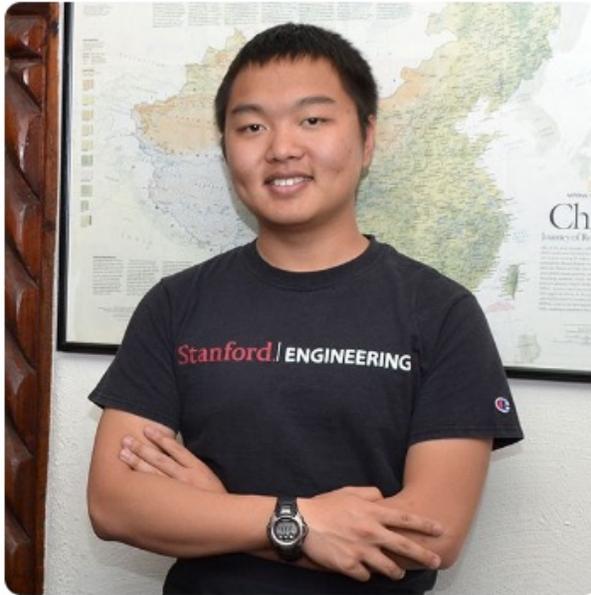


CS109: Probability for Computer Scientists

But first...
who are we?

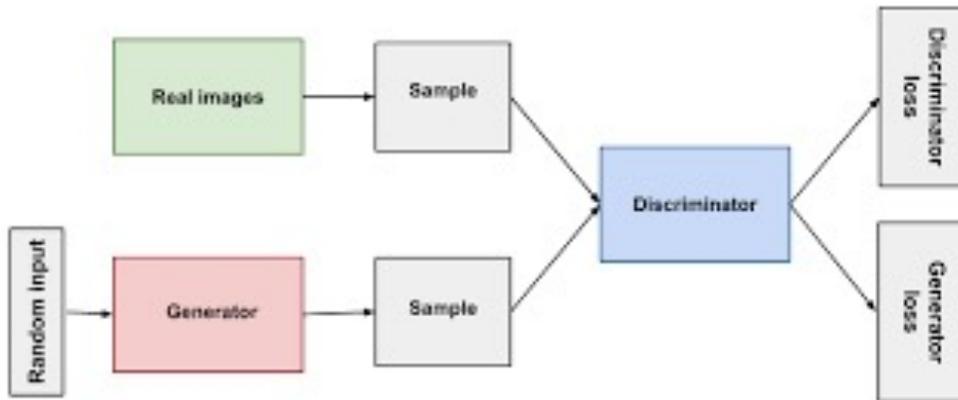
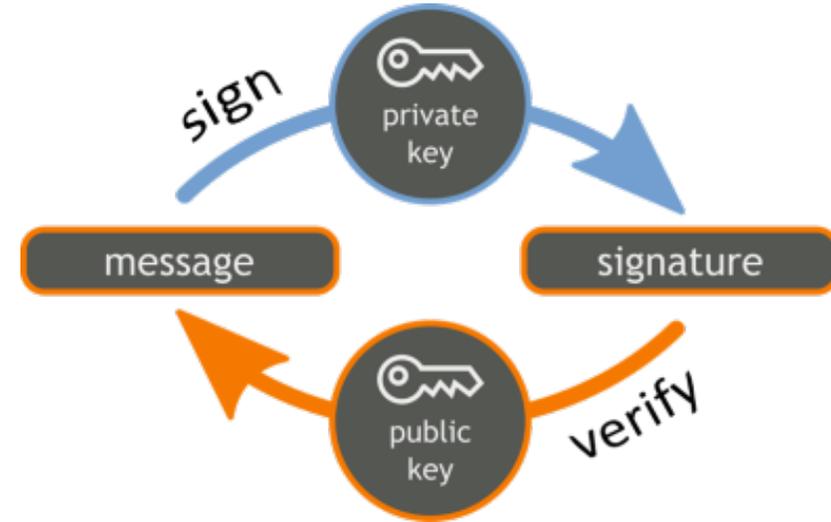
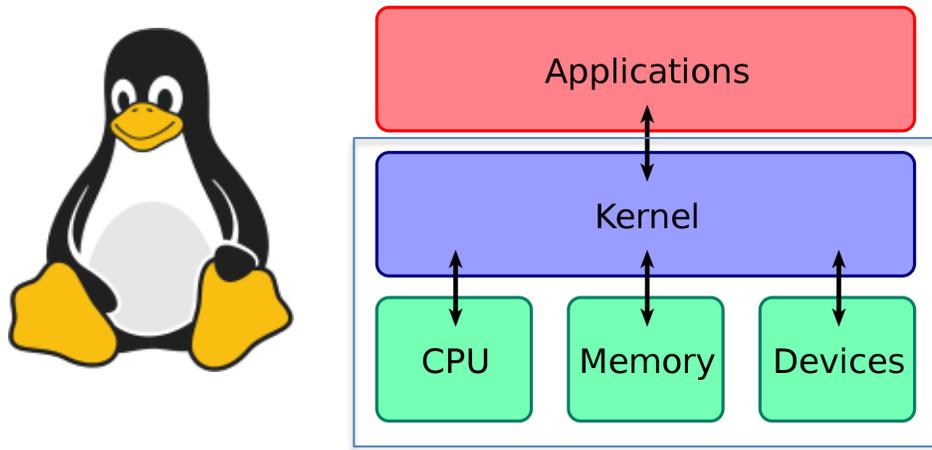
Amazing 109 Teaching Team



Kim, Piech, Song, CS109, Stanford University



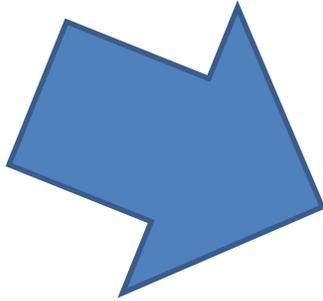
What Sort of Things Does Will Do?



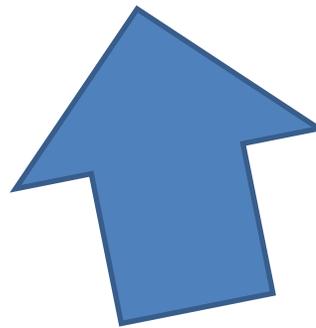
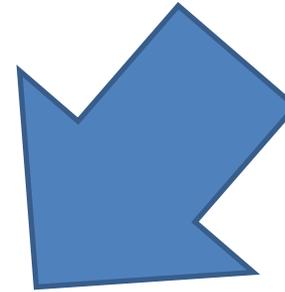
Course mechanics

(this is a light version. Please read the syllabus for details).

Essential Information



cs109.stanford.edu



Are you in the right place?

Prerequisites

What you really need:

CS106B/X (important):

- Recursion
- Hash Tables
- Binary Trees
- Programming

CS103 (not necessary):

- Proof techniques (induction)
- Set theory
- Math maturity

Math 51 or CME 100 (important, coreq ok)

- Multivariate differentiation
- Multivariate integration
- Basic facility with linear algebra (vectors)



Coding in CS109



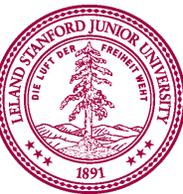
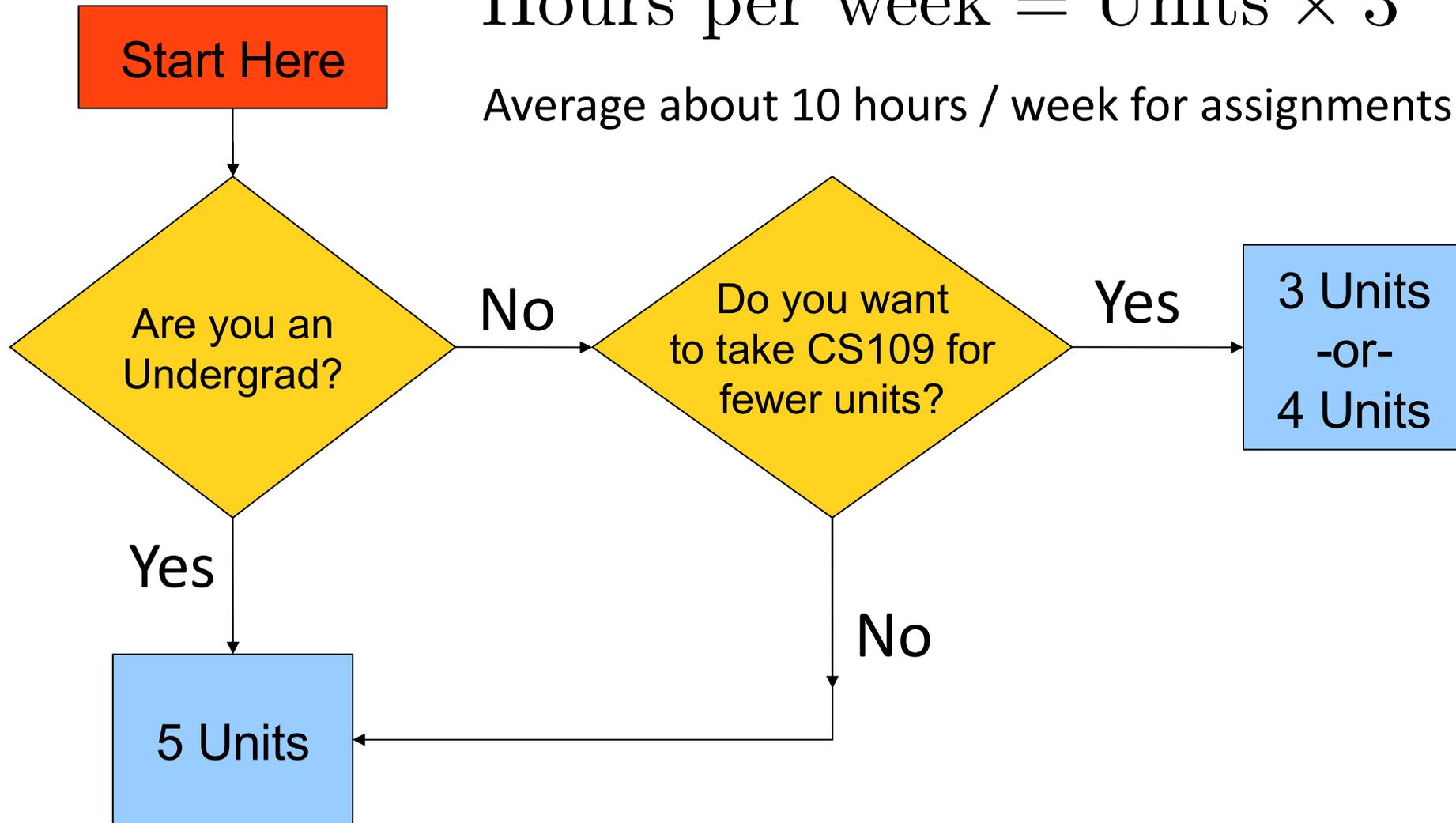
Review session on Thursday 4pm



CS109 Units

$$\text{Hours per week} = \text{Units} \times 3$$

Average about 10 hours / week for assignments



Class Breakdown

40%

6 Assignments

25%

Midterm

2 hour exam, July 25th, Time ~~TDB~~

7 - 9 p

30-35%

Final

3 hour exam, Aug ~~16th~~, 3:30pm

19

0-5%

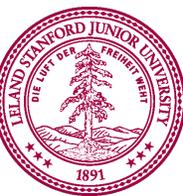
Optional Section Participation



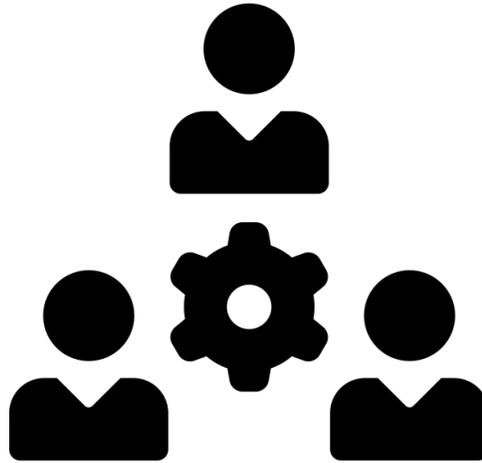
Is Class Online?



TLDR: Yes. We are recorded. Come to live class. It's a good time (and good for you)



Ask questions



Q&A forum
All announcements

Office hours
start on Thursday

Email cs109@cs.stanford.edu

From Last Year: Course Reader!

Probability for Computer Scienc

chrispiech.github.io/probabilityForComputerScientists/en/

Course Reader for CS109

Search book...

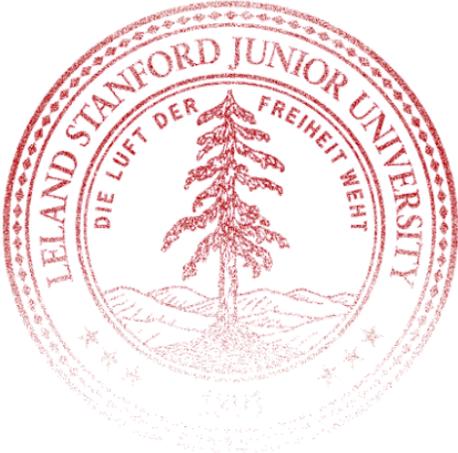
Part 1: Core Probability

- Counting
- Combinatorics
- Definition of Probability
- Equally Likely Outcomes
- Probability of **or**
- Conditional Probability
- Independence
- Probability of **and**
- Law of Total Probability
- Bayes' Theorem
- Log Probabilities
- Worked Examples
 - Enigma Machine
 - Serendipity
 - Bacteria Evolution
 - Many Coin Flips

Part 2: Random Variables

- Random Variables
- Probability Mass Functions
- Expectation
- Variance
- Bernoulli Distribution
- Binomial Distribution
- Poisson Distribution
- Continuous Distribution
- Normal Distribution

Course Reader for CS109



CS109
Department of Computer Science
Stanford University
December 2020
V 0.1.0.4

Acknowledgements: This book was written based on notes from Chris Piech for Stanford's CS109 course, Probability for Computer scientists using examples from Chris and Mehran Sahami. The course was originally designed by Mehran Sahami and followed the Sheldon Ross book Probability Theory from which we take inspiration. The course has since been taught by Lisa Yan, Jerry Cain and David Varodayan and their ideas and feedback have improved this reader. Special thanks to Robert Moss for drafting a PDF version.

I'm Curious





CS109: Probability for Computer Scientists

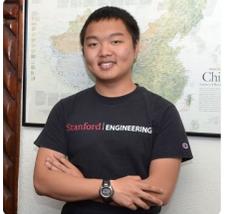
Summer 2023
Monday, Wednesday, Friday 3:00pm - 4:15pm in-person in [Skilling Auditorium](#)

WEEK 1 TODO
✓ Enjoy Summer!

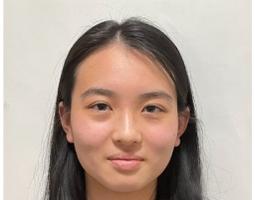
TEACHING TEAM
Co-Lecturer: Yunsung Kim



Co-Lecturer: Will Song



Awesome TAs:

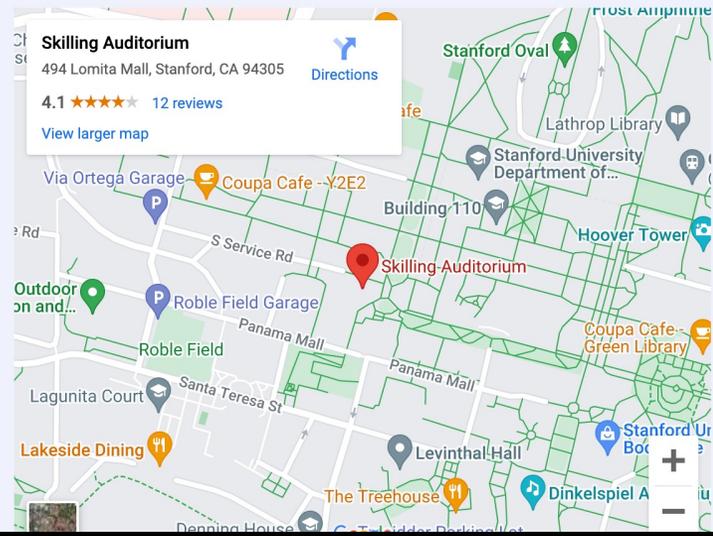


Schedule Week 1

Date	Lecture	Outside Class
Mon, June 26th	Counting	Enjoy the Sun!
Wed, June 28th	Combinatorics	
Fri, June 30th	What is Probability?	PSet 1 out

First Day of Class!

Welcome to CS109! We are looking forward to a fun quarter. Class starts Monday, June 26th at 3:00pm PT. We are looking forward to a great quarter.



RESOURCES

- Discussion
- Course Reader**
- Syllabus
- Schedule
- Office Hours

KEY DATES

- Midterm: July 25th
- Final: Aug 19th, 3:30p
- Last lecture: Aug 16th

COURSE VALUES

Everyone is welcome. Intellectual joy. Be kind. Be humane. Social connection. Learn by doing. Thrill of building. Adapt to new contexts.

CS 109 Application: Story of Modern AI

Modern AI = Probability + Programming

Brief History

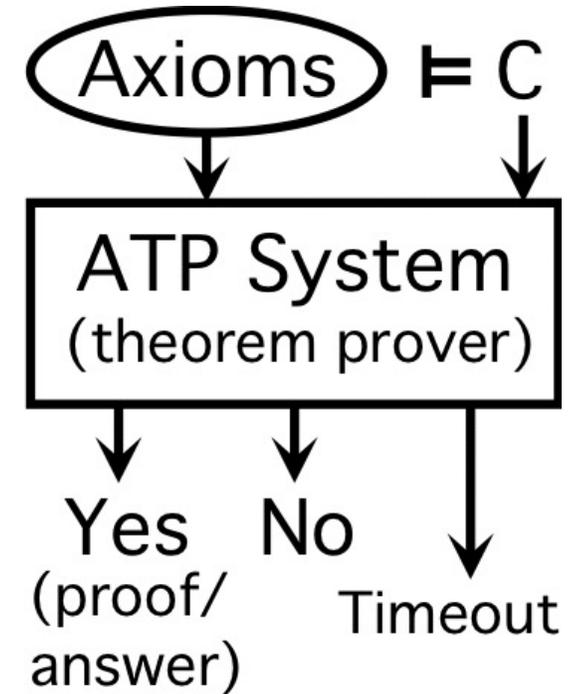


Early Optimism 1950s

1952



1955



Early Optimism 1950s

“Machines will be capable,
within twenty years, of doing
any work a man can do.”
–Herbert Simon, 1952



Underwhelming Results 1950s to 1980s

The spirit is willing but the flesh is weak.



(Russian)



The vodka is good but the meat is rotten.

The world is too complex



BRACE YOURSELVES

WINTER IS COMING



Something is going on in the world of AI

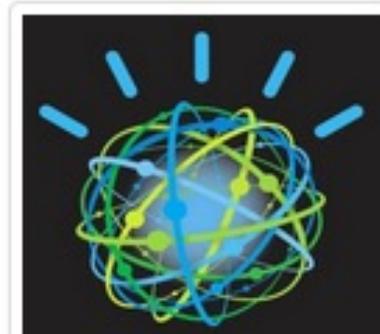
Big Milestones Part 1



1997 Deep Blue

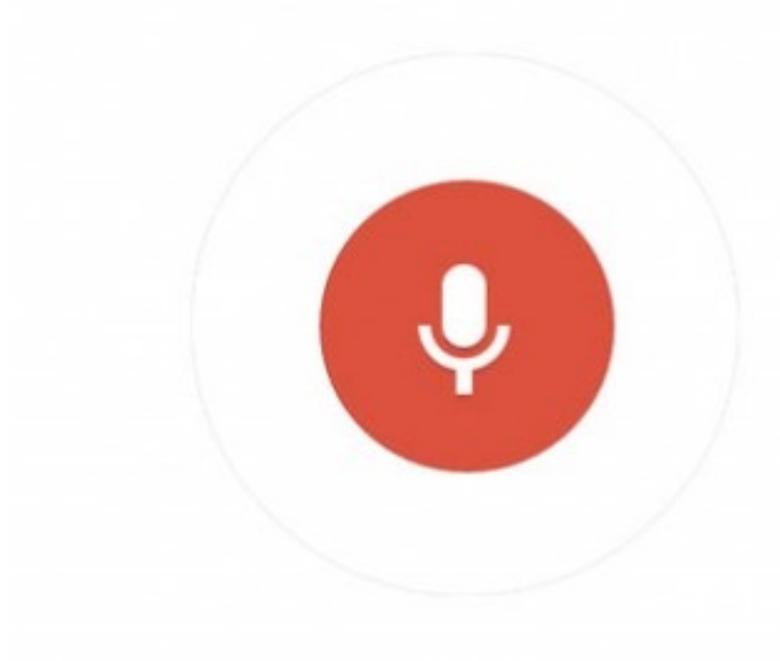


2005 Stanley



2011 Watson

We were told speech was 30 years out



Almost perfect...

Directions From A to B

2600 Columbia St to Mexico City

google.com/maps/dir/2600+Columbia+St,+Palo+Alto,+CA+94304,+USA/Mexico+City,+CDMX,+Mexico/@28.1116947,-119.664044...

Hotels Gas Rest stops More

Home (2600 Columbia St)
Mexico City, Mexico

Leave now Options

Send directions to your phone

Route	Time	Distance
via I-10 E	37 hr	2,191 miles
via México 15D	38 hr	2,207 miles
via I-5 S and I-10 E	38 hr	2,405 miles

Explore Mexico City

Restaurants Hotels Gas stations Parking Lots More

Map data ©2022 Google, INEGI United States Terms Privacy Send feedback 200 mi



3:31



Google Translate



English



Ukrainian



ENGLISH



Please translate this into Ukrainian.
Thank you



Camera



Conversation



Transcribe

UKRAINIAN



Будь ласка, переведіть це

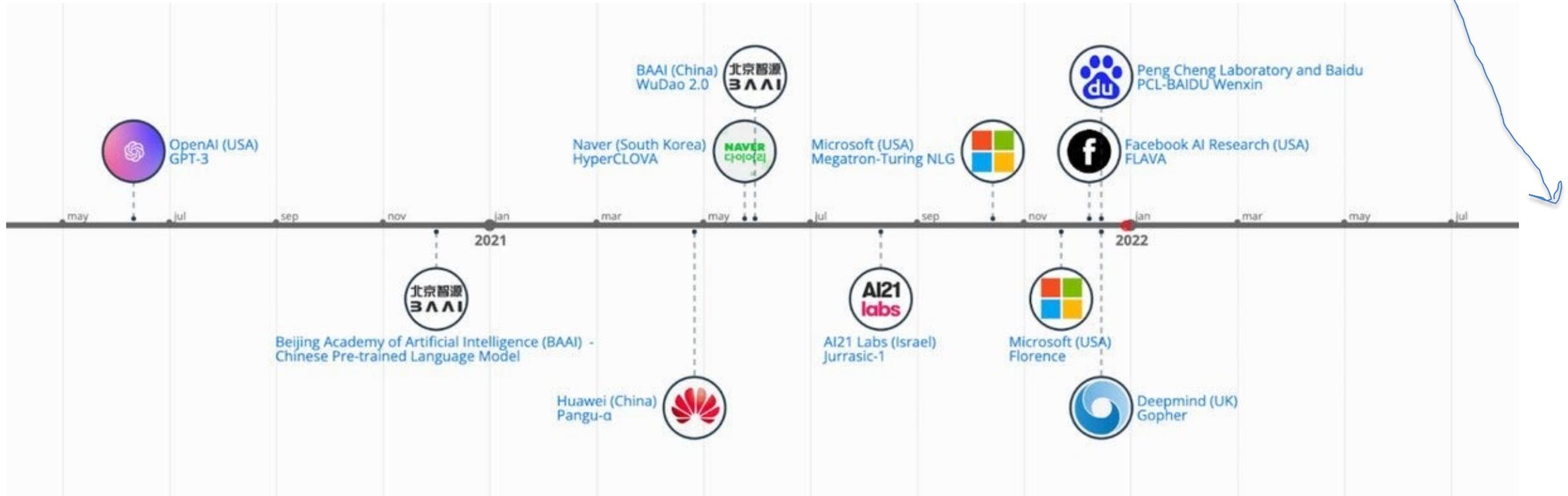
Self Driving Cars



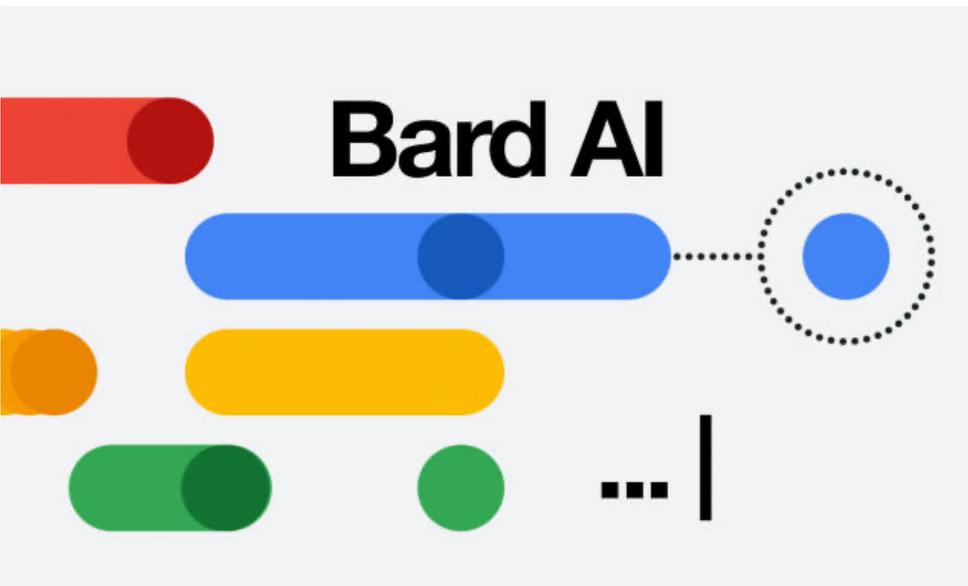
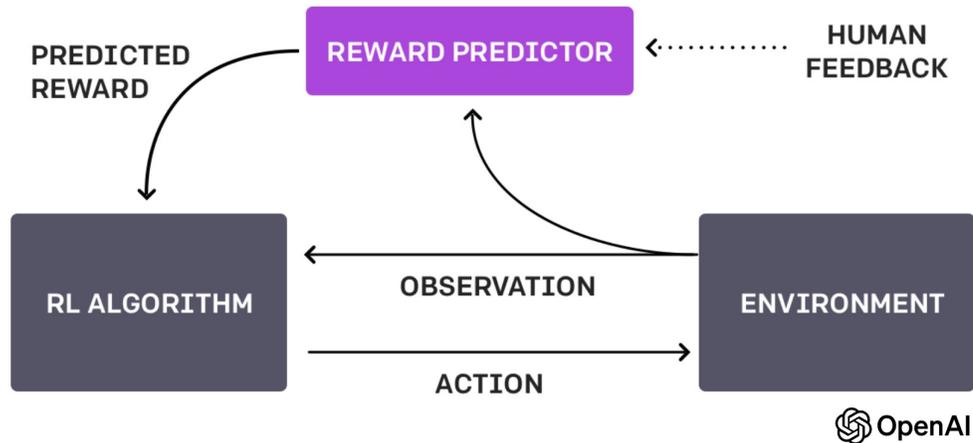
And then this year, everything changed, again

AI that (seems) to understand language

ChatGPT 11/2022 



Generative AI



JS How to make latte art?

To make latte art, follow these steps: 1. Prepare a shot of espresso and steam milk to a creamy texture. 2. Pour the steamed milk into a cup, holding it at an angle. 3. Start pouring slowly in the center, gradually moving the pitcher back and forth to create a base. 4. Once the cup is almost full, raise the pitcher slightly and create designs by pouring milk with controlled movements. 5. Use a toothpick or a latte art tool to refine the design. Practice is key, so keep experimenting with pouring techniques and patterns to improve your latte art skills.

"Computer Science and Probability merge, birthing the creative engine that fuels generative AI's boundless ingenuity." – ChatGPT



How is this done?

Focus on one problem

Computer Vision



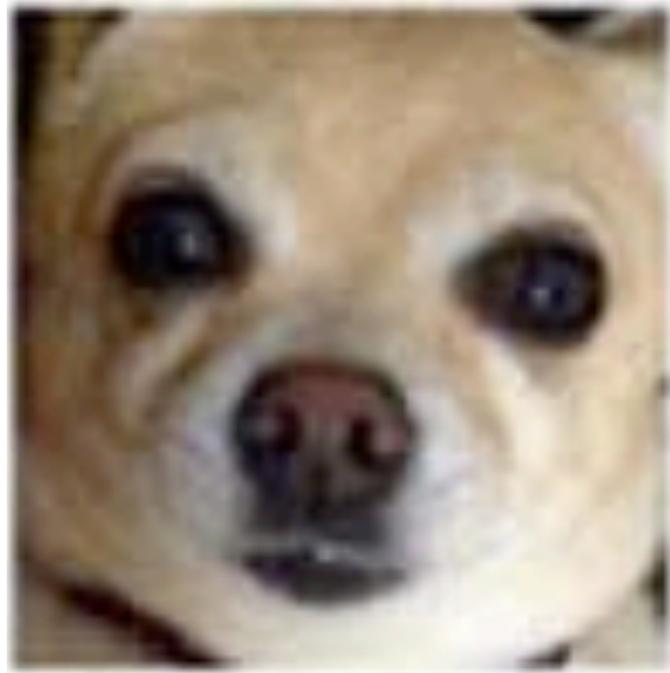
Chihuahua or muffin?

Kim, Piech, Song, CS109, Stanford University



Can you do it?

Chihuahua or Muffin?



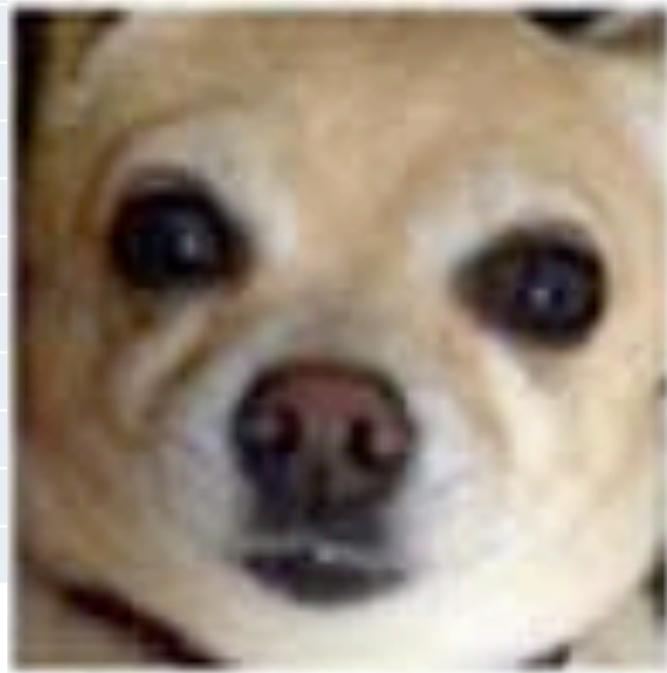
Chihuahua or Muffin?



How about now?

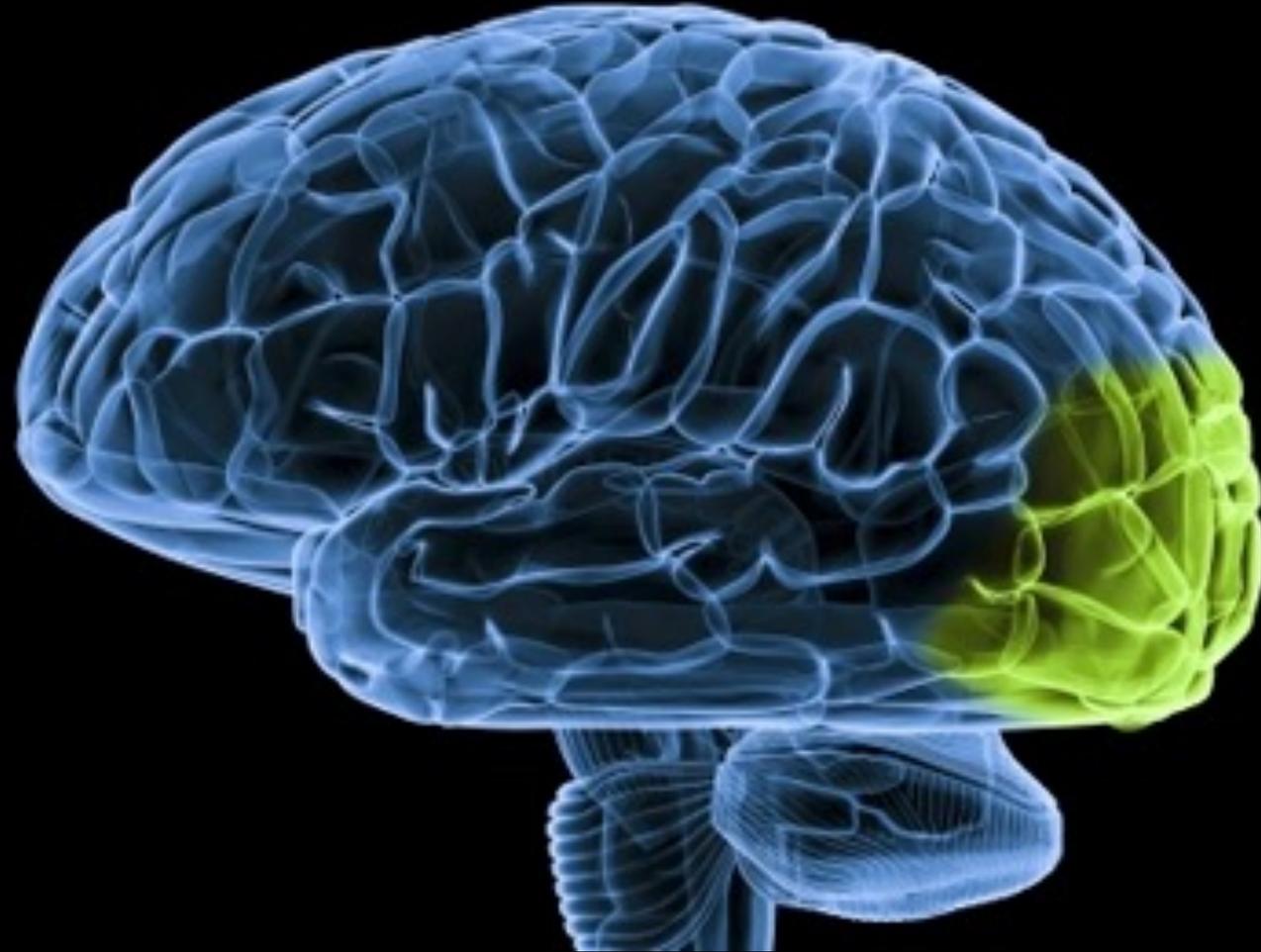
What a computer sees

0	0	1	0	1	0	1	0	0	0	1	1	1	0	1
1	0	0	1	0	1	1	1	0	1	0	0	0	0	0
1	1	1	0	1	0	0	1	1	0	0	1	0	1	0
1	1	1	1	1	0	0	0	0	0	1	1	0	1	1
0	0	0	1	1	0	0	1	0	0	0	1	1	1	0
1	0	0	1	1	0	0	0	1	0					
1	1	0	1	1	0	0	1	1	0					
1	0	1	0	0	1	0	0	1	0					
0	0	0	0	1	0	1	0	1	1					
0	1	1	0	0	0	0	0	1	1					
0	0	1	0	1	1	1	0	0	0					
0	1	1	1	0	1	0	0	1	0					
1	1	0	0	0	0	0	0	0	0					
0	0	0	0	0	0	0	0	1	1					
0	0	1	1	1	0	1	0	1	1					



What a human sees

Why is it easy for Humans?



About 30% of your cortex is used from vision
3% is used to process hearing







Make a Harry Potter Sorting Hat



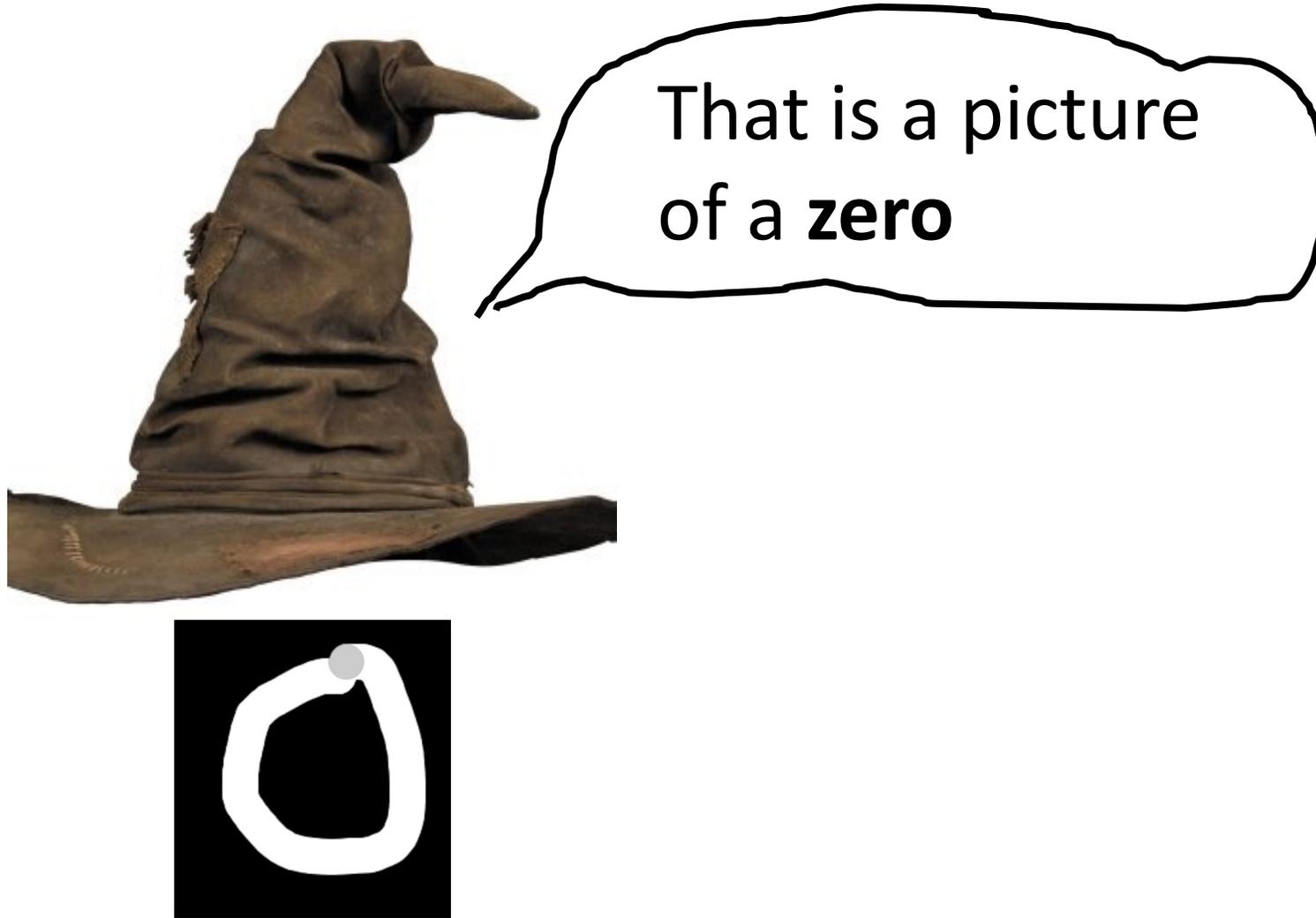
Kim, Piech, Song, CS109, Stanford University



Classification



Classification



Classification



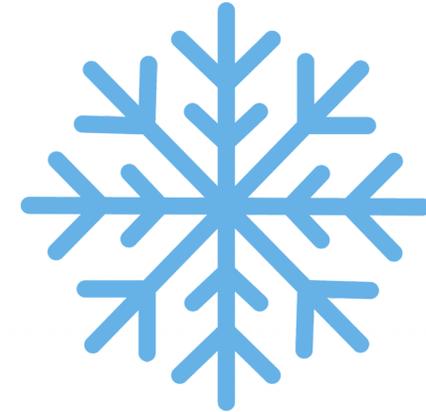
That is a picture
of a **zero**



* It doesn't have to be
correct all of the time



Very hard to Program



```
public class DigitDetector extends ConsoleProgram {  
  
    public void run() {  
        println("Todo: Write program");  
    }  
  
}
```



Perhaps there is an insight?

Two Great Ideas

1. Artificial Neurons

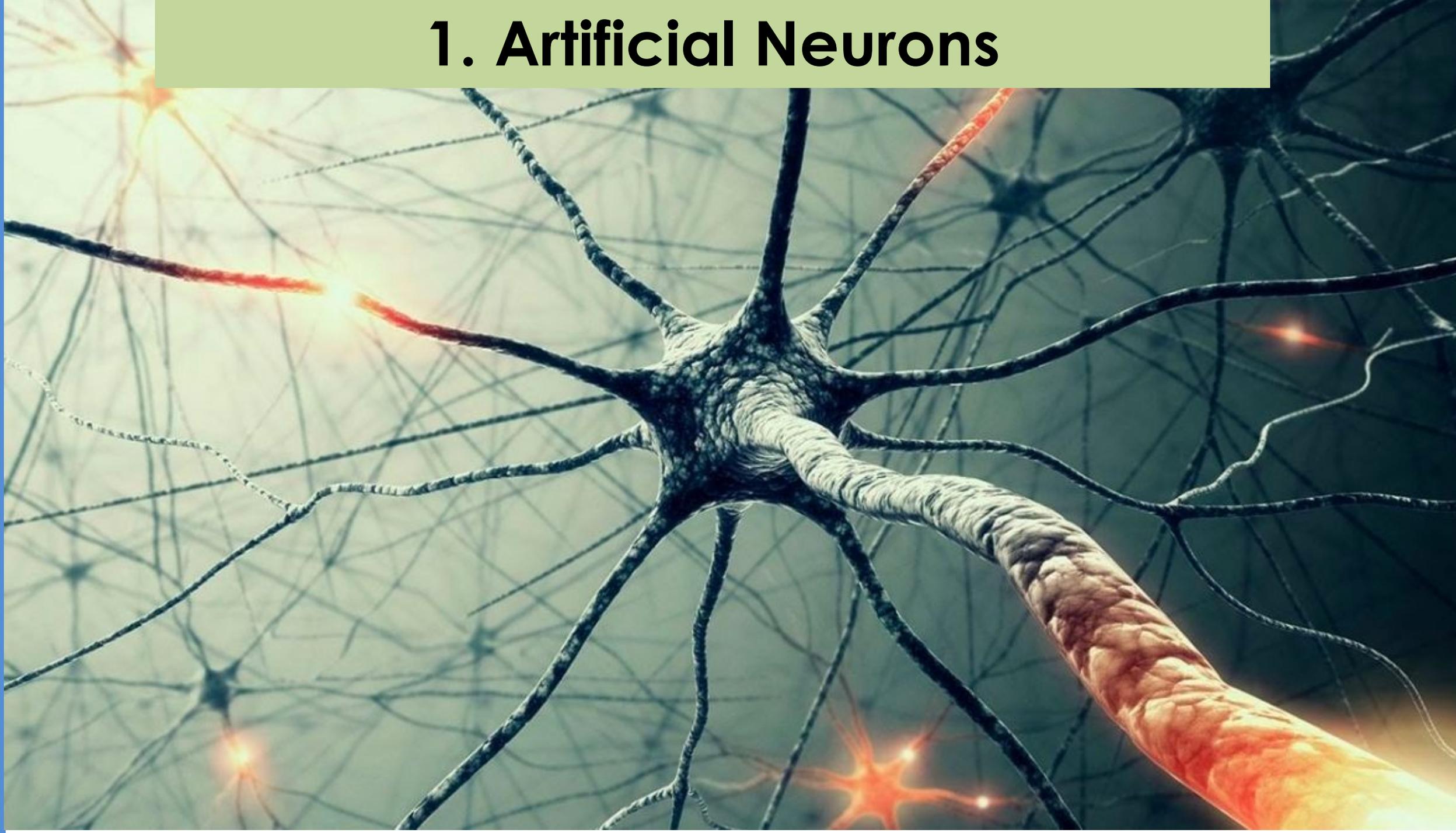
2. Learn by Example

Two Great Ideas

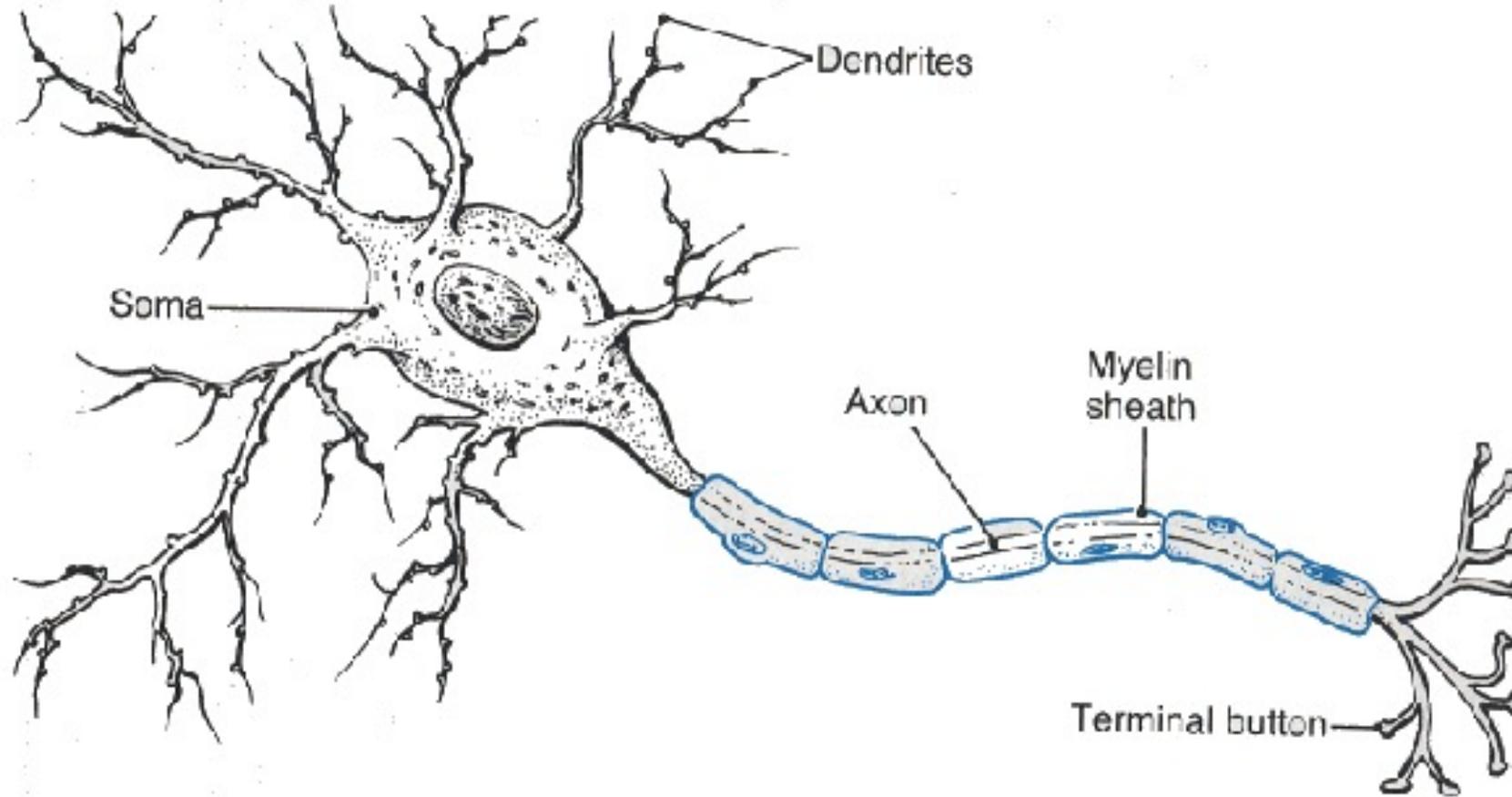
1. Artificial Neurons

2. Learn by Example

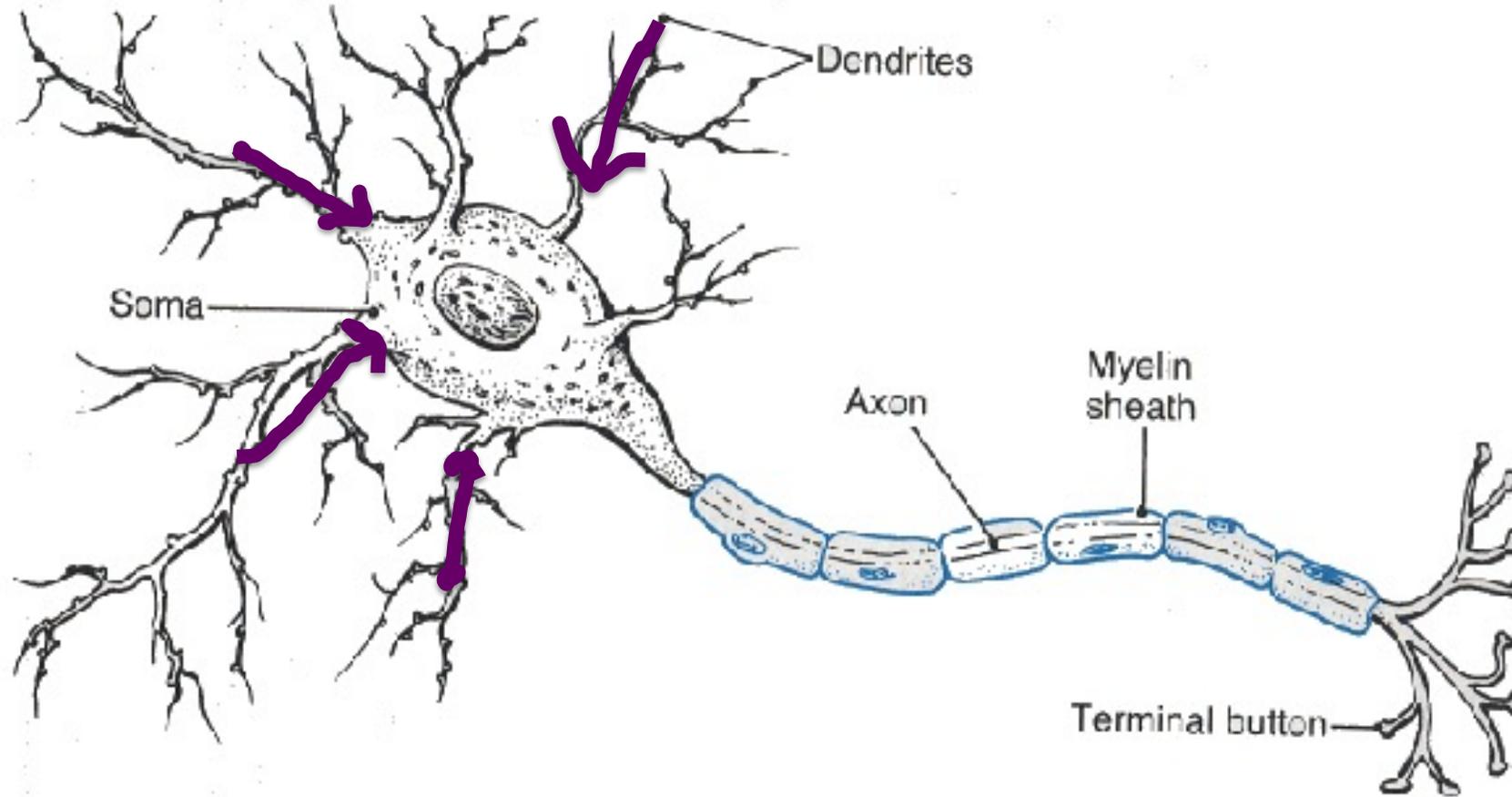
1. Artificial Neurons



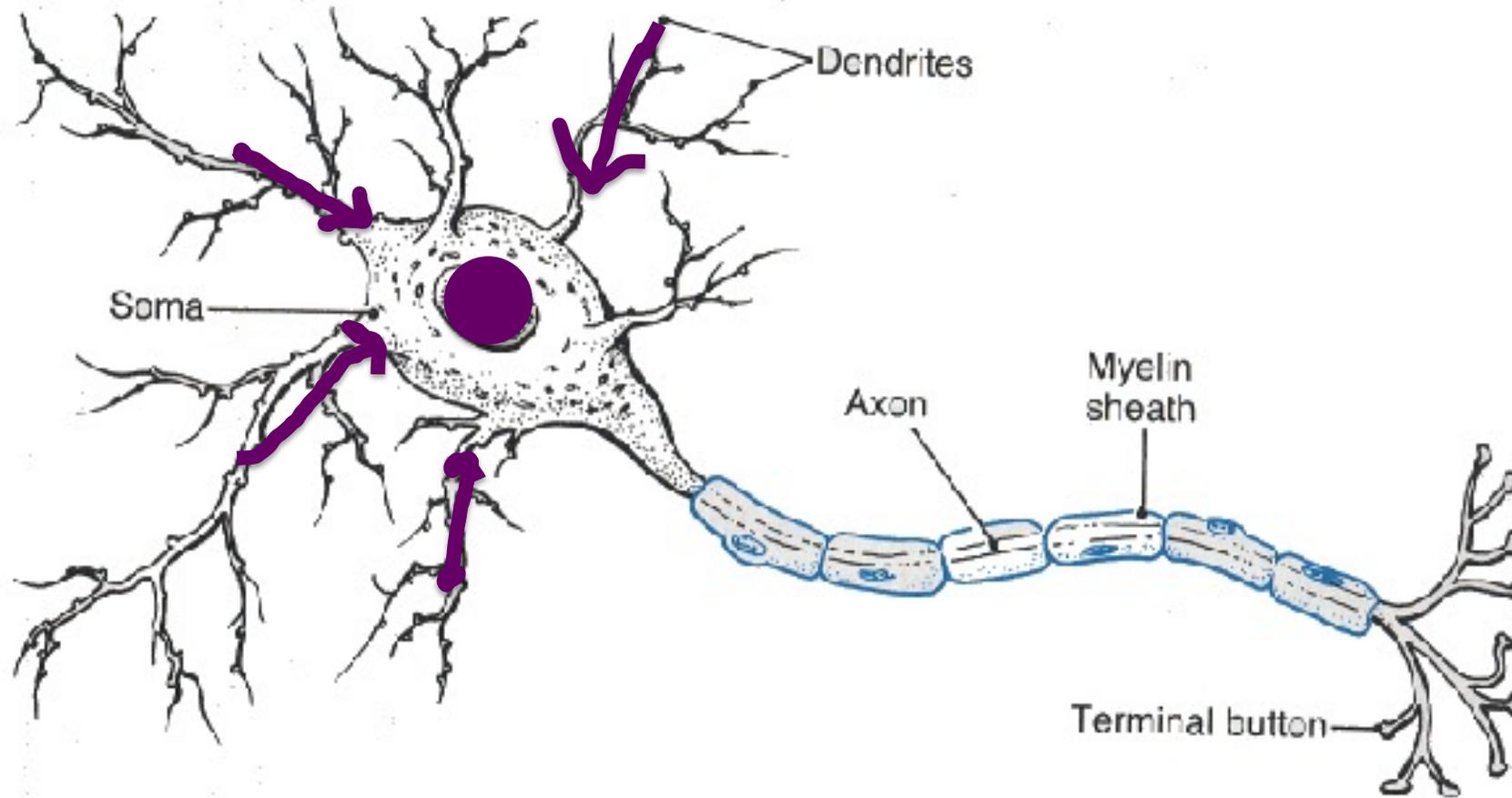
Neuron



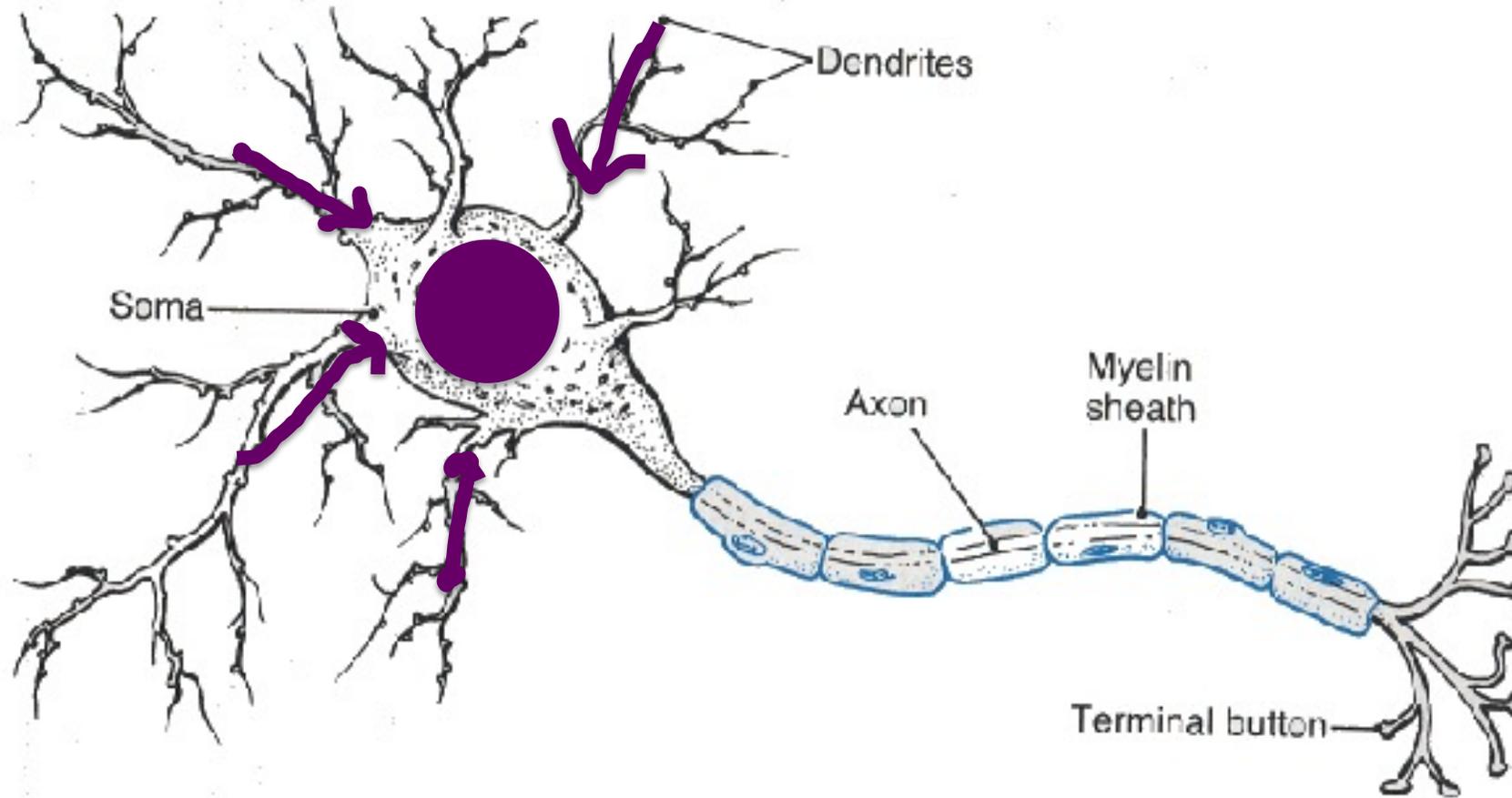
Neuron



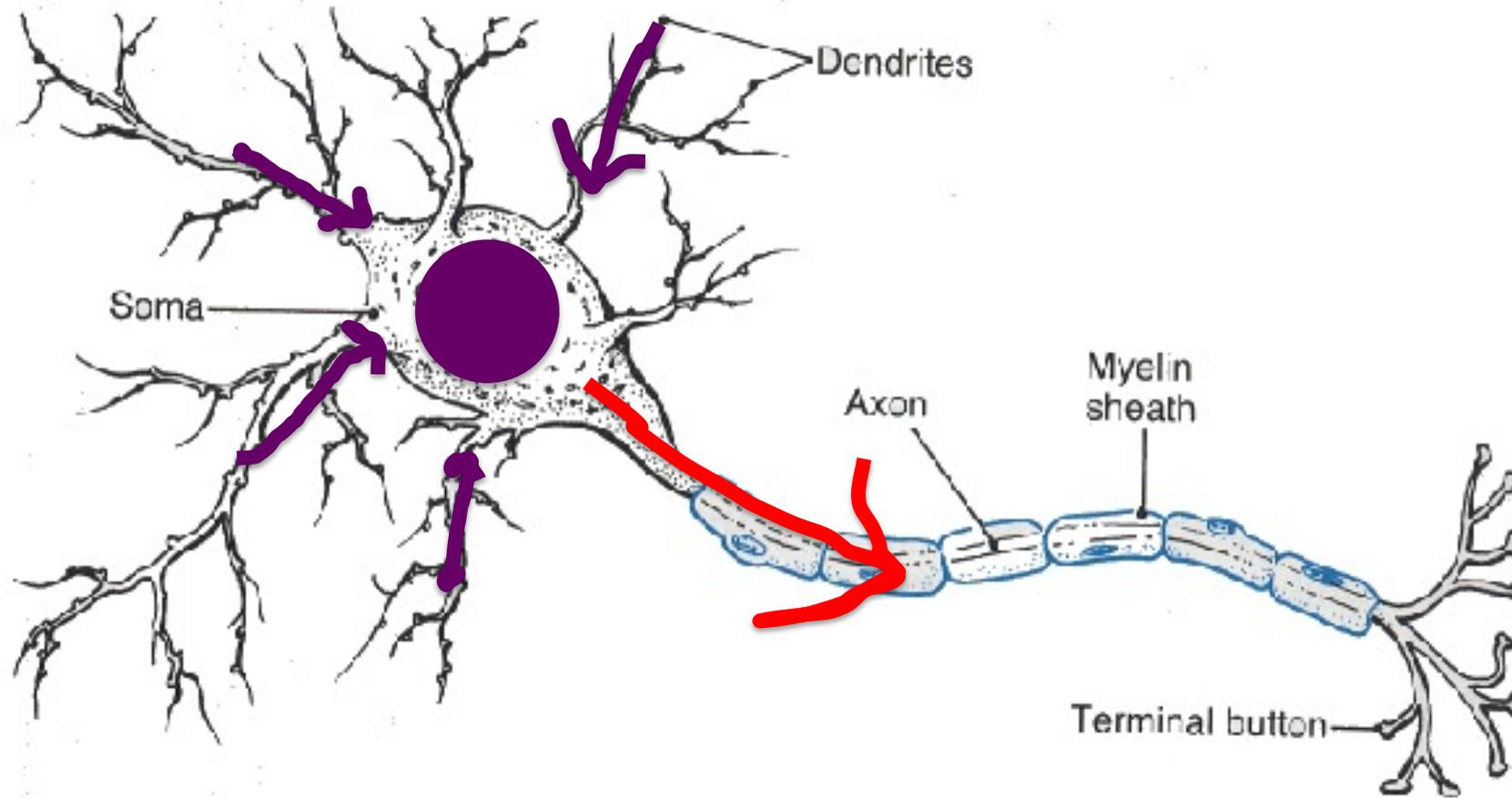
Neuron



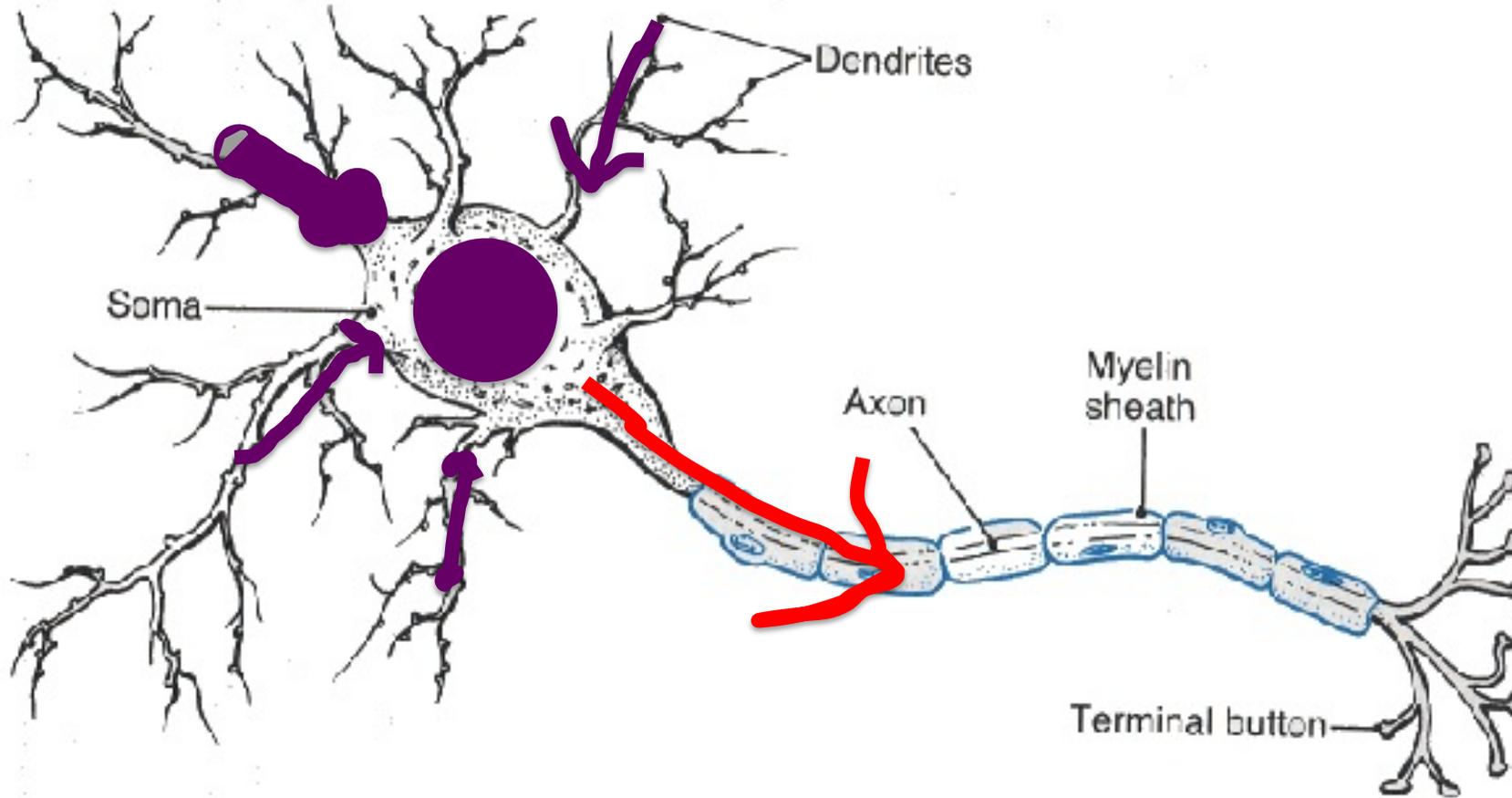
Neuron



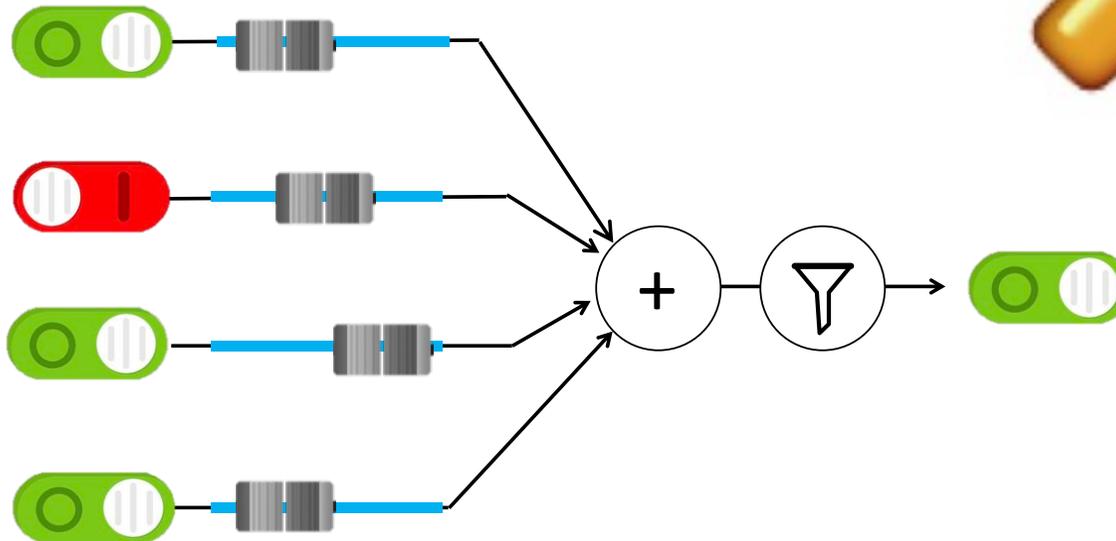
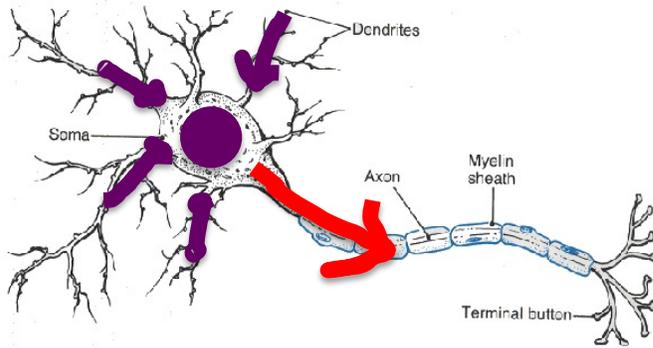
Neuron



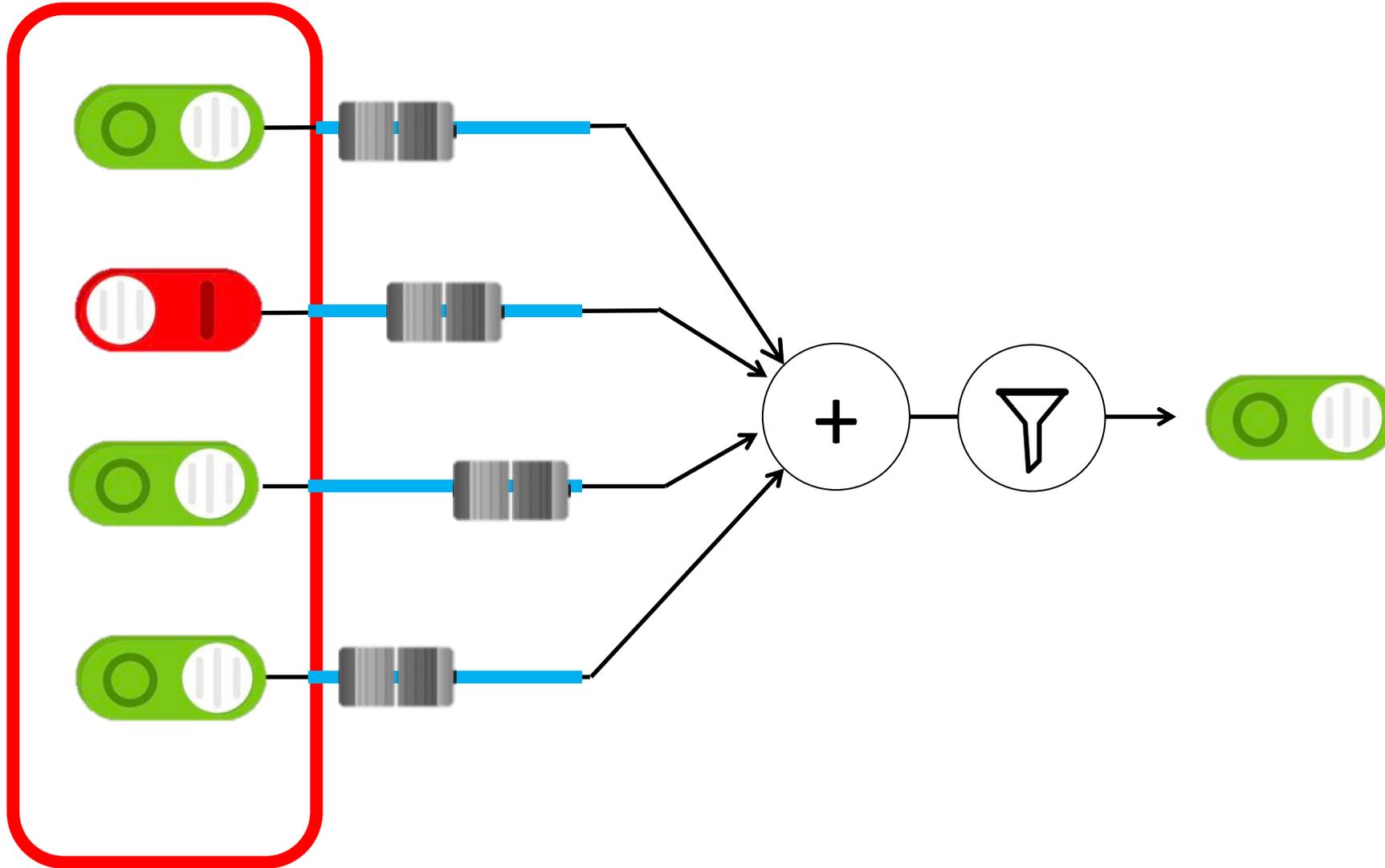
Some Inputs are More Important



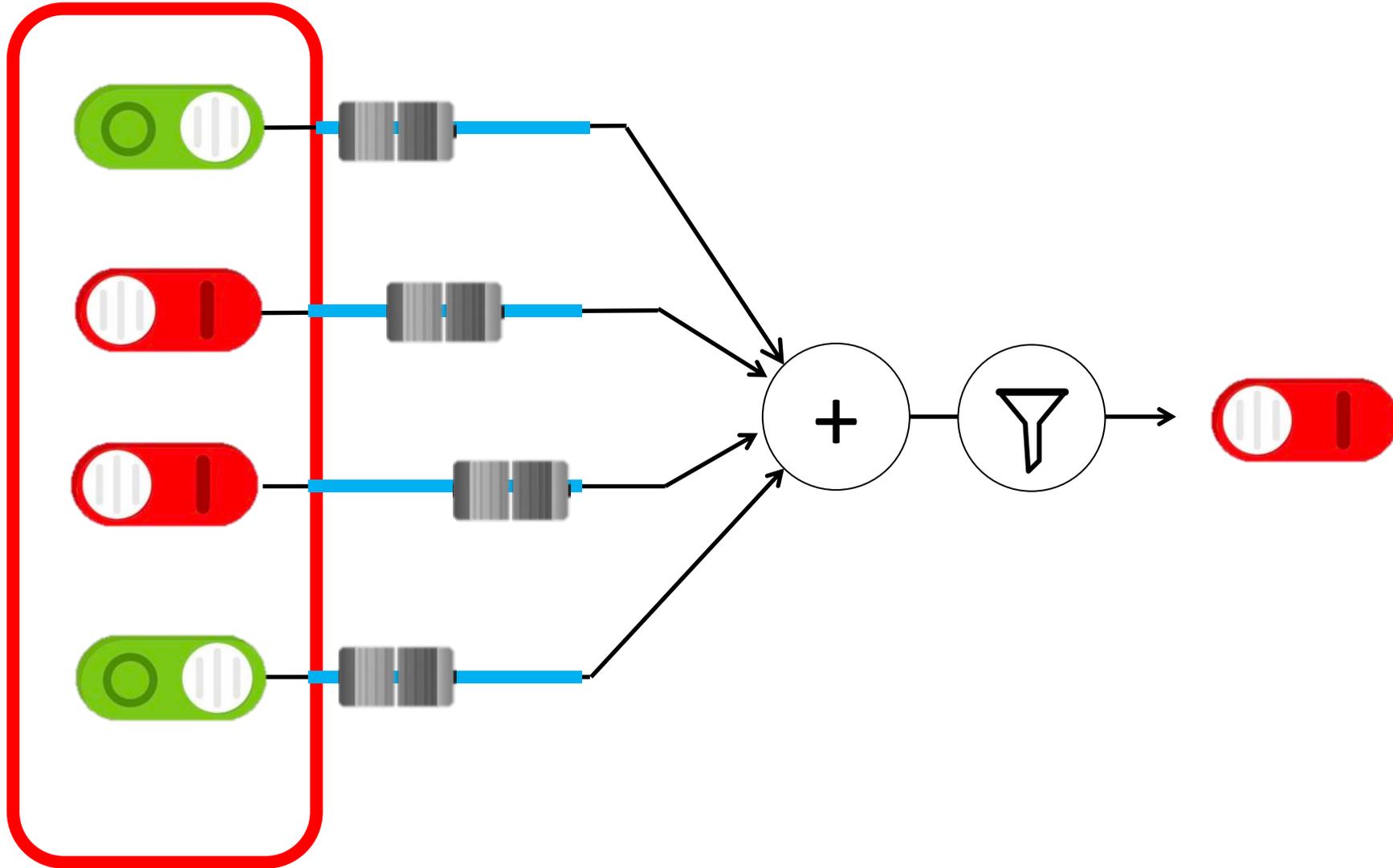
Artificial Neuron



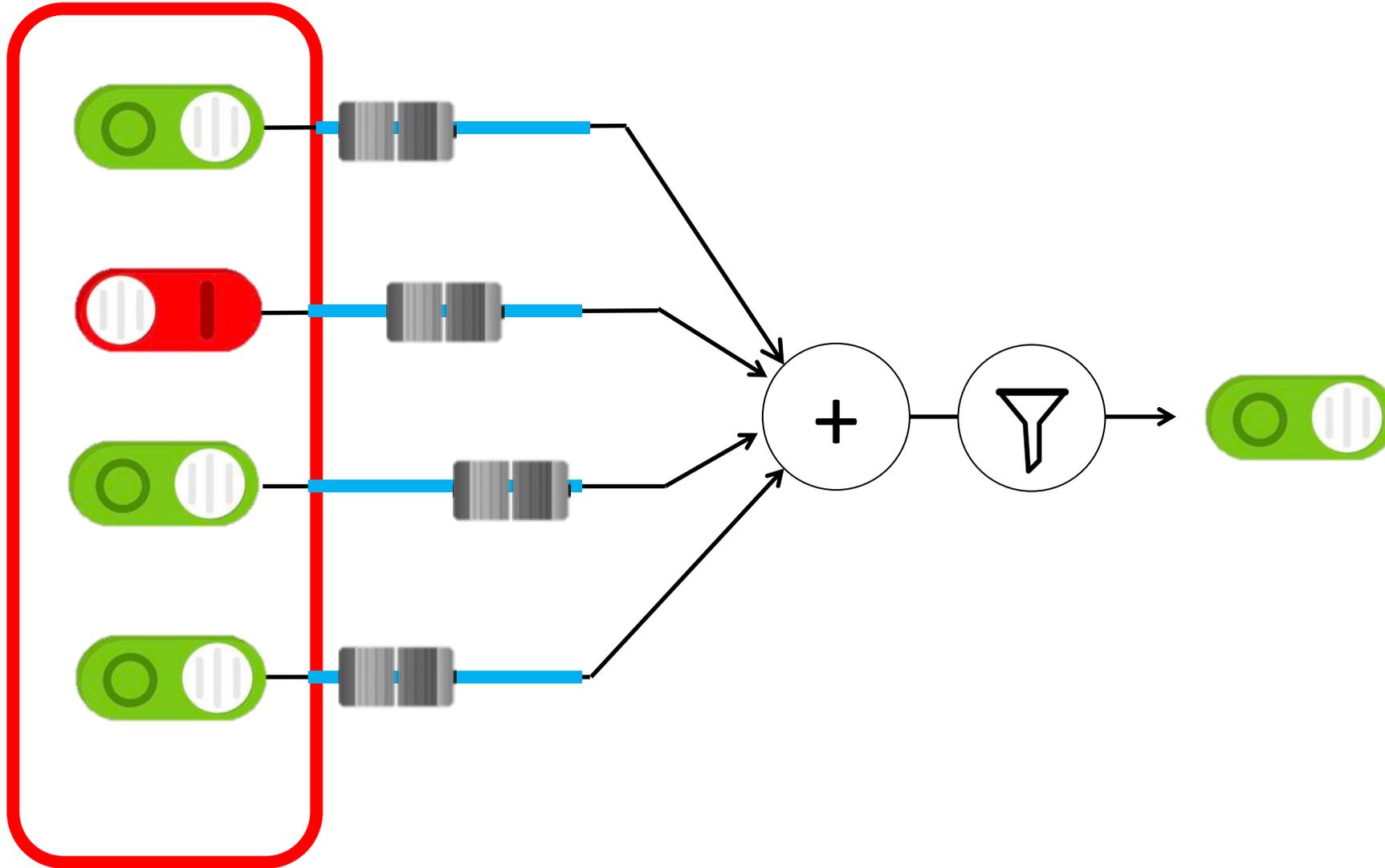
Inputs



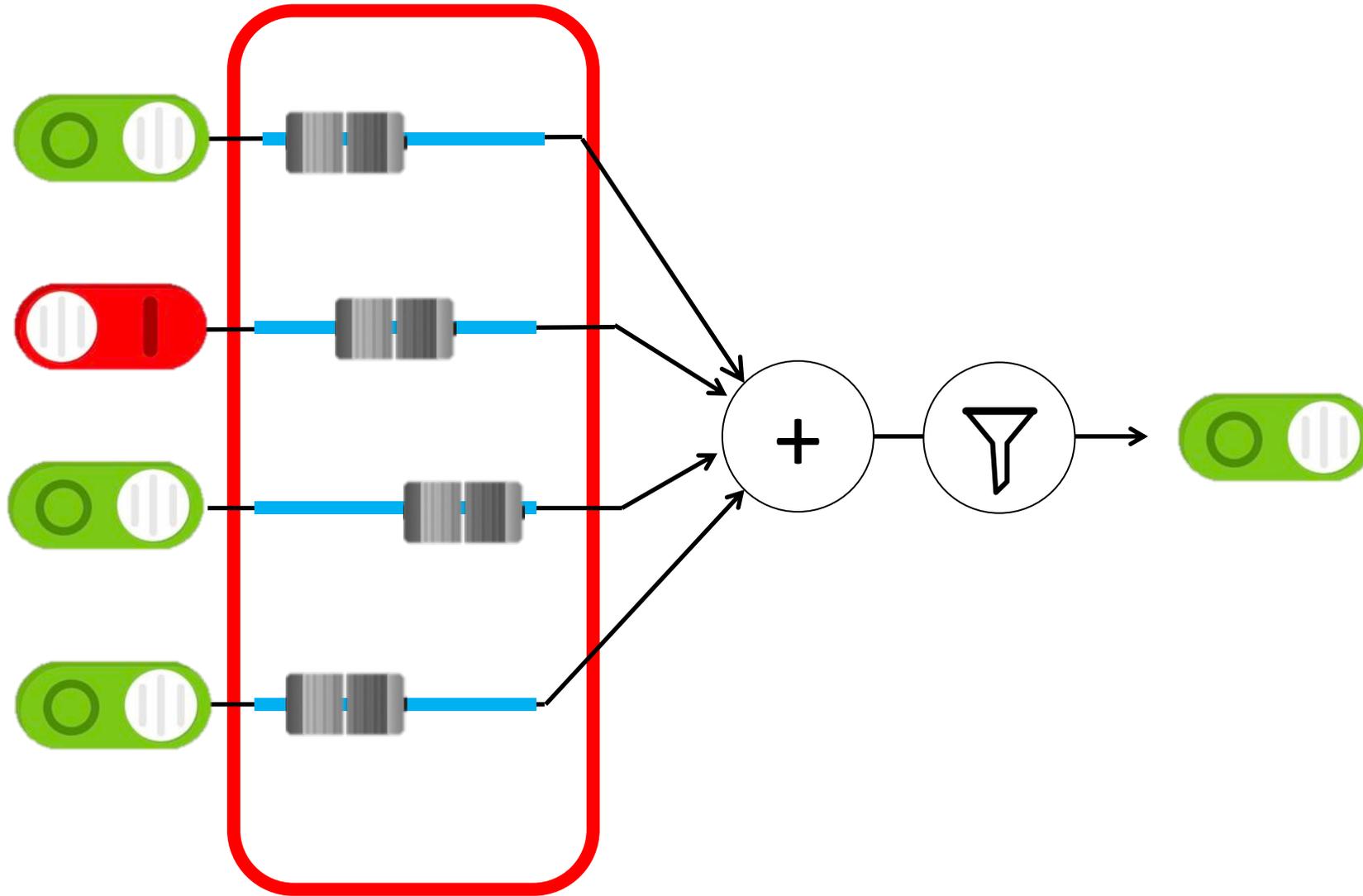
Inputs



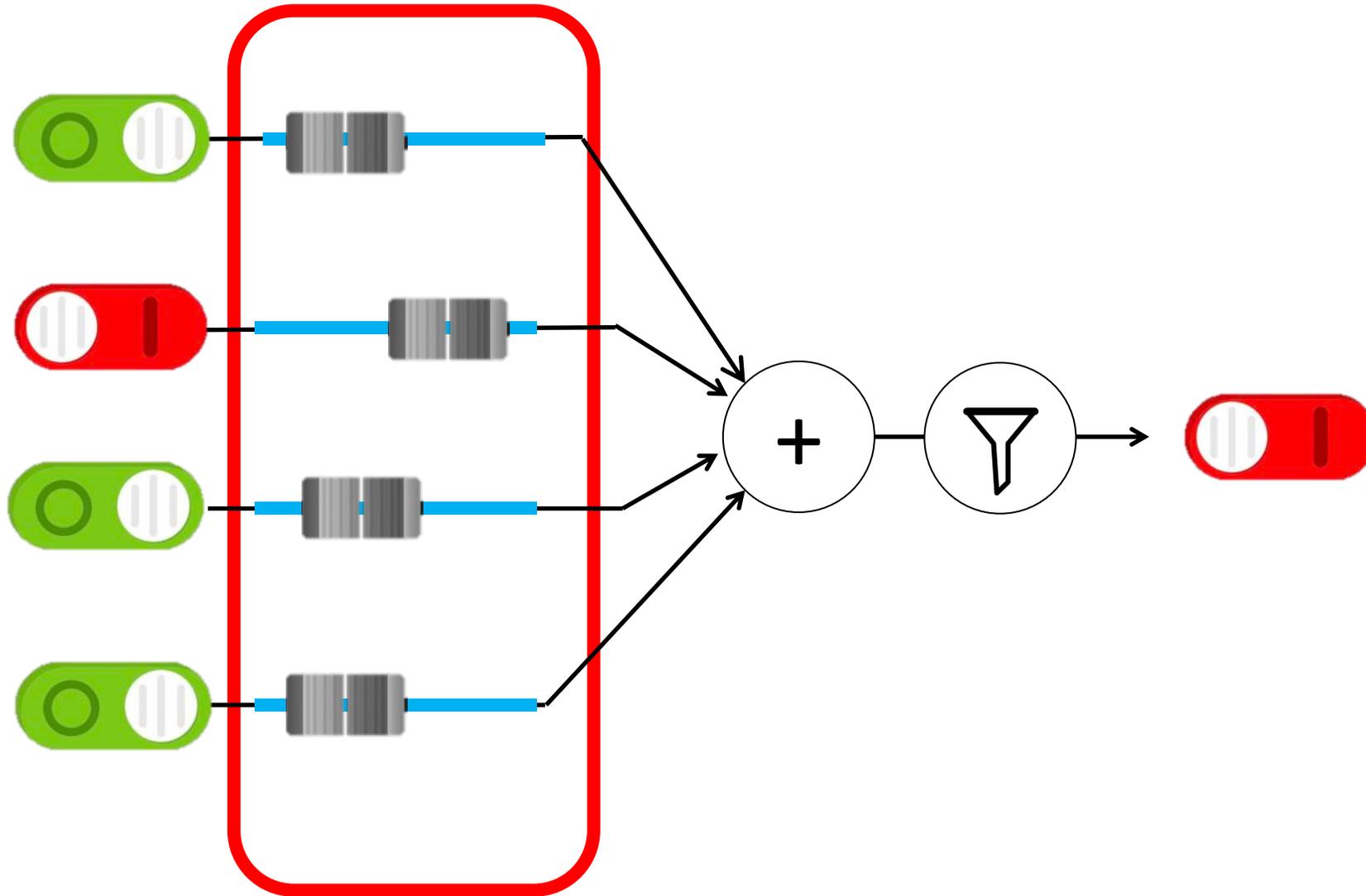
Inputs



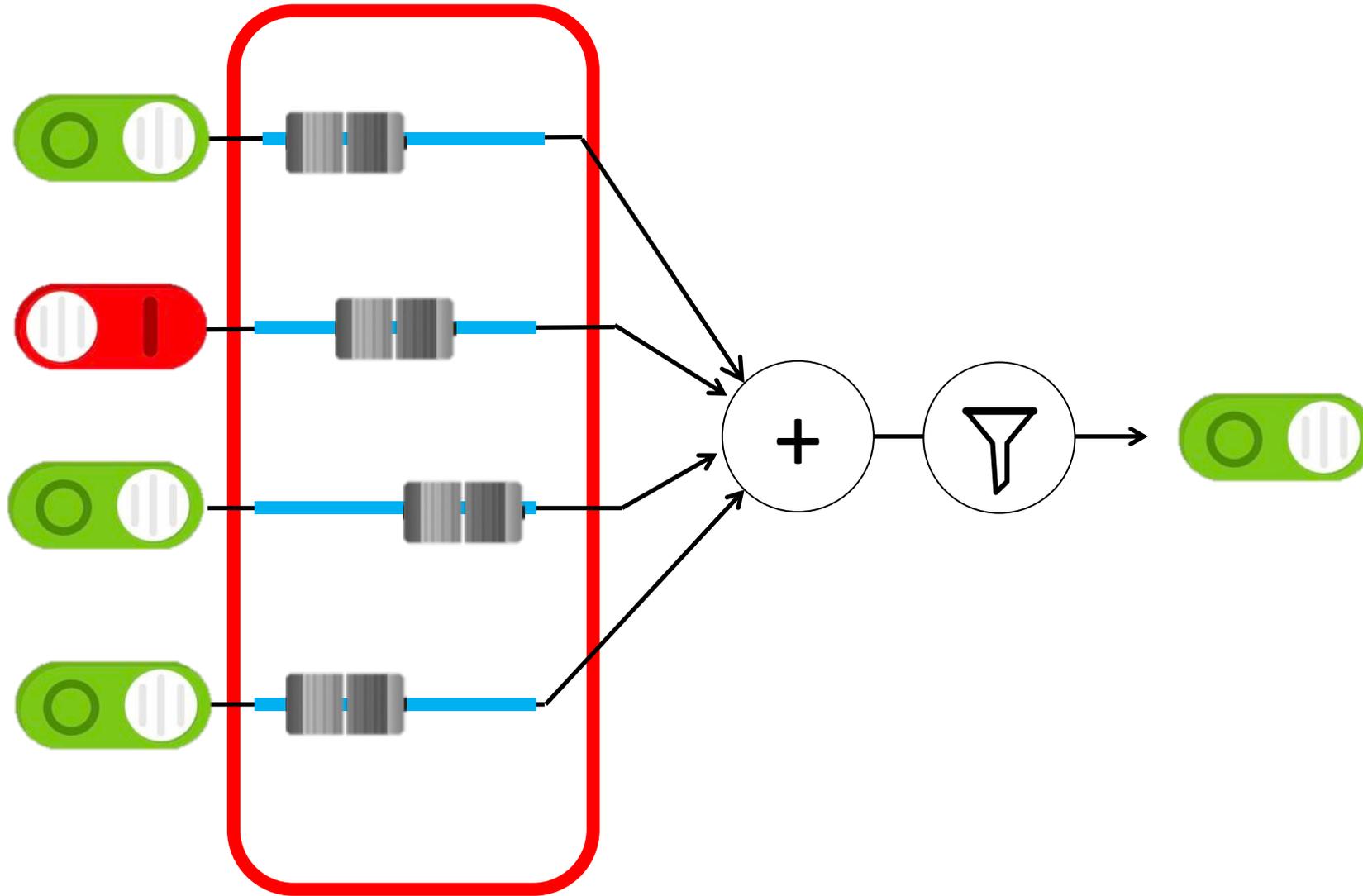
Weights



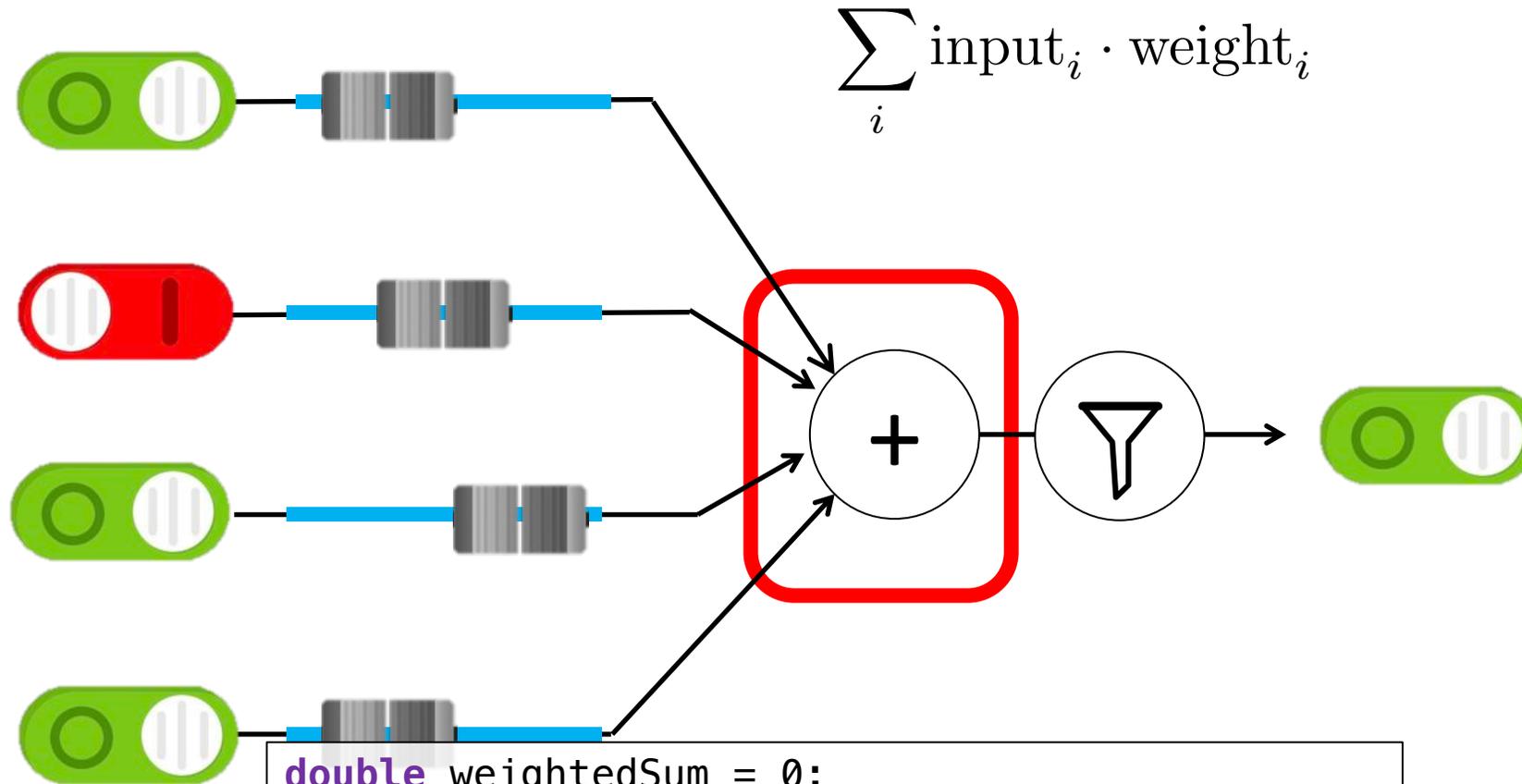
Weights



Weights



Weighted Sum

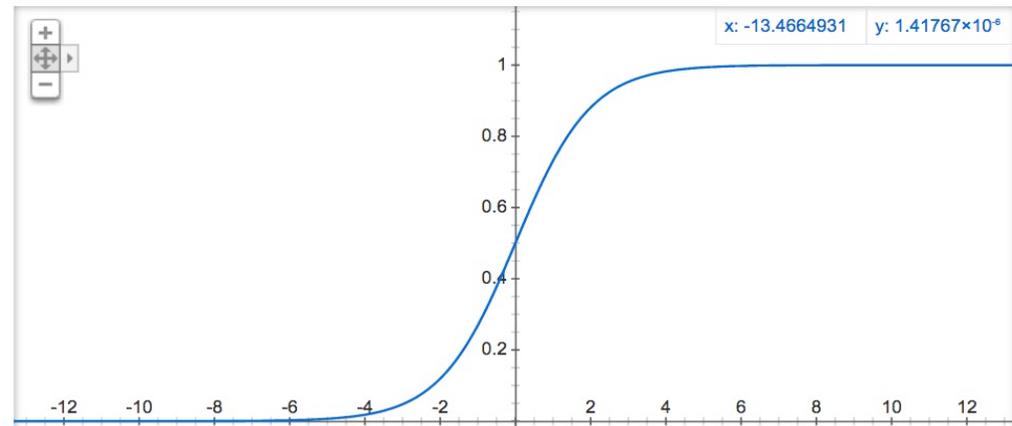
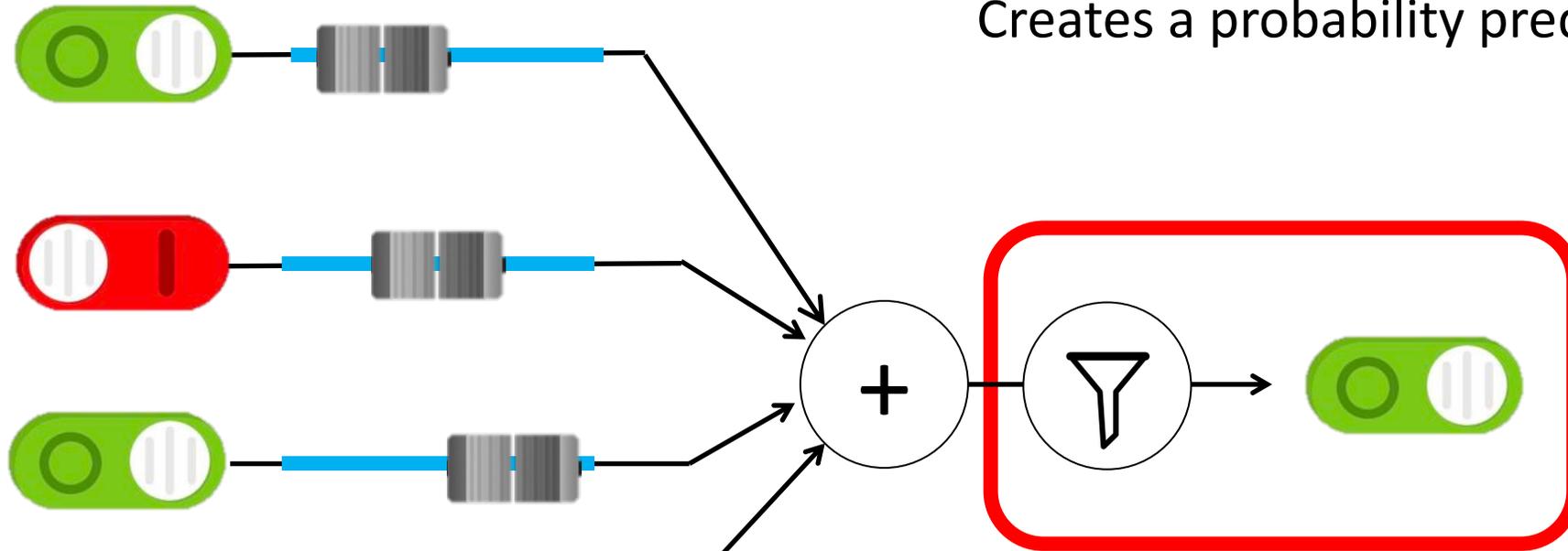


```
double weightedSum = 0;  
weightedSum += input0 * weight0;  
weightedSum += input1 * weight1;  
weightedSum += input2 * weight2;  
weightedSum += input3 * weight3;
```



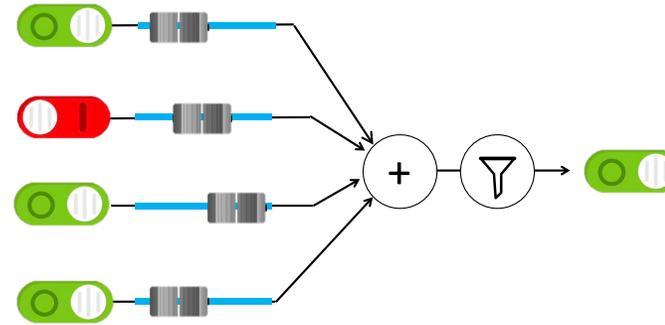
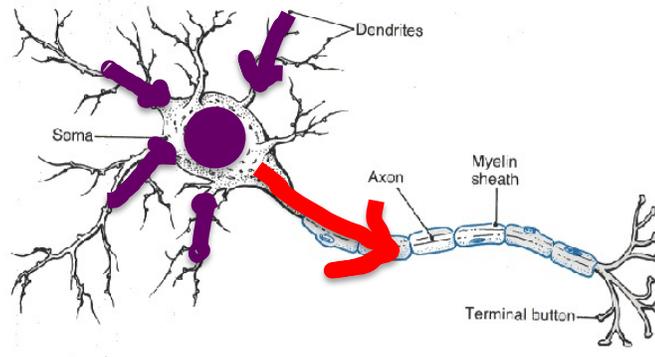
Filter and Output

Creates a probability prediction

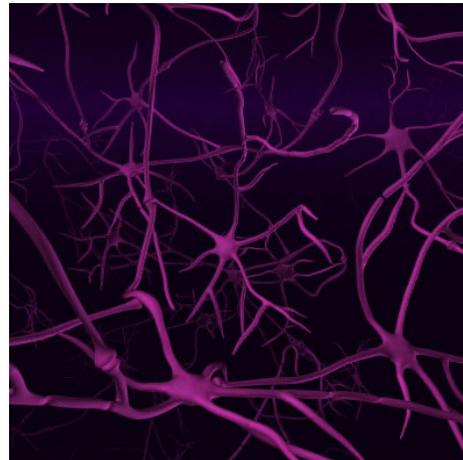


Biological Basis for Neural Networks

- A neuron



- Your brain



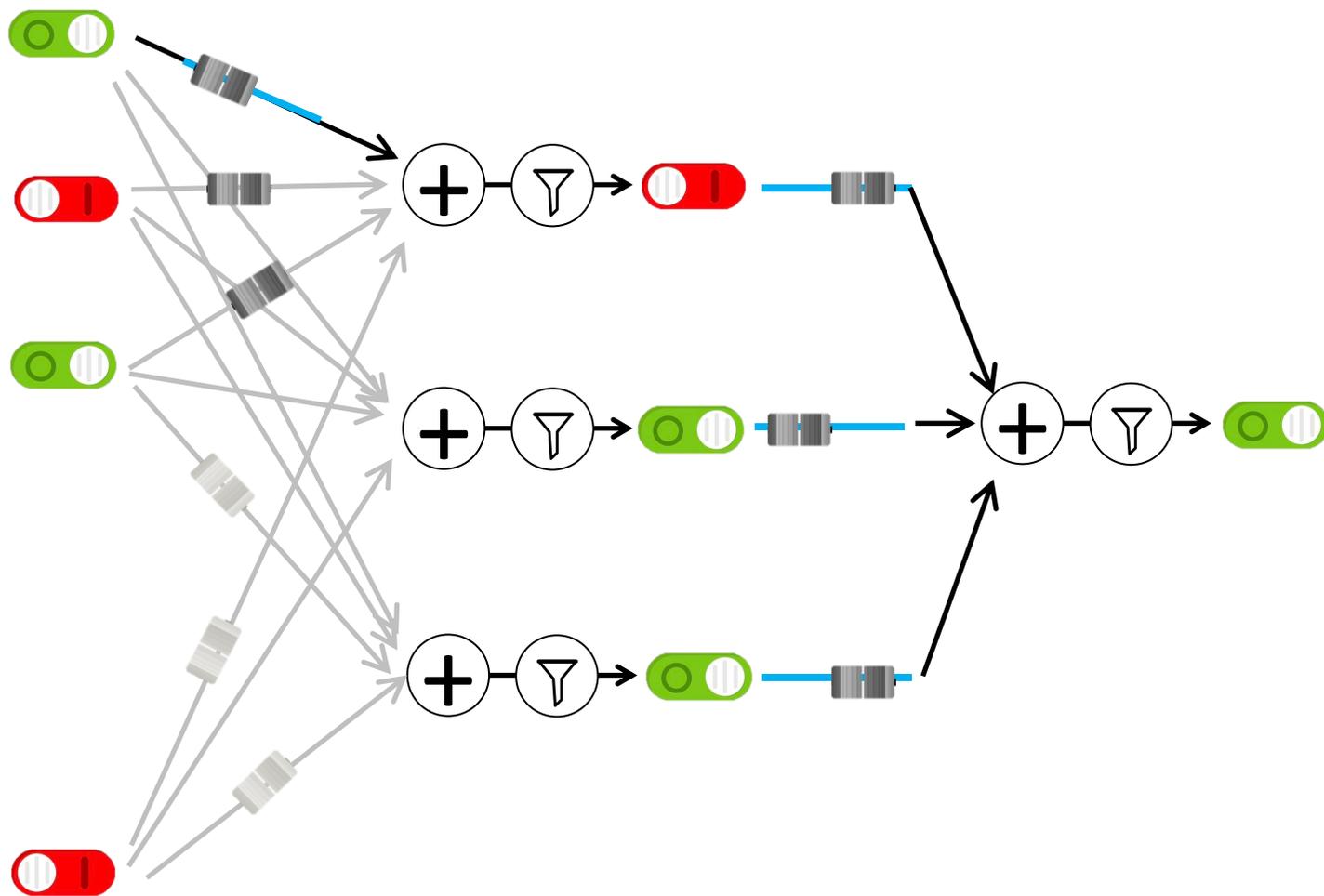
???

Actually, it's probably someone else's brain

Kim, Piech, Song, CS109, Stanford University

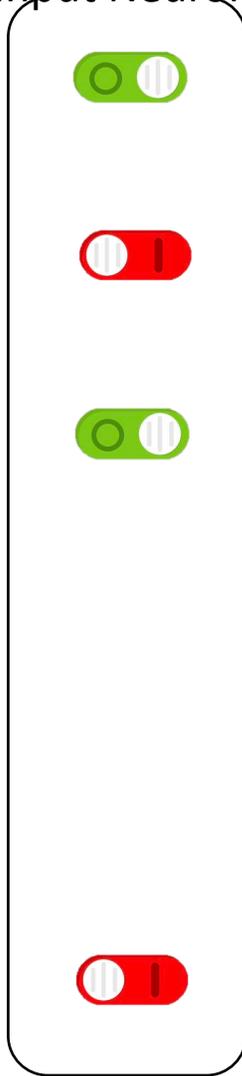


Put Many Together

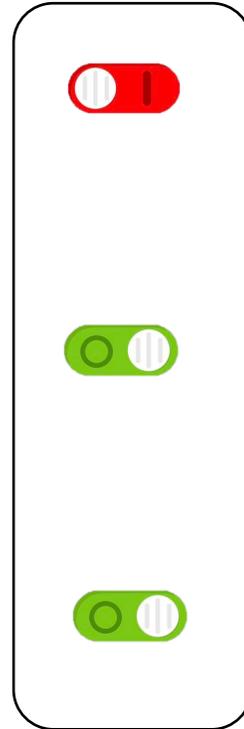


Put Many Together

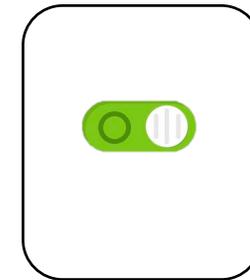
Input Neurons



Hidden Neurons



Output Neurons

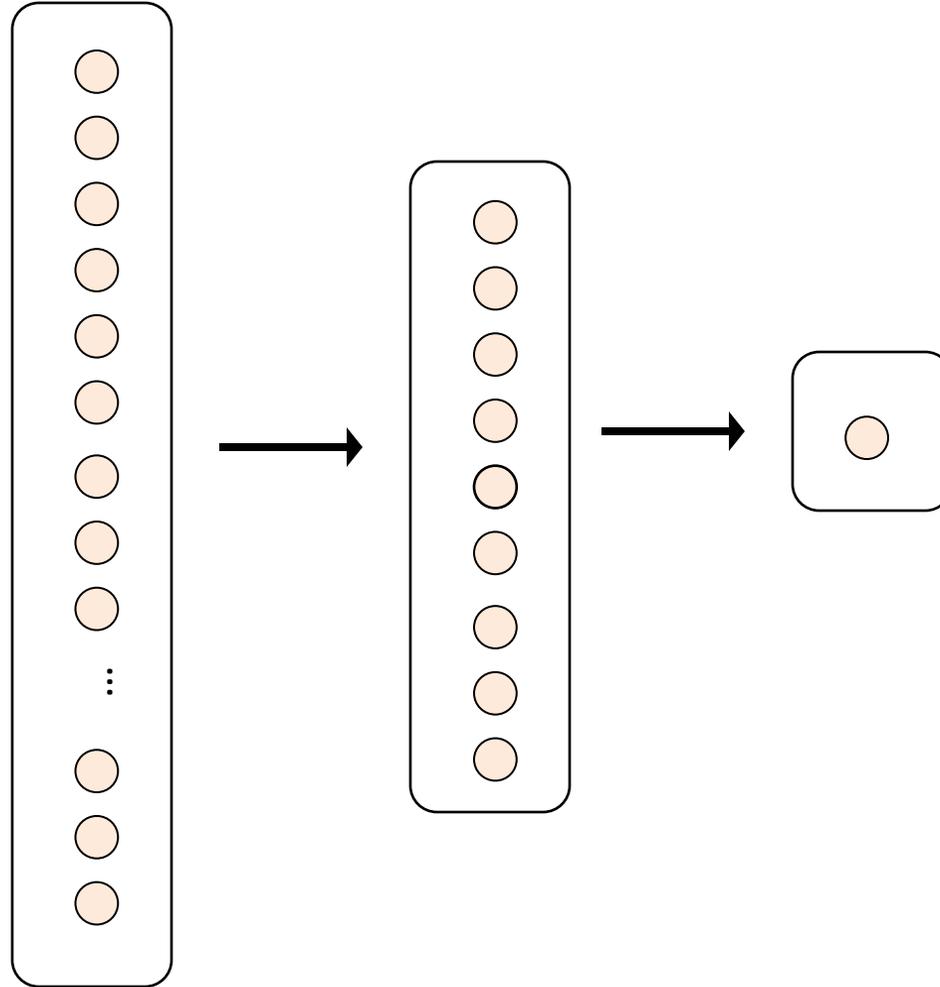


Making a Prediction

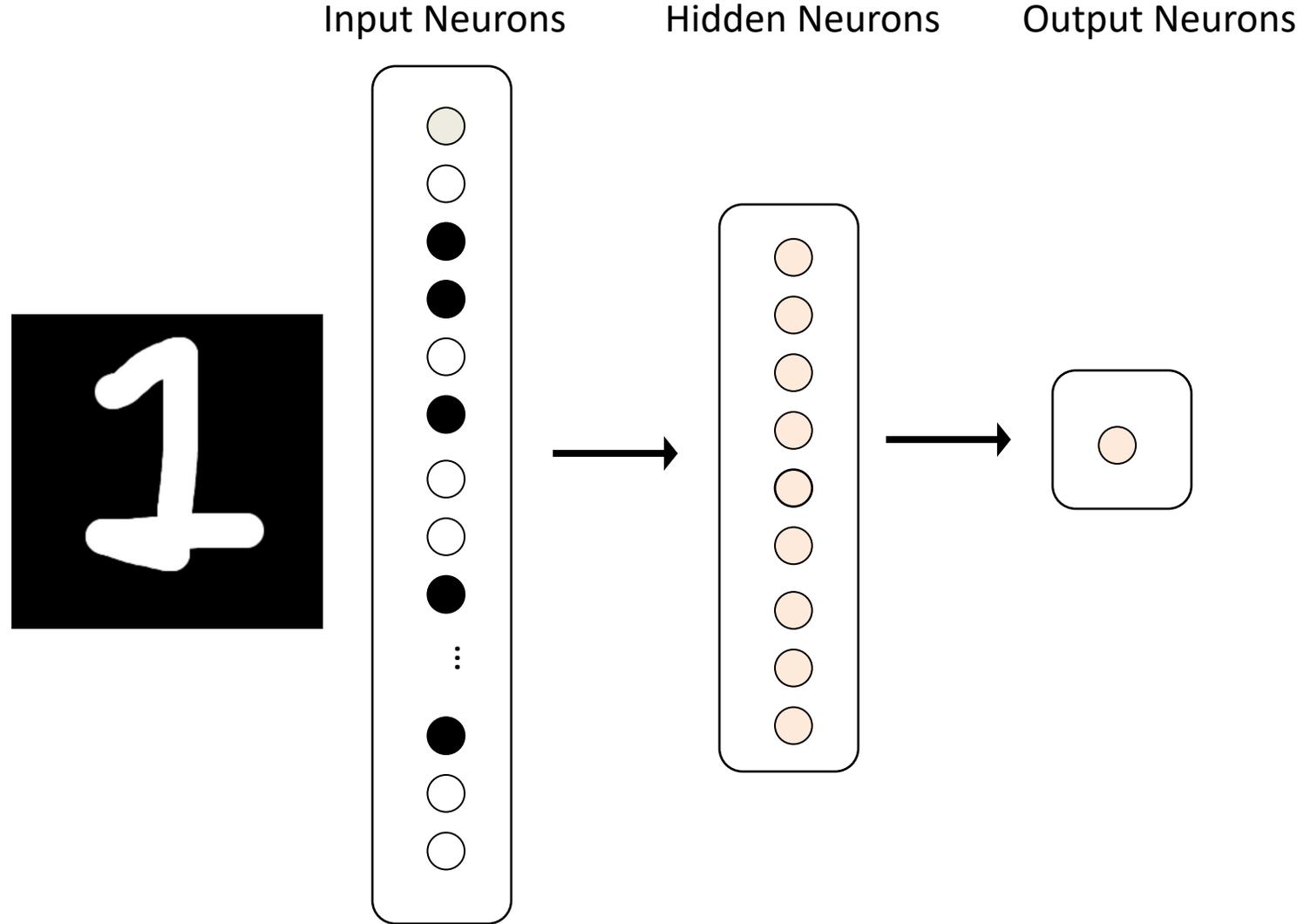
Input Neurons

Hidden Neurons

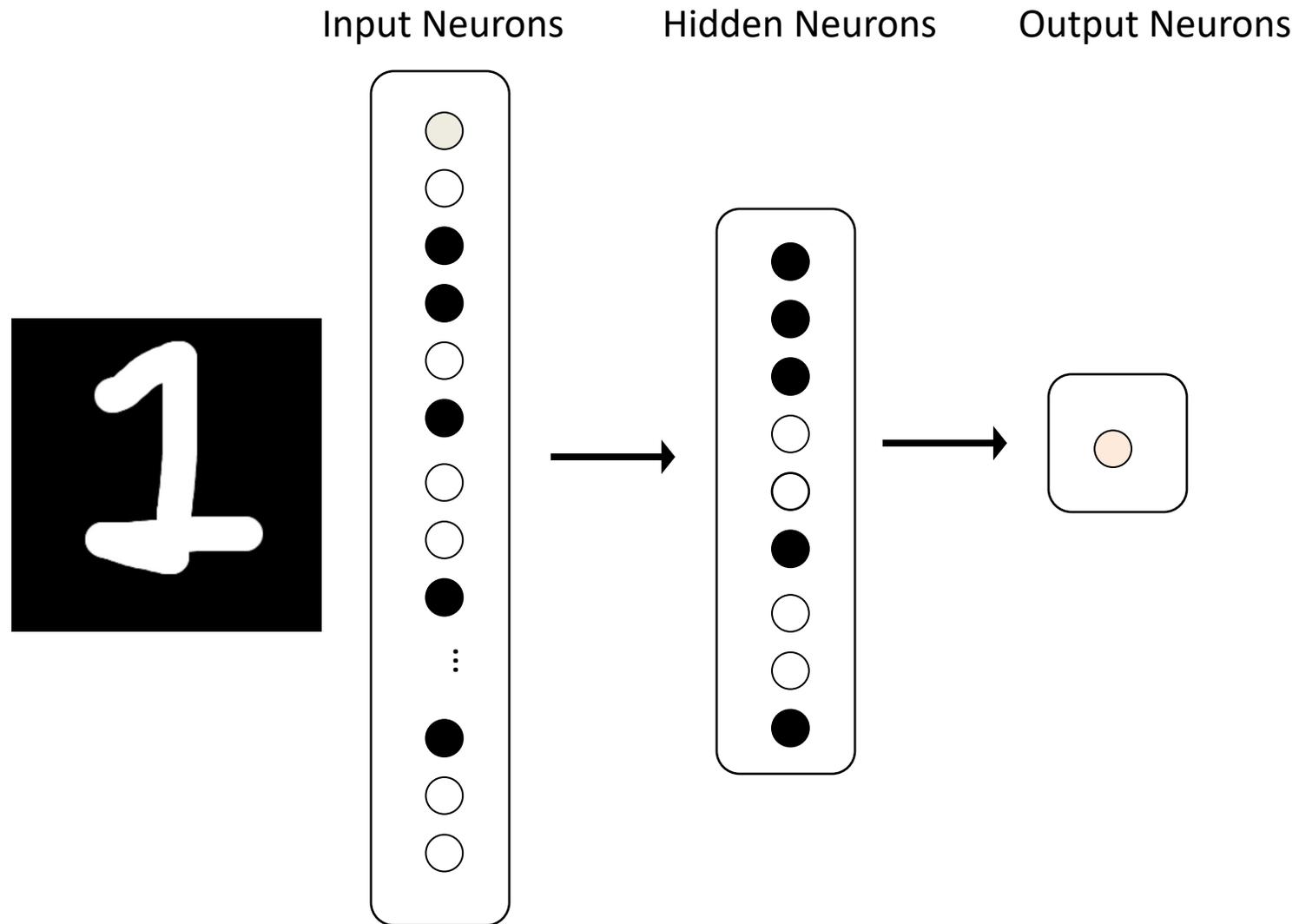
Output Neurons



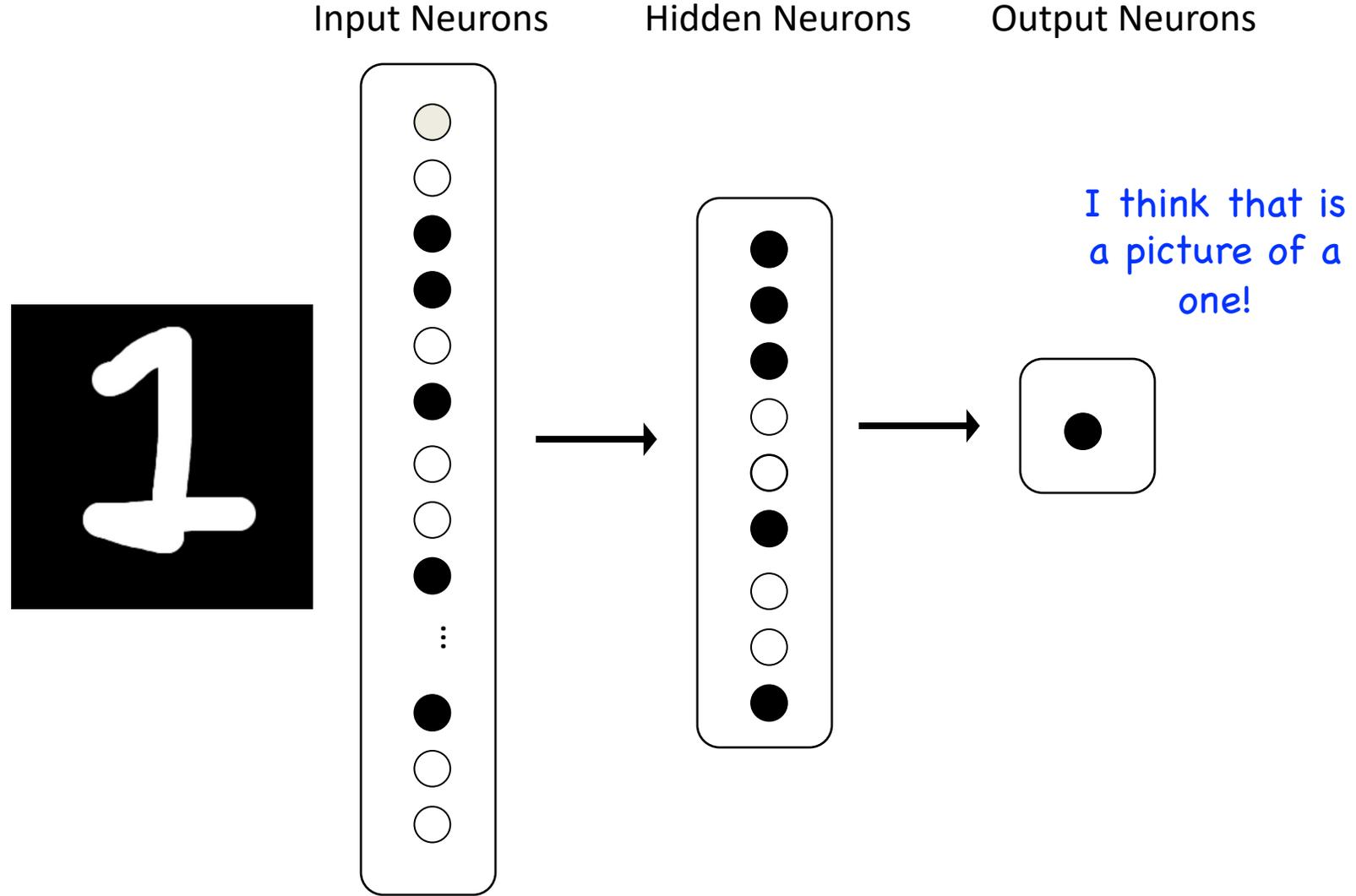
Making a Prediction



Making a Prediction

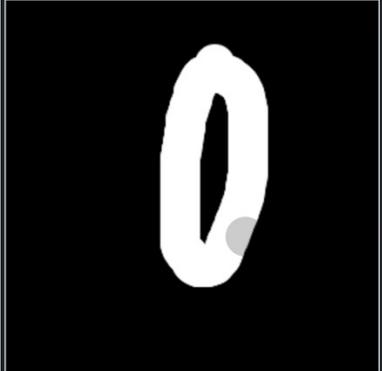


Making a Prediction



Demonstration

Draw your number here



X  

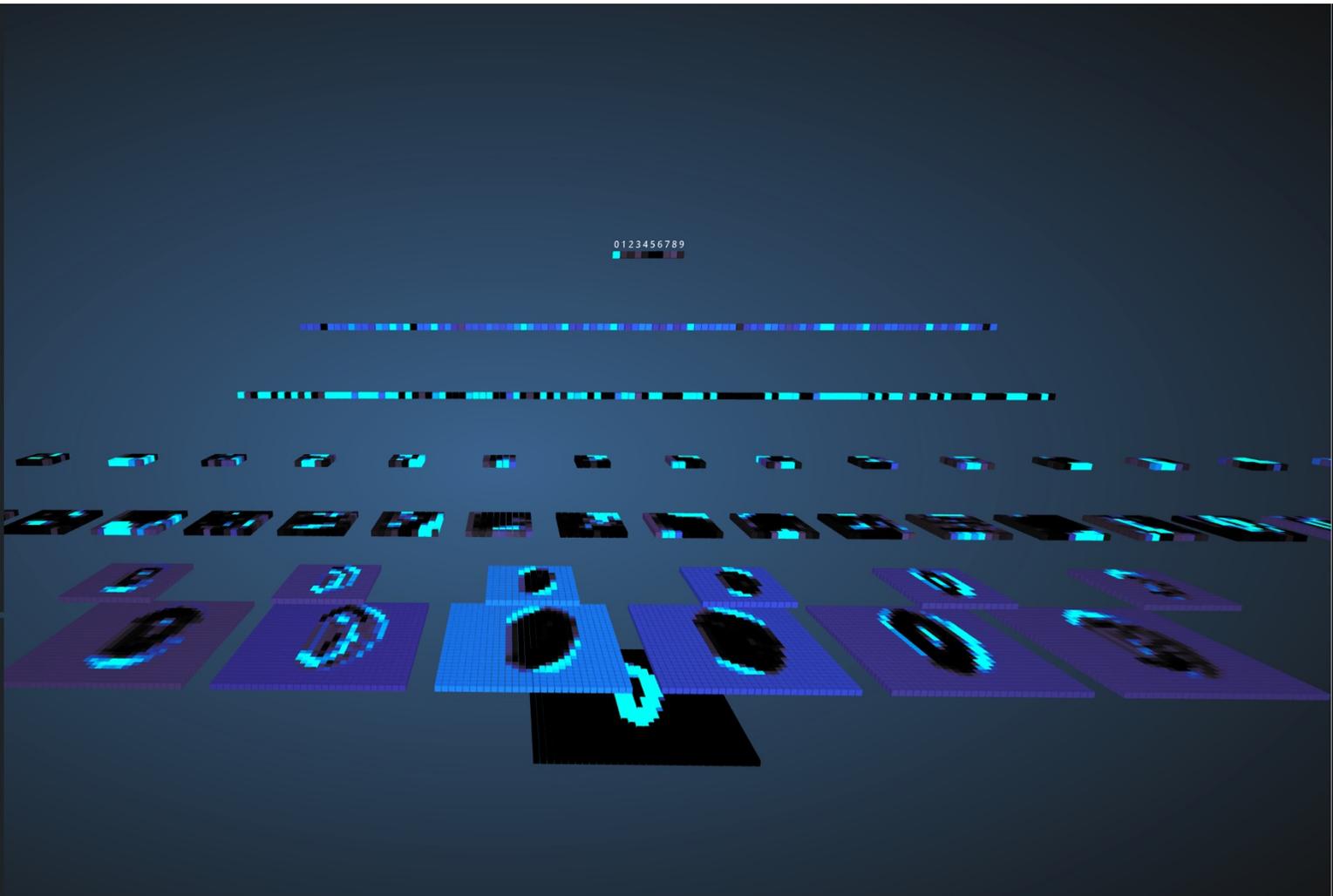
Downsampled drawing:

First guess:

Second guess:

Layer visibility

Input layer	Show
Convolution layer 1	Show
Downsampling layer 1	Show
Convolution layer 2	Show
Downsampling layer 2	Show



<http://scs.ryerson.ca/~aharley/vis/conv/>

Kim, Piech, Song, CS109, Stanford University



Interpret the Output as Prediction

Draw your number here



0 1 2 3 4 5 6 7 8 9



Downsampled drawing:

First guess: 3

Second guess: 3

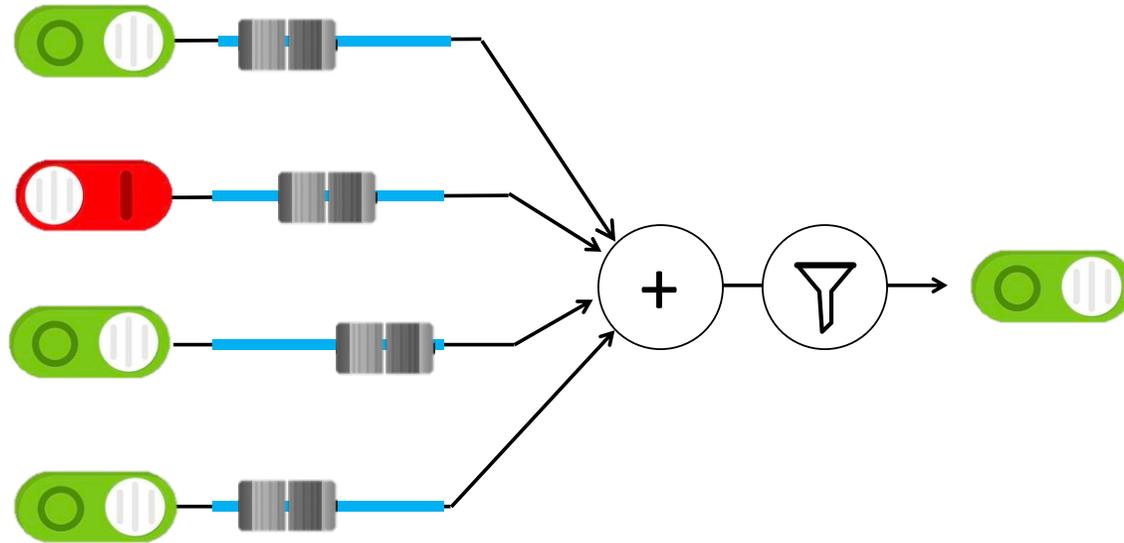
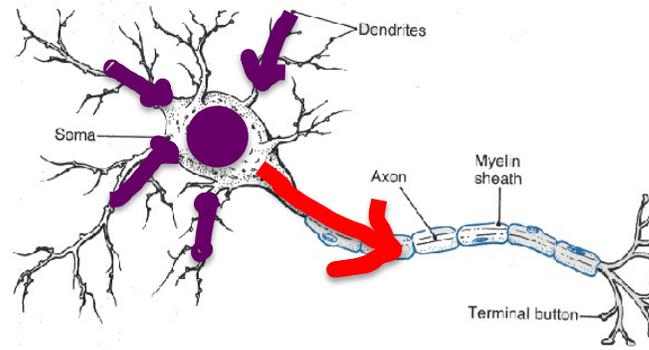
8

Layer visibility

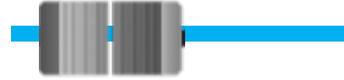
Input layer	Show
Convolution layer 1	Show
Downsampling layer 1	Show
Convolution layer 2	Show



Great Idea: Artificial Neurons



Where do Artificial
Neural Networks
get their
intelligence from?



Neural Networks get their intelligence from their sliders (parameters)

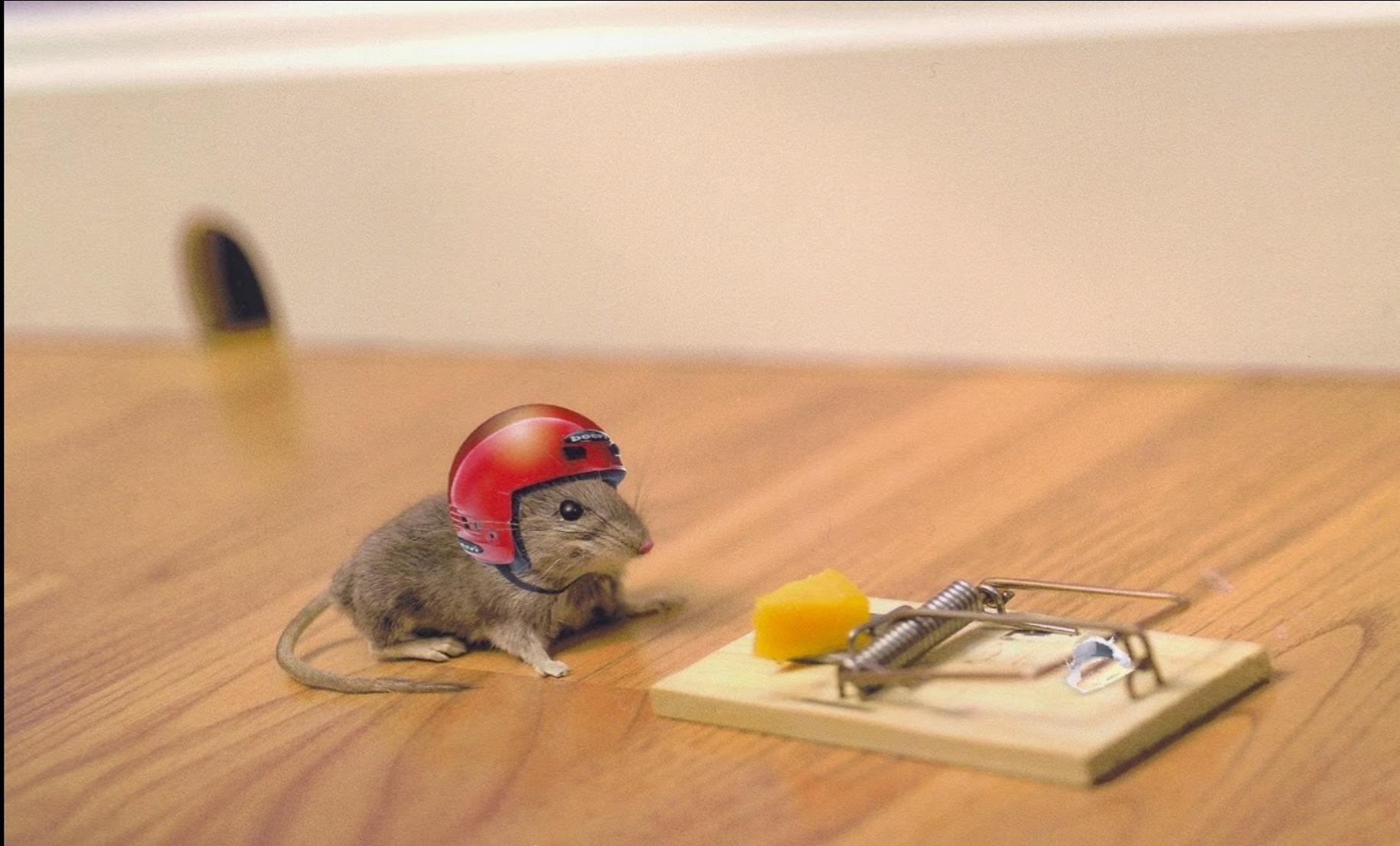


Two Great Ideas

1. Artificial Neurons

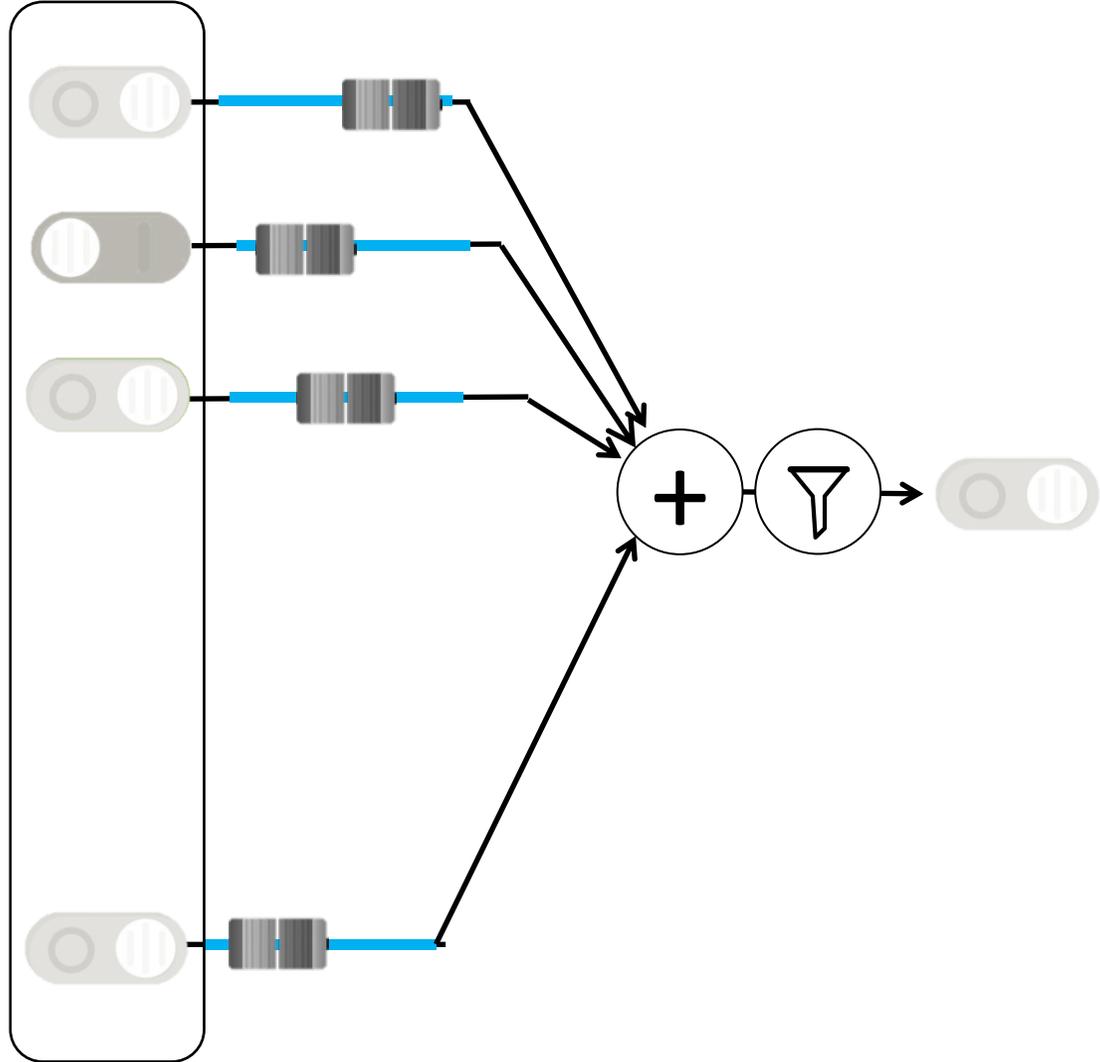
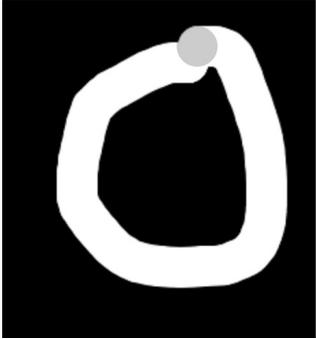
2. Learn by Example

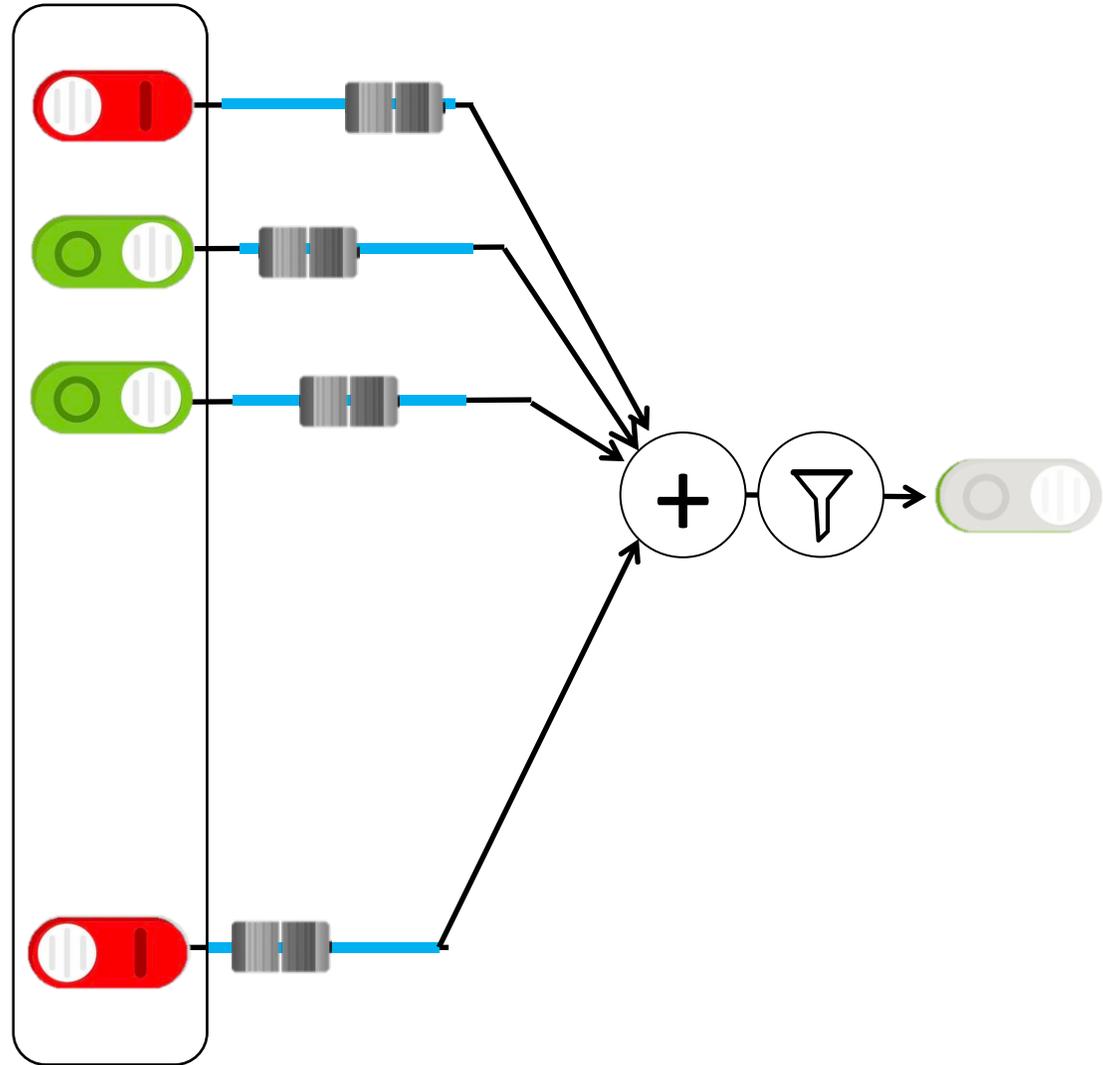
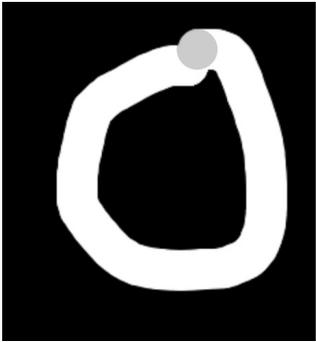
2. Learn From Experience

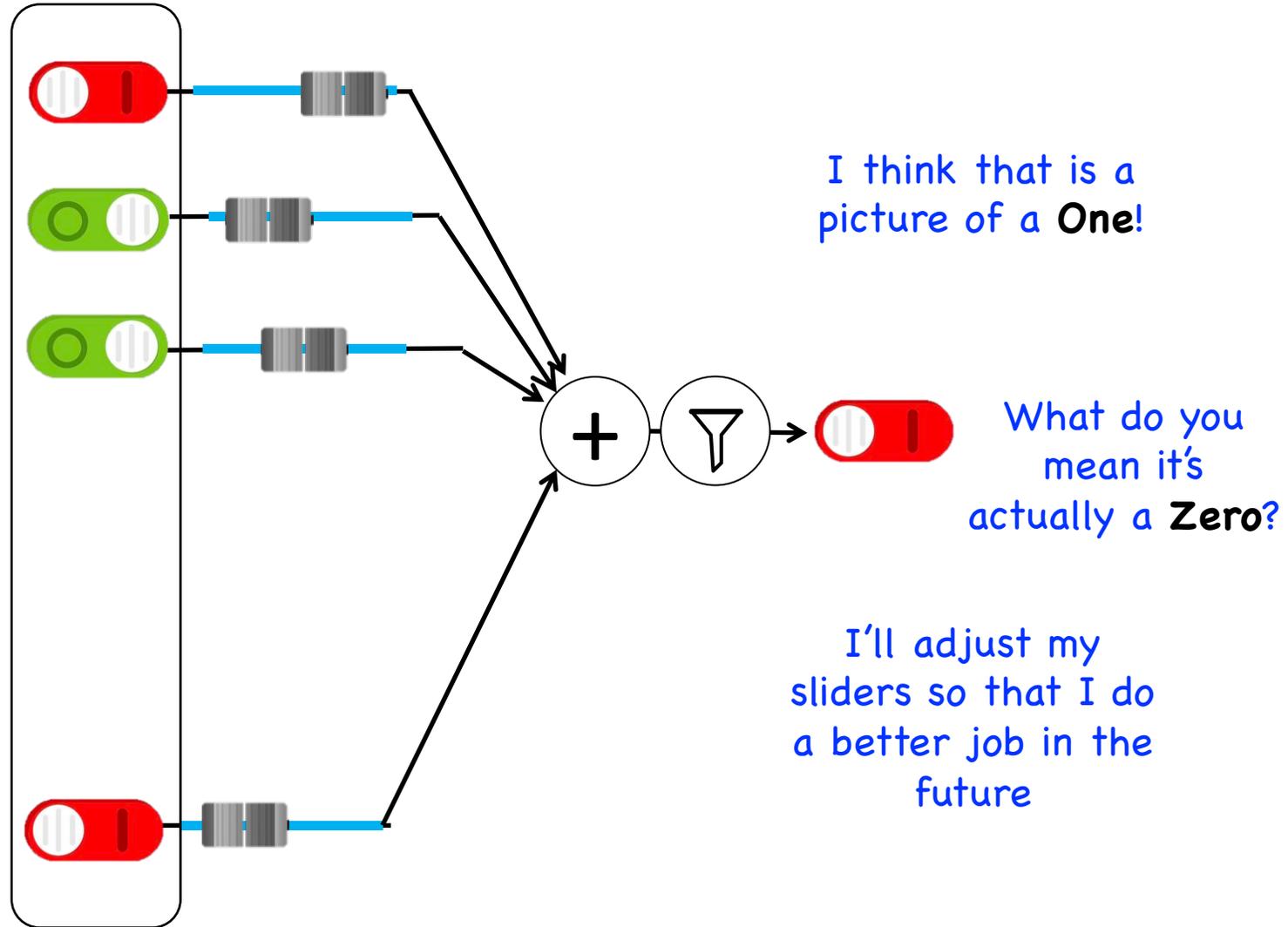
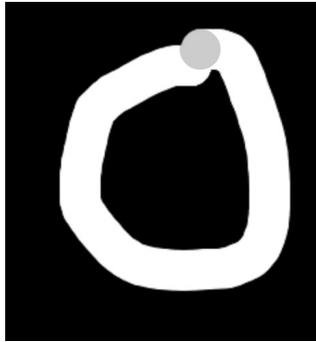


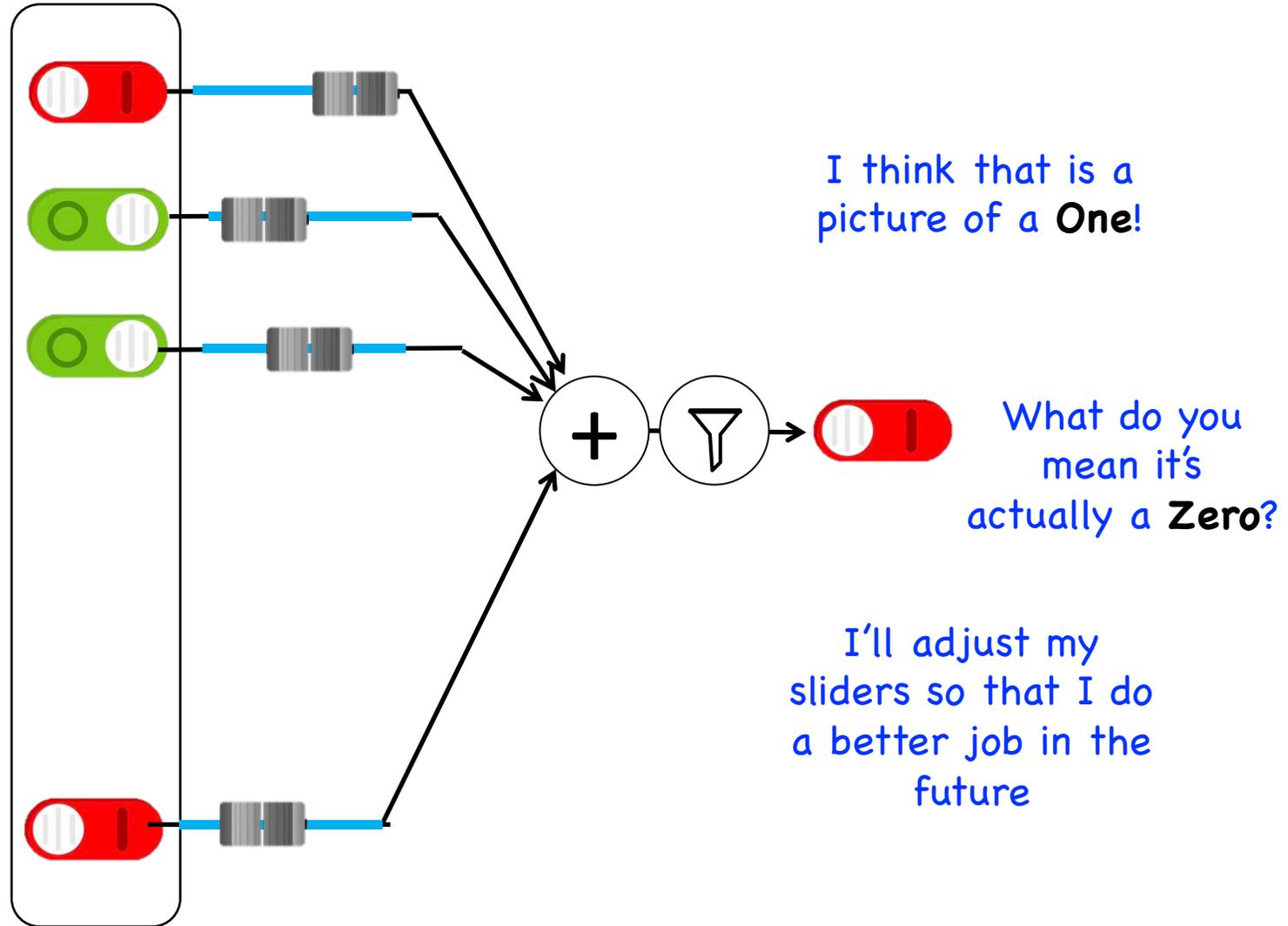
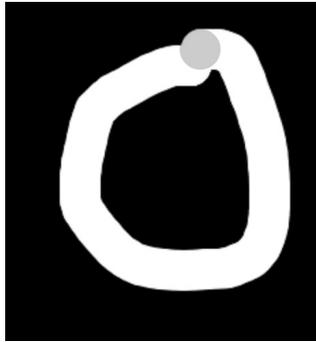
Learn by Example

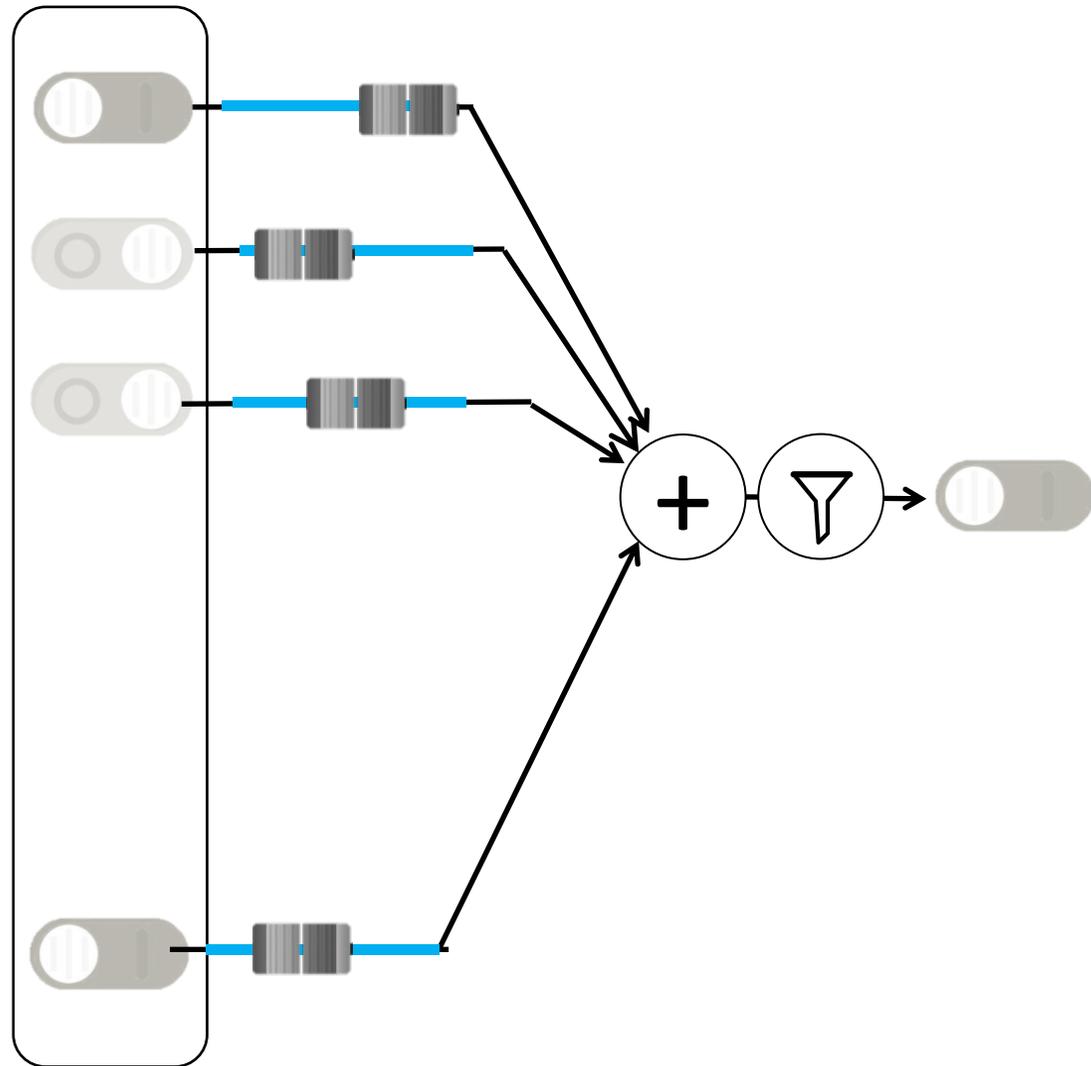


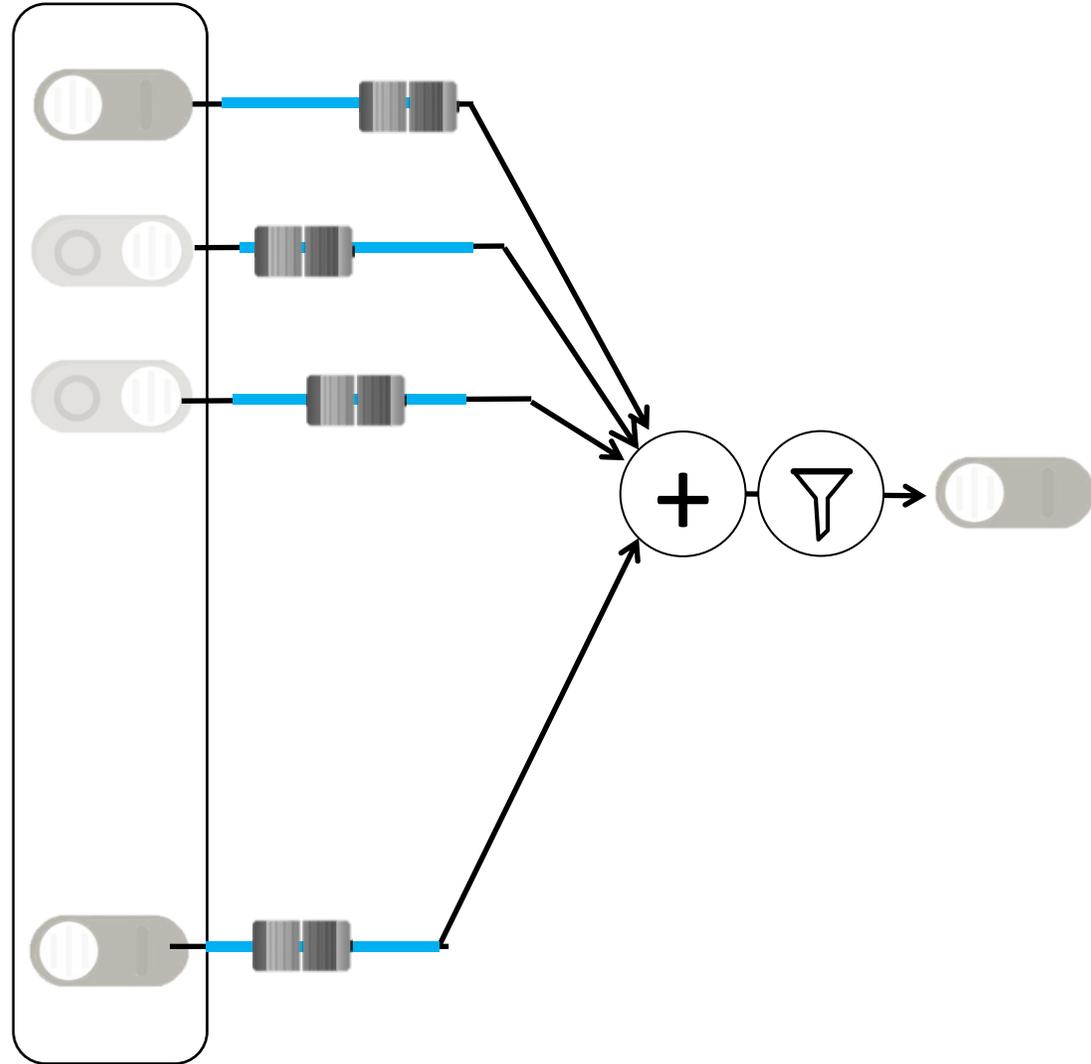


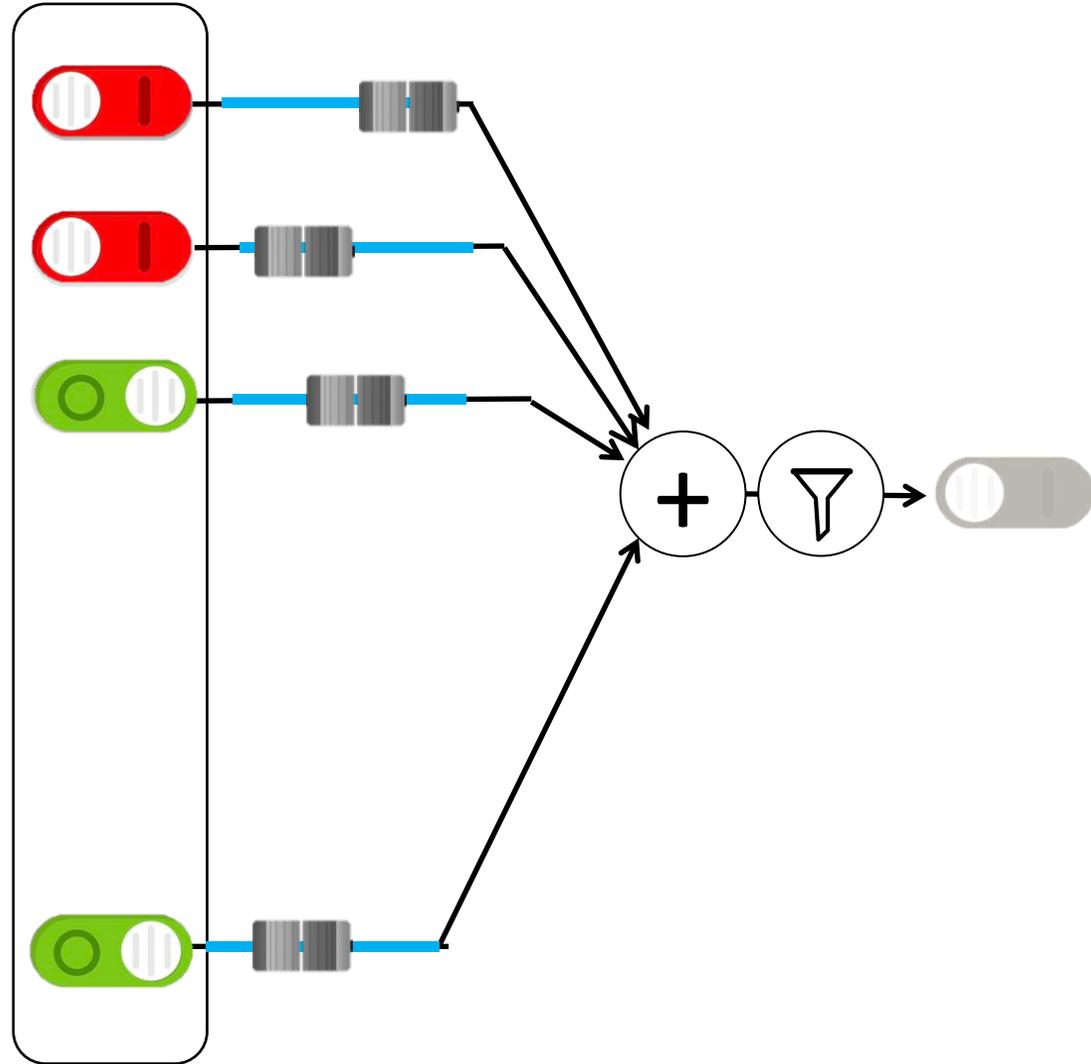


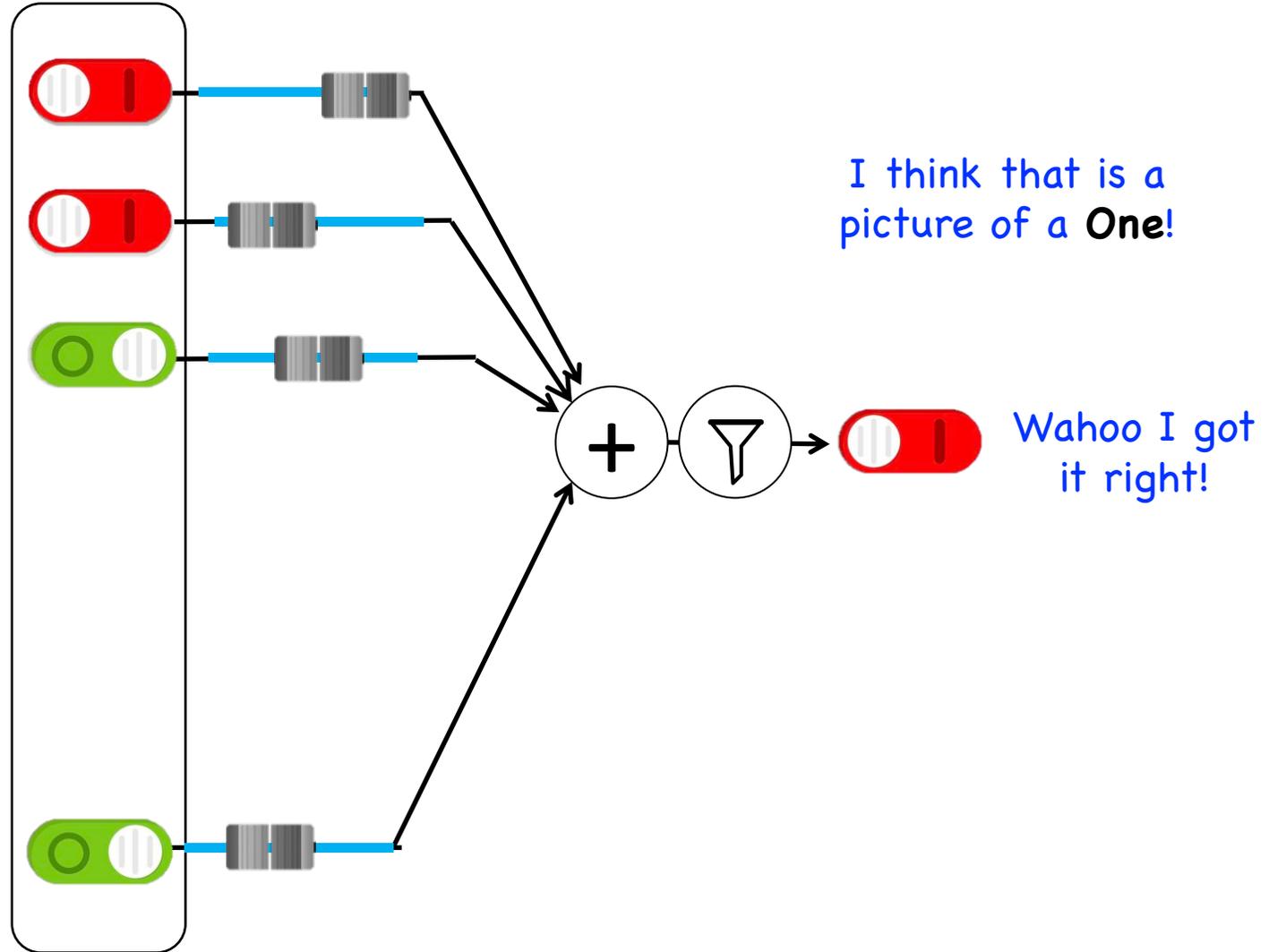


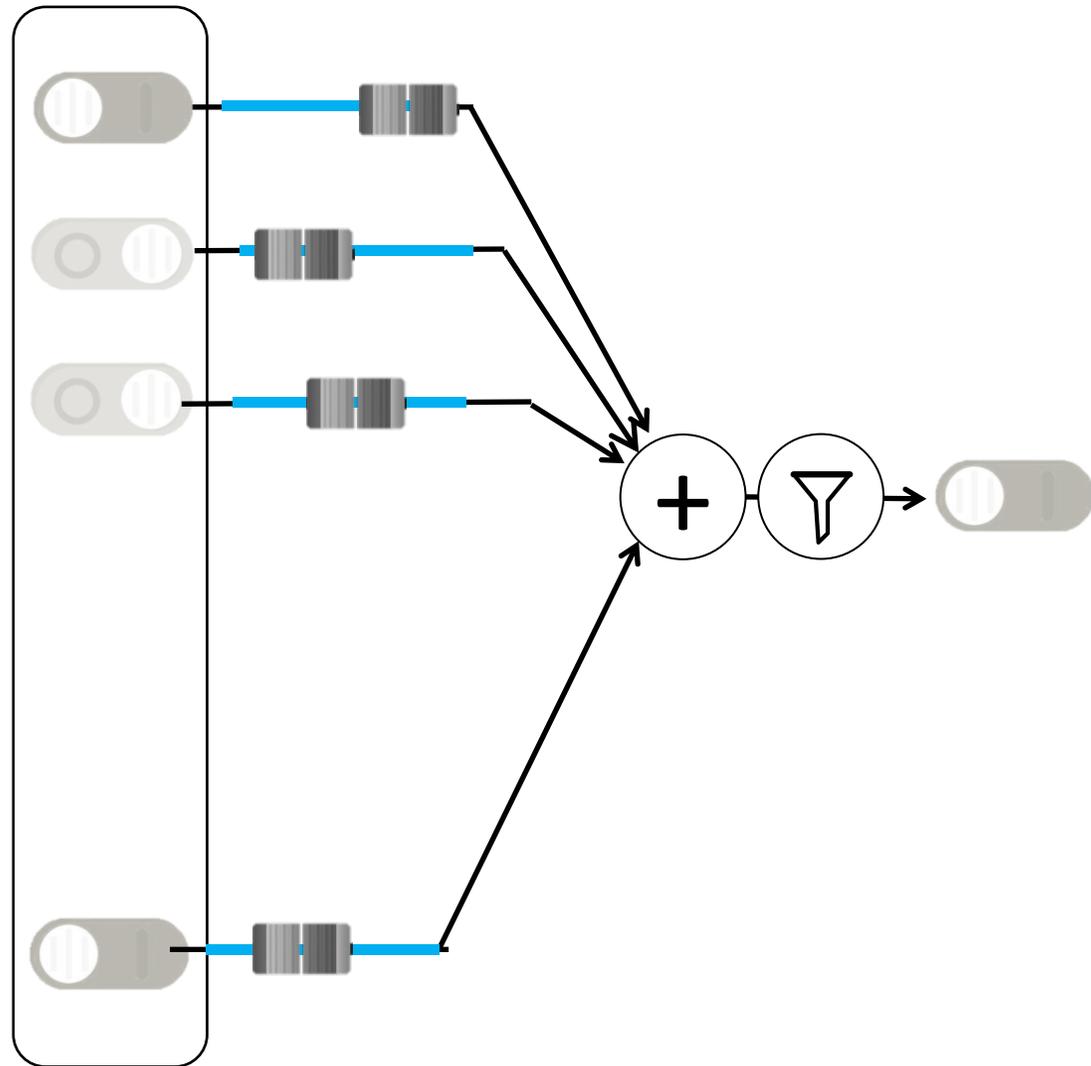


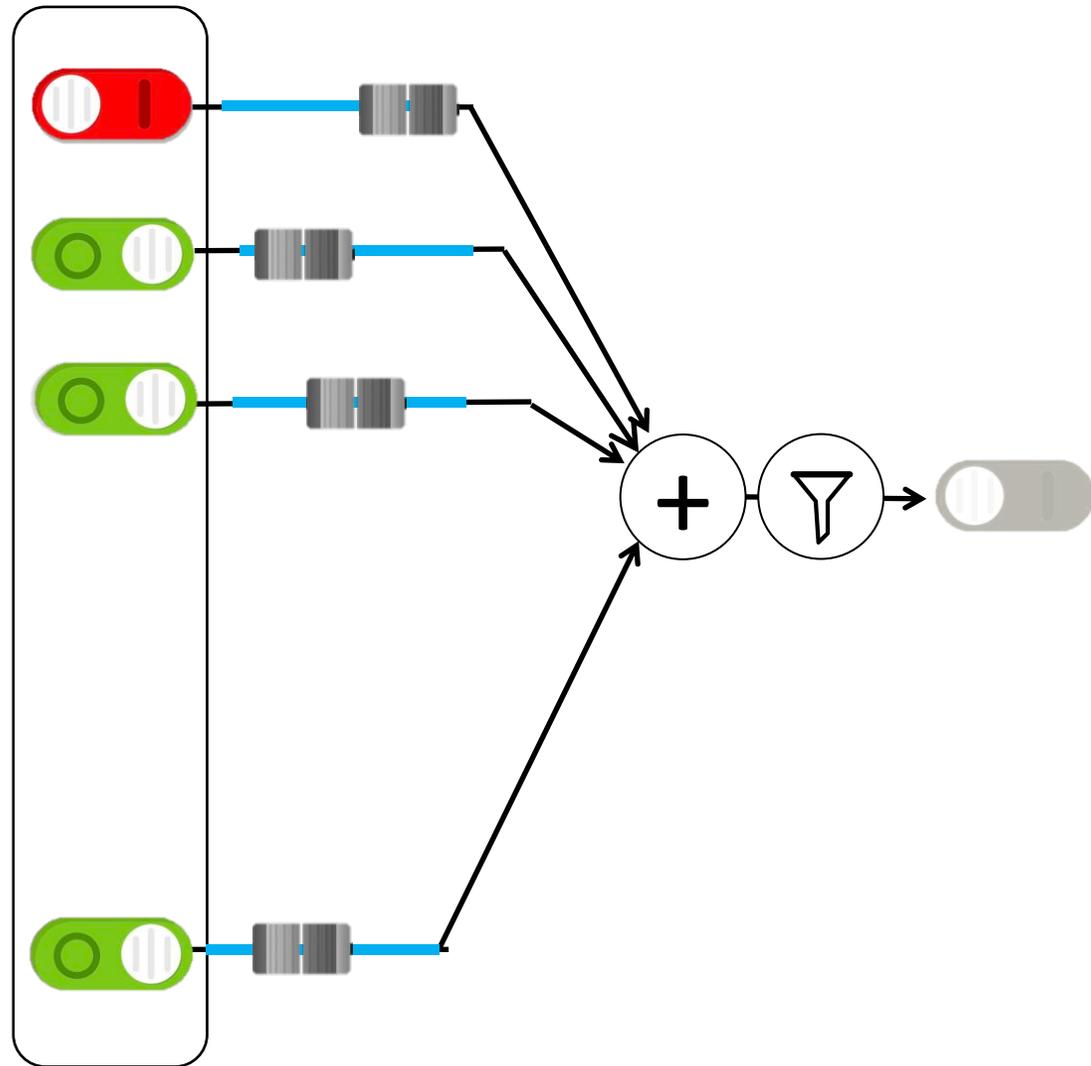


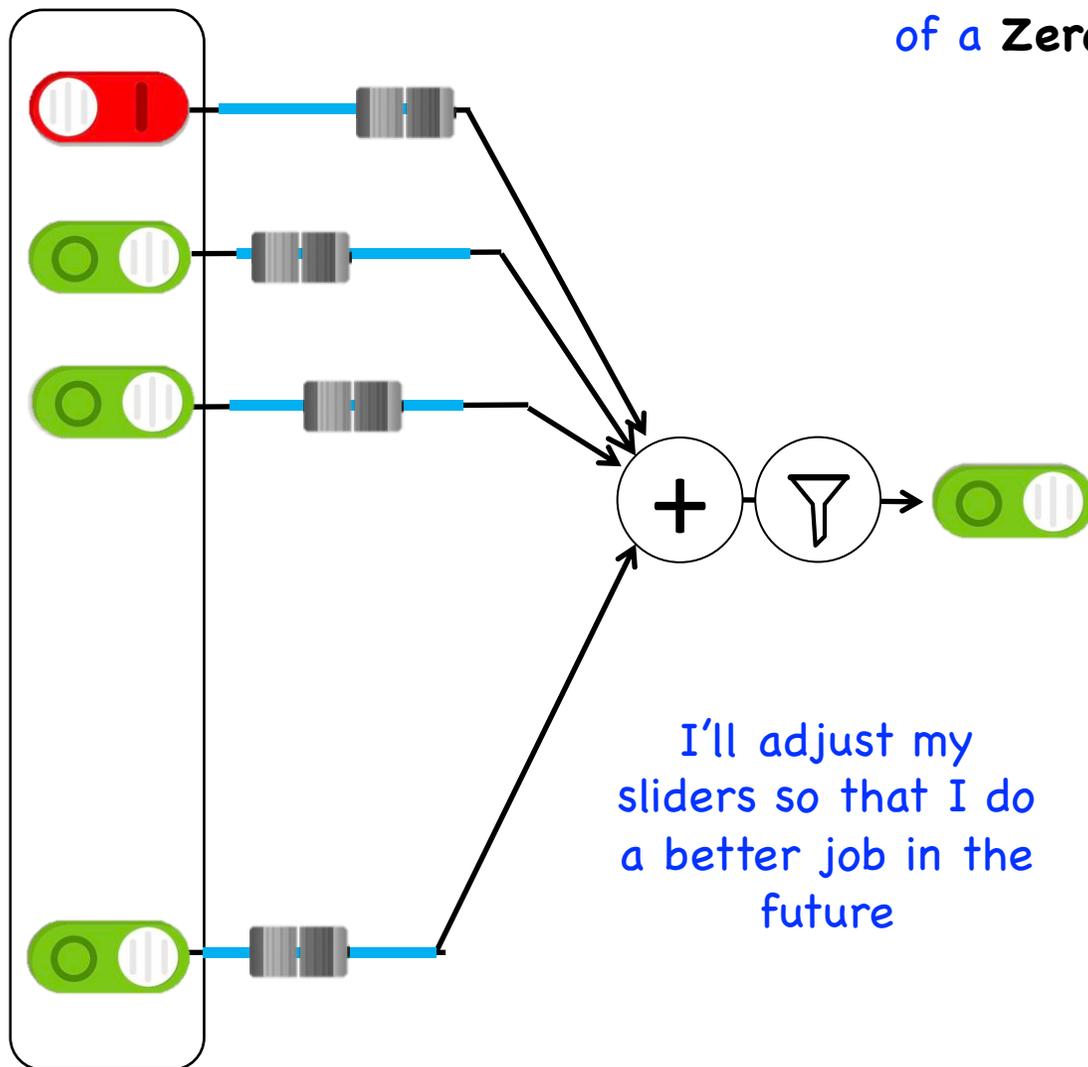










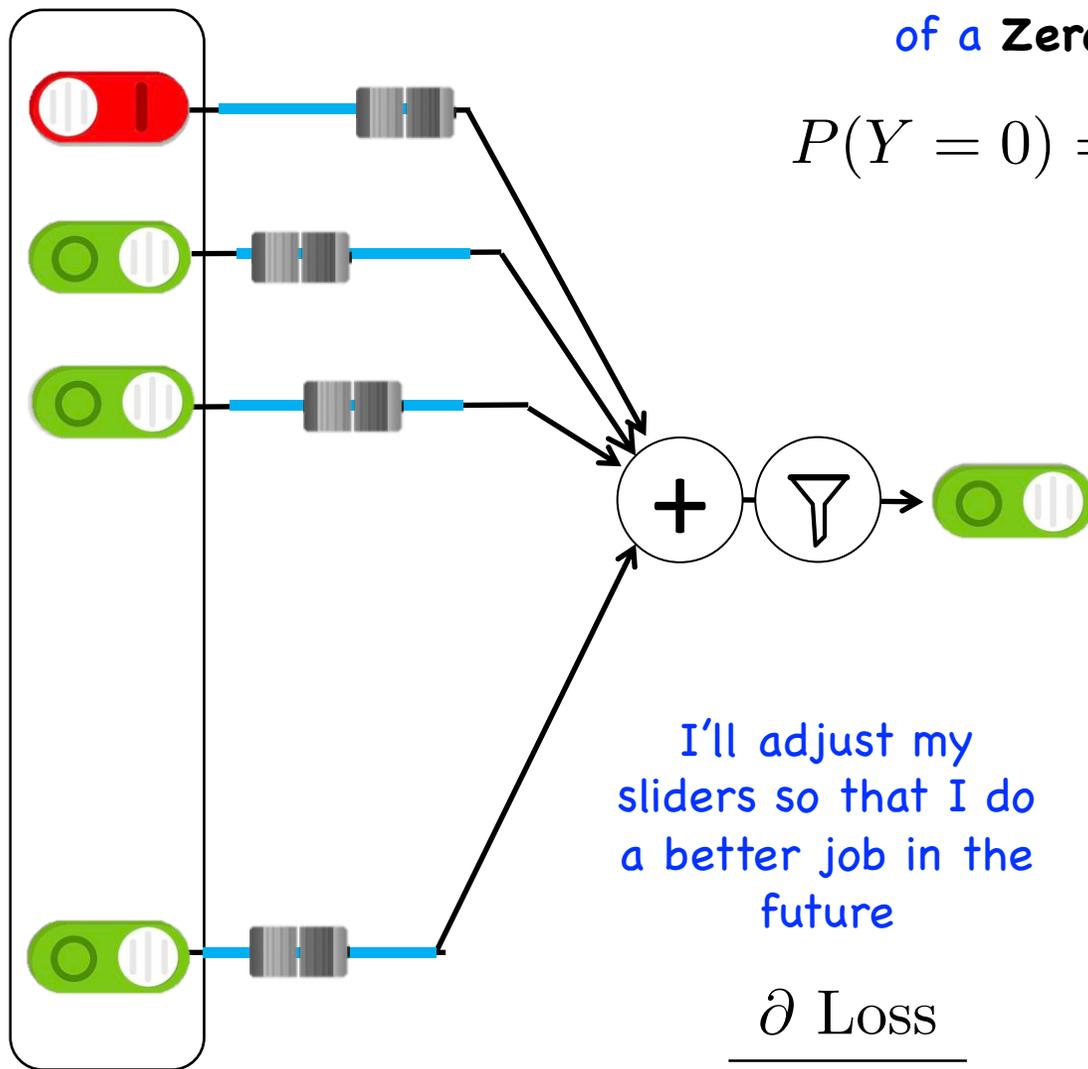


I think that is a picture of a **Zero**!

But it is actually a **Zero**

I'll adjust my sliders so that I do a better job in the future





I think that is a picture
of a **Zero!**

$$P(Y = 0) = 0.9$$

But it is
actually a **One**

$$\text{Loss} = 1$$

I'll adjust my
sliders so that I do
a better job in the
future

$$\frac{\partial \text{Loss}}{\partial \text{Slider}_i}$$



Update Neural Network

$$P(Y = 1|X = \mathbf{x}) = \hat{y} \qquad \hat{y} = \sigma \left(\sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})} \right)$$

For one datum

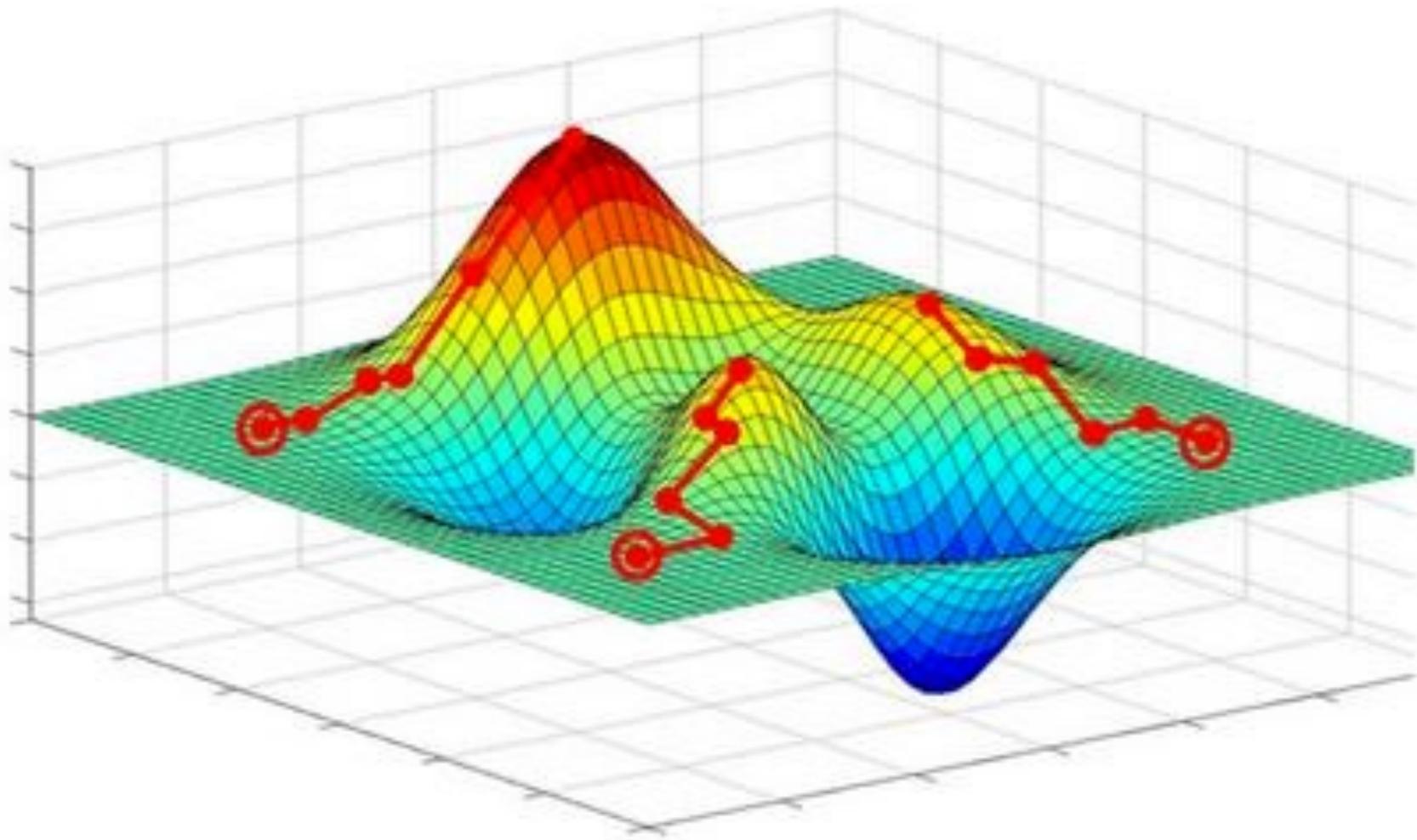
$$P(Y = y|X = \mathbf{X}) = (\hat{y})^y (1 - \hat{y})^{1-y}$$

For IID data

$$\begin{aligned} L(\theta) &= \prod_{i=1}^n P(Y = y^{(i)} | X = \mathbf{x}^{(i)}) \\ &= \prod_{i=1}^n (\hat{y}^{(i)})^{y^{(i)}} \cdot \left[1 - (\hat{y}^{(i)}) \right]^{(1-y^{(i)})} \end{aligned}$$



Gradient Descent



Walk uphill and you will find a local maxima
(if your step size is small enough)

Kim, Piech, Song, CS109, Stanford University



Gradient of Probability

$$\frac{\partial L}{\partial \theta_i^{(\hat{y})}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \theta_i^{(\hat{y})}}$$

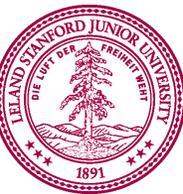
$$\hat{y} = \sigma \left(\sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})} \right)$$

$$\frac{\partial \hat{y}}{\partial \theta_i^{(\hat{y})}} = \sigma \left(\sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})} \right) \left[1 - \sigma \left(\sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})} \right) \right] \cdot \frac{\partial}{\partial \theta_i^{(\hat{y})}} \sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})}$$

$$= \hat{y} [1 - \hat{y}] \cdot \frac{\partial}{\partial \theta_i^{(\hat{y})}} \sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})}$$

$$= \hat{y} [1 - \hat{y}] \cdot h_i$$

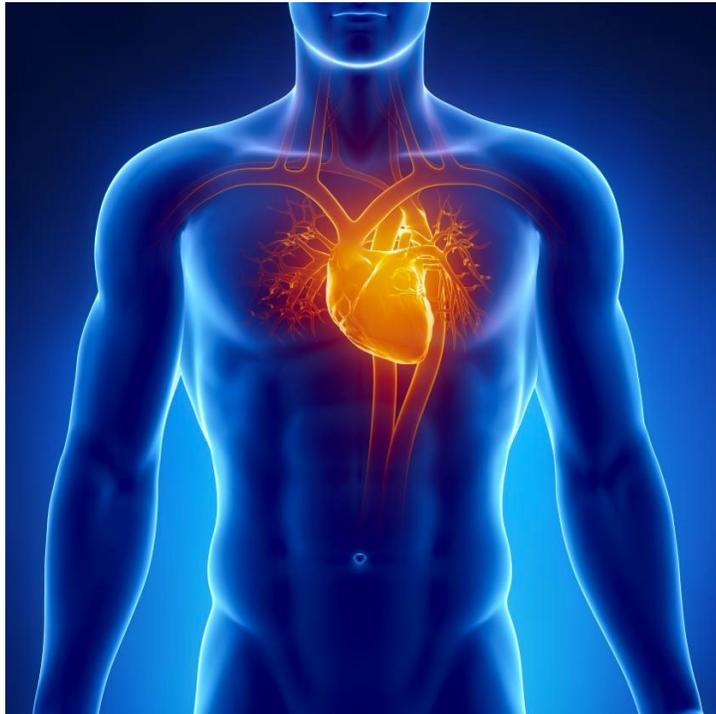
You will be able to do this.



Where you will be by the end of class

CS109: Theory Class focused on Applications

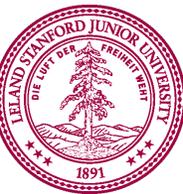
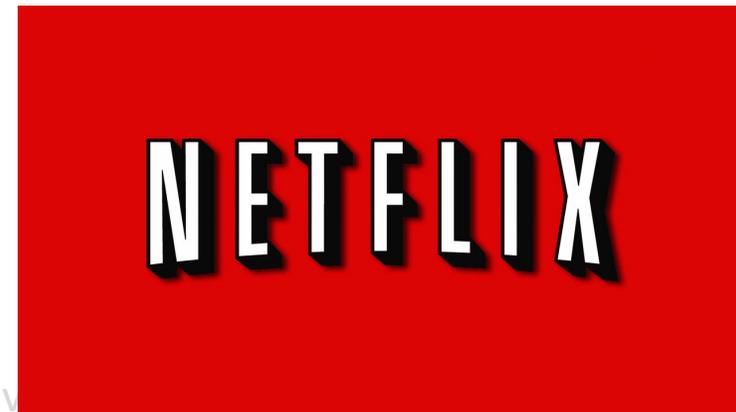
Heart Disease Detector



Ancestry Genetic Closeness



Netflix Recommender



End of Story

Except it isn't the end of the story...

Probability is more than just machine learning

Abundance of important problems



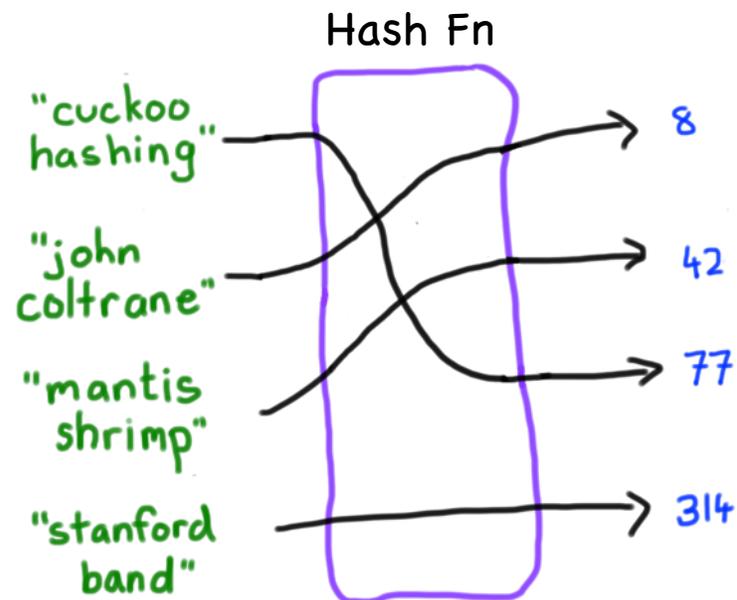
Algorithms and Probability

Eg Sorting

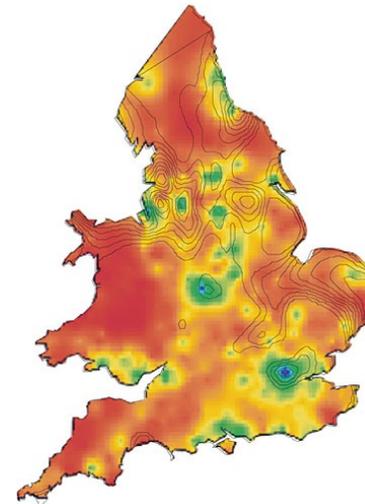


QuickSort

Eg HashMaps



Understanding the world and building tools



Autocomplete



dinosaurs we

[Advanced Search](#)
[Language Tools](#)

dinosaurs **websites for kids**
dinosaurs **we're back**
dinosaurs **webcomic**
dinosaurs **webquest**
dinosaurs **were made up by the cia to discourage time travel**
dinosaurs **website**
dinosaurs **went extinct**
dinosaurs **weight**
dinosaurs **we are scientists**
dinosaurs **weed episode**

Google Search I'm Feeling Lucky



Recommender Systems

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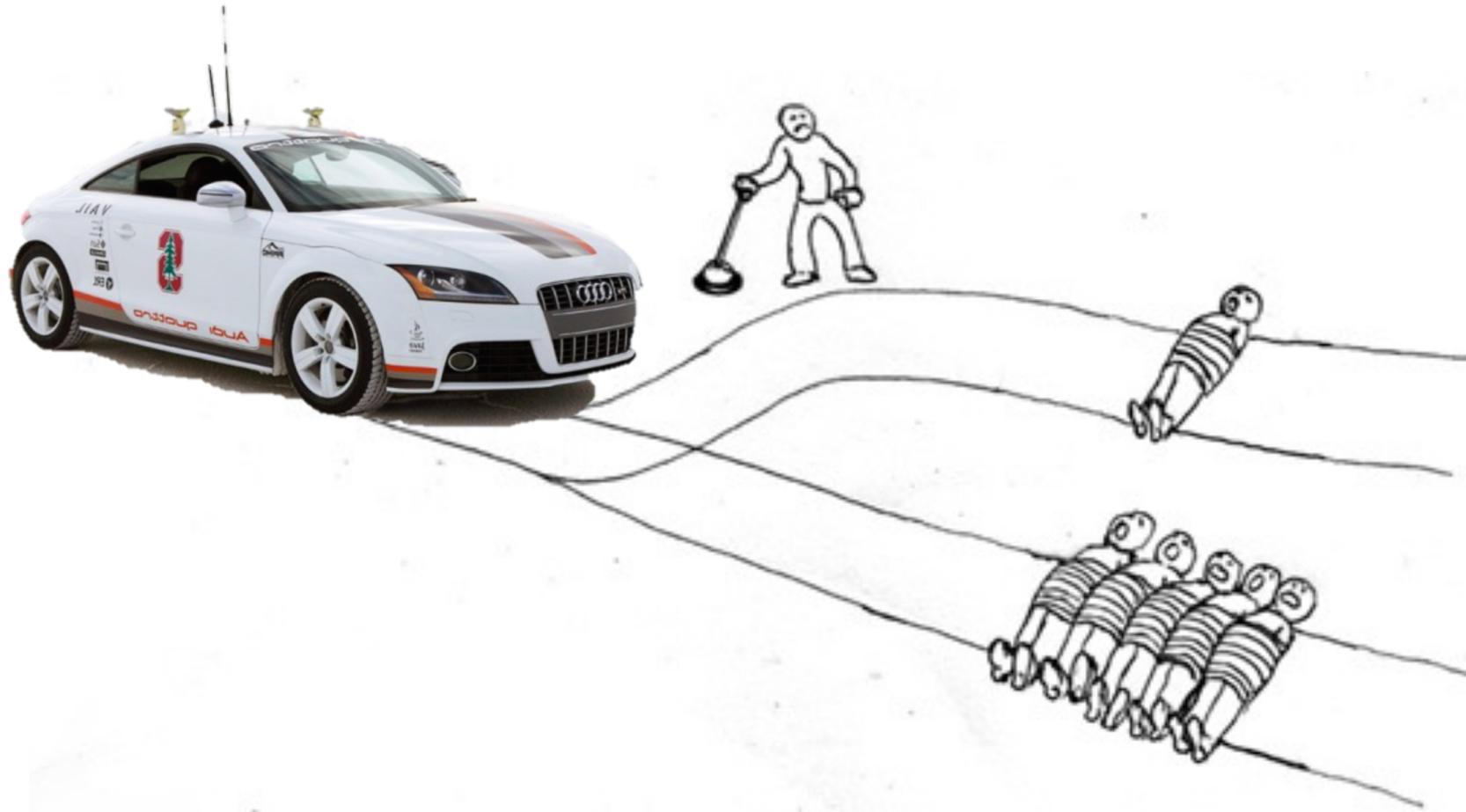
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★★★★☆ (5,186) \$19.79
- Harry Potter and the Order of the Phoenix (Book 5)** by J. K. Rowling
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- Harry Potter and the Half-Blood Prince (Book 6)** by J.K. Rowling
★★★★☆ (3,597) \$10.18
- The Tales of Beedle the Bard, Collector's Ed...** by J. K. Rowling
★★★★☆ (176)

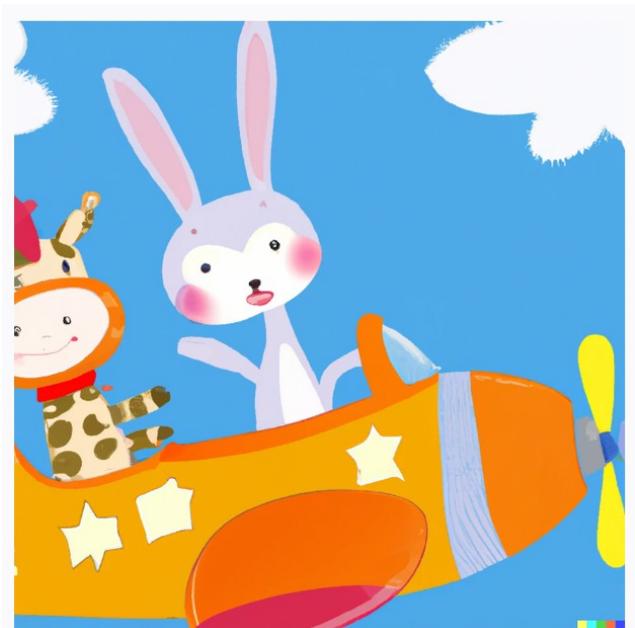


Philosophy and Ethics



Art and Probability

Create a probabilistic sample from the distribution of images, given a prompt



Most Desired Skill in Industry

Forbes Billionaires Innovation Leadership Money Consumer

30,575 views | Jan 29, 2018, 02:47pm

Data Scientist Is the Best Job In America According Glassdoor's 2018 Rankings

TWEET THIS

🐦 Data Scientist has been named the best job in America for three years running, with a median base salary of \$110,000 and 4,524 job openings.

🐦 DevOps Engineer is the second-best job in 2018, paying a median base salary of \$105,000 and 3,369 job openings.

f
t
in



- /Administration
- /Human Resources
- /Legal
- /Accounting
- /Finance
- /Marketing
- /Publicity
- /Promotion
- /Research
- /Business
- /Development
- /Engineering
- /Manufacturing
- /Planning

Job Score is based on:

- Earning potential
- Number of jobs
- Job satisfaction rating

“Data science and machine learning are generating more jobs than candidates right now, making these two areas the *fastest growing employment areas.*”

9.8 times more jobs than five years ago.

[LinkedIn's 2017 U.S. Emerging Jobs Report](#)



Most Desired Skill in Academia

Most CS PhD students list their highest desiderata upon graduation as:

“Better understanding of probability”



Learn Real Skills in CS109



But its not always intuitive

But Its NOT Always Intuitive



A patient has a
positive Zika test.

What is the probability they have zika?

-
- *0.8% of people have zika*
 - *Test has 90% positive rate for people with zika*
 - *Test has 7% positive rate for people without zika*

The right answer is 9%

Probability = Important + Needs Study

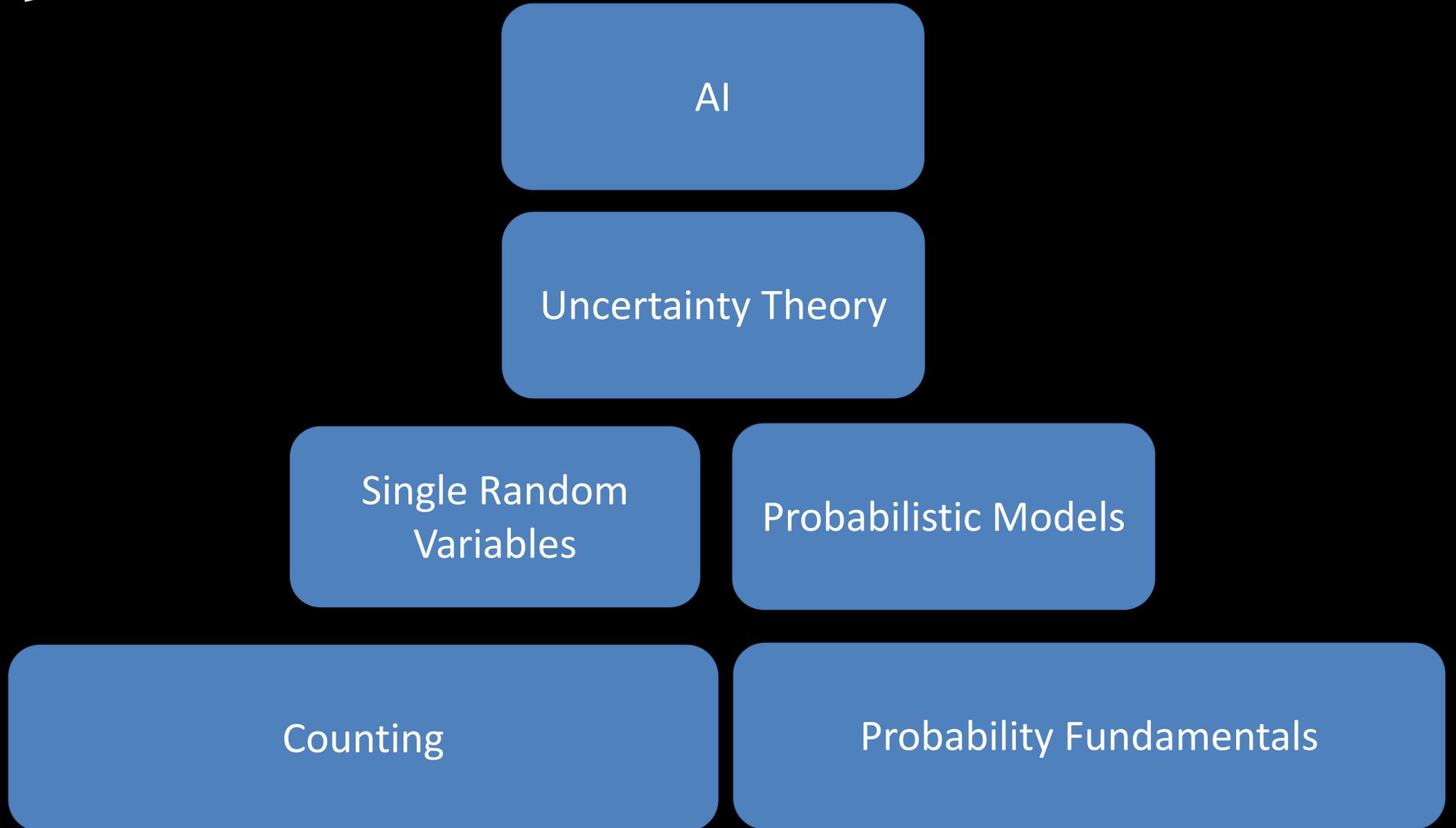
Delayed gratification

CS109 View of Probability

Teach you how to write programs
that most people are not able to write.

CS109 View of Probability

Teach you the theory you need to do the math that most people are not able to do.

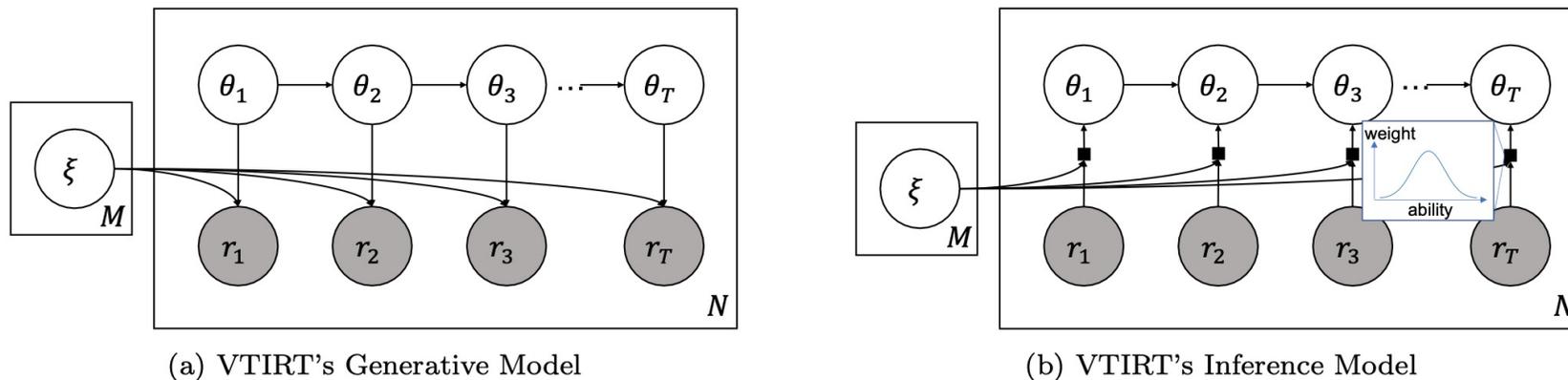


Lets dive in...



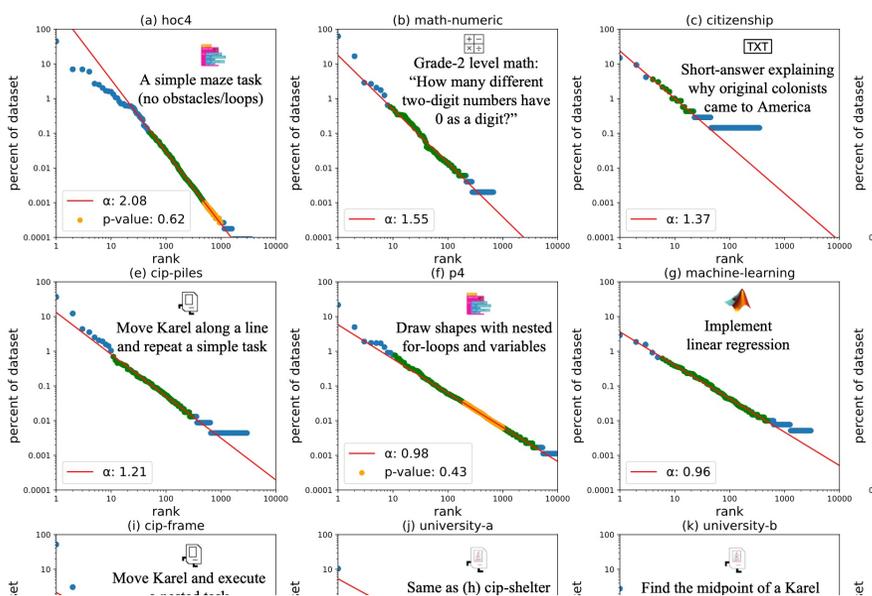
Counting I

What Sort of Thing Does Yunsung Research?

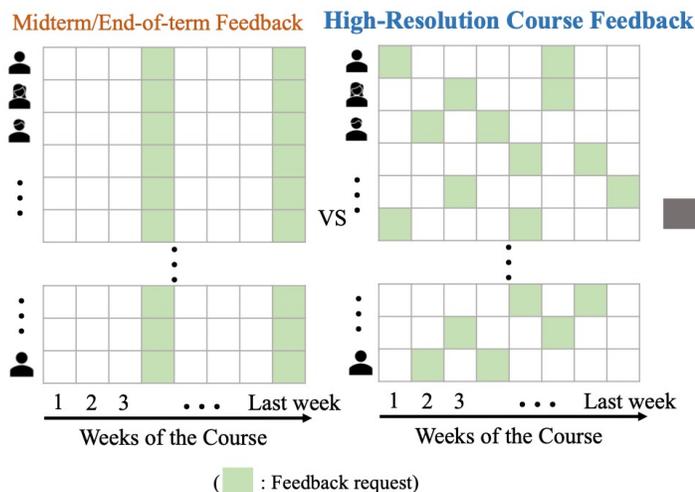


Probability
+
Education!

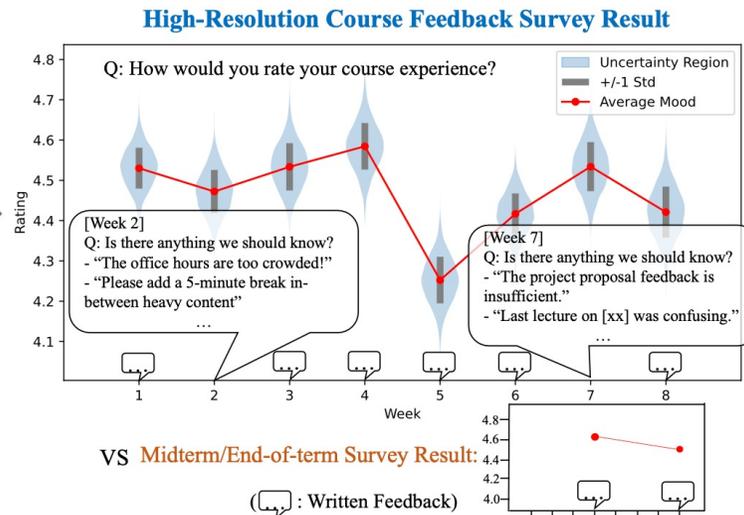
Figure 1: Graphical model view of VTIRT's generative model and inference model. Shaded nodes indicate observed variables, and arrows denote the direction of dependence. Squares denote the ability potentials in the form of a Gaussian density.



When Do We Request Student Feedback?

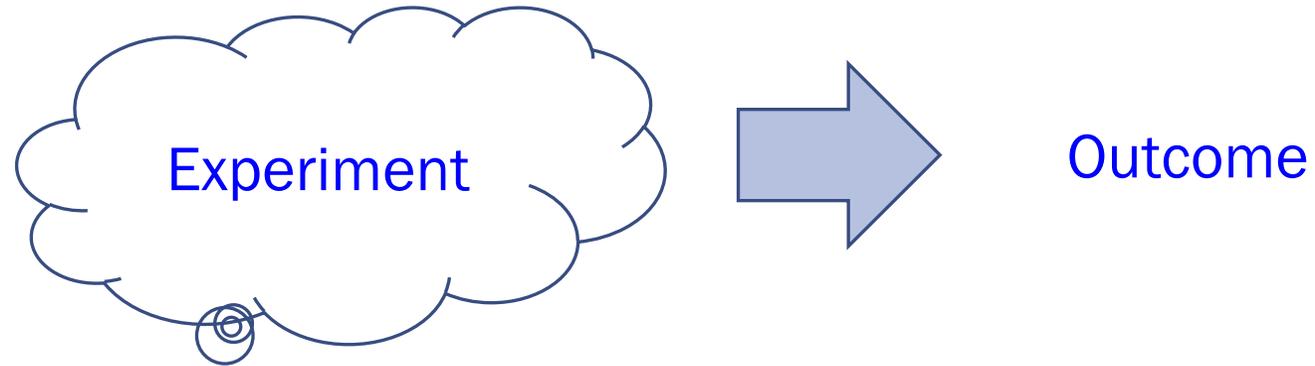


What Do Instructors Gain?



What is Counting?

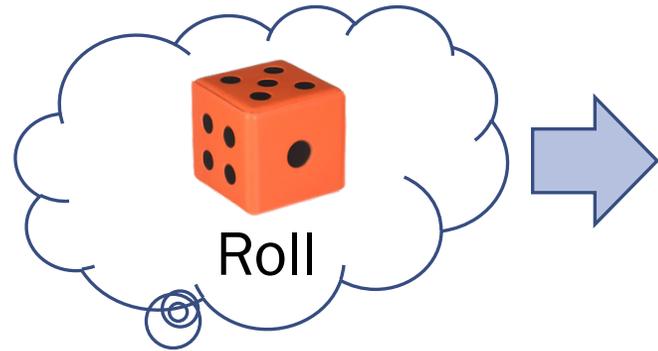
An experiment
in probability:



Counting:

How many possible **outcomes** satisfy some **event**?

What is Counting?



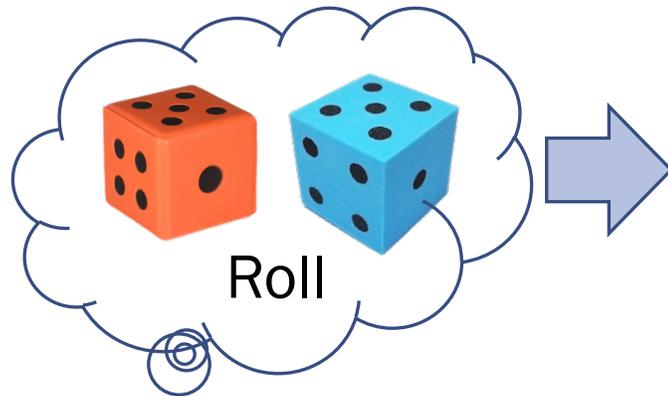
6

{1, 2, 3,
4, 5, 6}



3

{2, 4, 6}



36

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two steps, where

The first step (step A) can have one of m different outcomes, and the second step (step B) can have one of n different outcomes, and n is unaffected by outcome of first step.

Then the number of outcomes of the experiment is mn .

Two-step experiment



How Many Different Lock Patterns?



4-digit numeric combination lock

1st Digit



2nd Digit



3rd Digit



4th Digit

10 Choices per digit

$10^4=10,000$ total patterns

How Many Different Lock Patterns (2)?



Choices for 2nd digit depend on the choice of 1st digit...

4-digit numeric combination lock

BUT each digit used **exactly once**?

1st Digit



2nd Digit



3rd Digit



4th Digit

How Many Different Lock Patterns (2)?



For the step rule, only the number of possible next choices matters!

4-digit numeric combination lock

BUT each digit used **exactly once**?

1st Digit



2nd Digit



3rd Digit



4th Digit

$$10 \times 9 \times 8 \times 7 = 5040$$

Possible patterns

How Many Different Lock Patterns (3)?



3, 5, 3, _

3, 4, 6, _

4-digit numeric combination lock

BUT has **at least** one digit **repeated**?

1st Digit 10 Choices



2nd Digit 10 Choices



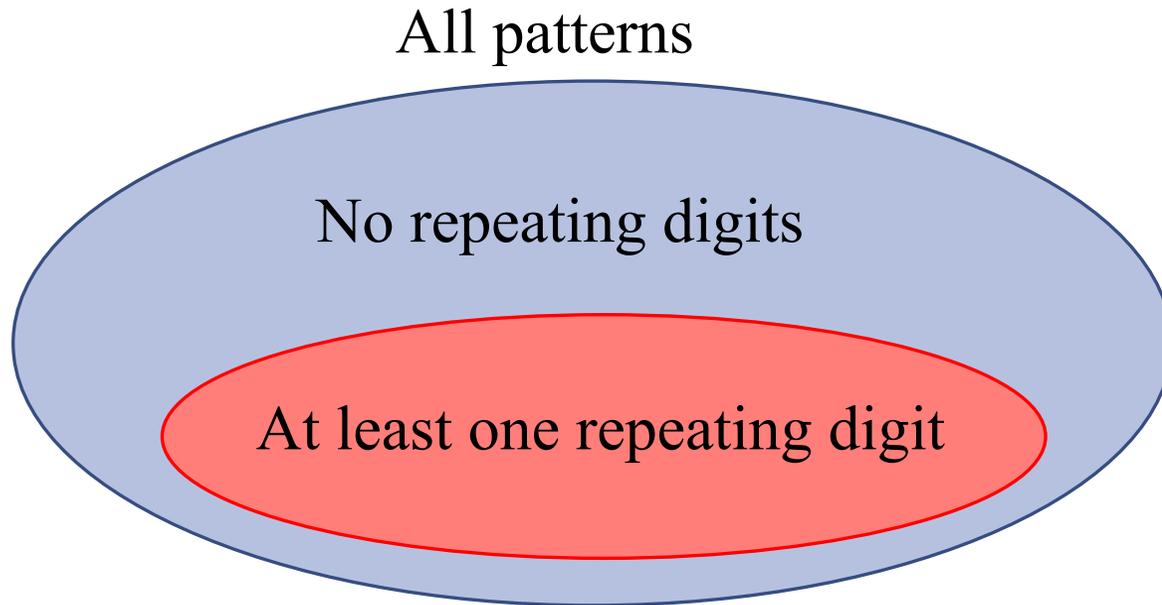
3rd Digit 10 Choices



4th Digit

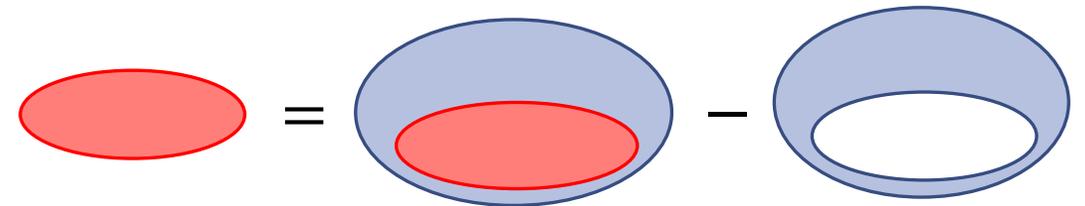


How Many Different Lock Patterns (3)?



4-digit numeric combination lock

BUT has **at least** one digit **repeated**?



$$\begin{aligned} & \# (\text{At least one repeating digit}) \\ &= \# (\text{All patterns}) - \# (\text{No repeating digit}) \\ &= 10,000 - 5,040 \\ &= 4,960 \end{aligned}$$



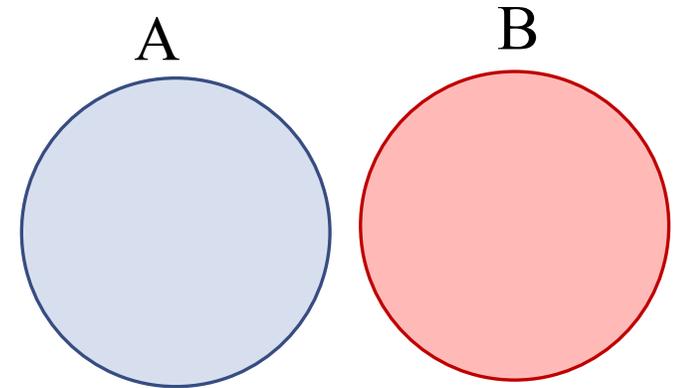
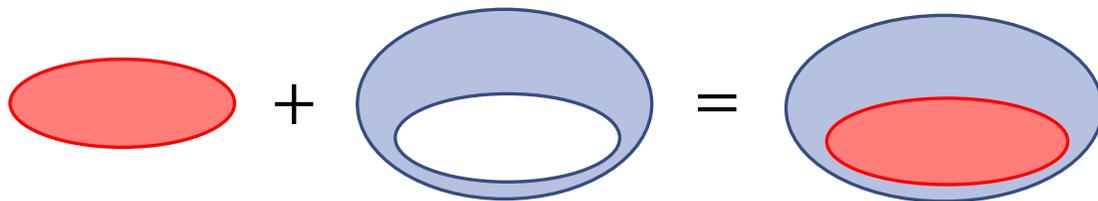
Complementary Counting

Sometimes it helps to “count away” complementary events

Sum Rule of Counting

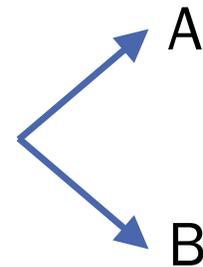
If the outcome of an experiment can be either from Set A , where $|A| = m$,
or Set B , where $|B| = n$,
where $A \cap B = \emptyset$,

Then the number of outcomes of the experiment is
 $|A| + |B| = m + n$.



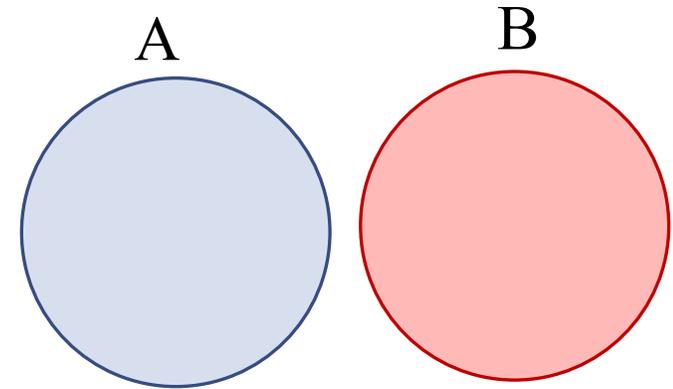
If $|A \cap B| = 0$,
 $|A \cup B| = |A| + |B|$

One experiment



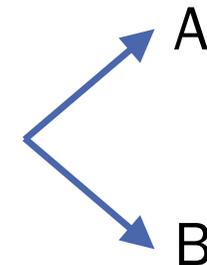
Sum Rule of Counting

If the outcome of an experiment can be either from Set A , where $|A| = m$,
or Set B , where $|B| = n$,
where $A \cap B = \emptyset$,
Then the number of outcomes of the experiment is
 $|A| + |B| = m + n$.



$$\text{If } |A \cap B| = 0, \\ |A \cup B| = |A| + |B|$$

One experiment



Can you derive the Step Rule from the Sum Rule?

How Many Bit Strings?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either **start with "01"** **OR start with "10"**. How many such strings are there?

Answer

$$\begin{aligned} N &= |A| + |B| \\ &= 2^4 + 2^4 \\ &= 32 \end{aligned}$$

2^4 start with 01

010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111

Set A

2^4 start with 10

100000
100001
100010
100011
100100
100101
100110
100111
101000
101001
101010
101011
101100
101101
101110
101111

3rd Digit



4th Digit



5th Digit



6th Digit

Set B

$$|A \cap B| = 0$$

How Many Bit Strings (2)?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either **start with "01"** **OR end with "10"**. How many such strings are there?

Answer



2^4 start with 01

010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111

Set *A*

2^4 end with 10

000010
000110
001010
001110
010010
010110
011010
011110
100010
100110
101010
101110
110010
110110
111010
111110

Set *B*

How Many Bit Strings (2)?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either **start with "01"** **OR end with "10"**. How many such strings are there?

2^4 start with 01

010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111

Set A

2^4 end with 10

000010
000110
001010
001110
010010
010110
011010
011110
100010
100110
101010
101110
110010
110110
111010
111110

Set B

Sum Rule of Counting

If the outcome of an experiment can be either from

Set A, where $|A| = m$,

or Set B, where $|B| = n$,

where $A \cap B = \emptyset$,

Then the number of outcomes of the experiment is

$$|A| + |B| = m + n.$$

How Many Bit Strings (2)?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Answer

2^4 start with 01

010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111

Set *A*

2^4 end with 10

000010
000110
001010
001110
010010
010110
011010
011110
100010
100110
101010
101110
110010
110110
111010
111110

Set *B*

How Many Bit Strings (2)?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Answer

2^4 start with 01

010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111

Set *A*

2^4 end with 10

000010
000110
001010
001110
010010
010110
011010
011110
100010
100110
101010
101110
110010
110110
111010
111110

Set *B*

How Many Bit Strings (2)?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Answer

$$\begin{aligned} N &= |A| + |B| - |A \text{ and } B| \\ &= 16 + 16 - 4 \\ &= 28 \end{aligned}$$

2^4 start with 01

010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111

Set *A*

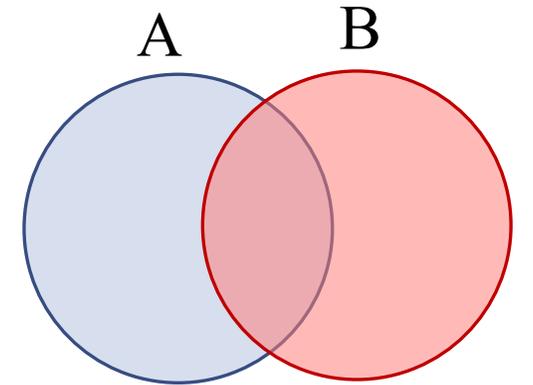
2^4 end with 10

000010
000110
001010
001110
010010
010110
011010
011110
100010
100110
101010
101110
110010
110110
111010
111110

Set *B*

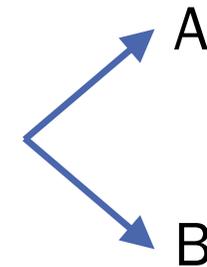
Or Rule of Counting (aka Inclusion/ Exclusion)

If the outcome of an experiment can be either from
Set A , where $|A| = m$,
or Set B , where $|B| = n$,
where $A \cap B$ *may not be empty*,
Then the number of outcomes of the experiment is
 $N = |A| + |B| - |A \cap B|$.



$$|A \cup B| = |A| + |B| - |A \cap B|$$

One experiment



Core Counting

Counting with steps

Definition: Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of m outcomes and the second part can result in one of n outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is $m \cdot n$.



Can you derive the Step Rule from the Sum Rule (or the OR rule)?

Counting with “or”

Definition: Inclusion Exclusion Counting

If the outcome of an experiment can either be drawn from set A or set B , and sets A and B may potentially overlap (i.e., it is not the case that A and B are mutually exclusive), then the number of outcomes of the experiment is $|A \text{ or } B| = |A| + |B| - |A \text{ and } B|$.

Challenge Problem

1. Strings

- How many *different* orderings of letters are possible for the string BOBA?

BOBA, ABOB, OBBA...



Incredible time. Incredible
school at which to study
probability!
Exciting.