

CS 106B

Lecture 18: Trees

Tuesday, August 1, 2017

Programming Abstractions
Summer 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:

Programming Abstractions in C++, Section 16.1



Today's Topics

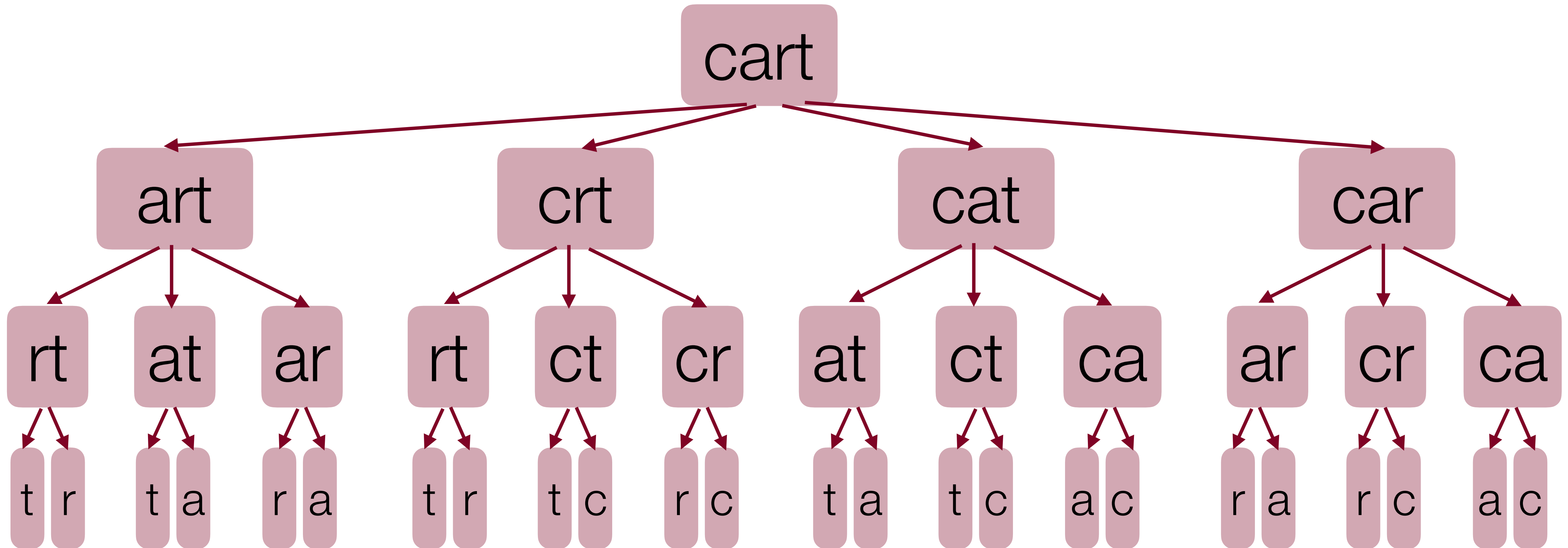
- Logistics
 - Chris's Office Hours cancelled from 5-6pm tonight (sorry!)
 - Additional office hours Wednesday morning, 9-10am
 - A note on the honor code.

- Introduction to Trees



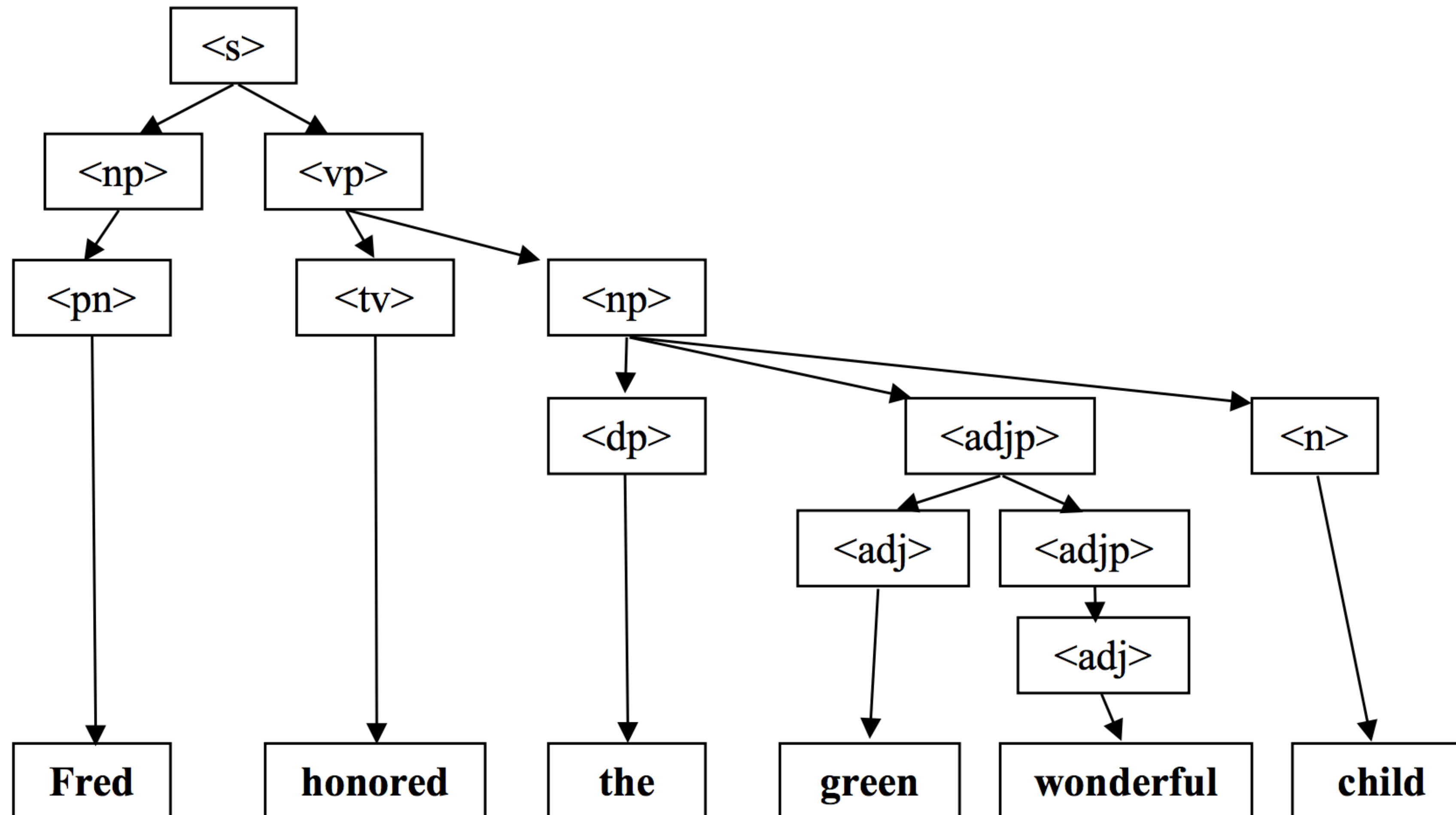
Trees

We have already seen trees in the class in the form of decision trees!



Trees

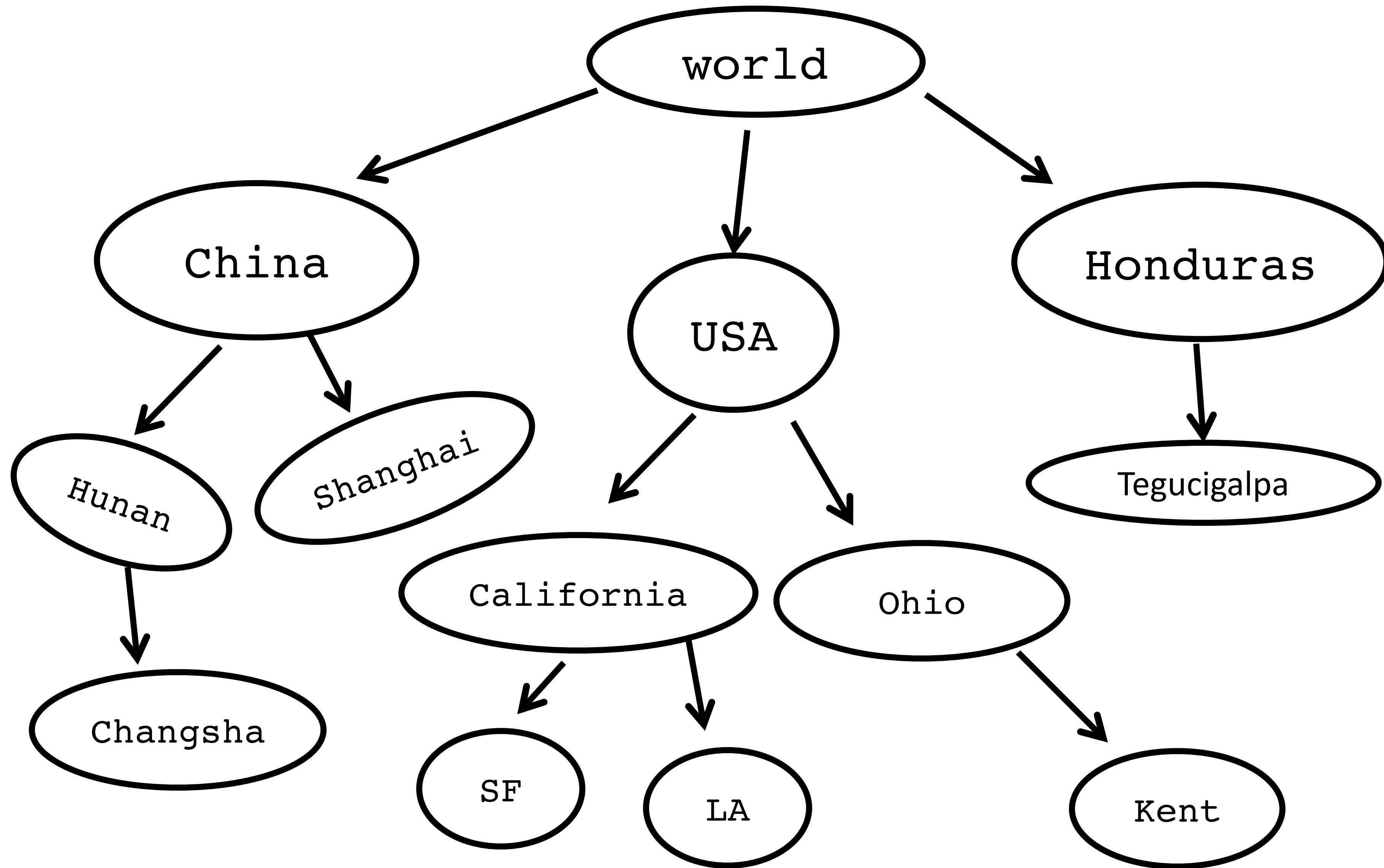
You've coded trees for recursive assignments!



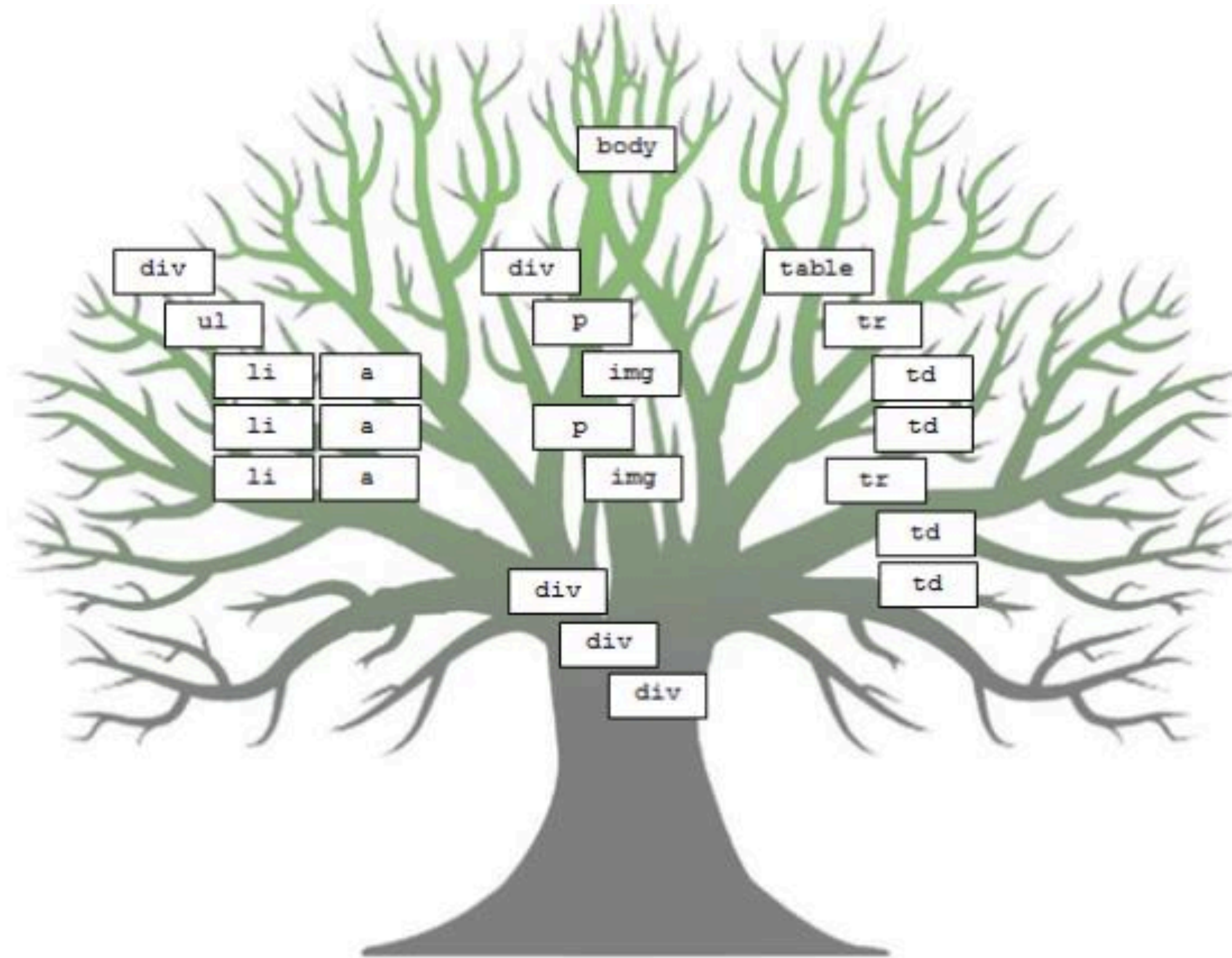
Random expansion from sentence.txt grammar for symbol "<s>"



Trees Can Describe Hierarchies

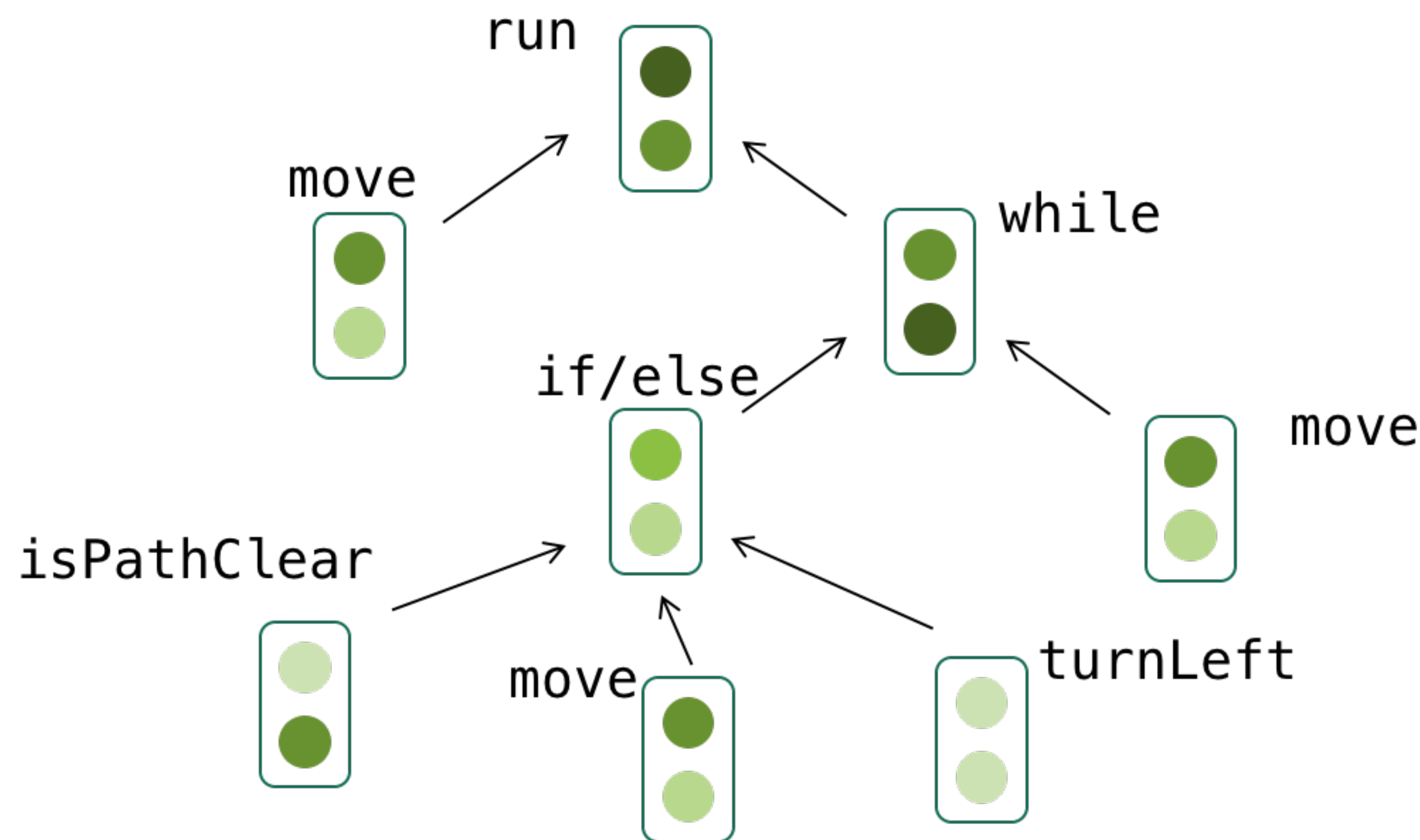


Trees Can Describe Websites (HTML)



Trees Can Describe Programs

```
// Example student solution
function run() {
  // move then loop
  move();
  // the condition is fixed
  while (notFinished()) {
    if (isPathClear()) {
      move();
    } else {
      turnLeft();
    }
  }
  // redundant
  move();
}
}
```



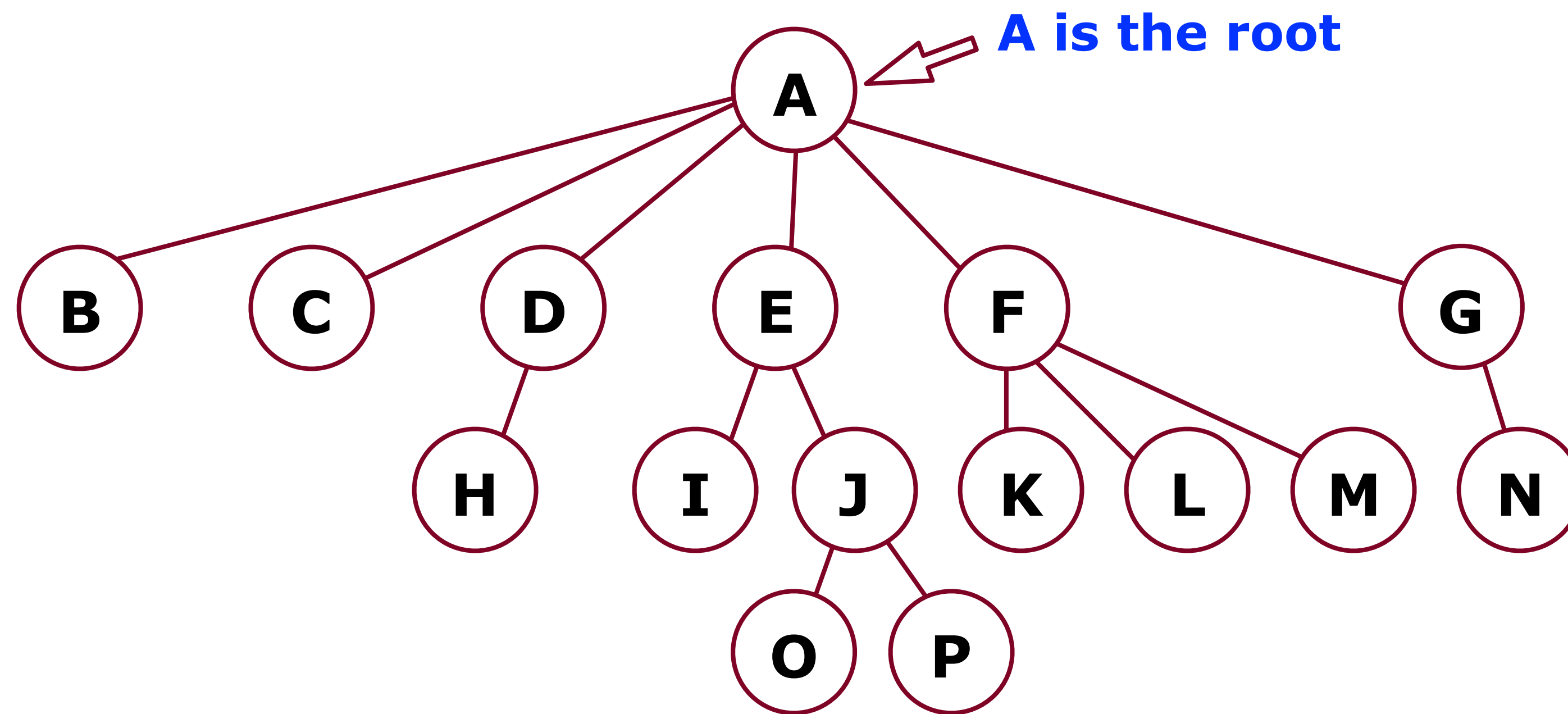
* This is a figure in an academic paper written by a recent CS106 student!



Trees are inherently recursive

What is a Tree (in Computer Science)?

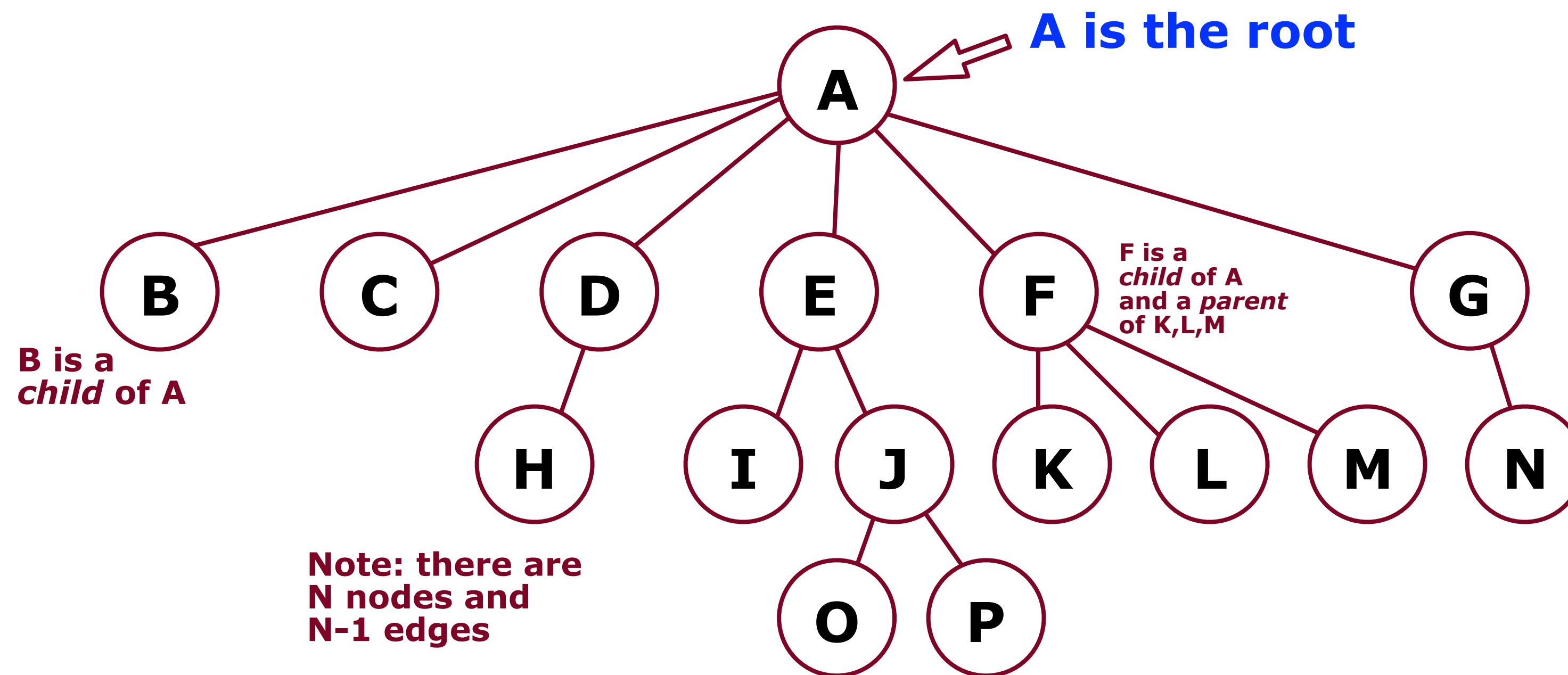
- A tree is a collection of **nodes**, which can be empty. If it is not empty, there is a “root” node, ***r***, and **zero or more non-empty subtrees**, **T_1, T_2, \dots, T_k** , whose roots are connected by a directed edge from ***r***.



Tree Terminology

What is a Tree (in Computer Science)?

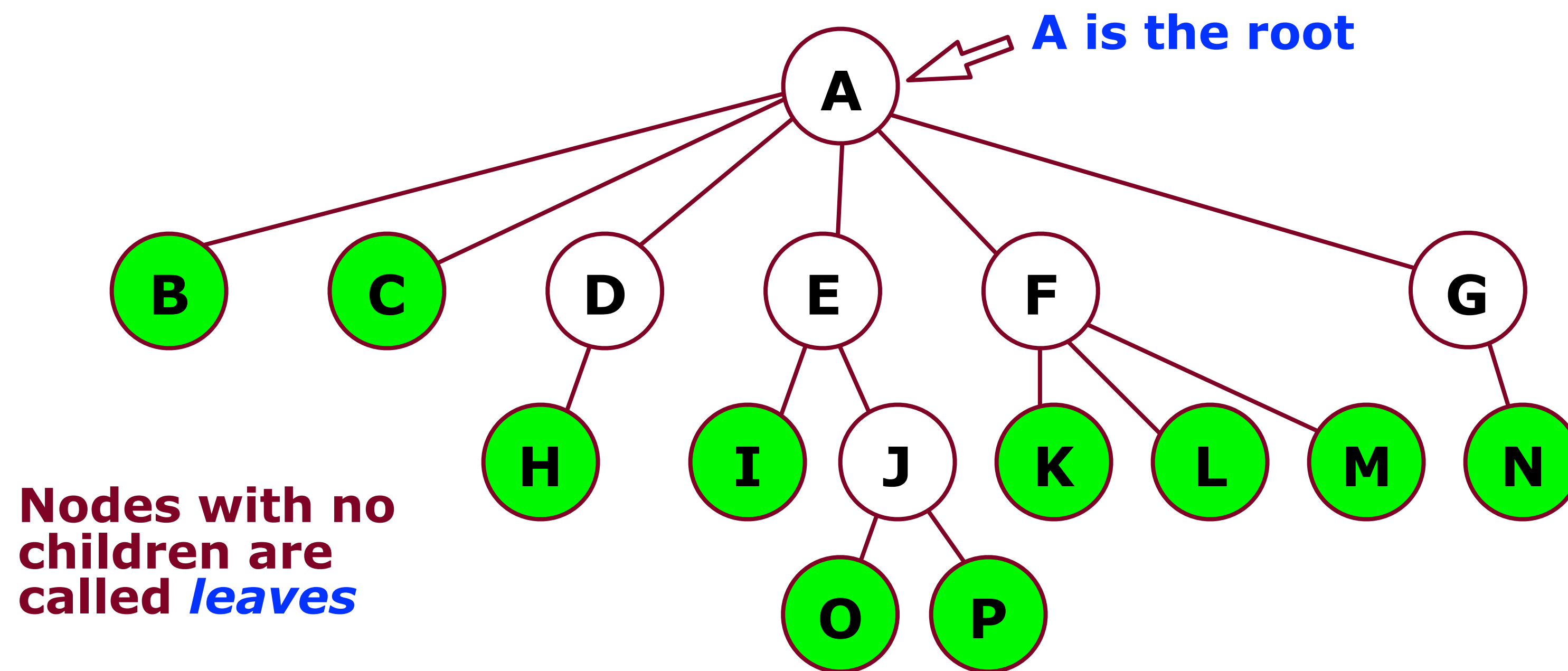
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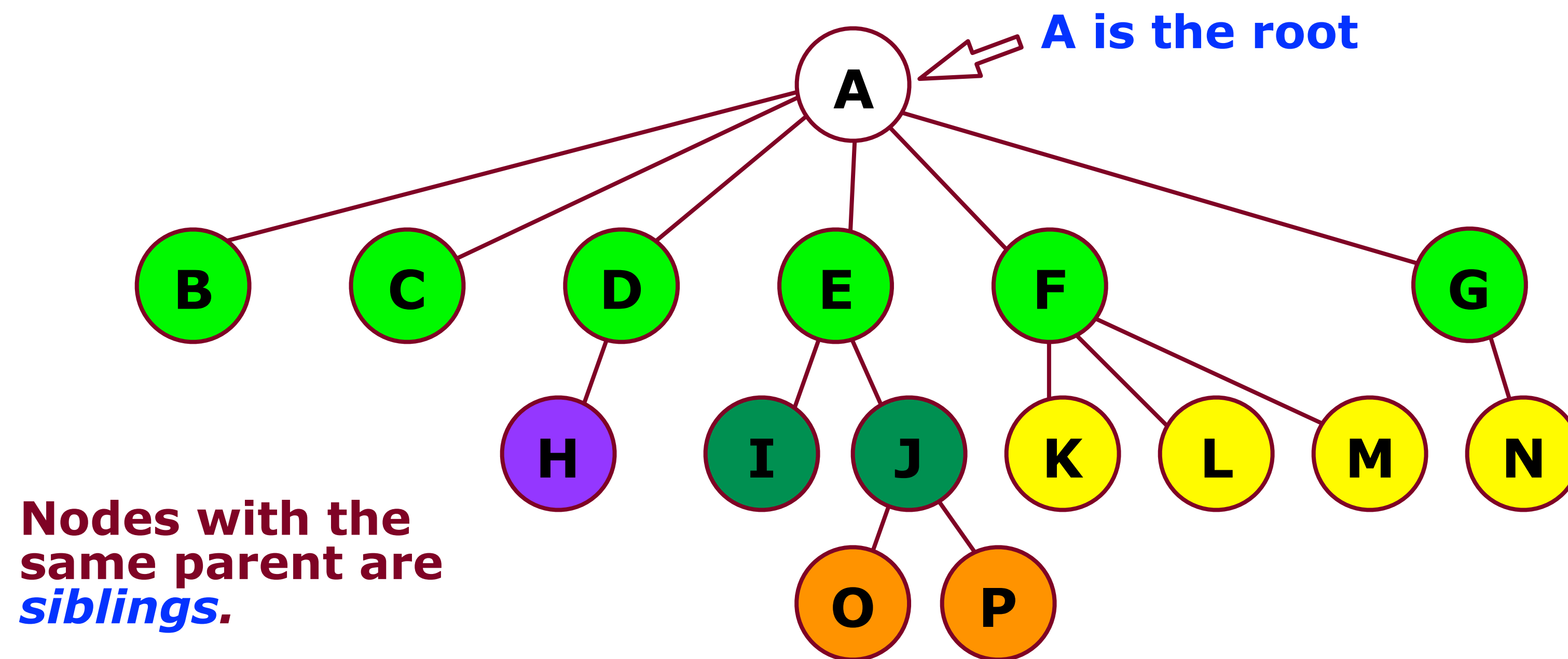
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Tree Terminology

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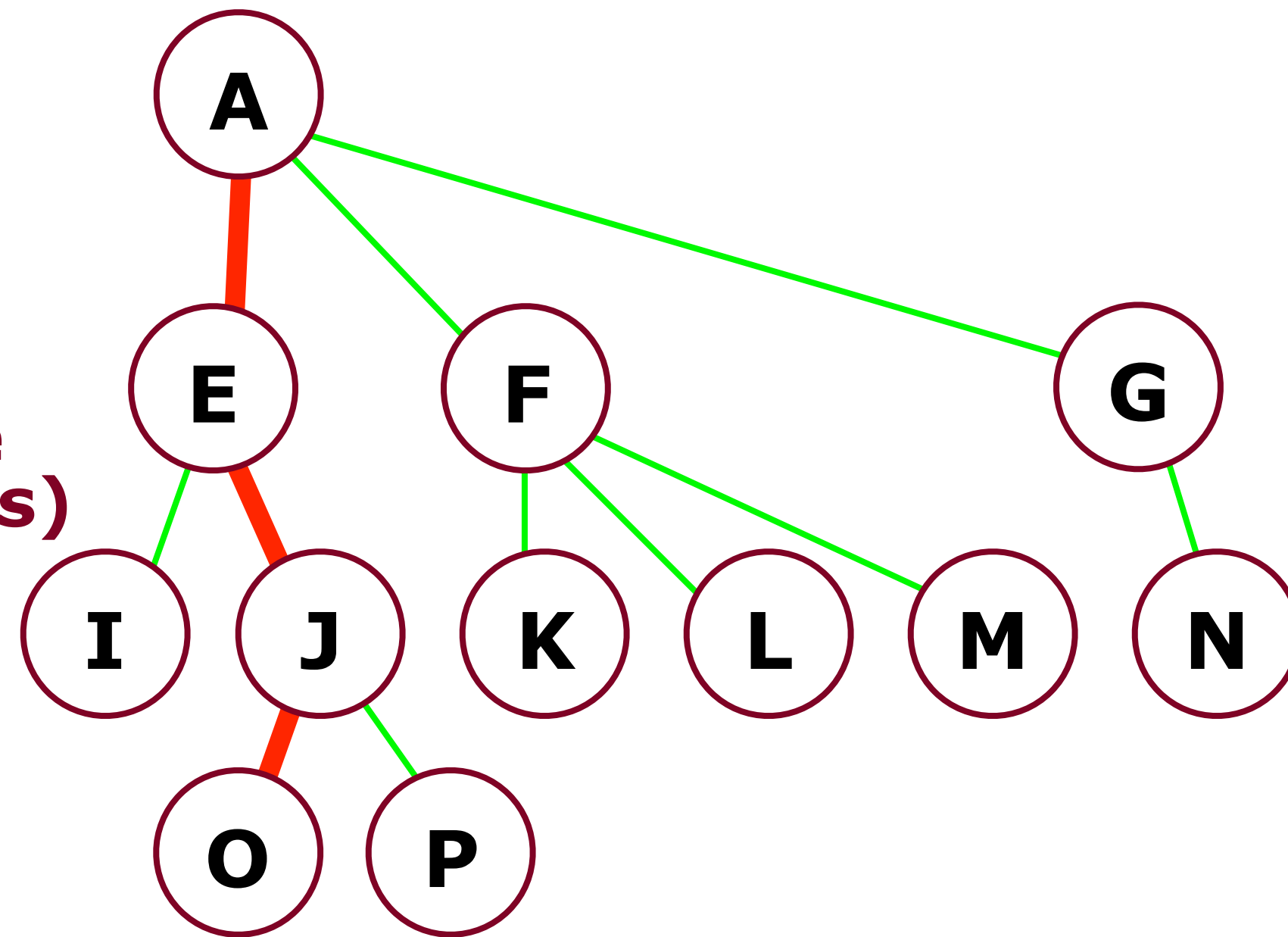
- A tree is a collection of **nodes**, which can be empty. If it is not empty, there is a “root” node, ***r***, and **zero or more non-empty subtrees**, **T_1, T_2, \dots, T_k** , whose roots are connected by a directed edge from ***r***.



Tree Terminology

We can define a *path* from a parent to its children.

The path A-E-J-O has a *length* of three (the number of edges)

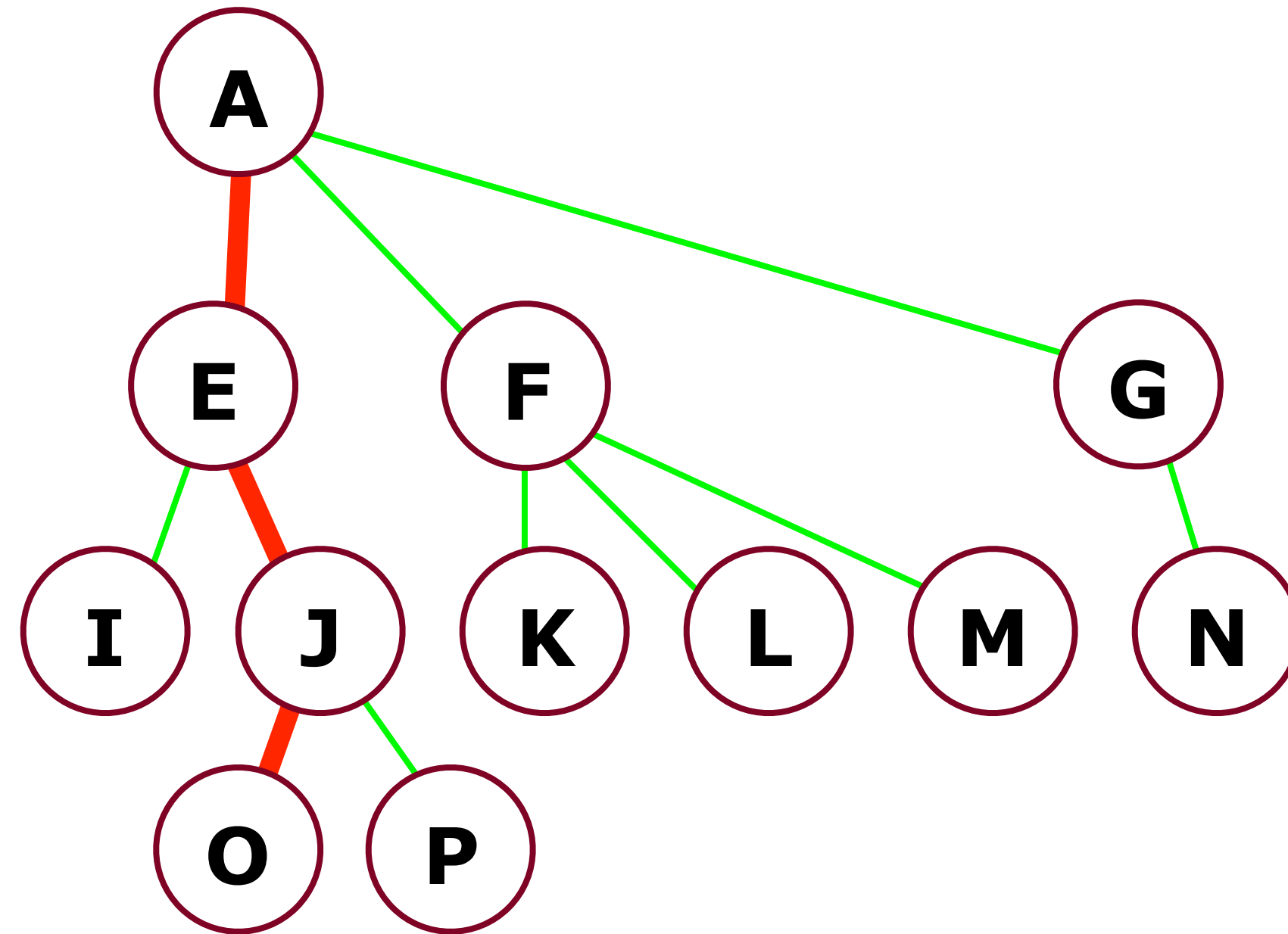


Tree Terminology



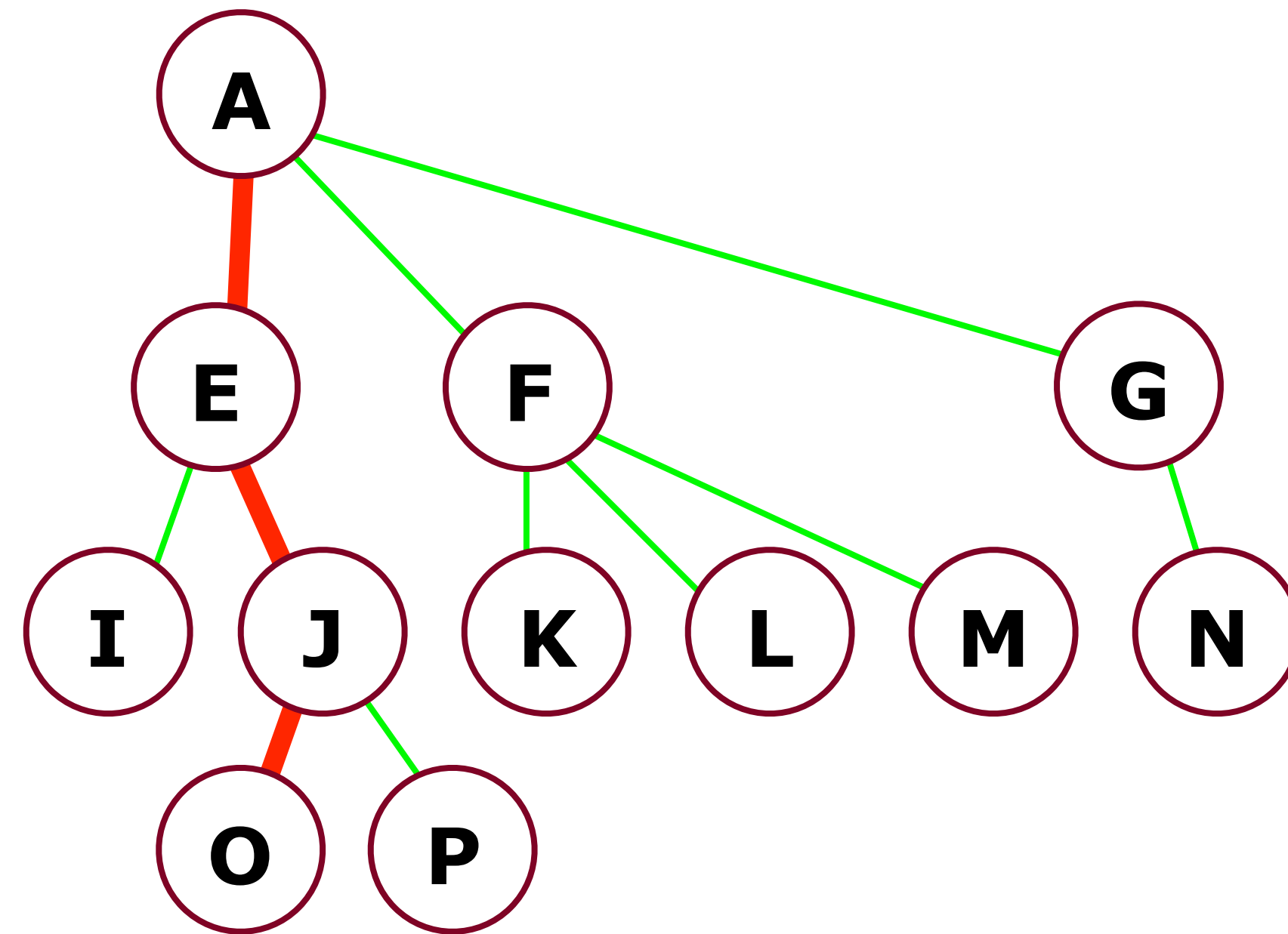
The **depth** of a node is the length from the root. The depth of node J is 2. The depth of the root is 0.

The **height** of a node is the longest path from the node to a leaf. The height of node F is 1. The height of all leaves is 0.



Tree Terminology

The **height** of a tree is the height of the root (in this case, the height of the tree is 3).



Tree Terminology

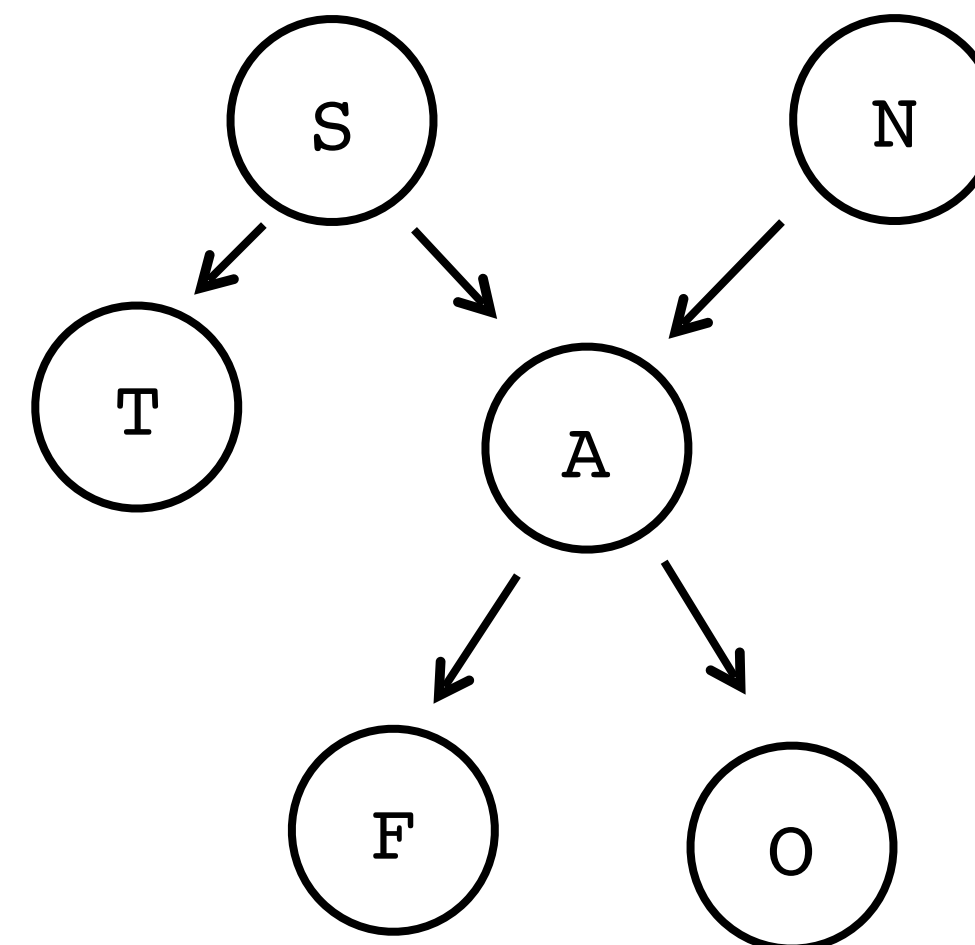
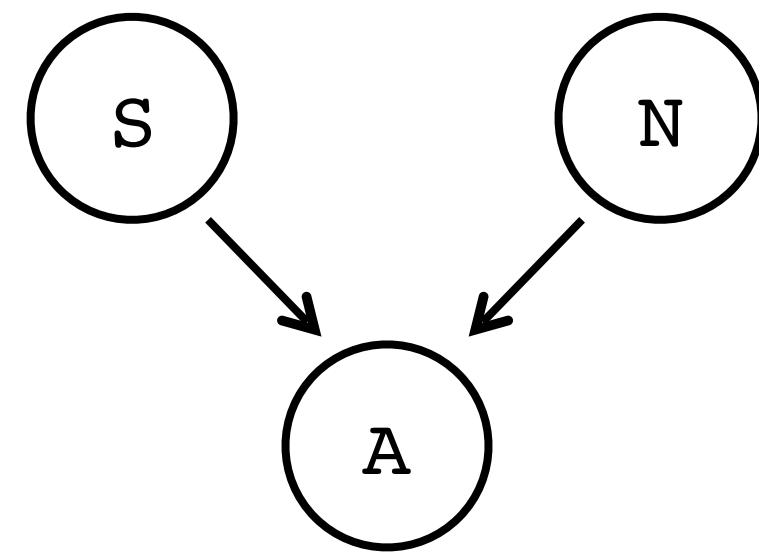
Trees can have only one parent, and cannot have *cycles*



Tree Terminology

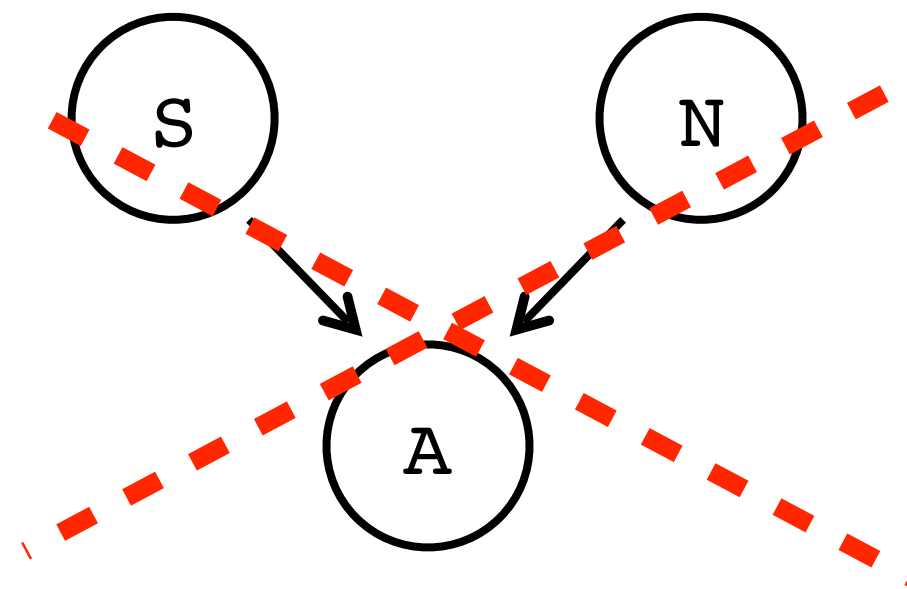


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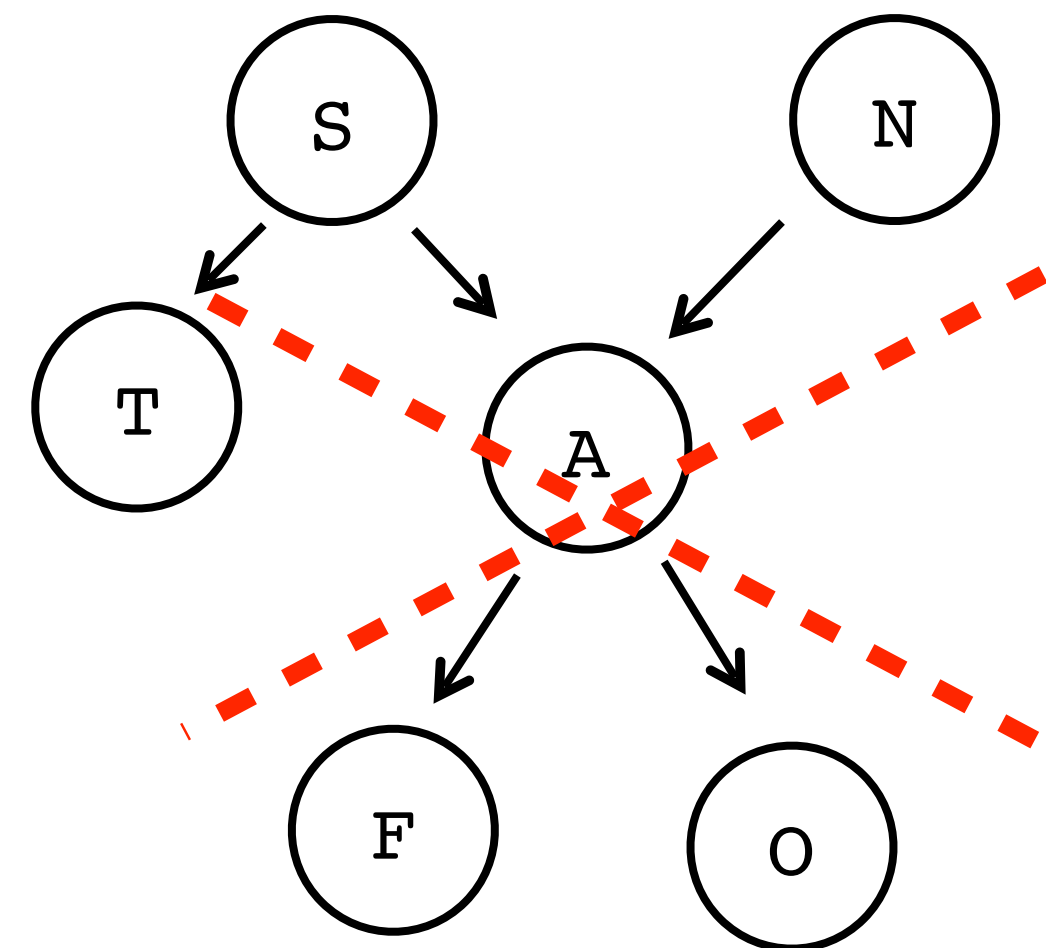


Tree Terminology

Trees can have only one parent, and cannot have *cycles*



Node A has two parents

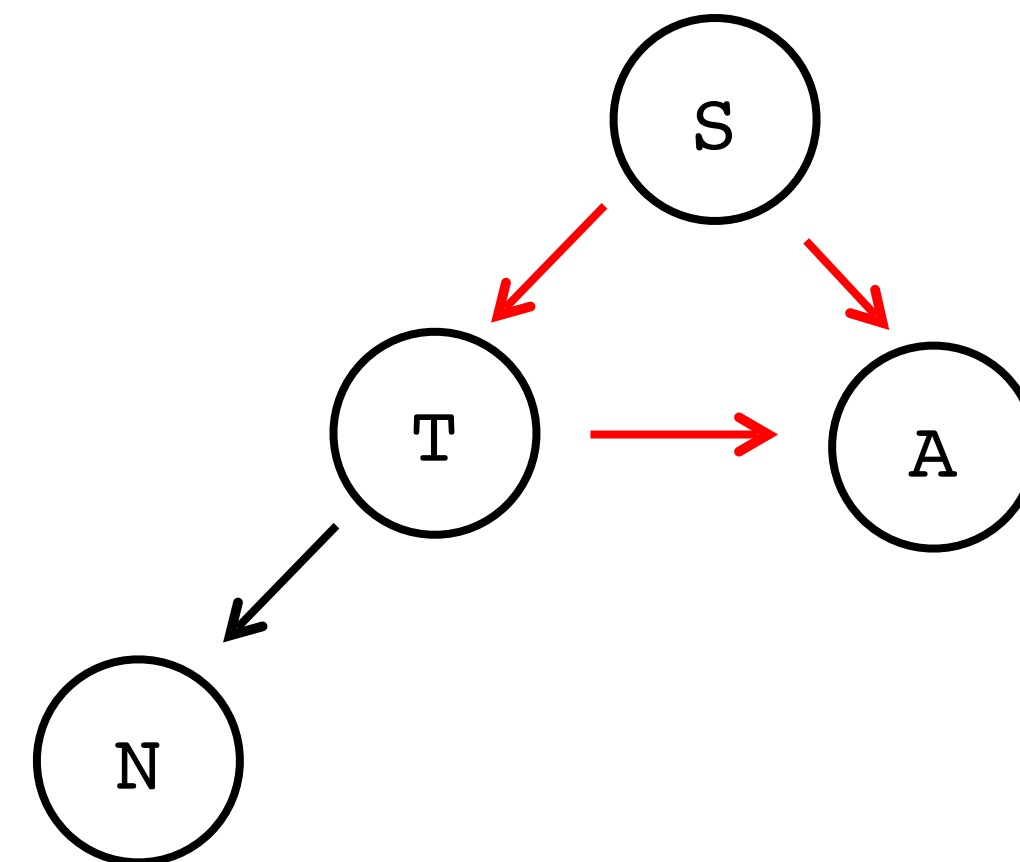
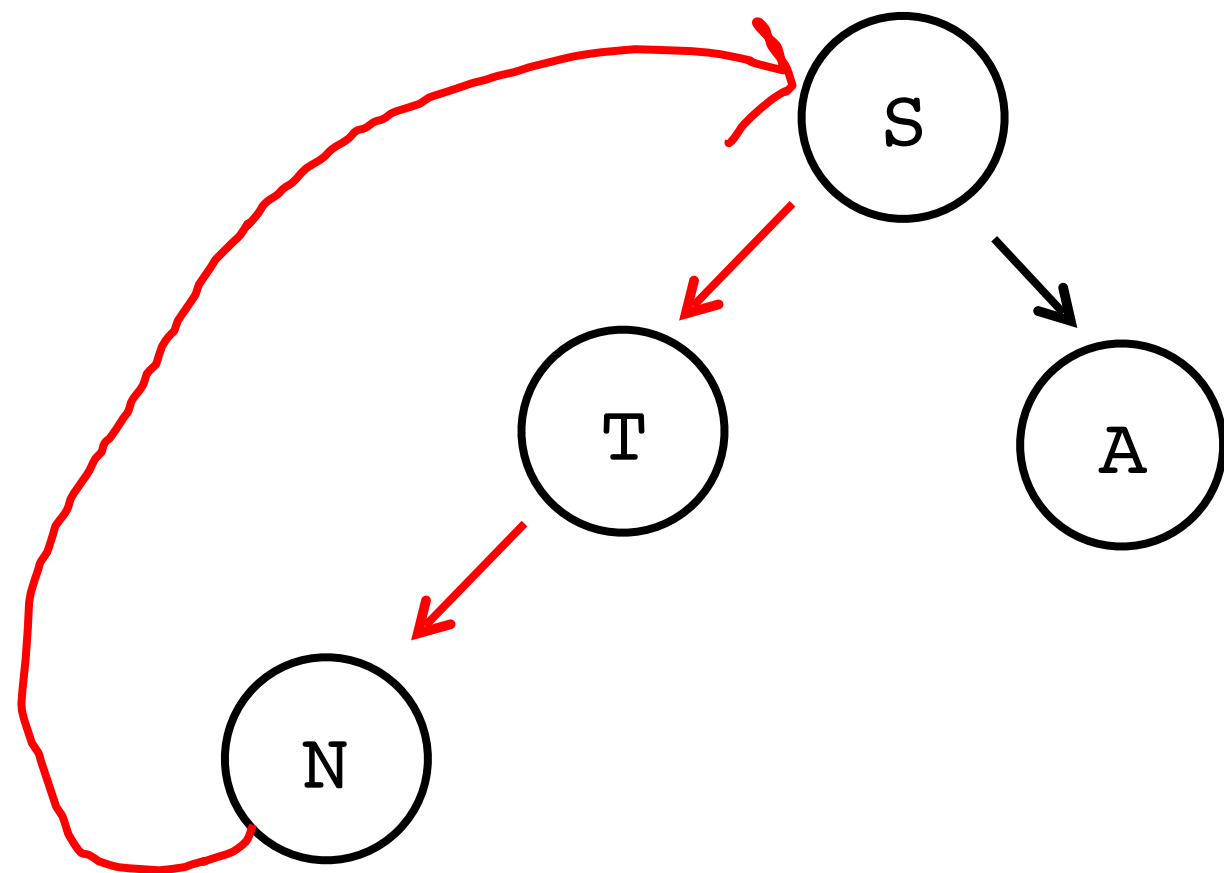


Node A has two parents



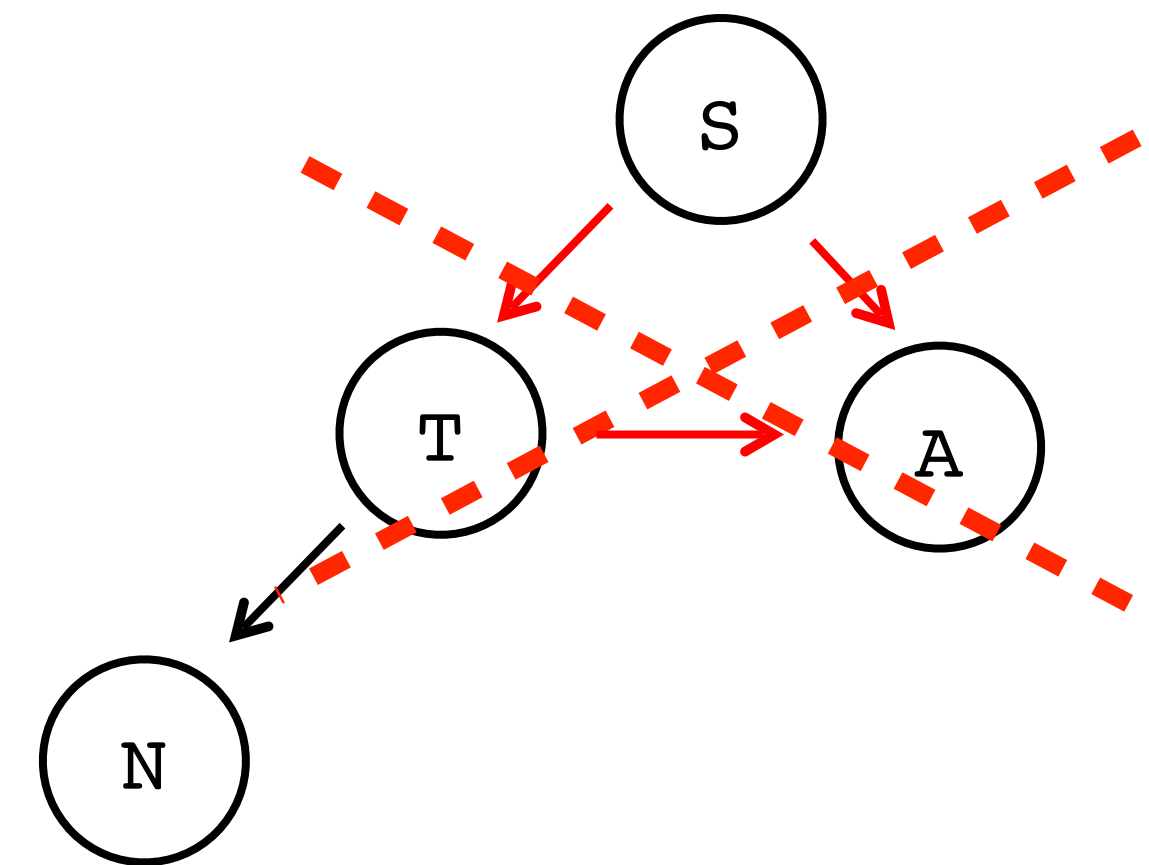
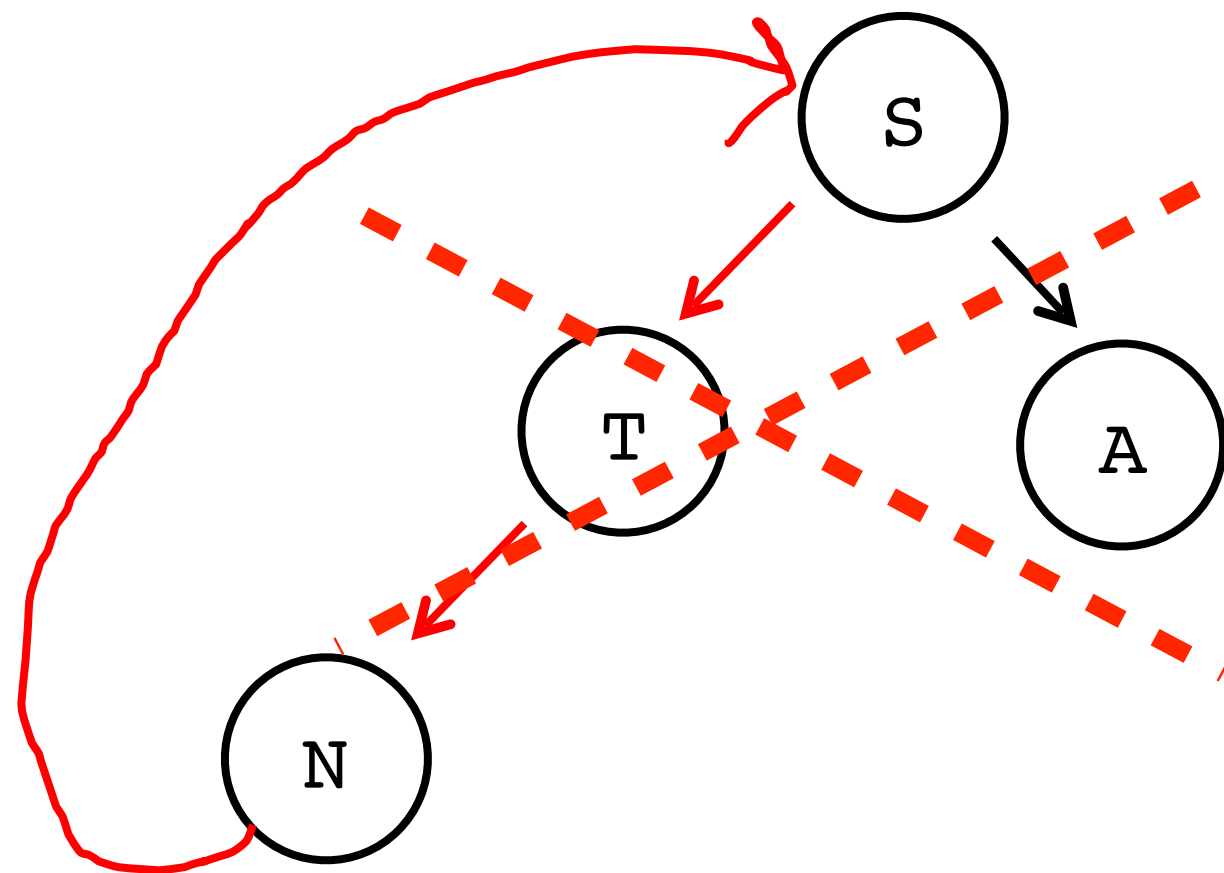
Tree Terminology

Trees can have only one parent, and cannot have *cycles*



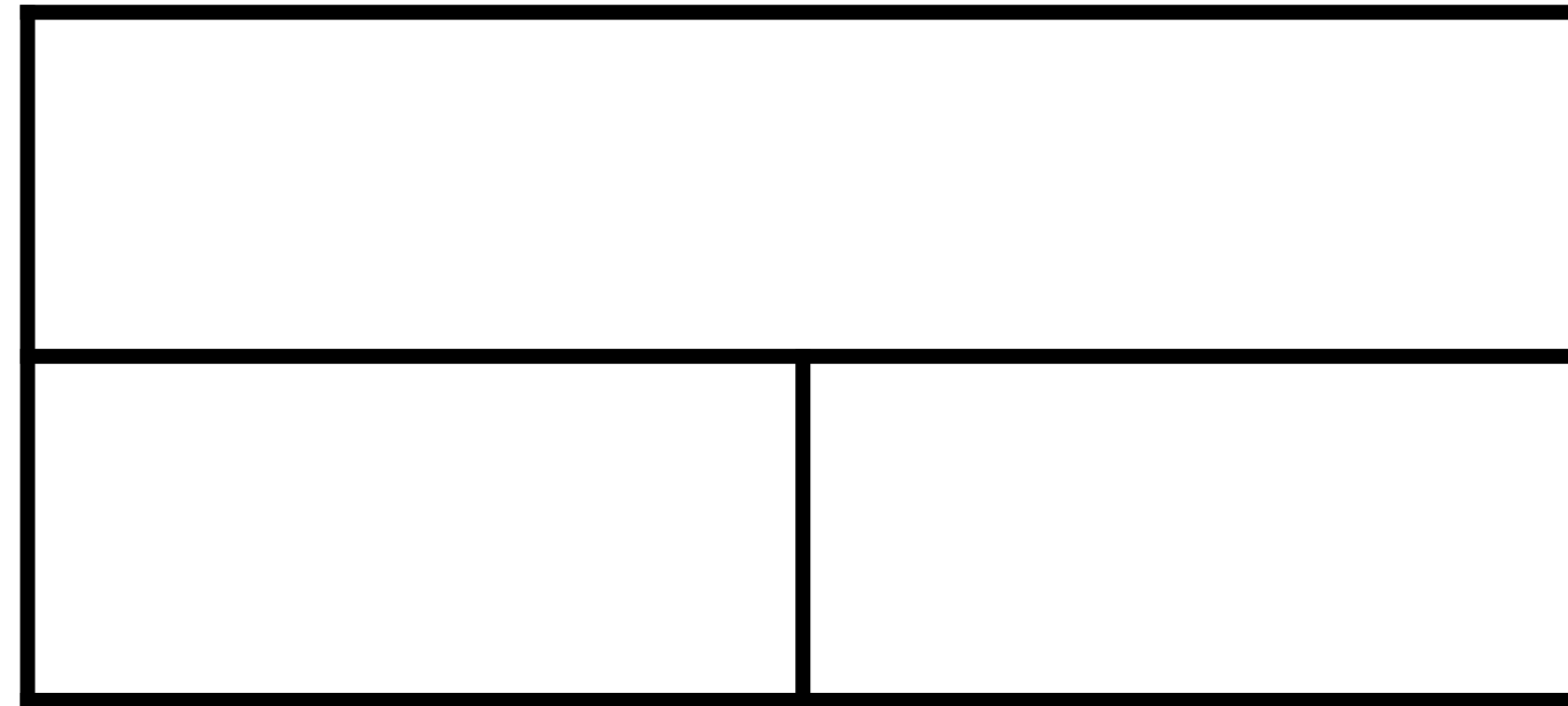
Tree Terminology

Trees can have only one parent, and cannot have *cycles*



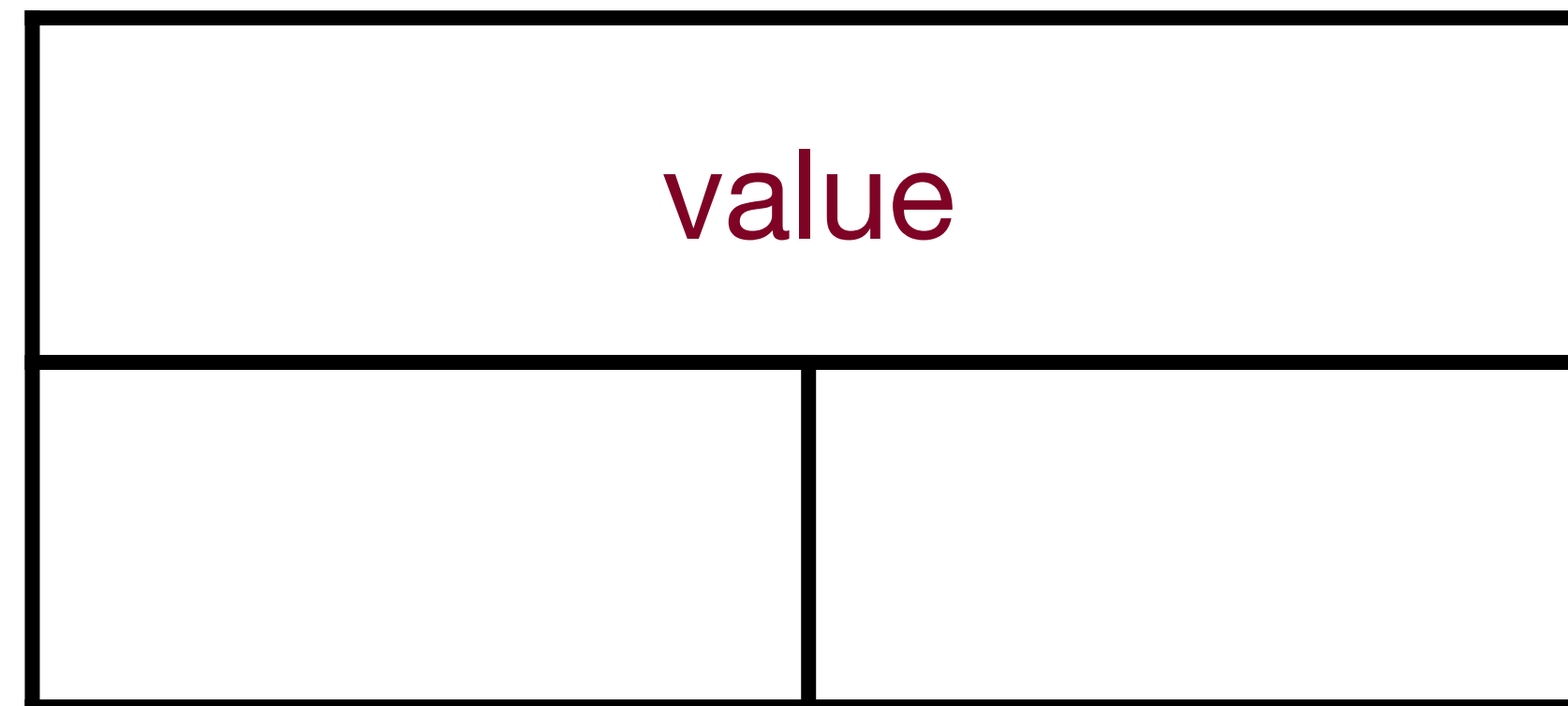
not a tree: the red edges make a cycle

How can we build trees programmatically?



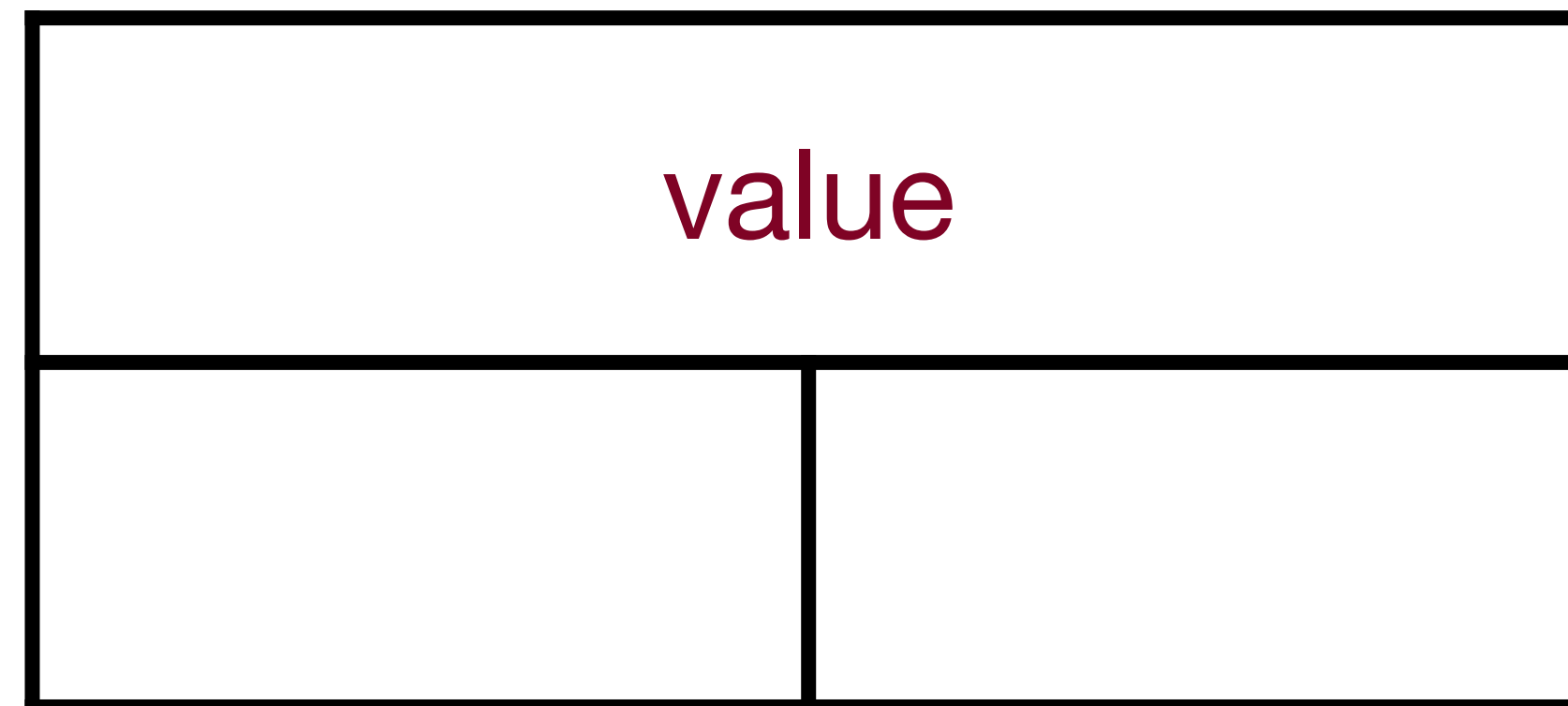
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Binary Tree:

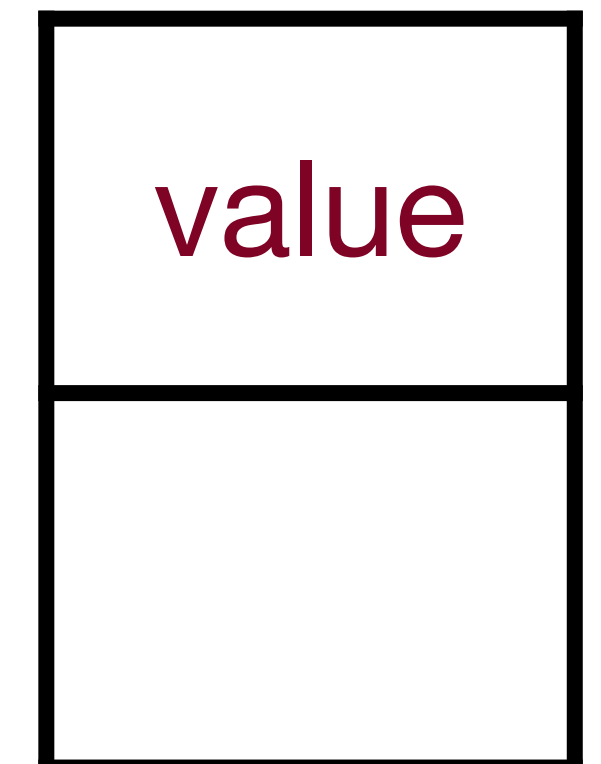


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Binary Tree:

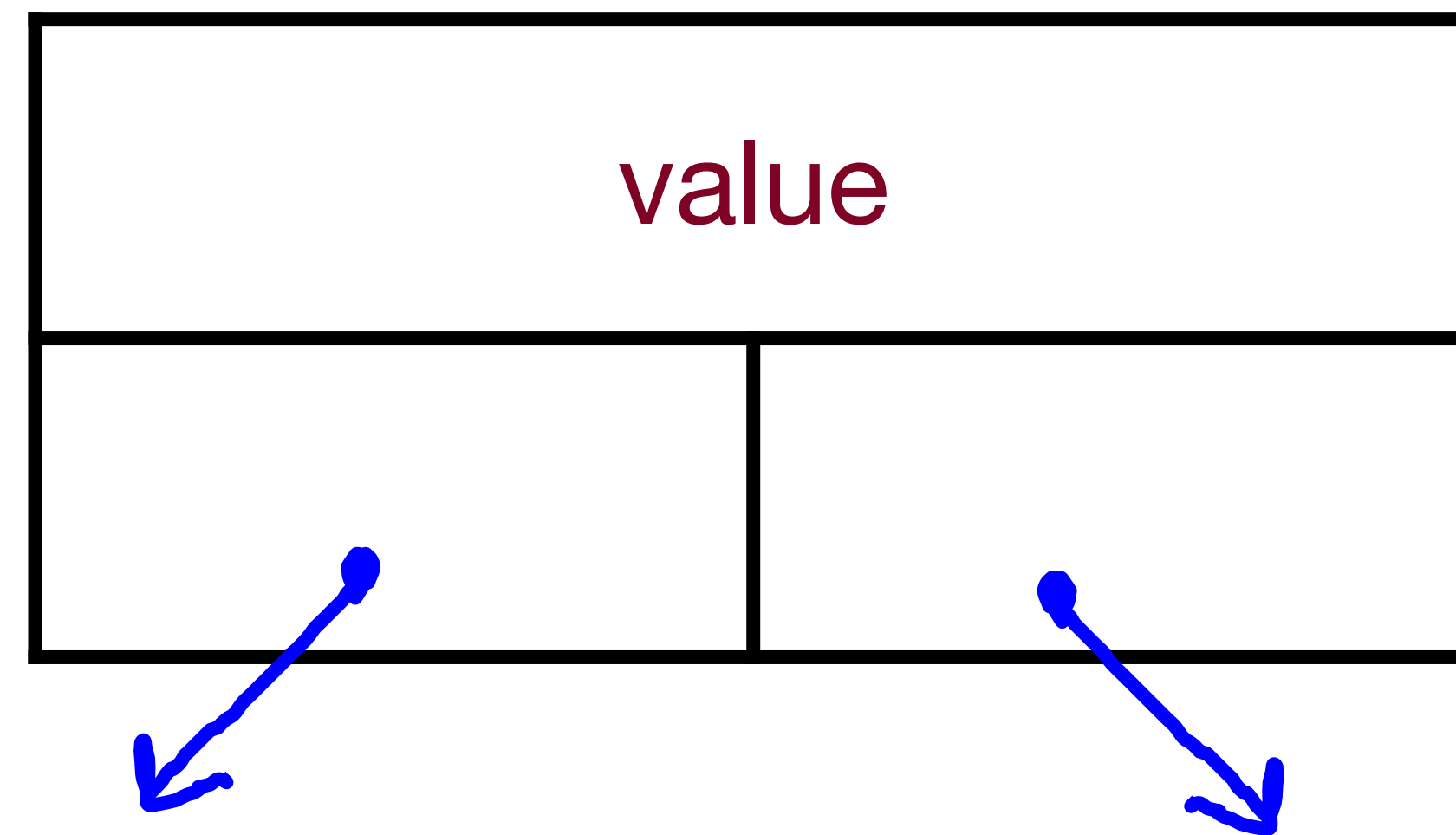


Linked List

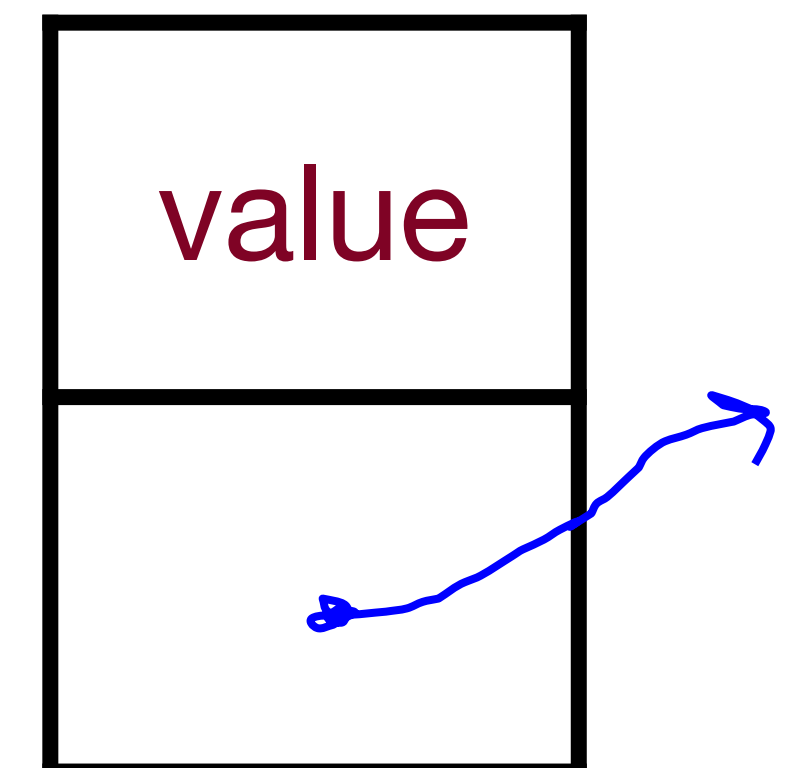


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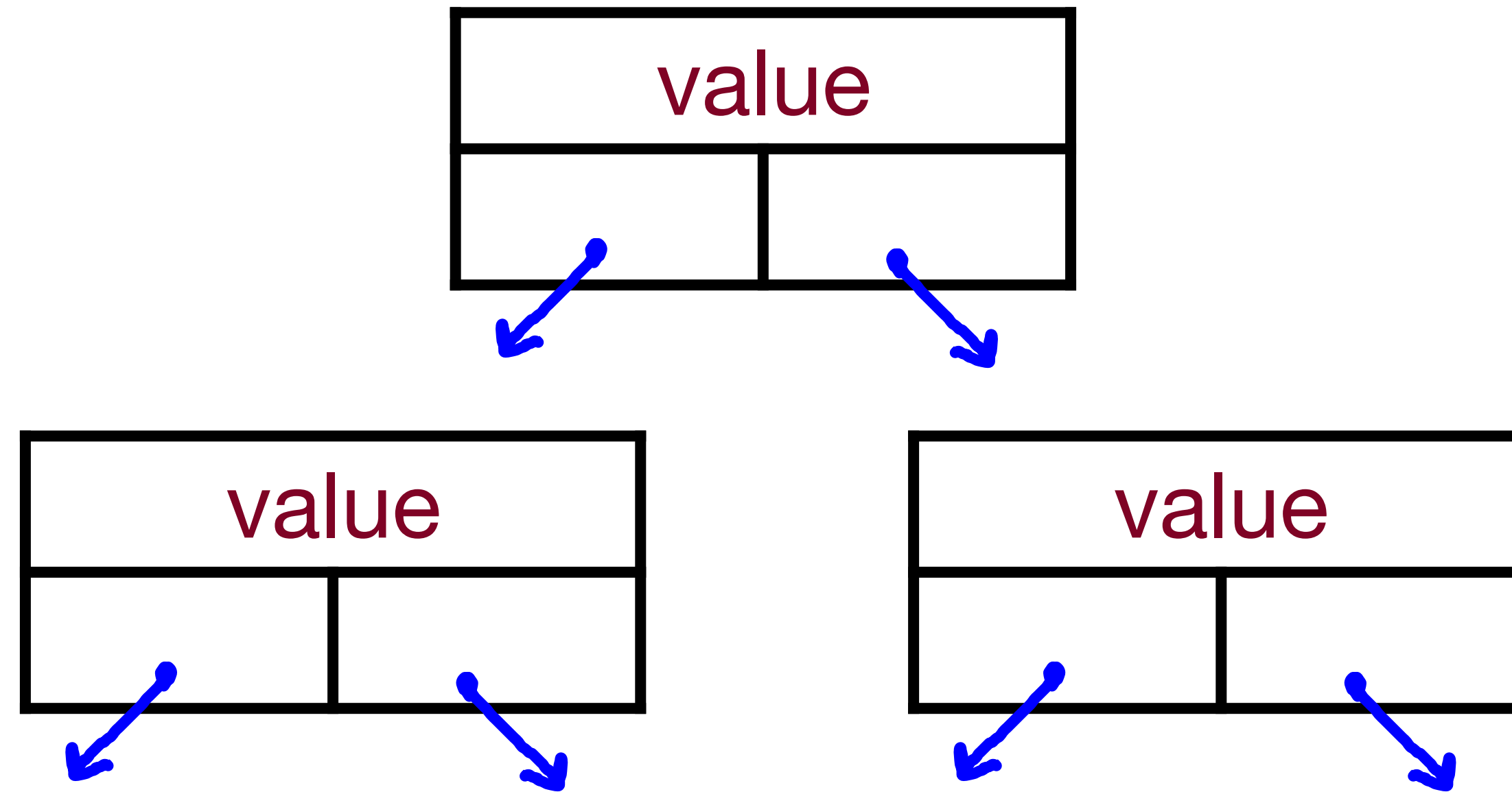


Linked List

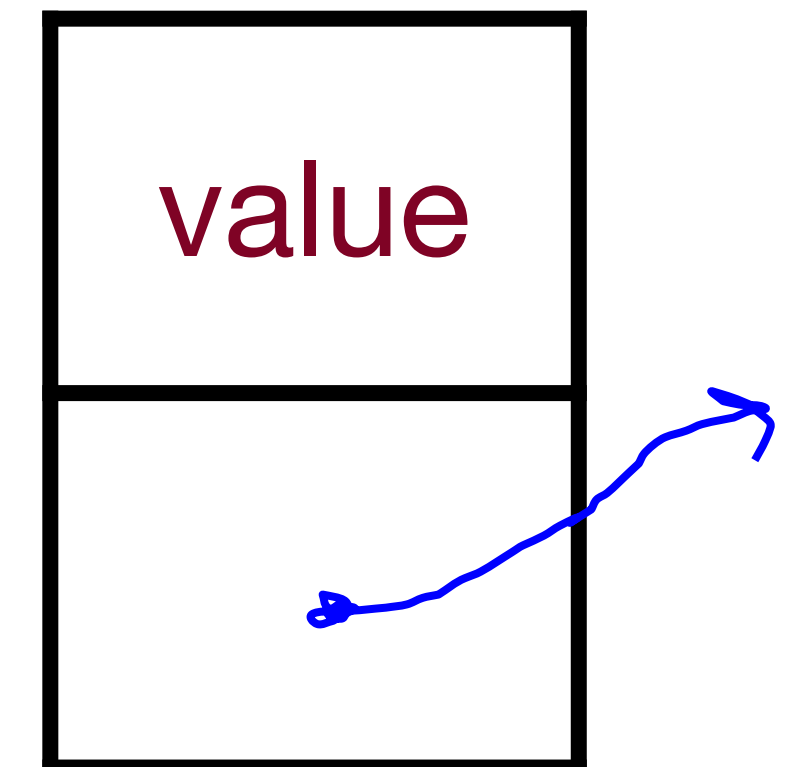


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Binary Tree:

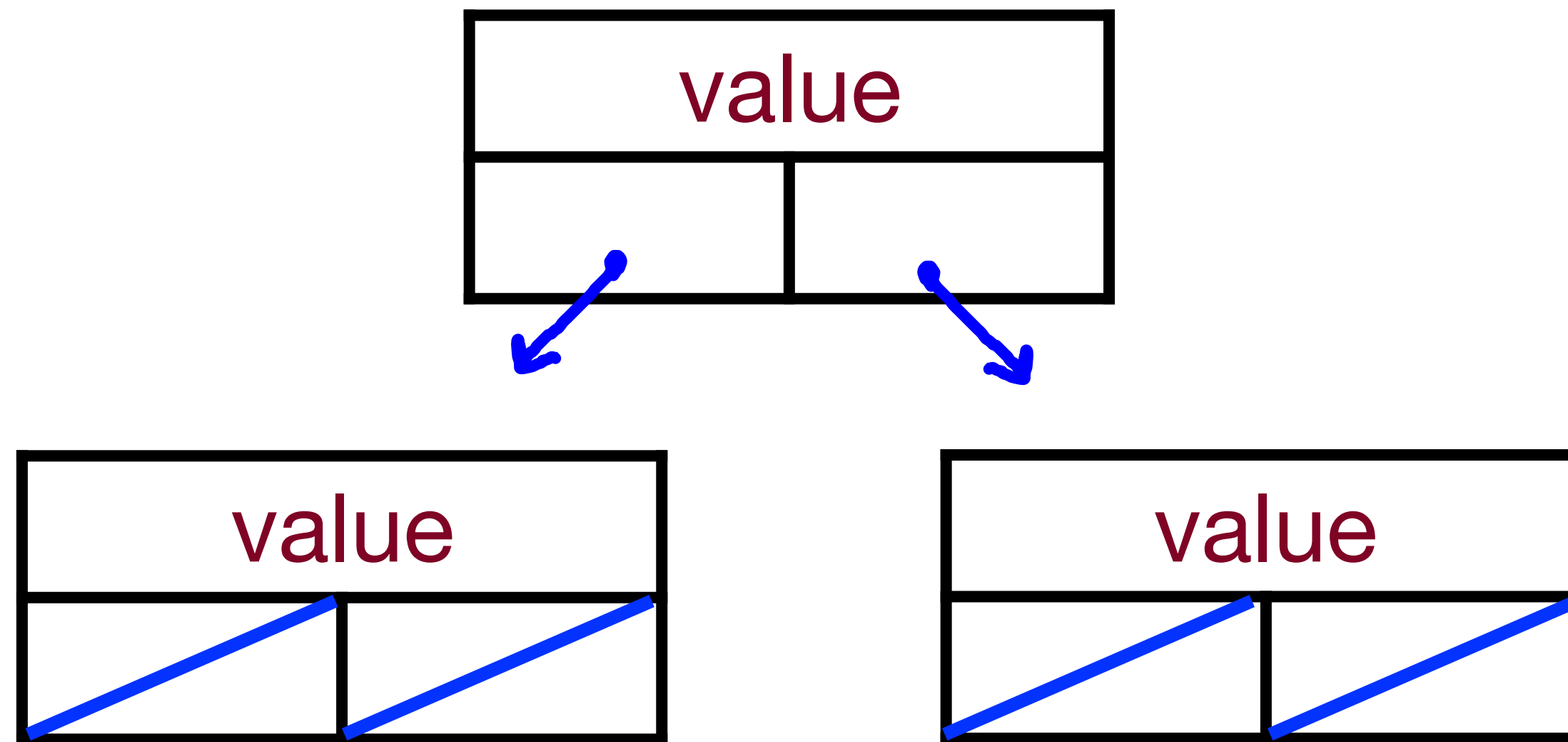


Linked List

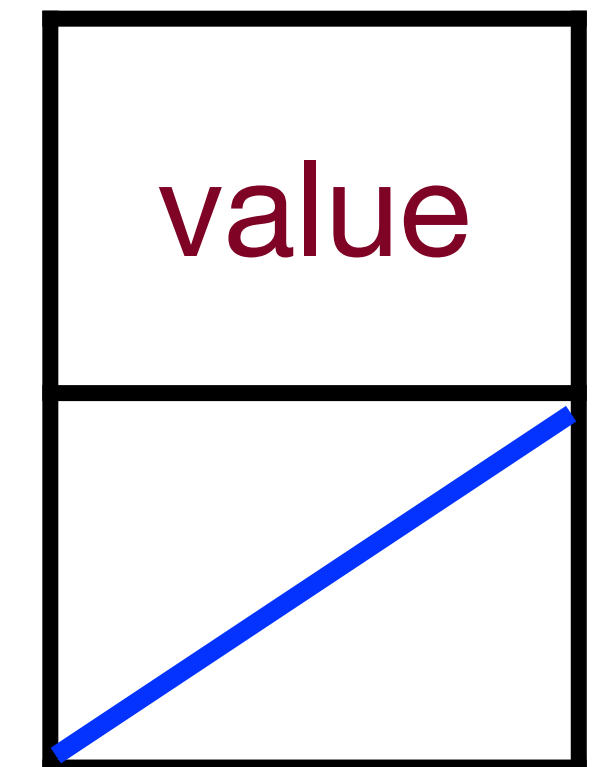


How can we build trees programmatically?

Binary Tree:



Linked List

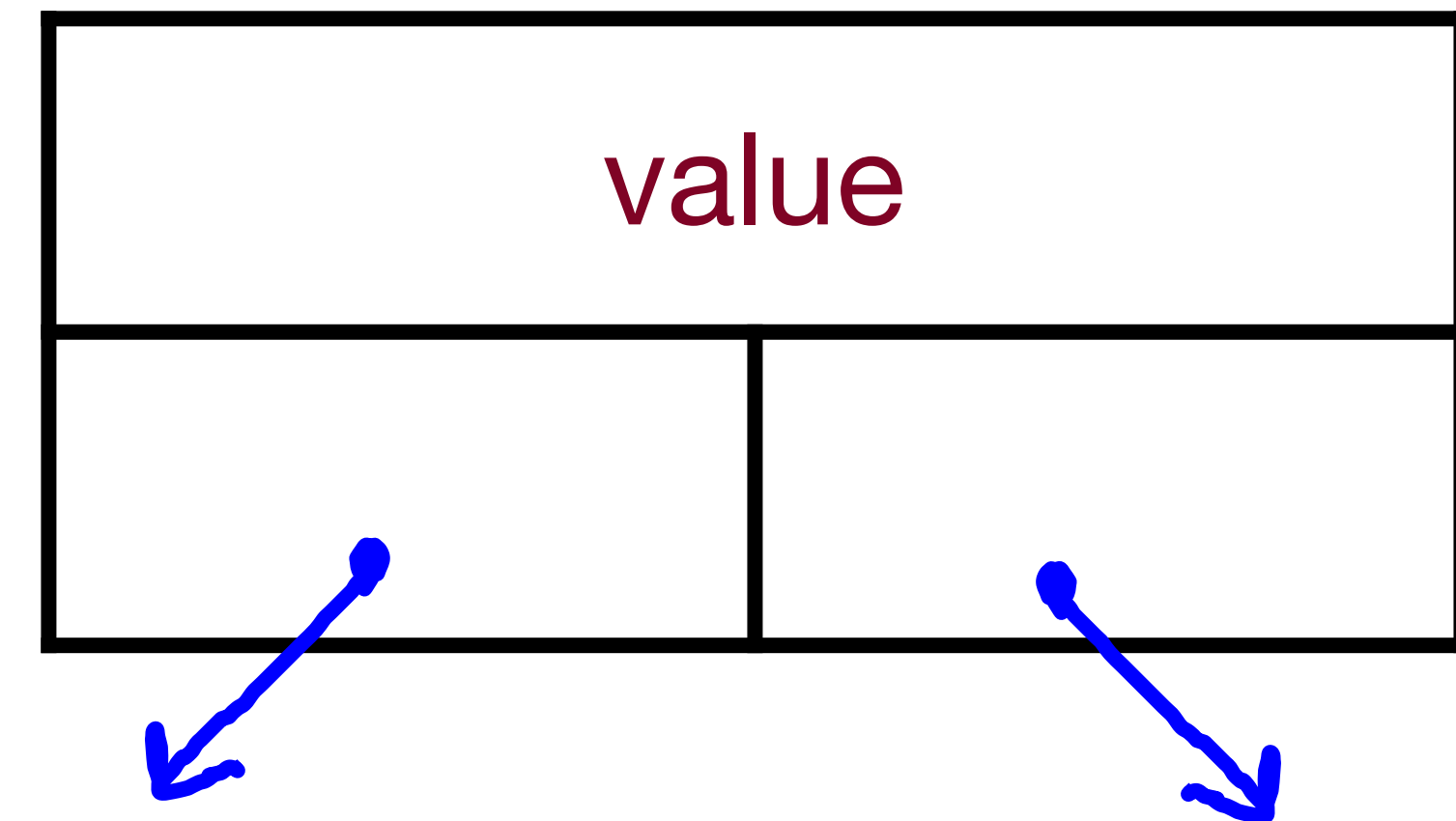


The Most Important Slide



Binary Tree:

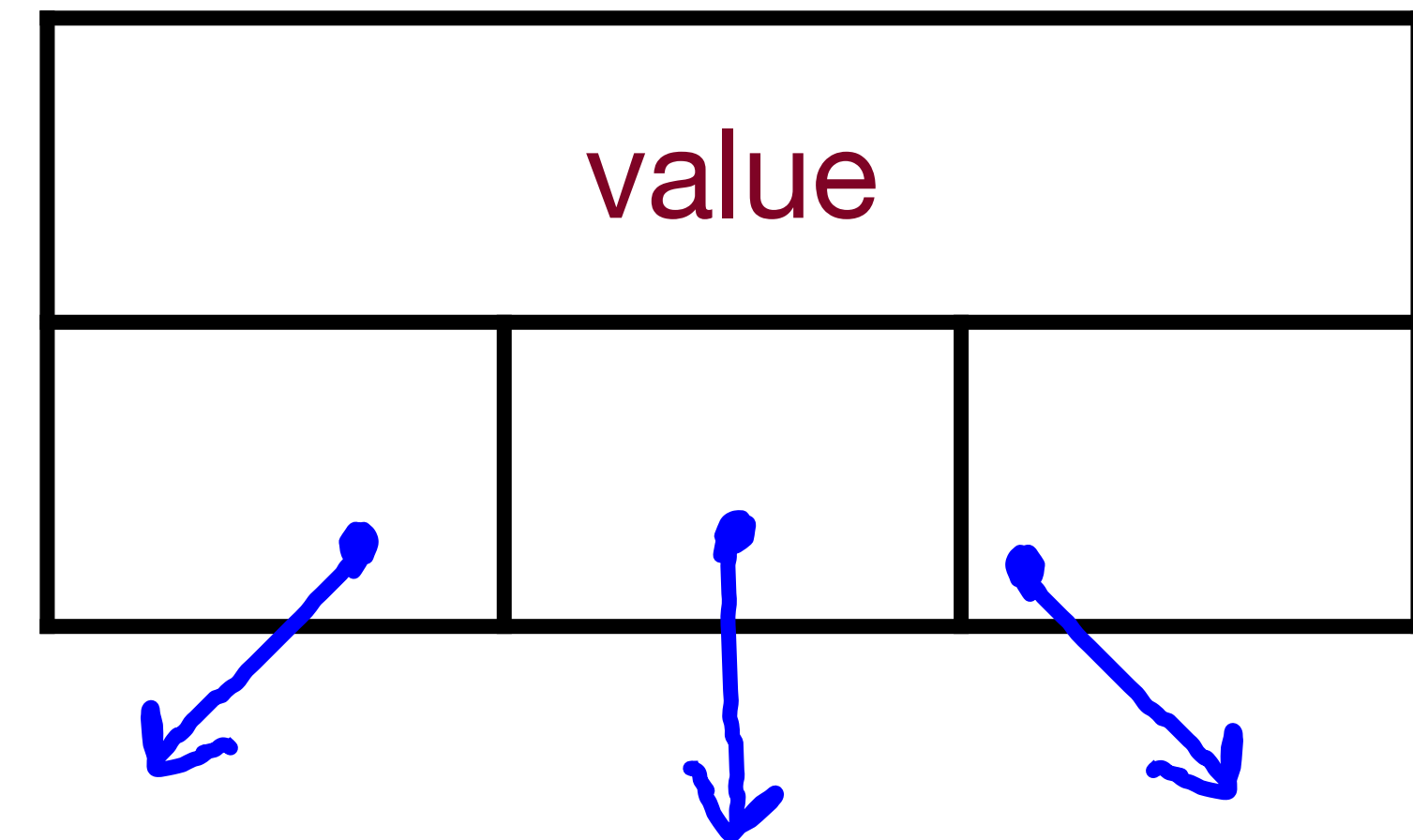
```
struct Tree {  
    string value;  
    Tree *left;  
    Tree *right;  
};
```



We Can Have Ternary Trees (or any number, n)

Ternary Tree:

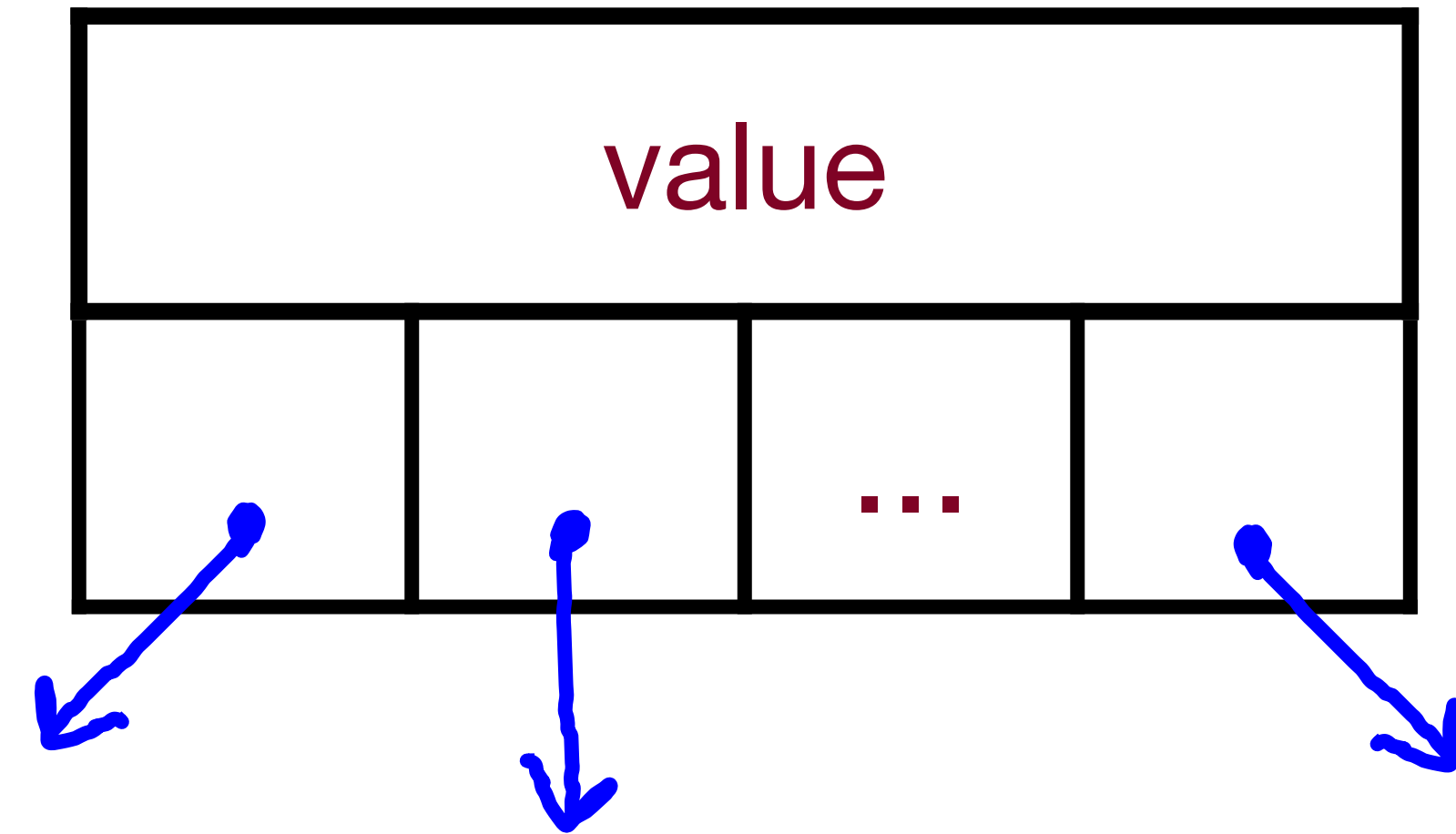
```
struct Tree {  
    string value;  
    Tree *left;  
    Tree *middle;  
    Tree *right;  
};
```



We Can Have Ternary Trees (or any number, n)

N-ary Tree:

```
struct Tree {  
    string value;  
    Vector<Tree *> children;  
};
```



Trees can be defined as either structs or classes

```
struct Tree {  
    string value;  
    Tree * left;  
    Tree * right;  
};
```

```
class Tree {  
private:  
    string value;  
    Vector<Tree *> children;  
};
```

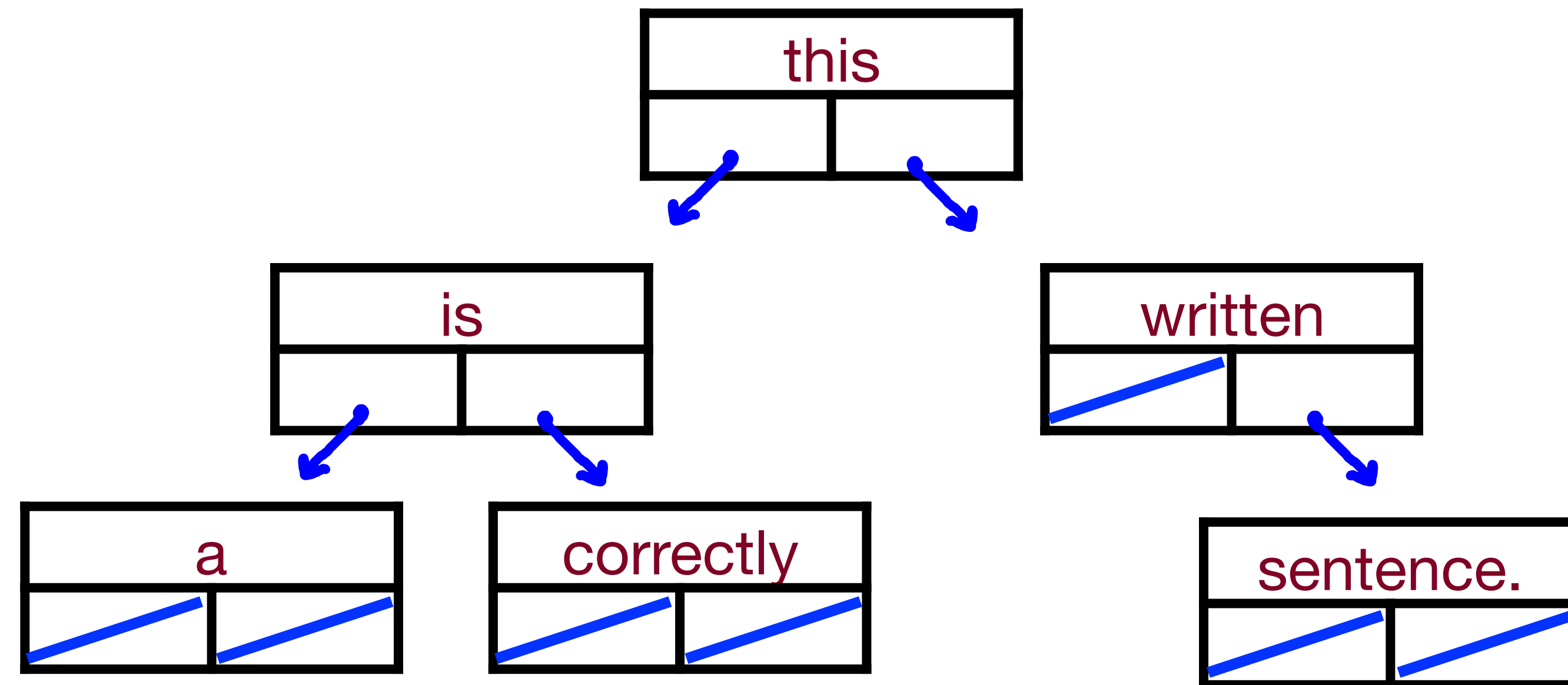


Let's write some code to "traverse" the tree

```
struct Tree {  
    string value;  
    Tree * left;  
    Tree * right;  
};
```

There are multiple ways to traverse the nodes in a binary tree:

1. Pre-order
2. In-order
3. Post-order
4. Level-order



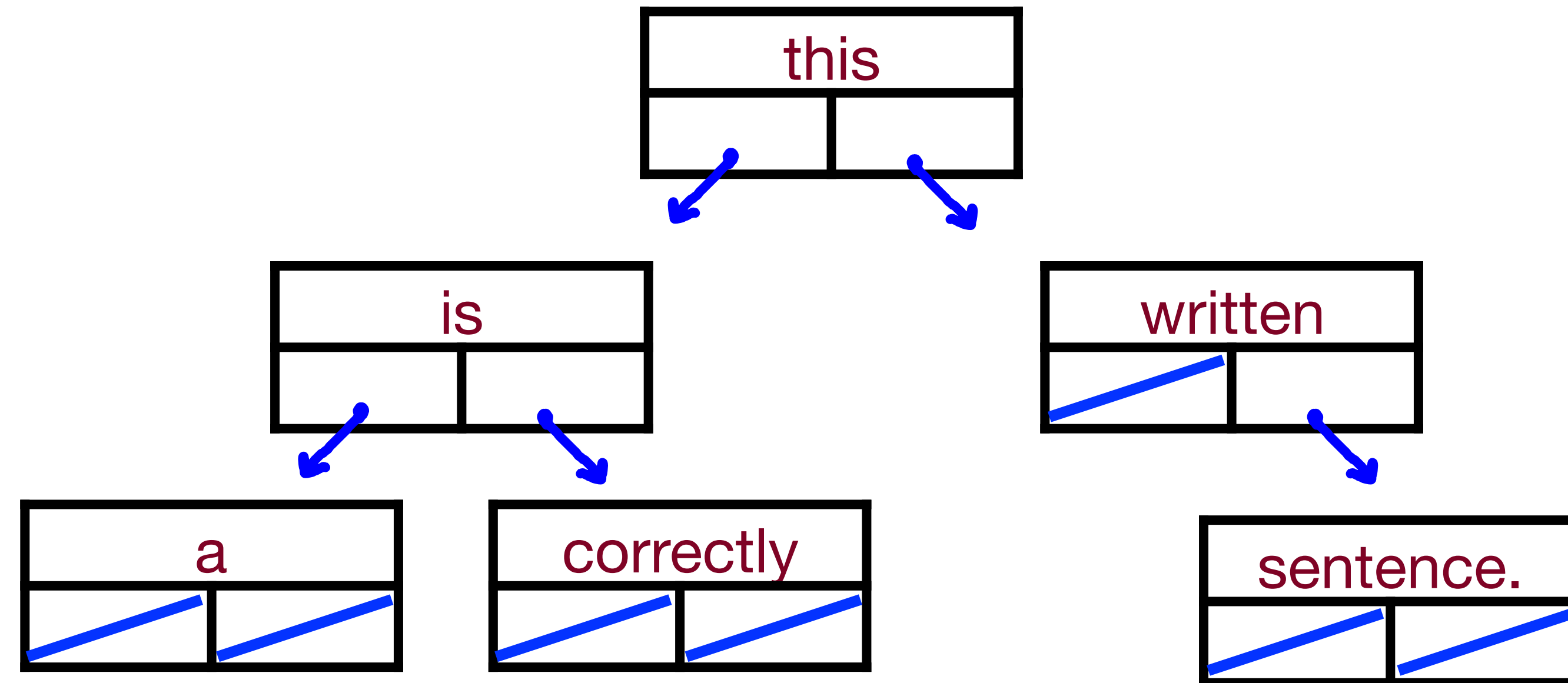
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1. Do something
2. Go left
3. Go right



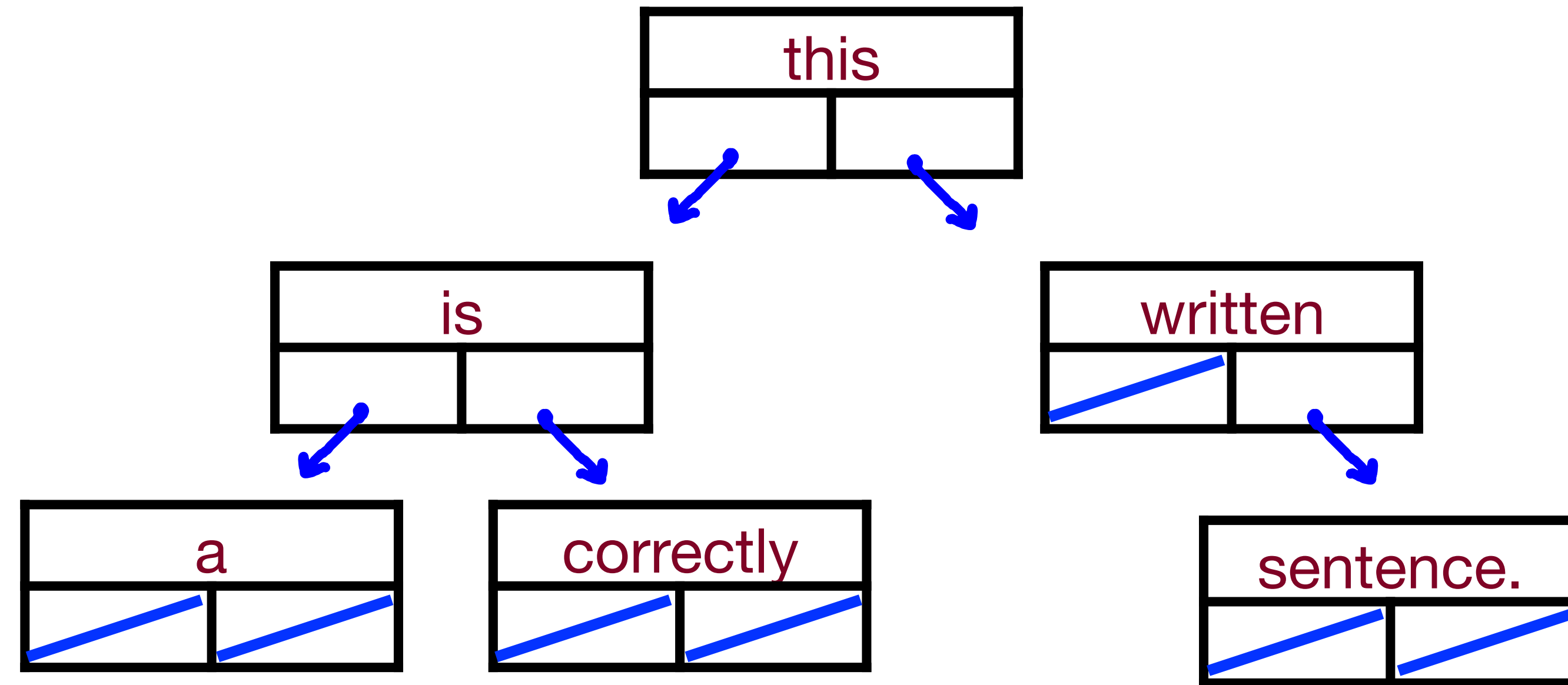
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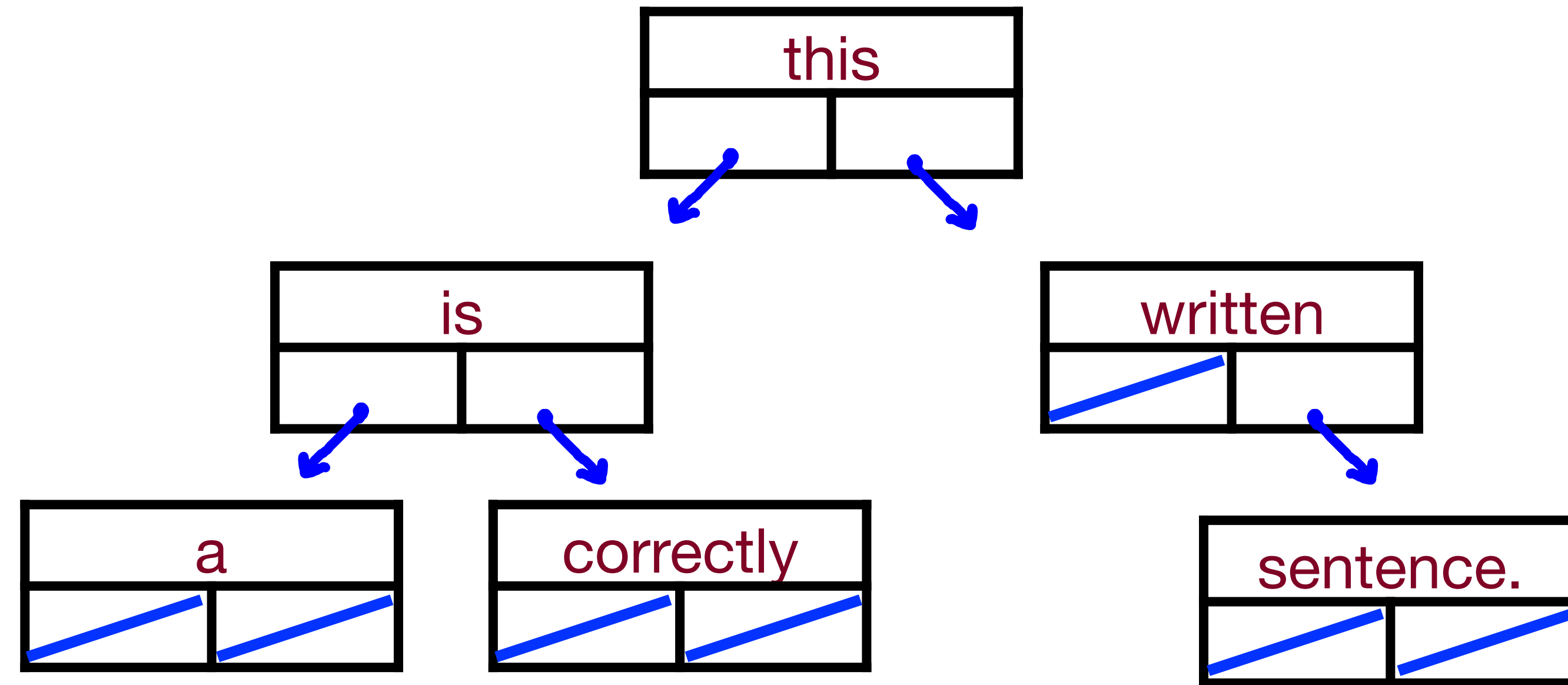
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2. Go right
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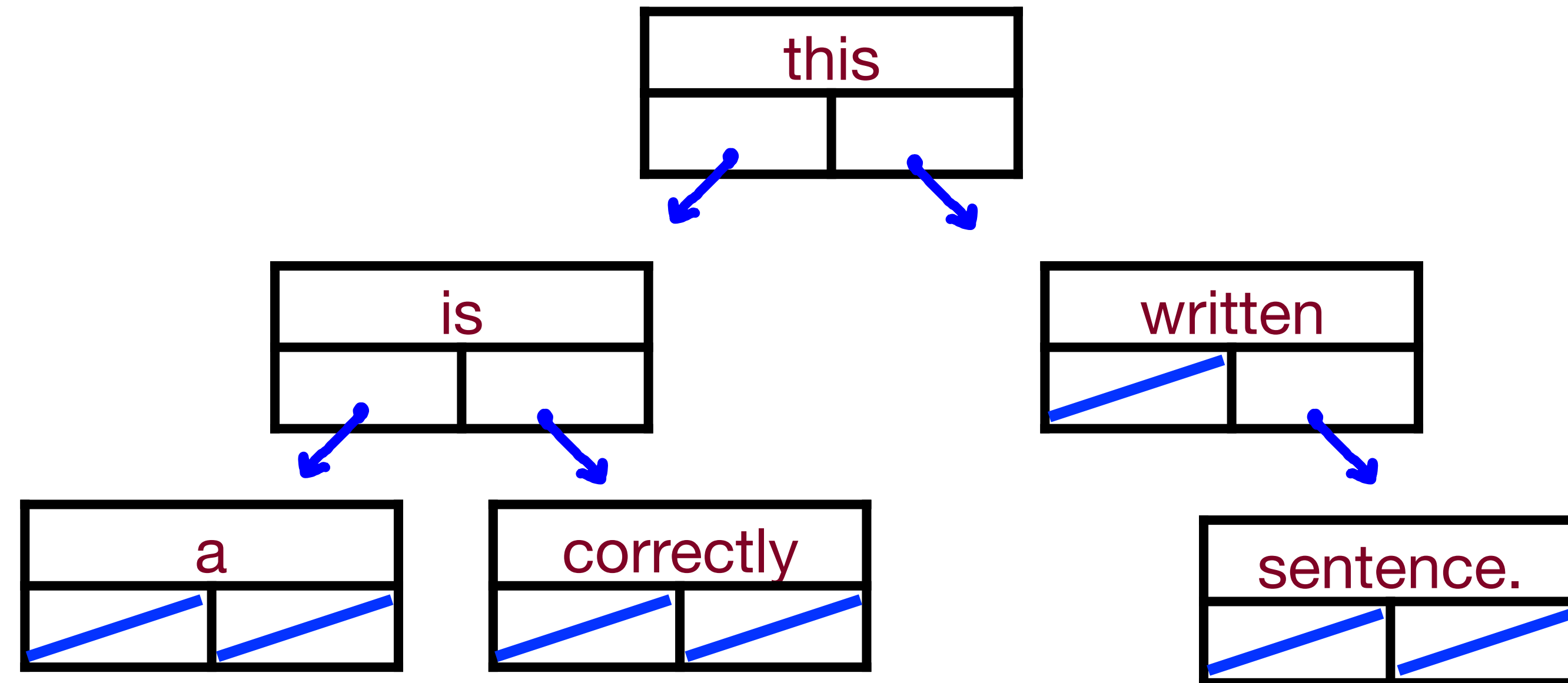


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```

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Hmm...can we do this recursively?

We want to print the levels: 0, 1, 2 from left-to-right order

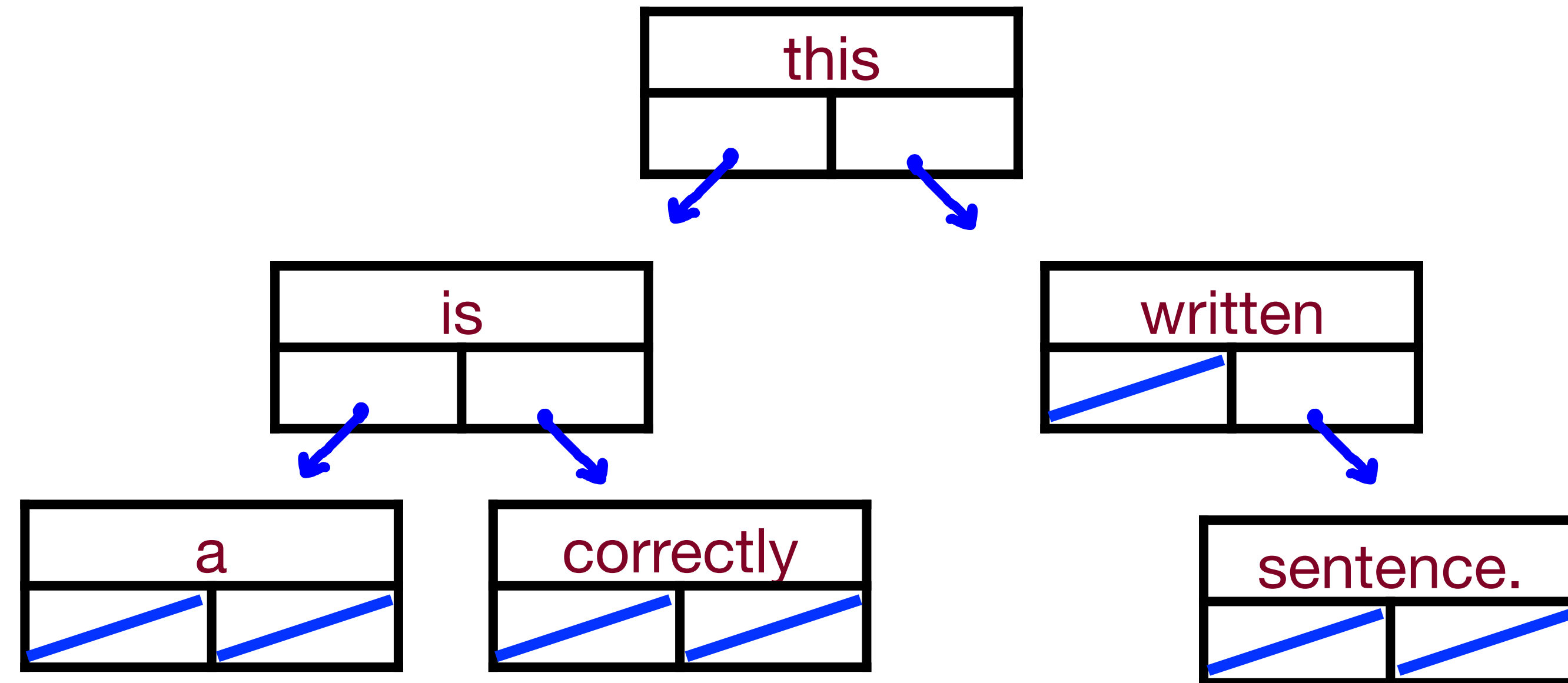


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Not easy recursively...let's use a queue!

1. Enqueue root
2. While queue is not empty:
 - a. dequeue node
 - b. do something with node
 - c. enqueue left child of node if it exists
 - d. enqueue right child of node if it exists

should look familiar...word ladder?



Let's write some code

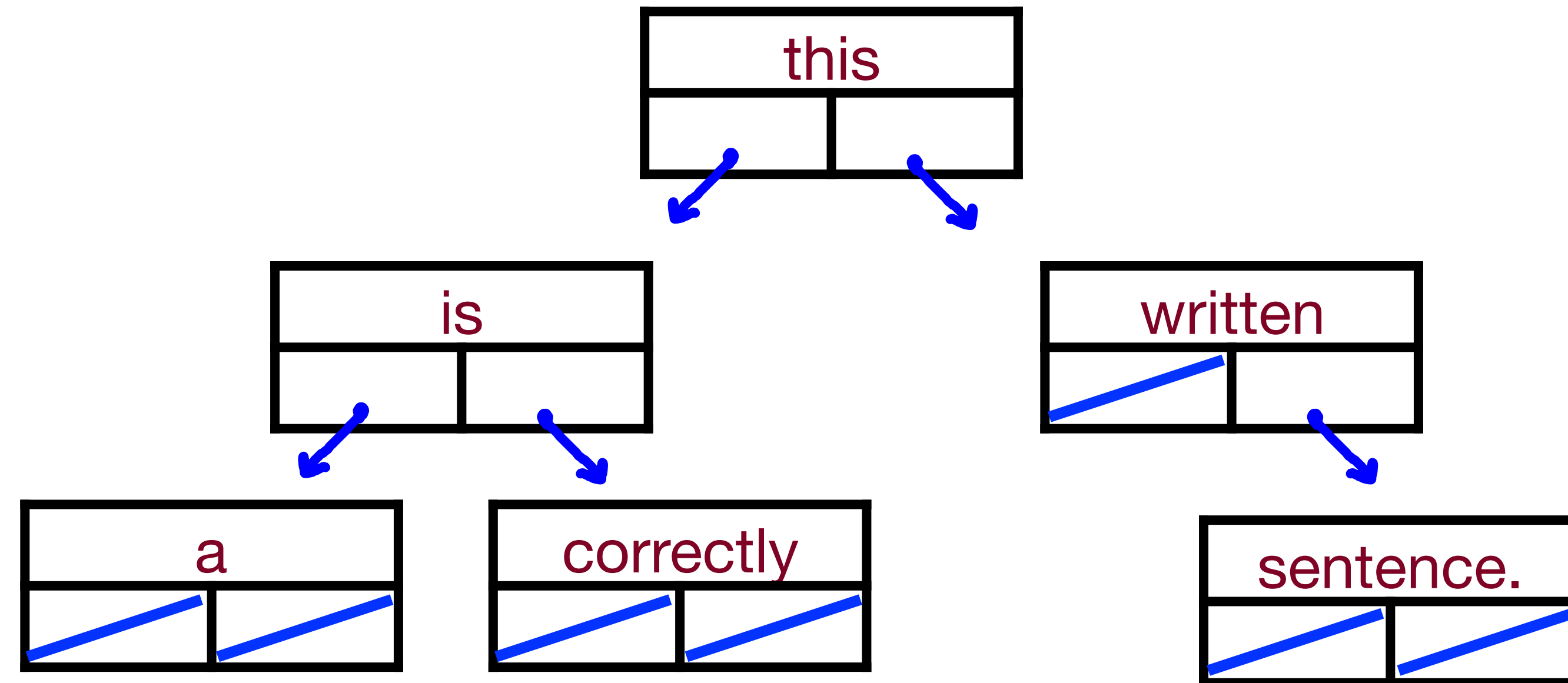
```
struct Tree {  
    string value;  
    Tree * left;  
    Tree * right;  
};
```

```
void preOrder(Tree * tree) {  
    if(tree == NULL) return;  
    cout<< tree->value <<" ";  
    preOrder(tree->left);  
    preOrder(tree->right);  
}
```

```
void inOrder(Tree * tree) {  
    if(tree == NULL) return;  
    inOrder(tree->left);  
    cout<< tree->value <<" ";  
    inOrder(tree->right);  
}
```

```
void postOrder(Tree * tree) {  
    if(tree == NULL) return;  
    postOrder(tree->left);  
    postOrder(tree->right);  
    cout<< tree->value <<" ";  
}
```

```
void levelOrder(Tree *tree) {  
    Queue<Tree *>treeQueue;  
    treeQueue.enqueue(tree);  
    while (!treeQueue.isEmpty()) {  
        Tree *node = treeQueue.dequeue();  
        cout << node->value << " ";  
  
        if (node->left != NULL) {  
            treeQueue.enqueue(node->left);  
        }  
        if (node->right != NULL) {  
            treeQueue.enqueue(node->right);  
        }  
    }  
}
```



References and Advanced Reading

- **References:**

- [https://en.wikipedia.org/wiki/Tree_\(data_structure\)](https://en.wikipedia.org/wiki/Tree_(data_structure))
- <http://pages.cs.wisc.edu/~vernon/cs367/notes/8.TREES.html>

- **Advanced Reading:**

- <http://www.cs.cmu.edu/~adamchik/15-121/lectures/Trees/trees.html>
- Great set of tree-type questions:
 - <http://cslibrary.stanford.edu/110/BinaryTrees.html>

