Programming Abstractions

CS106B

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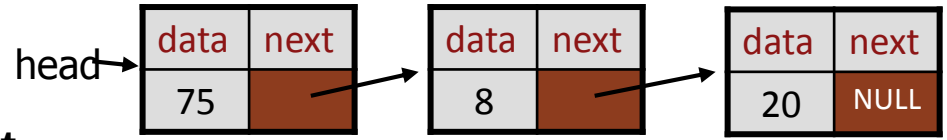
Topics:

- Priority Queue
  - Linked List implementation
    - Sorted
    - Unsorted
  - Heap data structure implementation

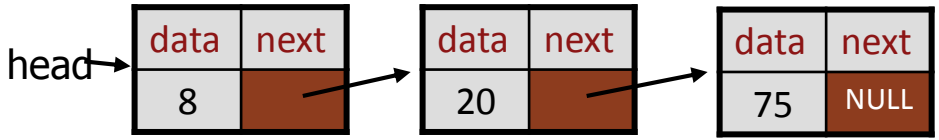
- **TODAY'S TOPICS NOT ON THE MIDTERM**

Some priority queue implementation options



Unsorted linked list

- Insert new element in front: O(1)
- Remove by searching list: O(N)



Sorted linked list

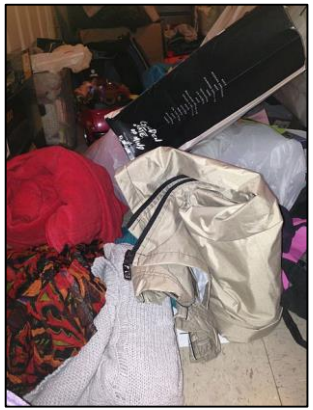
- Always insert in sorted order: O(N)
- Remove from front: O(1)

Priority queue implementations

We want the best of both

Fast add AND fast remove/peek

We will investigate trees as a way to get the best of both worlds



Fast add

+

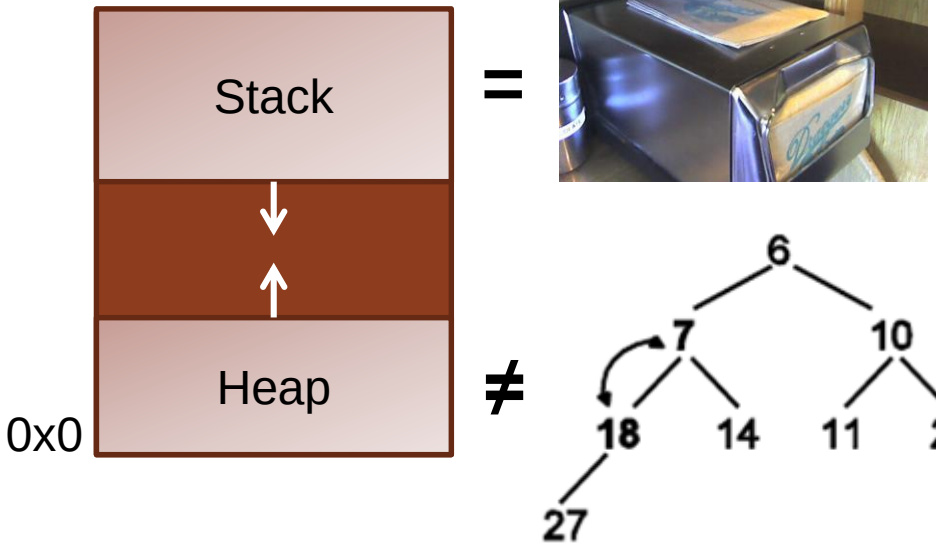
Fast remove/peek

=

Binary Heaps

Heap: not to be confused with the Heap!

- The Stack section of memory **is** a Stack like the ADT
- The Heap section of memory **has nothing to do** with the Heap structure.



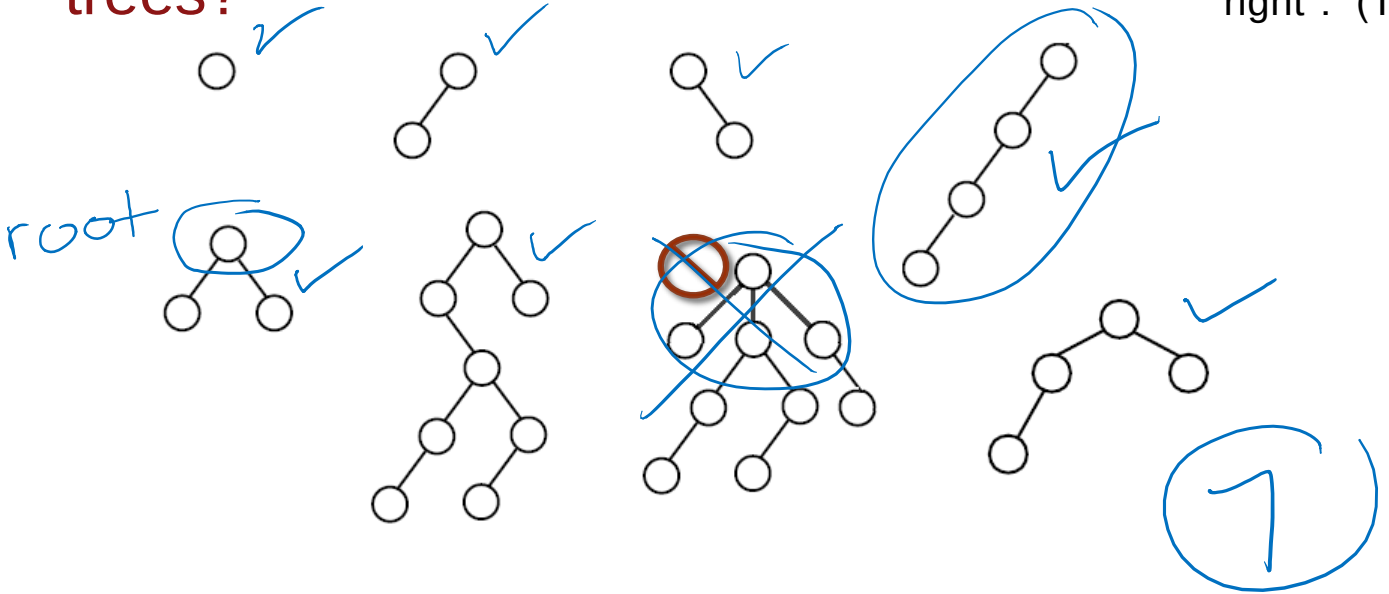
- Probably just happened to reuse the same word

Binary trees

A binary tree

“In computer science, a binary tree is a tree data structure in which each node has at most two child nodes, usually distinguished as "left" and "right".” (Thanks, Wikipedia!)

How many of these are valid binary trees?

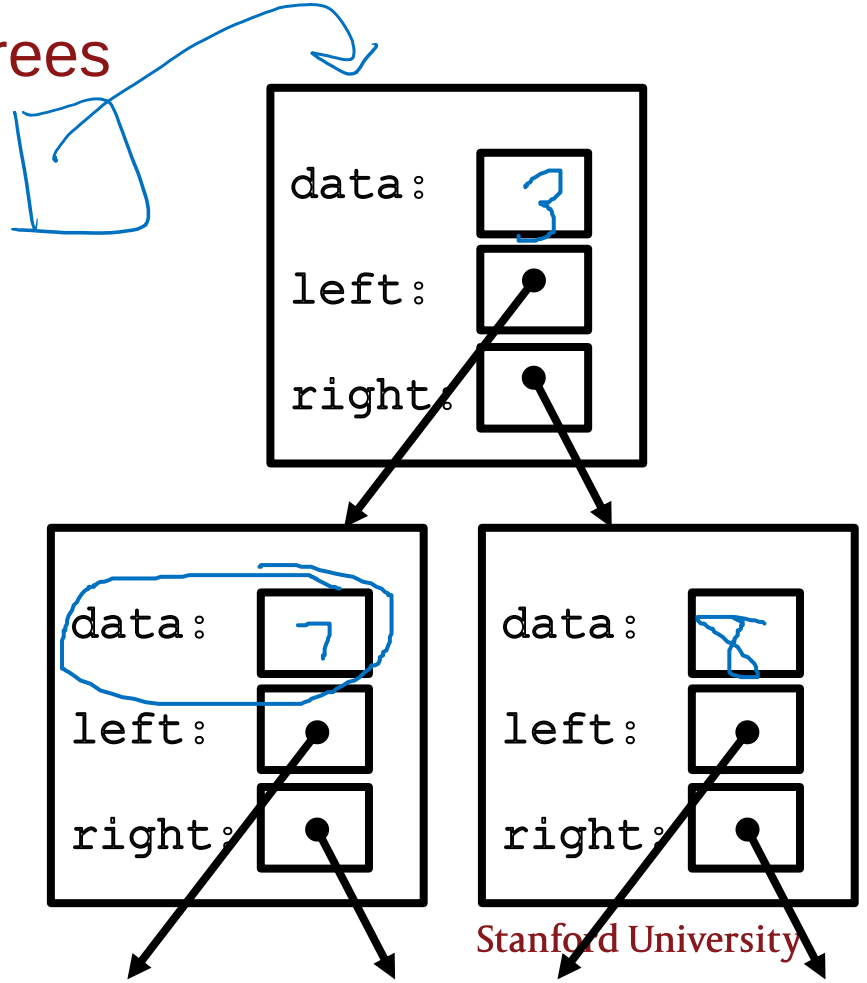


"In computer science, a binary tree is a tree data structure in which each node has at most two child nodes, usually distinguished as "left" and "right"." (Thanks, Wikipedia!)

A node struct for binary trees

- Similar to a linked list node, it contains **`data`**, and a pointer to the nearby elements
- A binary node tree has two child pointers, **`left`** and **`right`**

```
struct TreeNode {
    int data;
    TreeNode* left;
    TreeNode* right;
};
```



Heaps!

Binary Heaps*

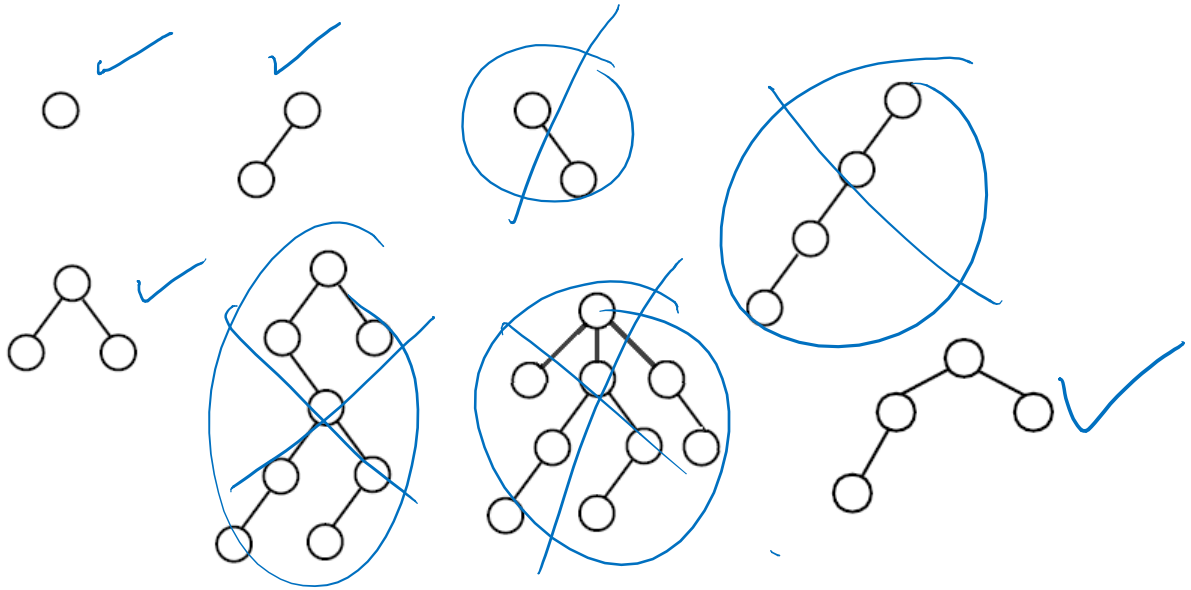
Binary heaps are **one kind** of binary tree

They have a few special restrictions, in addition to the usual binary tree:

- Must be **complete**
  - No “gaps”—nodes are filled in left-to-right on each level (row) of the tree
- Ordering of data must obey **heap property**
  - Min-heap version: a parent’s data is always **≤** both its children’s data
  - Max-heap version: a parent’s data is always **≥** both its children’s data

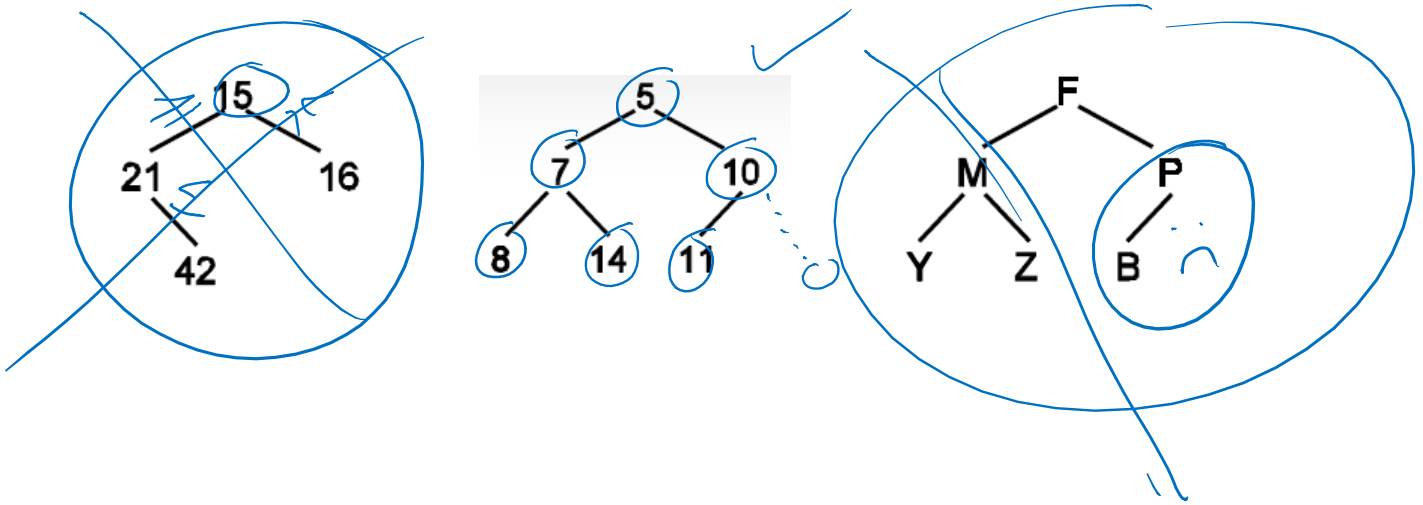
* There are other kinds of heaps as well. For example, binomial heap is extra credit on your assignment.

How many of these could be valid binary heaps?



A. 0-1
B. 2
C. 3
D. 4
E. 5-8

How many of these are valid min-binary-heaps?



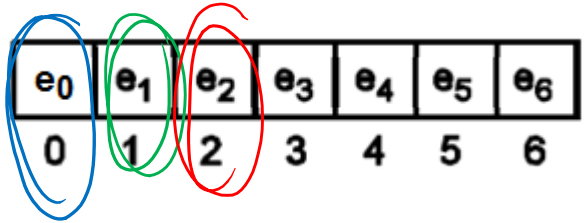
Binary heap in an array

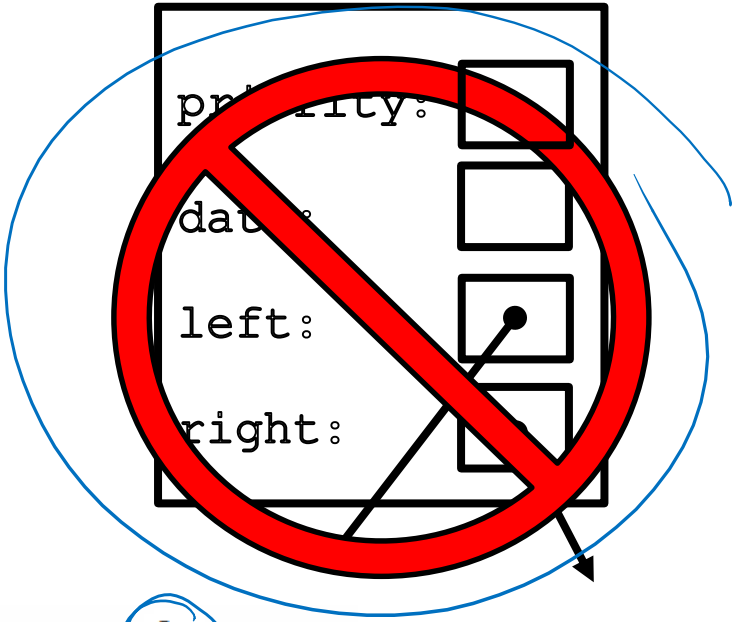
Binary heap in an array

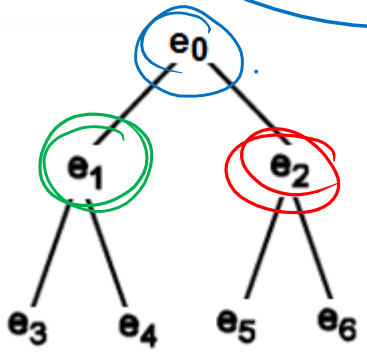
Binary heap is one special kind of binary tree, so we could use a node struct to represent it

However, … we actually do NOT typically use a node object to implement **heaps**

Because they have the special added constraint that they must be **complete**, they fit nicely into an array





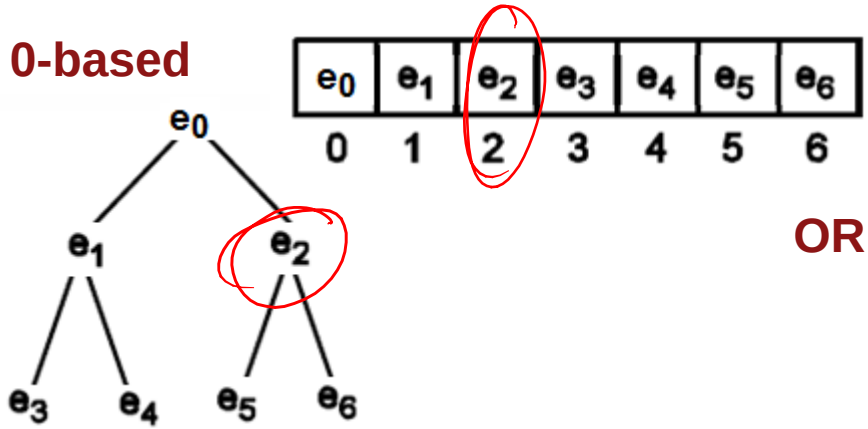


Two approaches:
Binary heap in an array

Wait, but the homework handout starts storing the elements at array index 1!

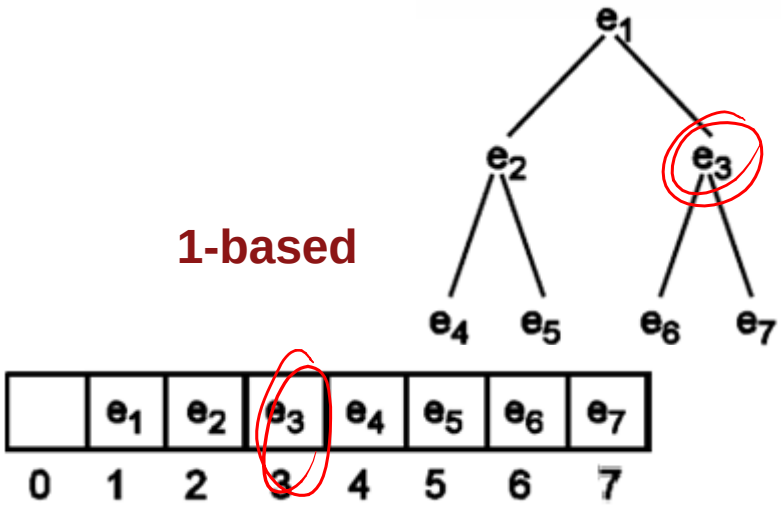
- Either way is ok for the assignment.
- You should understand both ways, so we're teaching both ways

0-based

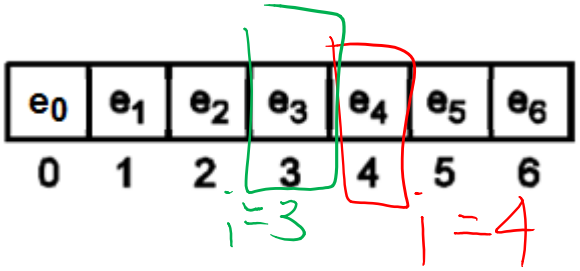


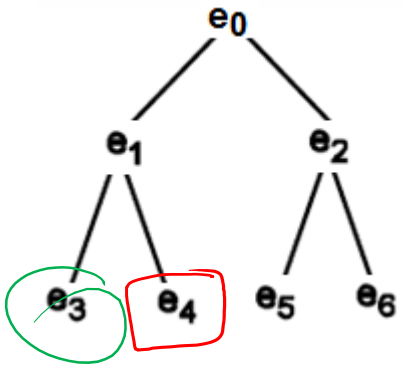
OR

1-based



Heap in an array



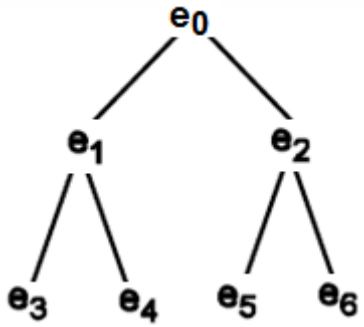


Pro tip: when testing, use both left and right child examples as test cases!

For a node in array index i:

- Q: The parent of that node is found where?
- A: at index:
  - A. i – 2
  - B. i / 2
  - C. (i – 1)/2
  - D. 2i

Fact summary: Binary heap in an array

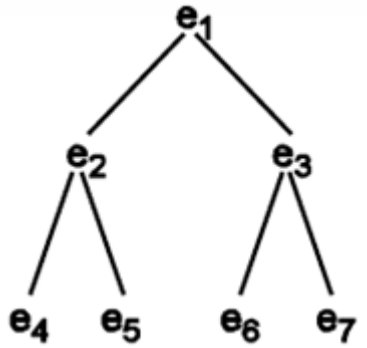


0-based:

For tree of height h, array length is 2^h-1

For a node in array index i:

- Parent is at array index: (i – 1)/2
- Left child is at array index: 2i + 1
- Right child is at array index: 2i + 2



1-based:

For tree of height h, array length is 2^h

For a node in array index i:

- Parent is at array index: i /2
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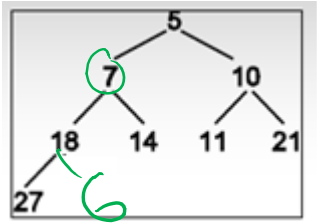
Binary heap enqueue and dequeue

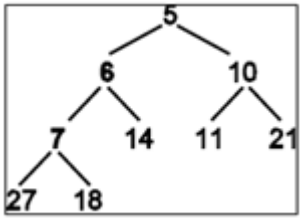
Binary heap enqueue (insert + “bubble up”)



Size=8, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	7	10	18	14	11	21	27	6	?	…	?

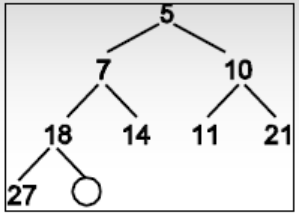
1 2 3 4



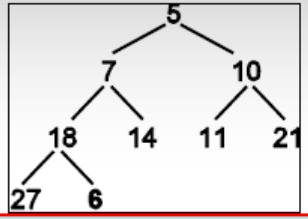
Size=9, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
5	6	10	7	14	11	21	27	18	?	…	?

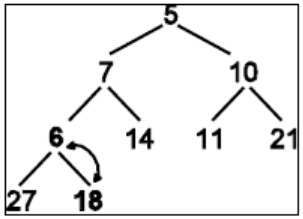
[Binary heap insert reference page]



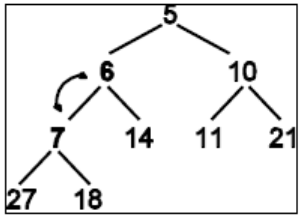
(a) A minheap prior to adding an element. The circle is where the new element will be put initially.



(b) Add the element, 6, as the new rightmost leaf. This maintains a complete binary tree, but may violate the minheap ordering property.

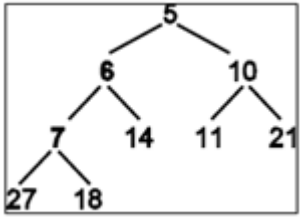


(c) "Bubble up" the new element. Starting with the new element, if the child is less than the parent, swap them. This moves the new element up the tree.



(d) Repeat the step described in (c) until the parent of the new element is less than or equal to the new element. The minheap invariants have been restored.

Binary heap dequeue (delete + “trickle down”)



Size=9, Capacity=15

0	1	2	3	4	5	6	7	8	9	…	14
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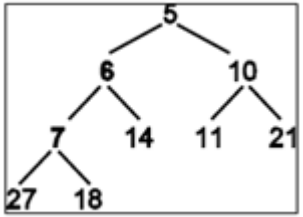
data

Binary heap dequeue (delete + “trickle down”)



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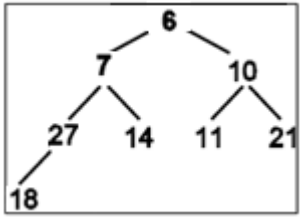
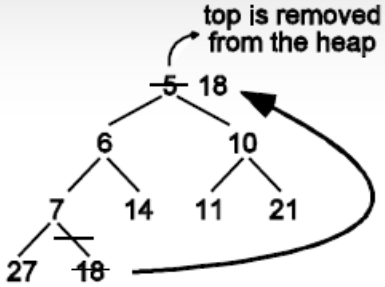


Size=8, Capacity=15

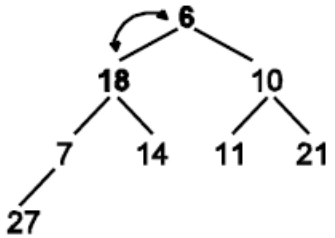
0	1	2	3	4	5	6	7	8	9	…	14
6	7	10	27	14	11	21	18	?	?	…	?

[Binary heap delete + “trickle-down” reference page]

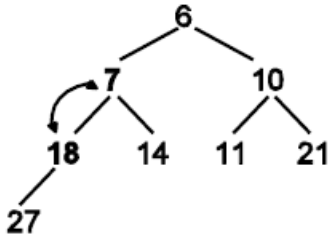
(a) Moving the rightmost leaf to the top of the heap to fill the gap created when the top element (5) was removed. This is a complete binary tree, but the minheap ordering property has been violated.



(b) “Trickle down” the element. Swapping top with the smaller of its two children leaves top’s right subtree a valid heap. The subtree rooted at 18 still needs fixing.



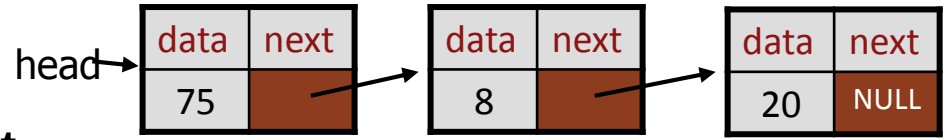
(c) Last swap. The heap is fixed when 18 is less than or equal to both of its children. The minheap invariants have been restored



Summary analysis

Comparing our priority queue options

Some priority queue implementation options



Unsorted linked list

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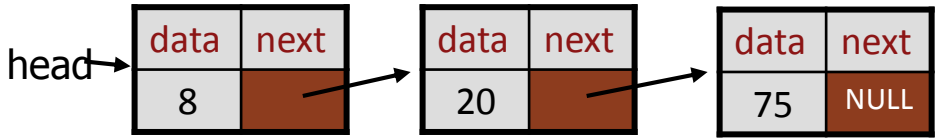
- Remove by searching list: O(N)



Sorted linked list

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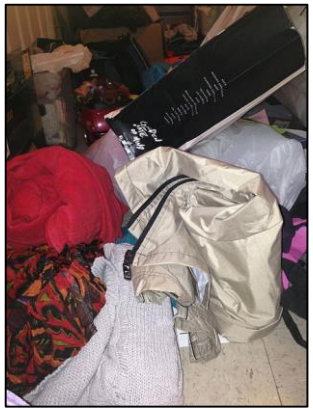
Priority queue implementations

We want the best of both

Fast add AND fast remove/peek

We will investigate trees as a way to get the best of both worlds



Fast add

+

Fast remove/peek

=

Review: priority queue implementation options

Unsorted linked list

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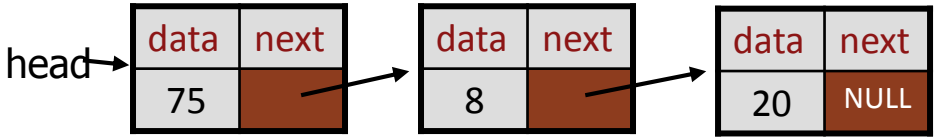

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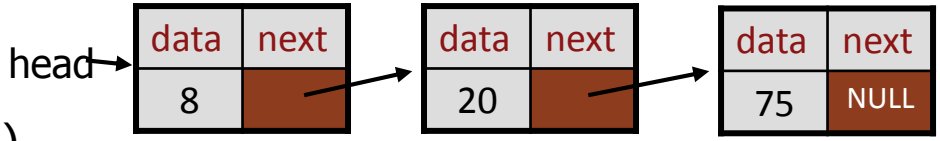
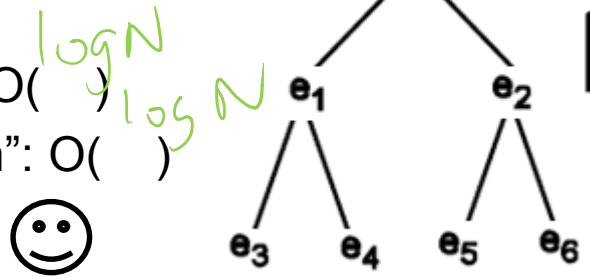
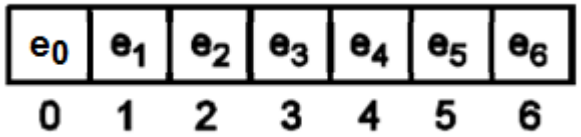


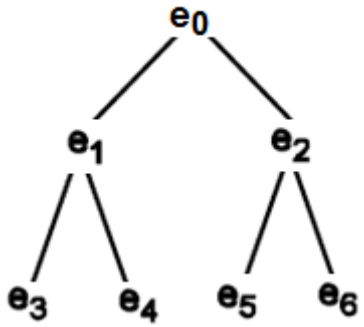
Binary heap

- Insert + "bubble up": O( )
- Delete + "trickle down": O( )





Fact summary: Binary heap in an array



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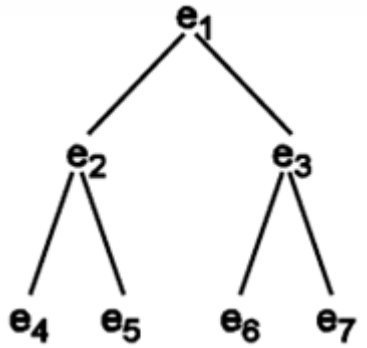

1-based:

For tree of height h, array length is 2^h

For a node in array index i:

- Parent is at array index: i /2
- Left child is at array index: 2i
- Right child is at array index: 2i + 1