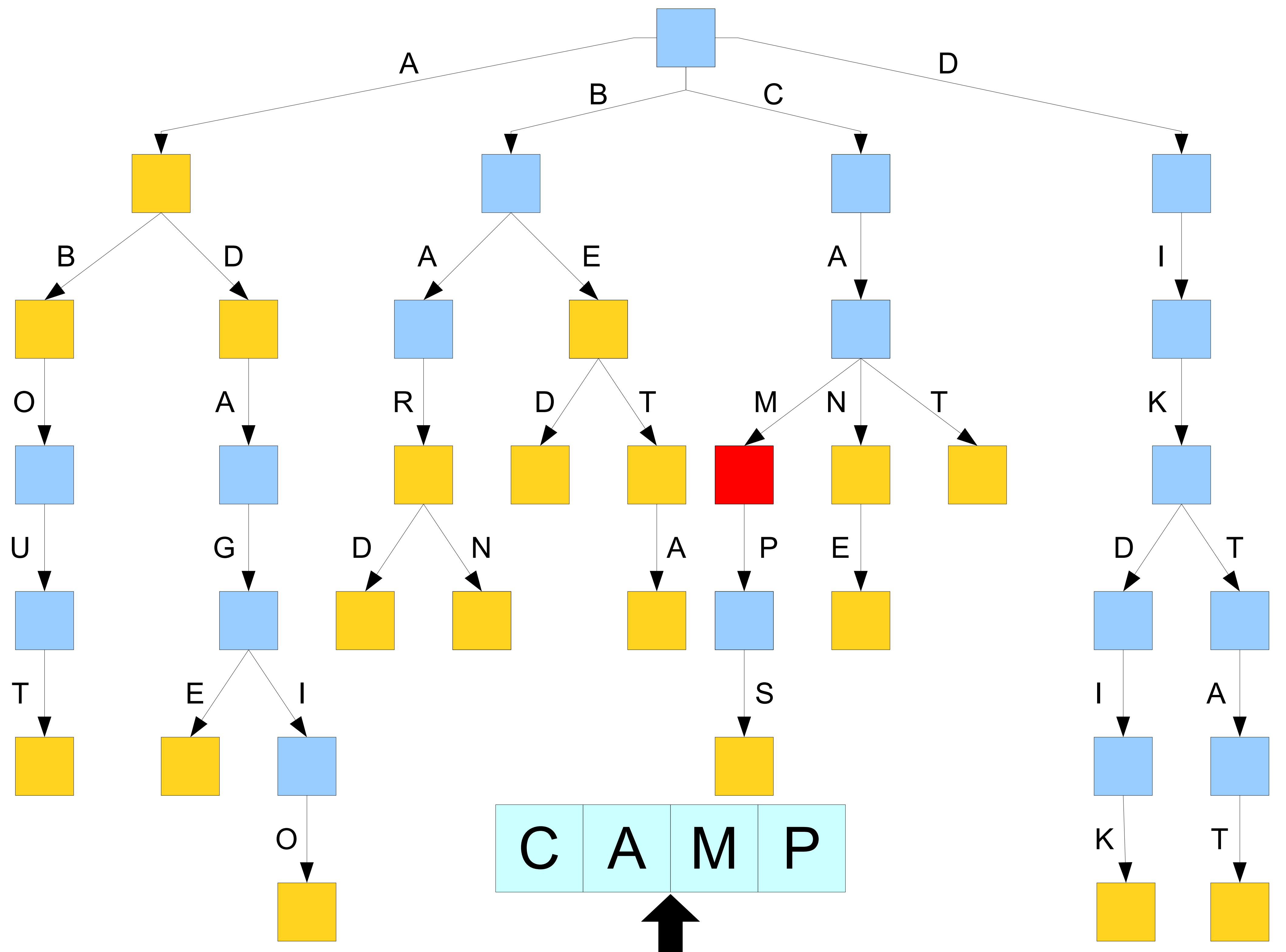
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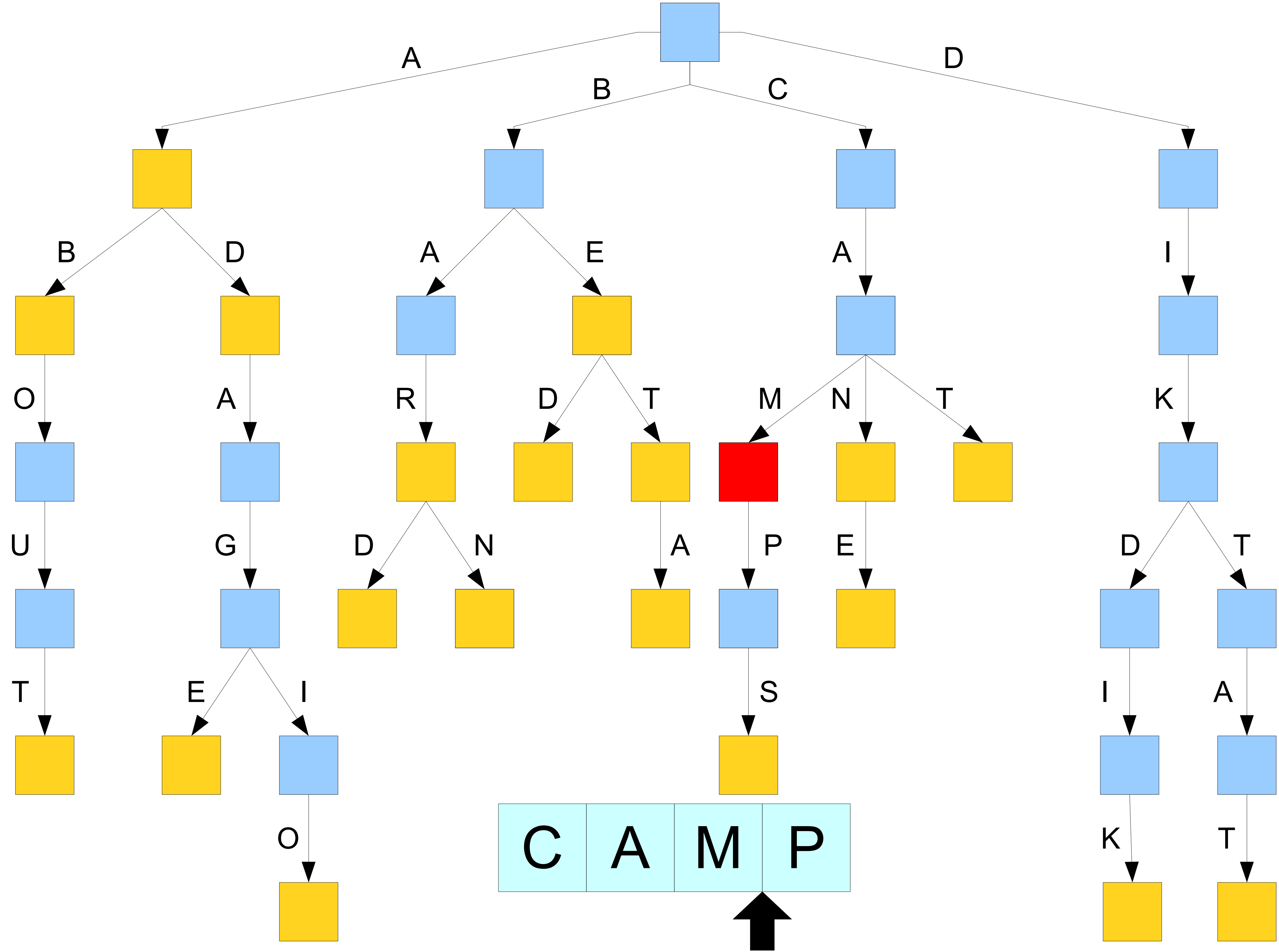
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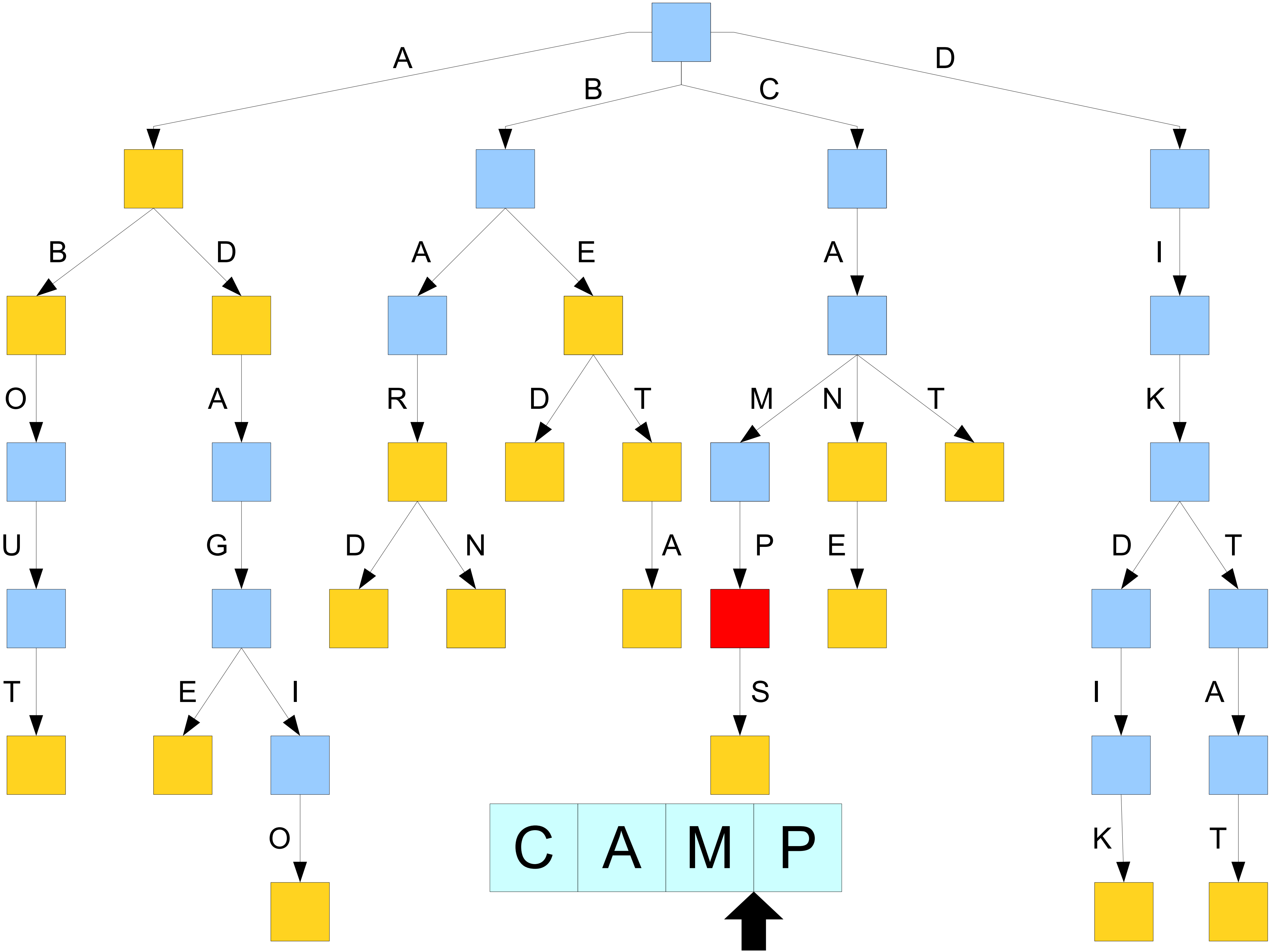
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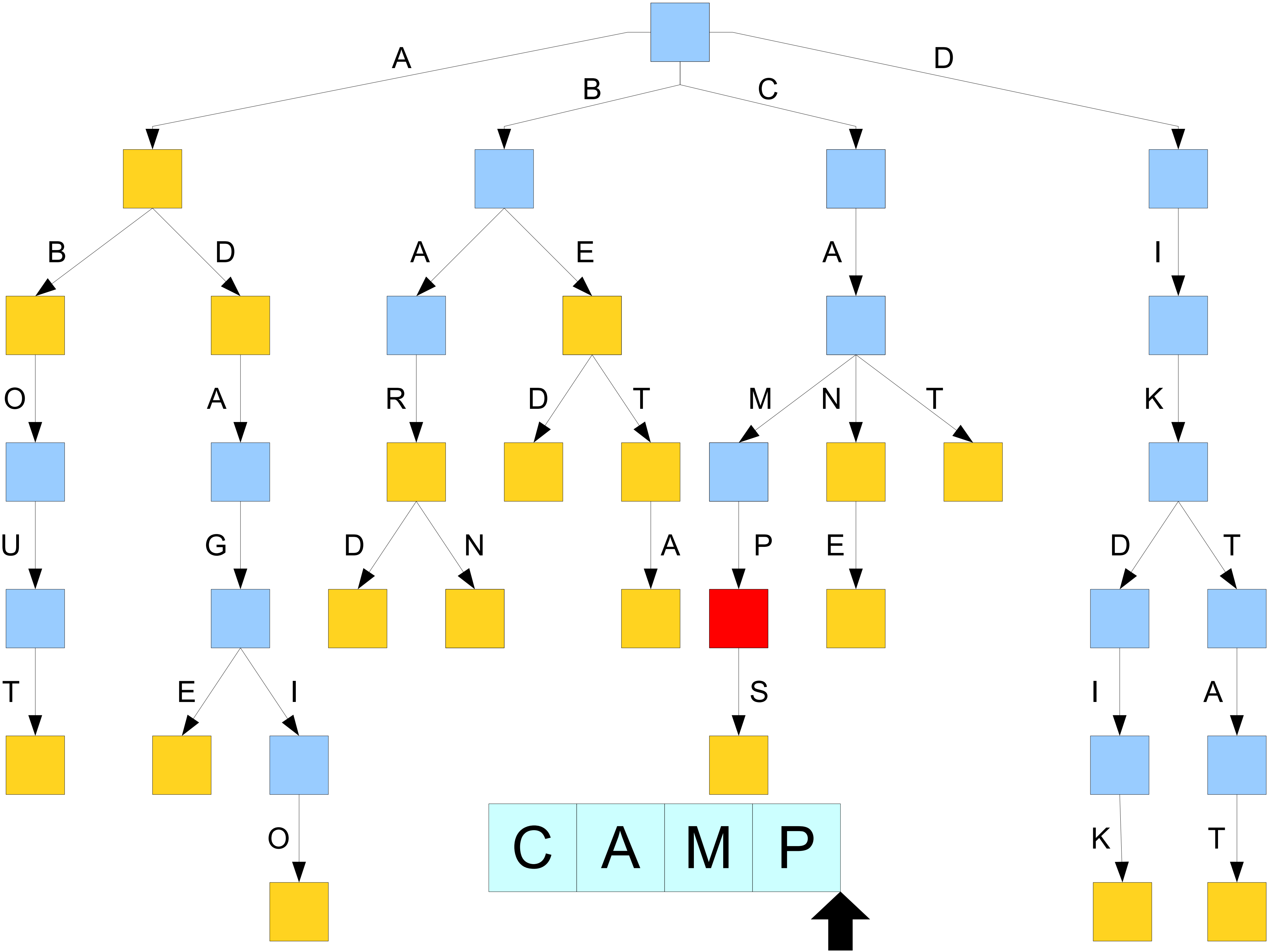
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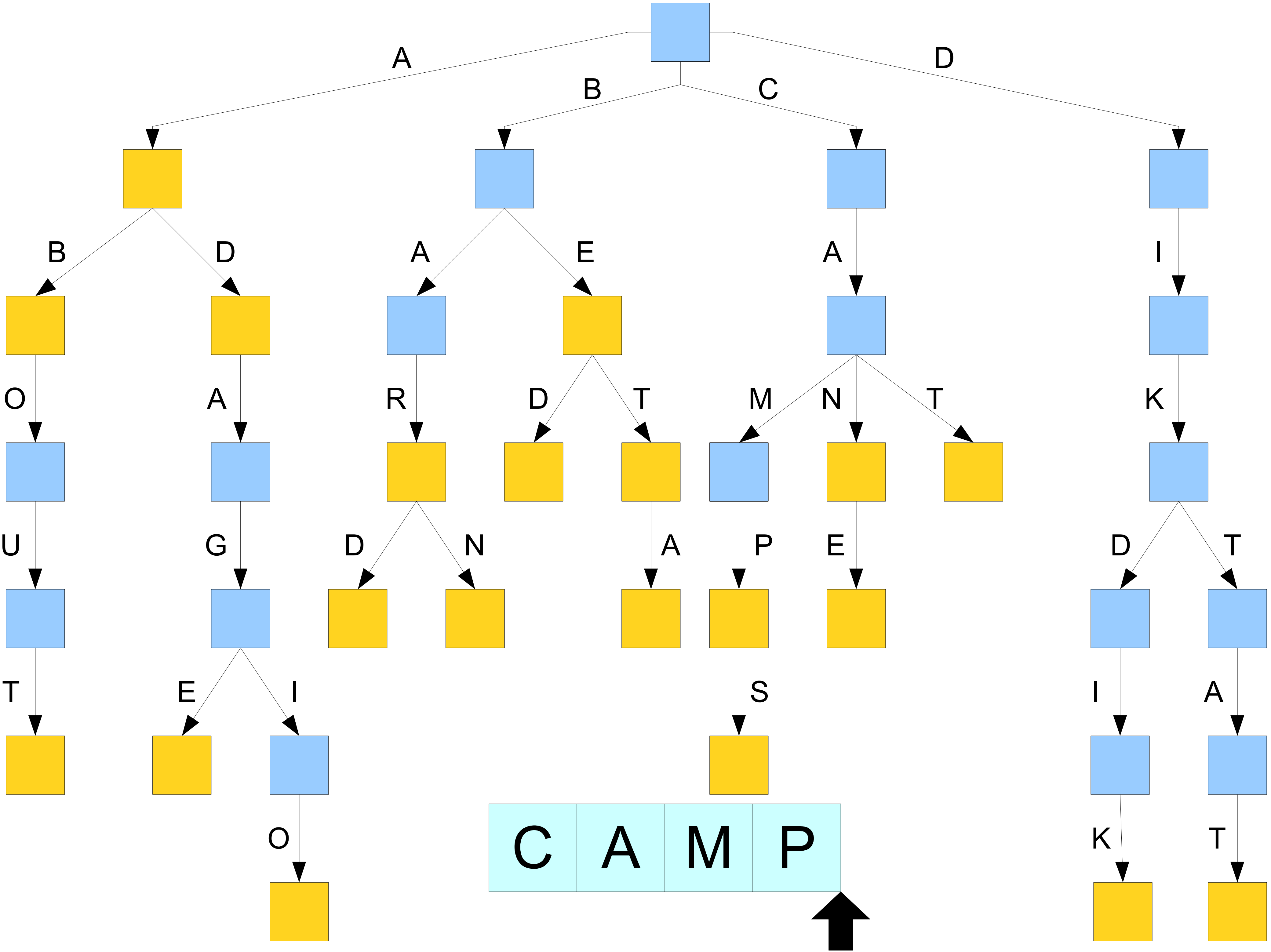
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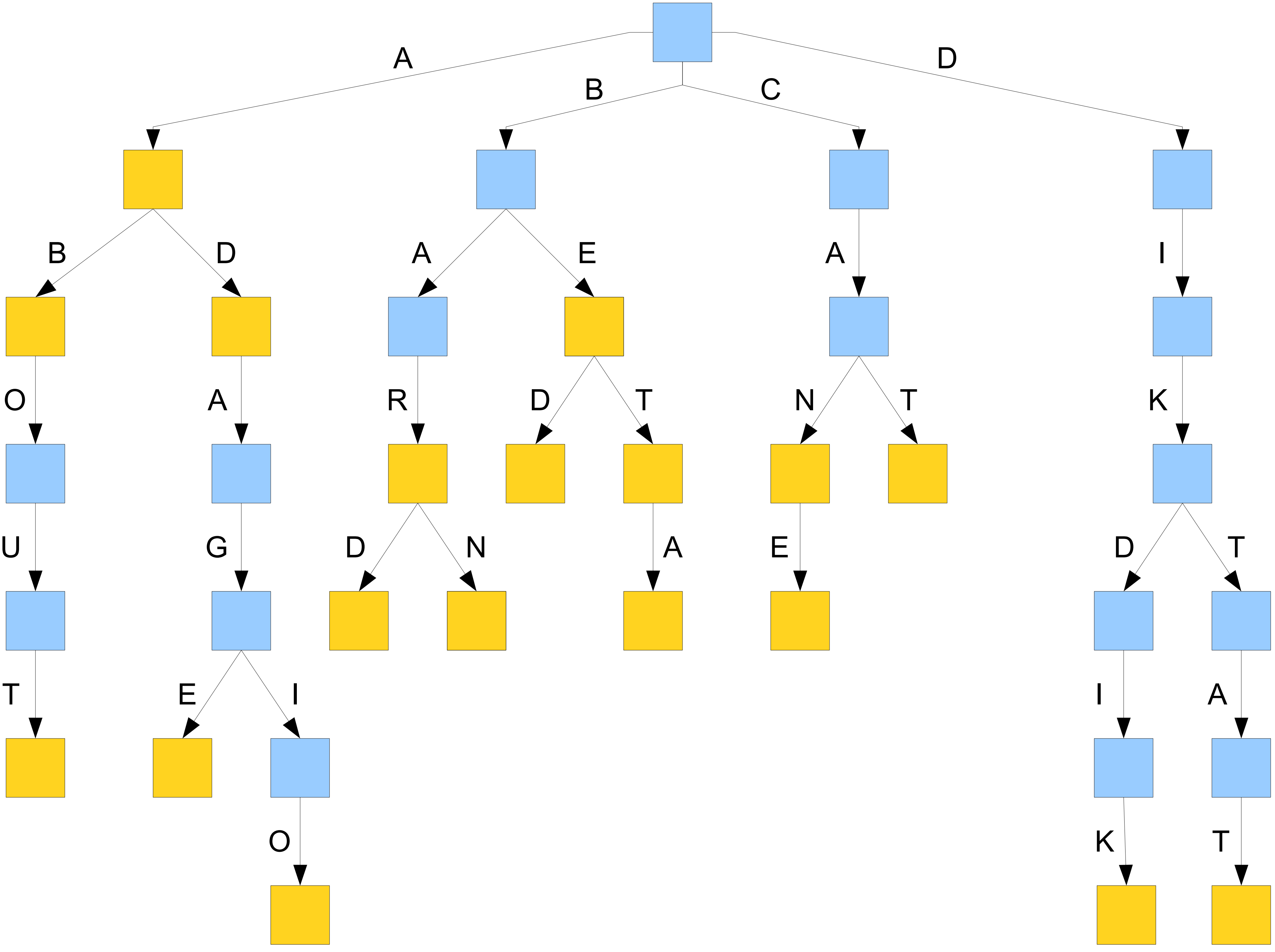
Let's trie coding up **`insert`**!

(OurLexicon.cpp/h)

# Analyzing the Trie

- How efficient are the operations on the trie?
- Every operation takes time proportional to the length of the string, which we'll denote ***L***.
- Time to add or look up an element is **O(*L*)**.
- Time to check if a prefix exists is **O(*L*)**.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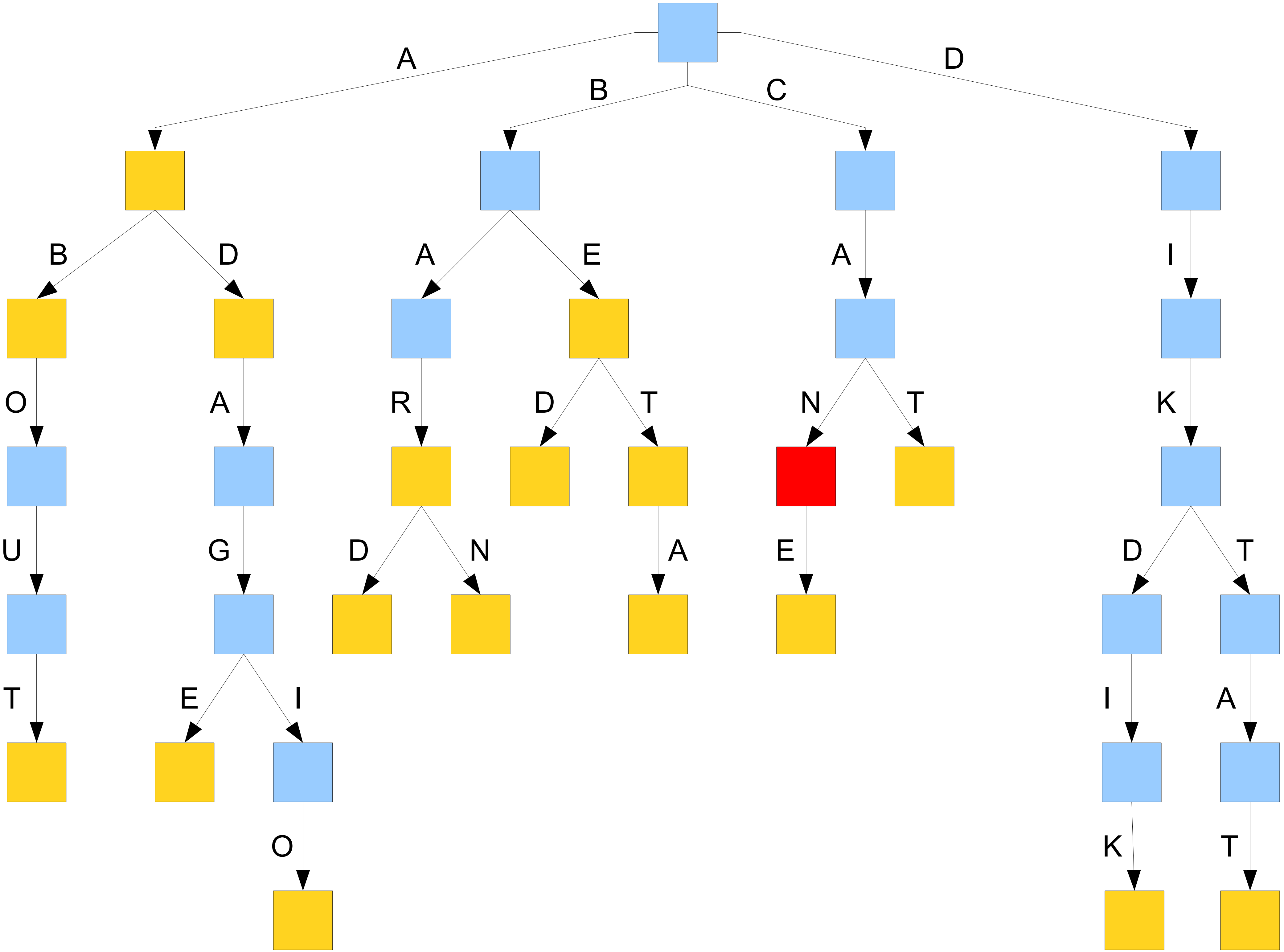
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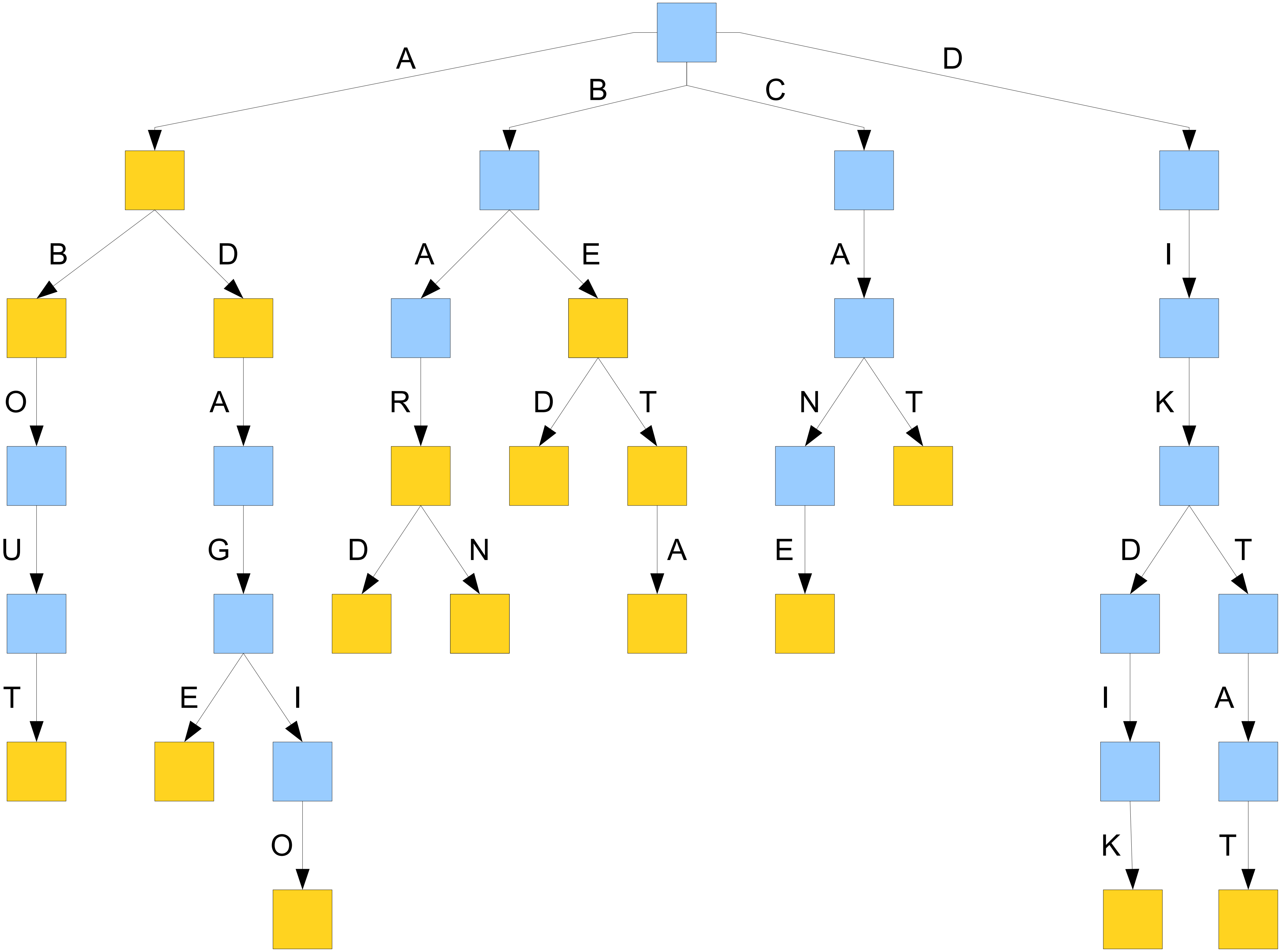
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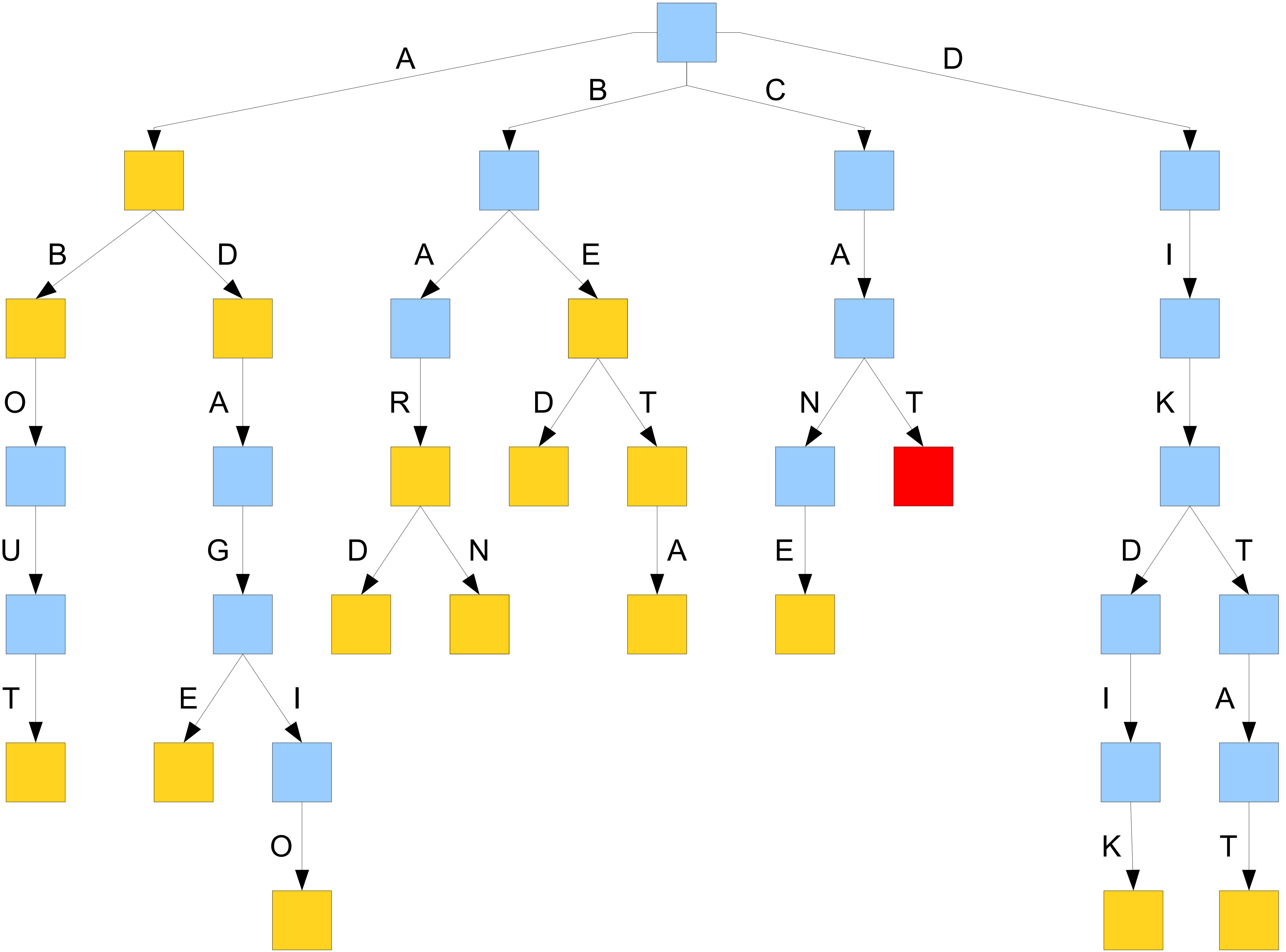
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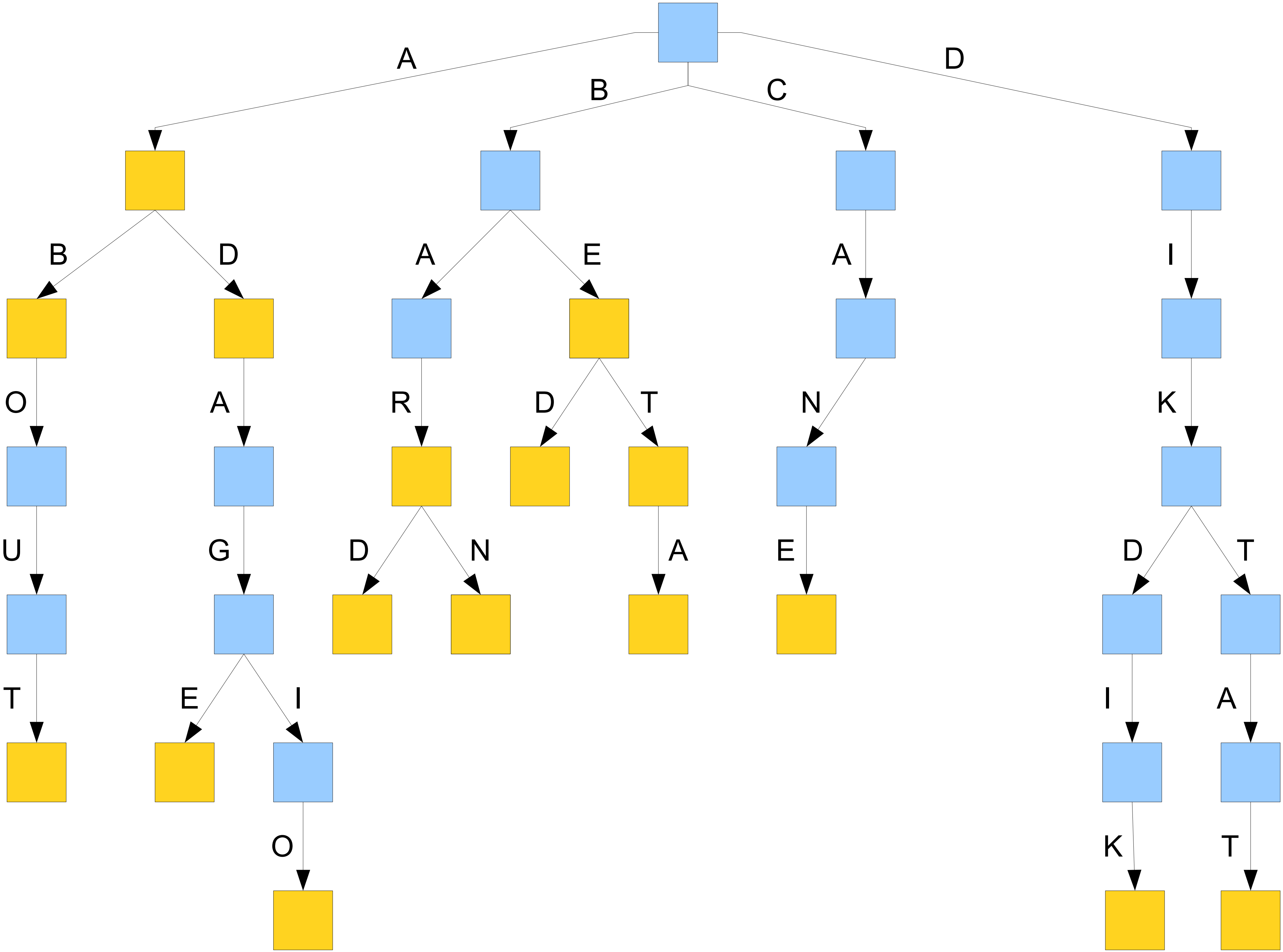
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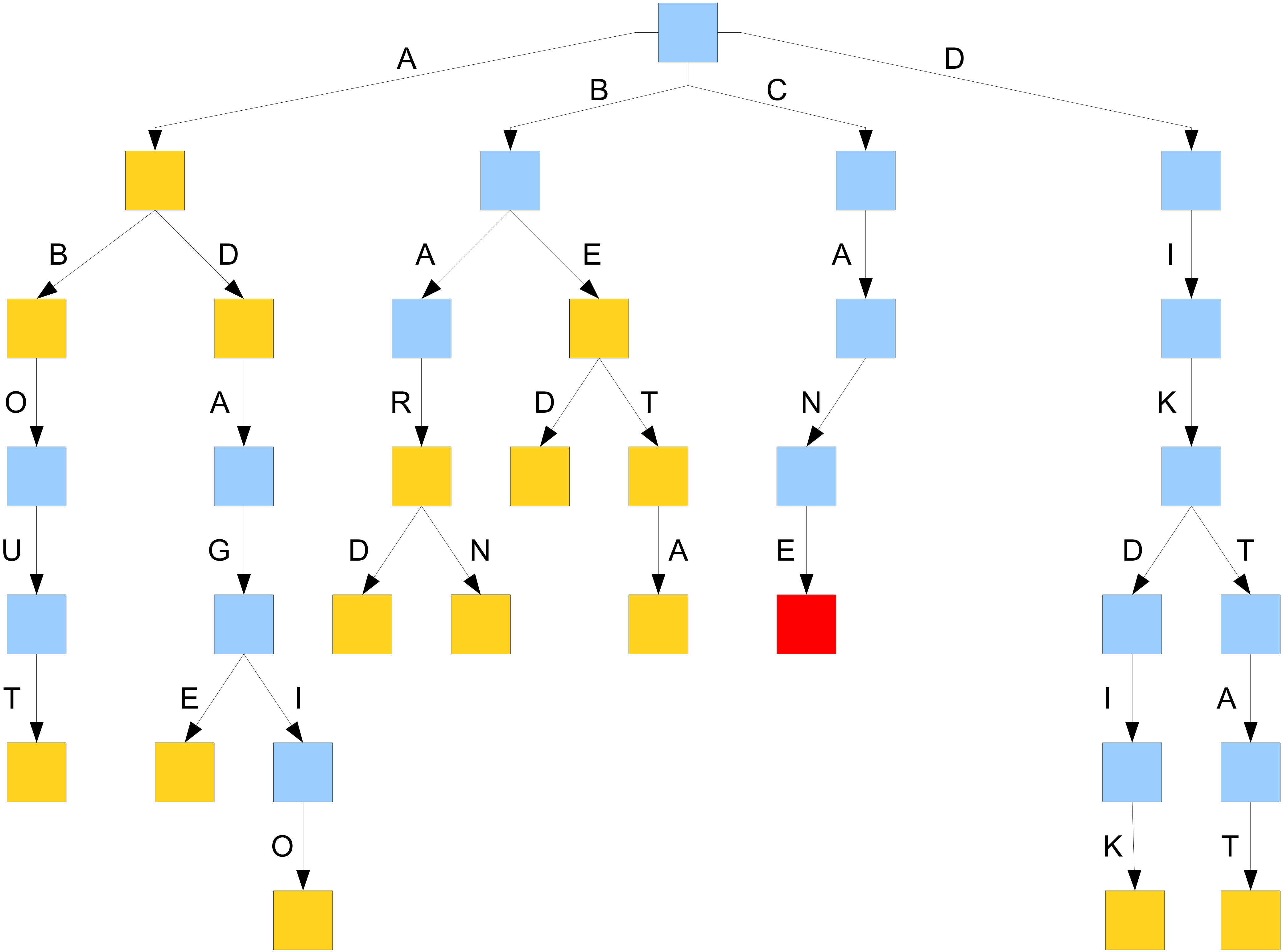
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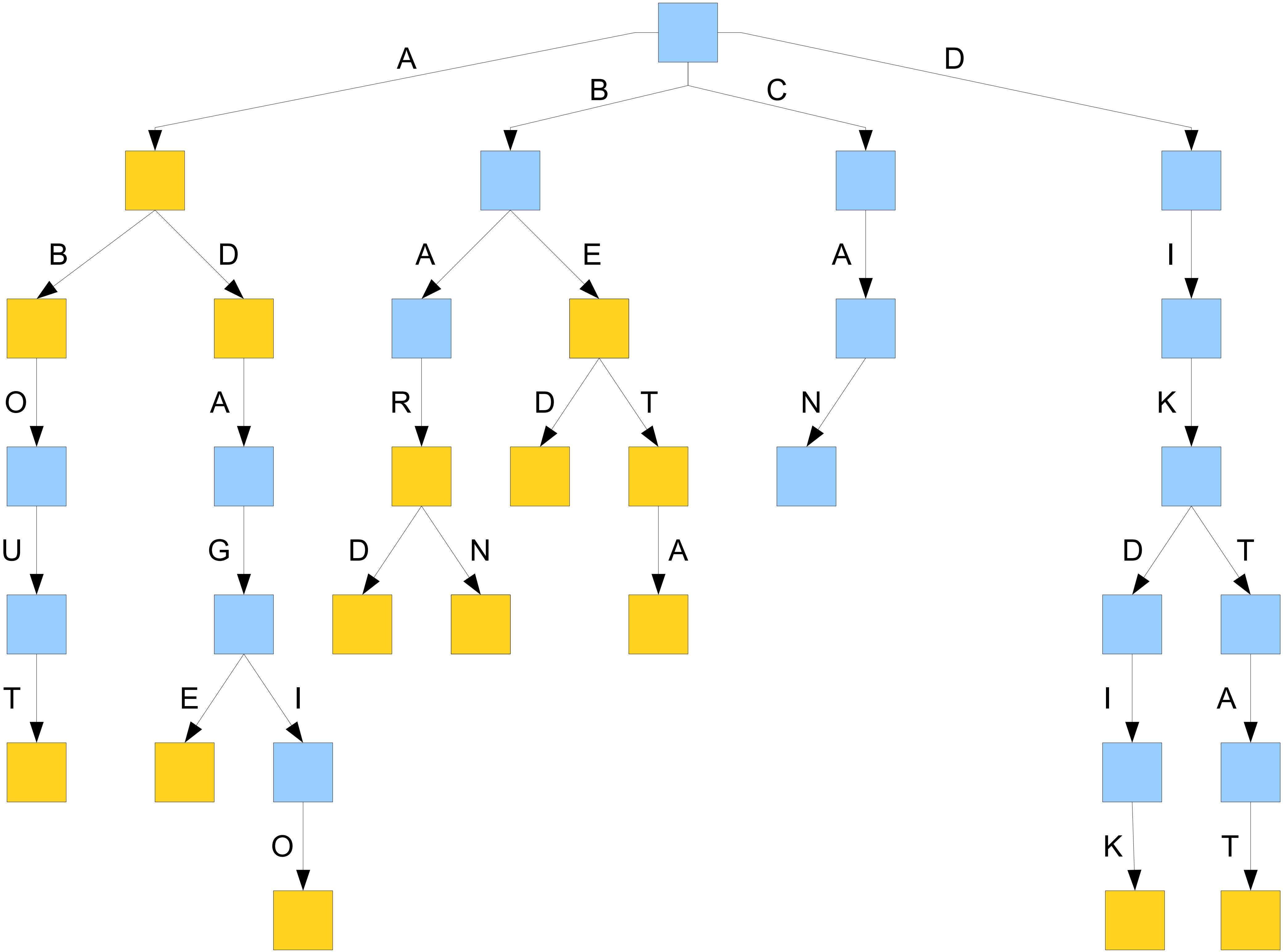
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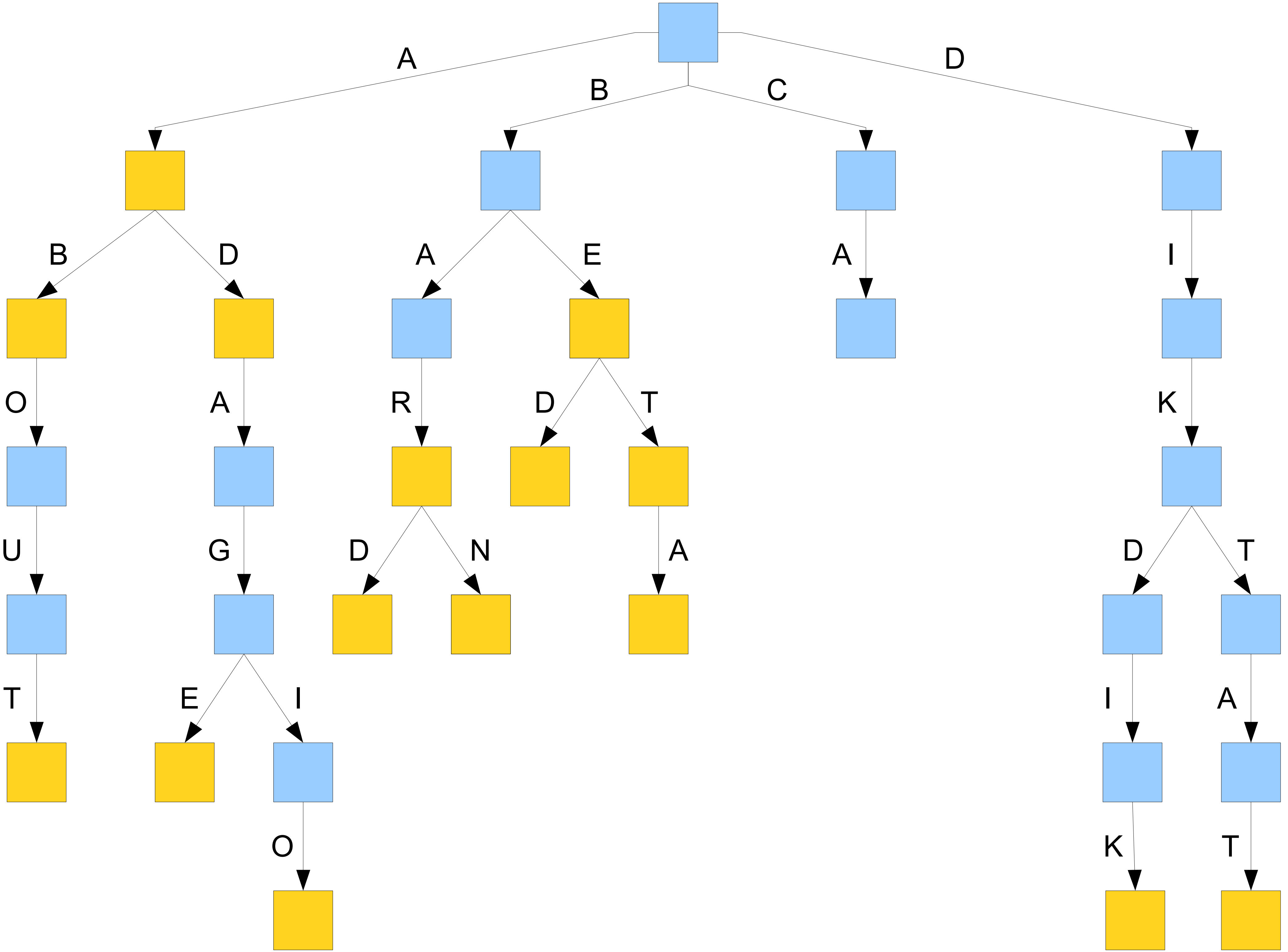
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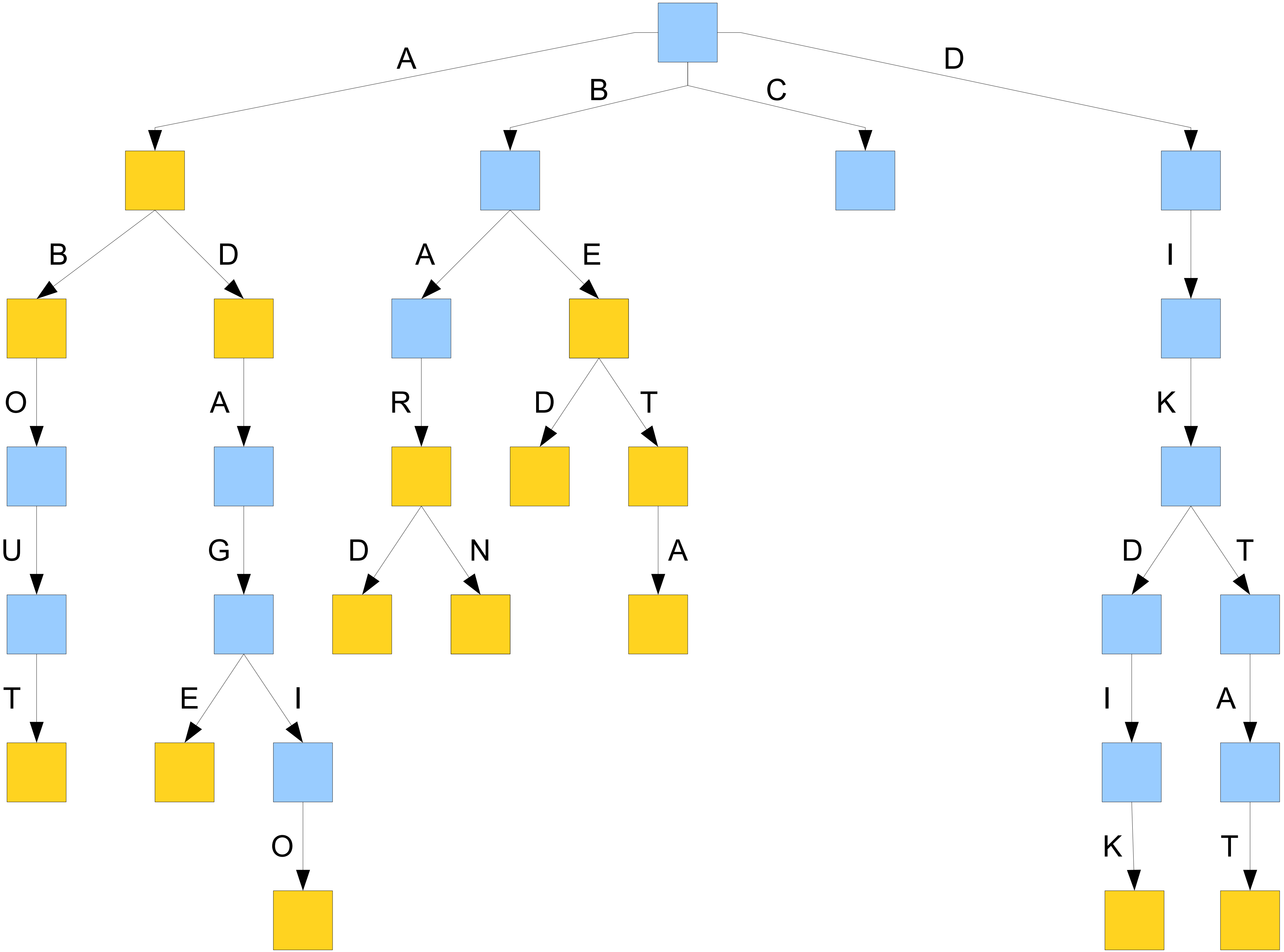
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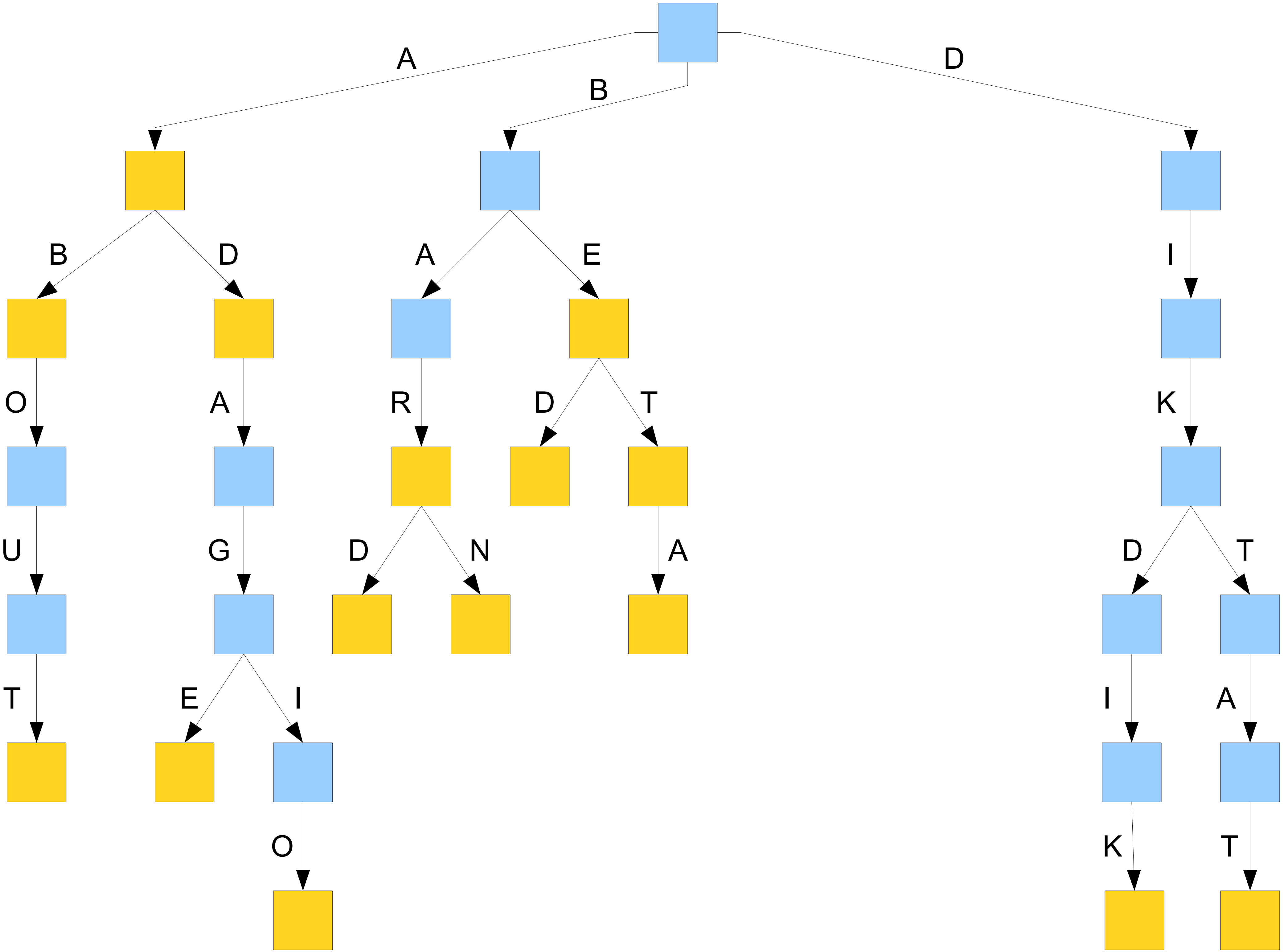

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# Removing from a Trie

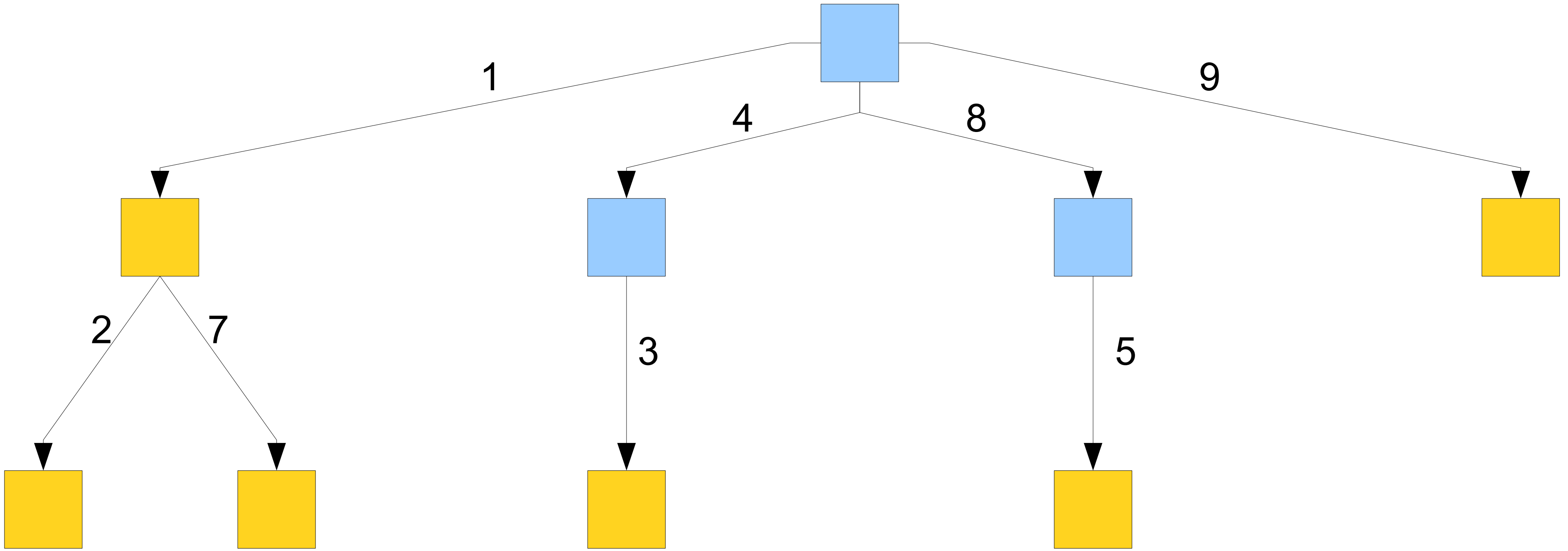
- Recurse until you reach the last node representing the word
- Mark the node as no longer containing a word.
- If the node has no children:
    - Remove that node.
        - **`delete`** and set equal to **`NULL`**
    - Repeat this process at the node one level higher up in the tree.

Let's trie coding up `Lexicon`!
(remove)
(OurLexicon.cpp/h)

# Other uses of Tries

- The Trie we wrote stores strings.  Could we use a Trie to store other data types as well?
  - Yes!

# Integer Trie



# Other uses of Tries

- We can also generalize Tries to any data type by branch on the binary representation of a piece of data
  - This is called a “Bitwise Trie”

# Tries: Pros and Cons

- Pros:

  1) **`containsPrefix()`** runs in **$O(L)$** time

  2) **`contains()`** runs in **$O(L)$** time

  3) Memory advantages by taking advantage of redundant prefixes.

     – e.g. All words that start with 'A' *share* a node representing the prefix “A”

# Tries: Pros and Cons

- Cons:
  - 1) Without redundant prefixes, Tries are super memory inefficient.
  - 2) Without some cool compression techniques that we'll go over in a couple lectures, Tries eat up a lot of memory.
    - It turns out, that these optimizations work best **when we don't modify the set of elements we are storing!**

# When do we use Tries?

- We generally use Tries when the following two properties hold:
  - Property 1: Lots of redundant prefixes
  - Property 2: The set of elements is *static* (the set of elements we want to store doesn't change over time)
- These two properties hold for languages

# Next Week

- **Graphs!**
  - How we represent and work with data with relationships (e.g. map data)