CS 106B, Lecture 23
The Power of DFS

Plan for Today

- Two different graph algorithms
  - Topological Sort
  - Bipartite Graph Matching
- Modify DFS for powerful results

Recap: Depth-First Search

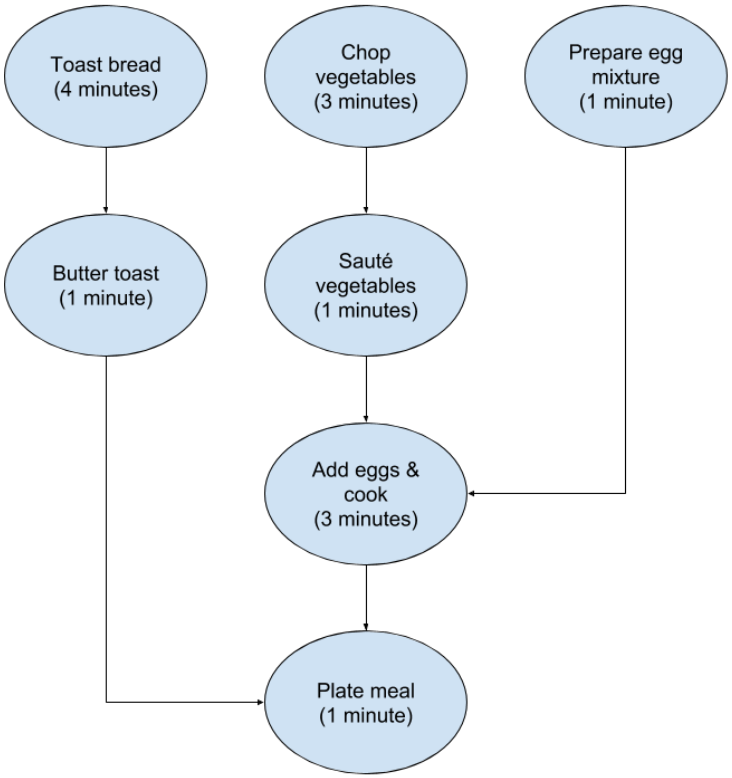
- Path-finding algorithm
- Pseudocode:

```
dfs from v1:
    mark v1 as seen.
    for each of v1's unvisited neighbors n:
        dfs(n)
```

- Can also run depth-first searching looking for a **specific** endpoint
  - Check out the "find all solutions" vs. "find one solution" pseudocode from recursive backtracking

A new problem

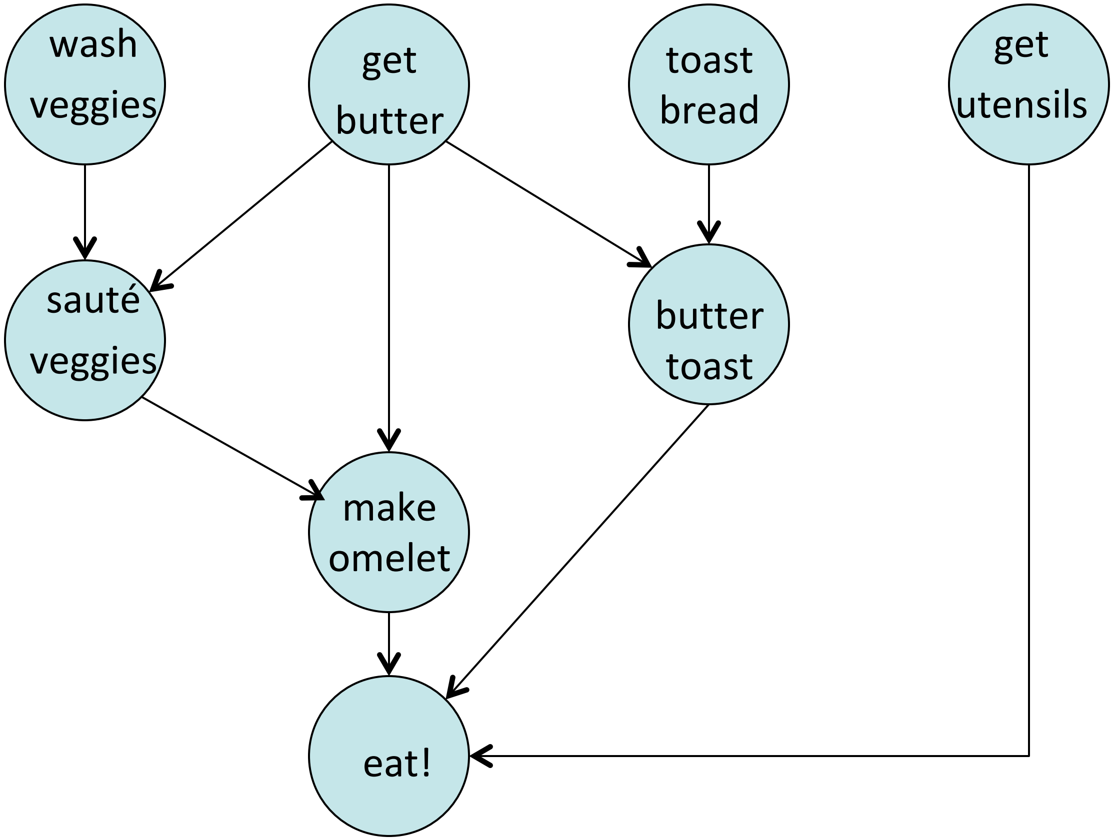
- In what order can you take the CS classes required for the major?
  - Some classes rely on other classes – you shouldn't take 106B until you've taken 106A
- Another example: you want to cook breakfast, but some steps must be done before others can begin. In what order should you perform the steps to cook breakfast?
- In what order should compilers compile code (with import statements)?
- What type of graphs are these?



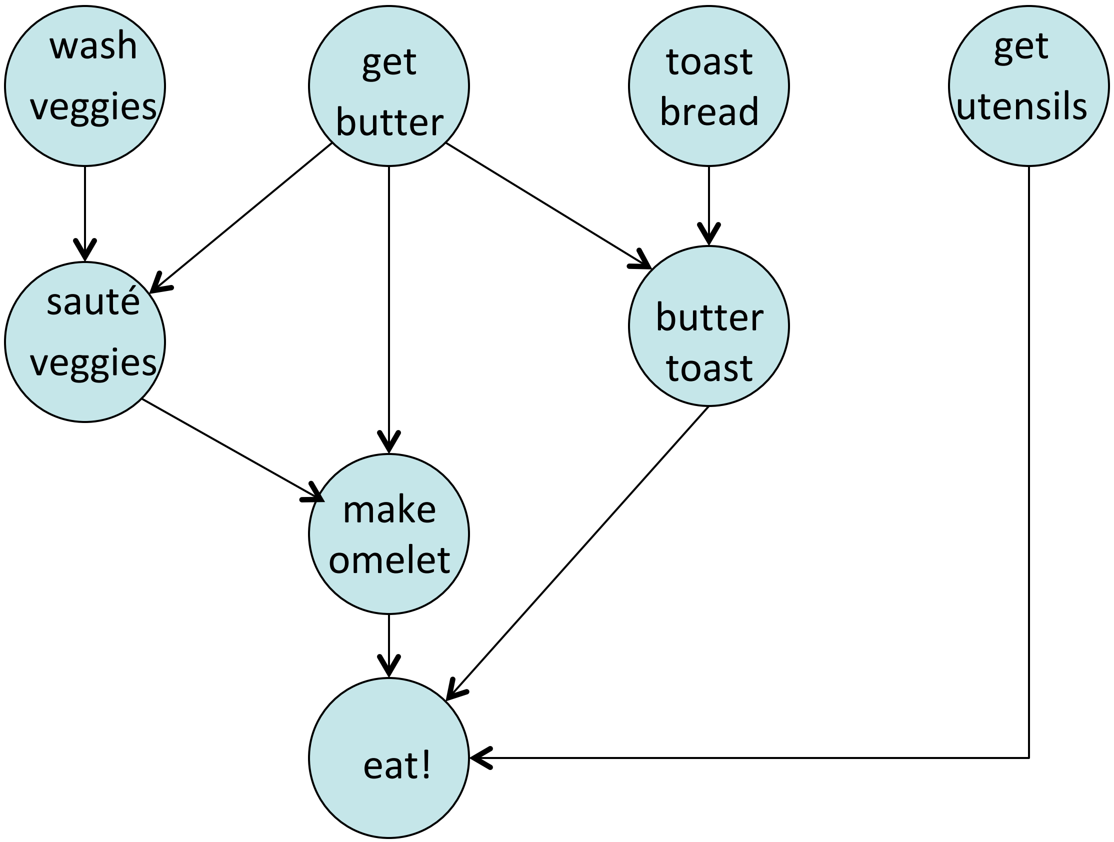
Topological Sort

- Want to order tasks such that every task's prerequisites appear before the task itself
- In other words, if 106A is a prerequisite for 106B, 106A should be before 106B in the ordering
- Such an ordering is a **topological ordering** and is created using **topological sort**
- Only works on **directed, acyclic graphs**
  - Prerequisite relationships are always directed
  - If the graph has cycles, no way to obey all the prerequisites

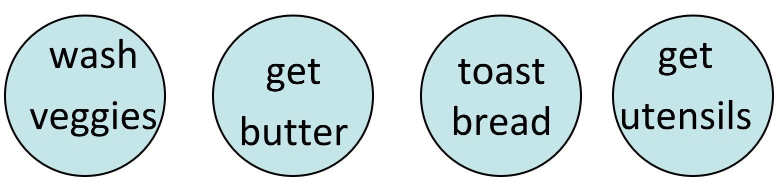
Topological Ordering



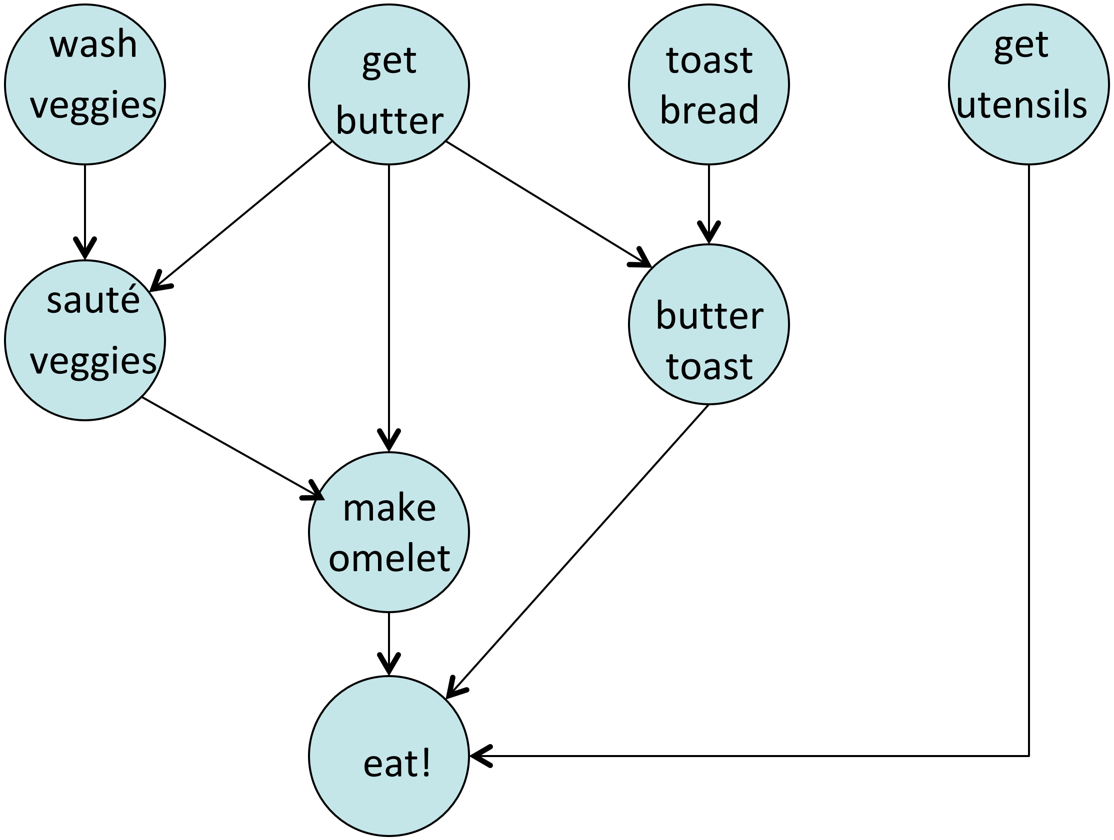
Topological Ordering



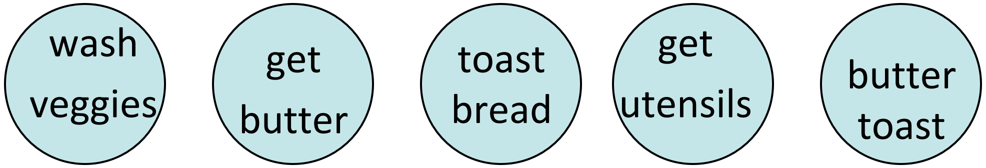
- Any of the top four tasks can be done in any order (no prerequisites)



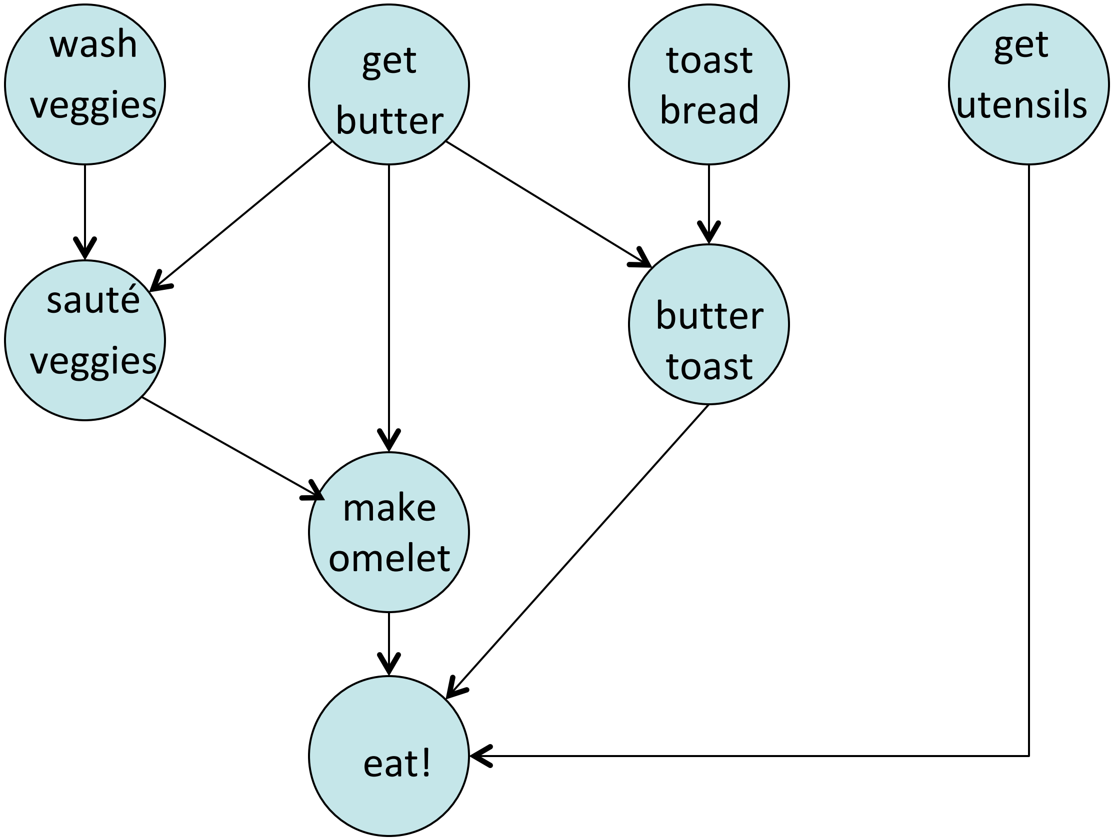
Topological Ordering



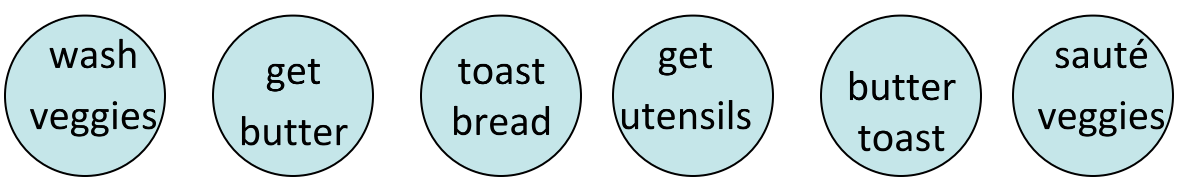
- Butter toast's prerequisites have all been met, so can do that next



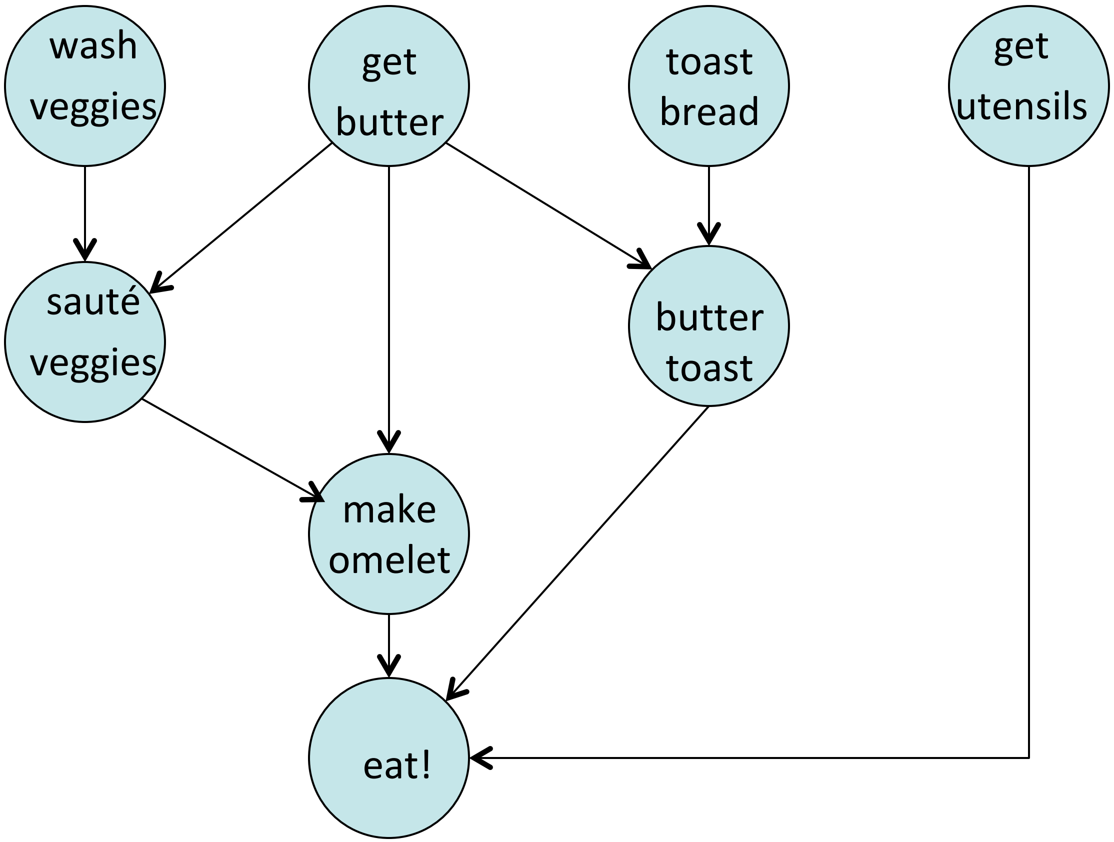
Topological Ordering



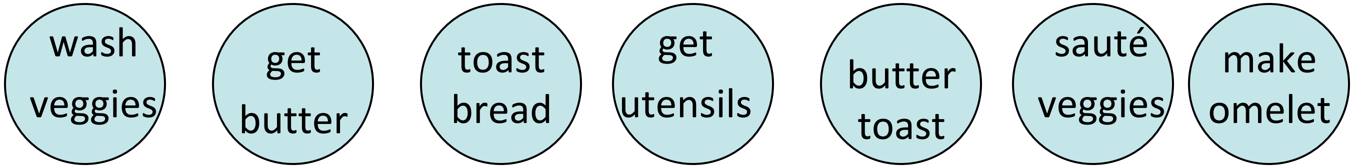
- Can sauté the vegetables since we've already washed the veggies and gotten butter



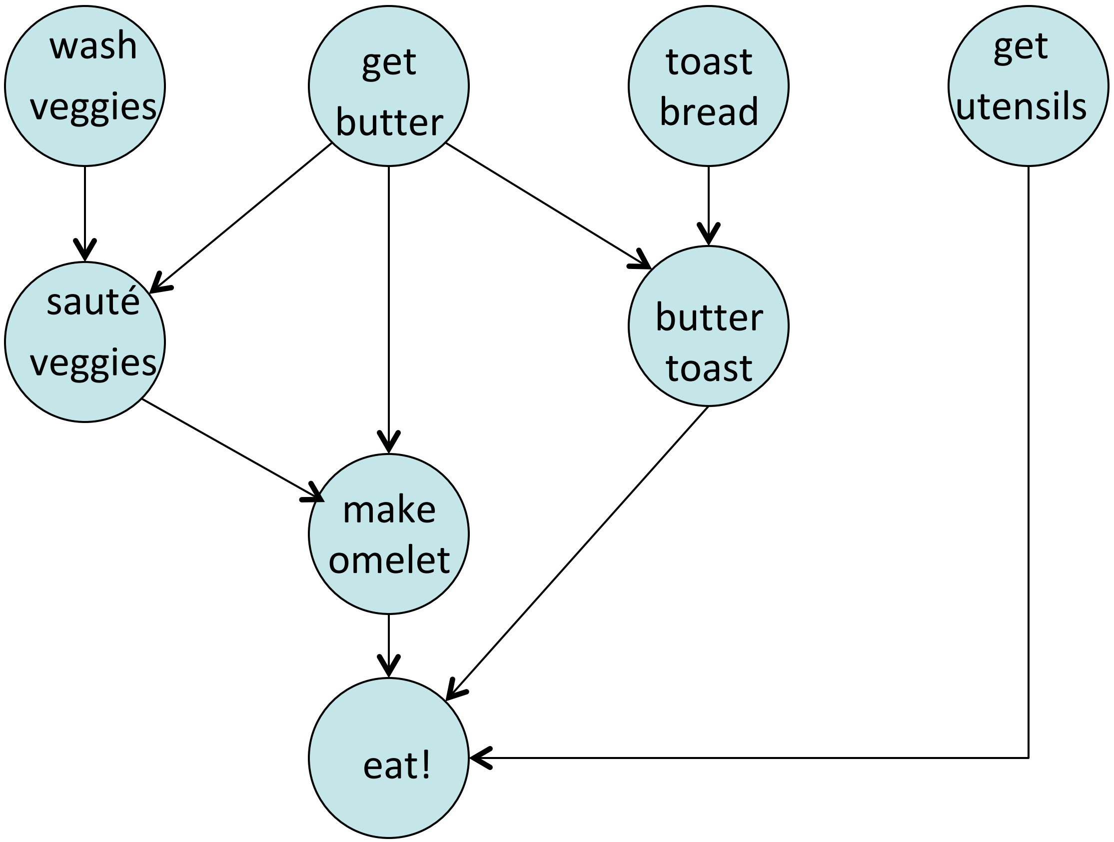
Topological Ordering



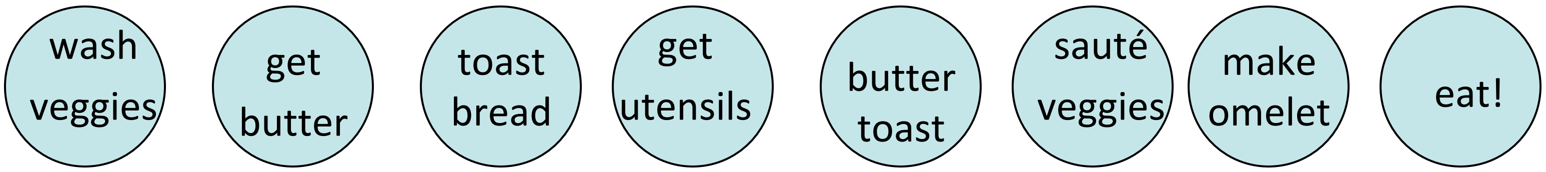
- Can make the omelet



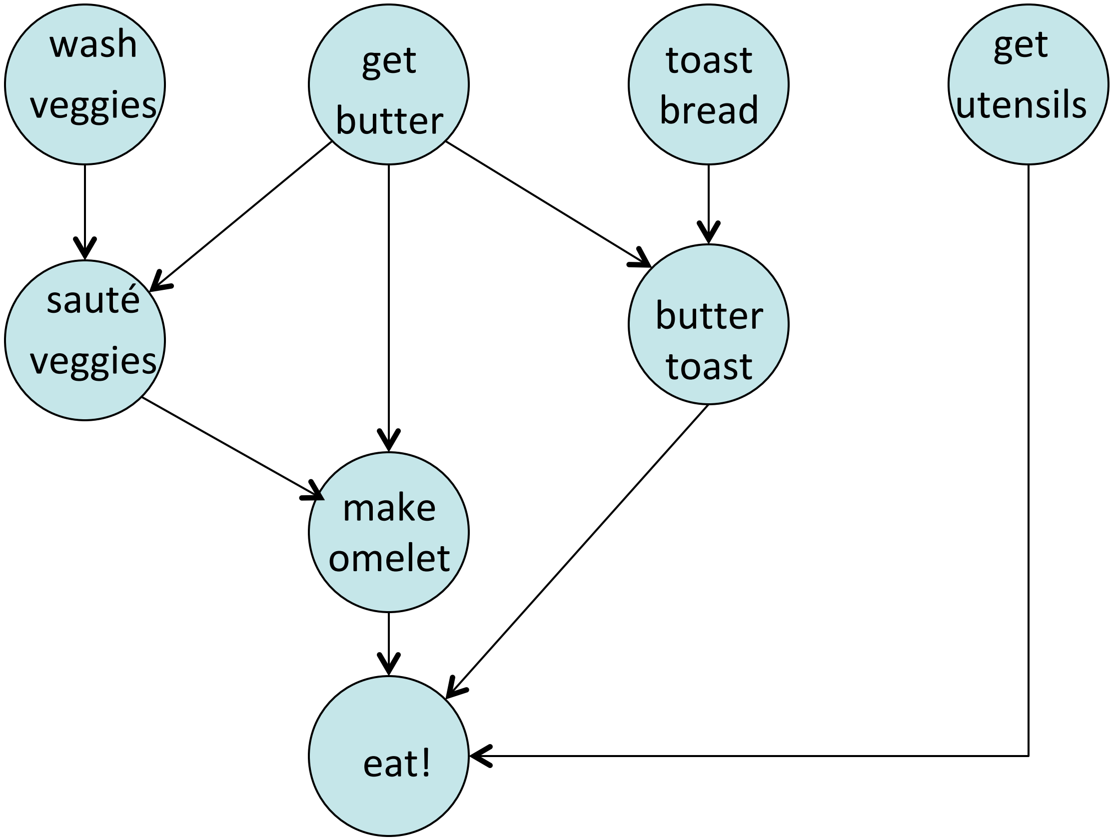
Topological Ordering



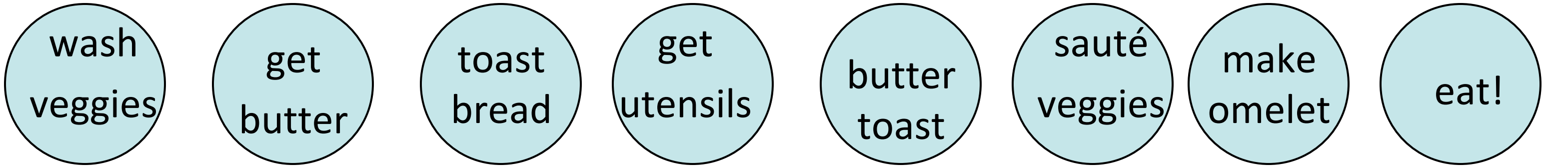
- Finally, we can eat!



Topological Ordering



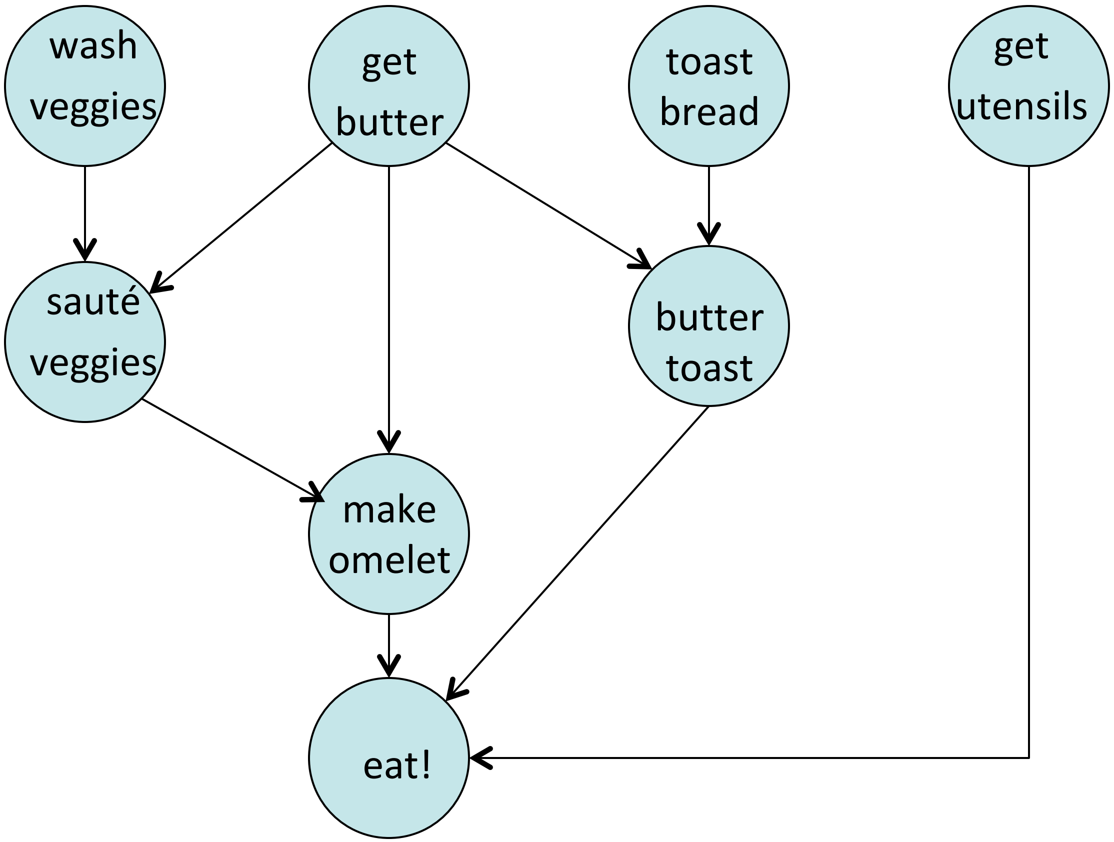
- This is just one topological ordering – what's another?



A Note About DFS

- In what order do we finish visiting nodes (do they turn grey in our example from Thursday) in DFS on a DAG?

DFS on a DAG


wash veggies
get butter
toast bread
get utensils
sauté veggies
butter toast
make omelet
eat!

Topological Sort with DFS

- Key observation: finishing visiting node *a* means we must have visited all nodes that have *a* as a prerequisite
- How could we modify DFS to return the topological ordering?
  - We'll need a Vector to maintain the order we traverse nodes
  - In what order should we add the nodes to the Vector? Where should we add the node (beginning/random place/end)?

Topological Sort Algorithm

For each *unvisited* node:

    run `TopoDFS(node)`

`TopoDFS(node):`

    if we've *seen* this node before while running DFS, there's a cycle!

    run TopoDFS on each of the node's neighbors

    add node to the front of the ordering

    node is now *visited*

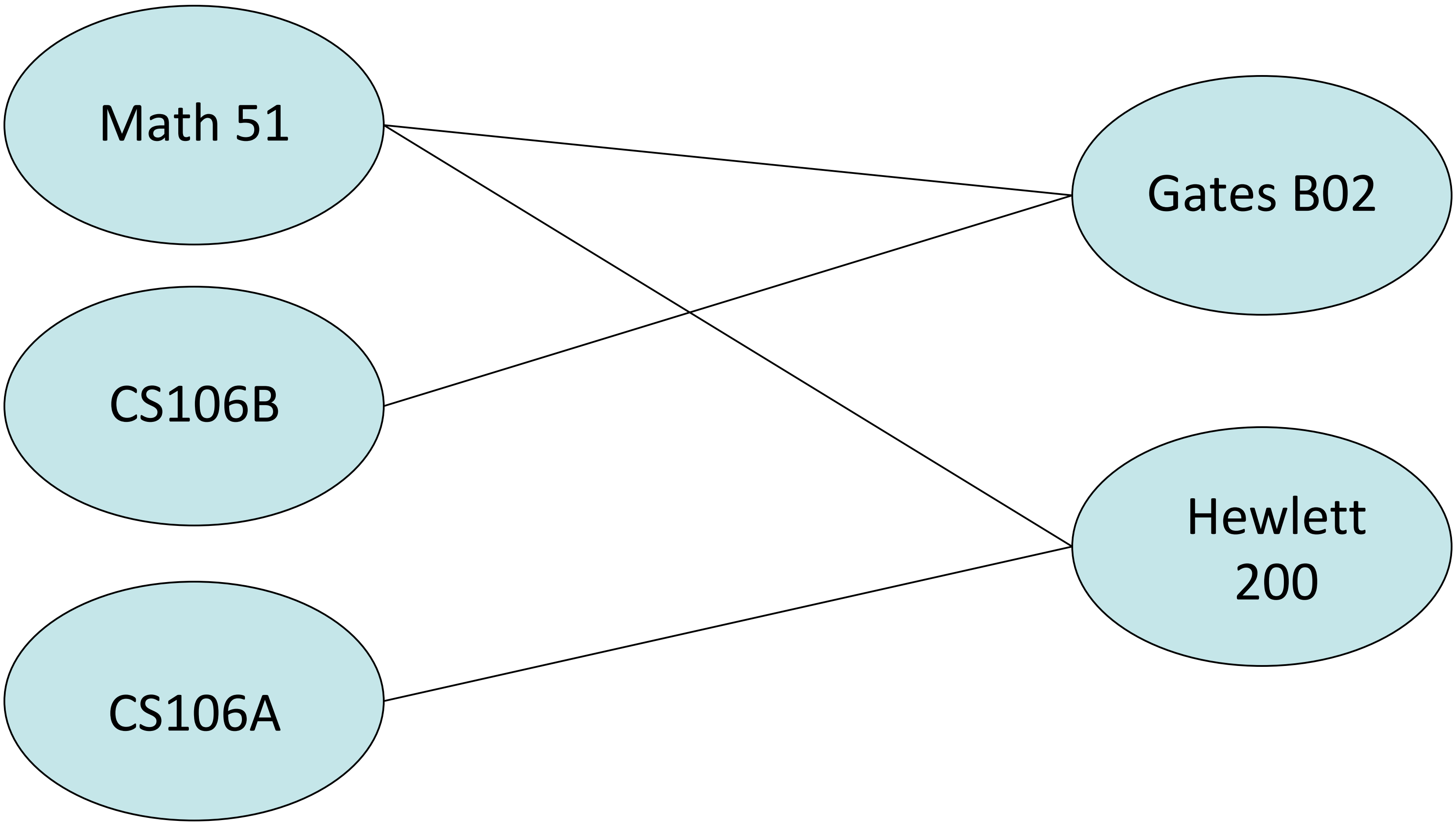
Announcements

- You should be working on Autocomplete
- Please give us feedback! cs198.stanford.edu
- Feel free to use seepluspl.us to help you understand trees or pointers. It's still in development, so be patient with quirks
- Course feedback:
  - You all like that I write code in class – we'll get back to doing that by the end of this week
  - It's a hard class, but you all are doing fantastically
    - Please ask questions on Piazza, come talk to me after class, email me for a meeting, etc. if you feel like you're falling behind or don't understand the material
  - We've set grading deadlines before each assignment is due – if you haven't received a grade from your SL by the time the next assignment is due, email them (we also tell them)

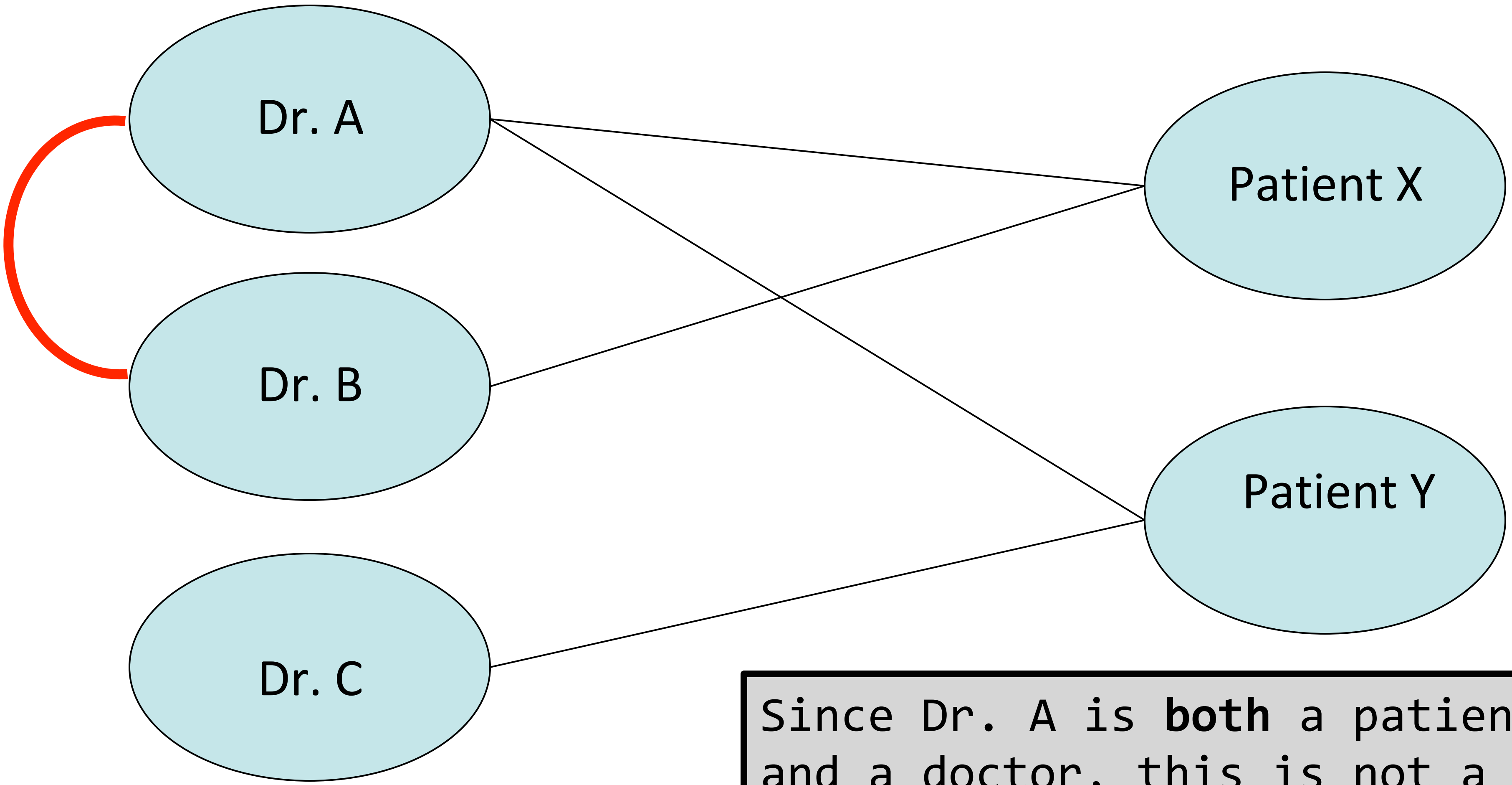
Another Type of Graph

- Sometimes, we want to model problems like assigning:
  - Doctors and patients
  - Students and classes
  - Classes and rooms
- Key properties:
  - we have two different types of nodes
  - all the relationships (edges) are between nodes of different types
    - e.g. a student is assigned to a class – no relationships between students or between classes
- A **bipartite graph** is a graph with two types of nodes (left-hand side and right-hand side), where all the (undirected) edges go from the LHS to the RHS

Bipartite Graphs



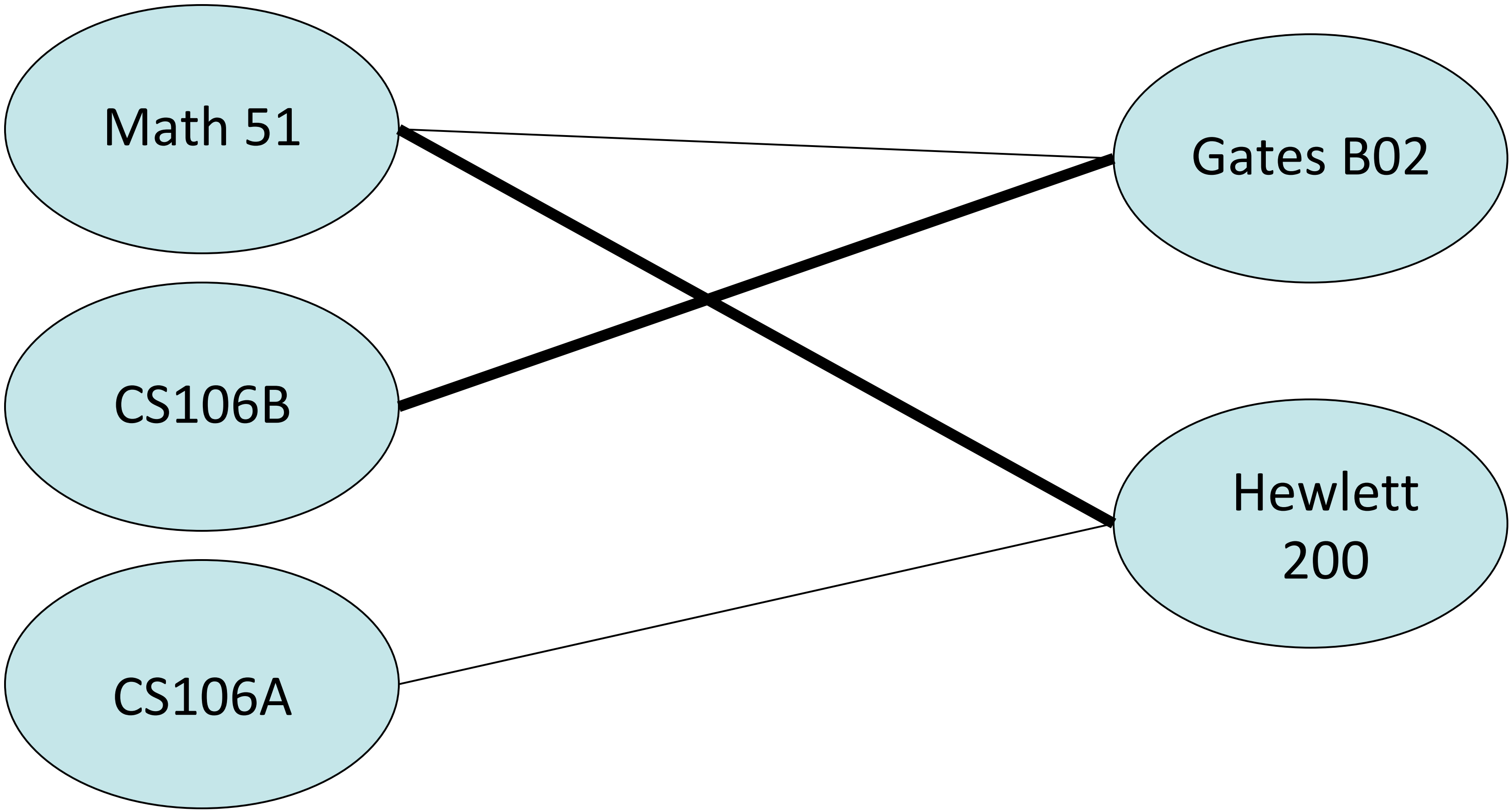
Not a Bipartite Graph



Since Dr. A is **both** a patient and a doctor, this is not a bipartite graph

Bipartite Graph Matching

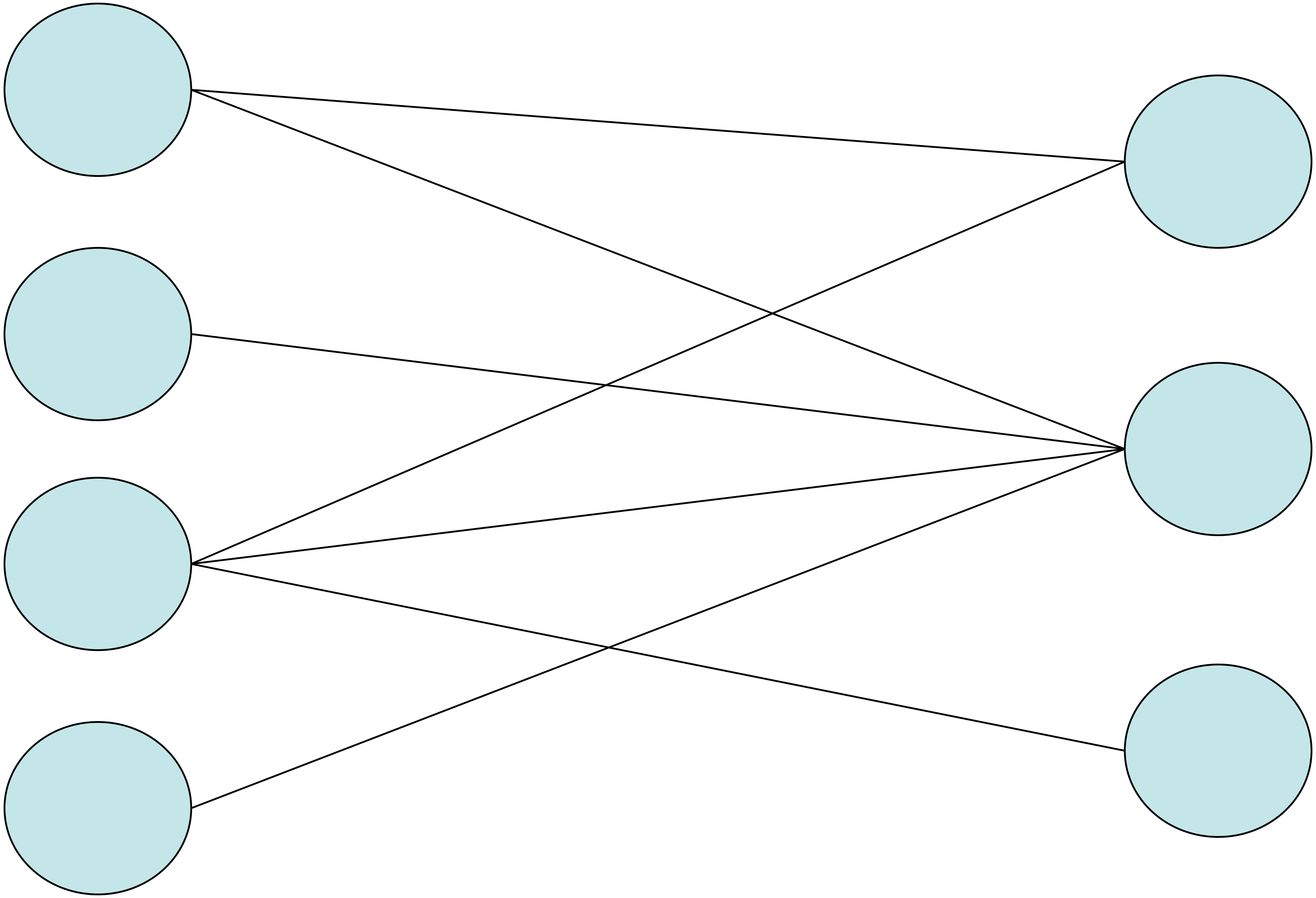
- A **matching** is a set of edges such that each node is connected to at most one edge
  - **Maximum matching**: largest such set of edges



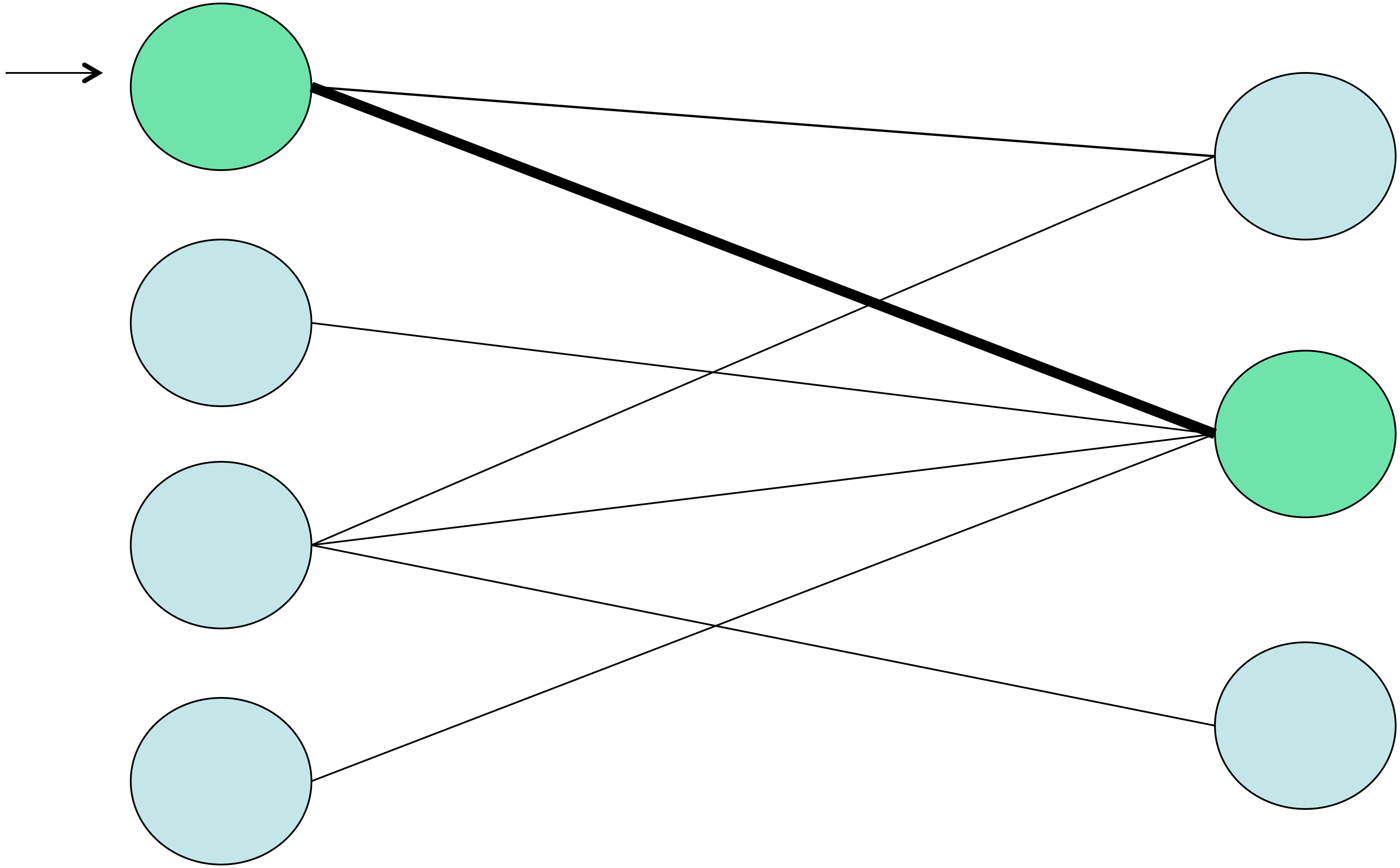
Matching Algorithm

- Start with an empty matching
- For each LHS node, either:
  - Match it to an unmatched RHS neighbor
  - Match it to a matched RHS neighbor and break the RHS neighbor's match, then try to match the newly unmatched LHS node. If you can't, keep the old matching
- How is this algorithm like depth-first search?

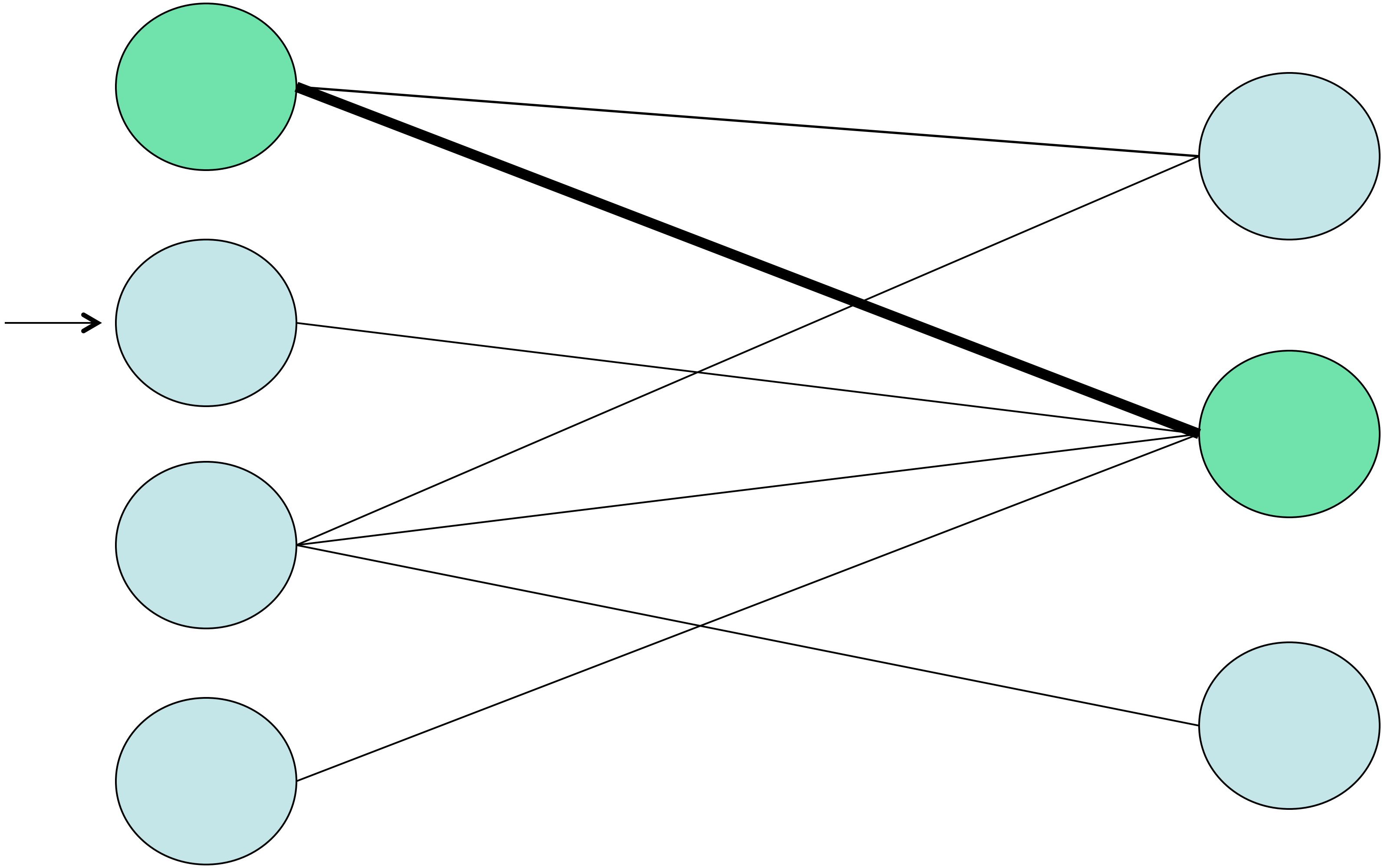
Bipartite Graph Algorithm



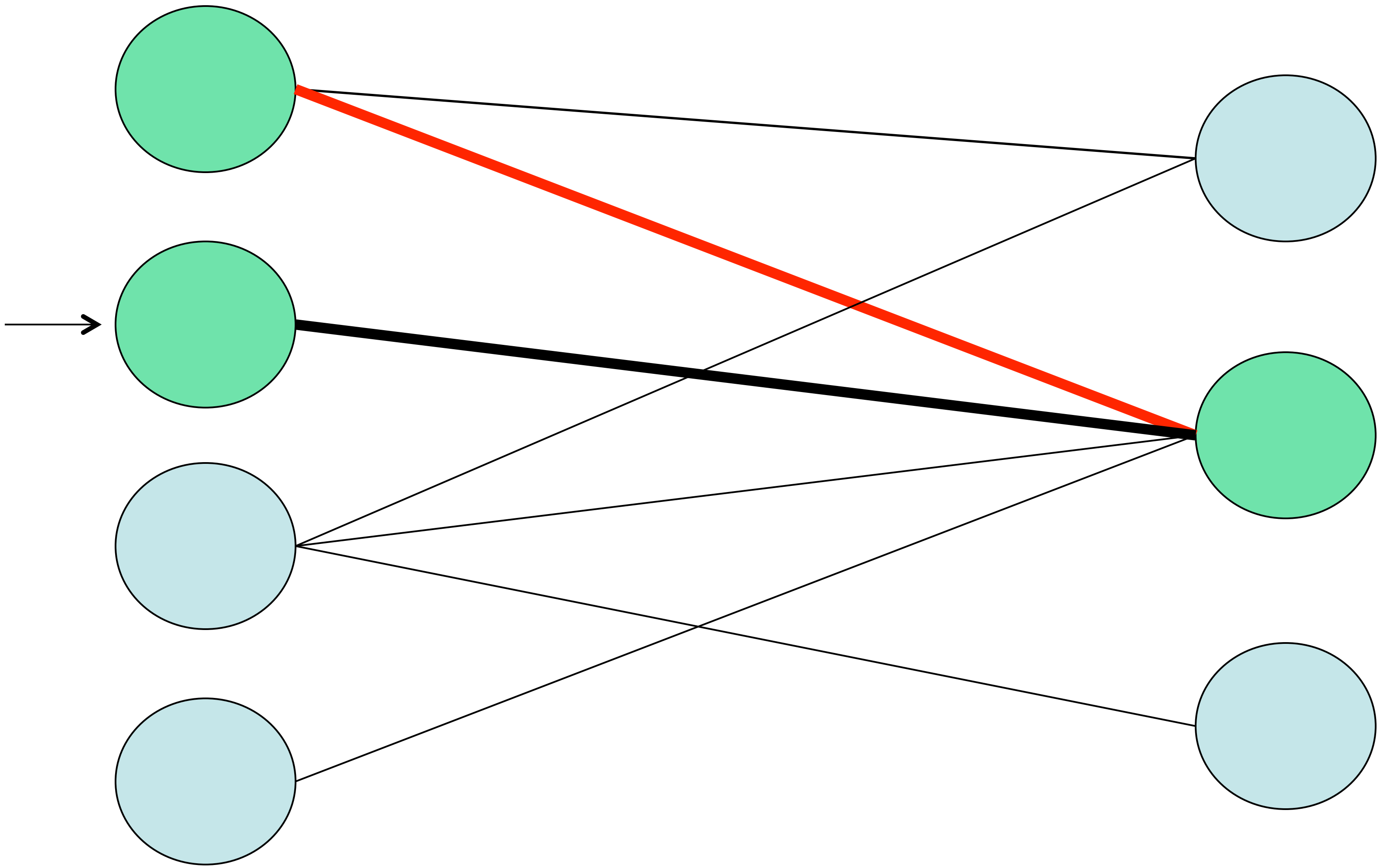
Bipartite Graph Algorithm



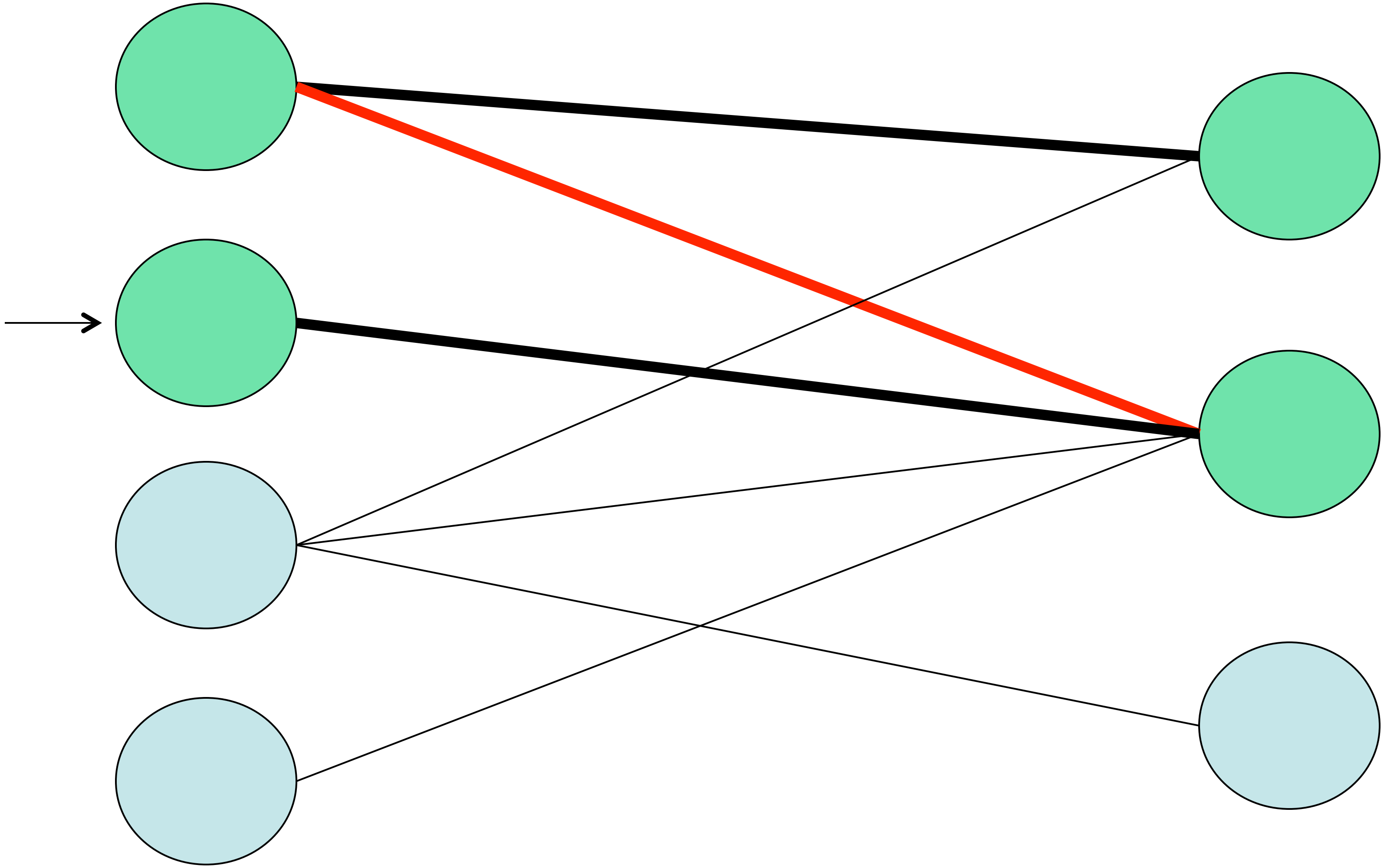
Bipartite Graph Algorithm



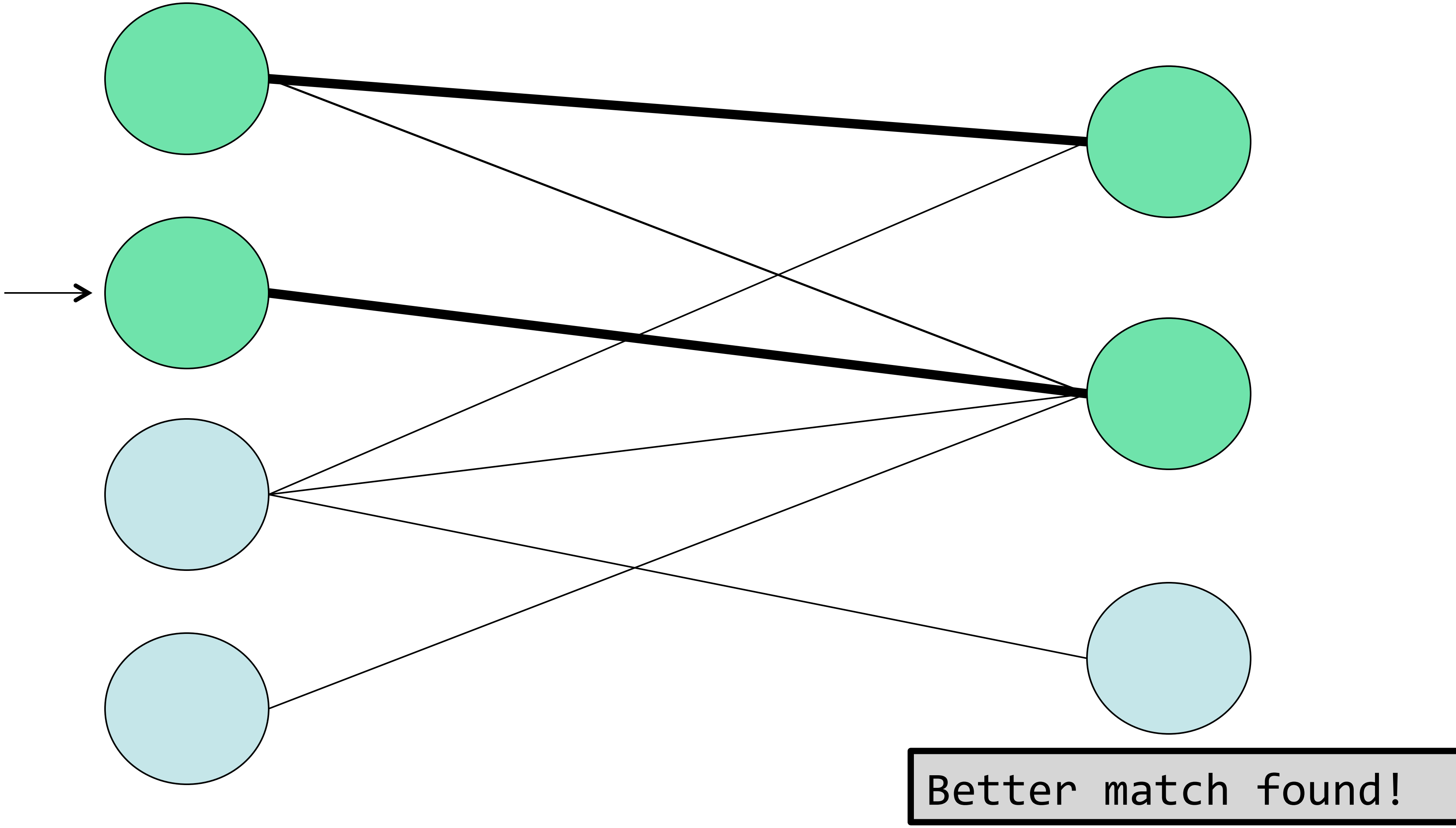
Bipartite Graph Algorithm



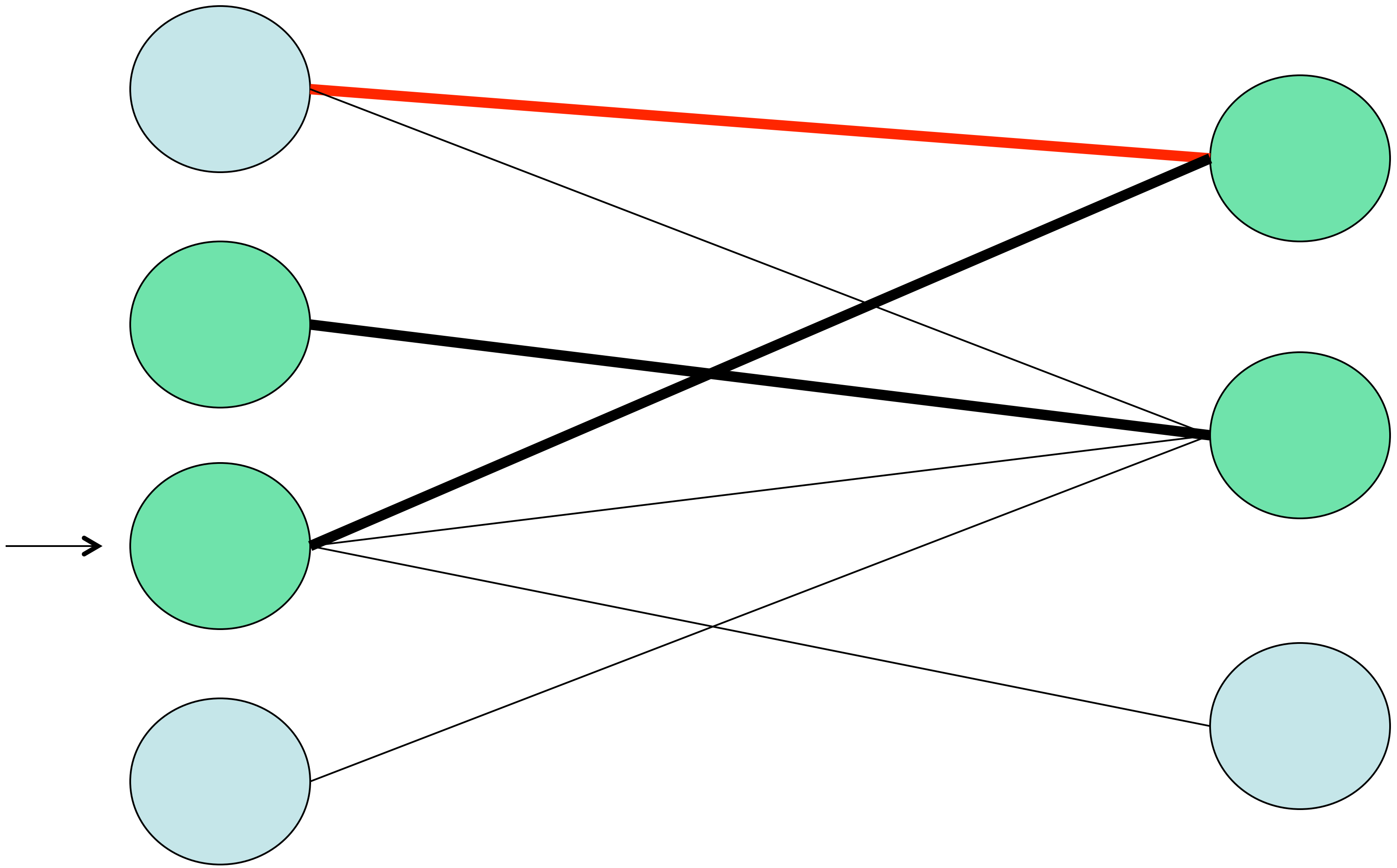
Bipartite Graph Algorithm



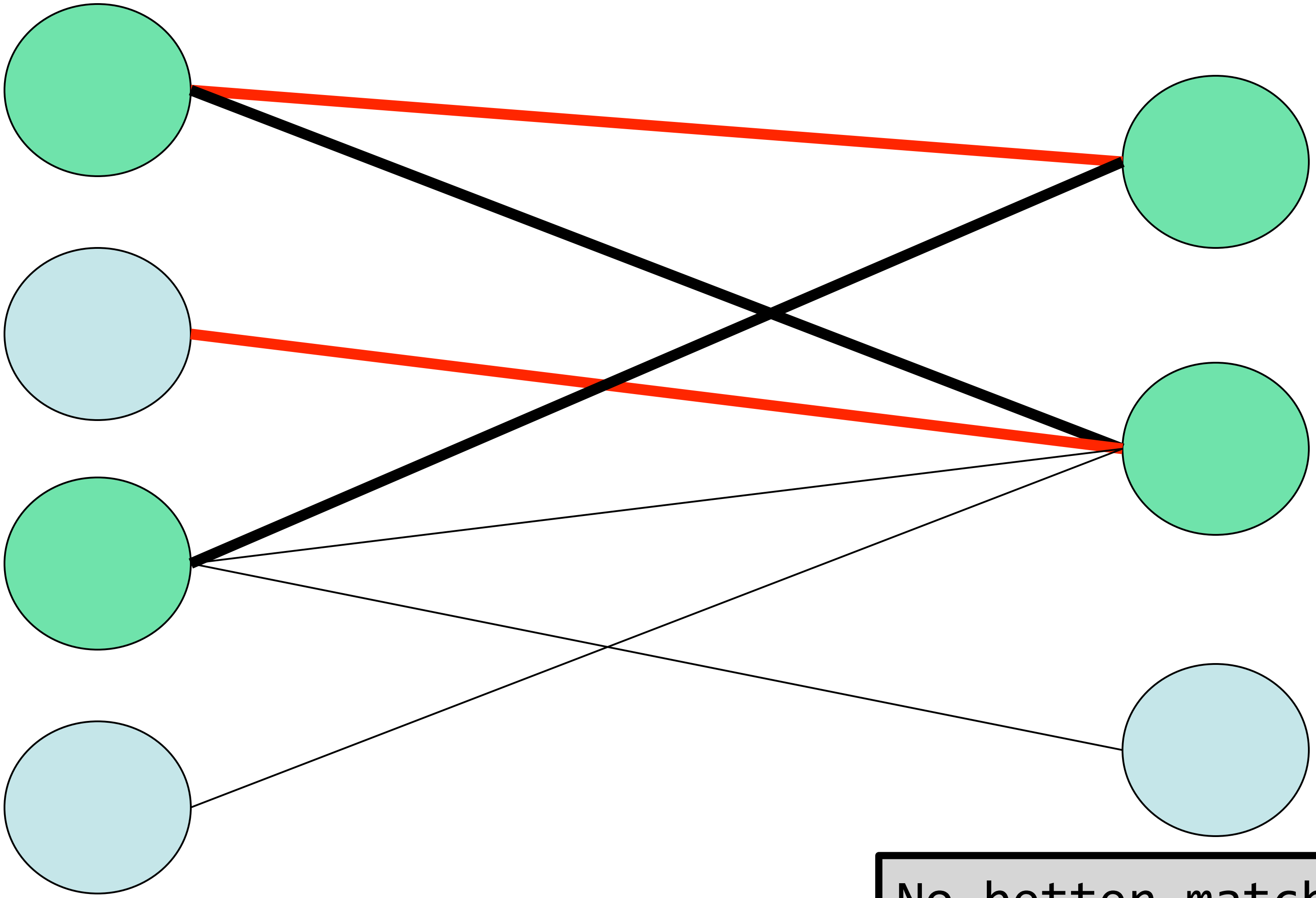
Bipartite Graph Algorithm



Bipartite Graph Algorithm

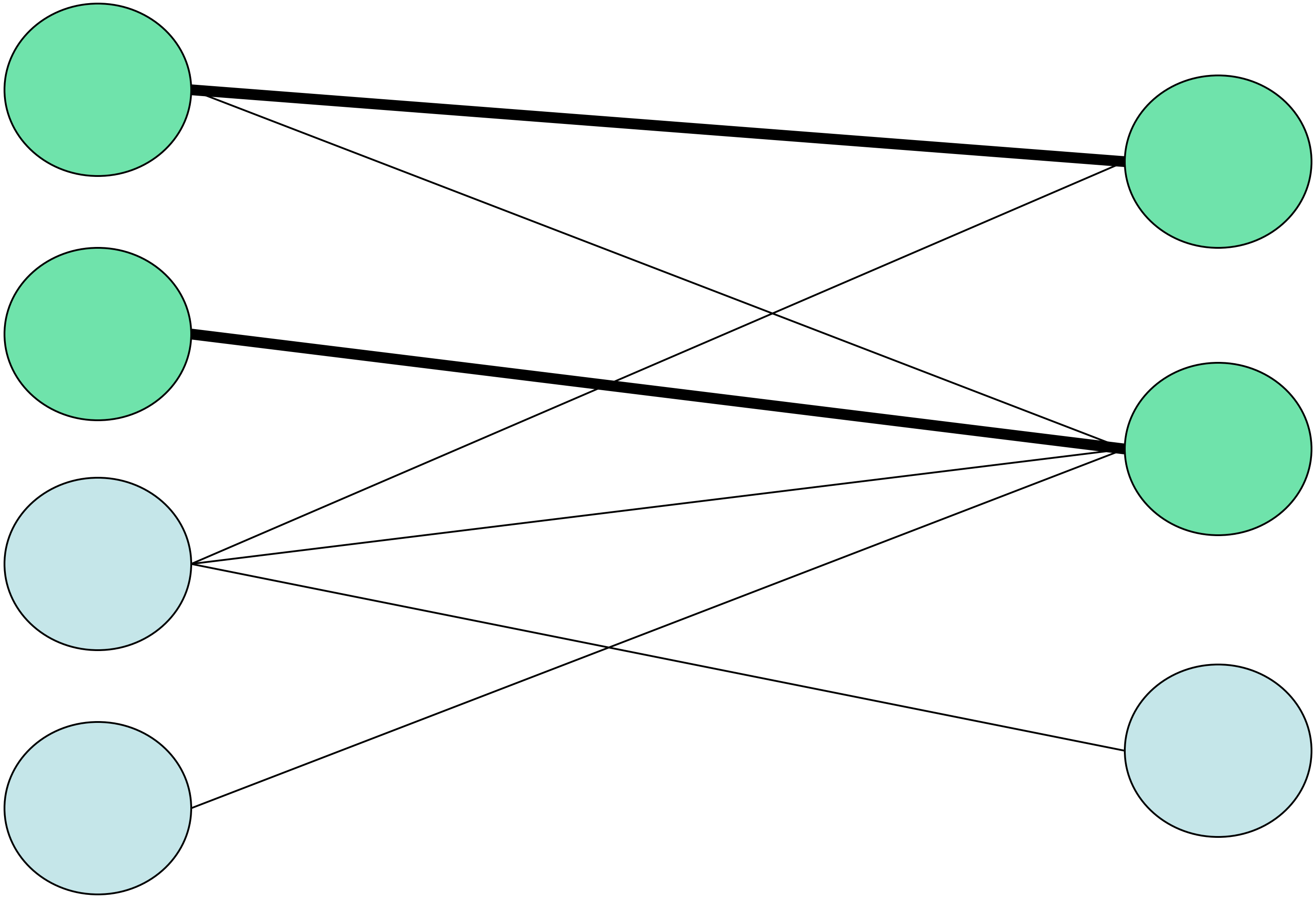


Bipartite Graph Algorithm

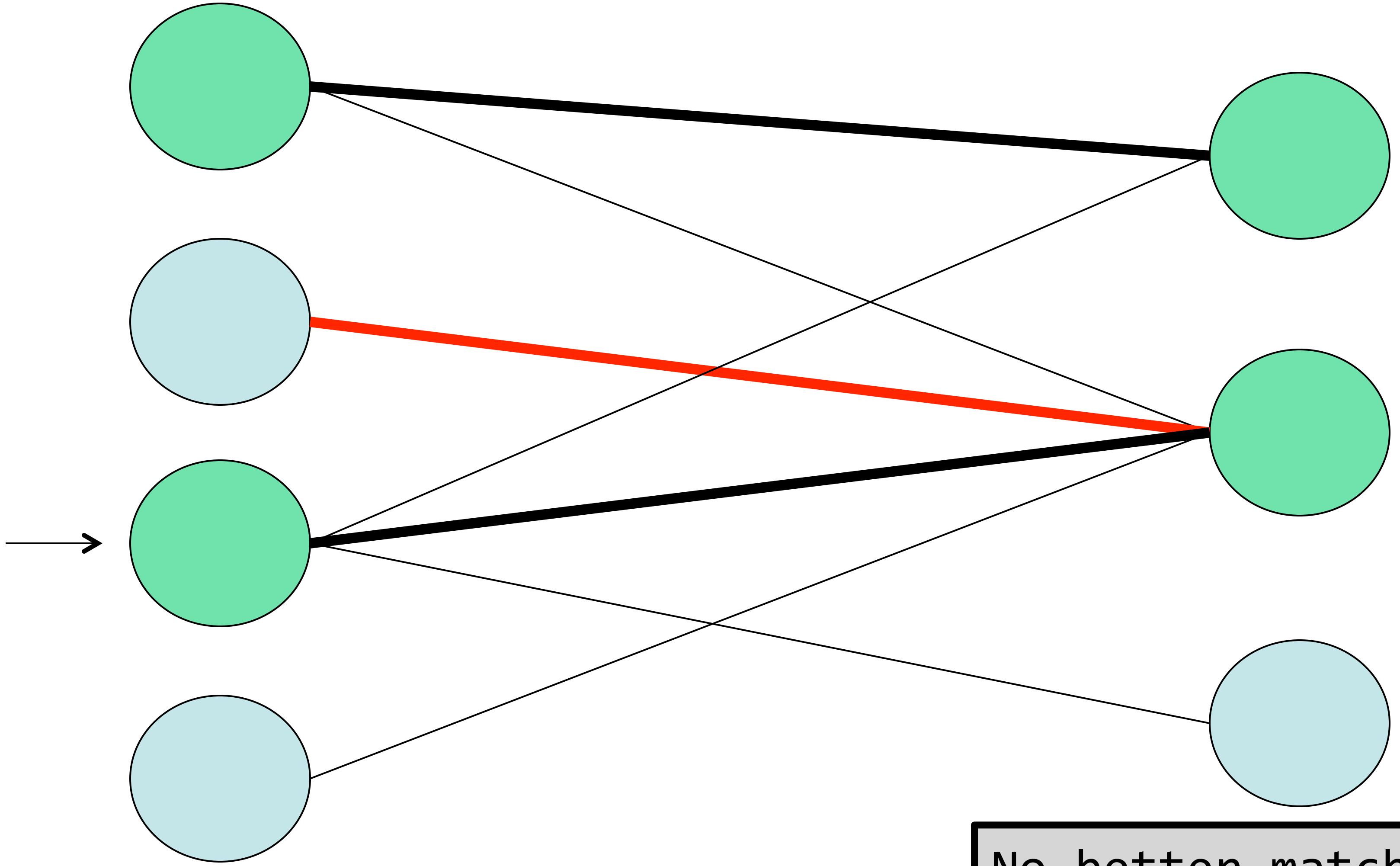


No better match found!

Bipartite Graph Algorithm

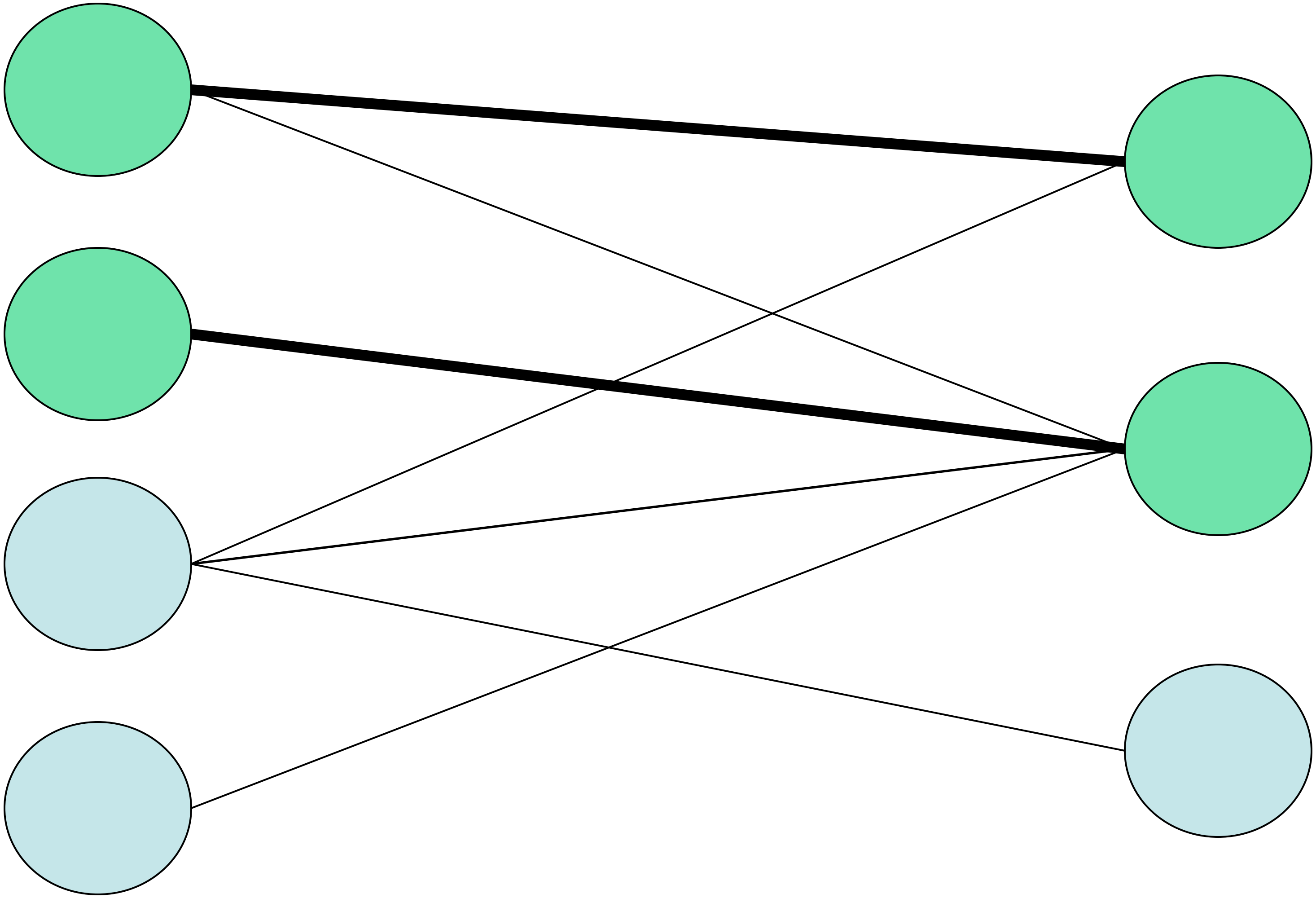


Bipartite Graph Algorithm

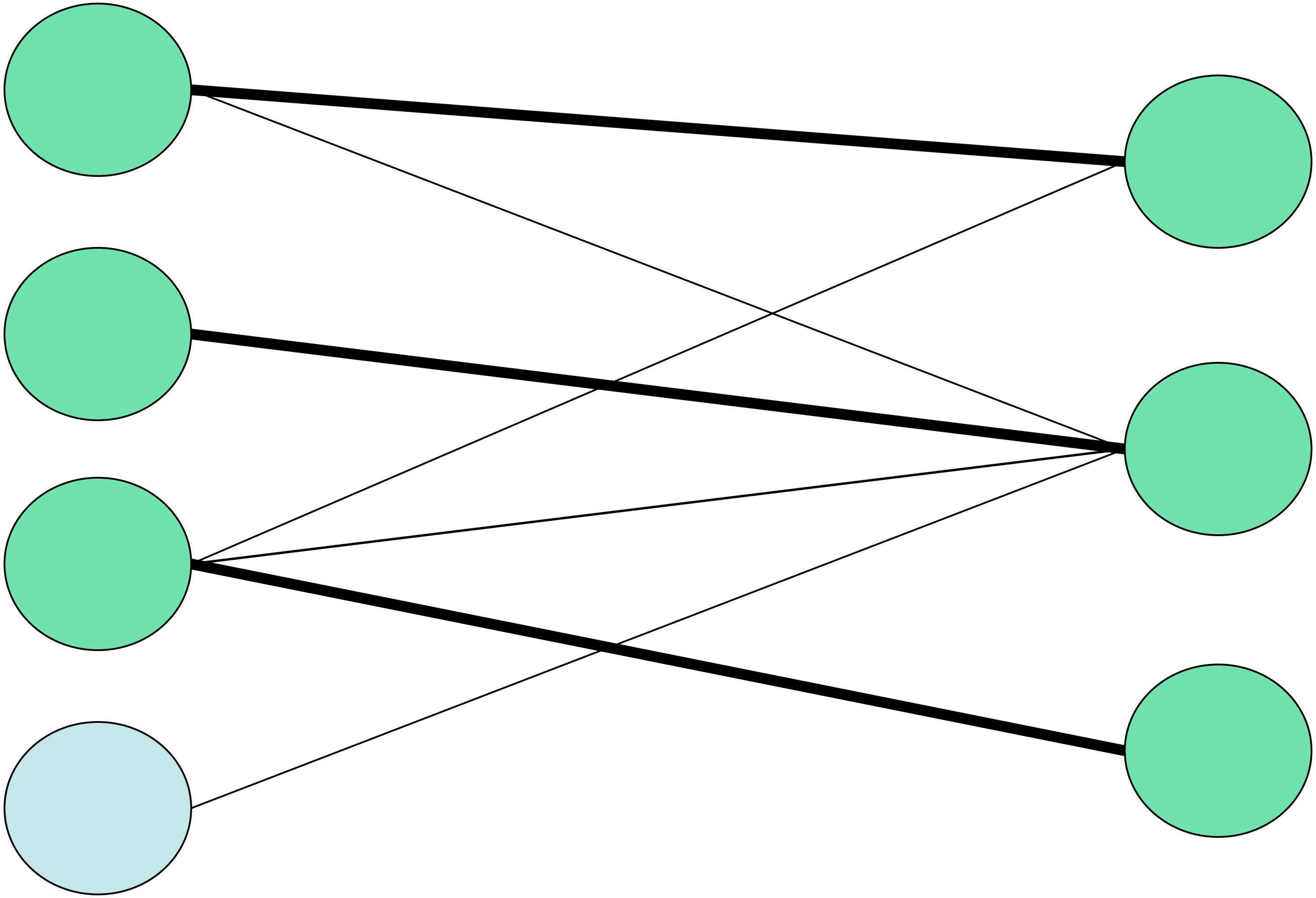


`No better match found!`

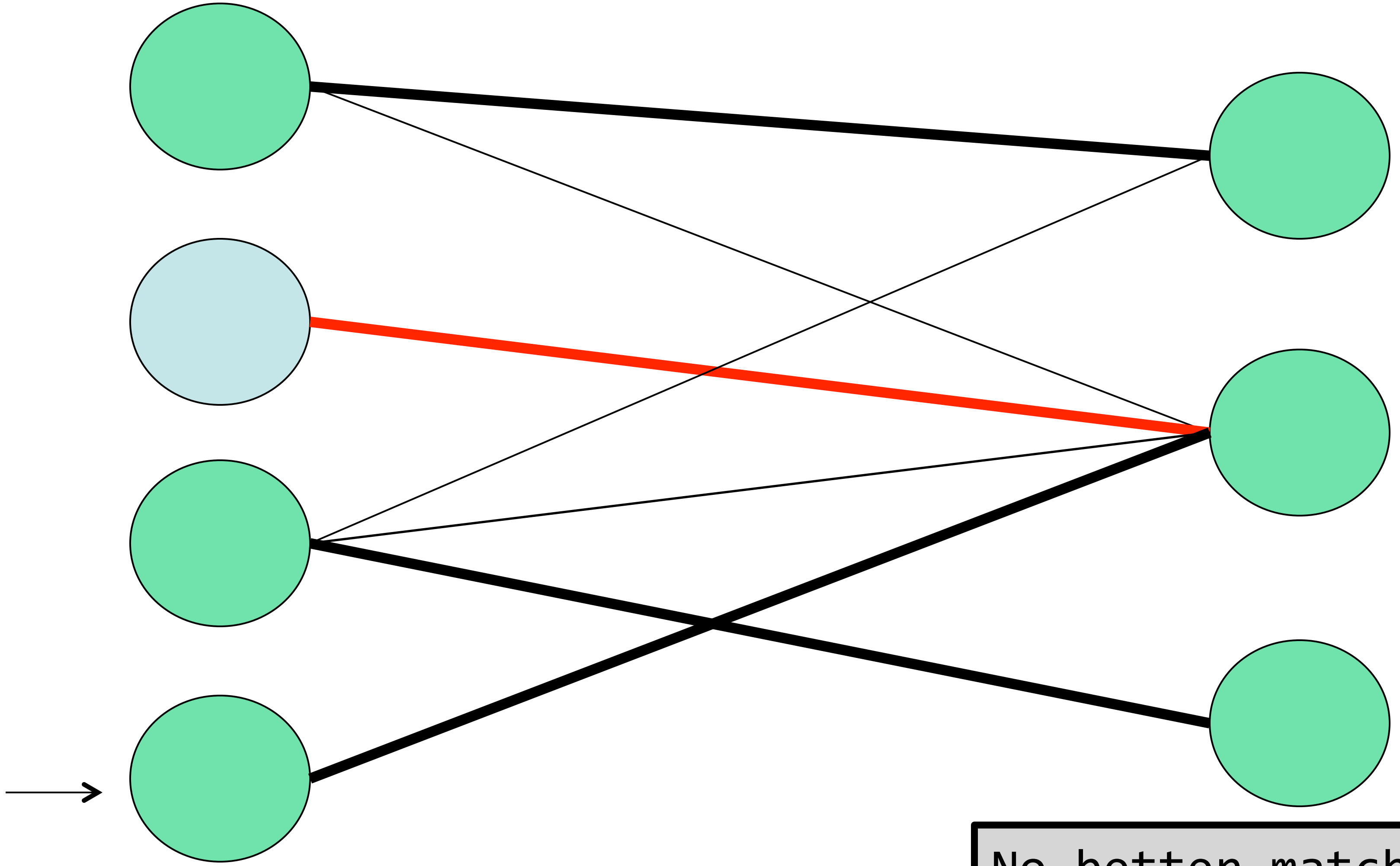
Bipartite Graph Algorithm



Bipartite Graph Algorithm

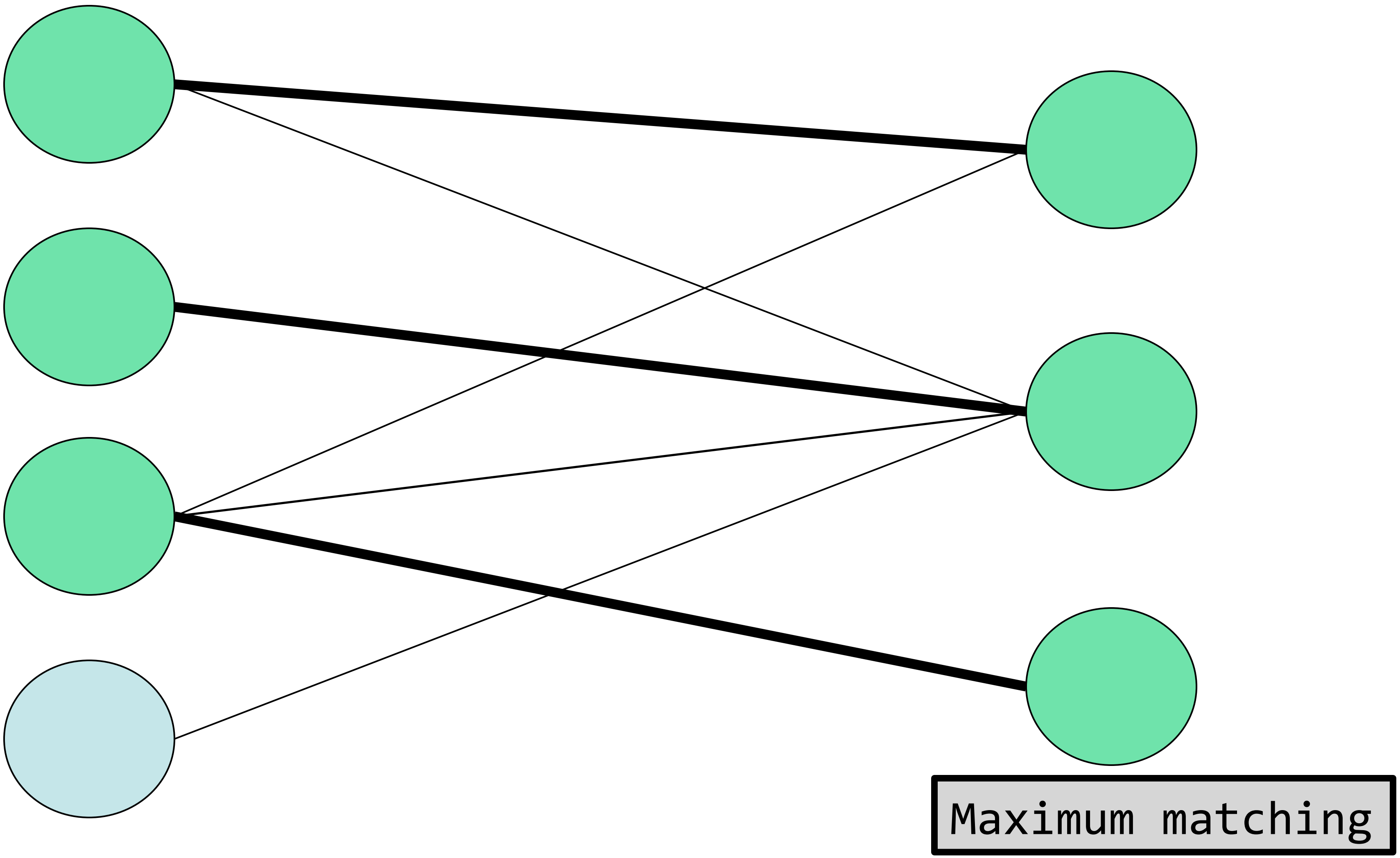


Bipartite Graph Algorithm

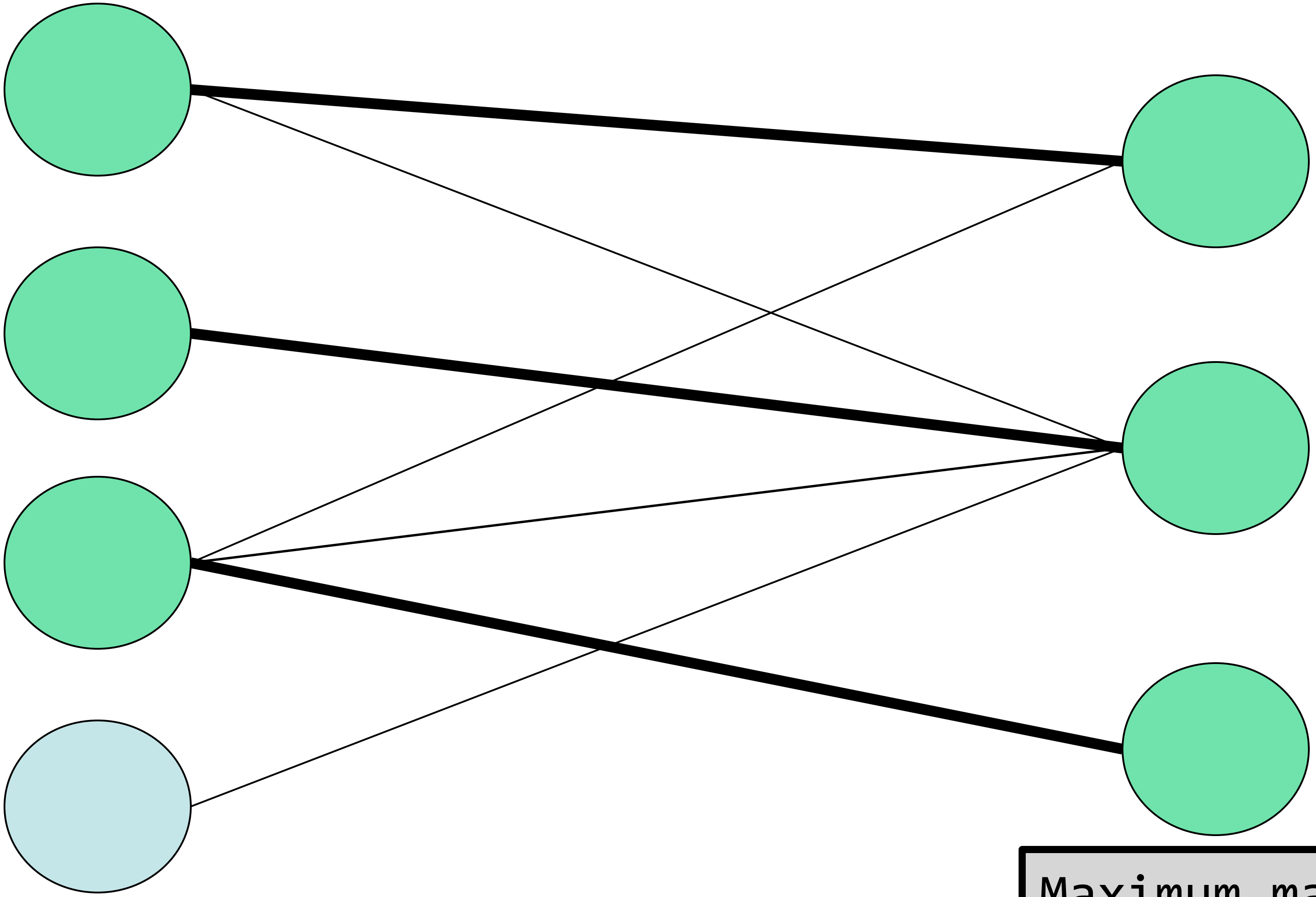


No better match found!

Bipartite Graph Algorithm



Bipartite Graph Algorithm



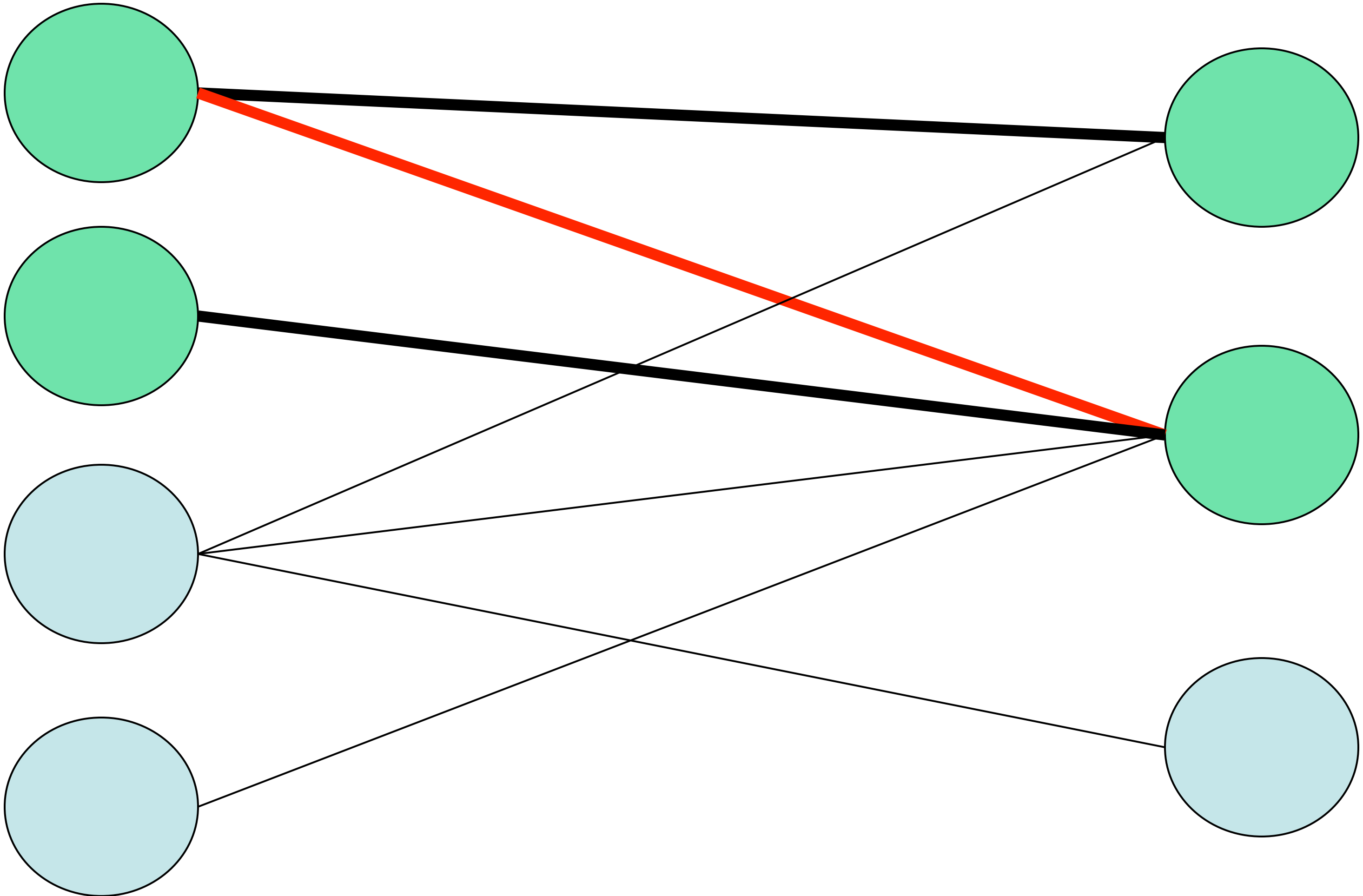
Maximum matching

Matching Algorithm

- Start with an empty matching
- For each LHS node, either:
  - Match it to an unmatched RHS neighbor
  - Match it to a matched RHS neighbor and break the RHS neighbor's match, then try to match the newly unmatched LHS node. If you can't, keep the old matching

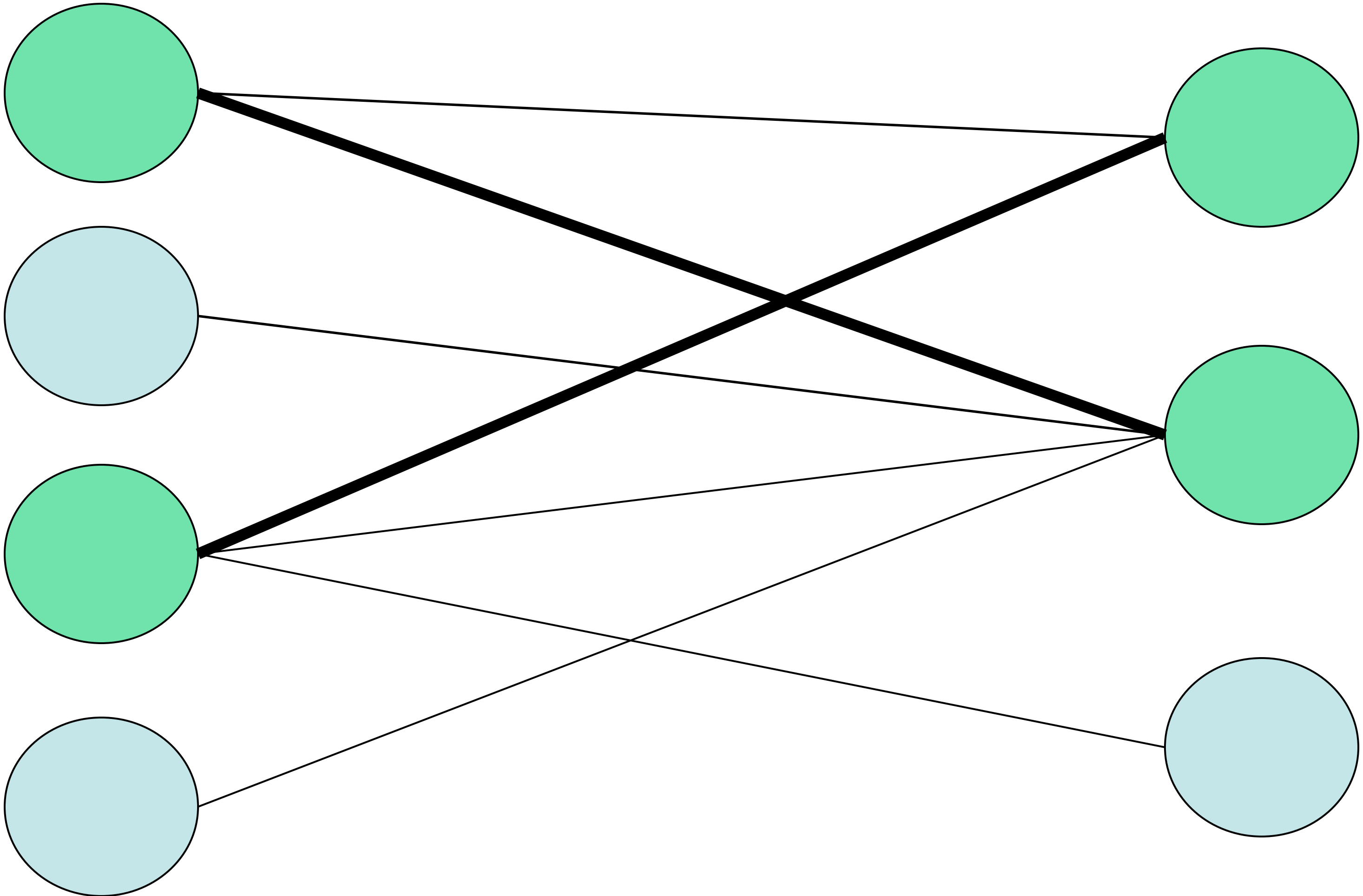
An observation

- Breaking an already made match and finding a better match means an **alternating path** from an unmatched LHS node to an unmatched RHS node



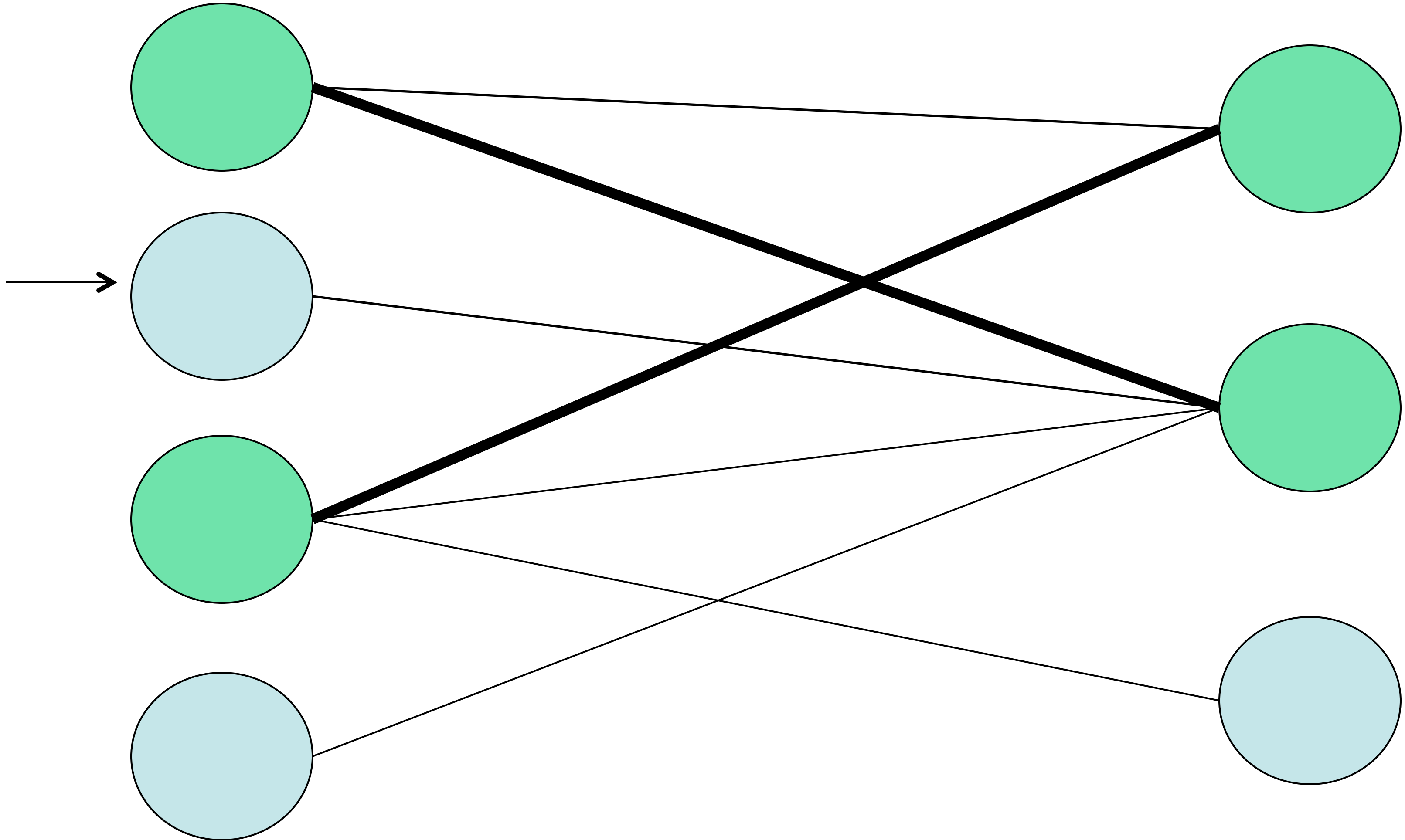
An observation

- Bigger example:



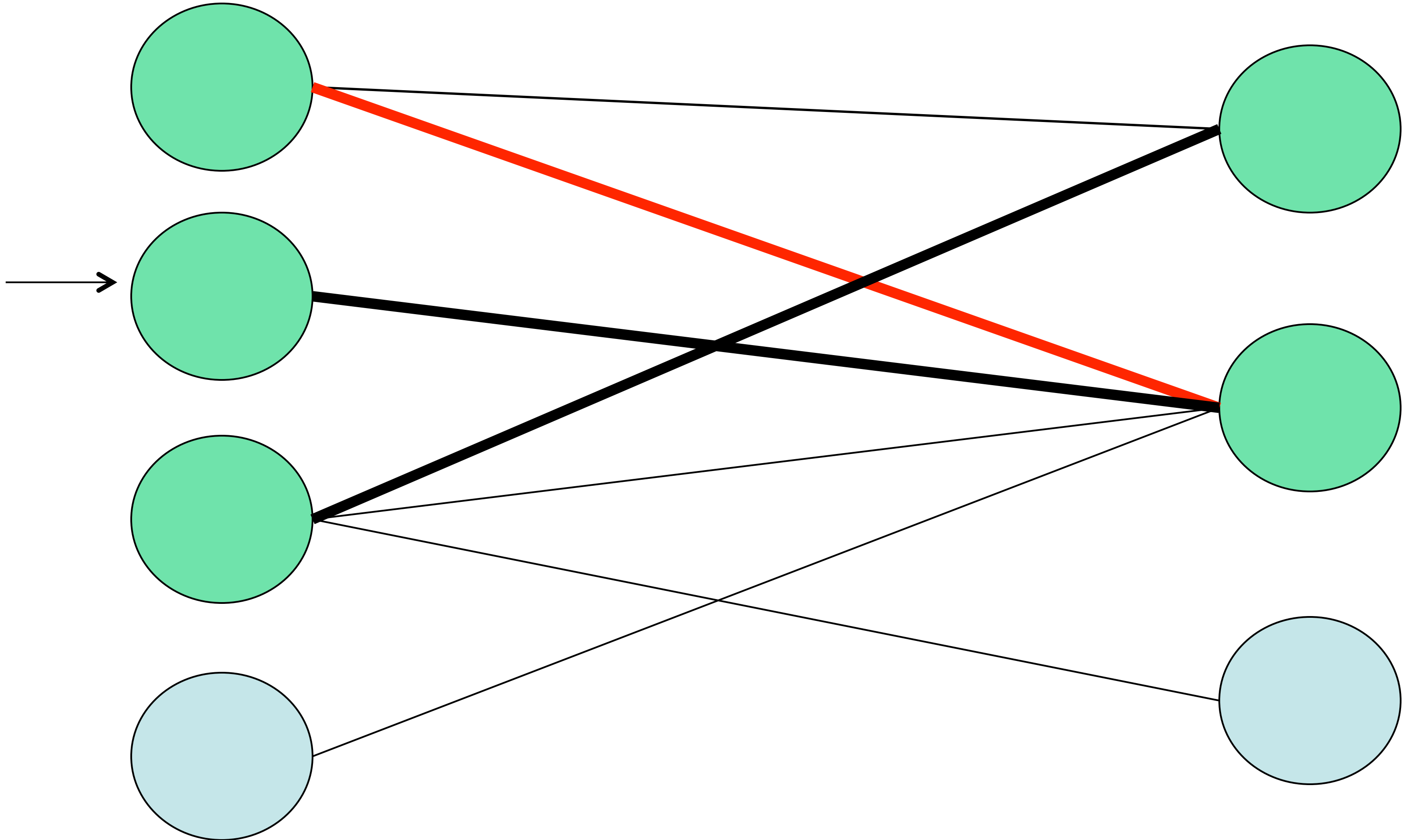
An observation

- Bigger example:



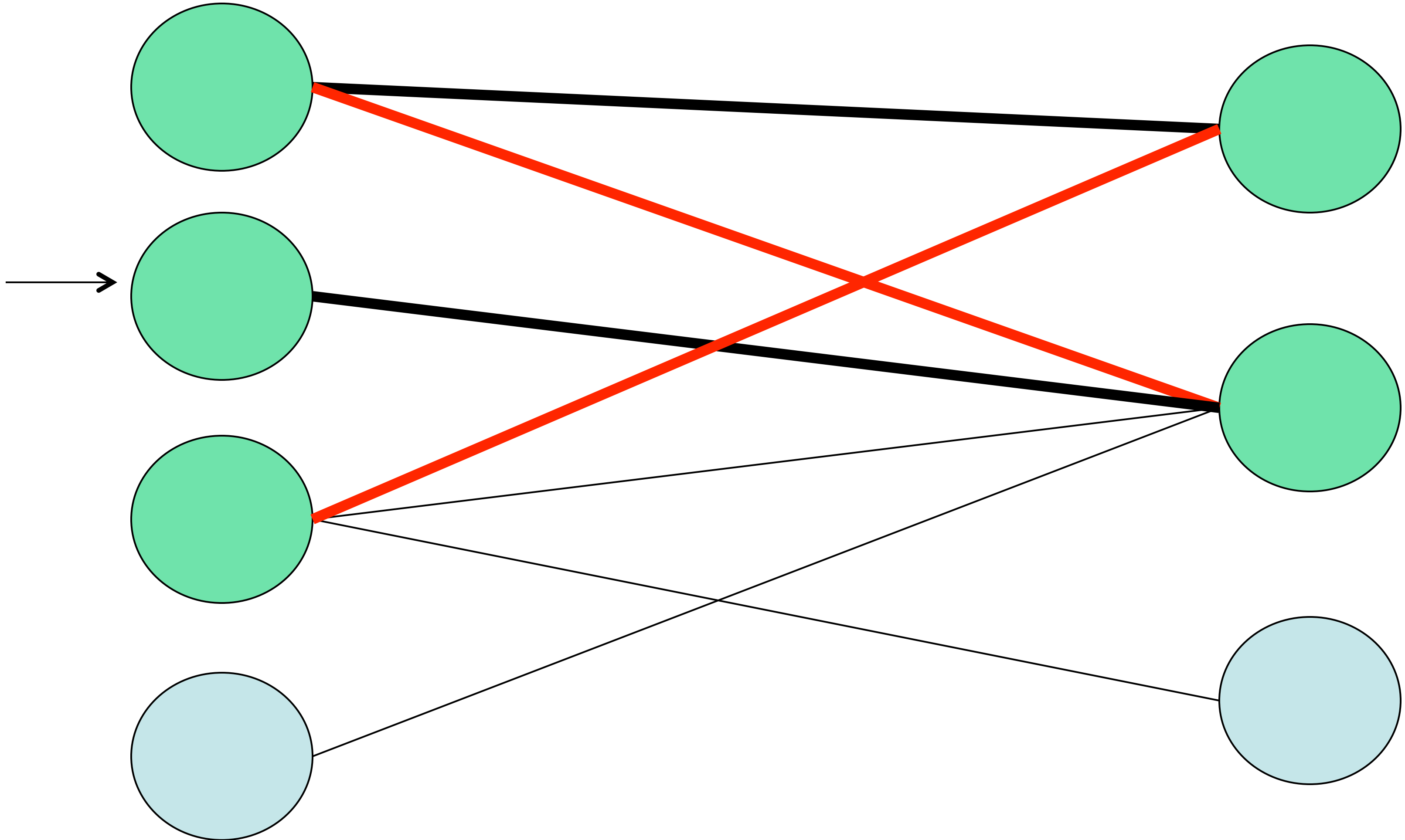
An observation

- Bigger example:



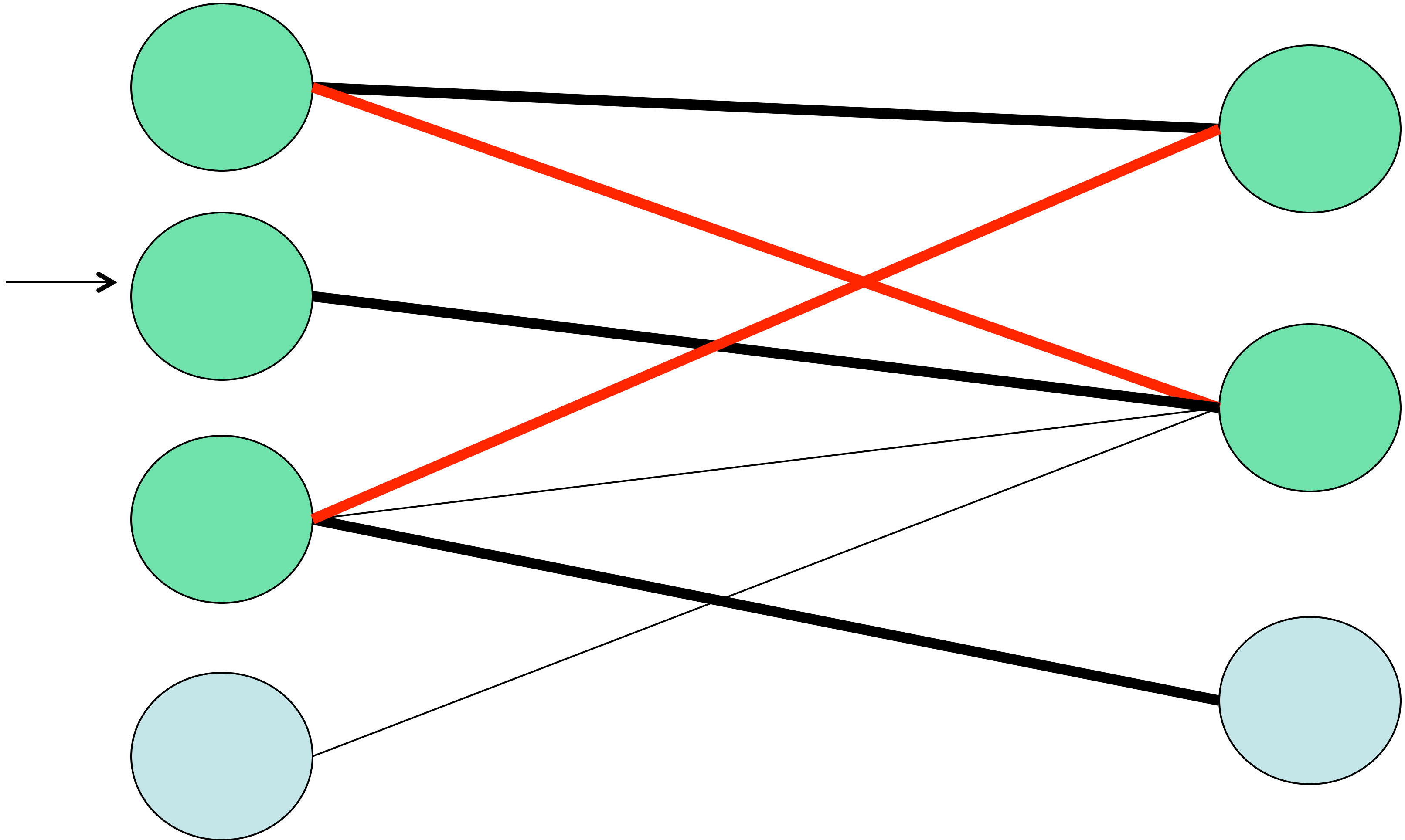
An observation

- Bigger example:



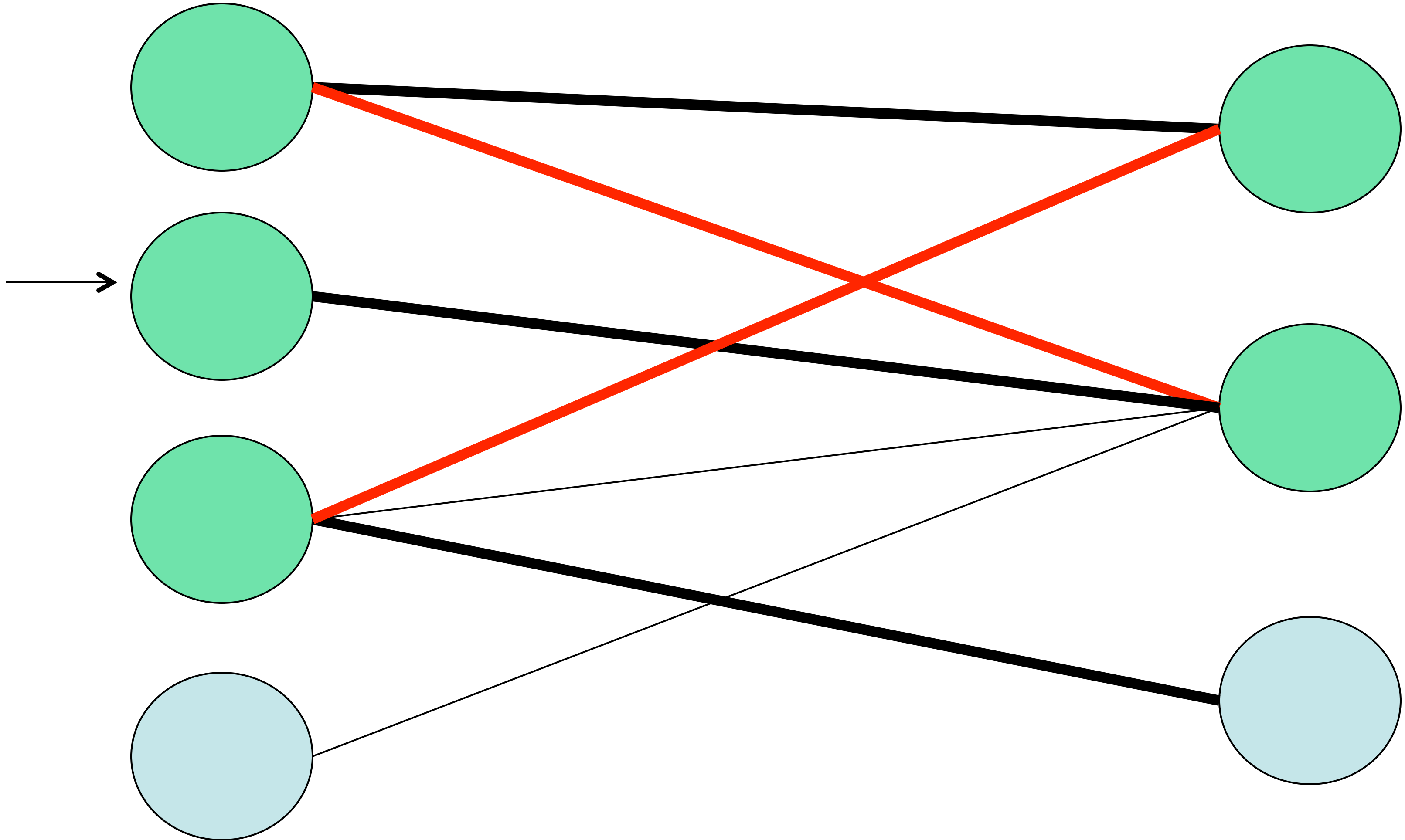
An observation

- Bigger example:



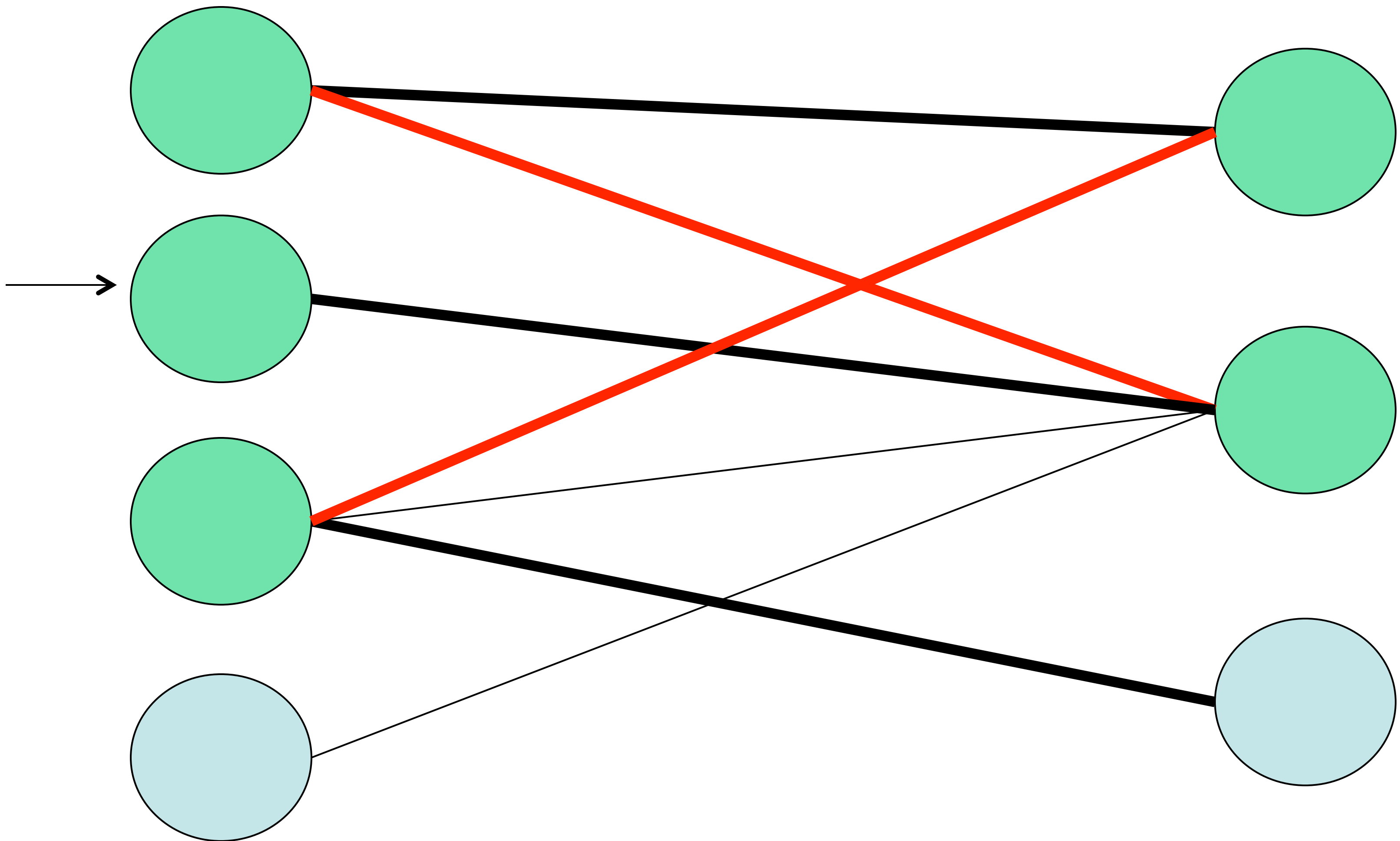
An observation

- Notice how we we **augmented** the alternating path by adding two new nodes (an unmatched LHS node and an unmatched RHS node)
  - The previous matching is now red, excluded from current matching



An observation

- The **black** edges are in the matching, and the **red** edges are not
  - **black** is LHS to RHS, **red** is RHS to LHS



Alternate Approach

- Start with an empty matching
- While possible:
  - Find an **alternating path** from an unmatched LHS node to an unmatched RHS, potentially by **augmenting** an existing alternating path
    - The **black** edges in such a path (from LHS to RHS) are included in the matching; the **red** edges (from RHS to LHS) are not
  - If no such path exists, we've found the maximum matching
- How do we find a path?