CS109: Probability for Computer Scientists

Gather around and let me tell you a story
that we all should know

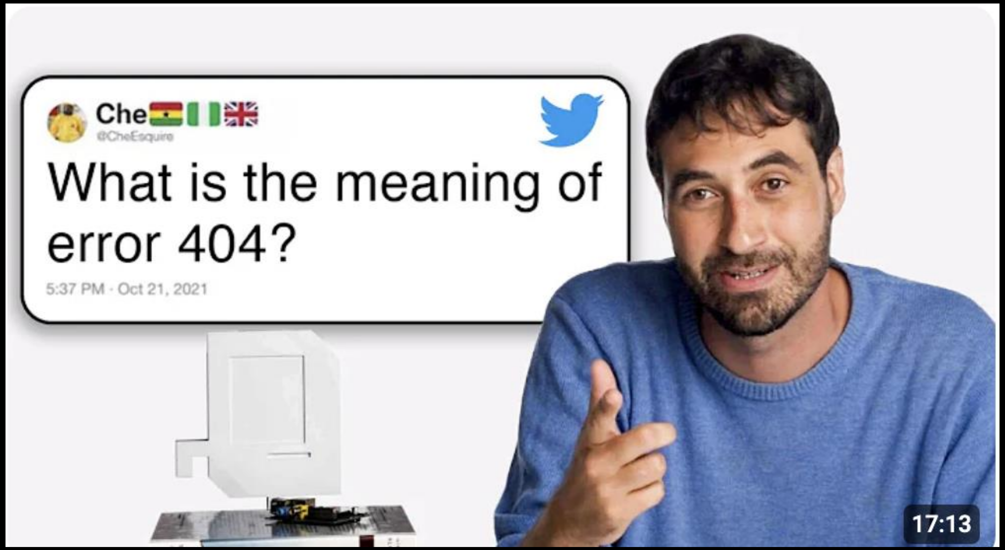
But first…
who am I?

Professor Chris Piech

Teaching at Stanford

8,000+ students over 10 years

CS106A	CS106B	CS109	CS221
Programming Methodologies	Programming Abstractions	Probability for Computer Scientists	Intro to Artificial Intelligence
CURRENT	LAST: FALL 2016	LAST: FALL 2018	LAST: SUM 2013



Code in Place: the course with the most teachers

From Three Places!

Grew up in Nairobi, Kuala Lumpur before Stanford!

Long History in CS109

I took the first CS109 back when I looked like this

Been teaching it since 2014

Stanford University

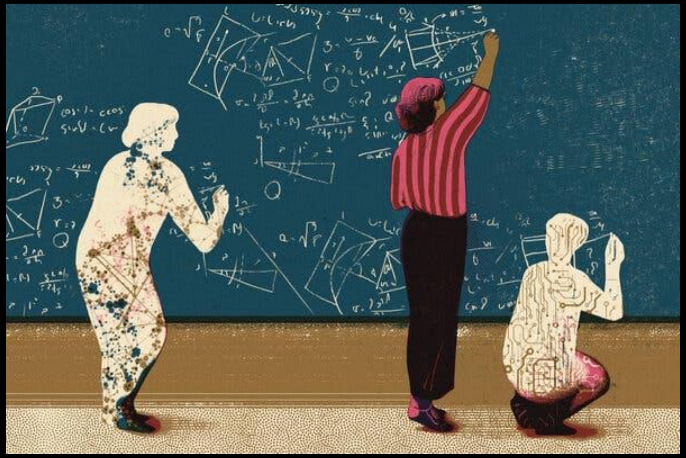
Computer Science Department

Stanford AI

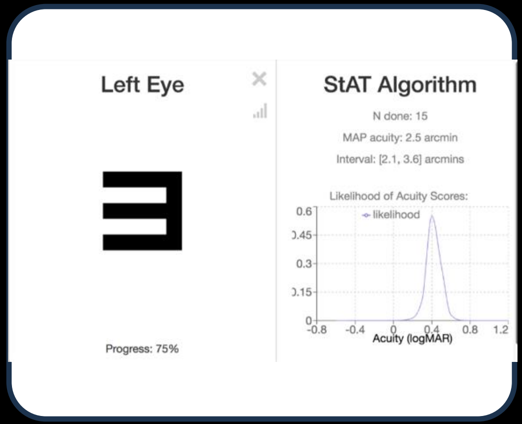
Piech Lab

Professor Chris Piech

Invented DKT, the algorithm used in Duolingo's Bird Brain

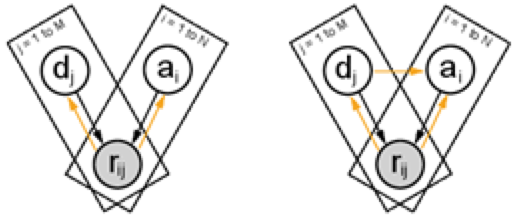


Invented Generative Grading, used in Code.org, featured in the New York Times

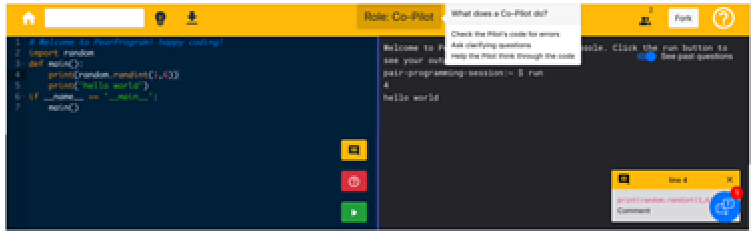


Created Stanford Acuity Test, Most Accurate Eye Exam!

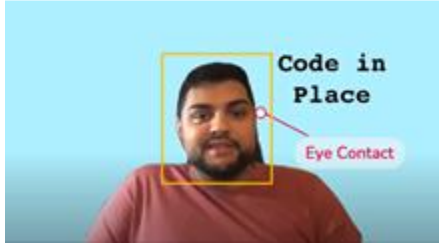
Computational Education (and medicine too)

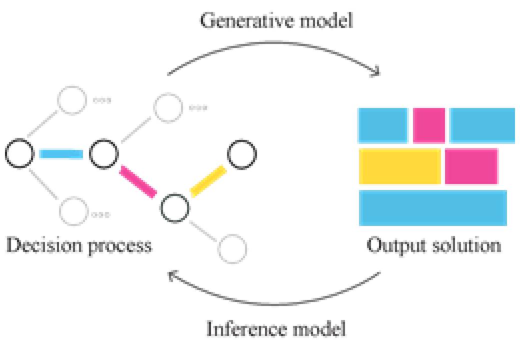


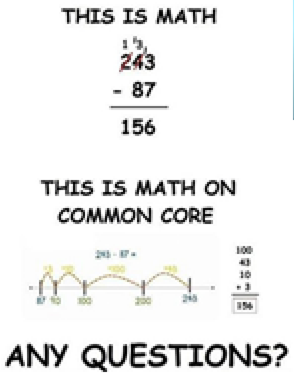
(a) (b)



NATIONAL EDUCATION
EQUITY LAB

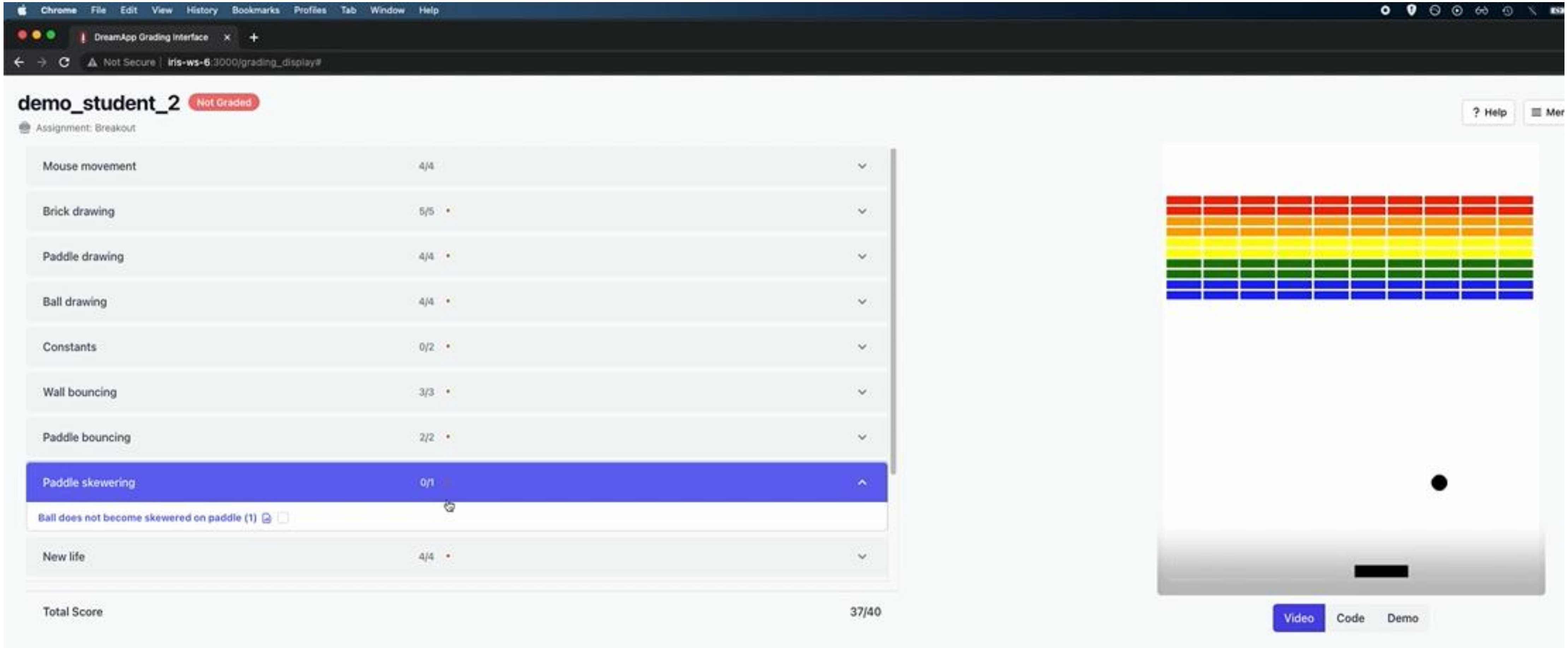






gradescope

Play to Grade



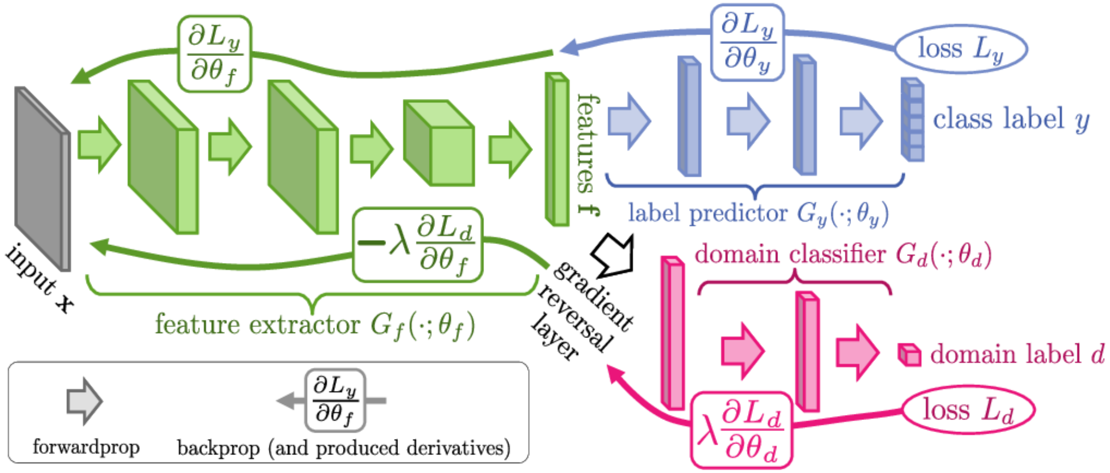
Best of STEM award for AI Tutor with Code.org

Research by CS109 Students

Fair AI with Adversarial Network

221 Citations

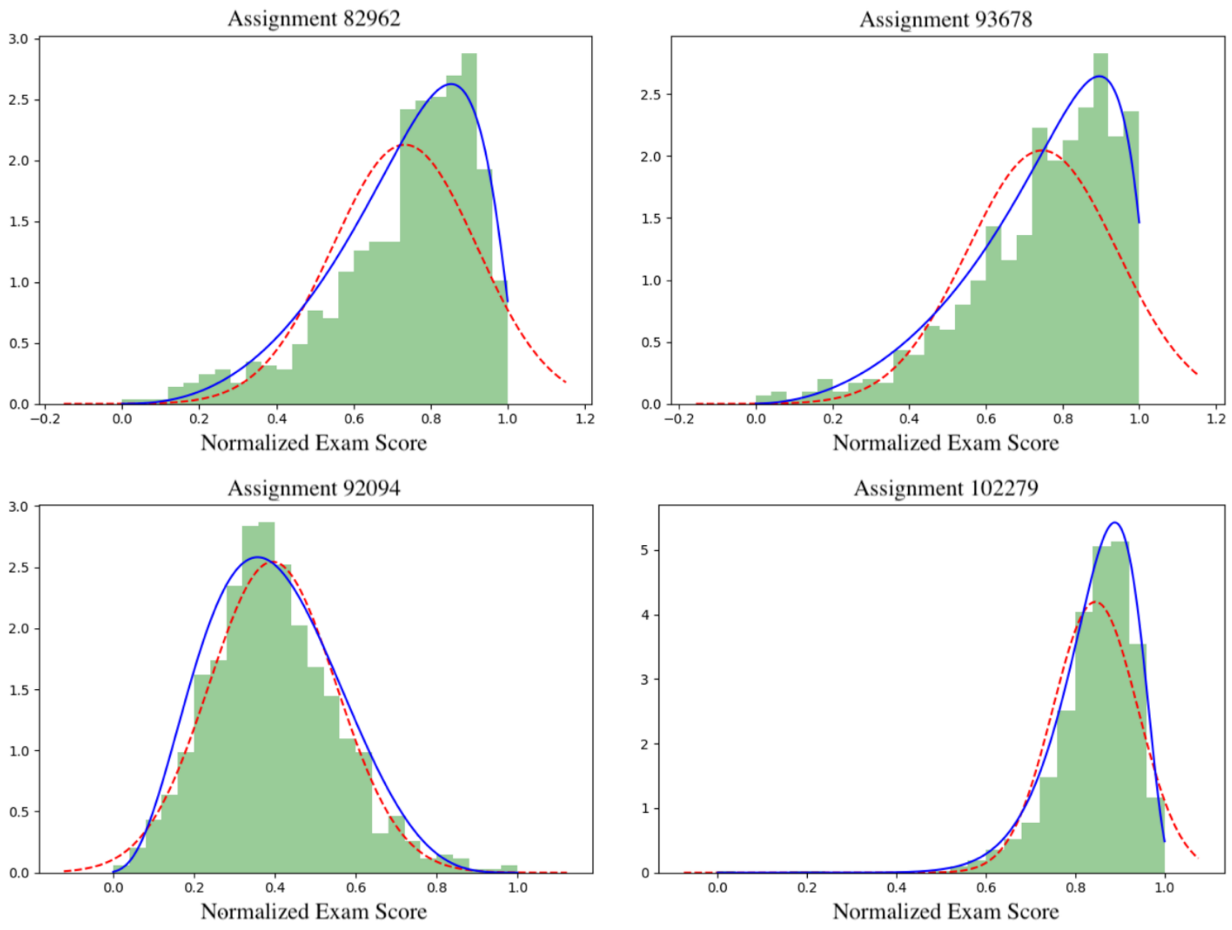
With undergrads Christina Wadsworth and Francesca Vera

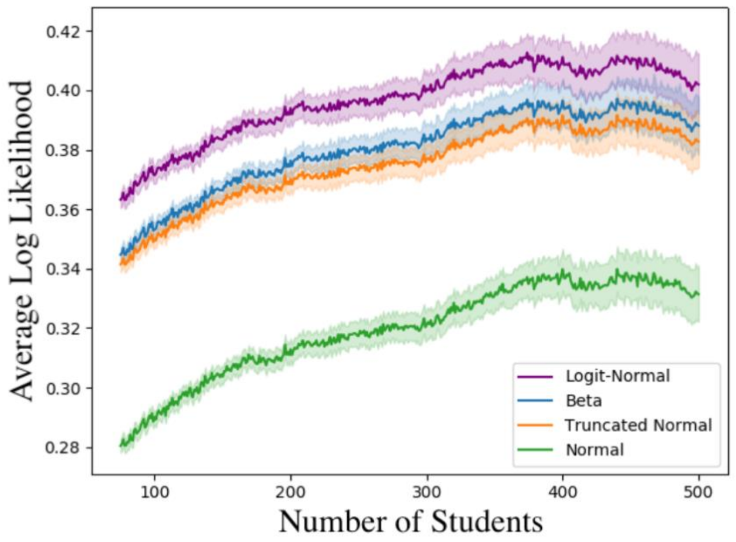


Model	Accuracy	FP Gap	FN Gap
COMPAS Scores (Our Test Set)	0.68	0.17	0.22
Our Recidivism Model	0.70	0.15	0.27
Our Chosen Adversarial Model	0.70	**0.01**	**0.02**
Bechavod et al. AVD Penalizers (2017)	0.65	**0.02**	**0.04**
Bechavod et al. SD Penalizers (2017)	0.66	**0.02**	**0.03**
Bechavod et al. Vanilla Regularized (2017)	0.67	0.20	0.30
Zafar et al. (2017)	0.66	**0.03**	0.11
Zafar et al. Baseline (2017)	0.66	**0.01**	0.09
Hardt et al. (2016)	0.65	**0.01**	**0.01**

Math question: Can you remove racism from a deep learning predictor?

Grades are Not Normal



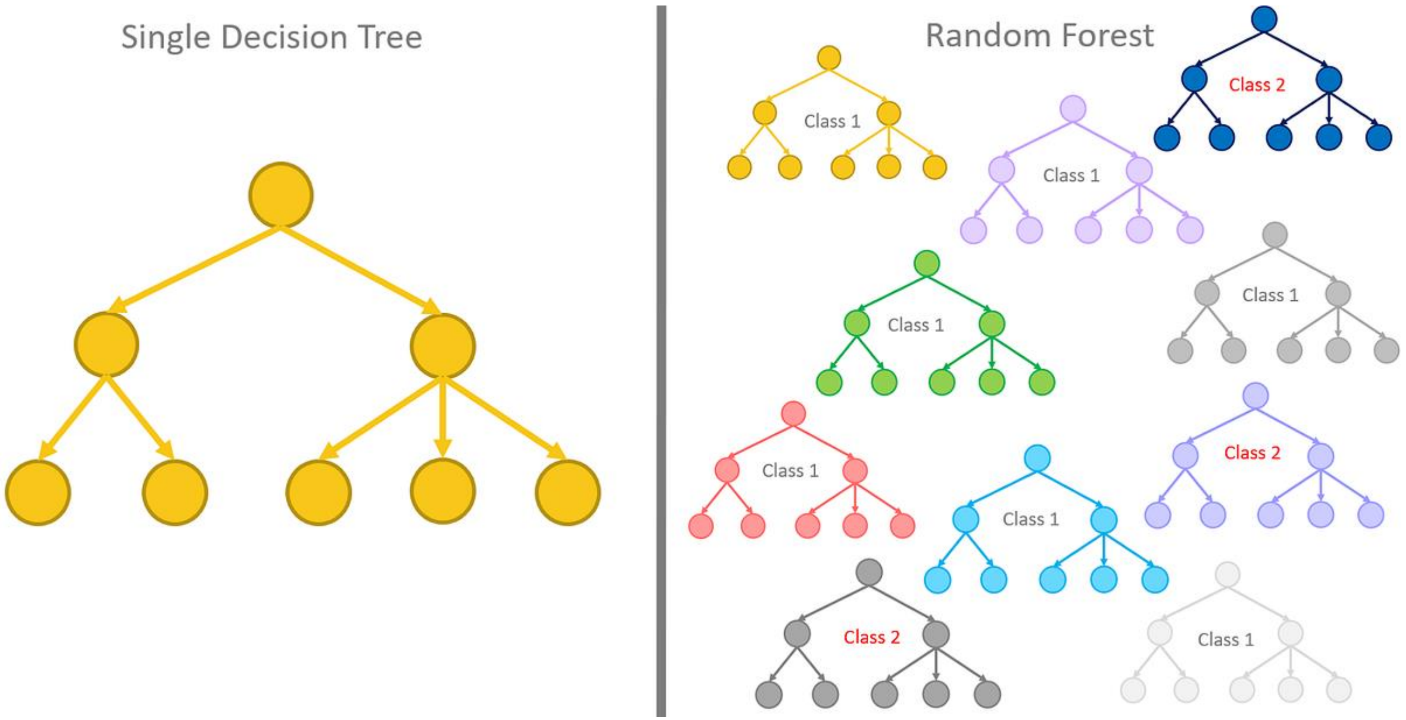


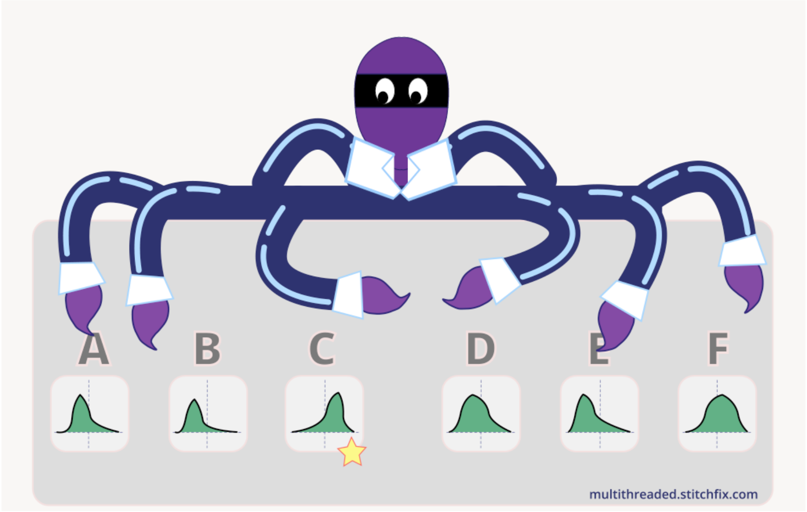
Noah Arthurs, CS109 Student

Math question: What is the generative story for grades in a typical classroom assignment?

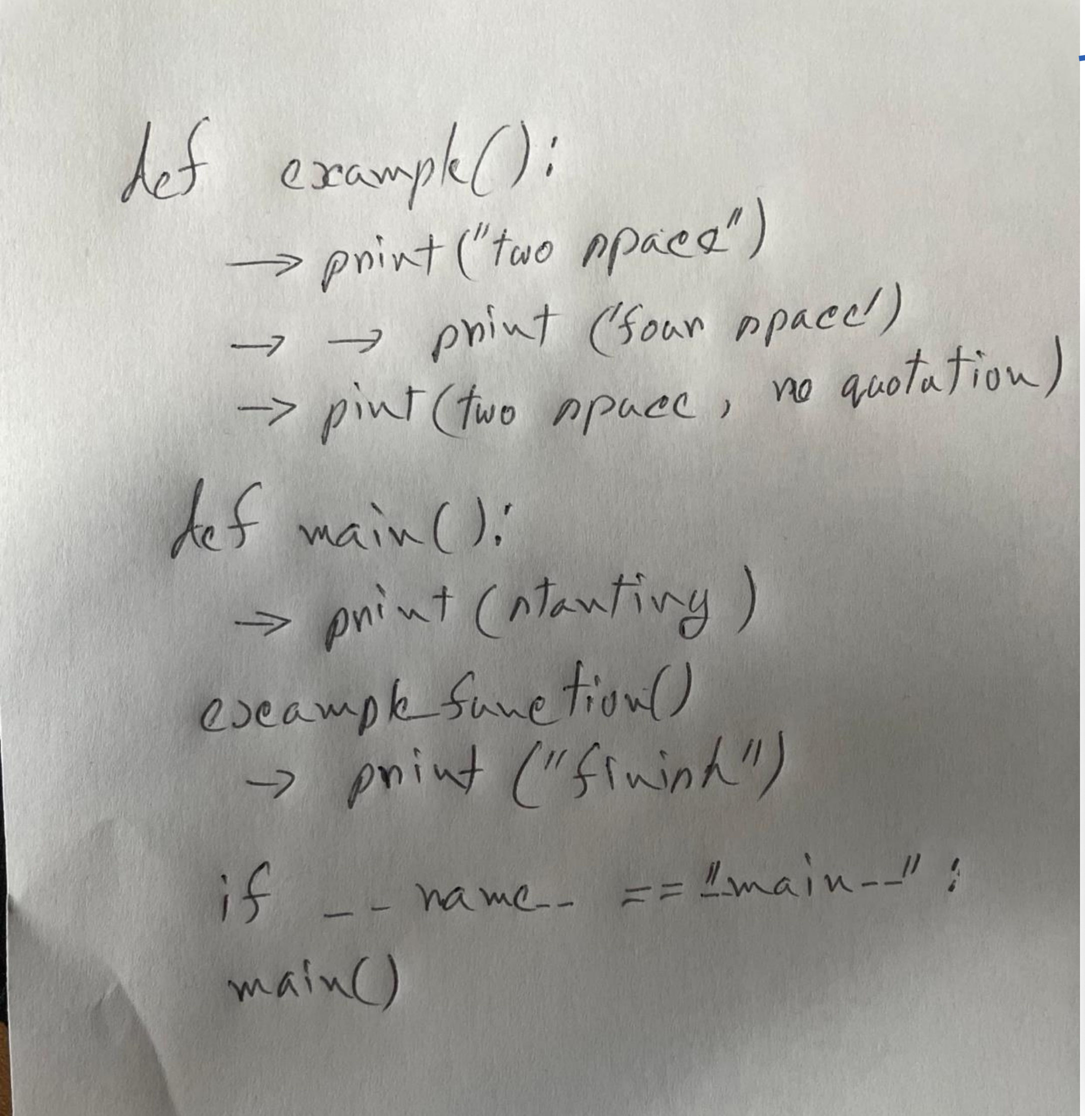
Analyzed 4,000 Gradescope Courses

Faster Forest Training Using Multi-Armed Bandits

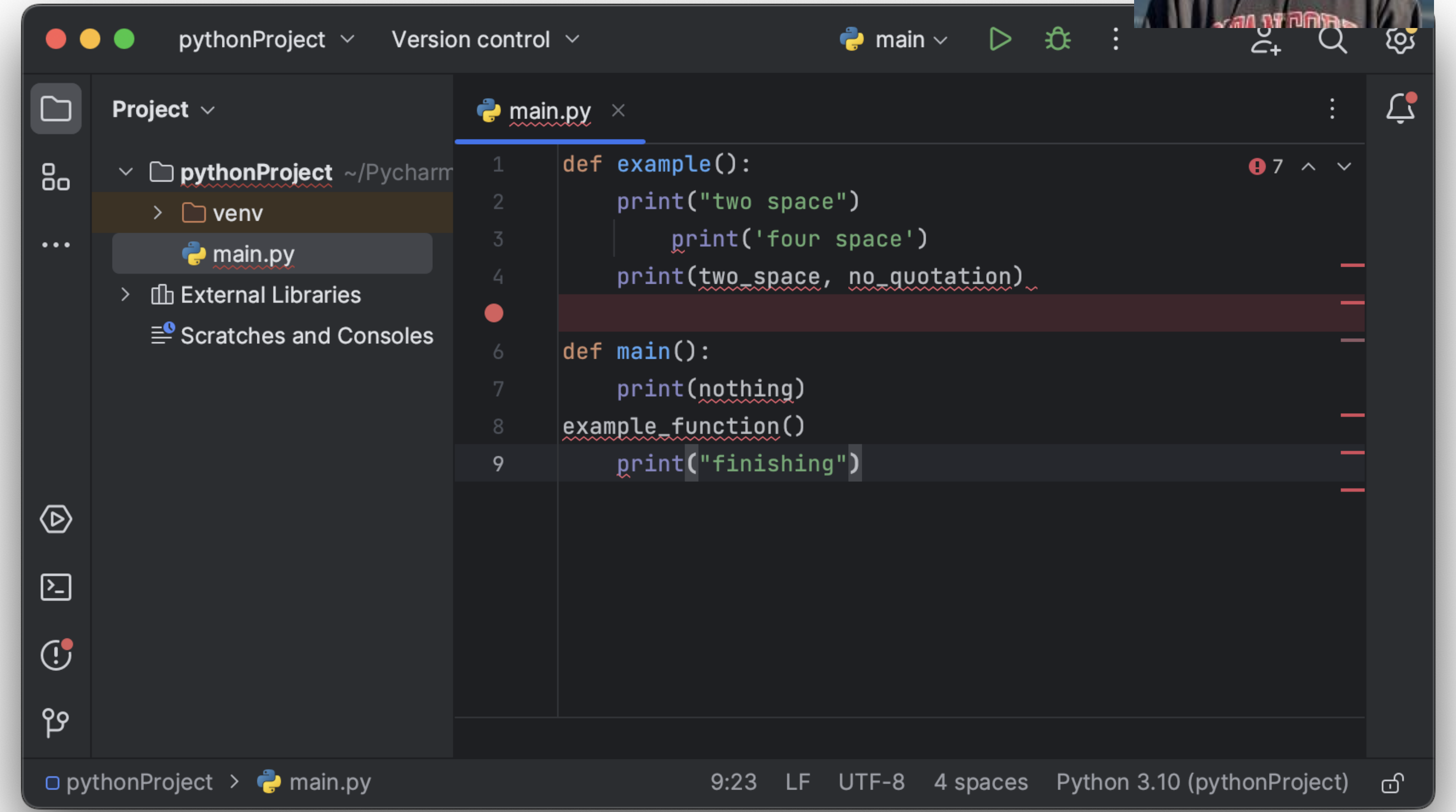




Math question: Can you speed up classic algorithms if you treat expensive computation as decisions under uncertainty?



Transcribed handwritten code



Math question: Can you relate pixel position to intended indentation using probability?

Undergraduate from CS109 Fall 23

Won Best Undergraduate Paper for L@S

So many things to love in this world

Most amazing family

Fantastic Teaching Team

Course mechanics

(this is a light version. Please read the handout for details).

Essential Information

cs109.stanford.edu

Are you in the right place?

Prerequisites

What you really need:

CS106B/X (important, coreq ok):

- Recursion
- Hash Tables
- Binary Trees
- Programming

CS103 (not necessary):

- Proof techniques (induction)
- Set theory
- Math maturity

Math 51 or CME 100 (important, coreq ok)

- Multivariate differentiation
- Multivariate integration
- Basic facility with linear algebra (vectors)

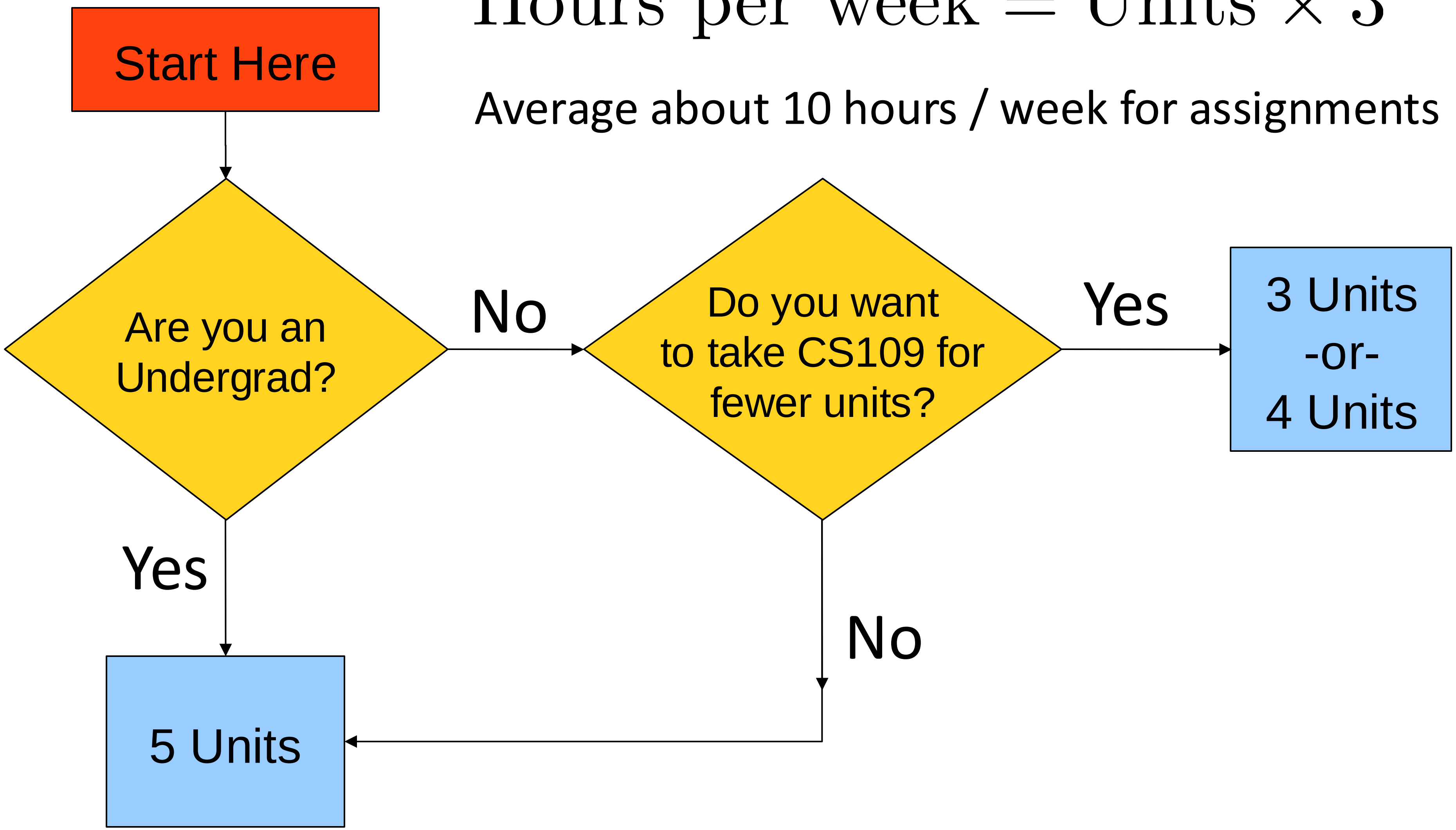
Coding in CS109

Review session on Friday

CS109 Units

$$\text{Hours per week} = \text{Units} \times 3$$

Average about 10 hours / week for assignments



Class Breakdown

40%	**6 Assignments**
20%	**Midterm** 2 hour exam, Oct 29th, 7pm
30%	**Final** 3 hour exam, Dec 10th, 8:30am
10%	**Participation** Section, PEP

New this year!

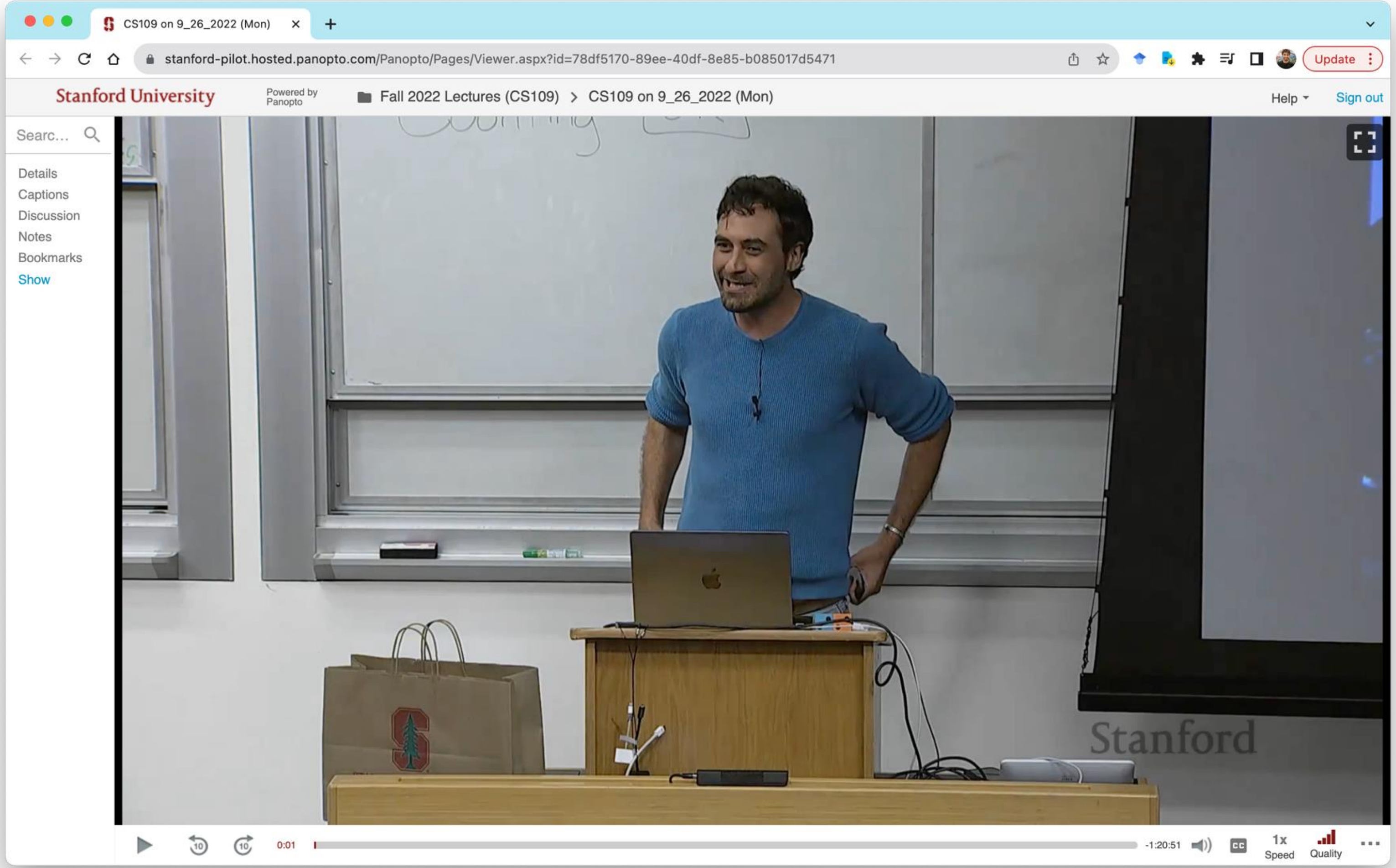
Personal Exam Prep (PEP)

- 15 mins
- Twice in the quarter
- 1:1 with a TA
- Week before each exam
- Participation graded
- Scheduled on Week 4

Song of the Quarter

[coming on Wednesday]

Is Class Online?



TLDR: Yes. Come to live class. It’s a good time (and good for you)

Come to Class

Section assumes you are caught up to Monday's lecture

"I met my husband in your CS109 lectures"
- Student from 3 years ago

Attending lecture strongly correlates with better grades

We have fun.

Ask questions

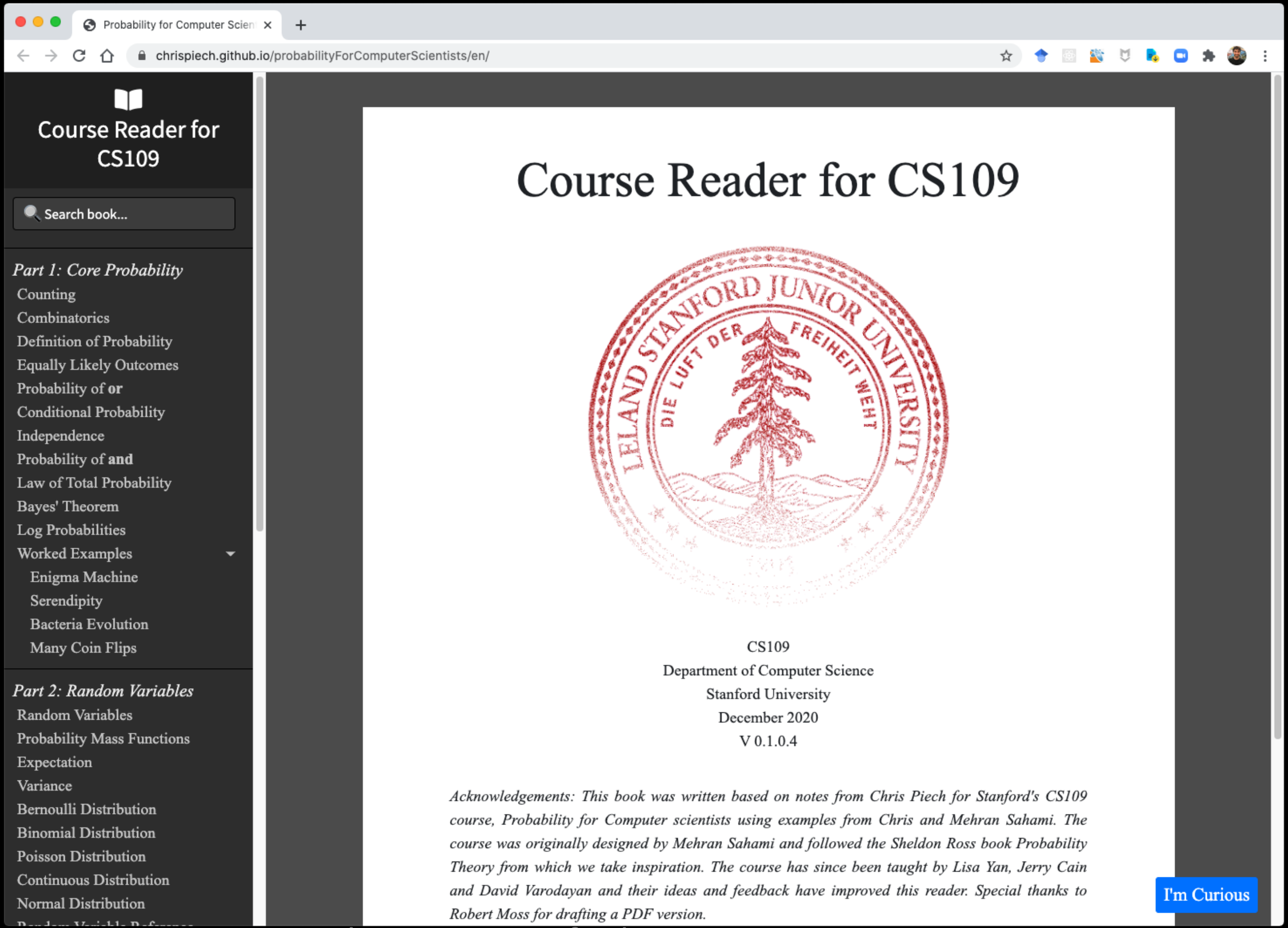
Q&A forum
All announcements

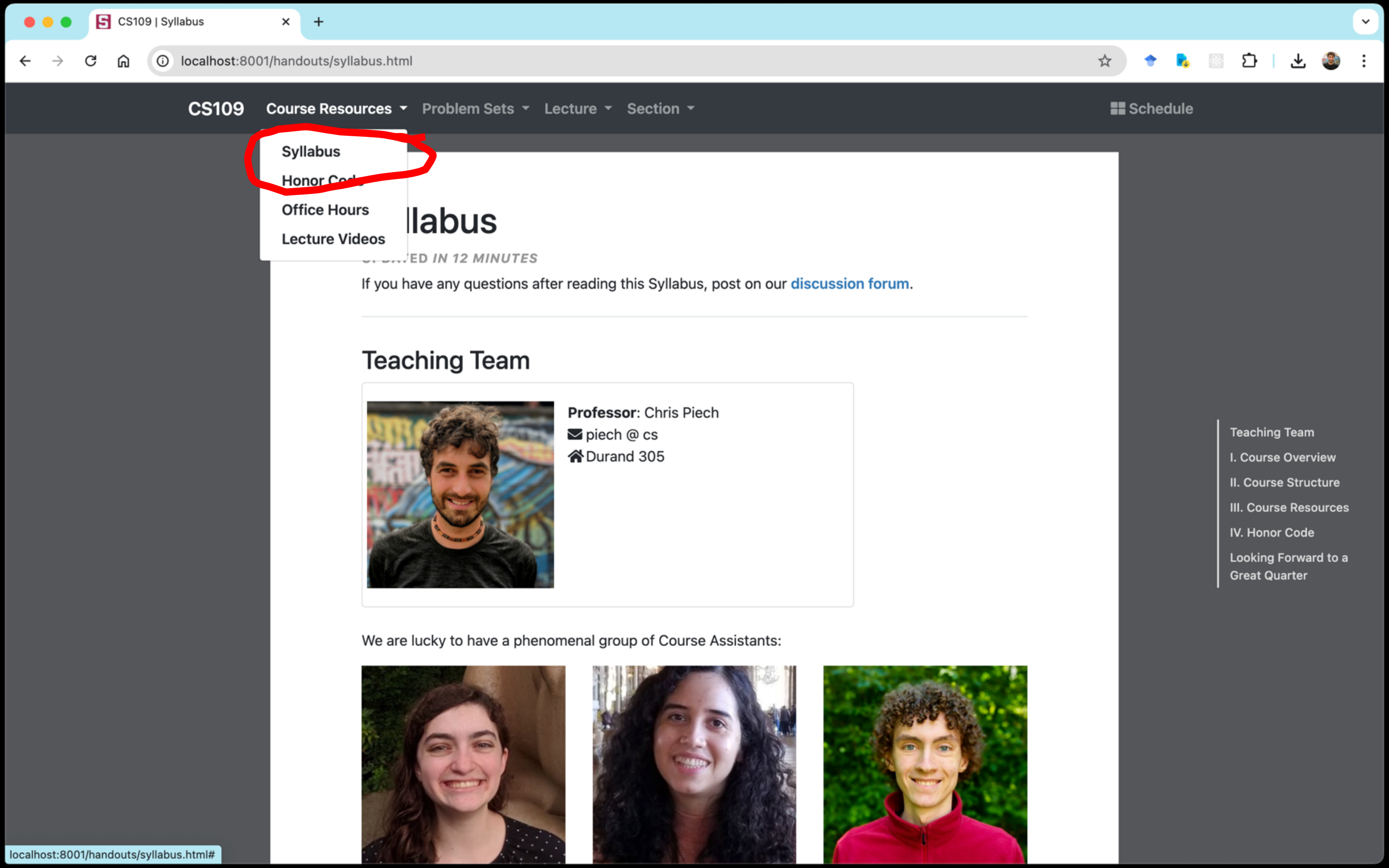
“Working” office hours
start on Saturday

Email cs109@cs.stanford.edu

Chris has 1:1 CS109 office hours

Chris Wrote a Course Reader!




CS109 | Syllabus
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CS109
Course Resources
Problem Sets
Lecture
Section
Schedule
Syllabus
Honor Cod
Office Hours
Lecture Videos
llabus
ED IN 12 MINUTES
If you have any questions after reading this Syllabus, post on our discussion forum.
Teaching Team
Professor: Chris Piech
piech @ cs
Durand 305
We are lucky to have a phenomenal group of Course Assistants:
Teaching Team
I. Course Overview
II. Course Structure
III. Course Resources
IV. Honor Code
Looking Forward to a Great Quarter
localhost:8001/handouts/syllabus.html#

Story of Modern AI

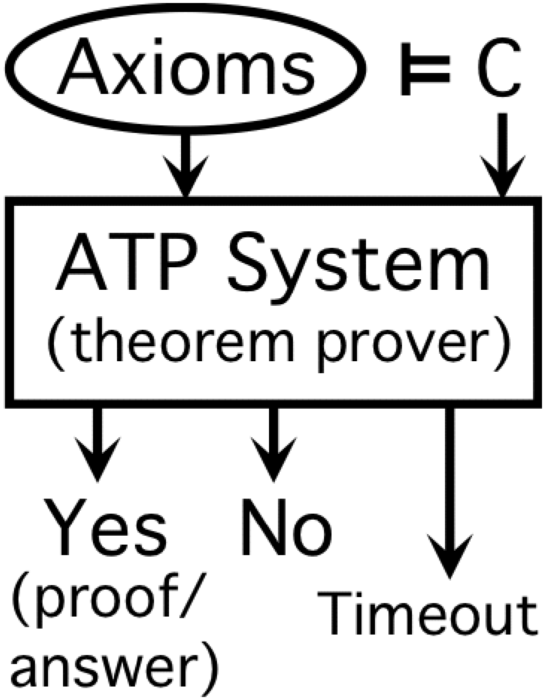
Modern AI
or, How we learned to combine probability and programming

Brief History

Early Optimism 1950s

1952

1955



Early Optimism 1950s

"Machines will be capable, within twenty years, of doing any work a man can do."
–Herbert Simon, 1952

Underwhelming Results 1950s to 1980s

The spirit is willing but the flesh is weak.

↓

(Russian)

↓

The vodka is good but the meat is rotten.

The world is too complex

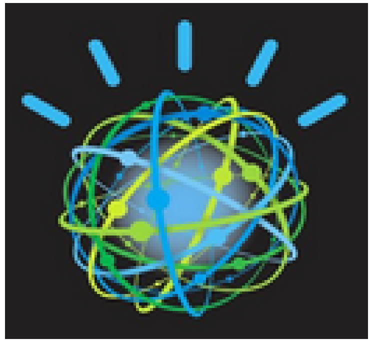

BRACE YOURSELVES
WINTER IS COMING

Something is going on in the world of AI

Big Milestones Part 1

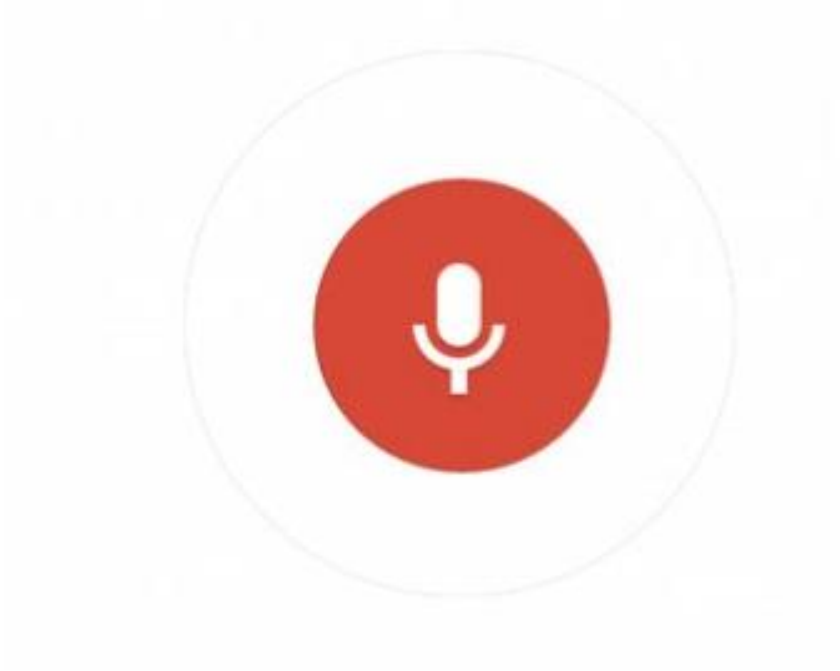
1997 Deep Blue

2005 Stanley



2011 Watson

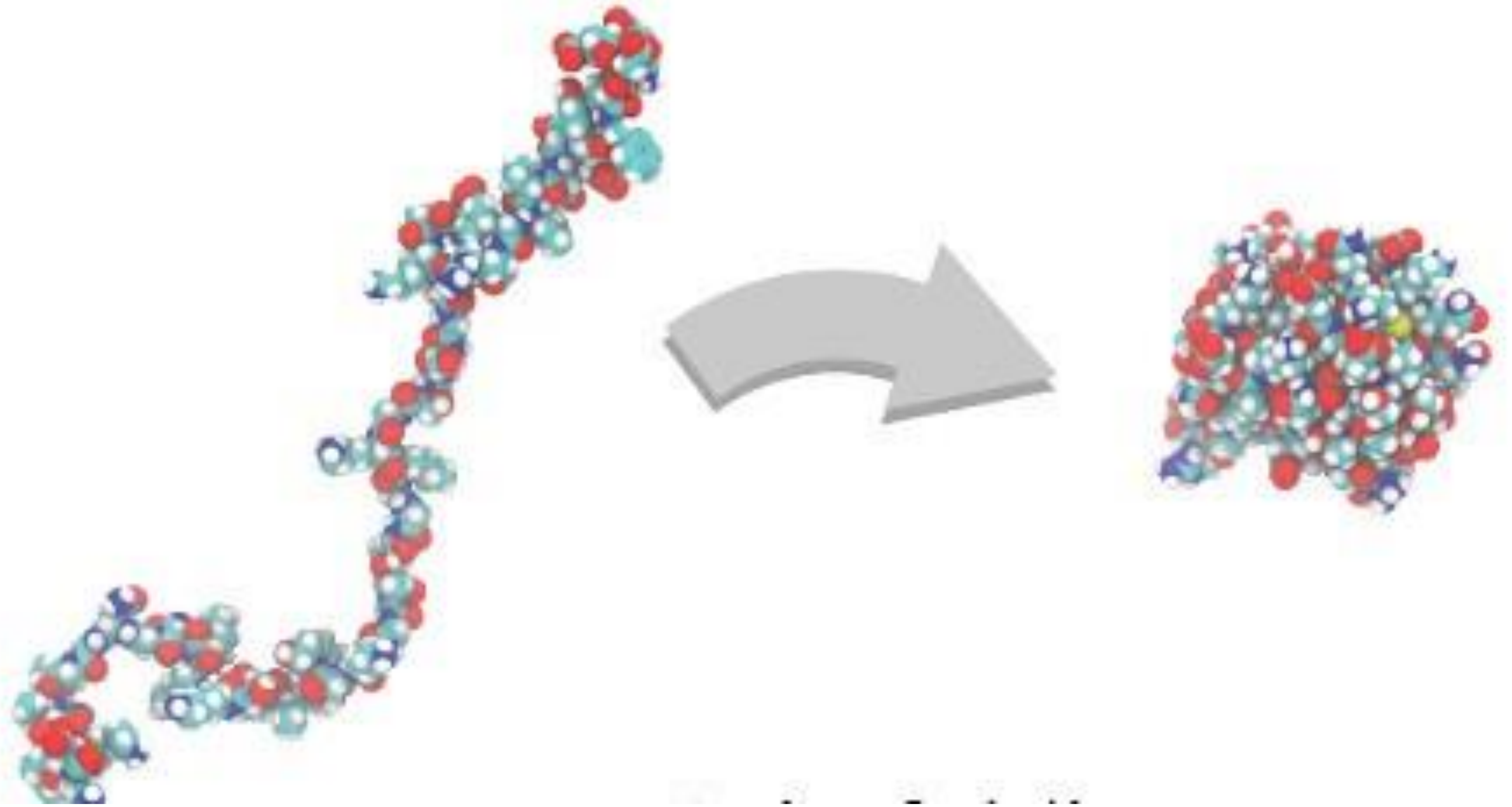
I was told speech was 30 years out



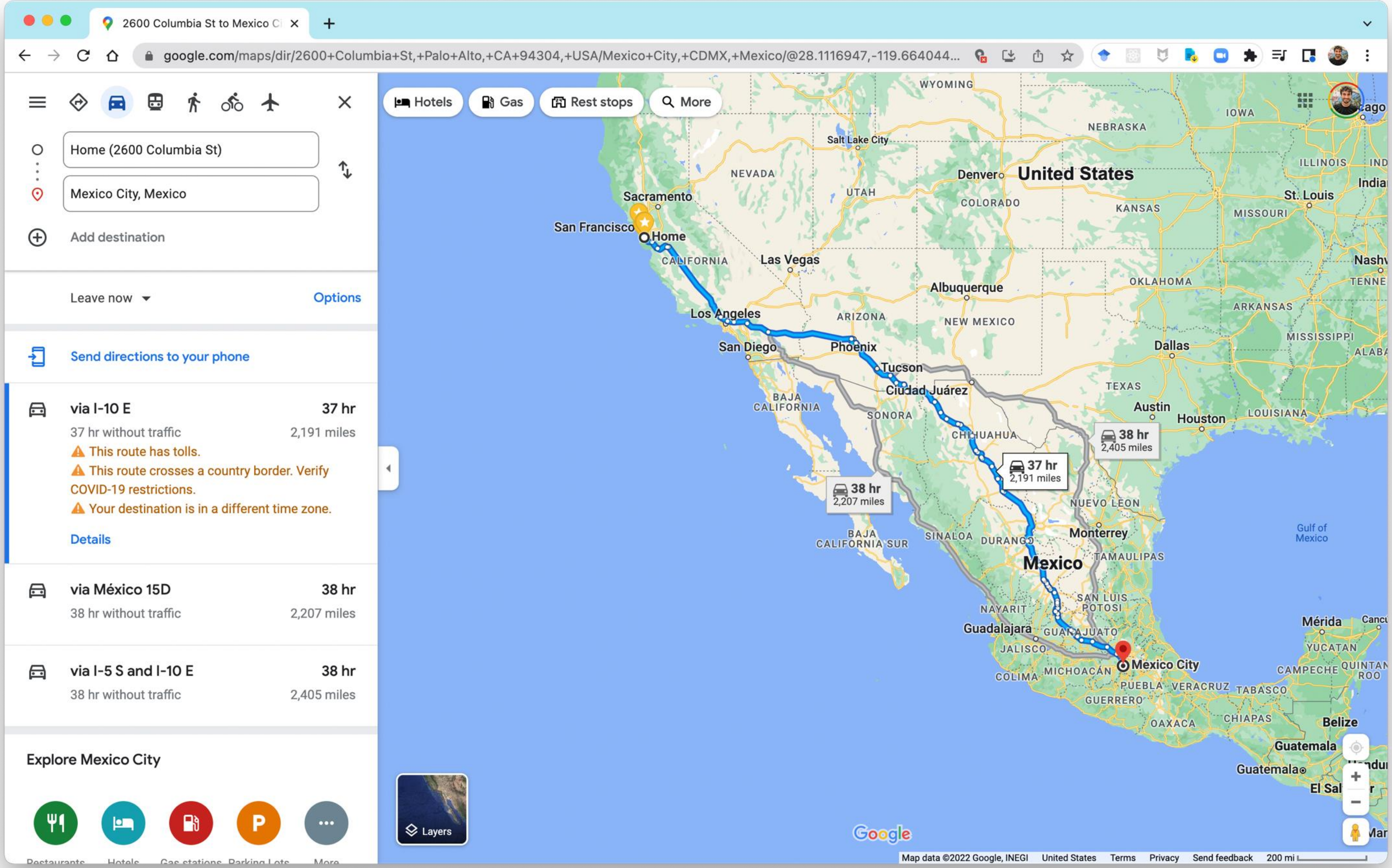
Almost perfect…

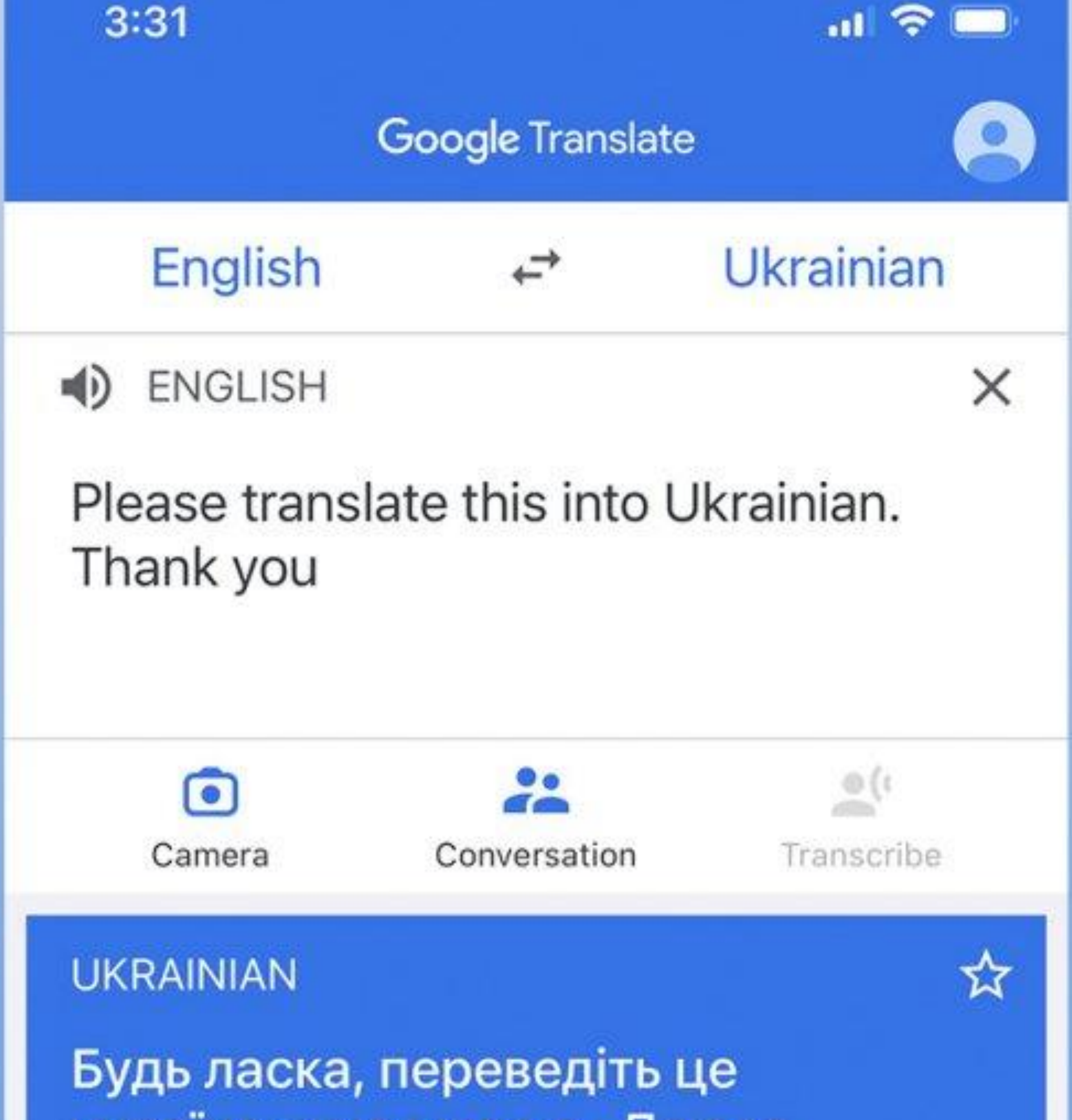
The last remaining board game

Protein Folding



Directions From A to B

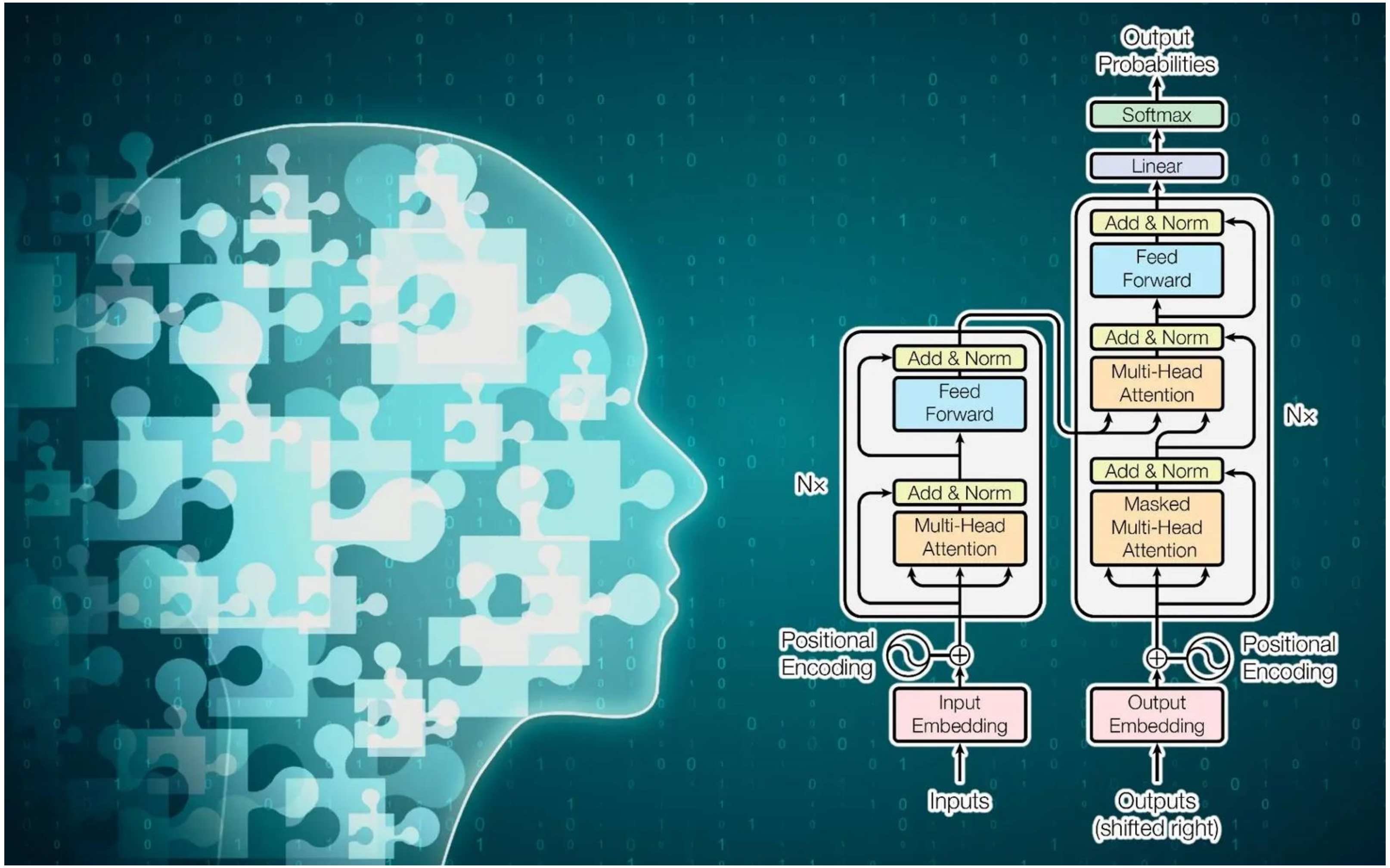



3:31
Google Translate
English
Ukrainian
ENGLISH
Please translate this into Ukrainian.
Thank you
Camera
Conversation
Transcribe
UKRAINIAN
Будь ласка, переведіть це

Self Driving Cars

And then last year, everything changed, again

AI that (seems) to understand language



DallE Images made by Freya (2 years old)

What is going on?

[suspense]

Focus on one problem

Computer Vision

Chihuahua or muffin?

Can you do it?

Chihuahua or Muffin?

Chihuahua or Muffin?

How about now?

What a computer sees

0	0	1	0	1	0	1	0	0	0	1	1	1	0	1
1	0	0	1	0	1	1	1	0	1	0	0	0	0	0
1	1	1	0	1	0	0	1	1	0	0	1	0	1	0
1	1	1	1	1	0	0	0	0	0	1	1	0	1	1
0	0	0	1	1	0	0	1	0	0	0	1	1	1	0
1	0	0	1	1	0	0	0	1	0					
1	1	0	1	1	0	0	1	1	0					
1	0	1	0	0	1	0	0	1	0					
0	0	0	0	1	0	1	0	1	1					
0	1	1	0	0	0	0	0	1	1					
0	0	1	0	1	1	1	0	0	0					
0	1	1	1	0	1	0	0	1	0					
1	1	0	0	0	0	0	0	0	0					
0	0	0	0	0	0	0	0	1	1					
0	0	1	1	1	0	1	0	1	1					

What a human sees

Why is it easy for Humans?

About 30% of your cortex is used from vision
3% is used to process hearing

1

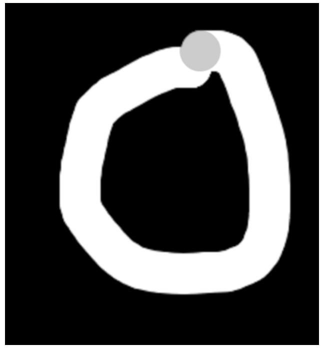
Make a Harry Potter Sorting Hat

Classification

That is a picture of a **one**

Classification

That is a picture of a **zero**



Classification

That is a picture of an **zero**

* It doesn't have to be correct all of the time

How about now?

What a computer sees

0	0	1	0	1	0	1	0	0	0	1	1	1	0	1
1	0	0	1	0	1	1	1	0	1	0	0	0	0	0
1	1	1	0	1	0	0	1	1	0	0	1	0	1	0
1	1	1	1	1	0	0	0	0	0	1	1	0	1	1
0	0	0	1	1	0	0	1	0	0					
1	0	0	1	1	0	0	0	1	0					
1	1	0	1	1	0	0	1	1	0					
1	0	1	0	0	1	0	0	1	0					
0	0	0	0	1	0	1	0	1	1					
0	1	1	0	0	0	0	0	1	1					
0	0	1	0	1	1	1	0	0	0					
0	1	1	1	0	1	0	0	1	0					
1	1	0	0	0	0	0	0	0	0					
0	0	0	0	0	0	0	0	1	1					
0	0	1	1	1	0	1	0	1	1					

What a human sees

Very hard to Program


```
class DigitDetector:

  def detect(raw_image):
    # Return a 0 or 1 based on the pixels of the image

    # TODO
```

Perhaps there is an insight?

Two Great Ideas

1. Artificial Neurons

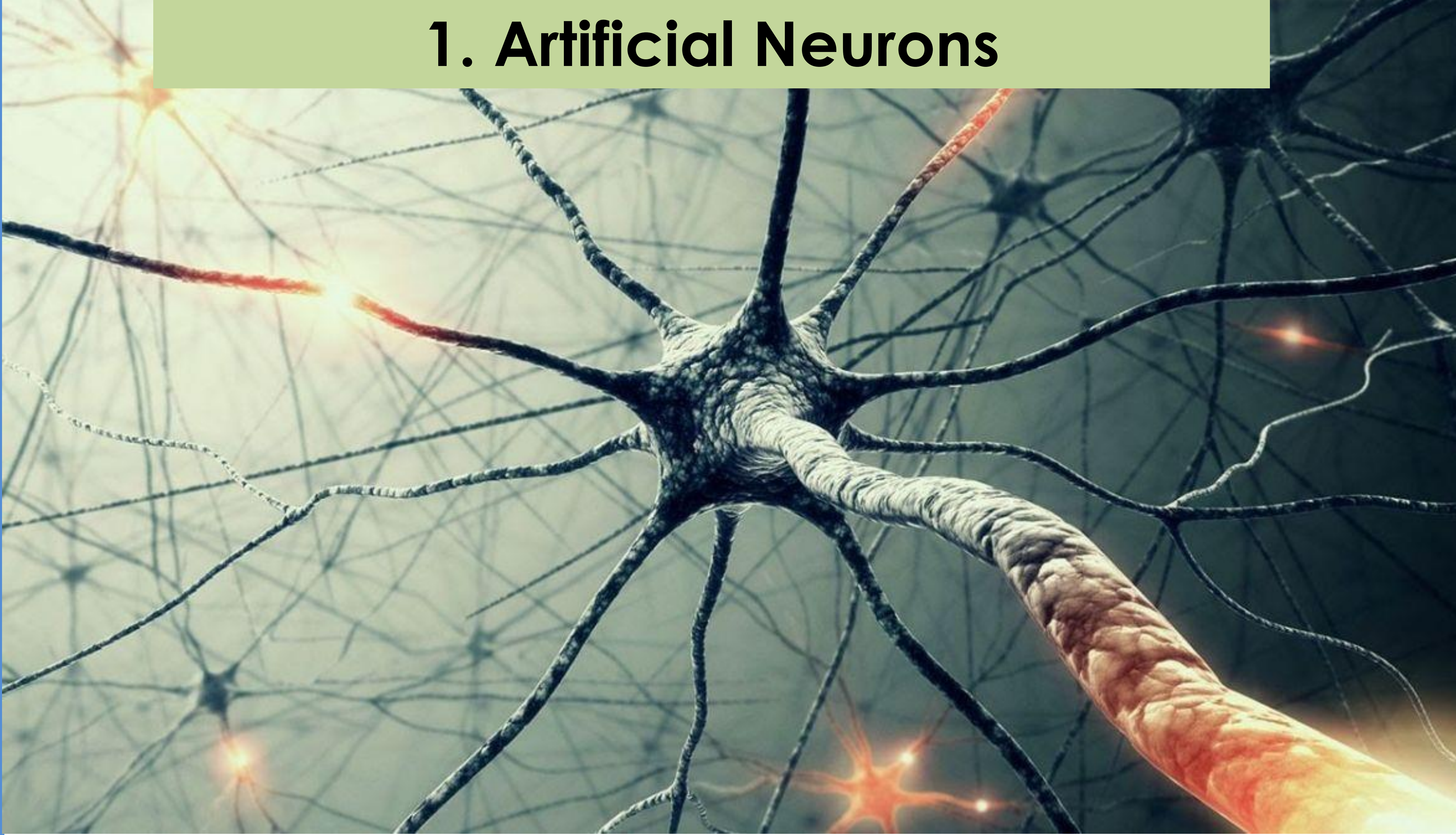
2. Learn by Example

Two Great Ideas

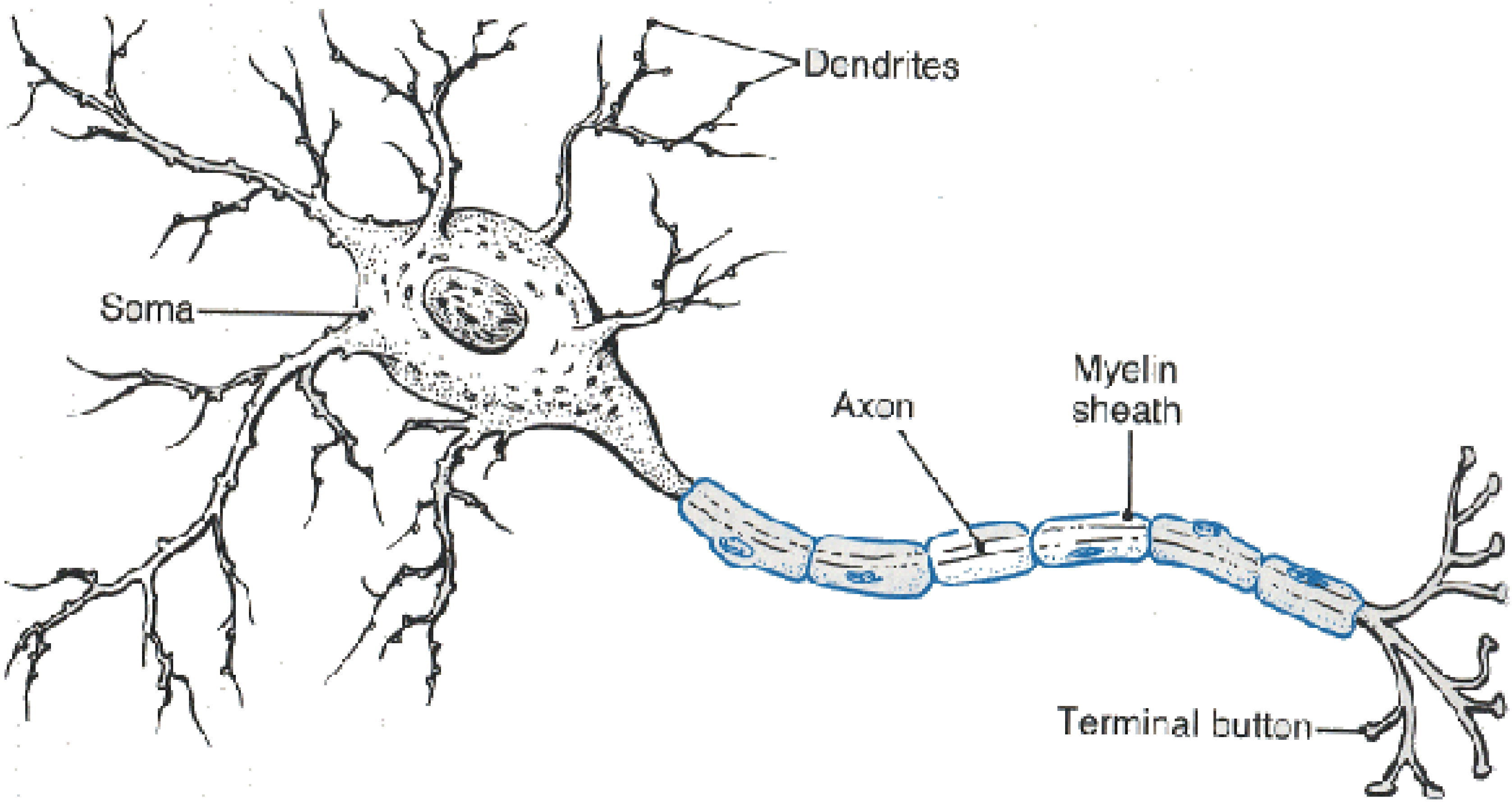
1. Artificial Neurons

2. Learn by Example

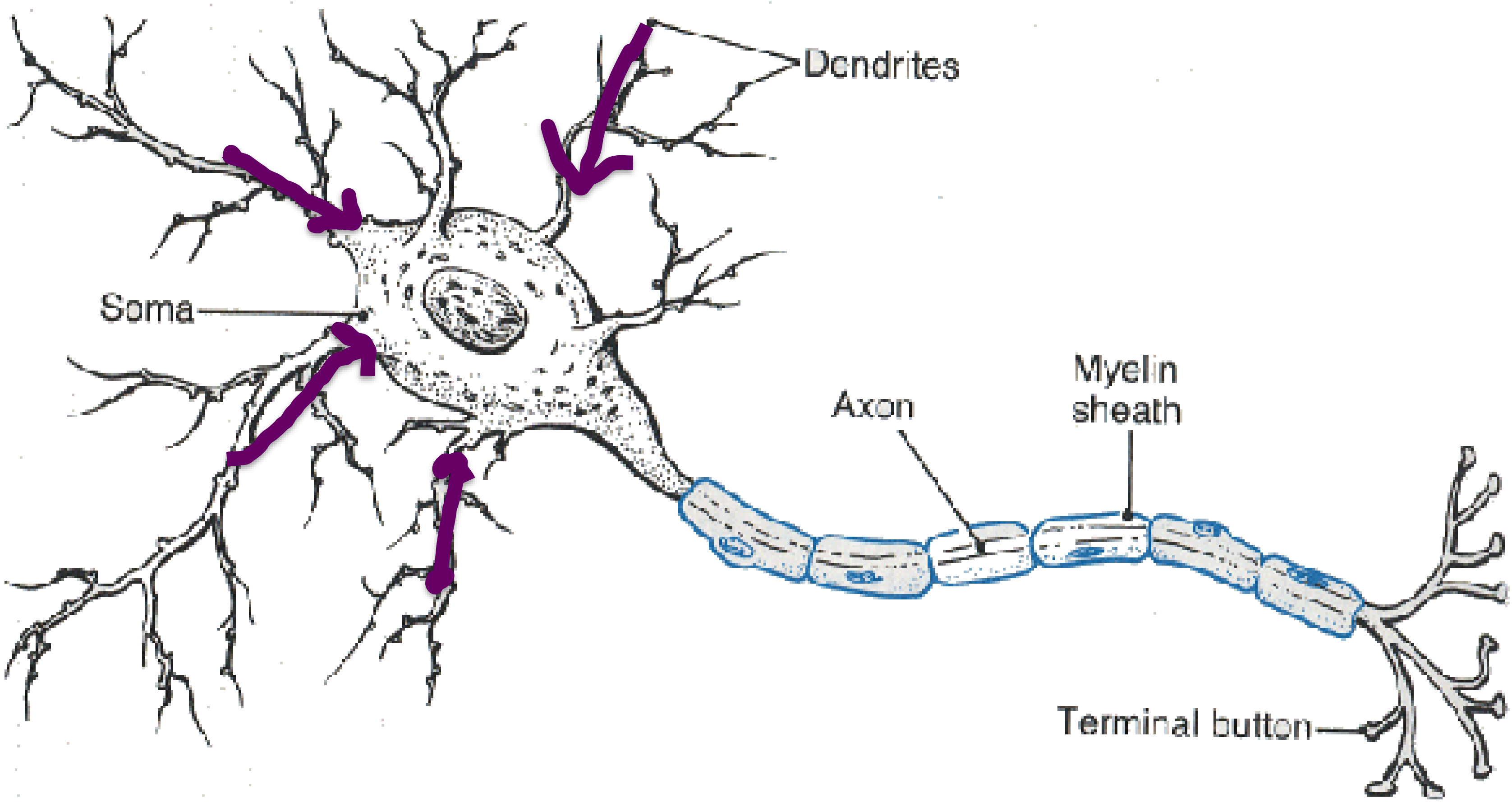
1. Artificial Neurons



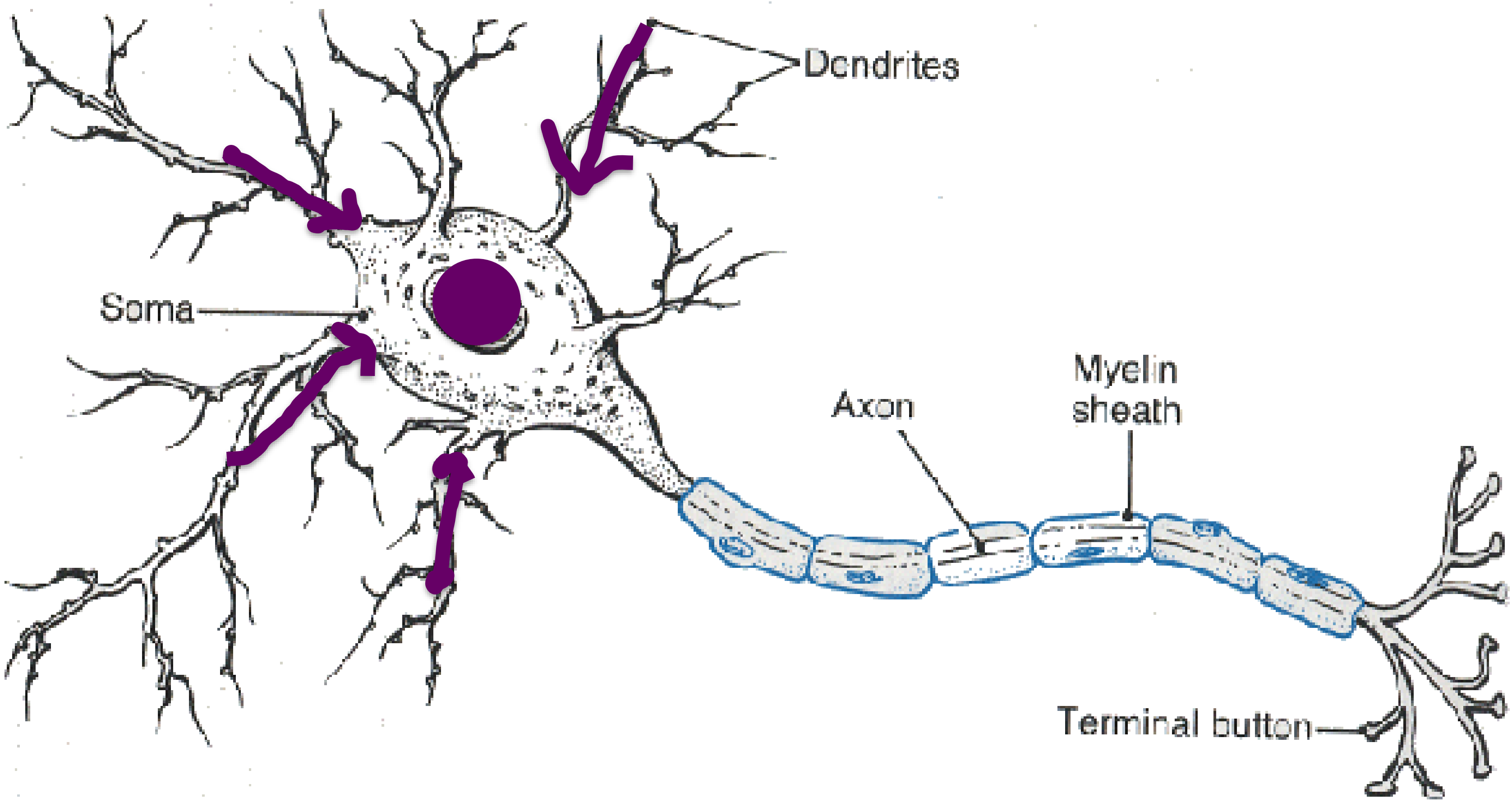
Neuron



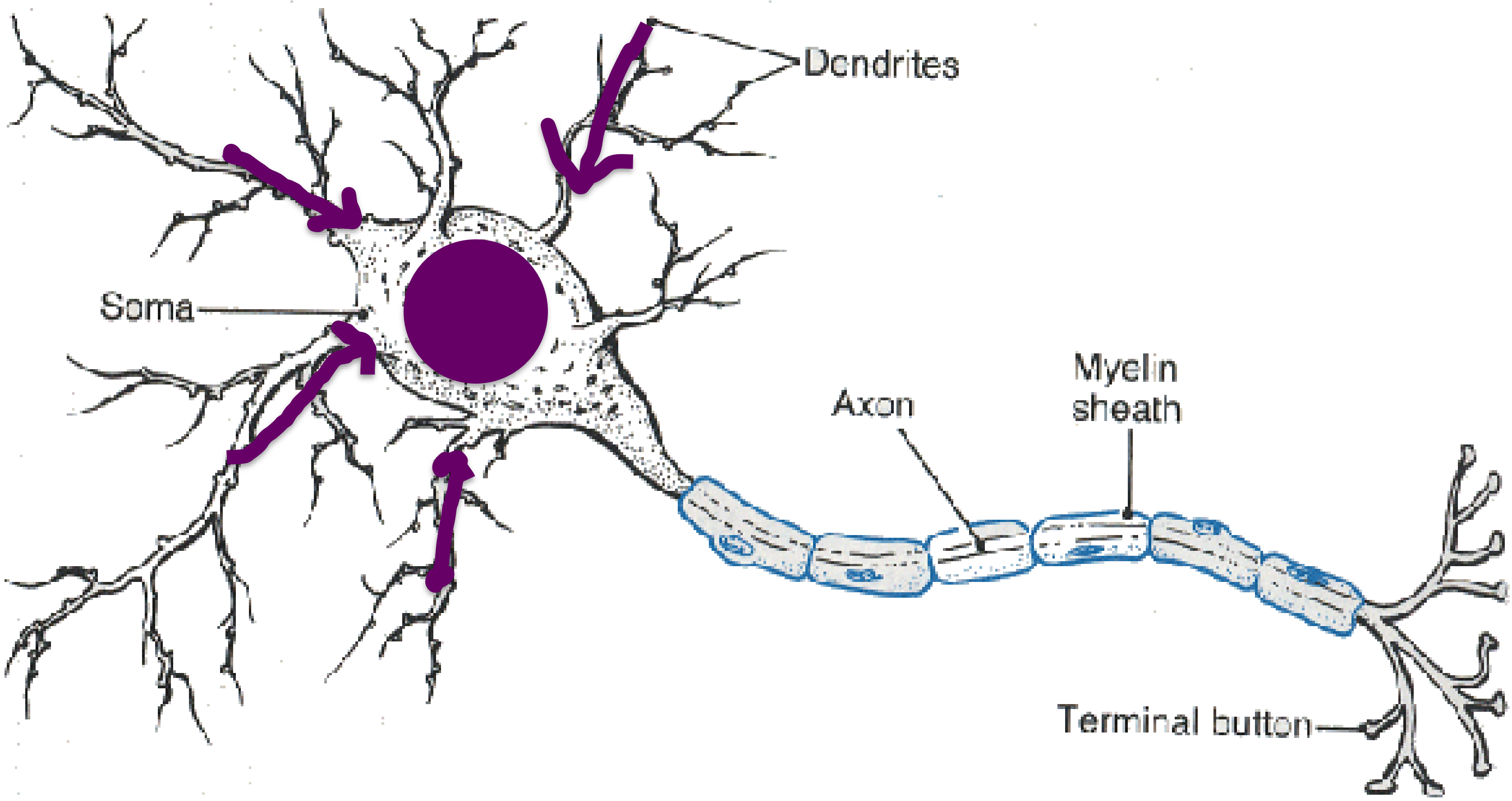
Neuron



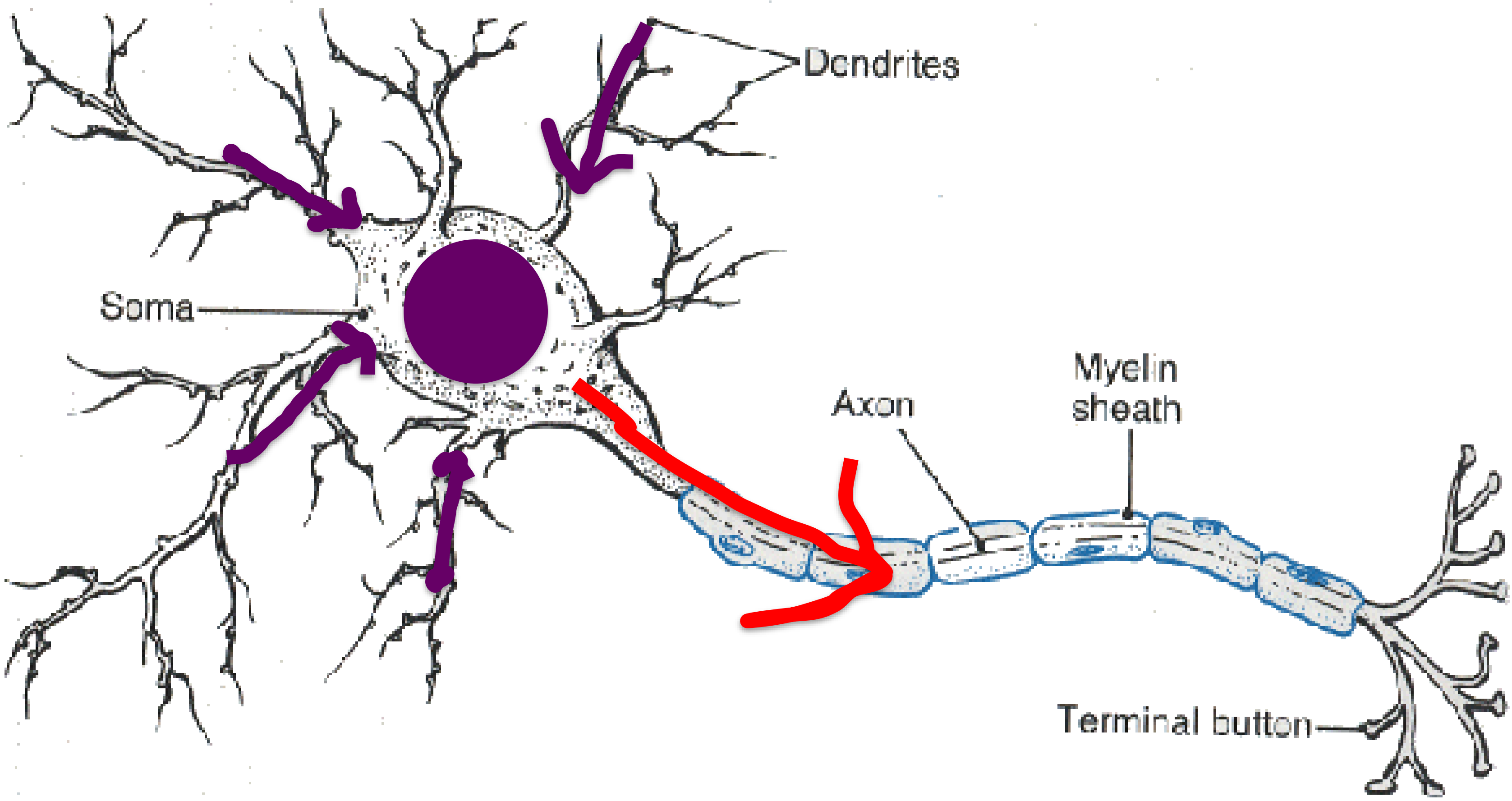
Neuron



Neuron

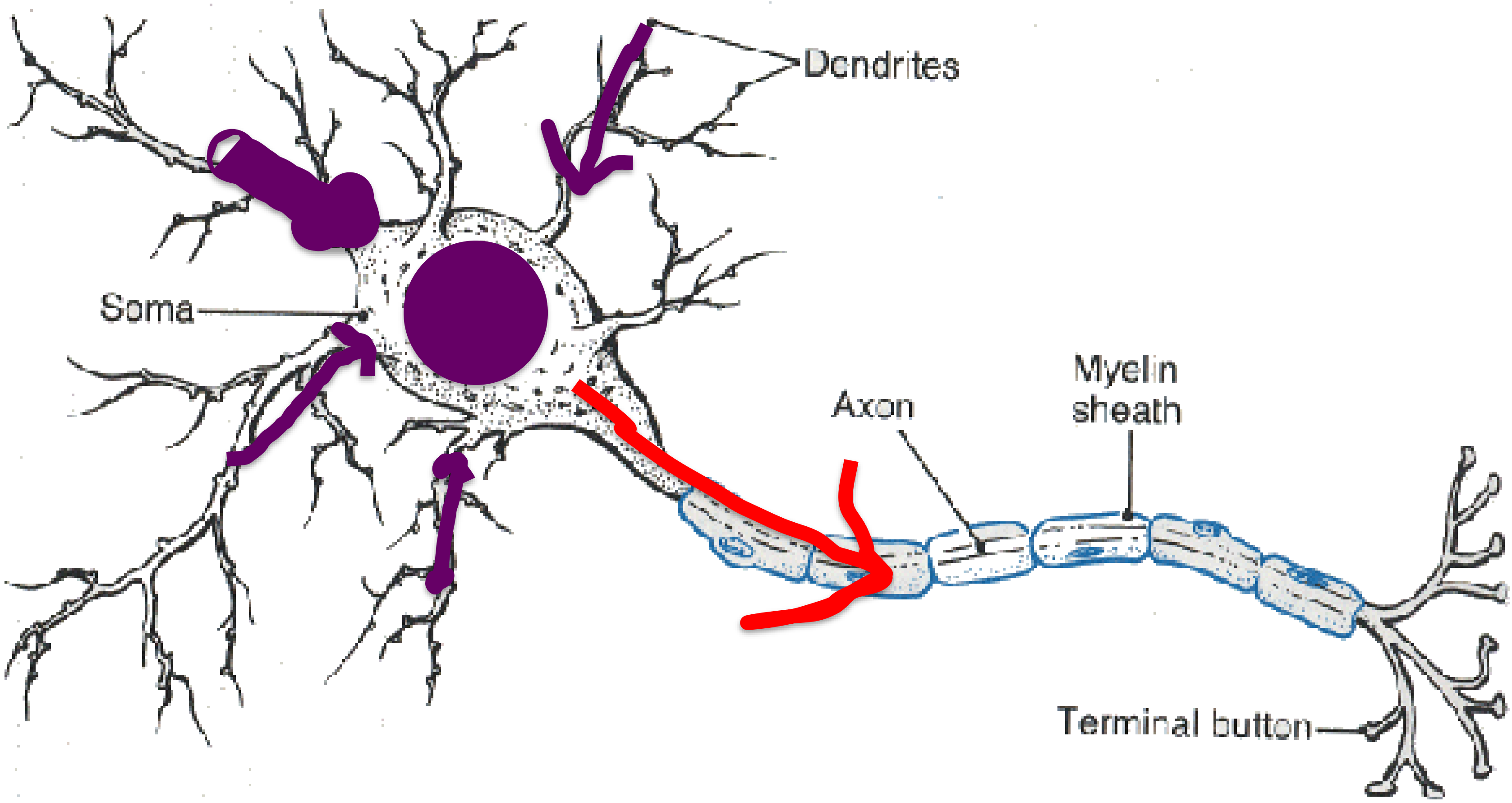


Neuron

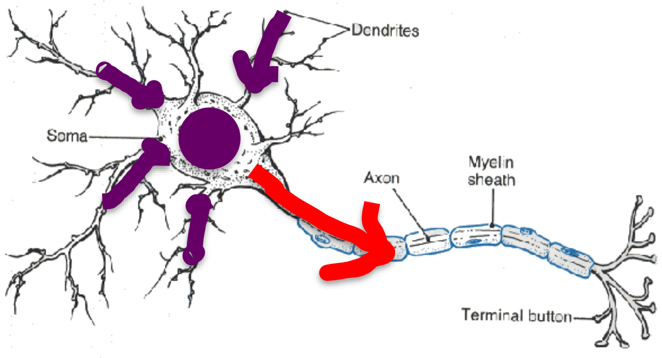


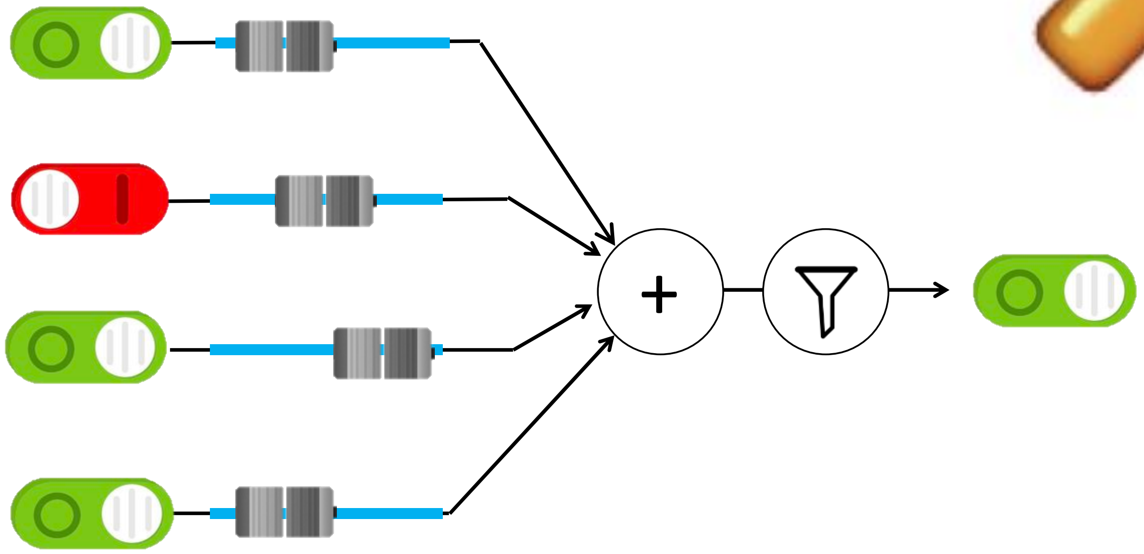
Piech

Some Inputs are More Important



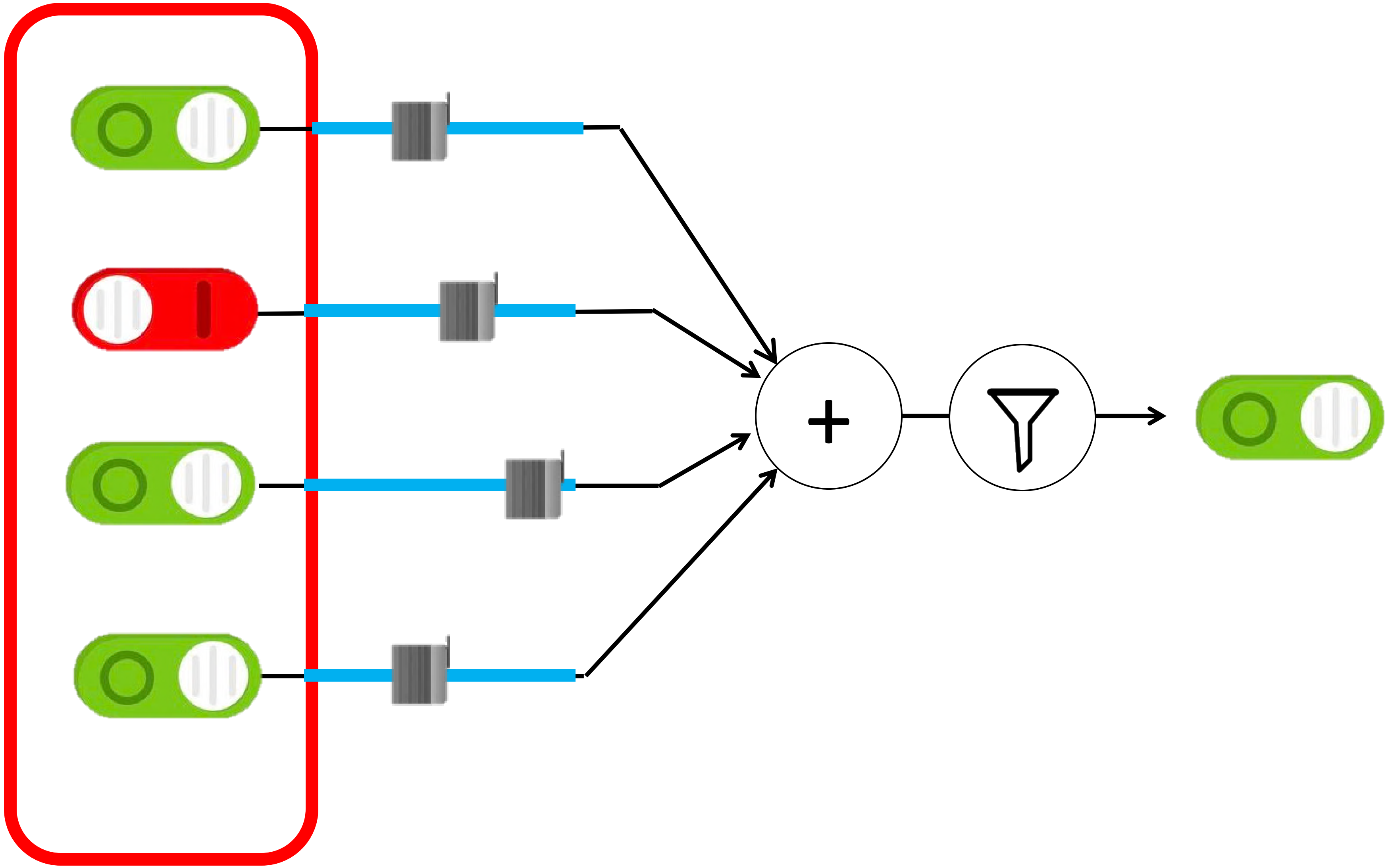
Artificial Neuron



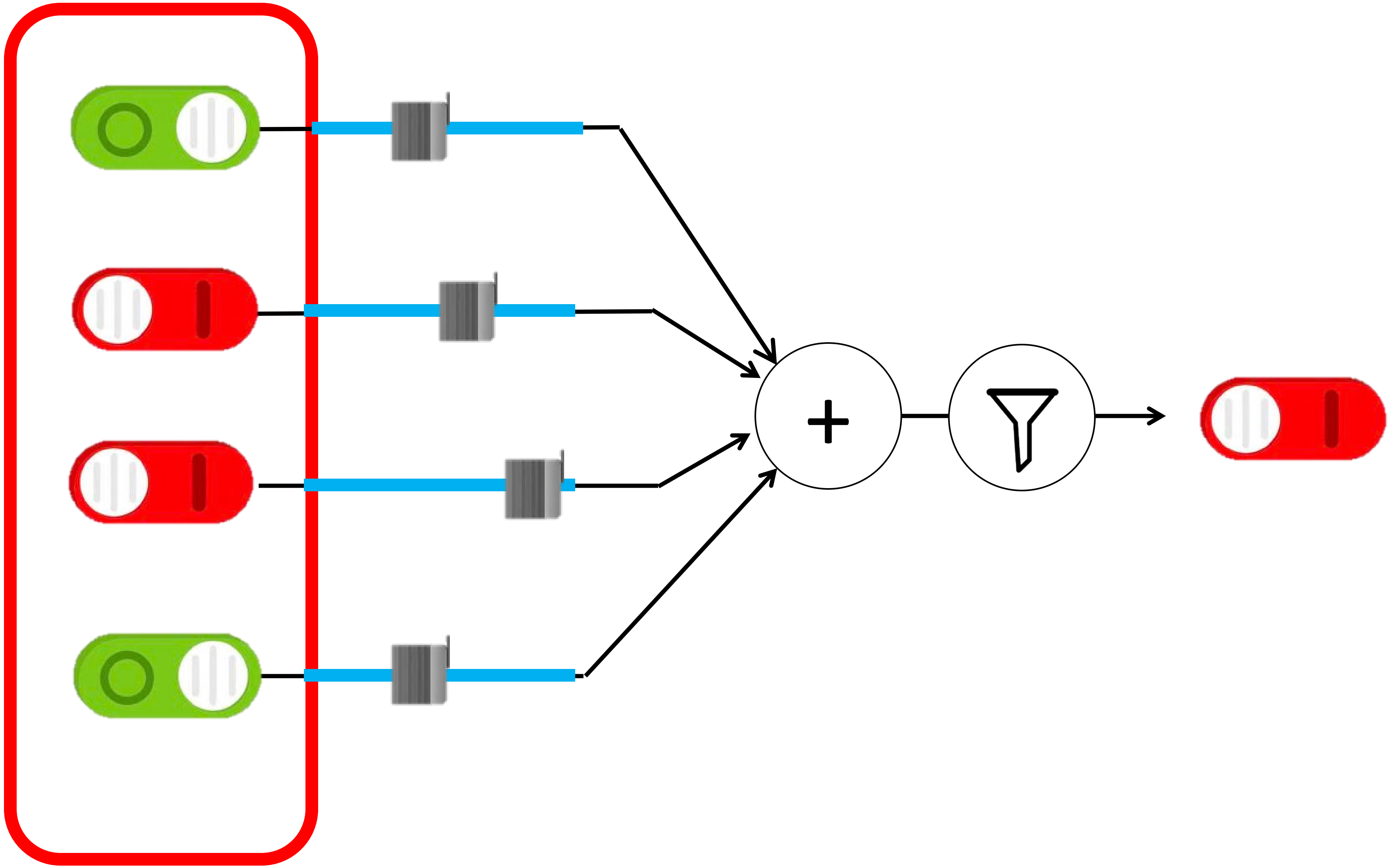


Piech

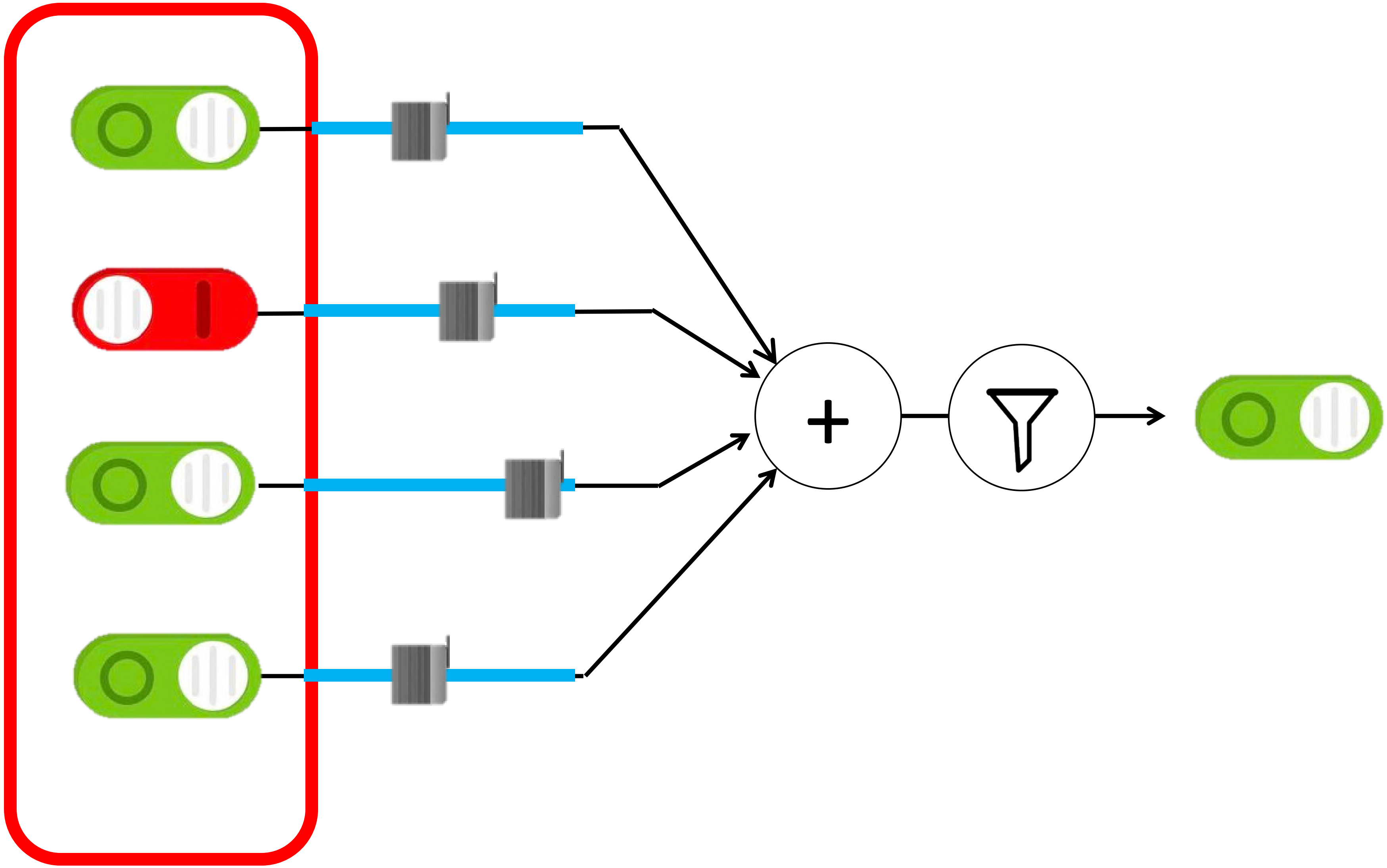
Inputs



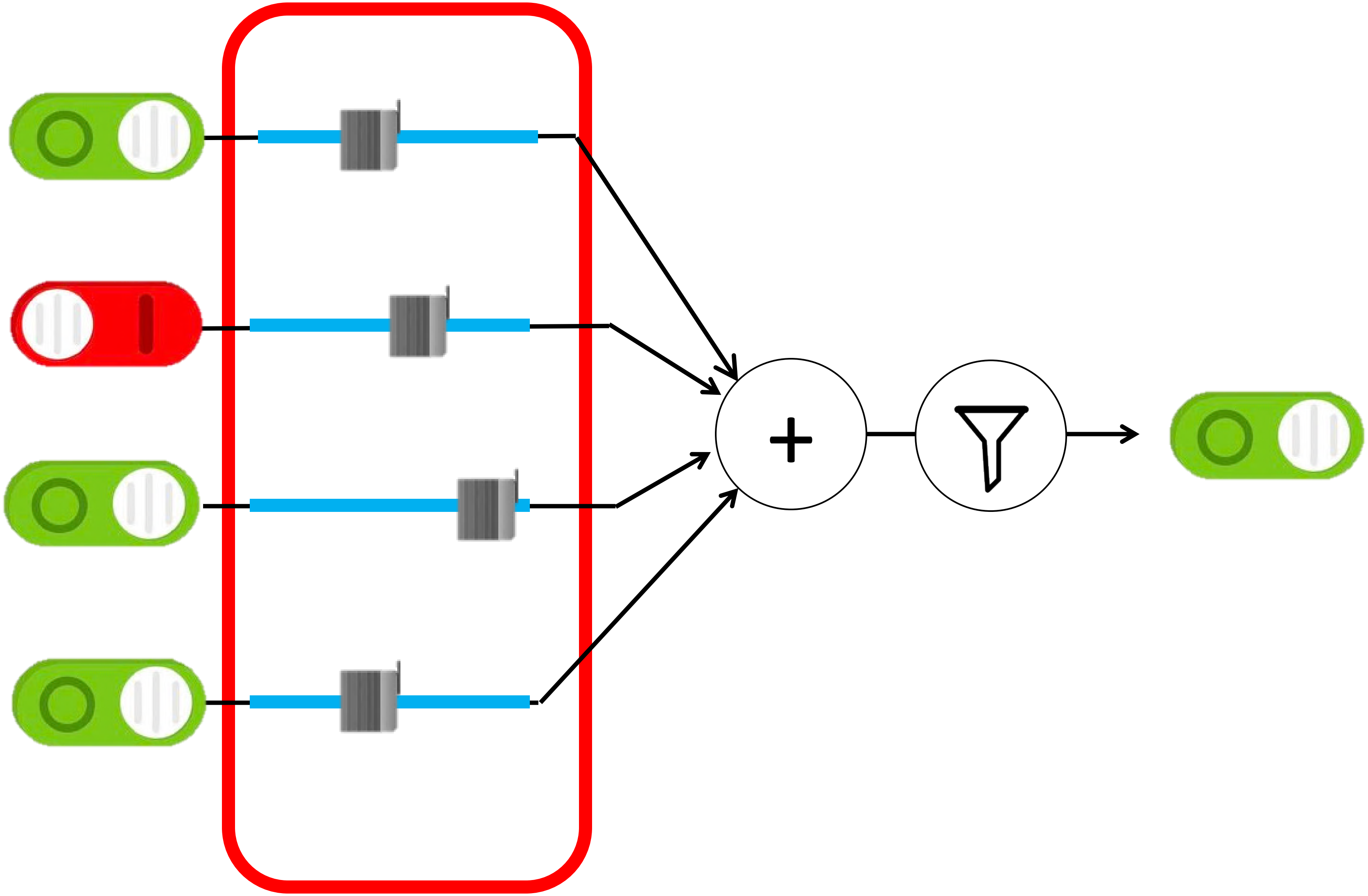
Inputs



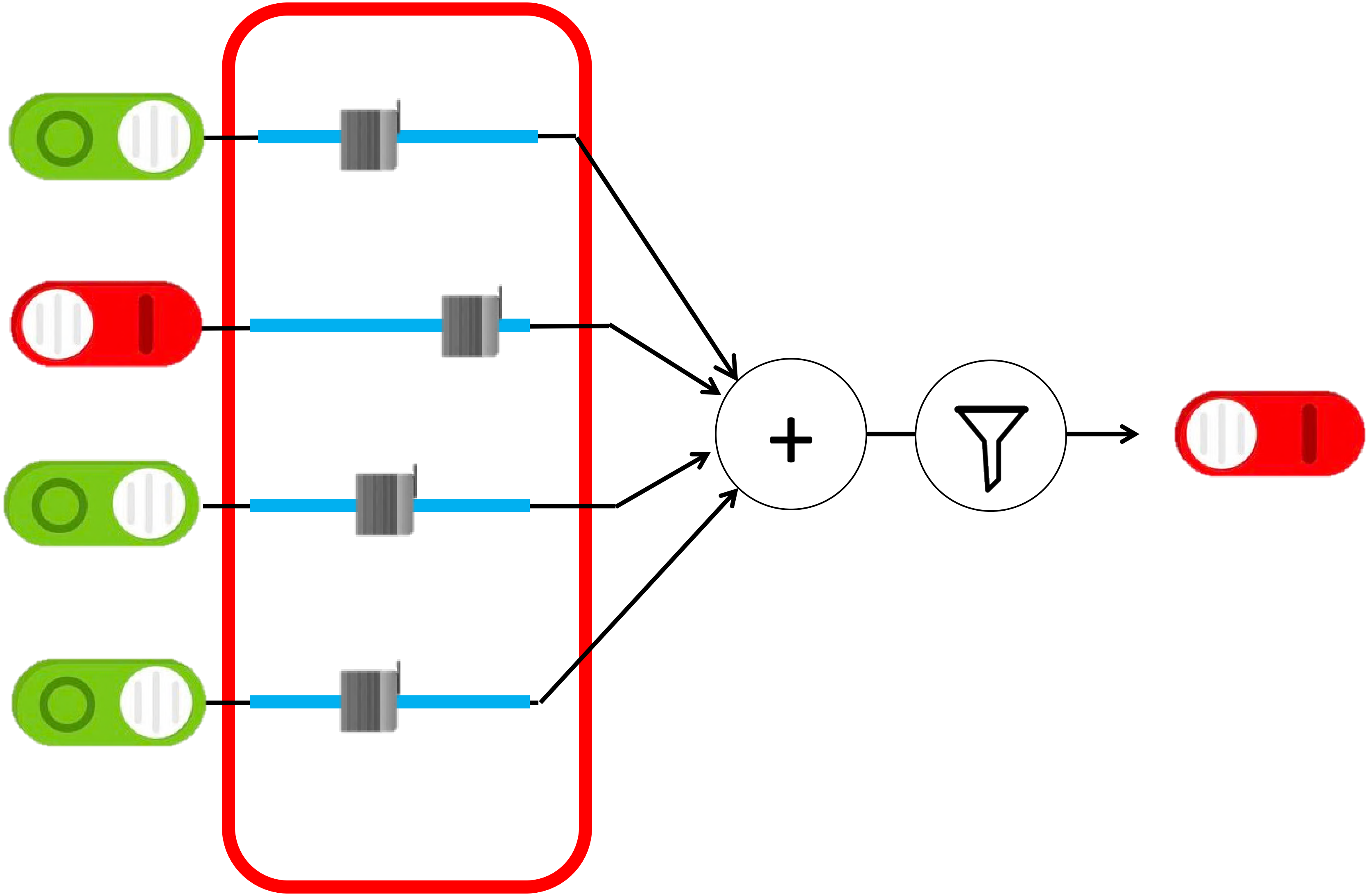
Inputs



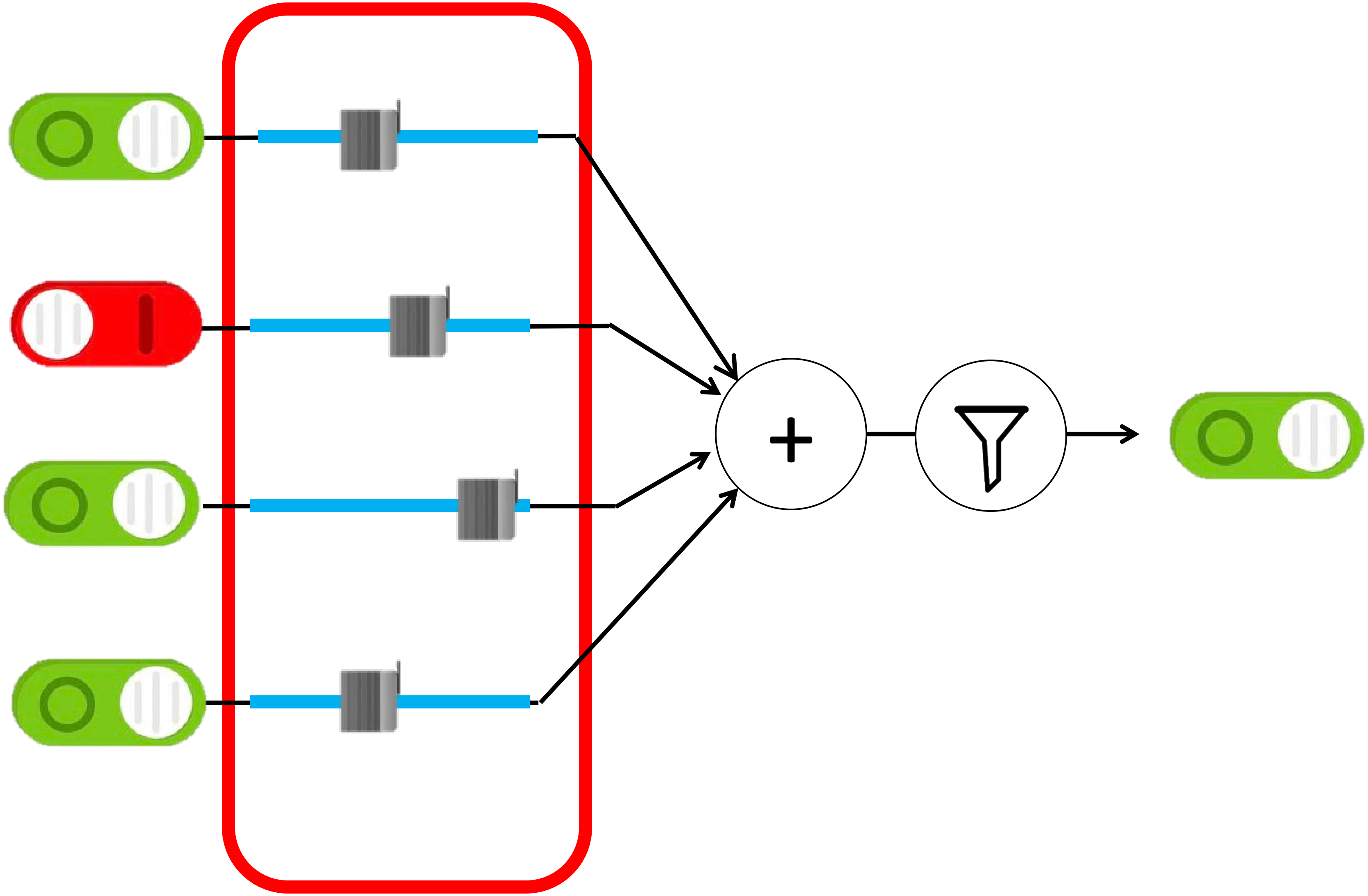
Weights



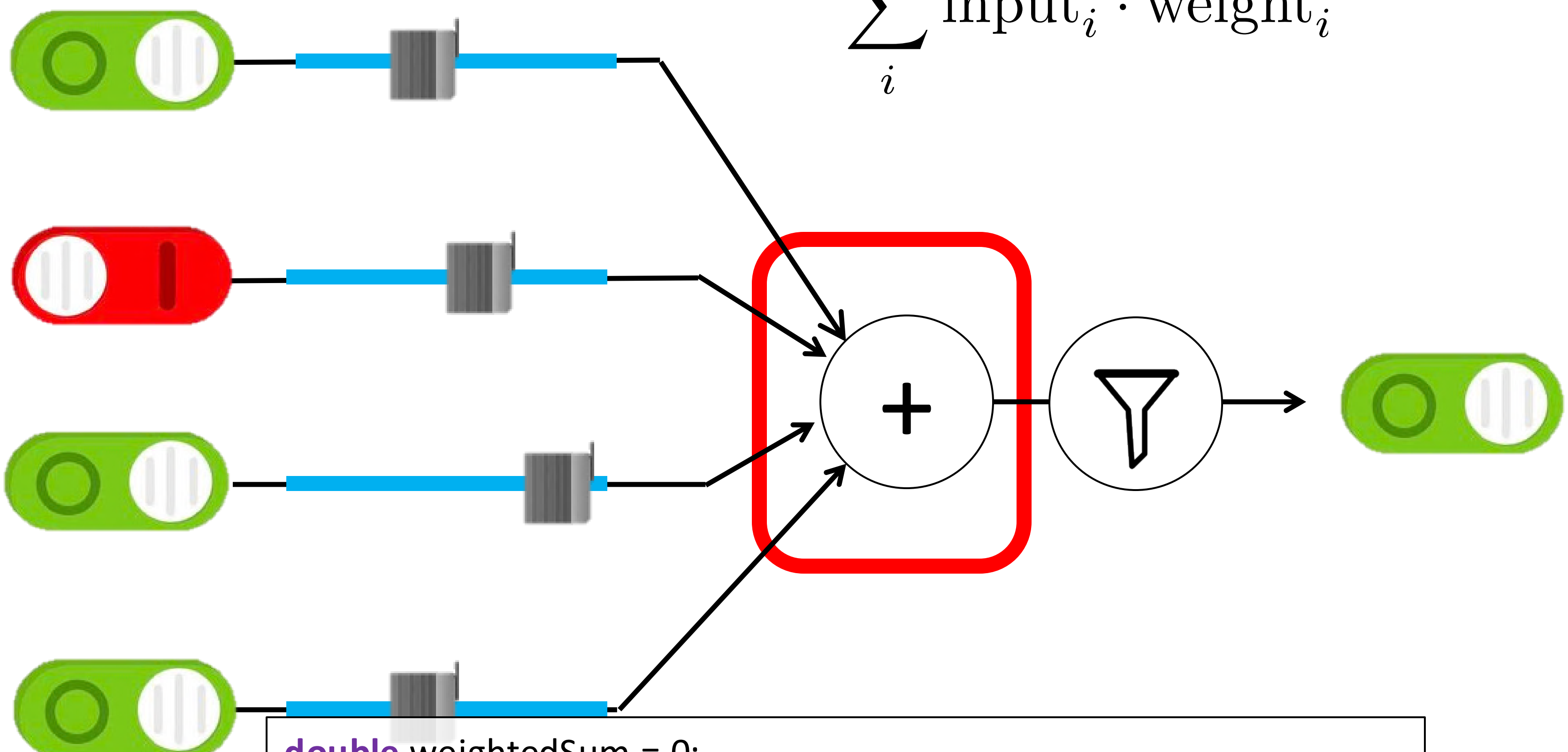
Weights



Weights



Weighted Sum

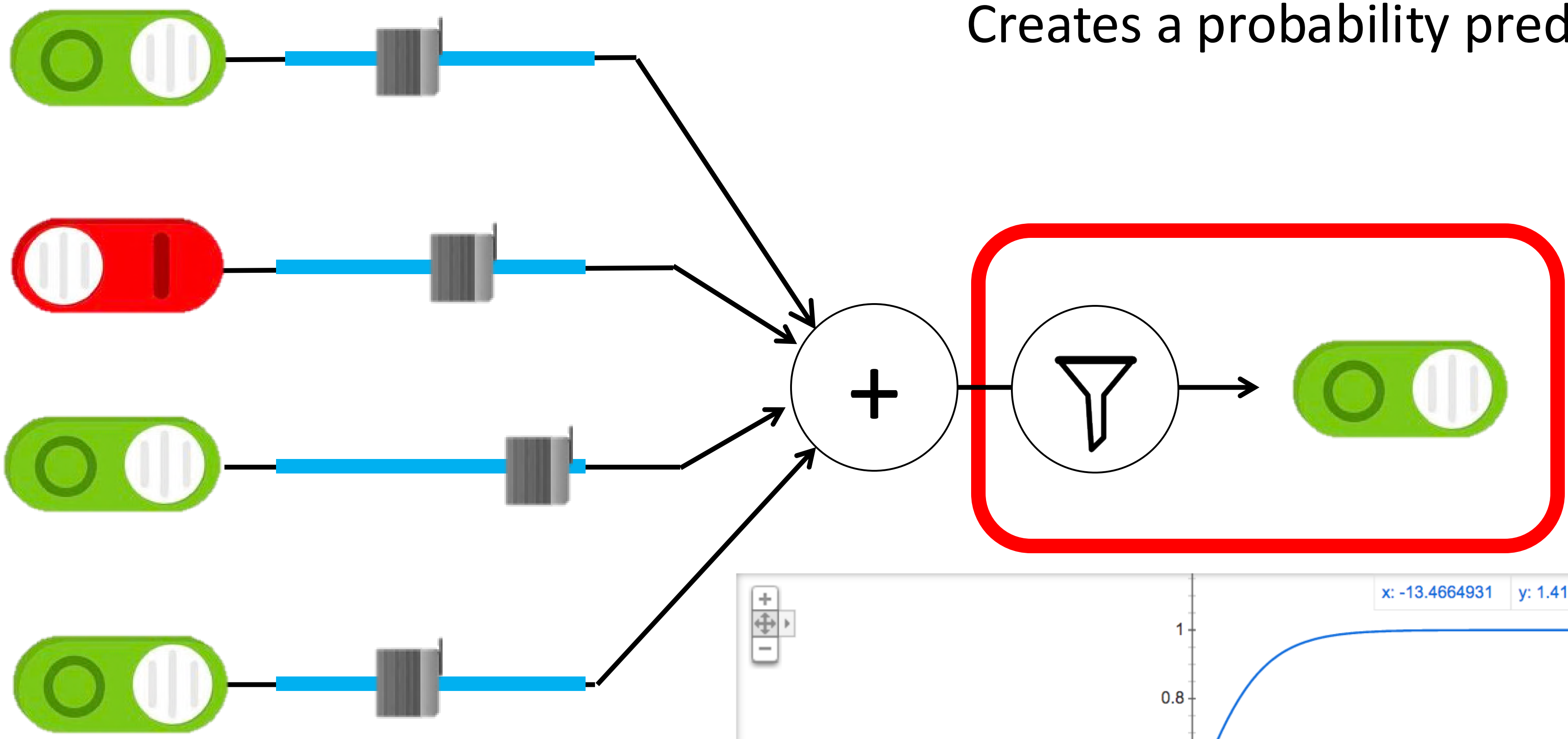


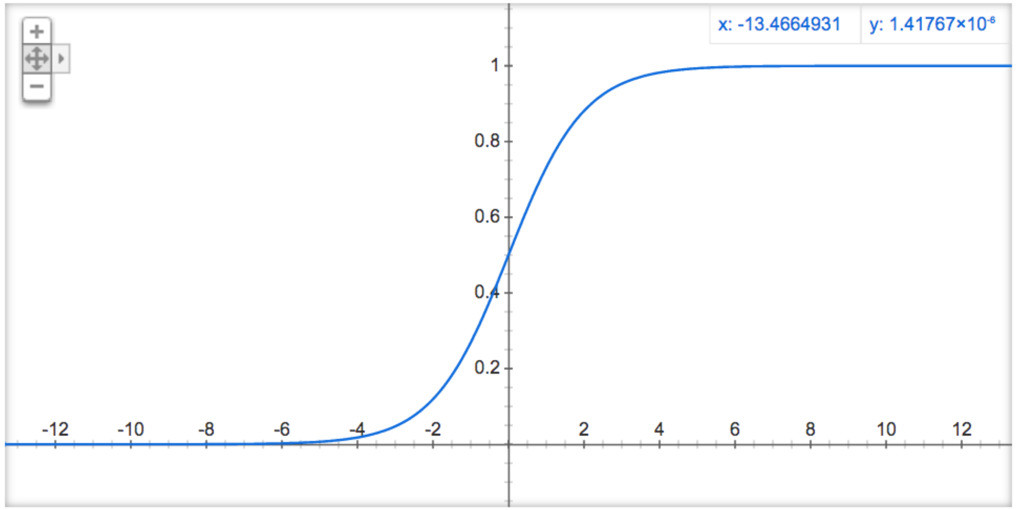
$$\sum_i \text{input}_i \cdot \text{weight}_i$$

```
double weightedSum = 0;
weightedSum += input0 * weight0;
weightedSum += input1 * weight1;
weightedSum += input2 * weight2;
weightedSum += input3 * weight3;
```

Filter and Output

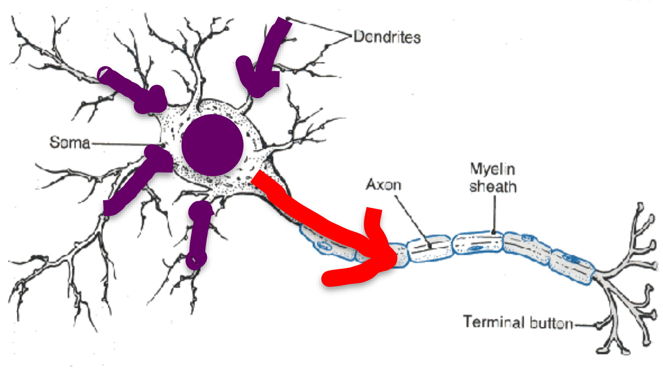
Creates a probability prediction

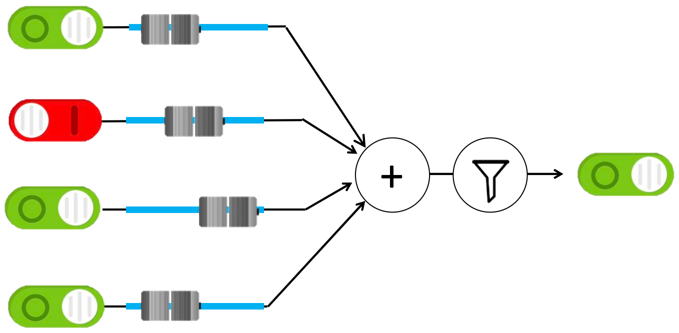




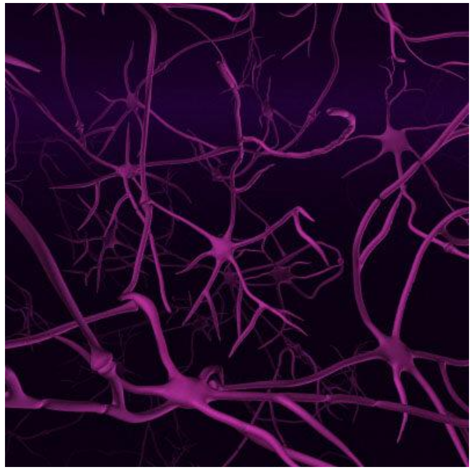
Biological Basis for Neural Networks

- A neuron





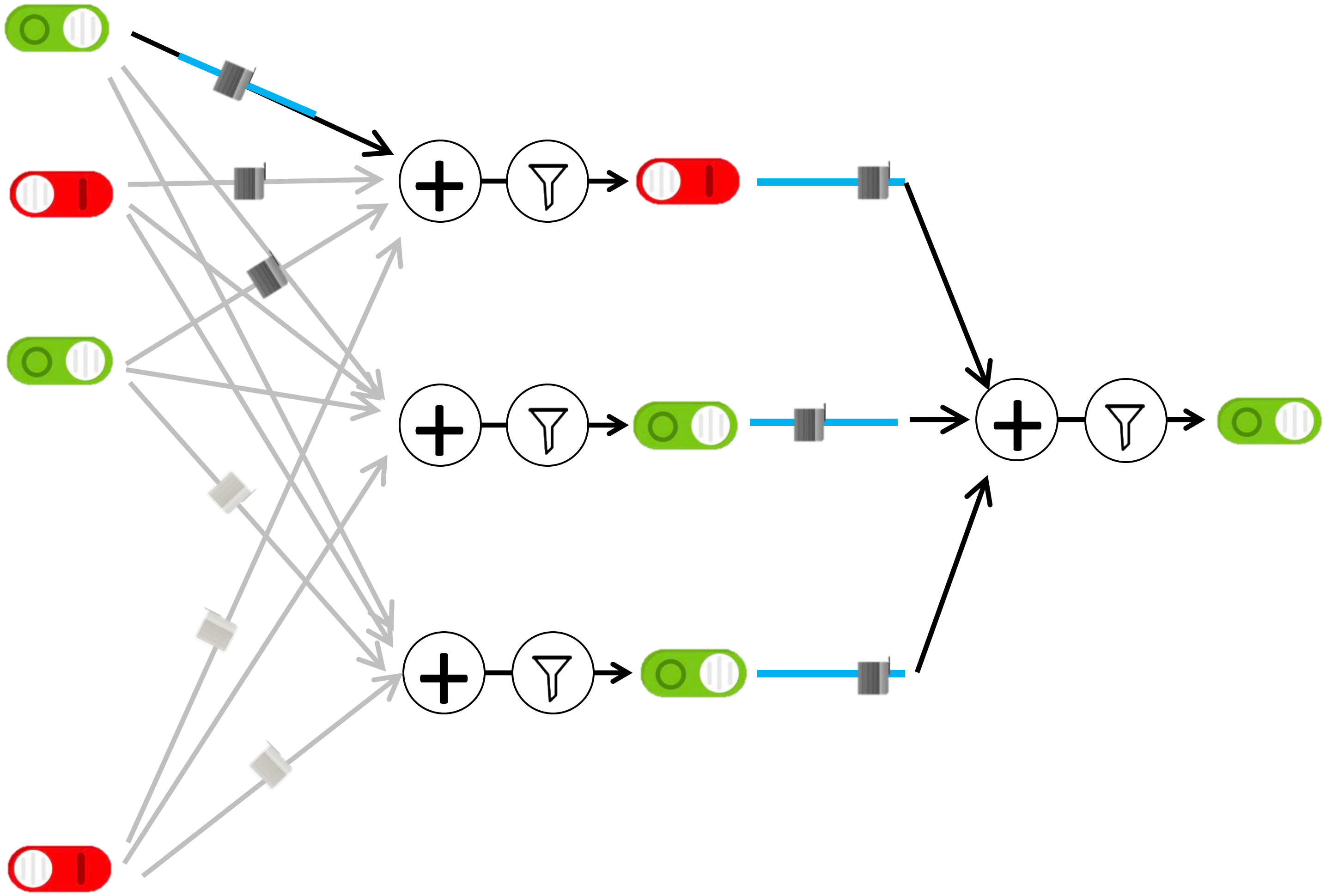
- Your brain



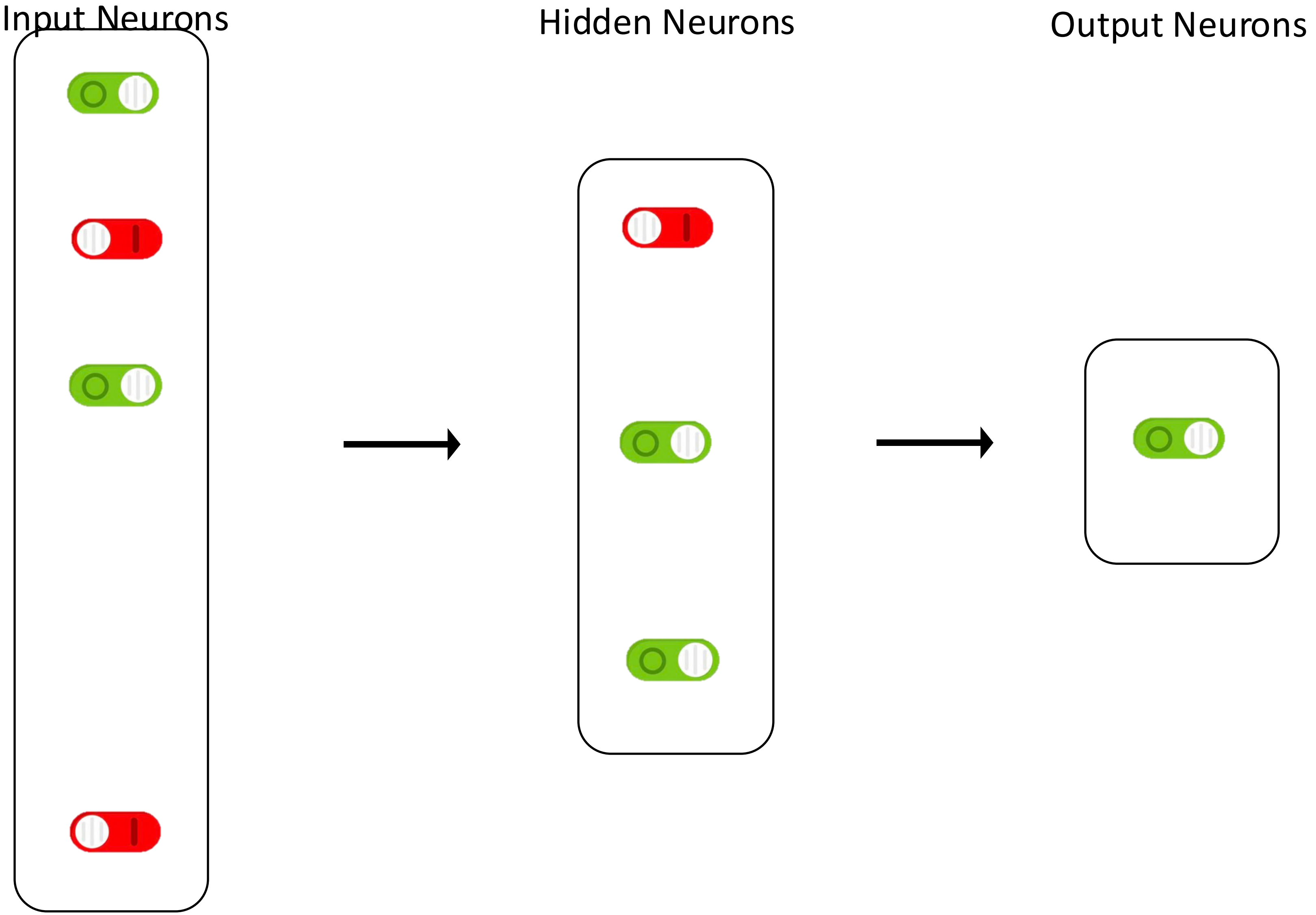
???

Actually, it's probably someone else's brain

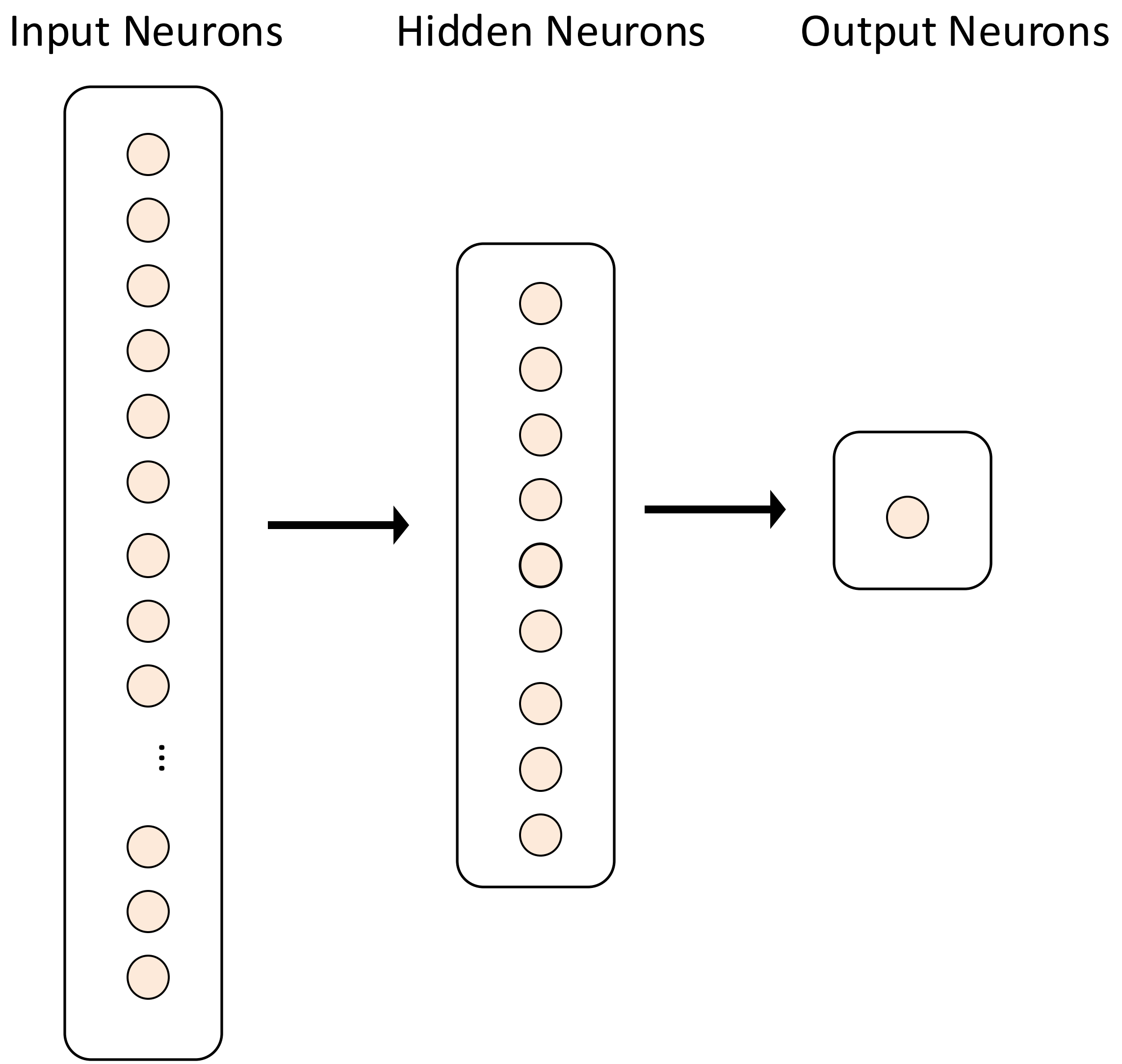
Put Many Together



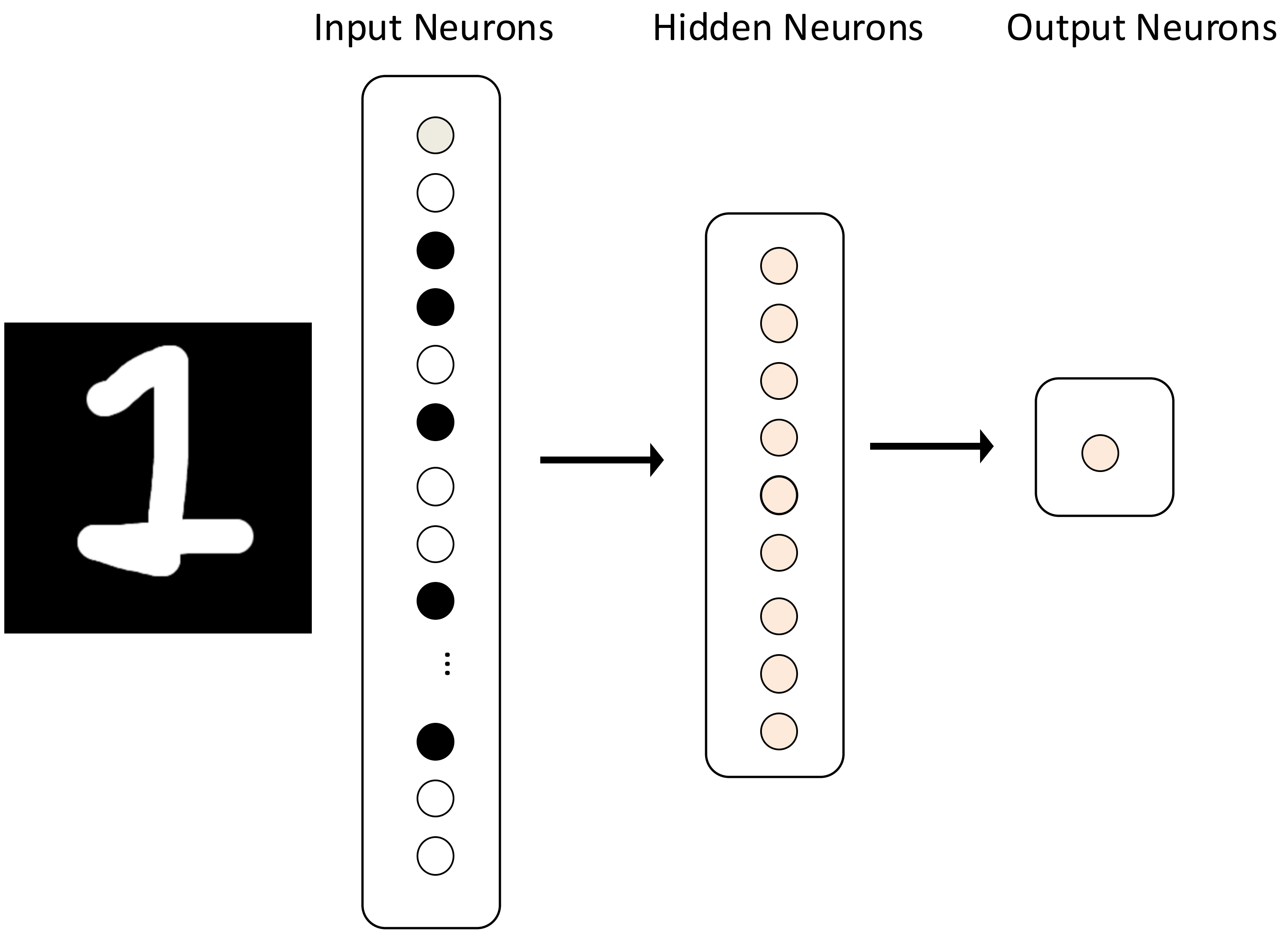
Put Many Together



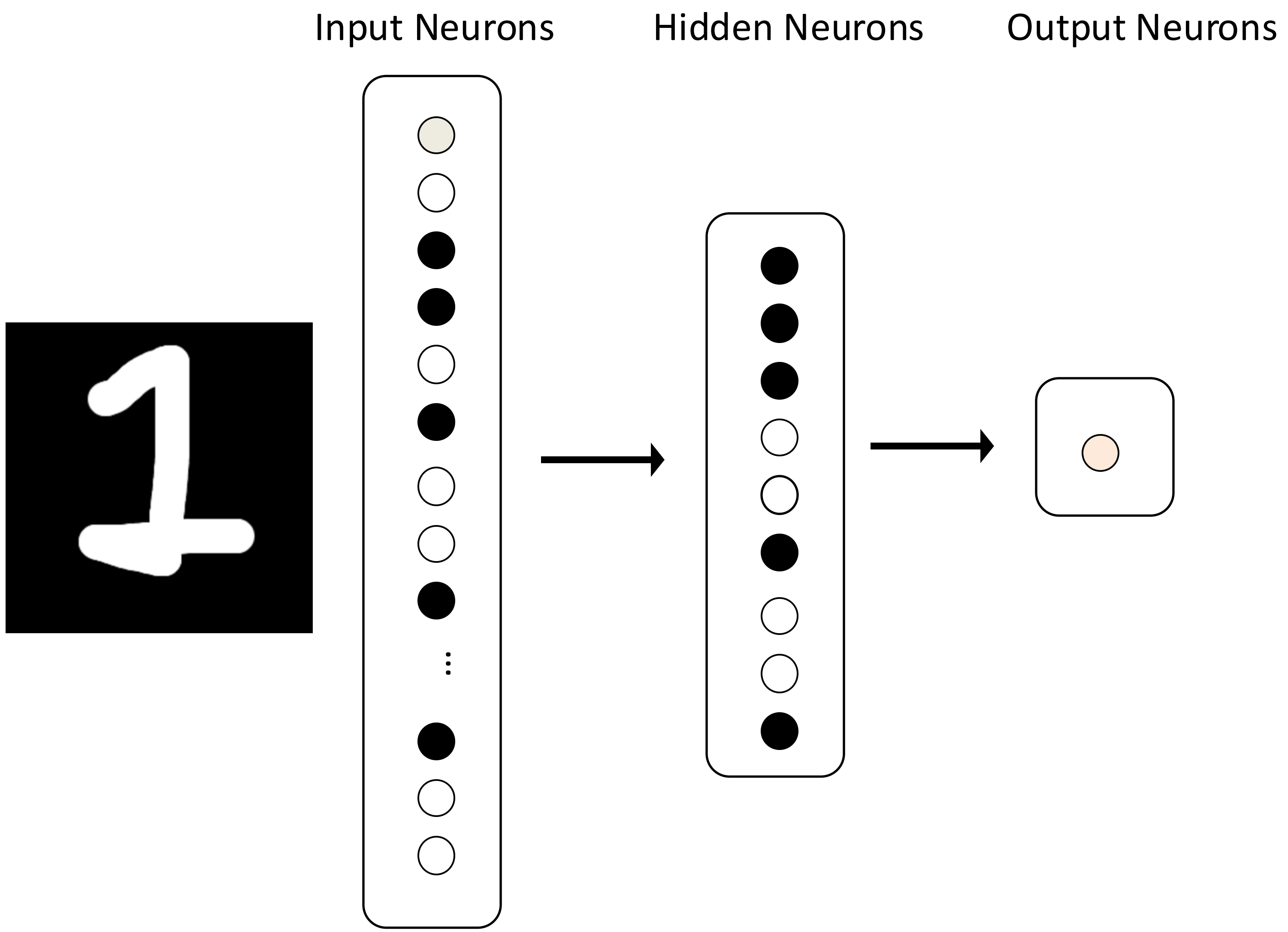
Making a Prediction



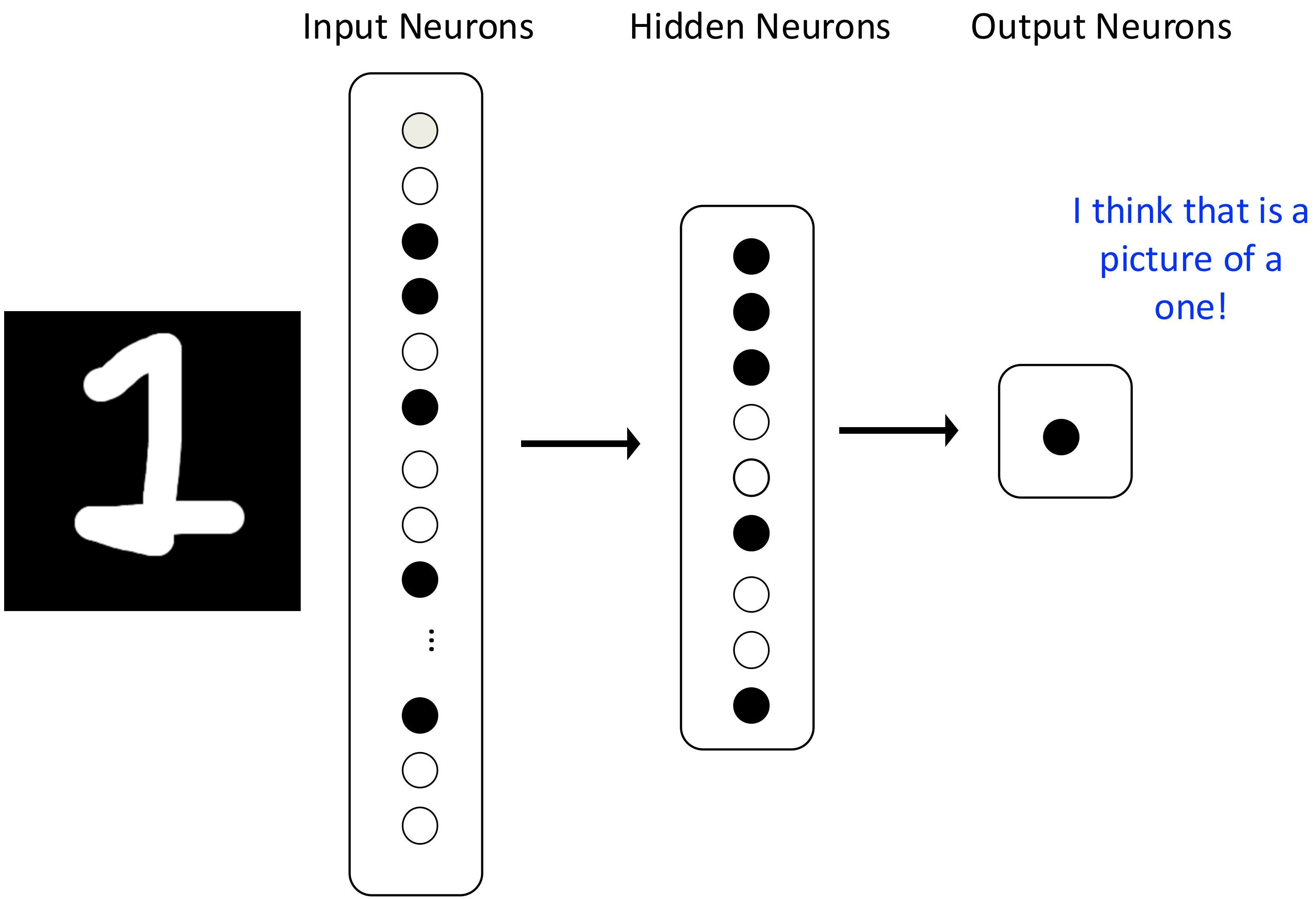
Making a Prediction



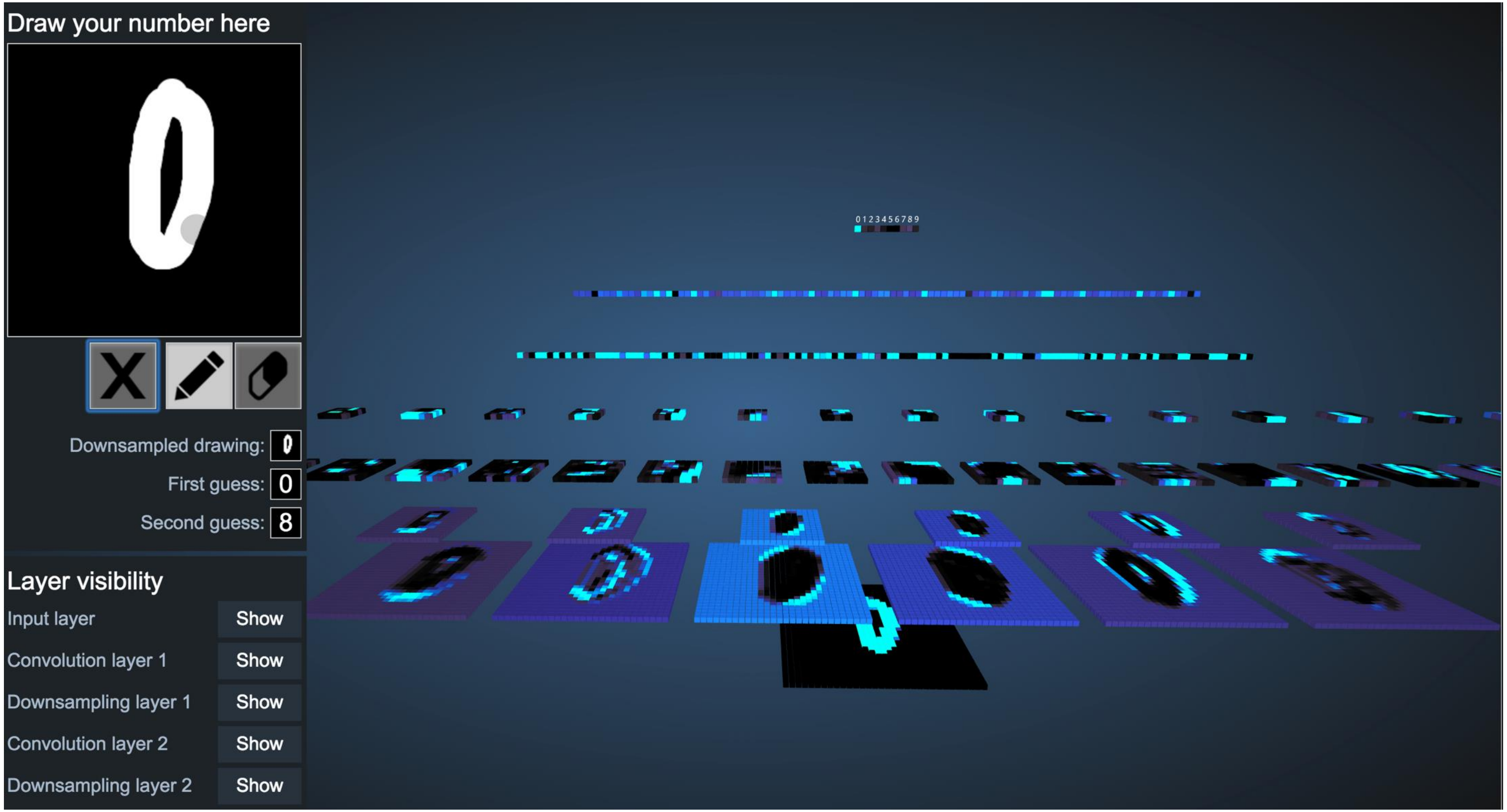
Making a Prediction



Making a Prediction

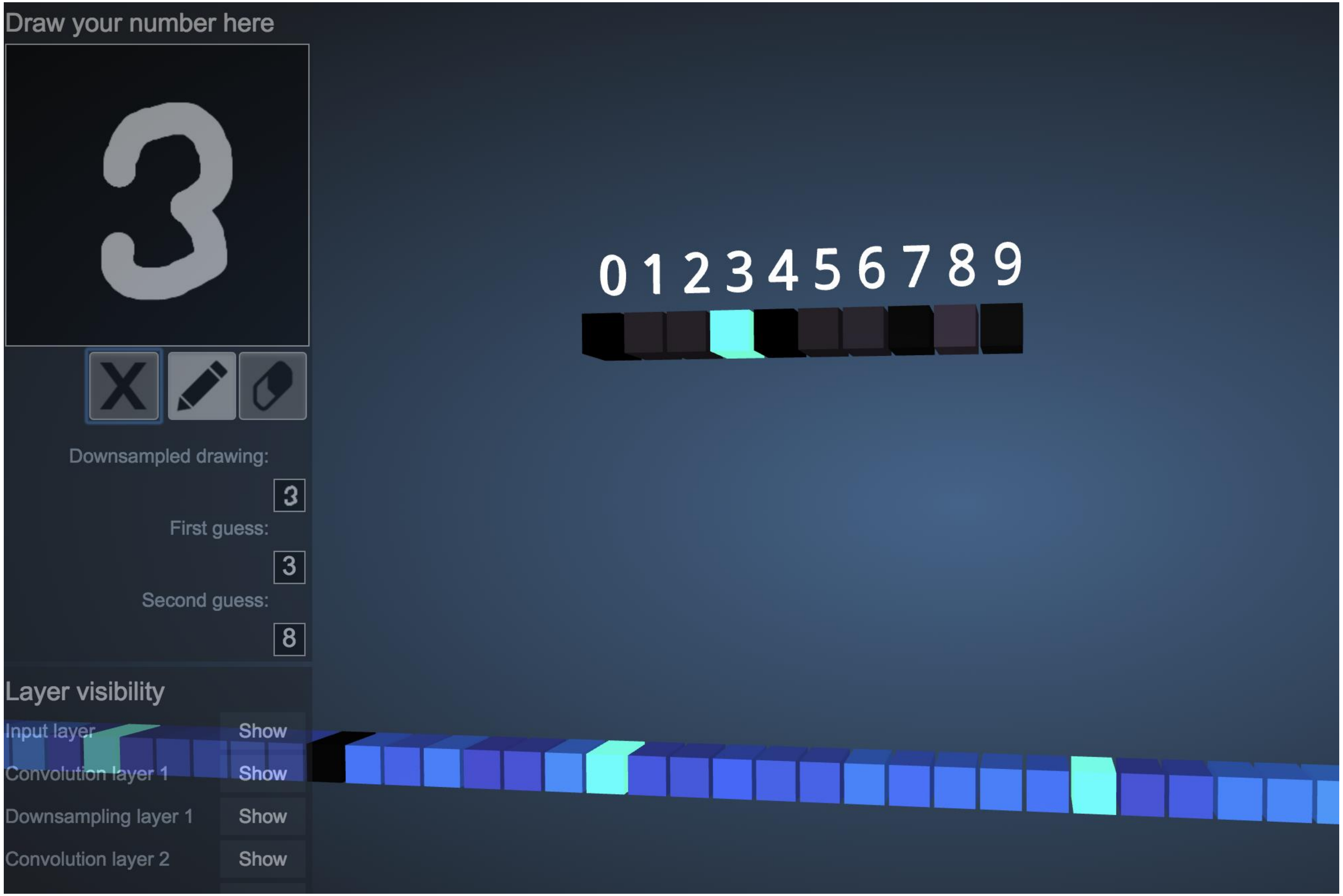


Demonstration

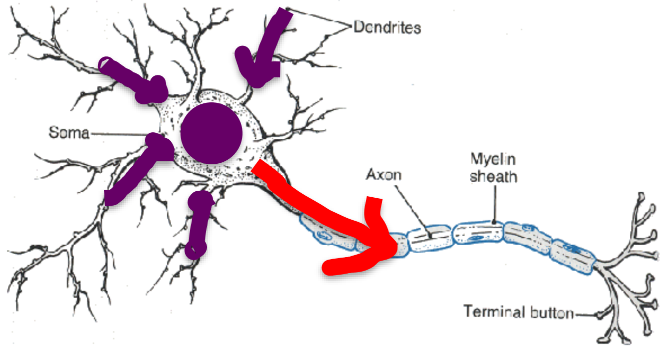


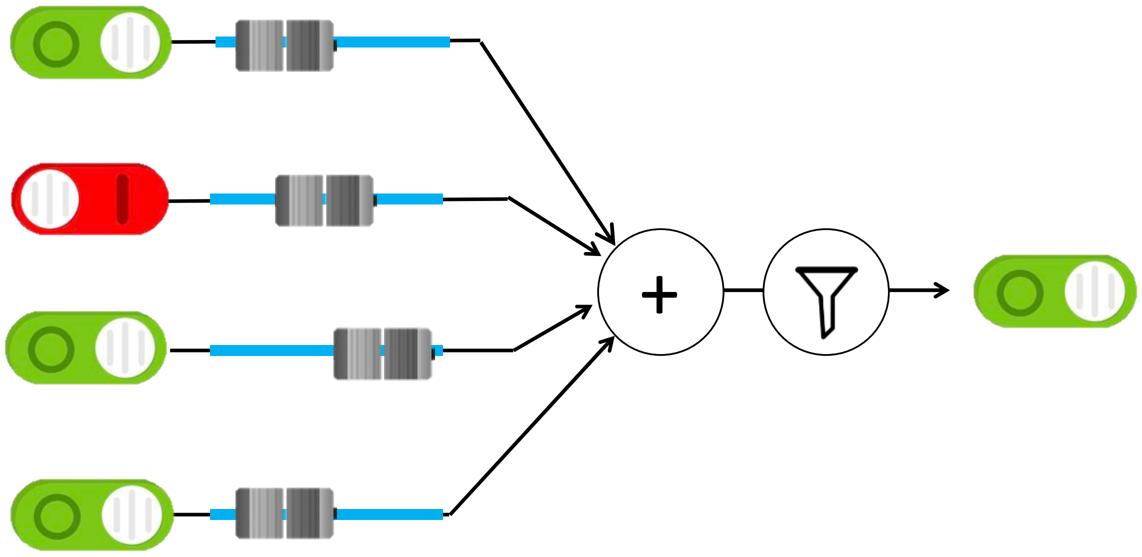
https://web.archive.org/web/20211117115916/https://www.cs.ryerson.ca/~aharley/vis/conv/

Interpret the Output as Prediction

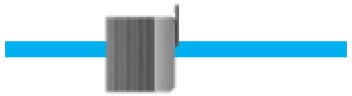


Great Idea: Artificial Neurons





Where do Artificial Neural Networks get their intelligence from?



Neural Networks get their intelligence from their sliders (parameters)

Two Great Ideas

1. Artificial Neurons

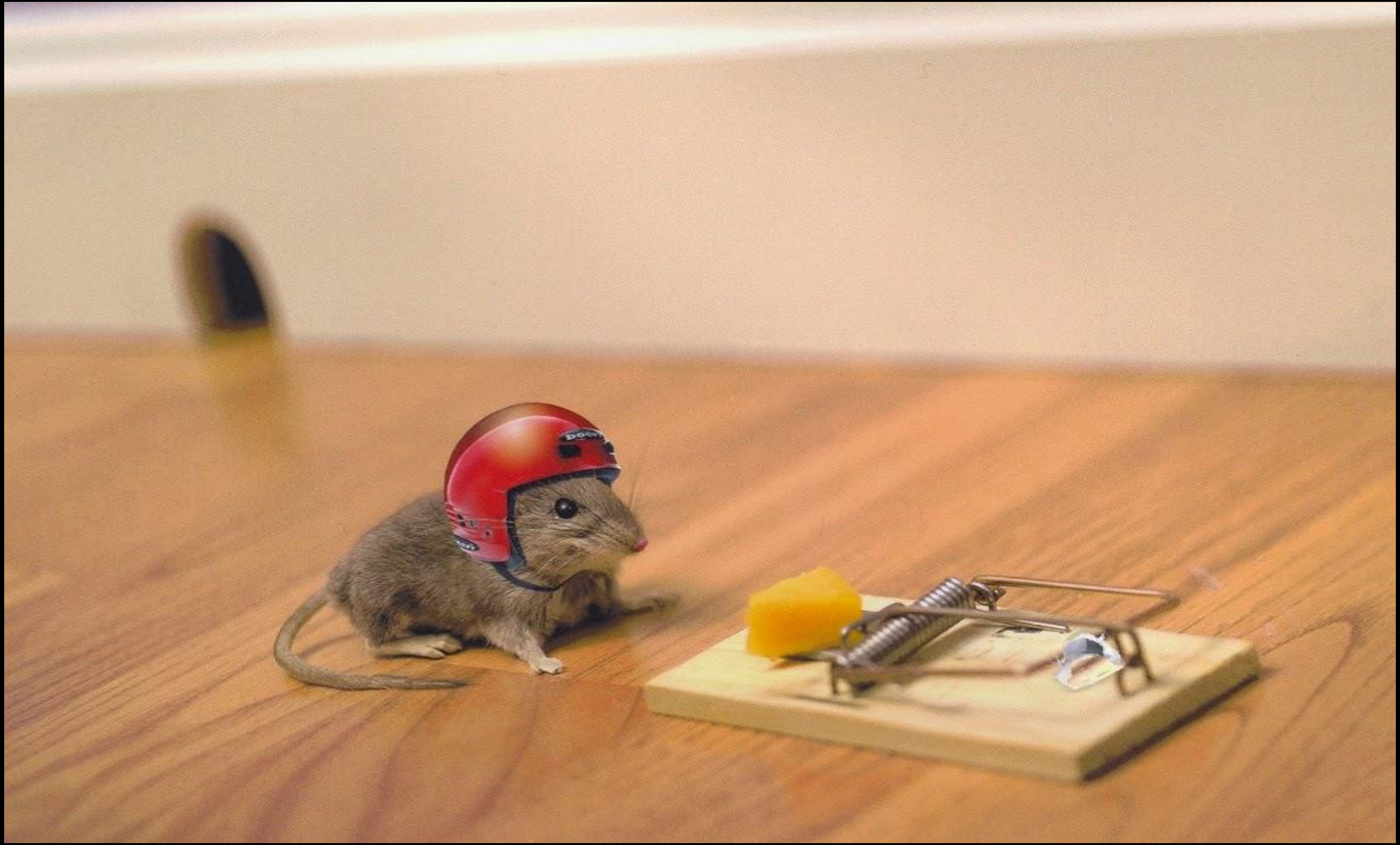
2. Learn by Example

Two Great Ideas

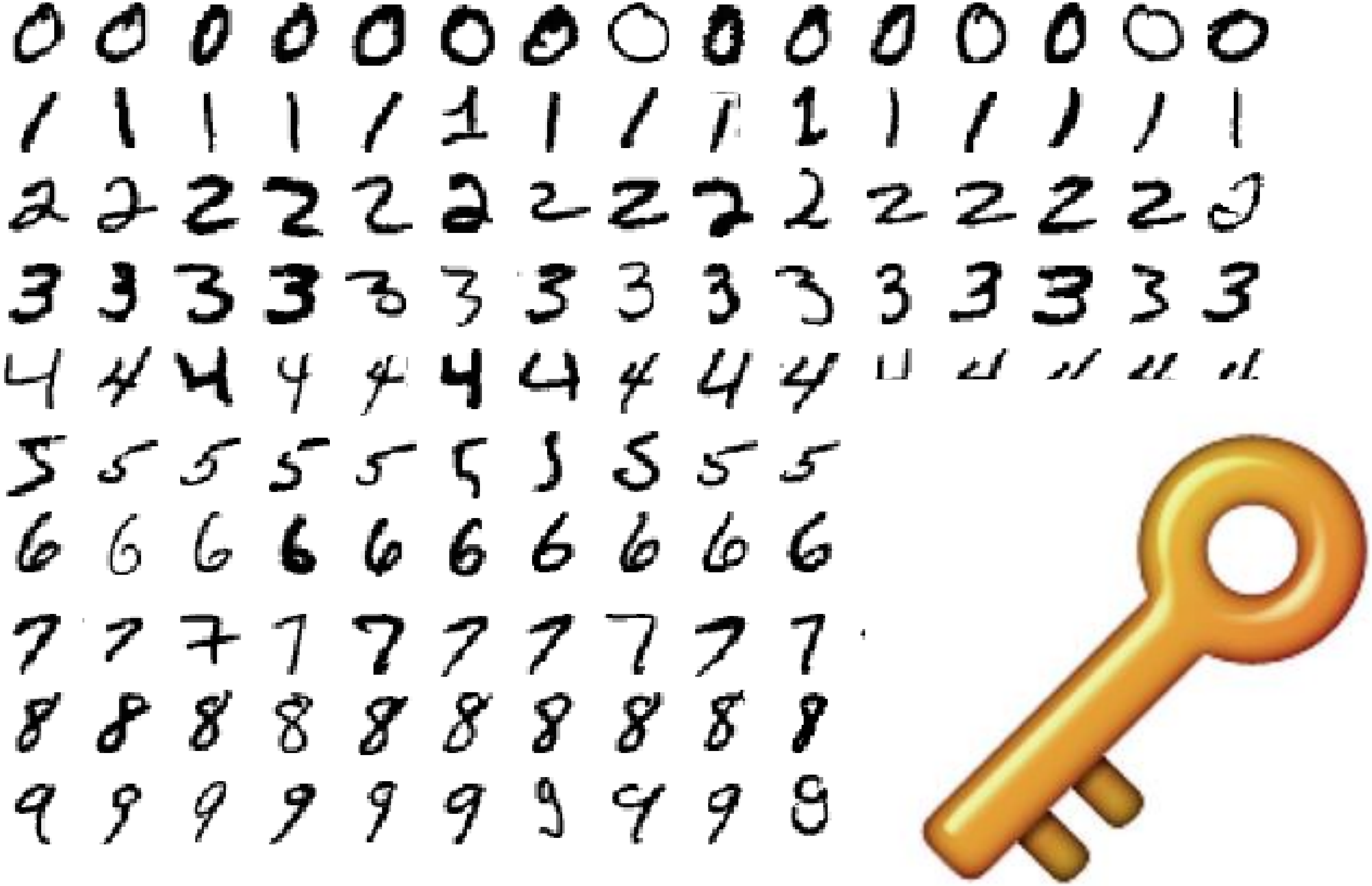
1. Artificial Neurons

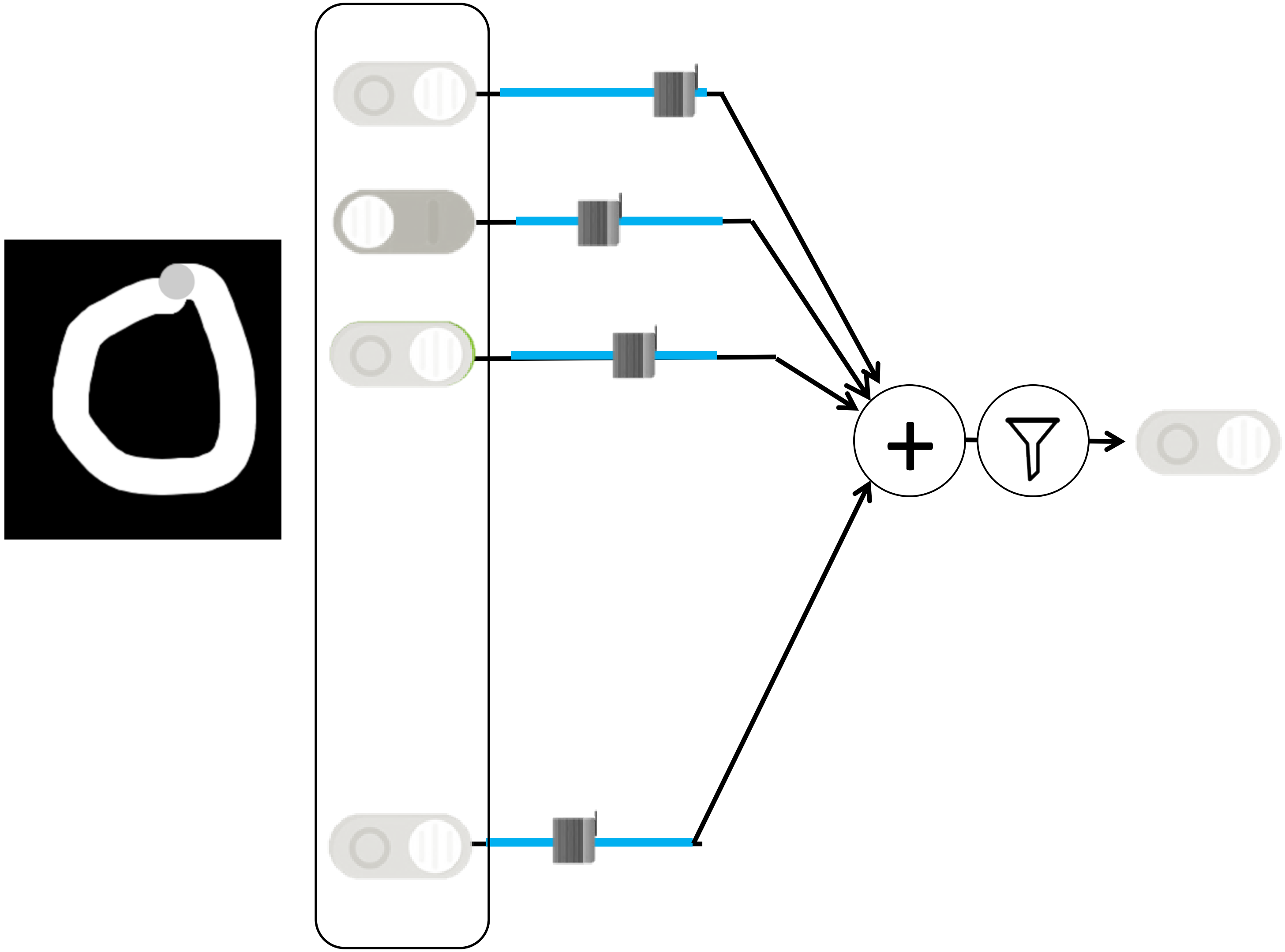
2. Learn by Example

2. Learn From Experience

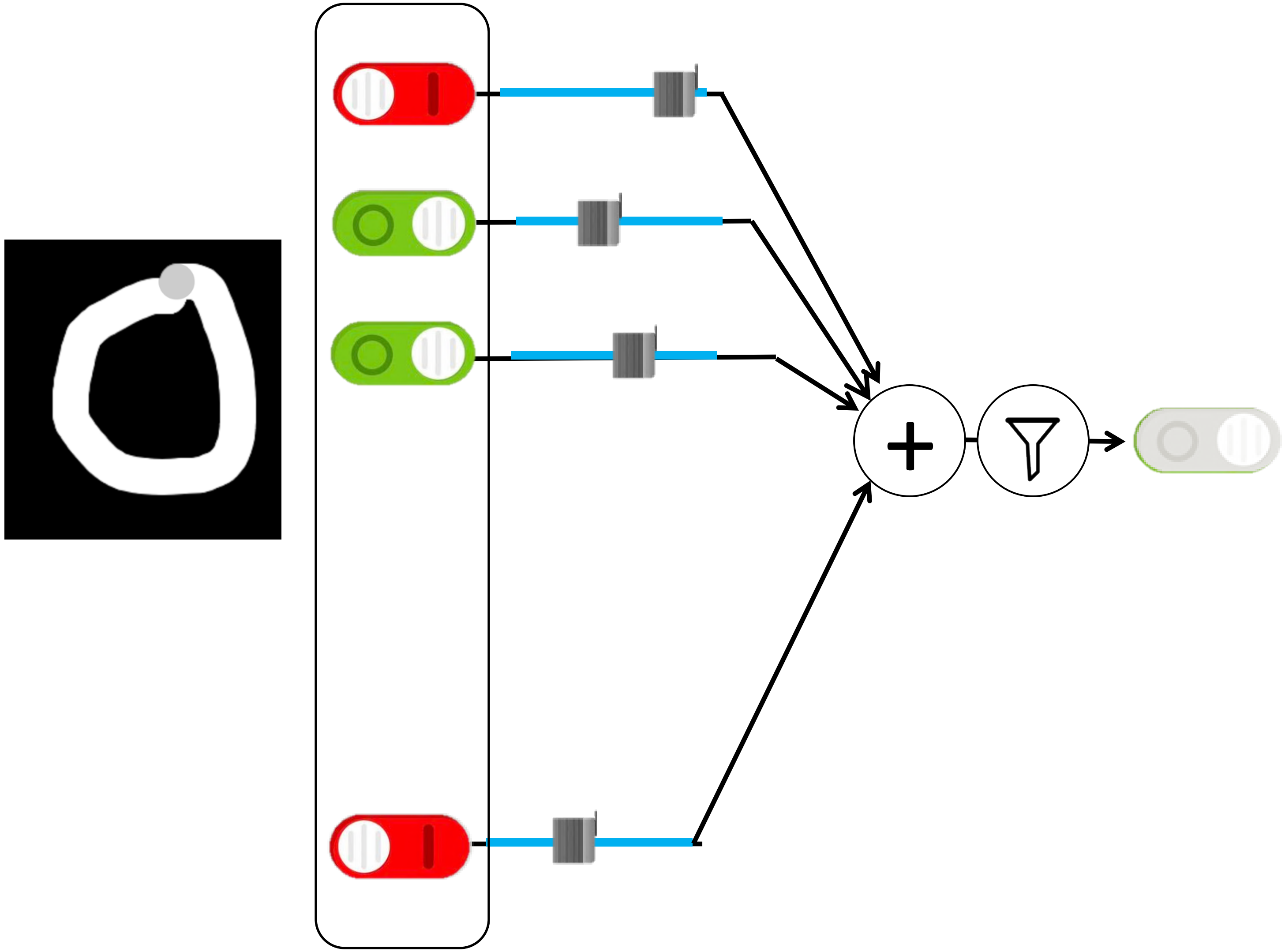


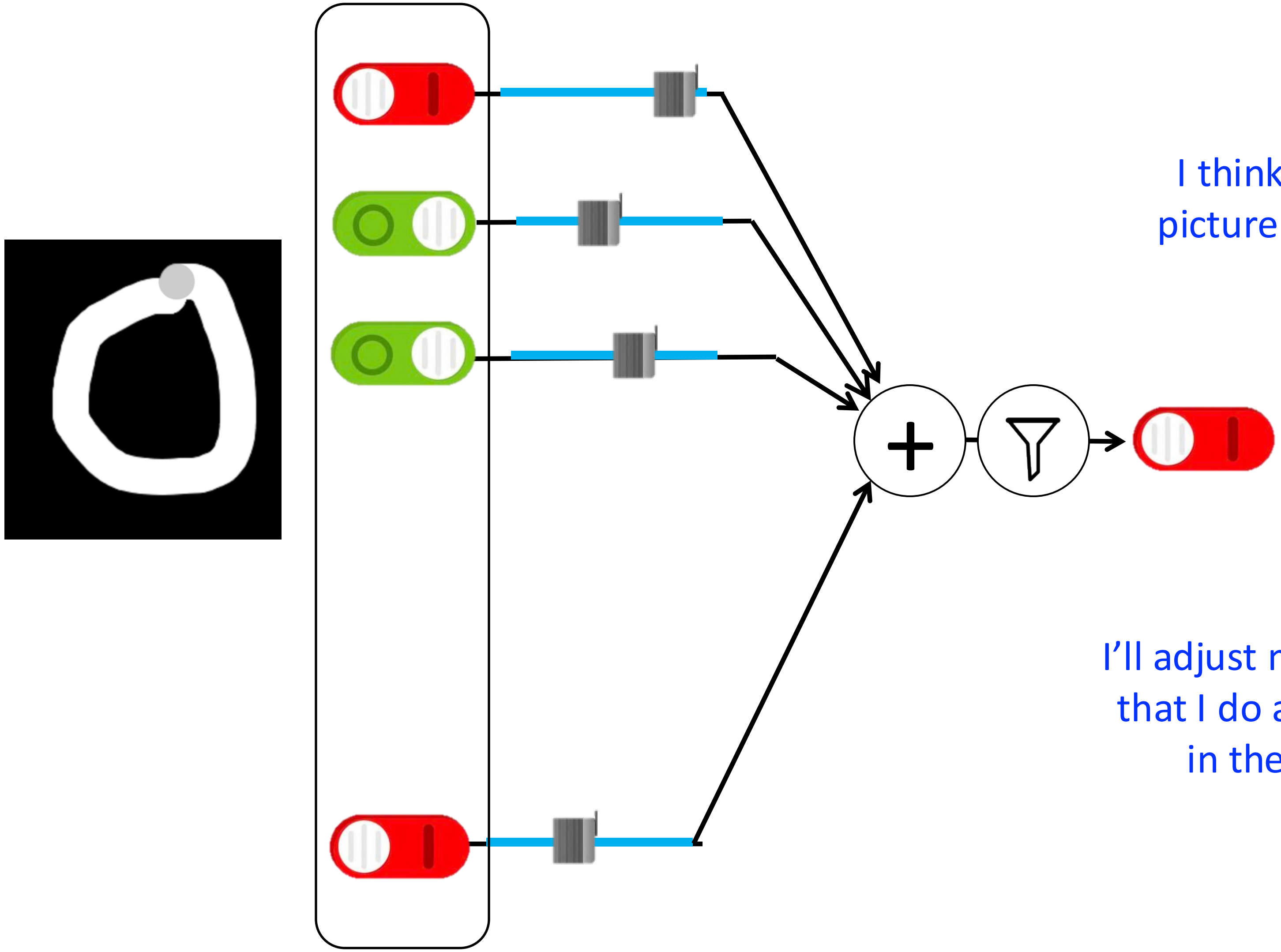
Learn by Example





Piech

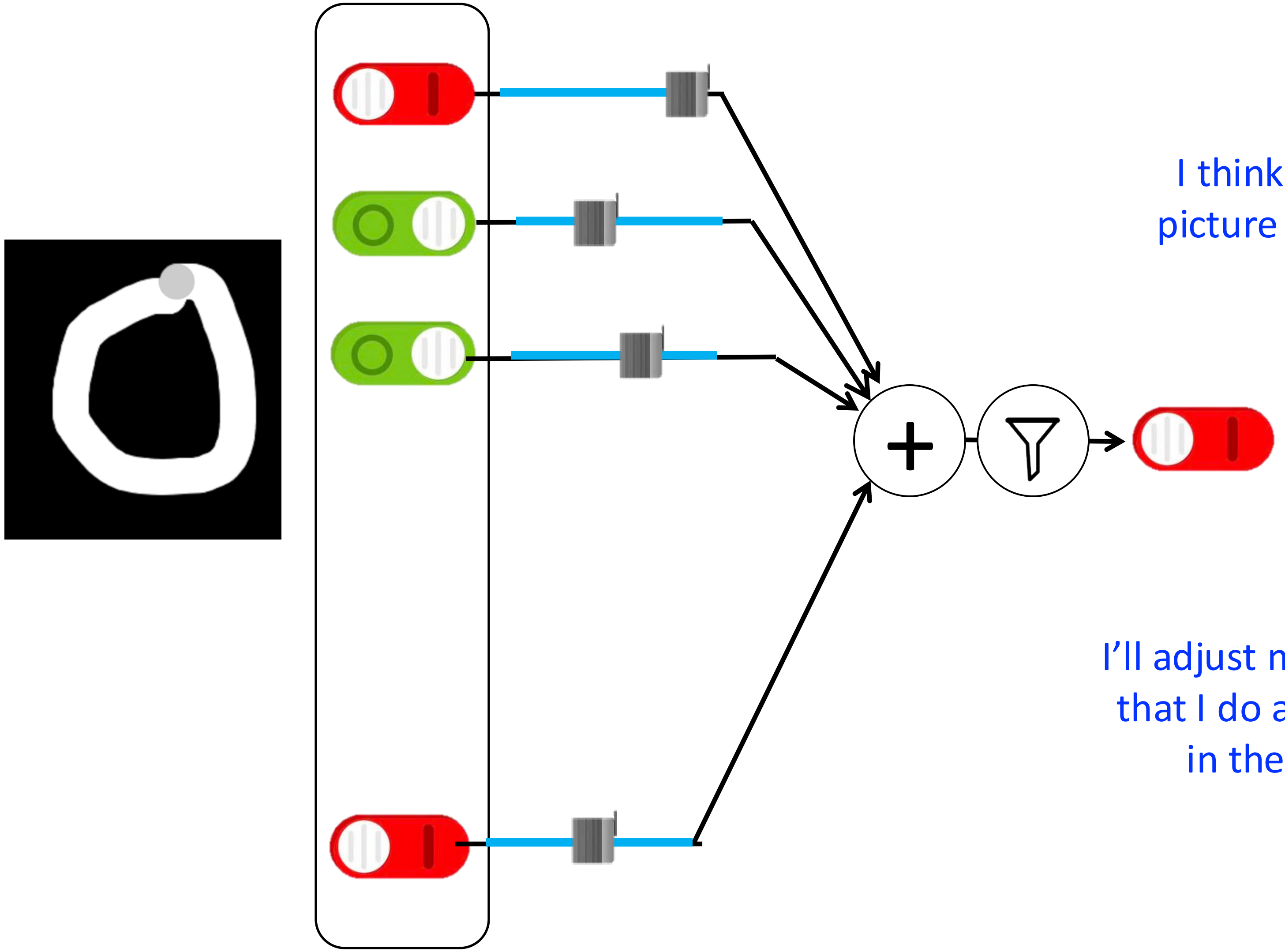




I think that is a
picture of a **One**!

What do you
mean it's actually
a **Zero**?

I'll adjust my sliders so
that I do a better job
in the future



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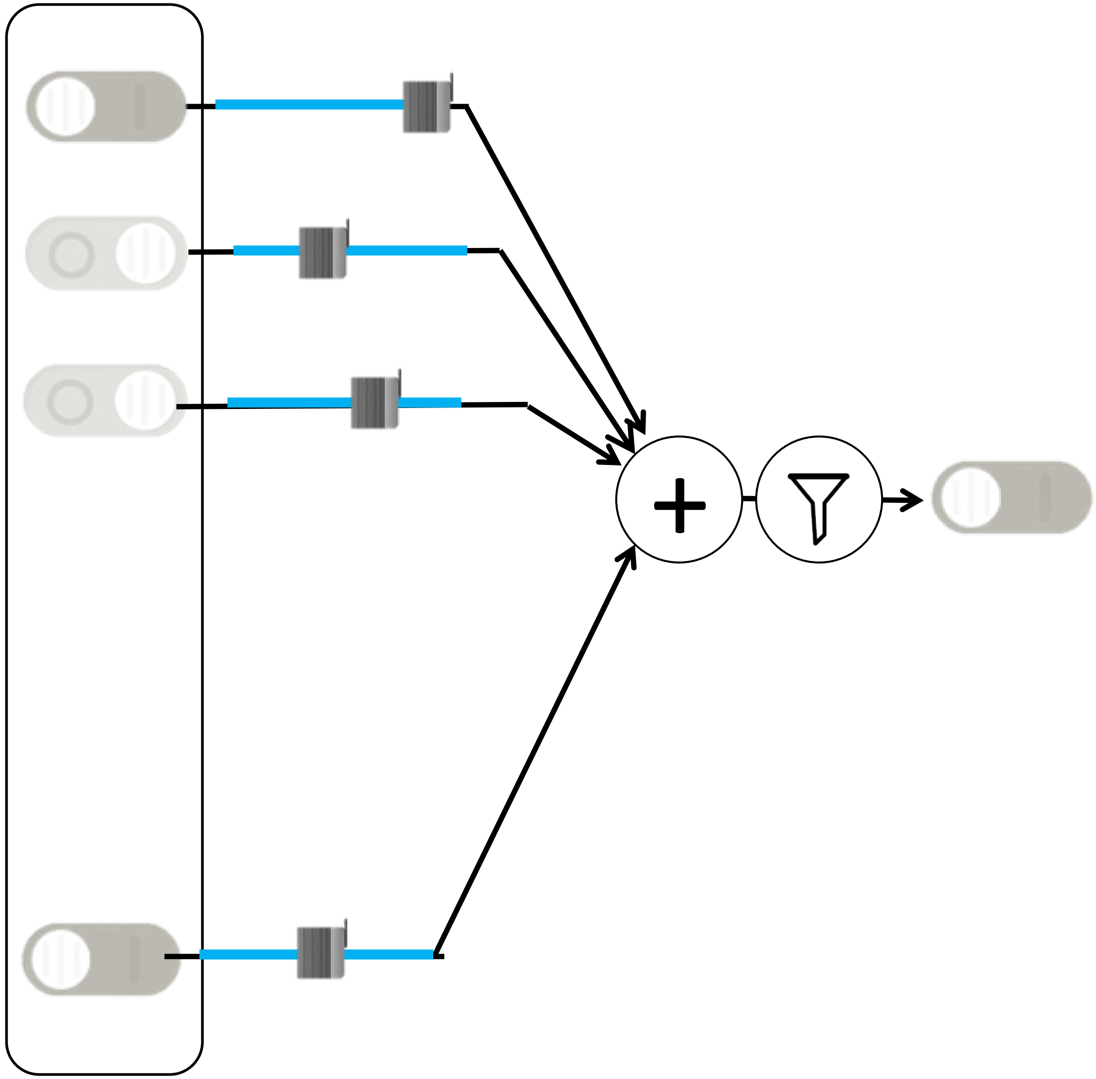
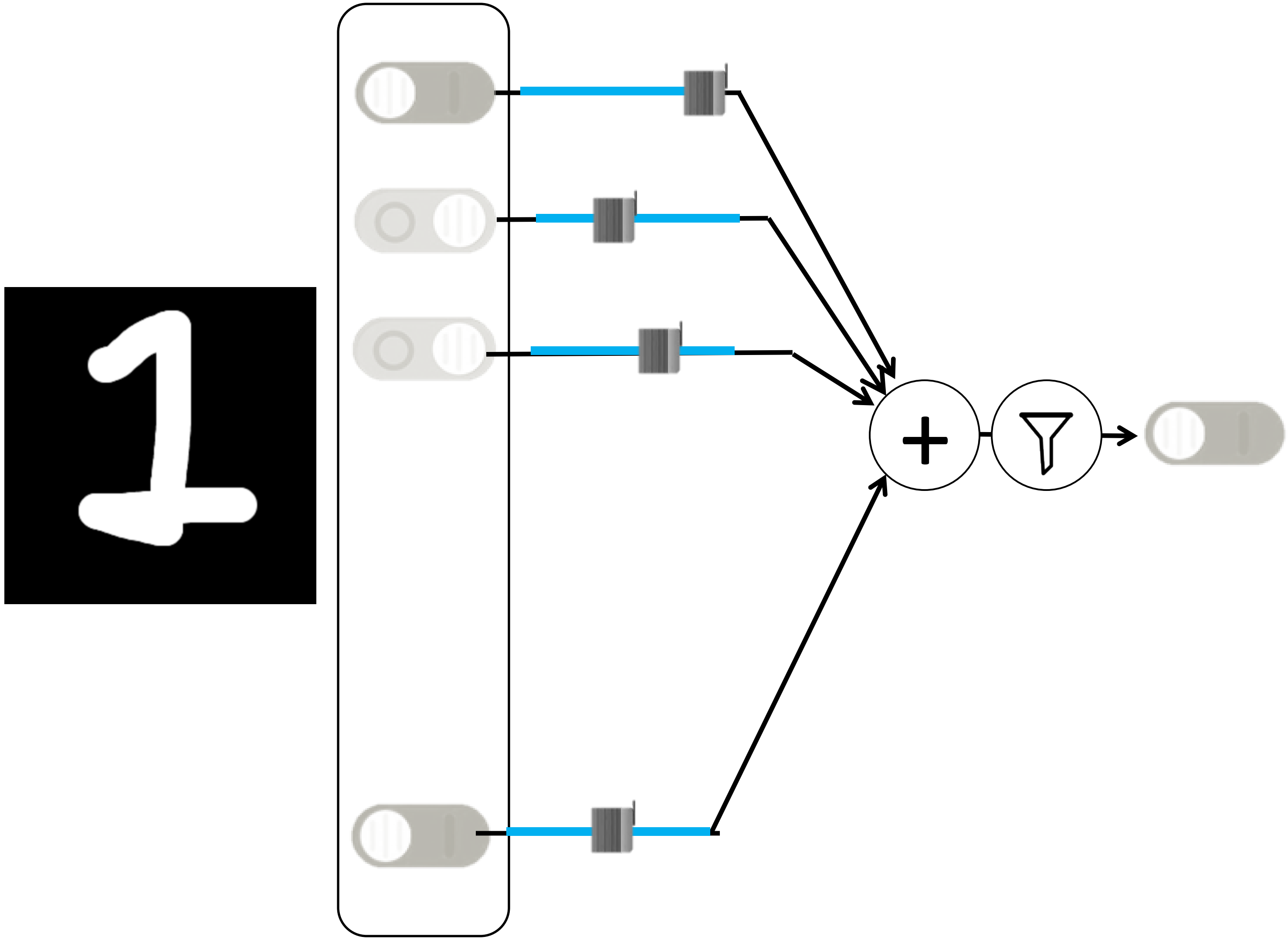
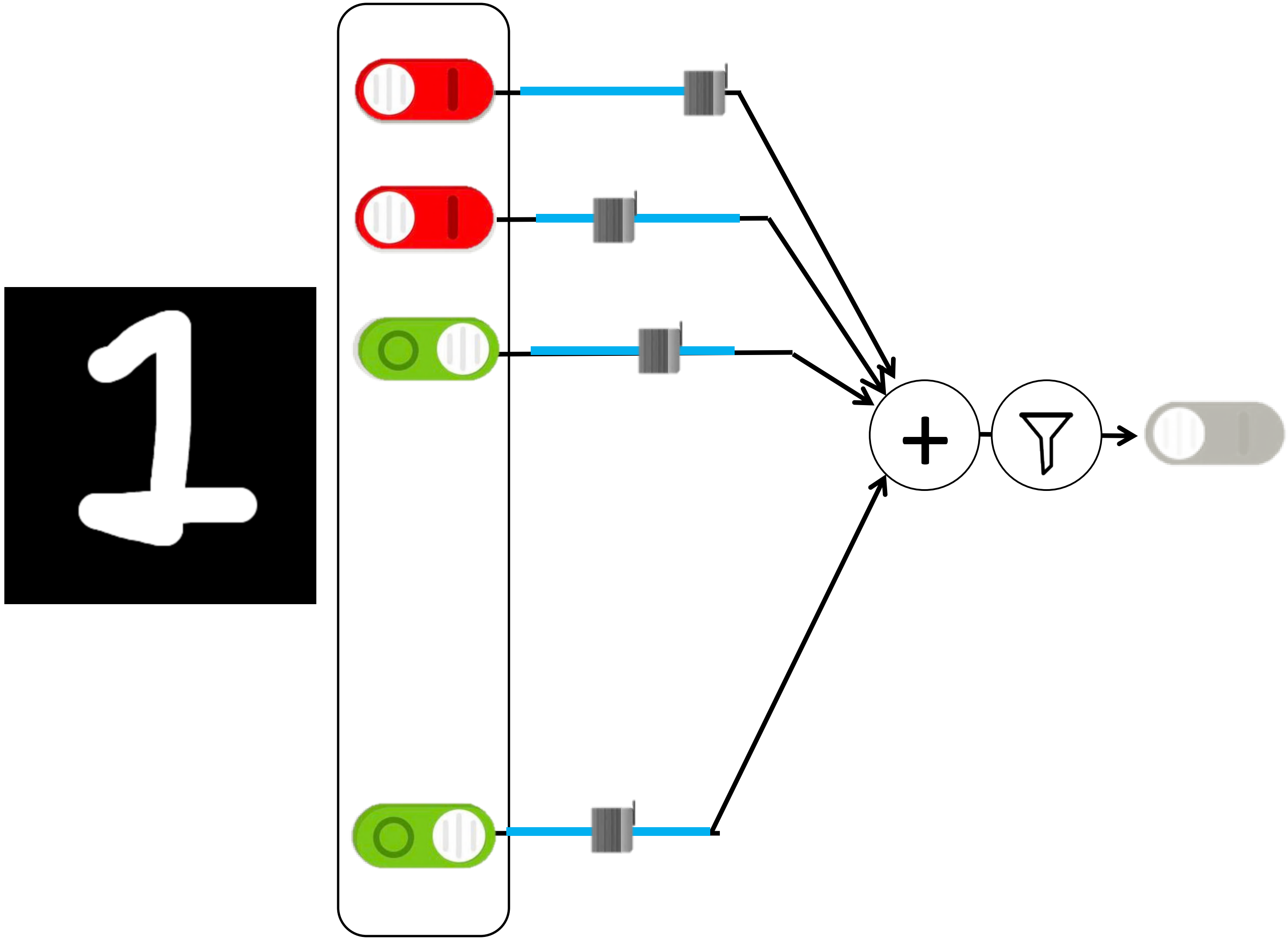
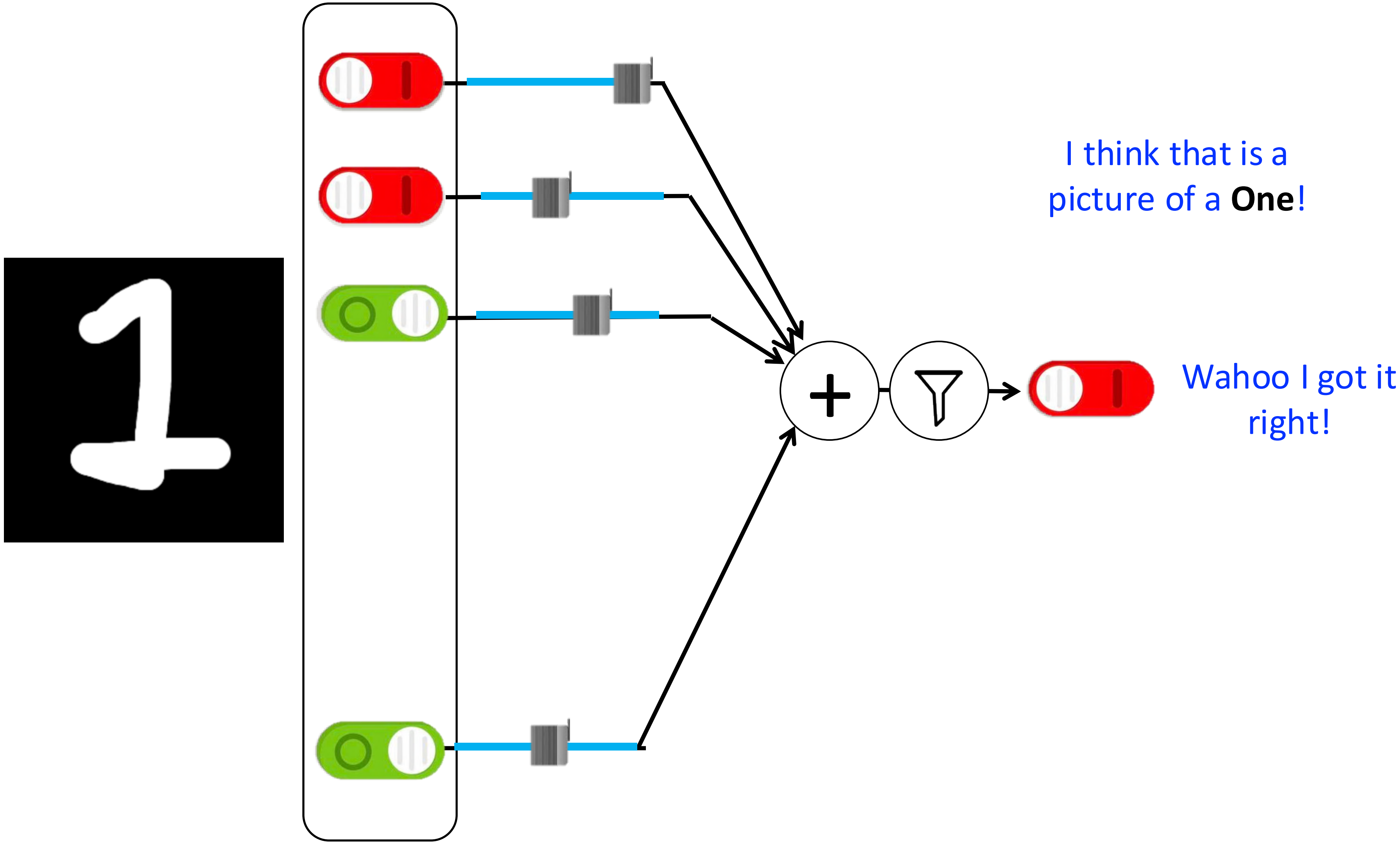
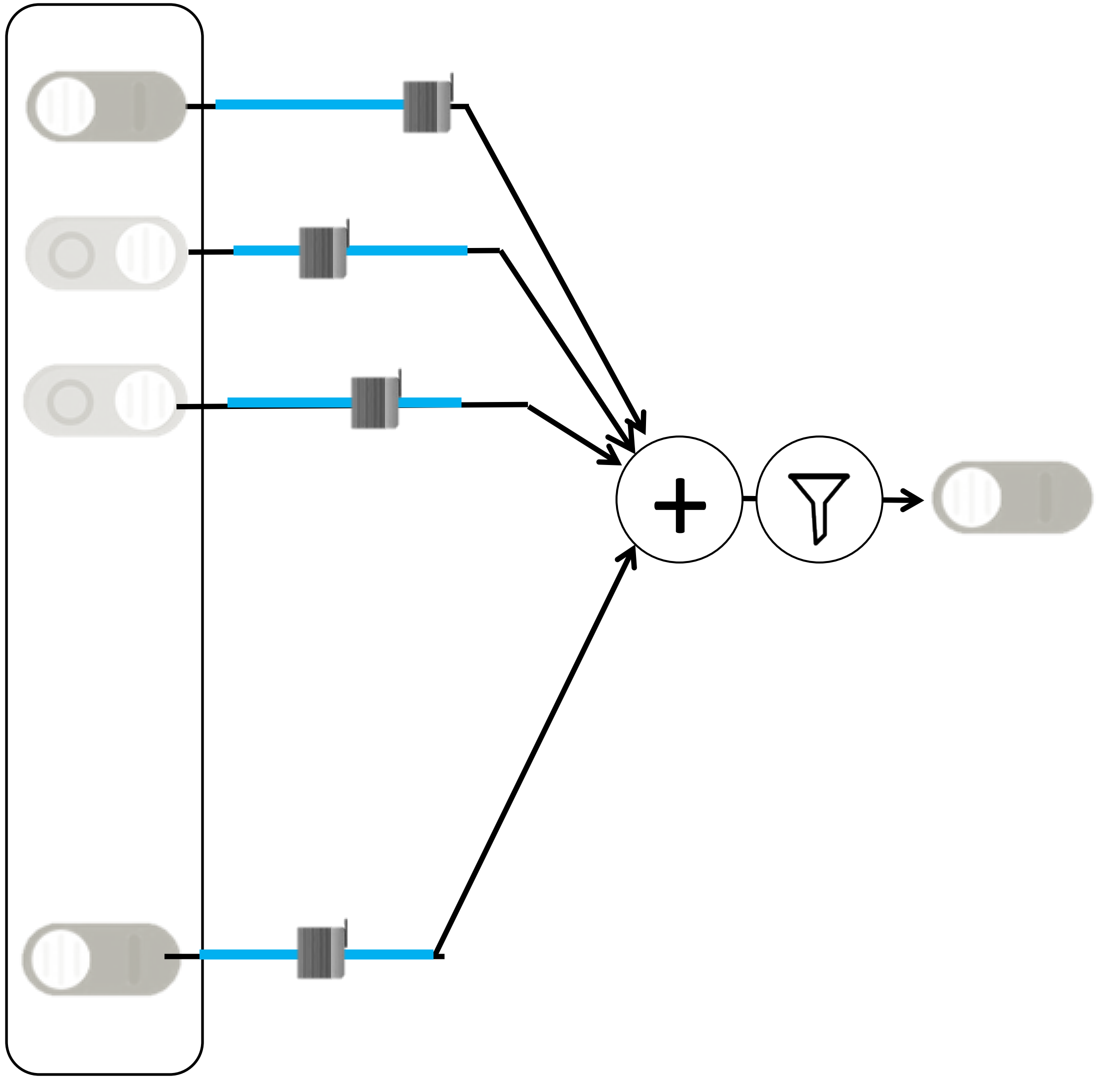
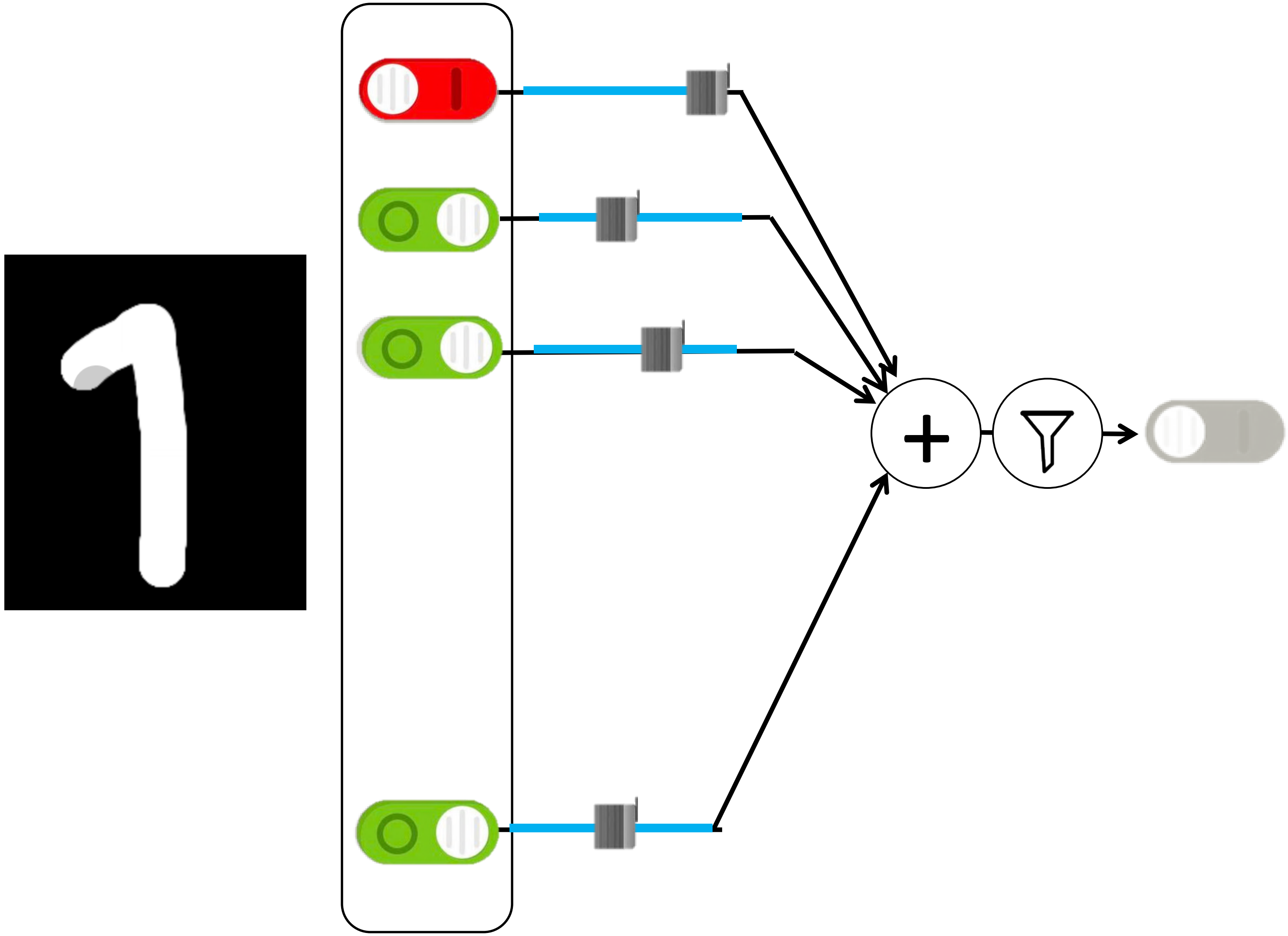




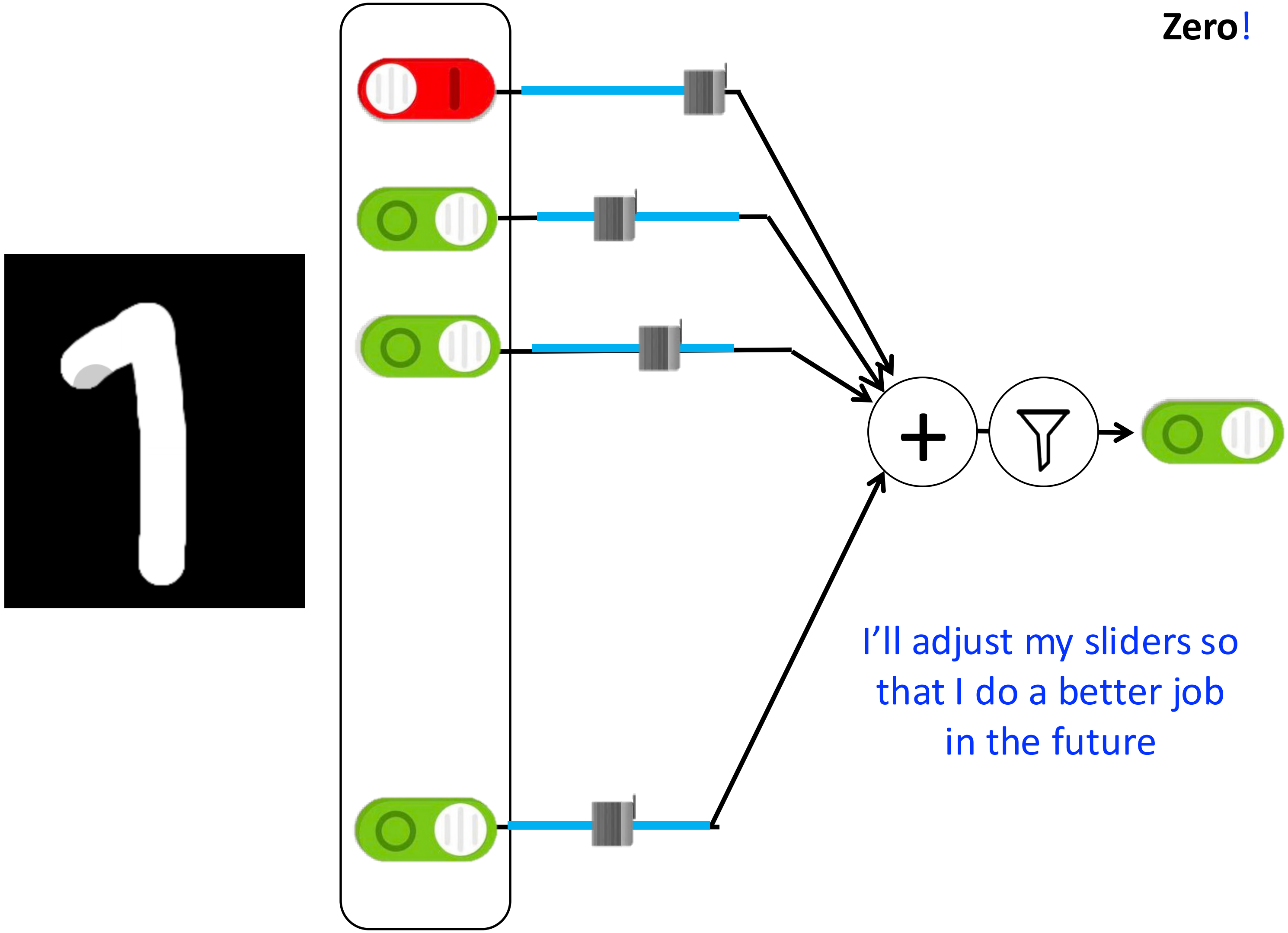



I think that is a picture of a **One**!
Wahoo I got it right!



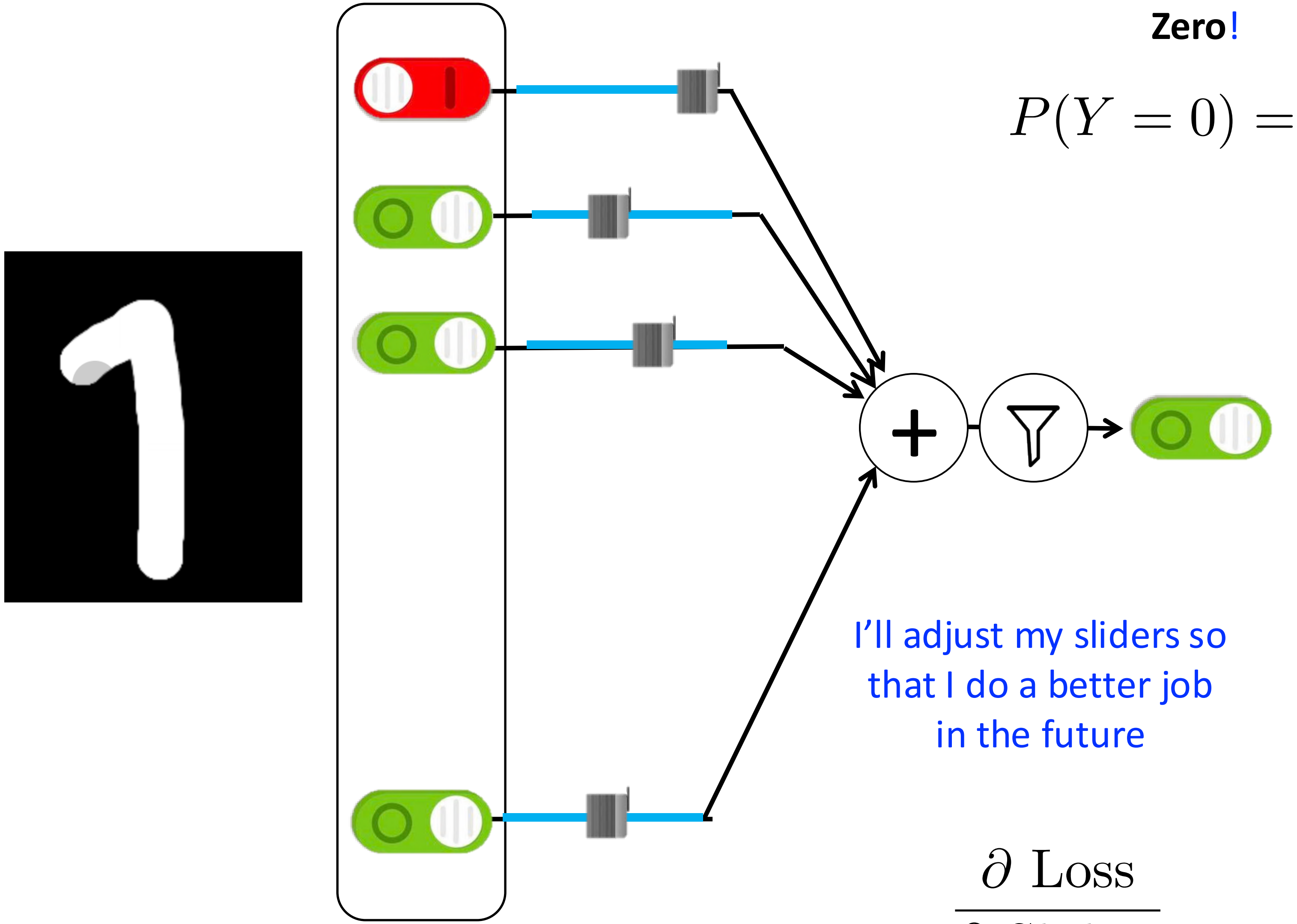


I think that is a picture of a **Zero**!



But it is actually a **Zero**

I'll adjust my sliders so that I do a better job in the future



I think that is a picture of a **Zero**!

$$P(Y = 0) = 0.9$$

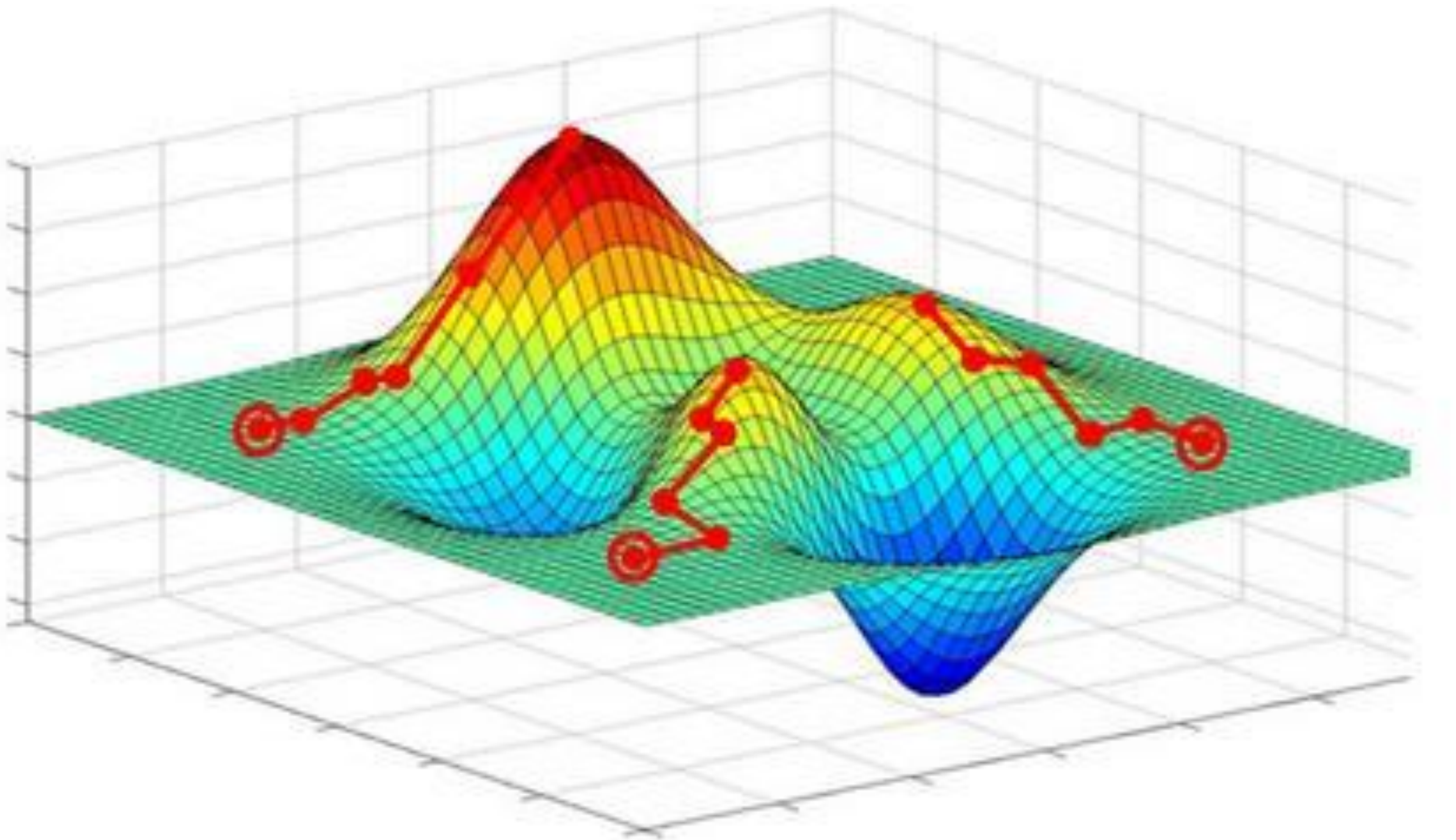
But it is actually a **Zero**

$$\text{Loss} = 1$$

I'll adjust my sliders so that I do a better job in the future

$$\frac{\partial\ \text{Loss}}{\partial\ \text{Slider}_i}$$

Gradient Descent



Walk uphill and you will find a local maxima
(if your step size is small enough)

Gradient of Probability

$$\frac{\partial L}{\partial \theta_i^{(\hat{y})}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \theta_i^{(\hat{y})}}$$

$$\hat{y} = \sigma\left(\sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})}\right)$$

$$\frac{\partial \hat{y}}{\partial \theta_i^{(\hat{y})}} = \sigma\left(\sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})}\right)\left[1 - \sigma\left(\sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})}\right)\right] \cdot \frac{\partial}{\partial \theta_i^{(\hat{y})}} \sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})}$$

$$= \hat{y}[1 - \hat{y}] \cdot \frac{\partial}{\partial \theta_i^{(\hat{y})}} \sum_{j=0}^{m_h} \mathbf{h}_j \theta_j^{(\hat{y})}$$

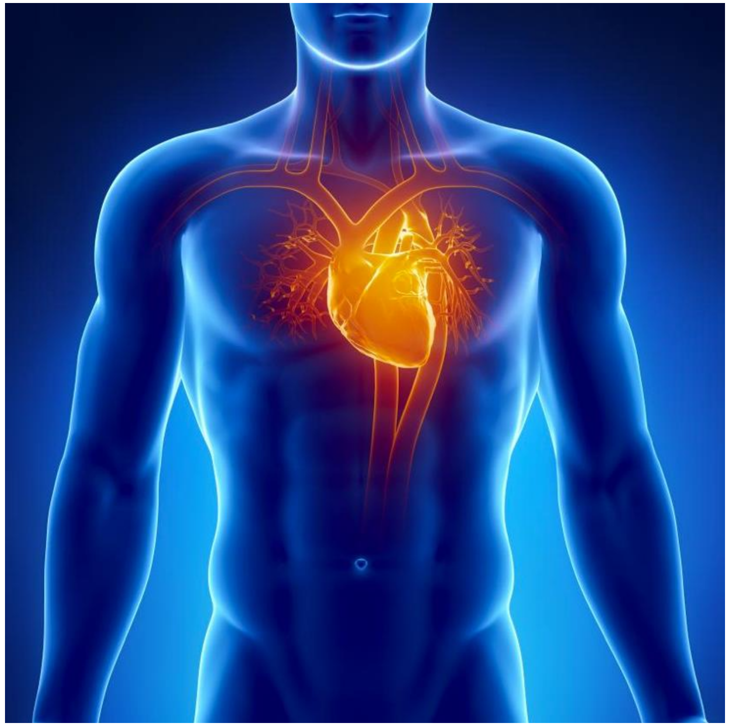
$$= \hat{y}[1 - \hat{y}] \cdot h_i$$

You will be able to do this.

Where you will be by the end of class

CS109: Theory Class focused on Applications

Heart



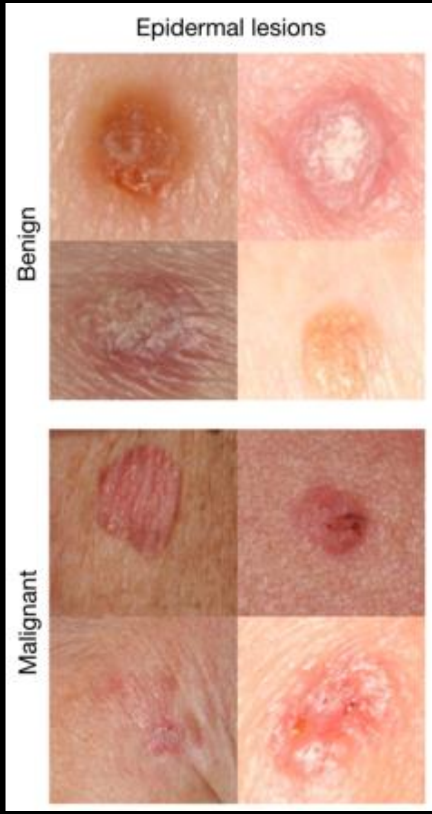
Ancestry

23andMe

Netflix

NETFLIX

Where is this Useful?



A machine learning algorithm performs **better than** the best dermatologists.

Developed recently, at Stanford.

Esteva, Andre, et al. "Dermatologist-level classification of skin cancer with deep neural networks." *Nature* 542.7639 (2017): 115-118.

What about Generative AI?

Who Invented Generative AI for Images?

Deep Unsupervised Learning using Nonequilibrium Thermodynamics by Jascha Sohl Dickstein

Dalle2. Prompt “a large lecture class at stanford learning probability for computer scientists in the style of vangough”

Deep Unsupervised Learning using Nonequilibrium Thermodynamics

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Abstract

A central problem in machine learning involves modeling complex data-sets using highly flexible families of probability distributions in which learning, sampling, inference, and evaluation are still analytically or computationally tractable. Here, we develop an approach that simultaneously achieves both flexibility and tractability. The essential idea, inspired by non-equilibrium statistical physics, is to systematically and slowly destroy structure in a data distribution through an iterative forward diffusion process. We then learn a reverse diffusion process that restores structure in data, yielding a highly flexible and tractable generative model of the data. This approach allows us to rapidly learn, sample from, and evaluate probabilities in deep generative models with thousands of layers or time steps, as well as to compute conditional and posterior probabilities under the learned model. We additionally release an open source reference implementation of the algorithm.

1. Introduction

Historically, probabilistic models suffer from a tradeoff between two conflicting objectives: *tractability* and *flexibility*. Models that are *tractable* can be analytically evaluated and easily fit to data (e.g. a Gaussian or Laplace). However, these models are unable to aptly describe structure in rich datasets. On the other hand, models that are *flexible* can be molded to fit structure in arbitrary data. For example, we can define models in terms of any (non-negative) function $\phi(\mathbf{x})$ yielding the flexible distribution $p(\mathbf{x}) = \frac{\phi(\mathbf{x})}{Z}$, where Z is a normalization constant. However, computing this normalization constant is generally intractable. Evaluating, training, or drawing samples from such flexible models typically requires a very expensive Monte Carlo process.

A variety of analytic approximations exist which ameliorate, but do not remove, this tradeoff–for instance mean field theory and its expansions (T, 1982; Tanaka, 1998), variational Bayes (Jordan et al., 1999), contrastive divergence (Welling & Hinton, 2002; Hinton, 2002), minimum probability flow (Sohl-Dickstein et al., 2011b;a), minimum KL contraction (Lyu, 2011), proper scoring rules (Gneiting & Raftery, 2007; Parry et al., 2012), score matching (Hyvärinen, 2005), pseudolikelihood (Besag, 1975), loopy belief propagation (Murphy et al., 1999), and many, many more. Non-parametric methods (Gershman & Blei, 2012) can also be very effective[1].

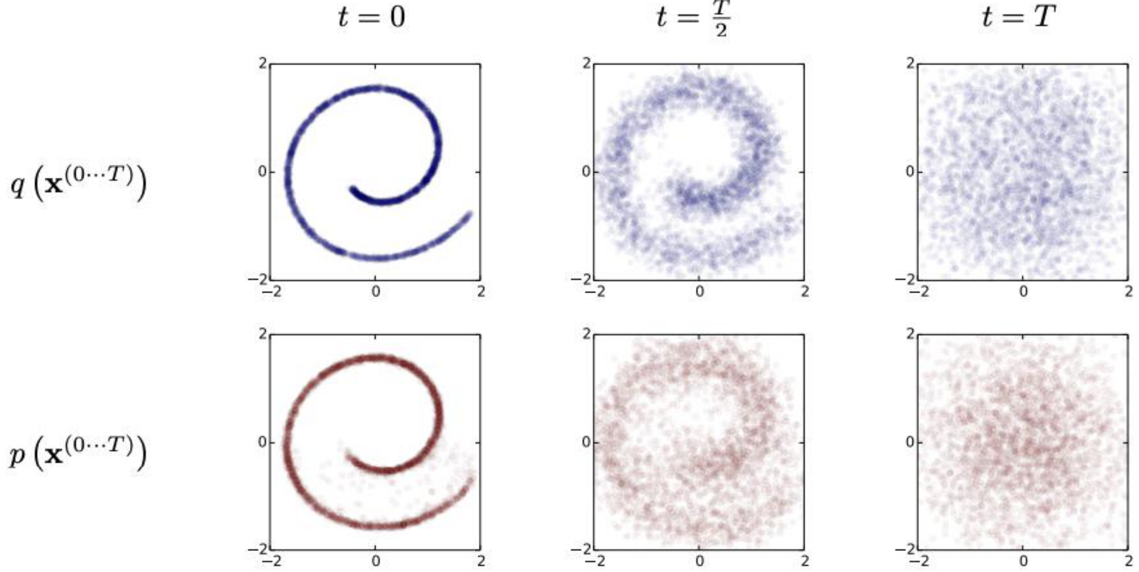
1.1. Diffusion probabilistic models

We present a novel way to define probabilistic models that allows:

1. extreme flexibility in model structure,
2. exact sampling,

[1]Non-parametric methods can be seen as transitioning smoothly between tractable and flexible models. For instance, a non-parametric Gaussian mixture model will represent a small

Proceedings of the 32nd International Conference on Machine Learning, Lille, France, 2015. JMLR: W&CP volume 37. Copy-



Will teach this at the end of the quarter

AI has constantly been revolutionized by people who understood probability theory.

End of Story

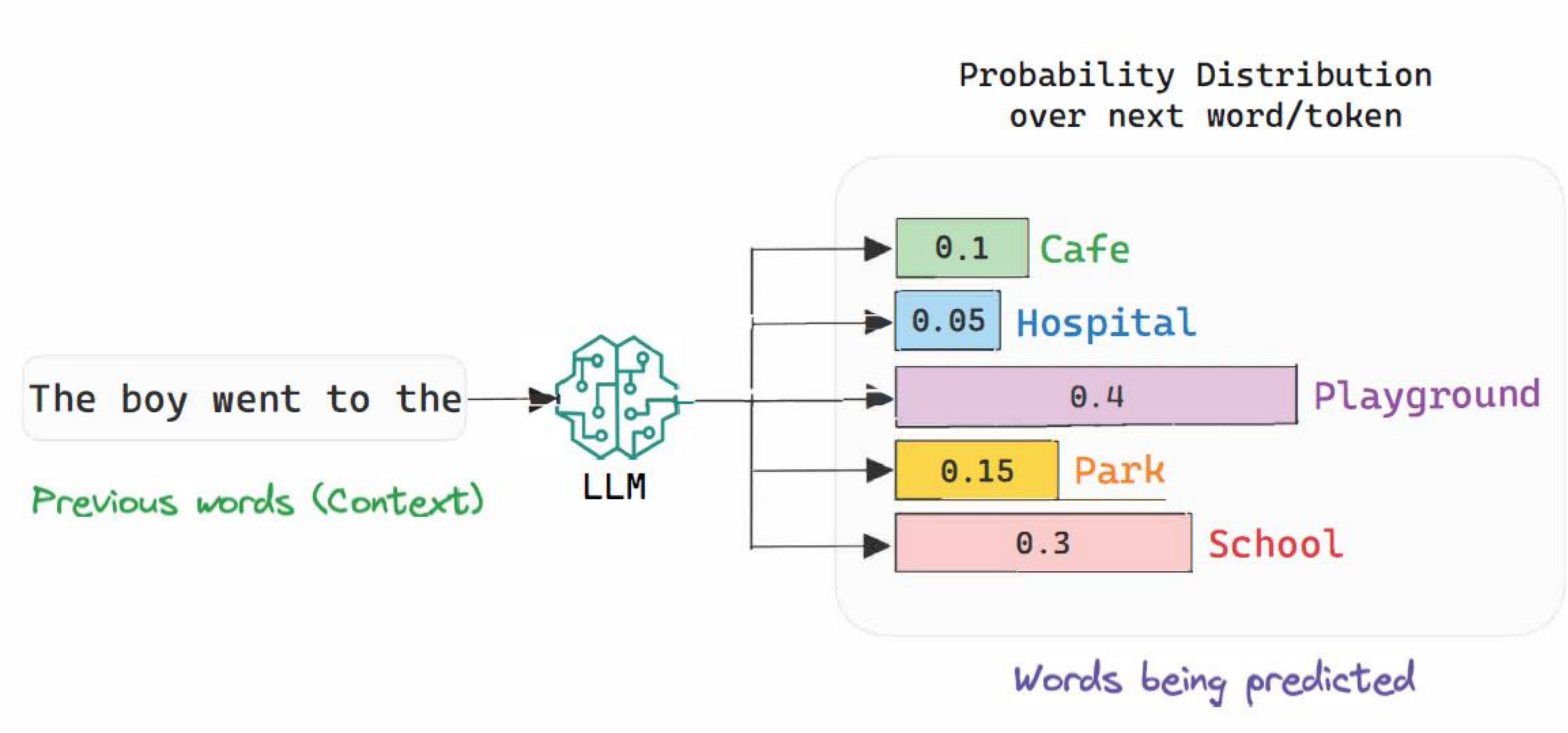
Except it isn't the end of the story…

Probability is more than just machine learning

Abundance of important problems

Many Open Problems

Why is beam search (a **probabilistic algorithm** that generates text) for LLMs (think **GPT4**) more effective with a small beam size?



Many Open Problems

Why is beam search (a **probabilistic algorithm** that generates text) for LLMs (think **GPT4**) more effective with a small beam size?

0.3 0.2

The boy went to the **school** **playground** **yesterday**

0.8

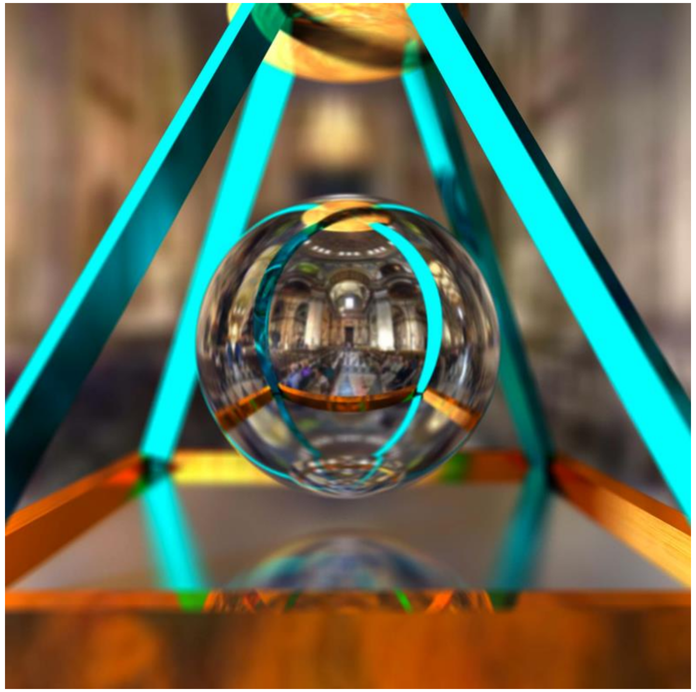
High beam size: Most likely sentence given the context

Small beam size: Greedy next word selection

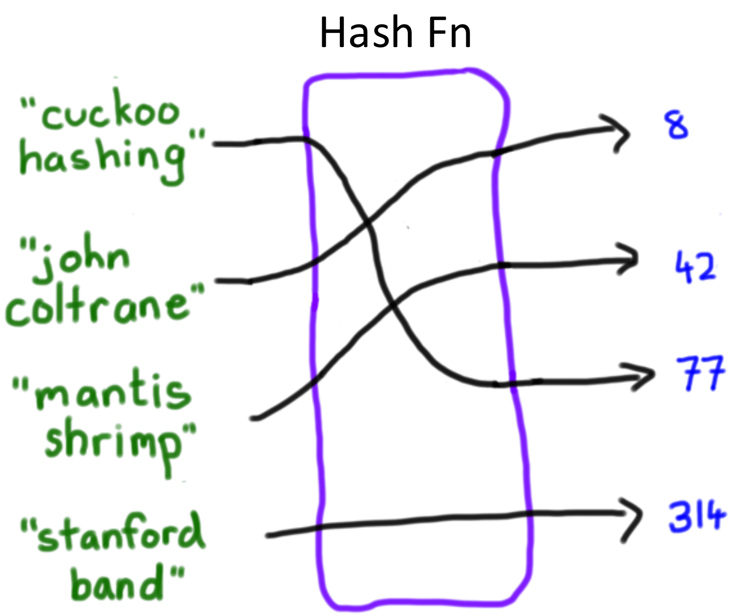
Medium beam size: ???

Algorithms and Probability

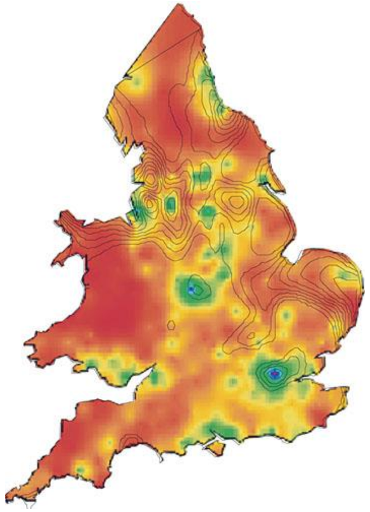
Eg Raytracing



Eg HashMaps

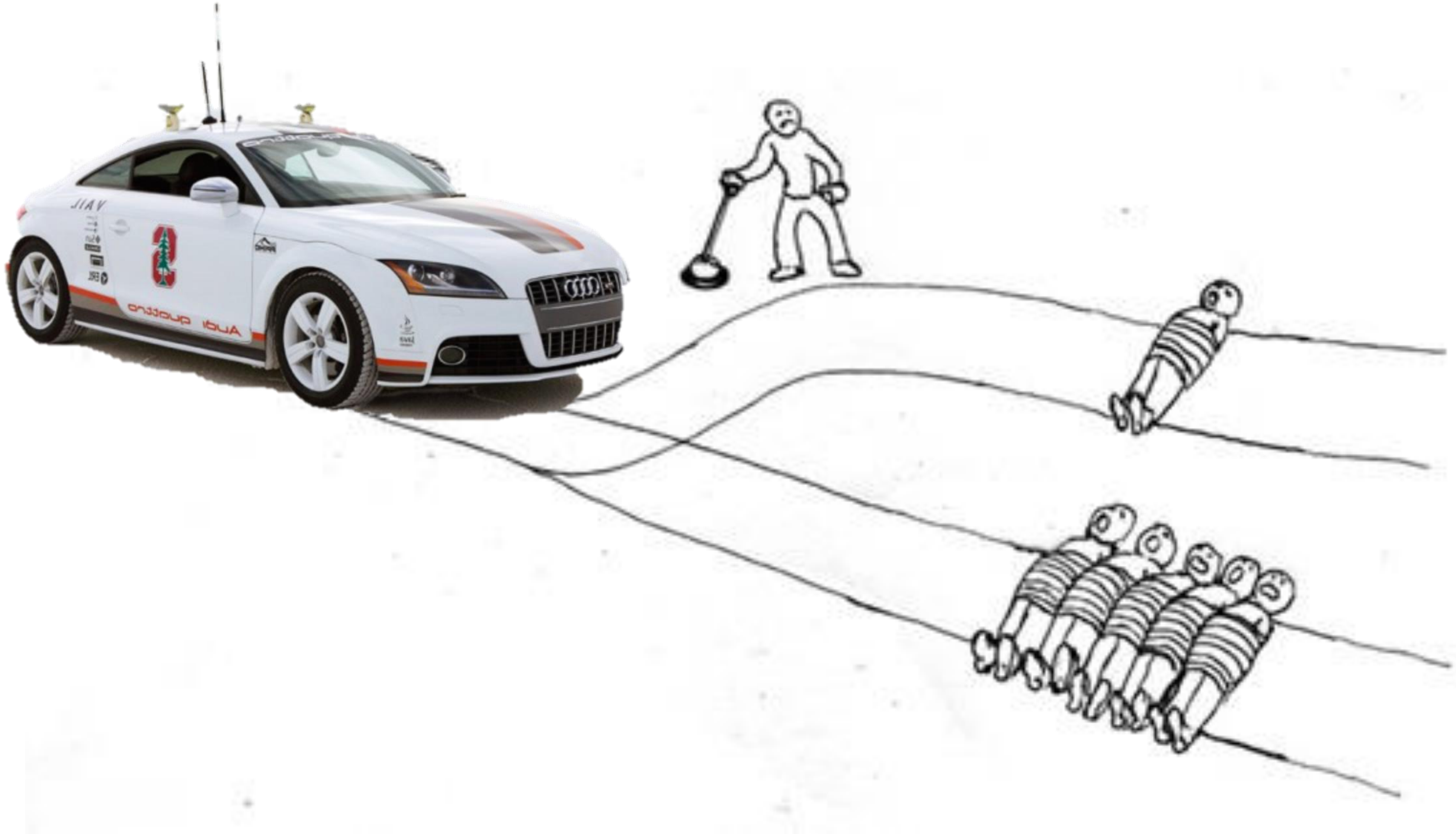


Understanding the world and building tools

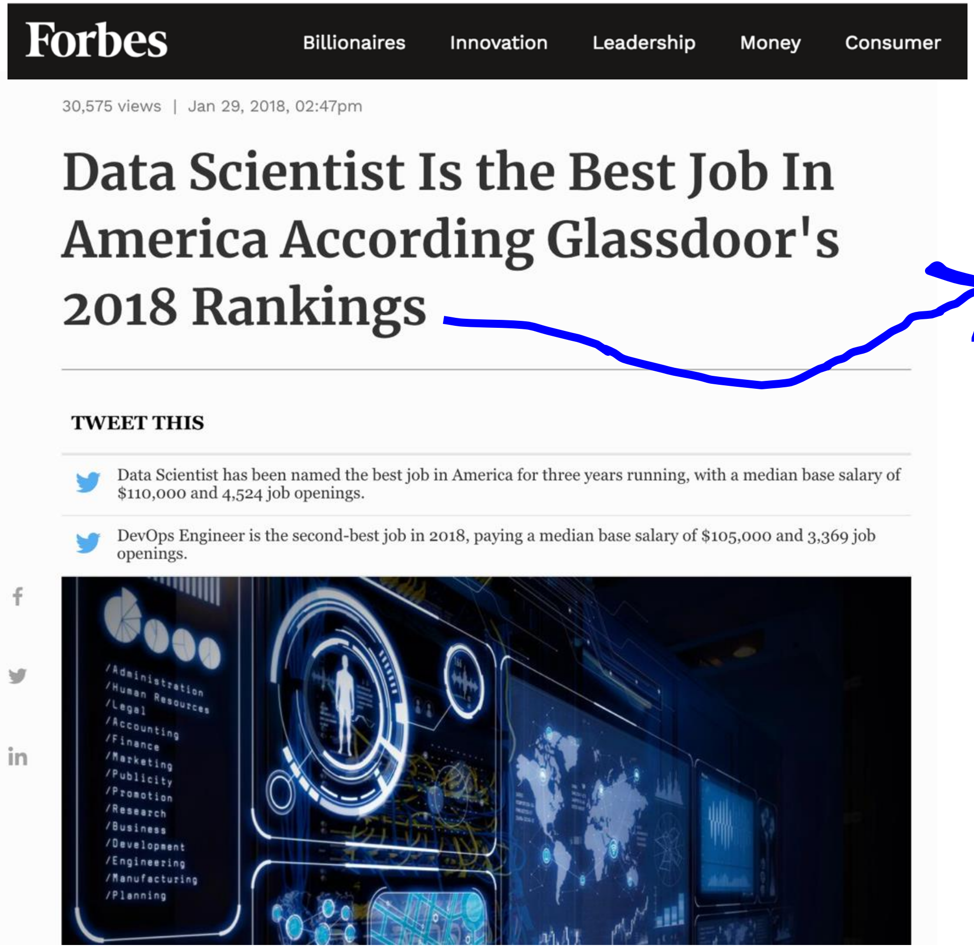


Recommender Systems

Philosophy and Ethics



Most Desired Skill in Industry



Forbes Billionaires Innovation Leadership Money Consumer

30,575 views | Jan 29, 2018, 02:47pm

Data Scientist Is the Best Job In America According Glassdoor's 2018 Rankings

TWEET THIS

Data Scientist has been named the best job in America for three years running, with a median base salary of $110,000 and 4,524 job openings.

DevOps Engineer is the second-best job in 2018, paying a median base salary of $105,000 and 3,369 job openings.

Job Score is based on:

- Earning potential
- Number of jobs
- Job satisfaction rating

"Data science and machine learning are generating more jobs than candidates right now, making these two areas the *fastest growing employment areas*."

9.8 times more jobs than five years ago.

[LinkedIn's 2017 U.S. Emerging Jobs Report]

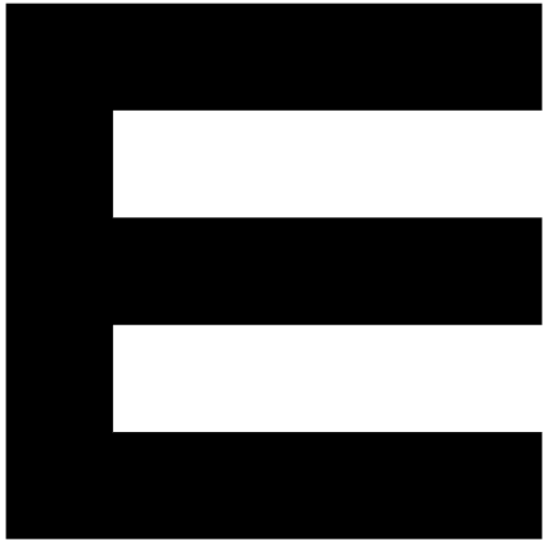
Most Desired Skill in Academia

Most CS PhD students list their highest desiderata upon graduation as:

"Better understanding of probability"

Learn Real Skills in CS109

Spring 2017



Patient sees a series of letters of different font size, and for each, answers correct or incorrect

You decide that the vision tests given by eye doctors could have more precise results if we used an approach inspired by logistic regression. In a vision test a user looks at a letter with a particular font size and either correctly guesses the letter or incorrectly guesses the letter.

You assume that the probability that a particular patient is able to guess a letter correctly is:

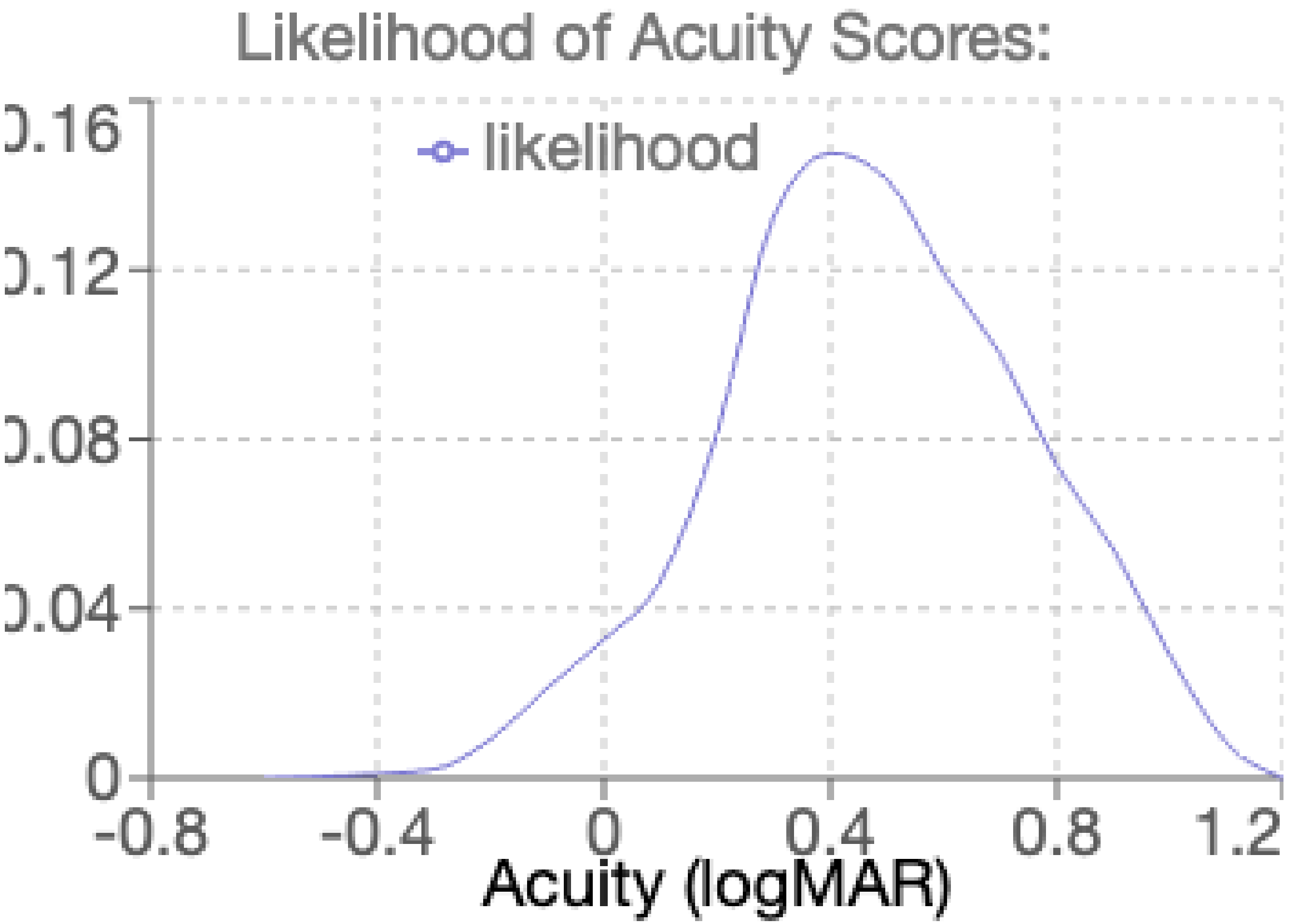
$$p = \sigma(\theta - f)$$

Where θ is the user's vision score and f is the font size of the letter.

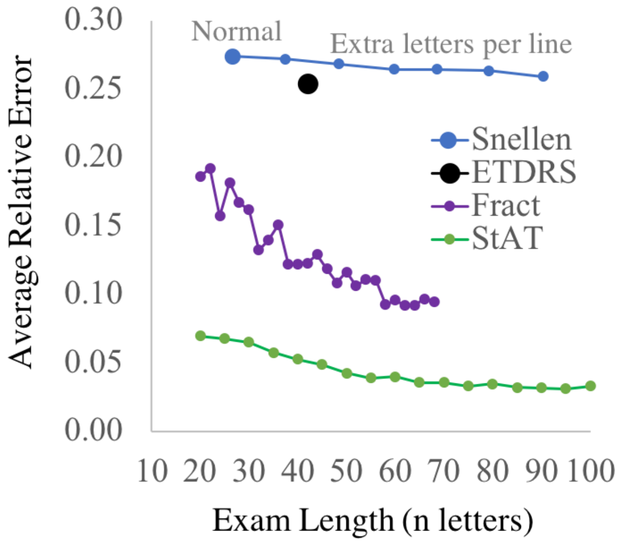
Explain how you could estimate a user's vision score (θ) based on their 20 responses $(f^{(1)}, y^{(1)}) \dots (f^{(20)}, y^{(20)})$, where $y^{(i)}$ is an indicator variable for whether the user correctly identified the ith letter and $f^{(i)}$ is the font size of the ith letter. Solve for any and all partial derivatives required by your answer.

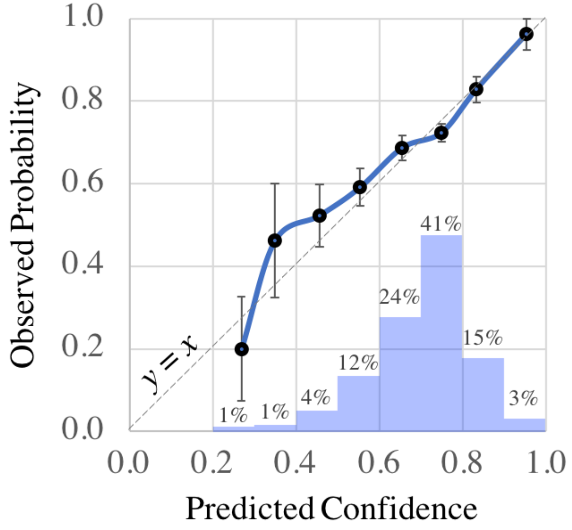
Learn Real Skills in CS109

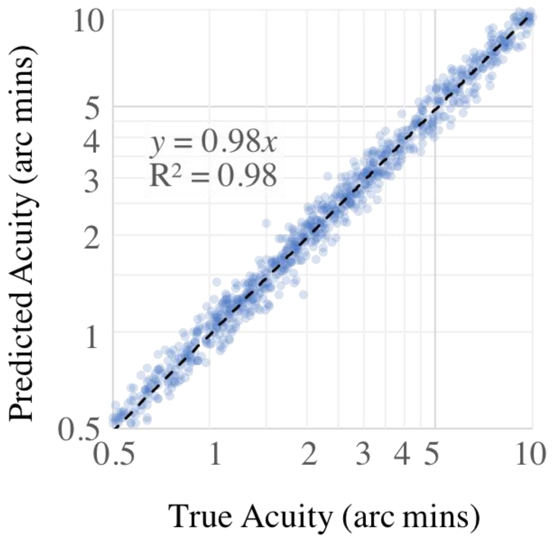
A patient has answered 20 “letter sizes” and got a few correct. What is your belief in how well they can see?



Now state of the art for eye exam theory







Learn Real Skills in CS109

The Stanford Acuity Test: A Precise Vision Test Using Bayesian Techniques and a Discovery in Human Visual Response

Chris Piech,*[1] Ali Malik,*[1] Laura M Scott, Robert T Chang,[2] Charles Lin[2]
[1]Department of Computer Science, Stanford University
[2] Department of Ophthalmology, Stanford University
{piech, malikali}@cs.stanford.edu, {rchang3, lincc}@stanford.edu

Abstract

Chart-based visual acuity measurements are used by billions of people to diagnose and guide treatment of vision impairment. However, the ubiquitous eye exam has no mechanism for reasoning about uncertainty and as such, suffers from a well-documented reproducibility problem. In this paper we make two core contributions. First, we uncover a new parametric probabilistic model of visual acuity response based on detailed measurements of patients with eye disease. Then, we present an adaptive, digital eye exam using modern artificial intelligence techniques which substantially reduces acuity exam error over existing approaches, while also introducing the novel ability to model its own uncertainty and incorporate prior beliefs. Using standard evaluation metrics, we estimate a 74% reduction in prediction error compared to the ubiquitous chart-based eye exam and up to 67% reduction compared to the previous best digital exam. For patients with eye disease, the novel ability to finely measure acuity from home could be a crucial part in early diagnosis. We provide a web implementation of our algorithm for anyone in the world to use. The insights in this paper also provide interesting implications for the field of psychometric Item Response Theory.

1 Introduction

Reliably measuring a person's visual ability is an essential component in the detection and treatment of eye diseases around the world. However, quantifying how well an individual can distinguish visual information is a surprisingly difficult task—without invasive techniques, physicians rely on chart-based eye exams where patients are asked visual questions and their responses observed.

Historically, vision has been evaluated by measuring a patient's *visual acuity*: a measure of the font size at which a patient can correctly identify letters shown a fixed distance away. Snellen, ... this statistic by ... asking the patie... dentify the size ... ters correct. Thi...

*Equal contrib... Copyright © 202... Intelligence (www...

treatment of patients; yet, it suffers from some notable shortcomings. Acuity exams such as these exhibit high variance in their results due to the large role that chance plays in the final diagnosis, and the approximation error incurred by the need to discretise letter sizes on a chart. On the other hand, digital exams can show letter of any size and can *adaptively* make decisions based on intelligent probabilistic models. As such they have potential to address the shortcomings of analog charts.

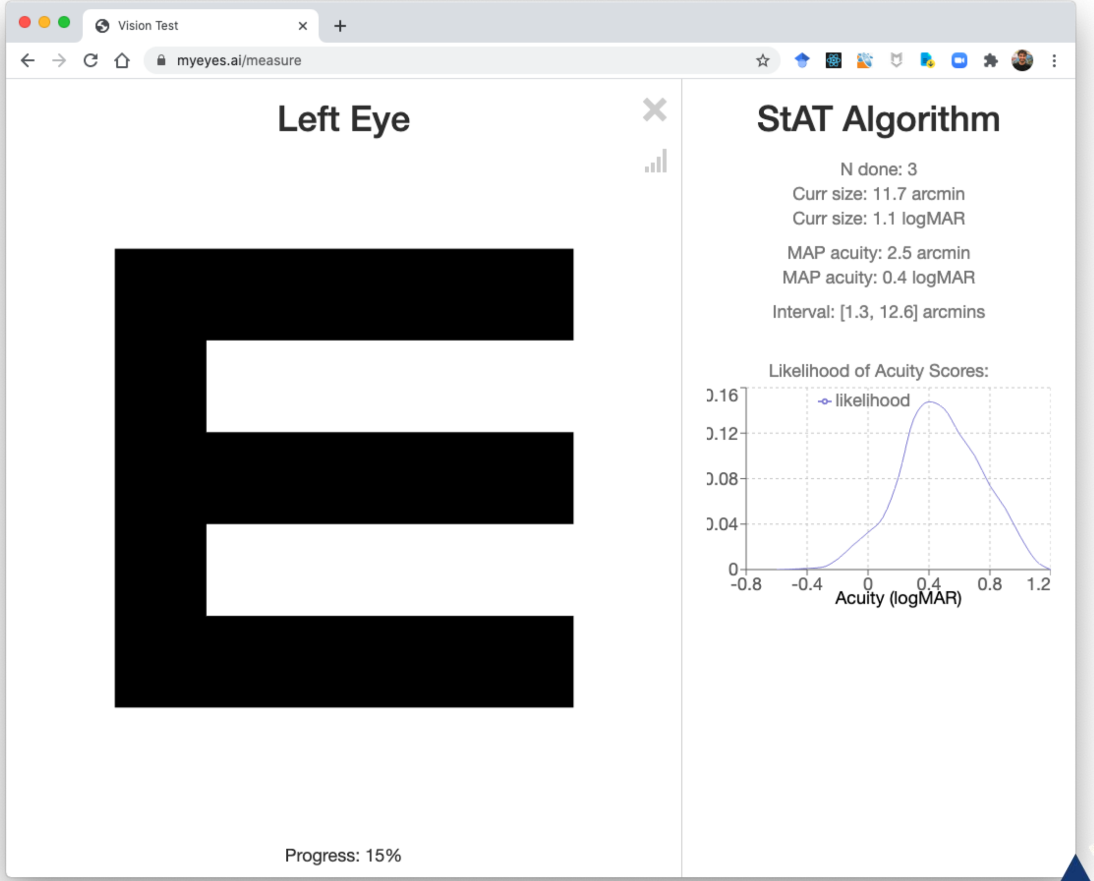
While promising, contemporary digital exams have yet to dramatically improve accuracy over traditional chart-based approaches. The current best digital exam uses a psychometric Item Response Theory (IRT) algorithm for both selecting the next letter size to query and for making a final prediction of acuity. Under simulation analysis, this digital exam results in a 19% reduction in error over traditional chart-based approaches. The separate fields of reinforcement learning and psychometric IRT have independently explored how to effectively make decisions under uncertainty. By merging the good ideas from both disciplines we can develop a much better visual acuity test.

In this paper we make two main contributions. First, we revisit the human Visual Response Function—a function relating the size of a letter to the probability of a person identifying it correctly—and discover that it follows an interpretable parametric form that fits real patient data. Second, we present an algorithm to measure a person's acuity which uses several Bayesian techniques common in modern artificial intelligence. The algorithm, called the Stanford Acuity Test (StACT)[1], has the following novel features:

1. Uses the new parametric form of the human Visual Response Function.
2. Returns a soft inference prediction of the patient's acuity, ... onfidence in the final

...ng algorithm to adap... ...wn to a user. This ef... ... acuity belief.

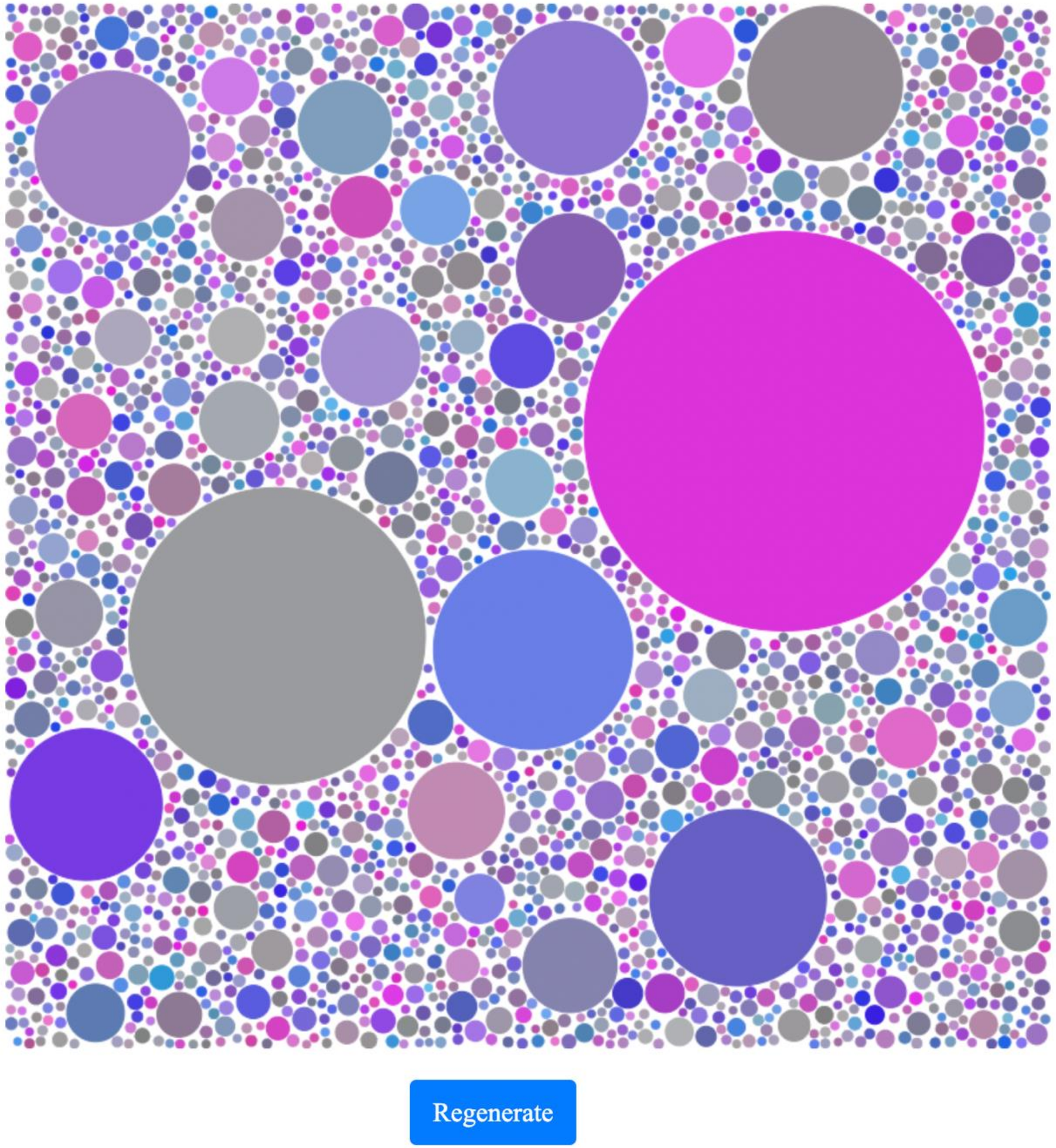
...CT, was named after ...ed. We continue in this



Science

THE LANCET

What is on a typical final?



1. Algorithmic Art
2. Lucky Events
3. Supply Chain Decision Making
4. P-Hacking
5. Chess.com Puzzle Ability
6. ML Calibration

https://chrispiech.github.io/probabilityForComputerScientists/en/examples/algorithmic_art/

Foundation for your future

But its not always intuitive

But Its not Always Intuitive

A patient has a positive Zika test.

What is the probability they have zika?

- *0.8% of people have zika*
- *Test has 90% positive rate for people with zika*
- *Test has 7% positive rate for people without zika*

The right answer is 9%

Probability = Important + Needs Study

Delayed gratification

CS109 View of Probability

Teach you how to write programs
that most people are not able to write.

CS109 View of Probability

Teach you the theory you need to do the math that most people are not able to do.

CS109

AI

Uncertainty Theory

Single Random Variables

Probabilistic Models

Counting

Probability Fundamentals

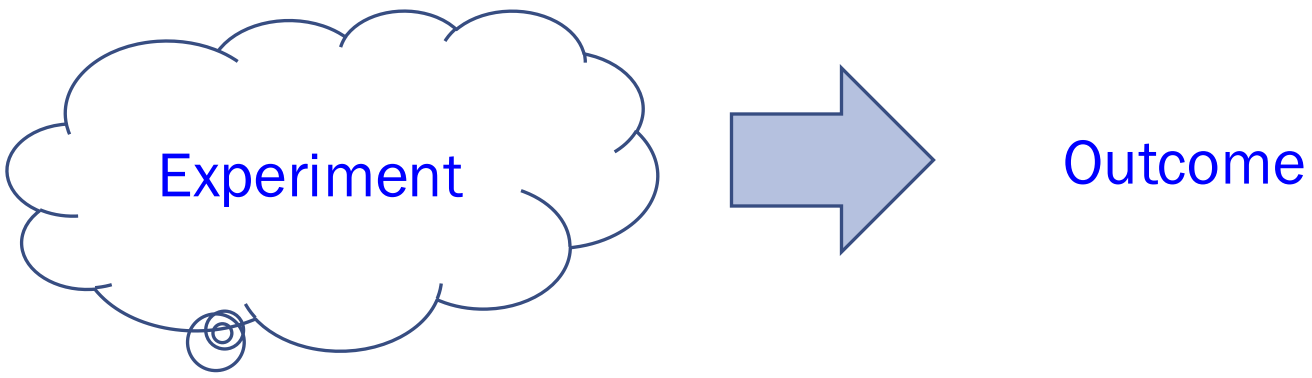
Lets dive in…

2 min pedagogic pause.

Counting I

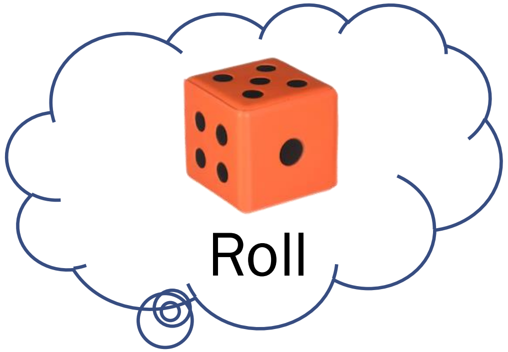
What is Counting?

An experiment
in probability:



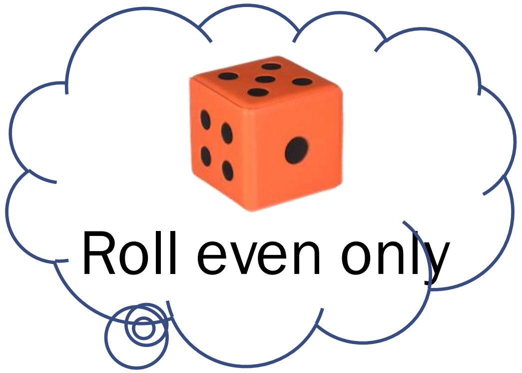
Counting: How many possible outcomes satisfy some event?

What is Counting?



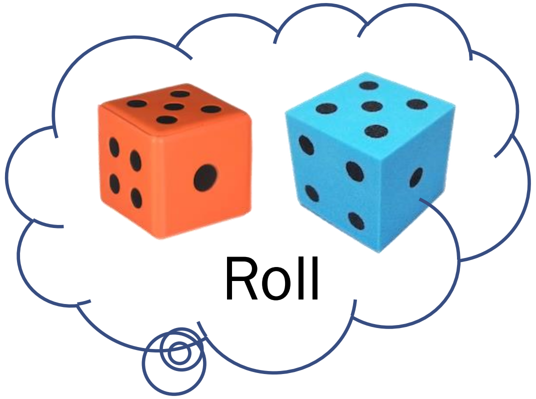
6

{1, 2, 3,
4, 5, 6}



3

{2, 4, 6}



36

{(1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Think about a generative story…

Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two steps, where

The first step's outcomes are from Set A, where $|A| = m$,
and the second step's outcomes are from Set B, where $|B| = n$,

and |B| is unaffected by outcome of first step.

Then the number of outcomes of the experiment is

$$|A||B| = mn.$$

Two-step experiment

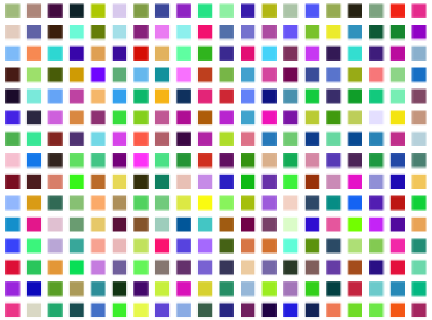
$\longrightarrow$ A $\longrightarrow$ B

[On board]

How Many Unique Images?

Each pixel can be one of 17 million distinct colors

(a) 12 million pixels



(b) 300 pixels

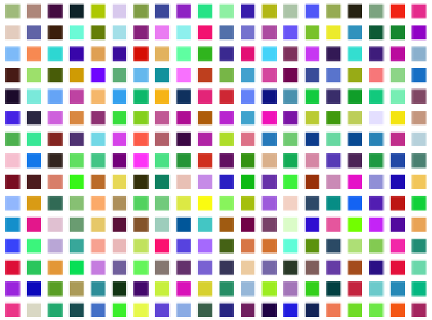
(c) 12 pixels

$$(17 \text{ million})^n$$


10^{80}

How Many Unique Images?

Each pixel can be one of 17 million distinct colors



(a) 12 million pixels

$$\approx 10^{86696638}$$

(b) 300 pixels

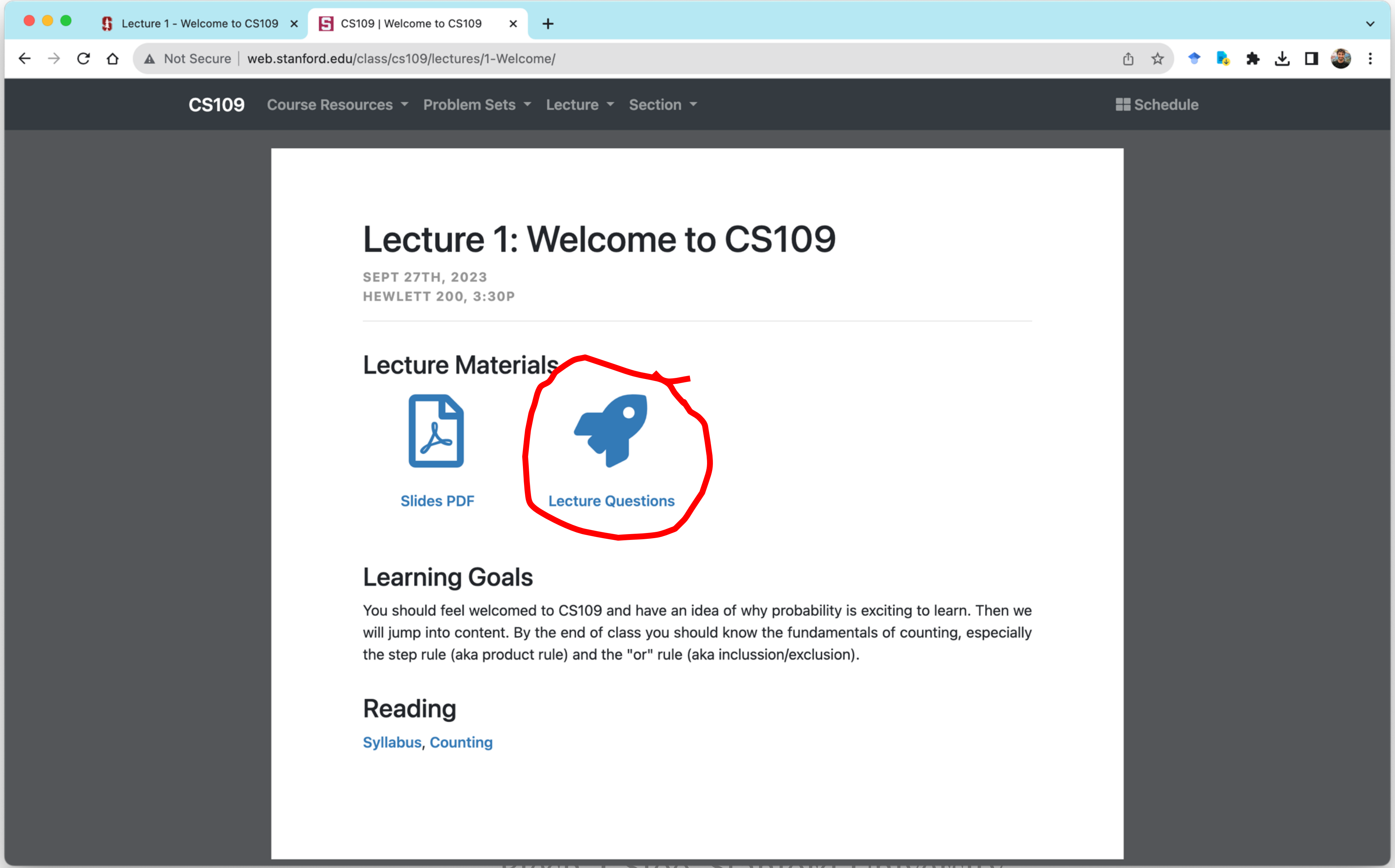
$$\approx 10^{2167}$$

(c) 12 pixels

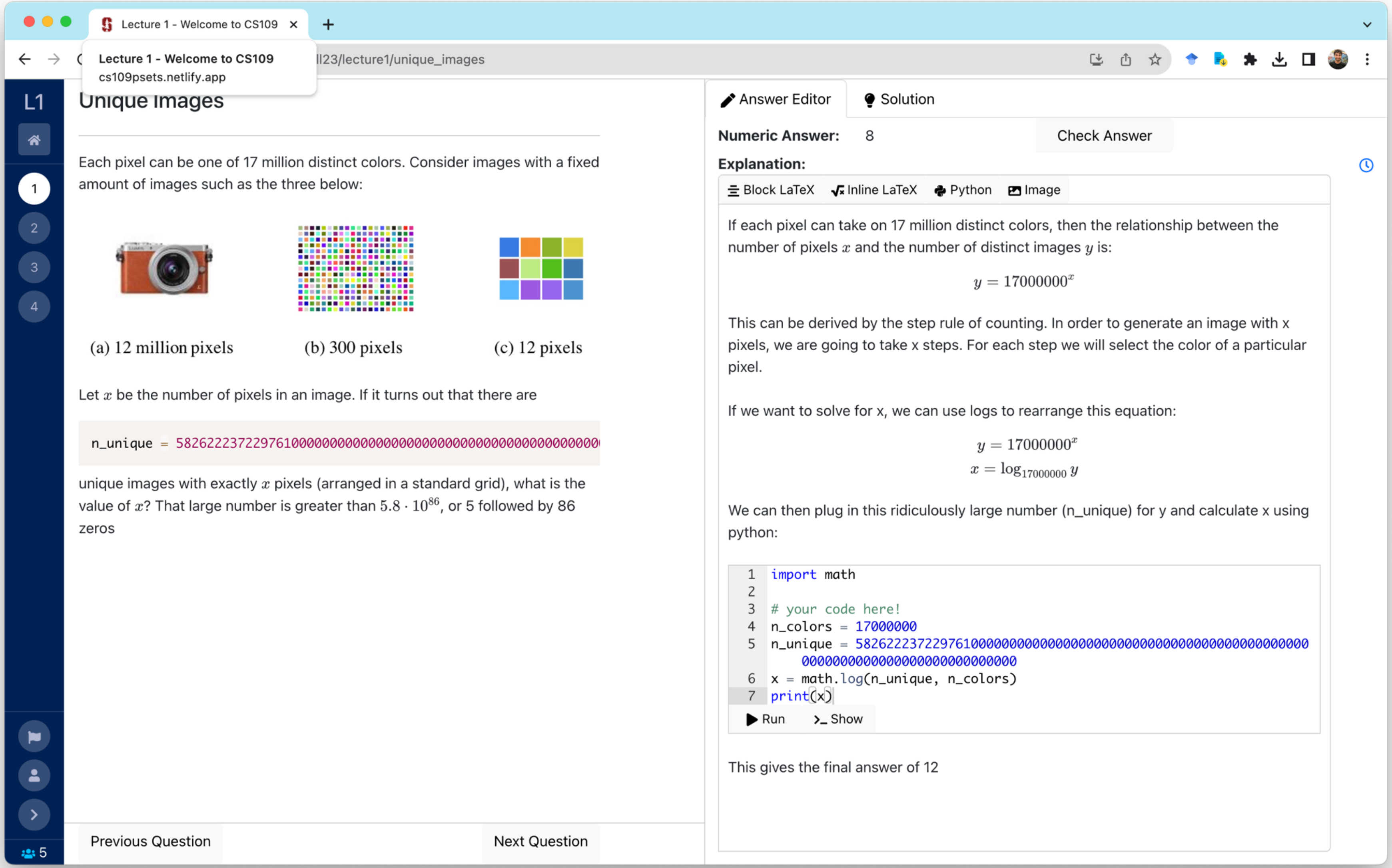
$$\approx 10^{86}$$

$(17 \text{ million})^n$

Follow Along Online



Follow Along Online



Sum Rule of Counting

If the outcome of an experiment can be either from

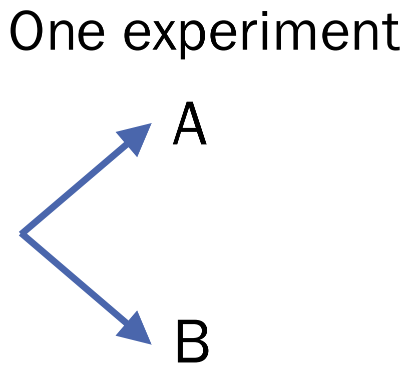
Set A, where $|A| = m$,

or Set B, where $|B| = n$,

where $A \cap B = \emptyset$,

Then the number of outcomes of the experiment is

$|A| + |B| = m + n$.



How many toys?

Question: All of Freya's toys are either Balls **OR** Plush Animals. She has 2 Balls and 3 Plush Animals. How many toys does she have?

Answer: 2 + 3


OFFICIAL
3
SIZE

How Many Bit Stings?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Answer

2^4 start with 01	2^4 end with 10
010000	000010
010001	000110
010010	001010
010011	001110
010100	010010
010101	010110
010110	011010
010111	011110
011000	100010
011001	100110
011010	101010
011011	101110
011100	110010
011101	110110
011110	111010
011111	111110
Set A	Set B

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010001	000110
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010011	001110
010100	**010010**
010101	**010110**
010110	**011010**
010111	**011110**
011000	100010
011001	100110
011010	101010
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Answer

$$N = |A| + |B| - |A \text{ and } B|$$
$$= 16 + 16 - 4$$
$$= 28$$

2^4 start with 01	2^4 end with 10
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010001	000110
010010	001010
010011	001110
010100	**010010**
010101	**010110**
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011001	100110
011010	101010
011011	101110
011100	110010
011101	110110
011110	111010
011111	111110
Set A	Set B

Or Rule of Counting (aka Inclusion/ Exclusion)

If the outcome of an experiment can be either from

Set A, where $|A| = m$,

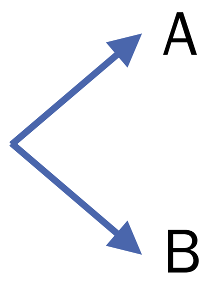
or Set B, where $|B| = n$,

where $A \cap B$ $may\ not\ be\ empty$,

Then the number of outcomes of the experiment is

$$N = |A| + |B| - |A \cap B|.$$

One experiment



Core Counting

Counting with steps

Definition: Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of m outcomes and the second part can result in one of n outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is $m \cdot n$.

Counting with "or"

Definition: Inclusion Exclusion Counting

If the outcome of an experiment can either be drawn from set A or set B, and sets A and B may potentially overlap (i.e., it is not the case that A and B are mutually exclusive), then the number of outcomes of the experiment is $|A \text{ or } B| = |A| + |B| - |A \text{ and } B|$.

Challenge Problem

1. Strings
   - How many *different* orderings of letters are possible for the string BOBA?

BOBA, ABOB, OBBA…

Incredible time. Incredible school at which to study probability!

Exciting.