CS 106X

Lecture 21: Hashing

Wednesday, March 1, 2017

Programming Abstractions (Accelerated)
Winter 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:
Programming Abstractions in C++, Chapter 15

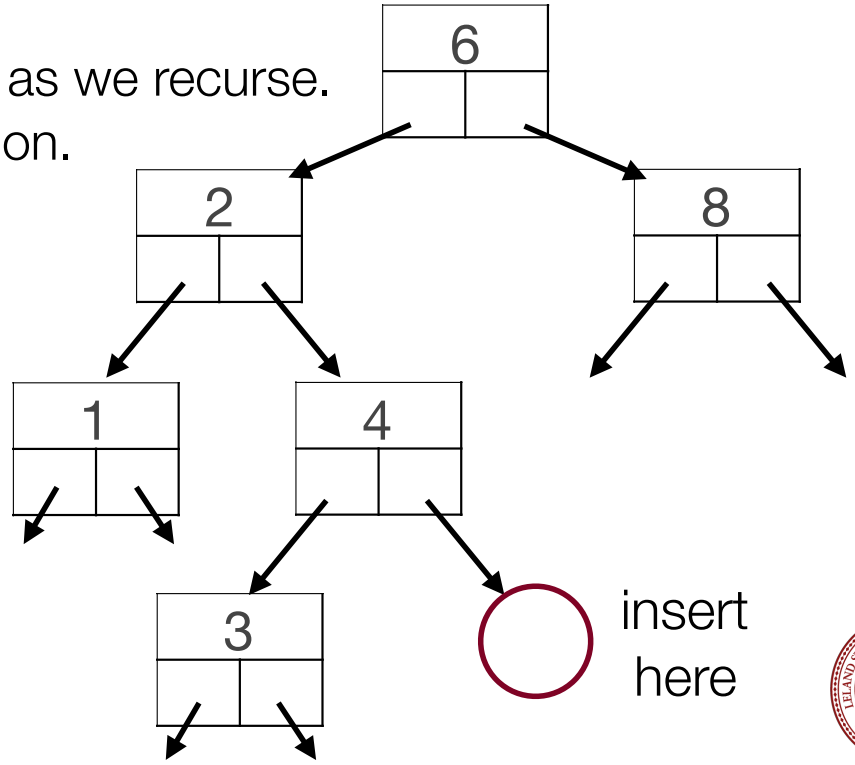
Today's Topics

- Logistics
  - Regrade requests due Friday
  - Meeting sign-up with Chris:
    - http://stanford.edu/~cgregg/cgi-bin/inperson/index.cgi
- Binary Search Trees: using references to pointers
- Hashing

Using References to Pointers

- To insert into a binary search tree, we must update the left or right pointer of a node when we find the position where the new node must go.
- In principle, this means that we could either
  1. Perform arms-length recursion to determine if the child in the direction we will insert is NULL, or
  2. Pass a *reference to a* pointer to the parent as we recurse.
- The second choice above is the cleaner solution.

```
set.insert(5)
```



Using References to Pointers

```
void StringSet::add(string s, Node* &node) {
    if (node == NULL) {
        node = new Node(s);
        count++;
    } else if (node->str > s) {
        add(s, node->left);
    } else if (node->str < s) {
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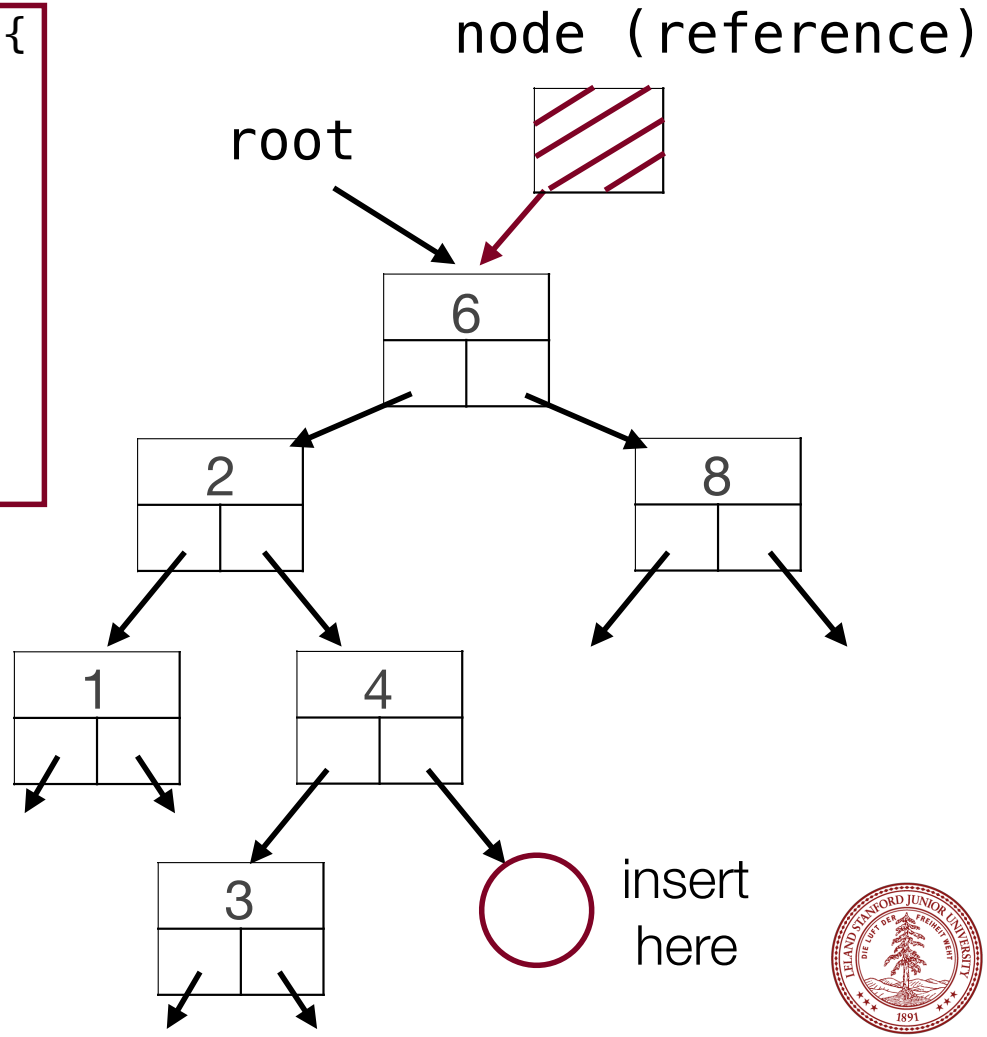

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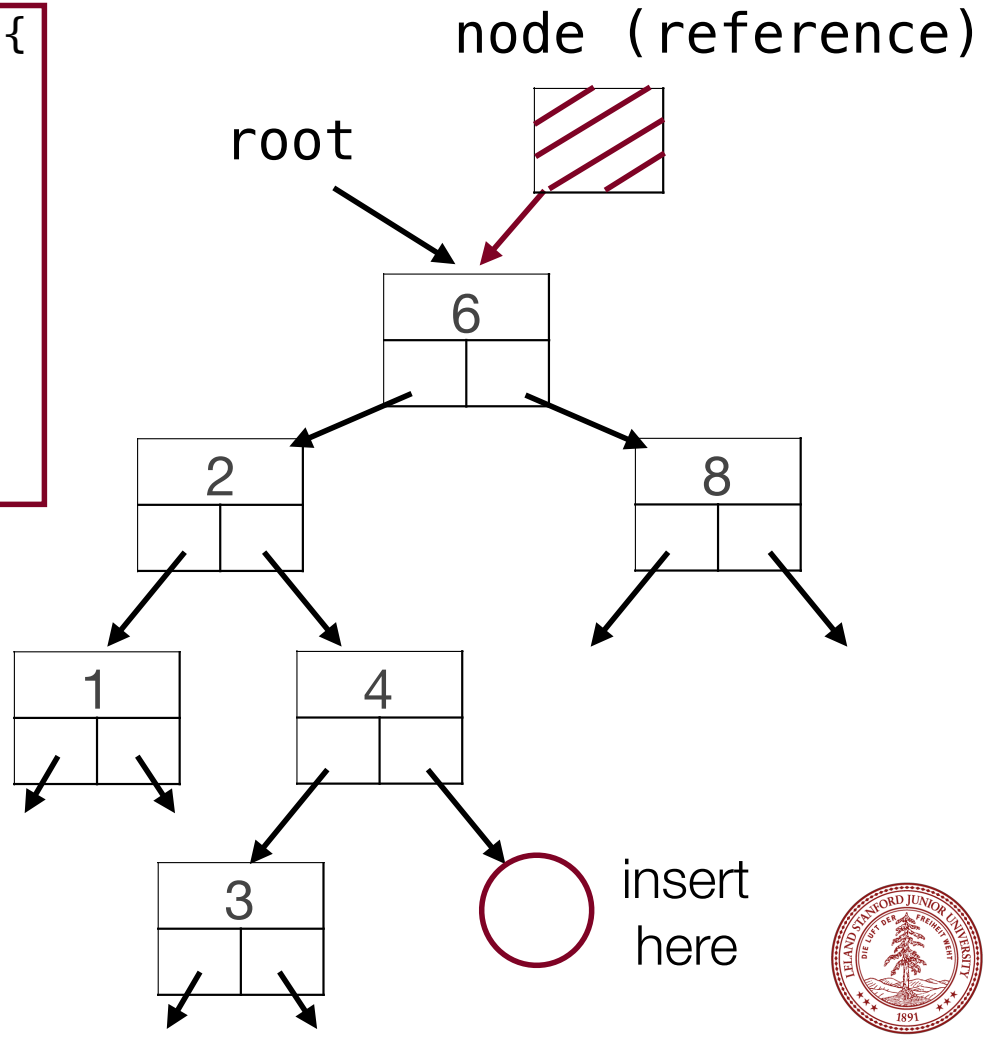

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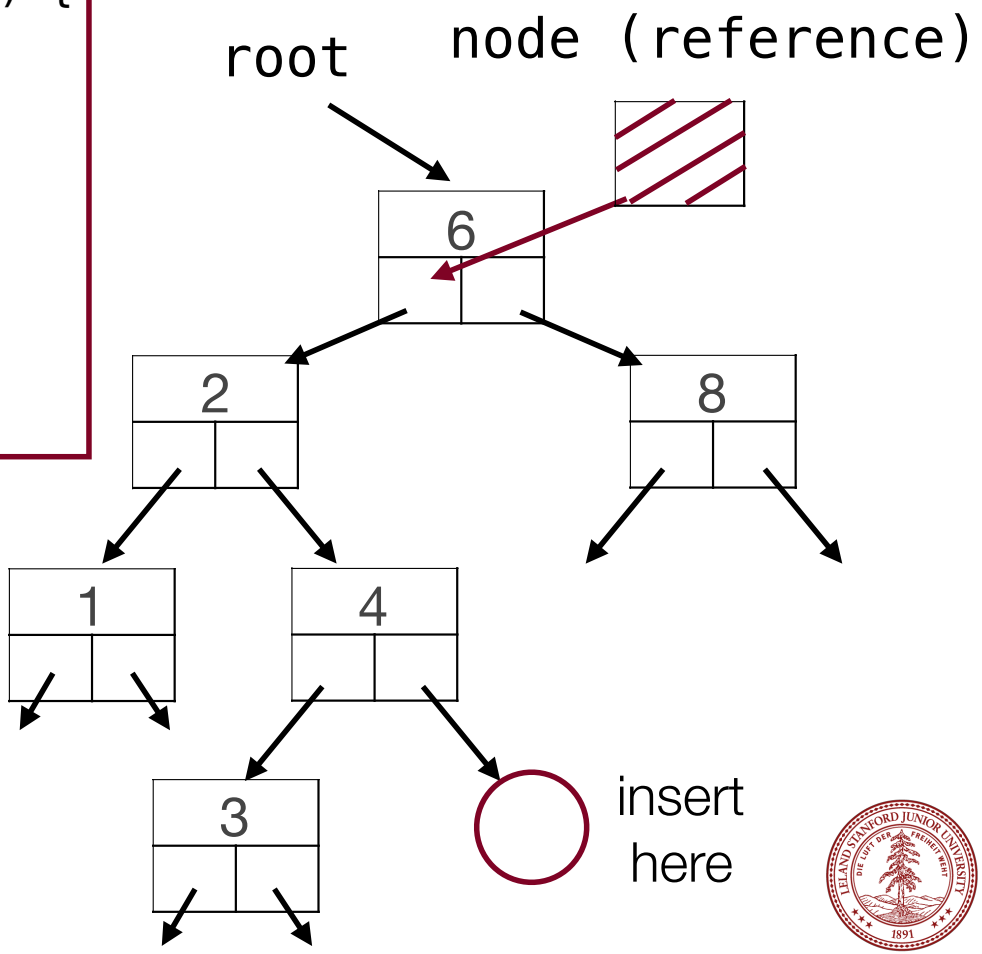

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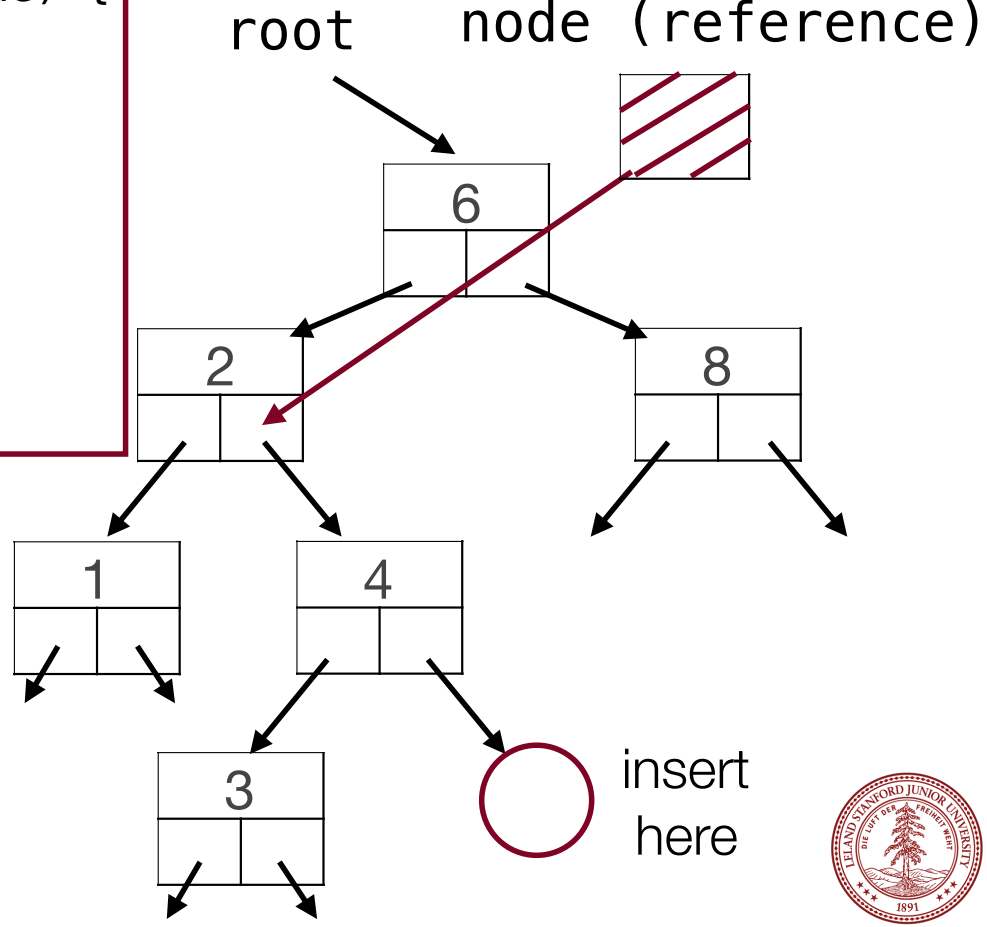

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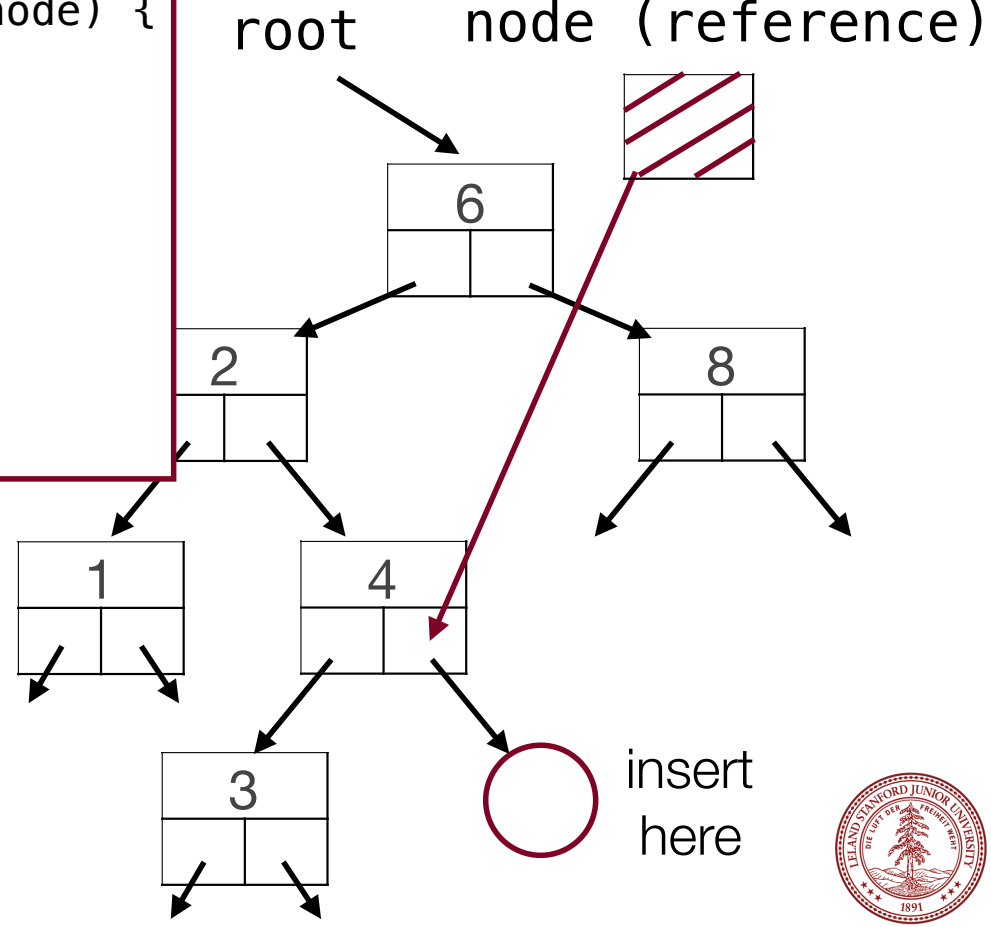

Using References to Pointers

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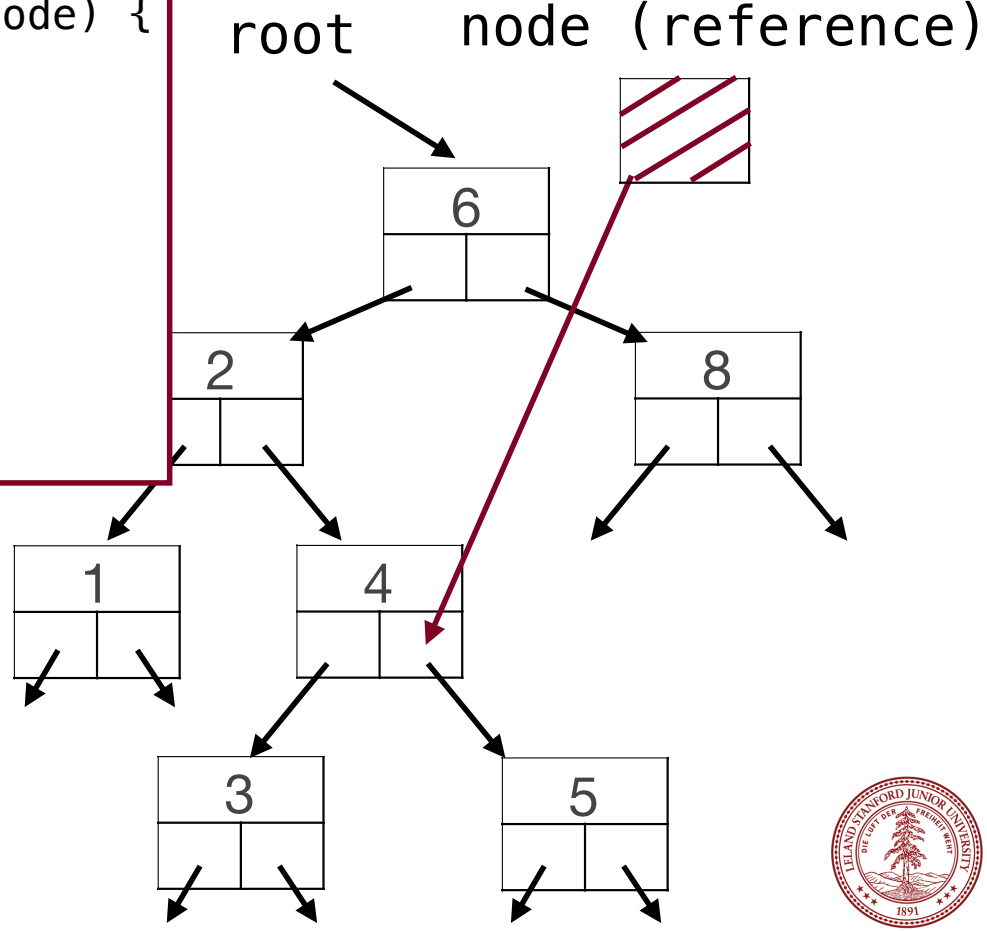


Hashing!

First, a cool program, written by Chris Piech

Hashing!

First, a cool program, written by Chris Piech

We'll see how this was written by the end of lecture!

Hashing!

What we *want* is a way to implement find(), insert(), and remove() in O(1) time (the "holy grail").

There is a completely different method than what we have discussed before for storing key/value pairs that can actually do this! The method is called *hashing*, and to perform hashing, you use a *hash function.*

The values returned by a hash function are called *hash values, hash codes,* or (simply), *hashes*.

Hashing!

Suppose you have:

2-letter words and their definitions

KEYS

VALUES

A word is the *key* that addresses the *value* (definition)
We want to store these in an *efficient* data structure.

You could do this in a set, but a set is O(log n) -- can we do better? Yes!

Hashing!

26 x 26 = 676 words

We want to insert a definition into the dictionary, which is going to be comprised of an array (or "buckets"):

function `hashCode()` maps each two-letter word (key) to 0..675

Hashing!

Possible definition for a Hash Function: Any algorithm that maps data to a number, and that is *deterministic.*

Example:
ox —> take each character, and treat it as a base-26 number. E.g., each character is assigned a number from 0-26:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Then, to get the hash for “ox”: 26*14 + 1*23 = 387
“at” : 26 * 0 + 1 * 19 = 19

Hashing!

Our current definition for a Hash Function: Any algorithm that maps data to a number, and that is *deterministic.*

Why does this help us with our goal for O(1) access?

If we can map two-letter words deterministically, then we can put their definitions into an array at that location, and have O(1) access!

e.g.,

Hashing!

Our current definition for a Hash Function: Any algorithm that maps data to a number, and that is *deterministic.*

The idea is that we can now hash a key, put the value into an array (O(1)), find the value with the hash (O(1)), and delete the value from the array (O(1)).

If the hash function is fast, then all the operations we want are also fast.

Let's write some code!

Hashing!

```
const int LETTERS = 26;
const int WORDS = LETTERS * LETTERS;

class Word {
public:
    // limited to 2-character words
    int hashCode() {
        return LETTERS * (word[0] - 'a')
        + (word[1] - 'a');
    }
    Word(string w) { // constructor
        word = w;
    }
private:
    string word;
};
```

```
class WordDictionary {
public:
    WordDictionary(){ // constructor
        defTable = new string[WORDS];
    }
    ~WordDictionary() { // destructor
        delete [] defTable;
    }
    void insert(Word w, string d) {
        defTable[w.hashCode()] = d;
    }
    string find(Word w){
        return defTable[w.hashCode()];
    }
private:
    string *defTable;
};
```

Hashing!

We've succeeded!
We have figured out a way to store key-value pairs with perfect O(1) access.

But wait…

Hashing!

What if we want to store *all* English words?

Word	Letters	Characteristics
Methionylthreonylthreonylglutaminylarginyl...isoleucine	189,819	Chemical name, disputed...
Methionylglutaminylarginyltyrosylglutamyl...serine	1,909	Longest published word
Lopadotemachoselachogaleokranioleipsano...pterygon	183	Longest word coined by a major author
Pneumonoultramicroscopicsilicovolcanoconiosis	45	Longest word in a major dictionary
Supercalifragilisticexpialidocious	34	Famous for being created for Mary Poppins
Pseudopseudohypoparathyroidism	30	Longest non-coined word in a major dictionary
Floccinaucinihilipilification	29	Longest unchallenged nontechnical word
Antidisestablishmentarianism	28	Longest non-coined and nontechnical word
Honorificabilitudinitatibus	27	Longest word in Shakespeare's works; longest word in the English language featuring alternating consonants and vowels.

Hashing!

What if we want to store *all* English words?

"The summit where Tamatea, the man with the big knees, the climber of mountains, the land-swallower who travelled about, played his nose flute to his loved one"

Hashing!

Supercalifragilisticexpialidocious

Would need an array with 26^{34} buckets...too big!

Doesn't even count uppercase!

English has ~700,000 words

p.s. $26^{34} \cong 10^{48}$, which is about the number of iron *atoms* in the Earth.

Hashing!

We need to conserve space. For a 700,000 word dictionary, we might only want to use a 800,000 element array (or much smaller, if we wanted only a subset of values)

Better definition for a Hash Function: Any algorithm that maps data to a number, that is *deterministic,* and that maps to a **fixed number of locations**.

But, remember, we need to store arbitrary words (i.e., we could add any word, of any length with the characters in our dictionary!)

Hashing!

Better Hash Function definition: Any algorithm that maps data to a number, that is *deterministic,* and that maps to a **fixed number of locations**.

A good method for mapping to a fixed number of locations is to use the modulus operator:

```
h(hashCode) = hashCode mod N
```

Where *N* is the length of the array we want to use.
We say we have "compressed" the hash.

Hash Tables

Using the compression function to place keys into a fixed array, we have created a *hash table*. A hash table maps a huge set of possible keys into N buckets by applying a *compression function* to each hash code.

```
h(hashCode) = hashCode mod N
```

Hash Codes and Compression Functions

Hash codes *must* be **deterministic**.
Hash codes *should* be **fast** and **distributed**

```
                                                compression
                                                function
key ---------->  hash code  ------------>  [0, N-1]
```

Birthday Hashing / Compressing: Hashing you!

a. decade of your birth year: (year / 10) % 10
b. last digit of your birth year: year % 10
c. last digit of your birth month: month % 10
d. last digit of your birth day: day % 10

Keys into Buckets

There were some problems with our birthday hashes. What was the biggest problem?

We need to consider how big our hash table array is, relative to the number of keys we want to store.

n : number of keys (words) stored

N : number of buckets in a table

a bit bigger than n

but much smaller than the number of possible keys

Keys into Buckets

As we saw in the birthday hashes, it is the case that we can map two keys to the same bucket. E.g.,

```
h(hashCode1) = h(hashCode2)
```

Keys into Buckets

Example:

- key space: integers
- table size: 10
- hashCode(K) = K mod 10
- Insert: 7, 18, 41, 34
- How do we **find** them?
  - Can we perform findMax()
  - or findMin()?
- What if we now try to add 54?

0	
1	41
2	
3	
4	34 54
5	
6	
7	7
8	18
9	

Problem?

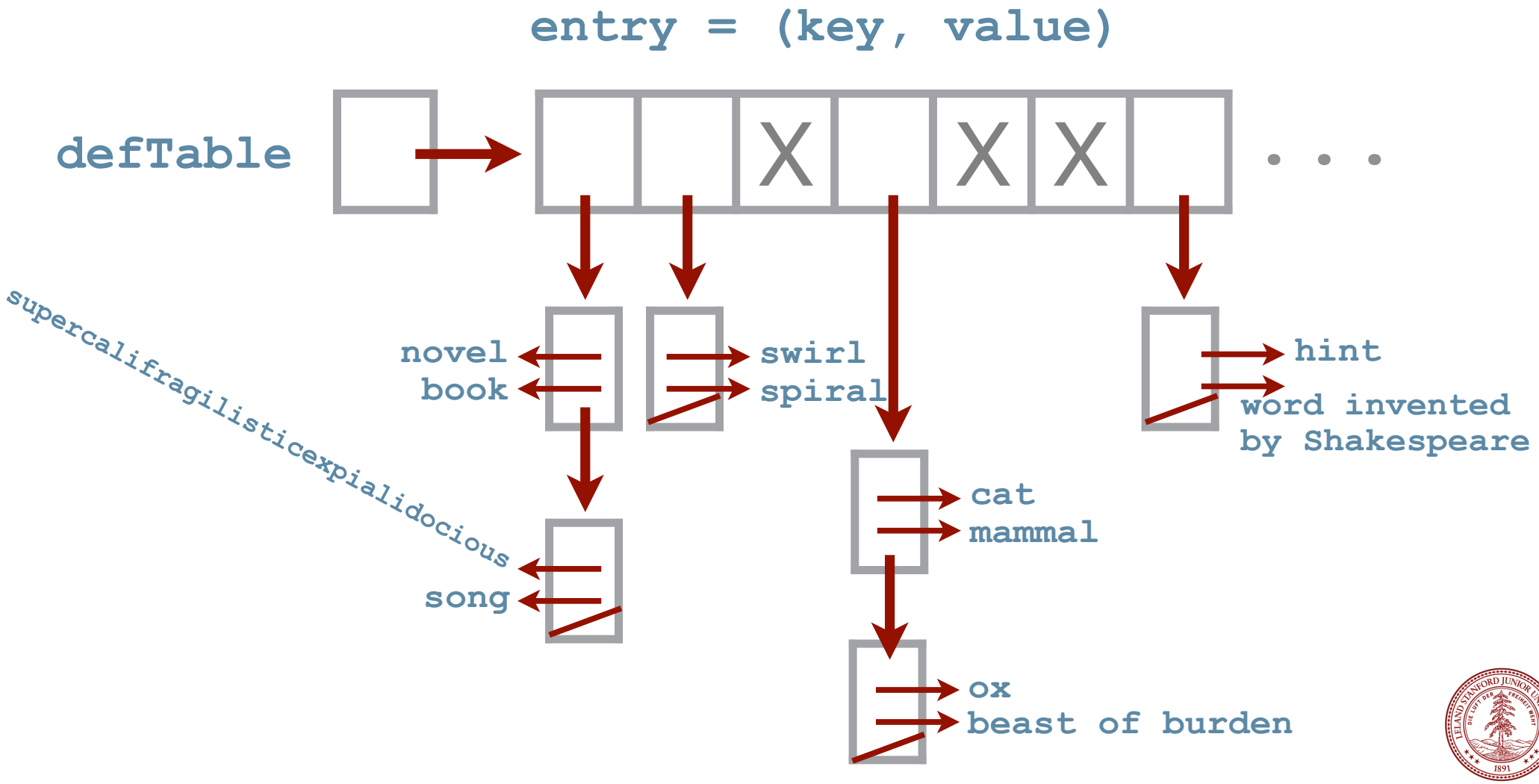
Handling Collisions

Chaining: each bucket references a linked list of entries, called a *chain*.

Q: How do we know which definition corresponds to which word?

A: Store each key in the table with its definition

Chaining


entry = (key, value)
defTable
X
X
X
. . .
novel
book
swirl
spiral
hint
word invented
by Shakespeare
supercalifragilisticexpialidocious
song
cat
mammal
ox
beast of burden

Load Factor

The *Load Factor* of a hash table:

$$\frac{n \text{ : number of keys (words) stored}}{N \text{ : number of buckets in a table}} = \frac{n}{N}$$

IF:

- The load factor stays low, and
- The hash code and compression function are "good," and
- No duplicate keys, THEN

Each operation takes O(1) time!

Load Factor

The *Load Factor* of a hash table:

$$\frac{n \text{ : number of keys (words) stored}}{N \text{ : number of buckets in a table}} = \frac{n}{N}$$

However, IF:

•The load factor gets big (n >> N), THEN

Each operation takes Θ(n) time. ☹

If your load factor gets to big, move your hash table to a bigger array (the penalty is worth it).

Hash Codes and Compression Functions

Hash codes *must* be **deterministic**.
Hash codes *should* be **fast** and **distributed**

Ideal Hash: Map each key to a random bucket.
Is an ideal hash collision-free?
What does your intuition say about 2500 keys in 1,000,000 buckets?

Even an ideal hash will not remove all collisions:

> "if 2,500 keys are hashed into a million buckets, even with a perfectly uniform random distribution, according to the birthday problem there is a 95% chance of at least two of the keys being hashed to the same slot." [Wikipedia, *Hash Table*]

Hash Codes and Compression Functions

What makes a good compression function?

Bad compression function:

- Suppose keys are ints
- `hashCode(i) = i` (hashCode is itself)
- Compression function `h(hashCode) = hashCode mod N`
- N = 10,000 buckets

Suppose keys are divisible by 4.
`h()` is divisible by 4 too!
Bad news: 3/4 of the buckets are never used!

The fix: make N prime. Now, once you take mod N, the numbers are not divisible by any number in particular.

Hash Codes and Compression Functions

A better compression function:

```
h(hashCode) = ((a * hashCode + b) mod p) mod N
```

Scrambles bits

a, b, p: positive integers
p is a large prime
p >> N

Now, N (buckets) doesn't need to be prime.

A Good Hashcode for Strings

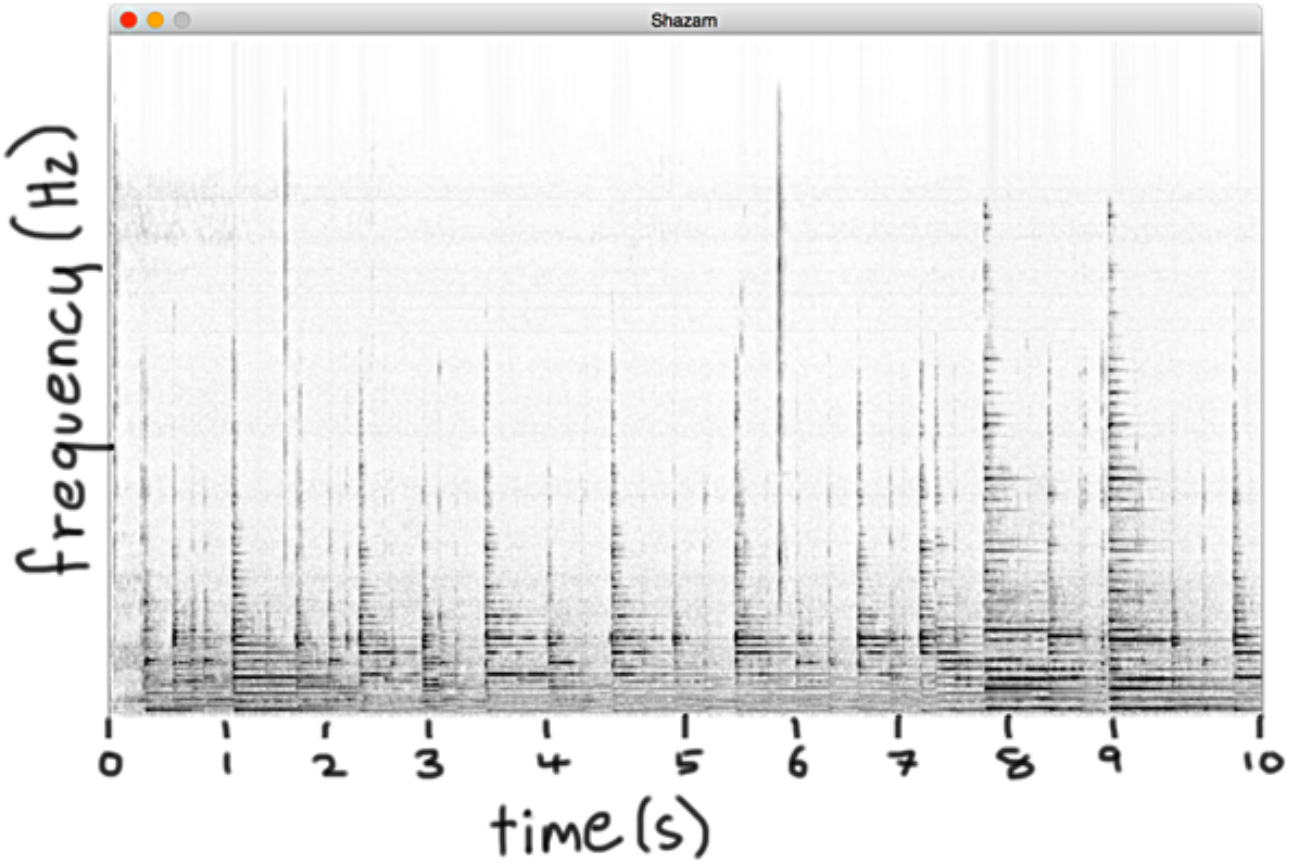
```
static int P = 16908799;
int hashCode(String key) {
    int hashVal = 0;
    for (int i=0; i<key.length(); i++) {
       hashVal = (127 * hashVal + key.charAt(i)) % P;
    }
    return hashVal;
}
```

Bad Hashcodes for Strings

1) Sum ASCII values of characters.
   - rarely exceeds 500 for most words
   - bunched up into 500 buckets
   - anagrams always collide!
2) Choose first three letters in a word, with 26^3 buckets.
   - lots of words that begin with the same three letters (e.g., “pre”) but many that don’t (e.g., “xgs”)
3) Suppose we change `P` in our previous hashCode() to 127.
   - bad because: (127 * hashVal) % 127 = 0.

Back to Hasham!

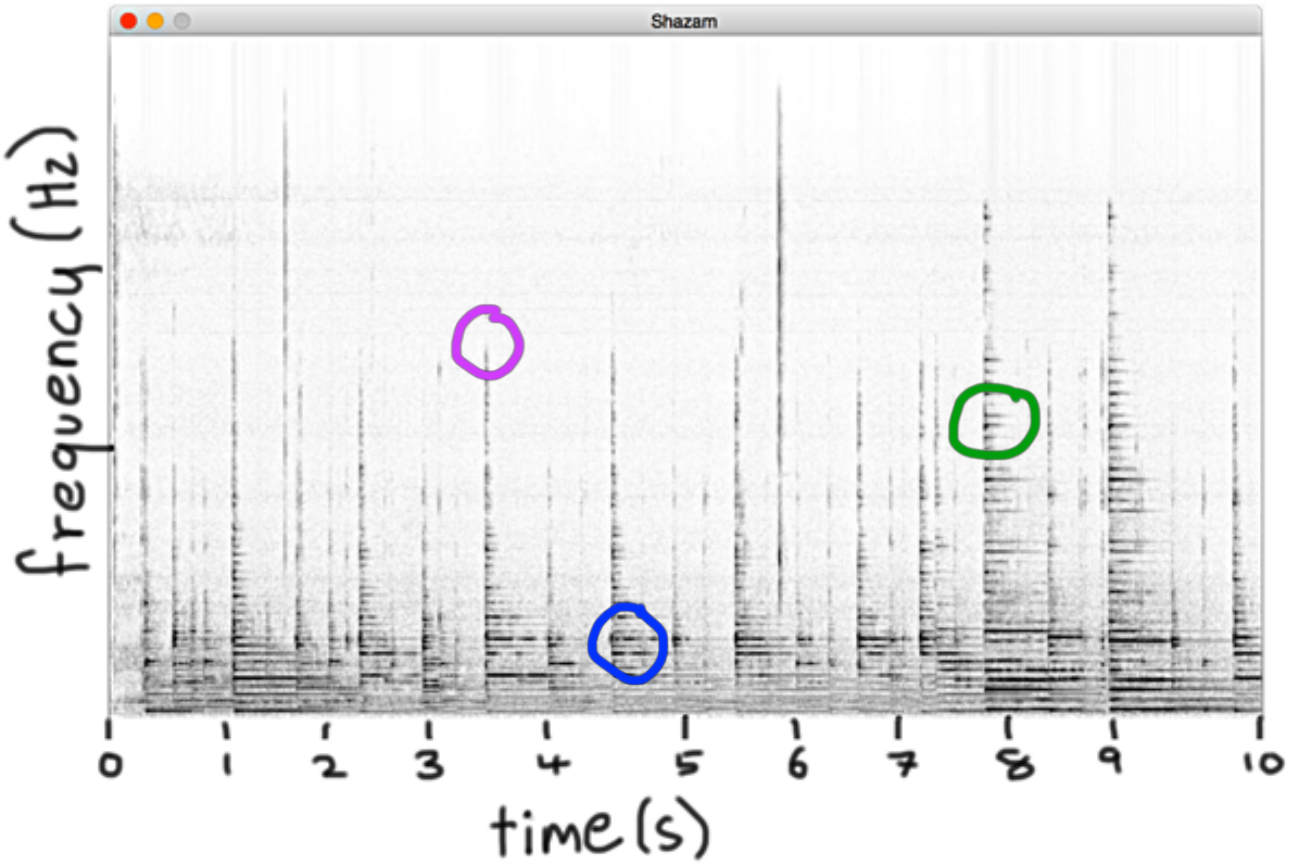
Hasham requires a large search space



Does anyone recognize this song?

Wang, A. An Industrial-Strength Audio Search Algorithm

Hasham requires a large search space



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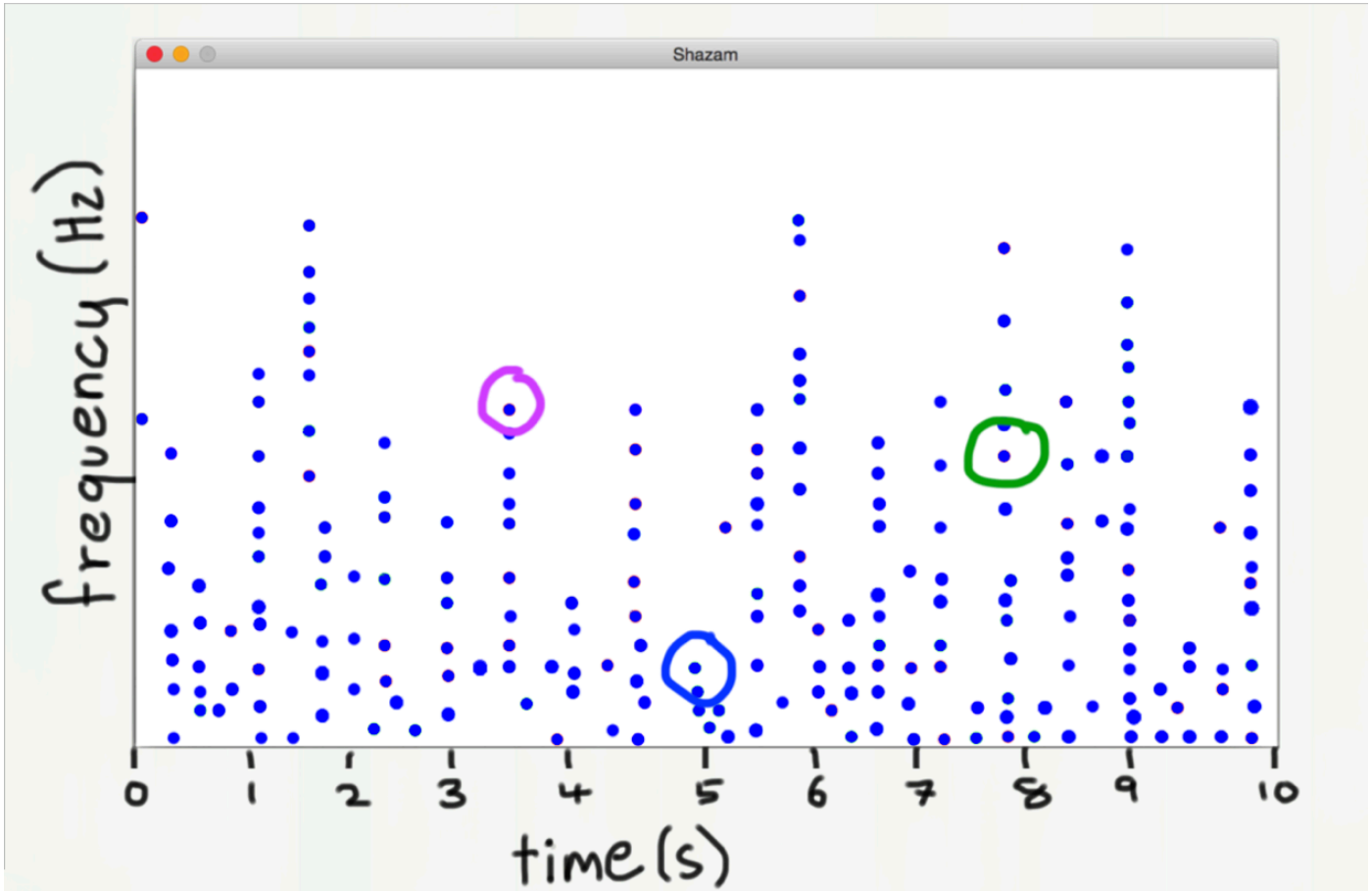
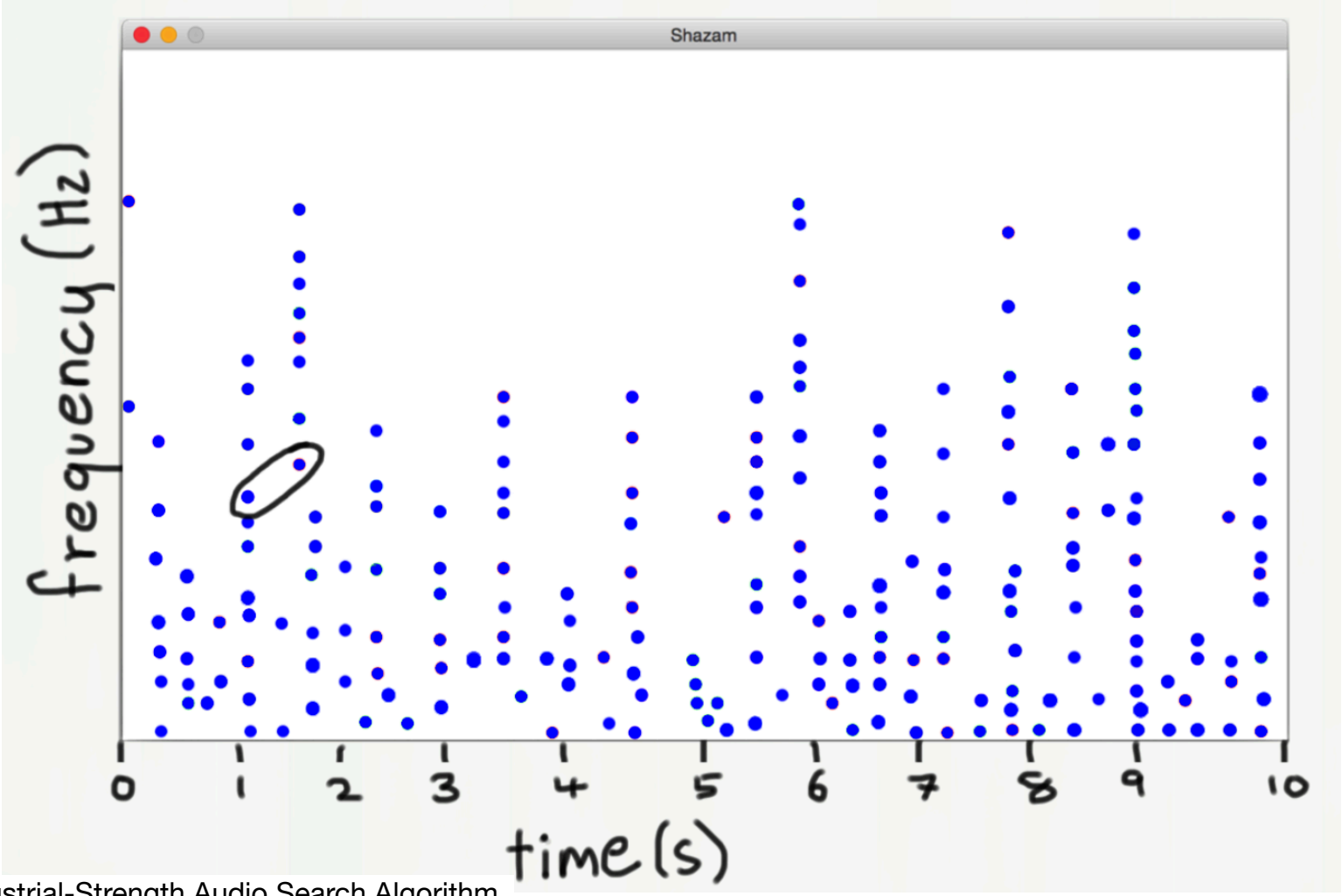


Wang, A. An Industrial-Strength Audio Search Algorithm

Hasham requires a large search space

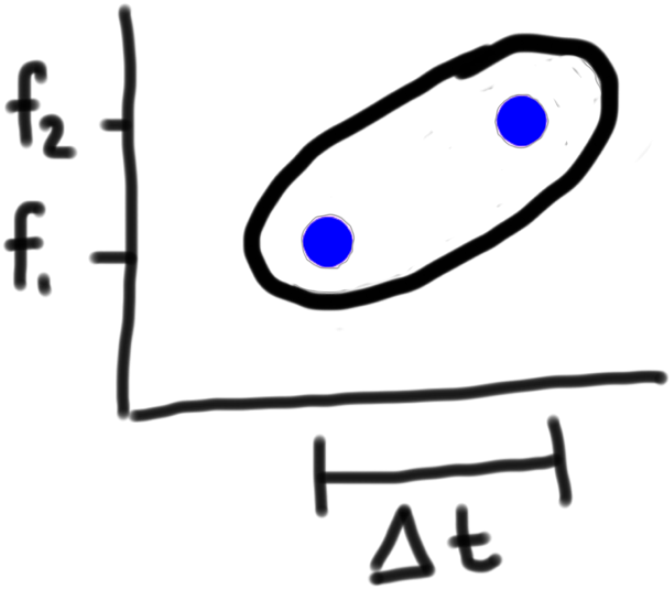
Should we hash the whole thing?
No.

Find Pairs of Notes



Wang, A. An Industrial-Strength Audio Search Algorithm

Note Pairs



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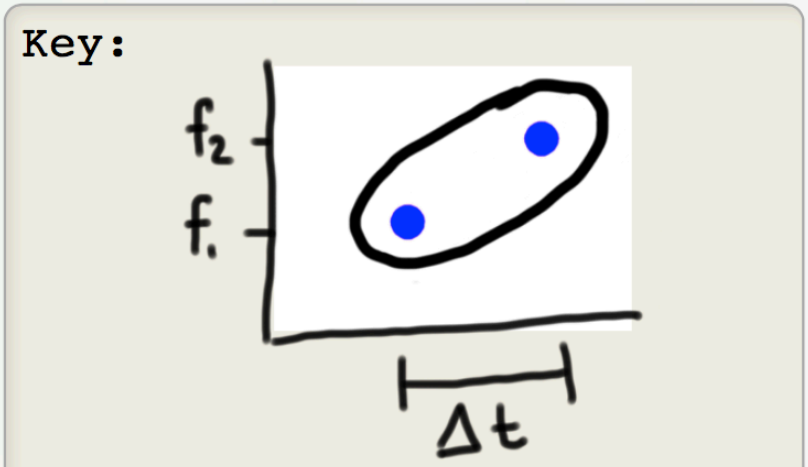

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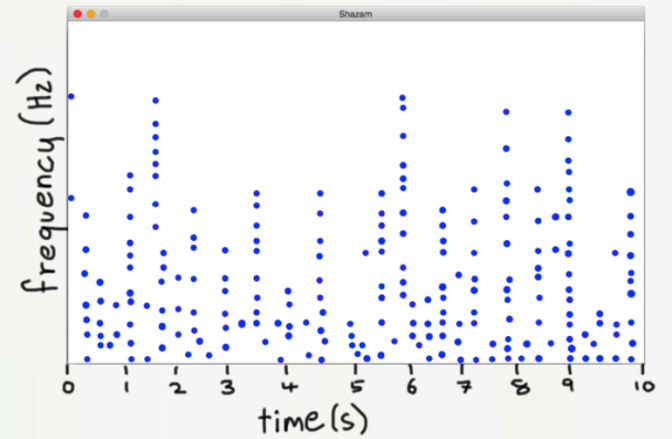
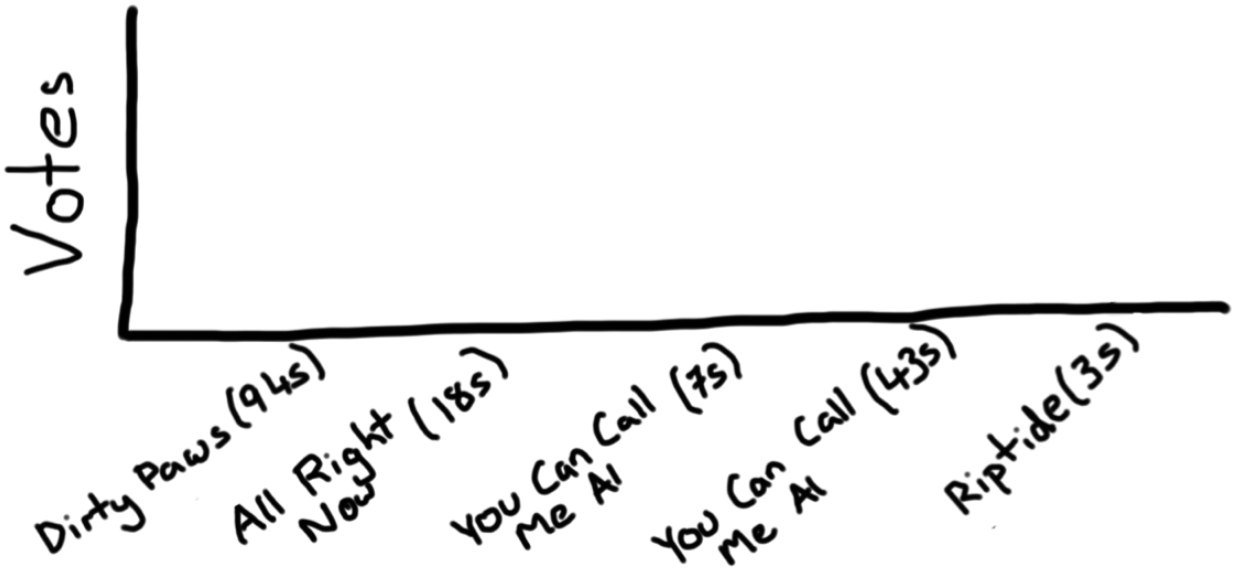
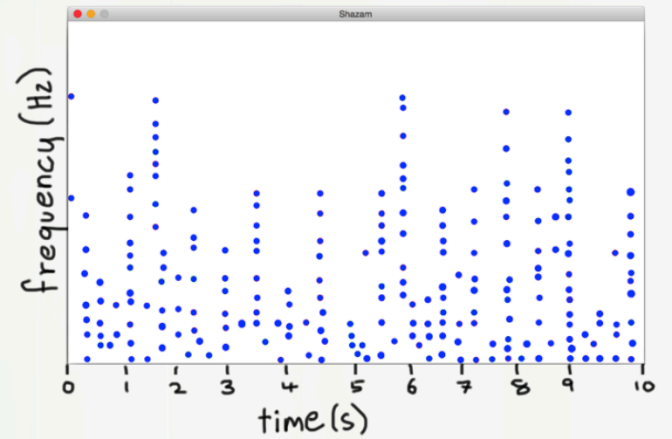

Shazam
frequency (Hz)
0
1
2
3
4
5
6
7
8
9
10
time (s)


Votes
Dirty Paws (94s)
All Right Now (185)
You Can Call Me Al (7s)
You Can Call Me Al (43s)
Riptide (35)

Note Pairs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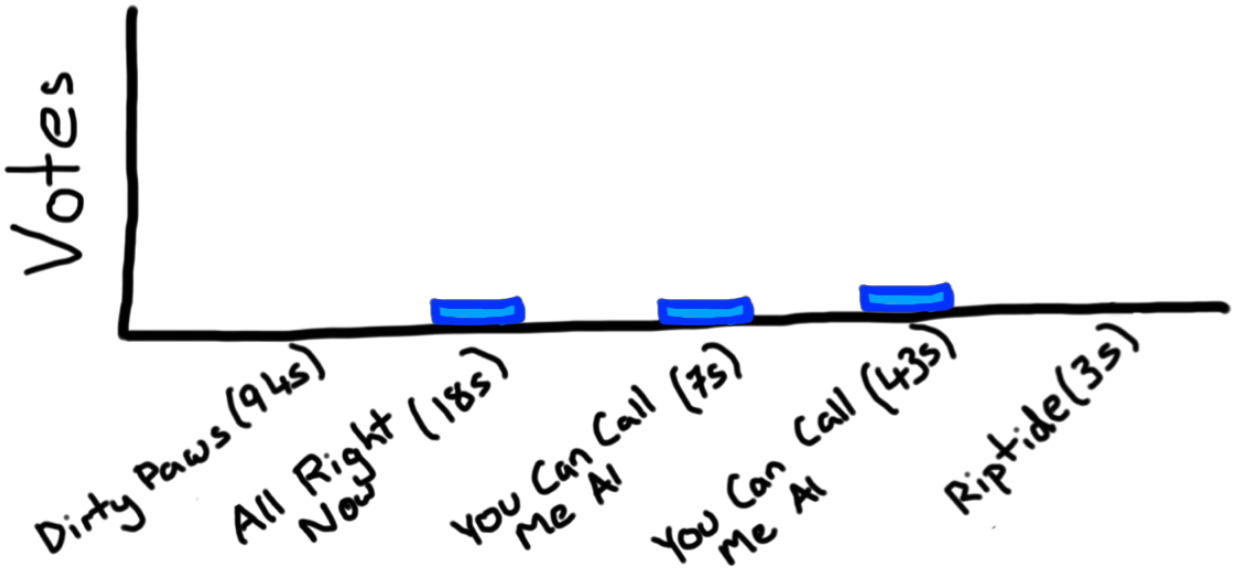
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You Can Call Me Al (435)
Riptide (35)

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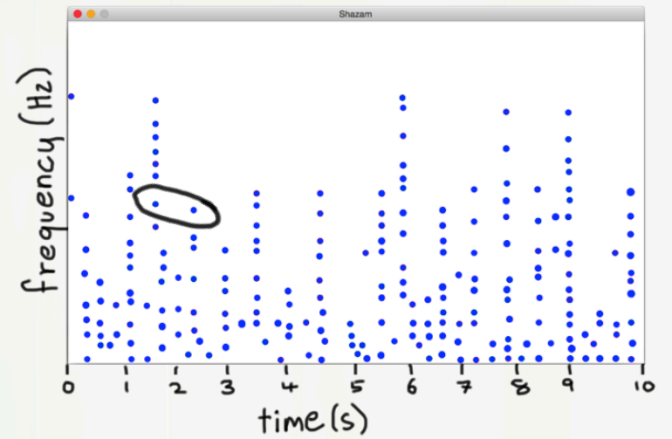
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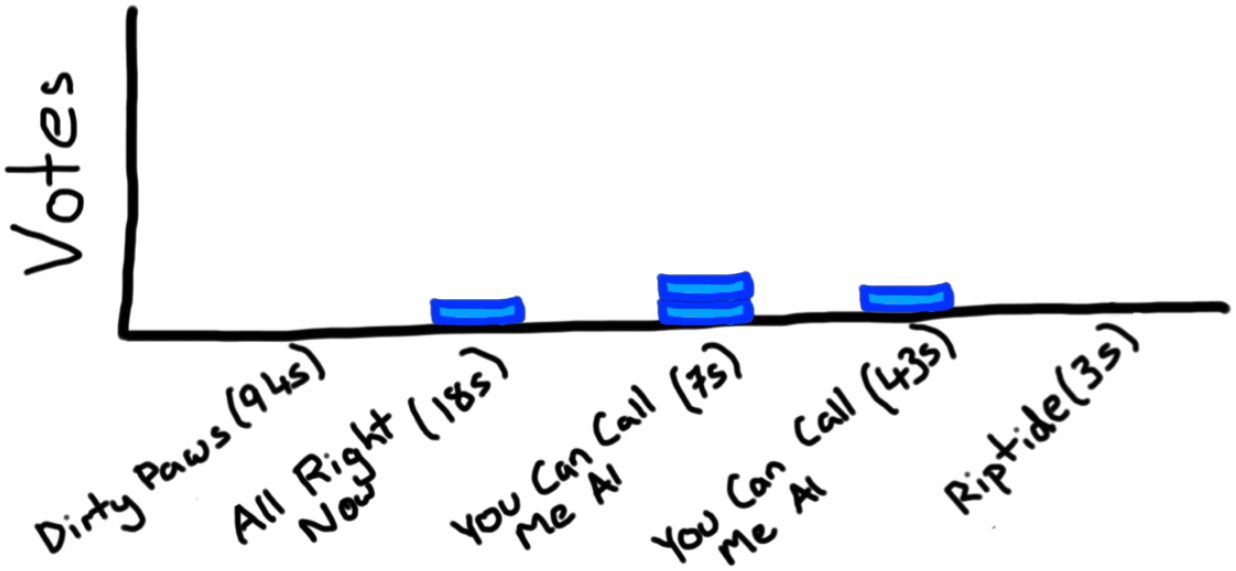
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You Can Call Me Al (43s)
Riptide (35s)

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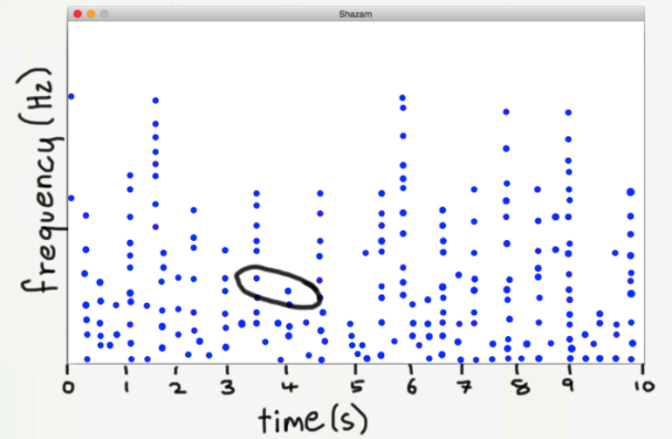
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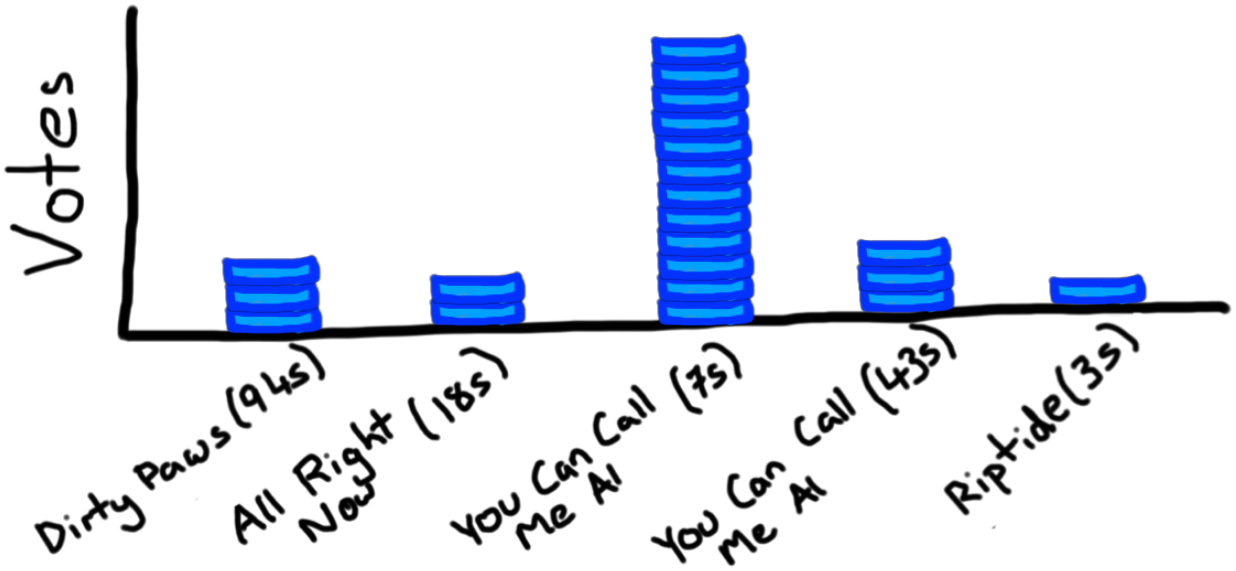
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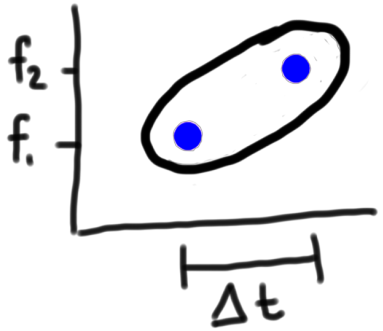


```
int hash(int f1, int f2, int timeDelta) {
    int p = 31;
      int pre = f1 + (p * f2) + (p * p * timeDelta);
      return pre % NUM_BUCKETS;
}
```

You Can Call Me Al – Paul Simon. 23s,

You Can Call Me Al – Paul Simon. 54s,

Message in a Bottle – Police. 92s

Extra Slides

Main for 2-character Dictionary

```
int main()
{
    WordDictionary wd;
    Word w1("ox");
    Word w2("at");

    // insert definitions
    wd.insert(w1,"bovine work animal");
    wd.insert(w2,"a place where something is");

    // find definitions for a word
    cout << wd.find(w1) << endl;
    cout << wd.find(w2) << endl;
}
```

References and Advanced Reading

- **References:**
  - Wikipedia Hash Function: http://en.wikipedia.org/wiki/Hash_function (very good)
  - Wikipedia Hash Table: http://en.wikipedia.org/wiki/Hash_table (very good)
  - Powerpoint: http://www.eecs.wsu.edu/~ananth/CptS223/Lectures/hashing.pdf
  - Shazam paper: https://www.ee.columbia.edu/~dpwe/papers/Wang03-shazam.pdf

- **Advanced Reading:**
  - MathWorld: http://mathworld.wolfram.com/HashFunction.html (good, short)
  - Youtube video: https://www.youtube.com/watch?v=MfhjkfocRR0 (good, 7min)

Extra Slides