Chris Piech
CS109

Section #2: Random Variables

Section #2
October 11, 2017

1. **Breaking Vegas**: Consider any even money bet (e.g., bet "Red" in roulette) where there is $p = 18/38$ that you with win \$Y and a $(1 - p)$ probability that you lose \$Y. Consider this algorithm for a series of bets:

   1. Let Y = \$1.
   2. Bet Y.
   3. If you win, then stop.
   4. If you lose, then set Y to be 2Y and goto step (2).

   What are your expected winnings when you stop? It will help to recall that the sum of a geometric series $a^0 + a^1 + a^2 + \cdots = \frac{1}{1-a}$ if $0 < a < 1$. Vegas breaks you: Why doesn't everyone do this?

2. **Sending Bits to Space**: When sending binary data to satellites (or really over any noisy channel) the bits can be flipped with high probabilities. In 1947 Richard Hamming developed a system to more reliably send data. By using Error Correcting Hamming Codes, you can send a stream of 4 bits with 3 redundant bits. If zero or one of the seven bits are corrupted, using error correcting codes, a receiver can identify the original 4 bits.

   Lets consider the case of sending a signal to a satellite where each bit is independently flipped with probability $p = 0.1$

   a. If you send 4 bits, what is the probability that the correct message was received (eg none of the bits are flipped).
   b. If you send 4 bits, with 3 Hamming error correcting bits, what is the probability that a correctable message was received?
   c. Instead of using Hamming codes, you decide to send 100 copies of each of the four bits. If for every single bit, more than 50 of the copies are not flipped, the signal will be correctable. What is the probability that a correctable message was received?

3. **Launch**: Laura is a software engineer at the startup Acc.io, which has just launched its new website. This week she is "on call": whenever something breaks, she gets a pager notification. She gets an average of one pager notification per 36 hours, independently and with the same frequency at all hours. What is the probability that Elena gets at least one notification between 1:00am and 7:00am? What is the standard deviation of the number of notifications?

4. **Slowing down hackers**: A hacker is trying to guess a password for your website. On each guess they have an independent 1/1000 probability of guessing correctly.

   a. What is the probability that the hacker guesses the password on her ith try?
   b. To slow down the hacker, you make your website wait 2^i seconds before responding to the ith login request. What is the expected amount of time before the hacker guesses correctly?