Recursive Backtracking 2

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Stanford CS 106B

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Based on slides created by Ashley Taylor, Marty Stepp, Chris Gregg, Keith Schwarz, Julie Zelenski, Jerry Cain, Eric Roberts, Mehran Sahami, Stuart Reges, Cynthia Lee, and others.

Outline

Exhaustive Search

- **Exhaustive search**: exploring every possible combination from a set of choices or values, often implemented recursively

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- Often the search space consists of many **decisions**, each of which has several available **choices**
- Example: When enumerating all 5-letter strings, each of the 5 letters is a decision, and each of those decisions has 26 possible choices.

Backtracking

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- Think of this as a "brute force" technique because it tries all paths or combinations

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Explore (decisions):
    If there are no more decisions to make:
        Stop
    Else:
        // Handle one decision here, and do the rest by recursion
        For each available choice C for this decision:
            Choose C
            Explore the remaining decisions that could follow C
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- Key tasks:

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- Key tasks:
  - Figure out appropriate smallest unit of work (decision)
  - Figure out how to enumerate all possible choices/options for it

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  3. Are we avoiding **arms-length** recursion?

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- More recursive backtracking practice
- Why "arms-length" recursion is not good, especially in backtracking
- Backtracking application: permutations

diceSum

diceSum Problem

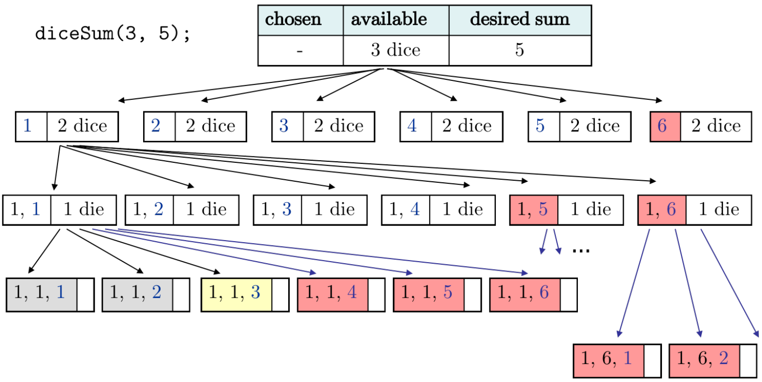
Write a function diceSum that accepts two integer parameters: a number of dice to roll, and a desired sum of all die values. Output all combinations of die values that add up to exactly that sum.

```
void diceSum(int dice, int desiredSum) {
    Vector<int> chosen;
    diceSumHelper(dice, desiredSum, chosen);
}
```

Initial Solution

```
void diceSumHelper(int dice, int desiredSum, Vector<int>&
    chosen) {
    if (dice == 0) {
        if (sumAll(chosen) == desiredSum) {
            cout << chosen << endl; // base case
        }
    } else {
        for (int i = 1; i <= 6; i++) {
            chosen.add(i);                 // choose
            diceSumHelper(dice - 1, desiredSum, chosen);
                // explore
            chosen.remove(chosen.size() - 1); // un-choose
        }
    }
}
```

Wasteful Recursion



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  - Sometimes the current sum is already **too low** – even rolling 6 for all remaining dice would exceed the desired sum
  - The code must **re-compute** the sum many times

Optimized diceSum

```
void diceSumHelper(int dice, int sum, int desiredSum,
    Vector<int>& chosen) {
    if (dice == 0) {
        if (sum == desiredSum) {
            cout << chosen << endl; // base case
        }
    } else if (sum + 1*dice <= desiredSum
            && sum + 6*dice >= desiredSum) {
        for (int i = 1; i <= 6; i++) {
            chosen.add(i);  // choose
            diceSumHelper(dice - 1, sum + i, desiredSum,
                chosen); // explore
            chosen.remove(chosen.size() - 1); // un-choose
        }
    }
}
```

Maze

"Escape Maze" exercise

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- Return `true` if able to escape, or `false` if not

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- "Mark" your path along the way
- "Taint" bad paths that do not work
- Do not explore the same path twice

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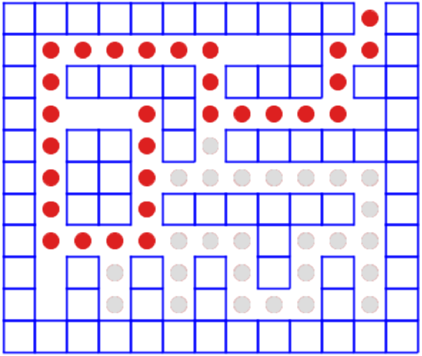

Maze class

```
#include "Maze.h"
```

Member name	Description
`m.inBounds(row, col)`	true if within maze boundaries
`m.isMarked(row, col)`	true if given cell is marked
`m.isOpen(row, col)`	true if given cell is empty (no wall or mark)
`m.isTainted(row, col)`	true if given cell has been tainted
`m.isWall(row, col)`	true if given cell contains a wall
`m.mark(row, col);`	sets given cell to be marked
`m.numRows(), m.numCols()`	returns dimensions of maze
`m.taint(row, col);`	sets given cell to be tainted
`m.unmark(row, col);`	sets given cell to be not marked if marked
`m.untaint(row, col);`	sets given cell to be not tainted if tainted

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- Base case

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- Base case
  1. What should we do in the **base case** when we're out of decisions?
     - *Return true or false*

Escape Maze solution

```
bool escapeMaze(Maze& maze, int row, int col) {
    if (!maze.inBounds(row, col)) return true;
    else if (!maze.isOpen(row, col)) return false;
    else {
        // recursive case: try to escape in 4 directions
        maze.mark(row, col);
        if (escapeMaze(maze, row - 1, col)
                || escapeMaze(maze, row + 1, col)
                || escapeMaze(maze, row, col - 1)
                || escapeMaze(maze, row, col + 1)) {
            return true;   // one of the paths worked!
        } else {
            maze.taint(row, col);
            return false; // all 4 paths failed; taint
        }
    }
}
```

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- **Arm's length recursion**: a poor style where unnecessary tests are performed before performing recursive calls
- Typically, the tests try to avoid making a call into what would otherwise be a base case
- Example: `escapeMaze` – our code recursively tries to explore up, down, left, and right. Some of those directions may lead to walls or off the board. Shouldn't we test before making calls in those directions?

"Arms-length" escapeMaze

```
// This code is bad. It uses arm's length recursion.
bool escapeMaze(Maze& maze, int r, int c) {
    maze.mark(row, col);
    // recursive case: check each one by arm's length
    if (maze.inBounds(r-1,c) && maze.isOpen(r-1, c))
        if (escapeMaze(r-1,c)) {return true; }
    if (maze.inBounds(r+1,c) && maze.isOpen(r+1, c))
        if (escapeMaze(r+1,c)) {return true; }
    if (maze.inBounds(r,c-1) && maze.isOpen(r,c-1))
        if (escapeMaze(r,c-1)) {return true; }
    if (maze.inBounds(r,c+1) && maze.isOpen(r,c+1))
        if (escapeMaze(r,c+1)) {return true; }
    maze.taint(row, col);
    return false; // all 4 paths failed; taint
}
```

escapeMaze better solution

```
bool escapeMaze(Maze& maze, int row, int col) {
    if (!maze.inBounds(row, col)) return true;
    else if (!maze.isOpen(row, col)) return false;
    else {
        // recursive case: try to escape in 4 directions
        maze.mark(row, col);
        if (escapeMaze(maze, row - 1, col)
               || escapeMaze(maze, row + 1, col)
               || escapeMaze(maze, row, col - 1)
               || escapeMaze(maze, row, col + 1)) {
            return true;   // one of the paths worked!
        } else {
            maze.taint(row, col);
            return false; // all 4 paths failed; taint
        }
    }
}
```

Permutations

"Permute Vector" exercise

- Write a function `permute` that accepts a Vector of strings as a parameter and outputs all possible rearrangements of the strings in that vector. The arrangements may be output in any order.

{a, b, c, d}	{b, a, c, d}	{c, a, b, d}	{d, a, b, c}
{a, b, d, c}	{b, a, d, c}	{c, a, d, b}	{d, a, c, b}
{a, c, b, d}	{b, c, a, d}	{c, b, a, d}	{d, b, a, c}
{a, c, d, b}	{b, c, d, a}	{c, b, d, a}	{d, b, c, a}
{a, d, b, c}	{b, d, a, c}	{c, d, a, b}	{d, c, a, b}
{a, d, c, b}	{b, d, c, a}	{c, d, b, a}	{d, c, b, a}

"Permute Vector" exercise

- Write a function `permute` that accepts a Vector of strings as a parameter and outputs all possible rearrangements of the strings in that vector. The arrangements may be output in any order.
- Example: if v contains {`"a", "b", "c", "d"`}, your function outputs these permutations:

{a, b, c, d}	{b, a, c, d}	{c, a, b, d}	{d, a, b, c}
{a, b, d, c}	{b, a, d, c}	{c, a, d, b}	{d, a, c, b}
{a, c, b, d}	{b, c, a, d}	{c, b, a, d}	{d, b, a, c}
{a, c, d, b}	{b, c, d, a}	{c, b, d, a}	{d, b, c, a}
{a, d, b, c}	{b, d, a, c}	{c, d, a, b}	{d, c, a, b}
{a, d, c, b}	{b, d, c, a}	{c, d, b, a}	{d, c, b, a}

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- We want to generate all possible sequences of decisions

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for (each possible first letter):
    for (each possible second letter):
        for (each possible third letter):
            ...
                print!
```

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- This is called a **depth-first search**

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  1. We generally iterate over decisions. What are we iterating over here? The iteration will be done by **recursion**.
     - *Iterating over strings left in the vector*
  2. What are the **choices** for each decision? Do we need a loop?
     - *Strings in the vector; use a for loop*
- Exploring
  1. How can we represent that choice? How should we **modify the parameters**?

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  1. What should we do in the **base case** when we're out of decisions?
     - *Print out permutation*

"Permute Vector" solution

```
// Outputs all permutations of the given vector.
void permute(Vector<string>& v) {
    Vector<string> chosen; permuteHelper(v, chosen);
}
void permuteHelper(Vector<string>& v, Vector<string>&
    chosen) {
    if (v.isEmpty()) cout << chosen << endl;
    else {
        for (int i = 0; i < v.size(); i++) {
            string s = v[i]; v.remove(i);
            chosen.add(s);                     // choose
            permuteHelper(v, chosen);          // explore
            chosen.remove(chosen.size() - 1); // un-choose
            v.insert(i, s);
        }
    }
}
```

Permute a string

- Write a function `permute` that accepts a string as a parameter and outputs all possible rearrangements of the characters in that string. The arrangements may be output in any order.

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- Write a function `permute` that accepts a string as a parameter and outputs all possible rearrangements of the characters in that string. The arrangements may be output in any order.
- Example: there are 6 permutations of "cat"

Permute a string – solution

```
// Outputs all permutations of the given string.
void permute(string s) {
    permute(s, "");
}
void permuteHelper(string s, string chosen = "") {
    if (s == "") {
        cout << chosen << endl; // base case: nothing left
    } else {
        // recursive case: choose each possible next letter
        for (int i = 0; i < s.length(); i++) {
            string rest = s.substr(0, i) + s.substr(i + 1);
            permuteHelper(rest, chosen + s[i]); //
                choose/explore
        }
    }
}
```