

CS 106B, Lecture 13

Recursive Backtracking

Plan for Today

- More backtracking!
 - Make sure to practice, in section, on CodeStepByStep, with the book
- Some notes on the midterm

"Arm's length" recursion

- Arm's length recursion: a poor style where unnecessary tests are performed before performing recursive calls
- Typically, the tests try to avoid making a call into what would otherwise be a base case
- Can lead to **functionality bugs** as well as **less readable code**
- Applies to all recursive code but **especially backtracking**

Backtracking Model

Choosing

1. We generally iterate over **decisions**. What are we iterating over here? What are the **choices** for each decision? Do we need a for loop?

Exploring

2. How can we *represent* that choice? How should we **modify the parameters** and **store our previous choices** (avoiding *arms-length* recursion)?
 - a) Do we need to use a **wrapper** due to extra parameters?
3. How should we **restrict** our choices to be valid?
4. How should we use the **return value** of the recursive calls? Are we looking for all solutions or just one?

Un-choosing

5. How do we **un-modify** the parameters from step 3? Do we need to explicitly un-modify, or are they copied? Are they un-modified at the same level as they were modified?

Base Case

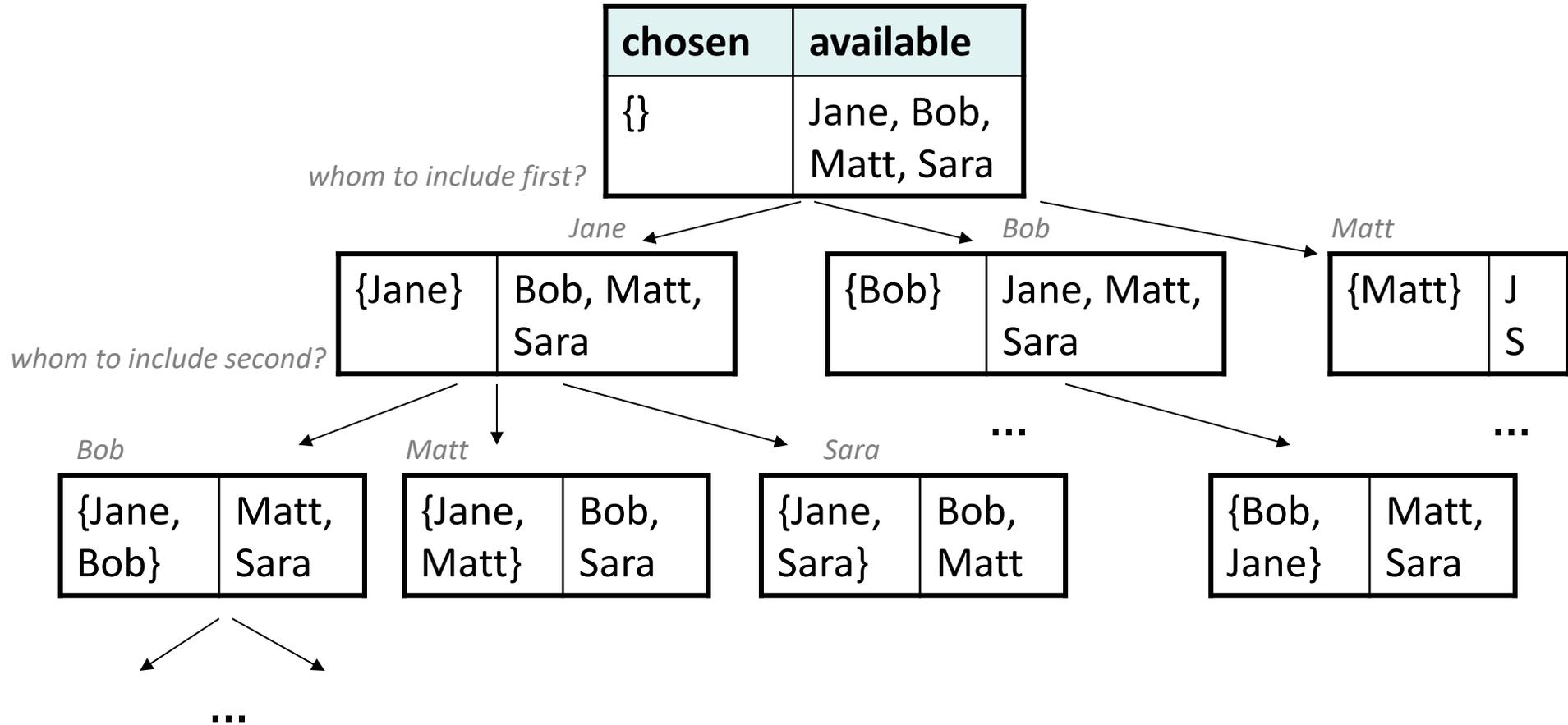
6. What should we do in the base case when we're **out of decisions** (usually return true)?
7. Is there a case for when there **aren't any valid choices left** or a "bad" state is reached (usually return false)?
8. Are the base cases ordered properly? Are we avoiding **arms-length** recursion?

Exercise: sublists

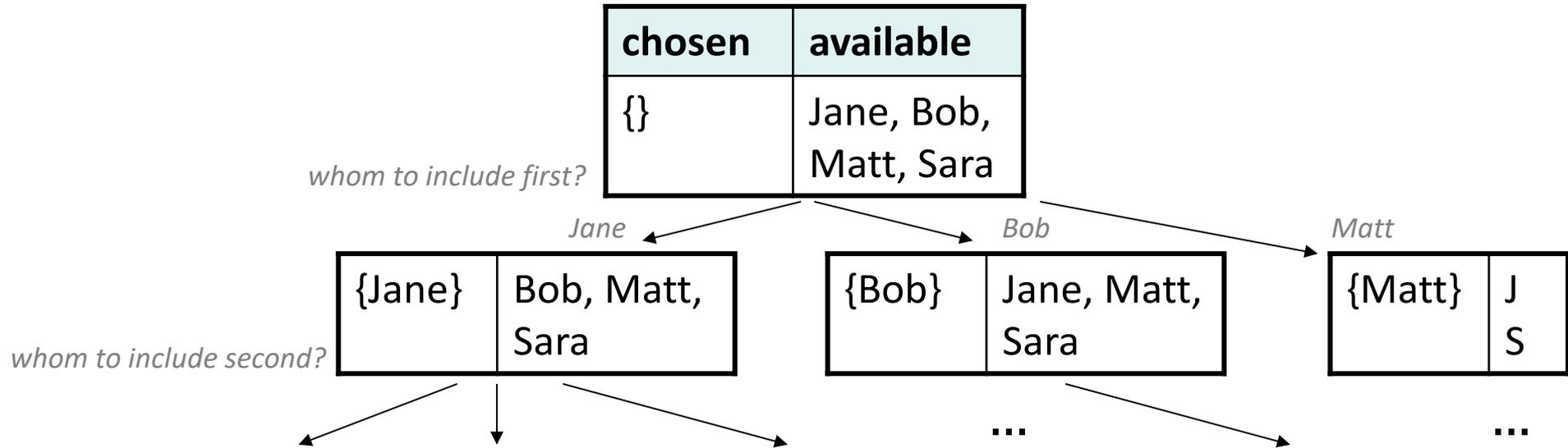
- Write a function **sublists** that finds every possible sub-list of a given vector. A sub-list of a vector V contains ≥ 0 of V 's elements.
 - Example: if V is {Jane, Bob, Matt, Sara}, then the call of **sublists**(V); prints:

```
{Jane, Bob, Matt, Sara}      {Bob, Matt, Sara}
{Jane, Bob, Matt}           {Bob, Matt}
{Jane, Bob, Sara}           {Bob, Sara}
{Jane, Bob}                  {Bob}
{Jane, Matt, Sara}          {Matt, Sara}
{Jane, Matt}                 {Matt}
{Jane, Sara}                 {Sara}
{Jane}                        {}
```

Decision tree?



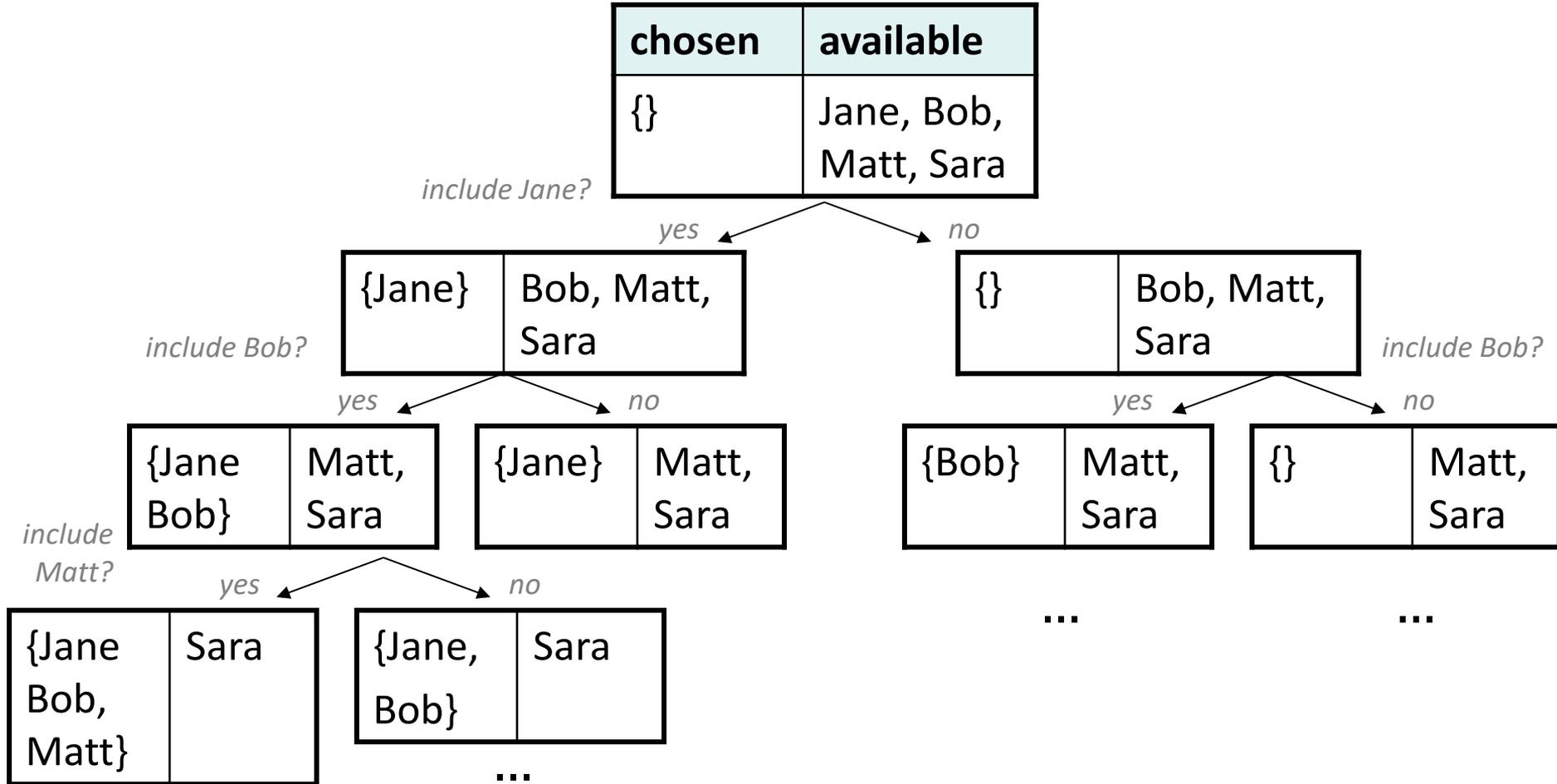
Wrong decision tree



Q: Why isn't this the right decision tree for this problem?

- A.** It does not actually end up finding every possible sublist.
- B.** It does find all sublists, but it finds them in the wrong order.
- C.** It does find all sublists, but it is inefficient.
- D.** None of the above

Better decision tree



- Each decision is: "Include Jane or not?" ... "Include Bob or not?" ...
 - The **order** of people chosen does not matter; only the **membership**.

Mental Model

- **Choose:** What decisions do we have to make? What are our choices?
- **Explore:** How should we modify our parameters after making a choice?
- **Un-Choose:** How do we revert our choice?
- **Base Case:** What should we do when we are out of decisions to make?

Mental Model

- **Choose:** What decisions do we have to make? What are our choices?
 - *Whether to include a person or not*
- **Explore:** How should we modify our parameters after making a choice?
 - *Build up a vector containing people chosen so far*
- **Un-Choose:** How do we revert our choice?
 - *Remove the person previously inserted into the vector*
- **Base Case:** What should we do when we are out of decisions to make?
 - *Print the result vector*

sublists solution

```
void sublists(Vector<string>& v) {
    Vector<string> chosen;
    sublistsHelper(v, 0, chosen);
}

void sublistsHelper(Vector<string>& v, int i,
                   Vector<string>& chosen) {
    if (i >= v.size()) {
        cout << chosen << endl;    // base case; nothing to choose
    } else {
        // there are two choices to explore:
        // the subset without i'th element, and the one with it

        sublistsHelper(v, i+1, chosen);    // choose/explore (without)

        chosen.add(v[i]);
        sublistsHelper(v, i+1, chosen);    // choose/explore (with)

        chosen.remove(chosen.size() - 1);    // "undo" our choice
    }
}
```

Announcements

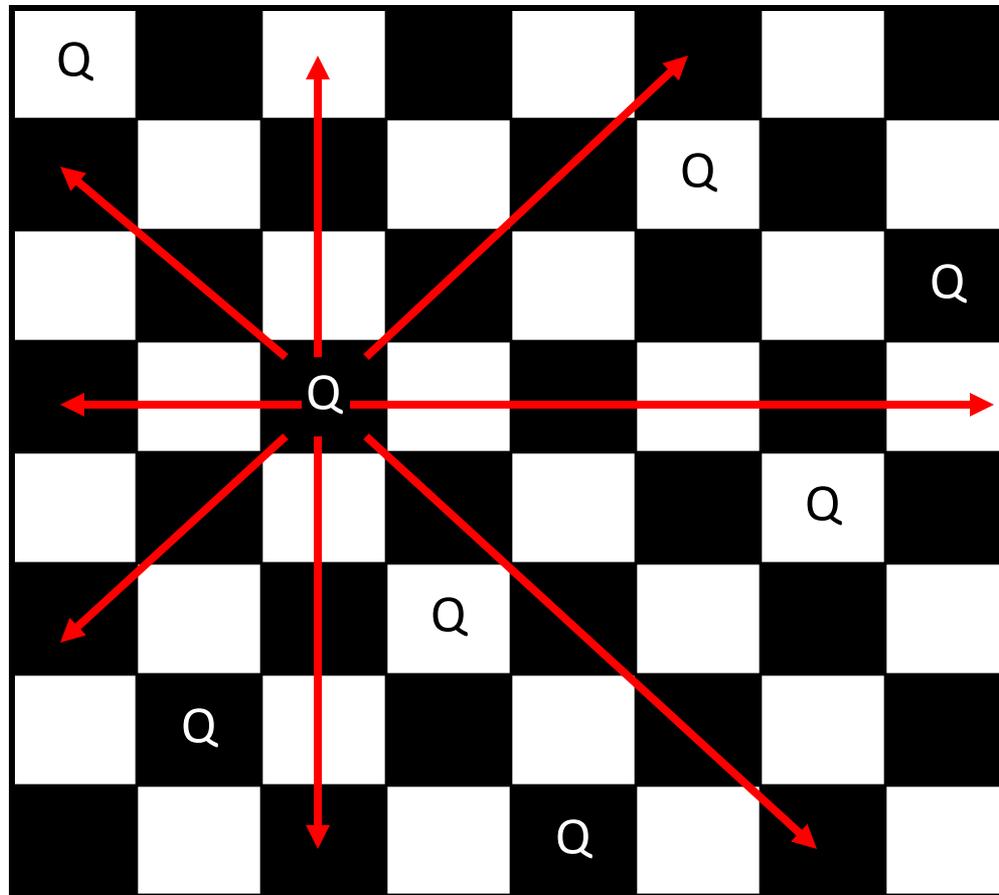
- Assignment 4 goes out tonight. You should receive Assn2 feedback by end of the day
 - A small part of Assn. 4 uses structs which are covered tomorrow.
- Exam logistics
 - Midterm review session in class on 7/23
 - Midterm is on Wednesday, July 24, from 7:00-9:00PM
 - Midterm info (list of topics covered and study tips) online:
<https://web.stanford.edu/class/cs106b/exams/midterm.html>
 - **Highly Recommended:** Complete assignment 4 (or parts of it) before the midterm – backtracking will be tested. Assignment 4 will not be due until July 25th though
 - Lectures 14 and 15 are NOT included on the midterm

Announcements

- Practice midterm is released. You need BlueBook to use it.
 - Download the file as a .json file
 - Open BlueBook, follow instructions
- Save your answers in a separate document to compare to the practice midterm answers.
 - Practice midterm answers will be released in a few days

The "8 Queens" problem

- Consider the problem of trying to place 8 queens on a chess board such that no queen can attack another queen.



Exercise

- Suppose we have a Board class with the following methods:

Member	Description
<code>Board b(size);</code>	construct empty board
<code>b.isSafe(row, column)</code>	true if a queen could be safely placed here (0-based)
<code>b.isValid()</code>	true if all current queens are safe
<code>b.place(row, column);</code>	place queen here
<code>b.remove(row, column);</code>	remove queen from here
<code>cout << b << endl;</code> <code>or b.toString()</code>	print/return a text display of the board state

- Write a function **solveQueens** that accepts a Board as a parameter and tries to place 8 queens on it safely.
 - Your method should return a board with the queens placed if it's possible.

Mental Model

- **Choose:** What decisions do we have to make? What are our choices?
- **Explore:** How should we modify our parameters after making a choice?
- **Un-Choose:** How do we revert our choice?
- **Base Case:** What should we do when we are out of decisions to make?

Naive algorithm

- for (each board square):
 - Place a queen there.
 - Try to place the rest of the queens.
 - Un-place the queen.

Q: How large is the solution space for this algorithm?

- A.** 64 choices
- B.** $64 * 8$
- C.** 64^8
- D.** $64 * 63 * 62 * 61 * 60 * 59 * 58 * 57$
- E.** none of the above

	0	1	2	3	4	5	6	7
0	Q
1
2
3
4
5
6
7

Better algorithm idea

- Observation: In a working solution, exactly 1 queen must appear in each row and in each column.

- Redefine a "choice" to be valid placement of a queen in a particular column.

- How large is the solution space now?

- $8 * 8 * 8 * \dots$

	0	1	2	3	4	5	6	7
0	Q					
1						
2		Q	...					
3			...					
4			Q					
5								
6								
7								

Mental Model

- **Choose:** What decisions do we have to make? What are our choices?
 - *Where in a column to place a queen*
- **Explore:** How should we modify our parameters after making a choice?
 - *Place the queen on the board, move on to the next column*
- **Un-Choose:** How do we revert our choice?
 - *Remove the queen that we placed previously*
- **Base Case:** What should we do when we are out of decisions to make?
 - *Return true*

8 Queens solution

```
// Recursively searches for a solutions to N queens
// on this board, starting with the given column.
// PRE: queens have been safely placed in columns 0 to (col-1)
bool solveHelper(Board& board, int col) {
    if (!board.isValid()) {
        return false;
    } else if (col >= board.size()) {
        return true; // base case: all columns placed
    } else {
        // recursive case: try to place a queen in this column
        for (int row = 0; row < board.size(); row++) {
            board.place(row, col); // choose
            if (solveHelper(board, col + 1)) { // explore
                return true;
            }
            board.remove(row, col); // un-choose
        }
    }
    return false;
}

bool solveQueens(Board& board) {
    solveHelper(board, 0);
}
```

Exercise: Dominoes

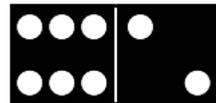
- Dominoes uses black tiles, each having 2 numbers of dots from 0-6. Players line up tiles to match dots.



- Given a class `Domino` with the following members:

```
int first()           // first dots value from 0-6
int second()          // second dots value from 0-6
void flip()           // inverts 1st/2nd
bool contains(int n)  // true if 1st and/or 2nd == n
string toString()     // e.g. "(3|5)"
```

- Write a function **`chainExists`** that takes a `Vector` of dominoes and a starting/ending dot value, and returns whether the dominoes can be made into a chain that starts/ends with those values.

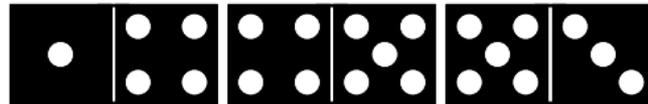


Domino chains

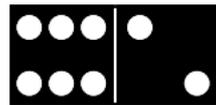
- Suppose we have the following dominoes:



- We can link them into a chain from 1 to 3 as follows:
 - Notice that the 3|5 domino had to be flipped.

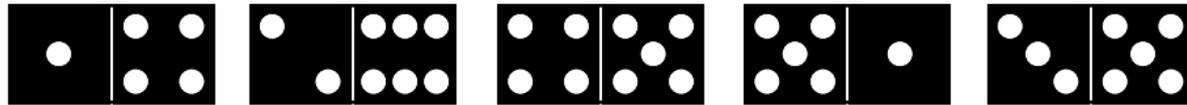


- We can "link" one domino into a "chain" from 6 to 2 as follows:



Enumerating choices

- If we have these dominoes, and we want a chain from 1 to 3:



Q: What are the "choices" your code should explore?

- A.** The numbers 0-6 that can appear on a domino.
- B.** The set of all of the dominoes above.
- C.** The set of dominoes above whose first number is 1.
- D.** The set of dominoes above whose second number is 3.
- E.** The set of dominoes above whose first or second number is 1.

hasChain pseudocode

```
function chainExists(dominoes, start, end):  
  if dominoes is empty: nothing to do.  
  if start == end:  
    if any domino in dominoes contains start, return true.  
  else:  
    for each domino d in dominoes:  
      if d contains start:  
        choose d.  
        if chainExists(dominoes): // explore remaining dominoes.  
          return true.  
        un-choose d.  
  
  return false. // no chain found
```

hasChain solution

```
bool chainExists(Vector<Domino>& dominoes, int start, int end) {
    if (start == end) { // base case
        for (Domino d : dominoes) {
            if (d.contains(start)) { return true; }
        }
        return false;
    } else {
        for (int i = 0; i < dominoes.size(); i++) {
            Domino d = dominoes[i];
            if (d.second() == start) {
                d.flip();
            }
            if (d.first() == start) {
                dominoes.remove(i); // choose
                if (d.second() == end || // explore
                    chainExists(dominoes, d.second(), end)) {
                    dominoes.insert(i, d);
                    return true;
                }
                dominoes.insert(i, d); // un-choose
            }
        }
        return false;
    }
}
```