

CS 106X

Lecture 24: Depth First and Breadth First Searching

Wednesday, March 8, 2017

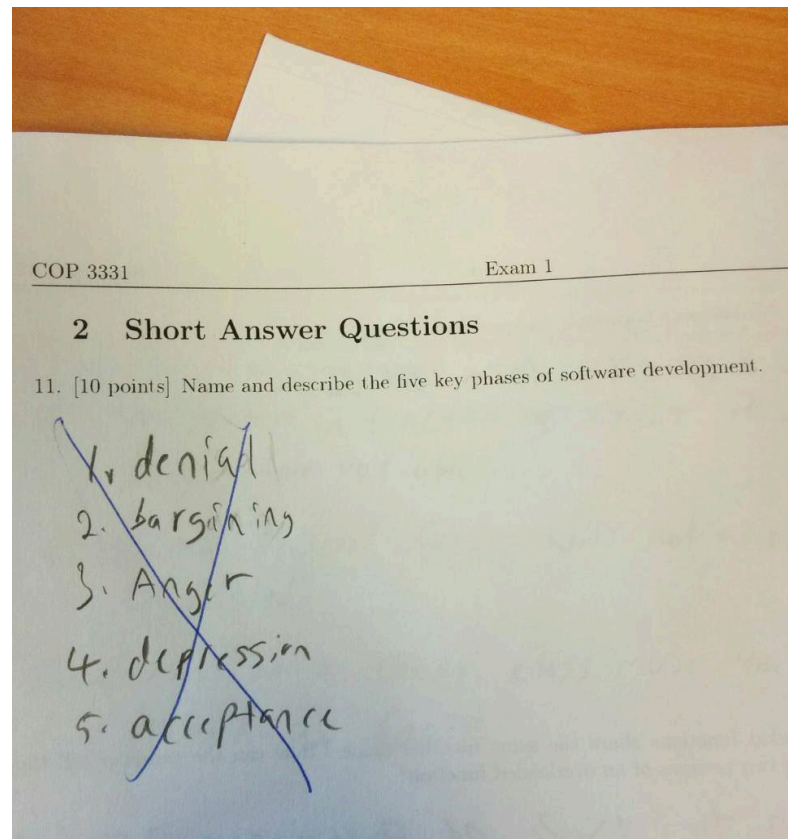
Programming Abstractions (Accelerated)
Winter 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:
Programming Abstractions in C++, Chapter 18.6



At this point in the quarter...



<https://i.redd.it/e5uylwsqzizx.jpg>

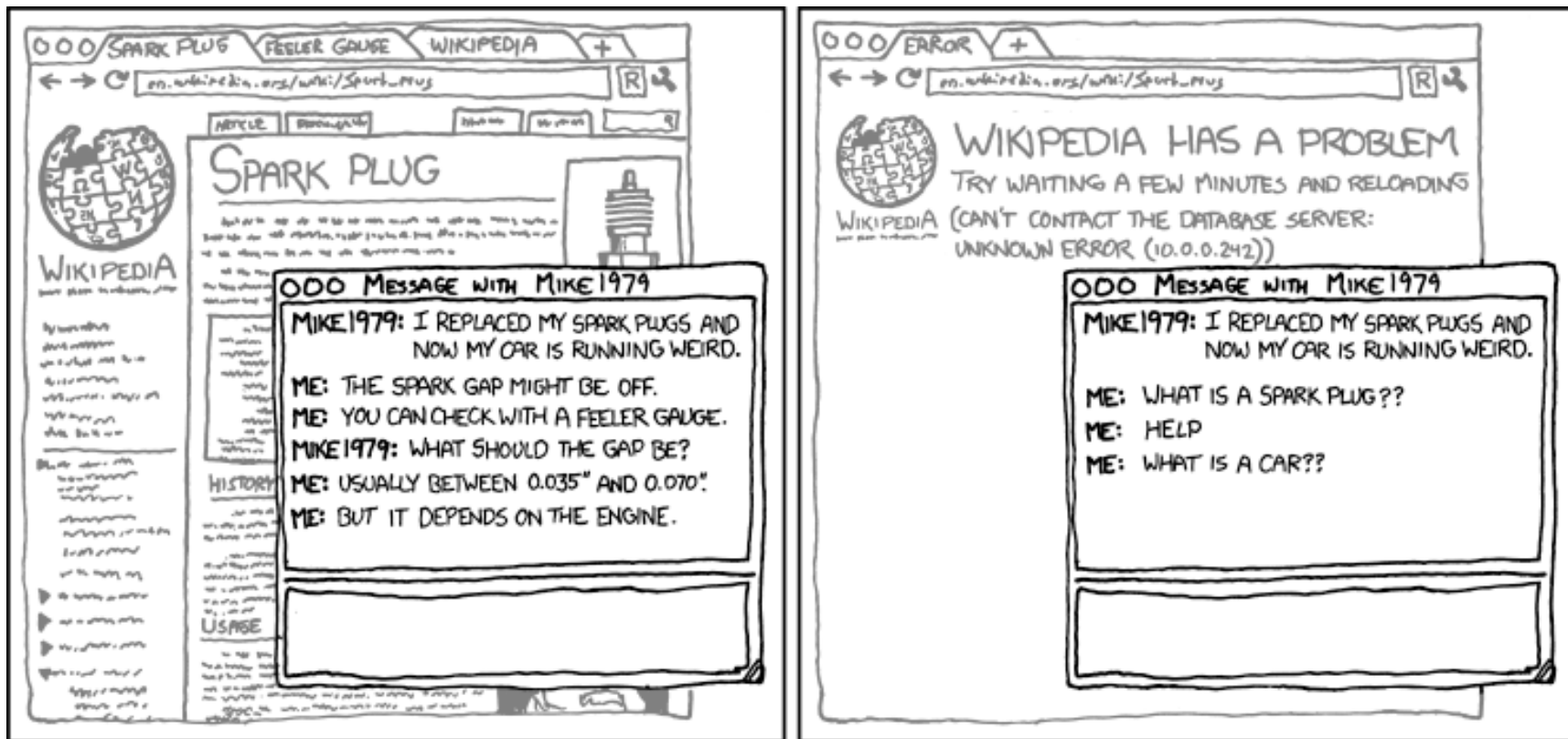


Today's Topics

- Logistics
 - Chris office hours canceled for Thursday.
 - Assignment 7: Will be due on the last Friday of classes, no late days allowed.
- More on Graphs (and a bit on Trees)
 - Depth First Search
 - Breadth First Search



Wikipedia

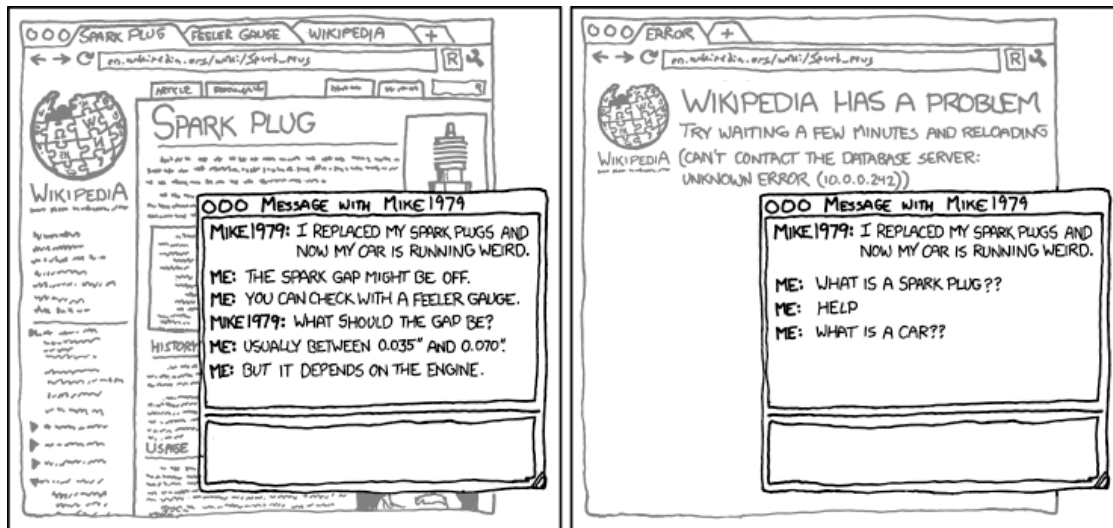


WHEN WIKIPEDIA HAS A SERVER OUTAGE, MY APPARENT IQ DROPS BY ABOUT 30 POINTS.

XKCD 903, Extended Mind, <http://xkcd.com/903/>



Wikipedia



WHEN WIKIPEDIA HAS A SERVER OUTAGE, MY APPARENT IQ DROPS BY ABOUT 30 POINTS.

When you hover over an XKCD comic, you get an extra joke:

Wikipedia trivia: if you take any article, click on the first link in the article text not in parentheses or italics, and then repeat, you will eventually end up at "Philosophy".

XKCD 903, Extended Mind, <http://xkcd.com/903/>



Wikipedia

Wikipedia trivia: if you take any article, click on the first link in the article text not in parentheses or italics, and then repeat, you will eventually end up at "Philosophy".

Is this true??

According to the Wikipedia article "Wikipedia:Getting to Philosophy" (so meta), (https://en.wikipedia.org/wiki/Wikipedia:Getting_to_Philosophy):

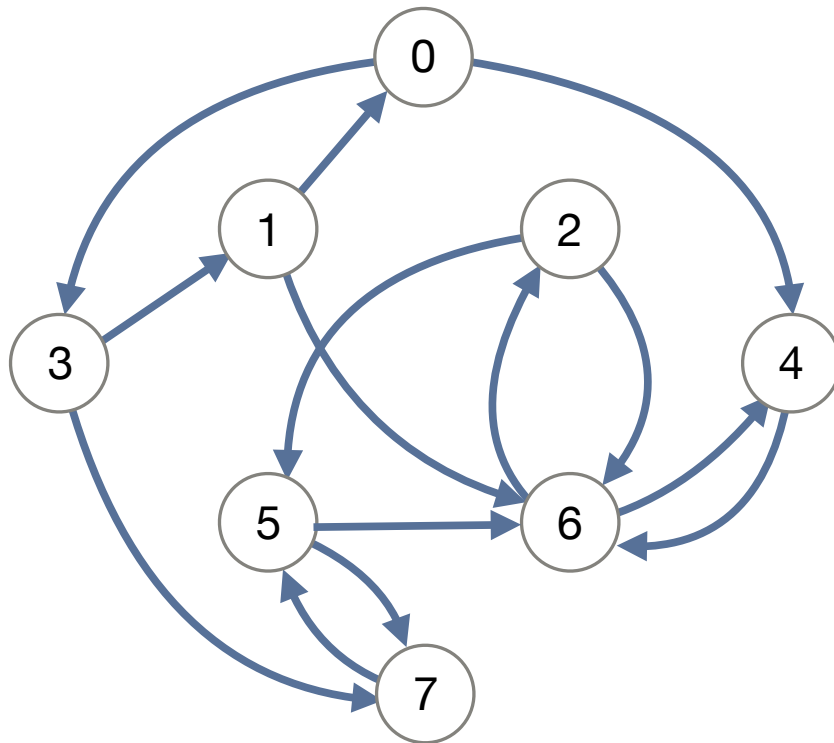
As of February 2016, 97% of all articles in Wikipedia eventually lead to the article Philosophy.

How can we find out? We shall see!



Graph Searching

Recall from the last couple of lectures that a *graph* is the "wild west of trees" — graphs relate *vertices* (nodes) to each other by way of *edges*, and they can be directed or undirected. Take the following directed graph:



A search on this graph starts at one vertex and attempts to find another vertex. If it is successful, we say there is a path from the start to the finish vertices.

What paths are there from 0 to 6?

0 → 4 → 6

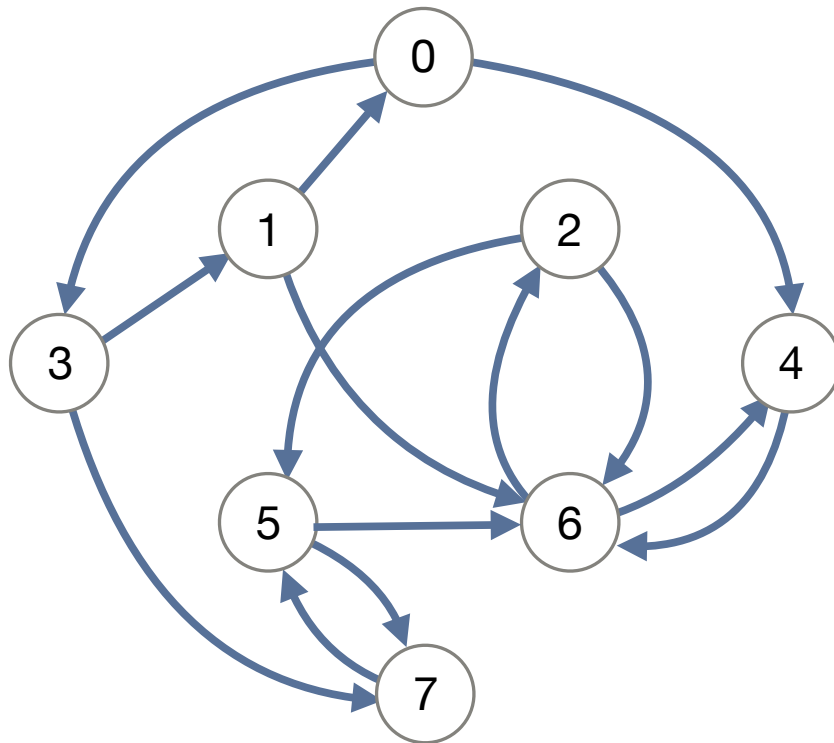
0 → 3 → 1 → 6

0 → 3 → 7 → 5 → 6



Graph Searching

What paths are there from 3 to 2?



3 → 1 → 6 → 2

3 → 7 → 5 → 6 → 2

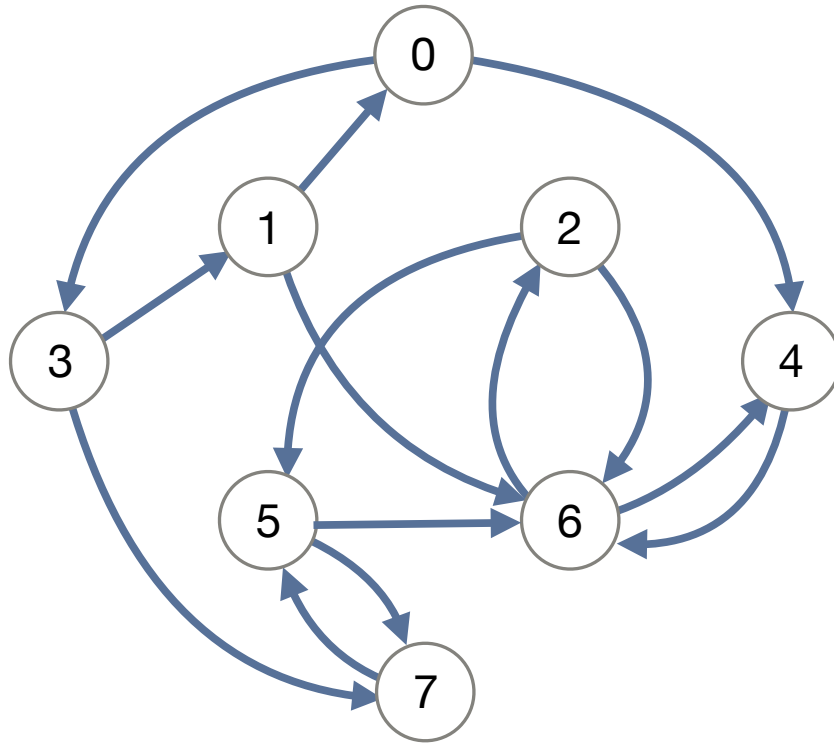
3 → 1 → 0 → 4 → 6 → 2



Graph Searching

What paths are there from 4 to 1?

None! :(

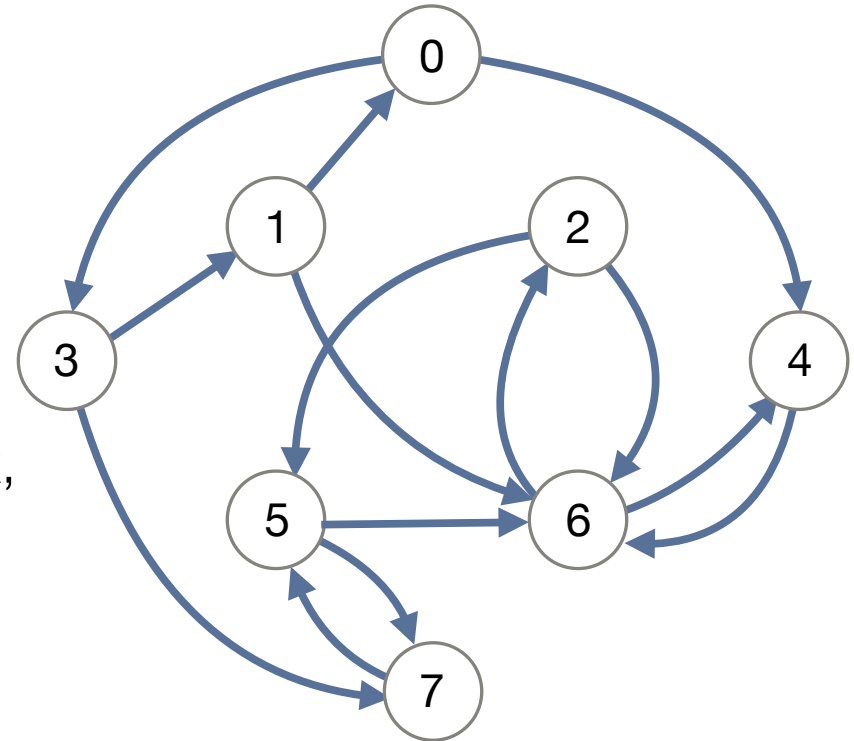


Graph Searching

We have different ways to search graphs:

- **Depth First Search:** From the start vertex, explore as far as possible along each branch before backtracking.
- **Breadth First Search:** From the start vertex, explore the neighbor nodes first, before moving to the next level neighbors.

Both methods have pros and cons — let's explore the algorithms.



Depth First Search (DFS)

From the start vertex, explore as far as possible along each branch before backtracking.

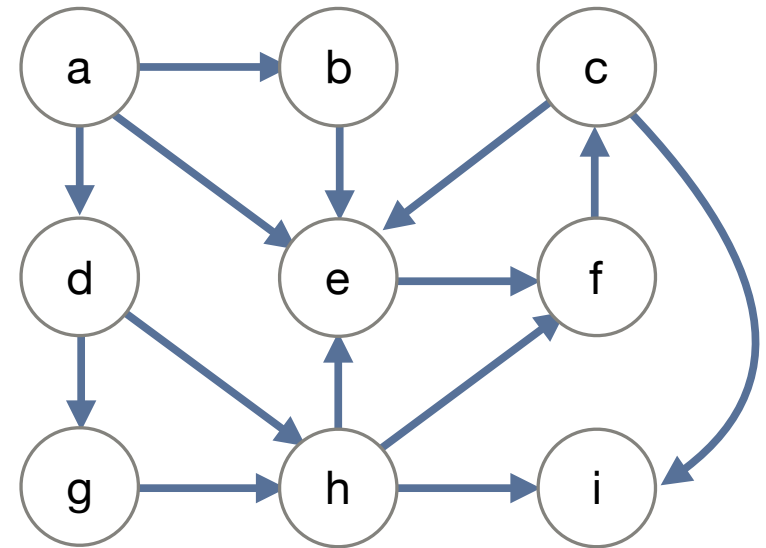
This is often implemented recursively. For a graph, you *must mark visited vertices*, or you might traverse forever (e.g., $c \rightarrow e \rightarrow f \rightarrow c \rightarrow e \dots$)

DFS from a to h (assuming a-z order) visits:

a
b
e
f
c
d i (dead end — back to c,f,e,b,a)
g
h

path found: a → d → g → h

Notice: not the shortest!



Depth First Search (DFS): Recursive pseudocode

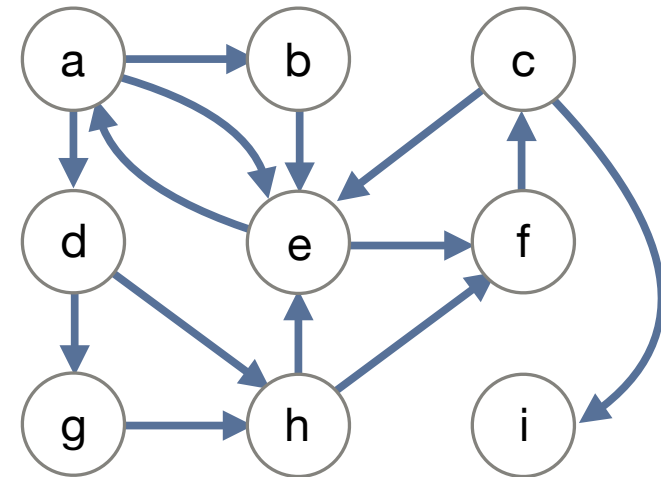
dfs from v_1 to v_2 :

base case: if at v_2 , found!

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n , v_2).



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

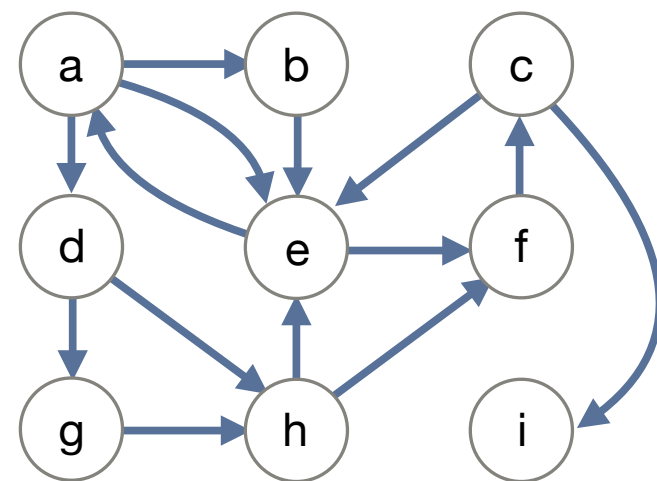
mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n , v_2).

Let's look at **dfs** from h to c :

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	false
i	false



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

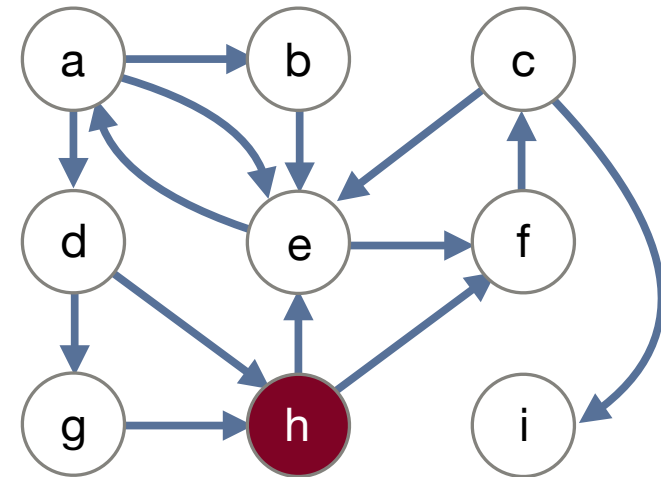
Let's look at **dfs** from h to c :

Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	true
i	false

call stack:

dfs(h,c)



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n , v_2).

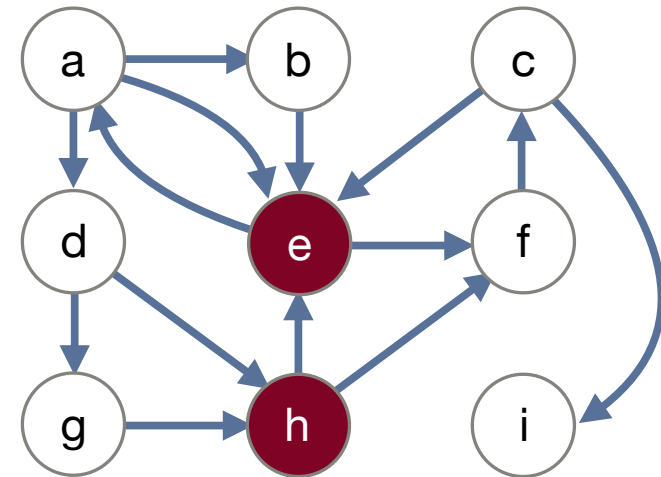
Let's look at **dfs** from h to c :

Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	true
f	false
g	false
h	true
i	false

call stack:

dfs(e,c)
dfs(h,c)



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

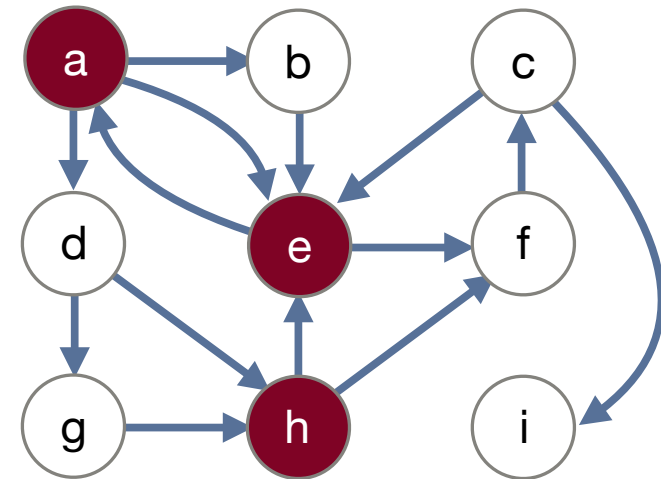
Let's look at **dfs** from h to c :

Vertex Map

Vertex	Visited?
a	true
b	false
c	false
d	false
e	true
f	false
g	false
h	true
i	false

call stack:

dfs(a,c)
dfs(e,c)
dfs(h,c)



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

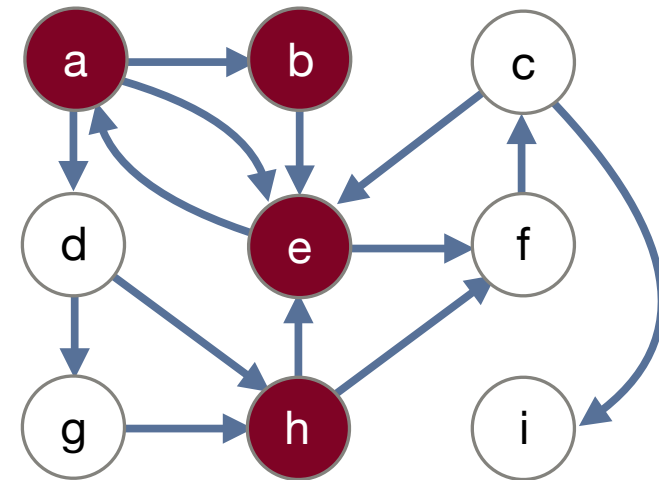
Let's look at **dfs** from h to c :

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	false
e	true
f	false
g	false
h	true
i	false

call stack:

dfs(b,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

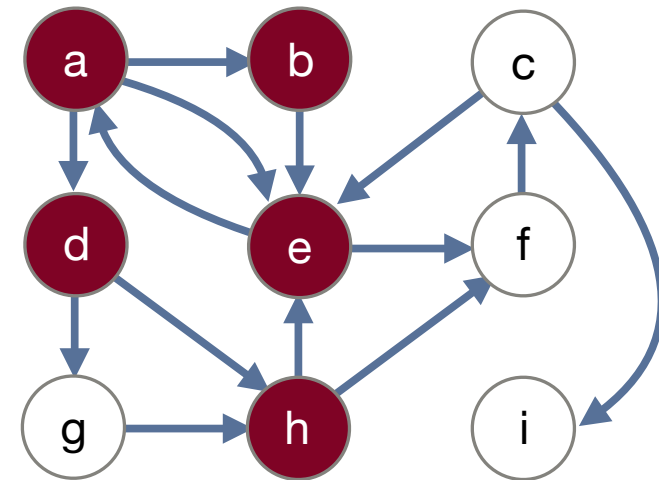
Let's look at **dfs** from h to c :

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	true
e	true
f	false
g	false
h	true
i	false

call stack:

dfs(d,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

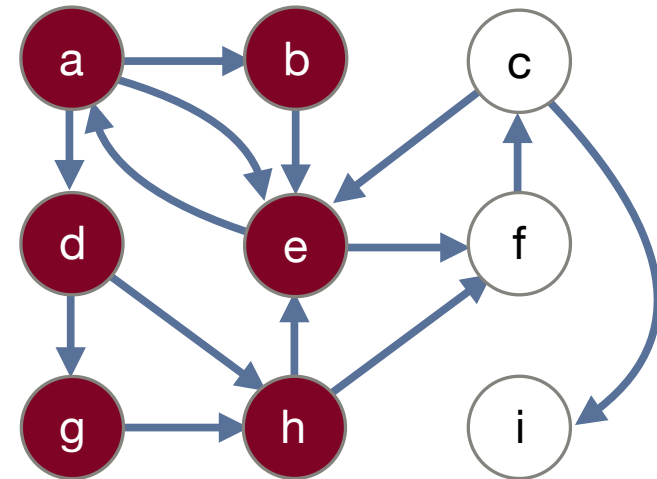
Let's look at **dfs** from h to c :

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	true
e	true
f	false
g	true
h	true
i	false

call stack:

dfs(g,c)
dfs(d,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

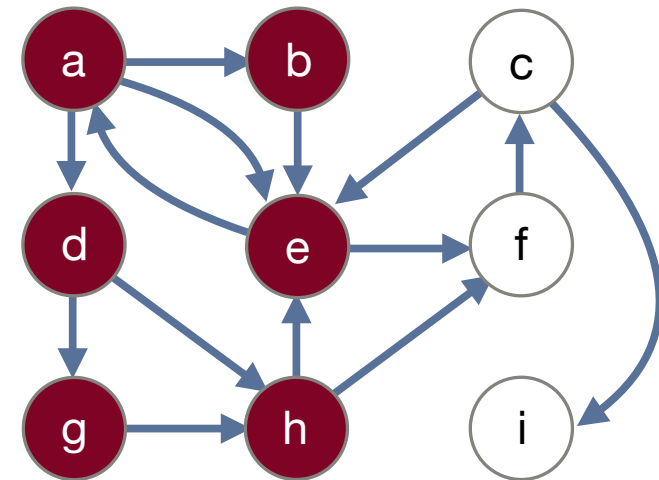
Let's look at **dfs** from h to c :

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false

call stack:

dfs(g,c)
dfs(d,c)
dfs(a,c)
dfs(e,c)
dfs(h,c)



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

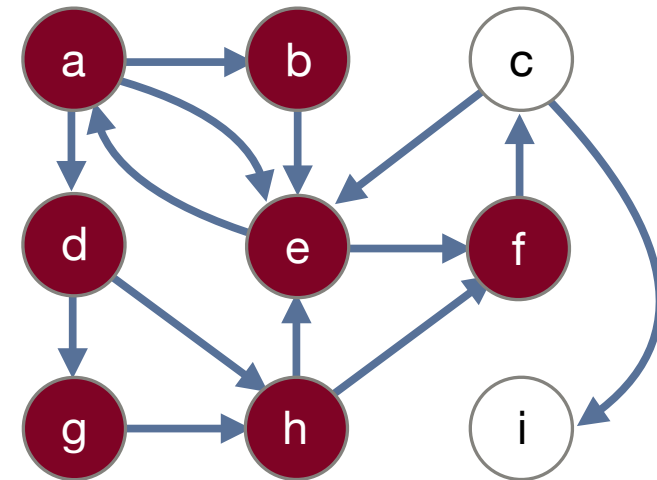
Let's look at **dfs** from h to c :

Vertex Map

Vertex	Visited?
a	true
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false

call stack:

dfs(f,c)
dfs(e,c)
dfs(h,c)



Depth First Search (DFS): Recursive pseudocode

dfs from v_1 to v_2 :

mark v_1 as visited.

for all edges from v_1 to its neighbors:

if neighbor n is unvisited, recursively call **dfs**(n, v_2).

Let's look at **dfs** from h to c :

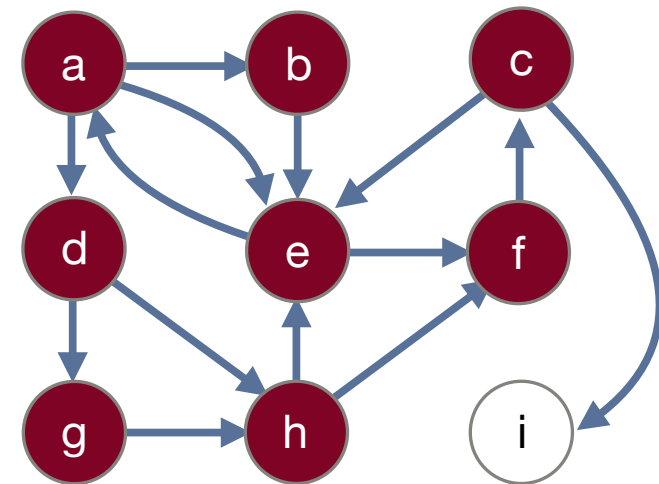
Vertex Map

Vertex	Visited?
a	true
b	true
c	true
d	true
e	true
f	true
g	true
h	true
i	false

call stack:

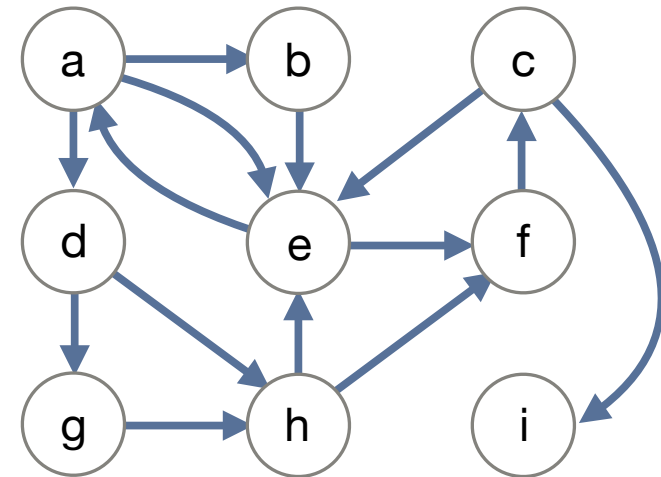
dfs(c,c)
dfs(f,c)
dfs(e,c)
dfs(h,c)

found!



Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2 :
create a stack, s
 $s.push(v_1)$
while s is not empty:
 $v = s.pop()$
 if v has not been visited:
 mark v as visited
 push all neighbors of v onto the stack



Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2 :

create a stack, s

$s.push(v_1)$

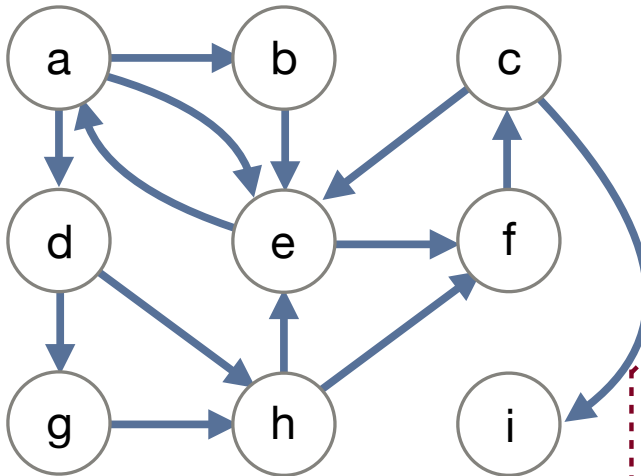
while s is not empty:

$v = s.pop()$

if v has not been visited:

mark v as visited

push all neighbors of v onto the stack

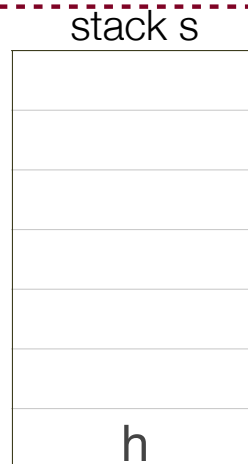


Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	false
i	false

Let's look at **dfs** from h to c :

push h



Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2 :

create a stack, s

$s.push(v_1)$

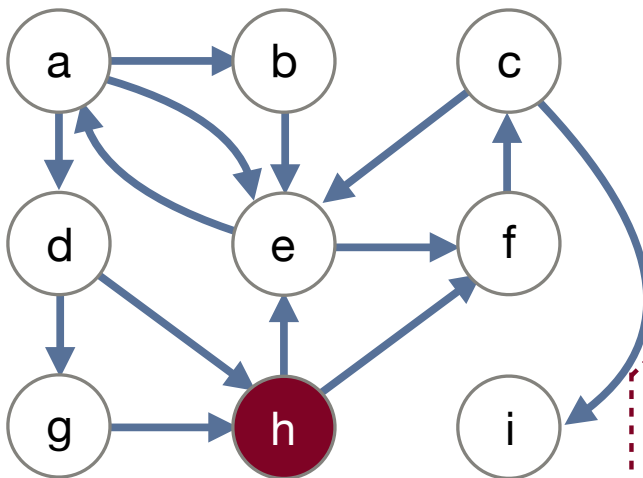
while s is not empty:

$v = s.pop()$

if v has not been visited:

mark v as visited

push all neighbors of v onto the stack



Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	true
i	false

Let's look at **dfs** from h to c :

in while loop:

$v = s.pop()$

$v: h$



Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2 :

create a stack, s

$s.push(v_1)$

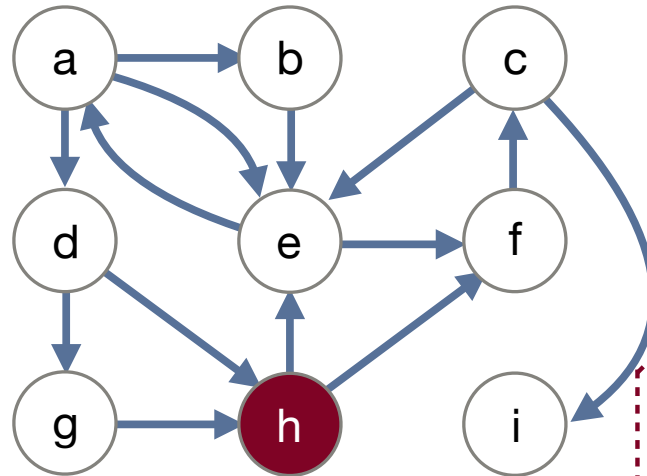
while s is not empty:

$v = s.pop()$

if v has not been visited:

mark v as visited

push all neighbors of v onto the stack



Vertex Map

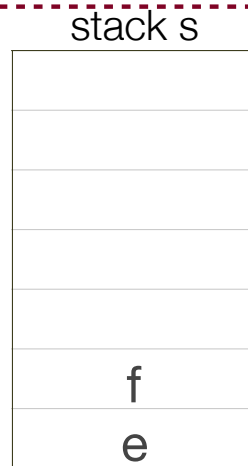
Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	false
g	false
h	true
i	false

Let's look at **dfs** from h to c :

in while loop:

push all

neighbors of h



Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2 :

create a stack, s

$s.push(v_1)$

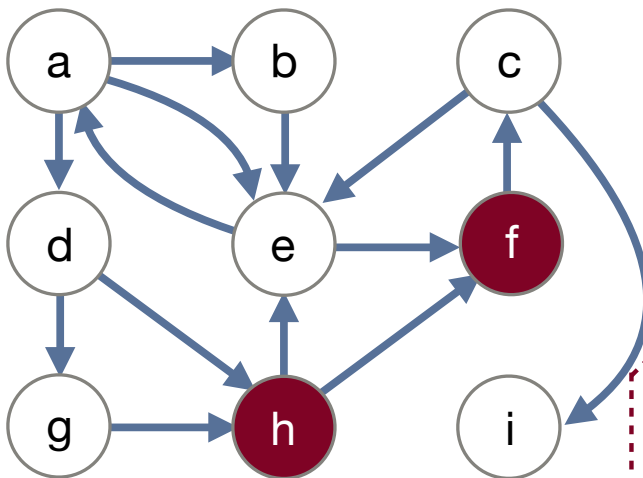
while s is not empty:

$v = s.pop()$

if v has not been visited:

mark v as visited

push all neighbors of v onto the stack



Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	true
g	false
h	true
i	false

Let's look at **dfs** from h to c :

in while loop:

$v = s.pop()$

v : f



Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2 :

create a stack, s

$s.push(v_1)$

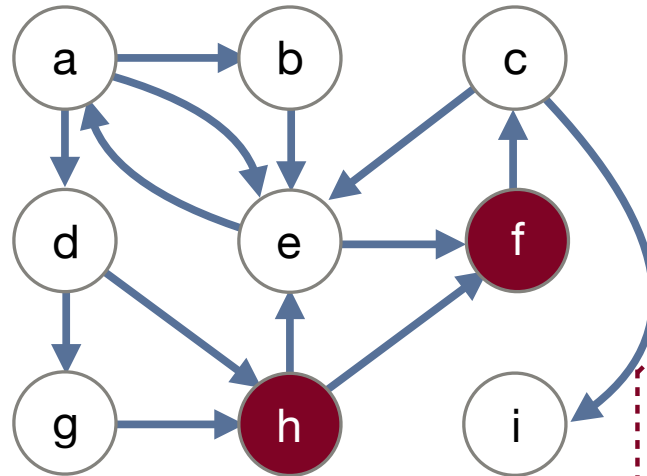
while s is not empty:

$v = s.pop()$

if v has not been visited:

mark v as visited

push all neighbors of v onto the stack



Vertex Map

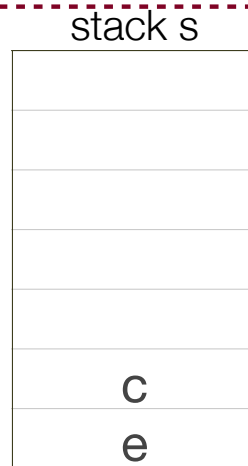
Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	true
g	false
h	true
i	false

Let's look at **dfs** from h to c :

in while loop:

push all

neighbors of f



Depth First Search (DFS): Iterative pseudocode

dfs from v_1 to v_2 :

create a stack, s

$s.push(v_1)$

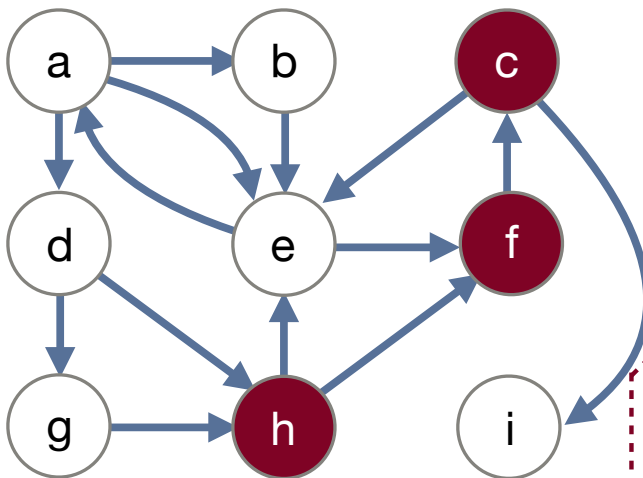
while s is not empty:

$v = s.pop()$

if v has not been visited:

mark v as visited

push all neighbors of v onto the stack



Vertex Map

Vertex	Visited?
a	false
b	false
c	false
d	false
e	false
f	true
g	false
h	true
i	false

Let's look at **dfs** from h to c :

in while loop:

$v = s.pop()$

v : c

found — stop!

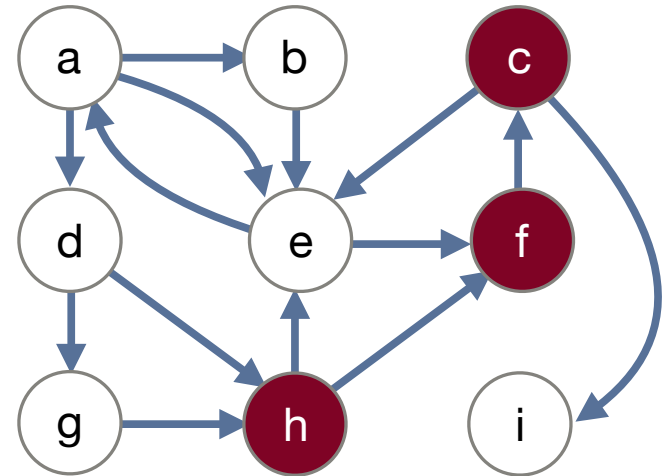


Depth First Search (DFS)

Both the recursive and iterative solutions to DFS were correct, but because of the subtle differences in recursion versus using a stack, they traverse the nodes in a different order.

For the h to c example, the iterative solution happened to be faster, but for different graphs the recursive solution may have been faster.

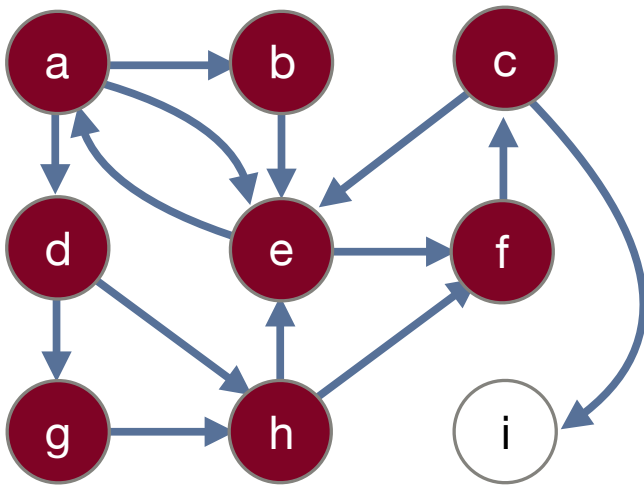
To retrieve the DFS path found, pass a collection parameter to each cell (if recursive) and choose-explore-unchoose (our old friend, recursive backtracking!)



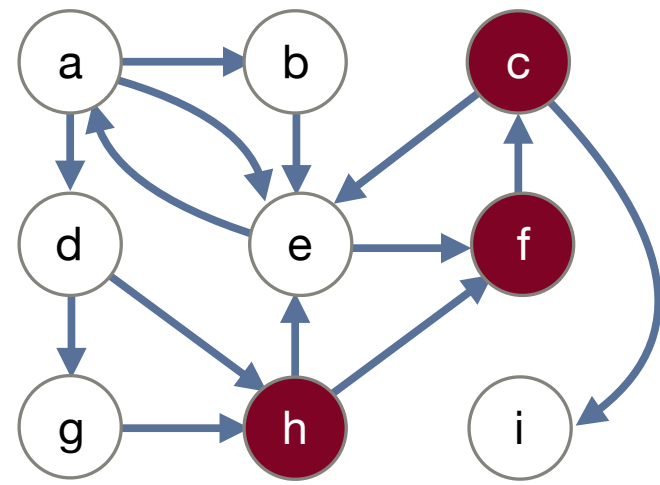
Depth First Search (DFS)

DFS is guaranteed to find a path if one exists.

It is *not* guaranteed to find the best or shortest path! (i.e., it is not optimal)



vs.



Breadth First Search (BFS)

- From the start vertex, explore the neighbor nodes first, before moving to the next level neighbors.

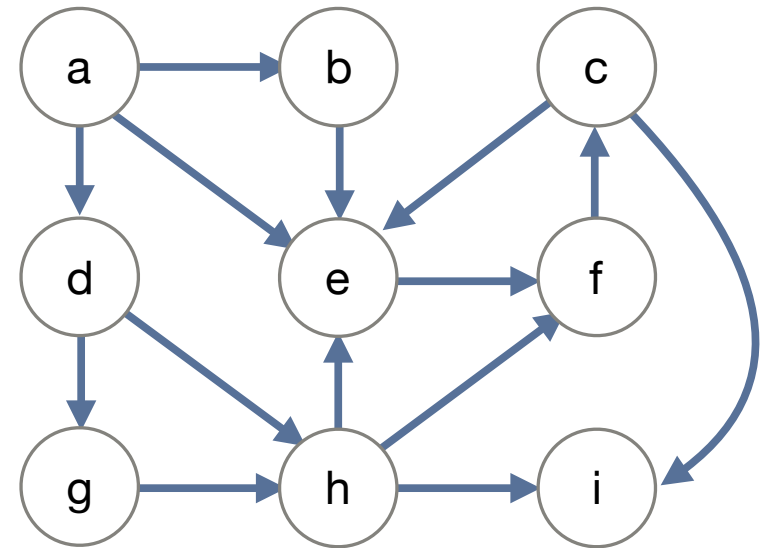
This *isn't easy to implement* recursively. The iterative algorithm is very similar to the DFS iterative, except that we use a queue.

BFS from a to i (assuming a-z order) visits:

a
a → b
a → d } neighbors of a
a → e }
a → d → g
a → d → h } neighbors of d
a → e → f
a → d → h → i

path: a → d → h → i

Notice: the shortest!



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :

create a queue of paths (a vector), q

$q.enqueue(v_1 \text{ path})$

while q is not empty and v_2 is not yet visited:

$path = q.dequeue()$

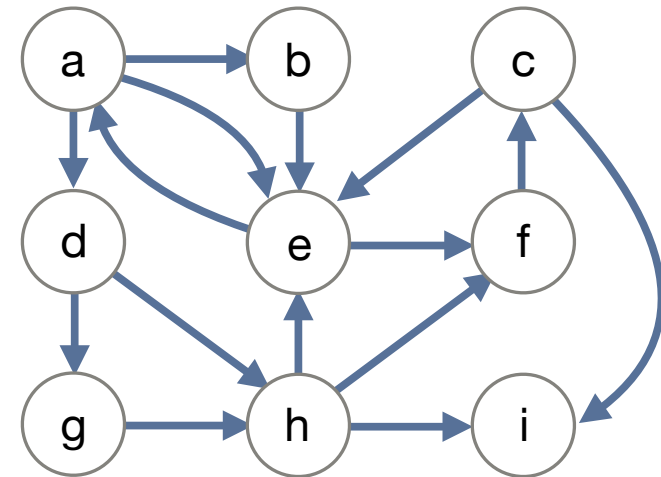
$v = \text{last element in path}$

 mark v as visited

 for each unvisited neighbor of v :

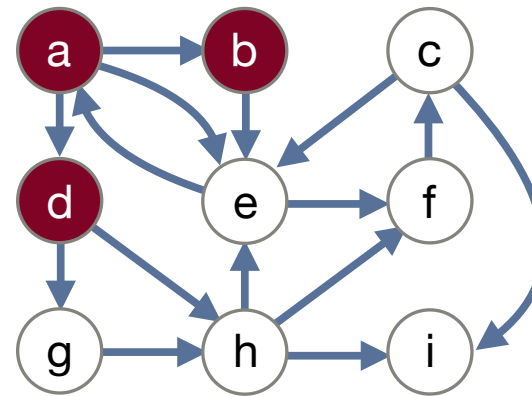
 make new path with v 's neighbor as last element

 enqueue new path onto q



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front
					adh	adg	abe	ae

in while loop:

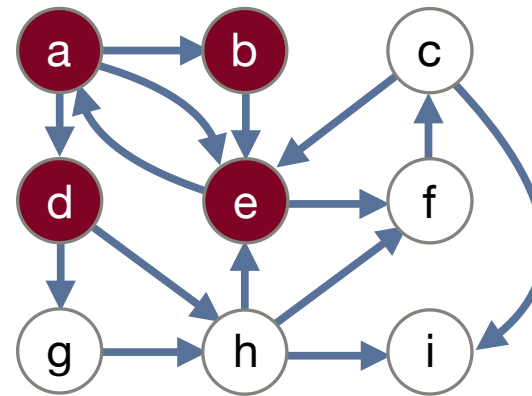
$curPath = q.dequeue()$ (path is ad)
 $v = \text{last element in curPath}$ (v is d)
 mark v as visited
 enqueue all unvisited neighbor paths onto q

Vertex	Visited?
a	false
b	true
c	false
d	true
e	false
f	false
g	false
h	false
i	false



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front
					abef	aef	adh	adg

in while loop:

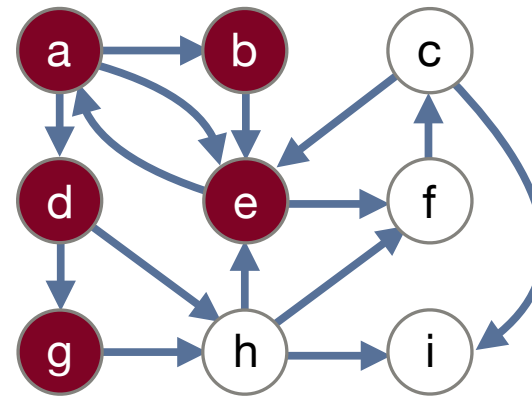
$curPath = q.dequeue()$ (path is abe)
 $v = \text{last element in curPath}$ (v is e)
 mark v as visited (already been marked)
 enqueue all unvisited neighbor paths onto q

Vertex	Visited?
a	false
b	true
c	false
d	true
e	true
f	false
g	false
h	false
i	false



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front
					adgh	abef	aef	adh

in while loop:

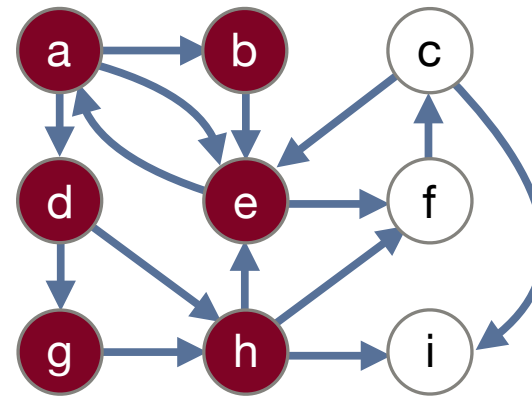
$curPath = q.dequeue()$ (path is adg)
 $v = \text{last element in } curPath$ (v is g)
 mark v as visited
 enqueue all unvisited neighbor paths onto q

Vertex	Visited?
a	false
b	true
c	false
d	true
e	true
f	false
g	true
h	false
i	false



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front
				adhi	adhf	adgh	abef	aef

in while loop:

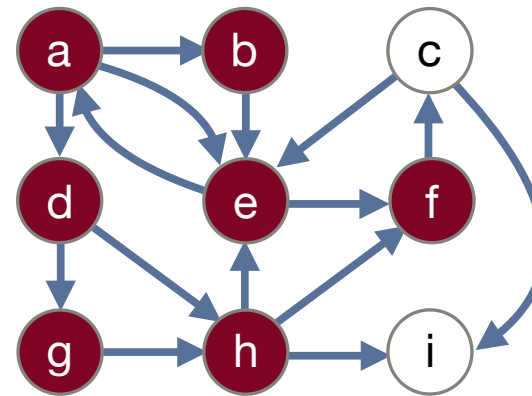
$curPath = q.dequeue()$ (path is adh)
 $v = \text{last element in curPath}$ (v is h)
 mark v as visited
 enqueue all unvisited neighbor paths onto q

Vertex	Visited?
a	false
b	true
c	false
d	true
e	true
f	false
g	true
h	true
i	false



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front
				aefc	adhi	adhf	adgh	abef

in while loop:

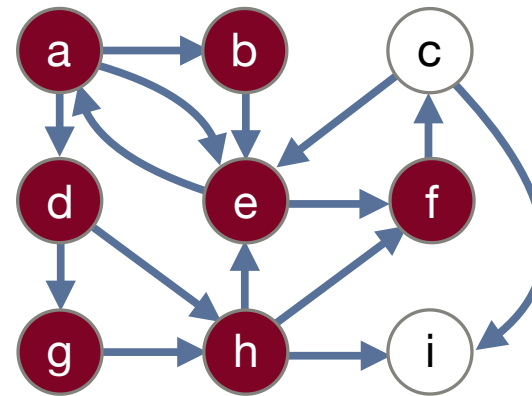
$curPath = q.dequeue()$ (path is aef)
 $v = \text{last element in curPath}$ (v is f)
 mark v as visited
 enqueue all unvisited neighbor paths onto q

Vertex	Visited?
a	false
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:									front
				abefc	aefc	adhi	adhf	adgh	

in while loop:

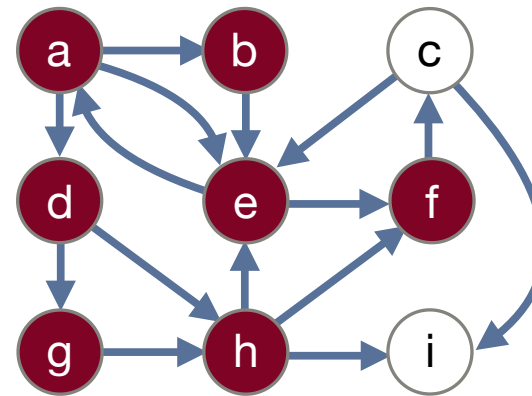
$curPath = q.dequeue()$ (path is abef)
 $v = \text{last element in curPath}$ (v is f)
 mark v as visited (already been marked)
 enqueue all unvisited neighbor paths onto q

Vertex	Visited?
a	false
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front
				adghi	abefc	aefc	adhi	adhf

in while loop:

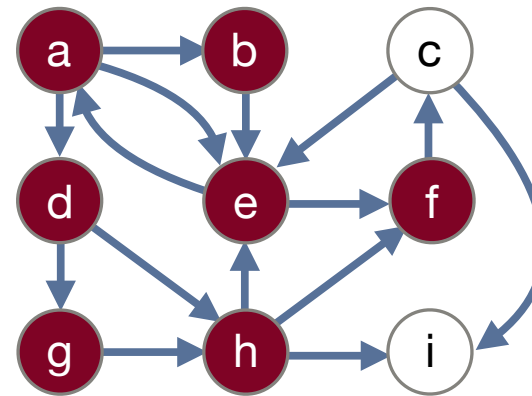
$curPath = q.dequeue()$ (path is adgh)
 $v = \text{last element in curPath}$ (v is h)
 mark v as visited (already been marked)
 enqueue all unvisited neighbor paths onto q

Vertex	Visited?
a	false
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front
				adhfc	adghi	abefc	aefc	adhi

in while loop:

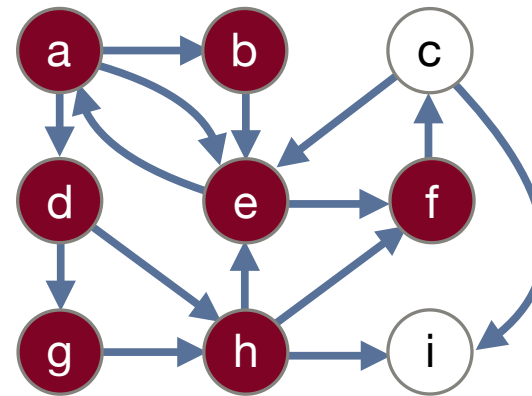
$curPath = q.dequeue()$ (path is adhf)
 $v = \text{last element in curPath}$ (v is f)
 mark v as visited (already been marked)
 enqueue all unvisited neighbor paths onto q

Vertex	Visited?
a	false
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false



Breadth First Search (BFS): Iterative pseudocode

bfs from v_1 to v_2 :
 create a queue of paths (a vector), q
 $q.enqueue(v_1 \text{ path})$
 while q is not empty and v_2 is not yet visited:
 $path = q.dequeue()$
 $v = \text{last element in path}$
 mark v as visited
 for each unvisited neighbor of v :
 make new path with v 's neighbor as last element
 enqueue new path onto q



Let's look at **bfs** from a to i:

queue:

								front	
					adhfc	adghi	abefc	aefc	adhi

in while loop:

$curPath = q.dequeue()$ (path is adhi)

$v = \text{last element in curPath}$ (v is i)

found!

Vertex	Visited?
a	false
b	true
c	false
d	true
e	true
f	true
g	true
h	true
i	false



Wikipedia: Getting to Philosophy



WIKIPEDIA
The Free Encyclopedia

So I downloaded Wikipedia...

It turns out that you *can* download Wikipedia, but it is > 10 Terabytes (!) uncompressed. The reason Wikipedia asks you for money every so often is because they have lots of fast computers with lots of memory, and this is expensive (so donate!)

But, the Internet is just a graph...so, Wikipedia pages are just a graph...let's just do the searching by taking advantage of this: download pages as we need them.



Wikipedia: Getting to Philosophy



WIKIPEDIA
The Free Encyclopedia

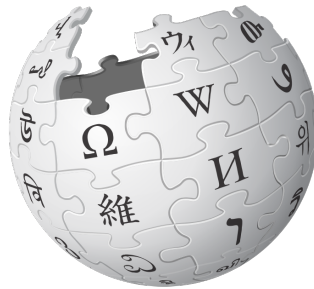
What kind of search is the "getting to philosophy" algorithm?

"Clicking on the first lowercase link in the main text of a Wikipedia article, and then repeating the process for subsequent articles, usually eventually gets one to the Philosophy article."

This is a depth-first search! To determine if a Wikipedia article will get to Philosophy, we just select the first link each time. If we ever have to select a second link (or if a first-link refers to a visited vertex), then that article doesn't get to Philosophy.



Wikipedia: Getting to Philosophy



WIKIPEDIA
The Free Encyclopedia

We can also perform a Breadth First Search, as well. How would this change our search?

A BFS would look at all links on a page, then all links for each link on the page, etc. This has the potential of taking a long time, but it will find a shortest path.



References and Advanced Reading

- **References:**

- Depth First Search, Wikipedia: https://en.wikipedia.org/wiki/Depth-first_search
- Breadth First Search, Wikipedia: https://en.wikipedia.org/wiki/Breadth-first_search

- **Advanced Reading:**

- Visualizations:
 - <https://www.cs.usfca.edu/~galles/visualization/DFS.html>
 - <https://www.cs.usfca.edu/~galles/visualization/BFS.html>

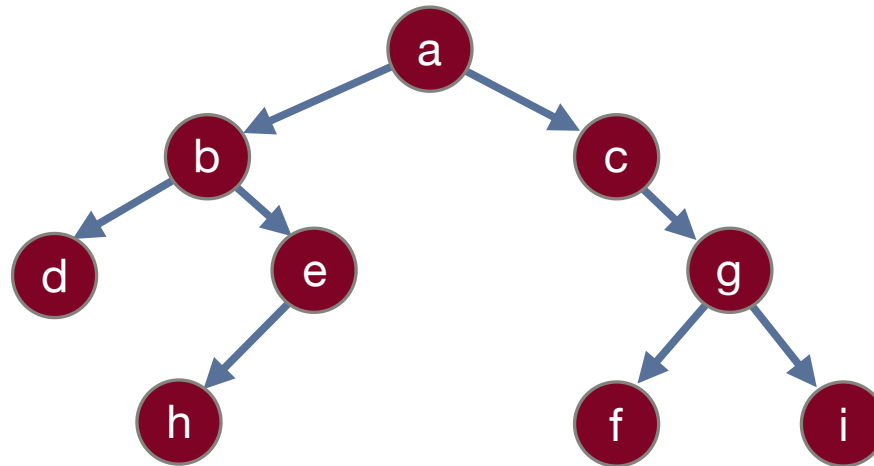


Extra Slides



Breadth First Search (BFS): Tree searching

A Breadth First Search on a tree will produce a "level order traversal":



Breadth First Search: a → b → c → d → e → g → h → f → i

This is necessary if we want to print the tree to the screen in a pretty way, such that it retains its tree-like structure.

