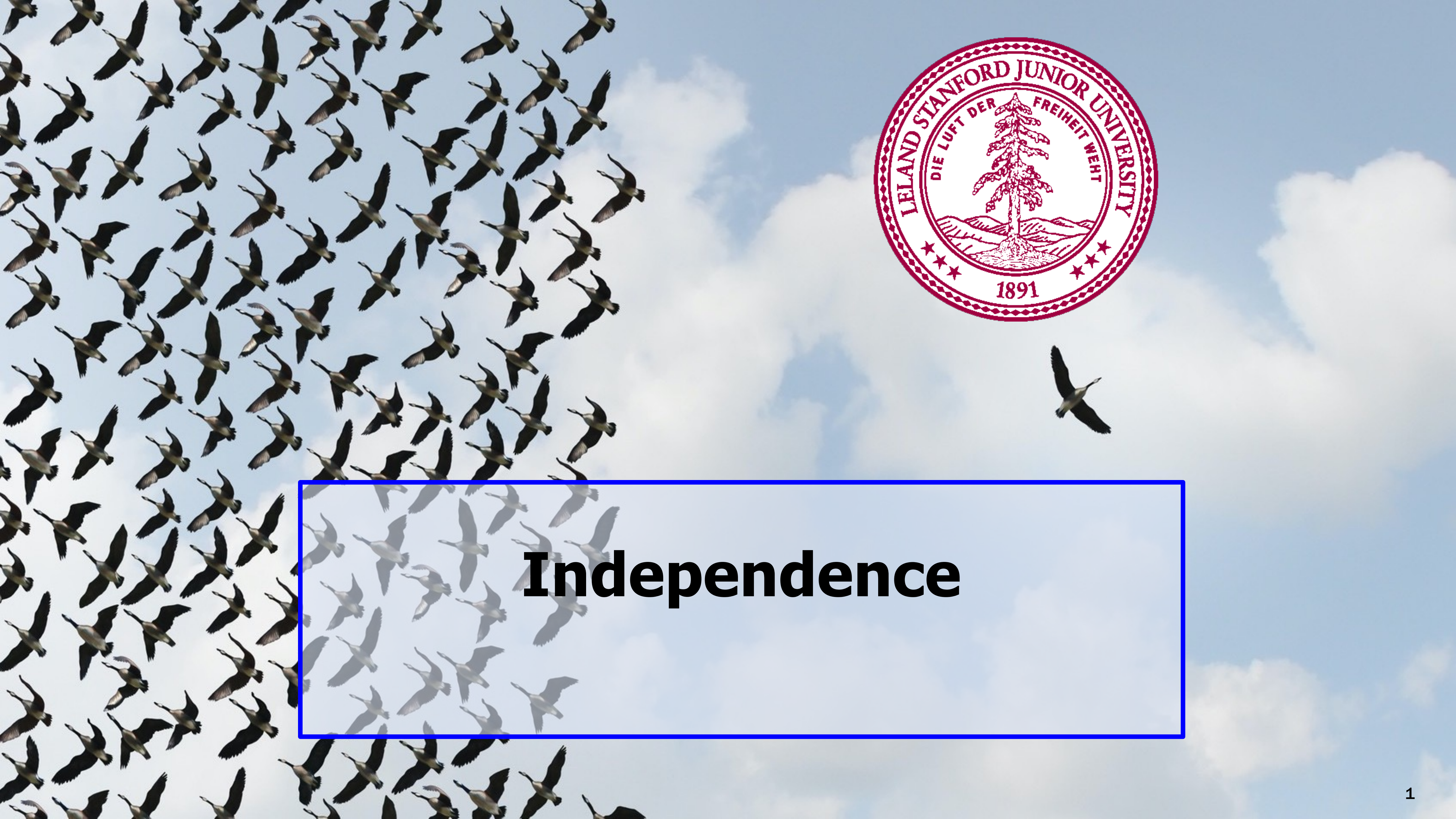


Independence

Announcements

- **Sections** start today. Wahoo! Enjoy.
- **PSet #1** is due Monday at Midnight. Recall grace period.

Please give us weekly feedback!

hrcf@cs.stanford.edu

Review

Review: Conditional Probability

$$p(AB) \quad \text{vs} \quad P(A|B)$$

$$P(AB) = P(A|B)P(B)$$

Notation

And	Or	Given
$P(E \text{ and } F)$	$P(E \text{ or } F)$	$P(E\|F)$
$P(E, F)$	$P(E \cup F)$	$P(E\|F, G)$
$P(EF)$		
$P(E \cap F)$		

Probability of E given F and G

Review: Chain Rule

Definition of conditional probability:

$$P(E|F) = \frac{P(EF)}{P(F)}$$

The Chain Rule:

$$\begin{aligned} P(EF) &= P(E|F)P(F) \\ &= P(F|E)P(E) \end{aligned}$$

Relationship Between Probabilities

$P(E \text{ and } F)$

Chain rule
(Product rule)

Definition of
conditional probability

$P(E|F)$

Law of Total
Probability

Bayes'
Theorem

$P(E)$

$P(F|E)$

Bayes' Theorem

$P(E|F) \Rightarrow P(F|E)$

<u>Thm</u> For any events E and F where $P(E) > 0$ and $P(F) > 0$,

$$\underbrace{P(F|E)}_{\text{posterior}} = \frac{\overbrace{P(E|F)}^{\text{likelihood}}\overbrace{P(F)}^{\text{prior}}}{\underbrace{P(E)}_{\text{normalization constant}}}$$

$$\underset{P(F)}{\text{prior}} \xrightarrow{\text{E happens}} \underset{P(F|E)}{\text{posterior}}$$

"Updating" your belief

Prior → Posterior

<u>Expanded form</u>:

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^C)P(F^C)}$$

SARS Virus Testing

A test is 98% effective at detecting SARS

- However, test has a "false positive" rate of 1%
- 0.5% of the world has SARS
- Let E = you test positive for SARS with this test
- Let F = you actually have SARS
- What is P(F | E)?

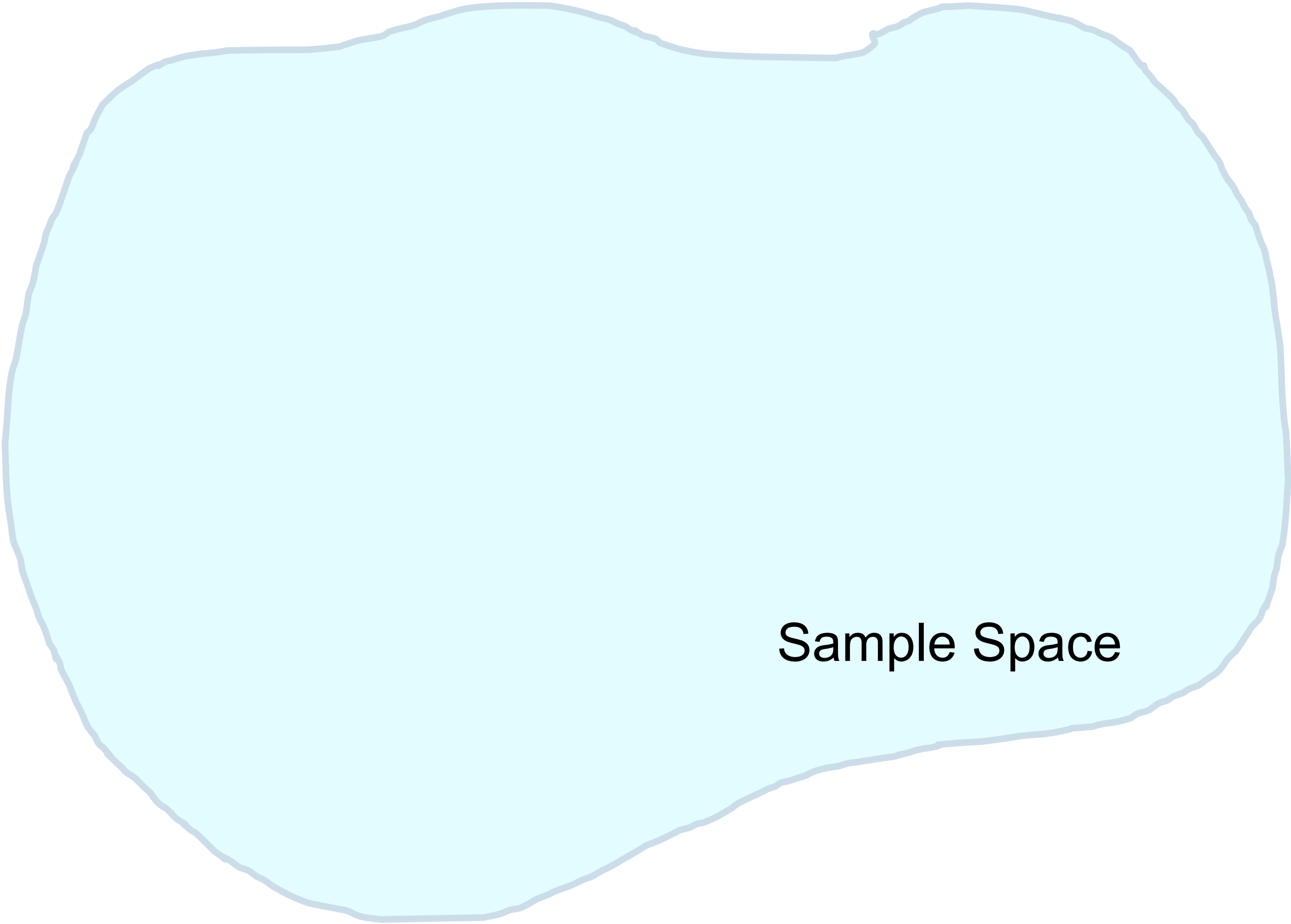
Solution:

$$P(F \mid E) = \frac{P(E \mid F)\, P(F)}{P(E \mid F)\, P(F) + P(E \mid F^c)\, P(F^c)}$$

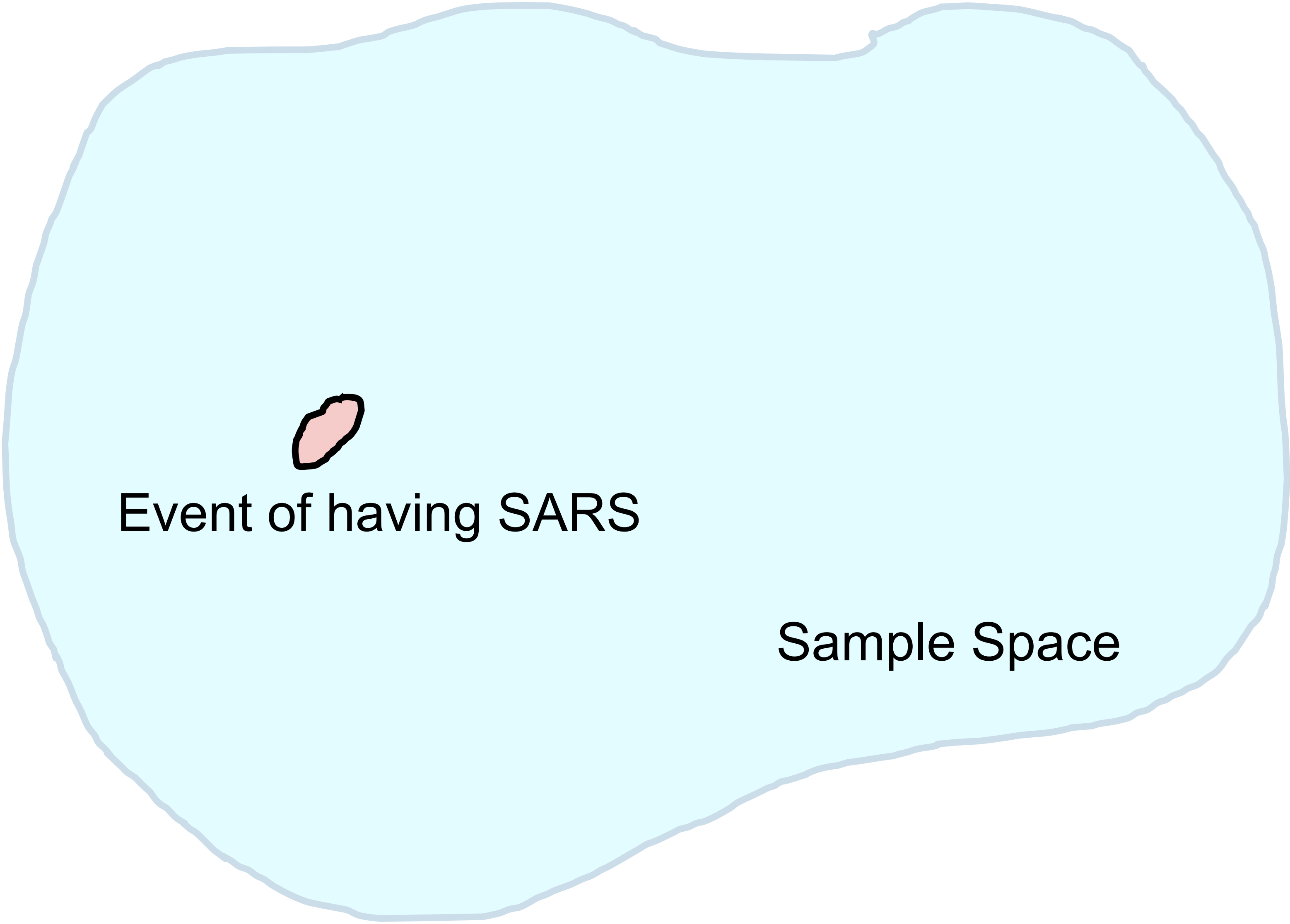
$$P(F \mid E) = \frac{(0.98)(0.005)}{(0.98)(0.005) + (0.01)(1 - 0.005)} \approx 0.330$$

Intuition Time

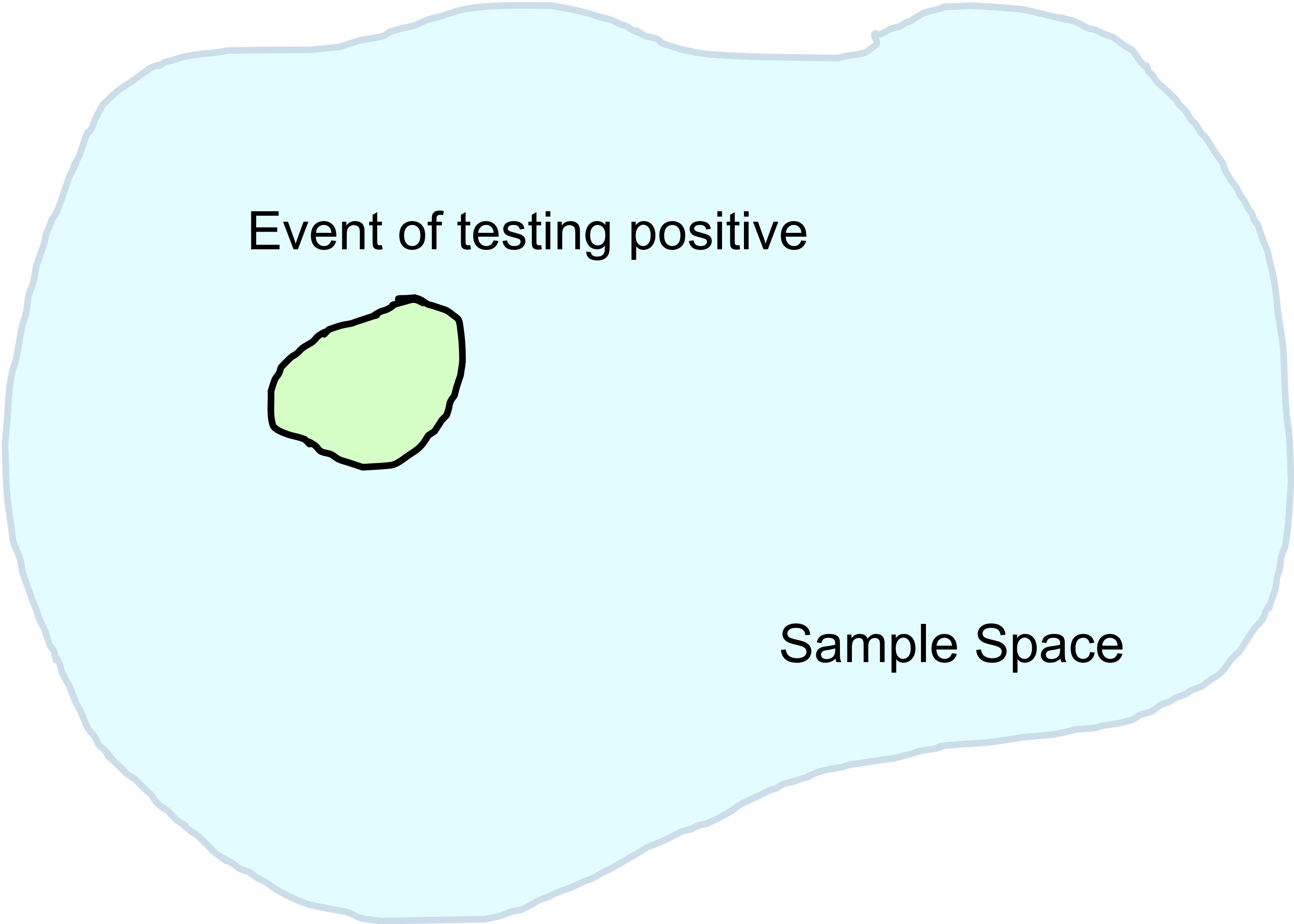
Bayes Thorem Intuition



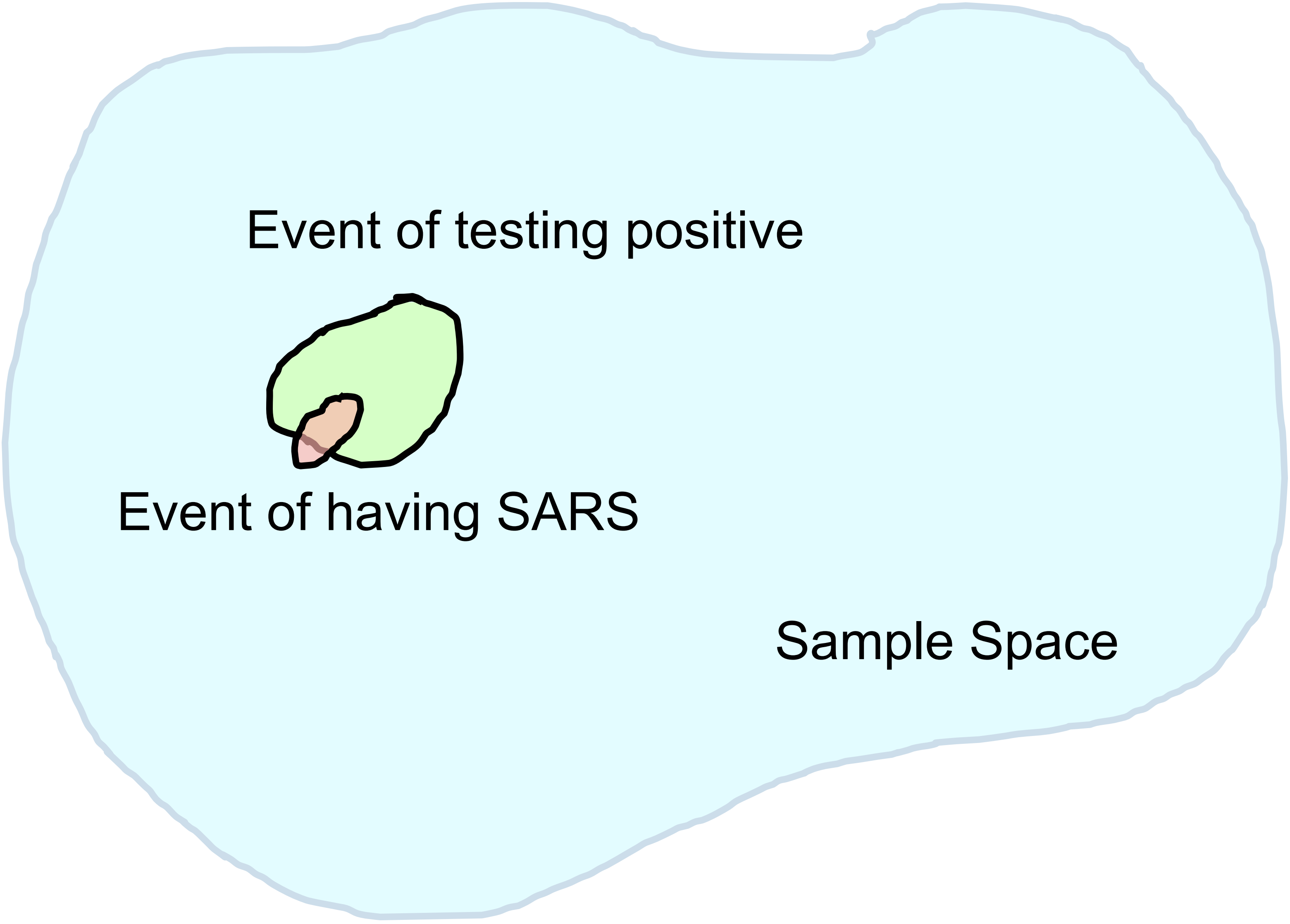
Bayes Thorem Intuition



Bayes Thorem Intuition

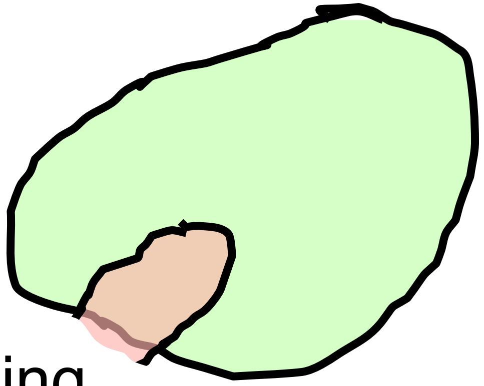


Bayes Thorem Intuition



Bayes Thorem Intuition

Conditioning on a positive result changes the sample space to this:



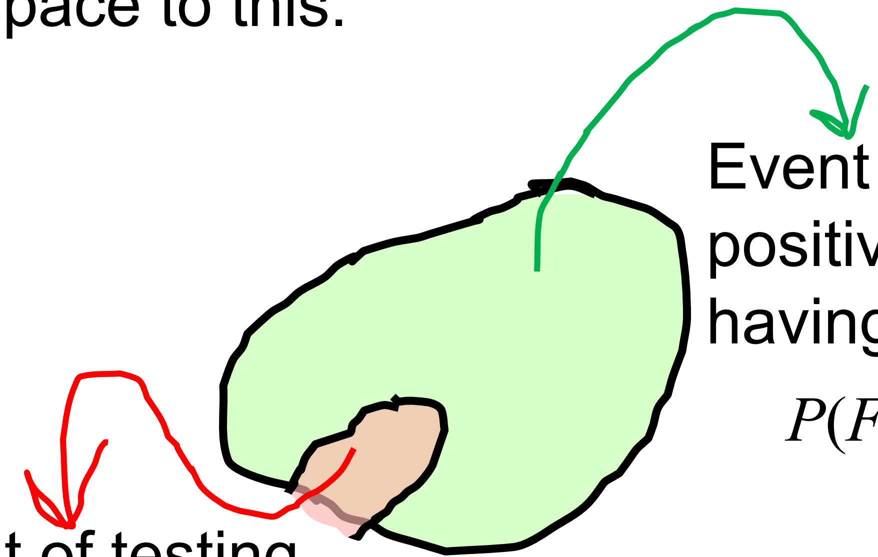
Event of testing positive and not having SARS

Event of testing positive and having SARS

≈ 0.330

Bayes Thorem Intuition

Conditioning on a positive result changes the sample space to this:



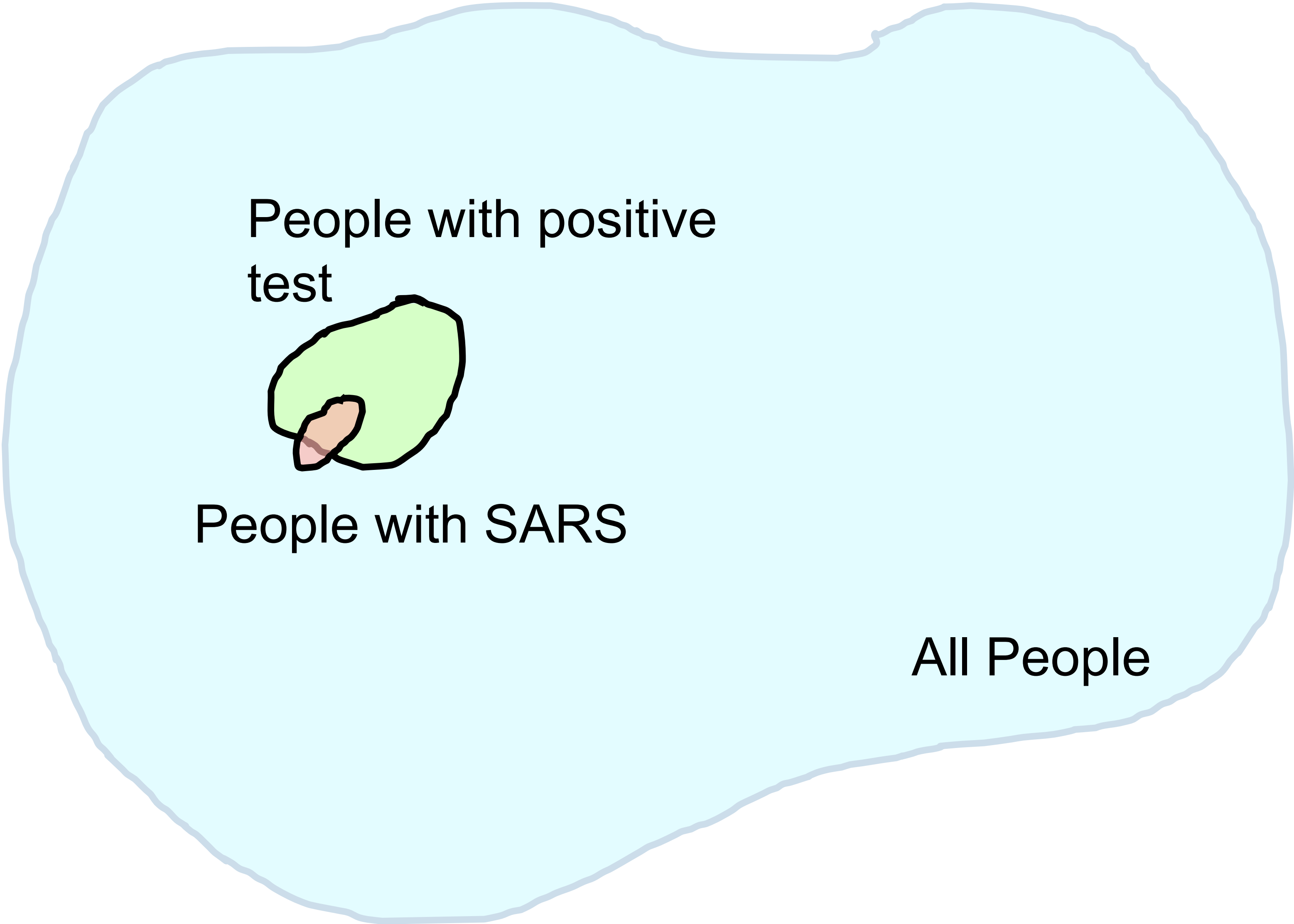
Event of testing positive and not having SARS

$P(F^c)P(E|F^c)$

Event of testing positive and having SARS $\mathrm{P}(F)P(E|F)$

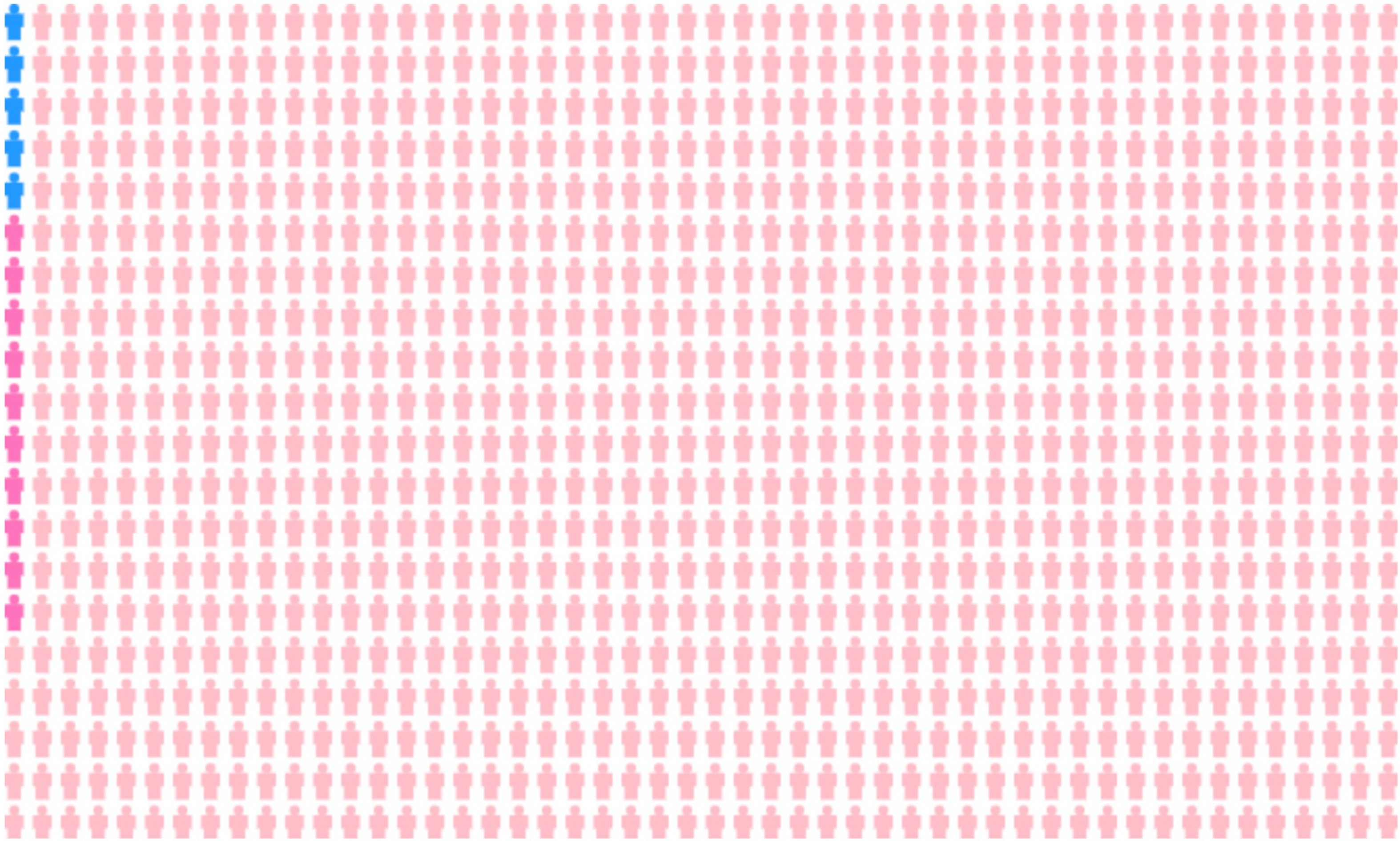
≈ 0.330

Bayes Thorem Intuition



Bayes Thorem Intuition

Say we have 1000 people:



5 have SARS and test positive, 985 **do not** have SARS and test negative.
10 **do not** have SARS and test positive.

≈ 0.333

Bayes Thorem Intuition

Conditioned on just those that test positive:

Notice that all the people with SARS are here,
but the group is still mainly folks without SARS

5 have SARS and test positive, 985 **do not** have SARS and test negative.
10 **do not** have SARS and test positive.

≈ 0.333

Why it is still good to get tested

	SARS +	SARS –
Test +	$0.98 = P(E \mid F)$	$0.01 = P(E \mid F^c)$
Test –	$0.02 = P(E^c \mid F)$	$0.99 = P(E^c \mid F^c)$

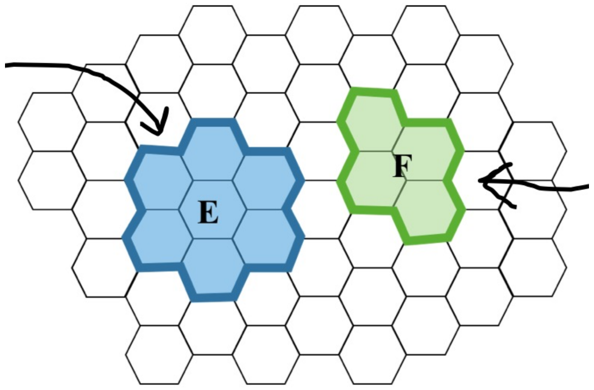
- Let E^c = you test <u>negative</u> for SARS with this test
- Let F = you actually have SARS
- What is $P(F \mid E^c)$?

$$P(F \mid E^c) = \frac{P(E^c \mid F)\,P(F)}{P(E^c \mid F)\,P(F) + P(E^c \mid F^c)\,P(F^c)}$$

$$P(F \mid E^c) = \frac{(0.02)(0.005)}{(0.02)(0.005) + (0.99)(1 - 0.005)} \approx 0.0001$$

End Review

Learning Goals of Today



Mutually Exclusive

$$P(A \text{ and } B) = 0$$

Makes ***OR*** easy:

$$P(A \text{ or } B) = P(A) + P(B)$$

Independent

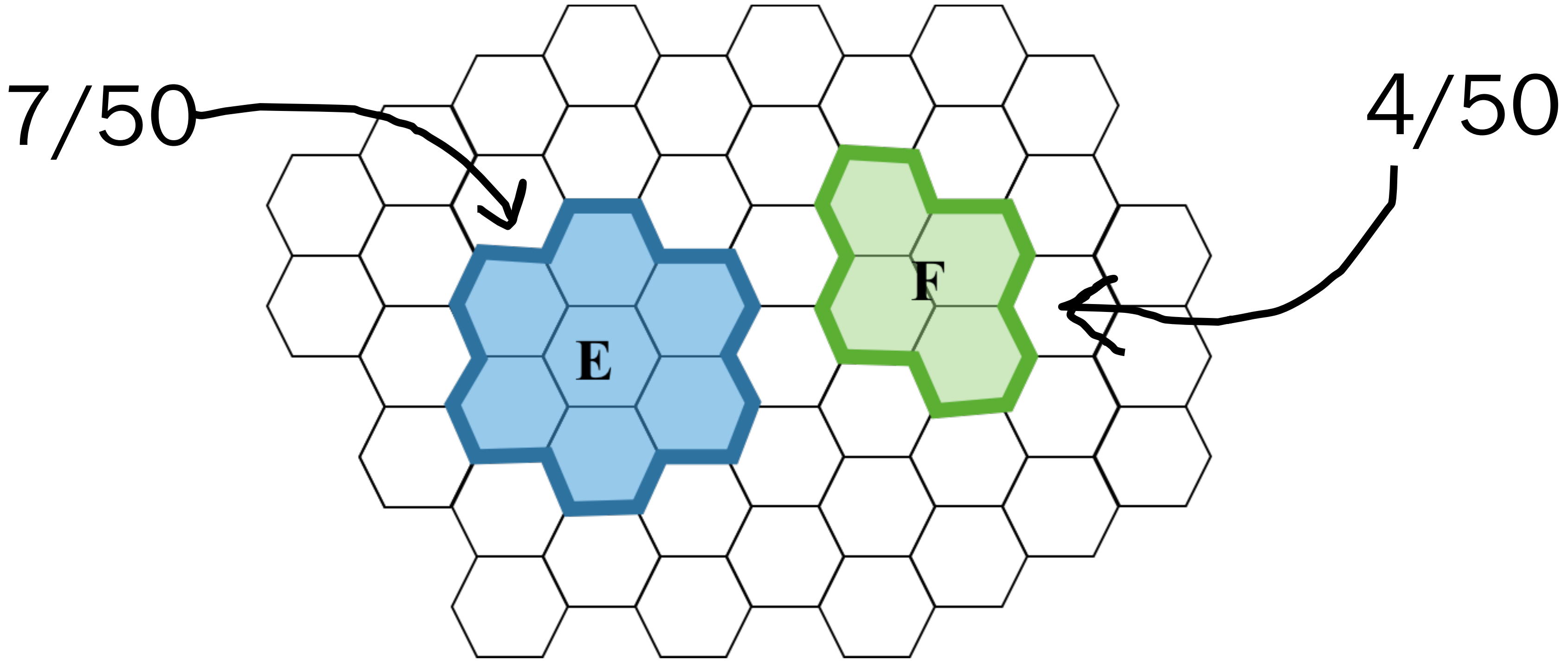
$$P(A) = P(A|B)$$

Makes ***AND*** easy:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Probability of “OR”

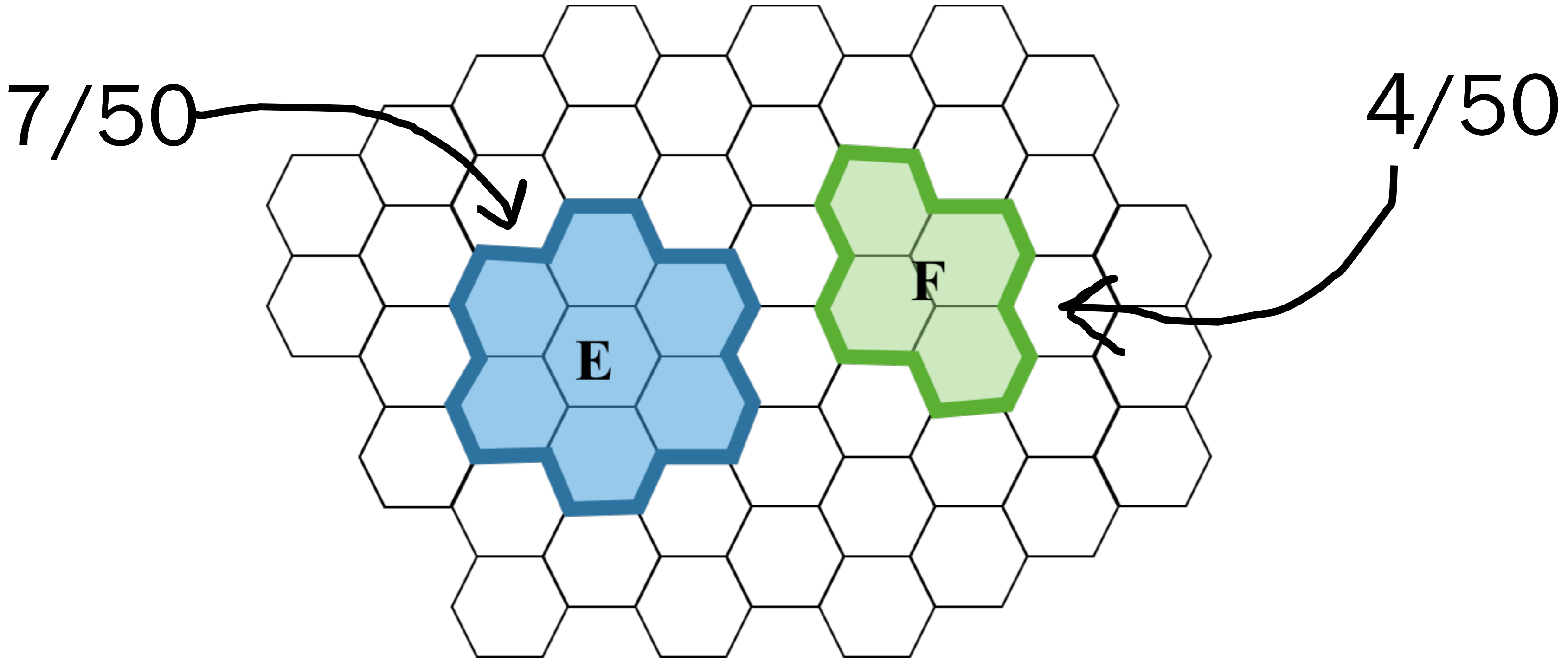
Review: OR with Mutually Exclusive Events



If events are mutually exclusive, probability of OR is simple:

$$P(E \cup F) = P(E) + P(F)$$

Review: OR with Mutually Exclusive Events

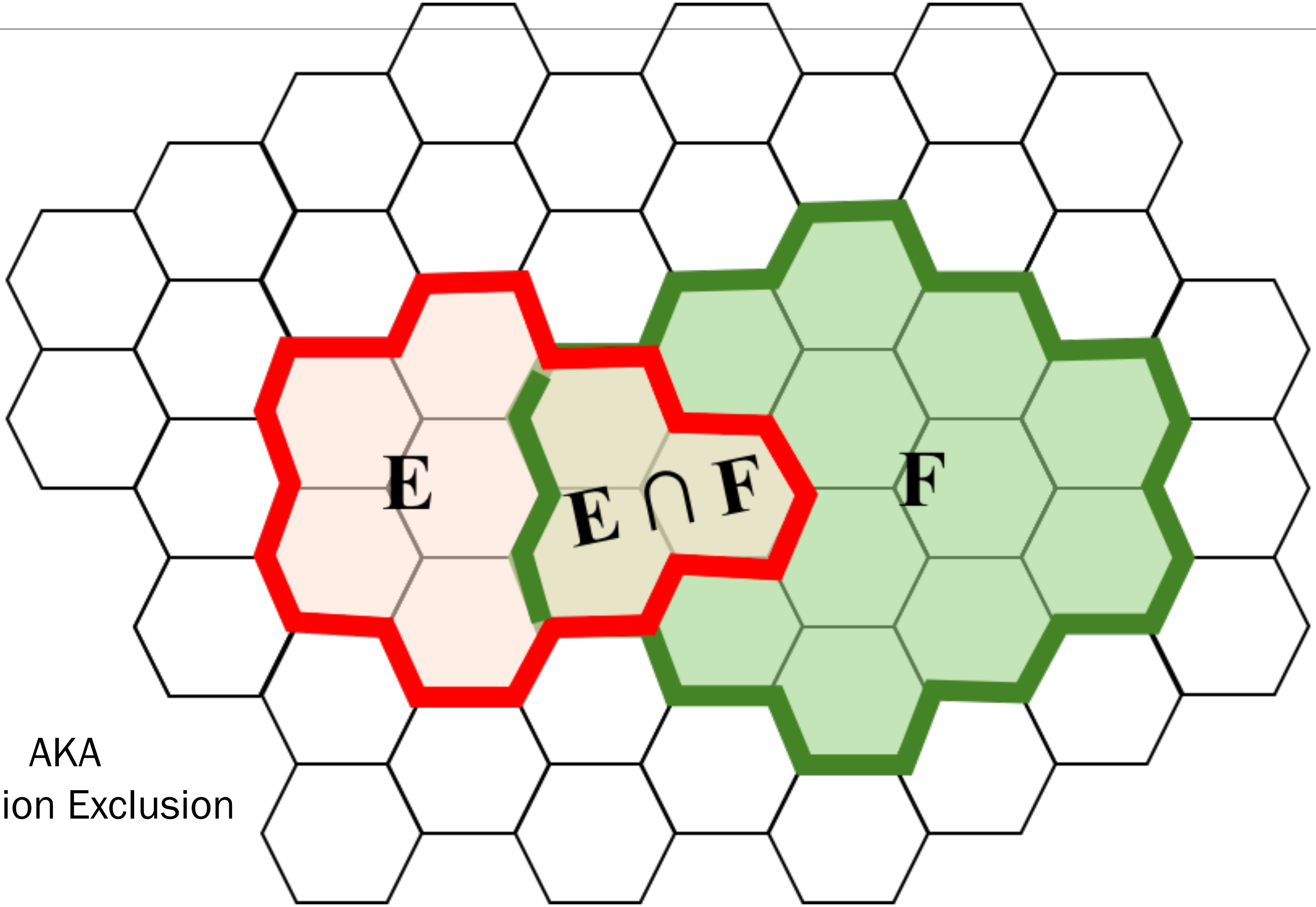


If events are mutually exclusive, probability of OR is simple:

$$P(E \cup F) = \frac{7}{50} + \frac{4}{5} = \frac{11}{50}$$

What about when they are not
Mutually exclusive?

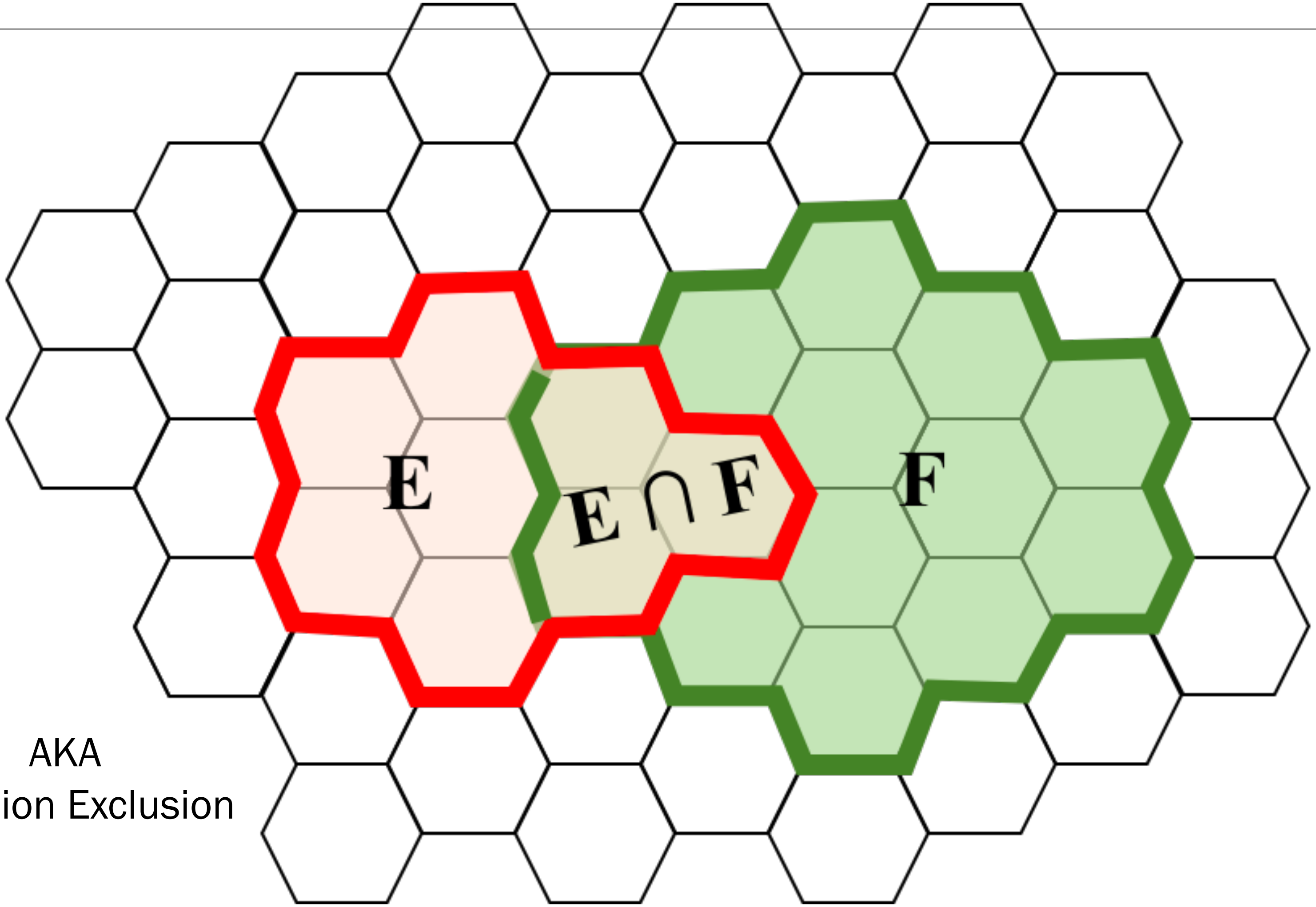
OR *without* Mutually Exclusive Events



AKA
Inclusion Exclusion

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

OR *without* Mutually Exclusive Events



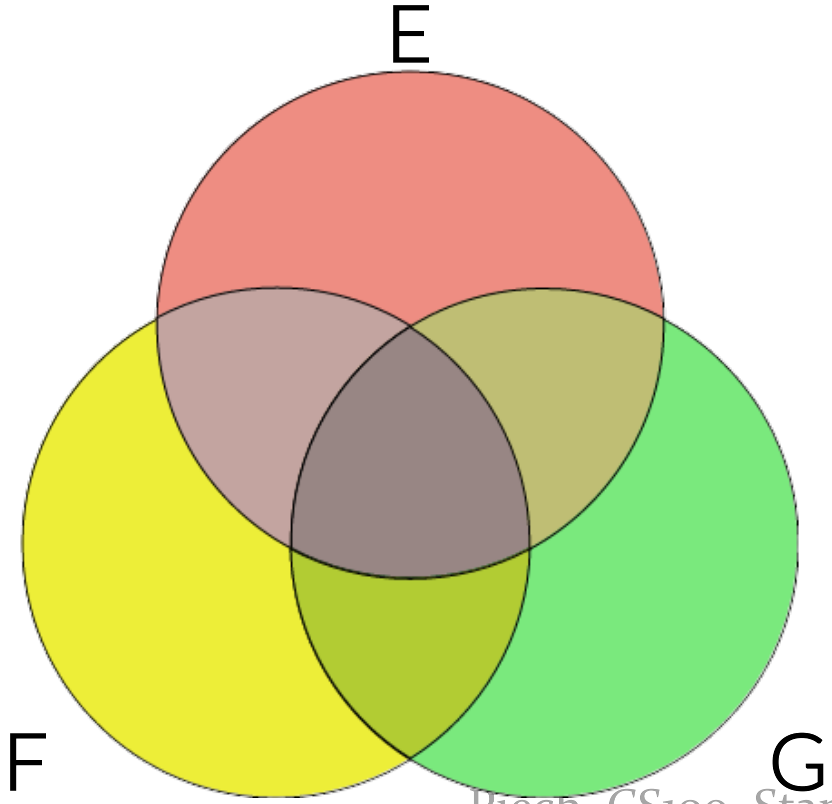
AKA
Inclusion Exclusion

$$P(E \cup F) = \frac{8}{50} + \frac{14}{50} - \frac{3}{50} = \frac{19}{50}$$

More than two sets?

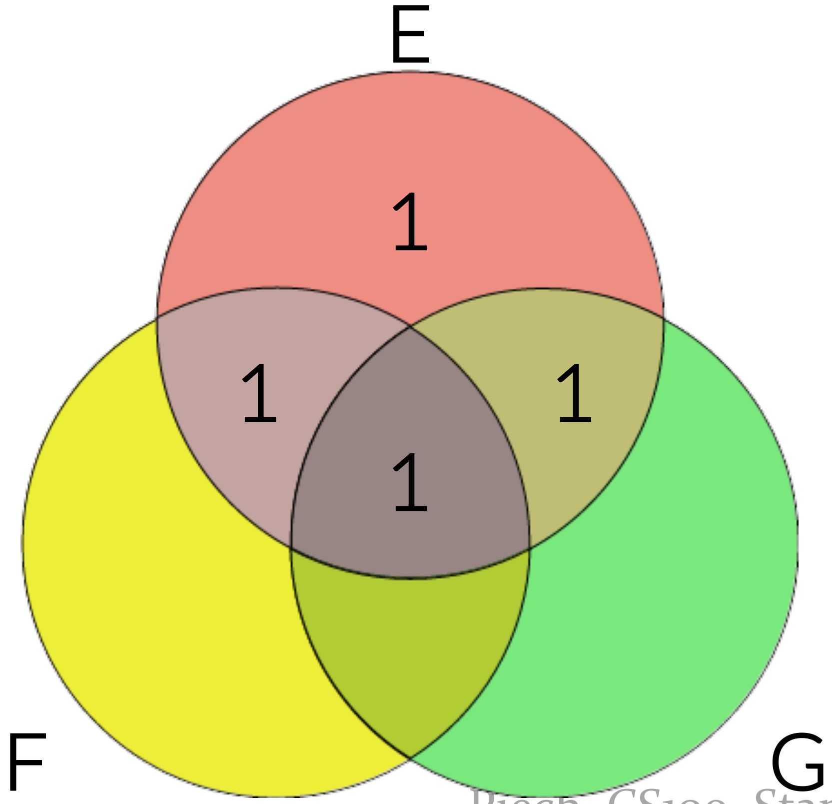
Inclusion / Exclusion with Three Events

$$P(E \cup F \cup G) =$$



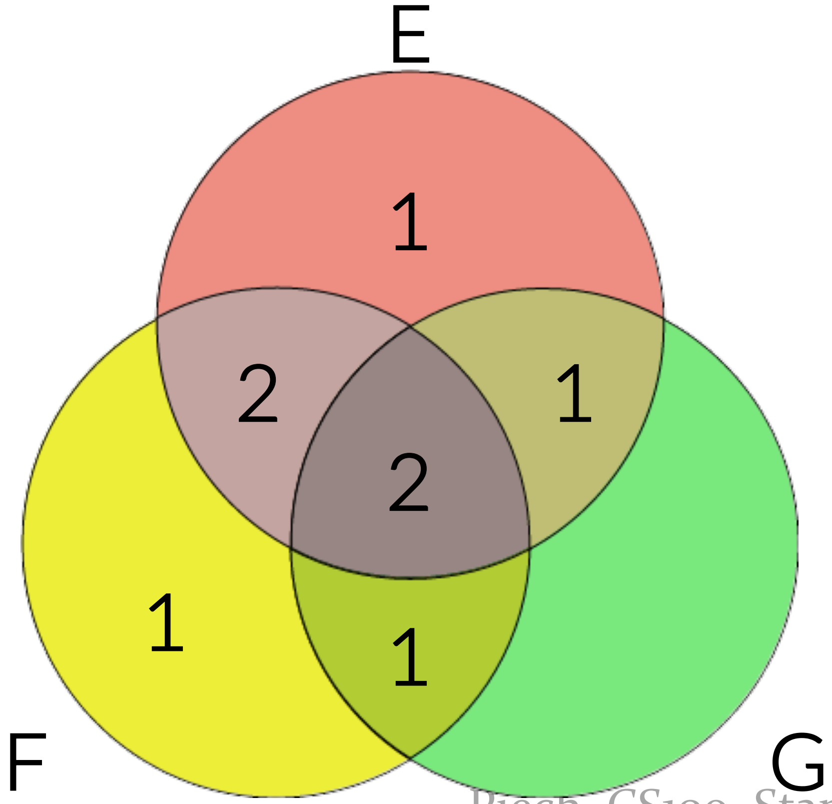
Inclusion / Exclusion with Three Events

$$P(E \cup F \cup G) = P(E)$$



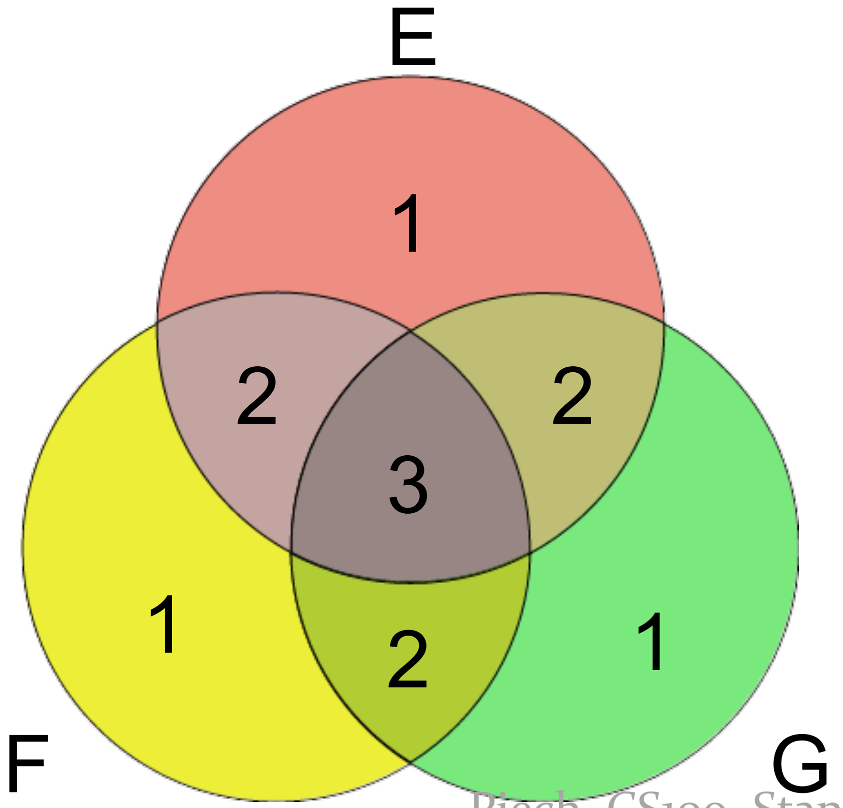
Inclusion / Exclusion with Three Events

$$P(E \cup F \cup G) = P(E) + P(F)$$



Inclusion / Exclusion with Three Events

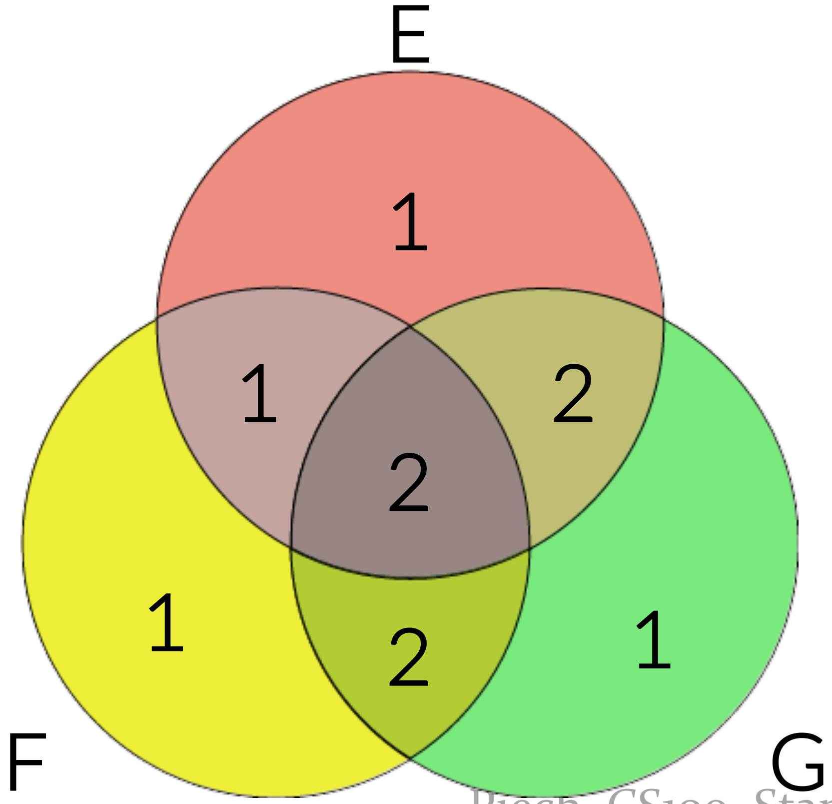
$$P(E \cup F \cup G) = P(E) + P(F) + P(G)$$



Inclusion / Exclusion with Three Events

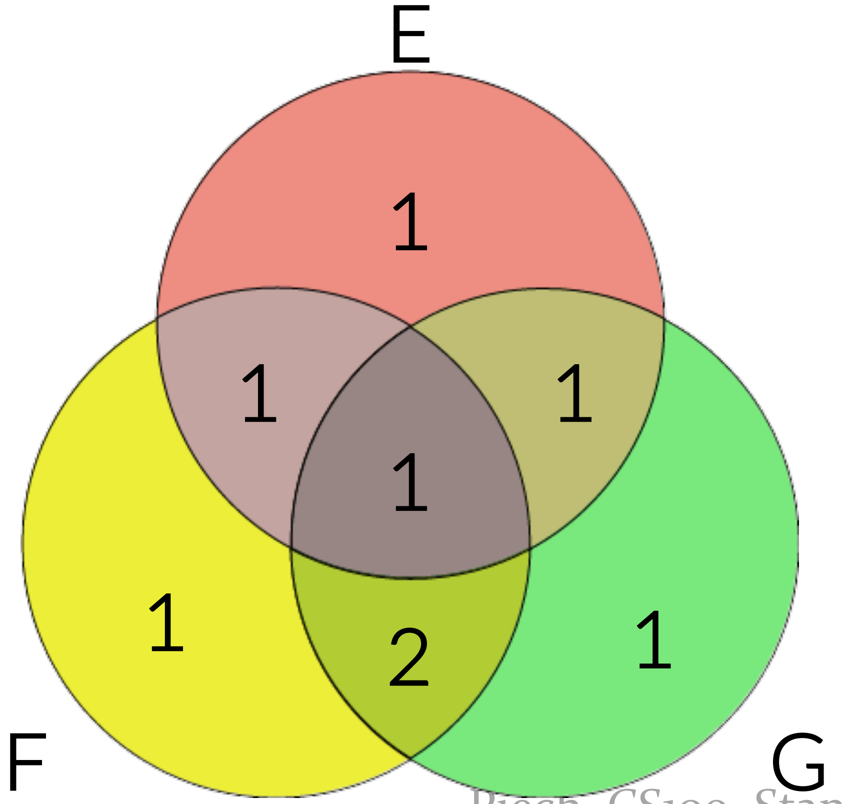
$$P(E \cup F \cup G) = P(E) + P(F) + P(G)$$

$$-P(EF)$$



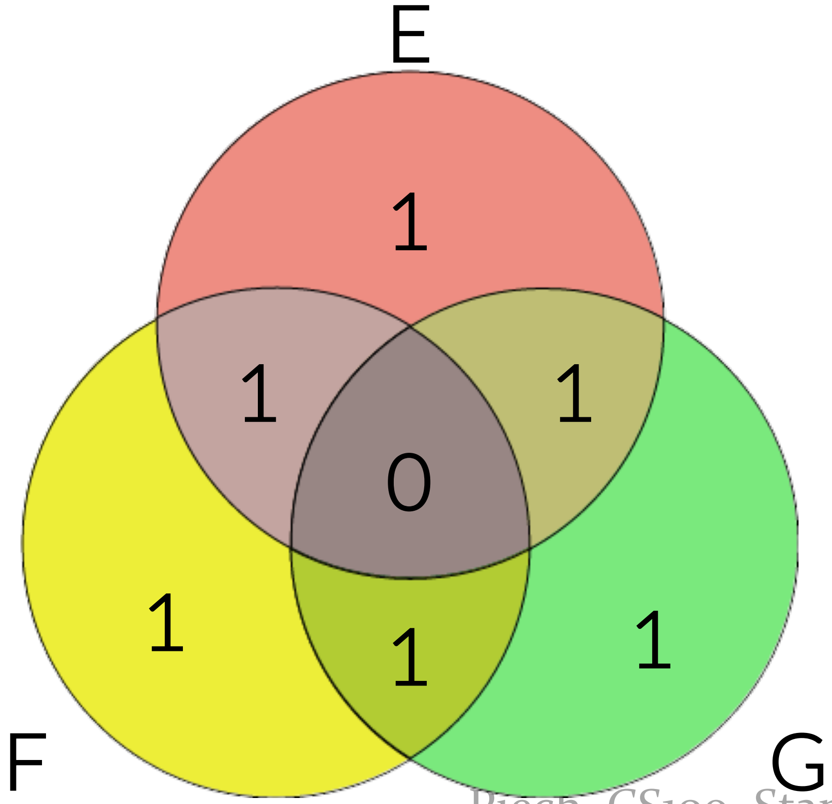
Inclusion / Exclusion with Three Events

$$P(E \cup F \cup G) = P(E) + P(F) + P(G)$$
$$-P(EF) - P(EG)$$



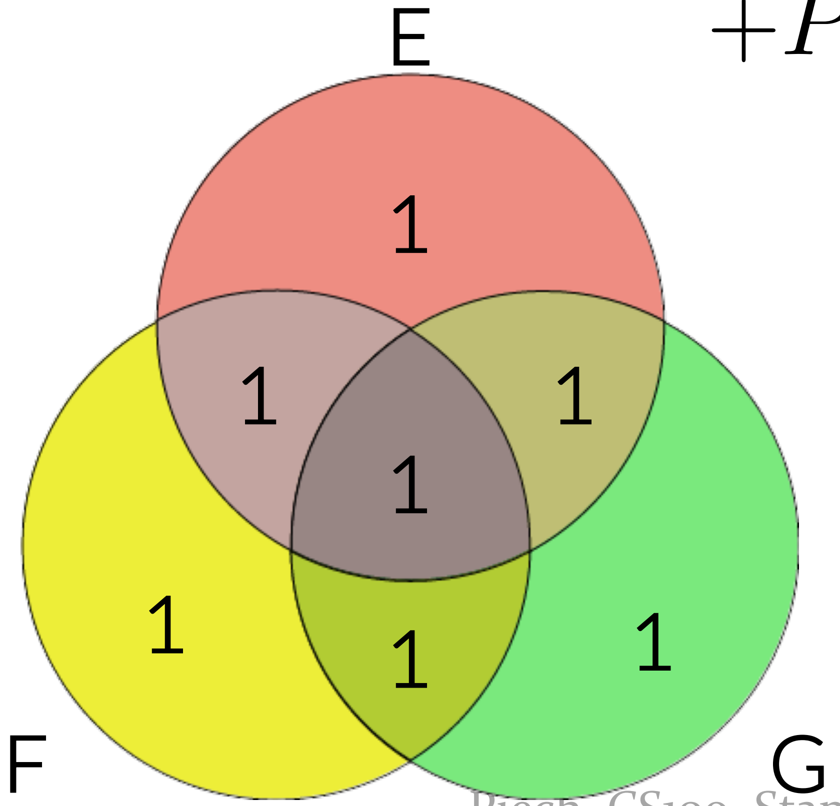
Inclusion / Exclusion with Three Events

$$P(E \cup F \cup G) = P(E) + P(F) + P(G)$$
$$-P(EF) - P(EG) - P(FG)$$



Inclusion / Exclusion with Three Events

$$P(E \cup F \cup G) = P(E) + P(F) + P(G)$$
$$-P(EF) - P(EG) - P(FG)$$
$$+P(EFG)$$



General Inclusion / Exclusion

$$P(E_1 \cup E_2 \cup \cdots \cup E_n) = \sum_{r=1}^{n} (-1)^{r+1} Y_r$$

Y_1 = Sum of all events on their own $\qquad \displaystyle\sum_i P(E_i)$

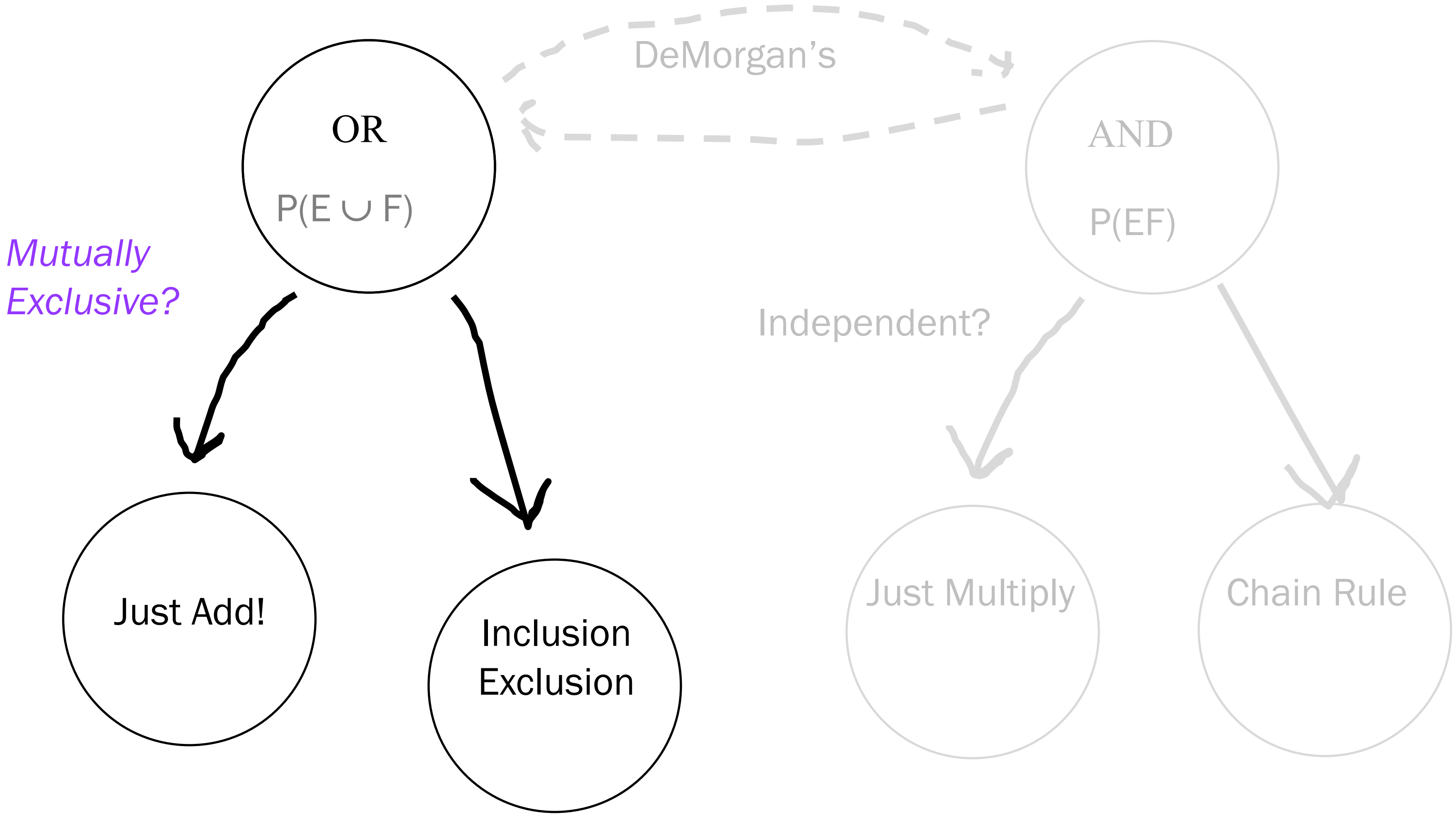
Y_2 = Sum of all pairs of events $\qquad \displaystyle\sum_{i,j} P(E_i \cap E_j) \quad \text{s.t.} i \neq j$

Y_3 = Sum of all triples of events $\qquad \displaystyle\sum_{i,j,k} P(E_i \cap E_j \cap E_k) \quad \text{s.t.} i \neq j, j \neq k, i \neq k$

* Where Y_r is the sum, for all combinations of r events, of the probability of the ~~union~~ those events.

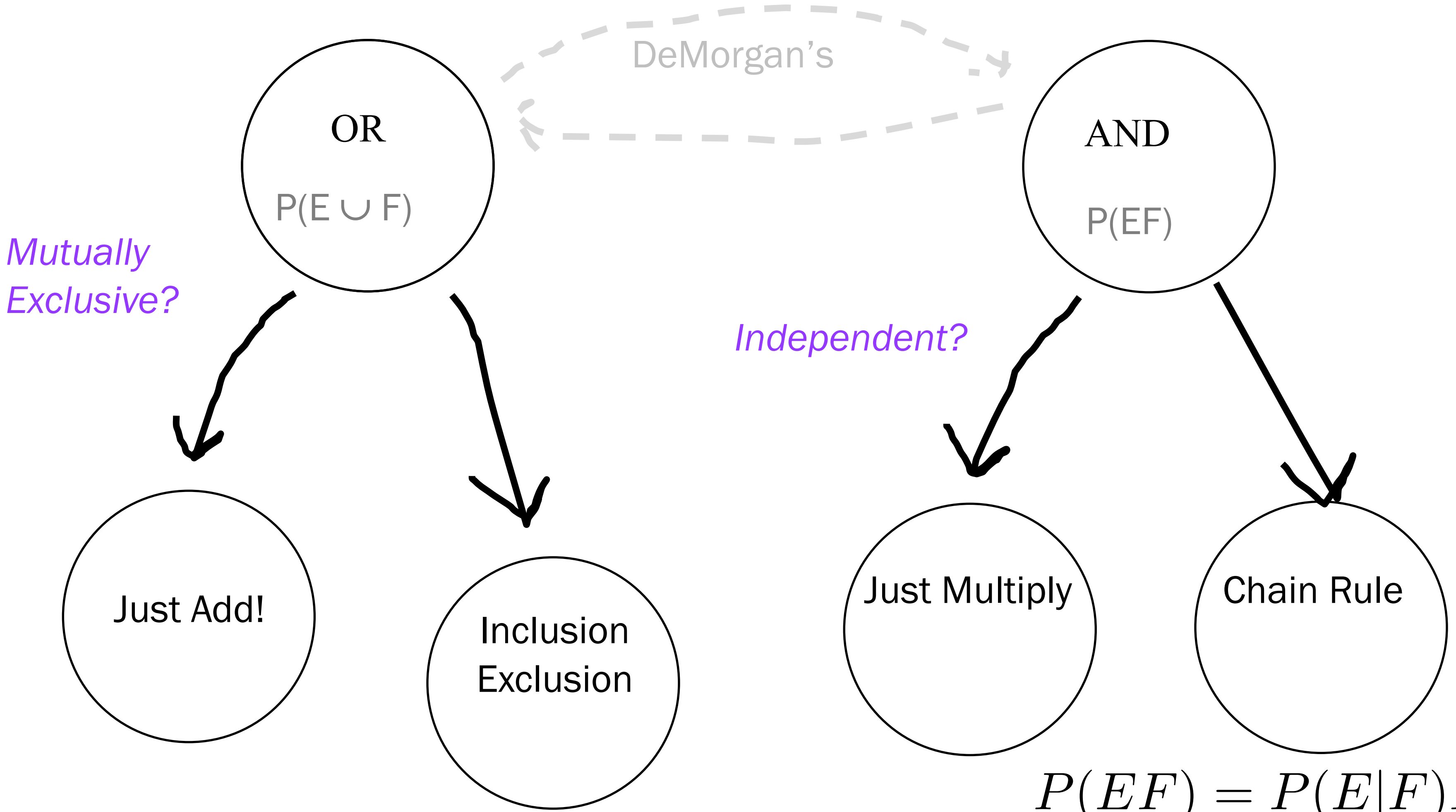
intersection

Today



DeMorgan's
OR
P(E ∪ F)
AND
P(EF)
Mutually Exclusive?
Independent?
Just Add!
Inclusion Exclusion
Just Multiply
Chain Rule

Today



$$P(EF) = P(E|F)P(F)$$

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

Probability of “AND”


We the People of the
insure domestic Tranquility provide for the common
and our Posterity, do ordain and establish this

Independence

Two events A and B are called **independent** if:

$$P(A) = P(A|B)$$

Knowing that event B happened, doesn't change our belief that A will happen.

Otherwise, they are called **dependent** events

Alternative Definition of Independence

Notation for *and*

$$P(A, B) = P(A) \cdot P(B|A)$$ Chain rule

$$= P(A) \cdot P(B)$$ Since B is independent of A

If you show this is true, you have proved the two events are independent!

Alternative Definition of Independence

Notation for *and*

$$P(A, B) = P(A) \cdot P(B|A)$$ Chain rule

$$= P(A) \cdot P(B)$$ Since B is independent of A

If you show this is true, you have proved the two events are independent!

If events are *independent* probability of AND is easy!

BUT!!

Always *start with* ***chain rule***

and then ask yourself:
"Does independence hold?"

Independent $\neq$ Mutually Exclusive

A,B Independent → $P(A,B) = P(A)P(B)$

A,B Mutually exclusive → $P(A,B) = 0$

Independence is reciprocal

If A is independent of B, then B is independent of A

$$P(A) = P(A|B) \qquad P(B|A) = P(B)$$

Proof:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Bayes' Thm.

$$= \frac{P(A)P(B)}{P(A)}$$

Because A is independent of B

$$= P(B)$$

Dice, our misunderstood friends

Roll two 6-sided dice, yielding values D_1 and D_2

- Let E be event: $D_1 = 1$
- Let F be event: $D_2 = 1$

What is P(E), P(F), and P(EF)?

- $P(E) = 1/6, \quad P(F) = 1/6, \quad P(EF) = 1/36$
- $P(EF) = P(E)\,P(F) \quad \rightarrow \quad$ E and F ***independent***

Let G be event: $D_1 + D_2 = 5$ $\{(1, 4), (2, 3), (3, 2), (4, 1)\}$

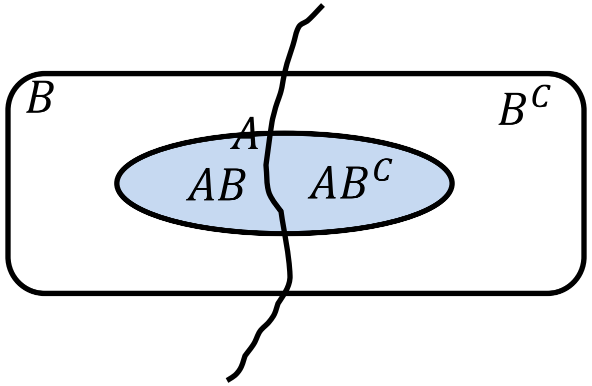
What is P(E), P(G), and P(EG)?

- $P(E) = 1/6, \quad P(G) = 4/36 = 1/9, \quad P(EG) = 1/36$
- $P(EG) \neq P(E)\,P(G) \quad \rightarrow \quad$ E and G ***dependent***

Independence of Complements

Given independent events A and B, prove that A and B^C are independent

We want to show that $P(AB^C) = P(A)P(B^C)$

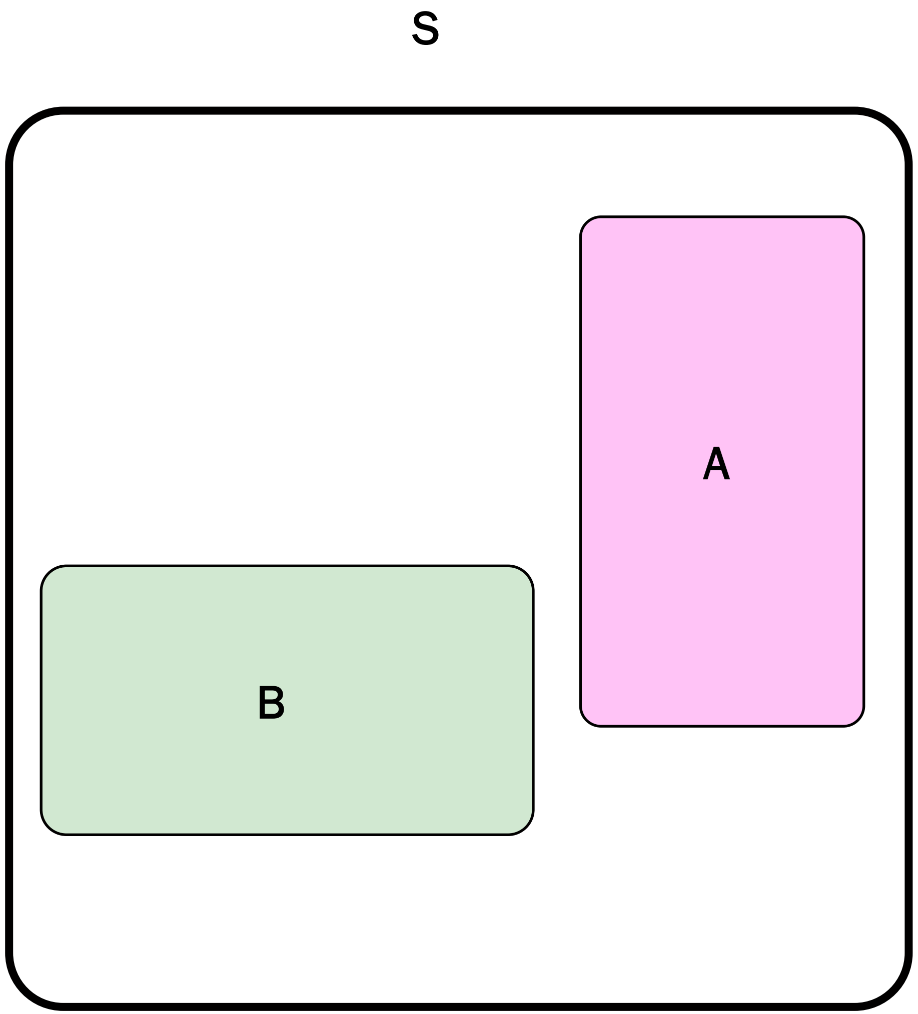


$$
\begin{aligned}
P(AB^C) &= P(A) - P(AB) && \text{By Total Law of Prob.} \\
&= P(A) - P(A)P(B) && \text{By independence} \\
&= P(A)[1 - P(B)] && \text{Factoring} \\
&= P(A)P(B^C) && \text{Since } P(B) + P(B^C) = 1
\end{aligned}
$$

So if A and B are independent A and B^C are also independent

What does independence look like?

Independence



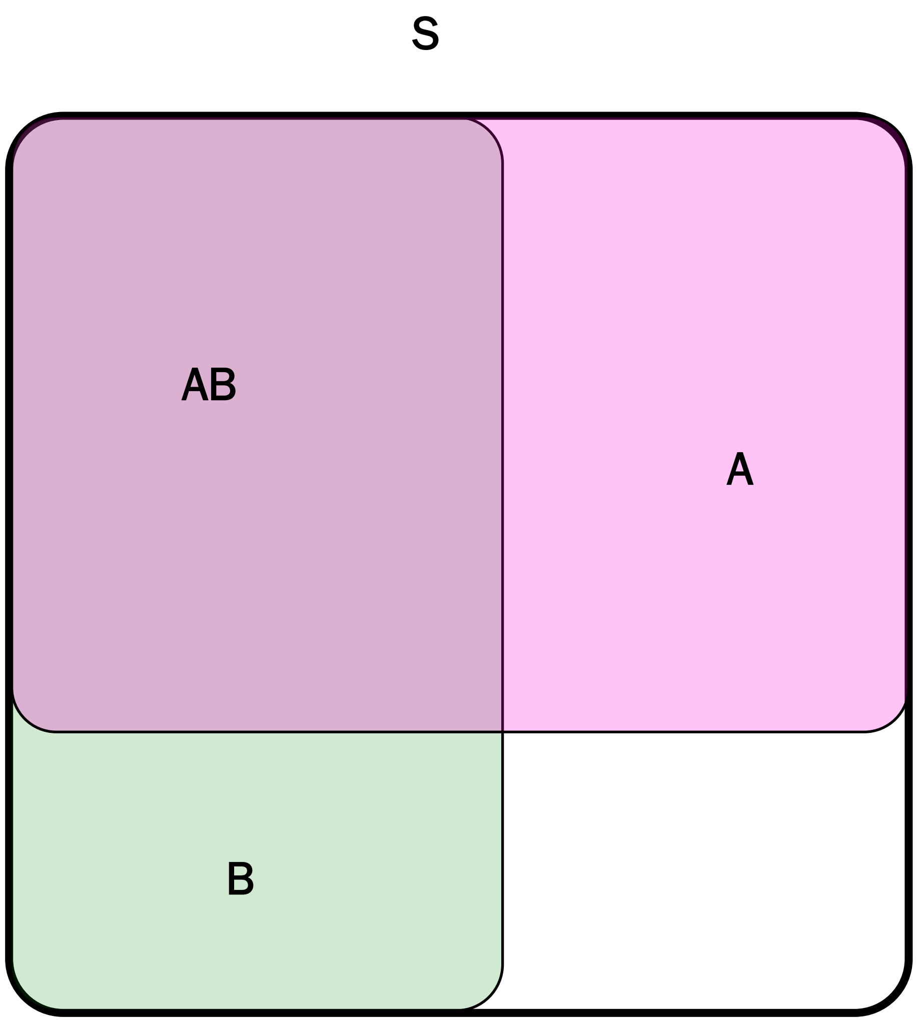
Independence Definition 1:

$$P(AB) = P(A)P(B)$$

0

$$\frac{|AB|}{|S|} = \frac{|A|}{|S|} \times \frac{|B|}{|S|}$$

Independence



Independence Definition 1:

$$P(AB) = P(A)P(B)$$

$$\frac{|AB|}{|S|} = \frac{|A|}{|S|} \times \frac{|B|}{|S|}$$

Independence Definition 2:

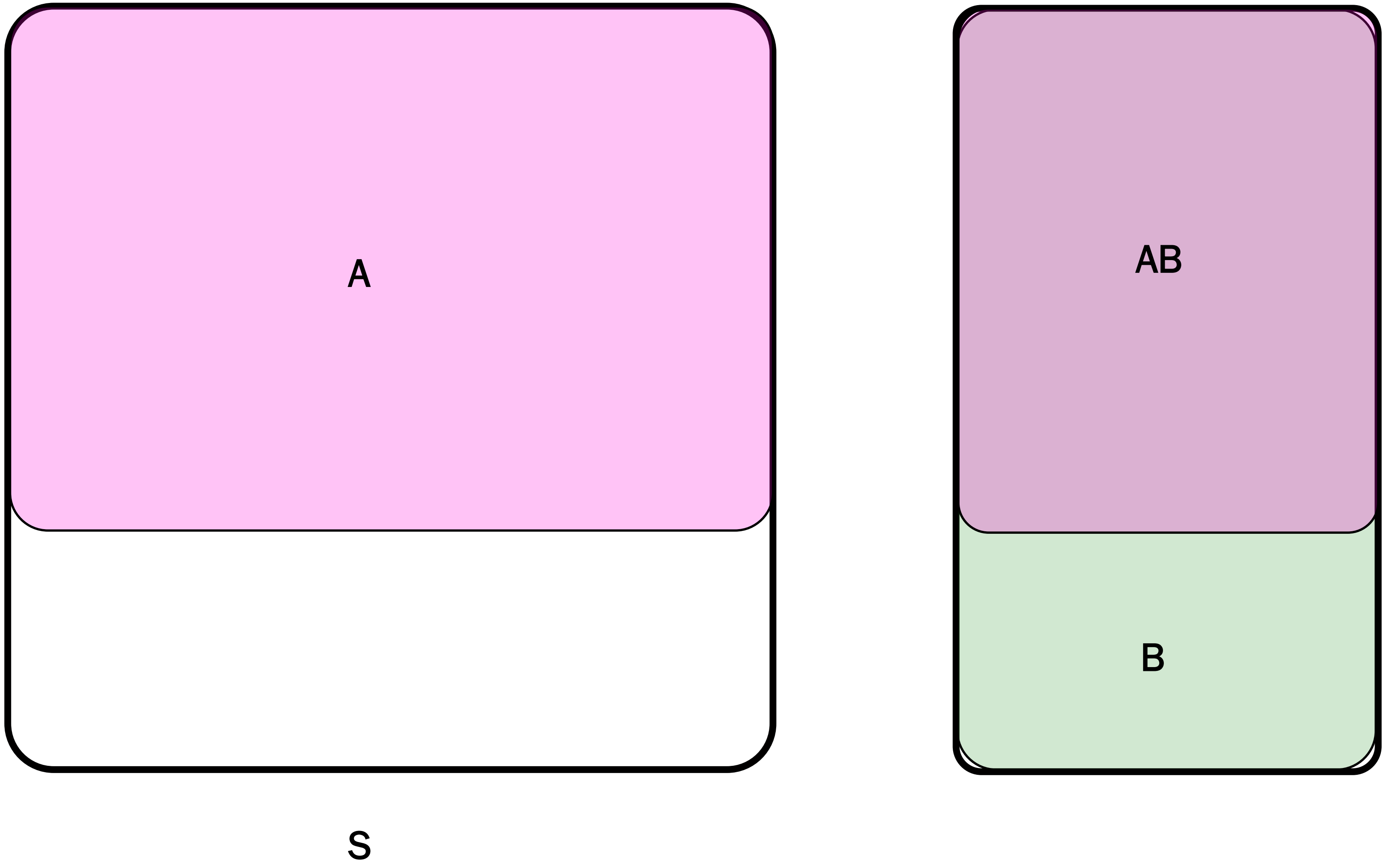
$$P(A|B) = P(A)$$

$$\frac{|AB|}{|B|} = \frac{|A|}{|S|}$$

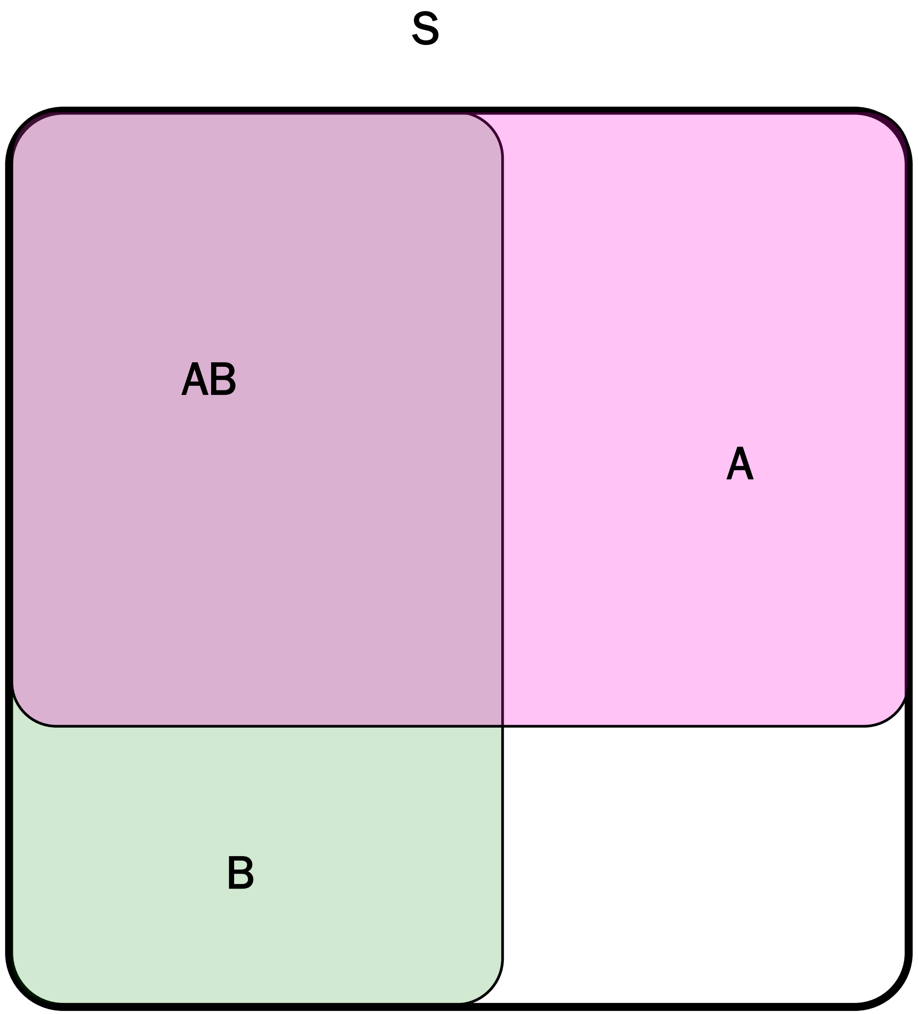
Independence

This ratio, P(A)...

... is the same as this one, P(A|B)



Independence



Independence Definition 1:

$$P(AB) = P(A)P(B)$$

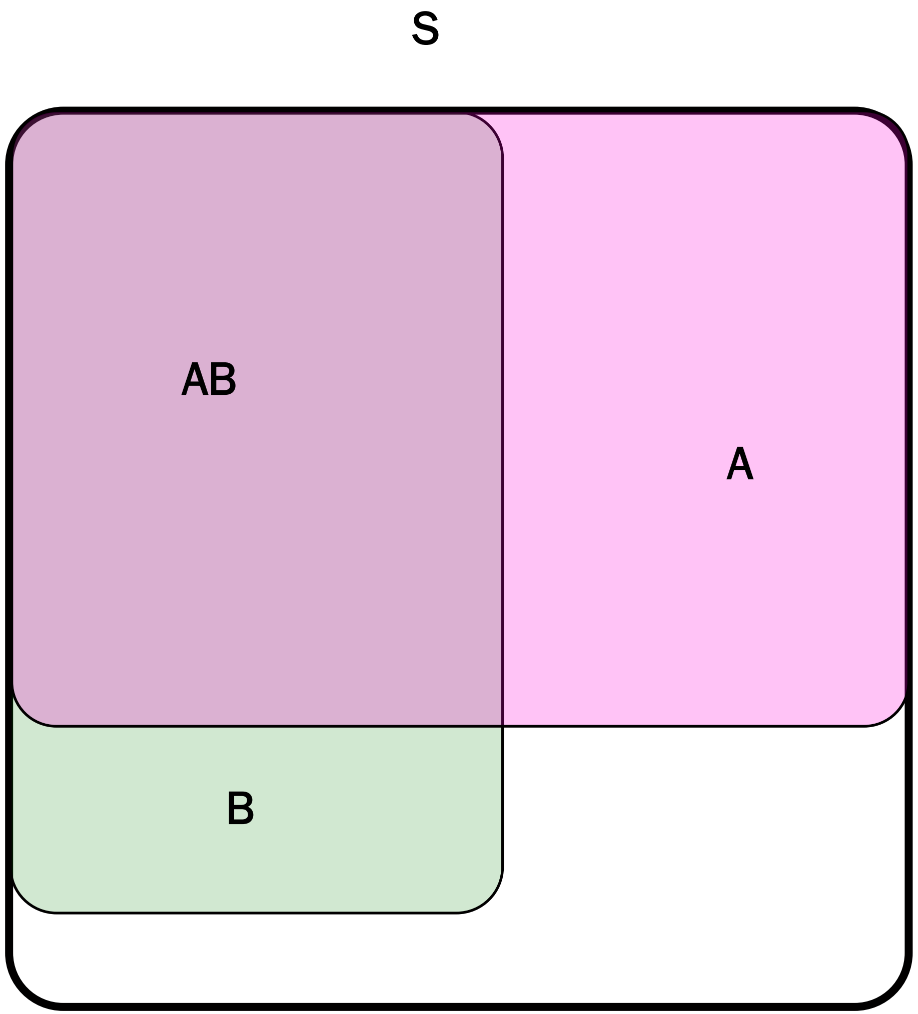
$$\frac{|AB|}{|S|} = \frac{|A|}{|S|} \times \frac{|B|}{|S|}$$

Independence Definition 2:

$$P(A|B) = P(A)$$

$$\frac{|AB|}{|B|} = \frac{|A|}{|S|}$$

Dependence



Independence Definition 1:

$$P(AB) = P(A)P(B)$$

$$\frac{|AB|}{|S|} = \frac{|A|}{|S|} \times \frac{|B|}{|S|}$$

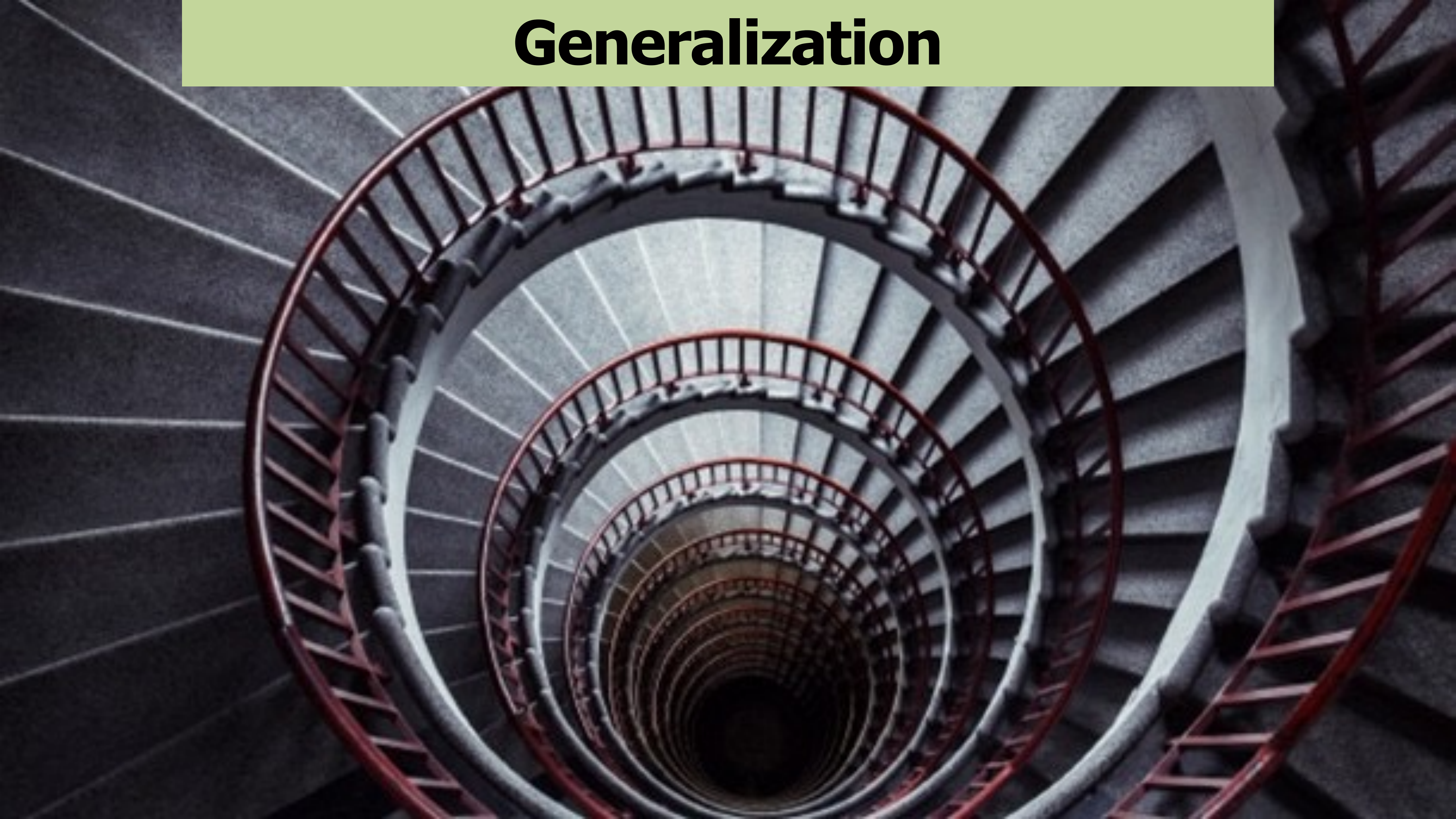
Independence Definition 2:

$$P(A|B) = P(A)$$

$$\frac{|AB|}{|B|} = \frac{|A|}{|S|}$$

End of visualization

Generalization



Generalized Independence (Mutual Independence)

General definition of Independence (Mutual Independence):

Events E_1 , E_2 , ..., E_n are **mutually independent** if **for every subset** with r elements (where $r \leq n$) it holds that:

$$P(E_{1'}E_{2'}E_{3'}\ldots E_{r'}) = P(E_{1'})P(E_{2'})P(E_{3'})\ldots P(E_{r'})$$

Example: outcomes of n separate flips of a coin are all independent of one another

- Each flip in this case is called a "trial" of the experiment

Two Dice (Pairwise Independent but not Mutually Independent)

Roll two 6-sided dice, yielding values D_1 and D_2

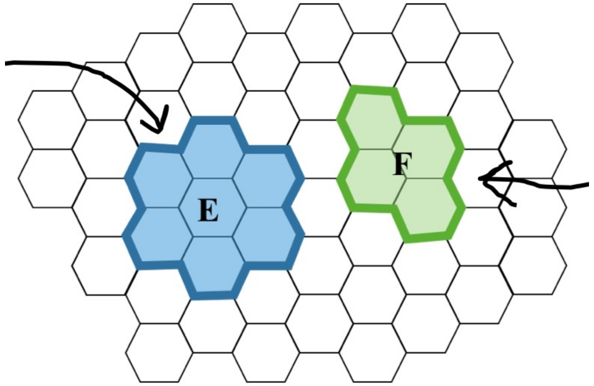
- Let E be event: $D_1 = 1$
- Let F be event: $D_2 = 6$
- Are E and F independent? Yes!

Let G be event: $D_1 + D_2 = 7$

- Are E and G independent? Yes!
- $P(E) = 1/6$, $P(G) = 1/6$, $P(E\,G) = 1/36$ [roll (1, 6)]
- Are F and G independent? Yes!
- $P(F) = 1/6$, $P(G) = 1/6$, $P(F\,G) = 1/36$ [roll (1, 6)]
- Are E, F and G independent? No!
- $P(EFG) = 1/36 \neq 1/216 = (1/6)(1/6)(1/6)$

New Ability

Properties of Pairs of Events



Mutually Exclusive

$$P(A \text{ and } B) = 0$$

also:

$$P(A \text{ or } B) = P(A) + P(B)$$

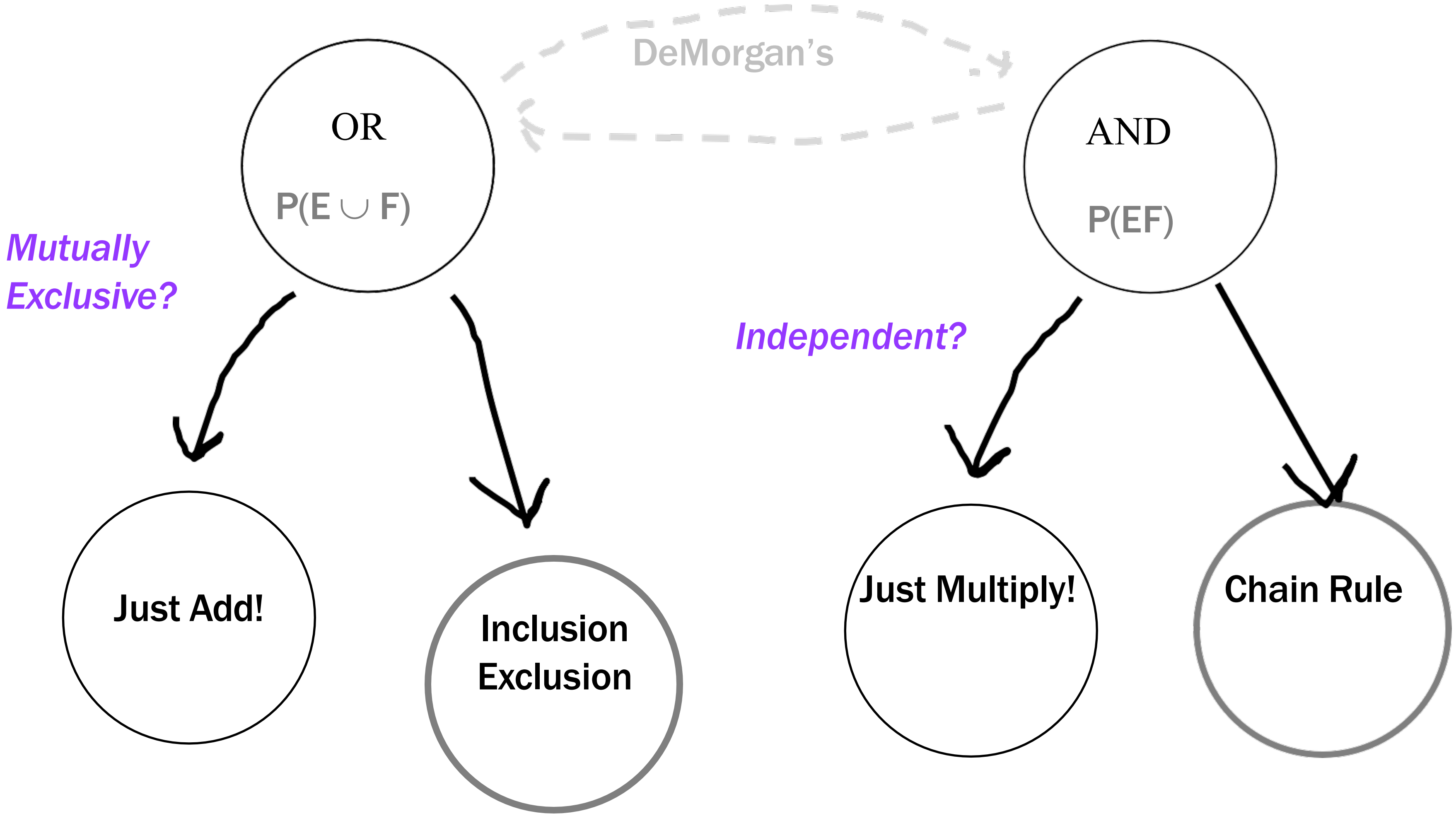
Independent

$$P(A) = P(A|B)$$

also:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Today


DeMorgan's
OR
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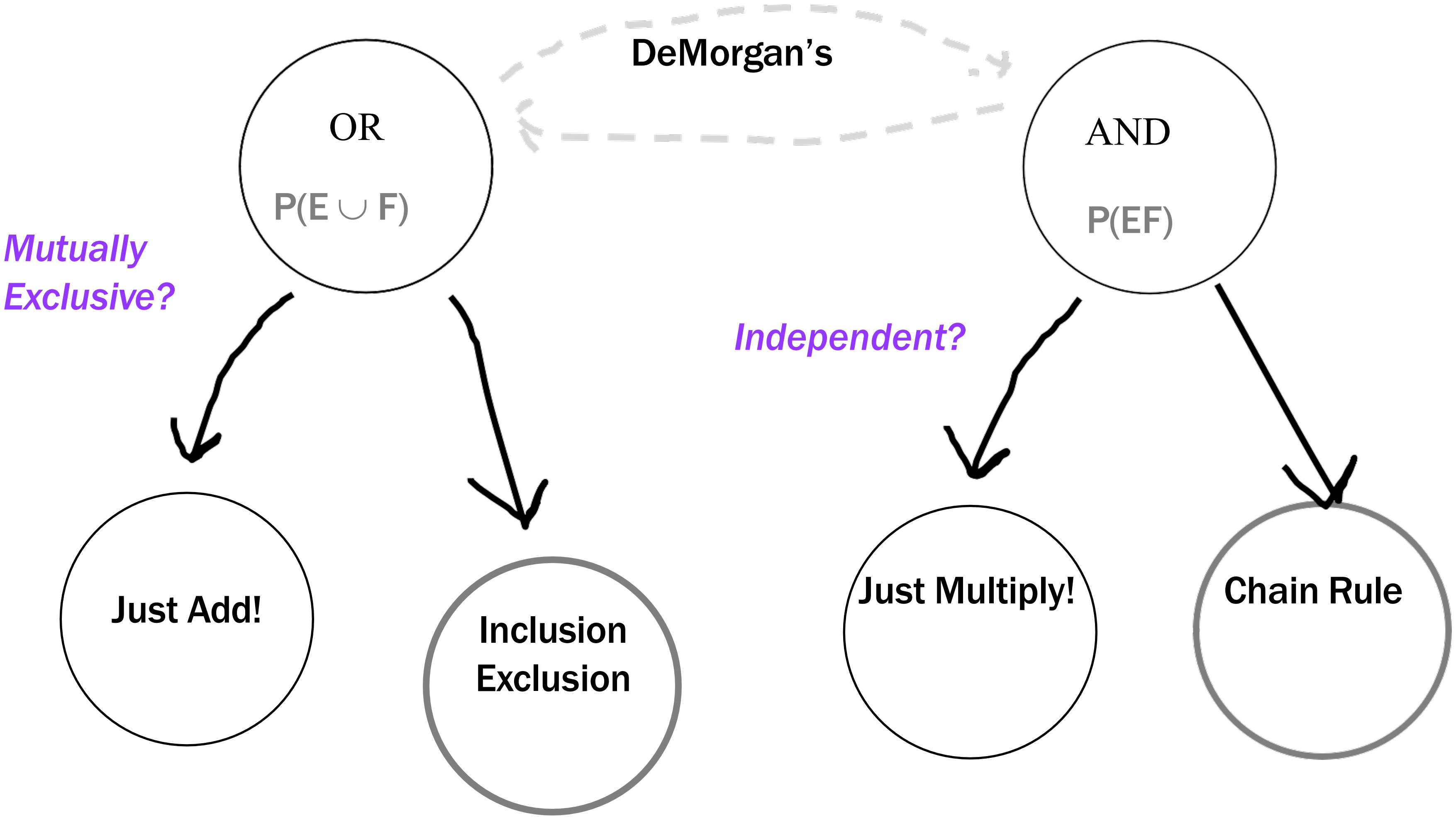
Chain Rule

Today



DeMorgan's
OR
P(E ∪ F)
AND
P(EF)
Mutually Exclusive?
Independent?
Just Add!
Inclusion Exclusion
Just Multiply!
Chain Rule

Think of the children as independent trials

Independence:
$P(A \text{ and } B) = P(A) \cdot P(B)$

Two parents both have an (A, a) gene pair.

- Each parent will pass on one of their genes (each gene equally likely) to a child.
- The probability of **any single child** having curly hair (the recessive trait) is 0.25, independent of other siblings.
- There are three children.

What is the probability that all three children have curly hair?

Let E_1, E_2, E_3 be the events that child 1, 2, and 3 have curly hair, respectively.

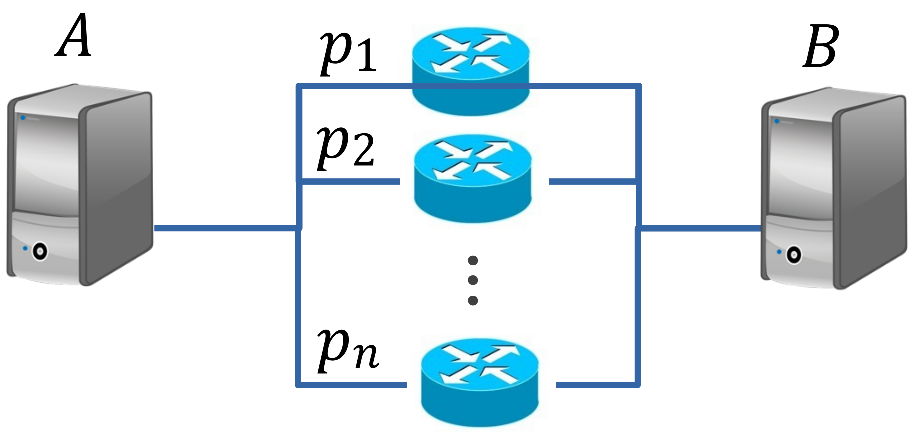
$$P(E_1E_2E_3) = P(E_1)P(E_2|E_1)P(E_3|E_1E_2)$$
$$= P(E_1)P(E_2)P(E_3)$$

Network reliability

Consider the following parallel network:

- n independent routers, each with probability p_i of functioning (where $1 \leq i \leq n$)
- E = functional path from A to B exists.

What is $P(E)$?

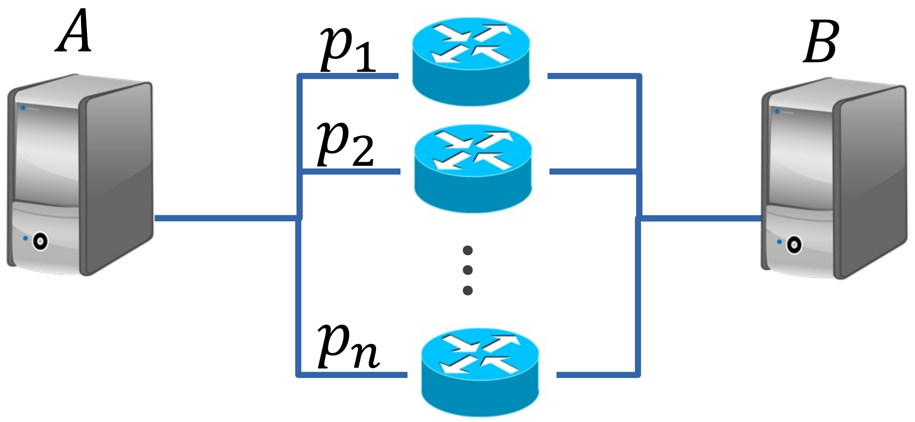


Network reliability

Consider the following parallel network:

- n independent routers, each with probability p_i of functioning (where $1 \le i \le n$)
- E = functional path from A to B exists.

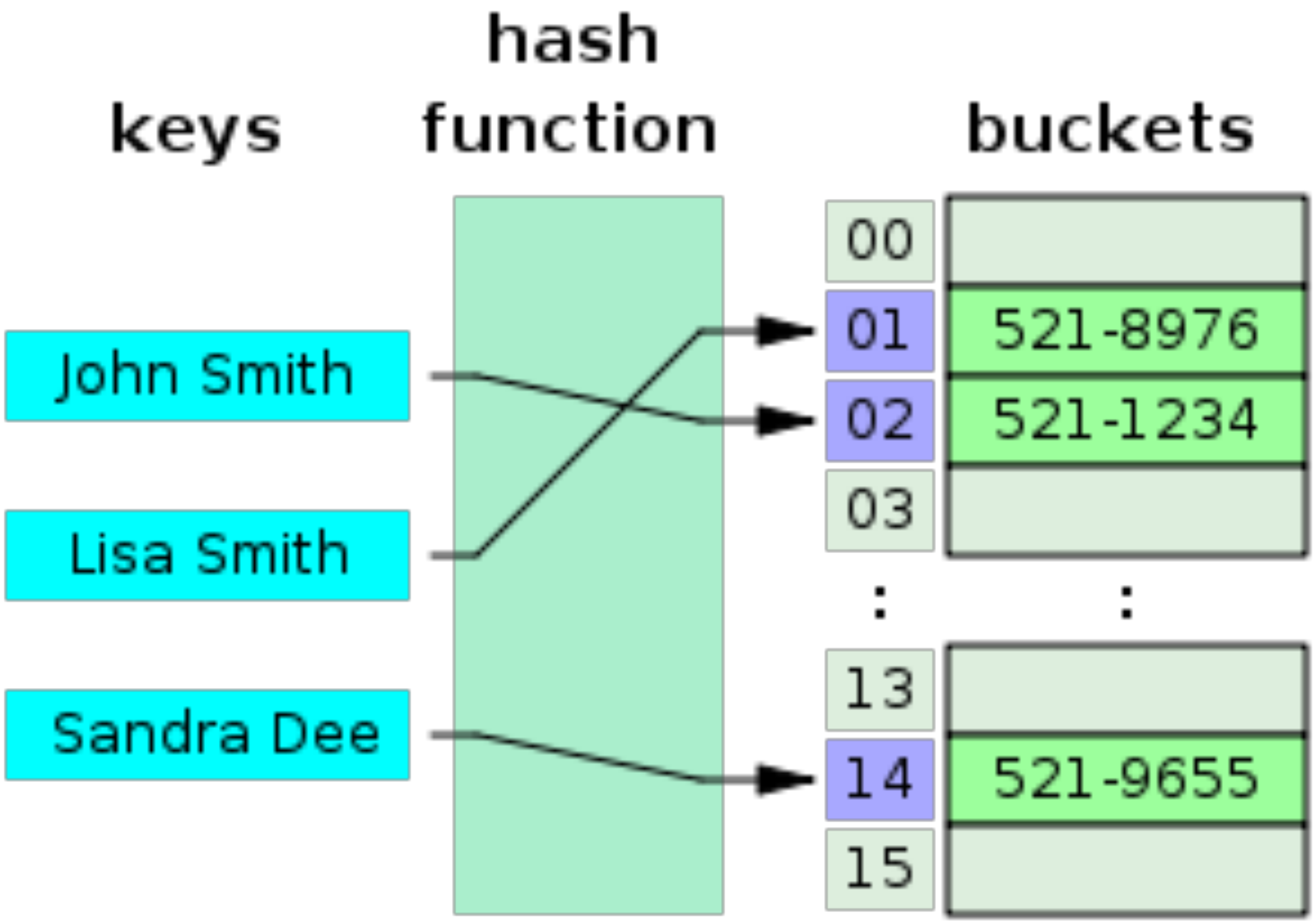
What is $P(E)$?



$$\begin{aligned} P(E) &= P(\geq 1 \text{ one router works}) \\ &= 1 - P(\text{all routers fail}) \\ &= 1 - (1 - p_1)(1 - p_2) \cdots (1 - p_n) \\ &= 1 - \prod_{i=1}^{n} (1 - p_i) \end{aligned}$$

≥ 1 with independent trials: take complement

Hash Tables


hash function
keys
buckets
John Smith
Lisa Smith
Sandra Dee
00
01
02
03
:
13
14
15
521-8976
521-1234
:
521-9655

Hash table fun

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is **independent** with probability p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has ≥ 1 string hashed into it?

Hash table fun

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has ≥ 1 string hashed into it?

Define: S_i **= string** i **hashes to bucket 1**
S_i^C **= string** i **doesn't hash to bucket 1**

$$P(S_i) = p_1$$
$$P(S_i^C) = 1 - p_1$$

Hash table fun

- m strings are hashed (not uniformly) into a hash table with n buckets.
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Define: S_i **= string i hashes to bucket 1**
S_i^C **= string i doesn't hash to bucket 1**

$$P(S_i) = p_1$$
$$P(S_i^C) = 1 - p_1$$

$$\begin{aligned}
P(E) &= P(S_1 \cup S_2 \cup \cdots \cup S_m) \\
&= 1 - P\left((S_1 \cup S_2 \cup \cdots \cup S_m)^C\right) && \textbf{Complement} \\
&= 1 - P(S_1^C S_2^C \cdots S_m^C) && \textbf{De Morgan's Law} \\
&= 1 - P(S_1^C)P(S_2^C)\cdots P(S_m^C) = 1 - \left(P(S_1^C)\right)^m && S_i \textbf{ independent trials} \\
&= 1 - (1 - p_1)^m
\end{aligned}$$

More hash table fun: Possible approach?

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has $\geq$ 1 string hashed into it?
2. E = at least 1 of buckets 1 to k has $\geq$ 1 string hashed into it?

$$P(E) \quad =$$

More hash table fun: Possible approach?

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has $\geq$ 1 string hashed into it?
2. E = at least 1 of buckets 1 to k has $\geq$ 1 string hashed into it?

$$P(E) \quad =$$

Define F_i = bucket i has at least one string in it

More hash table fun: Possible approach?

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has $\geq$ 1 string hashed into it?
2. E = at least 1 of buckets 1 to k has $\geq$ 1 string hashed into it?

$$P(E) \quad = P(F_1 \cup F_2 \cup \cdots \cup F_k)$$

Define F_i = bucket i has at least one string in it

⚠️ F_i events are *not mutually exclusive*! So we cannot just add.

More hash table fun: Possible approach?

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has $\geq$ 1 string hashed into it?
2. E = at least 1 of buckets 1 to k has $\geq$ 1 string hashed into it?

Define F_i = bucket i has at least one string in it

$$P(E) = P(F_1 \cup F_2 \cup \cdots \cup F_k)$$

$$= 1 - P\left((F_1 \cup F_2 \cup \cdots \cup F_k)^C\right)$$

$$= 1 - P\left(F_1^C F_2^C \cdots F_k^C\right)$$

$=$ 

F_i^C events are *dependent*! So we cannot just multiply.

More hash table fun: Possible approach?

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has $\geq$ 1 string hashed into it?
2. E = at least 1 of buckets 1 to k has $\geq$ 1 string hashed into it?

When your problem already implies independence of many processes, it may help to define events in terms of those processes.

(e.g., "buckets j has …" didn't lead to independent events whereas "string i is hashed to…" did)

The phrase "$\geq$ 1" implies a large OR which can get messy…
Try using complements to turn it into "each," "all," or "none".

More hash table fun: Possible approach?

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has ≥ 1 string hashed into it?
2. E = at least 1 of buckets 1 to k has ≥ 1 string hashed into it?

E^C = none of buckets 1 to k has ≥ 1 string hashed into it?

= each of (all) buckets 1 to k has no string hashed into it?

= each (all) string is hashed to buckets ≥ k+1

F_i's are Independent

Define F_i **= string** i **hashed to bucket** ≥ k+1

$$P(E) = 1 - P(E^C)$$
$$= 1 - P(F_1 F_2 \dots F_n)$$
$$= 1 - P(F_1)P(F_2)\dots P(F_n)$$
$$= 1 - (1 - p_1 - p_2 \dots - p_k)^n$$

$$P(F_i) = 1 - p_1 - p_2 \dots - p_k$$

The fun never stops with hash tables

- m strings are hashed (not uniformly) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has $\geq$ 1 string hashed into it? ✅
2. E = at least 1 of buckets 1 to k has $\geq$ 1 string hashed into it? ✅

Looking for another challenge? ☺

The fun never stops with hash tables

- m strings are hashed (unequally) into a hash table with n buckets.
- Each string hash is an **independent trial** w.p. p_i of getting hashed into bucket i.

What is $P(E)$ if

1. E = bucket 1 has ≥ 1 string hashed into it?
2. E = at least 1 of buckets 1 to k has ≥ 1 string hashed into it?
3. E = each of buckets 1 to k has ≥ 1 string hashed into it?

Hint: Use Part 2's event definition:

Define F_i = bucket i has at least one string in it

Hint: Try $k = 2$, then $k = 3$, then generalize.

The fun never stops with hash tables

Solution

- F_i = at least one string hashed into *i*-th bucket
- P(E) $= P(F_1 F_2 \ldots F_k) = 1 - P((F_1 F_2 \ldots F_k)^c)$

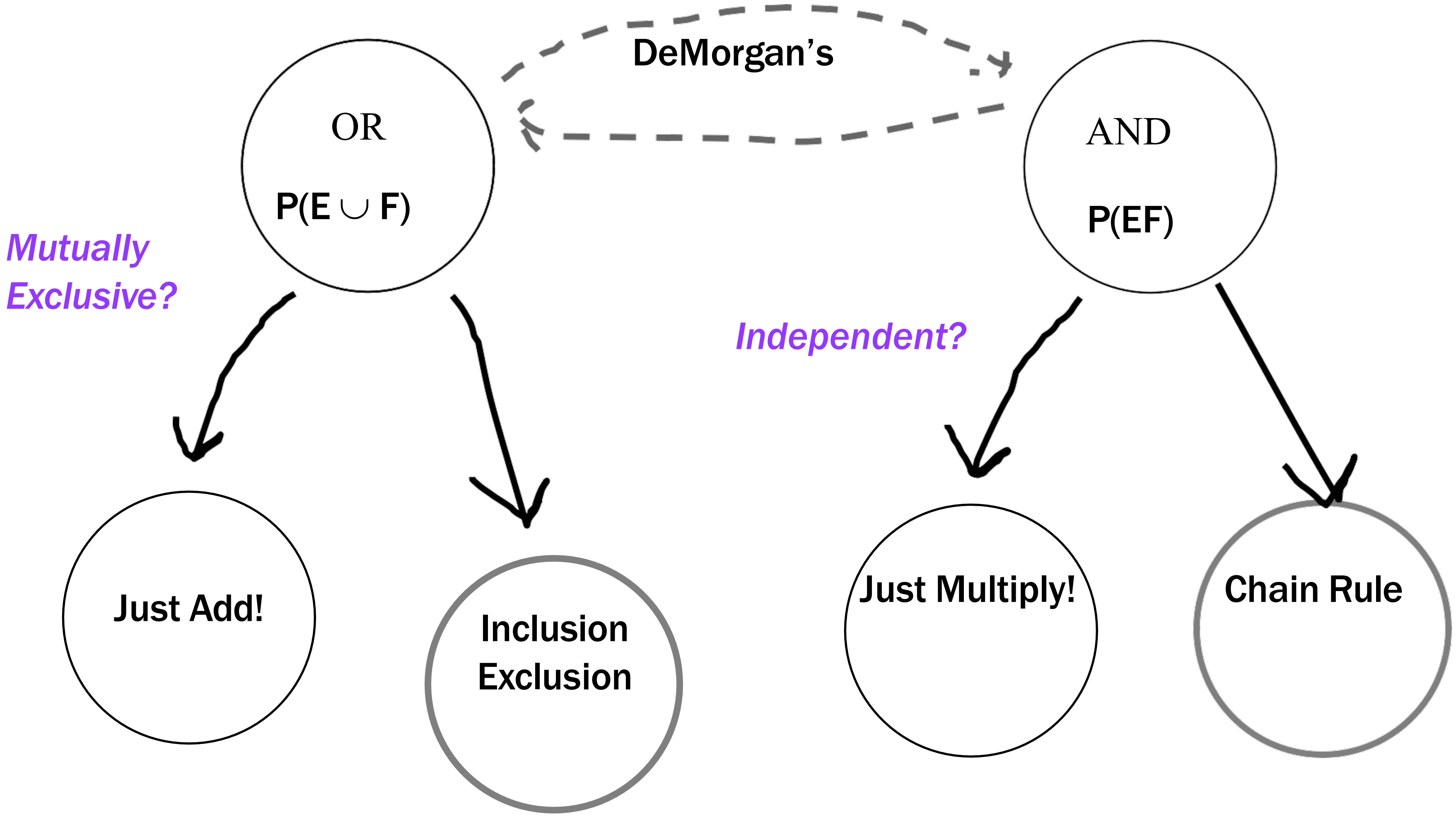
  $= 1 - P(F_1^c \cup F_2^c \cup \ldots \cup F_k^c)$ (DeMorgan's Law)

  $= 1 -$

where $$P\left(\bigcup_{i=1}^{k} F_i^c\right) = 1 - \sum_{r=1}^{k} (-1)^{(r+1)} \sum_{i_1 < \ldots < i_r} P(F_{i_1}{}^c F_{i_2}{}^c \ldots F_{i_r}{}^c)$$

$$P(F_{i_1}{}^c F_{i_2}{}^c \ldots F_{i_r}{}^c) = (1 - p_{i_1} - p_{i_2} - \ldots - p_{i_r})^m$$

Pedagogical Pause



The Most Important Core Probability Question

Say a coin comes up heads with probability p

- Flip the coin n times
- Each coin flip is an **independent** trial
- What is the probability of exactly k heads?

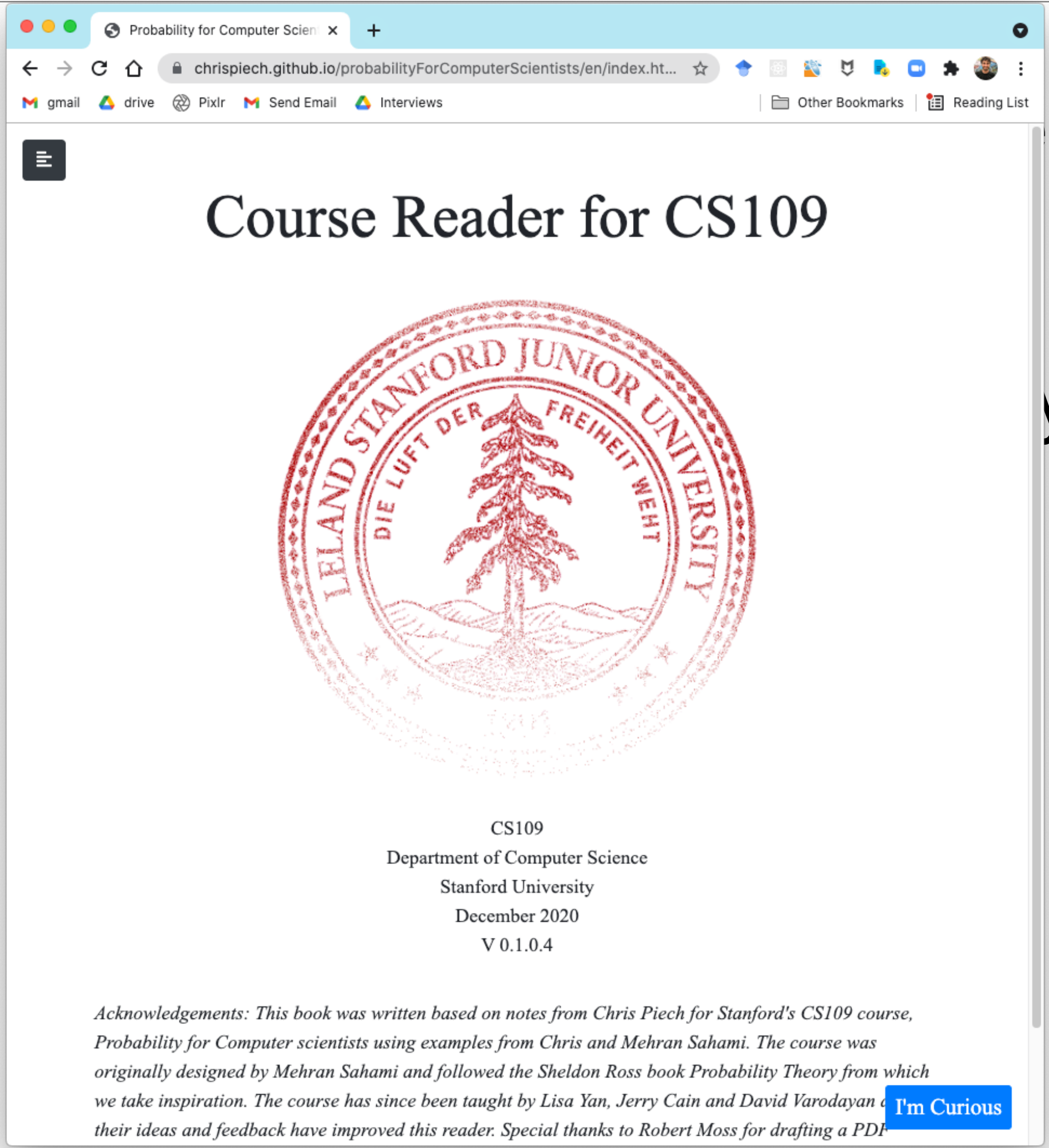
(Read the course reader!!)

The Most Important Core Probability Question

ads

dep

y of



Probability for Computer Scien

chrispiech.github.io/probabilityForComputerScientists/en/index.ht...

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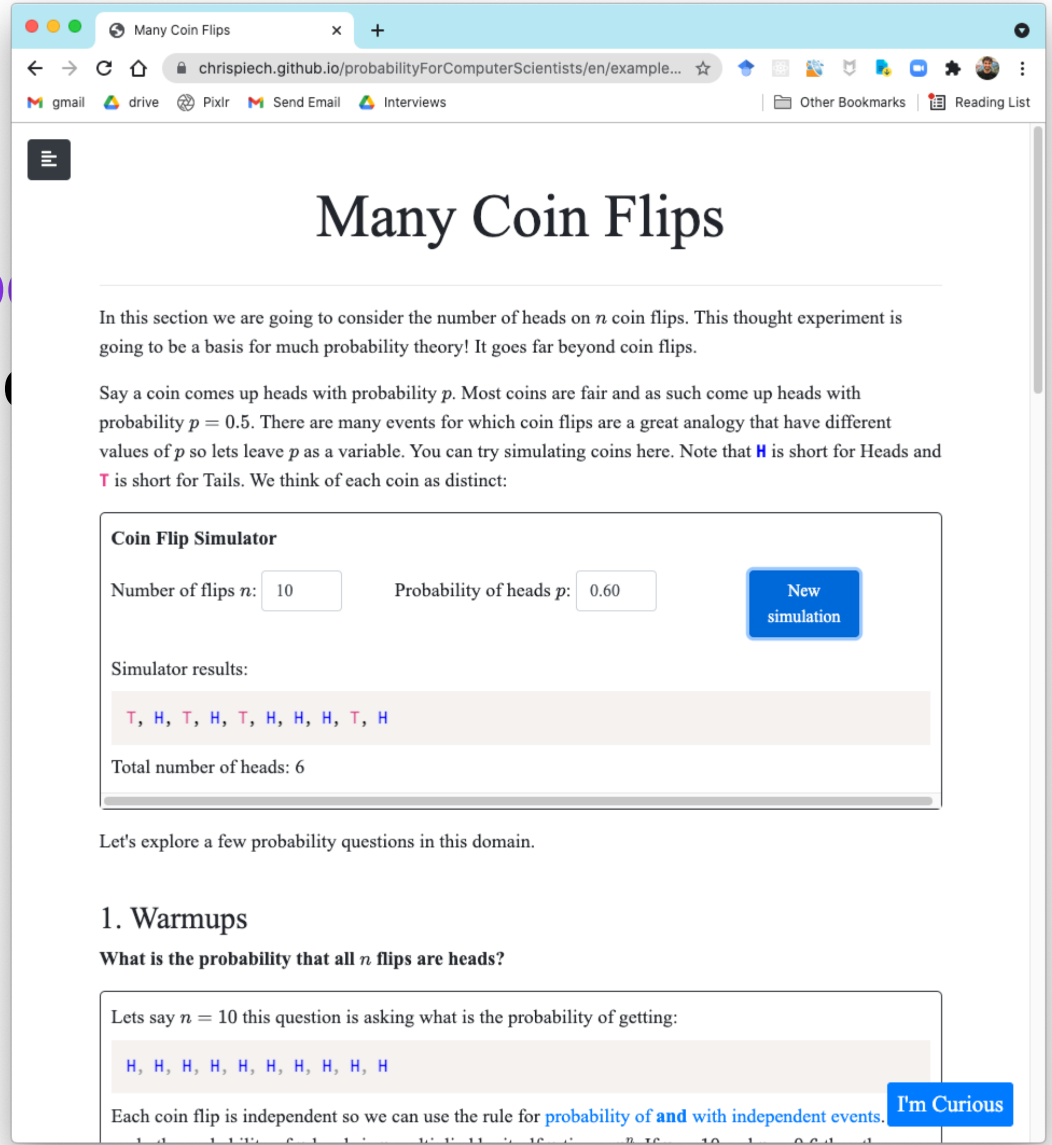
Course Reader for CS109

CS109
Department of Computer Science
Stanford University
December 2020
V 0.1.0.4

Acknowledgements: This book was written based on notes from Chris Piech for Stanford's CS109 course, Probability for Computer scientists using examples from Chris and Mehran Sahami. The course was originally designed by Mehran Sahami and followed the Sheldon Ross book Probability Theory from which we take inspiration. The course has since been taught by Lisa Yan, Jerry Cain and David Varodayan a their ideas and feedback have improved this reader. Special thanks to Robert Moss for drafting a PDF

I'm Curious



Many Coin Flips

chrispiech.github.io/probabilityForComputerScientists/en/example...

gmail drive Pixlr Send Email Interviews Other Bookmarks Reading List

Many Coin Flips

In this section we are going to consider the number of heads on n coin flips. This thought experiment is going to be a basis for much probability theory! It goes far beyond coin flips.

Say a coin comes up heads with probability p. Most coins are fair and as such come up heads with probability $p = 0.5$. There are many events for which coin flips are a great analogy that have different values of p so lets leave p as a variable. You can try simulating coins here. Note that **H** is short for Heads and **T** is short for Tails. We think of each coin as distinct:

Coin Flip Simulator

Number of flips n: 10 Probability of heads p: 0.60 New simulation

Simulator results:

T, H, T, H, T, H, H, H, T, H

Total number of heads: 6

Let's explore a few probability questions in this domain.

1. Warmups

What is the probability that all n flips are heads?

Lets say $n = 10$ this question is asking what is the probability of getting:

H, H, H, H, H, H, H, H, H, H

Each coin flip is independent so we can use the rule for probability of **and** with independent events.

I'm Curious

Stanford University

See you on Friday!!

(See the next slides if you want a small set theory review)

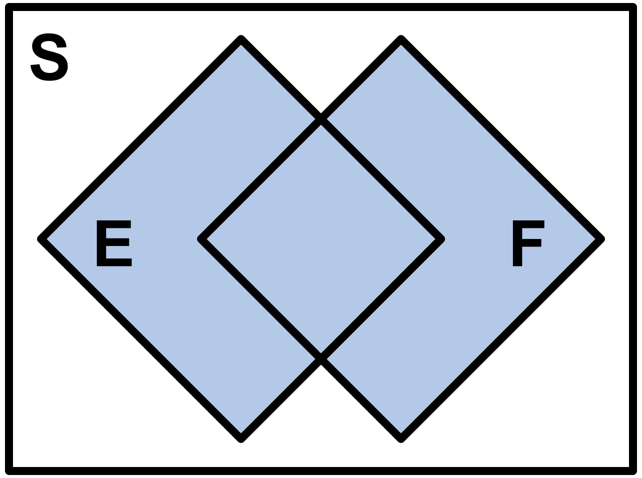
Sets Review

Sets Review

Say E and F are events in S

Event that is in E <u>or</u> F

E ∪ F



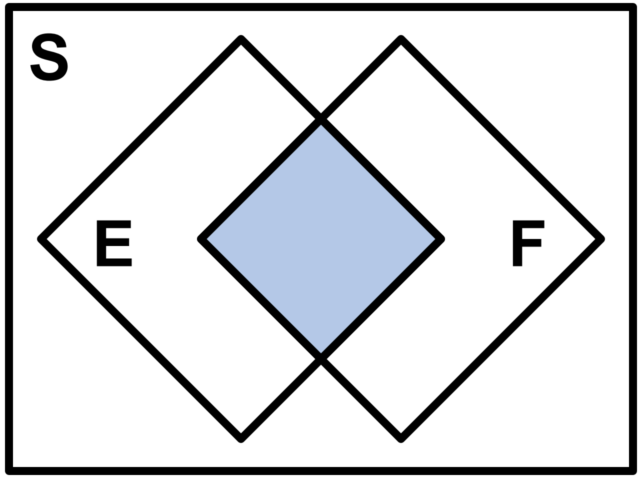
- S = {1, 2, 3, 4, 5, 6} die roll outcome
- E = {1, 2}   F = {2, 3}   E ∪ F = {1, 2, 3}

Sets Review

Say E and F are events in S

Event that is in E <u>and</u> F

$$E \cap F$$

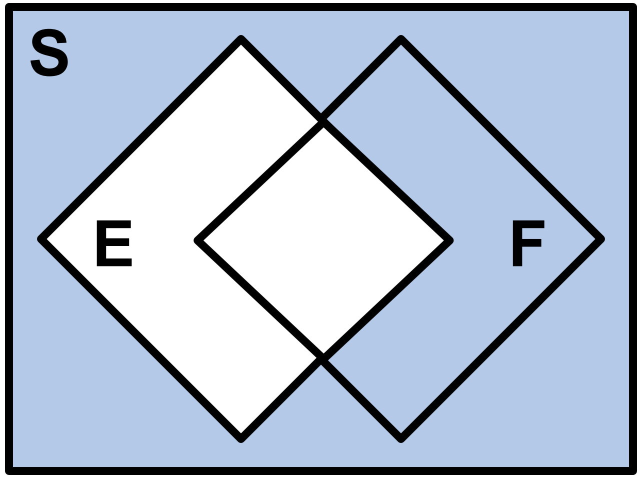


Sets Review

Say E and F are events in S

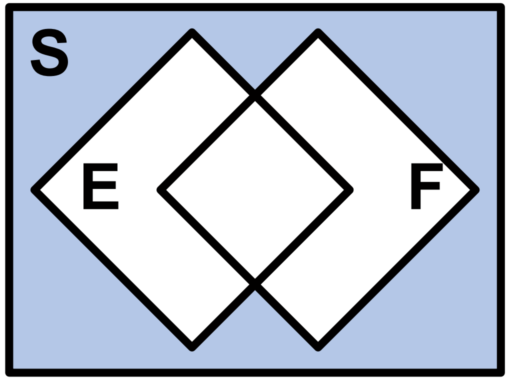
Event that is not in E (called complement of E)

E^c *or* ~E



Sets Review

Say E and F are subsets of S



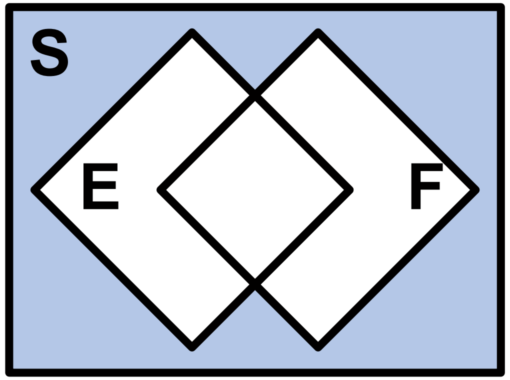
Which of these two is it?

a) $(E \text{ or } F)^C$

b) $(E^C \text{ and } F^C)$

Sets Review

Say E and F are subsets of S



Which of these two is it?

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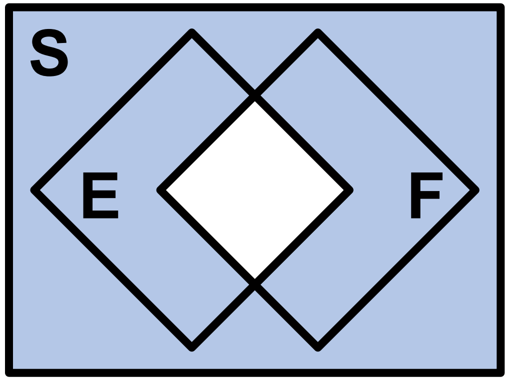
b) $(E^C \text{ and } F^C)$

Sets Review

Say E and F are subsets of S



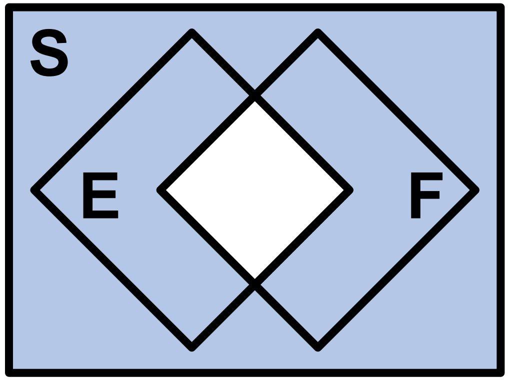
Which of these two is it?

a) $(E \text{ and } F)^C$

b) $(E^C \text{ or } F^C)$

Sets Review

Say E and F are subsets of S

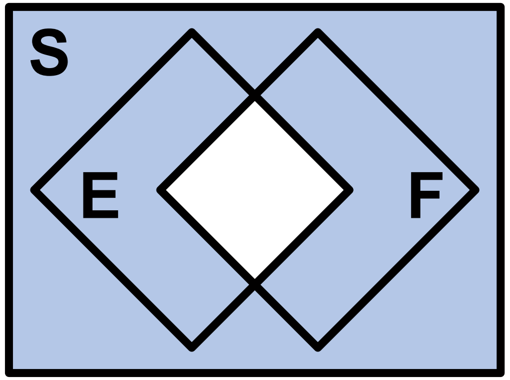


Which of these two is it?

a) $(E \text{ and } F)^C$ **=** b) $(E^C \text{ or } F^C)$

De Morgan's Laws

De Morgan's Law lets you alternate between AND and OR.

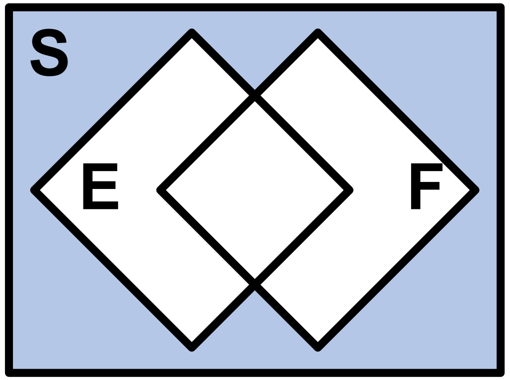


$(E \cap F)^C = E^C \cup F^C$

In probability:

$$P(E_1 E_2 \cdots E_n)$$
$$= 1 - P\left((E_1 E_2 \cdots E_n)^C\right)$$
$$= 1 - P(E_1^C \cup E_2^c \cup \cdots \cup E_n^c)$$

Great if E_i^C mutually exclusive!



$(E \cup F)^C = E^C \cap F^C$

In probability:

$$P(E_1 \cup E_2 \cup \cdots \cup E_n)$$
$$= 1 - P\left((E_1 \cup E_2 \cup \cdots \cup E_n)^C\right)$$
$$= 1 - P(E_1^c E_2^c \cdots E_n^c)$$

Great if E_i independent!

Augustin Demorgan

- British Mathematician who wrote the book “Formal Logic”in 1847