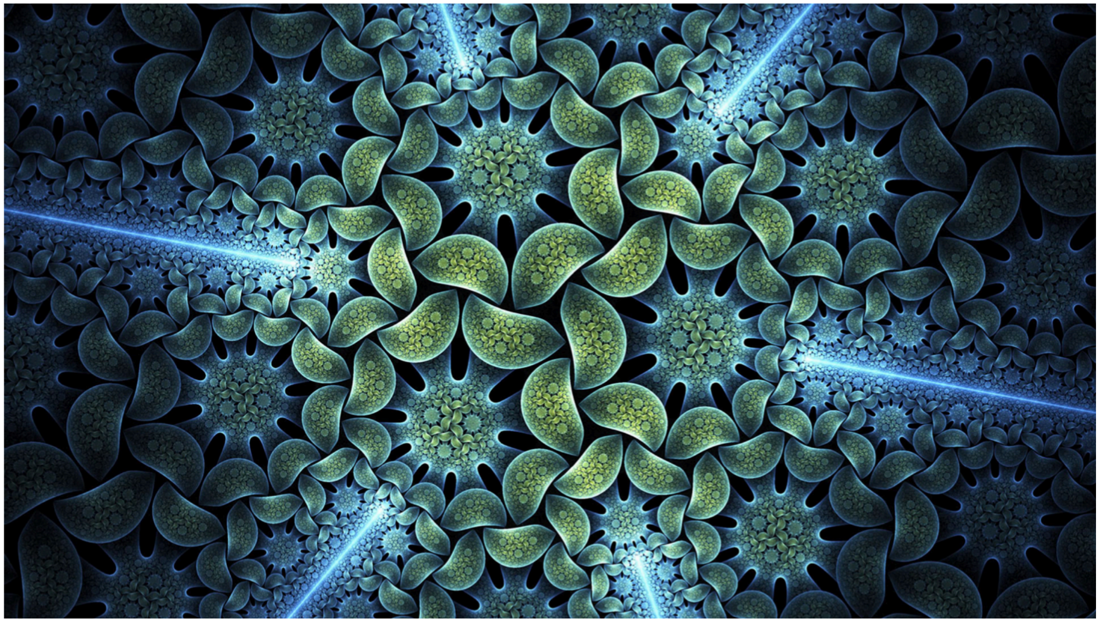
CS 106B
Lecture 8: Fractals

Wednesday, April 19, 2017

Programming Abstractions
Spring 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg



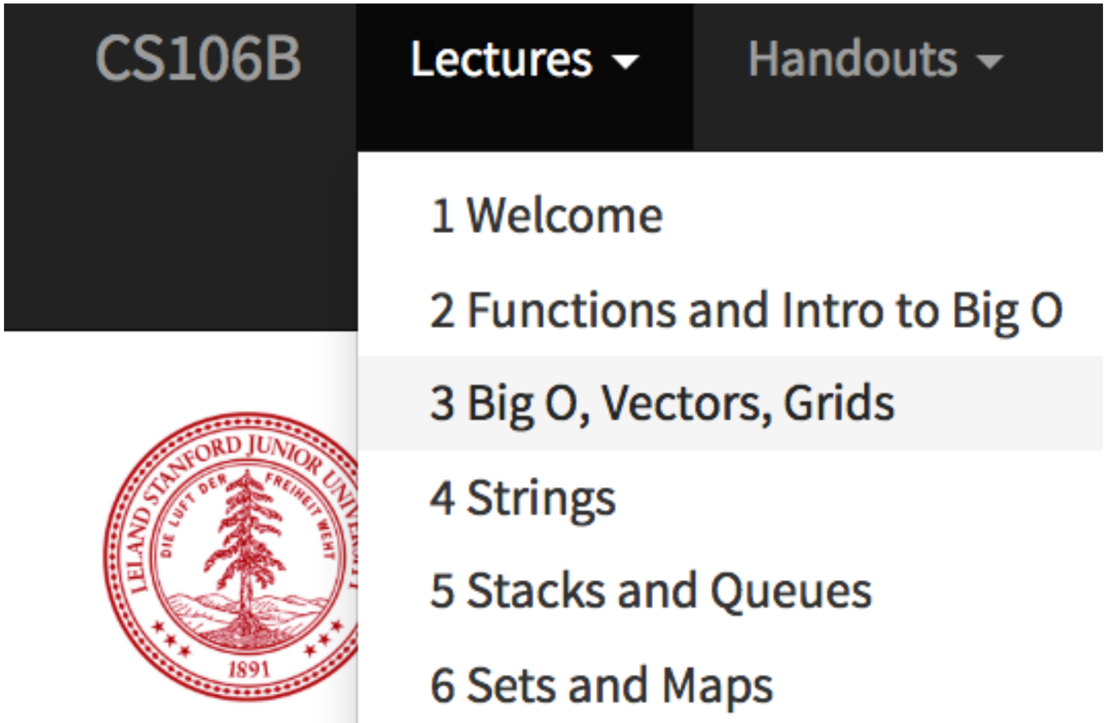
reading:
Programming Abstractions in C++, Chapter 5.4-5.6

Today's Topics

- Logistics:
  - ADTs Due Thursday April 20th, noon
  - Towers of Hanoi video featuring Keith Schwartz: https://www.youtube.com/watch?v=2SUvWfNJSsM
- Tiny Feedback
- Assignment 3: Recursion
  - Fractals
  - Grammar Solver
- A more detailed recursion example
- Fractals

Tiny Feedback

- **Could you please upload the .ppt of the classes and not only the .pdf?**
  - We've already been doing this! See the Lectures drop-down on the course web page:



CS106B Lectures ▾ Handouts ▾

1 Welcome

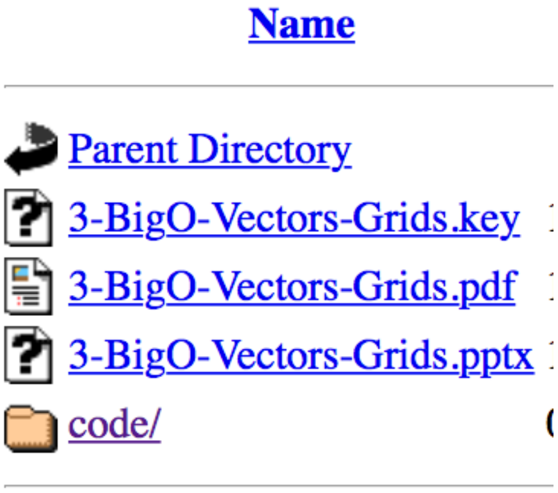
2 Functions and Intro to Big O

3 Big O, Vectors, Grids

4 Strings

5 Stacks and Queues

6 Sets and Maps



Name

Parent Directory

3-BigO-Vectors-Grids.key

3-BigO-Vectors-Grids.pdf

3-BigO-Vectors-Grids.pptx

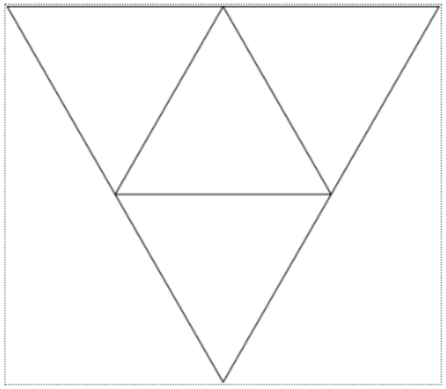
code/

Assignment 3: Recursion

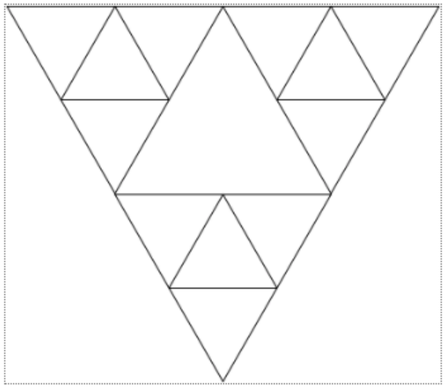
(1) Fractals and Graphics
(2) Grammar Solver

Assignment 3A: Fractals and Graphics

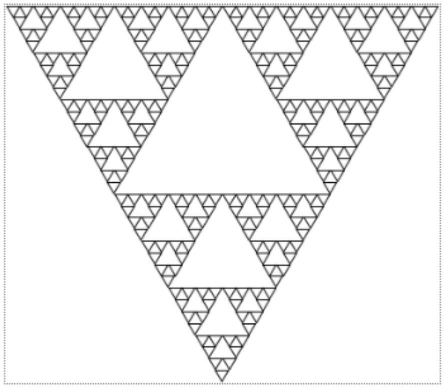
part 1
Sierpinski



Order-2

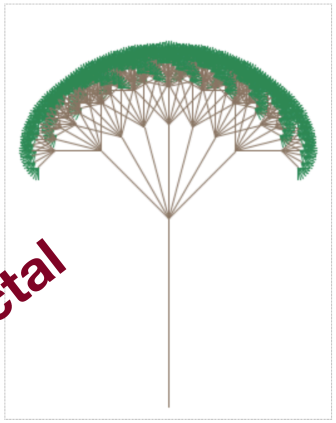


Order-3

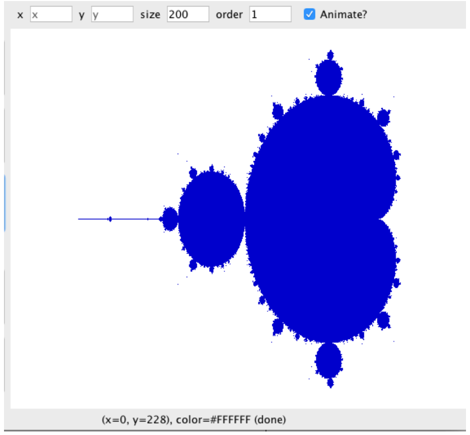


... Order-6

part 2
tree fractal



Order-5 tree fractal



x x y y size 200 order 1 ☑ Animate?

(x=0, y=228), color=#FFFFFF (done)

part 3
mandelbrot

Assignment 3B: Grammar Solver

write a function for generating random sentences from a grammar.

example describing a small subset of the English language. Non-terminal names such as <s>, <np> and <tv> are short for linguistic elements such as sentences, noun phrases, and transitive verbs:

```
<s>::=<np> <vp>
<np>::=<dp> <adjp> <n>|<pn>
<dp>::=the|a
<adjp>::=<adj>|<adj> <adjp>
<adj>::=big|fat|green|wonderful|faulty|subliminal|pretentious
<n>::=dog|cat|man|university|father|mother|child|television
<pn>::=John|Jane|Sally|Spot|Fred|Elmo
<vp>::=<tv> <np>|<iv>
<tv>::=hit|honored|kissed|helped
<iv>::=died|collapsed|laughed|wept
```

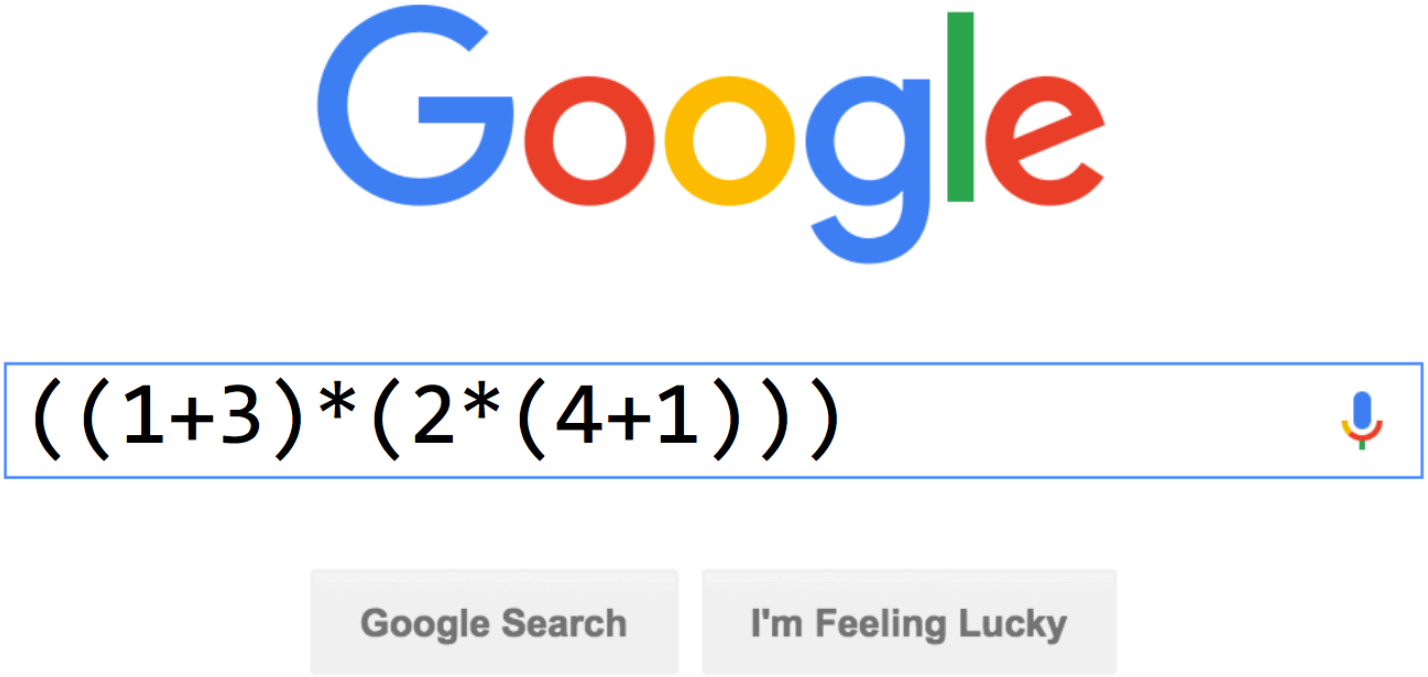
Three Musts of Recursion 🔑

1. Your code must have a case for all valid inputs

2. You must have a base case that makes no recursive calls

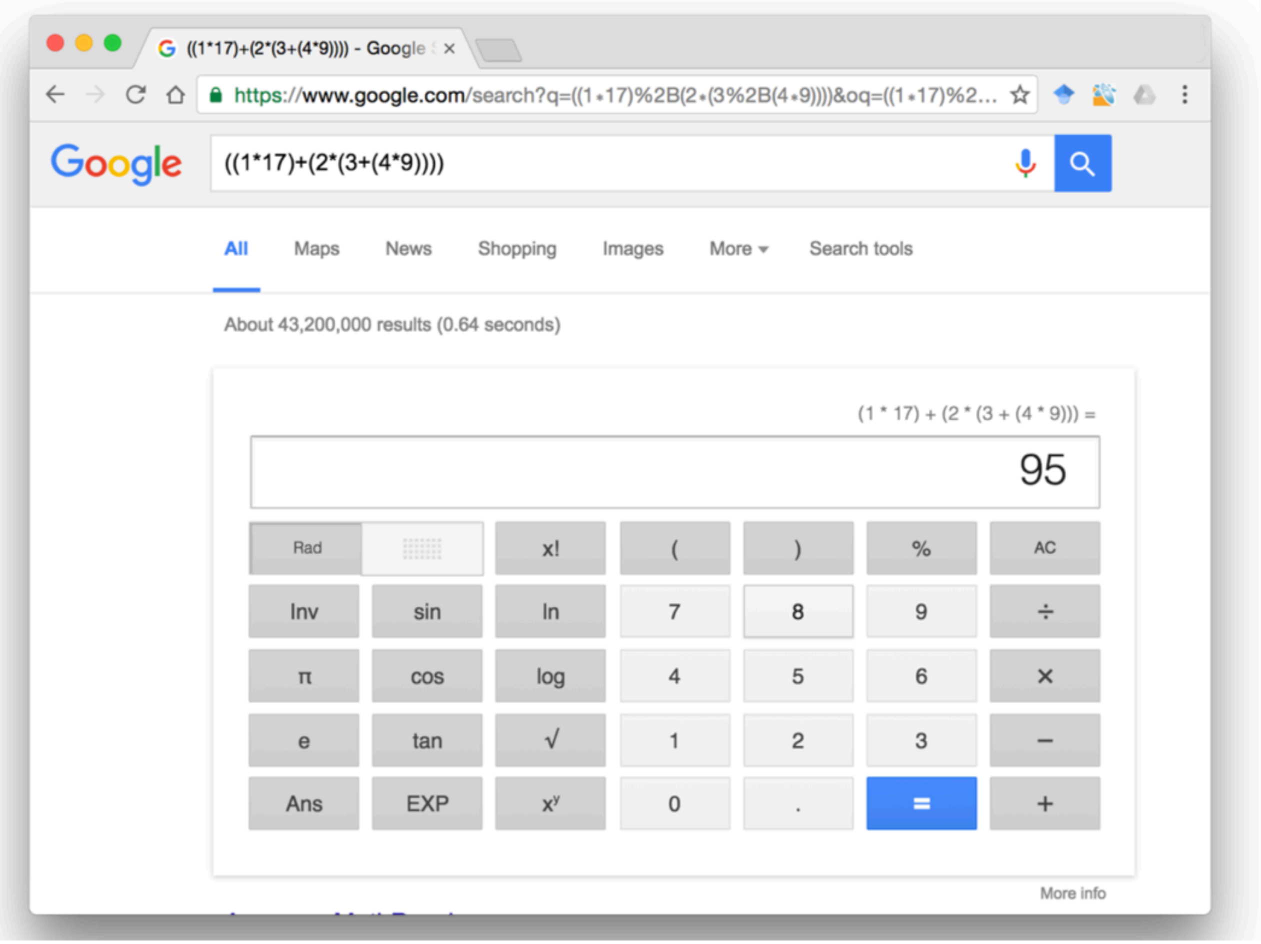
3. When you make a recursive call it should be to a simpler instance and make forward progress towards the base case.

Recursion Example



((1+3)*(2*(4+1)))

Recursion Example



((1*17)+(2*(3+(4*9)))) - Google

https://www.google.com/search?q=((1*17)%2B(2*(3%2B(4*9))))&oq=((1*17)%2...

((1*17)+(2*(3+(4*9))))

About 43,200,000 results (0.64 seconds)

(1 * 17) + (2 * (3 + (4 * 9))) =

95

More info

$$((1*17)+(2*(3+(4*9))))$$

$$\downarrow$$

$$95$$

Challenge

Implement a function which evaluates an expression string:

"((1+3)*(2*(4+1)))"

"(7+6)"

"(((4*(1+2))+6)*7)"

(only needs to implement * or +)

Anatomy of an Expression

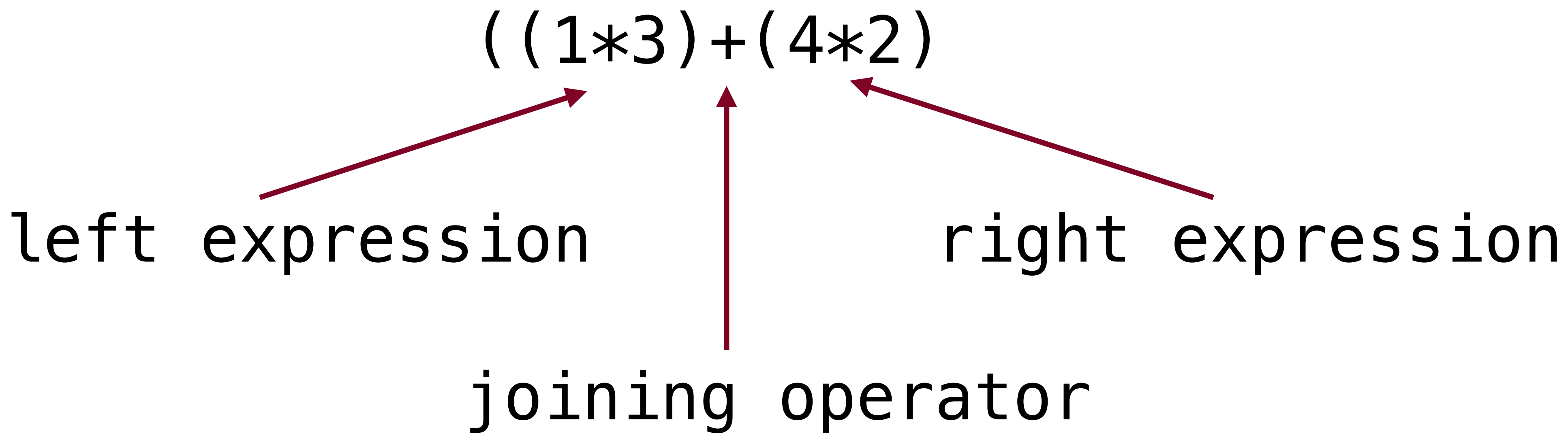
An expression is always one of these three things

```
               number

expression     (expression + expression)

               (expression * expression)
```

Anatomy of an Expression



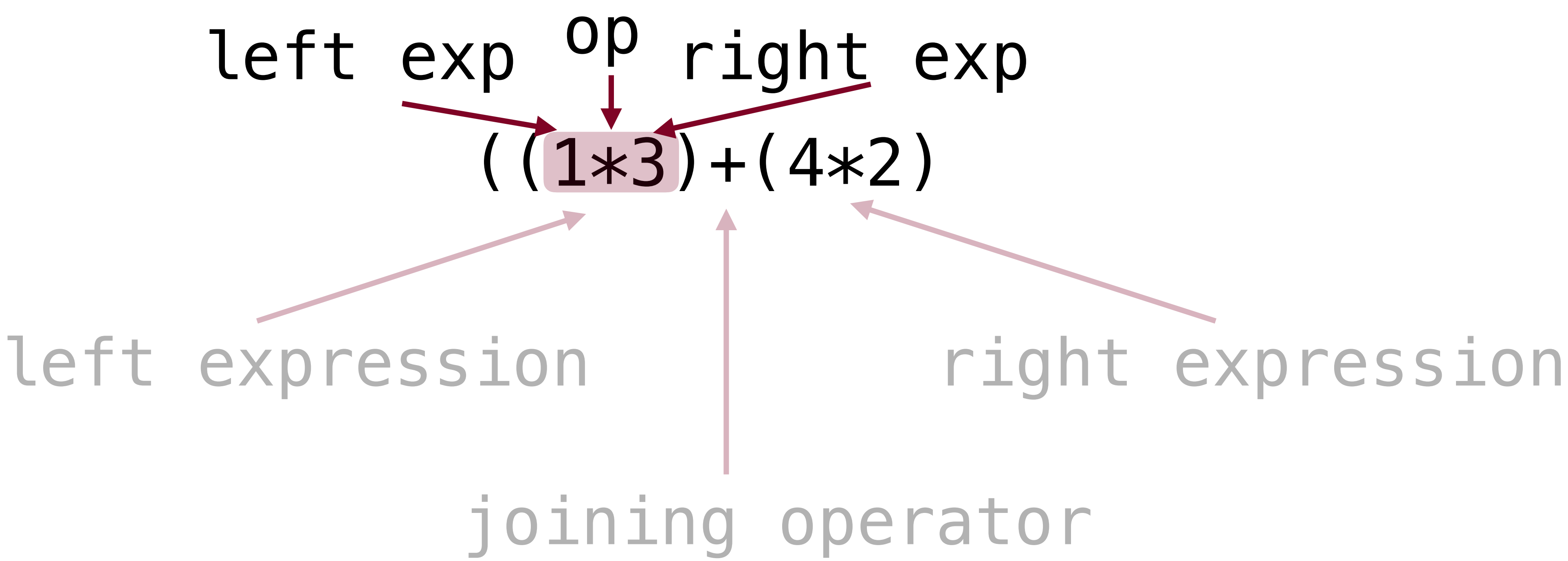
((1*3)+(4*2))

left expression

joining operator

right expression

Anatomy of an Expression



left exp

op

right exp

```
((1*3)+(4*2))
```

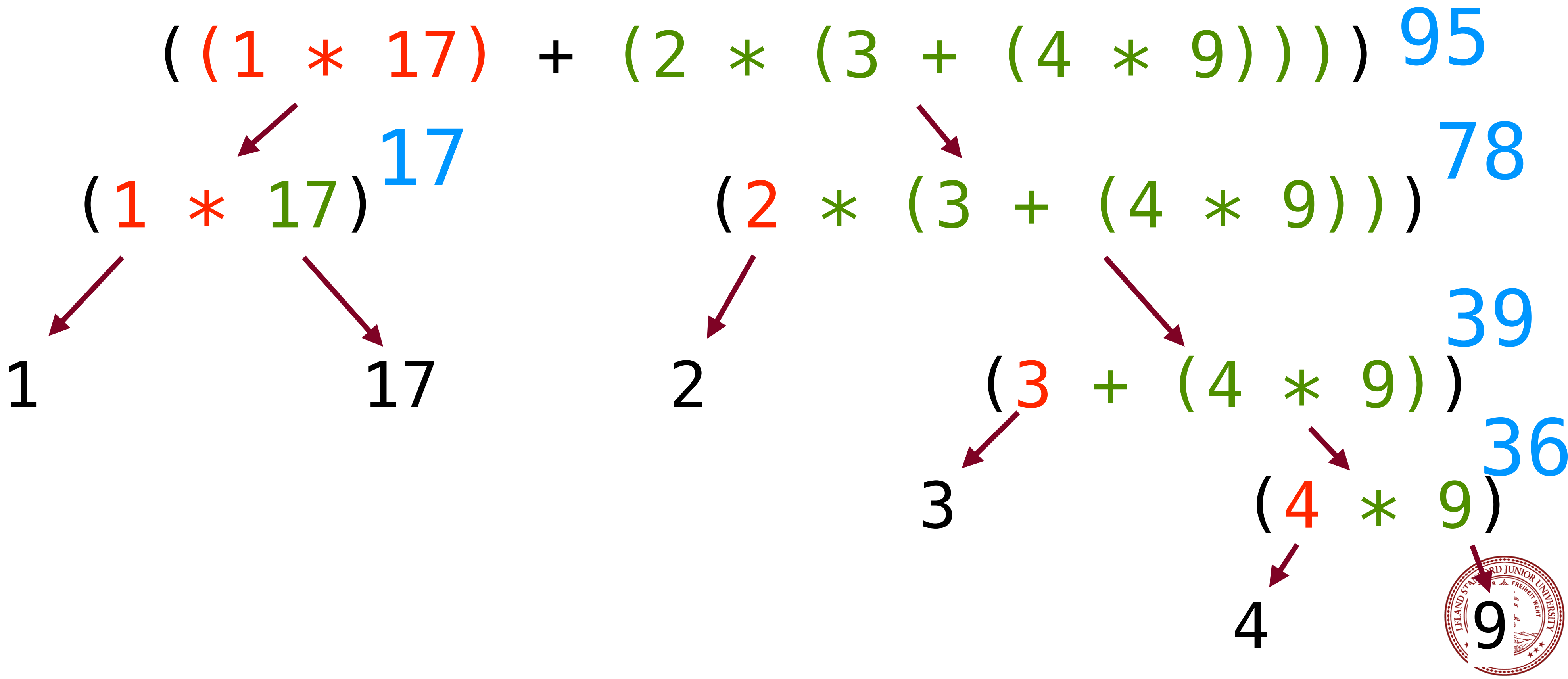
left expression

right expression

joining operator

Anatomy of an Expression

How do we evaluate ((1*17)+(2*(3+(4*9))))?



((1 * 17) + (2 * (3 + (4 * 9)))) 95

(1 * 17) 17

(2 * (3 + (4 * 9))) 78

1

17

2

(3 + (4 * 9)) 39

3

(4 * 9) 36

4

9

Is it Recursive? Yes!

((1*3)+(4+2))

The big instance of this problem is:

((1*3)+(4+2))

The smaller instances are:

(1*3) and (4+2)

Task

Write this function:

```
int evaluate(string exp);

"((1*3)+(4+2))" // returns 9
```

Using these library functions:

```
stringIsInteger(exp)
stringToInteger(exp)
```

And these exp helper functions:

```
//returns ‘+’
char op = getOperator(exp);
//returns “(1*3)”
string left = getLeftExp(exp);
//returns “(4+2)”
string right = getRightExp(exp);
```

Solution (Pseudocode)

`"((1*3)+(4+2))"`

```
int evaluate(expression):

• if expression is a number, return expression
• Otherwise, break up expression by its operator:
  • leftResult = evaluate(leftExpression)
  • rightResult = evaluate(rightExpression)
  • return leftResult operator rightResult
```

Solution

```
int evaluate(string exp) {
    if (stringIsInteger(exp)) {
        return stringToInteger(exp);
    } else {
        char op = getOperator(exp);
        string left = getLeftExp(exp);
        string right = getRightExp(exp);
        int leftResult = evaluate(left);
        int rightResult = evaluate(right);
        if (op == '+') {
            return leftResult + rightResult;
        } else if (op == '*') {
            return leftResult * rightResult;
        }
    }
}
```

exp = "((1*3)+(4*5)+2)"

op = '+'

left = "(1*3)"

right = "((4*5)+2)"

leftResult = 3

rightResult = 22

Helper Methods

Here is the key function behind the helper methods:

```
int getOppIndex(string exp){
    int parens = 0;
    // ignore first left paren
    for (int i = 1; i < exp.length(); i++) {
        char c = exp[i];
        if (c == '(') {
            parens++;
        } else if (c == ')') {
            parens--;
        }
        if (parens == 0 && (c == '+' || c == '*')) {
            return i;
        }
    }
}
```

By the way...

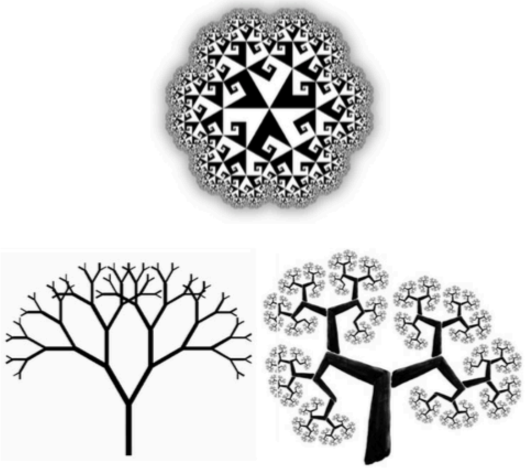
We could also have solved this with a stack!

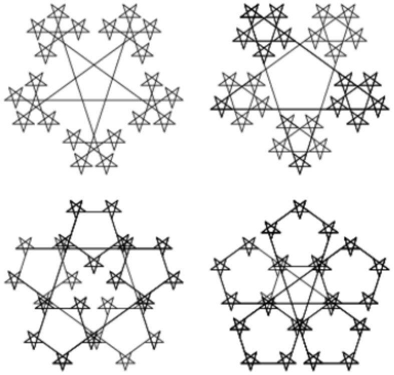
Today

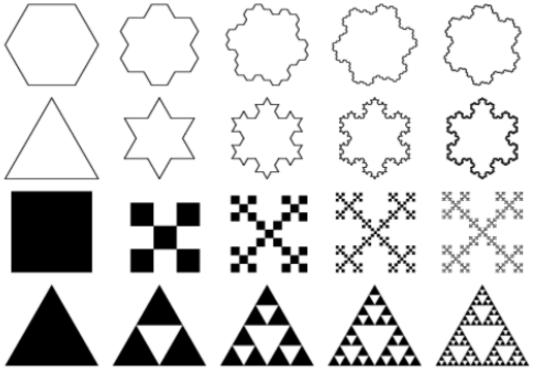
Recursion you can see

Fractal

fractal: A recurring graphical pattern. Smaller instances of the same shape or pattern occur within the pattern itself.



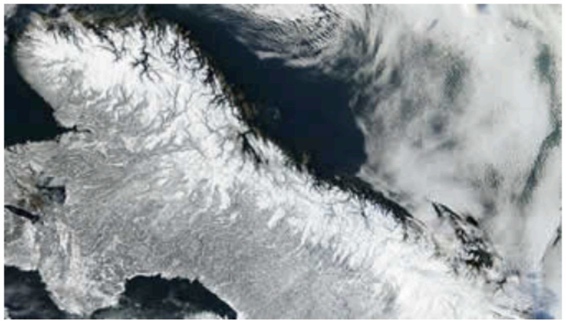




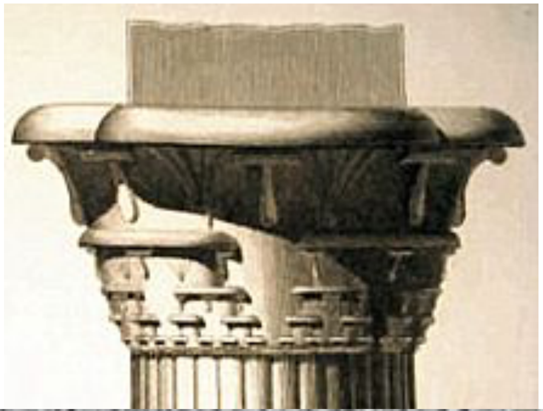
Fractal

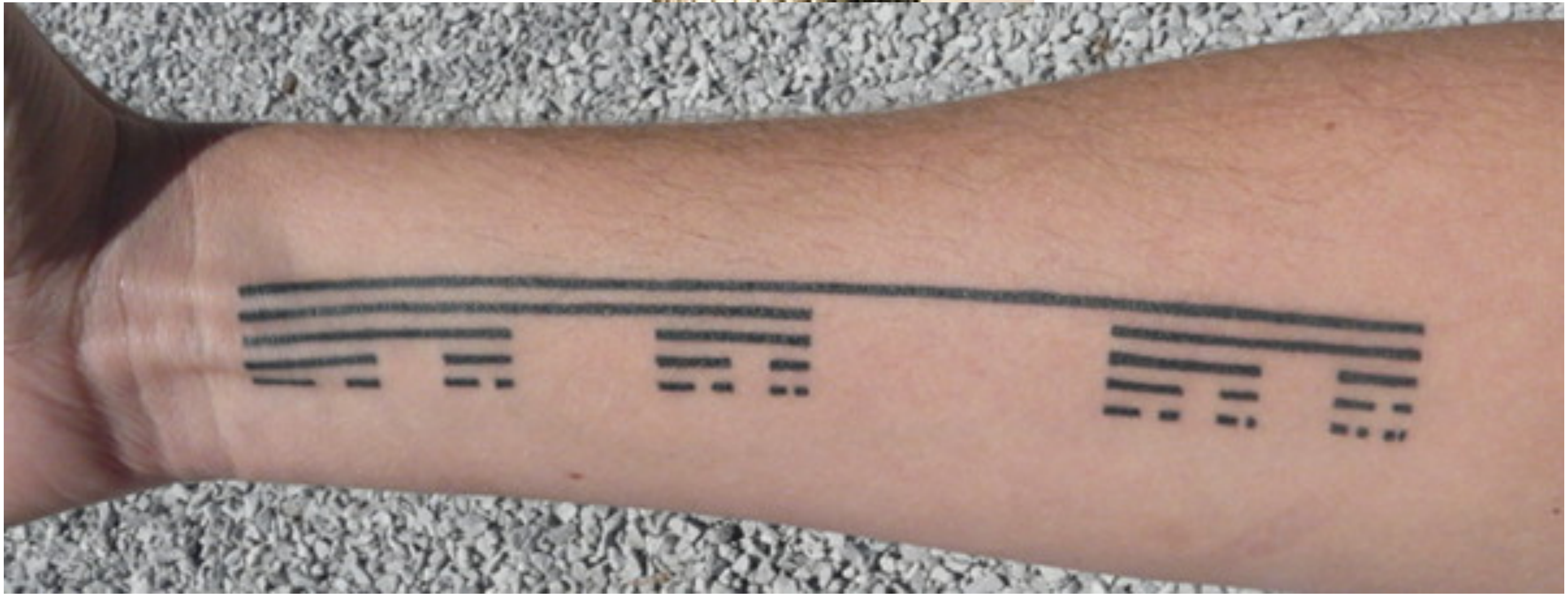
Many natural phenomena generate fractal patterns:

1. earthquake fault lines
2. animal color patterns
3. clouds
4. mountain ranges
5. snowflakes
6. crystals
7. DNA
8. ...

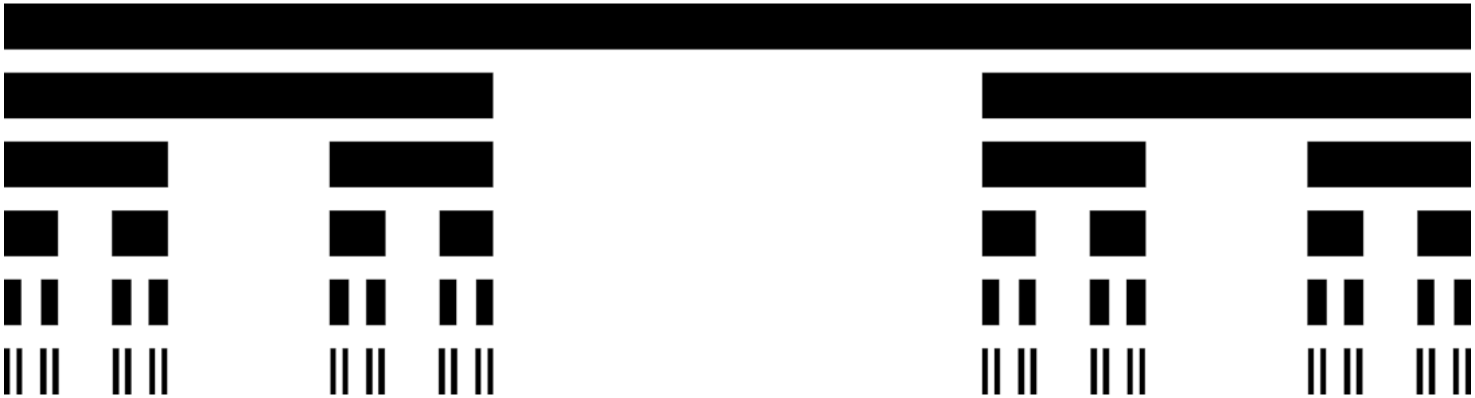


The Cantor Fractal



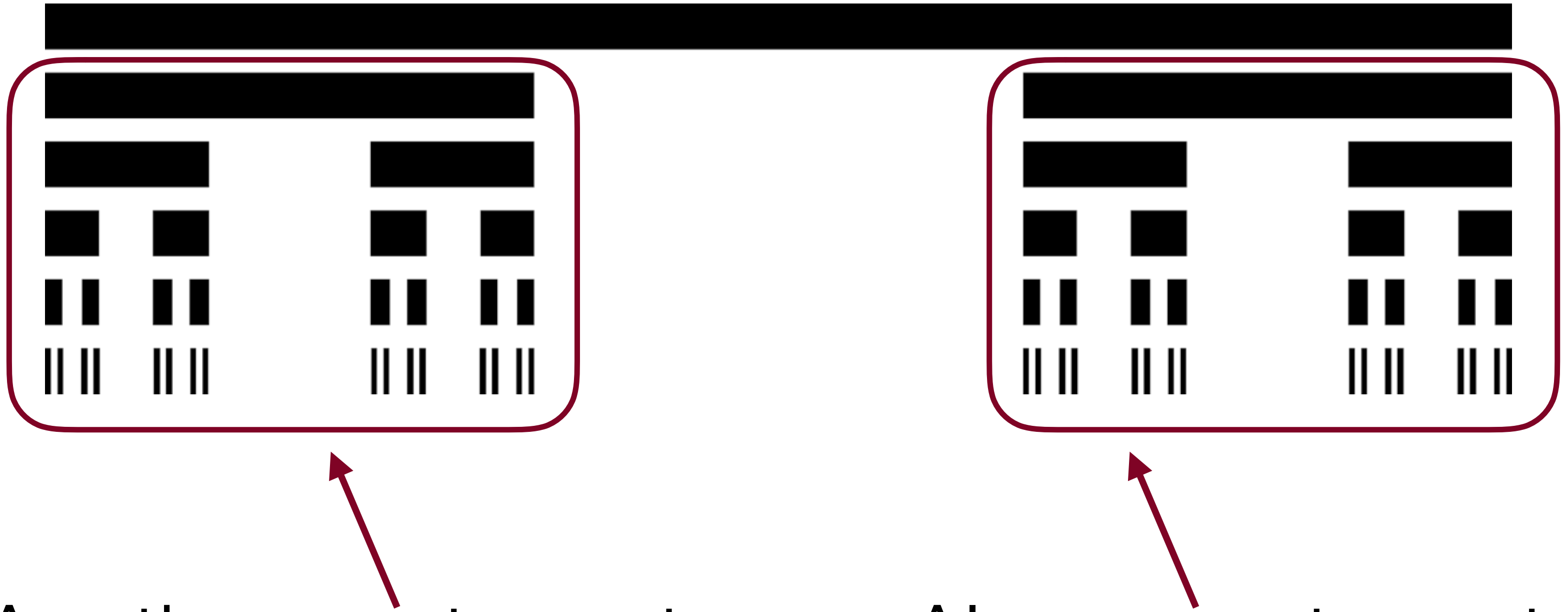


Cantor Fractal



Parts of a cantor set image … are Cantor set images

Cantor Fractal



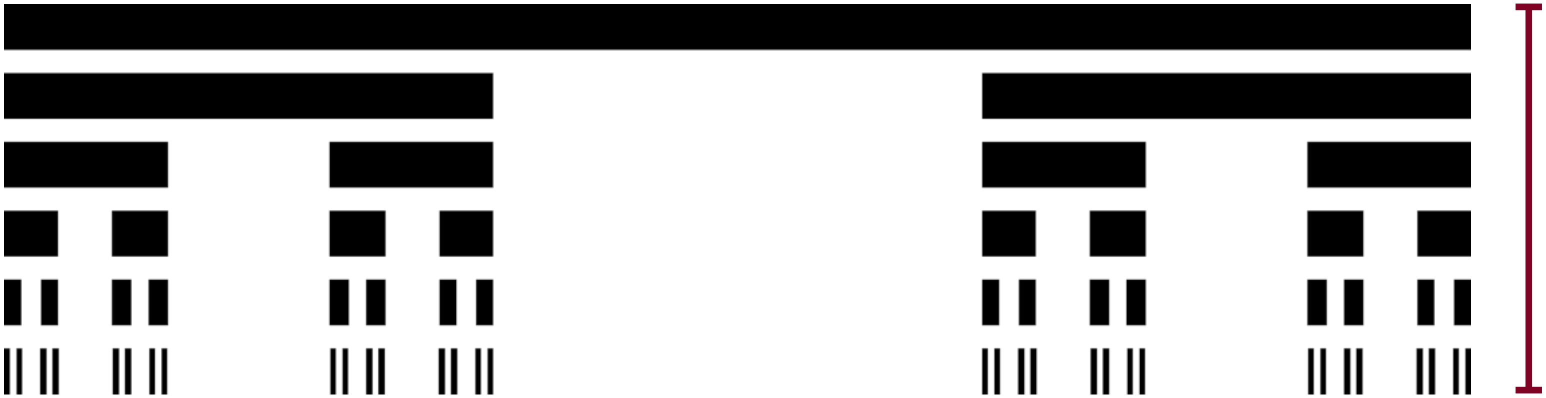
Start

End

Another cantor set

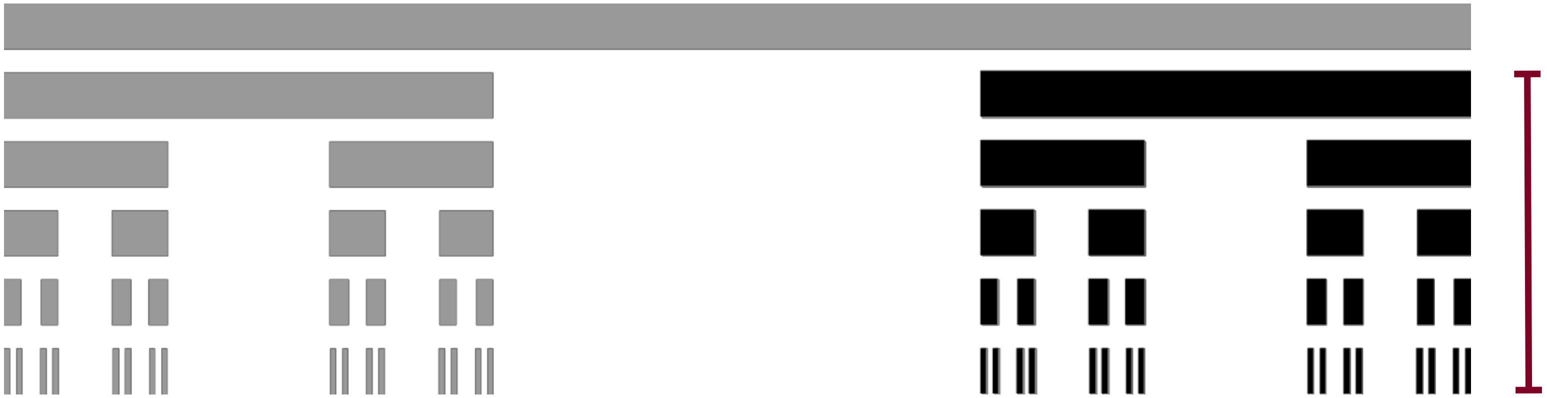
Also a cantor set

Levels of Cantor



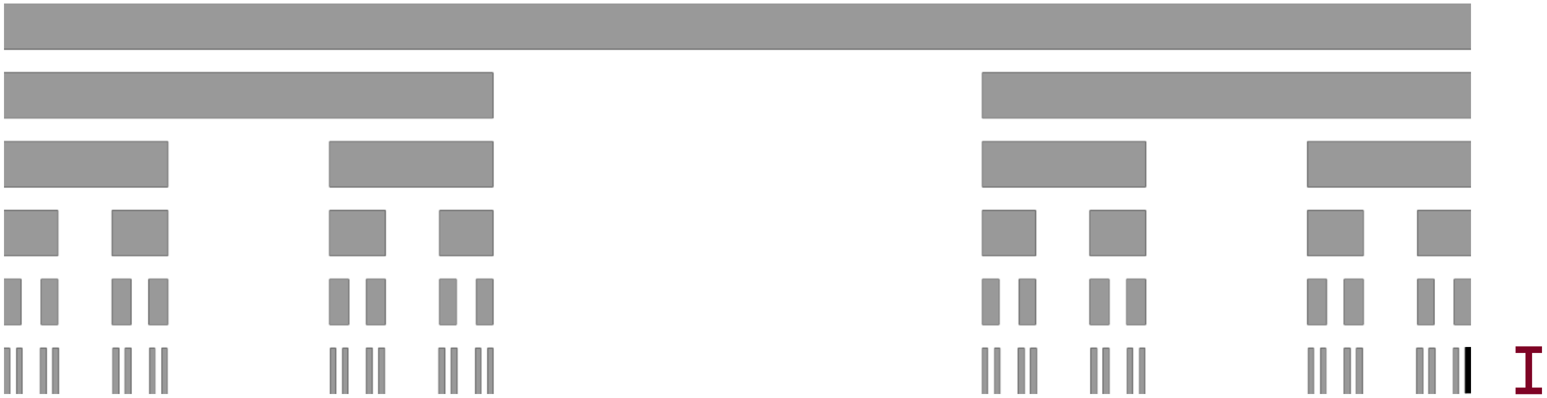
6 levels

Levels of Cantor



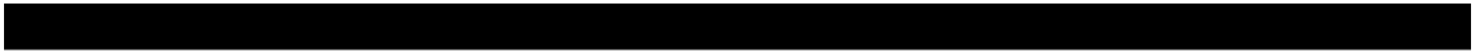
5 levels

Levels of Cantor



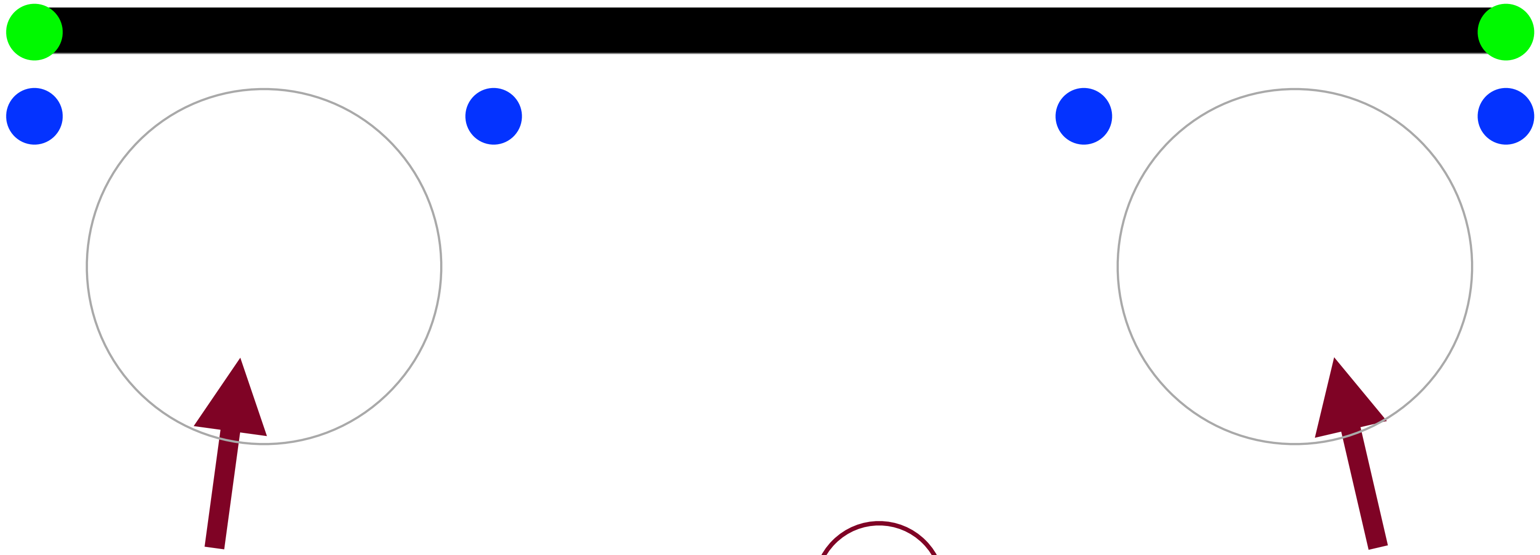
1 level

How to Draw a Level 1 Cantor



How to Draw a Level *n* Cantor

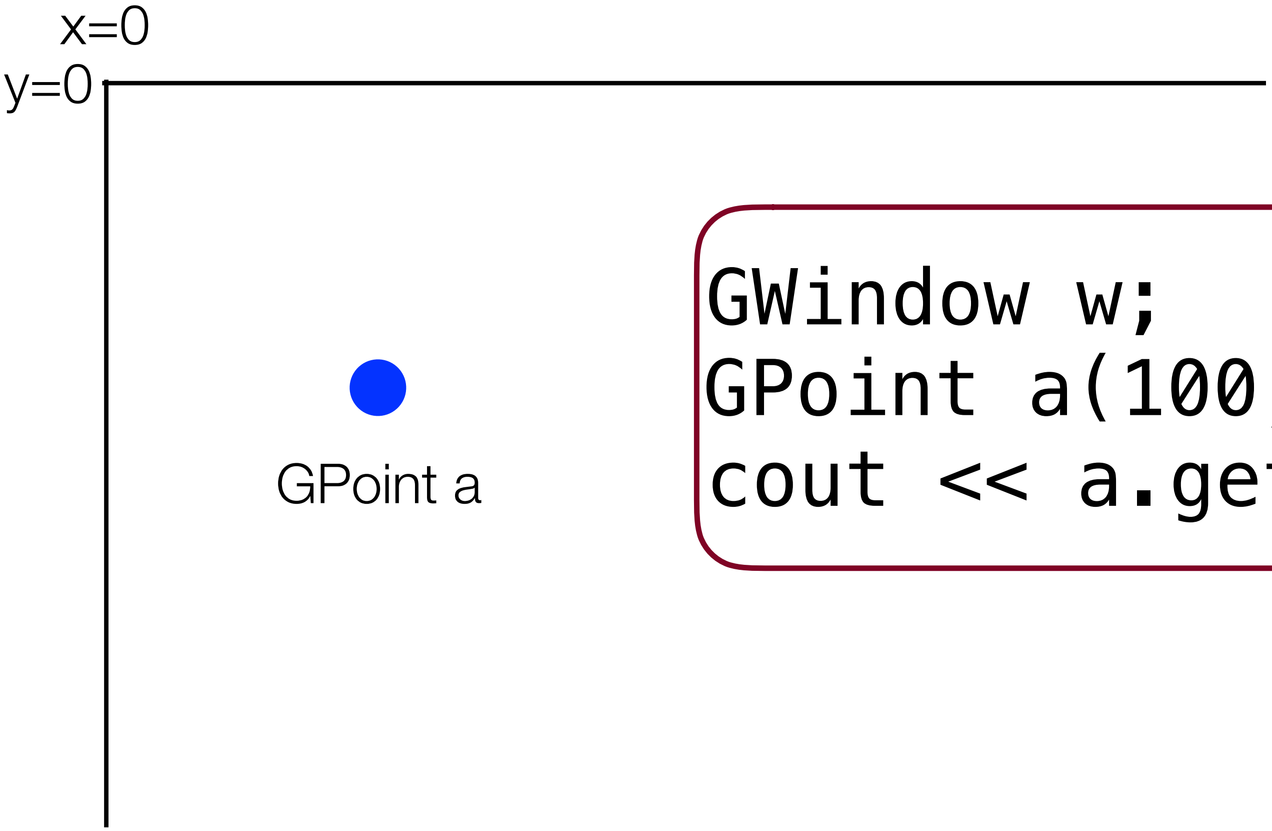
1 Draw a line from start to finish.



2 Draw a Cantor of size *n*-1

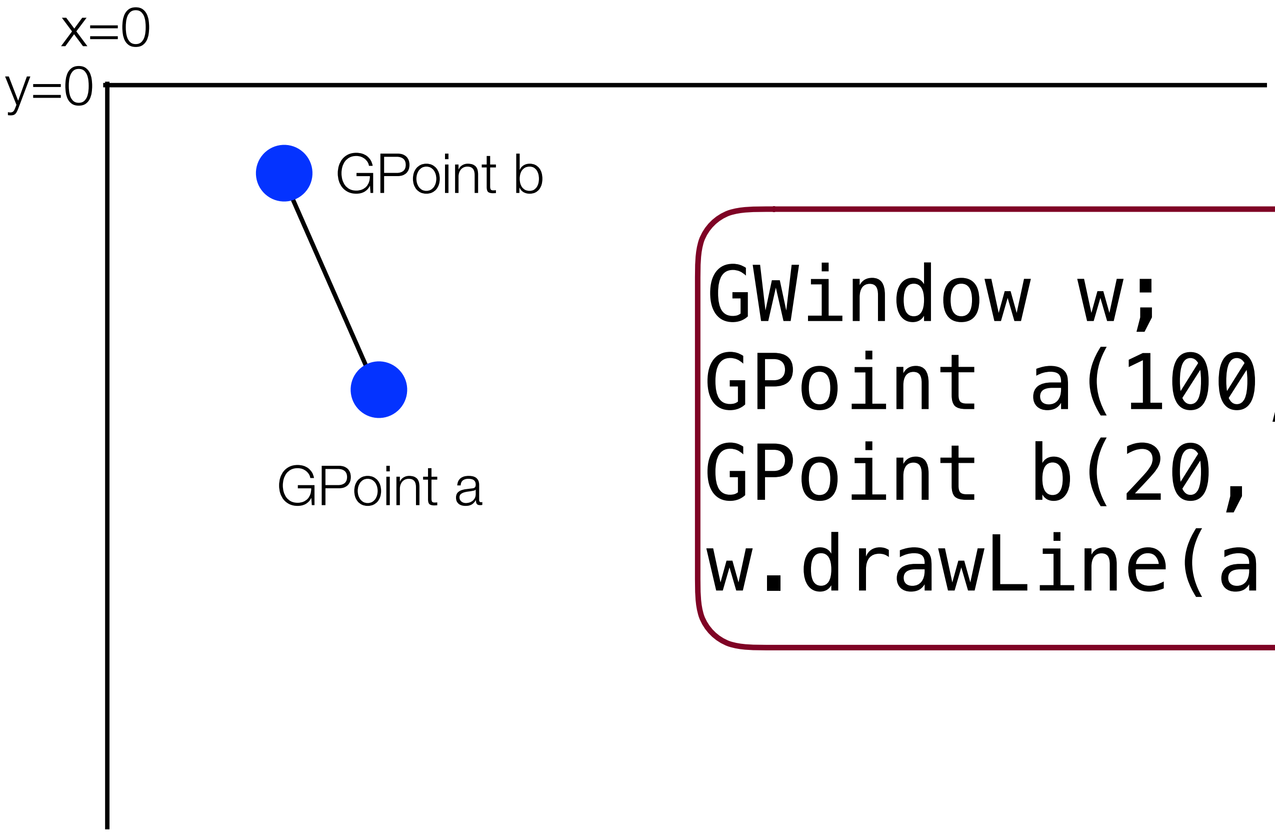
2 Draw a Cantor of size *n*-1

Graphics in C++ with the Stanford Libs: GPoint



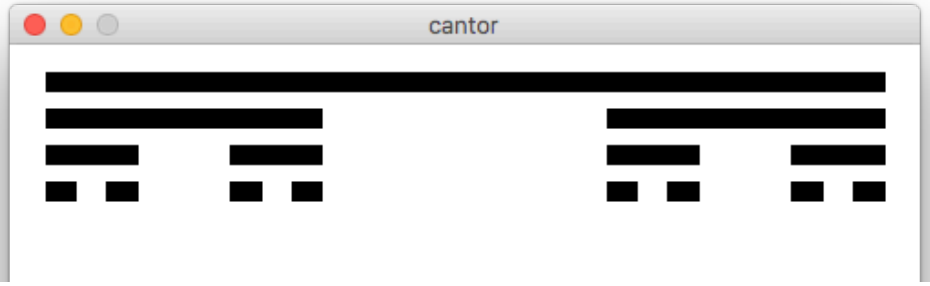
```
GWindow w;
GPoint a(100, 100);
cout << a.getX() << endl;
```

Graphics in C++ with the Stanford Libs: GPoint

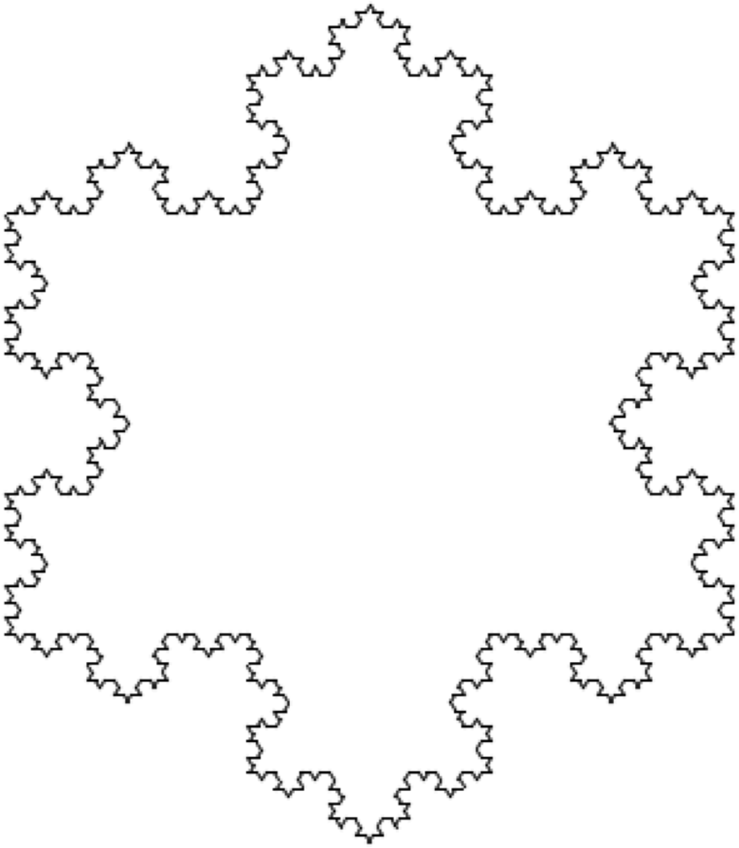


```
GWindow w;
GPoint a(100, 100);
GPoint b(20, 20);
w.drawLine(a, b);
```

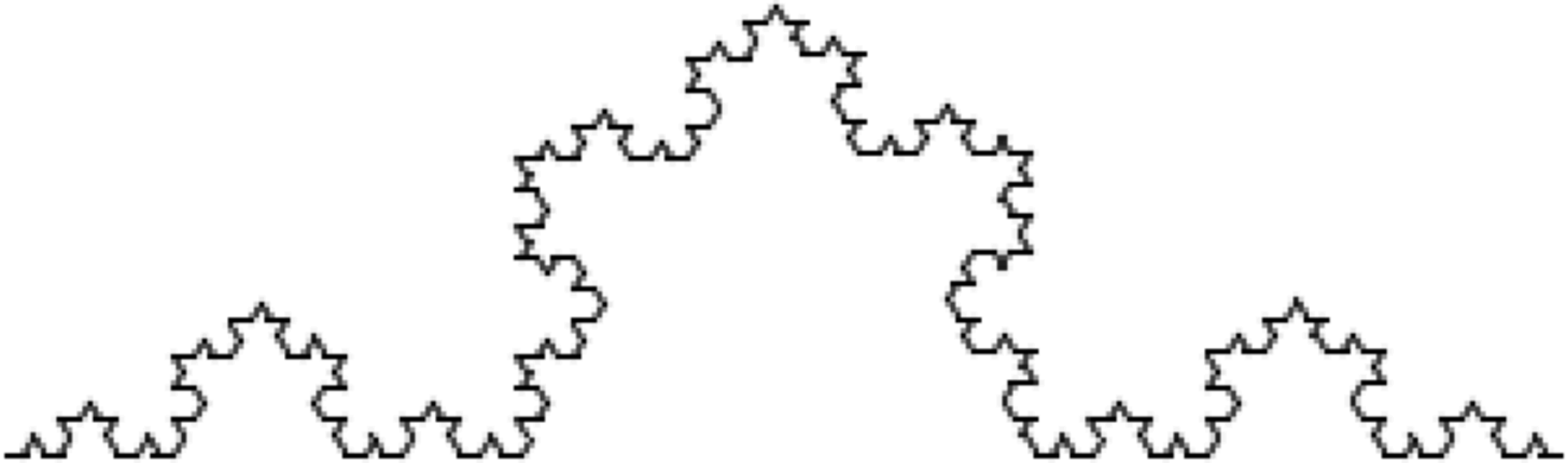
Cantor Fractal



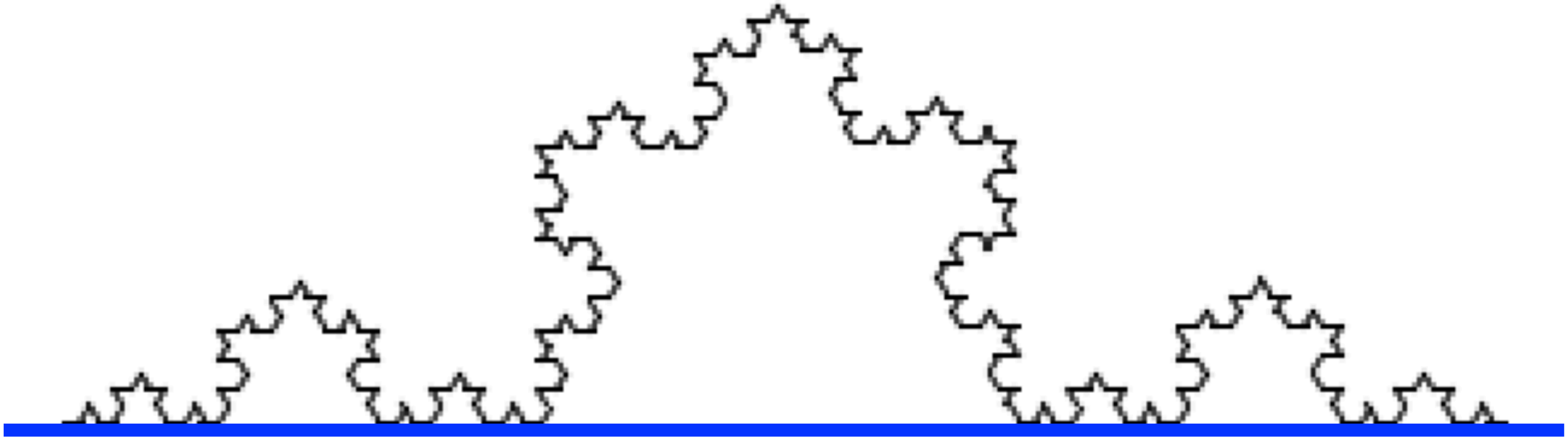
Snoflake Fractal



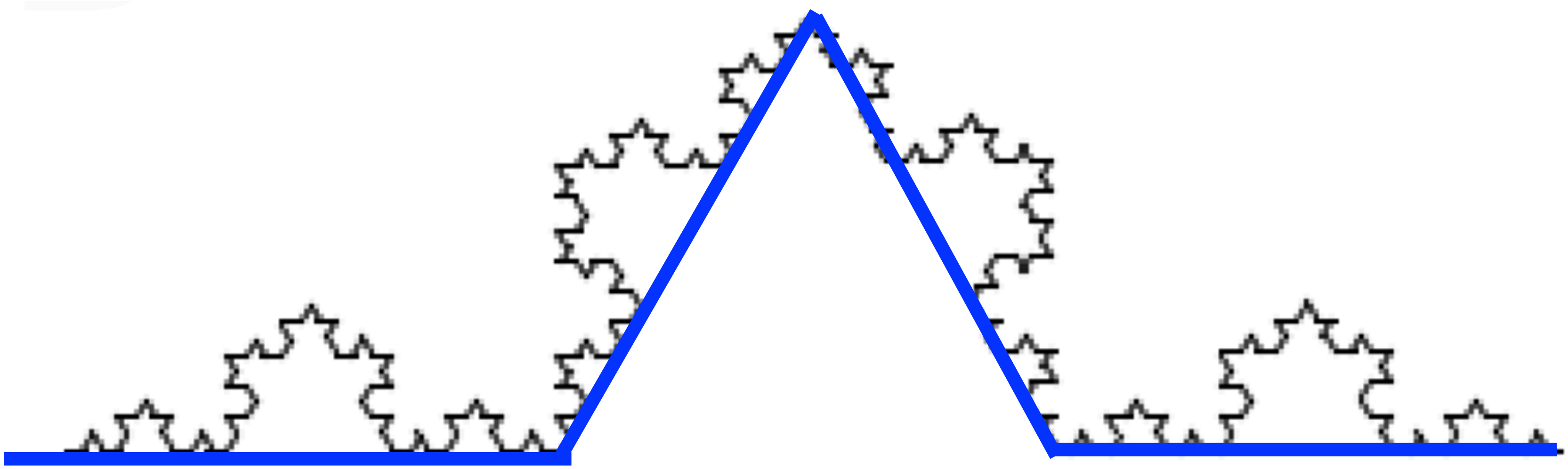
Snowflake Fractal



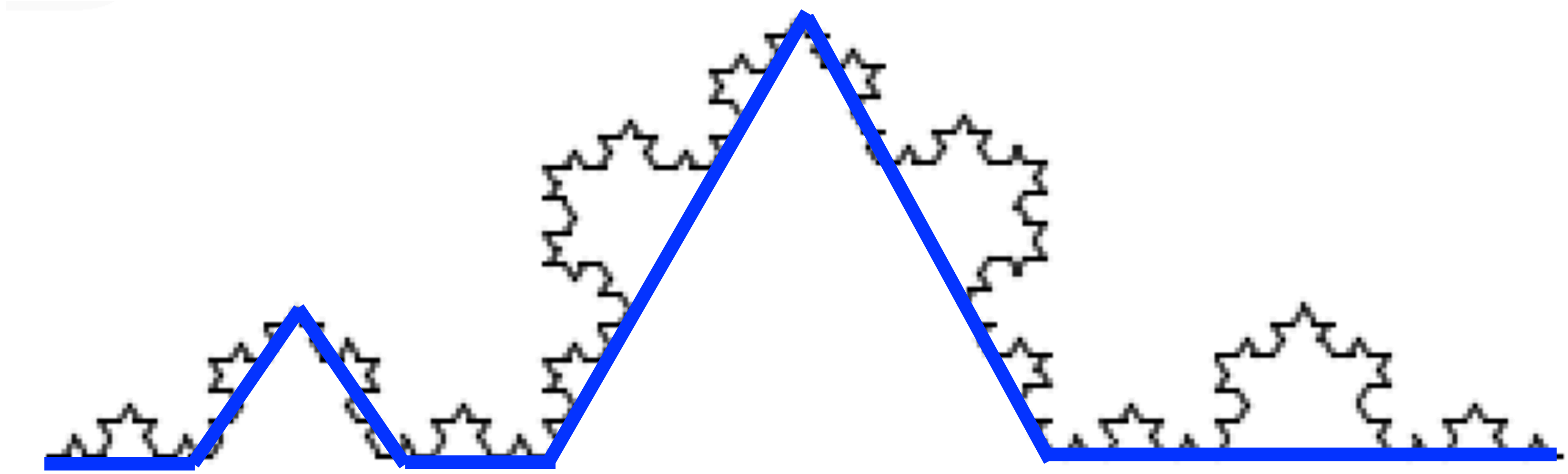
Depth 1 Snowflake Line



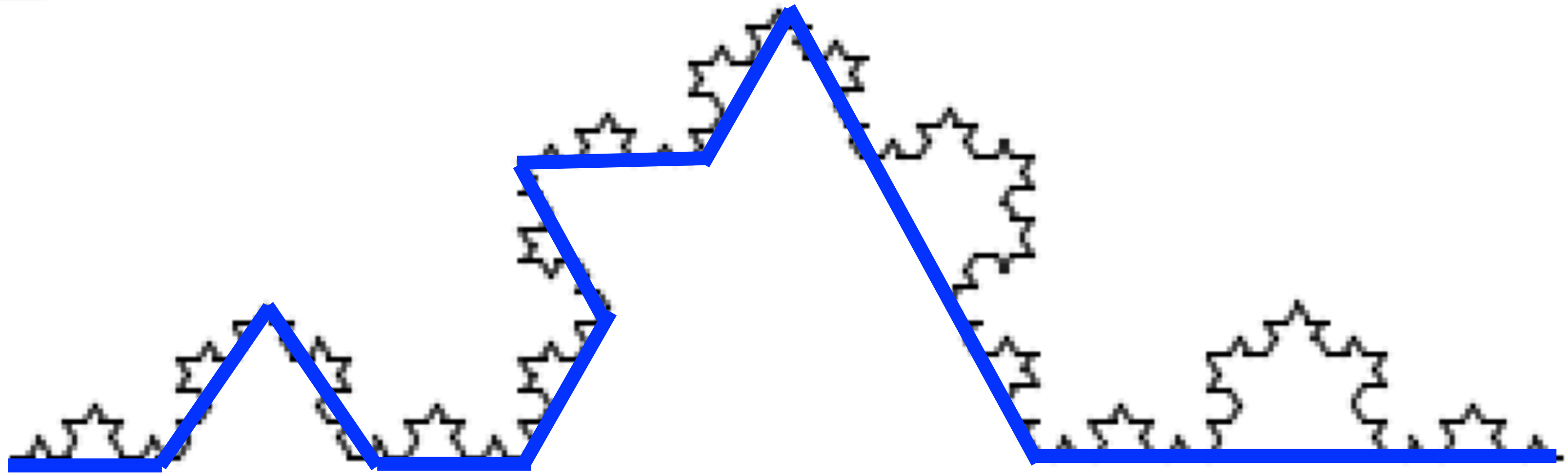
Depth 2 Snowflake Line



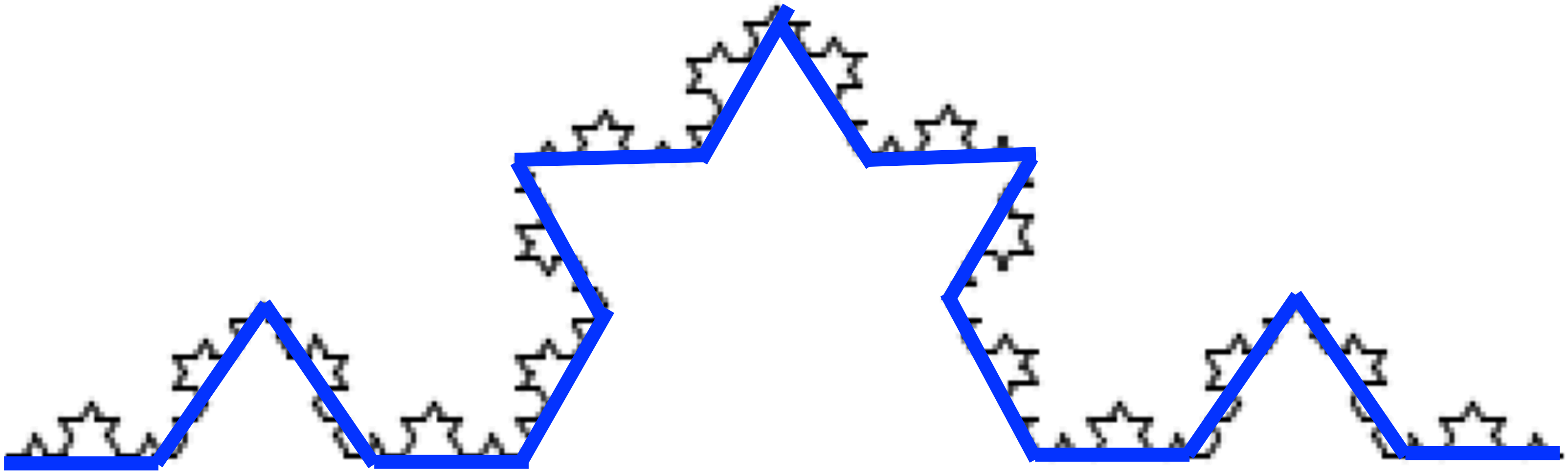
Depth 3 Snowflake Line (in progress)



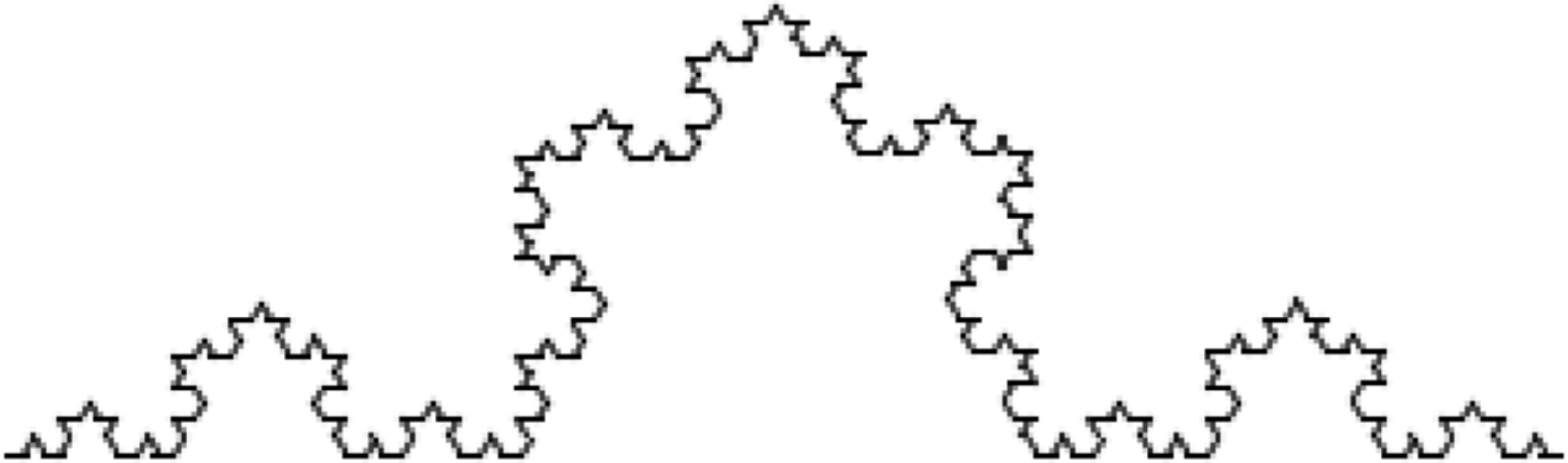
Depth 3 Snowflake Line (in progress)



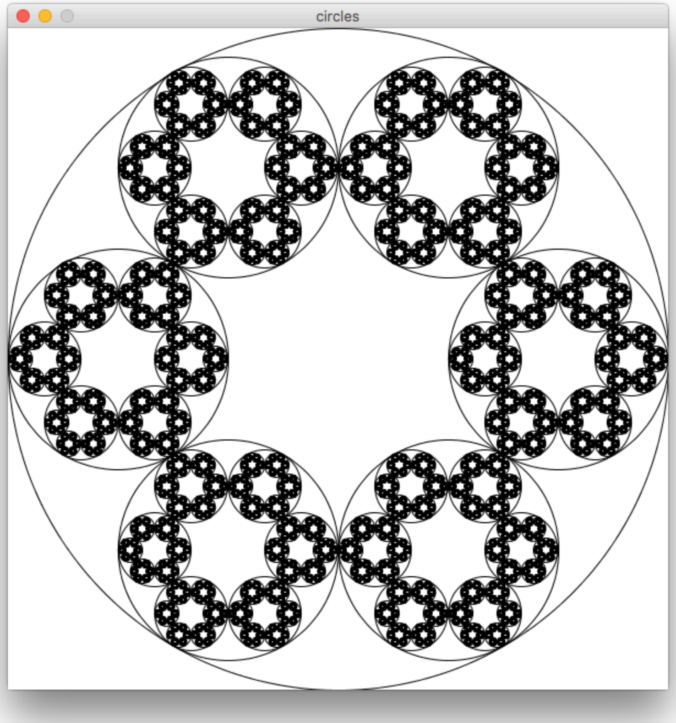
Depth 3 Snowflake Line (in progress)



Depth 3 Snowflake Line (in progress)



Another Example On the Website



Recap

- **Fractals**
  - Fractals are self-referential, and that makes for nice recursion problems!
  - Break the problem into a smaller, self-similar part, and don't forget your base case!

References and Advanced Reading

- **References:**
  - http://www.cs.utah.edu/~germain/PPS/Topics/recursion.html
  - Why is iteration generally better than recursion? http://stackoverflow.com/a/3093/561677

- **Advanced Reading:**
  - Tail recursion: http://stackoverflow.com/questions/33923/what-is-tail-recursion
  - Interesting story on the history of recursion in programming languages: http://goo.gl/P6Einb

Extra Slides