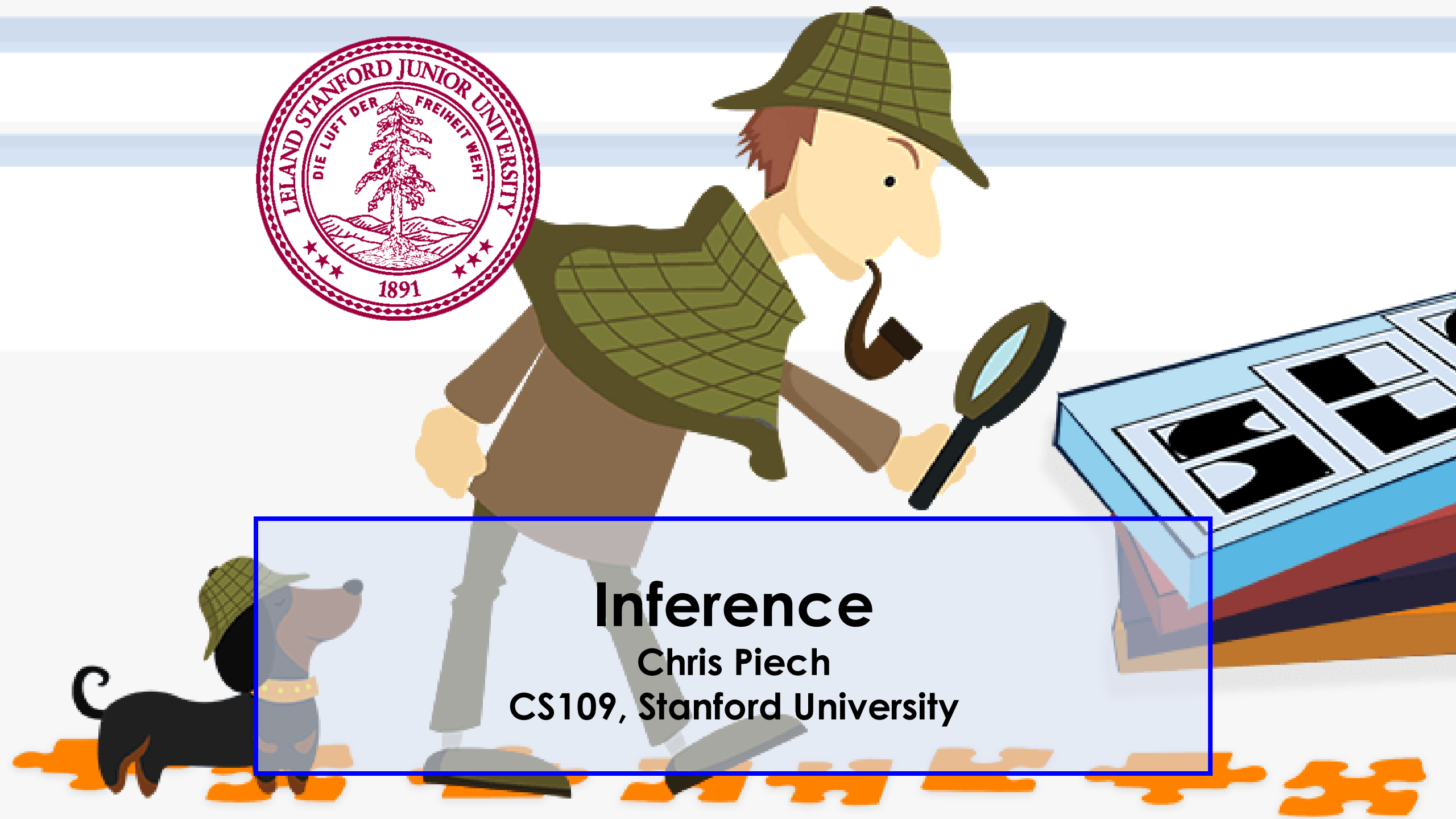
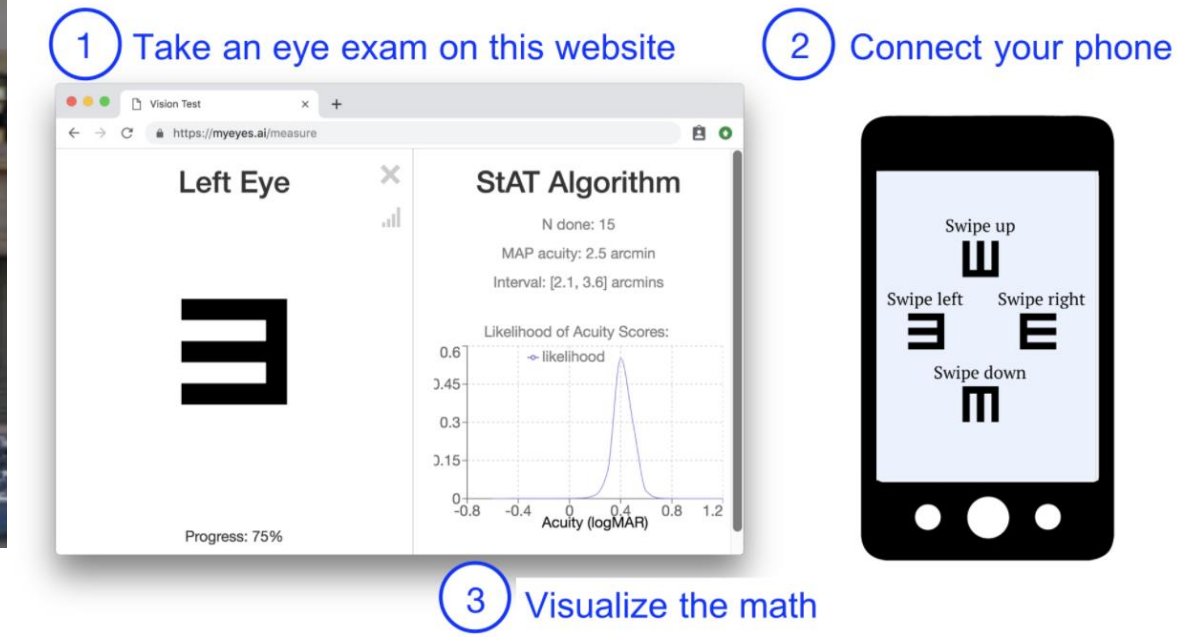


Inference

Chris Piech
CS109, Stanford University

Today: Stanford Eye Test



I always wanted to make this a class demo

Pset #4

Pset 4 - Probabilistic Models

cs109psets.netlify.app/fall23/pset4/probabilistic_supply_chain

PS4

1 2 3

Probabilistic Supply Chain

You are managing inventory of giant bolts in a company that produces airplanes (airplanes need giant bolts).

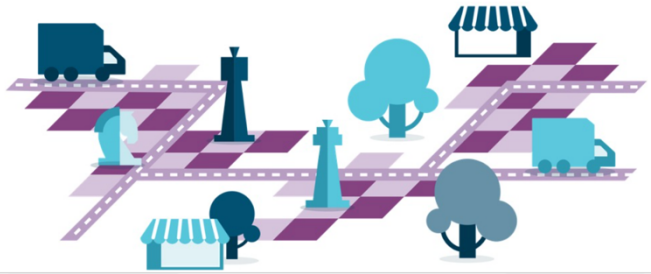
You have estimated the joint probability between supply of bolts, S (how many bolts you will have in the next week) and demand for bolts, D (how many bolts you will need in the next week while constructing airplanes).

You stored your estimate of the joint probability $P(S = s, D = d)$ in a variable `joint`. This variable is the full joint probability table, where the rows represent the S random variable and the columns represent D. Here are a few examples:

```
joint[0][1] # P(S = 0, D = 1)
joint[2][3] # P(S = 2, D = 3)
```

You may assume that both supply and demand are discrete, integer values in the range 0 to 5. You can't have negative bolts, and neither supply nor demand is ever more than 5. You can assume that all parameters passed to your functions are in the range 0 to 100.

Write code to compute the probability that you will be undersupplied. You are undersupplied when demand is greater than supply.



Previous Question | Next Question

138

Answer Editor | Solution

Numeric Answer: Enter your answer | Check Answer

Python:

```
joint = [
 [0.02, 0.01, 0.01, 0.01, 0.04, 0.02],
 [0.01, 0.02, 0.01, 0.02, 0.01, 0.01],
 [0.01, 0.01, 0.04, 0.04, 0.09, 0.01],
 [0.00, 0.01, 0.02, 0.09, 0.12, 0.04],
 [0.02, 0.01, 0.01, 0.06, 0.09, 0.03],
 [0.02, 0.02, 0.01, 0.01, 0.04, 0.01]
]

def main():
    print(joint)

if __name__ == '__main__':
    main()
```

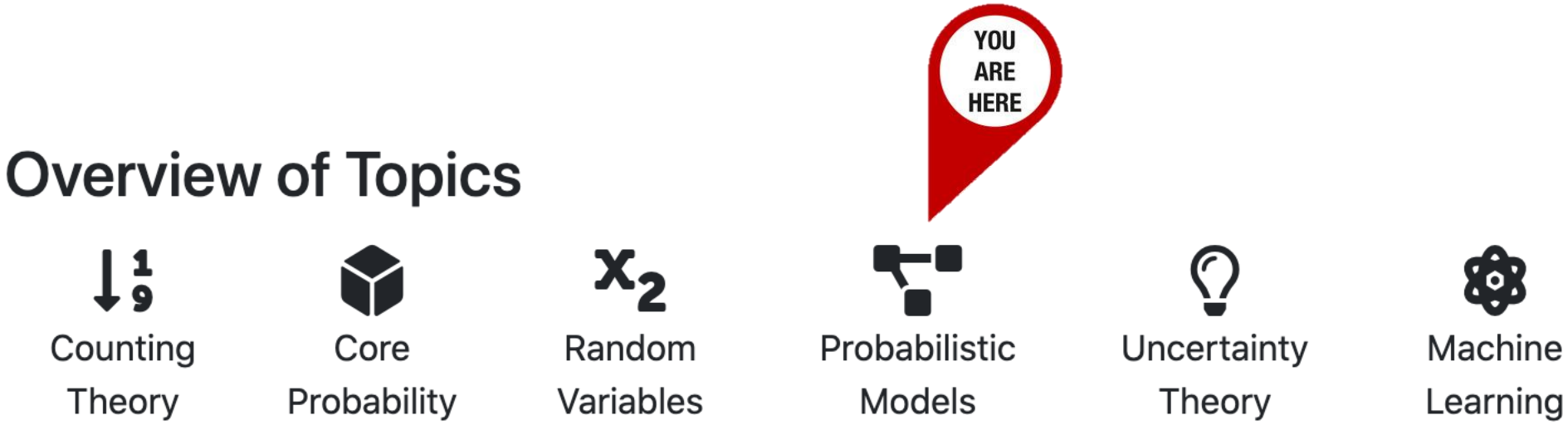
Run

Learning Goals

1. Combine Bayes Theorem and Random Variables

Review

Where are we in CS109?

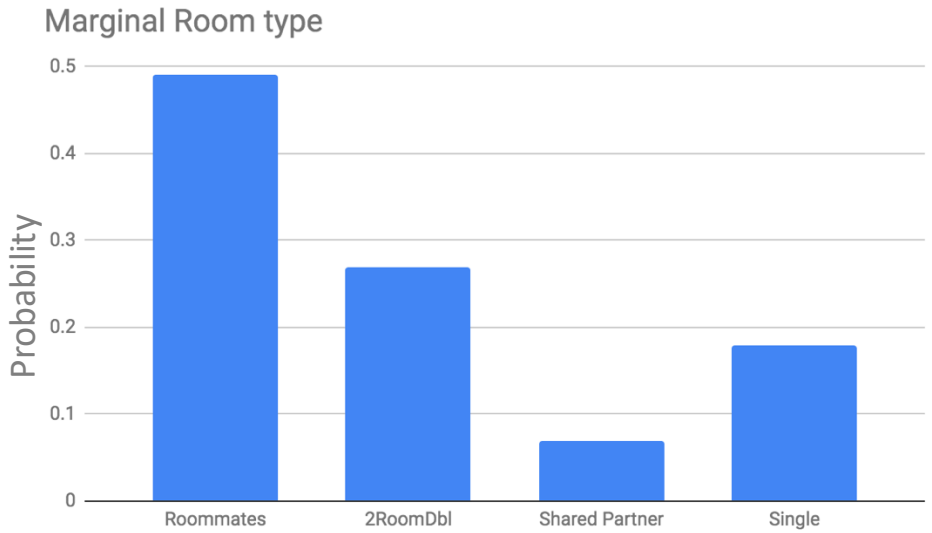


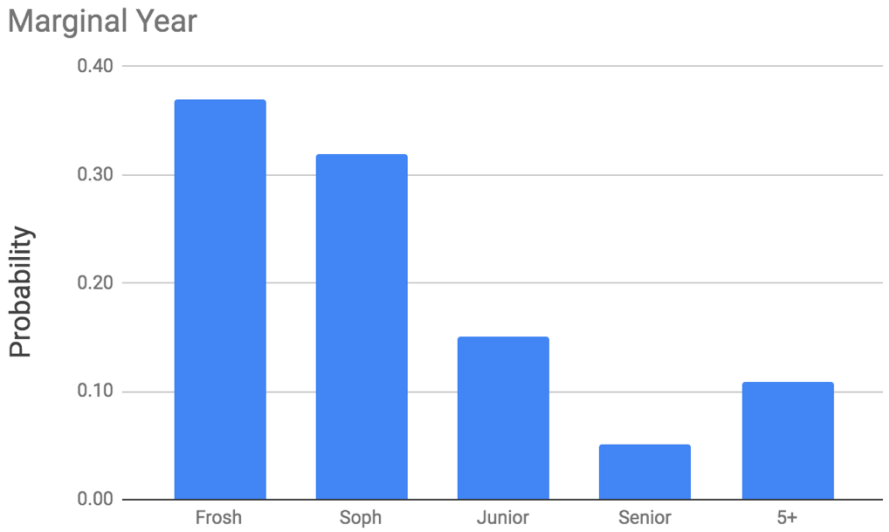
Probabilistic Models

Joint

	Roommates	2RoomDbl	Shared Partner	Single	
Frosh	0.30	0.07	0.00	0.00	0.37
Soph	0.12	0.18	0.00	0.03	0.32
Junior	0.04	0.01	0.00	0.10	0.15
Senior	0.01	0.02	0.02	0.01	0.05
5+	0.02	0.00	0.05	0.04	0.11
	0.49	0.27	0.07	0.18	1.00

Marginals





Today: Inference

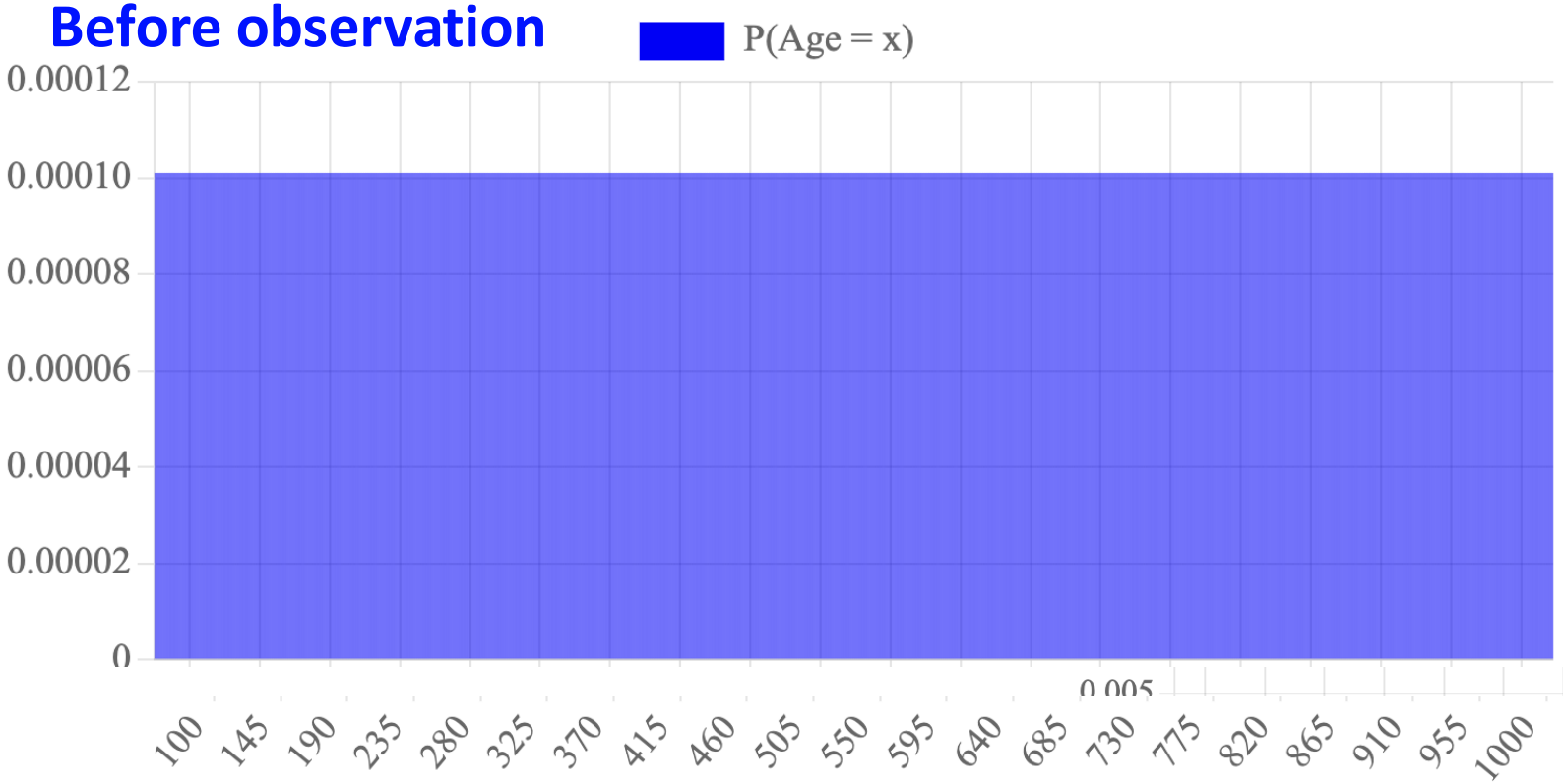
Inference *noun*

Updating one's belief about a random variable (or multiple) based on conditional knowledge regarding another random variable (or multiple) in a probabilistic model.

TLDR: conditional probability with random variables.

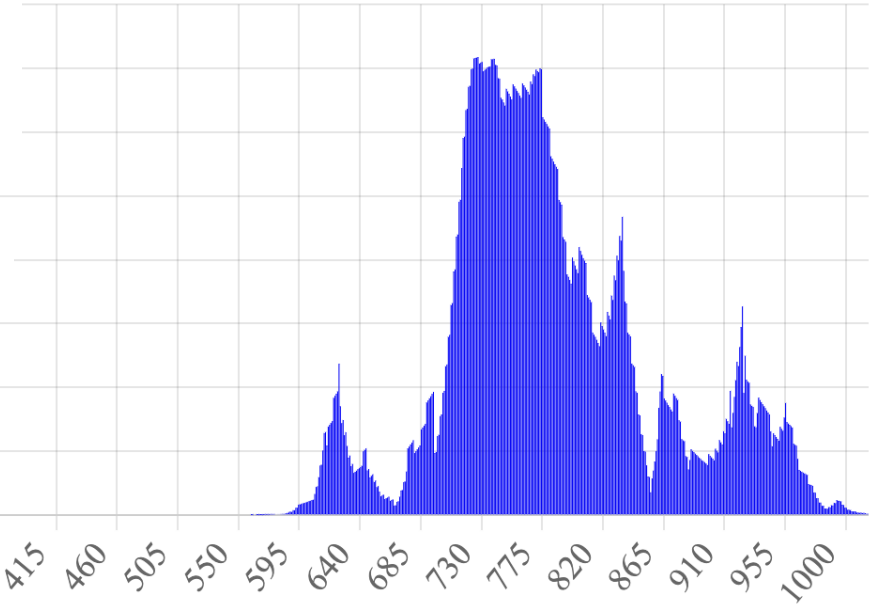
Update Belief PMF

Before observation



Observation

Remaining C14: 900



Bayesian Carbon Dating: Inference Overview

Let A be how many years old the sample is (A = 100 means the sample is 100 years old)
Let M be the observed amount of C14 left in the sample

$$P(A = i|M = 900) = \frac{P(M = 900|A = i)P(A = i)}{P(M = 900)}$$

$$= P(M = 900|A = i) \cdot P(A = i) \cdot K$$

Such that

$$K = \frac{1}{\sum_i P(M = 900|A = i)P(A = i)}$$

Probability of Having 900 Remain

$$P(M = 900|A = i)$$

There were originally 1000 C14 molecules.
Each molecule remains independently with equal probability p_i
What is the probability that 900 remain?

$$M \sim \text{Bin}(n = 1000, p = p_i)$$

$$P(M = 900|A = i) = \binom{1000}{900}(p_i)^{900} \cdot (1 - p_i)^{100}$$

Each molecules' time to live is exponential with λ = 1/8267

Let T be the time to decay for any one molecule

$$T \sim \text{Exp}(\lambda = 1/8267) \quad p_i = P(T > i) = 1 - P(T < i) = e^{-\frac{i}{8267}}$$

Inference as Code

$$P(A = i|M = 900) = P(M = 900|A = i) \cdot P(A = i) \cdot K$$

```
def update_belief(m = 900):
  """
  Returns a dictionary A, where A[i] contains the
  corresponding probability, P(A = i| M = 900).
  m is the number of C14 molecules remaining and i
  is age in years. i is in the range 100 to 10000
  """
  pr_A = {}
  n_years = 9901
  for i in range(100,10000+1):
    prior = 1 / n_years # P(A = i)
    likelihood = calc_likelihood(m, i) # P(M=m | A=i)
    pr_A[i] = prior * likelihood
  # implicitly computes the normalization constant
  normalize(pr_A)
  return pr_A
```

Bayes with variables

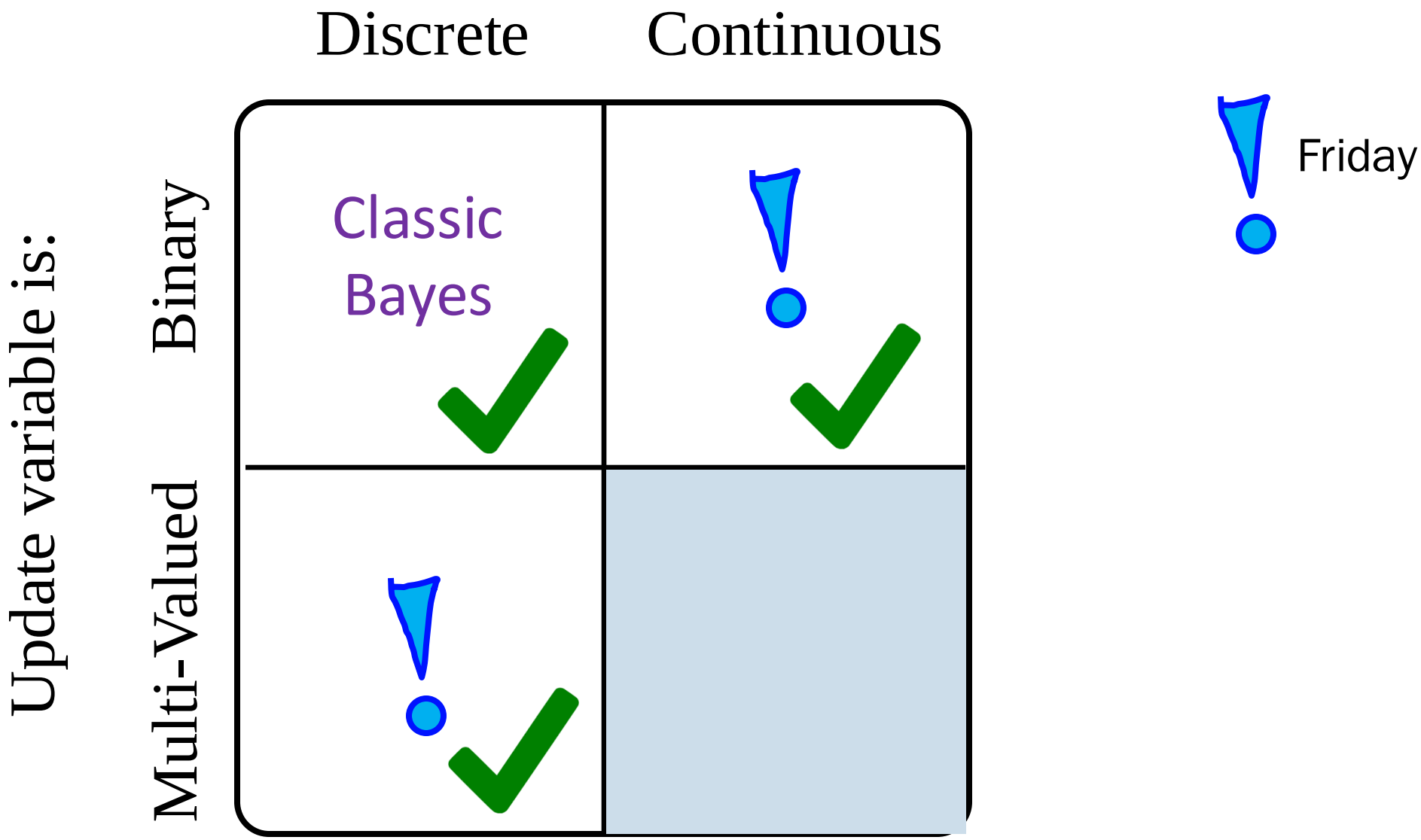
Can mix discrete and continuous

Why is inference hard?

Normalization term is trippy at first

Likelihood term can be complex

Your observation is:



Friday

Bayes + Continuous Random Variables

Let X be a **continuous** random variable

Let N be a **discrete** random variable

$$P(N = n|X = x) = \frac{P(X = x|N = n)P(N = n)}{P(X = x)}$$

$$P(N = n|X = x) = \frac{f(X = x|N = n) \cdot \epsilon \cdot P(N = n)}{f(X = x) \cdot \epsilon}$$

$$P(N = n|X = x) = \frac{f(X = x|N = n) \cdot P(N = n)}{f(X = x)}$$

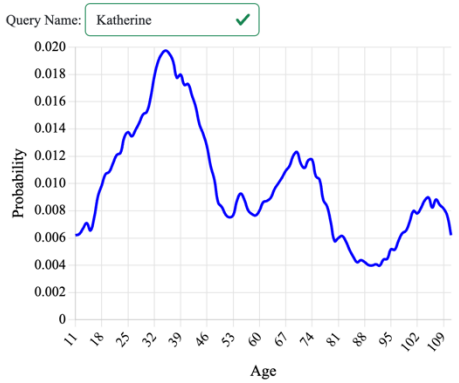
End Review

Today: Five New Real + Exciting Problems

Age from C14

Updated Delivery Prob

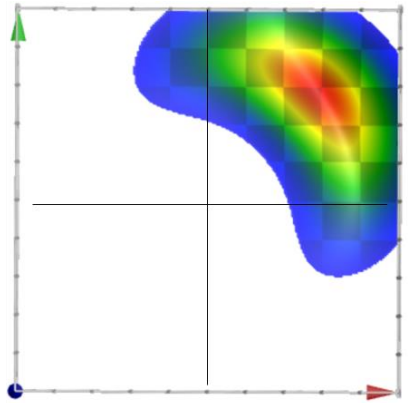
Age from Name



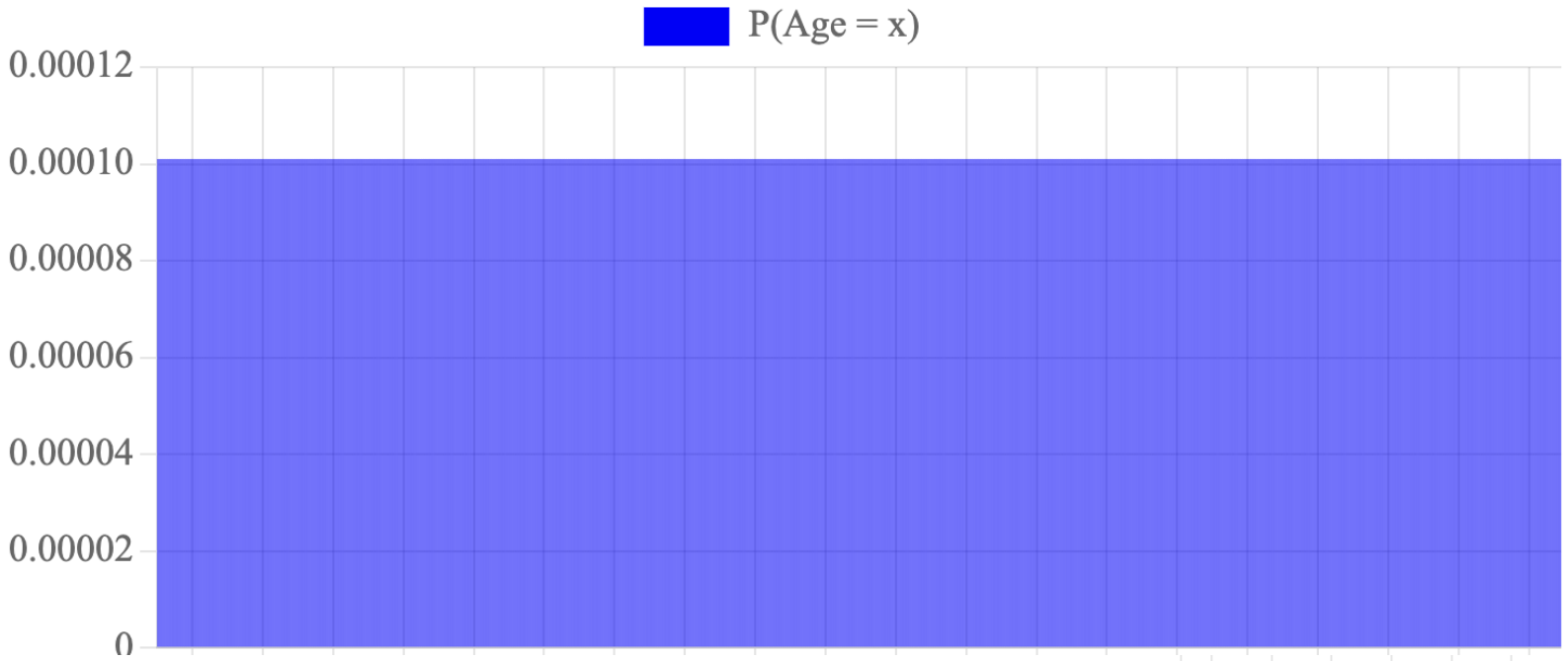
Hidden Chambers

Stanford Eye Test

Cellphone Tracking Cont.



Compare and Contrast Code: (1) Bayesian Carbon Dating

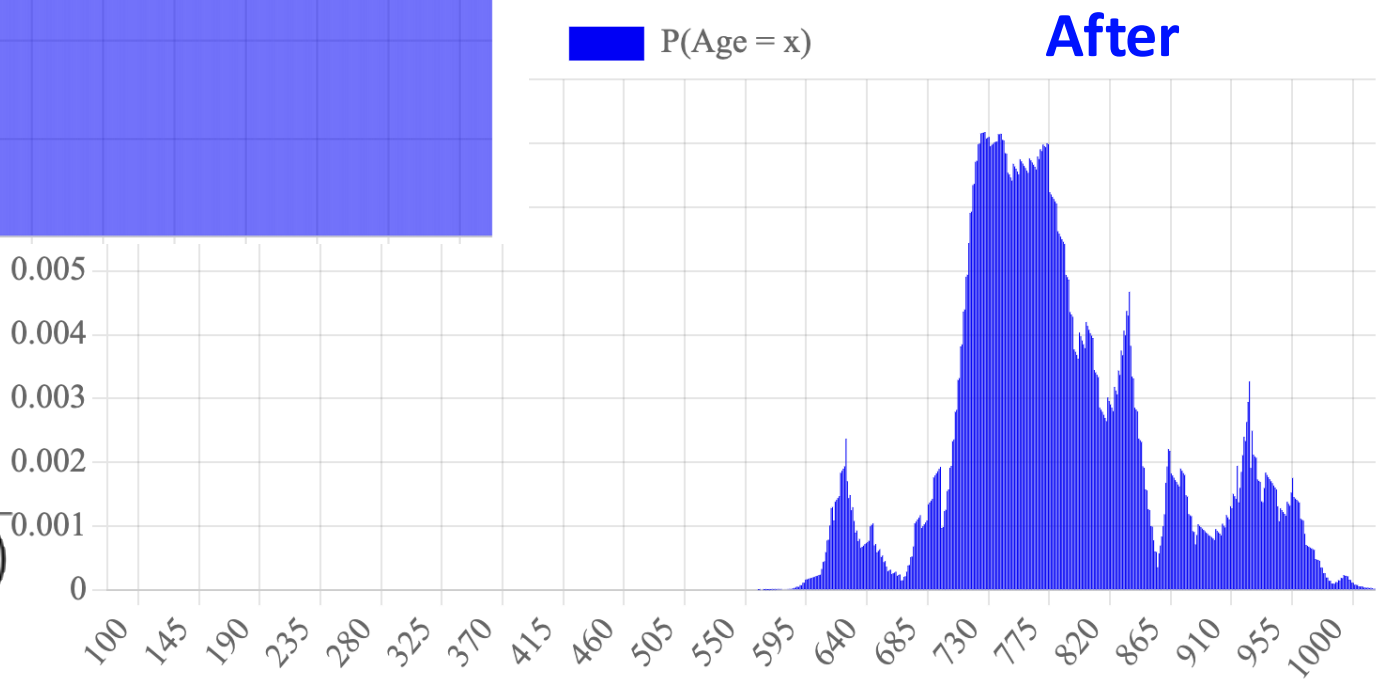


Observation

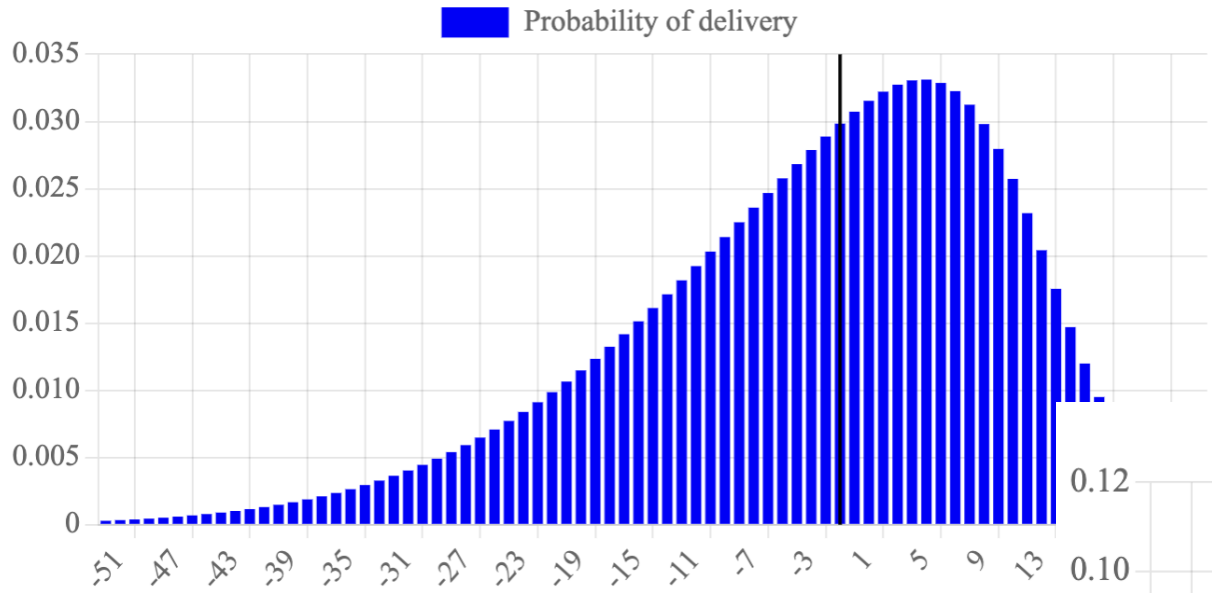
Remaining C14: 900

$$P(A = a|M = 900) = \frac{P(M = 900|A = a)P(A = a)}{\sum_i P(M = 900|A = i)P(A = i)}$$

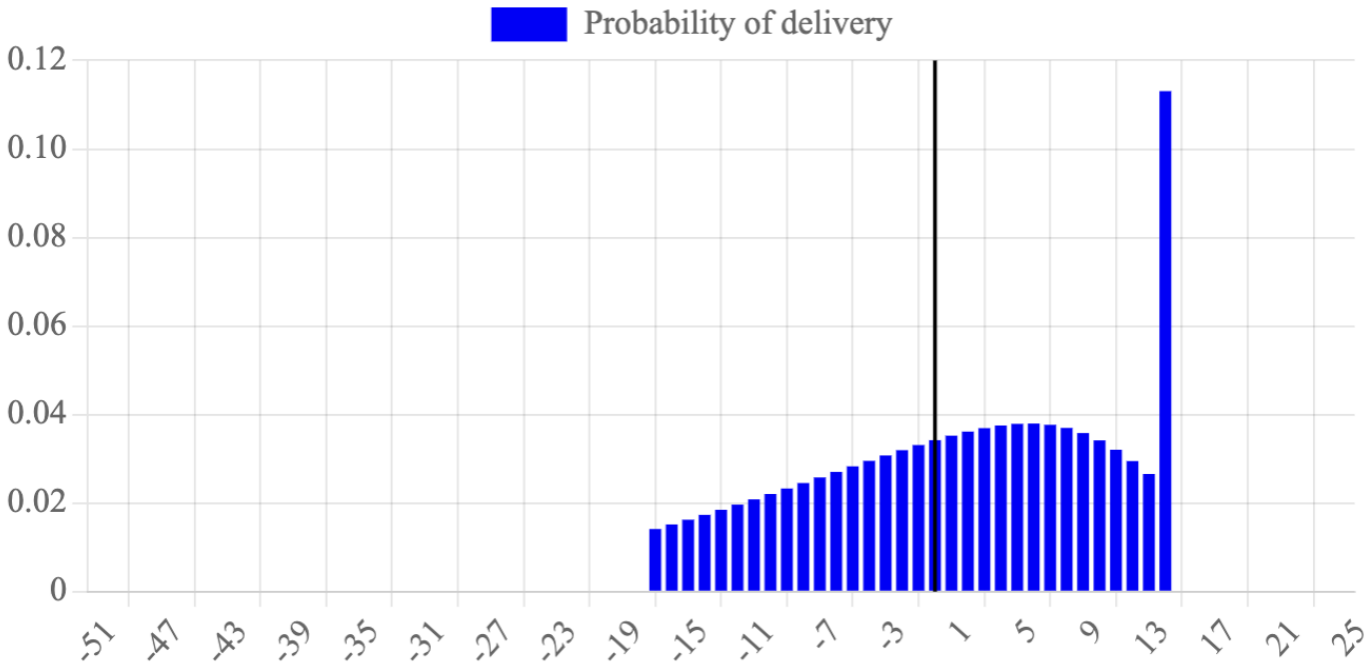
After



Compare and Contrast Code: (2) Baby Delivery



Its 19 days until the due date and no baby



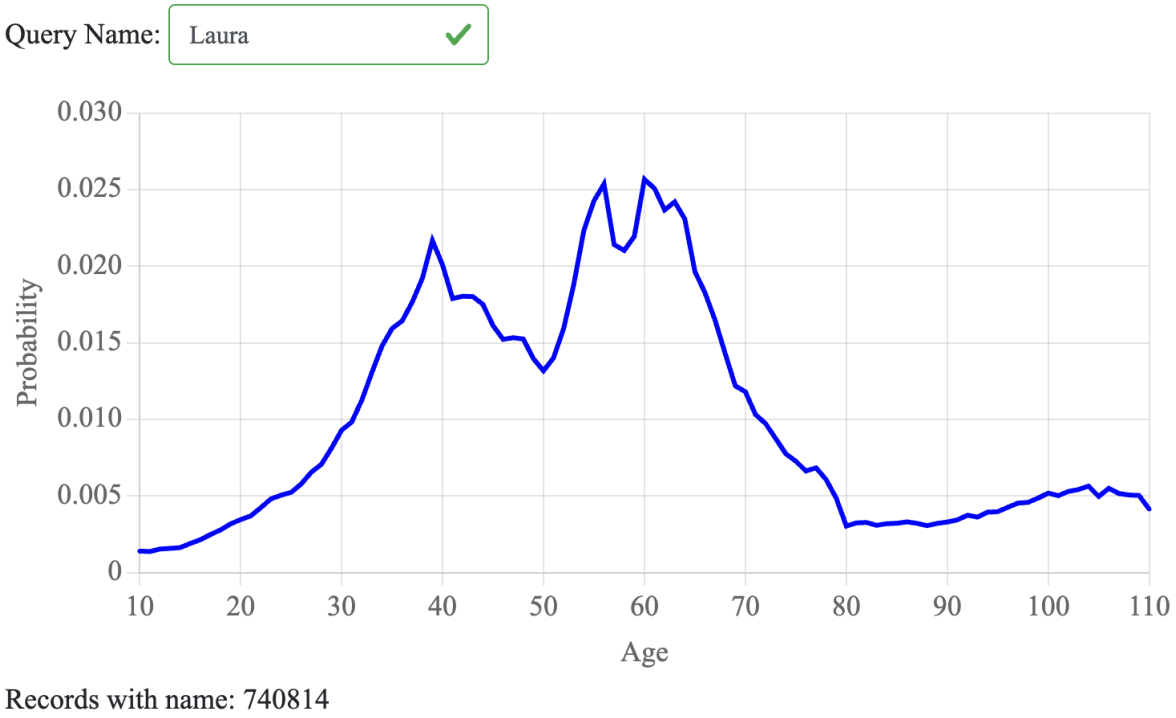
For each value *d:*

$$P(D = d|\text{no child so far})$$

$$= \frac{P(\text{no child so far}|D = d)P(D = d)}{P(\text{no child so far})}$$

Compare and Contrast Code: (3) Name To Age

$$\begin{aligned}
\mathrm{P}(B = 1950 | N = \text{Gary}) &= \frac{\mathrm{P}(N = \text{Gary}, B = 1950)}{\mathrm{P}(N = \text{Gary})} \\
&\approx \frac{\left(\frac{\text{count}(1950, \text{Gary})}{k}\right)}{\left(\frac{\sum_{y \in \text{years}} \text{count}(y, \text{Gary})}{k}\right)} \\
&\approx \frac{\text{count}(1950, \text{Gary})}{\sum_{y \in \text{years}} \text{count}(y, \text{Gary})}
\end{aligned}$$



```
def update_belief_carbon_dating(m = 900):
  # pr_A[i] is P(Age = i| m = 900).
  pr_A = {}
  for i in range(100,10000+1):
    prior = 1 / n_years # P(A = i)
    likelihood = calc_likelihood(m, i) #P(M=m|A=i)
    pr_A[i] = likelihood * prior
  # implicitly computes the normalization constant
  normalize(pr_A)
  return pr_A
```

```
def update_belief_baby(prior, today = 10):
  # pr_D[i] is P(D = i| No Baby Yet).
  pr_D = {}
  for i in range(-50,25):
    # P(NoBaby | D = i)
    likelihood = 0 if i < today else 1
    pr_D[i] = likelihood * prior[i]
  # implicitly computes the LOTP
  normalize(pr_D)
  return pr_D
```

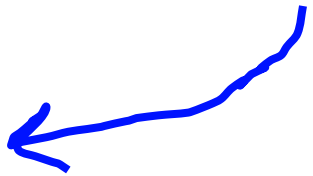
```
def update_belief_name_to_age(name = 'Laura'):
  # pr_age[i] is P(Age = i| name).
  # prob_name_and_age is just a counting from the US
  # Social Security database.
  pr_age = {}
  for i in range(10,110):
     pr_age[i] = calc_prob_name_and_age(name, i)
  # implicitly computes the normalization constant
  normalize(pr_age)
  return pr_age
```

What do you notice is the same. What is different?

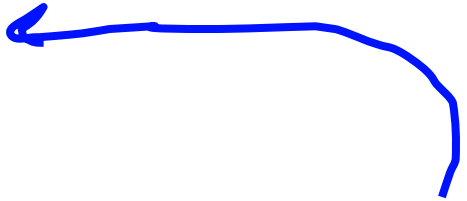
Normalize in Python

```
# list normalization
def normalize_list(data_list):
    total_sum = np.sum(data_list)
    return np.array(data_list) / total_sum

>>> norm = normalize_list([10, 20, 30, 40])
>>> np.sum(norm) # 1.0, always (within floating point error)
```

returned [0.1 0.2 0.3 0.4]



```
# dictionary normalization
def normalize_dict(data_dict):
    total_sum = sum(data_dict.values())
    return {key: value / total_sum for key, value in data.items()}

>>> norm = normalize_dict({'a': 100, 'b': 200, 'c': 300})
>>> np.sum(norm.values()) # 1.0, always (within floating point error)
```

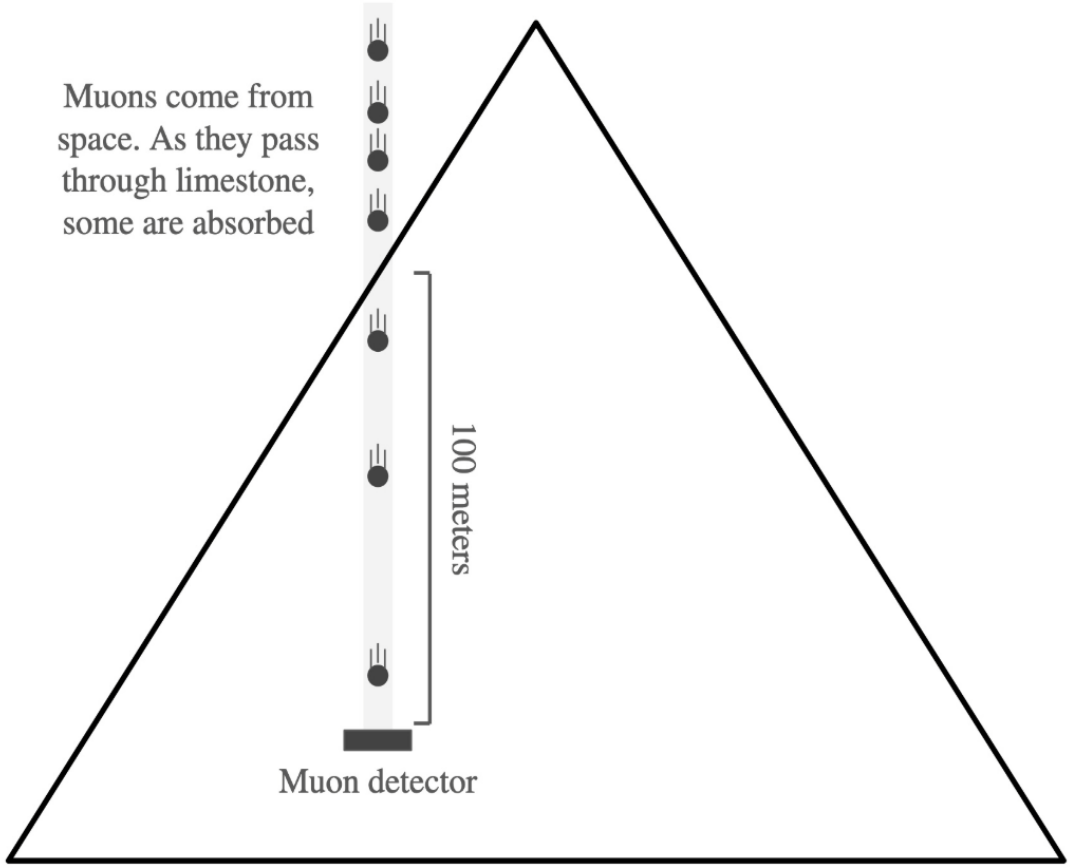


returned {'a': 0.166, 'b': 0.333, 'c': 0.5}

Are you ready
For Hidden Chambers???

Hidden Pyramid Chambers with Poisson + Bayes

Basics of Muography

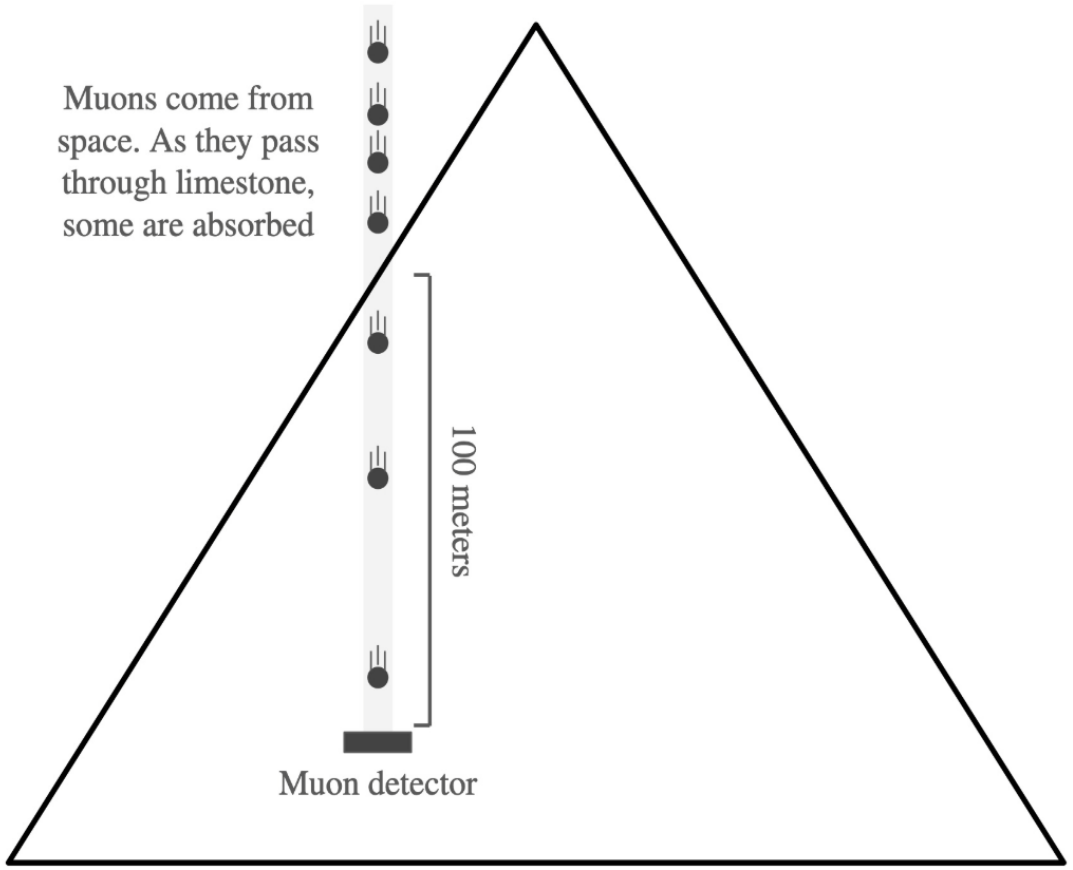


Beer Lambert Law

$$\lambda_x = 100 \cdot e^{-x/40}$$

Rate of muons depends on x, amount of limestone

a. (6 points) Imagine the entire 100 meter path is limestone. In that case, the rate of muons arriving per month on the detection plate is $100 \cdot e^{-100/40} = 8.2$. What is the probability that in one month you would observe 12 muons?

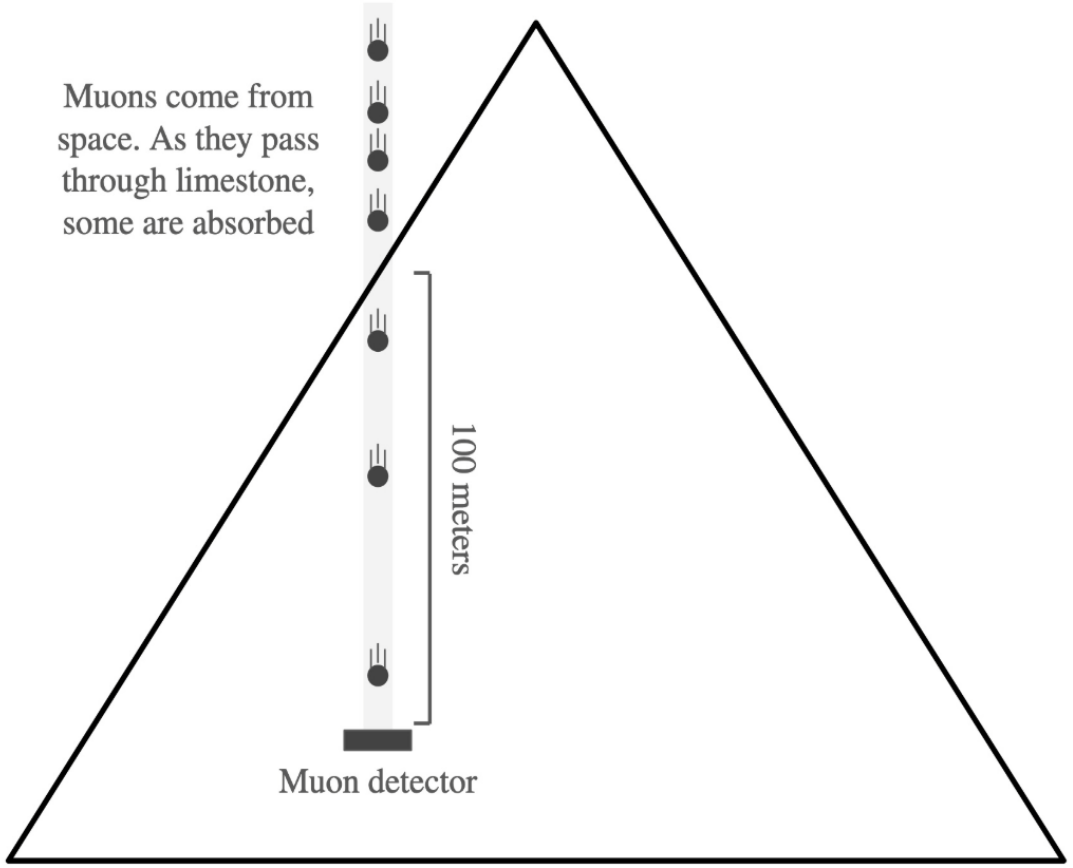


Beer Lambert Law

$$\lambda_x = 100 \cdot e^{-x/40}$$

Rate of muons depends on x, amount of limestone

b. (14 points) Let X be your belief in the meters of limestone above the detection plate. Your prior belief is that any number of meters from 0 to 100 is equally likely: $X \sim \text{Uni}(0, 100)$. After one month, your detection plate has been hit by 12 muons. What is your updated belief in X?
Recall: You may leave your answer with integrals or sums. You don't need to simplify for full credit.

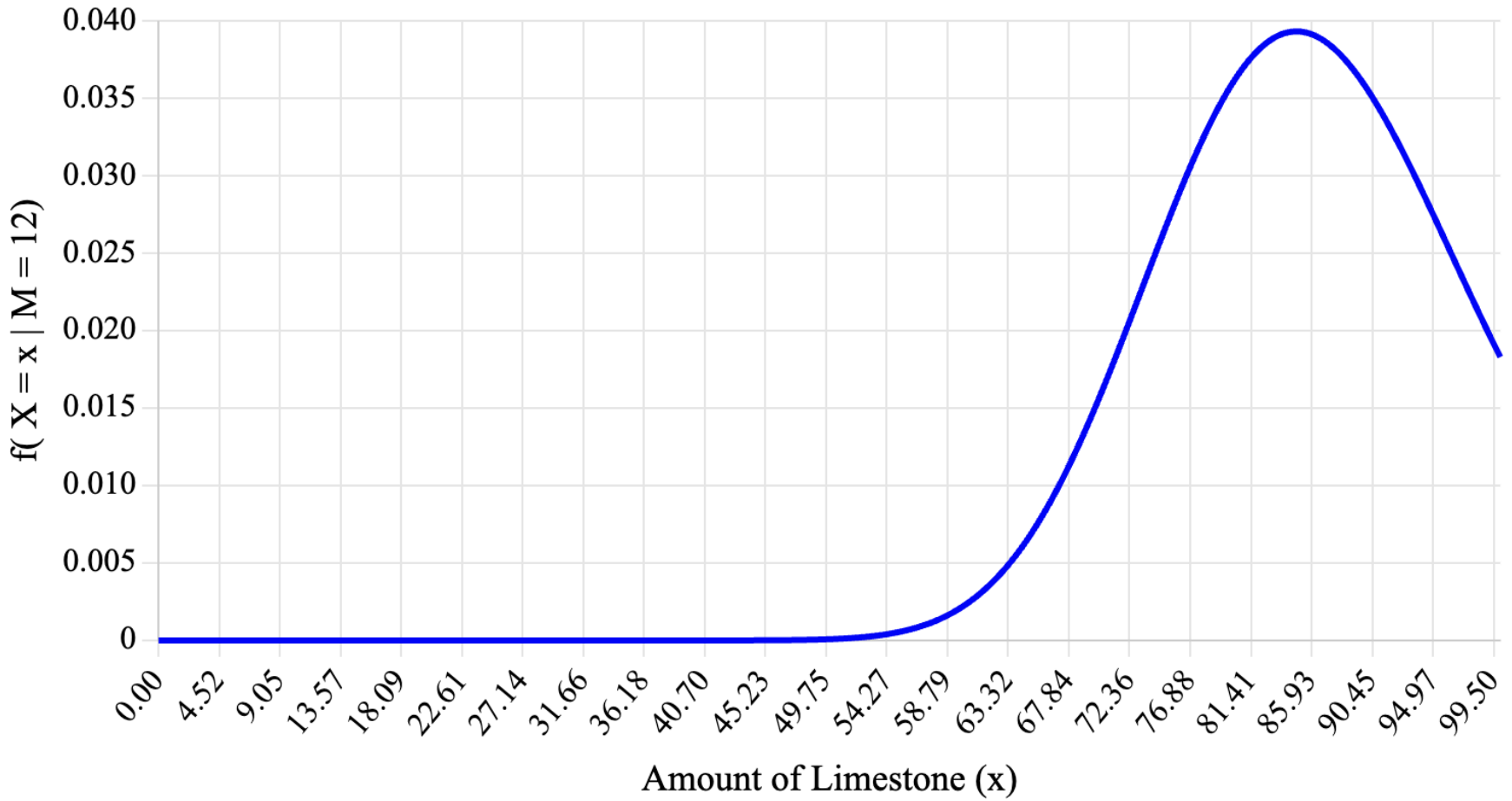


$$\lambda_x = 100 \cdot e^{-x/40}$$

Rate of muons depends on x, amount of limestone

Here is what that PDF equation looks like:

Number of muons (m): 12



Beyond Inference:
What hidden chambers make all the muon readings most likely?

Are you ready
For Stanford Acuity Test???

A Better Eye Test

https://www.thelancet.com/journals/lancet/article/PIIS0140-6736(21)02149-8/fulltext

https://www.science.org/content/article/eye-robot-artificial-intelligence-dramatically-improves-accuracy-classic-eye-exam

https://ojs.aaai.org/index.php/AAAI/article/view/5384/5240

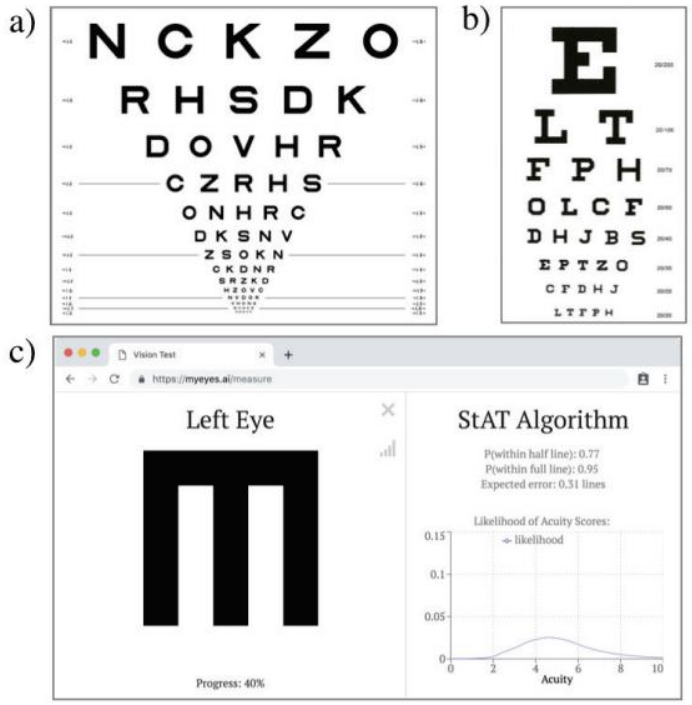
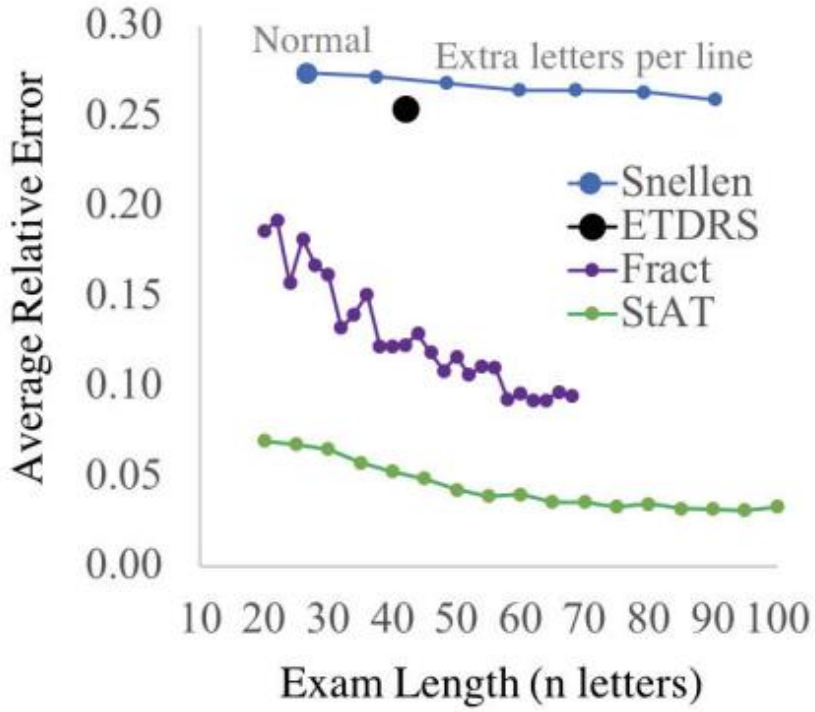
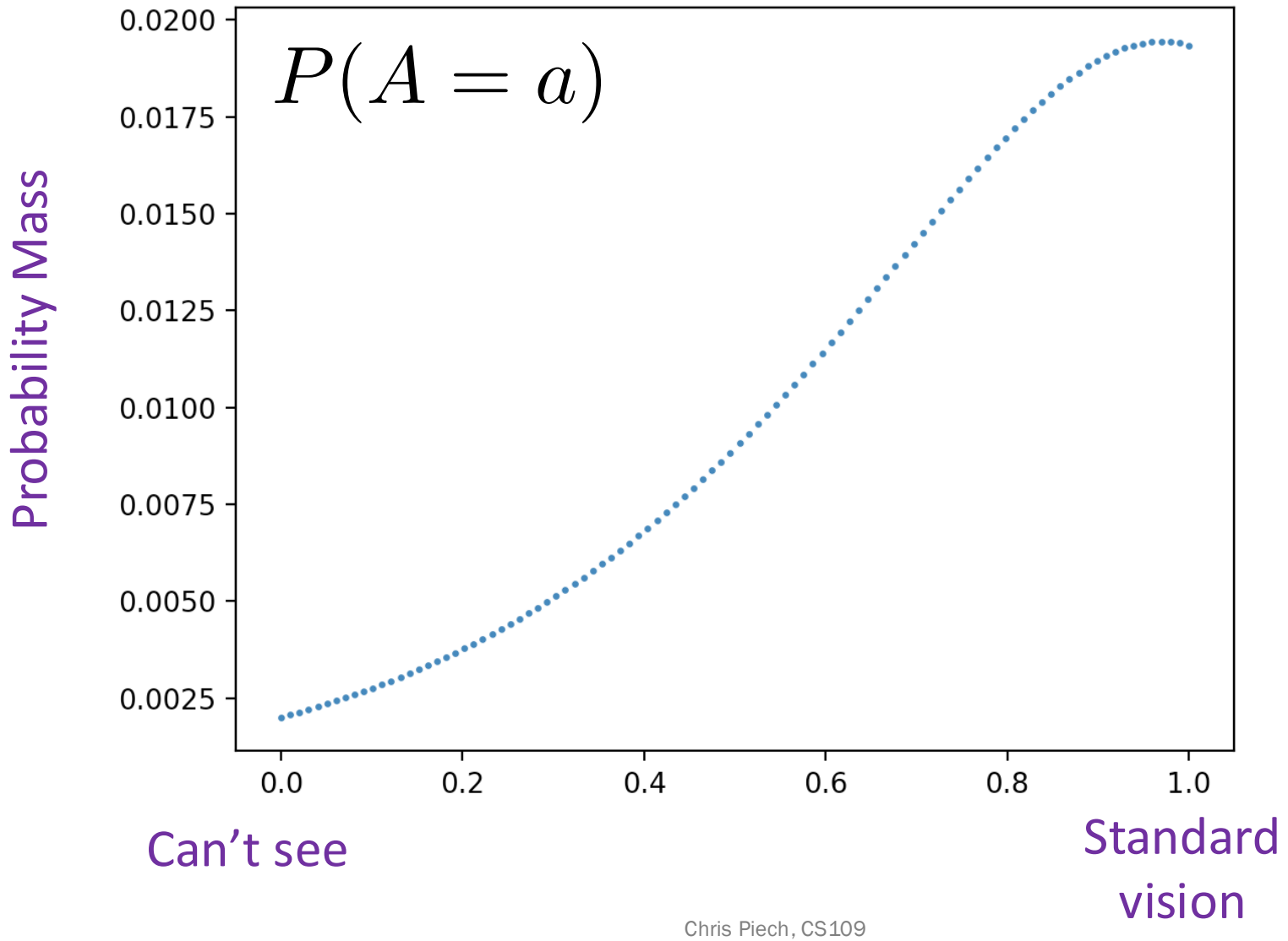


Figure 1: a) ETDRS, b) Snellen and c) StAT eye exams.



Prior Belief in Ability to See (Random Var A)



PMF is Actually Stored as a Dictionary

```
def main():
  belief = get_prior_belief()
```

a	P(A=a)
0.00	0.00198
0.01	0.00205
0.02	0.00211
0.03	0.00218
0.04	0.00225
0.05	0.00233
0.06	0.0024
0.07	0.00248
0.08	0.00256
0.09	0.00264
0.10	0.00273
0.11	0.00281
0.12	0.0029
0.13	0.00299
0.14	0.00309
0.15	0.00319
0.16	0.00329
0.17	0.00339
0.18	0.0035

a	P(A=a)
0.20	0.00372
0.21	0.00384
0.22	0.00396
0.23	0.00408
0.24	0.00421
0.25	0.00434
0.26	0.00447
0.27	0.00461
0.28	0.00475
0.29	0.00489
0.30	0.00504
0.31	0.00519
0.32	0.00535
0.33	0.00551
0.34	0.00567
0.35	0.00584
0.36	0.00601
0.37	0.00619
0.38	0.00637

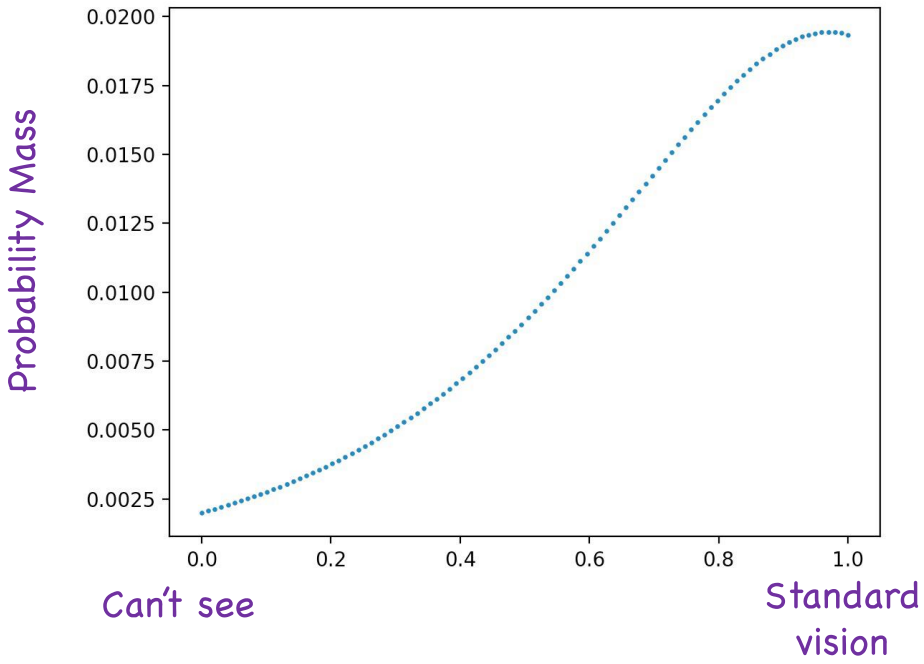
. . .

a	P(A=a)
0.80	0.01684
0.81	0.01708
0.82	0.01731
0.83	0.01753
0.84	0.01774
0.85	0.01795
0.86	0.01814
0.87	0.01832
0.88	0.01848
0.89	0.01864
0.90	0.01877
0.91	0.0189
0.92	0.019
0.93	0.01909
0.94	0.01916
0.95	0.01921
0.96	0.01924
0.97	0.01925
0.98	0.01924

Prior Belief in Ability to See (Random Var A)

```
belief = get_prior_belief()
```

As a graph



As a dictionary

a	P(A=a)
0.00	0.00198
0.01	0.00205
0.02	0.00211
0.03	0.00218
0.04	0.00225
0.05	0.00233
0.06	0.0024
0.07	0.00248
0.08	0.00256
0.09	0.00264
0.10	0.00273
0.11	0.00281
0.12	0.0029
0.13	0.00299
0.14	0.00309
0.15	0.00319
0.16	0.00329
0.17	0.00339
0.18	0.0035
0.19	0.00361

a	P(A=a)
0.20	0.00372
0.21	0.00384
0.22	0.00396
0.23	0.00408
0.24	0.00421
0.25	0.00434
0.26	0.00447
0.27	0.00461
0.28	0.00475
0.29	0.00489
0.30	0.00504
0.31	0.00519
0.32	0.00535
0.33	0.00551
0.34	0.00567
0.35	0.00584
0.36	0.00601
0.37	0.00619
0.38	0.00637
0.39	0.00655

...

a	P(A=a)
0.80	0.01684
0.81	0.01708
0.82	0.01731
0.83	0.01753
0.84	0.01774
0.85	0.01795
0.86	0.01814
0.87	0.01832
0.88	0.01848
0.89	0.01864
0.90	0.01877
0.91	0.0189
0.92	0.019
0.93	0.01909
0.94	0.01916
0.95	0.01921
0.96	0.01924
0.97	0.01925
0.98	0.01924
0.99	0.01921

Number or Dictionary?

belief

$$P(A = a|Y = 0) = \frac{P(Y = 0|A = a)P(A = a)}{P(Y = 0)}$$

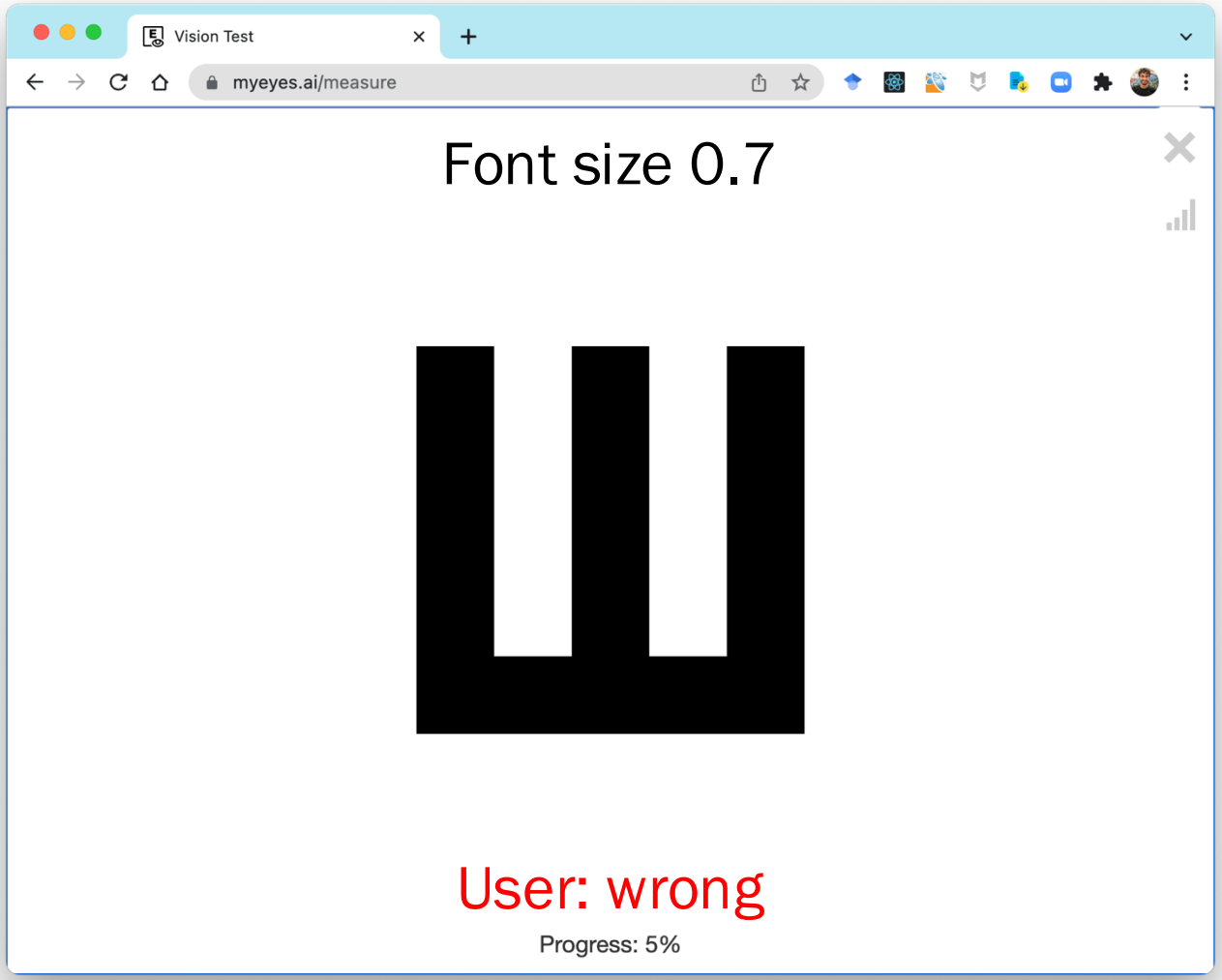
belief[a] = 0.001

Today: I am going to simplify the units of vision

Normally doctors measure ability to see in logarithmic units. To make today's demo easier to understand

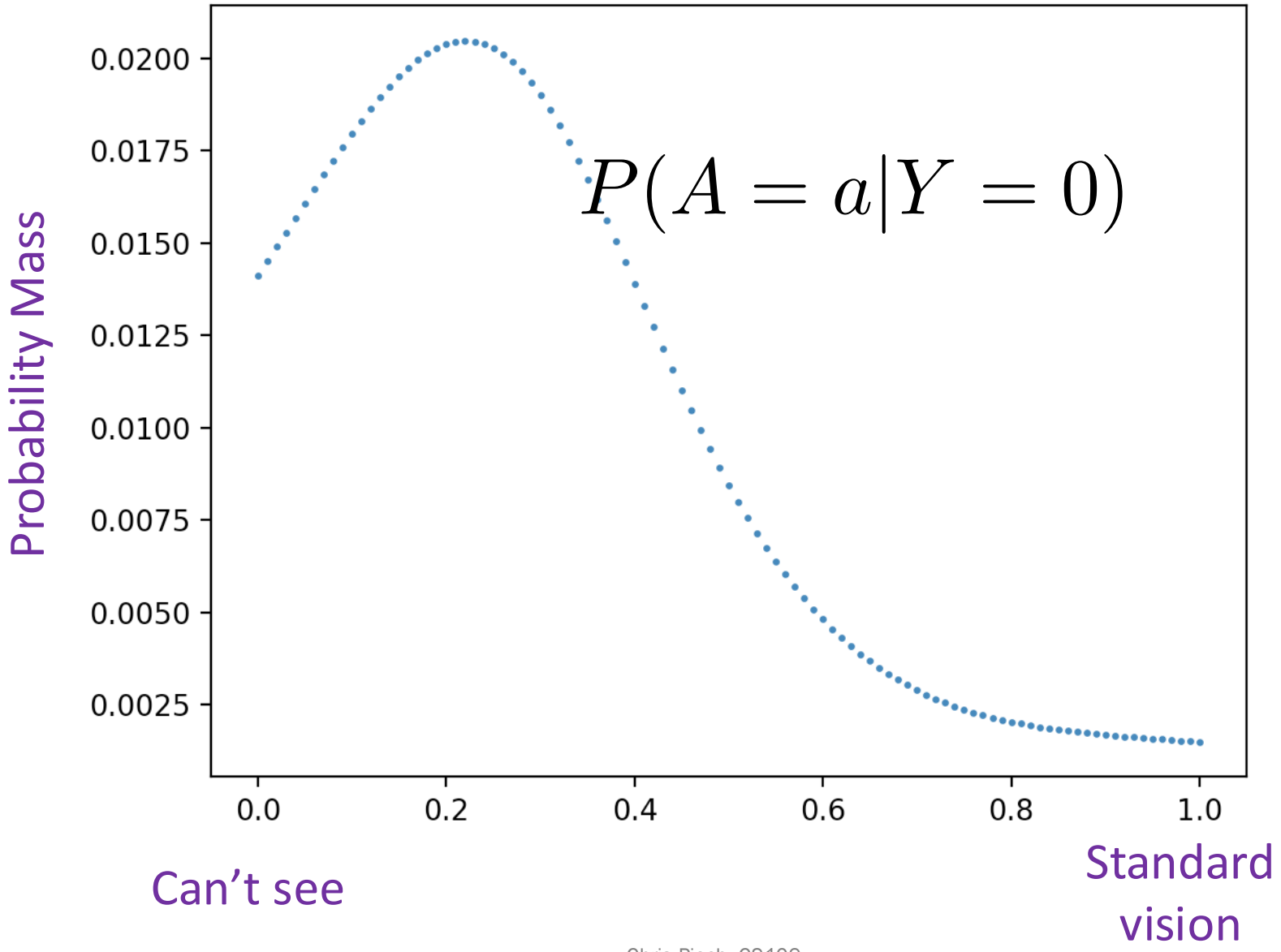
I have translated both onto a [0, 1] scale.
Where 0 means can't see and 1.0 is standard vision

The Patient is Shown One Letter and They Get it Wrong



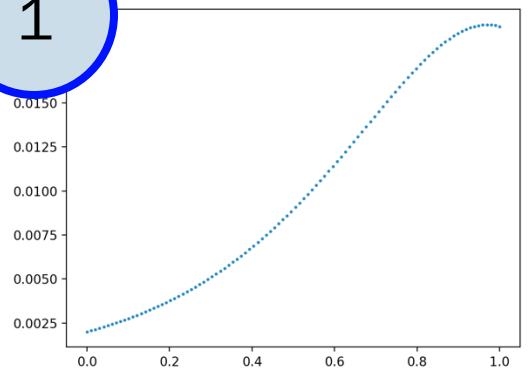
Observation $Y = 0$

Posterior Belief in Ability to See (Random Var A)



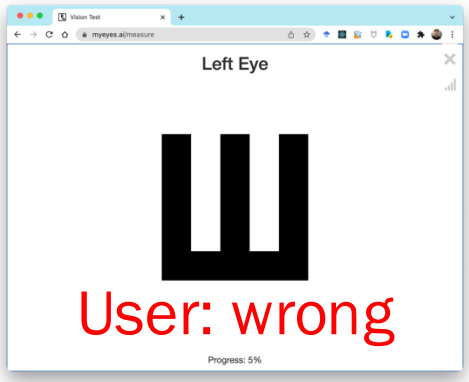
Bayes with Random Variables

1



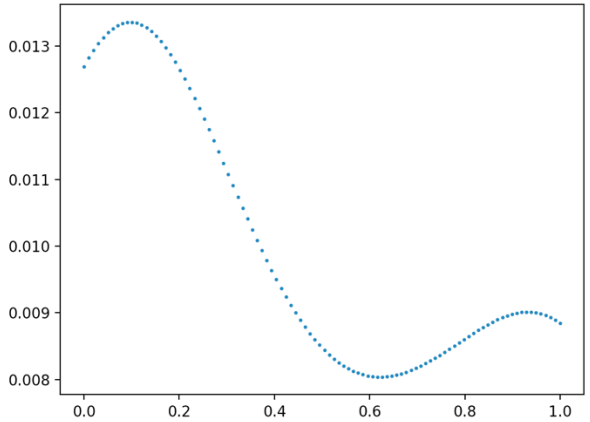
$$P(A = a)$$

2



Observation $Y = 0$
(At font size s_1)

3

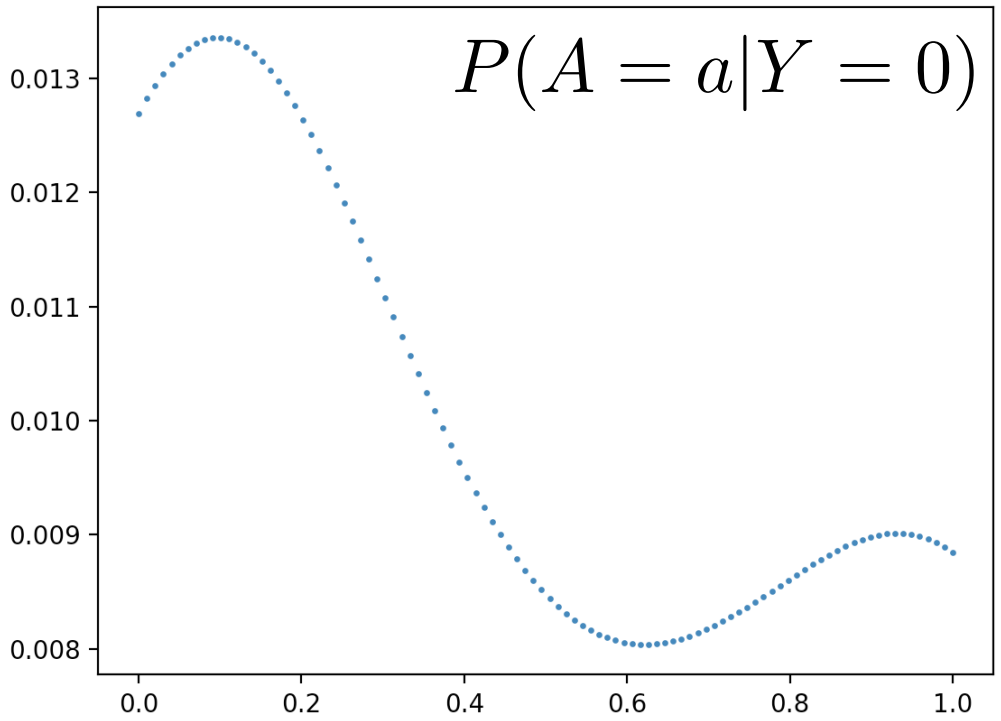


$$P(A = a|Y = 0)$$

$$P(A = a|Y = 0) = \frac{P(Y = 0|A = a)P(A = a)}{P(Y = 0)}$$

Inference on a non-bernoulli random variable

In plain English: run bayes for each value of a




```
# RV bayes as code
def update(belief, obs):
    for a in support:
      prior_a = belief[a]
      likelihood = calc_likelihood(a, obs)
      belief[a] = prior_a * likelihood
    normalize(belief)
```

likelihood

$$P(A = a|Y = 0) = \frac{P(Y = 0|A = a)P(A = a)}{P(Y = 0)}$$

Normalize???

```
# RV bayes as code
def update(belief, obs):
    for a in support:
      prior_a = belief[a]
      likelihood = calc_likelihood(a, obs)
      belief[a] = prior_a * likelihood
    normalize(belief)
```

In plain English: this is the numerator, summed over all values of A

$$P(A=a|Y=0) = \frac{P(Y=0|A=a)P(A=a)}{P(Y=0)}$$

$$= \frac{P(Y=0|A=a)P(A=a)}{\sum_{x \in A} P(Y=0, A=x)}$$

$$= \frac{P(Y=0|A=a)P(A=a)}{\sum_{x \in A} P(Y=0|A=x)P(A=x)}$$

Inference

In general Bayes theorem with a random variable is like the cellphone problem: multiple possible assignments to keep track of

Still true when some variables are continuous

Random Variables

Not all beliefs can be represented as a **function**. **Dictionary** / table is a great way to represent a random variable belief.

This is formally called non-parametric

Representing Continuous Variables

Dictionary can also be used to represent a discretization of a **continuous random var**

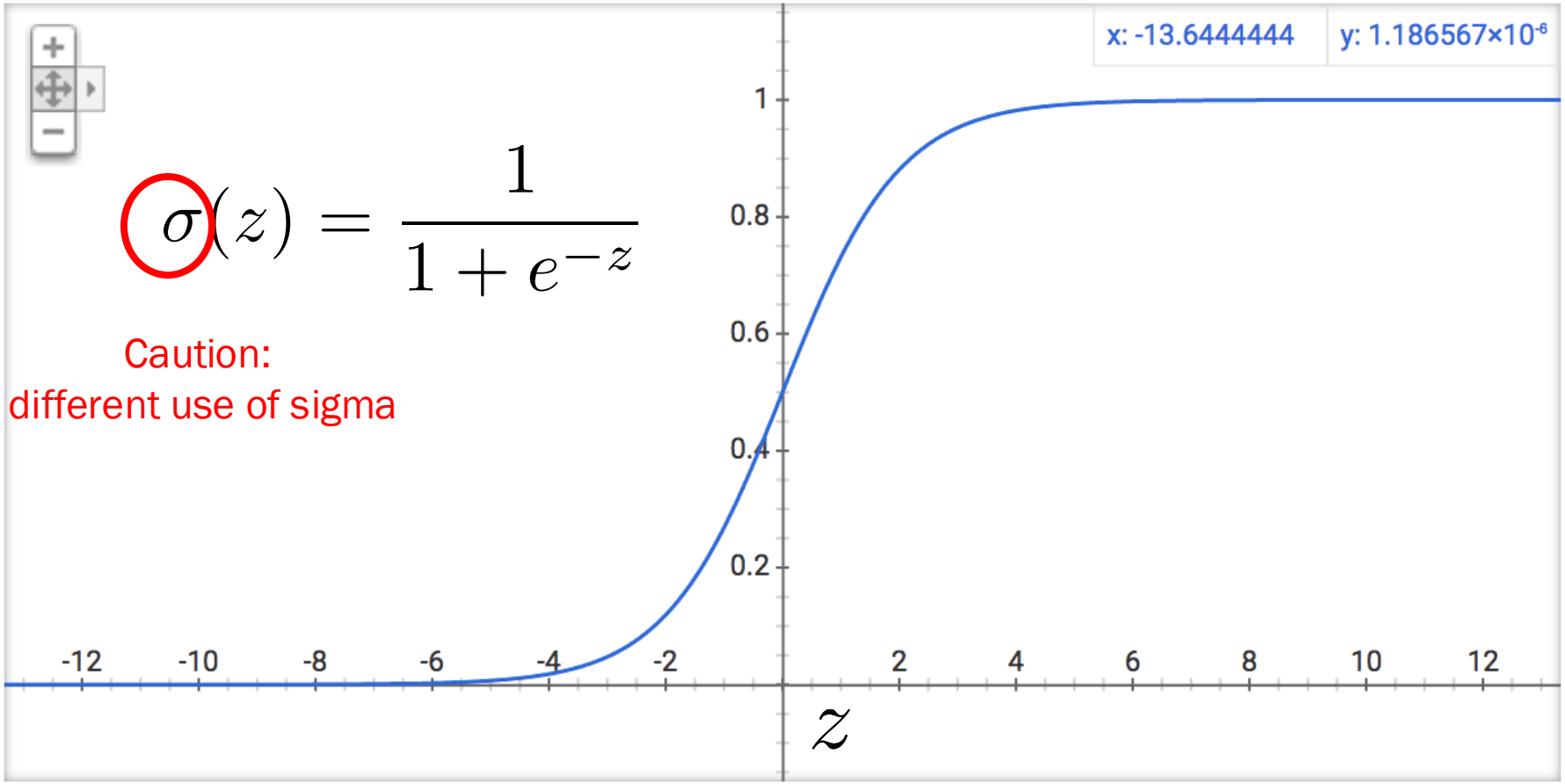
I do it all the time! Yay compute!

Aside: How to get

$$P(Y = 0|A = a)$$

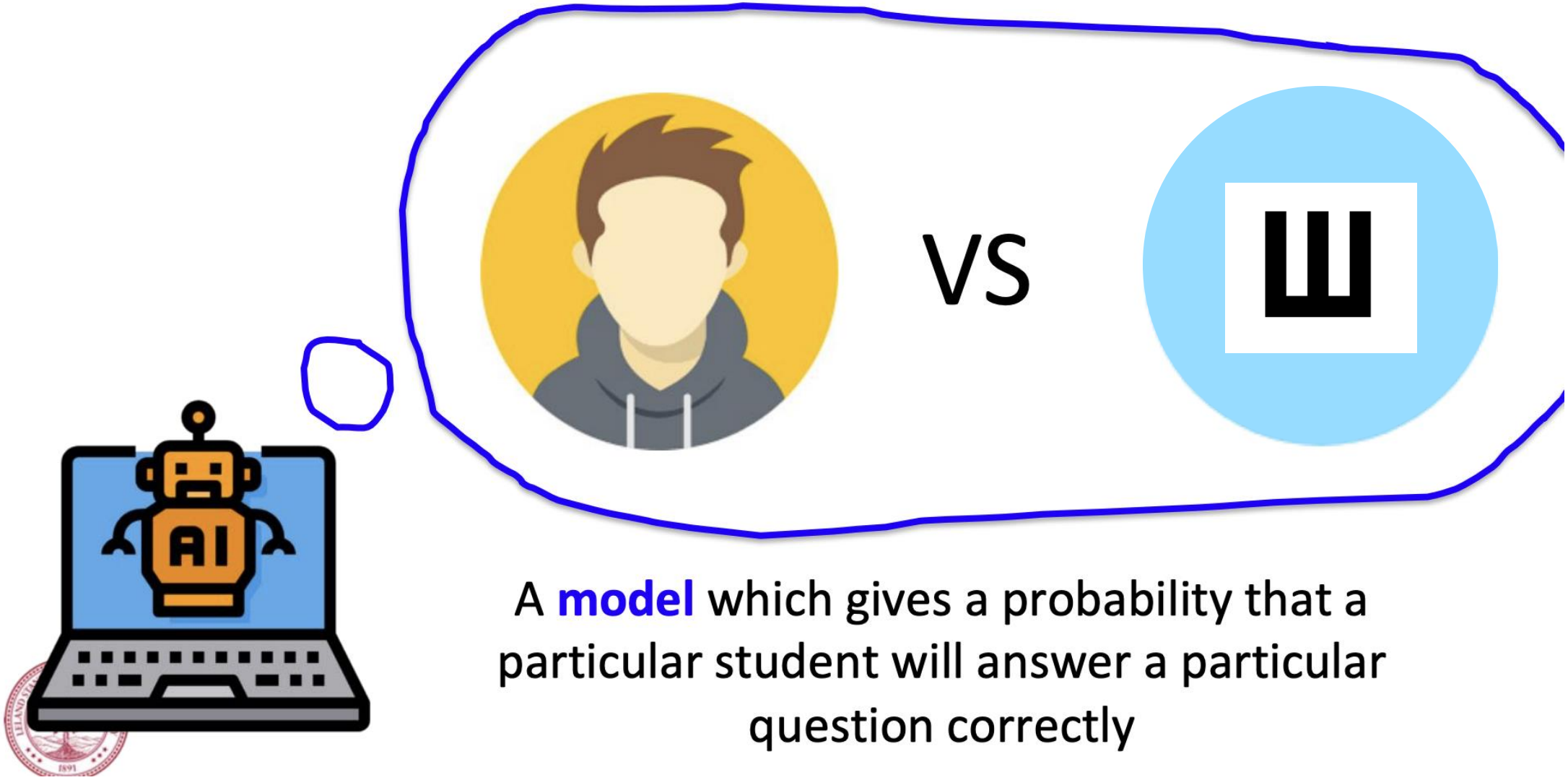
Mix of education theory and probability

Background: Sigmoid Function



The sigmoid function squashes z to be a number between 0 and 1

Likelihood: Item Response Theory



A **model** which gives a probability that a particular student will answer a particular question correctly

Likelihood: Item Response Theory

If a student with ability a
attempts an "item" with difficulty d

$$P(K = 1 | A = a) = \sigma(a - d)$$

Prob they know the answer

Squashing function

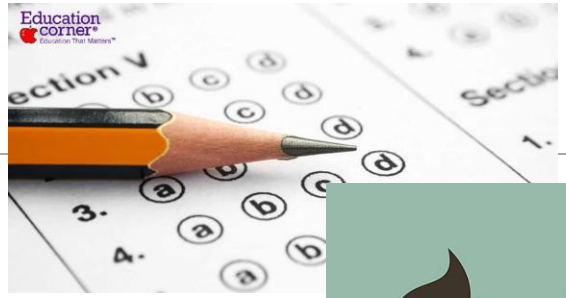
Ability of student i

Difficulty of problem j

Guess and Slip

Let Y = 1 be the event the answer is correct.
Let K = 1 be the event that the student knows the answer.
Let s be the probability of a slip

$$P(K = 1|A = a) = \sigma(a - d)$$



Guess

Slip

$$P(Y = 1|A = a) = P(Y = 1|K = 0, A = a) \cdot P(K = 0|A = a)$$
$$+ P(Y = 1|K = 1, A = a) \cdot P(K = 1|A = a)$$

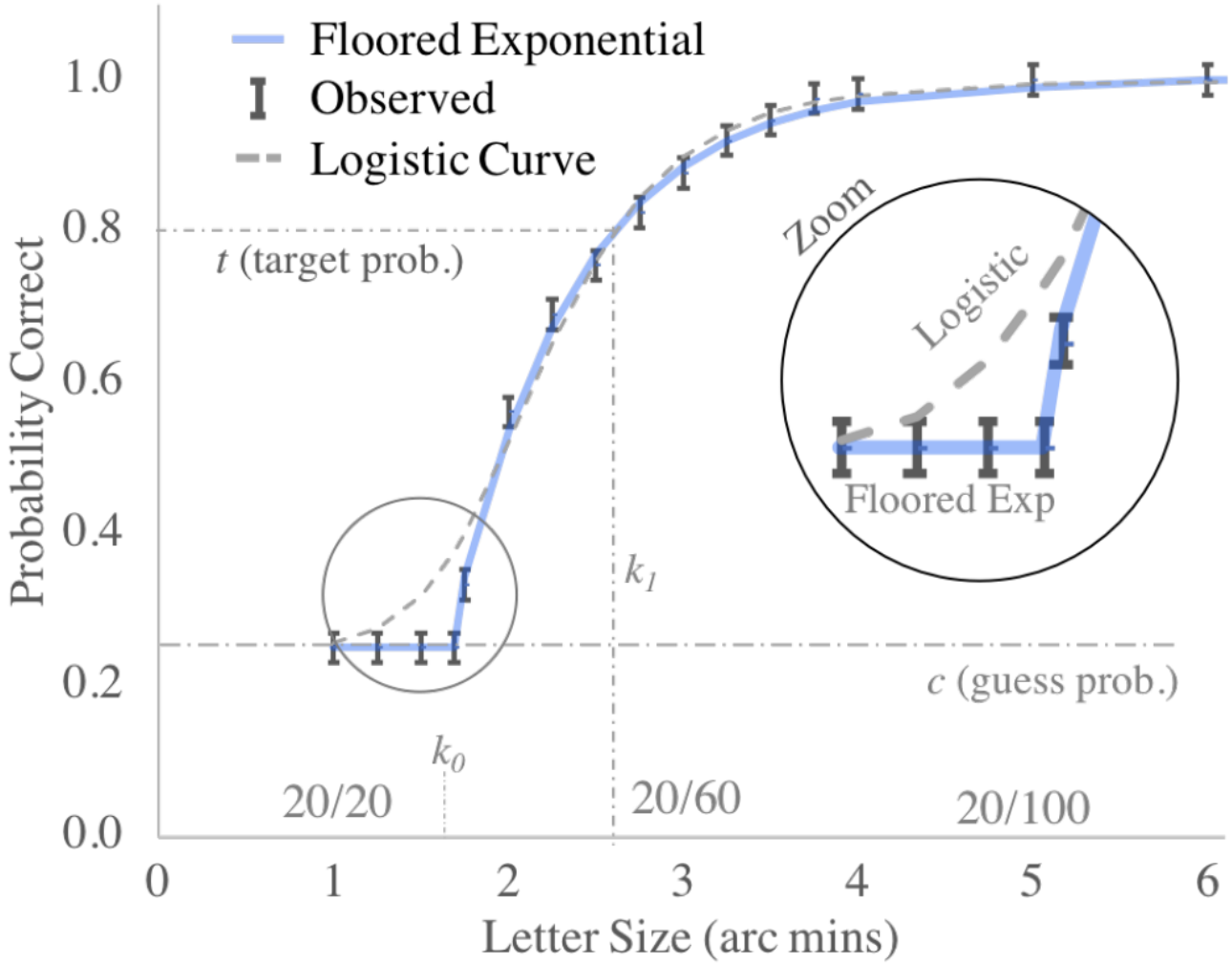
$$= \frac{1}{4} \cdot [1 - \sigma(a - d)] + (1 - s) \cdot \sigma(a - d)$$

$$= \frac{1}{4} + \left(\frac{3}{4} - s\right) \cdot \sigma(a - d)$$

Probability of Right and Wrong

$$P(Y = 0|A = a) = 1 - P(Y = 1|A = a)$$

We Actually Mapped out the Probabilities



End Aside: How to get

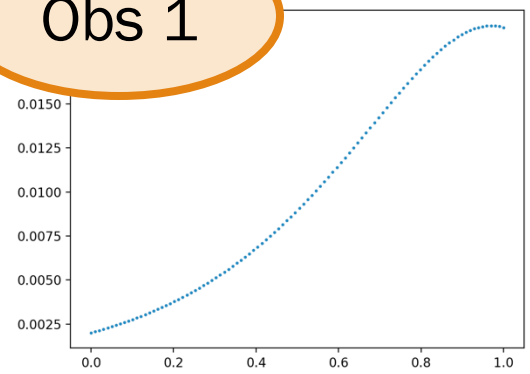
$$P(Y = 0|A = a)$$

Mix of education theory and probability

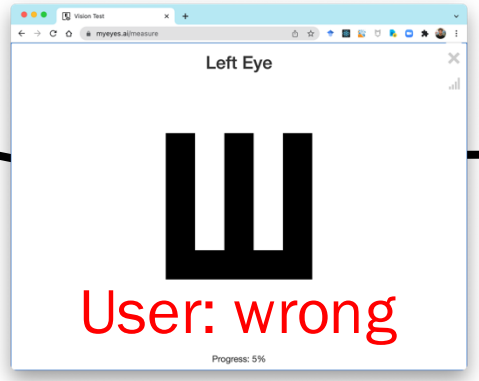
Multiple observations??

Multiple Observations

Obs 1

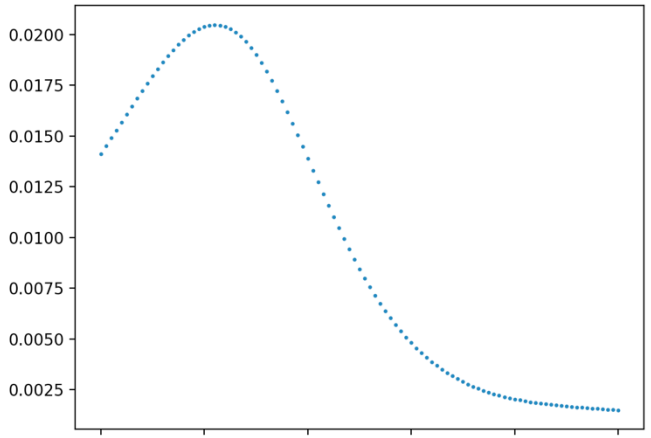


$P(A = a)$



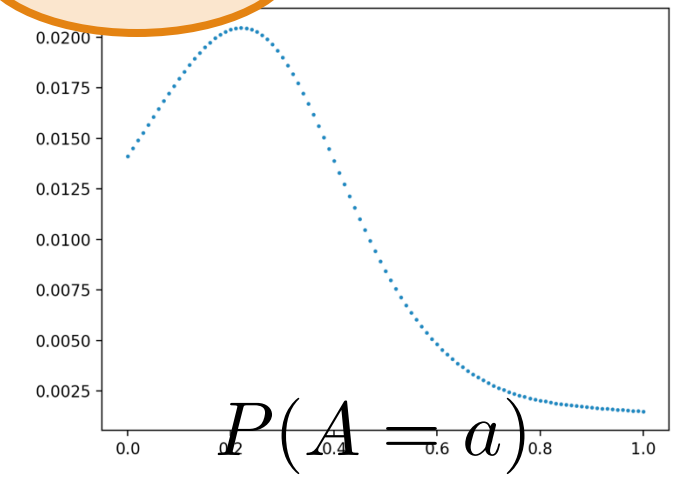
Observation $Y = 0$

(At font size 0.7)

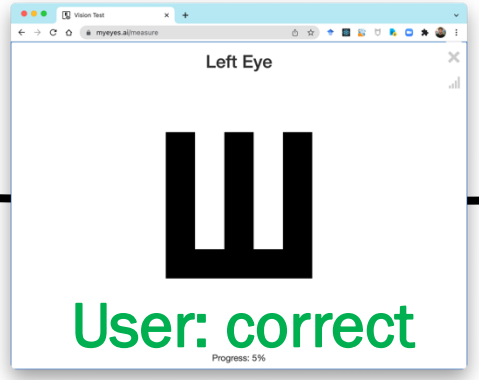


$P(A = a|Y = 0)$

Obs 2

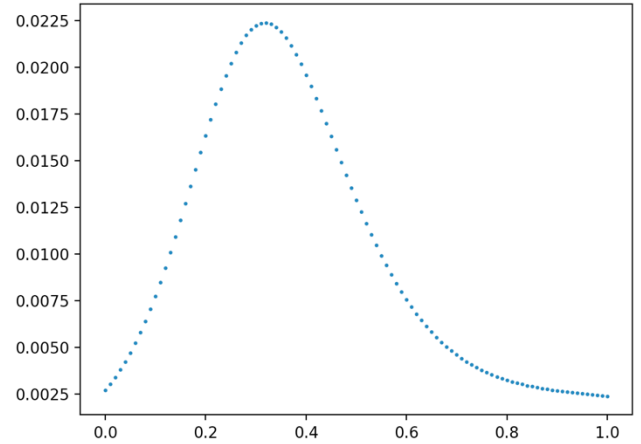


$P(A = a)$



Observation $Y = 0$

(At font size 0.8)



$P(A = a|Y = 1)$

Multiple Observations

Single Observation:

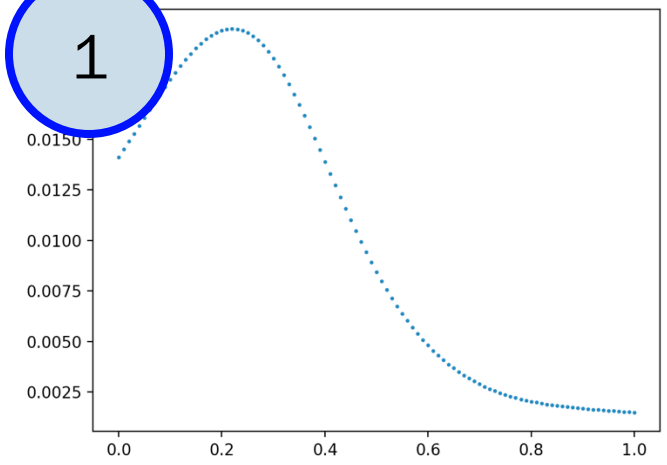
$$P(A = a|R_1) = P(R_1|A = a) \cdot P(A = a) \cdot K$$

Multiple Observations:

$$P(A = a|R_1, R_2) = P(R_1, R_2|A = a) \cdot P(A = a) \cdot K_2$$
$$= P(R_2|A = a) \cdot P(R_1|A = a) \cdot P(A = a) \cdot K_2$$

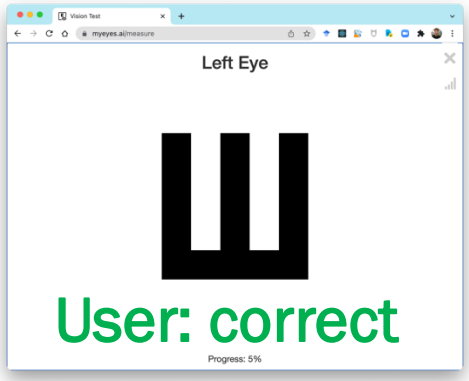
Posterior becomes new prior

1



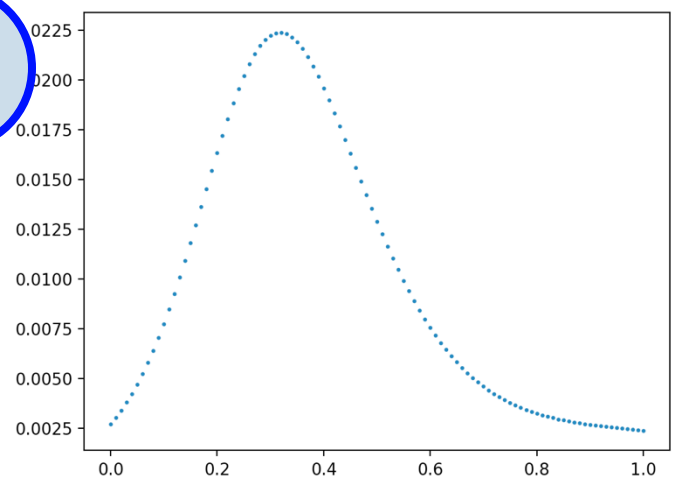
$P(A = a)$

2



Observation $Y = 1$
(At font size *0.8*)

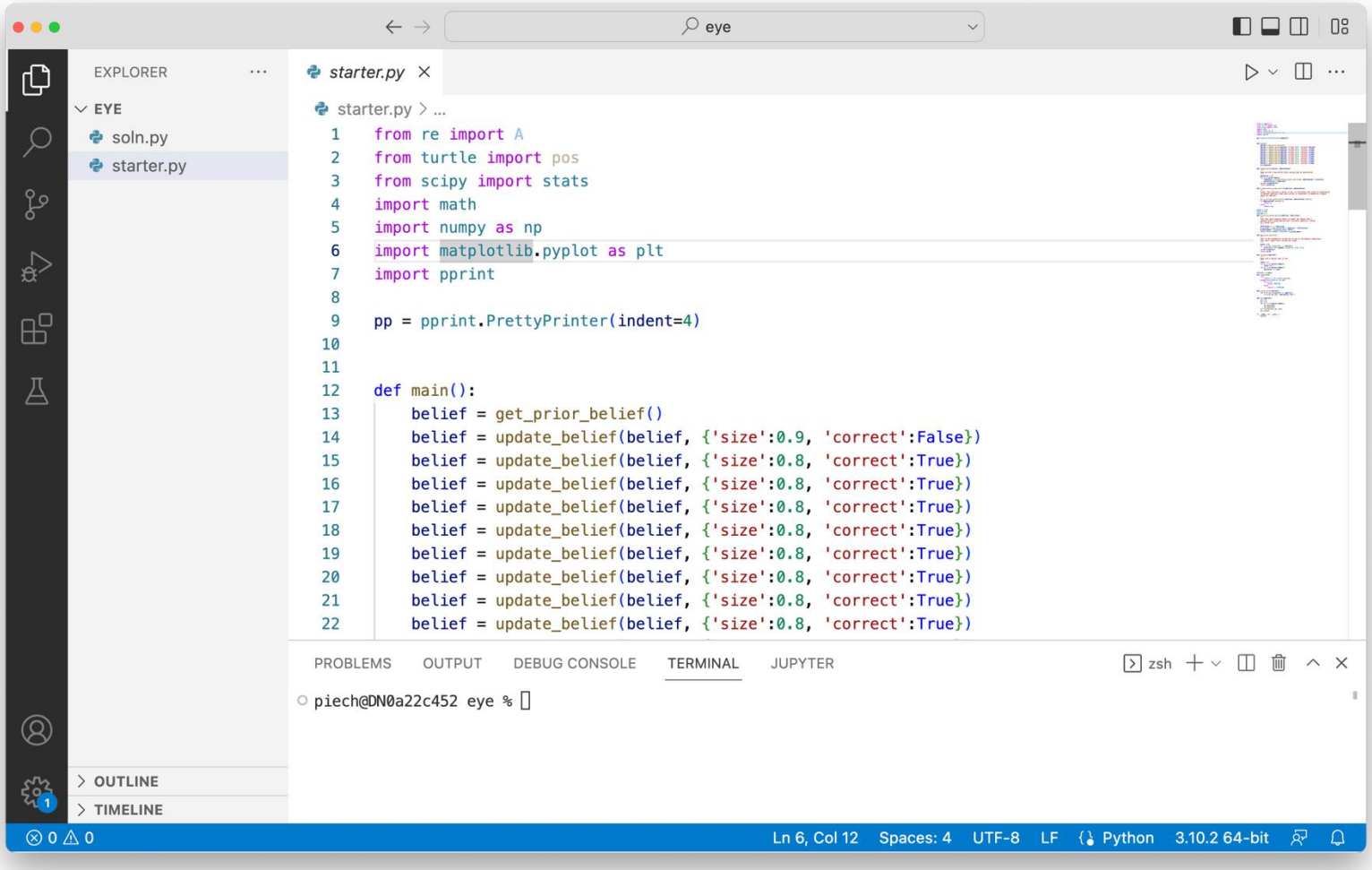
3



$P(A = a|Y = 1)$

$$P(A = a|Y = 1) = \frac{P(Y = 1|A = a)P(A = a)}{P(Y = 1)}$$

In the Code


eye
EXPLORER
EYE
soln.py
starter.py
starter.py
starter.py > ...
1 from re import A
2 from turtle import pos
3 from scipy import stats
4 import math
5 import numpy as np
6 import matplotlib.pyplot as plt
7 import pprint
8
9 pp = pprint.PrettyPrinter(indent=4)
10
11
12 def main():
13 belief = get_prior_belief()
14 belief = update_belief(belief, {'size':0.9, 'correct':False})
15 belief = update_belief(belief, {'size':0.8, 'correct':True})
16 belief = update_belief(belief, {'size':0.8, 'correct':True})
17 belief = update_belief(belief, {'size':0.8, 'correct':True})
18 belief = update_belief(belief, {'size':0.8, 'correct':True})
19 belief = update_belief(belief, {'size':0.8, 'correct':True})
20 belief = update_belief(belief, {'size':0.8, 'correct':True})
21 belief = update_belief(belief, {'size':0.8, 'correct':True})
22 belief = update_belief(belief, {'size':0.8, 'correct':True})
PROBLEMS
OUTPUT
DEBUG CONSOLE
TERMINAL
JUPYTER
zsh
piech@DN0a22c452 eye %
OUTLINE
TIMELINE
0 0
Ln 6, Col 12
Spaces: 4
UTF-8
LF
Python
3.10.2 64-bit

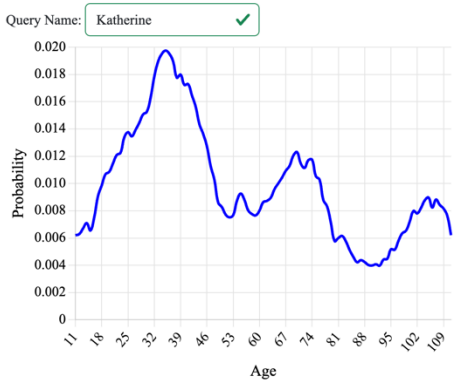
Beyond Inference:
How do you select the next size to show?

Today: Five New Real + Exciting Problems

Age from C14

Updated Delivery Prob

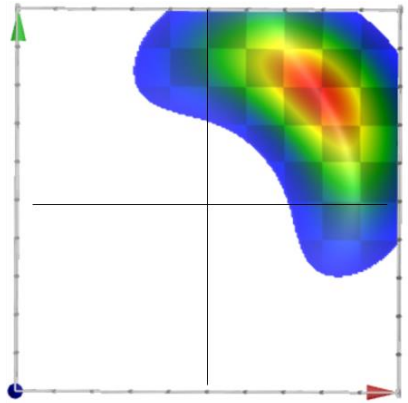
Age from Name



Hidden Chambers

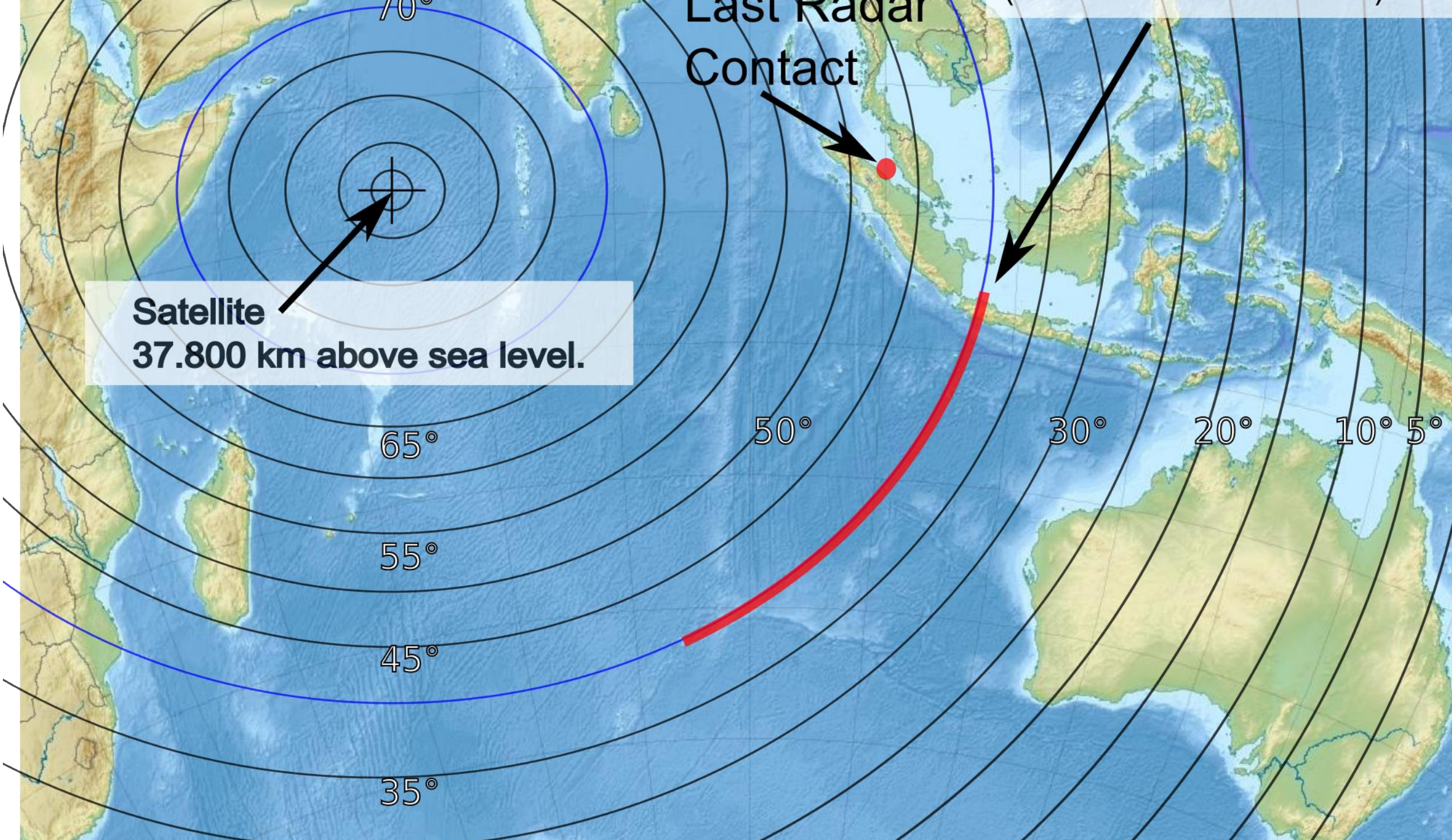
Stanford Eye Test

Cellphone Tracking Cont.



Are you ready
For Tracking in 2D Space???

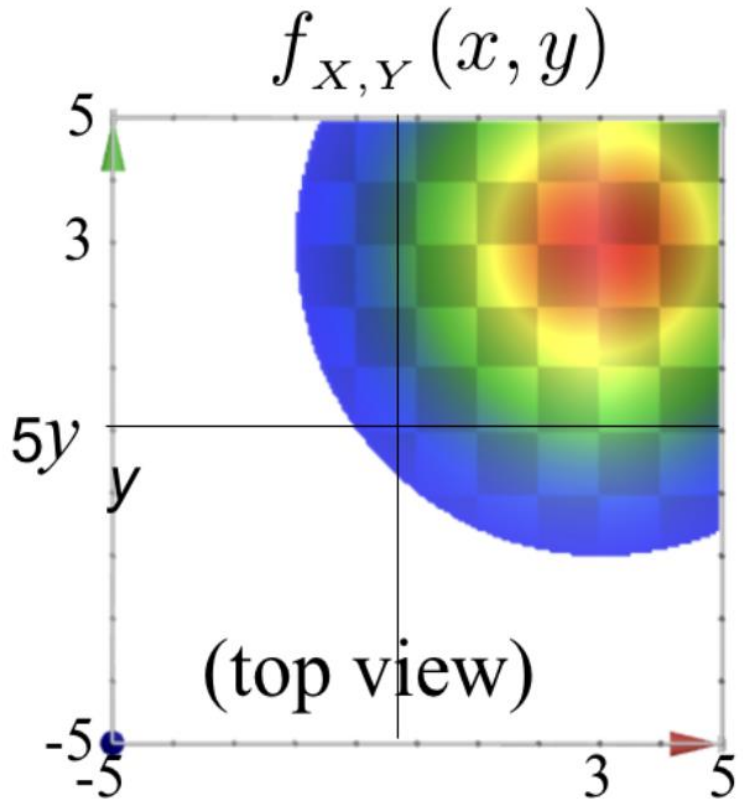
Tracking in 2D Space?

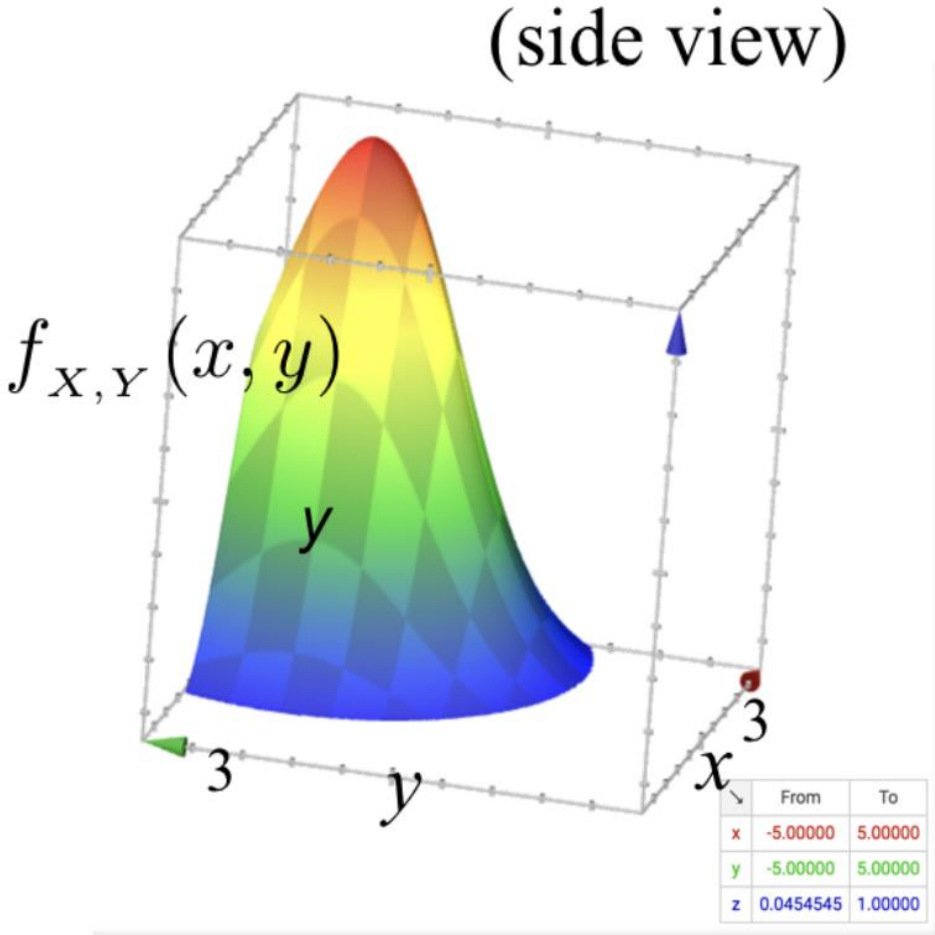

Last Radar
Contact
Satellite
37.800 km above sea level.
70°
65°
55°
50°
45°
35°
30°
20°
10°
5°


EXPERTS ONLY
Expect unmarked cliffs, rocks, avalanche debris and other alpine hazards.
GATE

Tracking Prior

Prior belief: $$f(X = x, Y = y) = \frac{1}{8 \cdot \pi} \cdot e^{-\frac{(x-3)^2 + (y-3)^2}{8}}$$

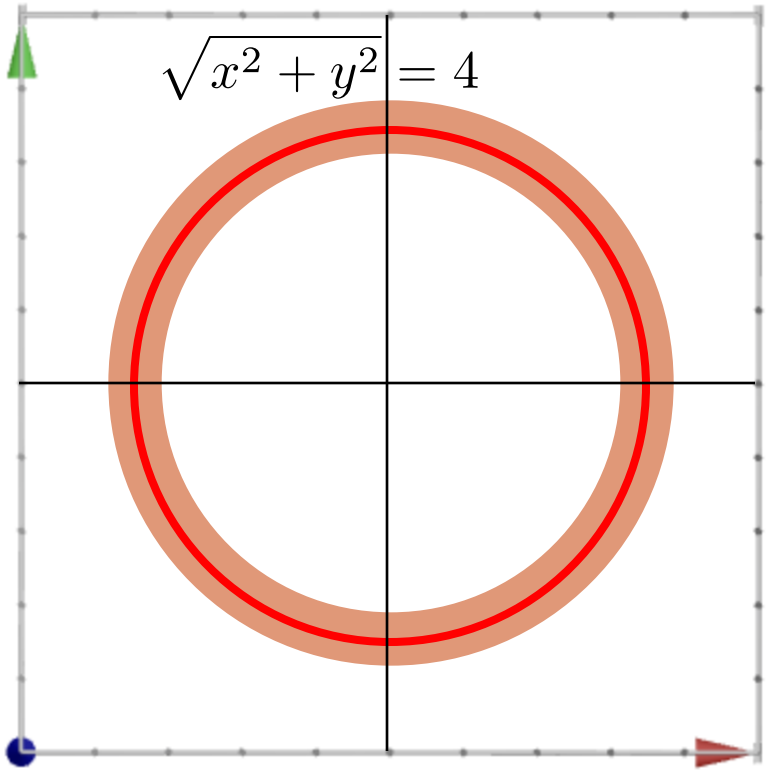


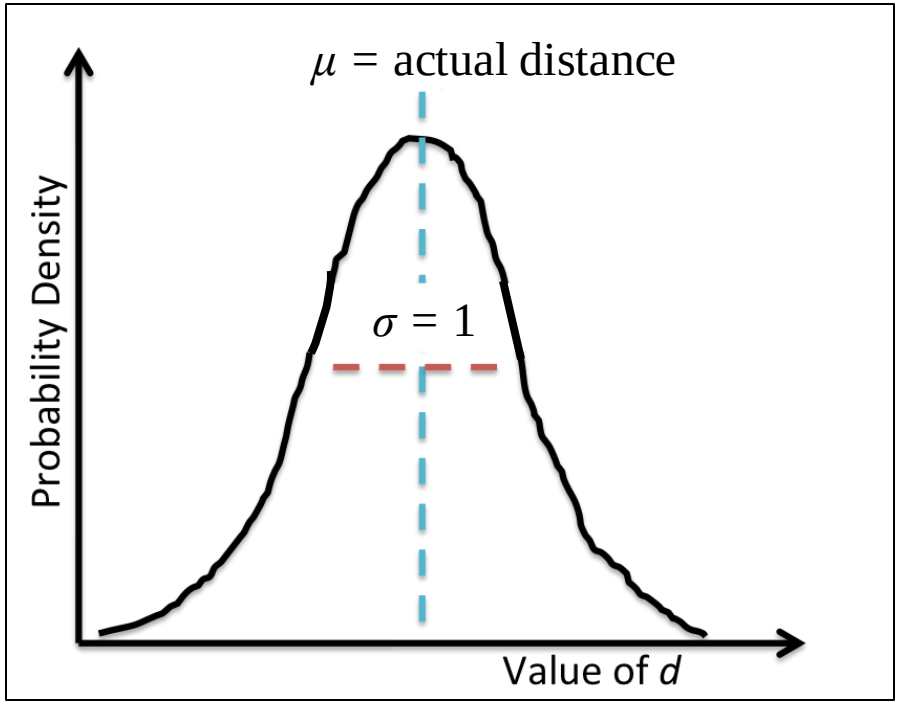


Tracking Observation

You now observe a noisy distance reading from a sensor at (0,0).
It says that your object is distance D = 4 away

$$D|X,Y \sim N(\mu = \sqrt{x^2 + y^2}, \sigma^2 = 1)$$

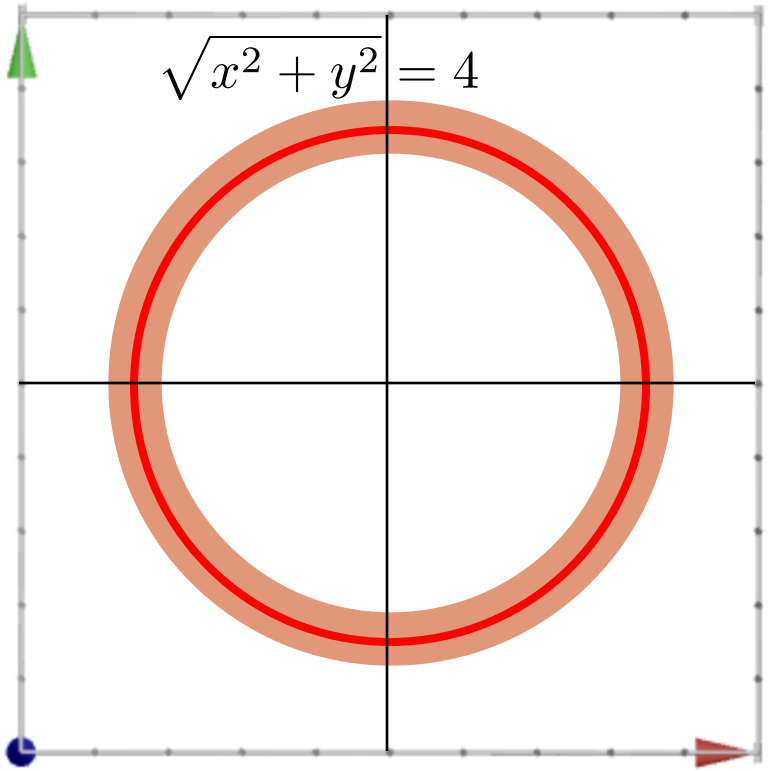


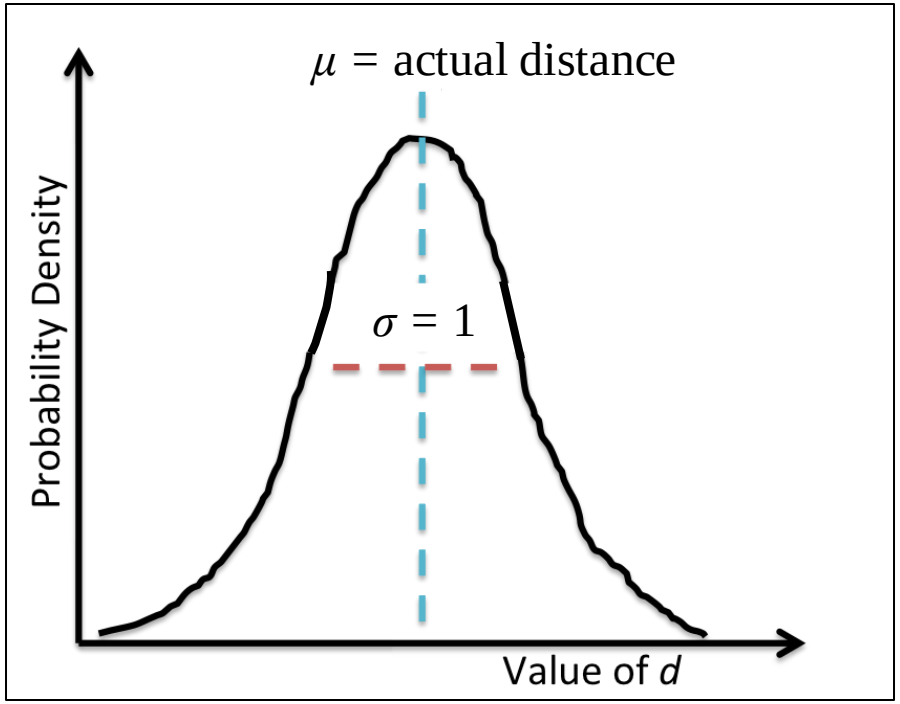


Tracking Observation

$$D|X,Y \sim N(\mu = \sqrt{x^2+y^2}, \sigma^2 = 1)$$

$$f(D = d \mid X = x, Y = y) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{\left(d-\sqrt{x^2+y^2}\right)^2}{2}}$$





Put it all Together

$$f(X = x, Y = y) = \frac{1}{8 \cdot \pi} \cdot e^{-\frac{(x-3)^2 + (y-3)^2}{8}}$$

$$f(X = x, Y = y \mid D = d) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{\left(d - \sqrt{x^2 + y^2}\right)^2}{2}}$$

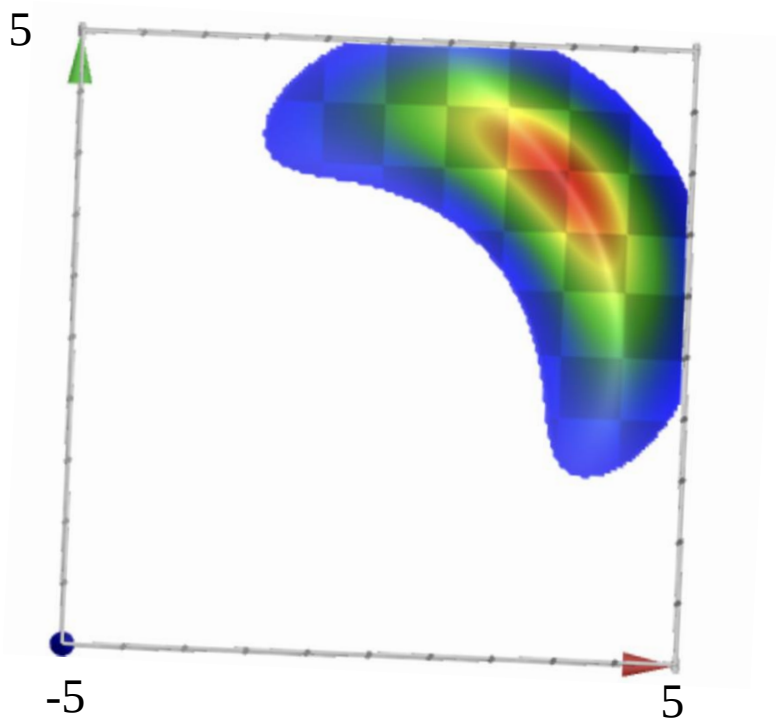
$$f(X = x, Y = y \mid D = 4) =$$

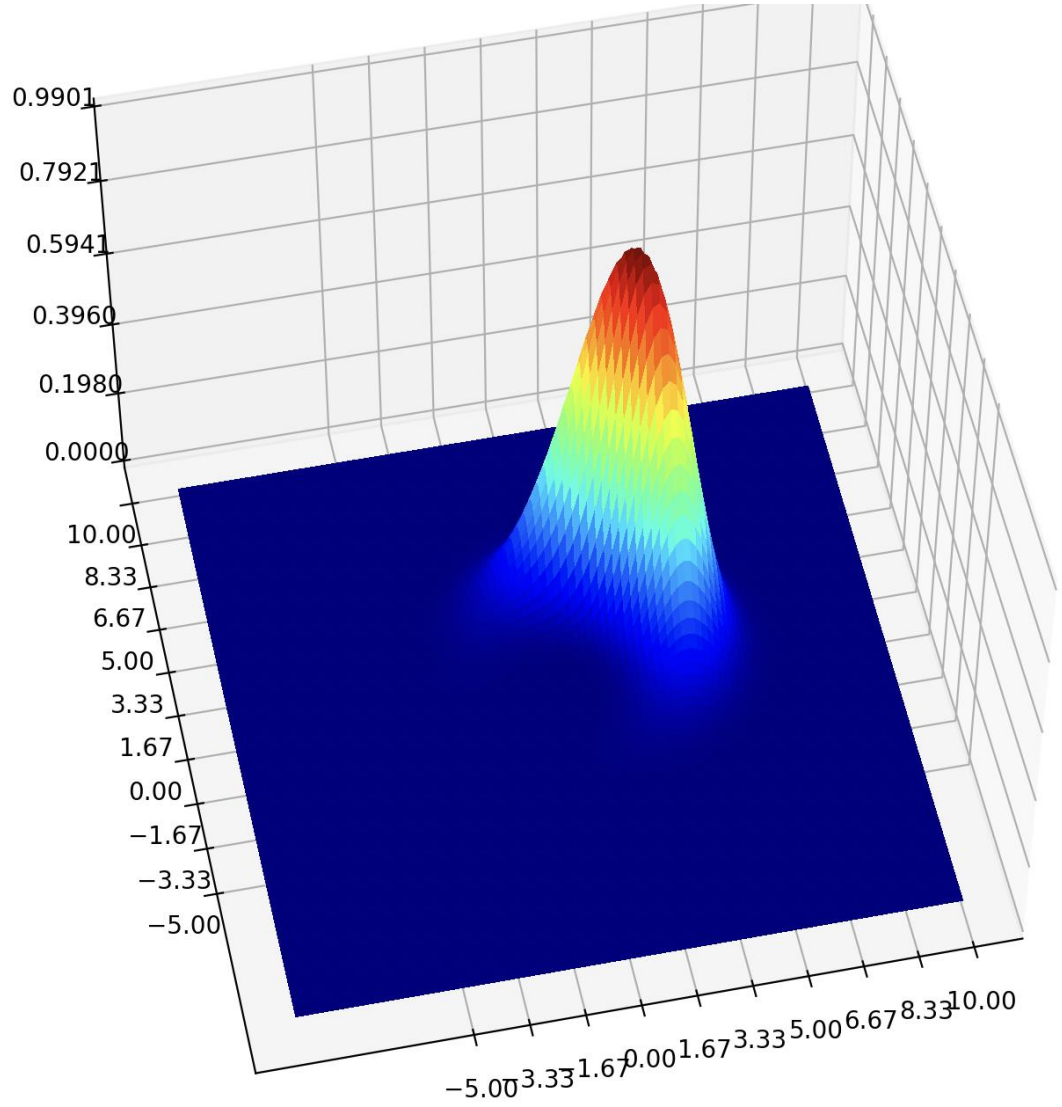
Put it all Together

$$
\begin{aligned}
& f(X = x, Y = y \mid D = 4) \\
&= \frac{f(D = 4 \mid X = x, Y = y) \cdot f(X = x, Y = y)}{f(D = 4)} && \text{Bayes using densities} \\
&= \frac{K_1 \cdot e^{-\frac{(4-\sqrt{x^2+y^2})^2}{2}} \cdot K_2 \cdot e^{-\frac{(x-3)^2+(y-3)^2}{8}}}{f(D = 4)} && \text{Substitute} \\
&= \frac{K_1 \cdot K_2}{f(D = 4)} \cdot e^{-\left[\frac{(4-\sqrt{x^2+y^2})^2}{2} + \frac{(x-3)^2+(y-3)^2}{8}\right]} && f(D = 4) \text{ is a constant w.r.t. } (x, y) \\
&= K_3 \cdot e^{-\left[\frac{(4-\sqrt{x^2+y^2})^2}{2} + \frac{(x-3)^2+(y-3)^2}{8}\right]} && K_3 \text{ is a new constant}
\end{aligned}
$$

Tracking Posterior

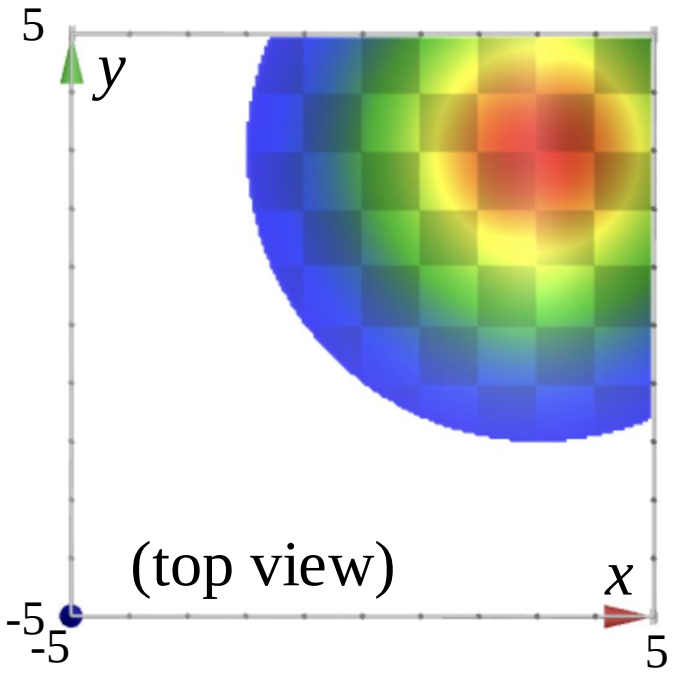
$$f(X = x, Y = y \mid D = 4)$$





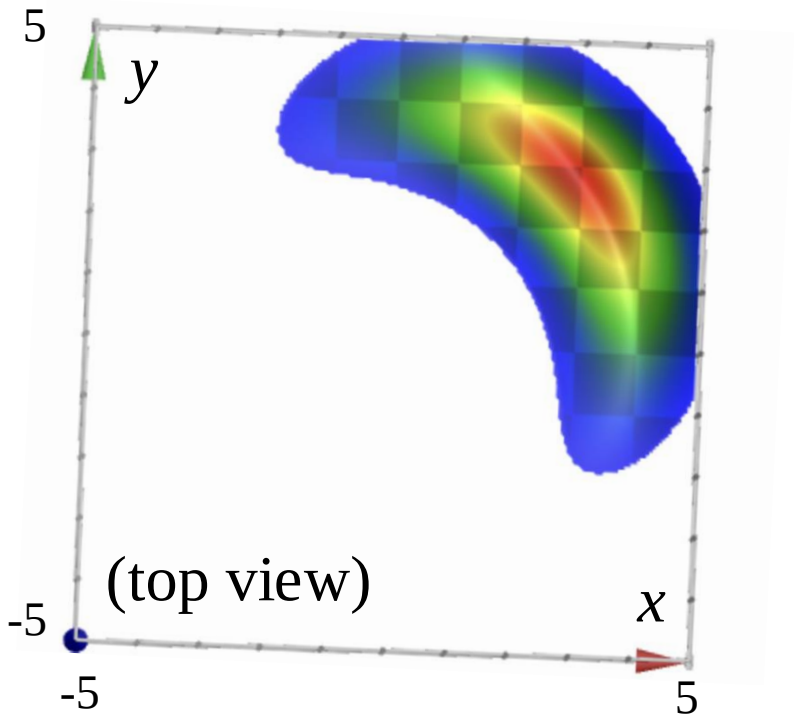
Tracking Posterior

$f(X = x, Y = y)$



Prior

$f(X = x, Y = y \mid D = 4)$



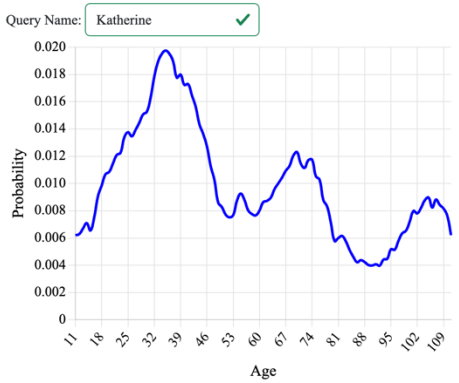
Posterior

Today: Five New Real + Exciting Problems

Age from C14

Updated Delivery Prob

Age from Name



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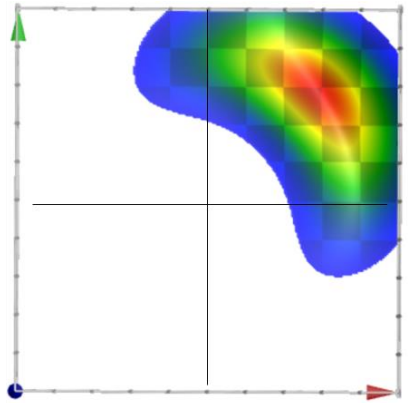
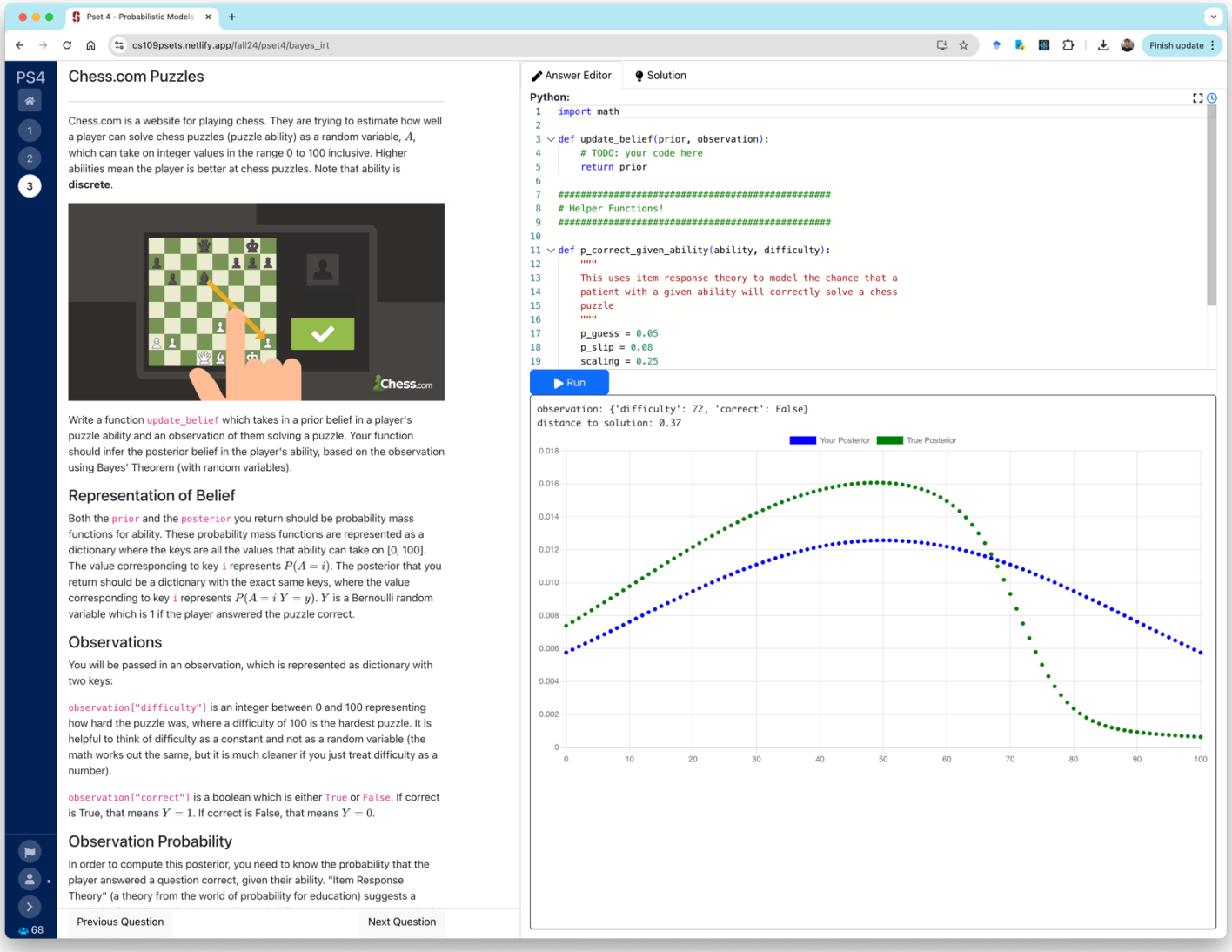
Cellphone Tracking Cont.



Practice!


Pset 4 - Probabilistic Models
cs109psets.netlify.app/fall24/pset4/bayes_irt
Finish update
PS4
Chess.com Puzzles
Chess.com is a website for playing chess. They are trying to estimate how well a player can solve chess puzzles (puzzle ability) as a random variable, A, which can take on integer values in the range 0 to 100 inclusive. Higher abilities mean the player is better at chess puzzles. Note that ability is discrete.
Write a function update_belief which takes in a prior belief in a player's puzzle ability and an observation of them solving a puzzle. Your function should infer the posterior belief in the player's ability, based on the observation using Bayes' Theorem (with random variables).
Representation of Belief
Both the prior and the posterior you return should be probability mass functions for ability. These probability mass functions are represented as a dictionary where the keys are all the values that ability can take on [0, 100]. The value corresponding to key i represents $P(A = i)$. The posterior that you return should be a dictionary with the exact same keys, where the value corresponding to key i represents $P(A = i|Y = y)$. Y is a Bernoulli random variable which is 1 if the player answered the puzzle correct.
Observations
You will be passed in an observation, which is represented as dictionary with two keys:
observation["difficulty"] is an integer between 0 and 100 representing how hard the puzzle was, where a difficulty of 100 is the hardest puzzle. It is helpful to think of difficulty as a constant and not as a random variable (the math works out the same, but it is much cleaner if you just treat difficulty as a number).
observation["correct"] is a boolean which is either True or False. If correct is True, that means $Y = 1$. If correct is False, that means $Y = 0$.
Observation Probability
In order to compute this posterior, you need to know the probability that the player answered a question correct, given their ability. "Item Response Theory" (a theory from the world of probability for education) suggests a
Previous Question
Next Question
Answer Editor
Solution
Python:
import math
def update_belief(prior, observation):
# TODO: your code here
return prior
##
Helper Functions!
##
def p_correct_given_ability(ability, difficulty):
"""
This uses item response theory to model the chance that a
patient with a given ability will correctly solve a chess
puzzle
"""
p_guess = 0.05
p_slip = 0.08
scaling = 0.25
Run
observation: {'difficulty': 72, 'correct': False}
distance to solution: 0.37
Your Posterior
True Posterior

See you Wednesday!