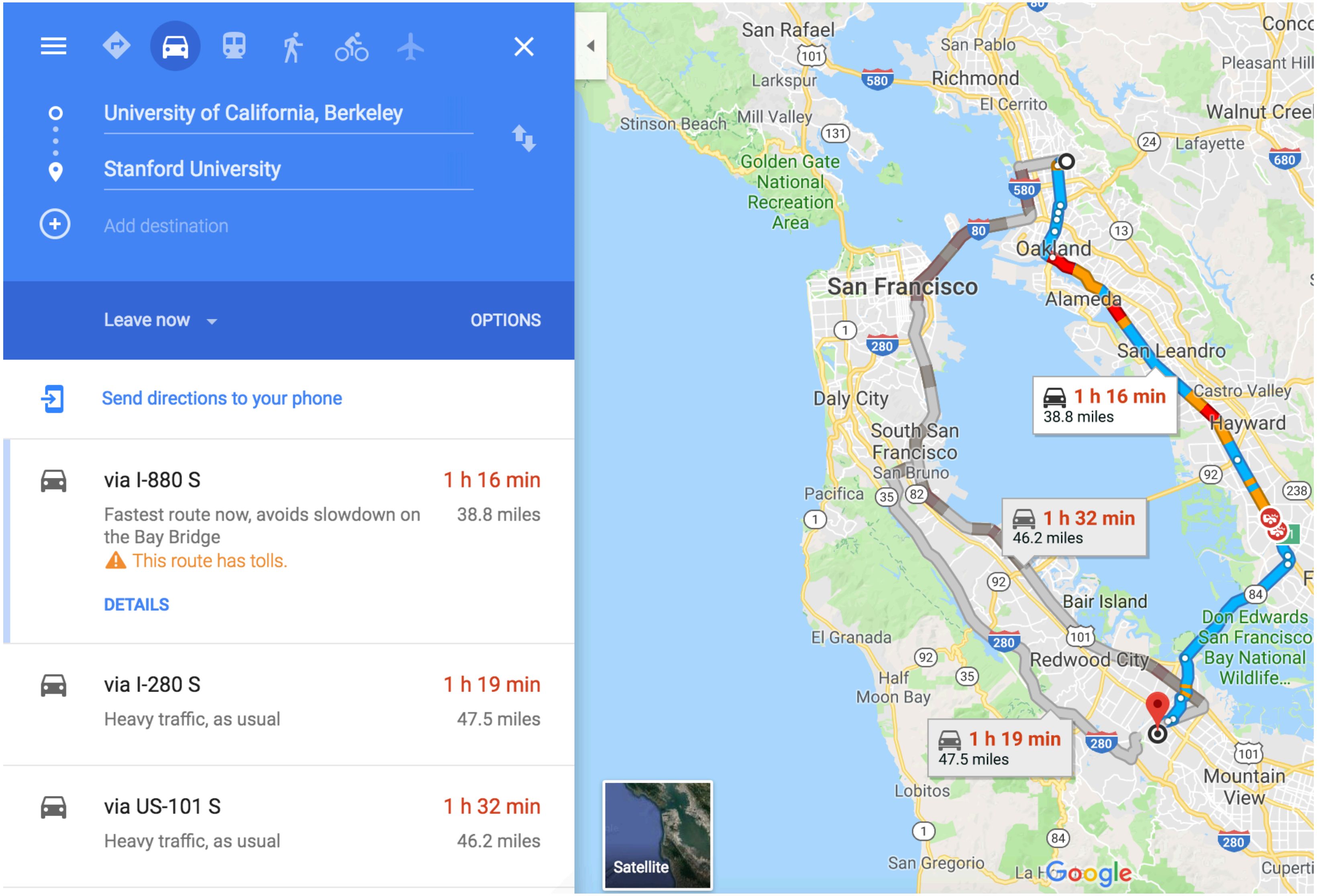
CS 106B, Lecture 22
Graphs

Plan for Today

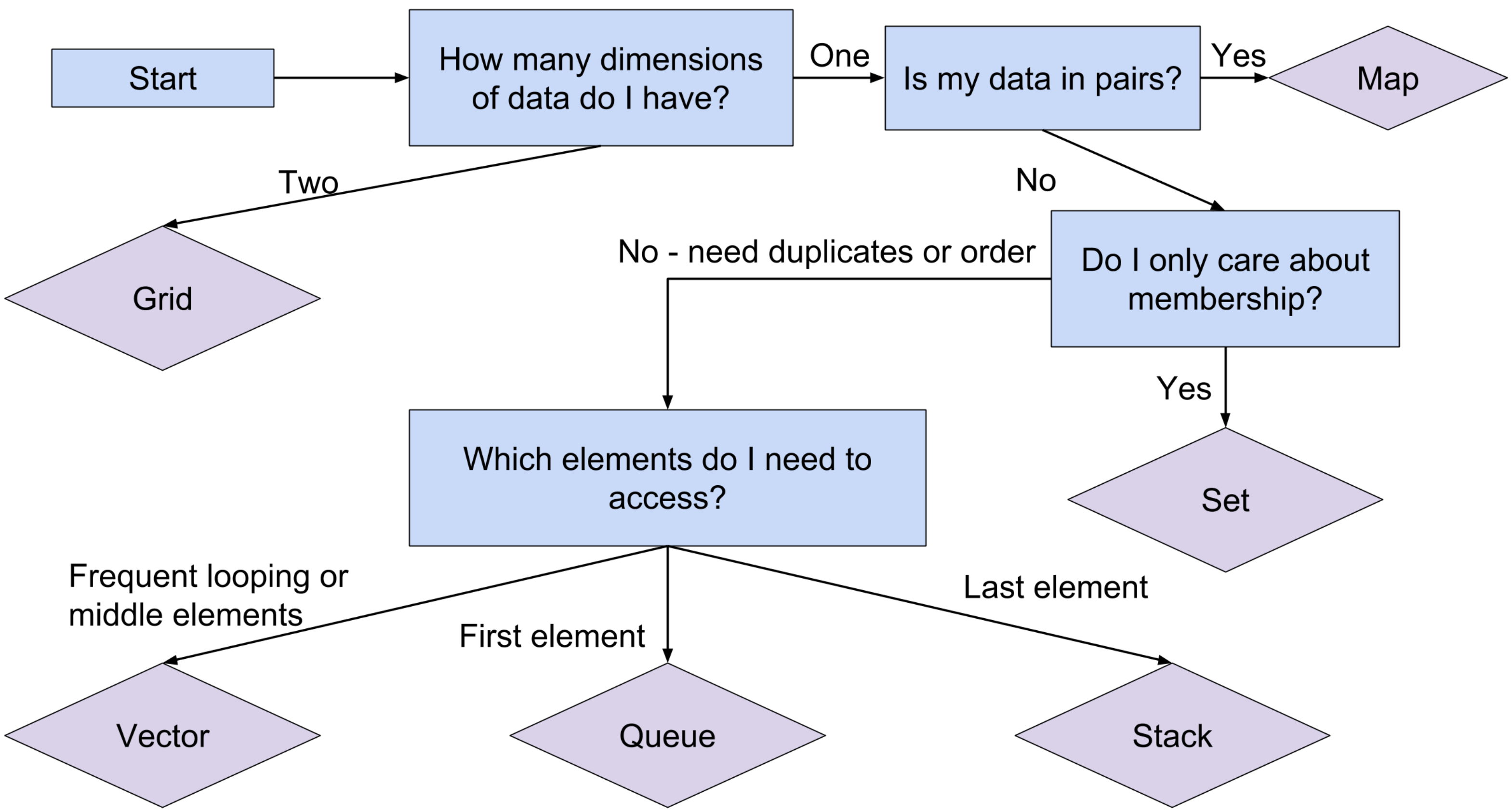
- Arguably the single most useful abstraction computer science: the graph
  - How to model problems using a graph
- Today and some of next week is algorithms to answer common graph questions
  - Learning these algorithms will help you solve very different problems more quickly

Google Maps

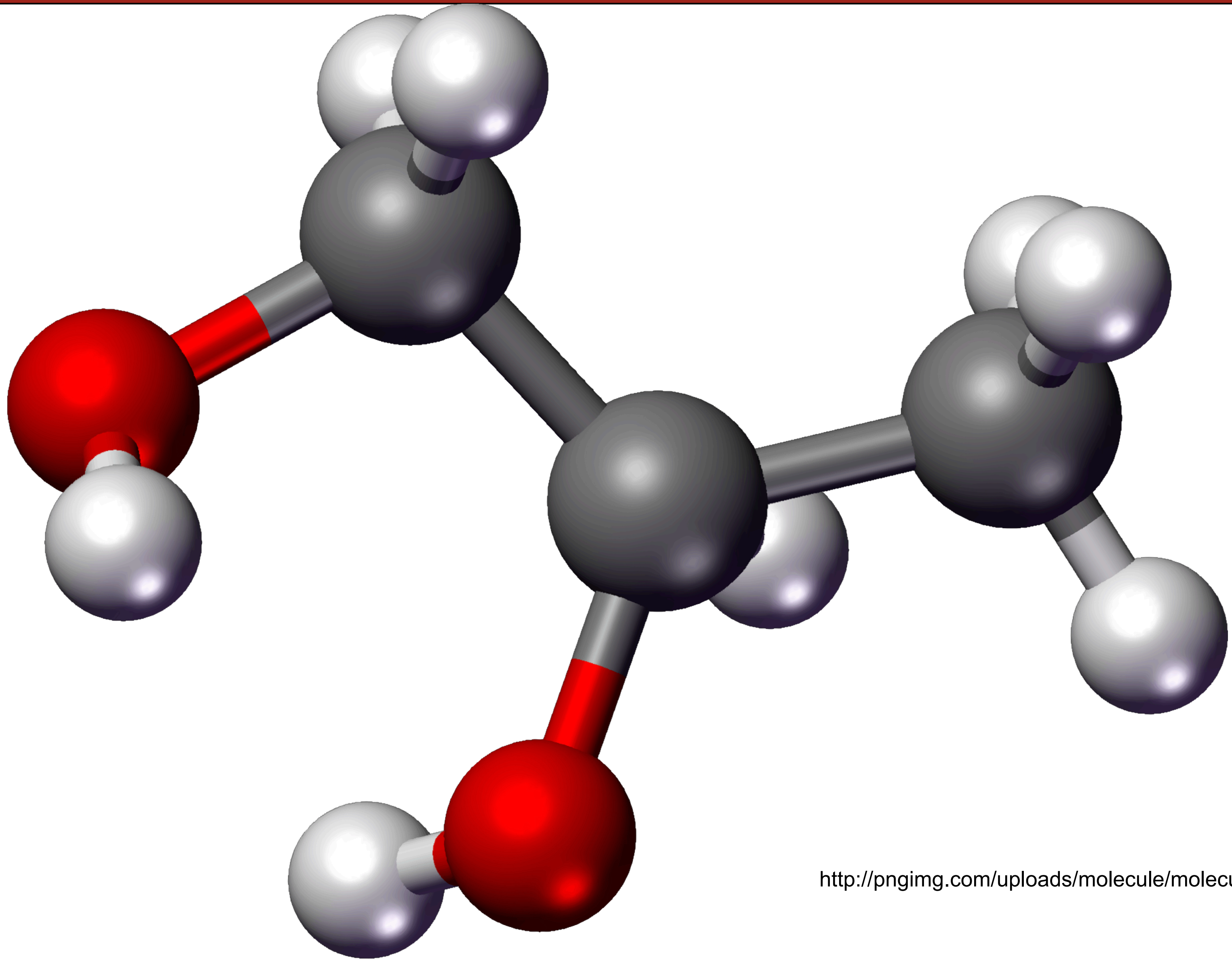


Source: https://www.google.com/maps

ADT Flowchart



Molecules



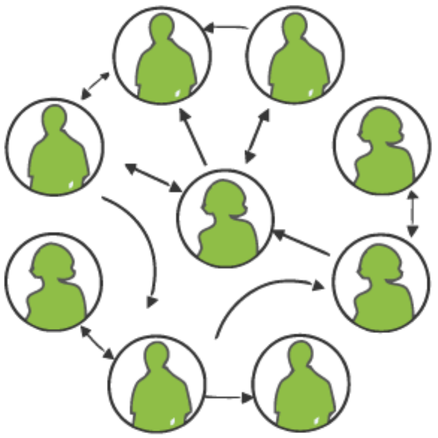
http://pngimg.com/uploads/molecule/molecule_PNG50.png

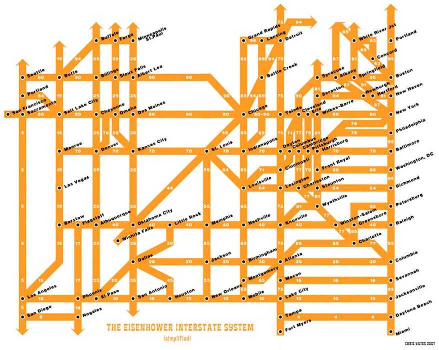
Introducing: The Graph

- A **graph** is a mathematical structure for representing relationships
- Consists of **nodes** (aka vertices) and **edges** (aka arcs)
  - **edges** are the relationships, **nodes** are the items that have the relationship
- Examples:
  - Map: cities (nodes) are connected by roads (edges)
  - Flowchart: questions and recommendations (nodes) are connected by answers (edges
  - Molecules: atoms (nodes) are connected by bonds (edges)

Graph examples

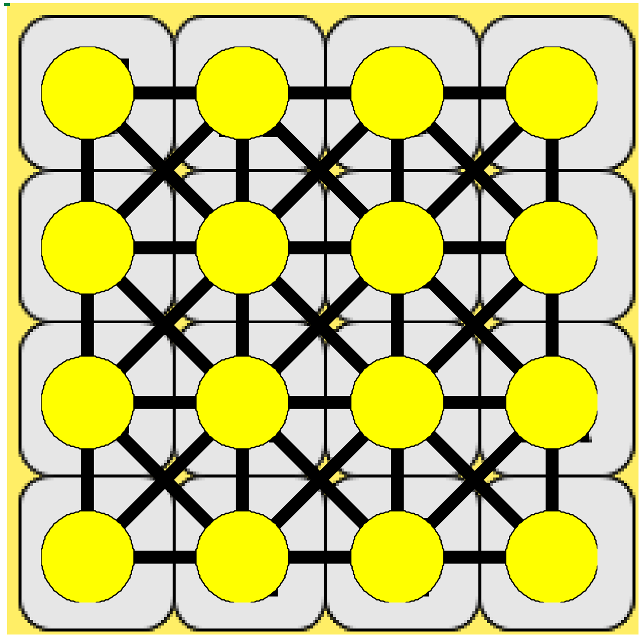
- For each, what are the nodes and what are the edges?
  - Web pages with links
  - Functions in a program that call each other
  - Airline routes
  - Facebook friends
  - Course pre-requisites
  - Family trees
  - Paths through a maze





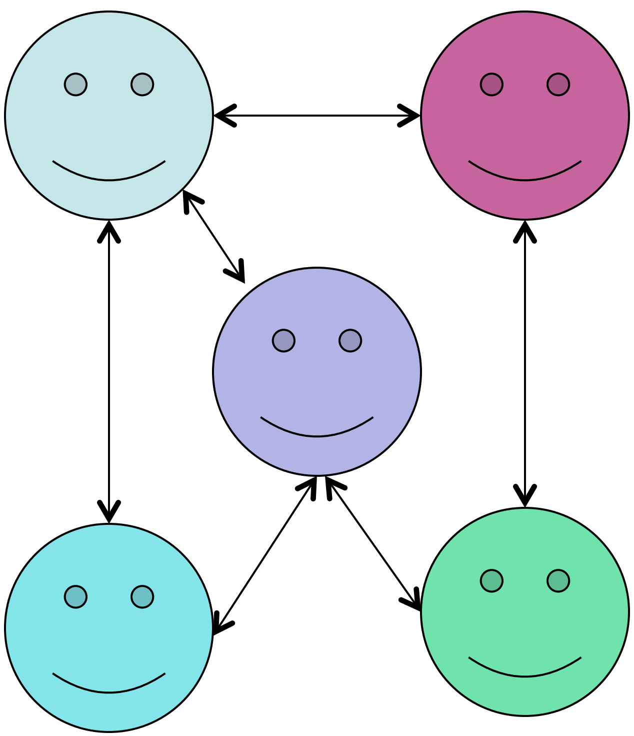
Boggle as a graph

- **Q:** If a Boggle board is a graph, what is a node? What is an edge?
  - **A.** Node = letter cube, Edge = Dictionary (lexicon)
  - **B.** Node = dictionary word; Edge = letter cube
  - **C.** Node = letter; Edge = between each letter that is part of a word
  - **D.** Node = letter cube; Edge = connection to neighboring cube
  - **E.** None of the above

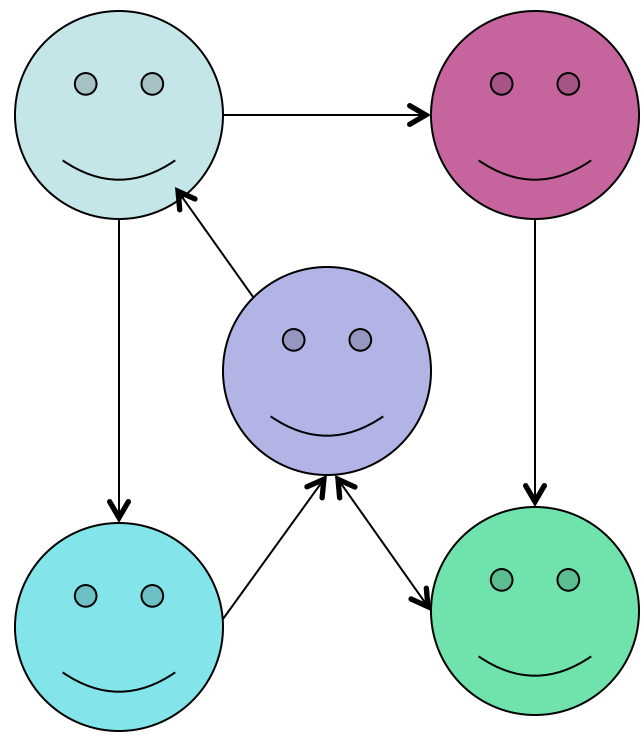


Undirected vs. Directed

- Some relationships are mutual
  - Facebook



- Some are one-way
  - Twitter
  - Doesn't mean that all relationships are non-mutual

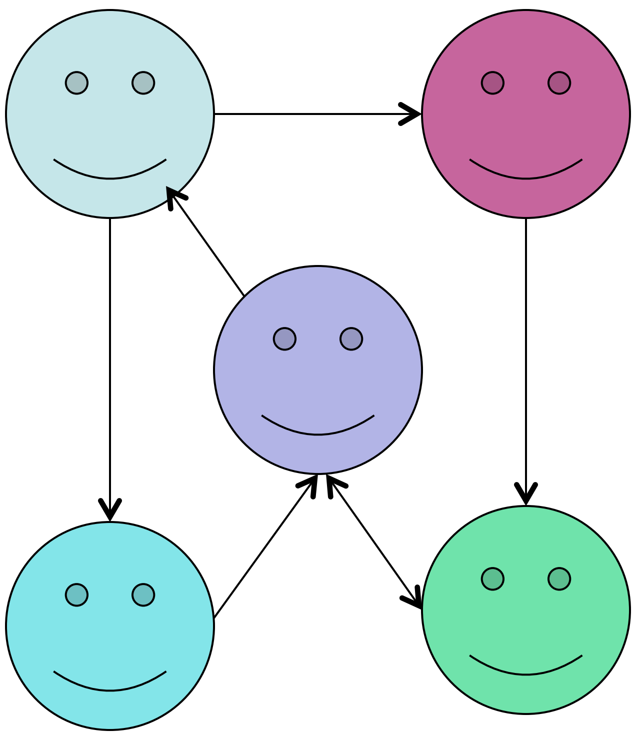


Representing Graphs

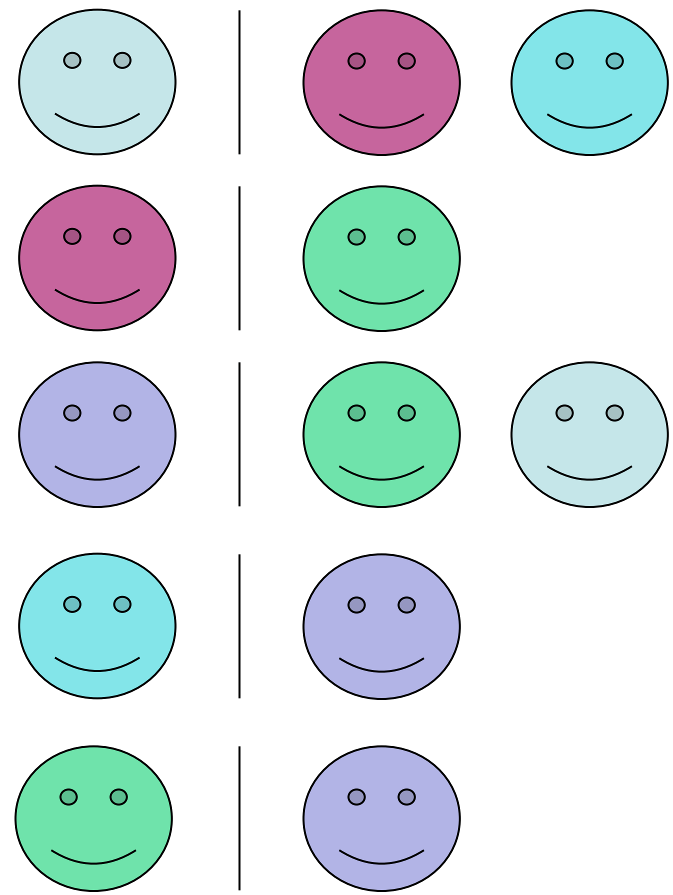
- Two main ways:
  - Have each node store the nodes it's connected to (**adjacency list**)
    - Enemies in problem 4 of the midterm
    - NGrams
    - Doctors without Orders
  - Have a list of all the edges/edges (**edge list**)
    - Similar to Marbles
- The choice depends on the problem you're trying to solve
- You can sometimes represent graphs implicitly instead of explicitly storing the edges and nodes
  - e.g. Boggle, WordLadder
  - draw a picture to see the graph more clearly!

Adjacency List

- Map<***Node***, Vector<***Node***>>
  - or `Map<`***`Node`***`, Set<`***`Node`***`>>`

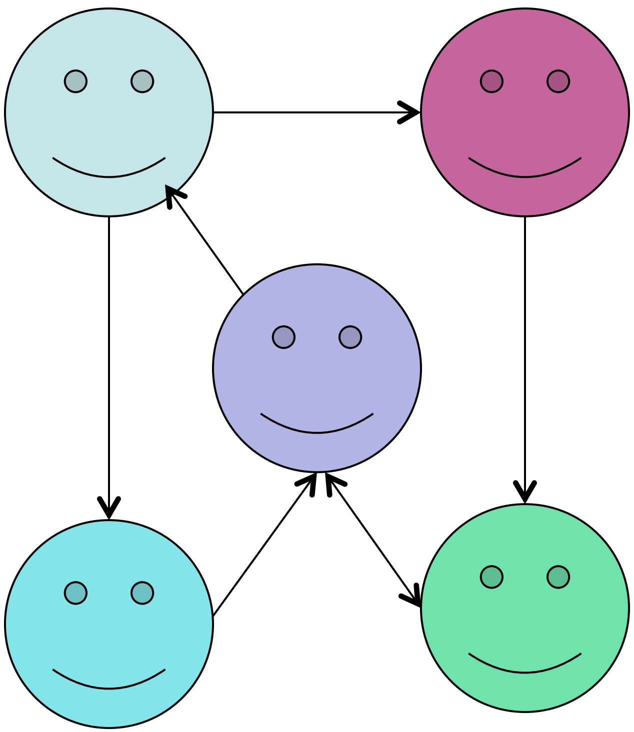


Node Set<***Node***>



Adjacency Matrix

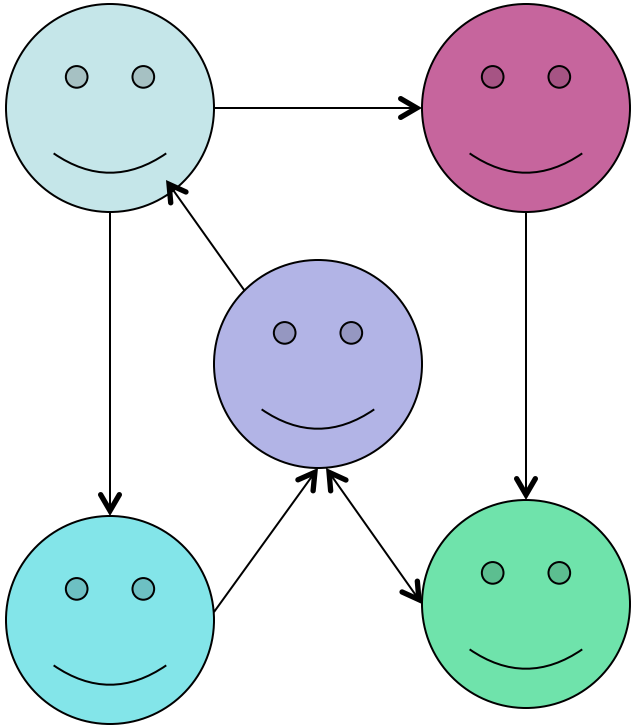
- Store a boolean grid, rows/columns correspond to nodes
  - Alternative to Adjacency List



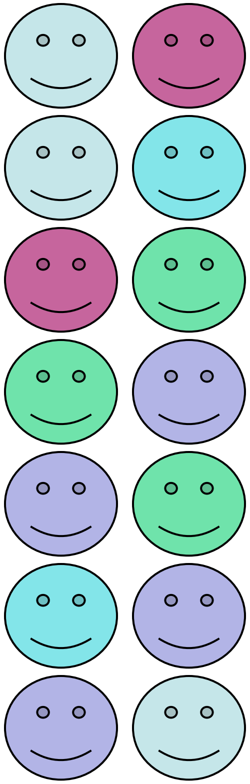
	F	T	F	T	F
	F	F	F	F	T
	T	F	F	F	T
	F	F	T	F	F
	F	F	T	F	F

Edge List

- Store a `Vector`<***Edge***> (or `Set`<***Edge***>)
  - ***Edge*** struct would have the two nodes

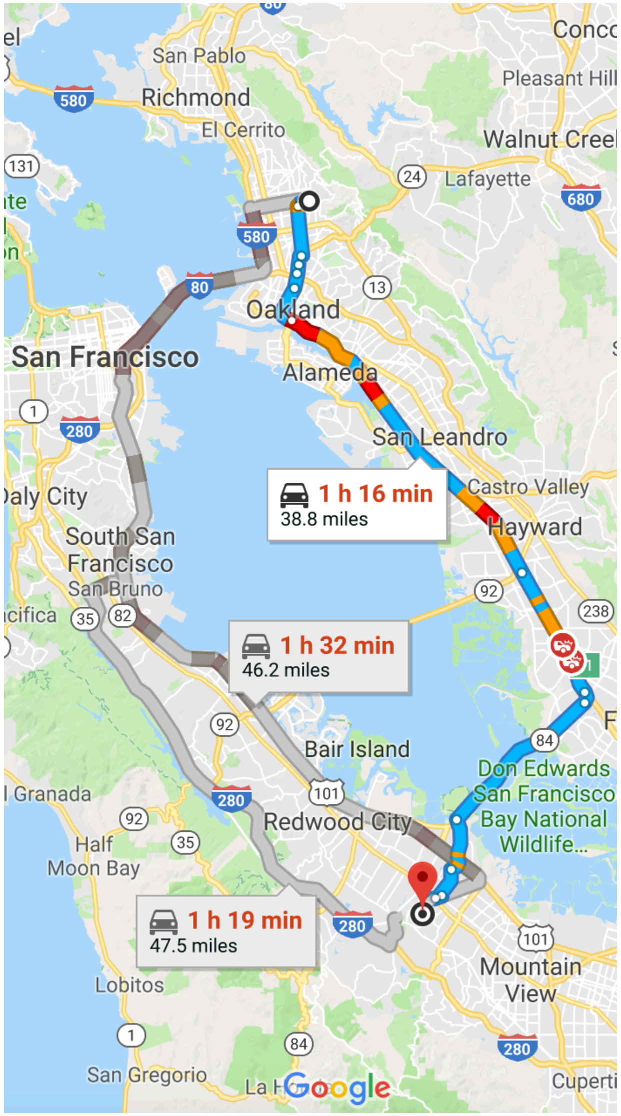


Vector<***Edge***>



Edge Properties

- Not all edges are created equally
  - Some have greater **weight**
- Real life examples:
  - Flight costs
  - Miles on a road
  - Time spent on a road
- Store a number with each edge corresponding to its weight

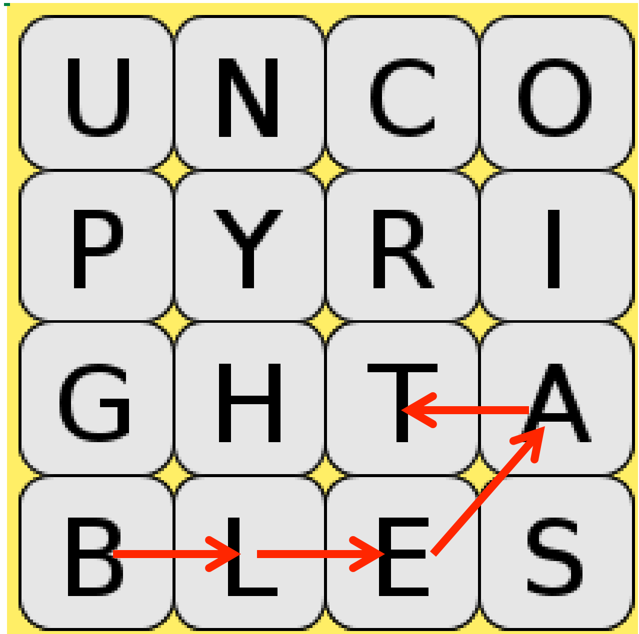


Source: https://www.google.com/maps

Paths

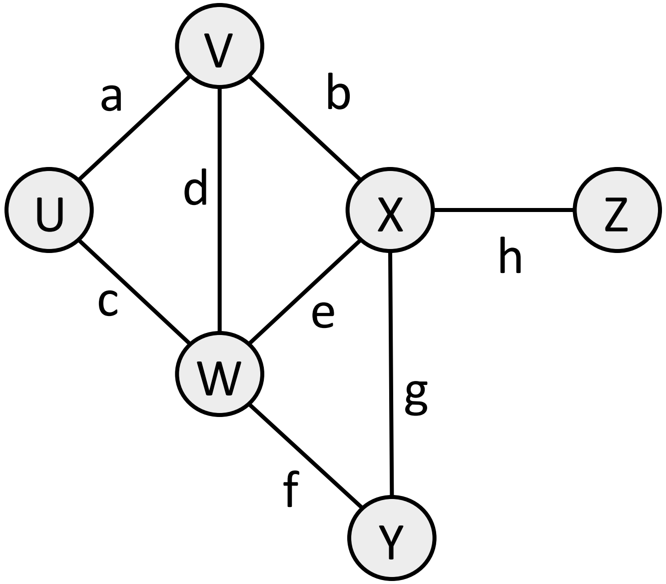
- I want a job at Google. Do I know anyone who works there? What about someone who knows someone?
- I want to find this word on a board made of letters "next to" each other (Boggle)
- A **path** is a sequence of nodes with edges between them connecting two nodes
  - Could store edges instead of nodes (why?)
  - You know Jane. Jane knows Sally. Sally knows knows Sergey Brin, the founder of Google, so the path is:

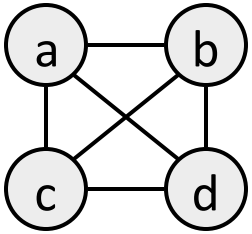
    You->Jane->Sally->Sergey

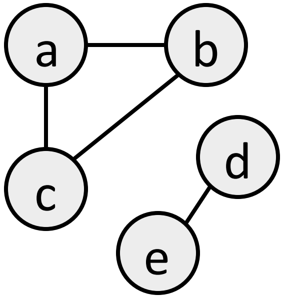


Other graph properties

- **reachable**: Vertex *u* is *reachable* from *v* if a path exists from *u* to *v*.
- **connected**: A graph is *connected* if every vertex is reachable from every other.
- **complete**: If every vertex has a direct edge to every other.



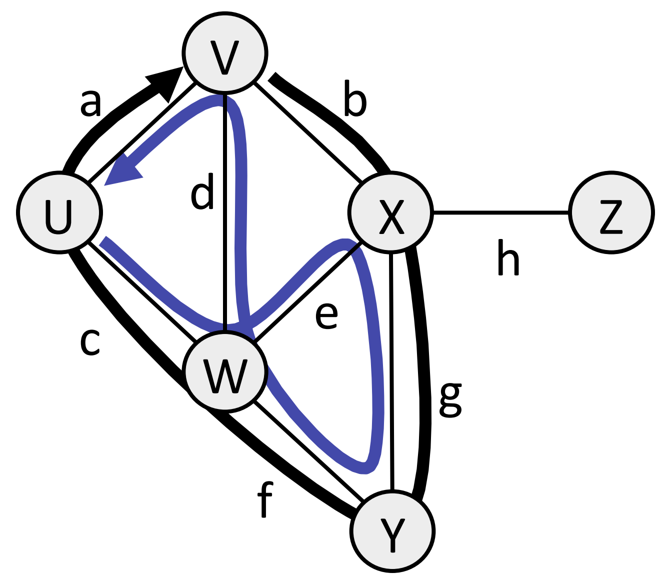




Loops and cycles

- **cycle**: A path that begins and ends at the same node.
  - example: {b, g, f, c, a} or {V, X, Y, W, U, V}.
  - example: {c, d, a} or {U, W, V, U}.
  - **acyclic graph**: One that does not contain any cycles.

- **loop**: An edge directly from a node to itself.
  - Many graphs don't allow loops.



Types of Graphs

- NGrams?
  - directed, weighted, cyclic, connected
- Boggle?
  - undirected, unweighted, cyclic, connected
- A molecule?
  - undirected, weighted, potentially cyclic, connected
- A map of flights?
  - directed, weighted, cyclic, perhaps not connected
- A tree?
  - directed, acyclic graph (not connected)
  - DAGs are especially important because of **topological sort**. More on that later!

Announcements

- You should be starting LineManager – it's hard.
- Please give us feedback! cs198.stanford.edu
- Feel free to use seepluspl.us to help you understand trees or pointers. It's still in development, so be patient with quirks
- Notes on course feedback:
  - If you have a question outside the scope of the class, please post on Piazza or come talk to me during OH! I don't want to stop your questions, but I sometimes have to make choices to ensure that I don't confuse other students or run out of time for material we need to cover.

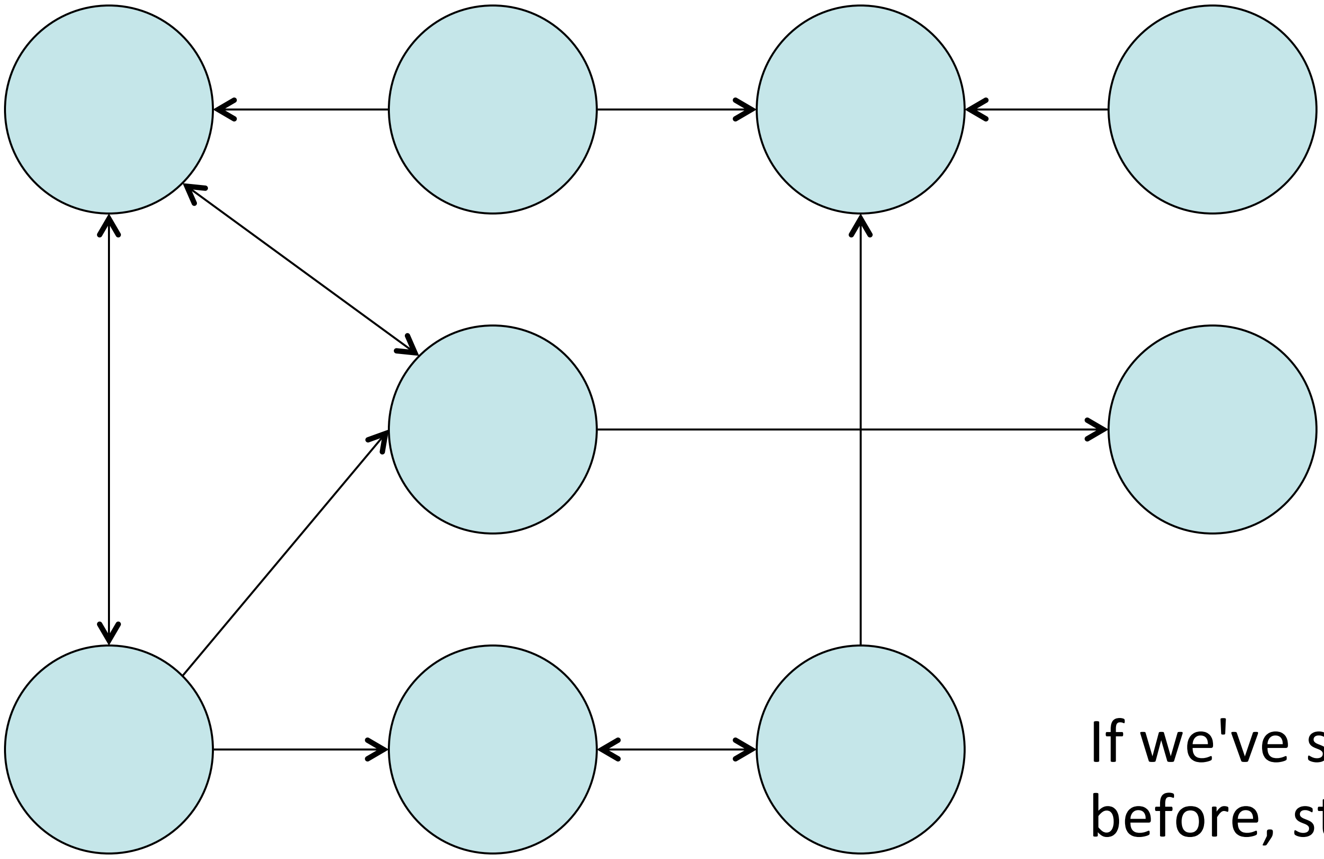
Working with Graphs

- We've seen how to model data with a graph
- There are lots of cool graph algorithms that make it easy to solve certain problems
  - Goal: know how to apply a model a problem as a graph and apply the relevant graph algorithm to it
- We'll spend most of the rest of this unit learning about graph algorithms

Finding Paths

- Easiest way: Depth-First Search (DFS)
  - Recursive backtracking!
- Finds a path between two nodes if it exists
  - Or can find all the nodes **reachable** from a node
    - Where can I travel to starting in San Francisco?
    - If all my friends (and their friends, and so on) share my post, how many will eventually see it?

DFS



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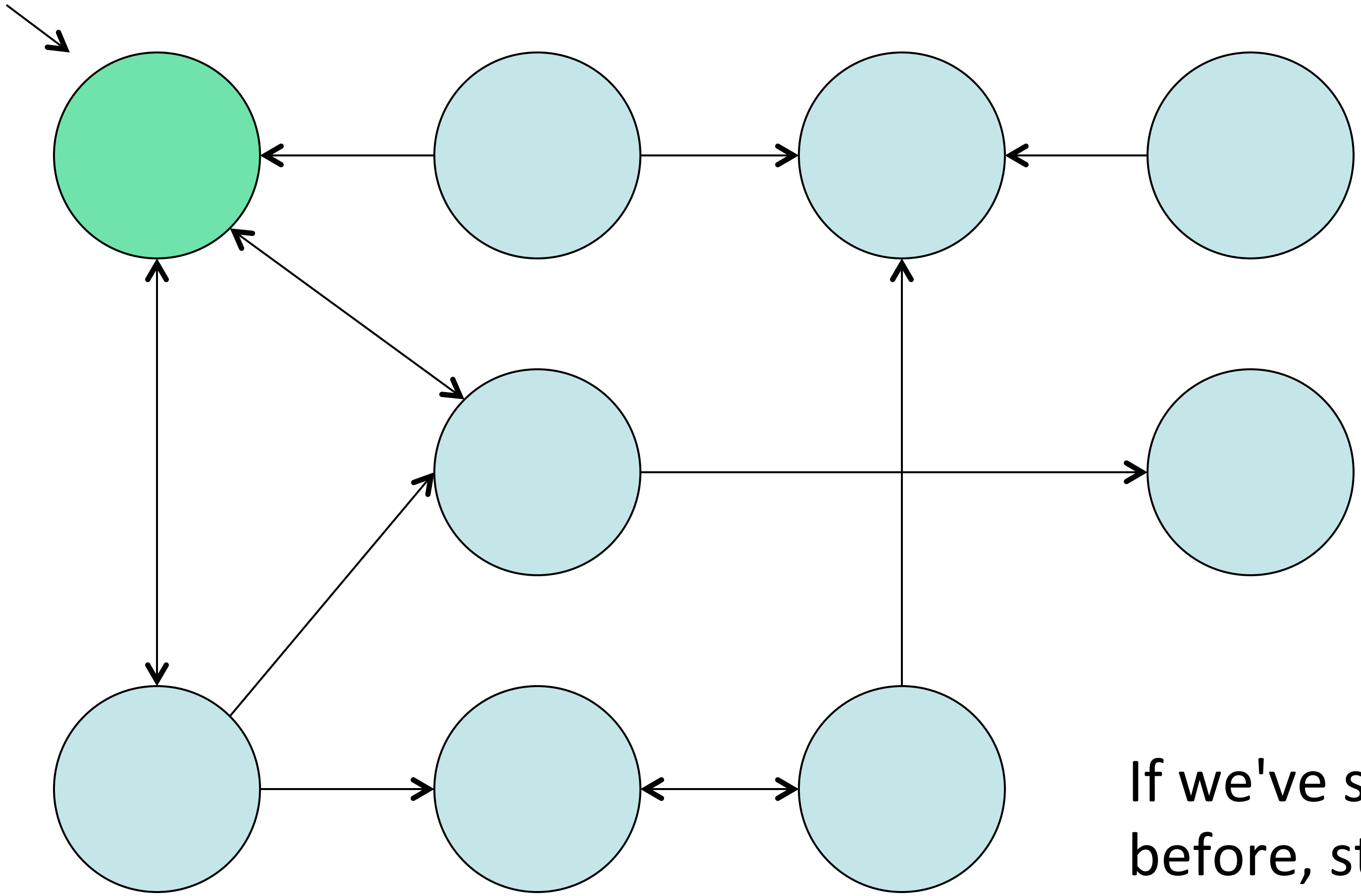

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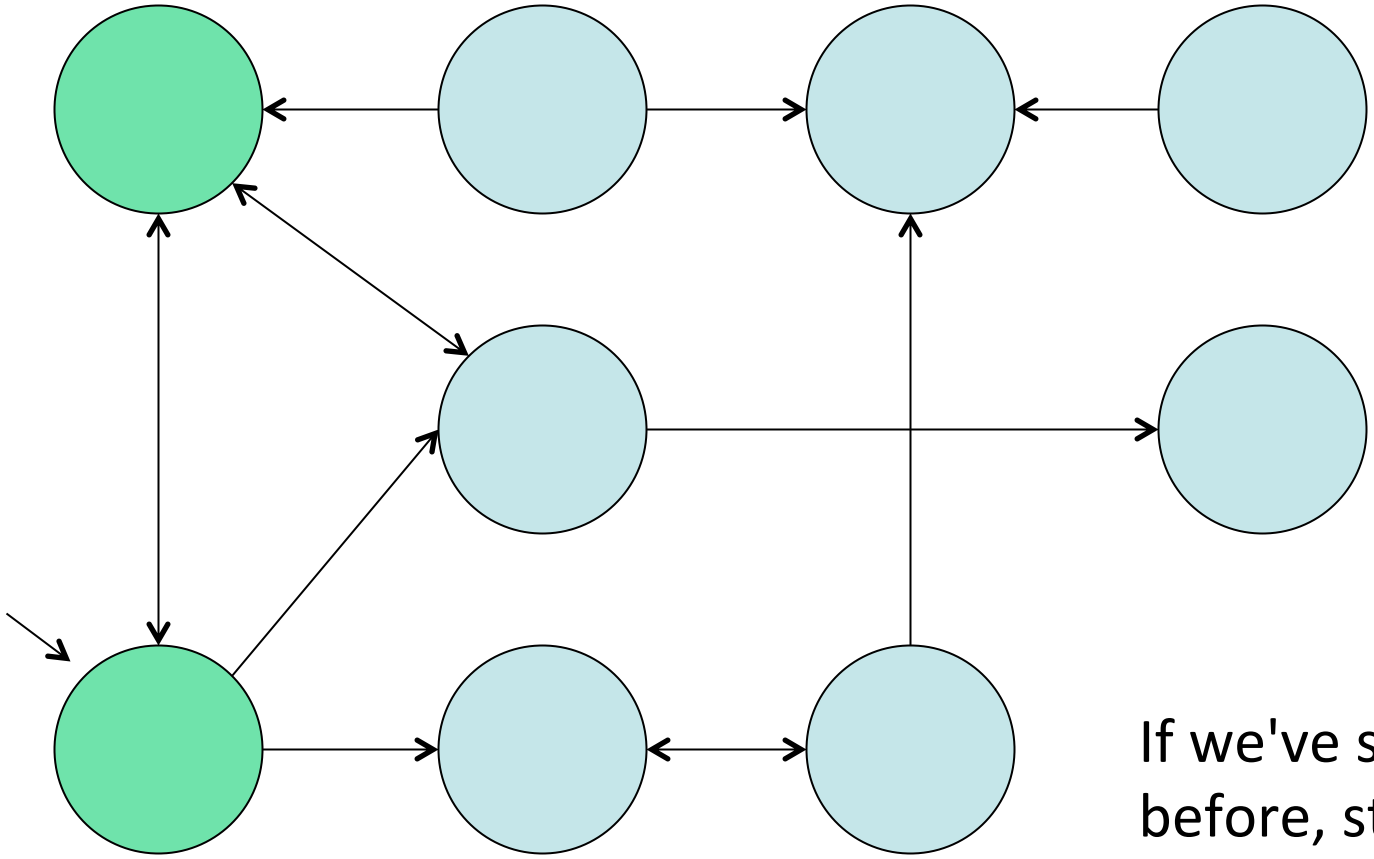

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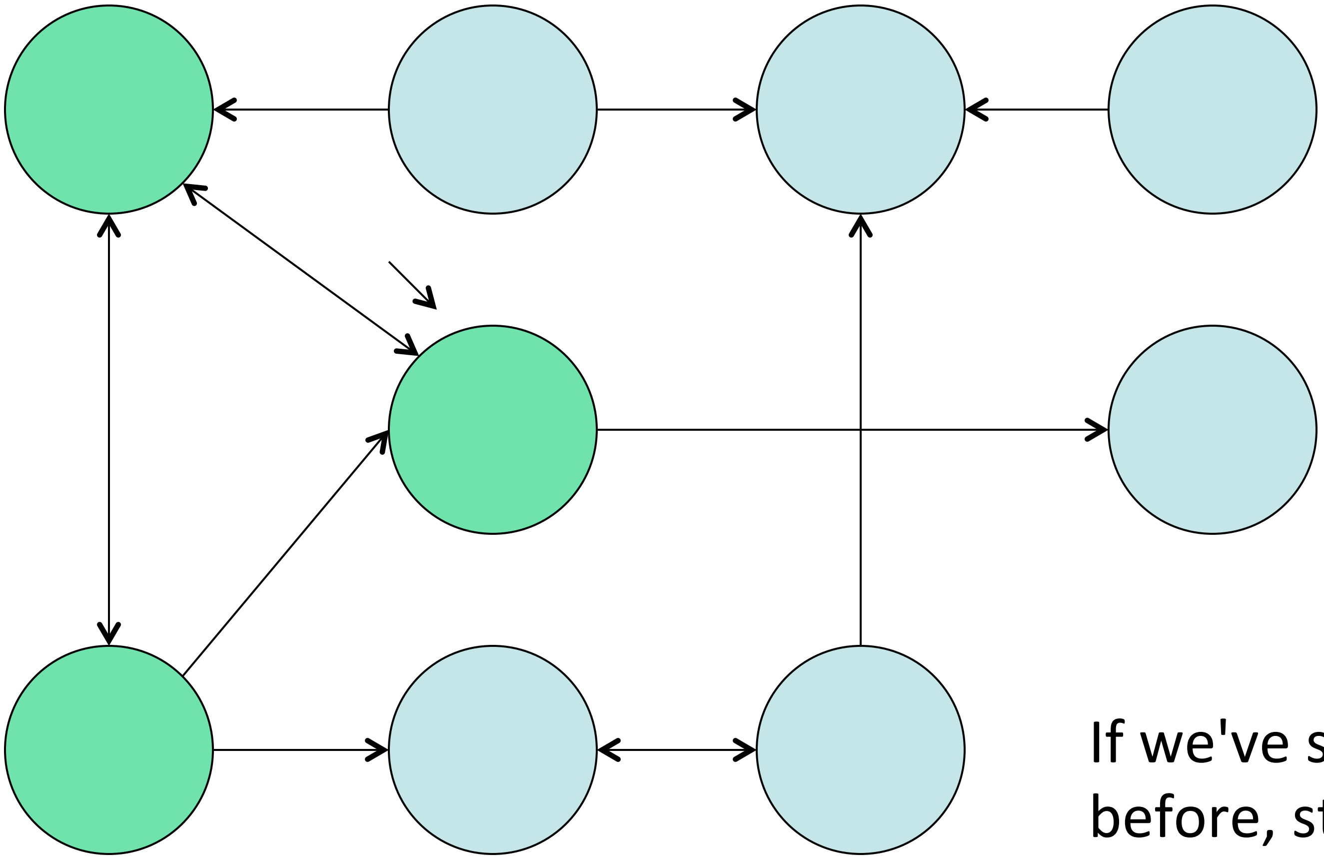

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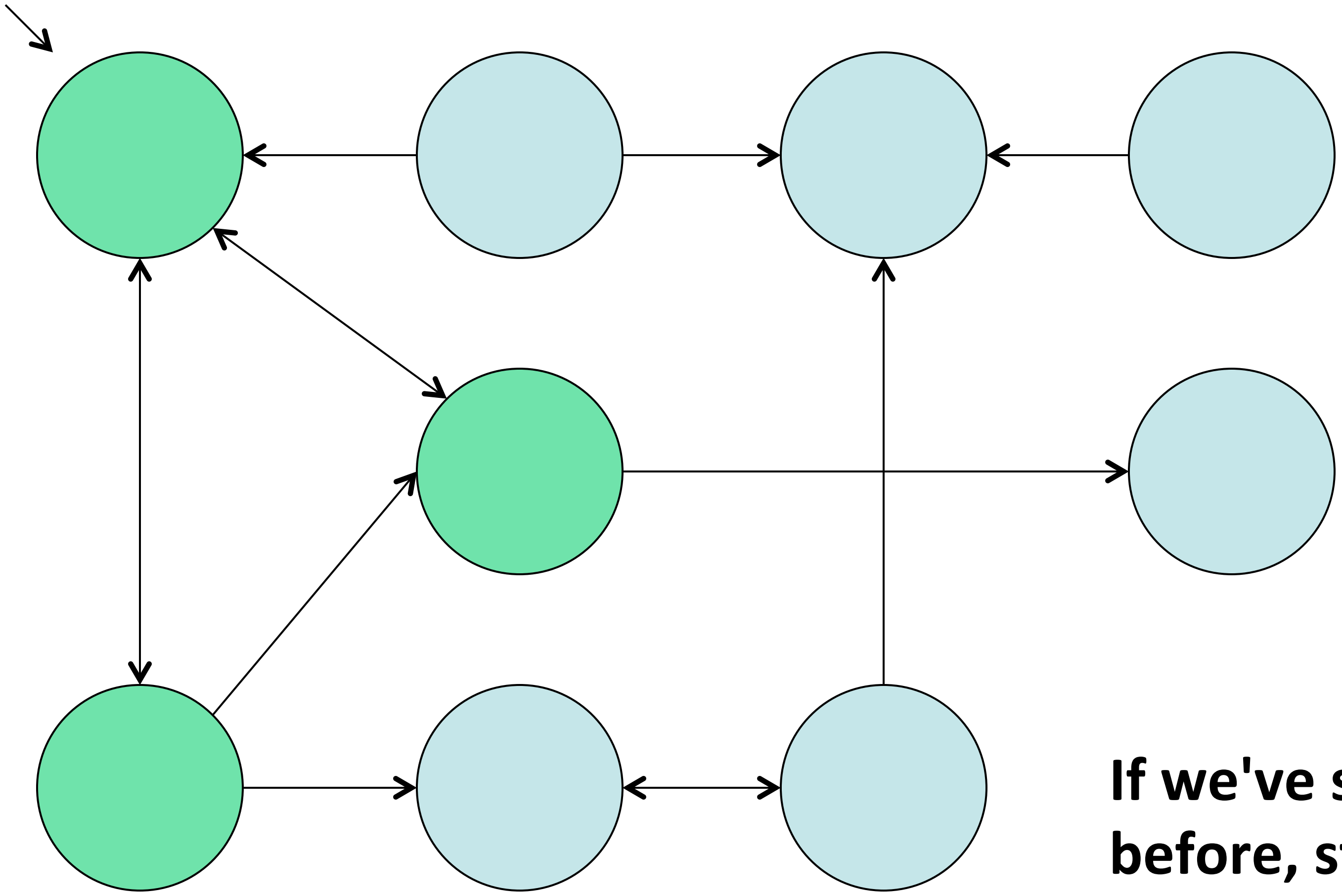

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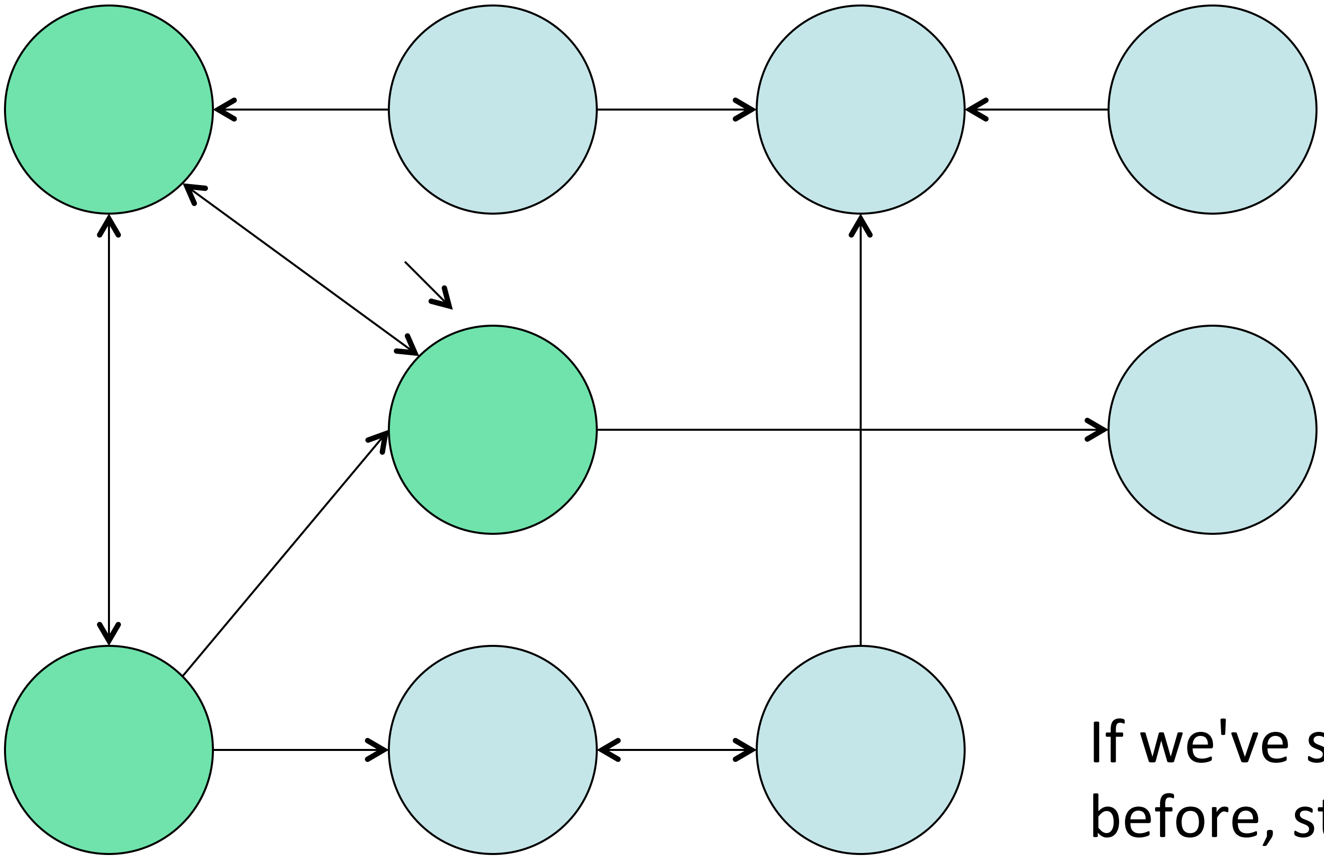

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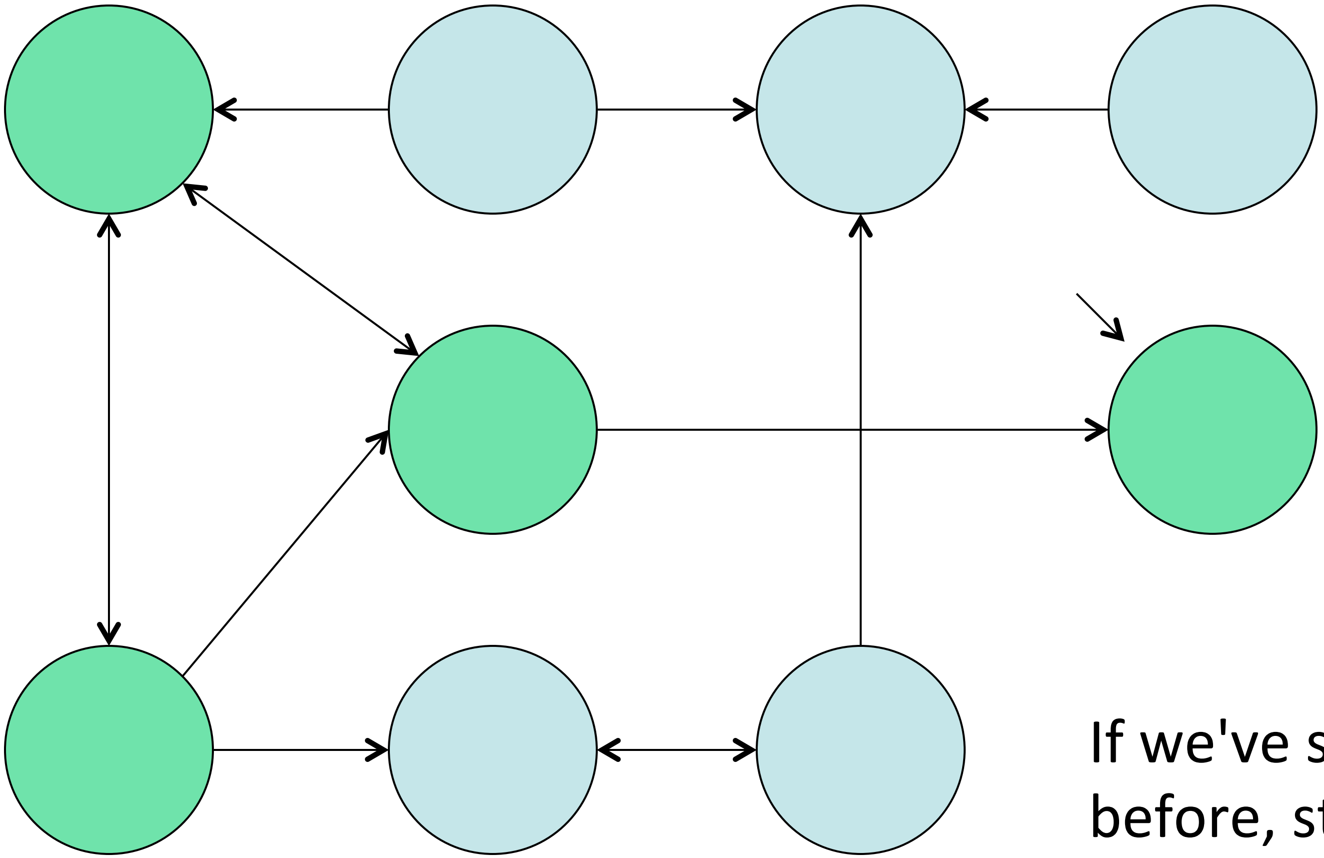

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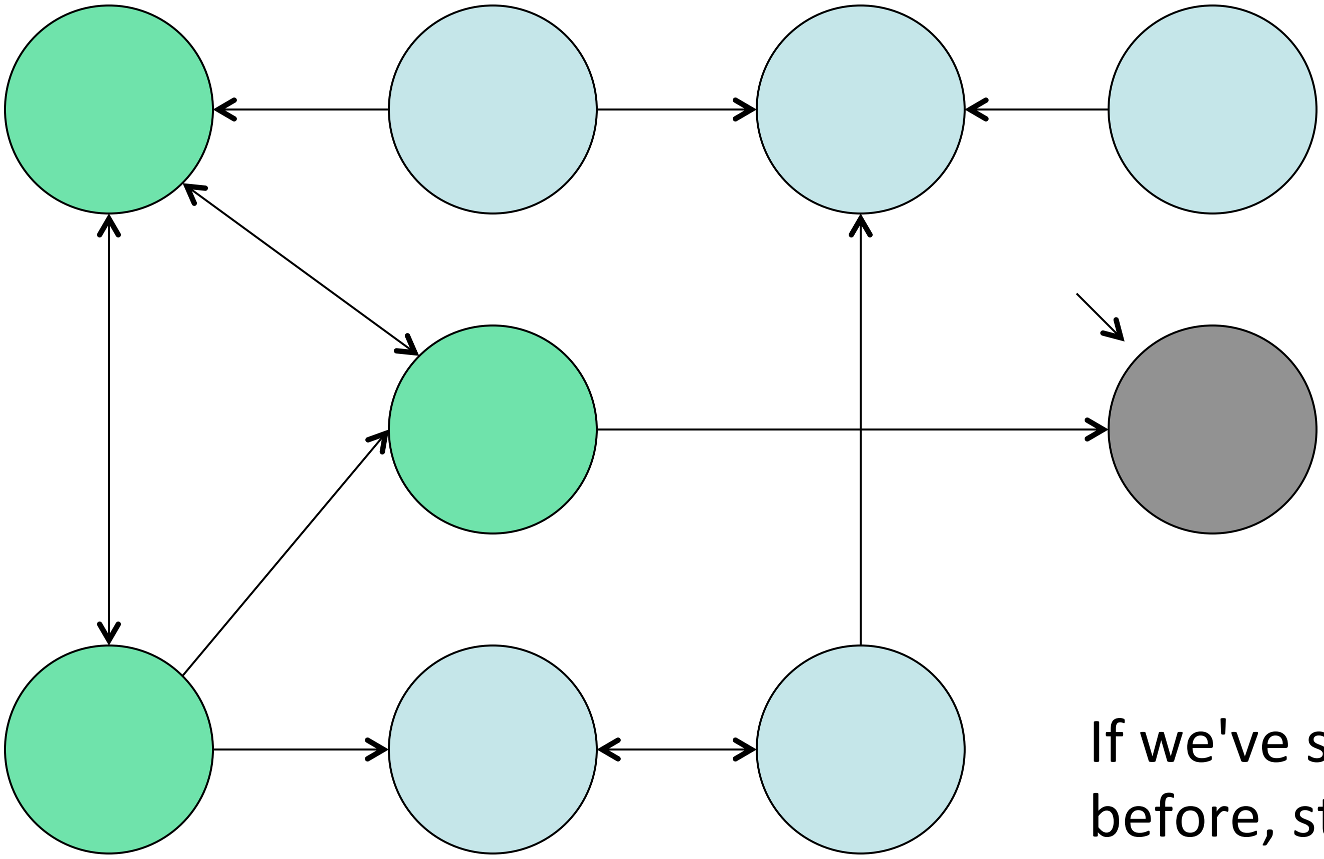

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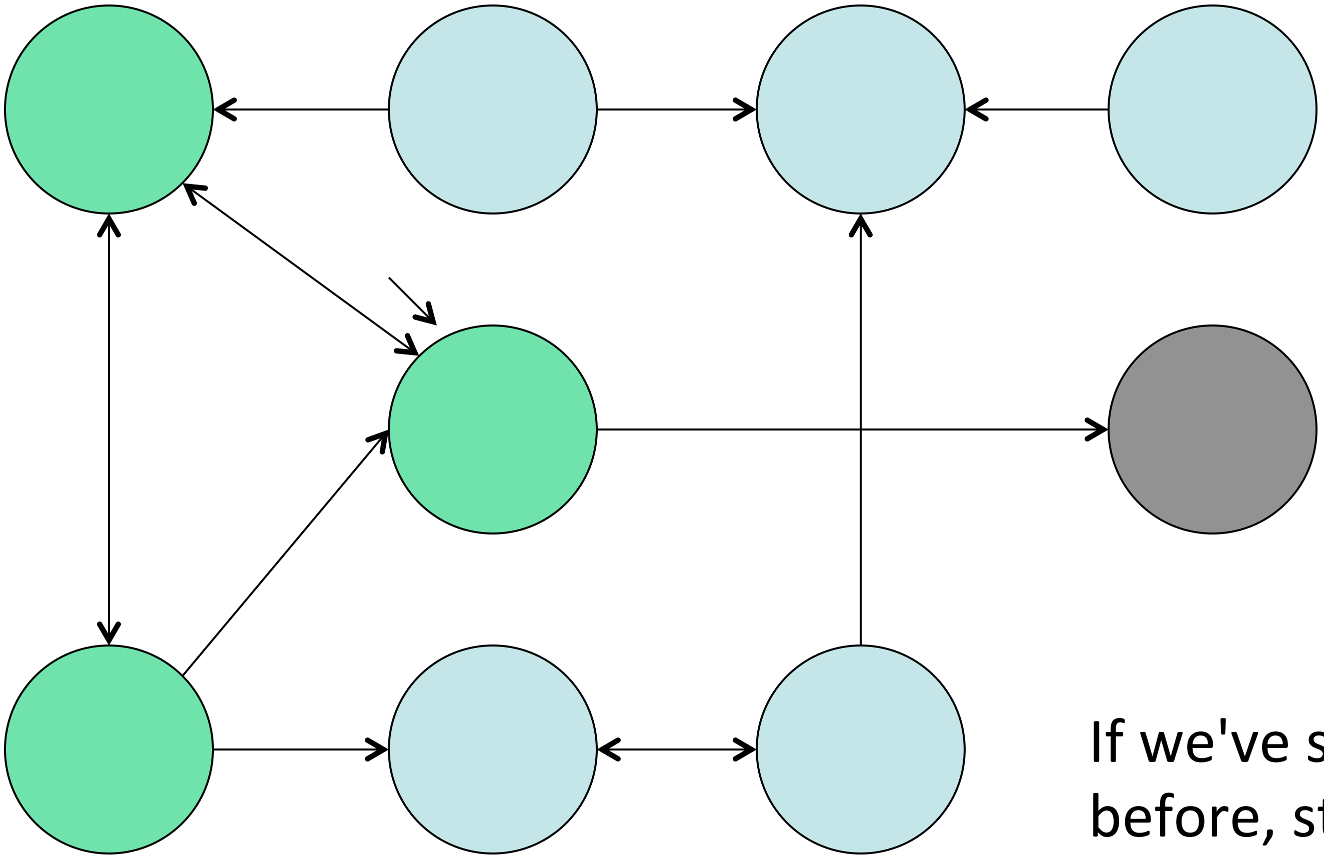

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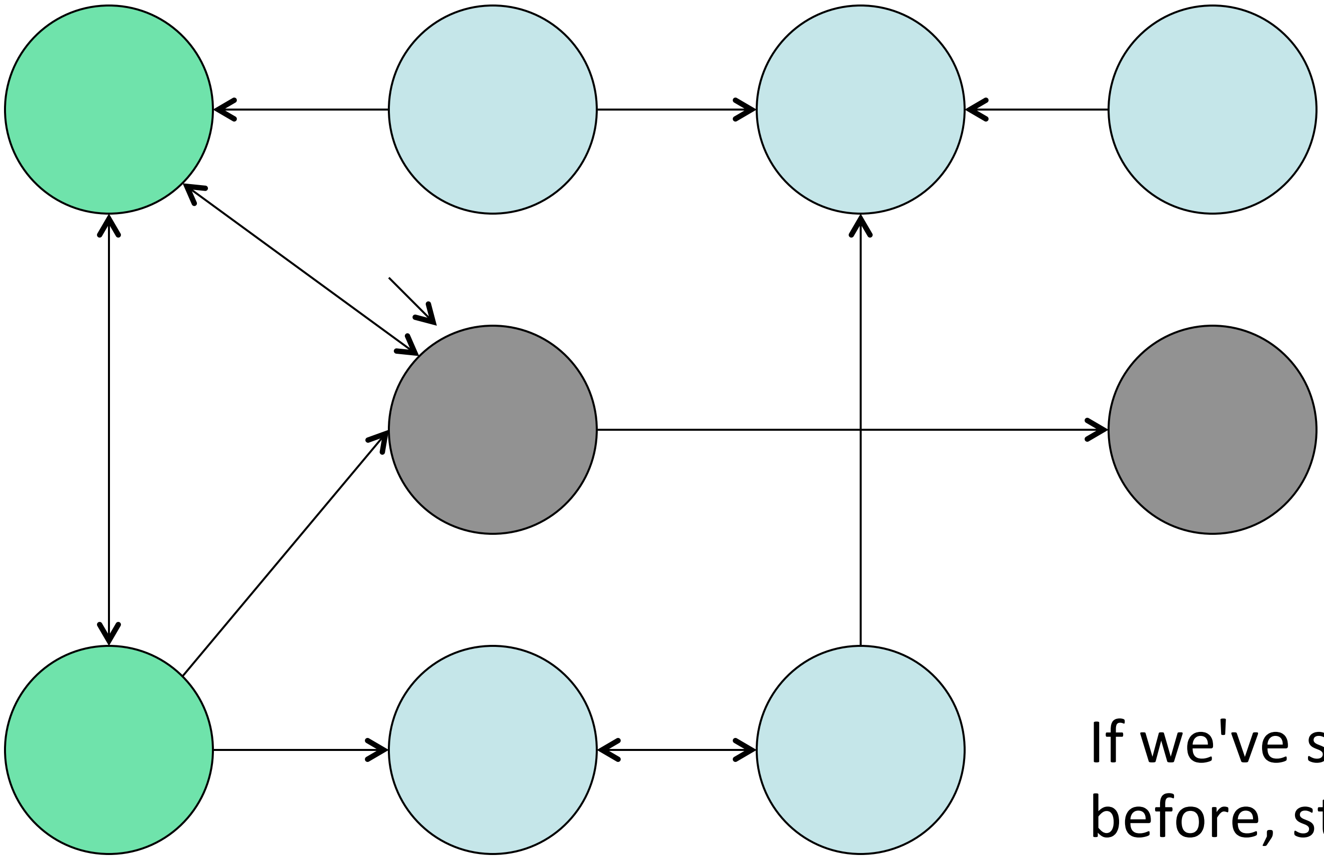

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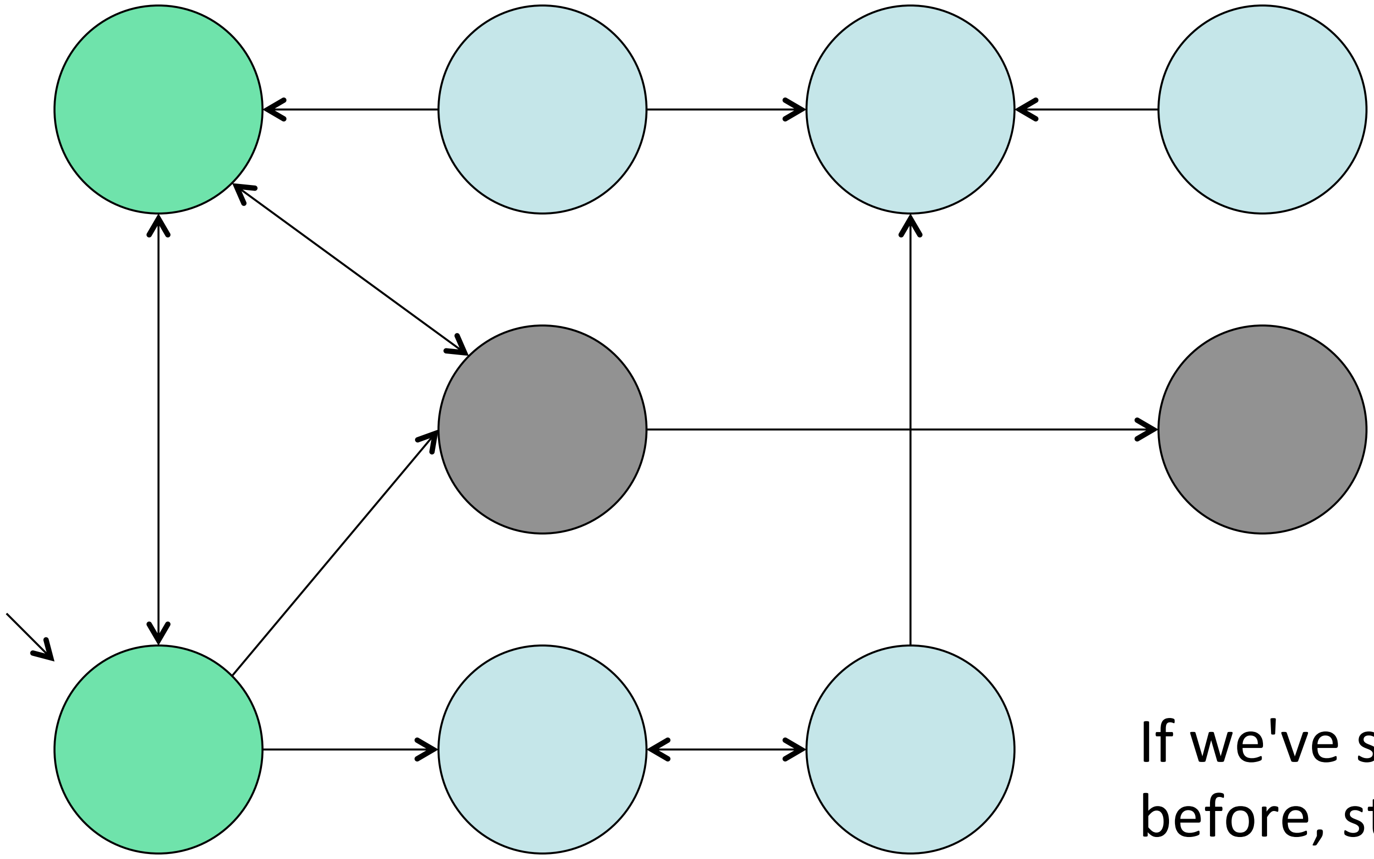

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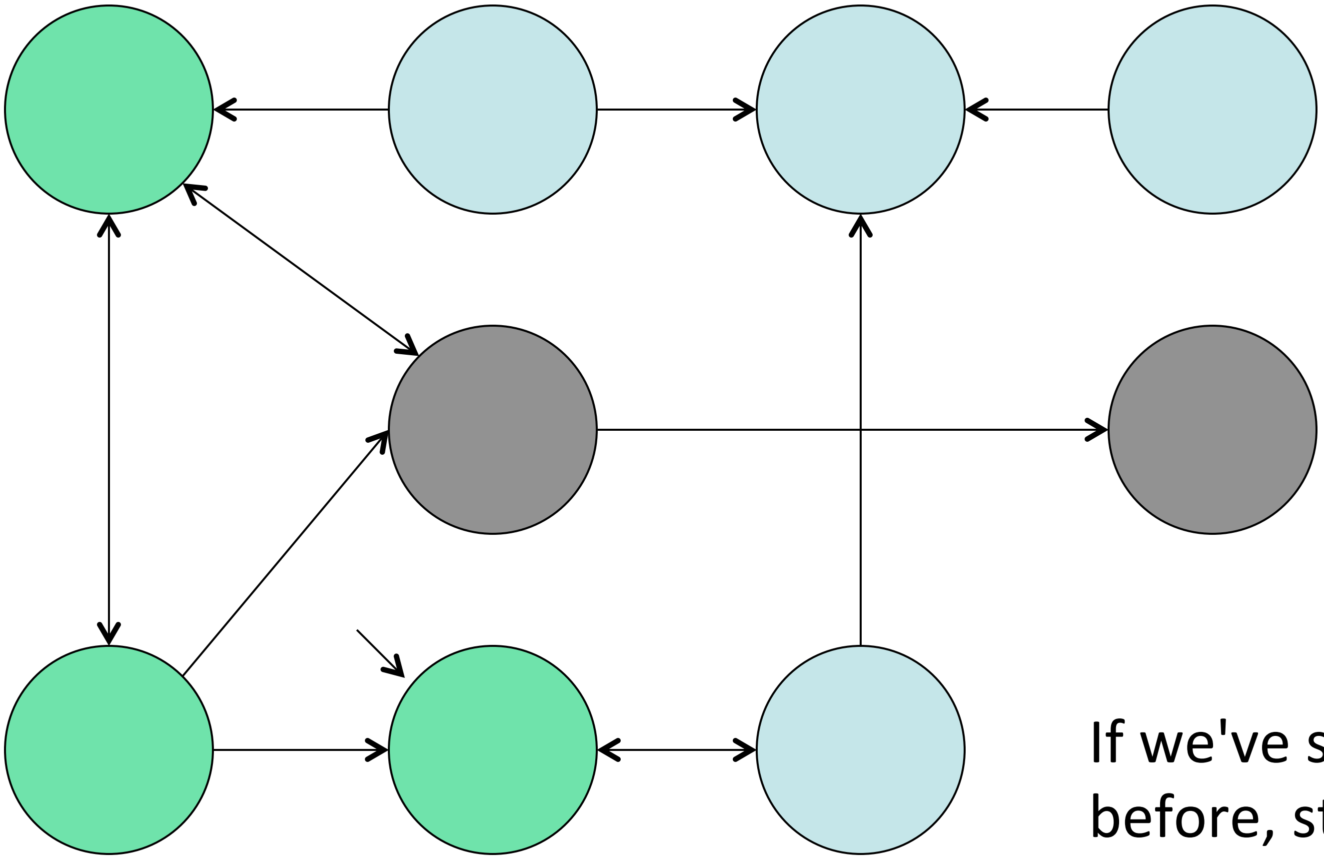

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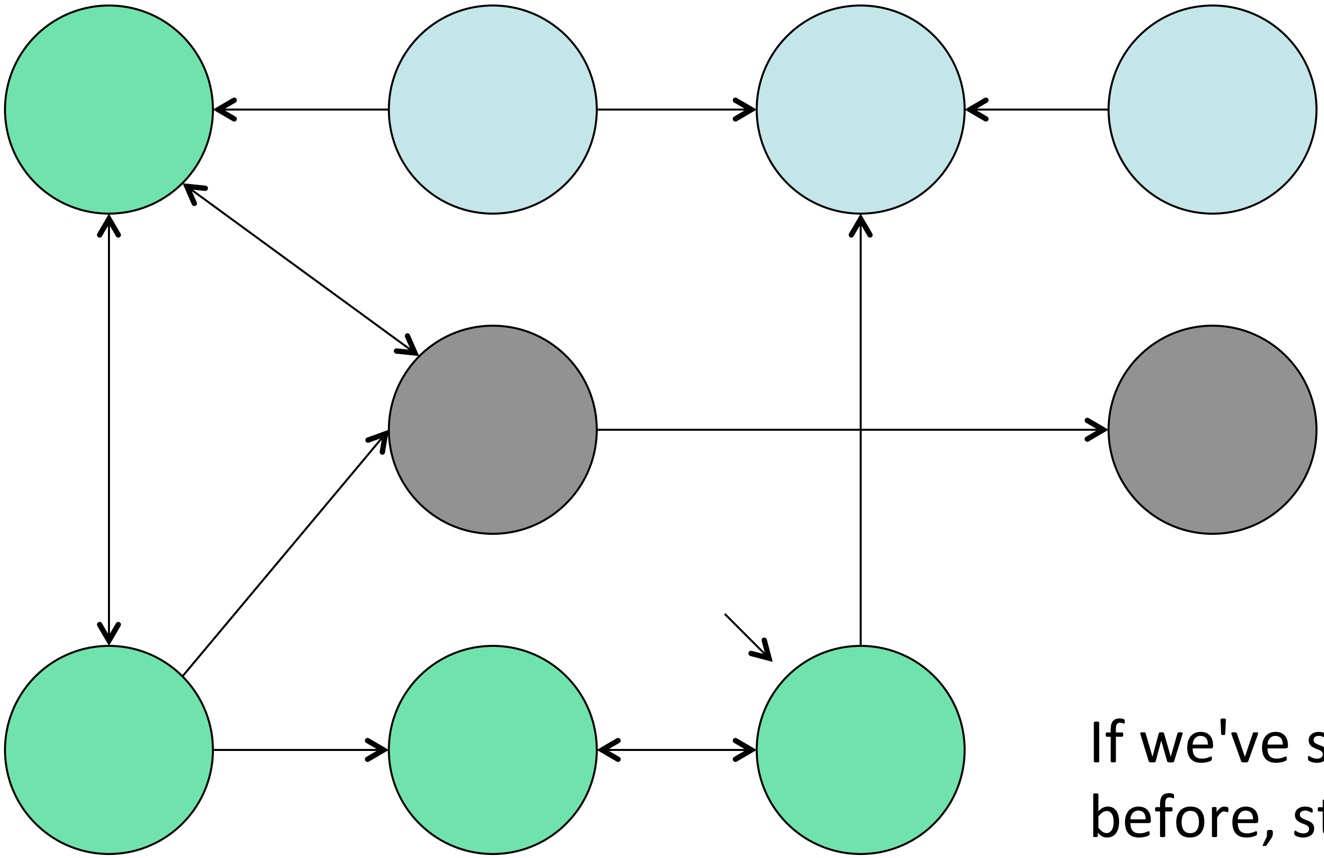

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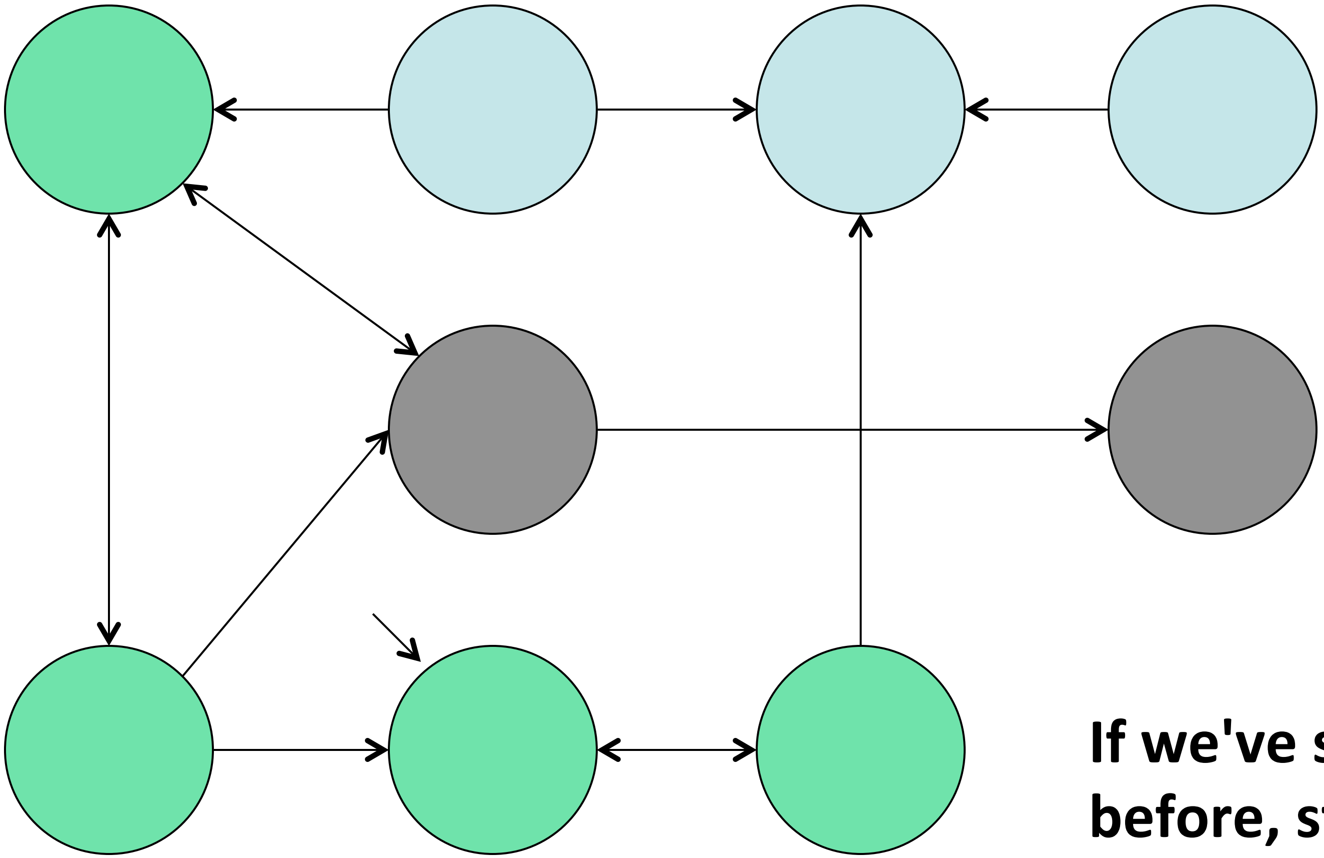

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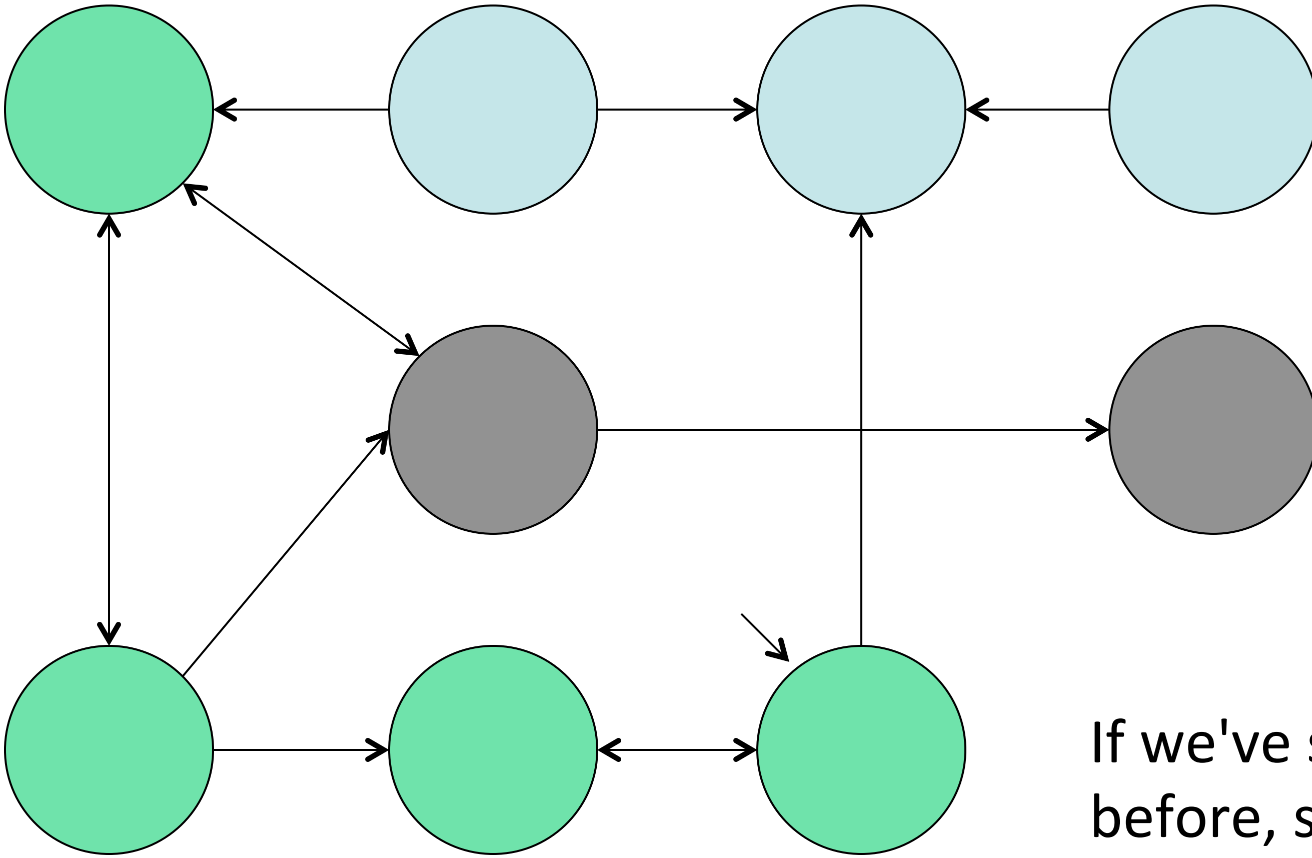

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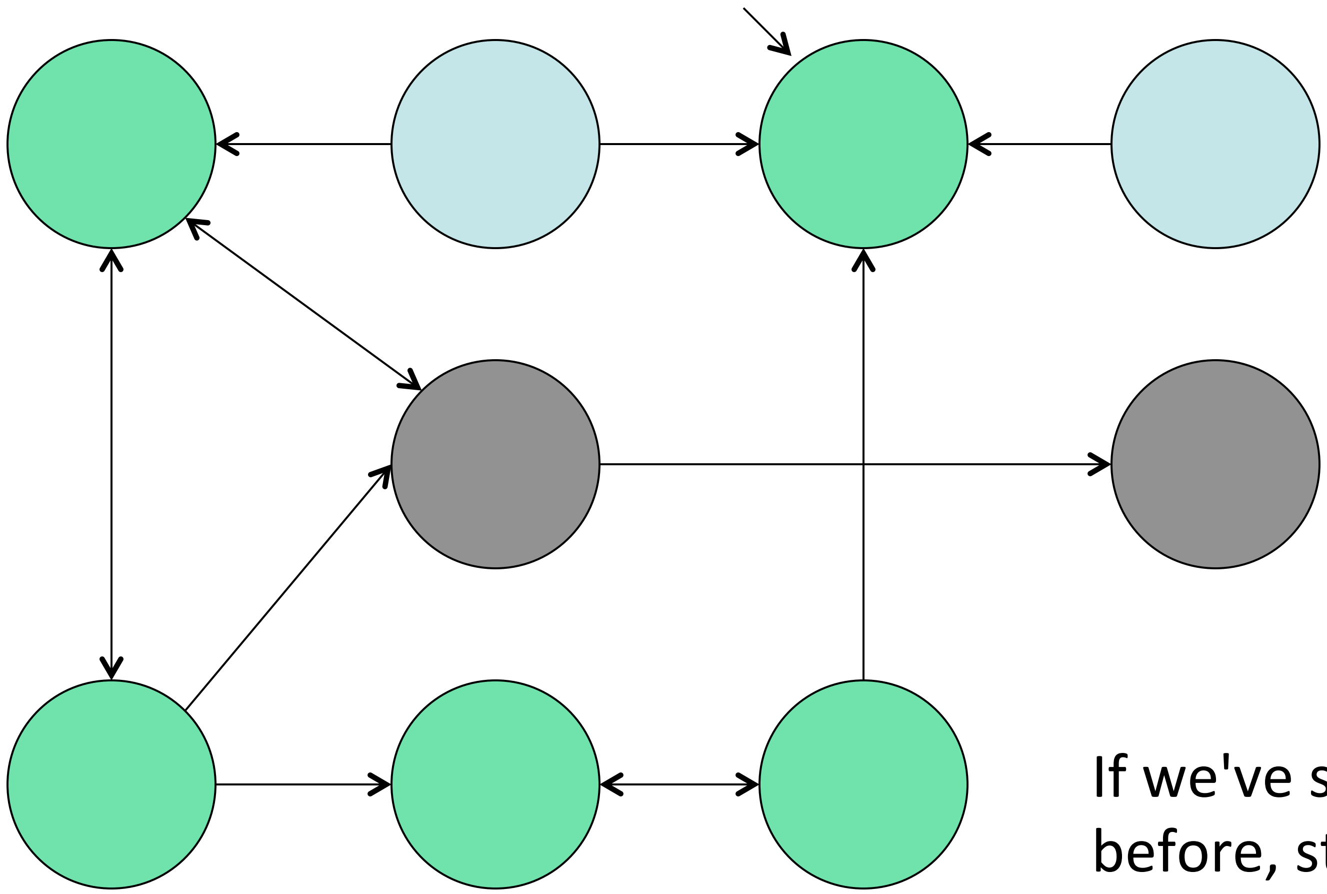

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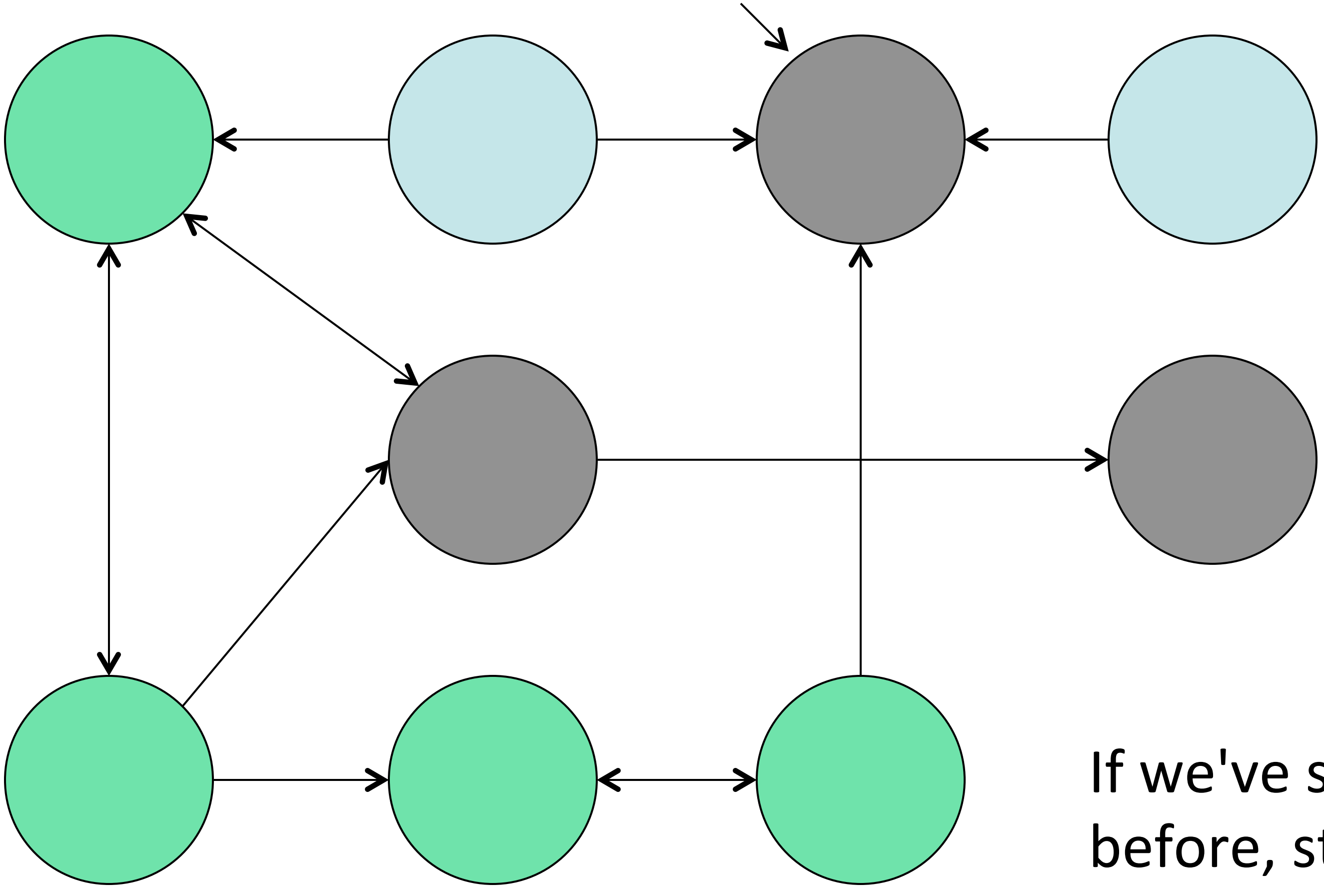

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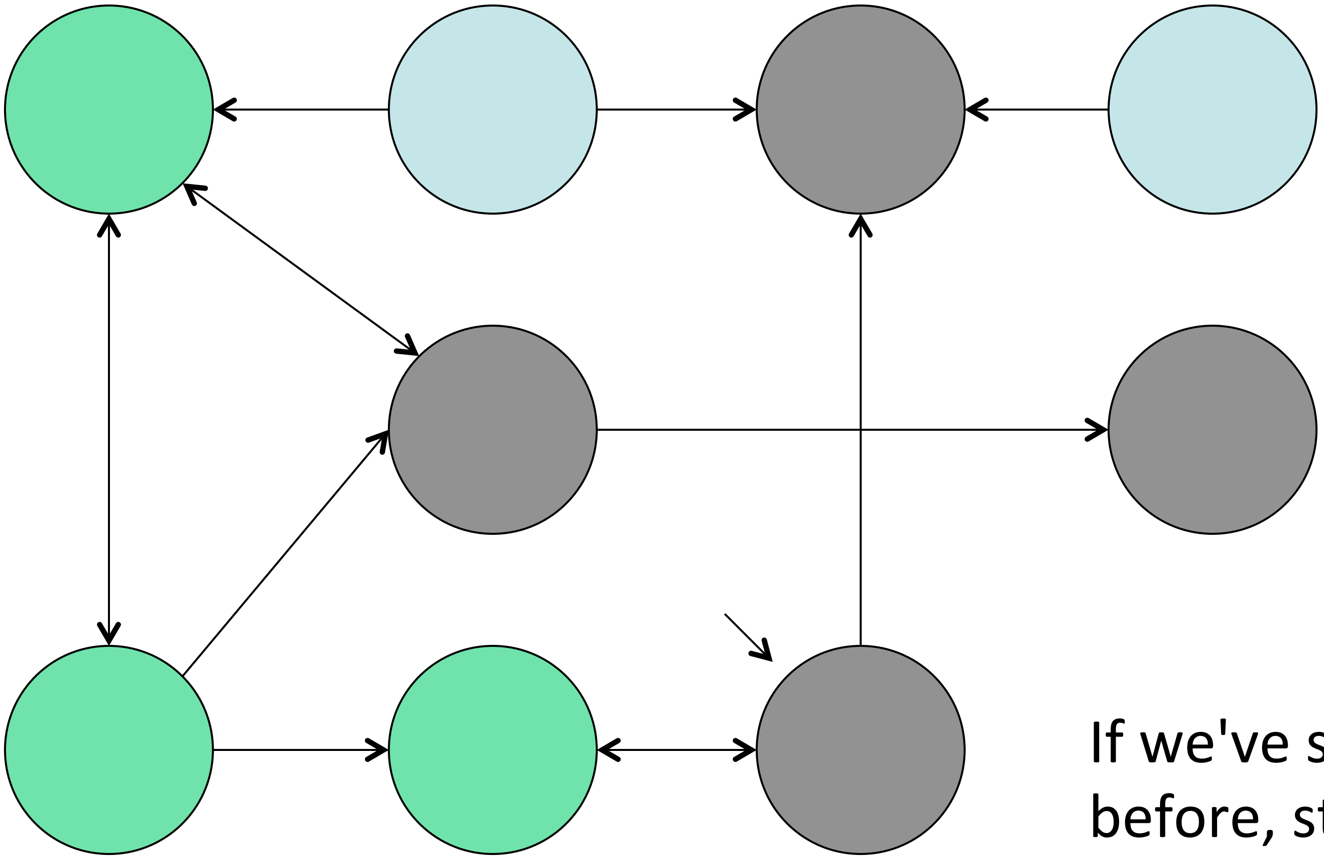

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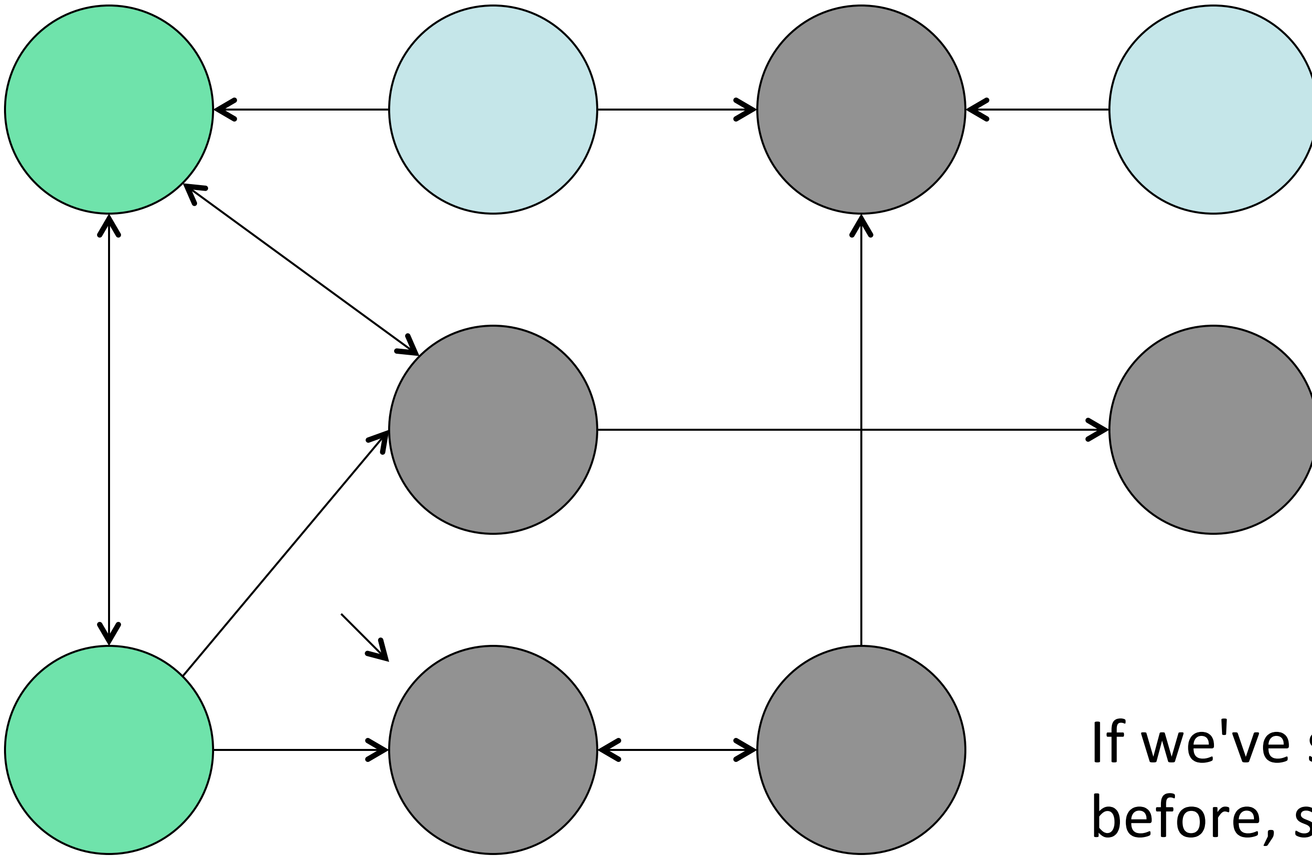

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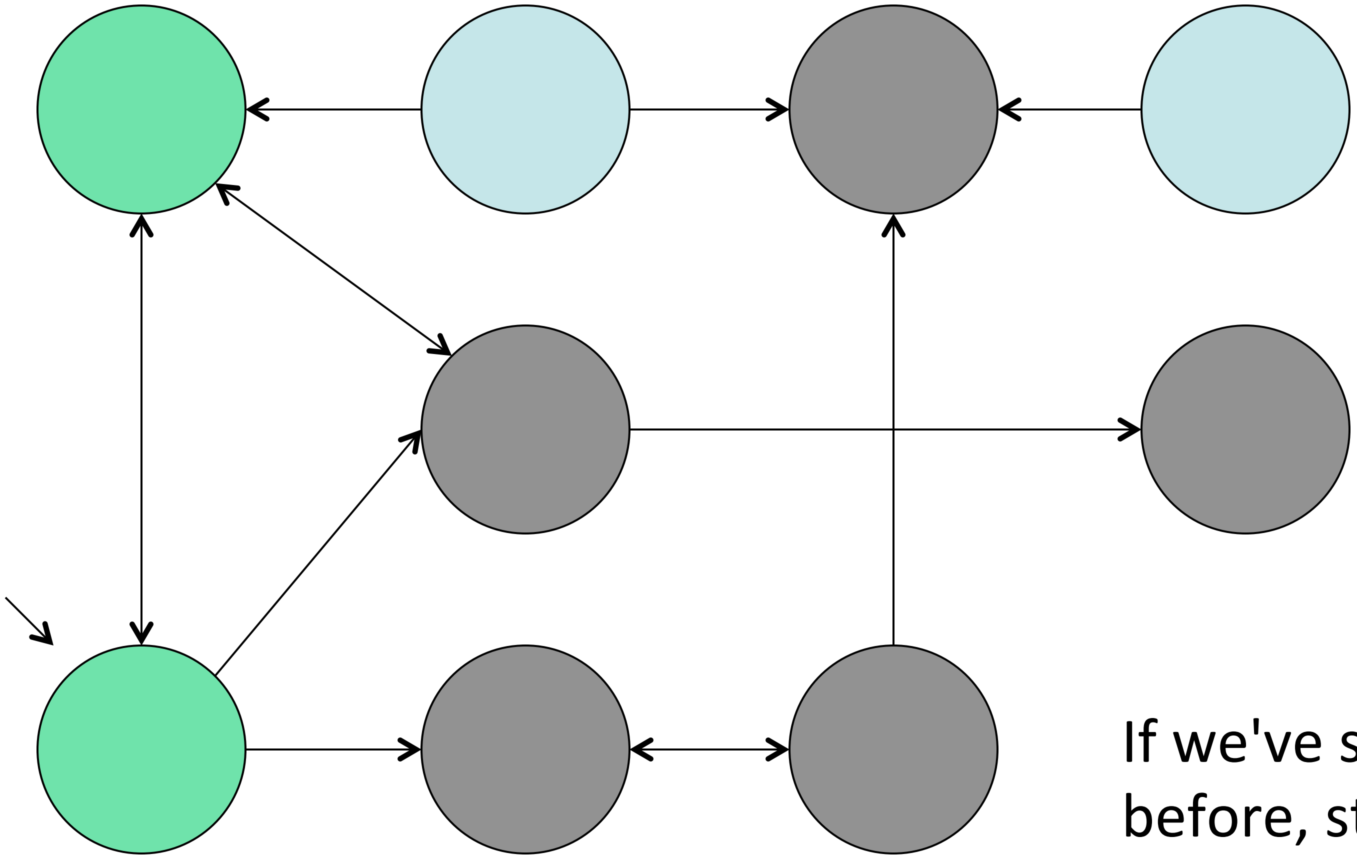

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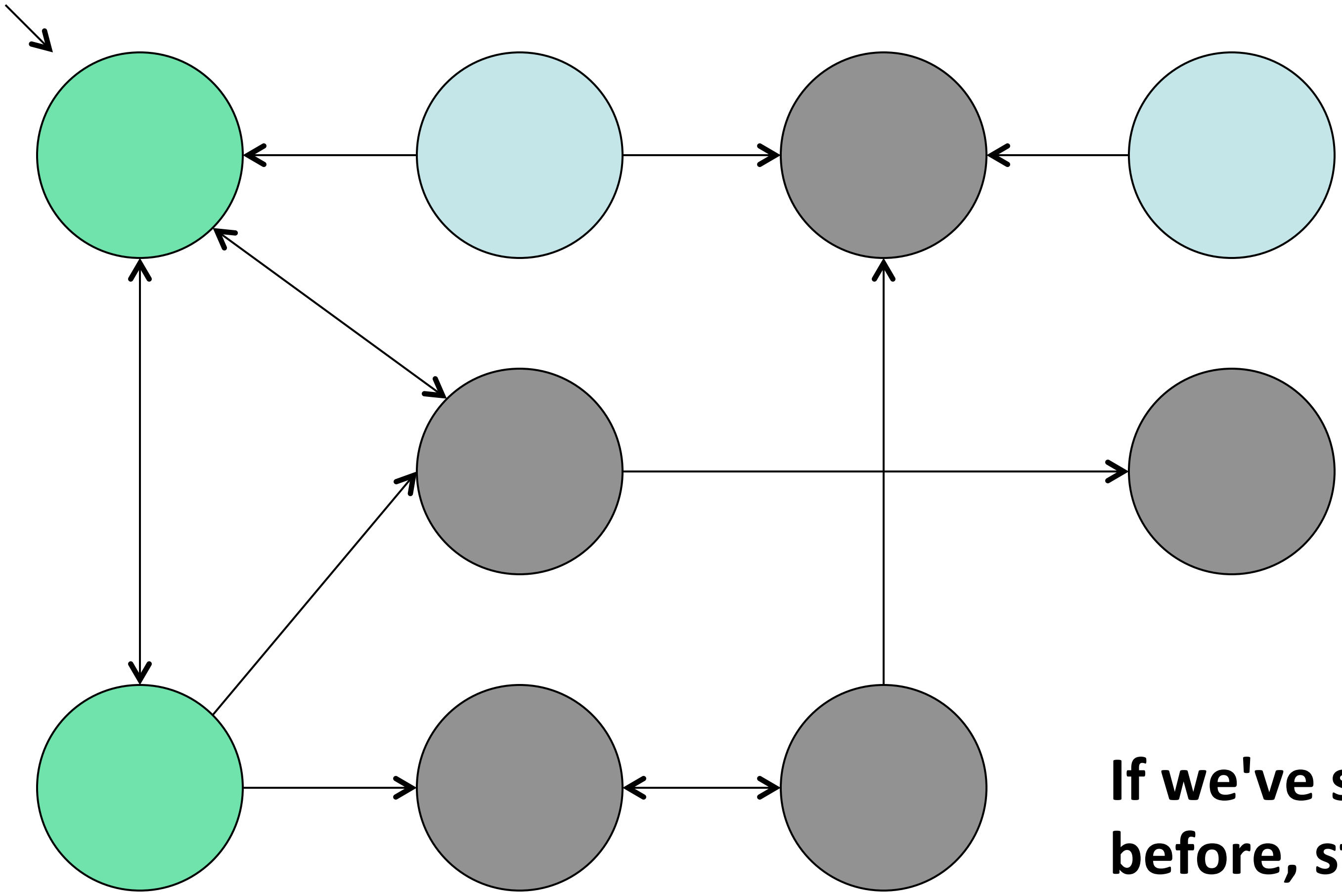

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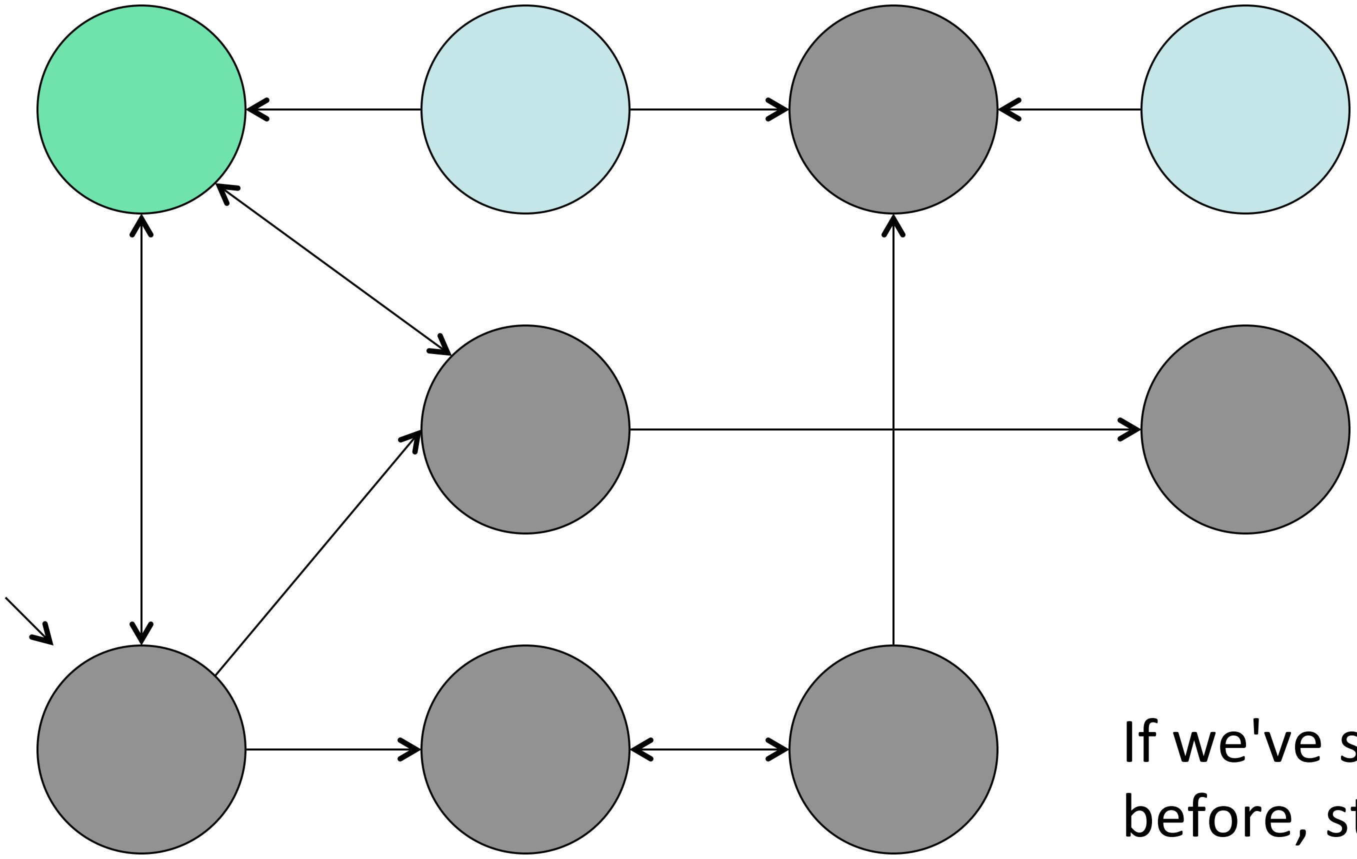

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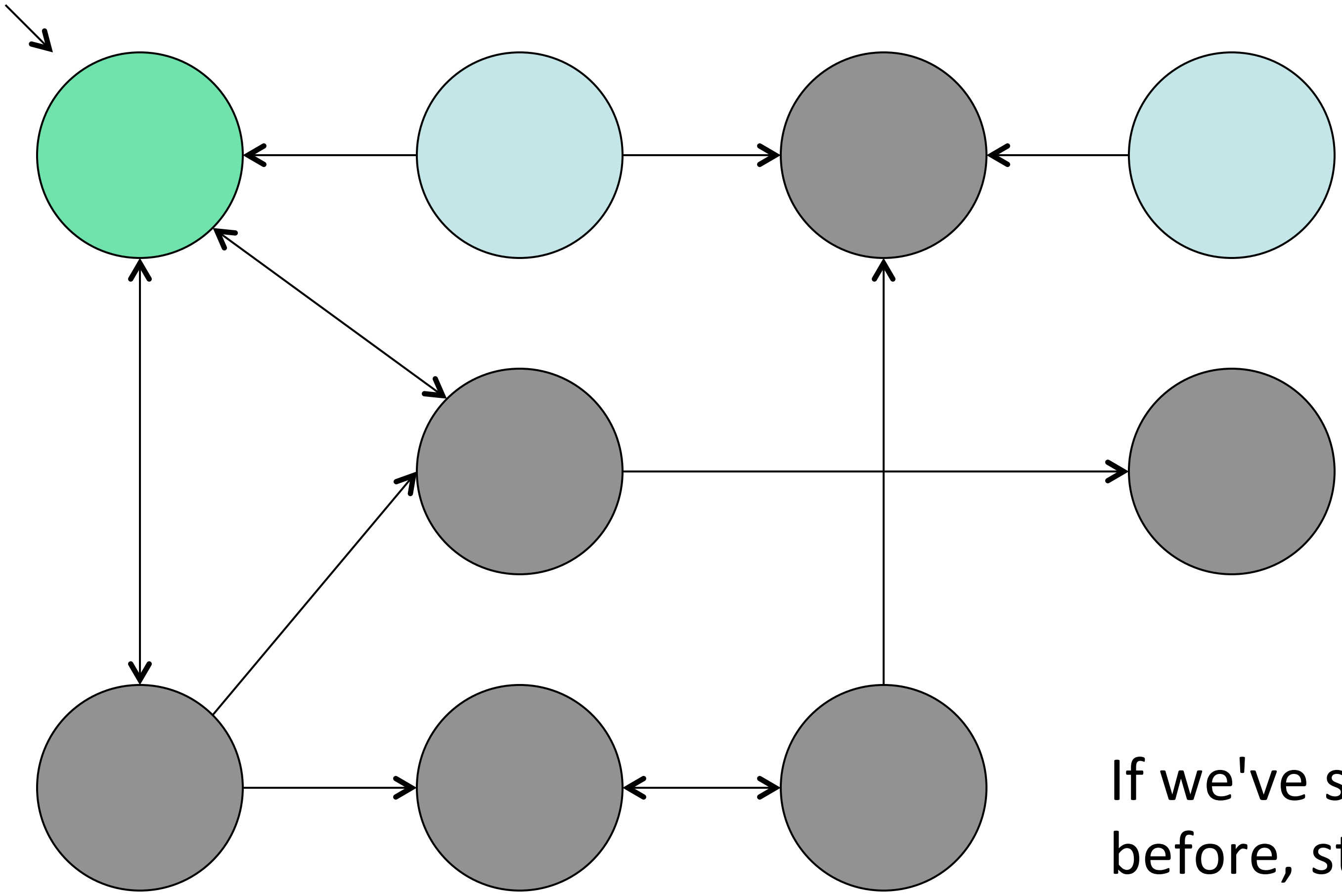

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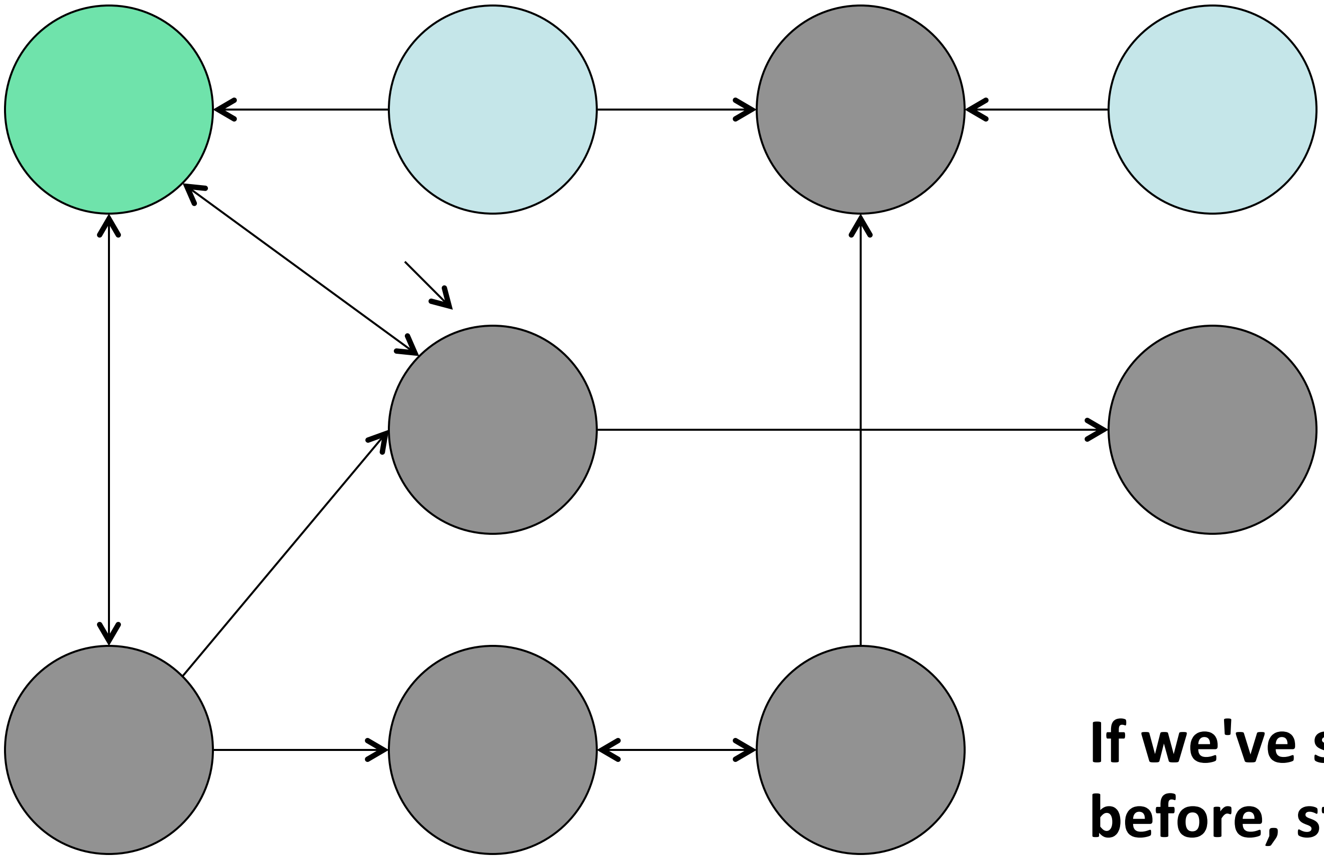

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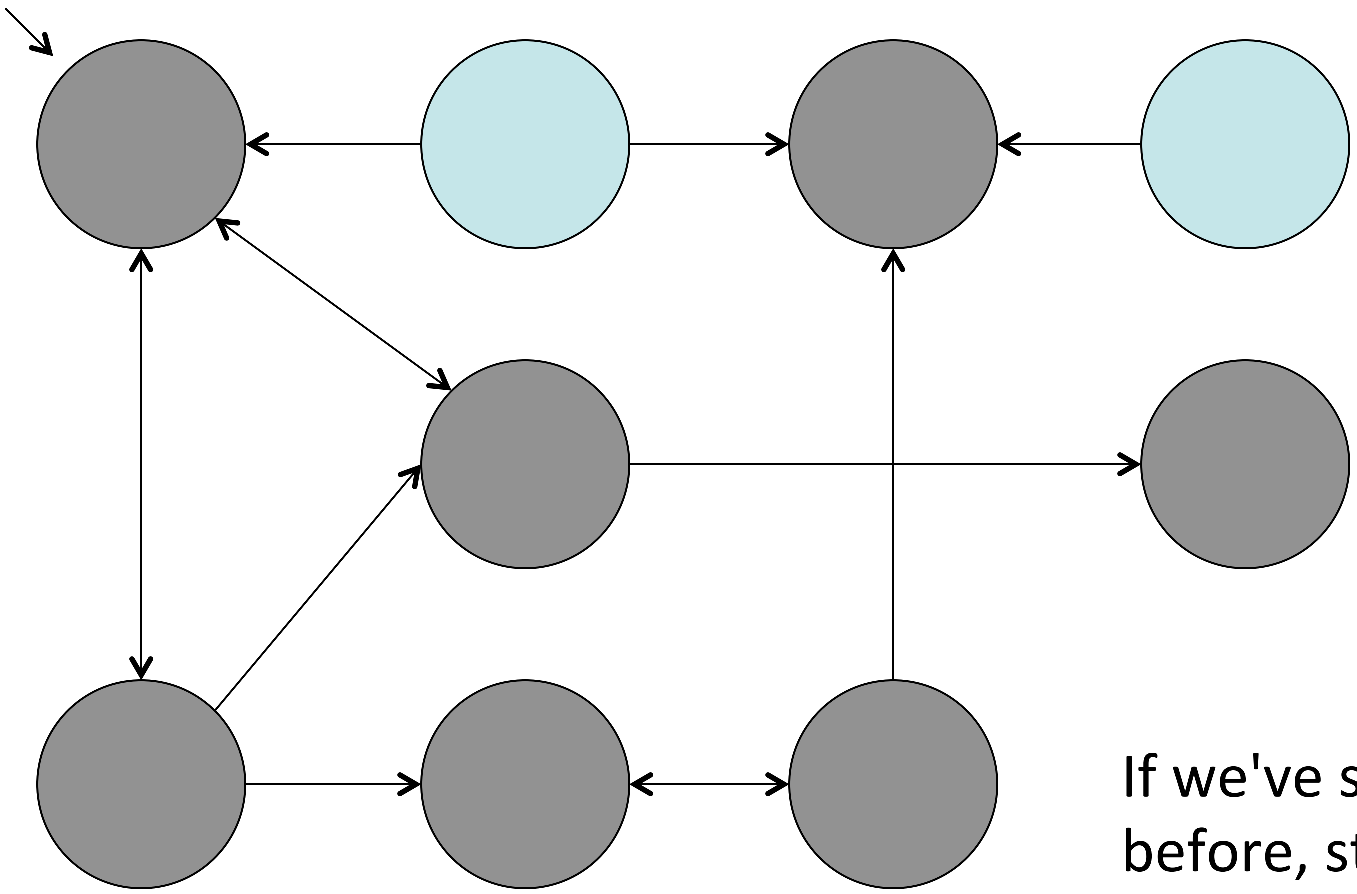

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DFS Details

- In an *n*-node, *m*-edge graph, takes O(*m* + *n*) time with an adjacency list
  - Visit each edge once, visit each node at most once
- Pseudocode:

```
dfs from v1:
    mark v1 as seen.
    for each of v1's unvisited neighbors n:
      dfs(n)
```

- How could we modify the pseudocode to look for a specific path?
  - Recursive Backtracking
  - Look at maze example from week 4

Finding *Shortest* Paths

- We can find paths between two nodes, but how can we find the **shortest** path?
  - Fewest number of steps to complete a task?
  - Least amount of edits between two words?
- When have we solved this problem before?

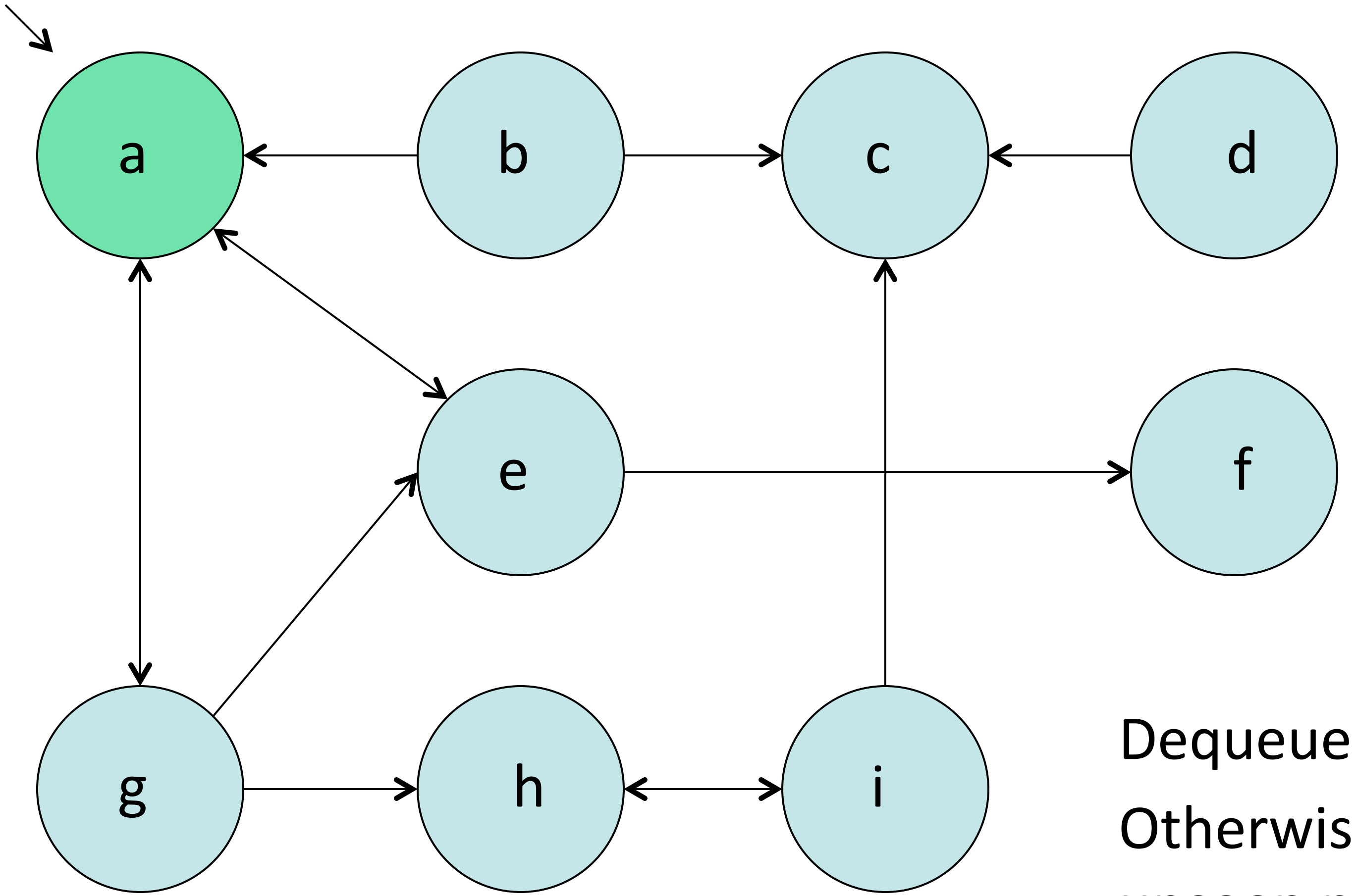
Breadth-First Search (BFS)

- Idea: processing a node involves knowing we need to visit all its neighbors (just like DFS)
- Need to keep a TODO list of nodes to process
- Which node from our TODO list should we process first if we want the shortest path?
  - The first one we saw?
  - The last one we saw?
  - A random node?

Breadth-First Search (BFS)

- Keep a Queue of nodes as our TODO list
- Idea: dequeue a node, enqueue all its neighbors
- Still will return the same nodes as reachable, just might have shorter paths

BFS

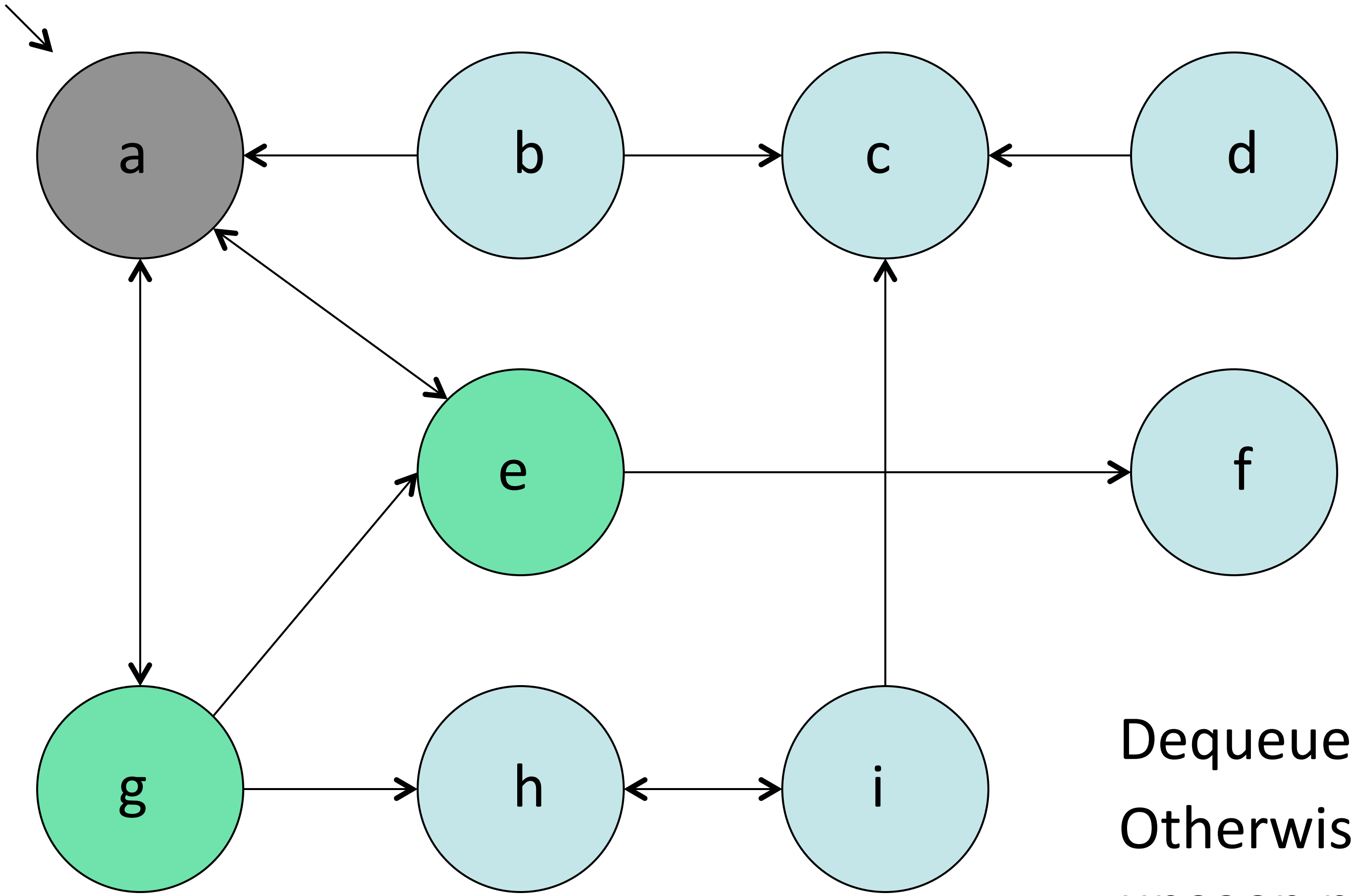


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: a

BFS



Dequeue a node

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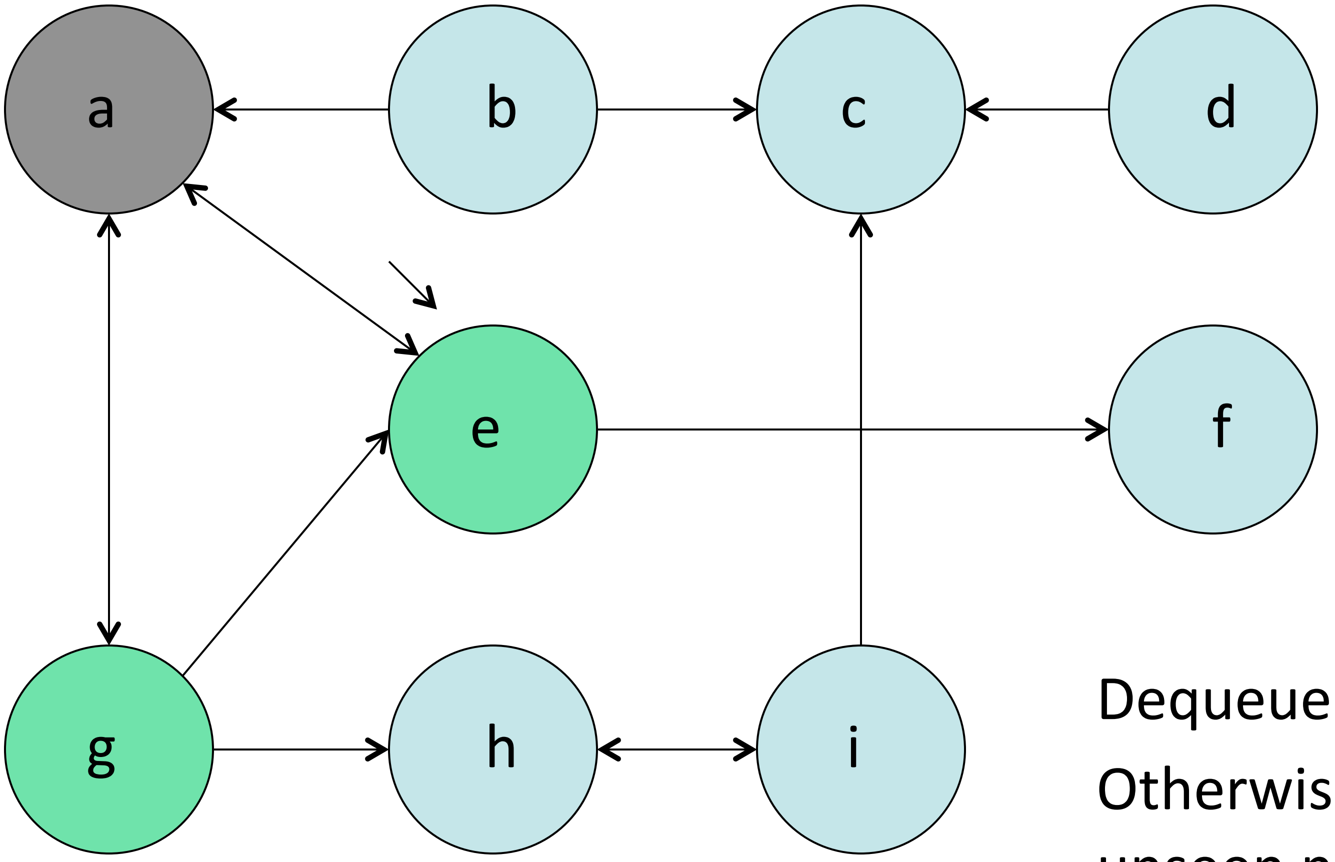
queue: e, g

BFS



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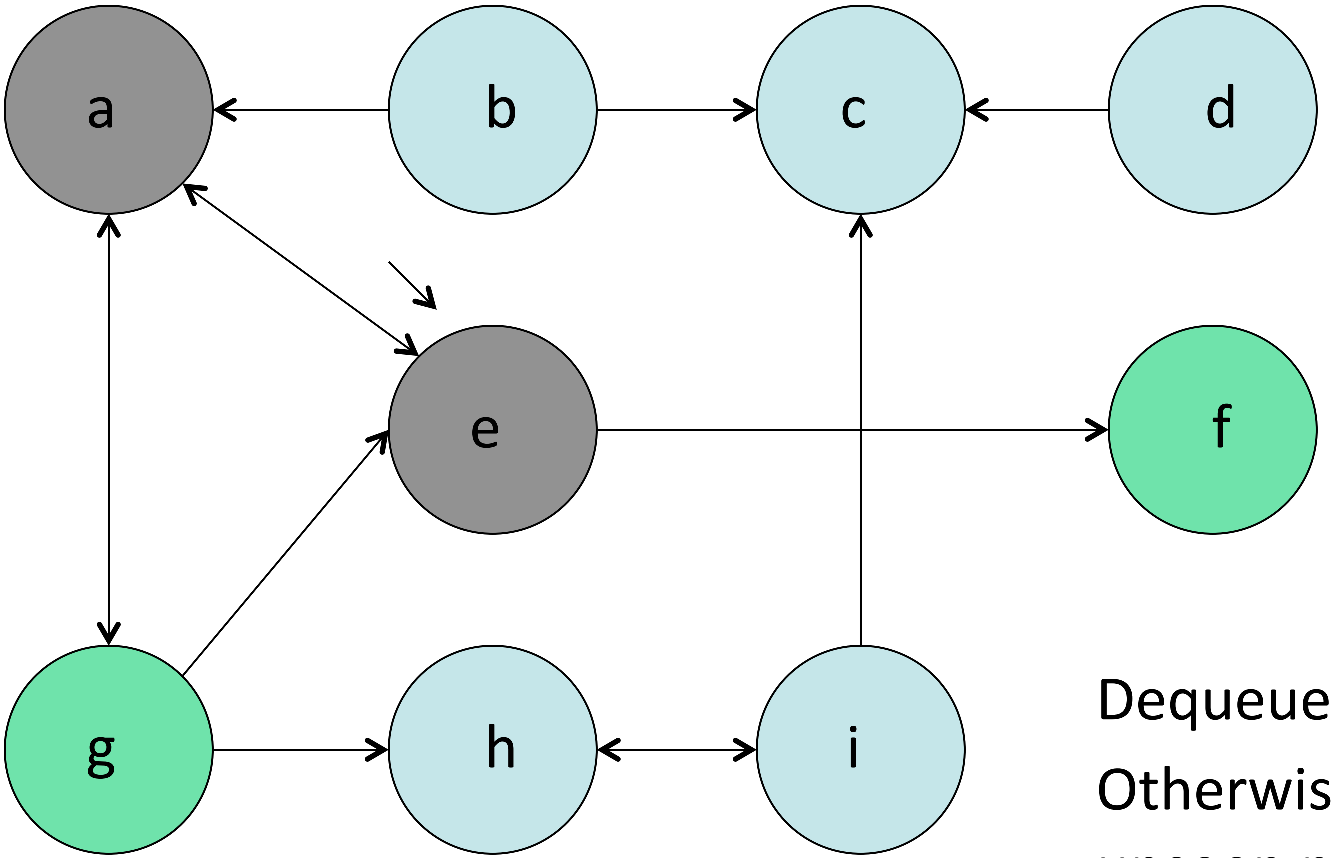
queue: e, g

BFS

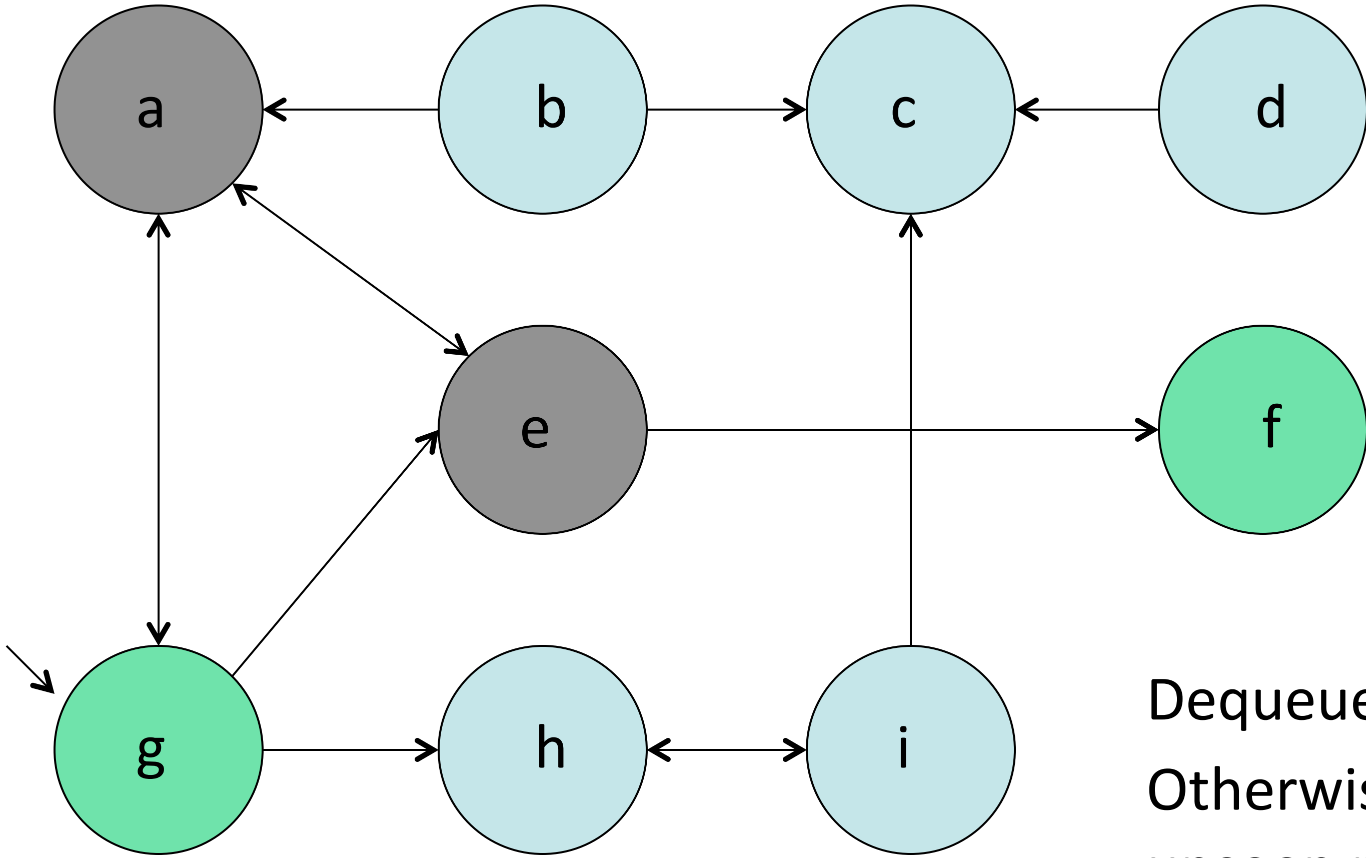


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: g, f

BFS



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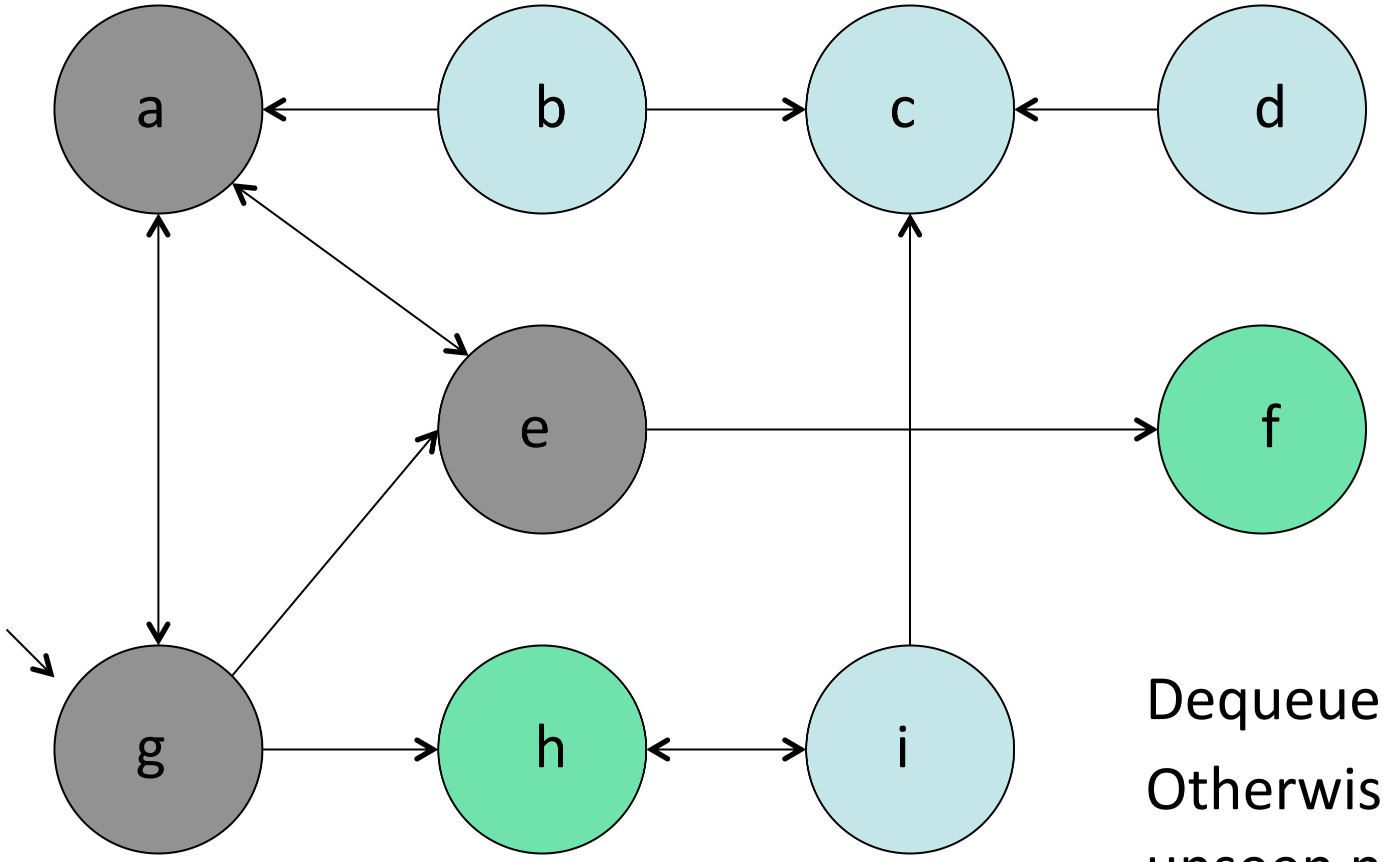
queue: g, f

BFS

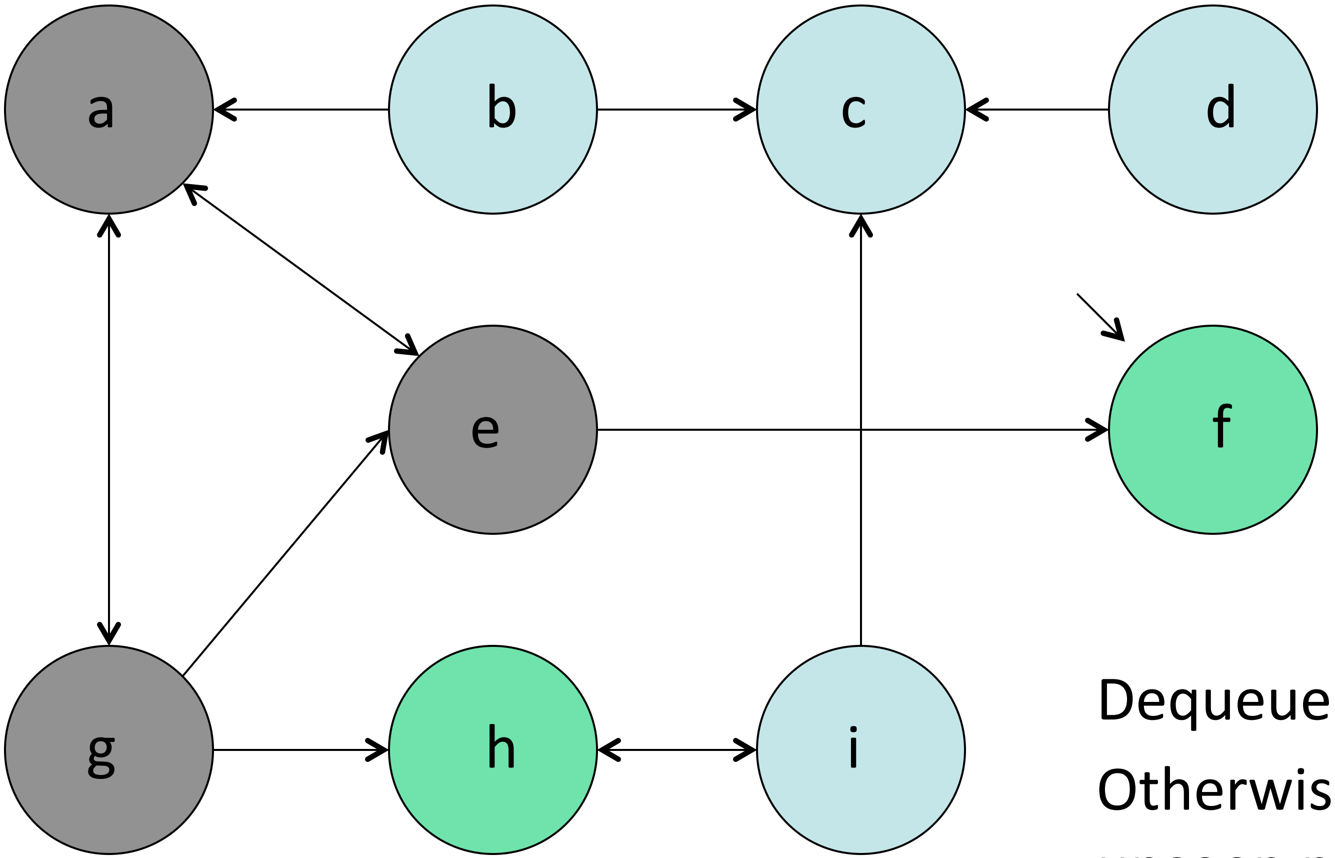


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: f, h

BFS

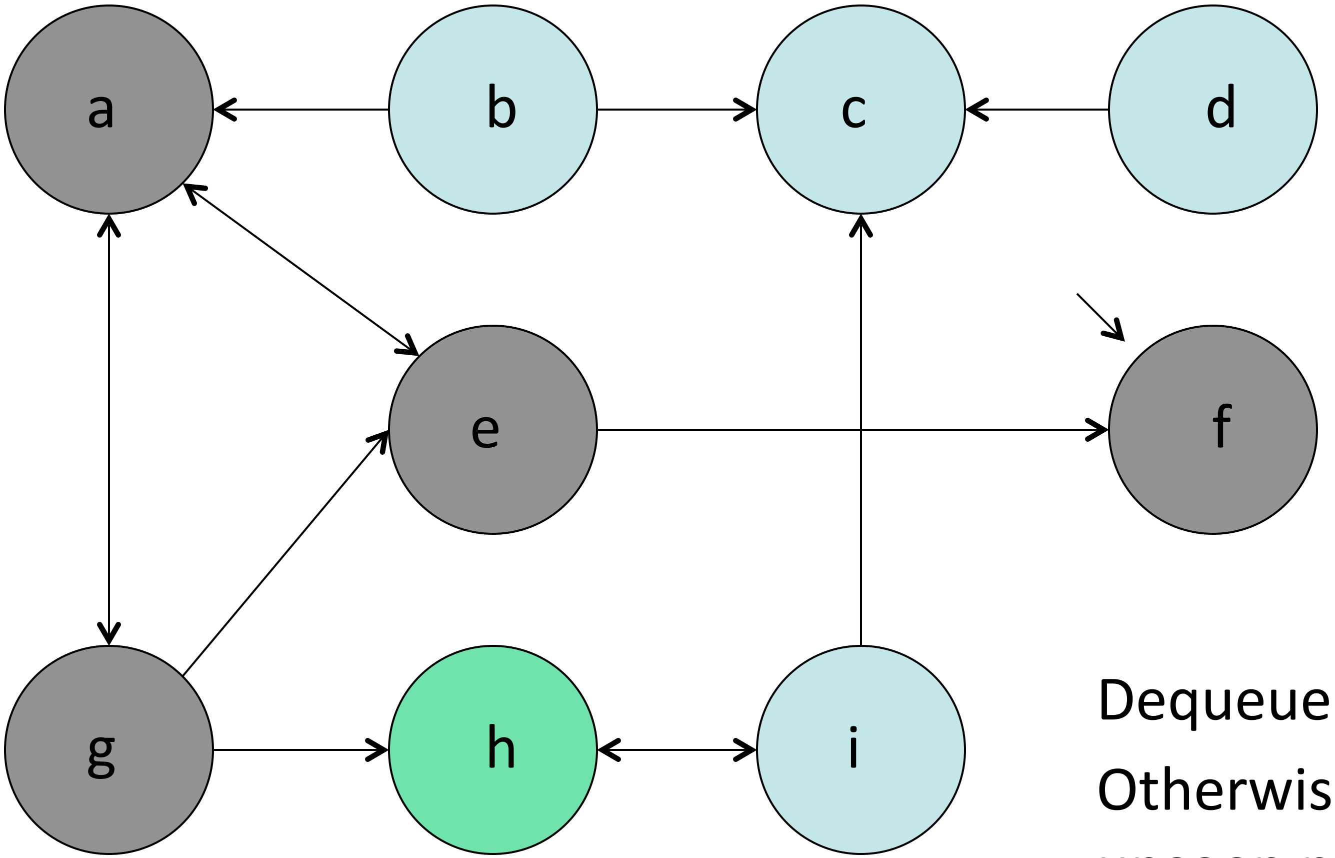


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: f, h

BFS

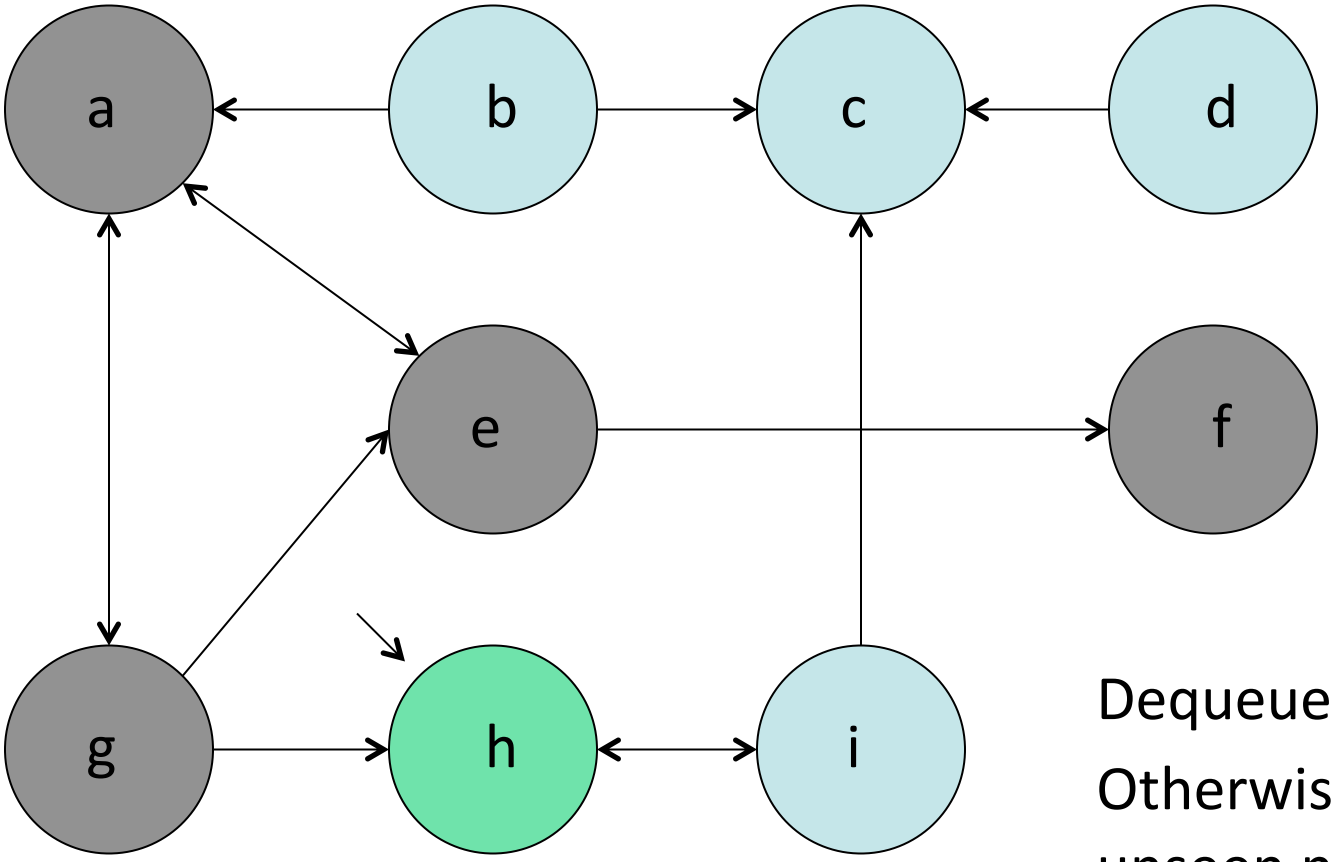


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: h

BFS

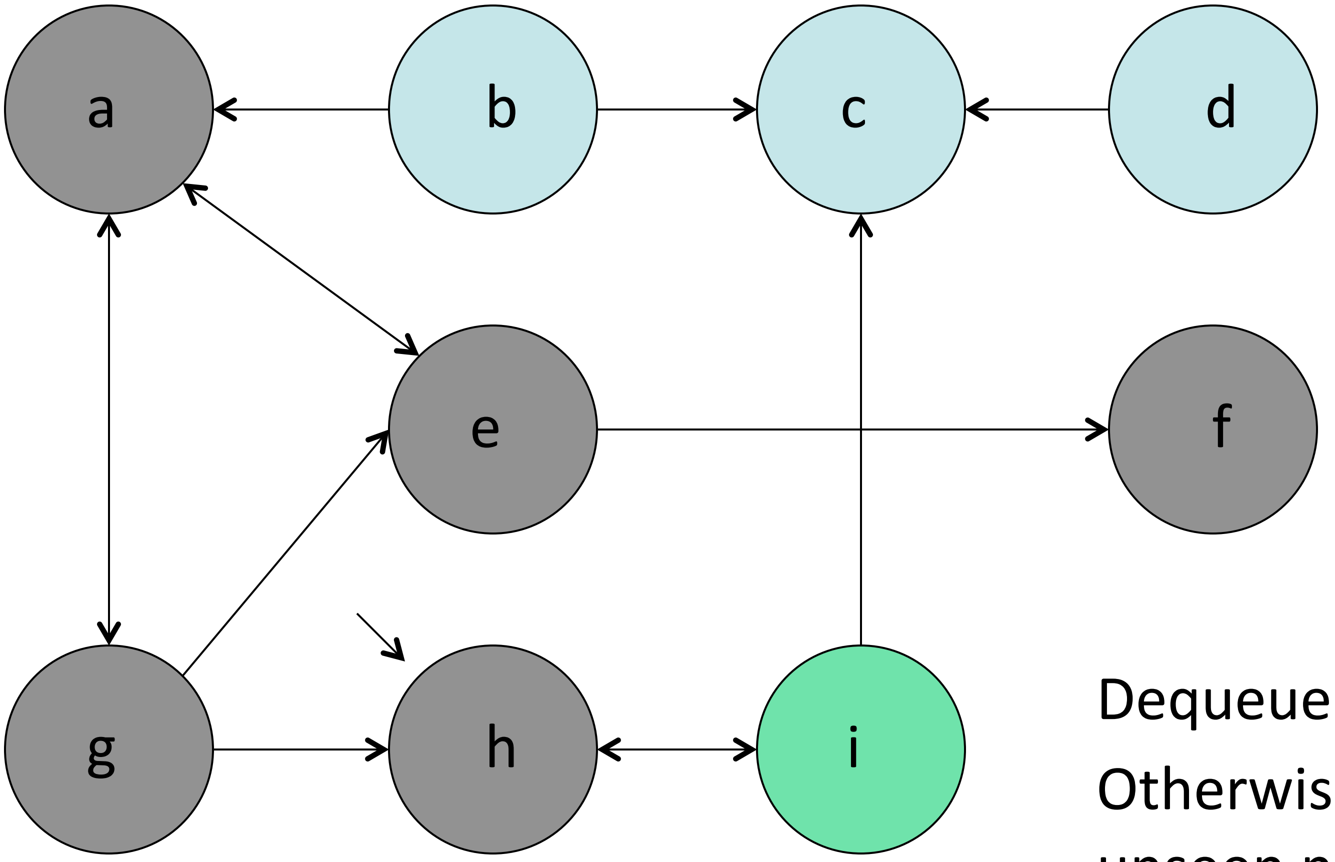


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: h

BFS

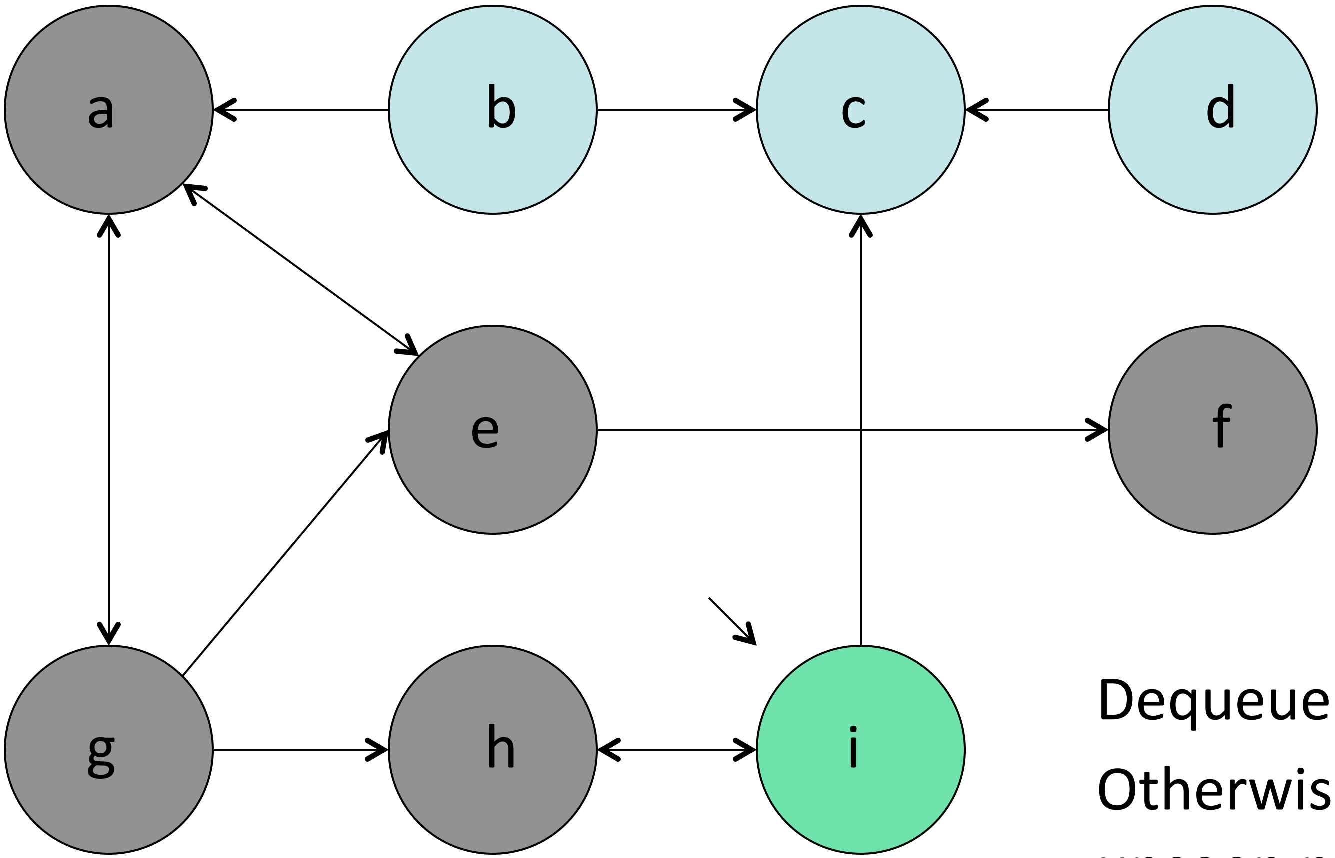


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: i

BFS

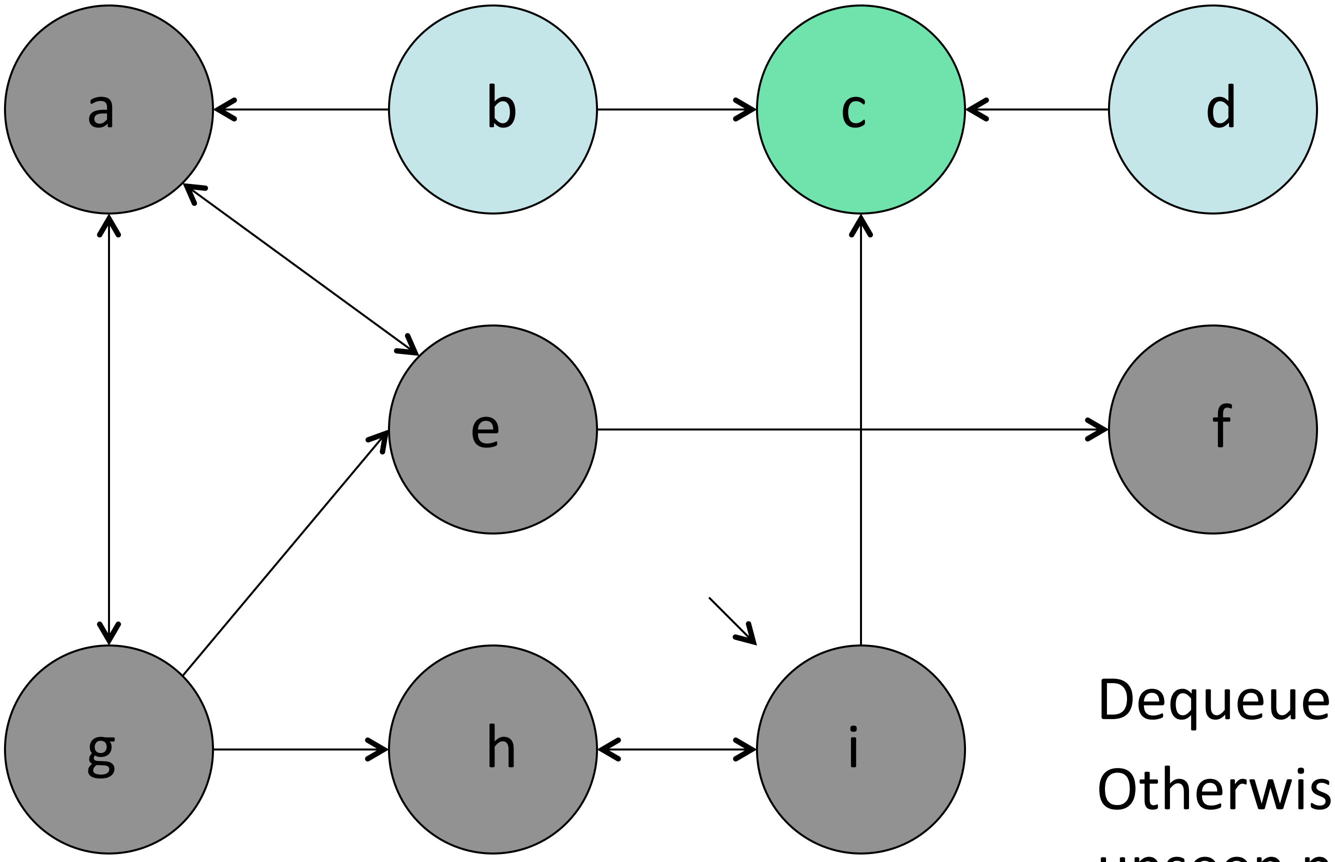


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: i

BFS

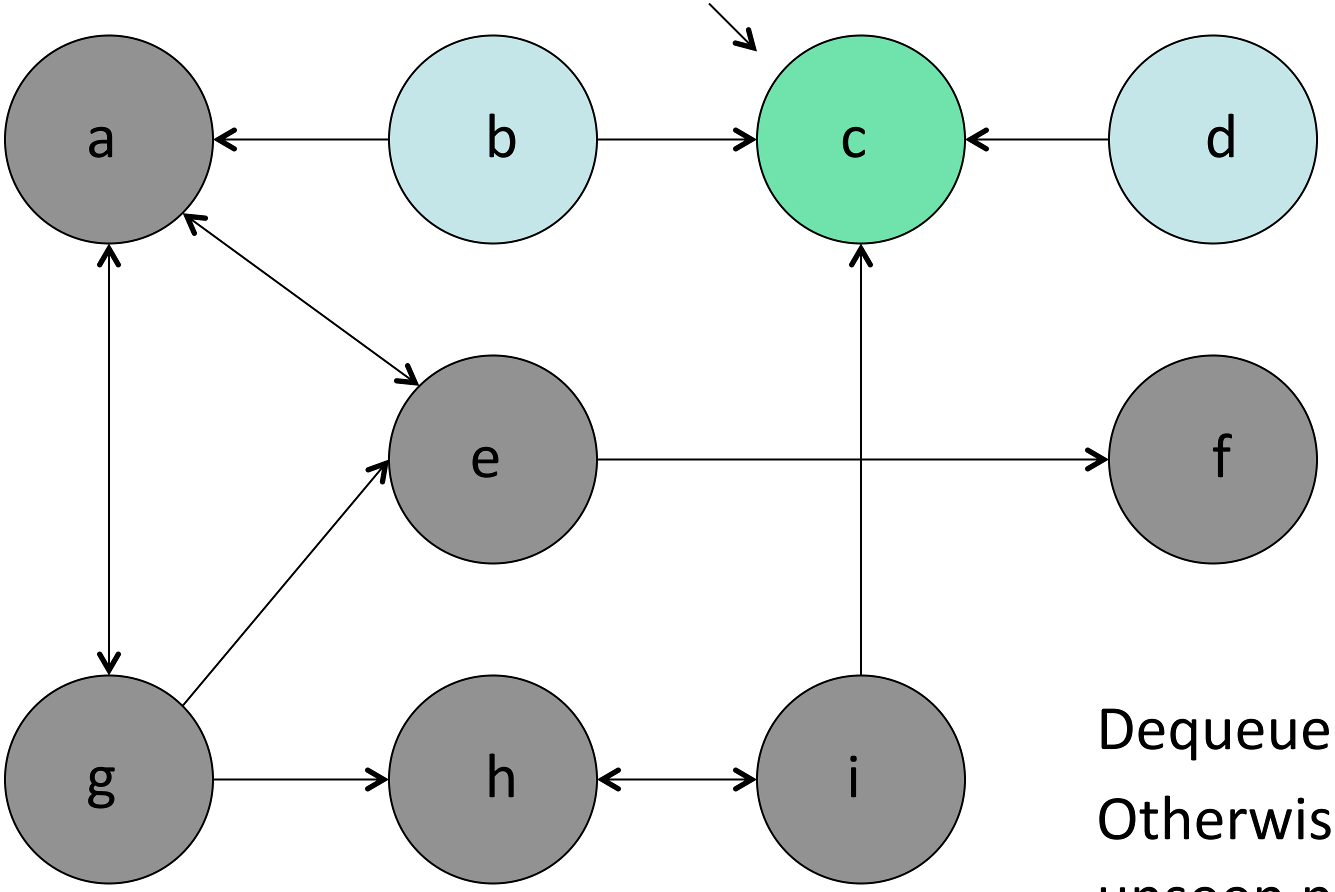


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: c

BFS

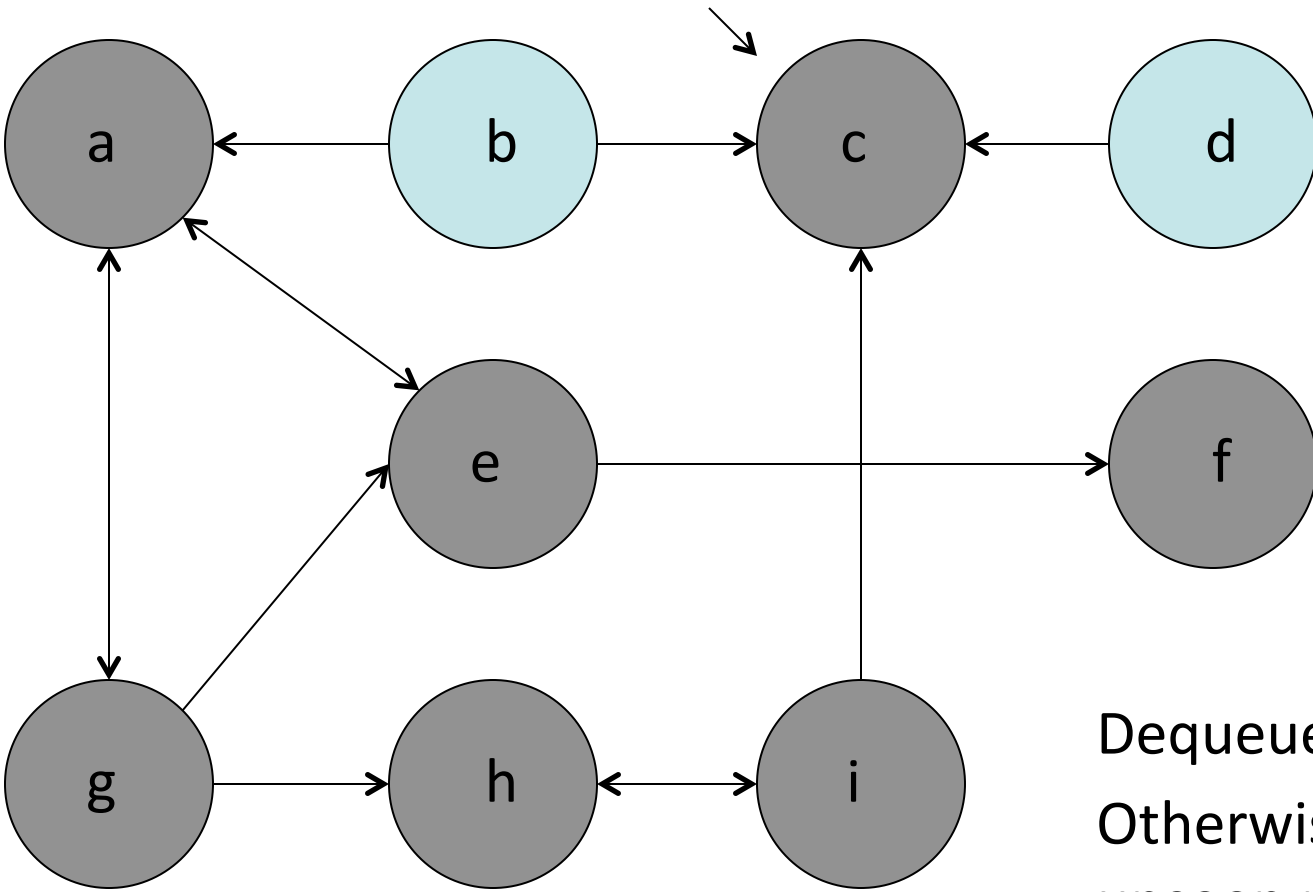


Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: c

BFS



Dequeue a node

Otherwise, add all its unseen neighbors to the queue

queue: c

BFS Details

- In an n-node, m-edge graph, takes $O(m + n)$ time with an adjacency list
  - Visit each edge once, visit each node at most once
- Pseudocode:

```
bfs from v1:
    add v1 to the queue.
    while queue is not empty:
      dequeue a node n
      enqueue n's unseen neighbors
```

- How could we modify the pseudocode to look for a specific path?