Section 1: Analytic Probability

Overview of Section Materials

The warmup questions provided are good review of lecture concepts. The section problems then help you to apply these concepts in more complex scenarios, more similar to what you will see on exams. In practice, there often isn't enough time in section to go through every problem on the handout. You can go over problems you don't work through in section on your own later, and TAs can help our in office hours or on Ed if you have questions about those.

Warmups

1. Fish Pond

Suppose there are 7 blue fish, 4 red fish, and 8 green fish in a large fishing tank. You drop a net into it and end up with 2 fish. What is the probability you get 2 blue fish?

2. Axioms of Probability

Decide whether each of the three statements below is true or false:

a. $P(A) + P(A^C) = 1$. Recall that A^C means A "complement" or "not" A

b. $P(A \cap B) + P(A \cap B^C) = 1$. Recall that $\cap$ means "and"

c. If $P(A) = 0.4$ and $P(B) = 0.6$ then it must be the case that $A = B^C$

3. Conditional Probability

What is the difference between these two terms $P(B|A)$ and $P(A \cap B)$? Imagine that B is the event that a student "correctly answer a multiple choice question" and A is the event that the same student "guesses randomly". Provide an explanation as well as a mathematical relationship between the two.

Problems

4. The Birthday Problem

When solving a counting problem, it can often be useful to come up with a generative process, a series of steps that "generates" examples. A correct generative process to count the elements of set A will (1) *generate every element of A* and (2) *not generate any element of A more than once*. If our process has the added property that (3) *any given step always has the same number of possible outcomes*, then we can use the product rule of counting.

Problem: Assume that birthdays happen on any of the 365 days of the year with equal likelihood (we'll ignore leap years).

a. What is the probability that of the n people in class, at least two people share the same birthday?

b. What is the probability that this class contains exactly one pair of people who share a birthday?

Extra Practice

5. Self Driving Car

A self driving car has a 60% belief that there is a motorcycle to its left based on all the information it has received up until this point in time. Then, it receives a new, independent report from its left camera. The camera reports that there is **no** motorcycle. What is the updated belief that there is a motorcycle to the left of the car? The camera is an imperfect instrument. When there is truly no motorcycle, the camera will report "no motorcycle" 90% of the time. When there actually is a motorcycle, the camera will report "no motorcycle" 5% of the time.

6. Flipping Coins

One thing that students often find tricky when learning combinatorics is how to figure out when a problem involves permutations and when it involves combinations. Naturally, we will look at a problem that can be solved with both approaches. Pay attention to what parts of your solution represent distinct objects and what parts represent indistinct objects.

Problem: We flip a fair coin n times, hoping to get k heads.

a. How many ways are there to get exactly k heads? Characterize your answer as a *permutation* of H's and T's.

b. For what x and y is your answer to part (a) equal to $\binom{x}{y}$? Why does this *combination* make sense as an answer?

c. What is the probability that we get exactly k heads?

7. Counting

The Inclusion Exclusion Principle for three sets is:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Explain why in terms of a venn-diagram.