Adding Random Variables

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Announcements

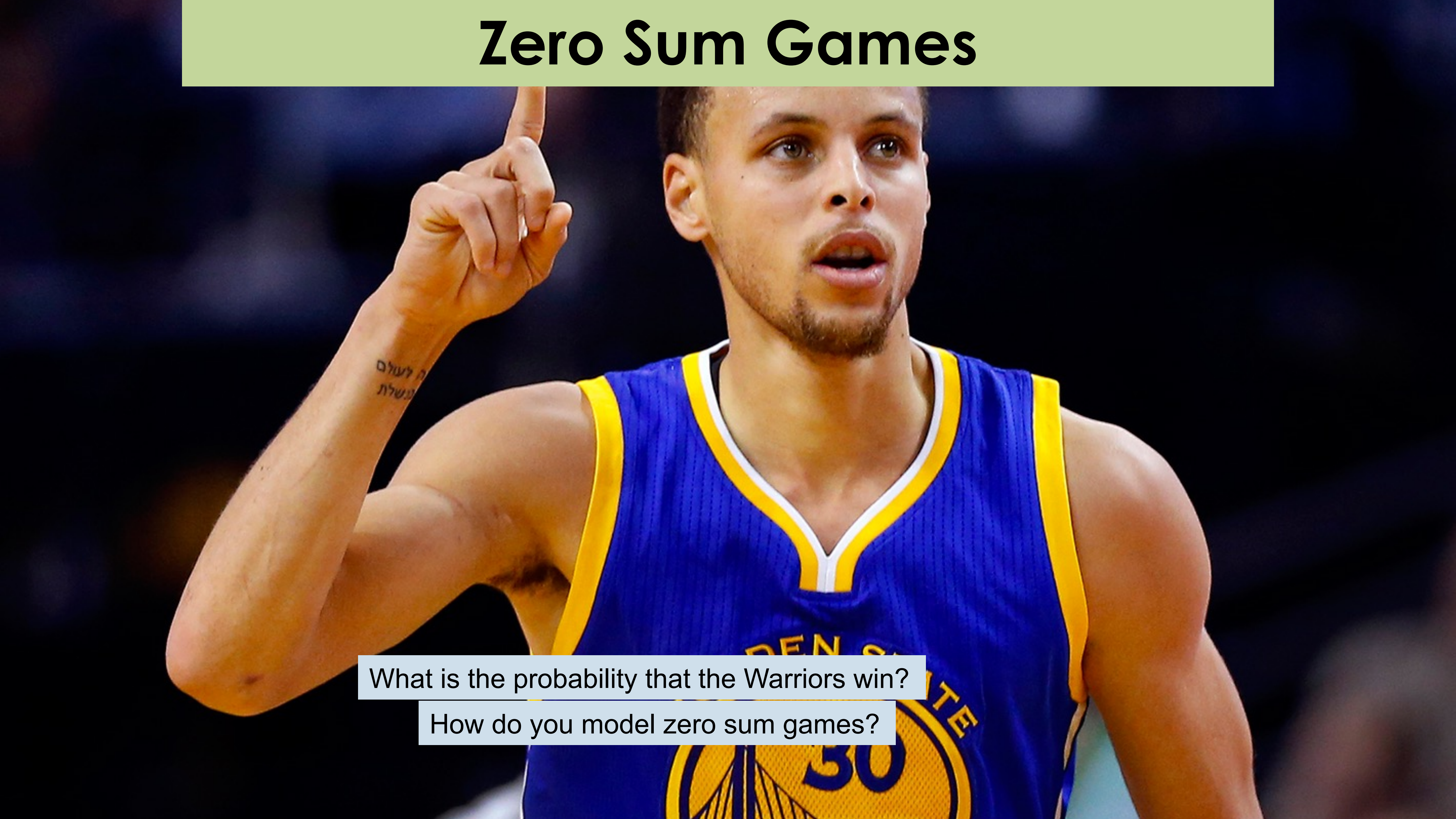
- Pset 4, the pros of finishing early
- Midterm results (end of class)

- Today's Goal:
    - Adding Independent Random Variable
    - Adding IID Random Variables
    - Intro To CLT.

What happens when you add random variables?

Why should you care?

Zero Sum Games



What is the probability that the Warriors win?

How do you model zero sum games?

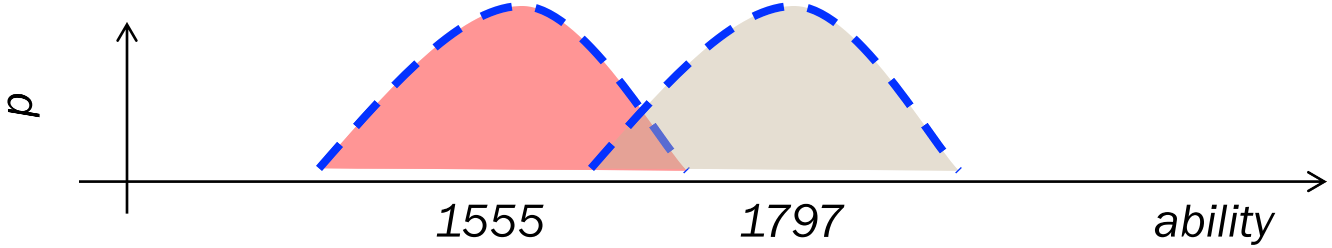
Motivating Idea: Zero Sum Games

How it works:

- Each team has an "ELO" score *S*, calculated based on their past performance.
- Each game, the team has ability $A \sim N(S, 200^2)$
- The team with the higher sampled ability wins.

Arpad Elo

$$A_B \sim \mathcal{N}(1555, 200^2) \qquad A_W \sim \mathcal{N}(1797, 200^2)$$



$$P(\text{Warriors win}) = P(A_W > A_B)$$

Motivating Idea: Zero Sum Games

$$A_W \sim \mathcal{N}(1797, 200^2)$$

$$A_B \sim \mathcal{N}(1555, 200^2)$$

$$P(\text{Warriors win}) = P(A_W > A_B)$$

How do we do this???

Some review

Independence

Recall the definition of independent events E and F:

$$P(EF) = P(E)P(F)$$

Two discrete random variables X and Y are **independent** if:

for all x, y:

$$P(X = x, Y = y) = P(X = x)P(Y = y)$$

Different notation, same idea:

$$p_{X,Y}(x, y) = p_X(x)p_Y(y)$$

- Intuitively: knowing value of X tells us nothing about the distribution of Y (and vice versa)
- If two variables are not independent, they are called **dependent**.

Sum of Two Die?

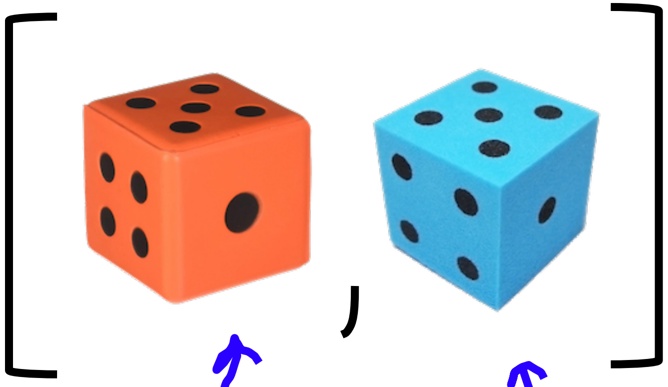
Roll two 6-sidex dice. What is P(sum = 7)?

S = {

[1,1]	[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
[2,1]	[2,2]	[2,3]	[2,4]	[2,5]	[2,6]
[3,1]	[3,2]	[3,3]	[3,4]	[3,5]	[3,6]
[4,1]	[4,2]	[4,3]	[4,4]	[4,5]	[4,6]
[5,1]	[5,2]	[5,3]	[5,4]	[5,5]	[5,6]
[6,1]	[6,2]	[6,3]	[6,4]	[6,5]	[6,6]

}

Each outcome



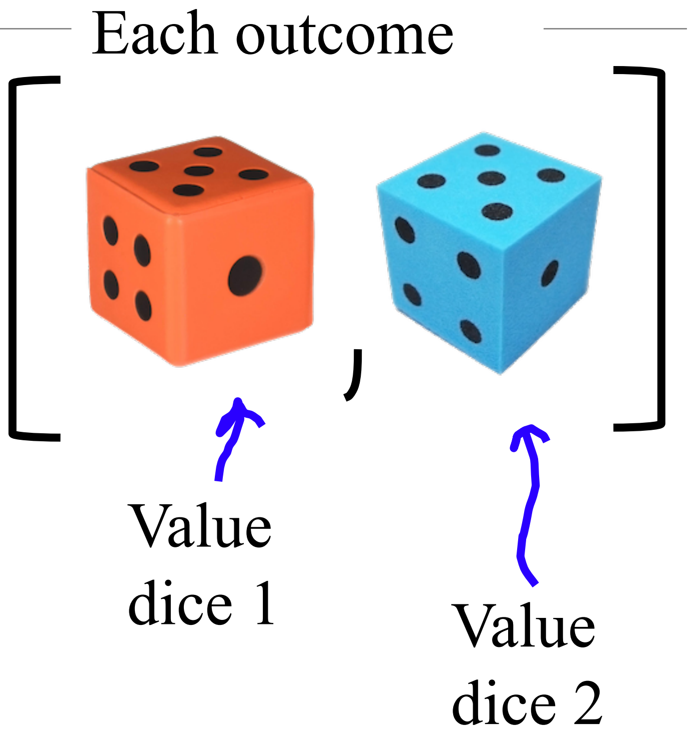
Value dice 1

Value dice 2

Sum of Two Die = 7?

Roll two 6-sidex dice. What is P(sum = 7)?

S = { [1,1] [1,2] [1,3] [1,4] [1,5] **[1,6]**
[2,1] [2,2] [2,3] [2,4] **[2,5]** [2,6]
[3,1] [3,2] [3,3] **[3,4]** [3,5] [3,6]
[4,1] [4,2] **[4,3]** [4,4] [4,5] [4,6]
[5,1] **[5,2]** [5,3] [5,4] [5,5] [5,6]
[6,1] [6,2] [6,3] [6,4] [6,5] [6,6] }



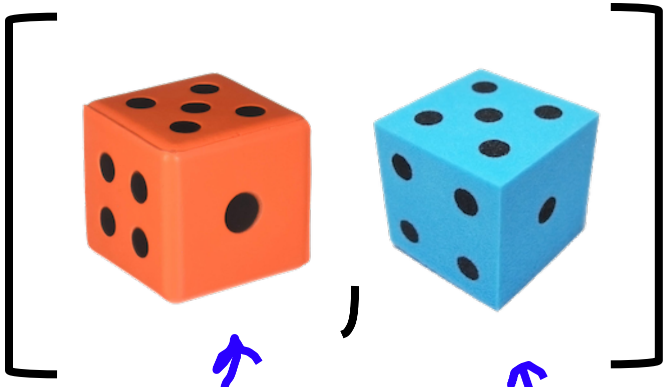
E = *in blue*

$$P(E) = \frac{|E|}{|S|} = \frac{6}{36} = 0.16\bar{6}$$

Sum of Two Die = 10?

Roll two 6-sidex dice. What is P(sum = 10)?

Each outcome



Value dice 1

Value dice 2

S = {

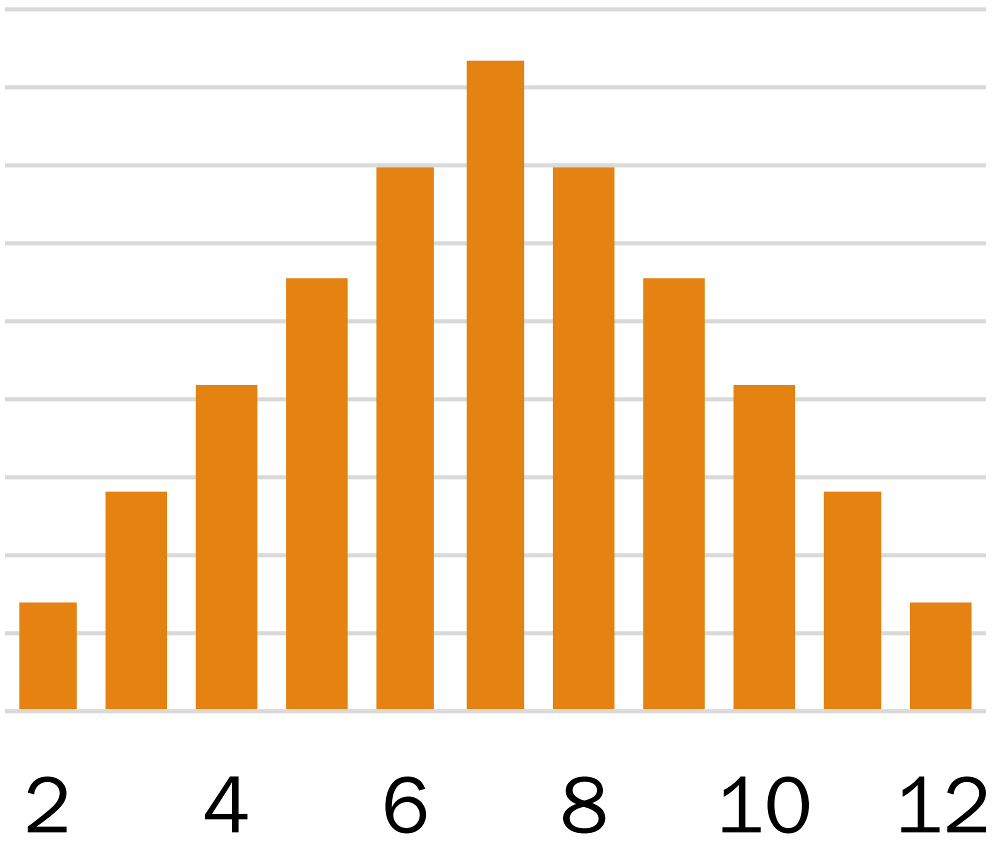
[1,1]	[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
[2,1]	[2,2]	[2,3]	[2,4]	[2,5]	[2,6]
[3,1]	[3,2]	[3,3]	[3,4]	[3,5]	[3,6]
[4,1]	[4,2]	[4,3]	[4,4]	[4,5]	**[4,6]**
[5,1]	[5,2]	[5,3]	[5,4]	**[5,5]**	[5,6]
[6,1]	[6,2]	[6,3]	**[6,4]**	[6,5]	[6,6] }

E = *in blue*

$$P(E) = \frac{|E|}{|S|} = \frac{3}{36} = 0.08\bar{3}$$

Sum of Two Dice

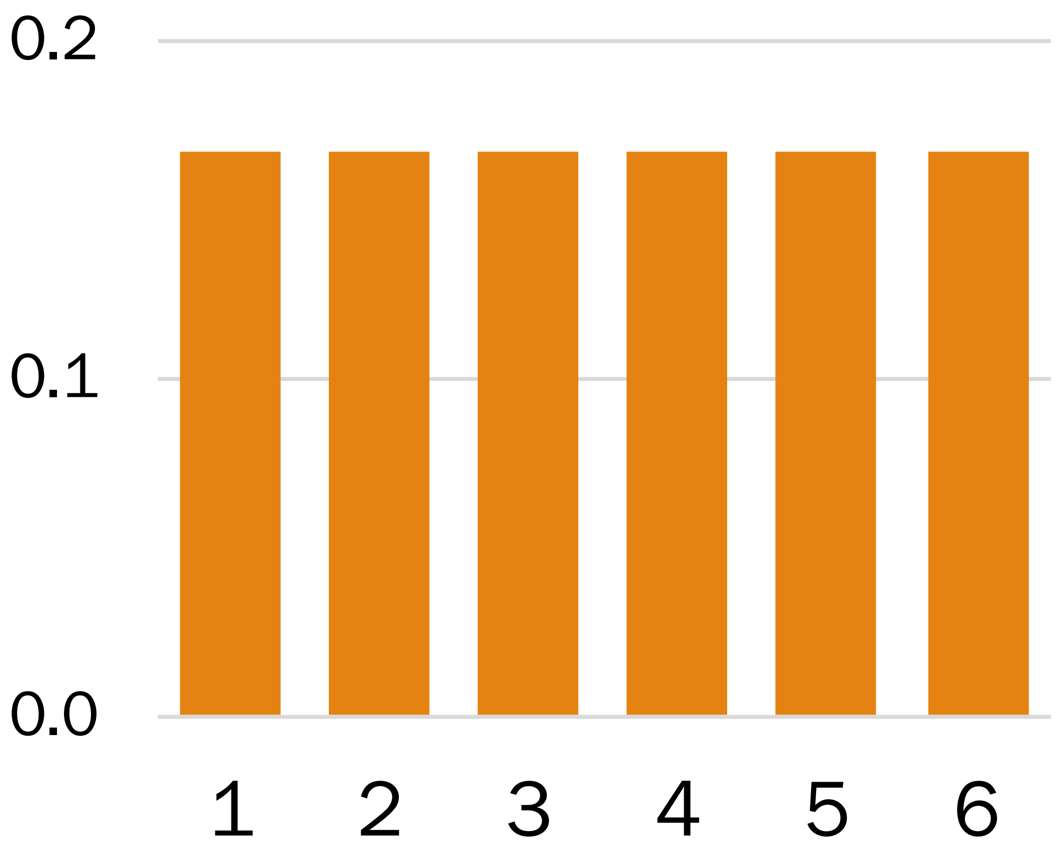
This is the PMF of the sum of two dice



Why is there more mass in the middle?

Sum of One Dice

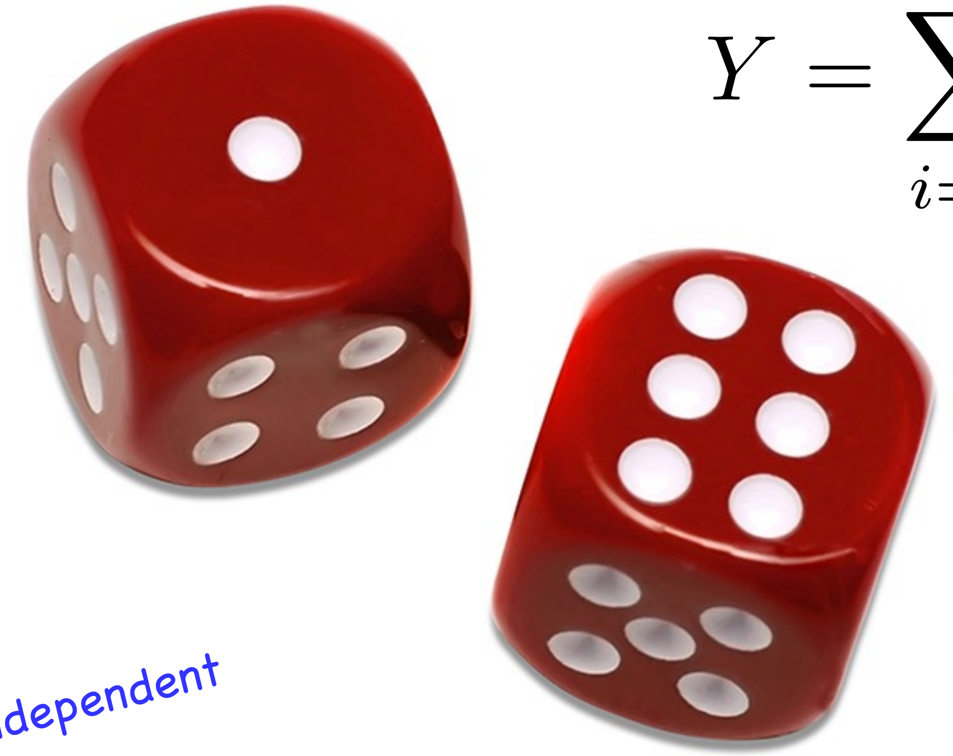
This is the PMF of the sum of one dice



So how did we do that?

Sum of Two Dice

$$Y = \sum_{i=0}^{2} X_i$$



X_i s are independent

X_i is the outcome of dice roll i

Sum of Two Dice

$$Y = \sum_{i=0}^{2} X_i$$

What's the distribution of Y?

$$P(Y = n) = P(X_1 + X_2 = n)$$

Bounds are over support of X_1

Apply Law of Total Probability on X_1

$$= \sum_{i=1}^{6} P(X_1 = i, X_2 + X_1 = n)$$

Substitute i for all instances of X_1

$$= \sum_{i=1}^{6} P(X_1 = i, X_2 + i = n)$$

Subtract the i

$$= \sum_{i=1}^{6} P(X_1 = i, X_2 = n - i)$$

Independence

$$= \sum_{i=1}^{6} P(X_1 = i)P(X_2 = n - i)$$

Convolution

What's the distribution of Y?

PMF of Y

Sum over supp of X_1

Fix one of the R.Vs

Multiply by resulting prob on other RV

$$P(Y = n) = \sum_{i=1}^{6} P(X_1 = i)P(X_2 = n - i)$$

This is 0 if $i > 6$

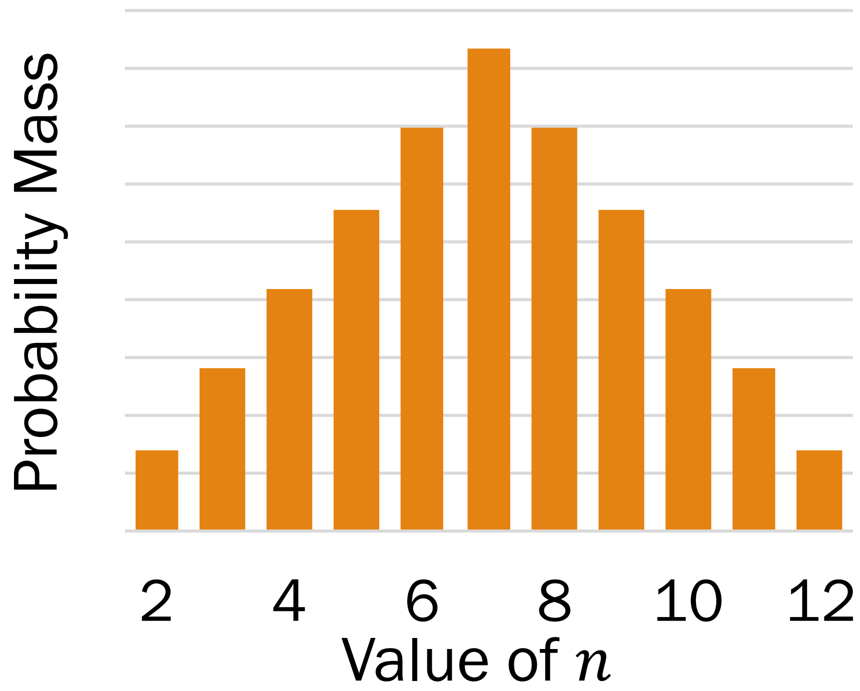
This is 0 if $n - i < 1$

Sum of Two Dice

What's the distribution of Y?

$$P(Y = n) = \sum_{i=1}^{6} P(X_1 = i)P(X_2 = n - i)$$

This is a valid PMF!



Test some points :P

Sanity Check

Plug in a point $n = 4$.

$$P(Y = 4) = \sum_{i=1}^{6} P(X_1 = i)P(X_2 = 4 - i) \quad = \frac{3}{36}$$

How many terms of that sum is non-zero?

$$P(X_1 = 1)P(X_2 = 4 - 1) = \frac{1}{36}$$

$$P(X_1 = 2)P(X_2 = 4 - 2) = \frac{1}{36}$$

$$P(X_1 = 3)P(X_2 = 4 - 3) = \frac{1}{36}$$

[1,1]	[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
[2,1]	[2,2]	[2,3]	[2,4]	[2,5]	[2,6]
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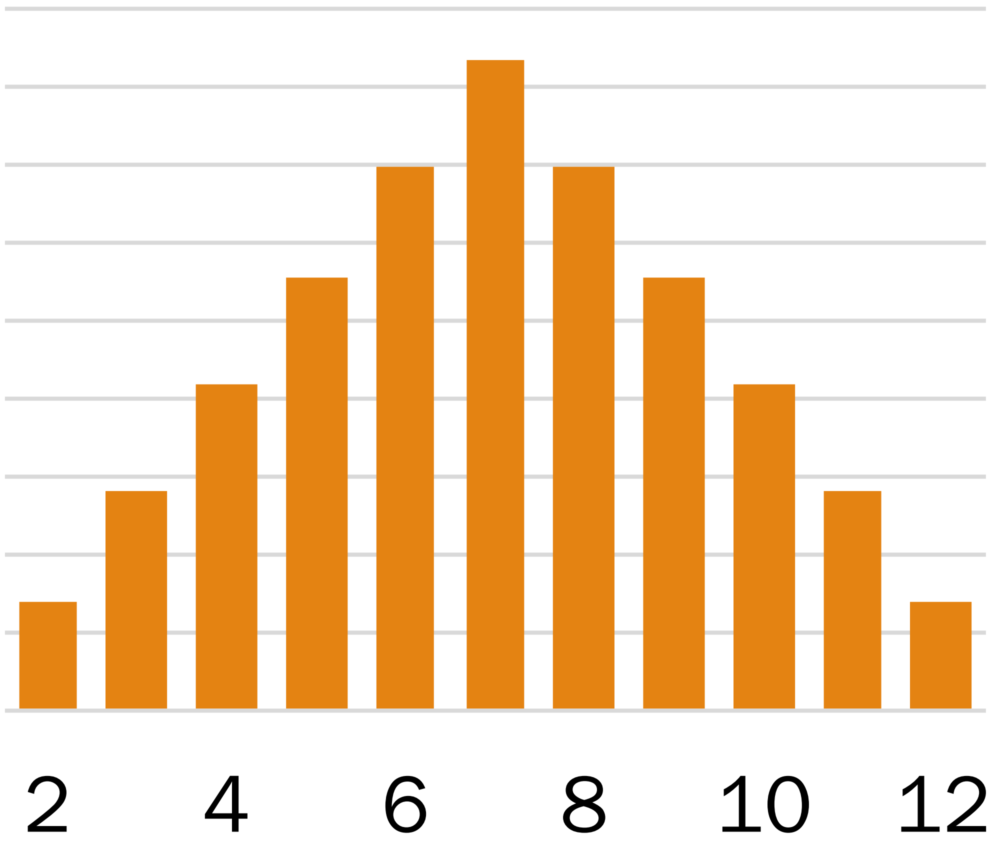
Sum of Two Dice

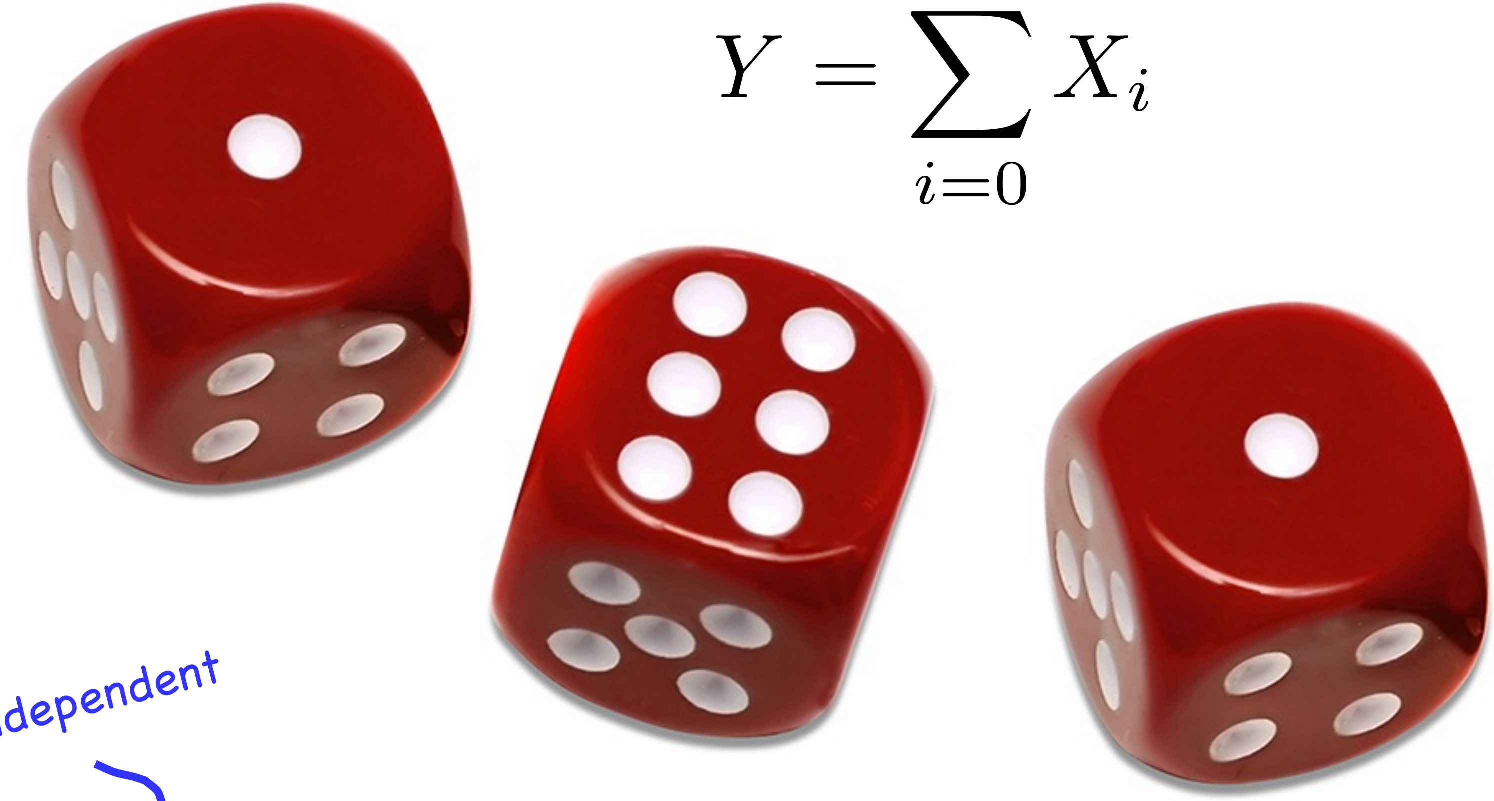
This is the PMF of the sum of two dice



Why is there more mass in the middle?

Sum of Three Dice

$$Y = \sum_{i=0}^{3} X_i$$

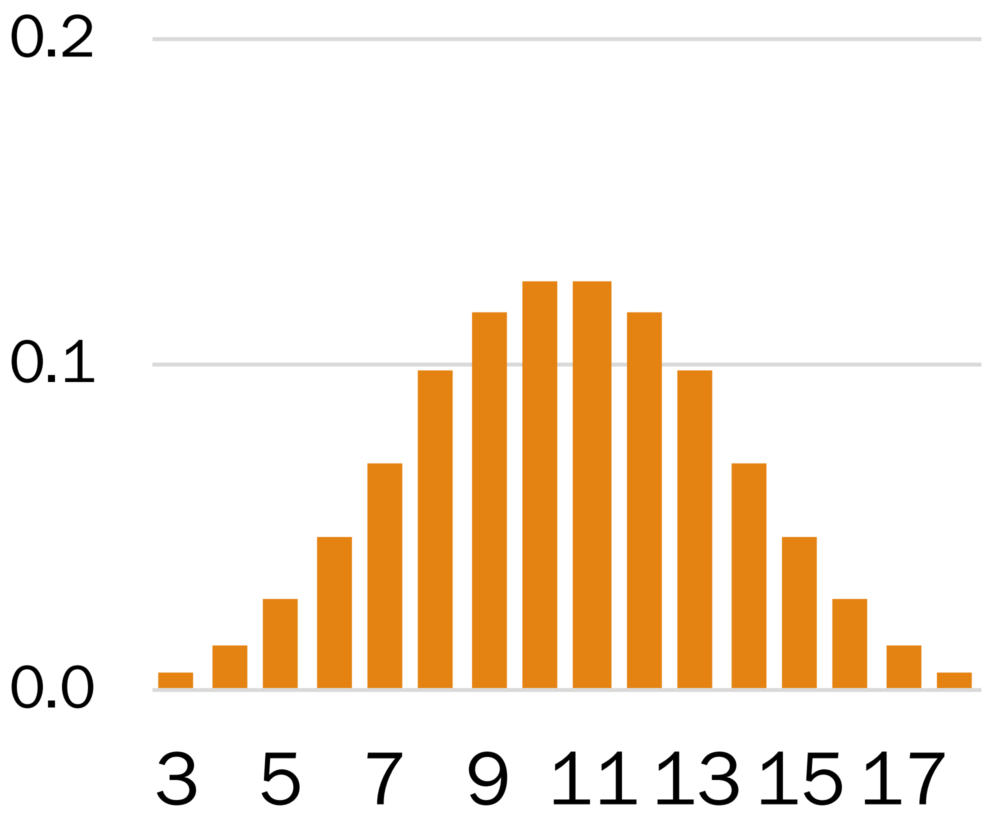


X_i s are independent

X_i is the outcome of dice roll *i*

Sum of Three Dice

This is the PMF of the sum of three dice



Why is there more mass in the middle?

Intuition on Convolution

The Insight to Convolution

Imagine a game
where each player *independently* scores between 0 and 100 points:

Let X be the amount of points you score.
Let Y be the amount of points your opponent scores.
Let's say you know P(X = x) and P(Y = y).

What is the probability of a tie?

$$P(\text{tie}) = \sum_{i=0}^{100} P(X = i, Y = i)$$

$$= \sum_{i=0}^{100} P(X = i)P(Y = i)$$

The Insight to Convolutions

What is the probability that X + Y = n?

$$P(X+Y=n)?$$

$$P(X+Y=n) = \sum_{i=0}^{n} P(X=i, Y=n-i)$$

X	Y	i	
0	n	0	$P(X=0, Y=n)$
1	n - 1	1	$P(X=1, Y=n-1)$
2	n - 2	2	$P(X=2, Y=n-2)$
	...		
n	0	n	$P(X=n, Y=0)$

The Insight to Convolutions

What is the probability that X + Y = n?

$$P(X+Y=n)?$$

Since this is the OR or mutually exclusive events

$$P(X+Y=n)=\sum_{k=0}^{n}P(X=k,Y=n-k)$$

If the random variables are independent

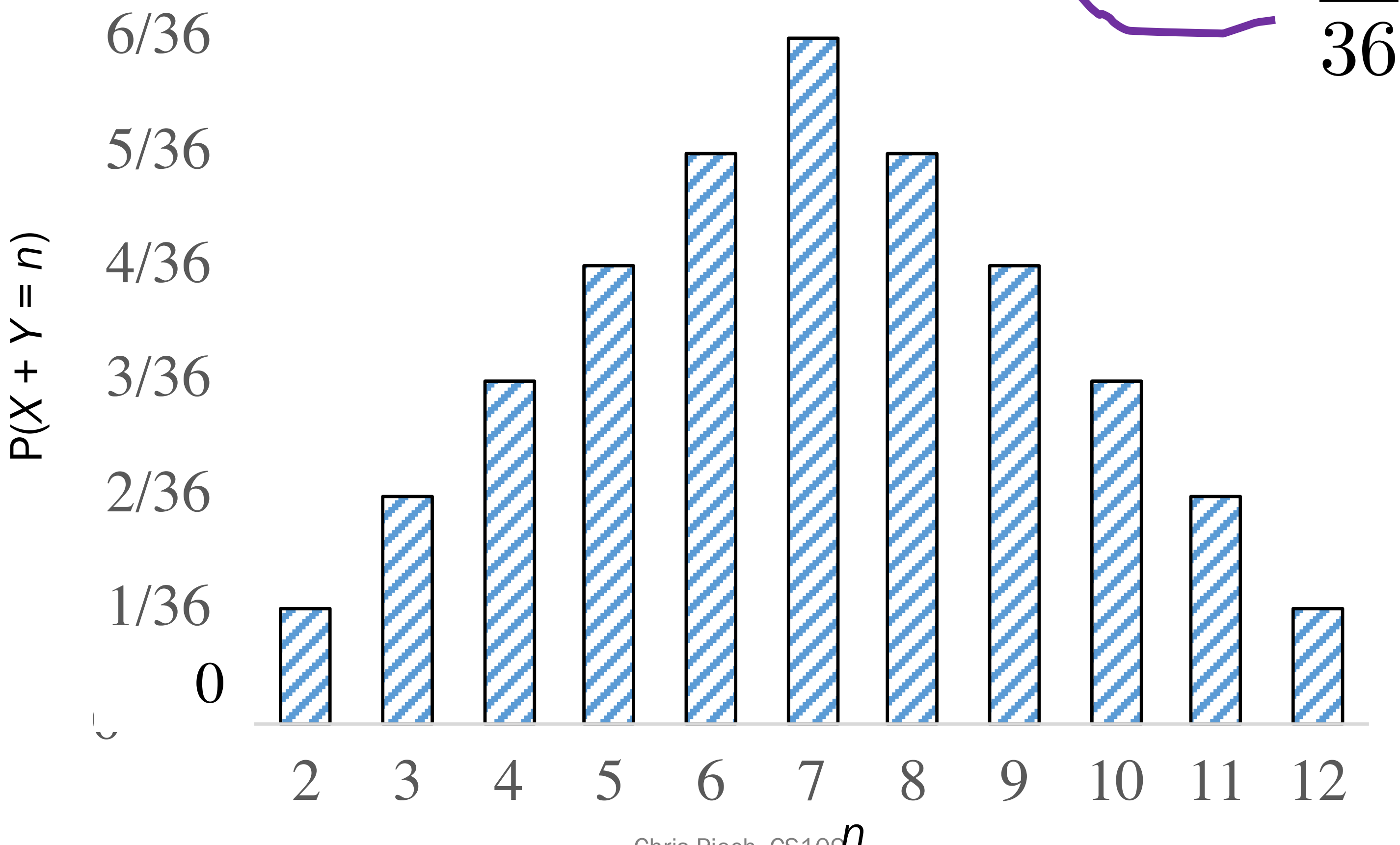
$$=\sum_{k=0}^{n}P(X=k)P(Y=n-k)$$

Sum of Two Dice

Let *X*+*Y* be the value of the sum of two dice
(aka two independent random variables)

$$P(X+Y=n) = \sum_{i=1}^{n-1} \mathrm{P}(X=i, Y=n-i)$$

$$\frac{1}{36}$$



Convolution: The fanciest way to say “adding random variables”

New Definition

IID Random Variables

- Consider *n* random variables X_1, X_2, ... X_n
  - X_i are all **independently** and **identically distributed** (I.I.D.)
  - All have the same PMF (if discrete) or PDF (if continuous)
  - All have the same expectation
  - All have the same variance

IID iid

Quick check

Are $X_1, X_2, \dots, X_n$ i.i.d. with the following distributions?

1. $X_i \sim \text{Exp}(\lambda)$, X_i independent

2. $X_i \sim \text{Exp}(\lambda_i)$, X_i independent

3. $X_i \sim \text{Exp}(\lambda)$, $X_1 = X_2 = \cdots = X_n$

4. $X_i \sim \text{Bin}(n_i, p)$, X_i independent

Quick check

Are $X_1, X_2, \ldots, X_n$ i.i.d. with the following distributions?

1. $X_i \sim \text{Exp}(\lambda),\ X_i$ independent ✅
2. $X_i \sim \text{Exp}(\lambda_i),\ X_i$ independent ❌ (unless λ_i equal)
3. $X_i \sim \text{Exp}(\lambda),\ X_1 = X_2 = \cdots = X_n$ ❌ dependent: $X_1 = X_2 = \cdots = X_n$
4. $X_i \sim \text{Bin}(n_i, p),\ X_i$ independent ❌ (unless n_i equal)
   Note underlying Bernoulli RVs are i.i.d.!

Sometimes Adding is Easy:

Sum of Independent Binomials

- Let X and Y be independent binomials with the same value for *p*:
  - $X \sim Bin(n_1, p)$ and $Y \sim Bin(n_2, p)$
  - $X + Y \sim Bin(n_1 + n_2, p)$

- Intuition:
  - X has n_1 trials and Y has n_2 trials
    - Each trial has same "success" probability p
  - Define Z to be $n_1 + n_2$ trials, each with success prob. p
  - $Z \sim Bin(n_1 + n_2, p)$, and also $Z = X + Y$

Sum of Independent Normals

- Let X and Y be independent random variables
  - $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$
  - $X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

- Generally, have *n* independent random variables $X_i \sim N(\mu_i, \sigma_i^2)$ for $i = 1, 2, ..., n$:

$$\left(\sum_{i=1}^{n} X_i\right) \sim N\left(\sum_{i=1}^{n} \mu_i,\ \sum_{i=1}^{n} \sigma_i^2\right)$$

Sum of Independent Poissons

- Let X and Y be independent random variables
  - $X \sim \text{Poi}(\lambda_1)$ and $Y \sim \text{Poi}(\lambda_2)$
  - $X + Y \sim \text{Poi}(\lambda_1 + \lambda_2)$

Virus Infections

- Say you are working with the WHO to plan a response to a the initial conditions of a virus:
  - Two exposed groups
  - P1: 50 people, each independently infected with $p = 0.1$
  - P2: 100 people, each independently infected with $p = 0.4$
  - Question: Probability of more than 40 infections?

Sanity check: Should we use the Binomial Sum-of-RVs shortcut?

A. YES!
B. NO!
C. Other/none/more

Virus Infections

- Say you are working with the WHO to plan a response to a the initial conditions of a virus:
  - Two exposed groups
  - P1: 50 people, each independently infected with $p = 0.1$
  - P2: 100 people, each independently infected with $p = 0.4$
  - A = # infected in P1 A ~ Bin(50, 0.1) ≈ X ~ N(5, 4.5)
  - B = # infected in P2 B ~ Bin(100, 0.4) ≈ Y ~ N(40, 24)
  - What is P(≥ 40 people infected)?
  - P(A + B ≥ 40) ≈ P(X + Y ≥ 39.5)
  - X + Y = W ~ N(5 + 40 = 45, 4.5 + 24 = 28.5)

$$P(W > 39.5) = 1 - P(X < 39.5)$$

$$= 1 - F_X(39.5) = 1 - \Phi(\frac{39.5 - 45}{\sqrt{28.5}}) \approx 0.8485$$

Linear Transform

Thinking of Y as a linear transform

$$X \sim N(\mu, \sigma^2)$$

$$Y = X + X = 2 \cdot X$$

$$Y \sim N(2\mu, 4\sigma^2)$$

Thinking of Y as the sum of independent normals

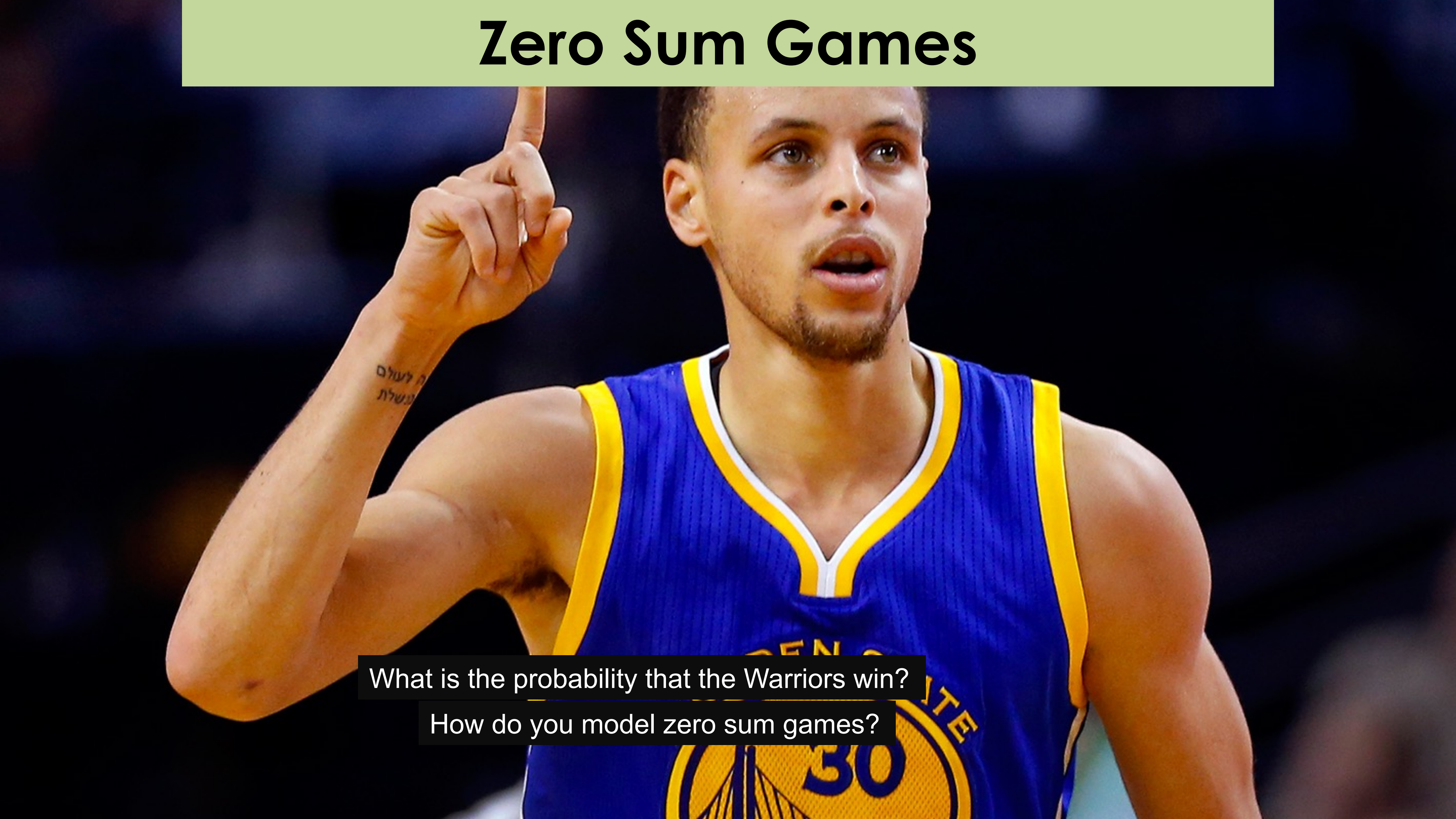
$$Y = X + X = 2 \cdot X$$

$$X + X \sim N(\mu + \mu, \sigma^2 + \sigma^2)$$

$$Y \sim N(2\mu, 2\sigma^2)$$

X is not independent of X

Zero Sum Games



What is the probability that the Warriors win?

How do you model zero sum games?

Gaussian Sampling and ELO ratings

What is the probability that the Warriors win?
How do you model zero-sum games?

Gaussian Sampling and ELO ratings

Each team has an ELO score S, calculated based on its past performance.

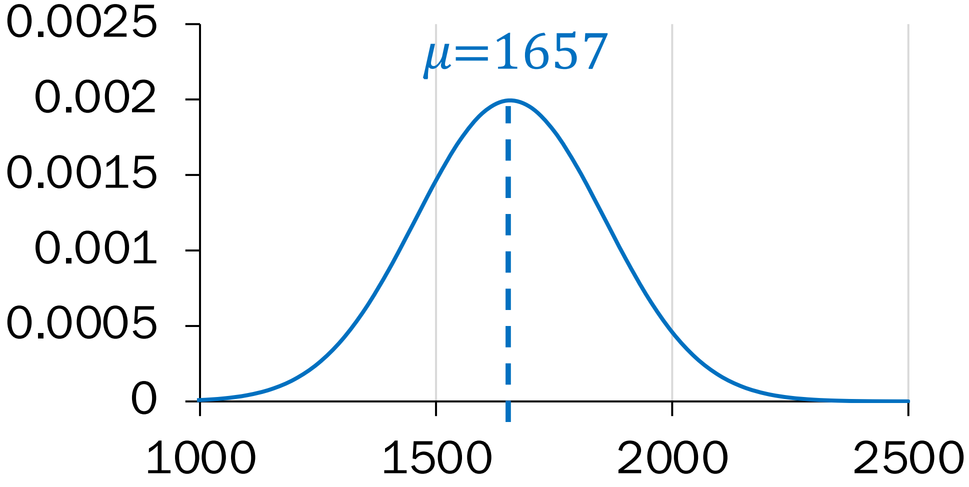
- Each game, a team has ability $A \sim \mathcal{N}(S, 200^2)$.
- The team with the higher sampled ability wins.

Arpad Elo

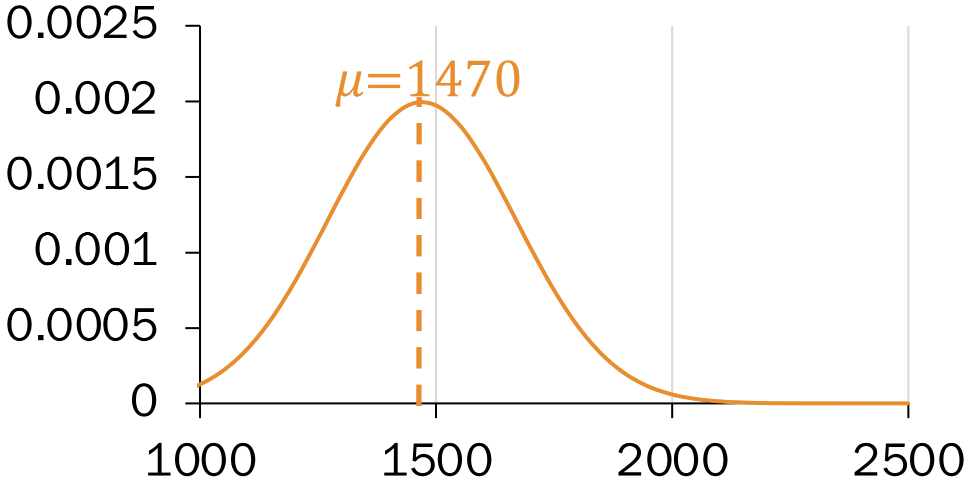
What is the probability that Warriors win this game?

Want: $P(\text{Warriors win}) = P(A_W > A_O)$

Warriors' $A_W \sim \mathcal{N}(S = 1657, 200^2)$



Opponent's $A_O \sim \mathcal{N}(S = 1470, 200^2)$



Probability of Winning a Game

$$A_W \sim N(1797, 200^2)$$

$$A_O \sim N(1555, 200^2)$$

$$P(\text{Warriors win}) = P(A_W > A_O)$$

$$P(\text{Warriors win}) = P(A_W - A_O > 0)$$

$$-A_O \sim N(-1555, 200^2)$$

$$D = A_W + (-A_O)$$

$$D \sim N(242, 2 \cdot 200^2)$$

$$P(D > 0) = 1 - F_D(0) \approx 0.804$$

We talked about sum of Binomial, Normal and Poisson…who's missing from this party?

Uniform.

Discrete Vs Continuous

Discrete

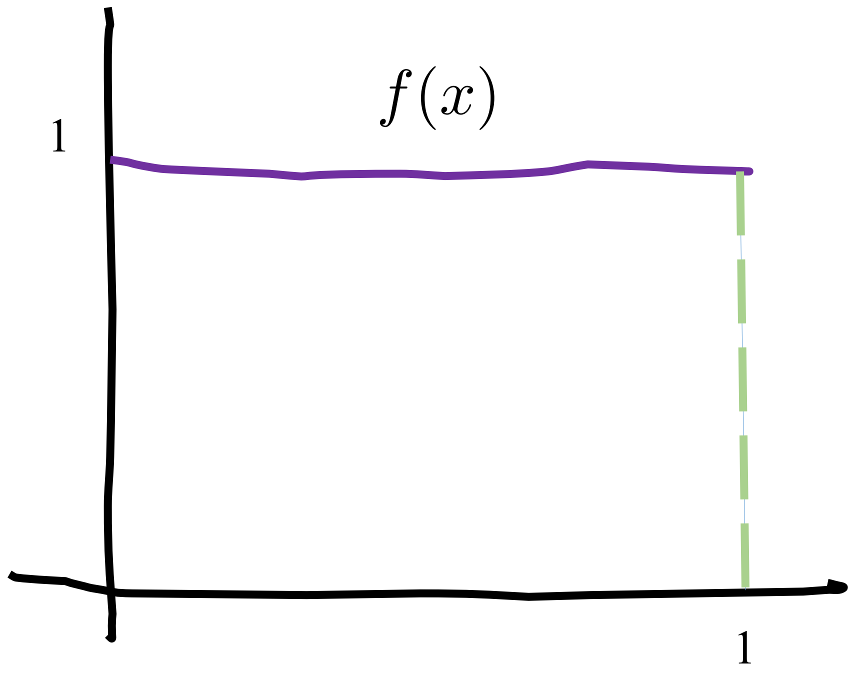
$$P(X+Y=a) = \sum_{y=-\infty}^{\infty} P(X=a-y)P(Y=y)\ dy$$

Continuous

$$f(X+Y=a) = \int_{y=-\infty}^{\infty} f(X=a-y)f(Y=y)\ dy$$

Sum of Independent Uniforms

- Let X and Y be independent random variables
  - X ~ Uni(0, 1) and Y ~ Uni(0, 1) → $f(x) = 1$ for $0 \le x \le 1$



For both X and Y

$1 < a < 2$

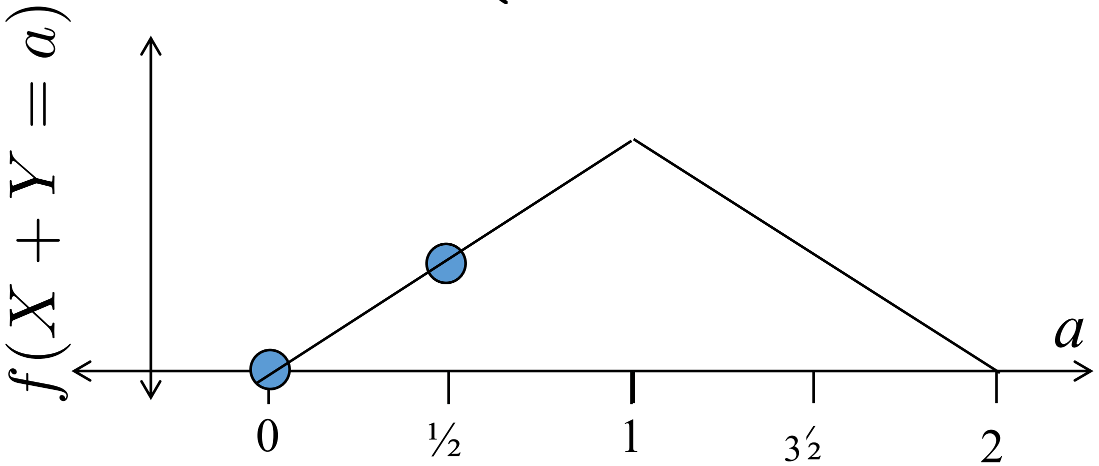
$X \sim \text{Uni}(0,1) \qquad Y \sim \text{Uni}(0,1)$

X and *Y* are independent

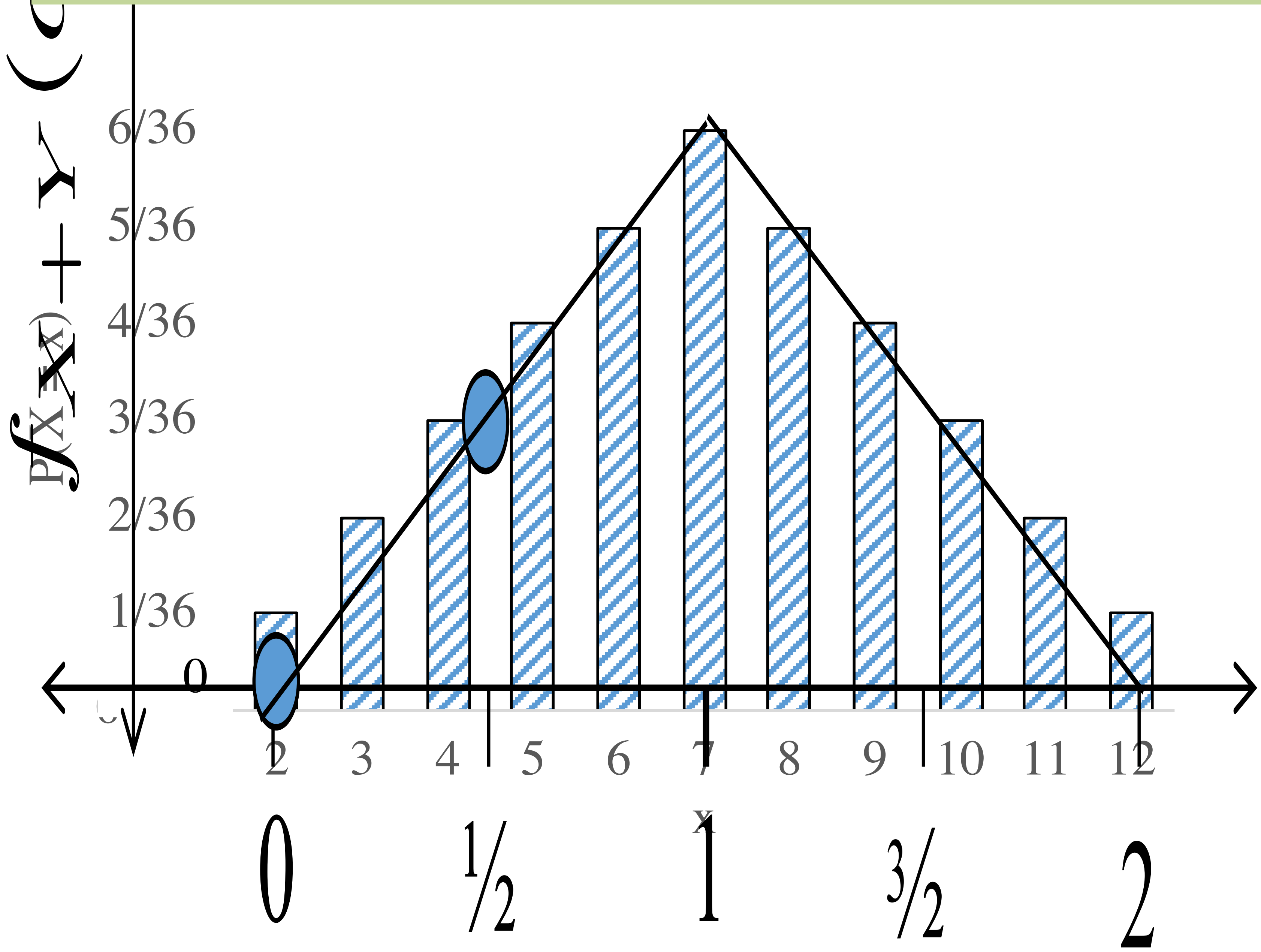
$$f(X+Y=a)?$$

$$f(X+Y=a) = \int_{y=-\infty}^{\infty} f(X=a-y)f(Y=y)\,dy$$

$$f(X+Y=a) = \begin{cases} a & 0 < a < 1 \\ 2-a & 1 < a < 2 \\ 0 & \text{otherwise} \end{cases}$$



Sum of Uniforms and Sum of Dice



Sum of 100 uniforms???

Were talking about the sum of uniforms

sum.py

```python
import random

def main():
    x = random.random()
    y = random.random()
    z = x + y
    print(z)

if __name__ == '__main__':
    main()
```

Sum of 100 poissons???

(drumroll)

Central Limit Theorem

Consider n independent and identically distributed (i.i.d) variables $X_1, X_2, \dots, X_n$ with $E[X_i] = \mu$ and $\text{Var}(X_i) = \sigma^2$.

As $n \to \infty$

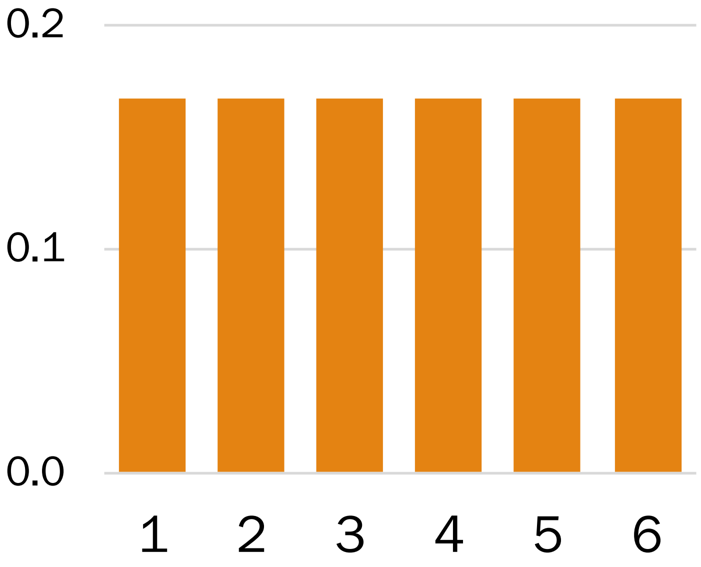
$$\sum_{i=1}^{n} X_i \sim \mathcal{N}(n\mu, n\sigma^2)$$

The sum of n i.i.d. random variables is normally distributed with mean $n\mu$ and variance $n\sigma^2$.

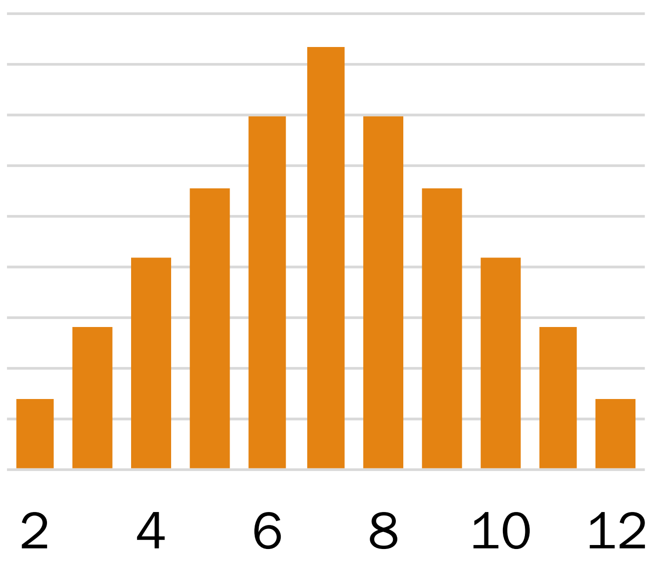
True happiness

Sum of dice rolls

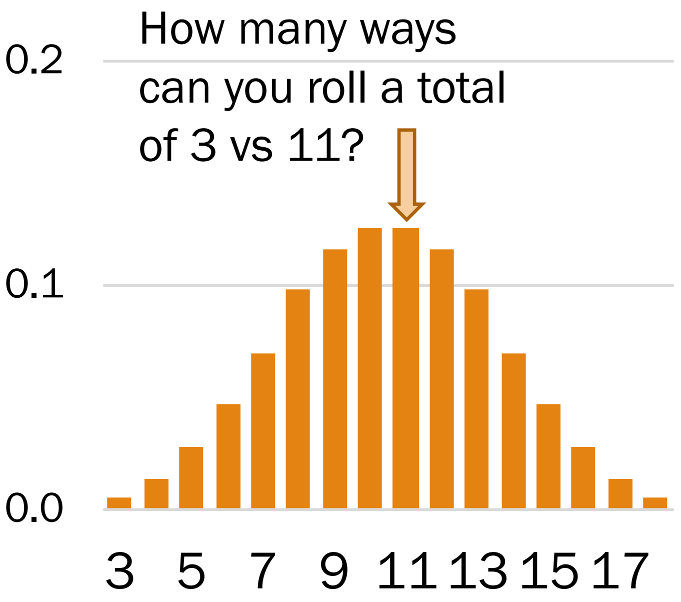
Roll n independent dice. Let X_i be the outcome of roll i. X_i are i.i.d.



$\sum_{i=1}^{1} X_i$ Sum of 1 die roll



$\sum_{i=1}^{2} X_i$ Sum of 2 dice rolls

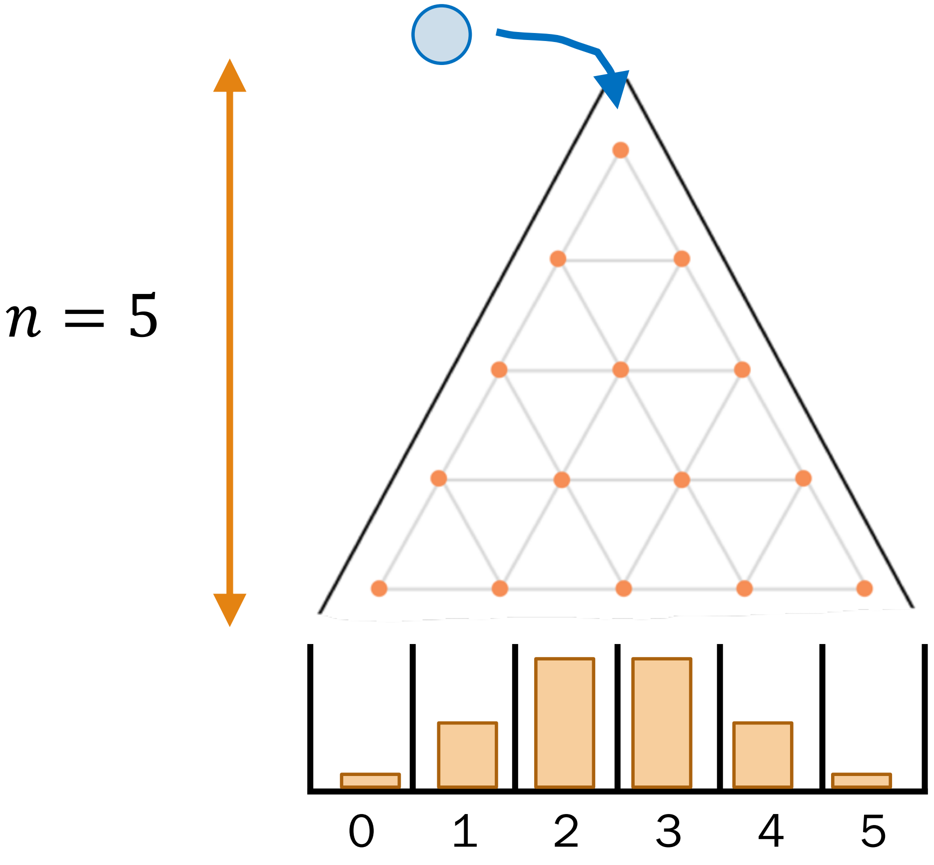


$\sum_{i=1}^{3} X_i$ Sum of 3 dice rolls

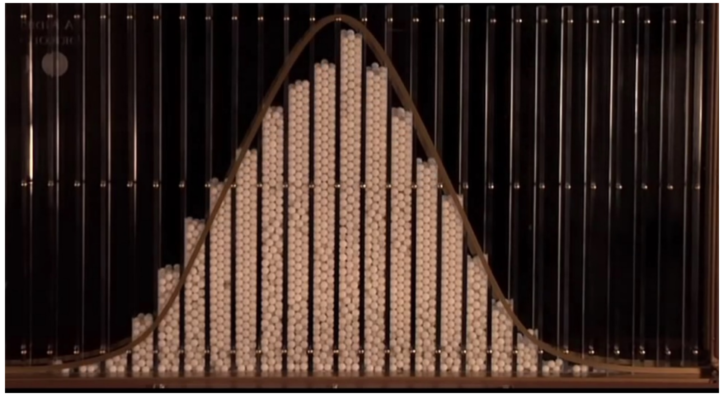
CLT explains a lot

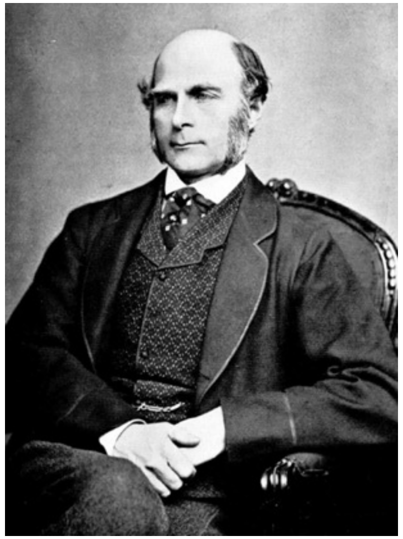
$$\sum_{i=1}^{n} X_i \sim \mathcal{N}(n\mu, n\sigma^2) \quad \text{As } n \to \infty$$

The sum of n **i.i.d.** random variables is normally distributed with mean $n\mu$ and variance $n\sigma^2$.



Galton Board, by Sir Francis Galton (1822-1911)

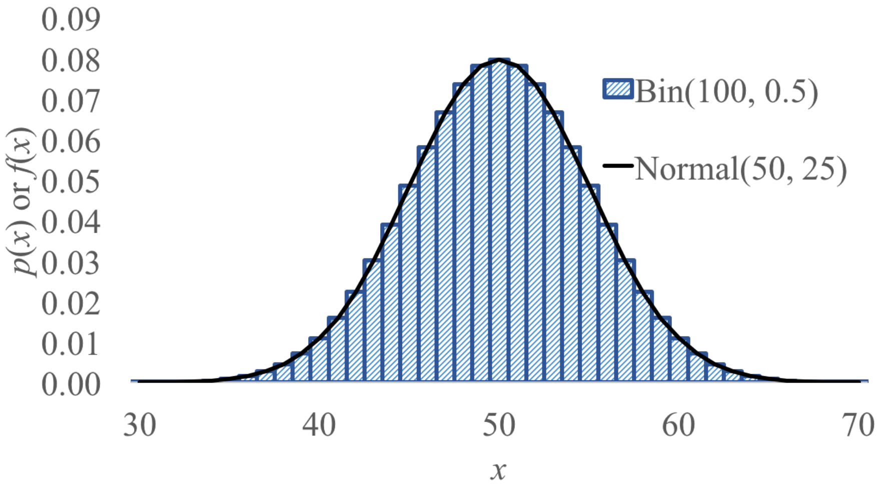




CLT explains a lot

$$\sum_{i=1}^{n} X_i \sim \mathcal{N}(n\mu, n\sigma^2)$$ As $n \to \infty$

The sum of n **i.i.d.** random variables is normally distributed with mean $n\mu$ and variance $n\sigma^2$.



Normal approximation of Binomial

Sum of i.i.d. Bernoulli RVs ≈ Normal

Proof:

Let $X_i \sim \text{Ber}(p)$ for $i = 1, \ldots, n$, where X_i are i.i.d.
$E[X_i] = p$, $\text{Var}(X_i) = p(1-p)$

$$X = \sum_{i=1}^{n} X_i \qquad (X \sim \text{Bin}(n,p))$$

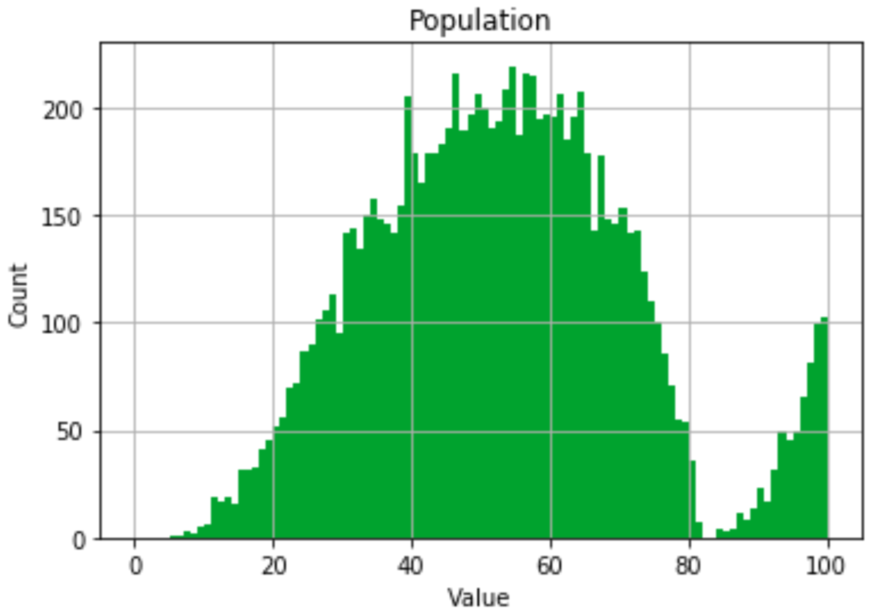
$$X \sim \mathcal{N}(n\mu, n\sigma^2) \qquad (\text{CLT, as } n \to \infty)$$

$$X \sim \mathcal{N}(np, np(1-p)) \qquad \text{(substitute mean, variance of Bernoulli)}$$

CLT explains a lot

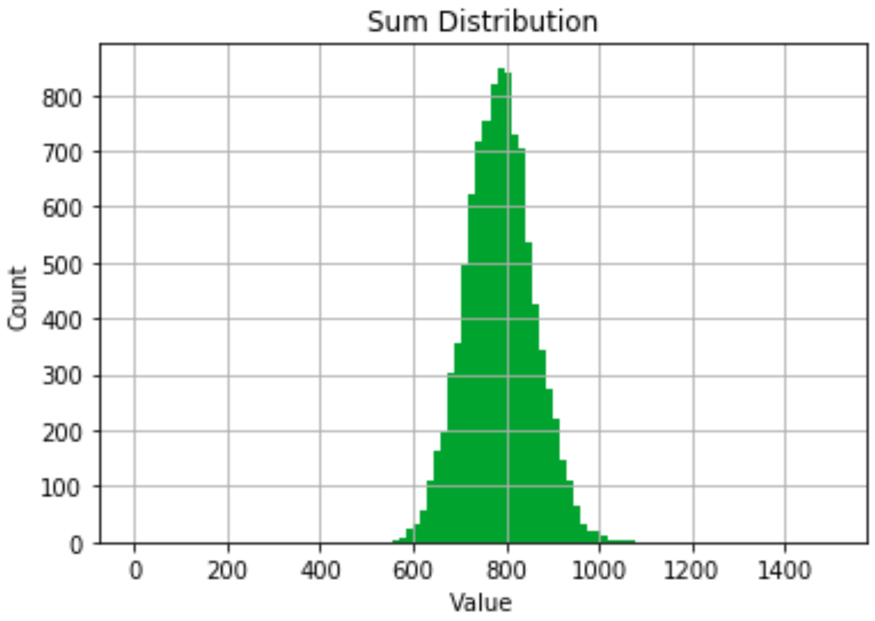
$$\sum_{i=1}^{n} X_i \sim \mathcal{N}(n\mu, n\sigma^2) \quad \text{As } n \to \infty$$

The sum of n i.i.d. random variables is normally distributed with mean $n\mu$ and variance $n\sigma^2$.



Distribution of X_i

Sample of size 15, sum values

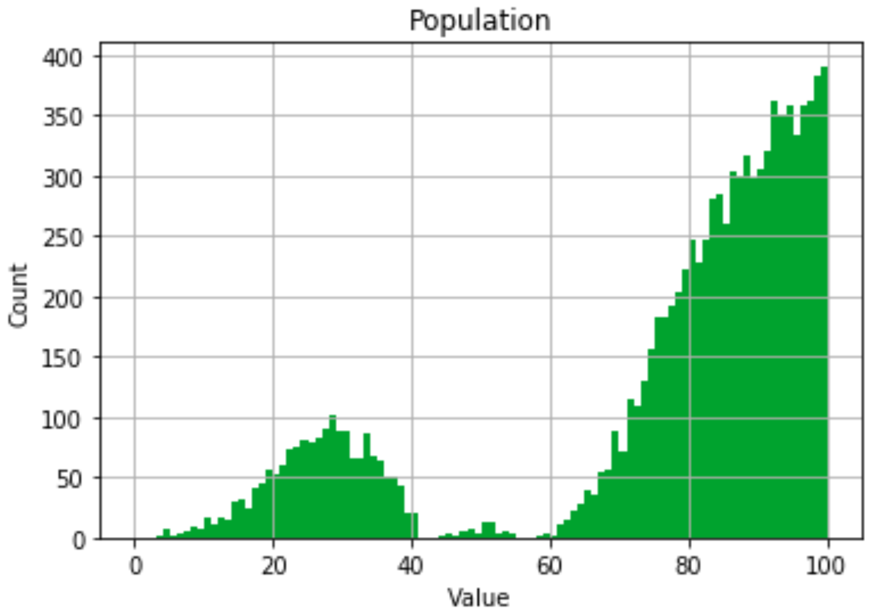


Distribution of $\sum_{i=1}^{15} X_i$

CLT explains a lot

$$\sum_{i=1}^{n} X_i \sim \mathcal{N}(n\mu, n\sigma^2) \quad \text{As } n \to \infty$$

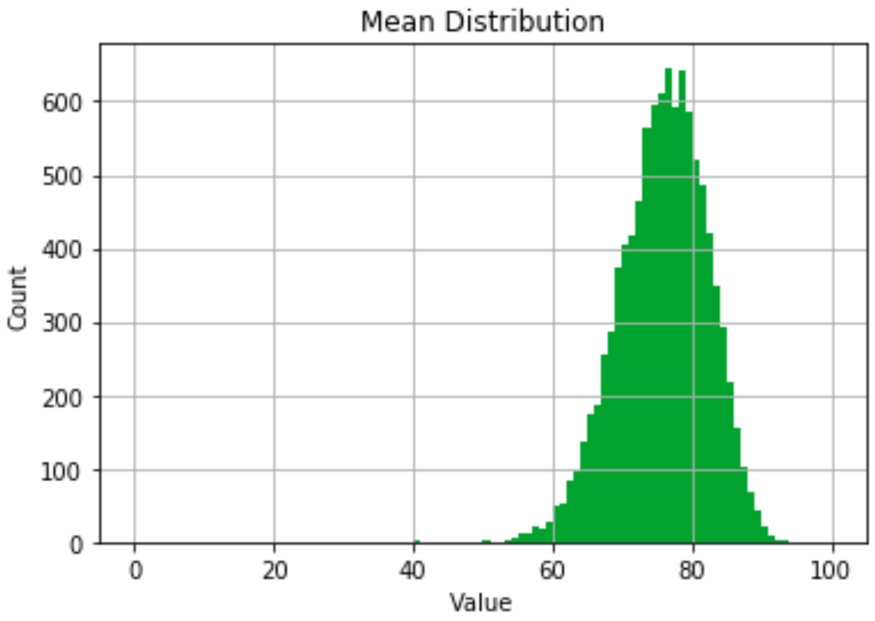
The sum of n i.i.d. random variables is normally distributed with mean $n\mu$ and variance $n\sigma^2$.



Distribution of X_i

Sample of size 15, average values

(sample mean)



Distribution of $\frac{1}{15}\sum_{i=1}^{15} X_i$

CLT example

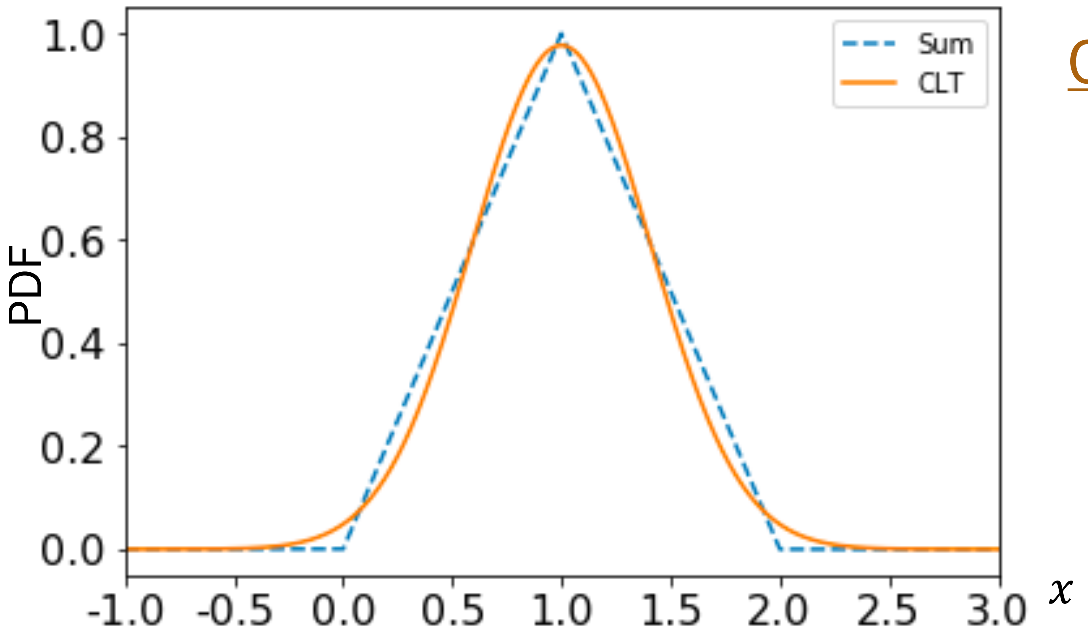
Sum of n independent Uniform RVs

Let $X = \sum_{i=1}^{n} X_i$ be sum of i.i.d. RVs, where $X_i \sim \text{Uni}(0,1)$.

$\mu = E[X_i] = 1/2$
$\sigma^2 = \text{Var}(X_i) = 1/12$

For different n, how close is the CLT approximation of $P(X \le n/3)$?

$n = 2$:



Exact $\quad P(X \le 2/3) \approx 0.2222$

CLT approximation

$$X \approx Y \sim \mathcal{N}(n\mu, n\sigma^2) \quad \Rightarrow Y \sim \mathcal{N}(1, 1/6)$$

$$P(X \le 2/3) \approx P(Y \le 2/3)$$

$$= \Phi\left(\frac{2/3 - 1}{\sqrt{1/6}}\right) \approx 0.2071$$

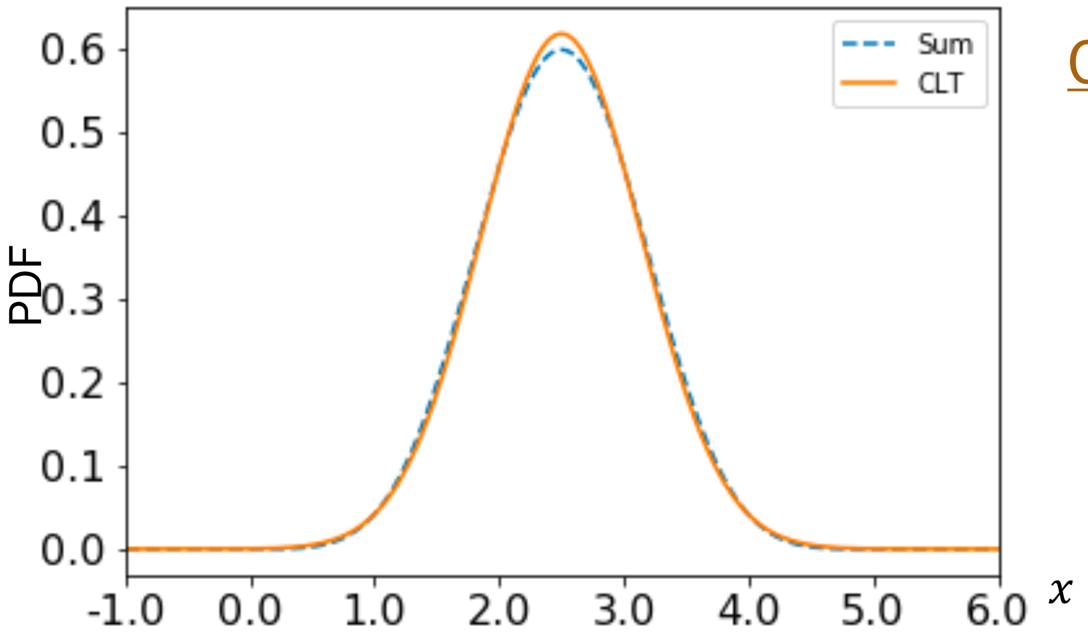
Sum of n independent Uniform RVs

Let $X = \sum_{i=1}^{n} X_i$ be sum of i.i.d. RVs, where $X_i \sim \text{Uni}(0,1)$.

$\mu = E[X_i] = 1/2$
$\sigma^2 = \text{Var}(X_i) = 1/12$

For different n, how close is the CLT approximation of $P(X \leq n/3)$?

$n = 5$:



Exact $\quad P(X \leq 5/3) \approx 0.1017$

CLT approximation

$$X \approx Y \sim \mathcal{N}(n\mu, n\sigma^2) \quad \Rightarrow Y \sim \mathcal{N}(5/2, 5/12)$$

$$P(X \leq 5/3) \approx P(Y \leq 5/3)$$

$$= \Phi\left(\frac{5/3 - 5/2}{\sqrt{5/12}}\right) \approx 0.0984$$

Sum of n independent Uniform RVs

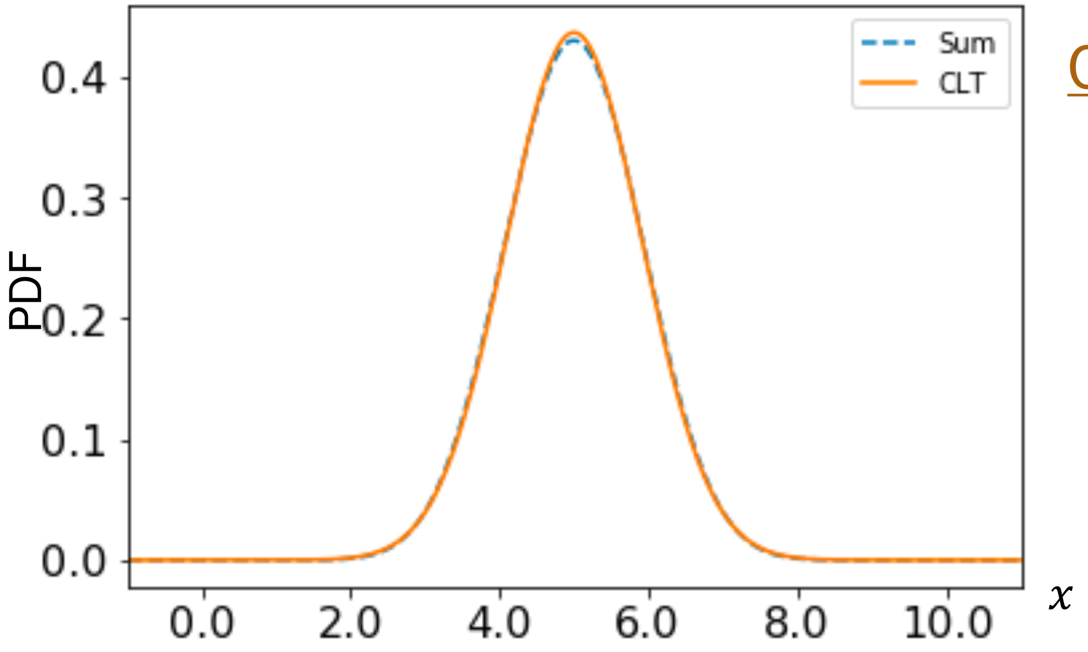
Let $X = \sum_{i=1}^{n} X_i$ be sum of i.i.d. RVs, where $X_i \sim \text{Uni}(0,1)$.

$\mu = E[X_i] = 1/2$

$\sigma^2 = \text{Var}(X_i) = 1/12$

For different n, how close is the CLT approximation of $P(X \le n/3)$?

$n = 10$:



Exact

$$P(X \le 10/3) \approx 0.0337$$

CLT approximation

$$X \approx Y \sim \mathcal{N}(n\mu, n\sigma^2) \quad \Longrightarrow Y \sim \mathcal{N}(5, 5/6)$$

$$P(X \le 10/3) \approx P(Y \le 10/3)$$

$$= \Phi\left(\frac{10/3 - 5}{\sqrt{5/6}}\right) \approx 0.0339$$

Sum of n independent Uniform RVs

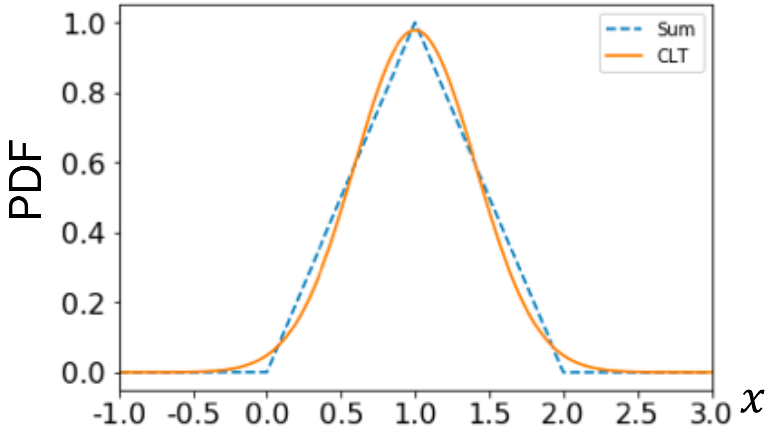
Let $X = \sum_{i=1}^{n} X_i$ be sum of i.i.d. RVs, where $X_i \sim \text{Uni}(0,1)$.

$\mu = E[X_i] = 1/2$

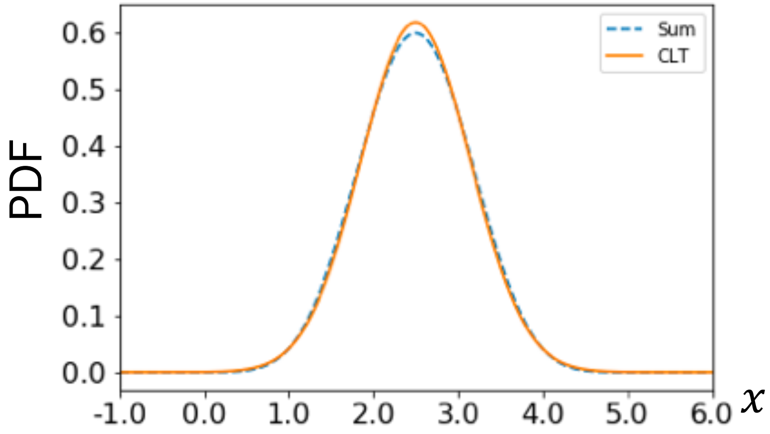
$\sigma^2 = \text{Var}(X_i) = 1/12$

For different n, how close is the CLT approximation of $P(X \leq n/3)$?

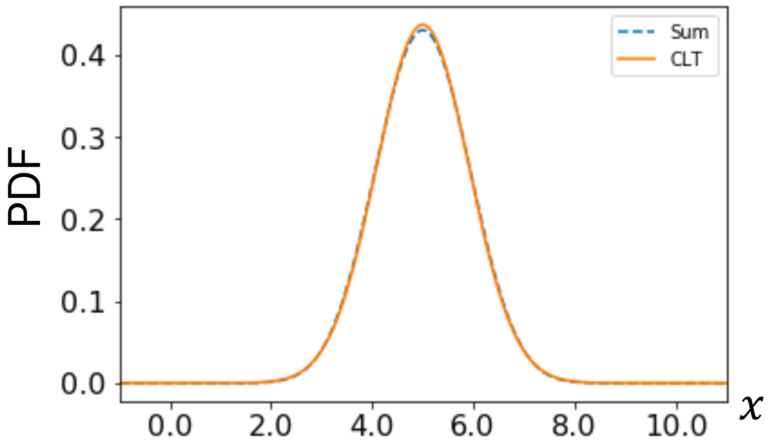
$n = 2$:



$n = 5$:



$n = 10$:



Most books will tell you that CLT holds if $n \geq 30$, but it can hold for smaller n depending on the distribution of your i.i.d. X_i's.

The sum of independent, identically distributed variables:

$$Y = \sum_{i=0}^{n} X_i$$

Is normally distributed:

$$Y \sim N(n\mu, n\sigma^2)$$

---

where $\mu = E[X_i]$

$$\sigma^2 = \text{Var}(X_i)$$

What about other functions?

Sum of iid? Normal

Average of iid?

Max of iid?

It's play time!

Sum of Dice

- You will roll 10 6-sided dice ($X_1, X_2, \ldots, X_{10}$)
  - X = total value of all 10 dice = $X_1 + X_2 + \ldots + X_{10}$
  - Win if: $X \leq 25$ or $X \geq 45$
  - Roll!

- And now the truth (according to the CLT)…

Sum of Dice

- You will roll 10 6-sided dice (X_1, X_2, …, X_{10})
  - X = total value of all 10 dice = $X_1 + X_2 + \ldots + X_{10}$
  - Win if: $X \le 25$ or $X \ge 45$

- Recall CLT: $X = \sum_i^n X_i \to N(n\mu, n\sigma^2)$ As $n \to \infty$
  - Determine $P(X \le 25 \text{ or } X \ge 45)$ using CLT:

$$\mu = E[X_i] = 3.5 \qquad \sigma^2 = \mathrm{Var}(X_i) = \frac{35}{12} \qquad X \approx N(35, 29.2)$$

$$1 - P(25.5 < X < 44.5) = 1 - P\left(\frac{25.5 - 35}{\sqrt{29.2}} < Z < \frac{44.5 - 35}{\sqrt{29.2}}\right)$$

$$\approx 1 - (2\Phi(1.76) - 1) \approx 2(1 - 0.9608) = 0.0784$$

Wonderful Form of Cosmic Order

I know of scarcely anything so apt to impress the imagination as the wonderful form of cosmic order expressed by the ”[Central limit theorem]". The law would have been personified by the Greeks and deified, if they had known of it. It reigns with serenity and in complete self-effacement, amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason. Whenever a large sample of chaotic elements are taken in hand and marshalled in the order of their magnitude, an unsuspected and most beautiful form of regularity proves to have been latent all along.

- Sir Francis Galton

Midterm Summer 2023…

We made it a bit hard…

But First

What we care about?

$$Knowledge!$$

What does the Midterm give us?

$$Test\ Score?$$

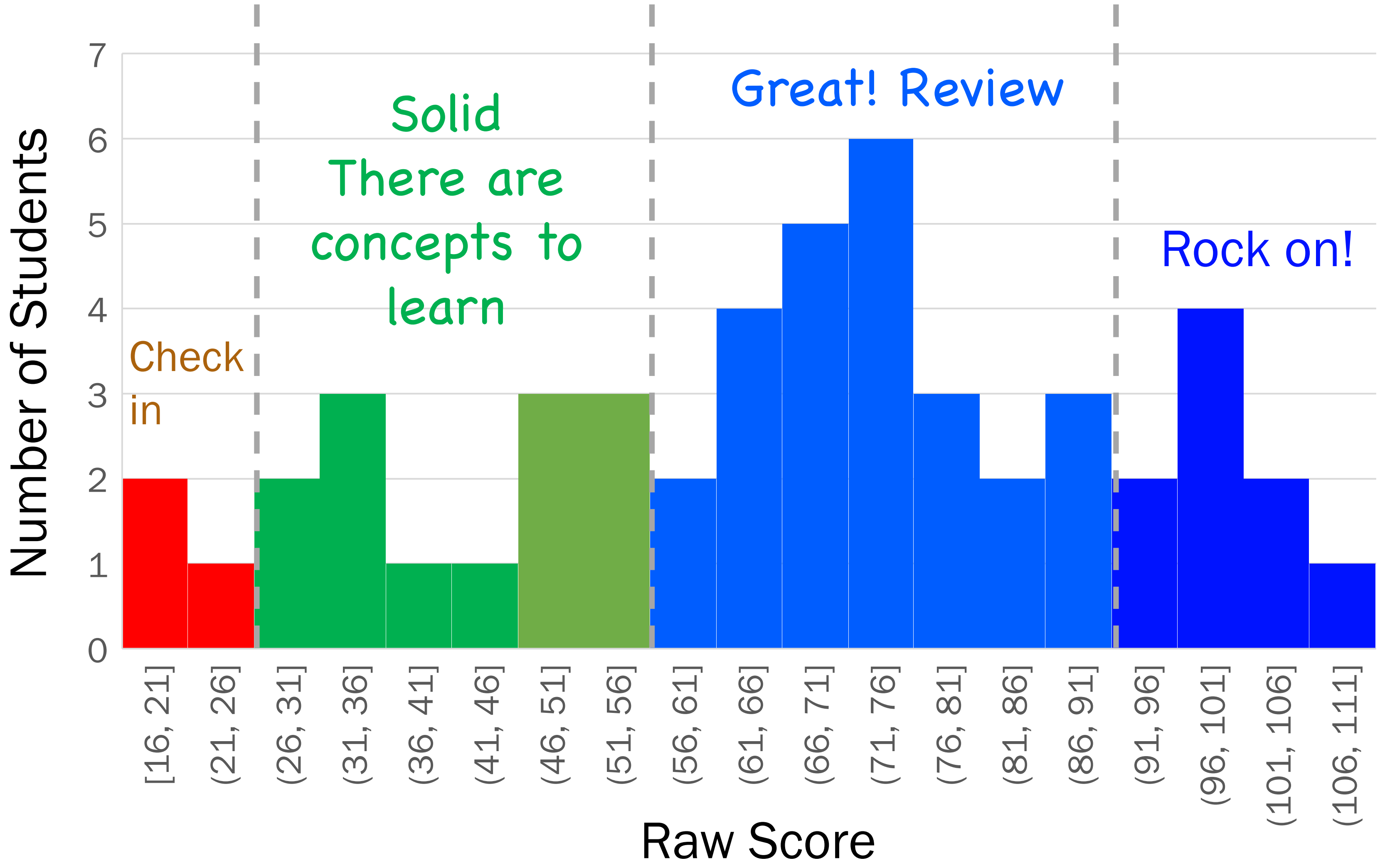
Just a Heuristic!

$$P(Knowledge \mid Test\ Score)$$

Plenty of ways to characterize this distribution!

- *Psets, Improvement on final!*

Grade Distribution



$$\mu = \frac{66.7}{120}$$

$$\sigma = \frac{23.88}{120}$$

Midterm Logistics

Regrade requests:

- Submit by Monday.
- Reserve the right to regrade the whole test

Grade is still under your control:

- Think of the midterm as a diagnostic. Found an area to improve?
- Staff are always looks for growth between the midterm and final!

What if you didn't get the score you wanted

We look for improvement between the midterm and final. We will substantially **down weight** the midterm for folks who show great improvement.