Continuous Variables

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CS109, Stanford University

Announcements

- Extra office hours. Thank you Tas
  - CS109 hasn't had this many OH since its start
  - Let the TAs know if you appreciate it
  - Some OH will be themed, look out!
- What is up with these checks on my pset?
- PSet 3 by tomorrow morning (PSet app takes time). Unrelated: I also fixed the app to reduce lag – let me know if you notice it.

What is a check in CS109?

Functionality and style grades for the assignments use the following scale:

The modal grade in CS109

✓+	Satisfies all requirements of the assignment. An "A"
✓	Meets most requirements, but with some problems. An "A-" / "B+"
✓−	Has more serious problems, such as not explaining work.
−	Progress was made.

If your overall score is a ✓+ you rocked the PSet.

Small nit: Avoid answers that are just equations

Numeric Answer: Enter your answer | Check Answer

Explanation:

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$$|E| = \binom{f}{1}\binom{u-f}{s-1} = 50 \cdot \binom{11950}{249}$$

A small group of students made this mistake on pset1. If you fix it, we will make sure it doesn't impact your overall grade

1906 Earthquake
Magnitude 7.8

ILL. No. 65. MEMORIAL ARCH, WITH CHURCH IN BACKGROUND, STANFORD UNIVERSITY, SHOWING TYPES OF CARVED WORK WITH THE SANDSTONE

Review

Binomial Random Variable

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The number of **successes**, in n independent **trials**, where each **trial** is a **success** with probability p:

Binomial Random Variable

Binomial Random Variable

Notation: $X \sim \text{Bin}(n, p)$

Description: Number of "successes" in n identical, independent experiments each with probability of success p.

Parameters: $n \in \{0, 1, \ldots\}$, the number of experiments.
$p \in [0, 1]$, the probability that a single experiment gives a "success".

Support: $x \in \{0, 1, \ldots, n\}$

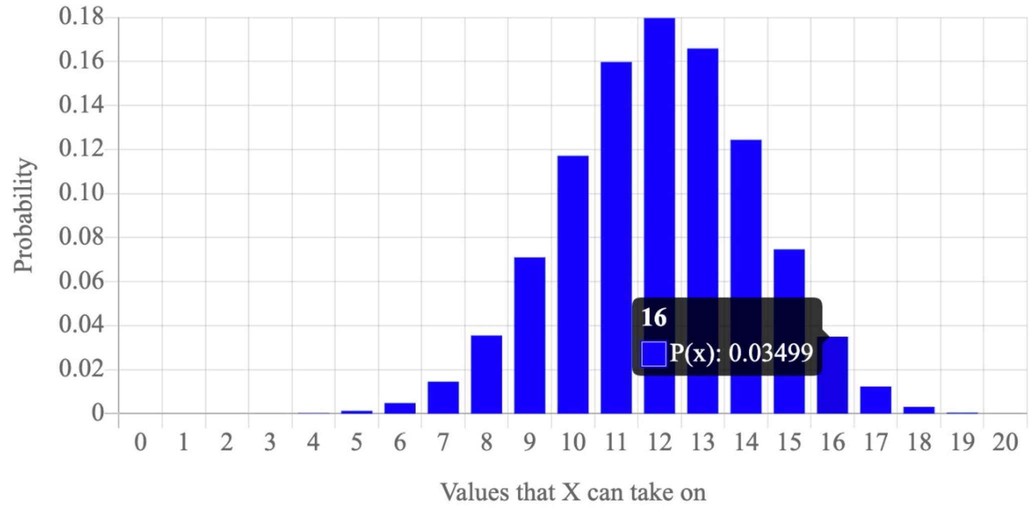
PMF equation: $\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Expectation: $\text{E}[X] = n \cdot p$

Variance: $\text{Var}(X) = n \cdot p \cdot (1-p)$

PMF graph:

Parameter n: 20 Parameter p: 0.60



Bernoulli Random Variable

Notation: $X \sim \text{Bern}(p)$

Description: A boolean variable that is 1 with probability p

Parameters: p, the probability that $X = 1$.

Support: x is either 0 or 1

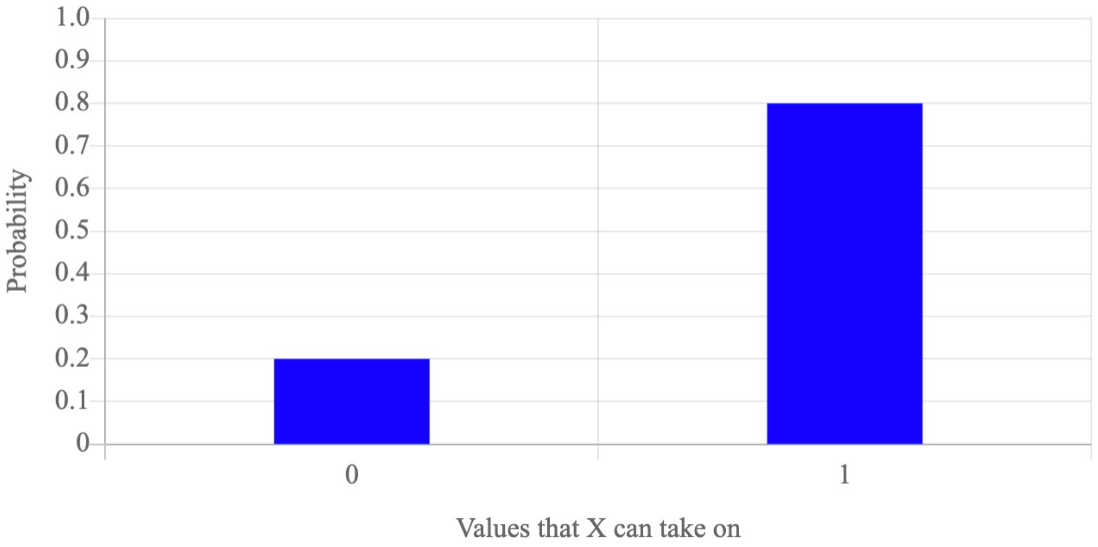
PMF equation: $\Pr(X = x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$

Expectation: $\text{E}[X] = p$

Variance: $\text{Var}(X) = p(1-p)$

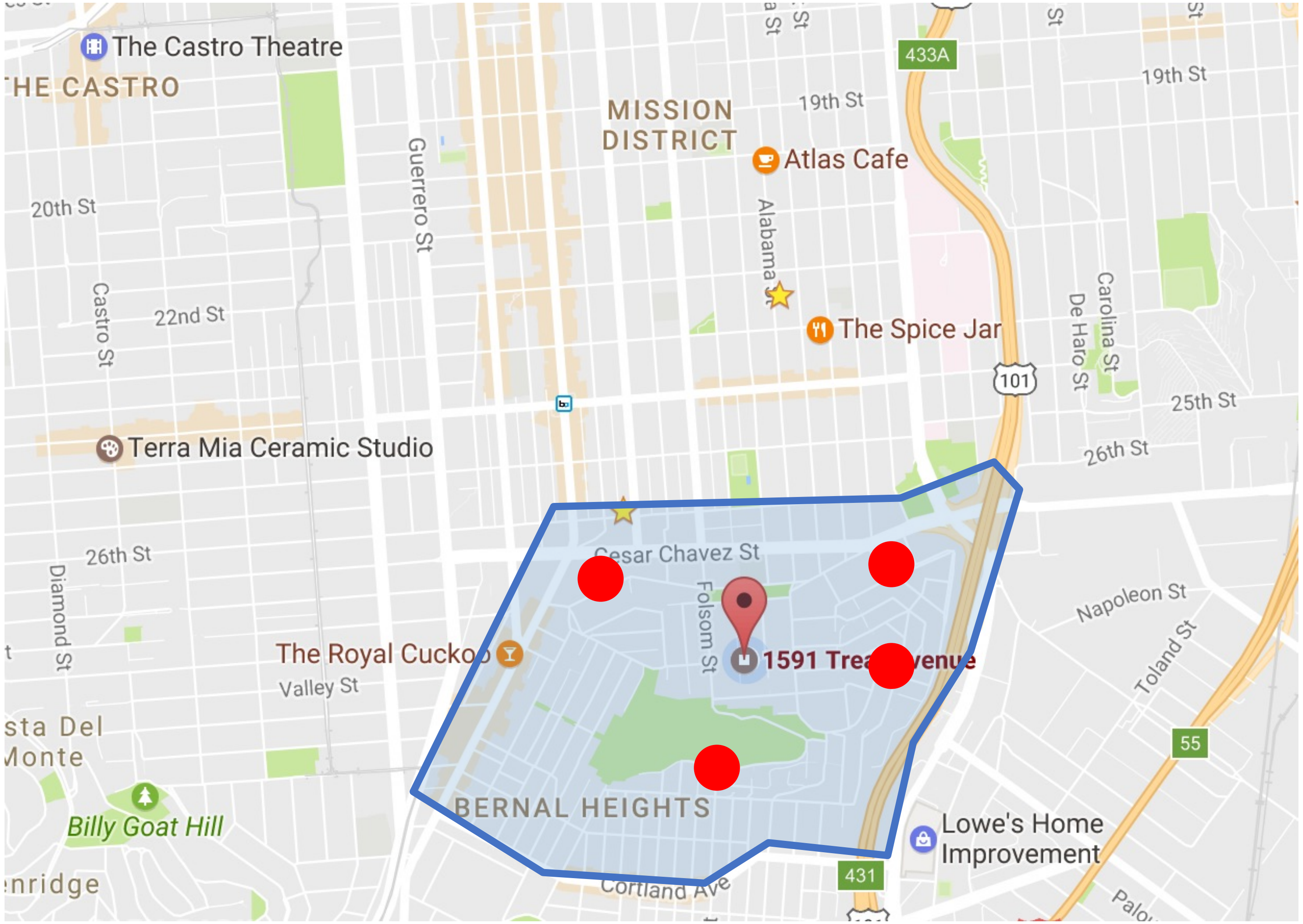
PMF graph:

Parameter p: 0.80



Poisson Random Variable

Probability of ***k* requests** from this area in the next 1 min



Poisson Random Variable

Poisson Random Variable

Notation: $X \sim \text{Poi}(\lambda)$

Description: Number of events in a fixed time frame if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.

Parameters: $\lambda \in \{0, 1, \ldots\}$, the constant average rate.

Support: $x \in \{0, 1, \ldots\}$

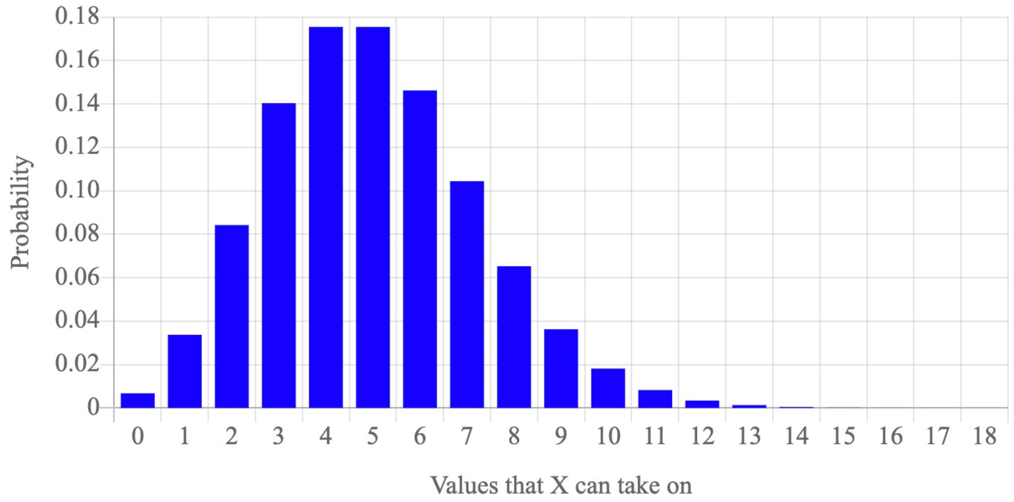
PMF equation: $\Pr(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$

Expectation: $\text{E}[X] = \lambda$

Variance: $\text{Var}(X) = \lambda$

PMF graph:

Parameter λ: 5



Learning Goals

1. Comfort using new discrete random variables
2. Integrate a density function (PDF) to get a probability
3. Use a cumulative function (CDF) to get a probability

Goal: Be Able to Use a New Random Variable

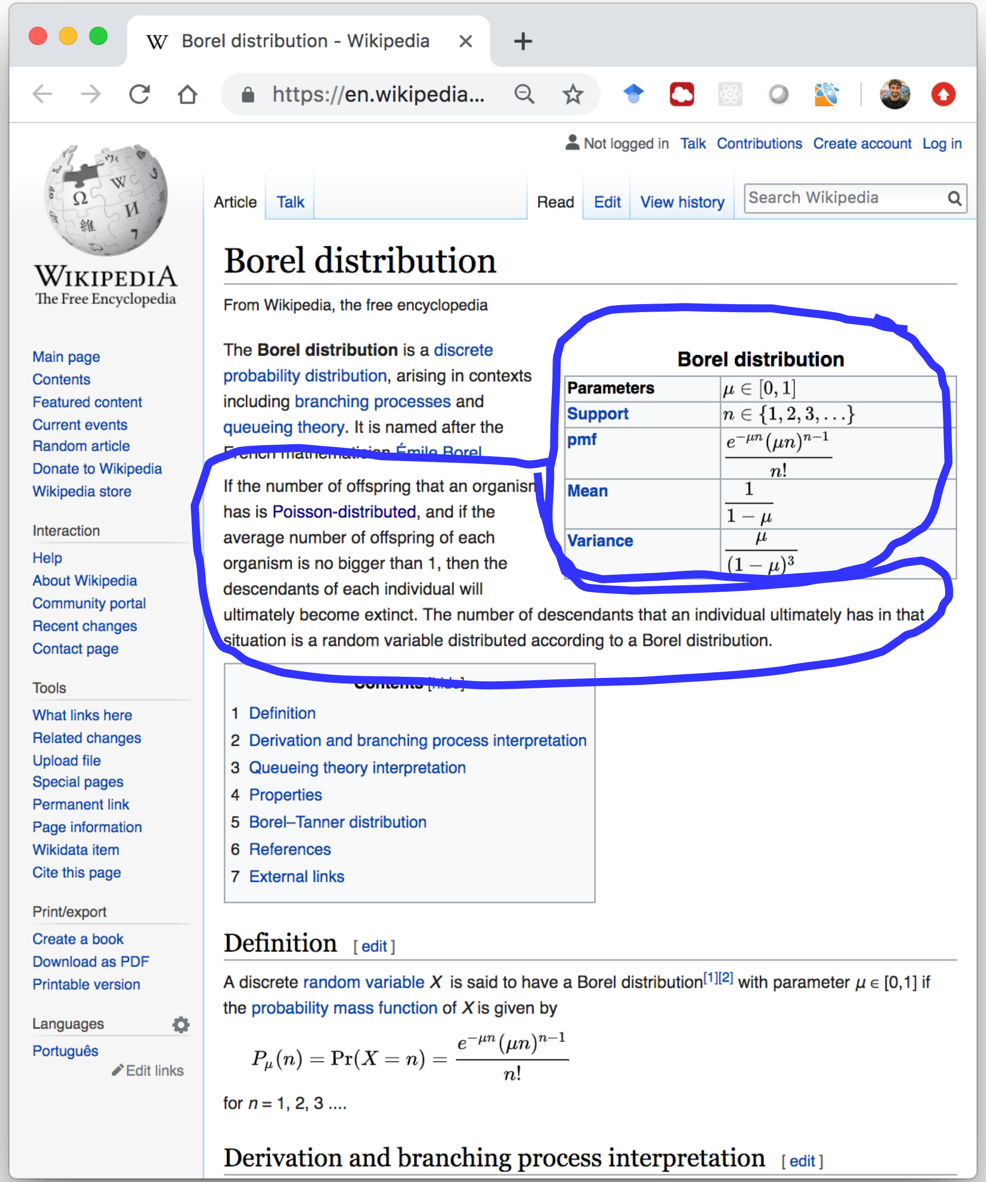
Don't have to derive all of the following distributions.
We want you to get a sense of how random variables work.

Goal: Be Able to Use a New Random Variable

You are learning about servers…

You read about the MD1 queue…

You find a paper that says the server "busy period" is distributed as a Borel with parameter μ = 0.2 …



Here are a few more Random Variables

	number of successes	time to get successes	
One trial	$X \sim \mathrm{Ber}(p)$	$X \sim \mathrm{Geo}(p)$	One success
	↑ $n = 1$	↑ $r = 1$	
Several trials	$X \sim \mathrm{Bin}(n, p)$	$X \sim \mathrm{NegBin}(r, p)$	Several successes
Interval of time	$X \sim \mathrm{Poi}(\lambda)$	$X \sim \mathrm{Exp}(\lambda)$	One success

Geometric Random Variable

X is **Geometric** Random Variable: X ~ Geo(*p*)

- X is number of independent trials until first success
- *p* is probability of success on each trial
- X takes on values 1, 2, 3, …, with probability:

$$P(X = n) = (1-p)^{n-1}p$$

$$E[X] = \frac{1}{p}$$

$$\text{Var}(X) = \frac{1-p}{p^2}$$

Negative Binomial Random Variable

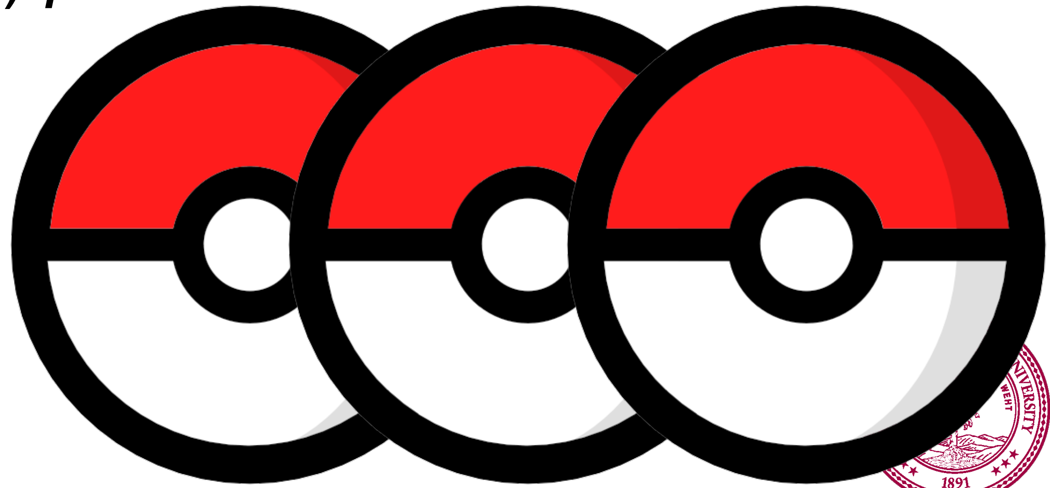
X is **<u>Negative Binomial</u>** RV: X ~ NegBin(r, p)

- X is number of independent trials until r successes
- p is probability of success on each trial
- X takes on values r, r + 1, r + 2…, with probability:

$$P(X = n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}, \text{ where } n = r, r+1, \ldots$$

- E[X] = r/p Var(X) = $r(1 - p)/p^2$

Note: Geo(p) ~ NegBin(1, p)



Geometric and Negative Binomial

Geometric Random Variable

Notation: $X \sim \text{Geo}(p)$

Description: Number of experiments until a success. Assumes independent experiments each with probability of success p.

Parameters: $p \in [0, 1]$, the probability that a single experiment gives a "success".

Support: $x \in \{1, \ldots, \infty\}$

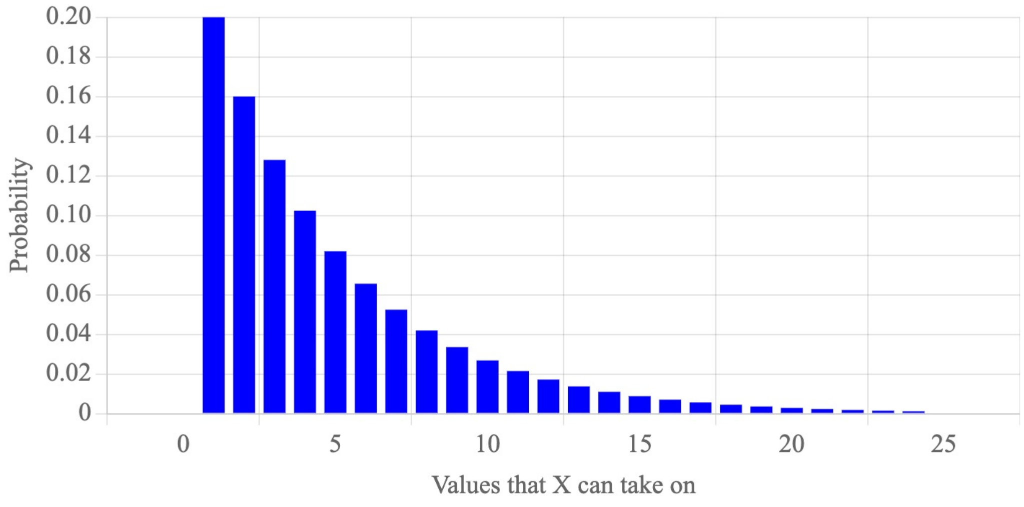
PMF equation: $P(X = x) = (1-p)^{x-1}p$

Expectation: $E[X] = \frac{1}{p}$

Variance: $\text{Var}(X) = \frac{1-p}{p^2}$

PMF graph:

Parameter p: 0.20



Negative Binomial Random Variable

Notation: $X \sim \text{NegBin}(r, p)$

Description: Number of experiments until r successes. Assumes each experiment is independent with probability of success p.

Parameters: $r > 0$, the number of success we are waiting for.
$p \in [0, 1]$, the probability that a single experiment gives a "success".

Support: $x \in \{r, \ldots, \infty\}$

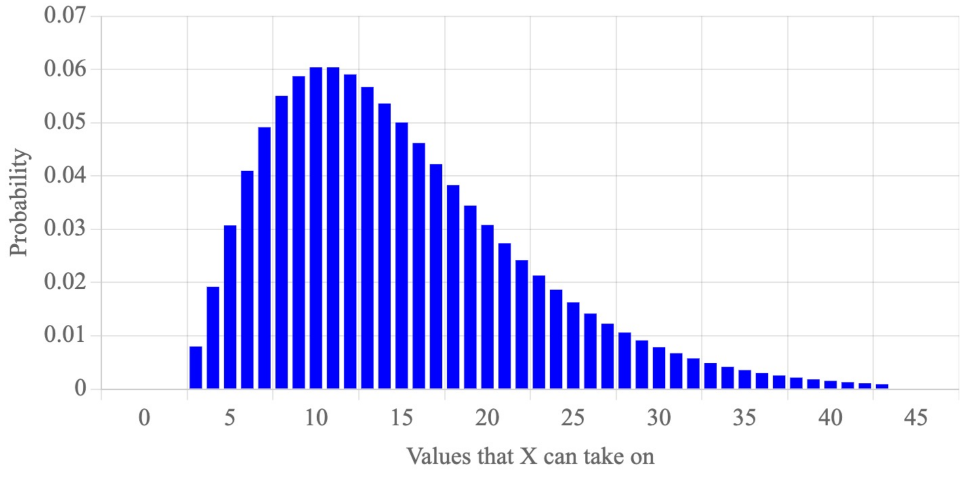
PMF equation: $P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$

Expectation: $E[X] = \frac{r}{p}$

Variance: $\text{Var}(X) = \frac{r \cdot (1-p)}{p^2}$

PMF graph:

Parameter r: 3 Parameter p: 0.20



Discrete Distributions

Bernoulli:

- indicator of coin flip X ~ Ber(*p*)

Binomial:

- # successes in *n* coin flips X ~ Bin(*n*, *p*)

Poisson:

- # successes in a fixed interval of time X ~ Poi(λ)

Geometric:

- # coin flips until success X ~ Geo(*p*)

Negative Binomial:

- # trials until *r* successes X ~ NegBin(*r*, *p*)

Dating at Stanford

Each person you date has a 0.2 probability of being someone you spend your life with. What is the average number of people one will date? What is the standard deviation? **Your meta goal: what steps would you take to answer this question?**

Equity in the Courts

Berghuis v. Smith

If a group is underrepresented in a jury pool, how do you tell?

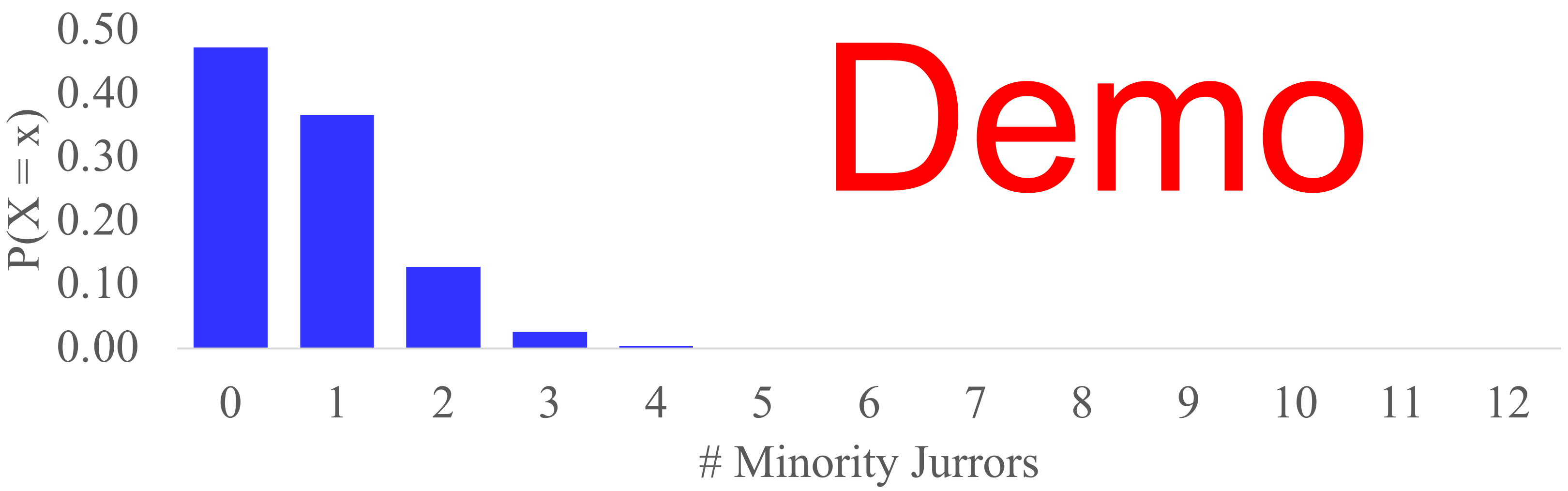
Justice Breyer [Stanford Alum] opened the questioning by invoking the binomial theorem. He hypothesized a scenario involving **"an urn with a thousand balls, and sixty are blue, and nine hundred forty are purple, and then you select them at random… twelve at a time."** According to Justice Breyer and the binomial theorem, if the purple balls were under represented jurors then **"you would expect… something like <u>a third to a half</u> of juries would have at least one minority person"** on them.

Justin Breyer Meets CS109

Approximation using Binomial distribution

- Assume P(blue ball) constant for every draw = 60/1000
- X = # blue balls drawn. $X \sim \text{Bin}(12, 60/1000 = 0.06)$
- $P(X \geq 1) = 1 - P(X = 0) \approx 1 - 0.4759 = 0.5240$

In Breyer's description, should actually expect just <u>over half</u> of juries to have at least one non-white person on them



Demo

Bitcoin Mining

SHA-256 Hash(Data *Fixed* Salt *Choice*

Number that looks like random bits

You “mine a bitcoin” if, for given data D, you find a salt number N such that Hash(D, N) produces a string that starts with g zeroes.

You "mine a bitcoin" if, for given data D, you find a number N such that Hash(D, N) produces a string that starts with g zeroes.

(a) What is the probability that the first number you try will produce a bit string which starts with g zeroes (in other words you mine a bitcoin)?

(b) How many different numbers do you expect to have to try until you mine five bitcoins?

You “mine a bitcoin” if, for given data D, you find a number N such that Hash(D, N) produces a string that starts with g zeroes.

(a) What is the probability that the first number you try will produce a bit string which starts with g zeroes (in other words you mine a bitcoin)?

$$\frac{1}{2}^{g}$$

Call this answer p_a

(b) How many different numbers do you expect to have to try until you mine five bitcoins?

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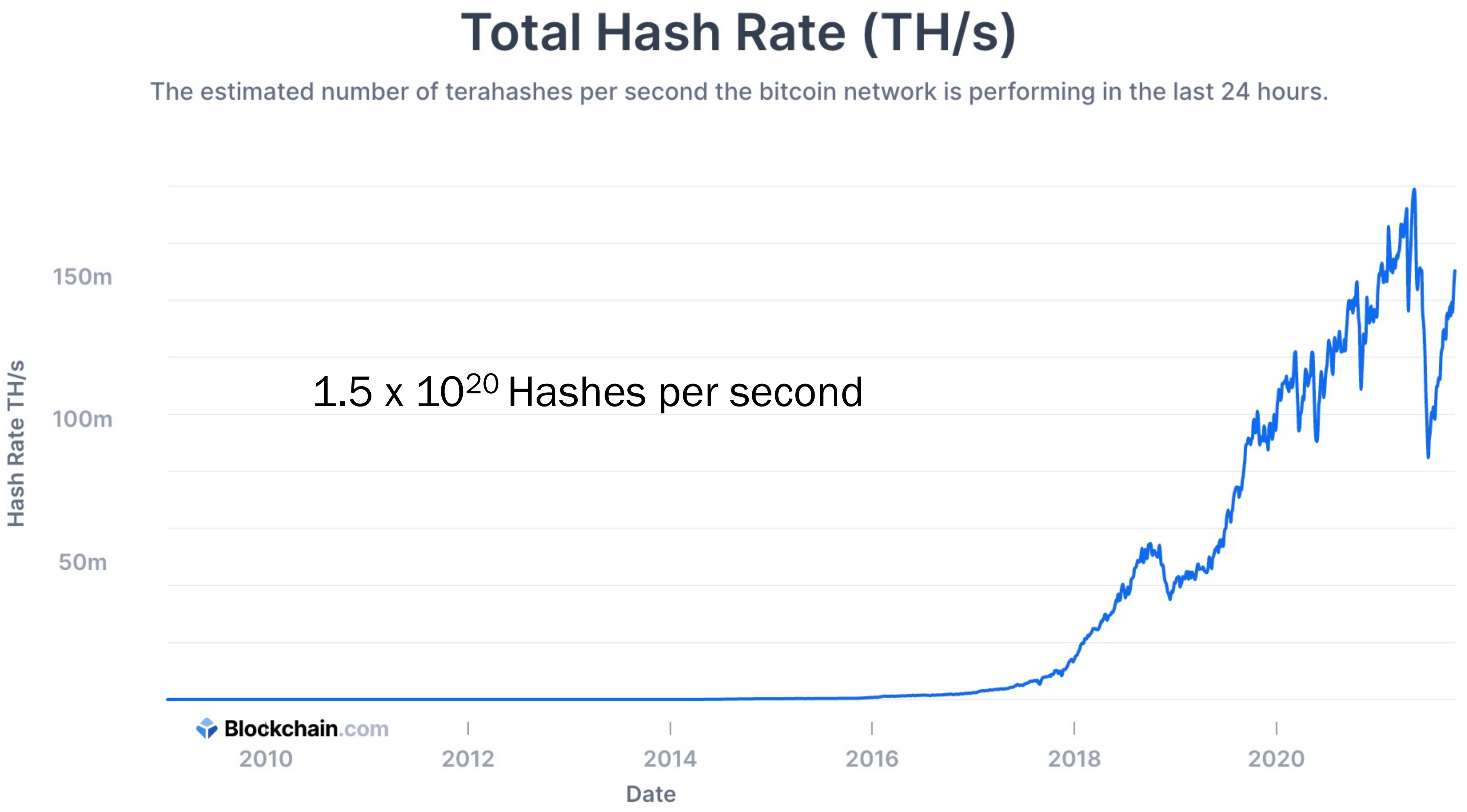
(b) How many different numbers do you expect to have to try until you mine five bitcoins?

Let Y be the number of tries until you mine 5 bitcoins. $Y \sim \text{NegBin}(r = 5, p = p_a)$

$$\text{E}[Y] = \frac{r}{p} = \frac{5}{p_a}$$

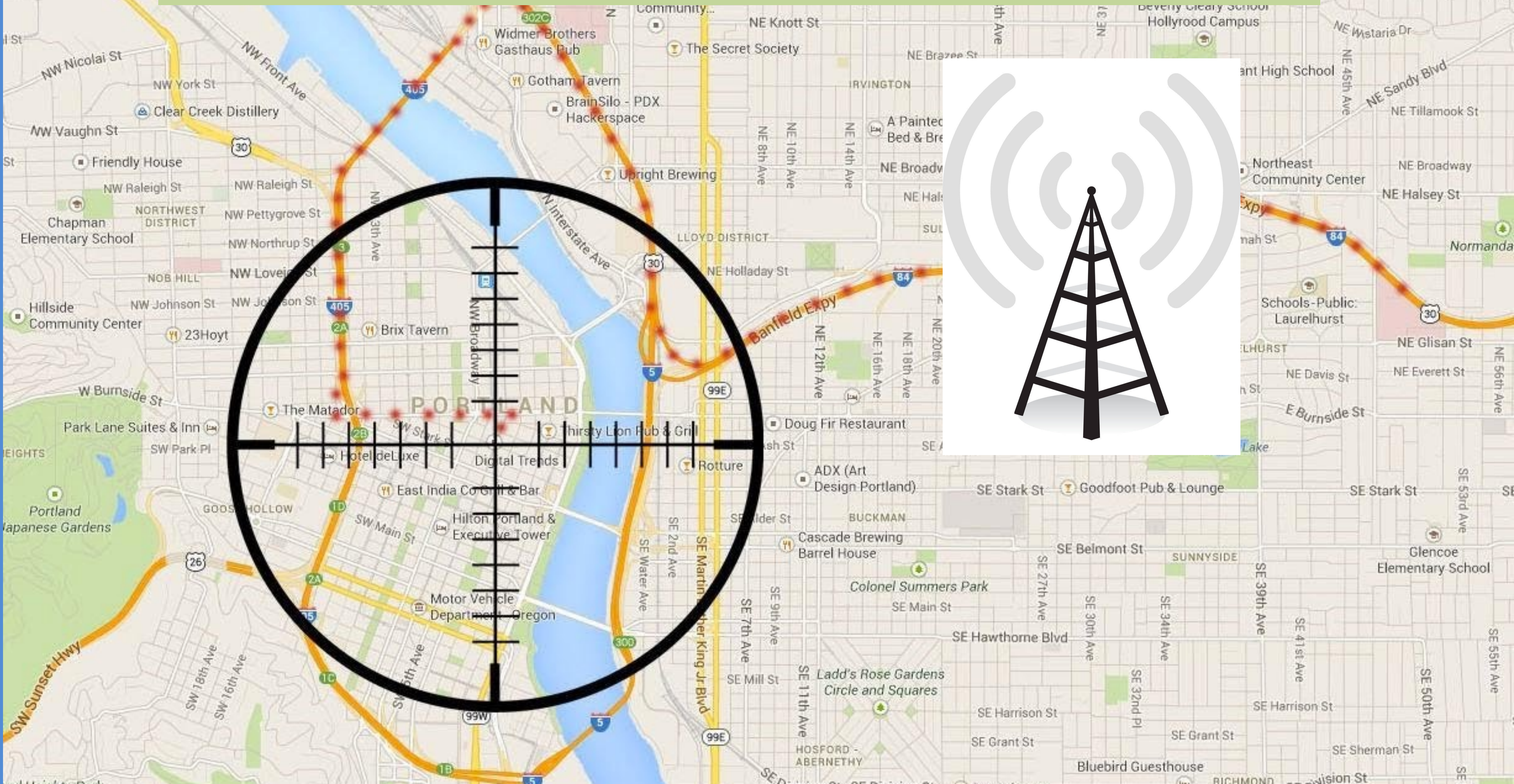
Bitcoin Mining Real Life



Pedagogic Pause

Big hole in our knowledge

Not all values are discrete



`random()?`

Poisson

Say the average rate of earthquakes is 1 every 100 years.

We can talk about the probability distribution of different numbers of earthquakes next year.

We can't talk about the probability distribution of the amount of time until the next earthquake.

Riding the Marguerite

Riding the Marguerite

You are running to the bus stop. You don't know exactly when the bus arrives. You have a distribution of probabilities.

You show up at 2:20pm.

What is P(wait < 5 minutes)?

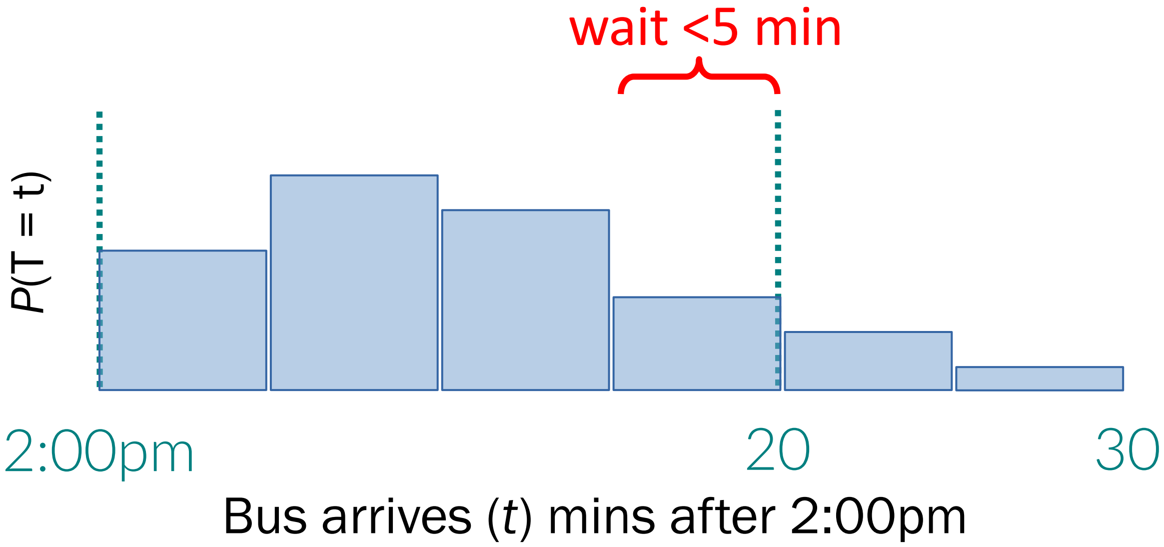
What is the probability that the bus arrives at:
2:17pm and 12.12333911102389234 seconds?

Riding the Marguerite

You are running to the bus stop. You don't know exactly when the bus arrives. You have a distribution of probabilities.

You show up at 2:15pm.

What is P(wait < 5 minutes)?



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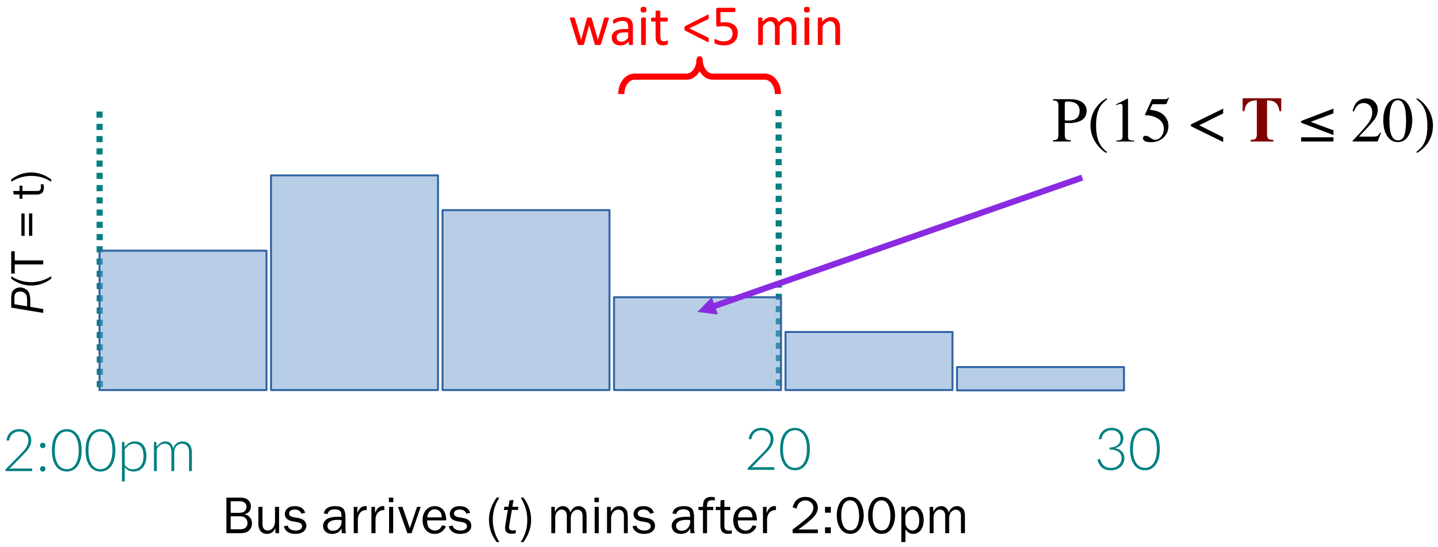

image: Haha169

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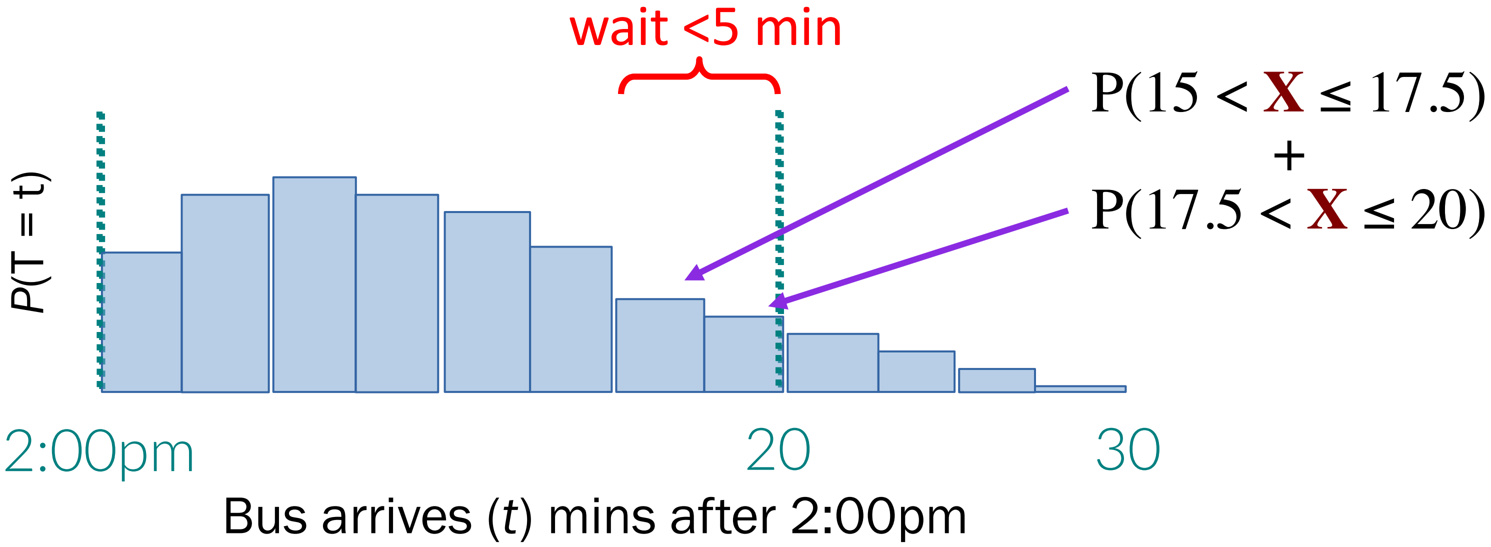

image: [Haha169]()

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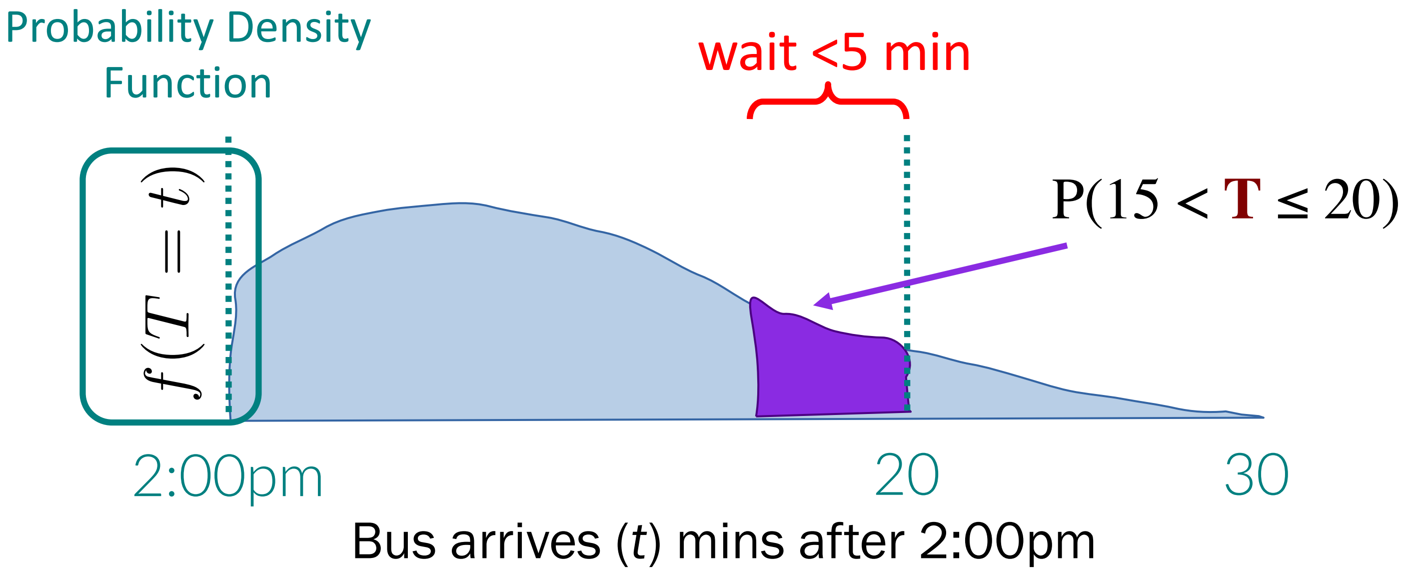

image: [Haha169]

Probability Density Function

The **probability density function** (PDF) of a continuous random variable represents the relative likelihood of various values.

Units of probability *divided by units of X*.
Integrate it to get probabilities!

$$P(a < X < b) = \int_{x=a}^{b} \boxed{f(X = x)} dx$$

Probability Density Function

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Integrate it to get probabilities!

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This is another way to write the PDF

Probability Density Function

The **probability density function** (PDF) of a continuous random variable represents the relative likelihood of various values.

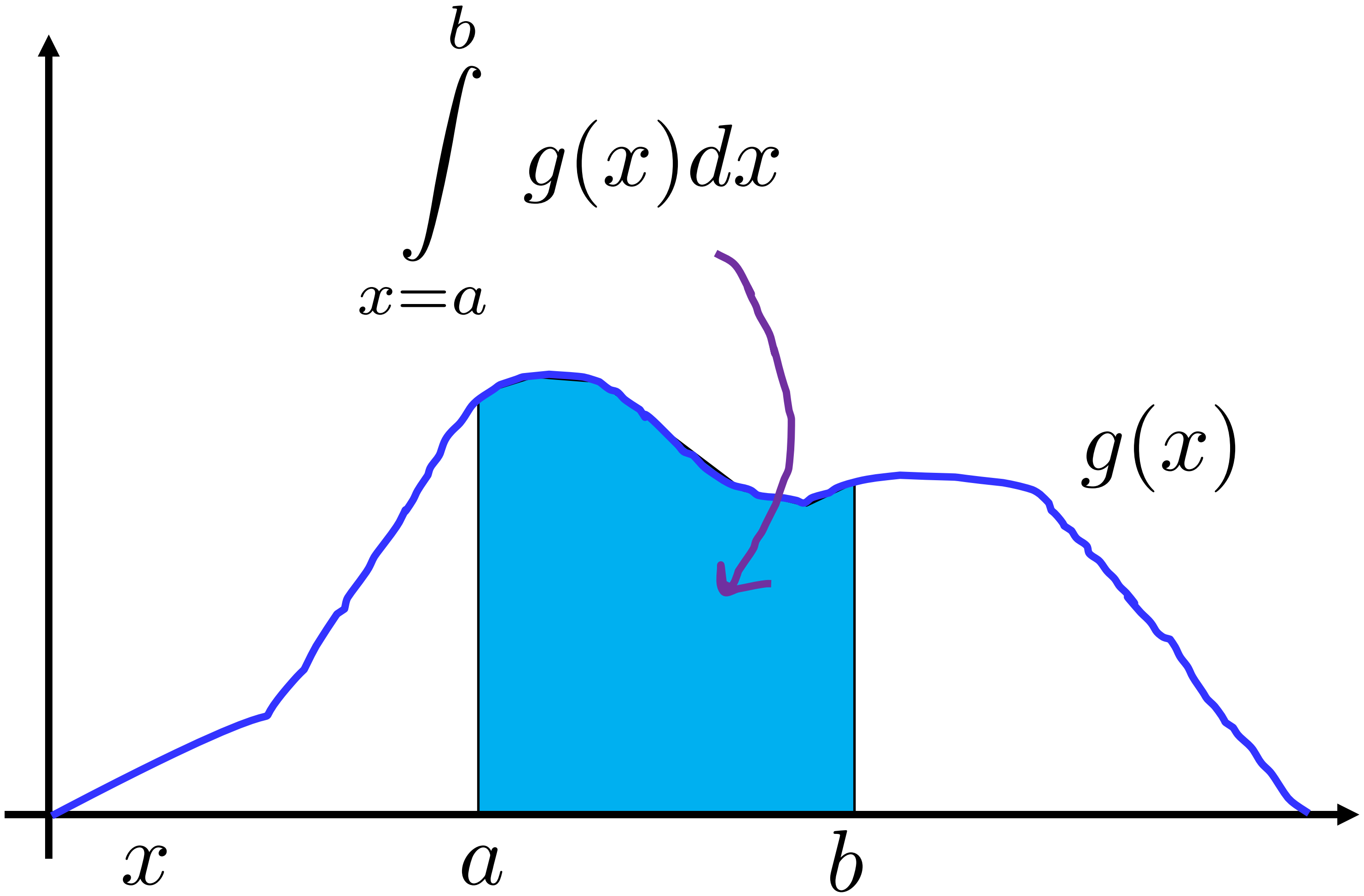
Units of probability *divided by units of X*.
Integrate it to get probabilities!

$$P(a < X < b) = \int_{x=a}^{b} f(X = x)dx$$

Integrals!

*loving, not scary

Integrals



Riding the Marguerite

You are running to the bus stop. You don't know exactly when the bus arrives. You have a distribution of probabilities.

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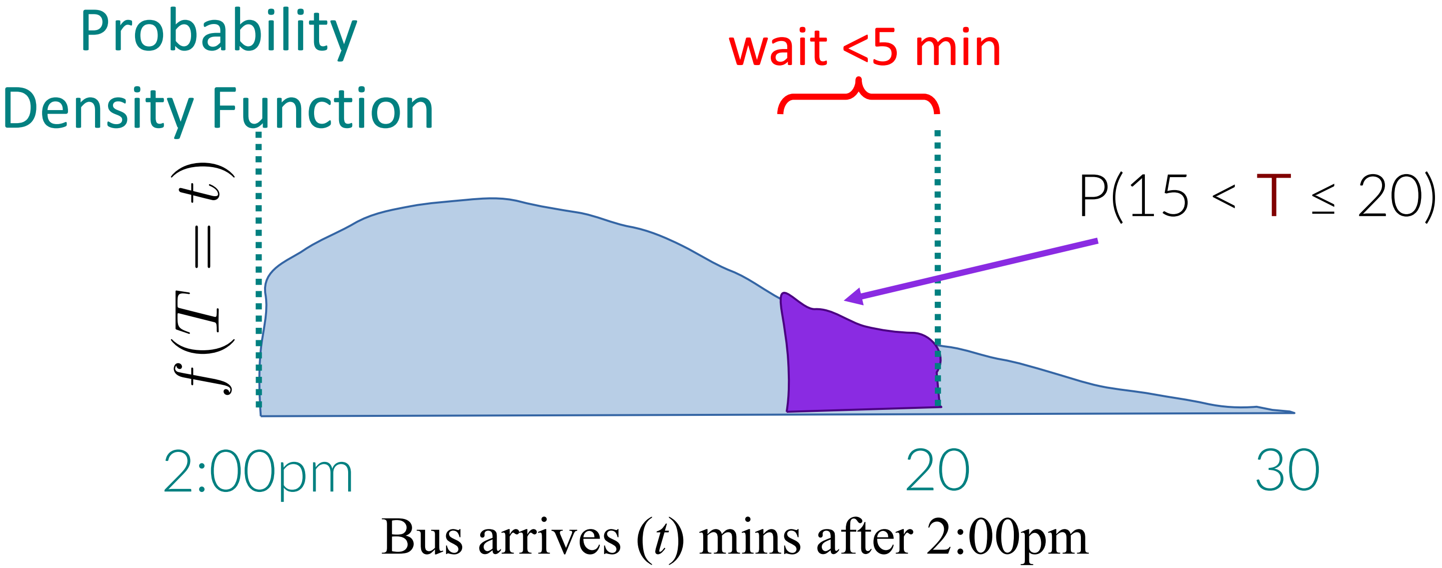


image: Haha169

Properties of PDFs

The integral of a PDF gives a probability. Thus:

$$0 \leq \int_{x=a}^{b} f(X = x)\, dx \leq 1$$

$$\int_{x=-\infty}^{\infty} f(X = x)\, dx = 1$$

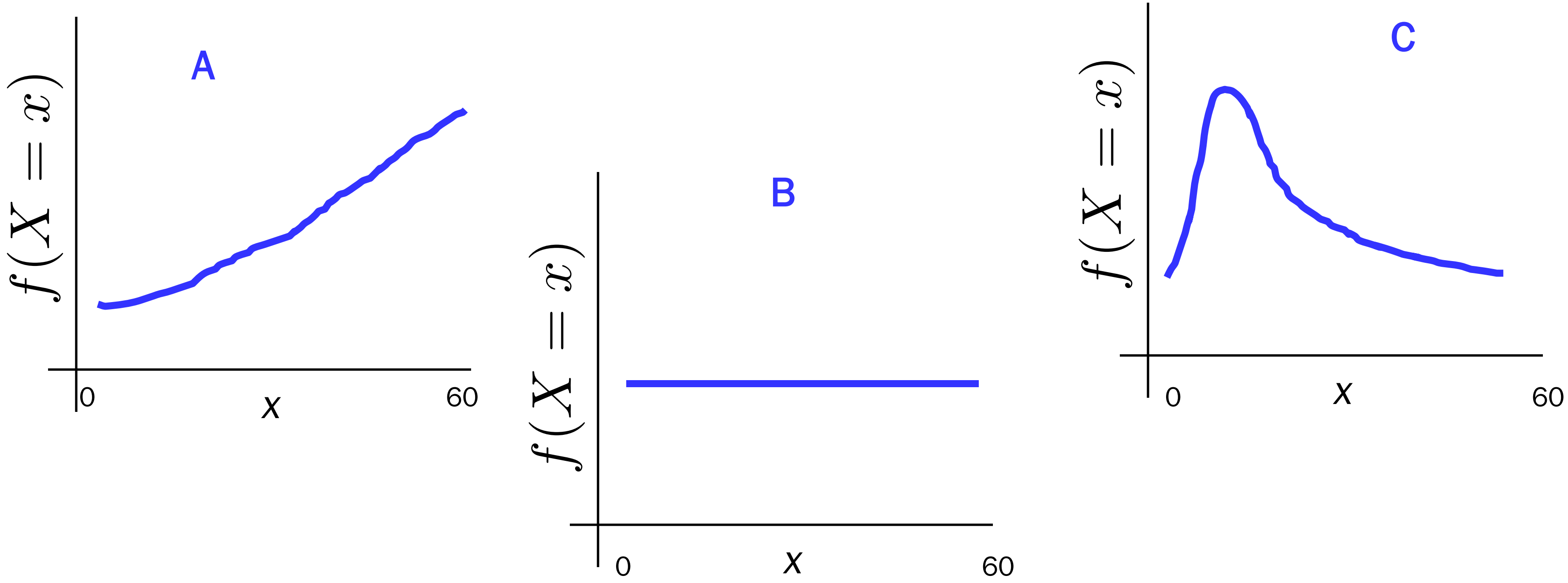
What do you get if you integrate over a probability *density* function?

A probability!

Probability Density Function

Probability density functions articulate ***relative*** belief.

Let X be a random variable which is the # of minutes after 2pm that the bus arrives at the stop:

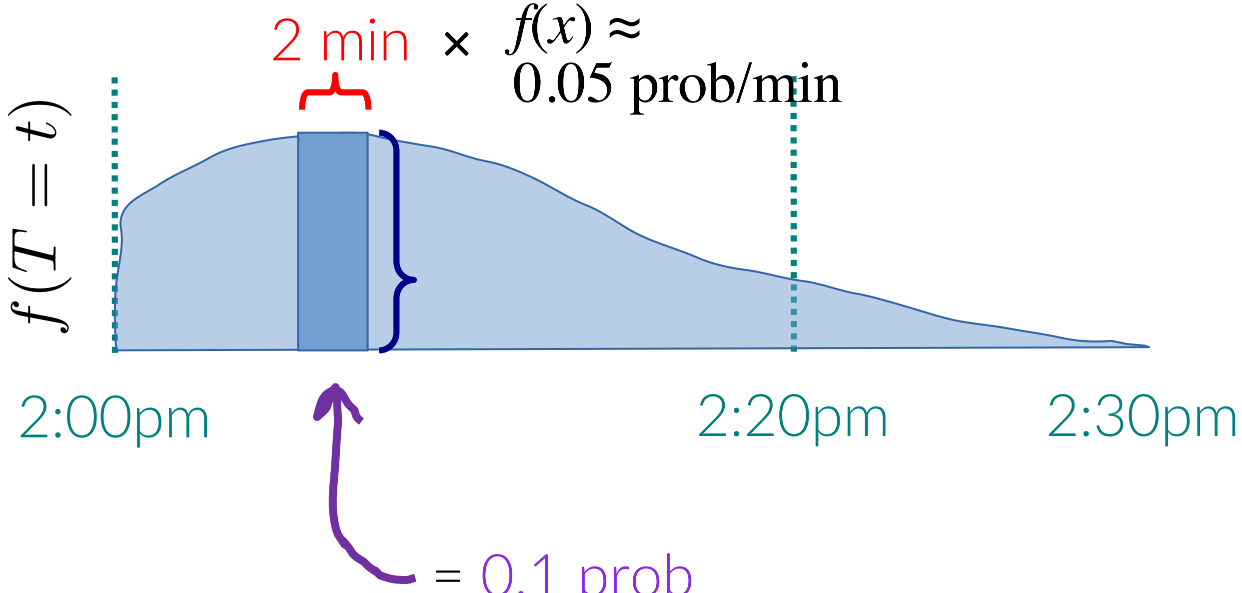


Which of these represent that you think the arrival is more likely to be close to 3:00pm

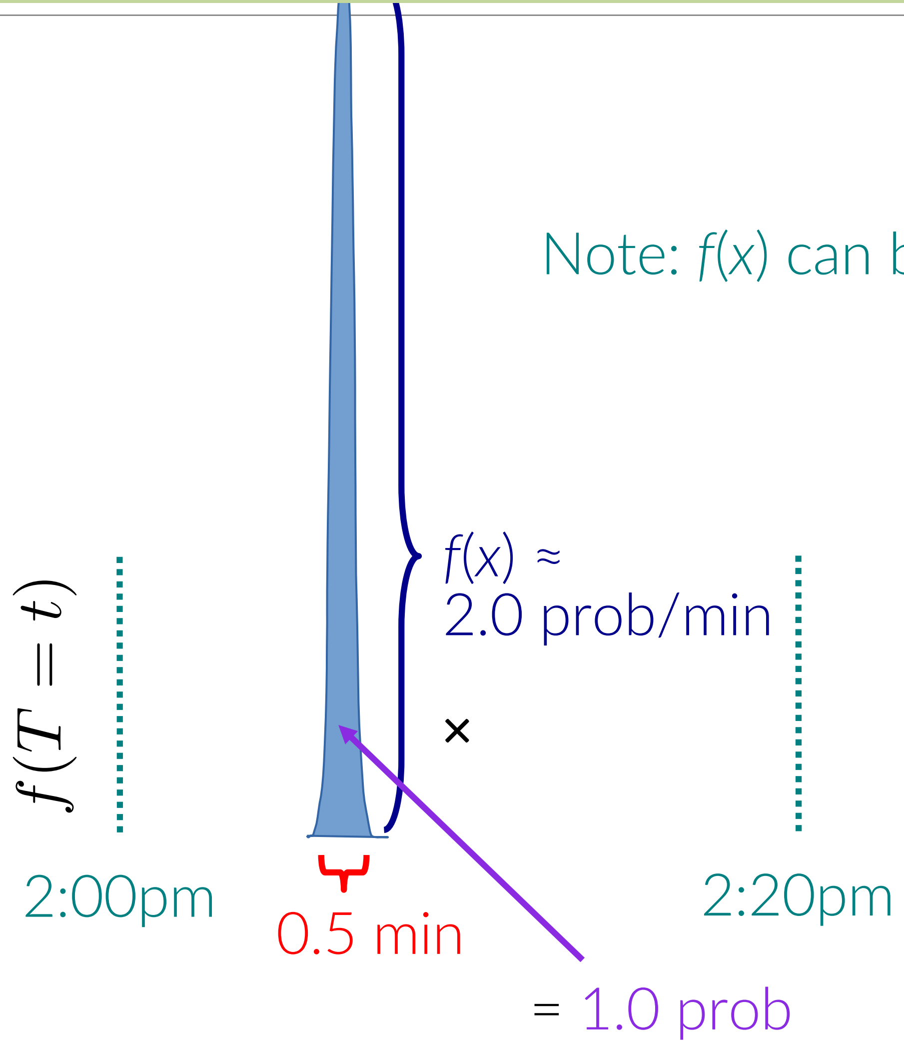
The ratio of probability densities is meaningful

$f(X = x)$ is *Not* a Probability

Rather, it has "units" of:
probability divided by units of X.



$f(X = x)$ is *Not* a Probability



Note: $f(x)$ can be **greater than 1**!

$f(X = x)$ vs $P(X = x)$

"The probability that a discrete random variable X takes on the value little *x*."

$$P(X = x)$$

Aka the PMF

"The *derivative* of the probability that a continuous random variable X takes on the value little *x*."

$$f(X = x)$$

Aka the PDF

They are both measures of how likely X is to take on the value x.
Sometimes called the distribution function. Sometimes called the likelihood functions.

Quantum Example

Consider a random 5000x5000 matrix, where each element in the matrix is Uniform(0,1). What is the probability that a selected eigenvalue (λ) of the matrix is greater than 0?*

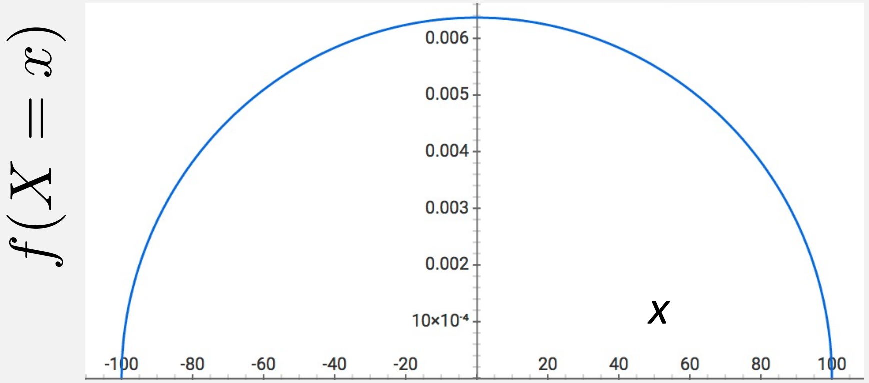
* With help from Wigner, Chris is going to rephrase this problem

Rephrased as a Standard Continuous Problem

Let X be a continuous random variable[1]

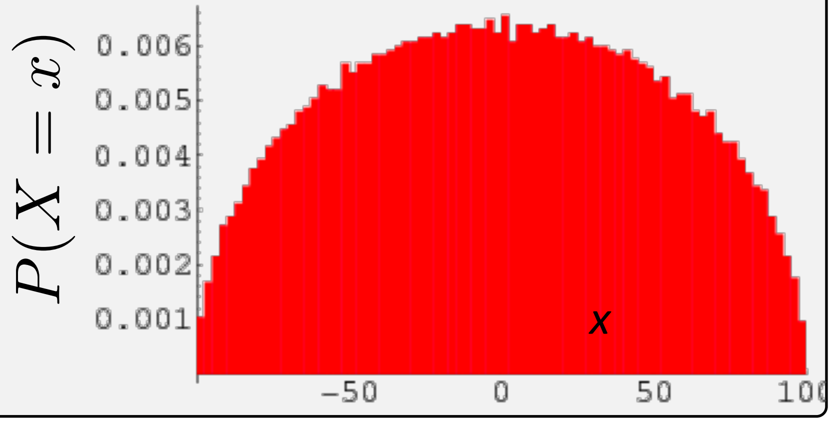
Theory

$$f(X = x) = \frac{1}{15708}\sqrt{100^2 - x^2}$$



Practice

From simulations



$$P(X > 0) = ?$$

[1] X represents the eigenvalue of a 5000x5000 matrix of uniform random variables

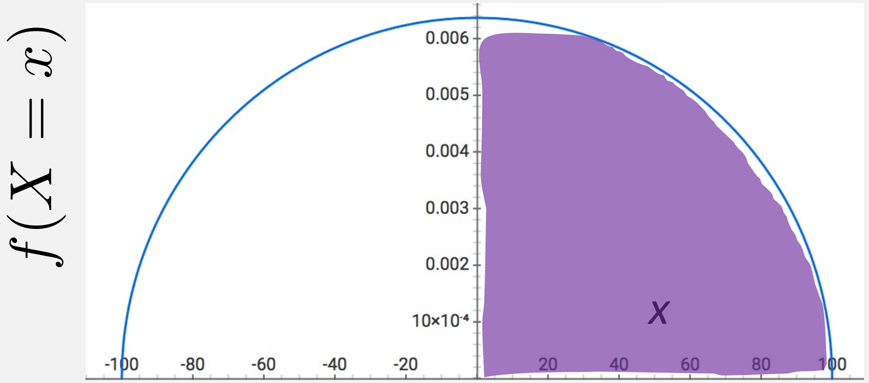
https://en.wikipedia.org/wiki/Wigner_semicircle_distribution

Simple Example from Quantum Physics

Let X be a continuous random variable[1]

Theory

$$f(X = x) = \frac{1}{15708}\sqrt{100^2 - x^2}$$



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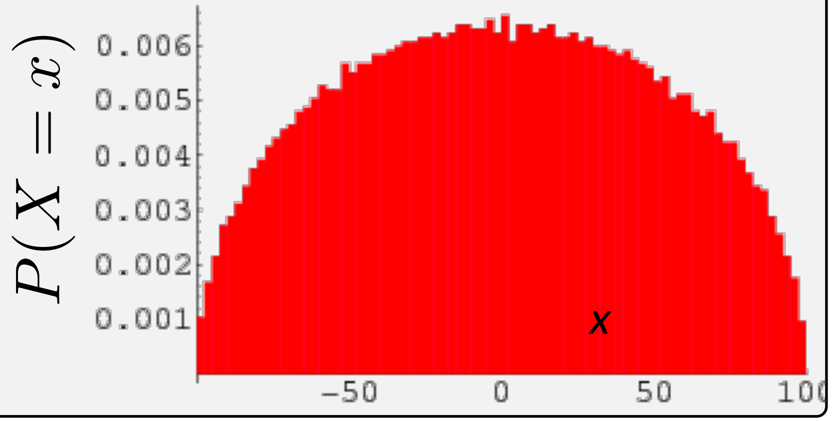


Approach #1: Integrate over the PDF

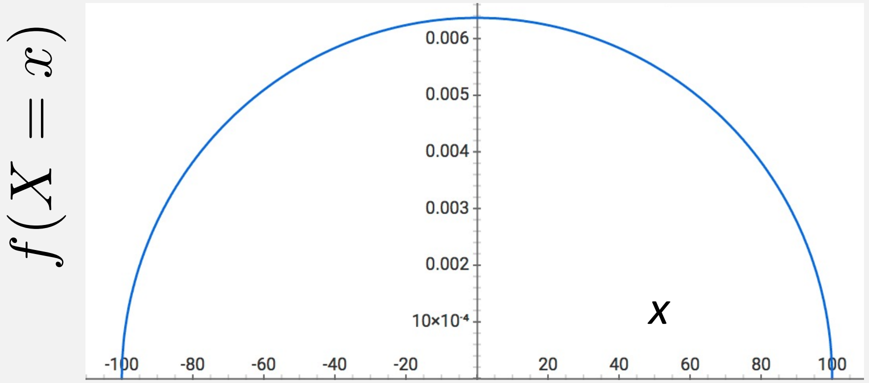
$$P(X > 0) = \int_{0}^{100} f(X = x)\, dx$$

Simple Example from Quantum Physics

Let X be a continuous random variable[1]

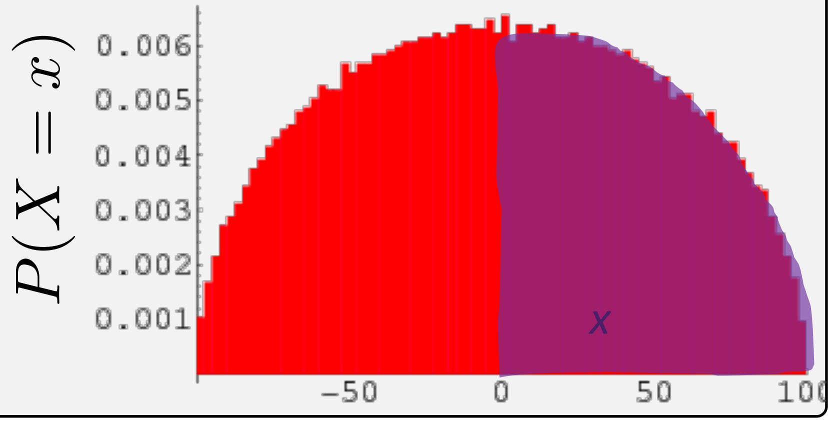
Theory

$$f(X = x) = \frac{1}{15708}\sqrt{100^2 - x^2}$$



Practice

From simulations



Approach #2: Discrete Approximation

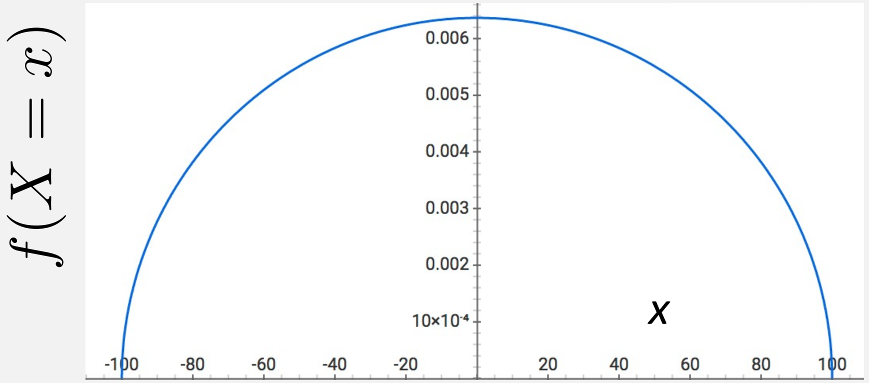
$$P(X > 0) \approx \sum_{i=0}^{100} P(X = i)$$

Simple Example from Quantum Physics

Let X be a continuous random variable[1]

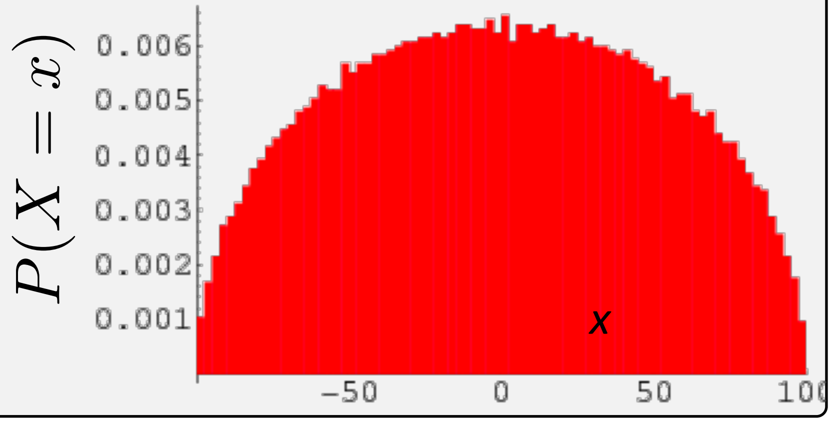
Theory

$$f(X = x) = \frac{1}{15708}\sqrt{100^2 - x^2}$$



Practice

From simulations



Approach #3: Know Semi-Circles

$$P(X > 0) = \frac{1}{2}$$

What do you get if you integrate over a probability *density* function?

A probability!

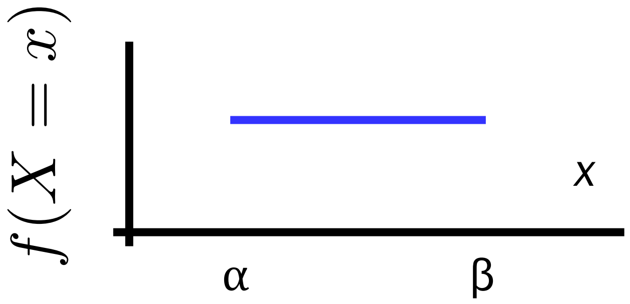
Uniform Random Variable

A **uniform** random variable is **equally likely** to be any value in an interval.

$$X \sim \text{Uni}(\alpha, \beta)$$

Probability Density

$$f(X = x) = \begin{cases} \frac{1}{\beta - \alpha} & \alpha \le x \le \beta \\ 0 & \text{otherwise} \end{cases}$$



Properties

$$E[X] = \frac{\beta - \alpha}{2}$$

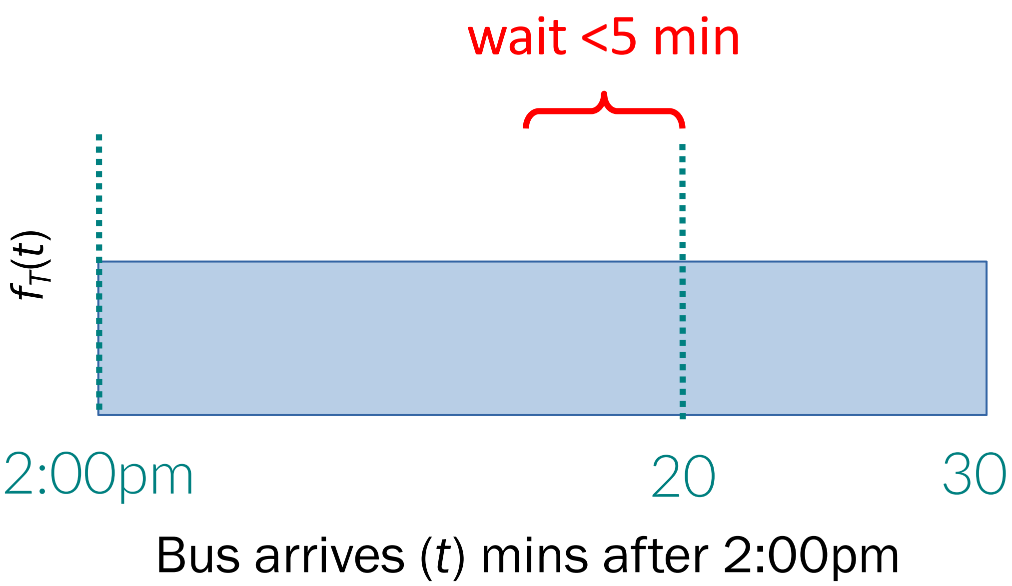
$$\text{Var}(X) = \frac{(\beta - \alpha)^2}{12}$$

Uniform Bus

You are running to the bus stop. You don't know exactly when the bus arrives. **You believe all times between 2 and 2:30 are equally likely.**

You show up at 2:15pm. What is P(wait < 5 minutes)?

$$T \sim \text{Uni}(\alpha = 0, \beta = 30)$$



$$P(\text{Wait} < 5) = \int_{15}^{20} \frac{1}{\beta - \alpha}\, dx$$

$$= \frac{x}{\beta - \alpha}\Big|_{15}^{20}$$

$$= \frac{x}{30 - 0}\Big|_{15}^{20} = \frac{5}{30}$$

Expectation and Variance

For <u>discrete</u> RV *X*:

$$E[X] = \sum_x x \cdot p(X = x)$$

$$E[g(X)] = \sum_x g(x) \cdot p(X = x)$$

$$E[X^n] = \sum_x x^n \cdot p(X = x)$$

For <u>continuous</u> RV *X*:

$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \cdot f_X(x)$$

$$E[X^n] = \int_{-\infty}^{\infty} x^n \cdot f_X(x)$$

For <u>both</u> discrete and continuous RVs:

$$E[aX + b] = aE[X] + b$$

$$\text{Var}(X) = E[(x - \mu)^2] = E[X^2] - (E[X]^2)$$

$$\text{Var}(aX + b) = a^2\text{Var}(X)$$

Expectation of Uniform $X \sim \text{Uni}(\alpha, \beta)$

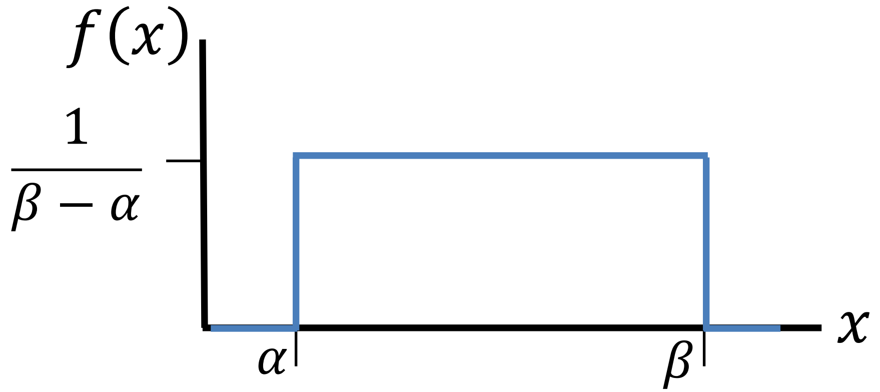
$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\ dx$$

$$= \int_{\alpha}^{\beta} x \cdot \frac{1}{\beta - \alpha}\ dx$$

$$= \frac{1}{\beta - \alpha}\left[\frac{1}{2}x^2\right]_{\alpha}^{\beta}$$

$$= \frac{1}{\beta - \alpha}\left[\frac{\beta^2}{2} - \frac{\alpha^2}{2}\right]$$

$$= \frac{1}{2}\frac{1}{\beta - \alpha}(\beta + \alpha)(\beta - \alpha)$$



just average
the start
and end!

$$= \frac{1}{2}(\alpha + \beta)$$

Exponential Random Variable

Consider an experiment that lasts a duration of time until success occurs.

<u>def</u> An **Exponential** random variable X is the amount of time until success.

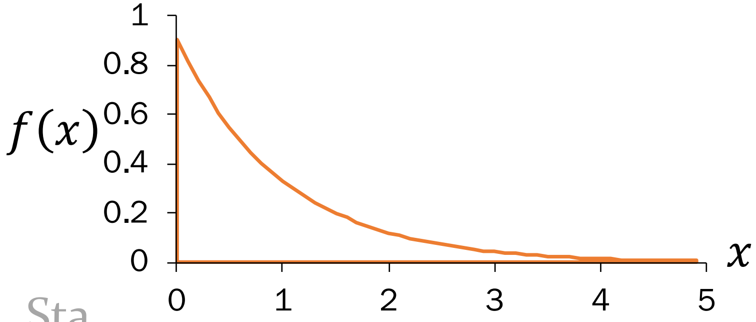
$$X \sim \text{Exp}(\lambda)$$

Support: $[0, \infty)$

PDF	$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$
Expectation	$E[X] = \frac{1}{\lambda}$
Variance	$\text{Var}(X) = \frac{1}{\lambda^2}$

Examples:

- Time until next earthquake
- Time for request to reach web server
- Time until end of cell phone contract



1906 Earthquake
Magnitude 7.8

ILL. No. 65. MEMORIAL ARCH, WITH CHURCH IN BACKGROUND, STANFORD UNIVERSITY, SHOWING TYPES OF CARVED WORK WITH THE SANDSTONE

How Many Earthquakes

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the probability of **zero major earthquakes magnitude next year?**

X = Number of major earthquakes next year

$$X \sim \text{Poi}(\lambda = 0.002)$$

$$P(X = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = \frac{0.002^0 e^{-0.002}}{0!} \approx 0.998$$

*In California, according to the USGS, 2015 ty

How Long Until the Next Earthquake

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the probability of **a major earthquake in the next 30 years?**

Y = Years until the next earthquake of magnitude 8.0+

$$Y \sim \text{Exp}(\lambda = 0.002)$$

$$f_Y(y) = \lambda e^{-\lambda y}$$
$$= 0.002^{-0.002y}$$

$$P(Y < 30) = \int_0^{30} 0.002 e^{-0.002y} \, dy$$

*In California, according to the USGS, 2015

ty

Integral Review

$$\int e^{cx} \, dx = \frac{1}{c} e^{cx}$$

How Long Until the Next Earthquake

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the probability of **a major earthquake in the next 30 years?**

Y = Years until the next earthquake of magnitude 8.0+

$$Y \sim \text{Exp}(\lambda = 0.002)$$

$$f_Y(y) = \lambda e^{-\lambda y}$$
$$= 0.002^{-0.002y}$$

$$P(Y < 30) = \int_0^{30} 0.002 e^{-0.002y} \; dy$$
$$= 0.002 \Big[-500 e^{-0.002y} \Big]_0^{30}$$
$$= \frac{500}{500}(-e^{-0.06} + e^0) \qquad \approx 0.06$$

*In California, according to the USGS, 2015

How Long Until the Next Earthquake

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the **expected number of years until the next earthquake?**

Y = Years until the next earthquake of magnitude 8.0+

$$Y \sim \text{Exp}(\lambda = 0.002)$$

$$E[Y] = \frac{1}{\lambda} = \frac{1}{0.002} = 500$$

How Long Until the Next Earthquake

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the **standard deviation of years until the next earthquake?**

Y = Years until the next earthquake of magnitude 8.0+

$$Y \sim \text{Exp}(\lambda = 0.002)$$

$$\text{Var}(Y) = \frac{1}{\lambda^2} = \frac{1}{0.002^2} = 250,000 \text{ years}^2$$

$$\text{Std}(Y) = \sqrt{\text{Var}(X)} = 500 \text{ years}$$

*In California, according to the USGS, 2015

Is there a way to avoid integrals?

Cumulative Density Function

A cumulative density function (CDF) is a "closed form" equation for the probability that a random variable is less than a given value

$$F(x) = P(X < x)$$

If you learn how to use a cumulative density function, you can avoid integrals!

$$F_X(x)$$

This is also shorthand notation for the PMF

Cumulative Density Function

$$F(x) = P(X < x)$$

$$x = 2$$

$$0.03125$$

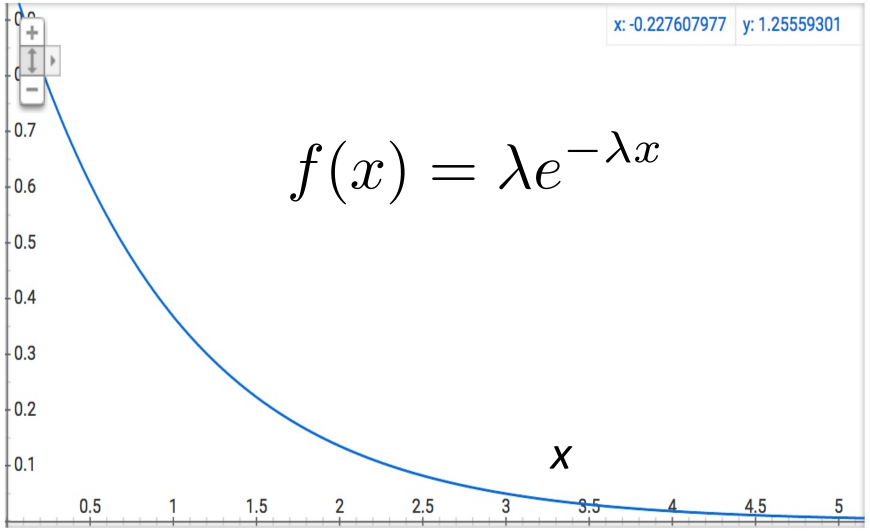
CDF of an Exponential

$$F_X(x) = 1 - e^{-\lambda x}$$

$$\begin{aligned} P(X < x) &= \int\limits_{y=-\infty}^{x} f(y)\, dy \\ &= \int\limits_{y=0}^{x} \lambda e^{-\lambda y}\, dy \\ &= \frac{\lambda}{\lambda}\Big[-e^{-\lambda y}\Big]_0^x \\ &= [-e^{-\lambda x}] - [-e^{\lambda 0}] \\ &= 1 - e^{-\lambda x} \end{aligned}$$

Using CDF Example. X is Exp(λ = 1)

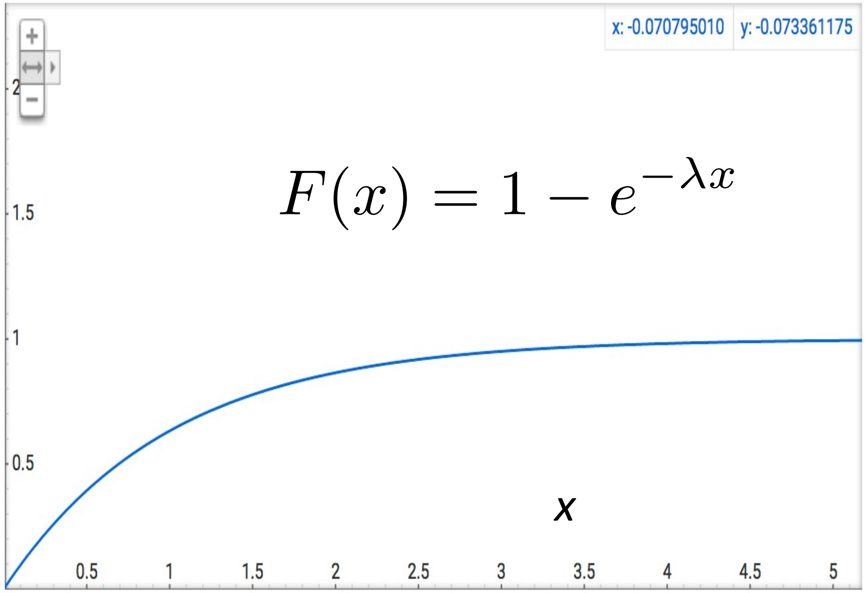
Probability density function



Cumulative density function

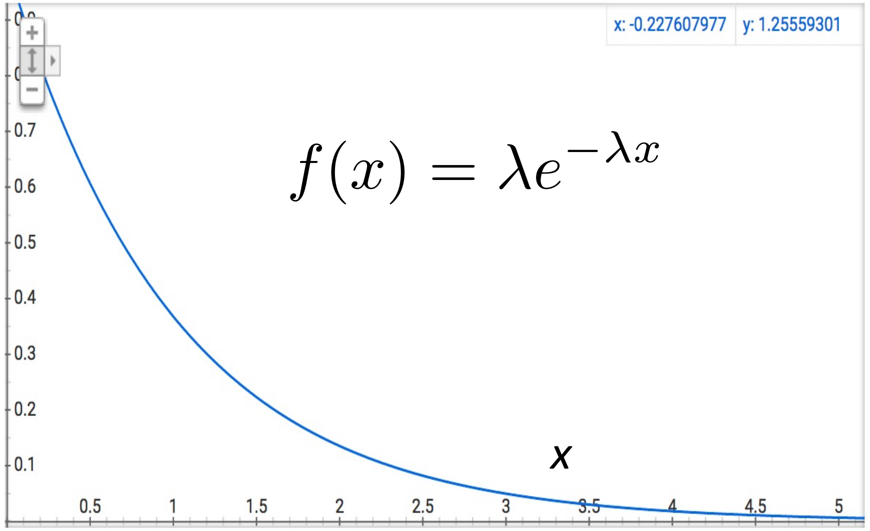
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp(λ = 1)

Probability
density
function

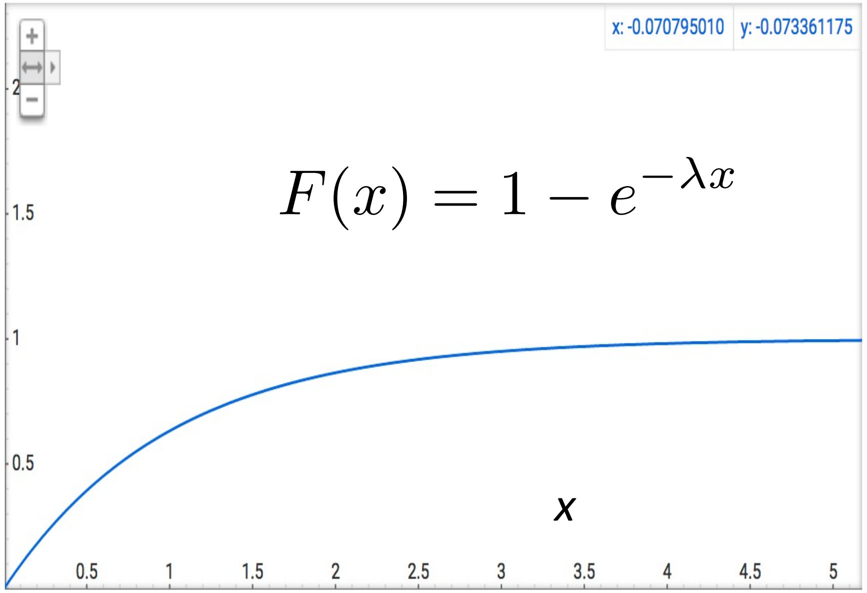


P(X < 2)

Cumulative
density function

$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp(λ = 1)

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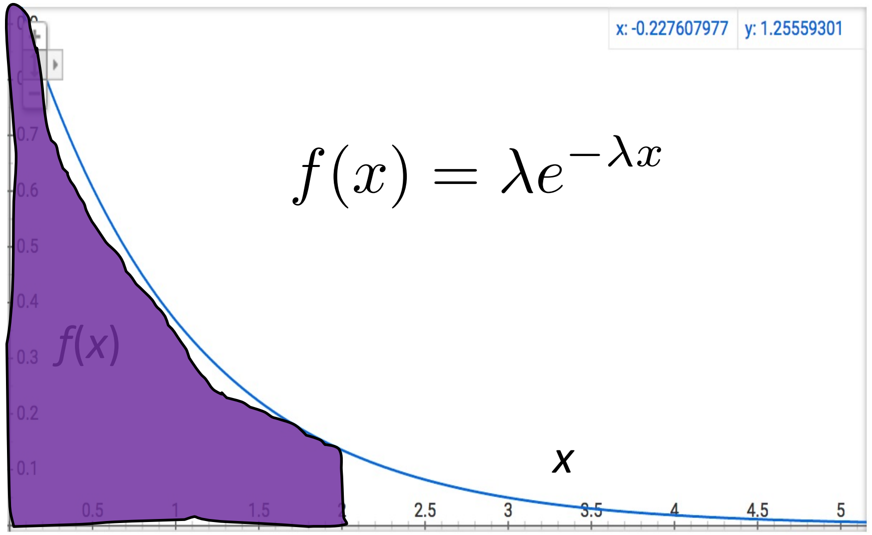


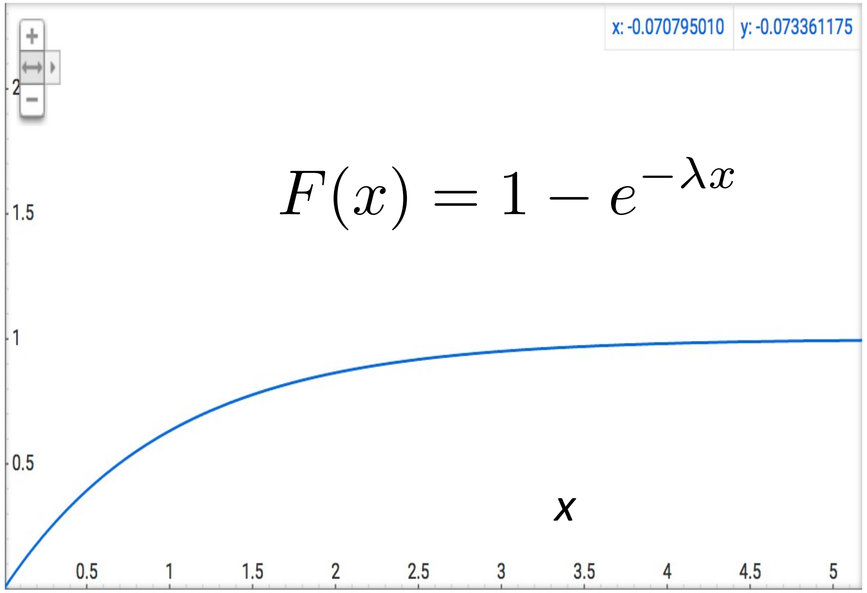
P(X < 2)

$$= \int_{x=-\infty}^{2} f(x)\, dx$$

Cumulative density function

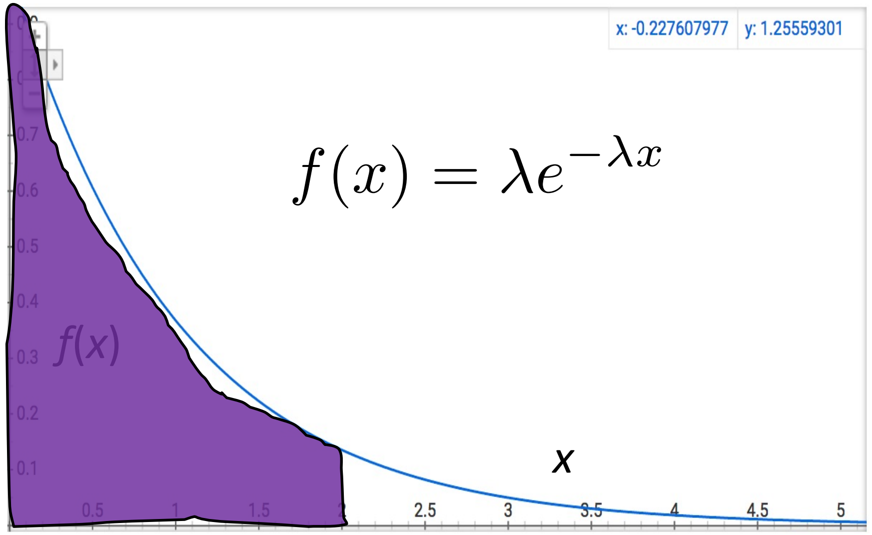
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp(λ = 1)

Probability density function



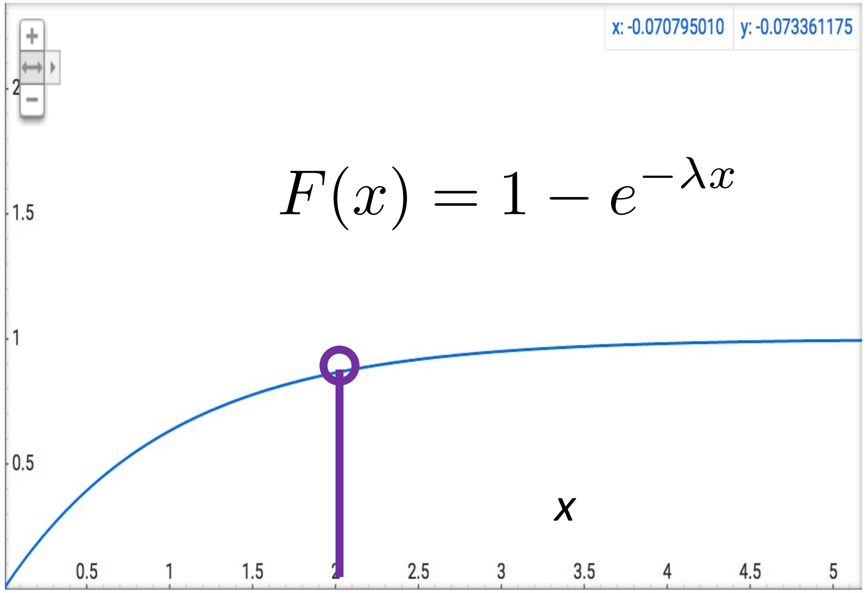
P(X < 2)

$$= \int_{x=-\infty}^{2} f(x)\, dx$$

Cumulative density function

$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



or

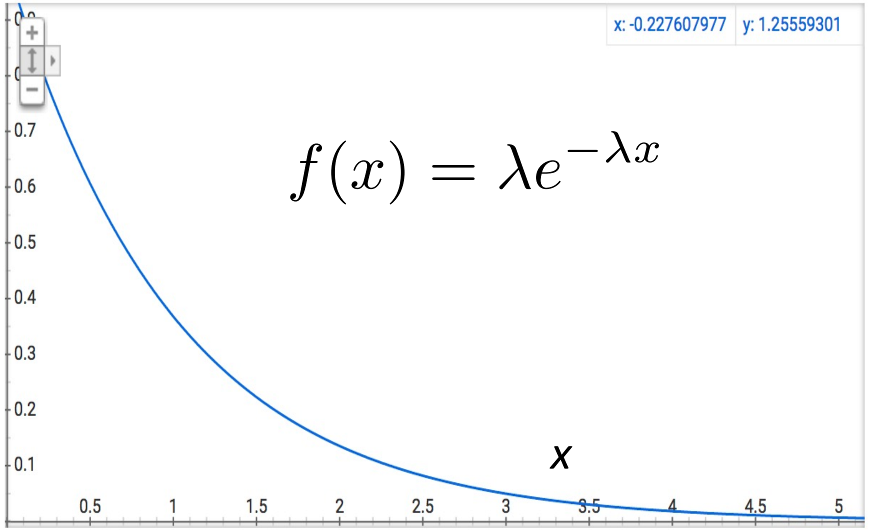
$$= F(2)$$

$$= 1 - e^{-2}$$

$$\approx 0.84$$

Using CDF Example. X is Exp(λ = 1)

Probability density function

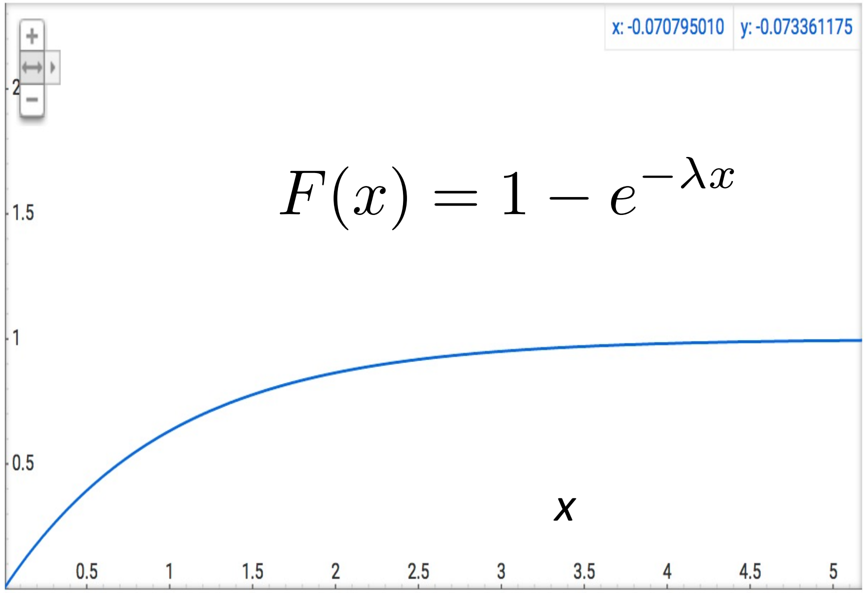


P(X > 1)

Cumulative density function

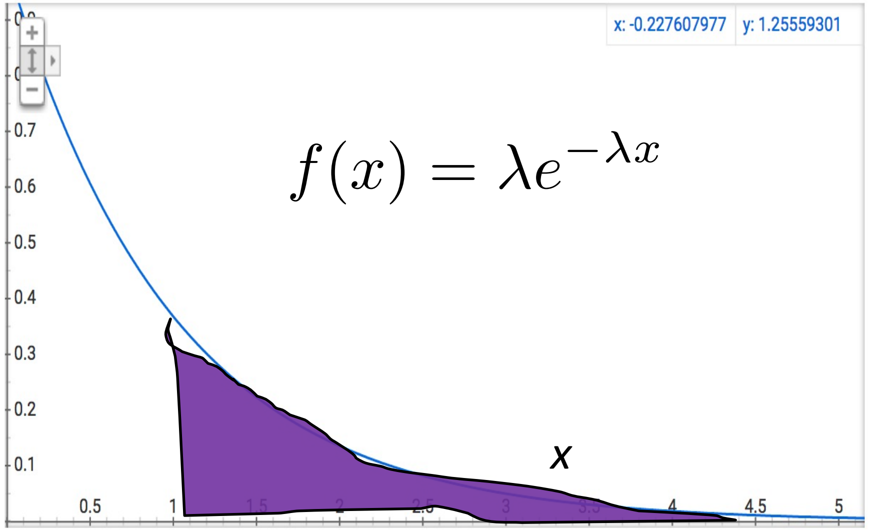
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp(λ = 1)

Probability
density
function



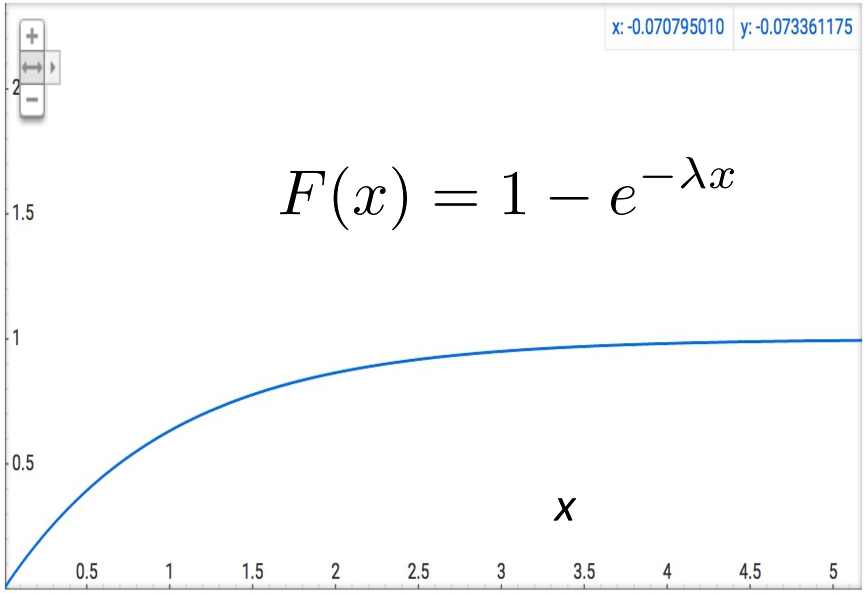
P(X > 1)

$$= \int_{x=1}^{\infty} f(x)\ dx$$

Cumulative
density function

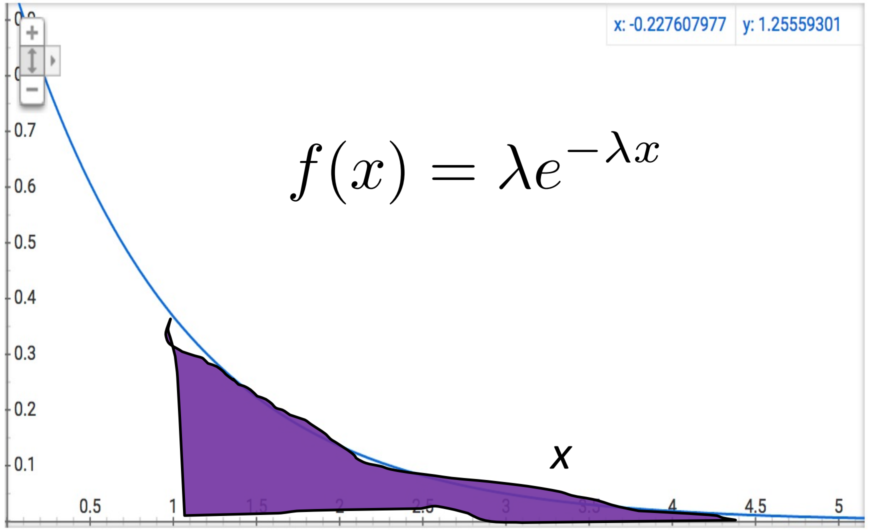
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\ dy$$



Using CDF Example. X is Exp(λ = 1)

Probability density function



P(X > 1)

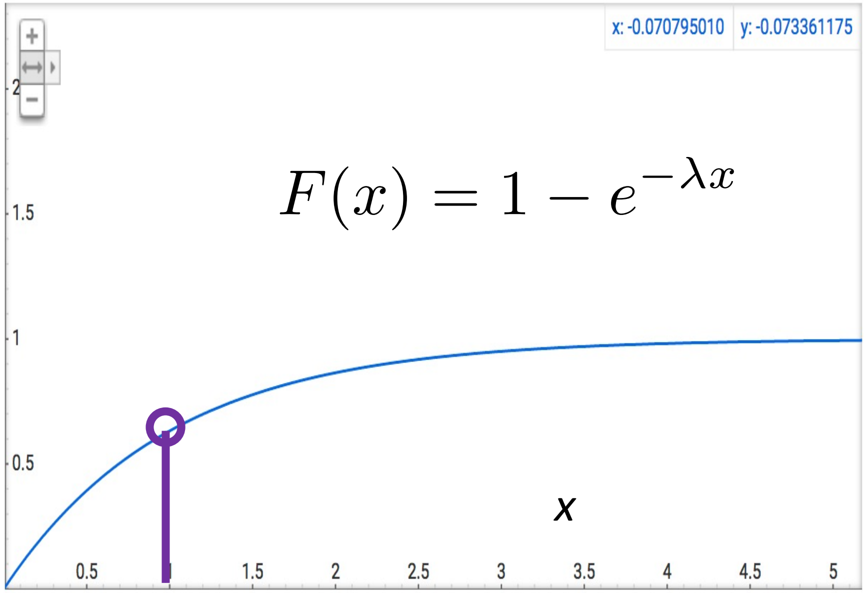
$$= \int_{x=1}^{\infty} f(x)\, dx$$

or

Cumulative density function

$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



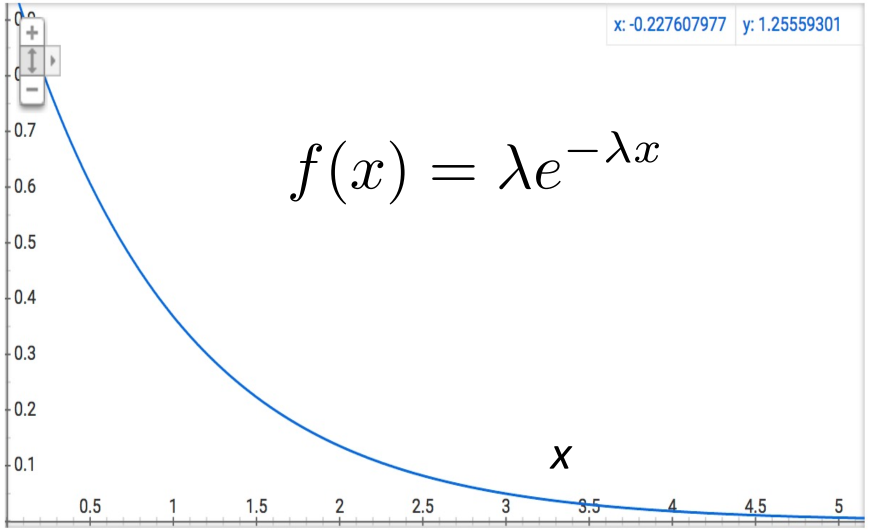
$$= 1 - F(1)$$

$$= e^{-1}$$

$$\approx 0.37$$

Using CDF Example. X is Exp(λ = 1)

Probability
density
function

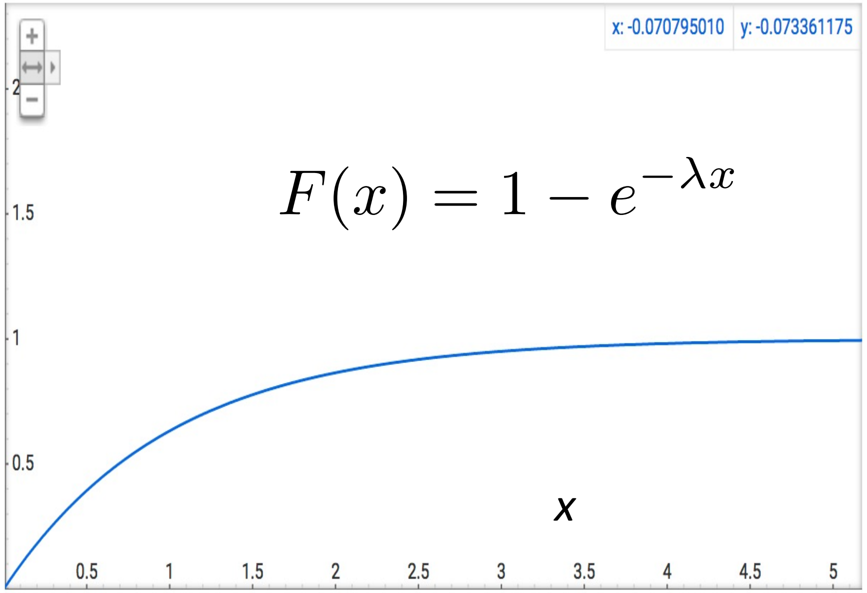


P(1 < X < 2)

Cumulative
density function

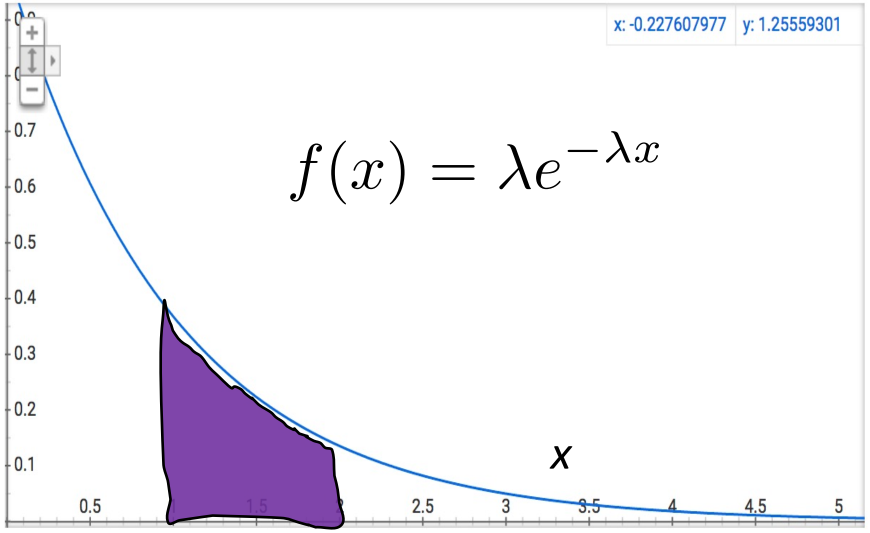
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp(λ = 1)

Probability density function



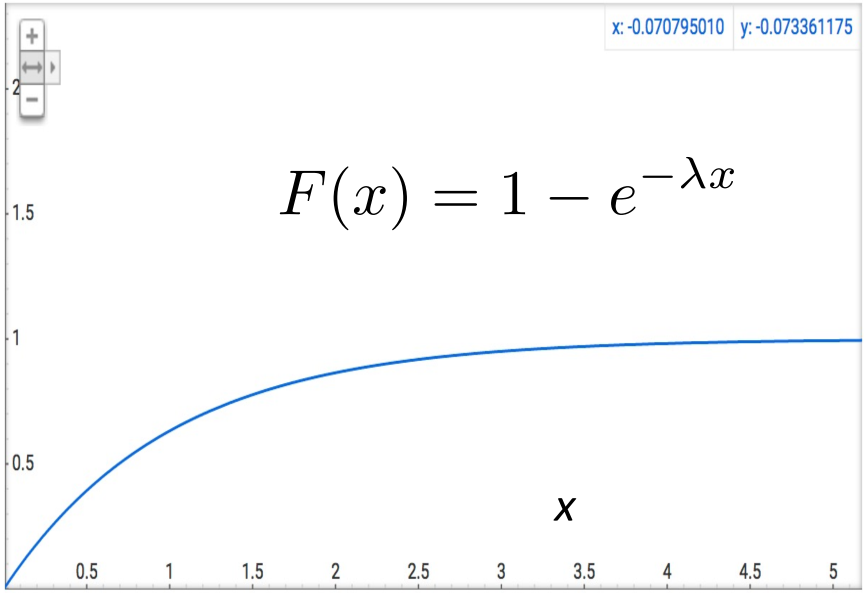
P(1 < X < 2)

$$= \int_{x=1}^{2} f(x)\, dx$$

Cumulative density function

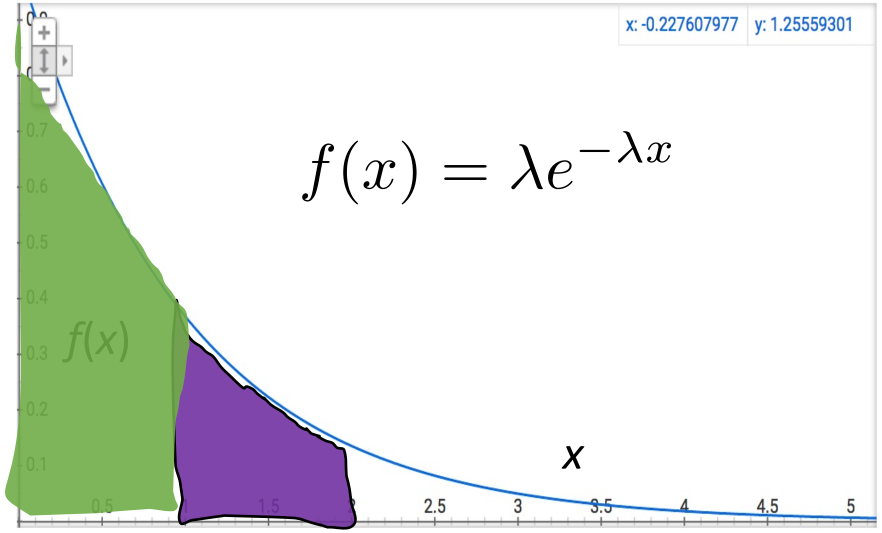
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp(λ = 1)

Probability density function



P(1 < X < 2)

$$= \int_{x=1}^{2} f(x)\, dx$$

or

$$= F(2) - F(1)$$

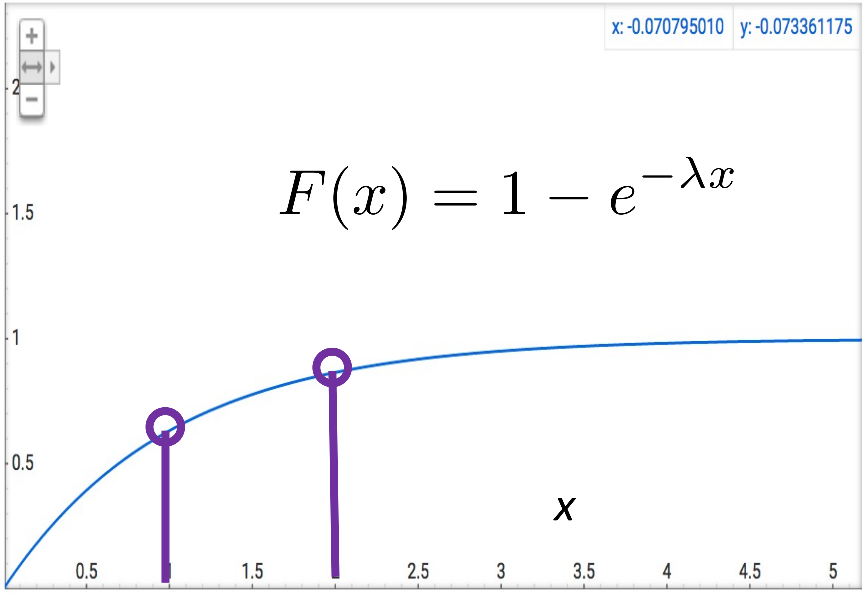
$$= (1 - e^{-2}) - (1 - e^{-1})$$

$$\approx 0.23$$

Cumulative density function

$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Probability of Earthquake in Next 4 Years?

Based on historical data, earthquakes of magnitude 8.0+ happen at a **rate of 0.002** per year*. What is the probability of **an major earthquake in the next 4 years?**

Y = Years until the next earthquake of magnitude 8.0+

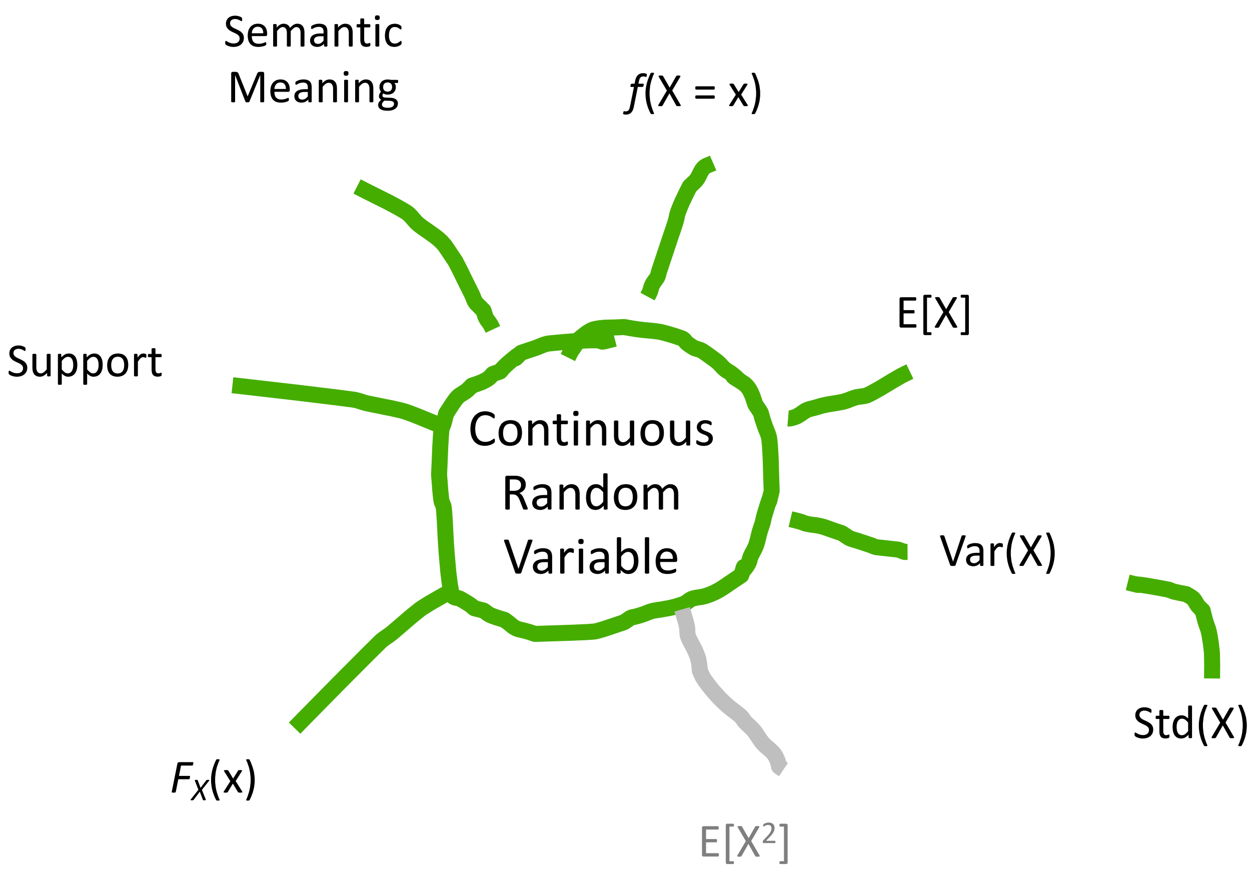
$$Y \sim \text{Exp}(\lambda = 0.002) \qquad F(y) = 1 - e^{-0.002y}$$

$$\begin{aligned} P(Y < 4) &= F(4) \\ &= 1 - e^{-0.002 \cdot 4} \\ &\approx 0.008 \end{aligned}$$

Feeling lucky?

*According to USGS, 2015

Properties for Continuous Random Variable



Here are a few more Random Variables

	number of **successes**	**time** to get successes	
One trial	$X \sim \text{Ber}(p)$	$X \sim \text{Geo}(p)$	One success
	$n = 1$	$r = 1$	
Several trials	$X \sim \text{Bin}(n, p)$	$X \sim \text{NegBin}(r, p)$	Several successes
Interval of time	$X \sim \text{Poi}(\lambda)$	$X \sim \text{Exp}(\lambda)$	One success

That is all folks!