Problem Set 1 is out

Problem:

Imagine you have a robot (🤖) that lives on an 7 x 8 grid (it has 7 rows and 8 columns). The robot starts in cell (1, 1) and can take steps either to the right or down (no left or up steps). How many distinct paths can the robot take to the destination (빨간 사각형) in cell (7, 8)?

Check your answer

Insert LaTeX

Submission is automatic!
You can use LaTeX to type fancy math

\begin{aligned}
\quad P(E) & = \sum_{i=0}^{n} e^i \\
\quad & = 0.25
\end{aligned}

There’s a handout on the course website to help you get started!
Late Policy

Two types of extensions:
1. Grace period (1 hour)
2. Full extension (2 lecture days later)

You grant them to yourself in the psetapp.

But CS109 is a fast class, and we don’t want you to fall behind – after additional extensions past two, we start to cap the pset grade.
Python Review Session

Friday at 5pm PT with Joel (Zoom)

Find links and recordings on the course website
How To Get Attendance Credit

Find today’s lecture here

Then click here

Answer each lecture’s concept check question in the psetapp – linked on lecture page
Section Signups Are Due Tonight
Back to Learning!
The Journey of CS109:

- Counting
- Core Probability
- Random Variables
- Probabilistic Models
- Uncertainty Theory
- Machine Learning
- Counting
CS109: From Counting to Machine Learning

- Counting Theory
- Core Probability
- Random Variables
- Probabilistic Models
- Uncertainty Theory
- Machine Learning
Last Lecture: Step Rule of Counting

**Definition:** Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of \( m \) outcomes and the second part can result in one of \( n \) outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is \( m \cdot n \).

\[
\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), \\
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), \\
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), \\
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), \\
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), \\
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}
\]

**Tip:** Associate every rule with example problems that fit the rule.
The “Or” Rule, Part 1

One experiment

A

B

- If the outcome of an experiment can be either from
  - Set $A$, where $|A| = m$,
  - or Set $B$, where $|B| = n$,
  - where $A \cap B = \emptyset$ (no overlap)
- Then the number of outcomes of the experiment is
  - $|A| + |B| = m + n$. 
How Many Bit Strings?

**Problem**: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?
How Many Bit Strings?

**Problem**: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Tip: Whenever you’re stuck starting a counting problem, try writing out example outcomes.
**Problem**: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?
Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Set A
2⁴ start with 01
010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111

Set B
2⁴ end with 10
000010
000110
001010
001110
010010
010110
011010
011110
100010
100110
101010
101110
110010
110110
111010
111110

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Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?
How Many Bit Strings?

Problem: A 6-bit string (made of 1s and 0s) is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Answer

\[ N = |A| + |B| - |A \text{ and } B| \]

\[ = 16 + 16 - 4 \]

\[ = 28 \]
The Real “Or” Rule (aka Inclusion/Exclusion)

One experiment

A

B

• If the outcome of an experiment can be either from
  • Set $A$,
  • **or** Set $B$,
  • where $A \cap B$ **might not be empty**,
• Then the number of outcomes of the experiment is
  • $N = |A| + |B| - |A \cap B|$.
The Core Counting Rules

Counting with steps

**Definition:** Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of \( m \) outcomes and the second part can result in one of \( n \) outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is \( m \cdot n \).

Counting with “or”

**Definition:** Inclusion Exclusion Counting

If the outcome of an experiment can either be drawn from set \( A \) or set \( B \), and sets \( A \) and \( B \) may potentially overlap (i.e., it is not the case that \( A \) and \( B \) are mutually exclusive), then the number of outcomes of the experiment is \( |A \text{ or } B| = |A| + |B| - |A \text{ and } B| \).
Permutations
Counting Possible Orderings

How many letter orderings are possible for the following string?

CS109
All Possible Orderings Of Characters

<table>
<thead>
<tr>
<th>CS109</th>
<th>CS190</th>
<th>CS019</th>
<th>CS091</th>
<th>CS910</th>
<th>CS901</th>
<th>C1S09</th>
<th>C1S90</th>
</tr>
</thead>
<tbody>
<tr>
<td>C10S9</td>
<td>C109S</td>
<td>C19S0</td>
<td>C190S</td>
<td>C0S19</td>
<td>C0S91</td>
<td>C01S9</td>
<td>C019S</td>
</tr>
<tr>
<td>C09S1</td>
<td>C091S</td>
<td>C9S10</td>
<td>C9S01</td>
<td>C91S0</td>
<td>C910S</td>
<td>C90S1</td>
<td>C901S</td>
</tr>
<tr>
<td>SC109</td>
<td>SC190</td>
<td>SC019</td>
<td>SC091</td>
<td>SC910</td>
<td>SC901</td>
<td>S1C09</td>
<td>S1C90</td>
</tr>
<tr>
<td>S10C9</td>
<td>S109C</td>
<td>S19C0</td>
<td>S190C</td>
<td>S0C19</td>
<td>S0C91</td>
<td>S01C9</td>
<td>S019C</td>
</tr>
<tr>
<td>S09C1</td>
<td>S091C</td>
<td>S9C10</td>
<td>S9C01</td>
<td>S91C0</td>
<td>S910C</td>
<td>S90C1</td>
<td>S901C</td>
</tr>
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<td>1CS90</td>
<td>1C0S9</td>
<td>1C09S</td>
<td>1C9S0</td>
<td>1C90S</td>
<td>1SC09</td>
<td>1SC90</td>
</tr>
<tr>
<td>1S0C9</td>
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<td>1S9C0</td>
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<td>10S9C</td>
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<td>19C0S</td>
<td>19SC0</td>
<td>19S0C</td>
<td>190CS</td>
<td>190SC</td>
</tr>
<tr>
<td>0CS19</td>
<td>0CS91</td>
<td>0C1S9</td>
<td>0C19S</td>
<td>0C9S1</td>
<td>0C91S</td>
<td>0SC19</td>
<td>0SC91</td>
</tr>
<tr>
<td>0S1C9</td>
<td>0S19C</td>
<td>0S9C1</td>
<td>0S91C</td>
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<td>01C9S</td>
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<td>09C1S</td>
<td>09SC1</td>
<td>09S1C</td>
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<tr>
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<td>9CS01</td>
<td>9C1S0</td>
<td>9C10S</td>
<td>9C0S1</td>
<td>9C01S</td>
<td>9SC10</td>
<td>9SC01</td>
</tr>
<tr>
<td>9S1C0</td>
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<td>9S01C</td>
<td>91CS0</td>
<td>91C0S</td>
<td>91SC0</td>
<td>91S0C</td>
</tr>
<tr>
<td>910CS</td>
<td>910SC</td>
<td>90CS1</td>
<td>90C1S</td>
<td>90SC1</td>
<td>90S1C</td>
<td>901CS</td>
<td>901SC</td>
</tr>
</tbody>
</table>
Tip: Think about a generative story...
Counting Possible Orderings
Counting Possible Orderings

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

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Counting Possible Orderings

(5 options)

9

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

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Counting Possible Orderings

(5 options)  (4 options)

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

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Counting Possible Orderings

(5 options) (4 options) (3 options) (2 options) (1 option)

9 C 0 S 1

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

Piech & Cain, CS109, Stanford University
Counting Possible Orderings

(5 options)  (4 options)  (3 options)  (2 options)  (1 option)

9 C 0 S 1

Step 1:
Chose 1st character

Step 2:
Chose 2nd character

Step 3:
Chose 3rd character

Step 4:
Chose 4th character

Step 5:
Chose 5th character

Answer: 5! 5 × 4 × 3 × 2 × 1
Possible Orderings Are Called Permutations

**Permutation:** any *ordered* arrangement of objects.

The number of unique orderings (permutations) of *n distinct* objects is:

\[ n! = n \times (n - 1) \times (n - 2) \times \cdots \times 2 \times 1 \]
Practice: Unique 6-digit passcodes with six smudges

How many unique 6-digit passcodes are possible if each passcode uses six distinct numbers?
Practice: Unique 6-digit passcodes with six smudges

How many unique 6-digit passcodes are possible if each passcode uses six distinct numbers?

6! = 720 passcodes
Challenge: Unique 6-digit passcodes with six mystery smudges

How many unique passcodes are possible if each passcode is any 6 unique digits?
Challenge: Unique 6-digit passcodes with **six mystery** smudges

How many unique passcodes are possible if each passcode is *any* 6 unique digits?

\[
10 \times 9 \times 8 \times 7 \times 6 \times 5 = \frac{10!}{4!} = 151200 \text{ passcodes}
\]
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on \( n \) objects

- Sort objects (permutations)
- Choose \( k \) objects (combinations)
- Put objects in \( r \) buckets

Distinct (distinguishable)

\( n! \)
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on $n$ objects

- Sort objects (permutations)
  - Distinct (distinguishable)
    - $n!$
- Choose $k$ objects (combinations)
  - Some distinct
- Put objects in $r$ buckets
Permutations With Some Distinct Objects

How many different orderings of letters are possible for the string “boba”?

boba, abob, obba...
Permutations With Some Distinct Objects

How many different orderings of letters are possible for the string “boba”?

boba  obba  bboa  abob
boab  obab  bbao  abbo
bboa  obba  boba  aobb
bbao  obab  boab  aobb
baob  oabb  babo  abbo
babo  oabb  baob  abob

Are there $4! = 24$ unique permutations?
Permutations With Some Distinct Objects

How many different orderings of letters are possible for the string “boba”? Are there $4! = 24$ unique permutations? No! We’re overcounting by a multiple of 2.
Permutations With Some Distinct Objects

How many different orderings of letters are possible for the string BOBA?

\[
\frac{4!}{2} = 12
\]

Tip: Starting with a number that you know over- or under-counts, then correcting the difference, is a perfectly valid strategy.
Can We Code It? Yes We Can!

```python
import itertools

def list_outcomes():
    letters = ['b','o','b','a']
    perms = set(itertools.permutations(letters))
    for perm in perms:
        perm_as_string = ''.join(perm)
        print(perm_as_string)

import math

def count_outcomes():
    numerator = math.factorial(4)
    denominator = math.factorial(2)
    print(numerator / denominator)
```

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Back To Unique Bit Strings: How Many Permutations?

1 0 1 0 0 0
Back To Unique Bit Strings: How Many Permutations?

1 0 1 0 0
How many ways can we sort coke cans?
How many ways can we sort $n$ distinct objects?
How many ways can we sort $n$ distinct objects?

# of permutations = 5!  $5 \times 4 \times 3 \times 2 \times 1$
Then, how do we sort semi-distinct objects?

All Distinct

Some Indistinct

5! is an overcount... How do we correct it?

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General approach to counting permutations

When there are $n$ objects, and

- $n_1$ are the same (indistinguishable or indistinct),
- $n_2$ are the same,
- ...
- $n_r$ are the same,

The number of unique permutations is

$$\frac{n!}{n_1! n_2! \cdots n_r!}$$
General approach to counting permutations

When there are $n$ objects, and

- $n_1$ are the same (indistinguishable or indistinct),
- $n_2$ are the same,
- ...  
- $n_r$ are the same,

The number of unique permutations is

$$\frac{n!}{n_1! n_2! \cdots n_r!}$$

For each group of indistinct objects, divide by the overcounted permutations.
Permutations of Semi-Distinct Objects

Coke  Coke0  Coke  Coke0  Coke0

Order $n$ semi-distinct objects $\frac{n!}{n_1!n_2!\cdots n_r!}$

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Permutations of Semi-Distinct Objects

Order $n$ semi-distinct objects $\frac{n!}{n_1!n_2!\cdots n_r!}

$\frac{5!}{2!3!} = 10$
Boss Battle: String Permutations

How many letter orderings are possible for the following string?

MISSISSIPPI
How many letter orderings are possible for the following string?

MISSISSIPPI

\[
\frac{11!}{1!4!4!2!} = 34,650
\]
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on $n$ objects

- Sort objects (permutations)
  - Distinct (distinguishable)
    - $n!$
  - Some distinct
    - $\frac{n!}{n_1! n_2! \cdots n_r!}$
- Choose $k$ objects (combinations)
- Put objects in $r$ buckets

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Unique 6-digit passcodes with **six** smudges

How many unique 6-digit passcodes are possible if each passcode uses **six** distinct numbers?

$$6! = 720 \text{ passcodes}$$
Unique 6-digit passcodes with five smudges

How many unique 6-digit passcodes are possible if each passcode uses exactly five distinct numbers?
How many unique 6-digit passcodes are possible if each passcode uses exactly five distinct numbers?

Steps:
1. Choose digit to repeat
   - 5 outcomes
2. Create passcode
   - (sort 6 digits: 4 distinct, 2 indistinct)
How many unique 6-digit passcodes are possible if each passcode uses exactly five distinct numbers?

Steps:
1. Choose digit to repeat 5 outcomes
2. Create passcode (sort 6 digits:
   4 distinct, 2 indistinct)

\[5 \times \frac{6!}{2!} = 1,800\] passcodes

Order $n$ semi-distinct objects \(\frac{n!}{n_1!n_2! \cdots n_r!}\)
Combinations
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on $n$ objects

Sort objects (permutations)
Distinct (distinguishable)
$n!$

Choose $k$ objects (combinations)
Some distinct
$n!$

Put objects in $r$ buckets
Distinct

\[
\frac{n!}{n_1! n_2! \cdots n_r!}
\]

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Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?
Think about a generative story...
Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?

1. $n$ people get in line

   $n!$ ways
Combinations with cake

There are \( n = 20 \) people.

How many ways can we choose \( k = 5 \) people to get cake?

1. \( n \) people get in line
2. Put first \( k \) in cake group

\( n! \) ways

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Combinations with cake

There are $n = 20$ people.

How many ways can we **choose** $k = 5$ people to get cake?

1. $n$ people get in line
2. Put first $k$ in cake group

$n!$ ways
Combinations with cake

There are \( n = 20 \) people.

How many ways can we choose \( k = 5 \) people to get cake?

1. \( n \) people get in line
   
   \( n! \) ways

2. Put first \( k \) in cake group

3. Un-order the cake group
   
   \( k! \) different permutations

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Combinations with cake

There are \( n = 20 \) people.
How many ways can we choose \( k = 5 \) people to get cake?

1. \( n \) people get in line
   \[ n! \text{ ways} \]

2. Put first \( k \) in cake group

3. Un-order the cake group
   \[ k! \text{ different permutations} \]

4. Un-order the non-cake group
   \[ (n - k)! \text{ different permutations} \]

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Combinations

A combination is an unordered selection of $k$ objects from a set of $n$ distinct objects.

The number of ways of making this selection is

$$\frac{n!}{k! (n-k)!} = n! \times \frac{1}{k!} \times \frac{1}{(n-k)!}$$
Combinations

A combination is an unordered selection of $k$ objects from a set of $n$ distinct objects.

The number of ways of making this selection is

$$\frac{n!}{k! (n-k)!} = n! \times \frac{1}{k!} \times \frac{1}{(n-k)!}$$

1. Order $n$ distinct objects
2. Correct for overcounting: any ordering of the chosen group is the same
3. Correct for overcounting: any ordering of the not-chosen group is the same
Combinations

A combination is an unordered selection of $k$ objects from a set of $n$ distinct objects.

The number of ways of making this selection is

$$\frac{n!}{k! \ (n-k)!} = n! \times \frac{1}{k!} \times \frac{1}{(n-k)!} = \binom{n}{k}$$
Practice: Combinations of Books

How many ways are there to choose 3 books from a set of 6 distinct books?

Choose $k$ of $n$ distinct objects

$$\binom{n}{k} = \frac{n!}{k! (n-k)!}$$

Piech & Cain, CS109, Stanford University
Practice: Combinations of Books

How many ways are there to choose 3 books from a set of 6 distinct books?

\[
\binom{6}{3} = \frac{6!}{3!3!} = 20 \text{ ways}
\]
Can We Code It? Yes We Can

How many unique hands of 5 cards are there in a 52 card deck?
Can We Code It? Yes We Can

How many unique hands of 5 cards are there in a 52 card deck?

\[ \binom{52}{5} \]
Can We Code It? Yes We Can

How many unique hands of 5 cards are there in a 52 card deck?

def main():
    total = math.comb(52, 5)
    print(total)

def main():
    cards = make_deck()
    all_hands = itertools.combinations(cards, 5)
    for hand in all_hands:
        print(hand)

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Balls In Bins
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on $n$ objects

- Sort objects (permutations)
  - Distinct (distinguishable)
    - Distinct: $n!$
  - Some distinct
    - Some distinct: $\frac{n!}{n_1! n_2! \cdots n_r!}$

- Choose $k$ objects (combinations)
  - Distinct
    - Distinct: $\binom{n}{k}$

- Put objects in $r$ buckets
  - Distinct
  - Indistinct

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Balls in bins Hash tables and distinct strings

How many ways are there to hash $n$ distinct strings to $r$ buckets?
Balls in bins Hash tables and distinct strings

How many ways are there to hash $n$ distinct strings to $r$ buckets?

Steps:
1. Bucket 1$^{\text{st}}$ string – $r$ choices
2. Bucket 2$^{\text{nd}}$ string – $r$ choices
   ... 
$n$. Bucket $n^{\text{th}}$ string – $r$ choices
Balls in bins: Hash tables and distinct strings

How many ways are there to hash $n$ distinct strings to $r$ buckets?

**Steps:**
1. Bucket 1\textsuperscript{st} string – $r$ choices
2. Bucket 2\textsuperscript{nd} string – $r$ choices
   ...
$n$. Bucket $n$\textsuperscript{th} string – $r$ choices

$\boxed{r^n}$ outcomes
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on \( n \) objects

- Sort objects (permutations)
  - Distinct (distinguishable)
    - \( n! \)
  - Some distinct
    - \( \frac{n!}{n_1!n_2! \cdots n_r!} \)

- Choose \( k \) objects (combinations)
  - Distinct
    - \( \binom{n}{k} \)

- Put objects in \( r \) buckets
  - Distinct
    - \( r^n \)
  - Indistinct
Servers and \textit{indistinct} requests

How many ways are there to distribute $n$ \textit{indistinct} web requests to $r$ servers?
Servers and indistinct requests

How many ways are there to distribute $n$ indistinct web requests to $r$ servers?

What does one outcome look like?

- Server 1 has $x_1$ requests,
- Server 2 has $x_2$ requests,
- ...  
- Server $r$ has $x_r$ requests

constraint: $\sum_{i=1}^{r} x_i = n$
Flowers In Vases

How many ways can we put \( n = 5 \) indistinct flowers in \( r = 3 \) distinct vases?
What does one possible outcome look like? Can we build a generative story?
What does one possible outcome look like? Can we build a generative story?

Goal: order $n$ indistinct objects and $r - 1$ indistinct dividers.
Flowers In Vases

What does one possible outcome look like? Can we build a generative story?
Start by treating objects and dividers as distinct.

**Goal:** order $n$ indistinct objects and $r - 1$ indistinct dividers.
Flowers In Vases

What does one possible outcome look like? Can we build a generative story?

Start by treating objects and dividers as distinct.

1. Order $n$ distinct objects and $r - 1$ distinct dividers

   \[(n + r - 1)!\]

Goal: order $n$ indistinct objects and $r - 1$ indistinct dividers.

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Flowers In Vases

What does one possible outcome look like? Can we build a generative story? Start by treating objects and dividers as distinct.

1. Order \( n \) distinct objects and \( r - 1 \) distinct dividers

\[(n + r - 1)!\]

2. Un-order \( n \) objects

\[
\frac{1}{n!}
\]

**Goal:** order \( n \) indistinct objects and \( r - 1 \) indistinct dividers.
Flowers In Vases

What does one possible outcome look like? Can we build a generative story?

Start by treating objects and dividers as distinct.

1. Order \( n \) distinct objects and \( r - 1 \) distinct dividers

\[(n + r - 1)!\]

2. Un-order \( n \) objects

\[\frac{1}{n!}\]

3. Un-order \( r - 1 \) dividers

\[\frac{1}{(r - 1)!}\]

**Goal:** order \( n \) indistinct objects and \( r - 1 \) indistinct dividers.
The Divider Method: Indistinct Balls In Bins

The number of ways to distribute $n$ indistinct objects into $r$ buckets is

\[(n + r - 1)! \times \frac{1}{n!} \times \frac{1}{(r-1)!} = \binom{n + r - 1}{r - 1}\]

This is equivalent to the number of ways to permute $n + r - 1$ objects, such that $n$ are indistinct objects, and $r - 1$ are indistinct dividers.
Integer solutions to equations

How many integer solutions are there to the following equation:

\[ x_1 + x_2 + \cdots + x_r = n, \]

where for all \( i \), \( x_i \) is an integer such that \( 0 \leq x_i \leq n \)?
Combinatorics: Formulas For Common Counting Tasks

Counting tasks on $n$ objects

- **Sort objects (permutations)**
  - Distinct (distinguishable)
    - $n!$
  - Some distinct
    - $\frac{n!}{n_1!n_2!\cdots n_r!}$

- **Choose $k$ objects (combinations)**
  - Distinct
    - $\binom{n}{k}$

- **Put objects in $r$ buckets**
  - Distinct
    - $r^n$
  - Indistinct
    - $\binom{n + r - 1}{r - 1}$

Piech & Cain, CS109, Stanford University
You’re ready for the first 6 problems now!
Want to go deeper? Cool examples in the course reader

Enigma Machine

One of the very first computers was built to break the Nazi “enigma” codes in WW2. It was a hard problem because the “enigma” machine, used to make secret codes, had so many unique configurations. Every day the Nazis would choose a new configuration and if the Allies could figure out the daily configuration, they could read all enemy messages. One solution was to try all configurations until one produced legible German. This begs the question: How many configurations are there?

The WW2 machine built to search different enigma configurations.

The enigma machine has three rotors. Each rotor can be set to one of 26 different positions. How
Up next: going from counts to probabilities...