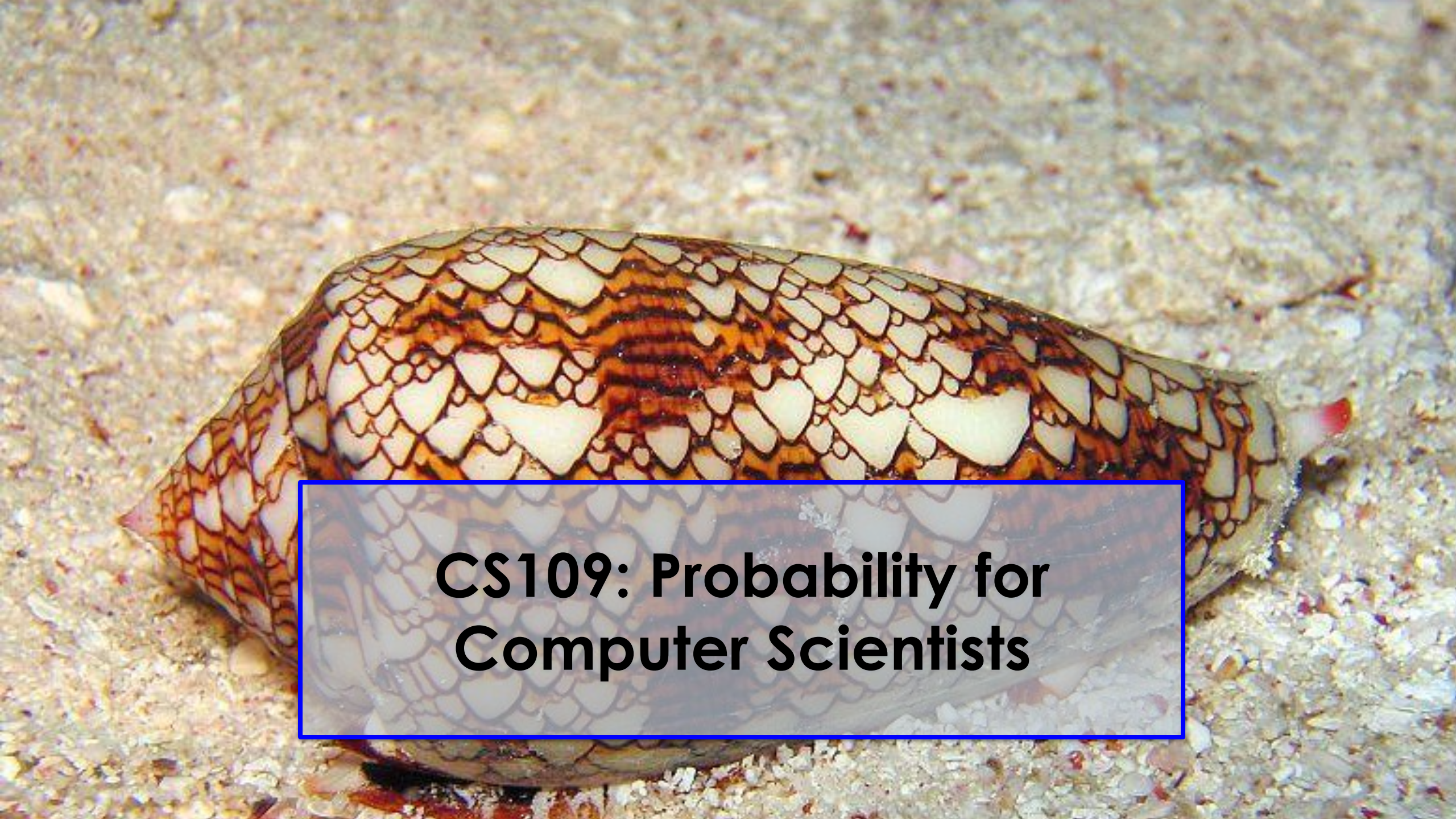
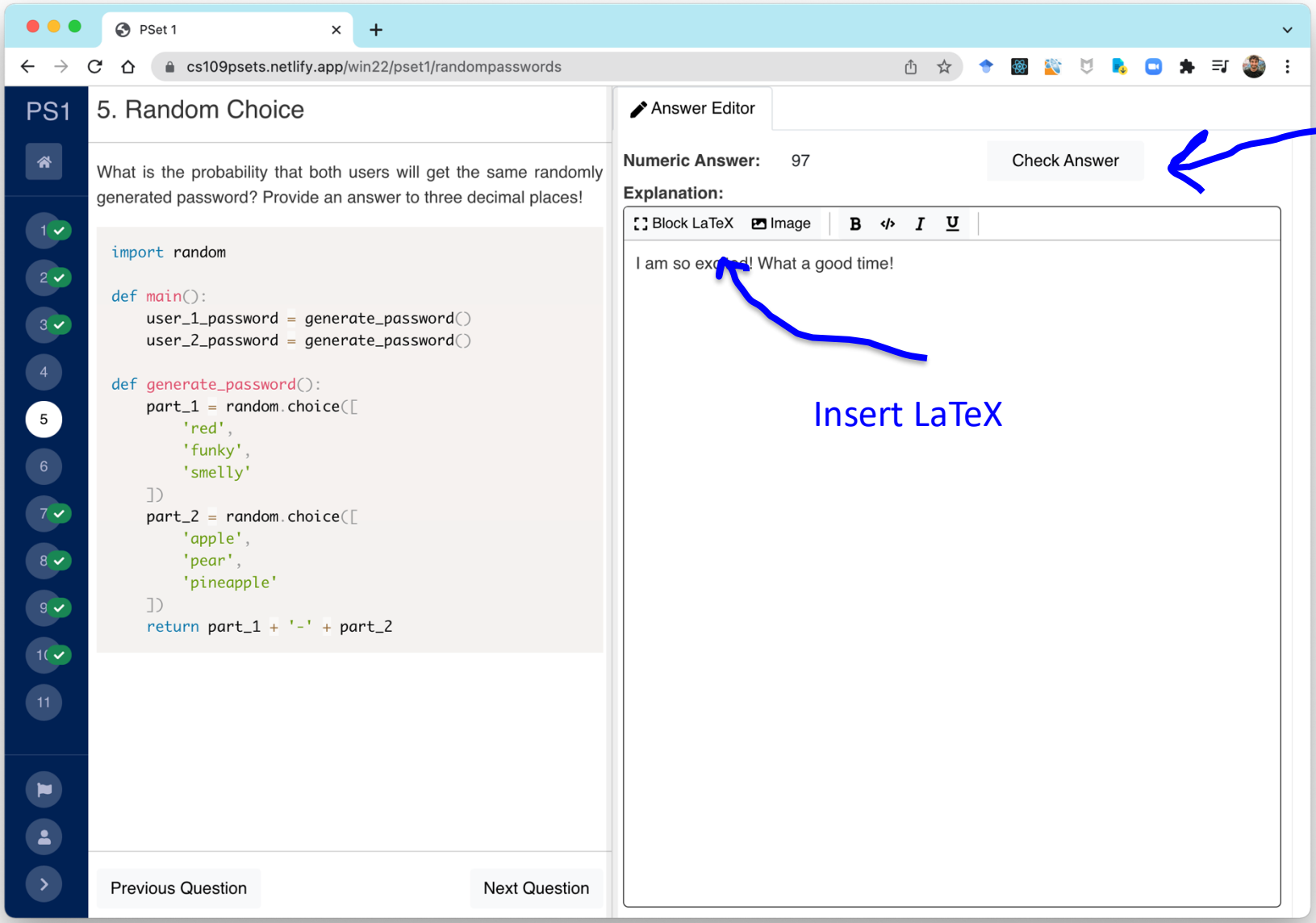


CS109: Probability for Computer Scientists

Problem Set #1 is out

newish



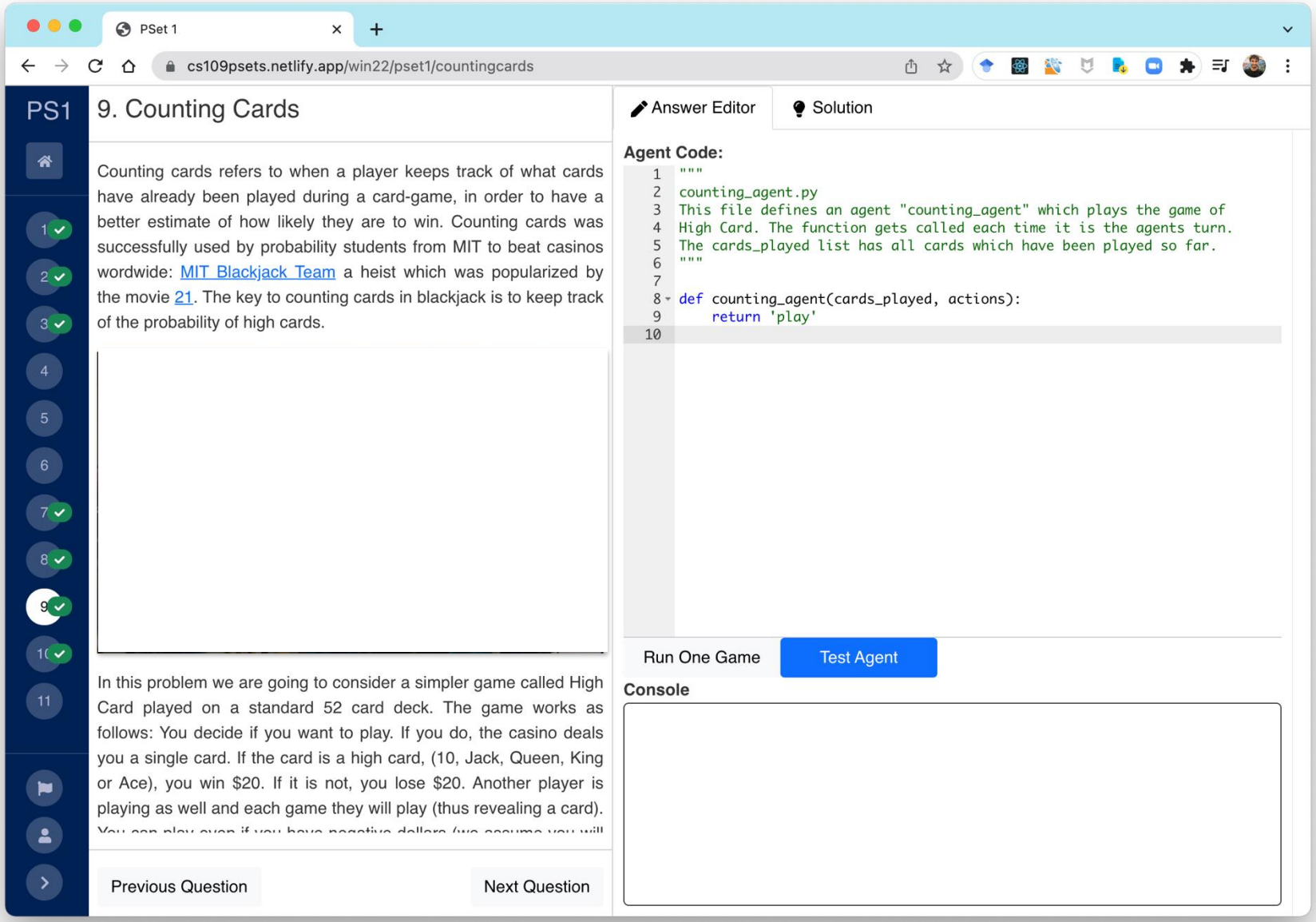
Auto Submission

Check your answer

Insert LaTeX

Write an Agent

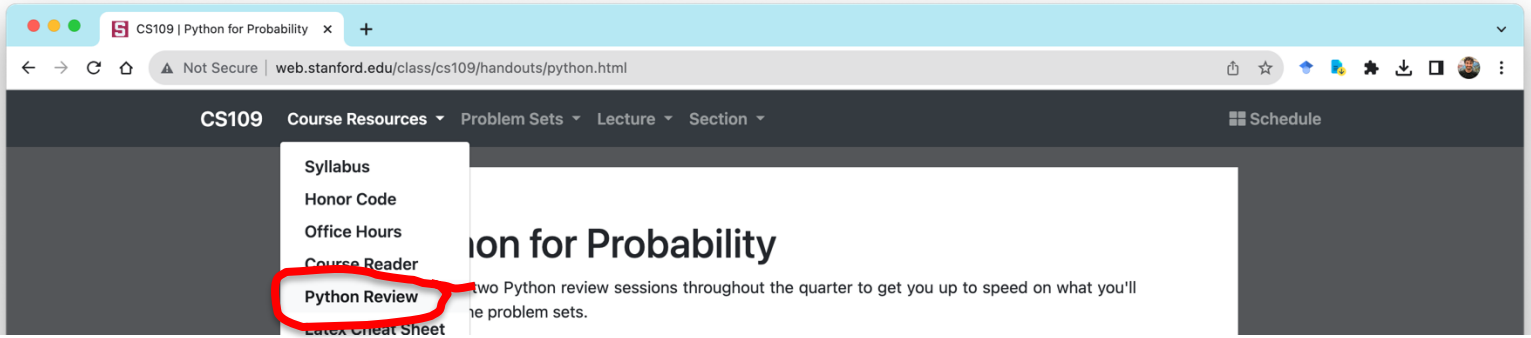
newish



Python Review Session

Friday at 4:30-5:30pm PT, recorded

Find links, recordings, and setup here

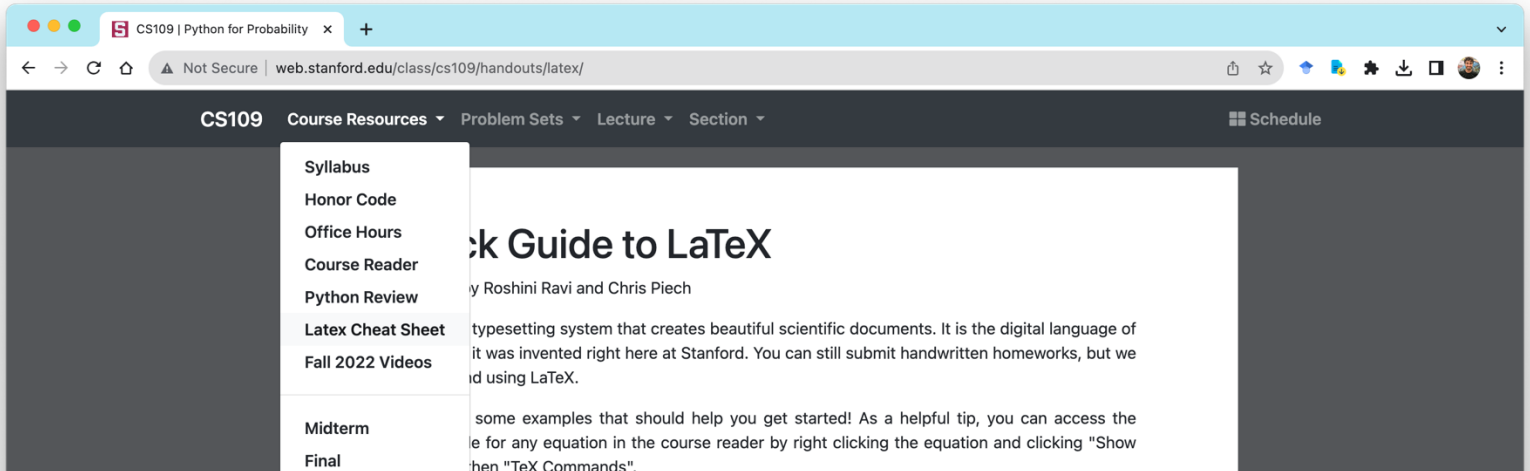


Learn LaTex

```
\begin{aligned}
P(E) 
&= \sum_{i=0}^n e^{i}\\
&= 0.25
\end{aligned}
```

$$P(E) = \sum_{i=0}^{n} e^i = 0.25$$

Done



Inline LaTex

new

Explanation:

Block LaTeX | Inline LaTeX | Python | Image

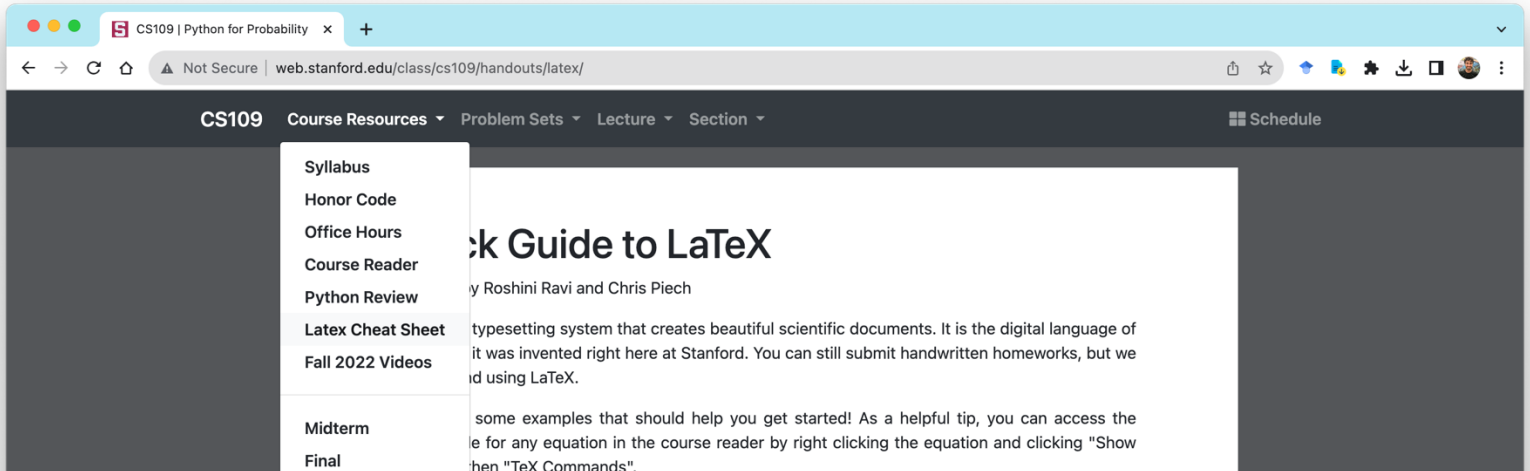
This is an example of inline latex. Let $$Y$

When you type the closing $

Explanation:

Block LaTeX | Inline LaTeX | Python | Image

This is an example of inline latex. Let Y



What Makes for a Good Answer

PS1

1 2 2c 3 4 5a 5b 6 7 8 9 10 11 12

Etude: Possible Answer Keys (a)

A true-false exam has 15 questions, and precisely three of the answers are false.

Figure: An example of a true-false test to be clear the questions are ordered and that each question can only be answered in one of two ways.

How many different answer keys are possible? In general, you should always show your work for all problems ever submitted for any problem on any problem set so we know how you arrived at your final answer.

Answer Editor | Solution

Answer editing: Off

Explanation:

There exist 15 questions, of which we need to choose 3 to be false and the remaining 12 to be true. The true questions will be indistinguishable from each other, and the false questions will similarly be indistinct. That means we select some subset of size 3 from a set of size 15, and that, by definition, is $\binom{15}{3} = 455$. You can also think of this count as all of the ways to order 15 objects, where each objects is in one of two indistinct groups (trues and falses).

If you notice a bug?

It should be robust, but things can happen.

Let me know: send an email to cpiech@stanford.edu or message me on slack. I need your email and the approximate time you encountered the bug.

Honor Code

Always remember: You need to be able to recreate your ability on an exam. And in the real world. This is a foundation course. **Cheating in CS109 is cheating yourself and your friends**.

Talk to your friends about the **concepts**, not the solution. Words must be your own.

Practice the **art of teaching**. Three most important things to know:

1. Do not give away the answer
2. Always be respectful
3. Know what you don't know

Improved Cadence

#	Weekday	Date	Topic	Notes
Week 1				
1	Monday	Jan 6	Counting	
2	Wednesday	Jan 8	Combinatorics	
3	Friday	Jan 10	What is Probability?	
Week 2				
4	Monday	Jan 13	Conditional Probability and Bayes	
5	Wednesday	Jan 15	Independence	PSet 1 Due
6	Friday	Jan 17	Random Variables and Binomial	
Week 3				
-	Monday	Jan 20	No Class (MLK Jr Day)	
7	Wednesday	Jan 22	Moments	
8	Friday	Jan 24	Poisson	PSet 2 Due
Week 4				
9	Monday	Jan 27	Continuous Random Variables	
10	Wednesday	Jan 29	Normal Distribution	
11	Friday	Jan 31	Probabilistic Models	PSet 3 Due
Week 5				
12	Monday	Feb 3	Inference	PEP 1
13	Wednesday	Feb 5	Inference II	
14	Friday	Feb 7	General Inference	PSet 4 Due

You are here

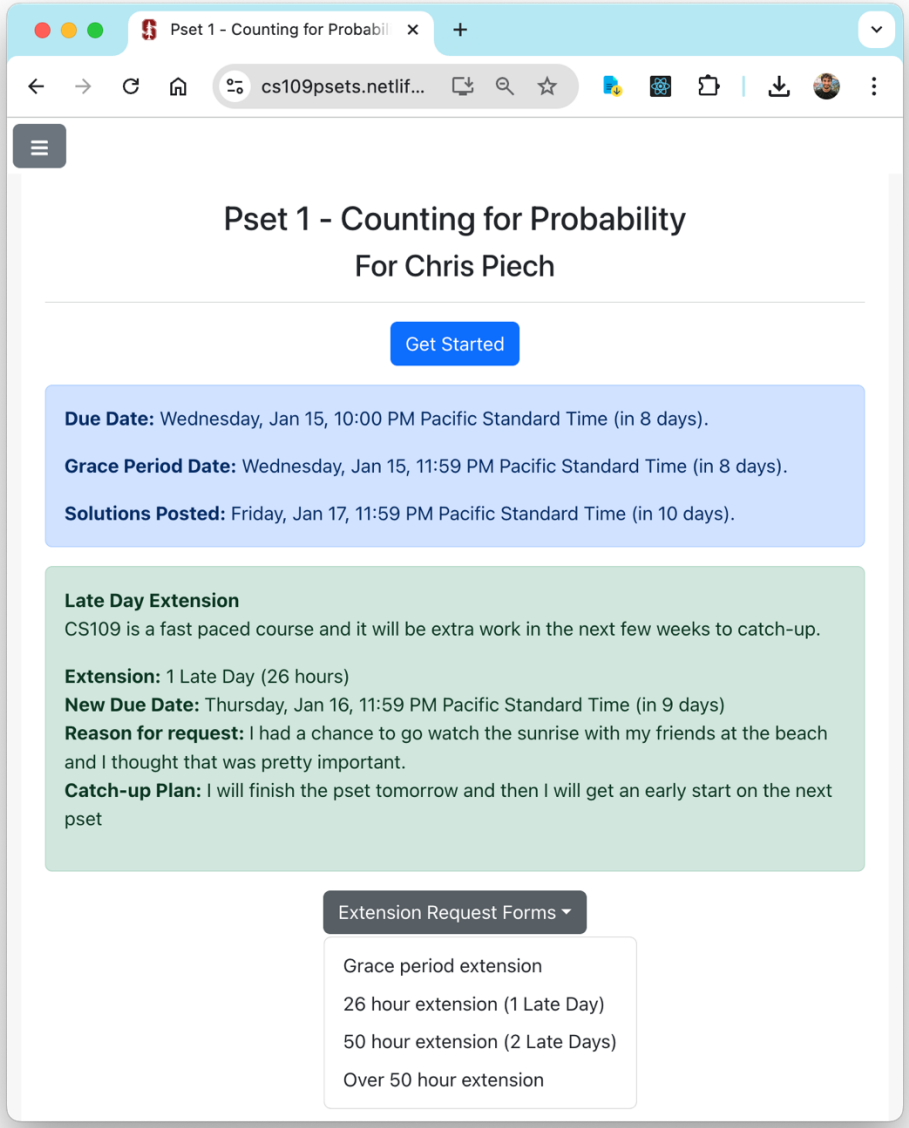
Done with counting early

Core probability early

Random Vars

Chance to Practice Inference

Late Policy (5 Late Days!)



Three types of extensions:

1. Grace period (2 hours)
2. Can take up to 2 late days (50 hours)
3. After the **hard** deadline (> 2 late days)

You give them to yourself. Need to talk to us if:

A. You need more than 5 late days / qtr
B. Extension past the hard deadline

But CS109 is a fast class. If you want a long extension I want you to be intentional about how you are going to catch up. Not all late days are created equal (especially before exams).

Section Signups Open Thursday

Class Theme Song Starts Thursday

Chose the top 16 freshest songs. Song recommendations open Thursday and close Sunday.

Then we start running a probabilistic exploration algorithm that will give you three songs at a time to rate.

We could have our top 16 songs by the midterm…


Above
&Beyond

Review

CS109: From Counting to Machine Learning

Counting Theory

Core Probability

Random Variables

Probabilistic Models

Uncertainty Theory

Machine Learning

Core Counting

Counting with steps

Definition: Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of m outcomes and the second part can result in one of n outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is $m \cdot n$.

Scales nicely to many steps

Counting with "or"

Definition: Inclusion Exclusion Counting

If the outcome of an experiment can either be drawn from set A or set B, and sets A and B may potentially overlap (i.e., it is not the case that A and B are mutually exclusive), then the number of outcomes of the experiment is $|A \text{ or } B| = |A| + |B| - |A \text{ and } B|$.

Only scales nicely if "mutually exclusive"

Core Counting

Counting with steps

Definition: Step Rule of Counting (aka Product Rule of Counting)

If an experiment has two parts, where the first part can result in one of m outcomes and the second part can result in one of n outcomes regardless of the outcome of the first part, then the total number of outcomes for the experiment is $m \cdot n$.

Scales nicely to many steps

Fantastic Question

Counting with steps

Definition: Step Rule of Counting with Many Events

If an experiment has k parts, where the first step has n_1 outcomes and the ith step has n_i outcomes (regardless of the result of any earlier steps), then the total number of outcomes of the experiment is:

$$\text{Number of Outcomes} = \prod_{i=1}^{k} n_i$$

Scales nicely to many steps

What does this big pi mean?

Fantastic Question

$$\sum_{i=1}^{k} n_i$$

```
total = 0
for i in range(1, k+1):
  total += n_i
```

$$\prod_{i=1}^{k} n_i$$

```
total = 1
for i in range(1, k+1):
  total *= n_i
```

Sometimes we are lazy…

$$\prod_{i} n_i$$

How Many Bit Stings?

Problem: A 6-bit string is sent over a network. The valid set of strings recognized by the receiver must either start with "01" or end with "10". How many such strings are there?

Answer

$$N = |A| + |B| - |A \text{ and } B|$$
$$= 16 + 16 - 4$$
$$= 28$$

2^4 start with 01

010000
010001
010010
010011
010100
010101
010110
010111
011000
011001
011010
011011
011100
011101
011110
011111

Set A

2^4 end with 10

000010
000110
001010
001110
010010
010110
011010
011110
100010
100110
101010
101110
110010
110110
111010
111110

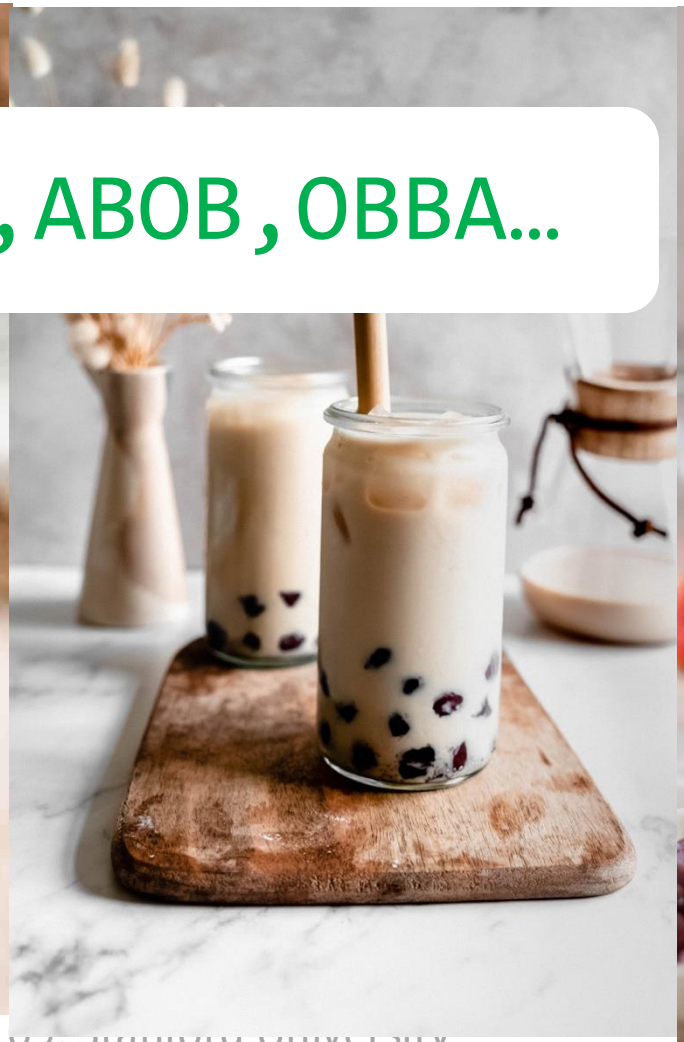
Set B

End Review

BOBA

- How many *different* orderings of letters are possible for the string **BOBA**?

BOBA, ABOB, OBBA…



Permutations I

Orderings of Letters

How many letter orderings are possible for the following strings?

1. `CHRIS`

This is Jerry's dog, Doris. She puts her little Doris paw up to her chin when she's thinking.

Orderings of Letters

chirs	crish	hicrs	hsirc	irchs	rcish	rschi	shirc
chisr	crshi	hicsr	hsrci	ircsh	rcshi	rscih	shrci
chris	crsih	hircs	hsric	irhcs	rcsih	rshci	shric
chrsi	cshir	hirsc	ichrs	irhsc	rhcis	rshic	sichr
chsir	cshri	hiscr	ichsr	irsch	rhcsi	rsich	sicrh
chsri	csihr	hisrc	icrhs	irshc	rhics	rsihc	sihcr
cihrs	csirh	hrcis	icrsh	ischr	rhisc	schir	sihrc
cihsr	csrhi	hrcsi	icshr	iscrh	rhsci	schri	sirch
cirhs	csrih	hrics	icsrh	ishcr	rhsic	scihr	sirhc
cirsh	hcirs	hrisc	ihcrs	ishrc	richs	scirh	srchi
cishr	hcisr	hrsci	ihcsr	isrch	ricsh	scrhi	srcih
cisrh	hcris	hrsic	ihrcs	isrhc	rihcs	scrih	srhci
crhis	hcrsi	hscir	ihrsc	rchis	rihsc	shcir	srhic
crhsi	hcsir	hscri	ihscr	rchsi	risch	shcri	srich
crihs	hcsri	hsicr	ihsrc	rcihs	rishc	shicr	srihc

Orderings of letters

Step 1:
Chose first letter

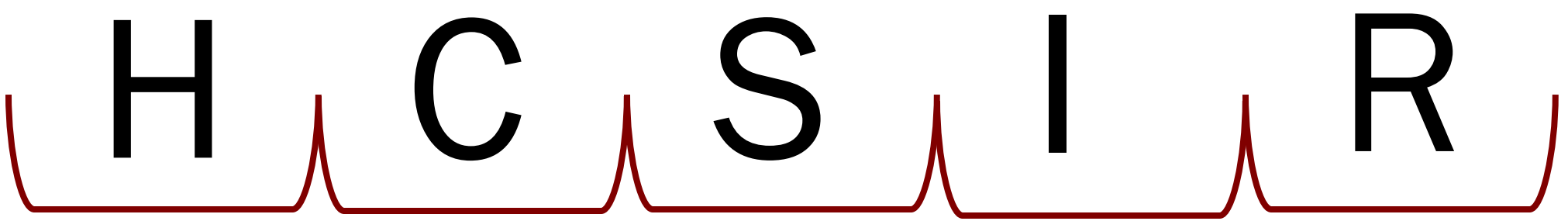
Step 2:
Chose 2nd letter

Step 3:
Chose 3rd letter

Step 4:
Chose 4th letter

Step 5:
Chose 5th letter

Orderings of letters



Step 1:
Chose first letter
(5 options)

Step 2:
Chose 2nd letter
(4 options)

Step 3:
Chose 3rd letter
(3 options)

Step 4:
Chose 4th letter
(2 options)

Step 5:
Chose 5th letter
(1 option)

Permutations

A permutation is an ordered arrangement of objects.

The number of unique orderings (permutations) of n distinct objects is

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1.$$

Unique 6-digit passcodes with six smudges

How many unique 6-digit passcodes are possible if a phone password uses each of **six** distinct numbers?

Unique 6-digit passcodes with six smudges

How many unique 6-digit passcodes are possible if a phone password uses each of **six** distinct numbers?

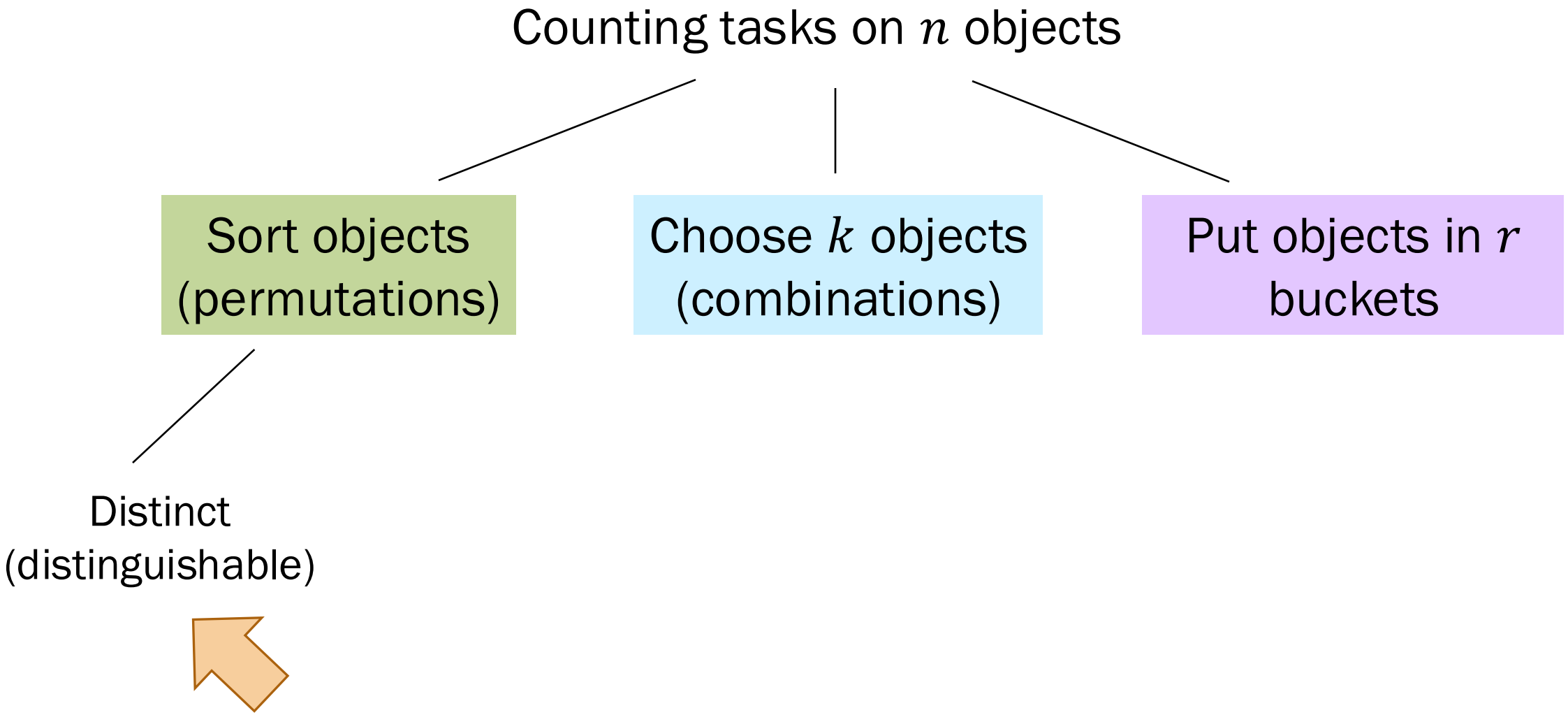
$$\text{Total} = 6! = 720 \text{ passcodes}$$

What if you had no smudges, but you knew it was still 6 digits long?

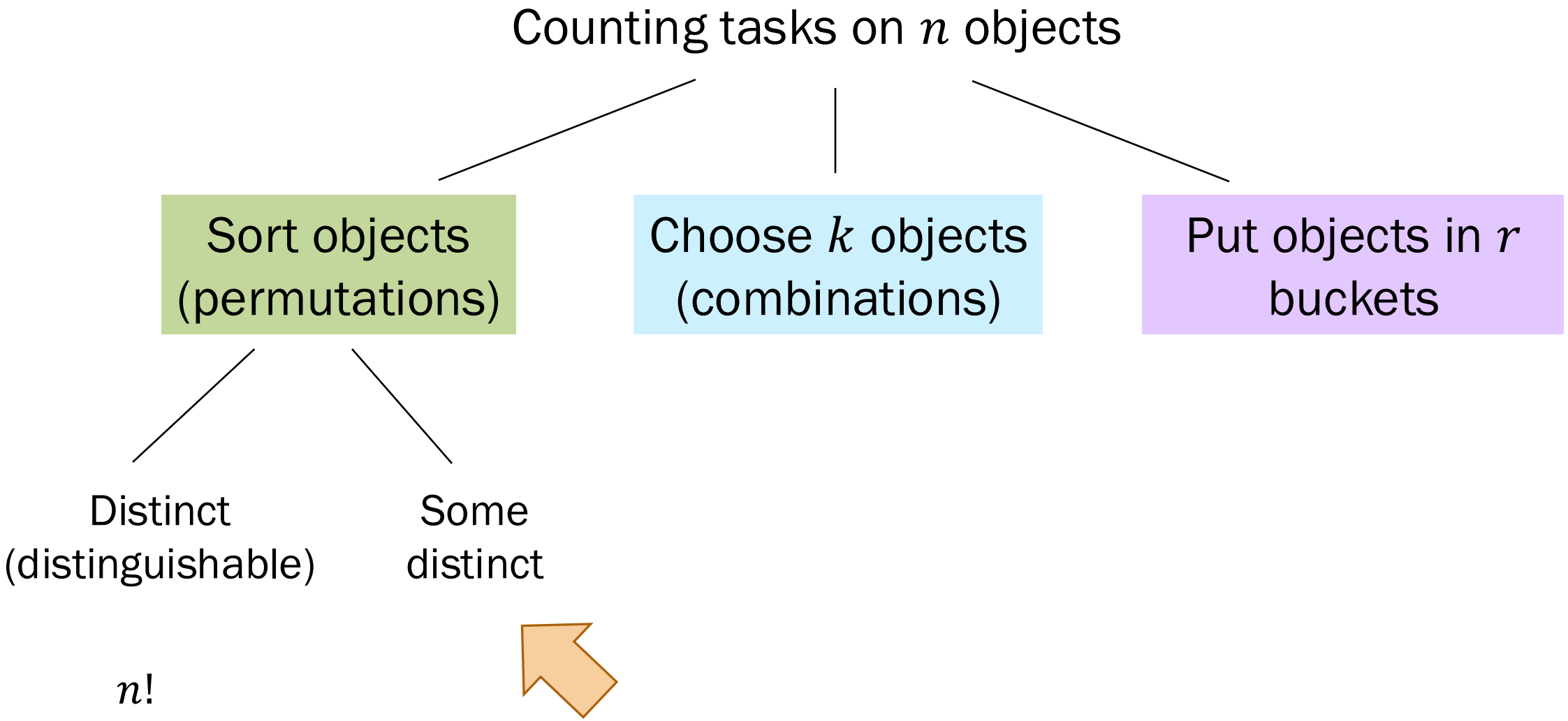
$$\text{Total} = 10^6 = 1 \text{ million passcodes}$$

Permutations II

Summary of Combinatorics



Summary of Combinatorics



Unique Bit Strings

1,0,1,0,0

Sort n distinct objects

Sort n distinct objects

Sort n distinct objects

1st 2nd 3rd 4th 5th

Steps:

1. Choose 1st can — 5 options
2. Choose 2nd can — 4 options

…

5. Choose 5th can — 1 option

$$\text{Total} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

How many ways can we sort coke cans!

Coke1

Coke0

Coke1

Coke0

Coke0

Sort n distinct objects

# of permutations =

Sort semi-distinct objects

Order n distinct objects	$n!$

All distinct

Ayesha Tim Irina Joey Waddie

Some indistinct

Coke Tim Coke Joey Waddie

Sort semi-distinct objects

How do we find the number of permutations considering some objects are indistinct?

By the product rule of counting (aka step rule), permutations of distinct objects is a two-step process:

permutations of distinct objects = permutations considering some objects are indistinct × Permutations of just the indistinct objects

Sort semi-distinct objects

How do we find the number of permutations considering some objects are indistinct?

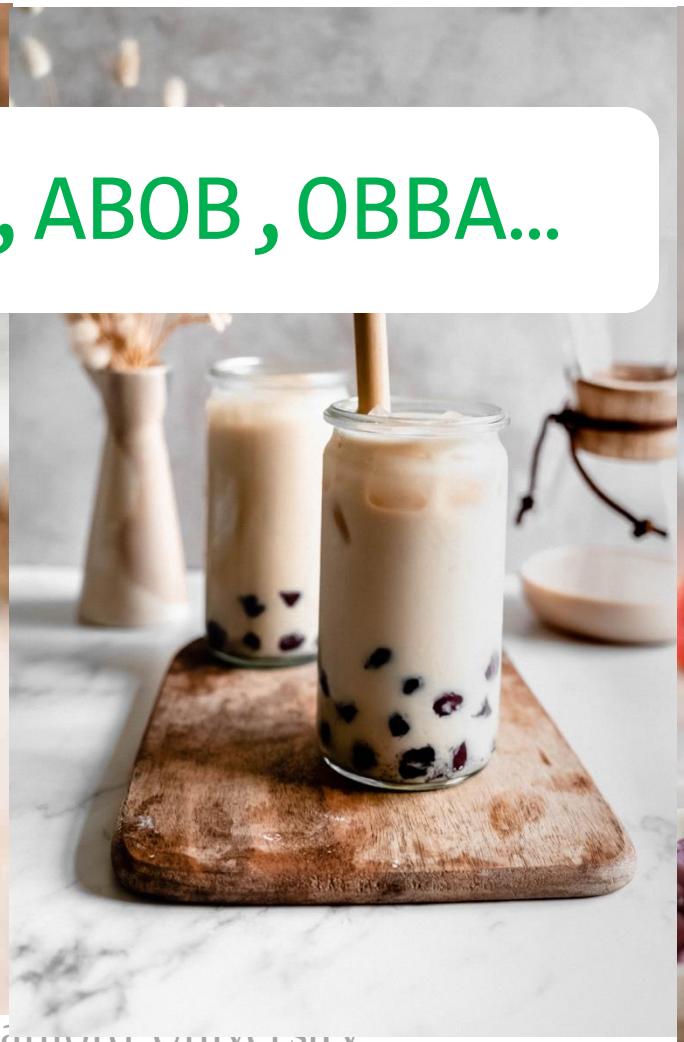
By the product rule of counting (aka step rule), permutations of distinct objects is a two-step process:

$$\frac{\text{permutations of distinct objects}}{\text{Permutations of just the indistinct objects}} = \text{permutations considering some objects are indistinct}$$

BOBA

How many *different* orderings of letters are possible for the string **BOBA**?

BOBA, ABOB, OBBA...



Strings

Is this just a regular permutation? 4! = 24 unique permutations!

boba
boab
bboa
bbao
baob
babo
obba
obab
obba
obab
oabb
oabb

bboa
bbao
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Strings

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aobb
abbo
abob

To the Code!

baob
bbao
obba
oabb
boab
babo
abbo
aobb
boba
abob
bboa
obab

```
import itertools

def main():
  letters = ['b','o','b','a']
  perms = set(itertools.permutations(letters))
  for perm in perms:
    pretty_perm = "".join(perm)
    print(pretty_perm)
```

```
import math

def main():
  n = math.factorial(4)
  d = math.factorial(2)
  print(n / d)
```

General approach to counting permutations

When there are n objects such that

n_1 are the same (indistinguishable or indistinct), and
n_2 are the same, and
…
n_r are the same,

The number of unique orderings (permutations) is

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}.$$

For each group of indistinct objects,
Divide by the overcounted permutations.

Sort semi-distinct objects

Order n semi-distinct objects $\frac{n!}{n_1! n_2! \cdots n_r!}$

How many permutations?

Strings

Order n semi-distinct objects $\frac{n!}{n_1! n_2! \cdots n_r!}$

How many letter orderings are possible for the following strings?

1. BOBA

2. MISSISSIPPI

This is Jerry's dog, Doris. She puts her little Doris paw up to her chin when she's thinking.

Strings

Order n semi-distinct objects $\frac{n!}{n_1!\,n_2!\cdots n_r!}$

How many letter orderings are possible for the following strings?

1. BOBA $= \frac{4!}{2!} = 12$

2. MISSISSIPPI $= \frac{11!}{1!4!4!2!} = 34{,}650$

Unique 6-digit passcodes with five smudges

Order n semi-distinct objects $\frac{n!}{n_1!\,n_2!\cdots n_r!}$

How many unique 6-digit passcodes are possible if a phone password uses each of **five** distinct numbers?

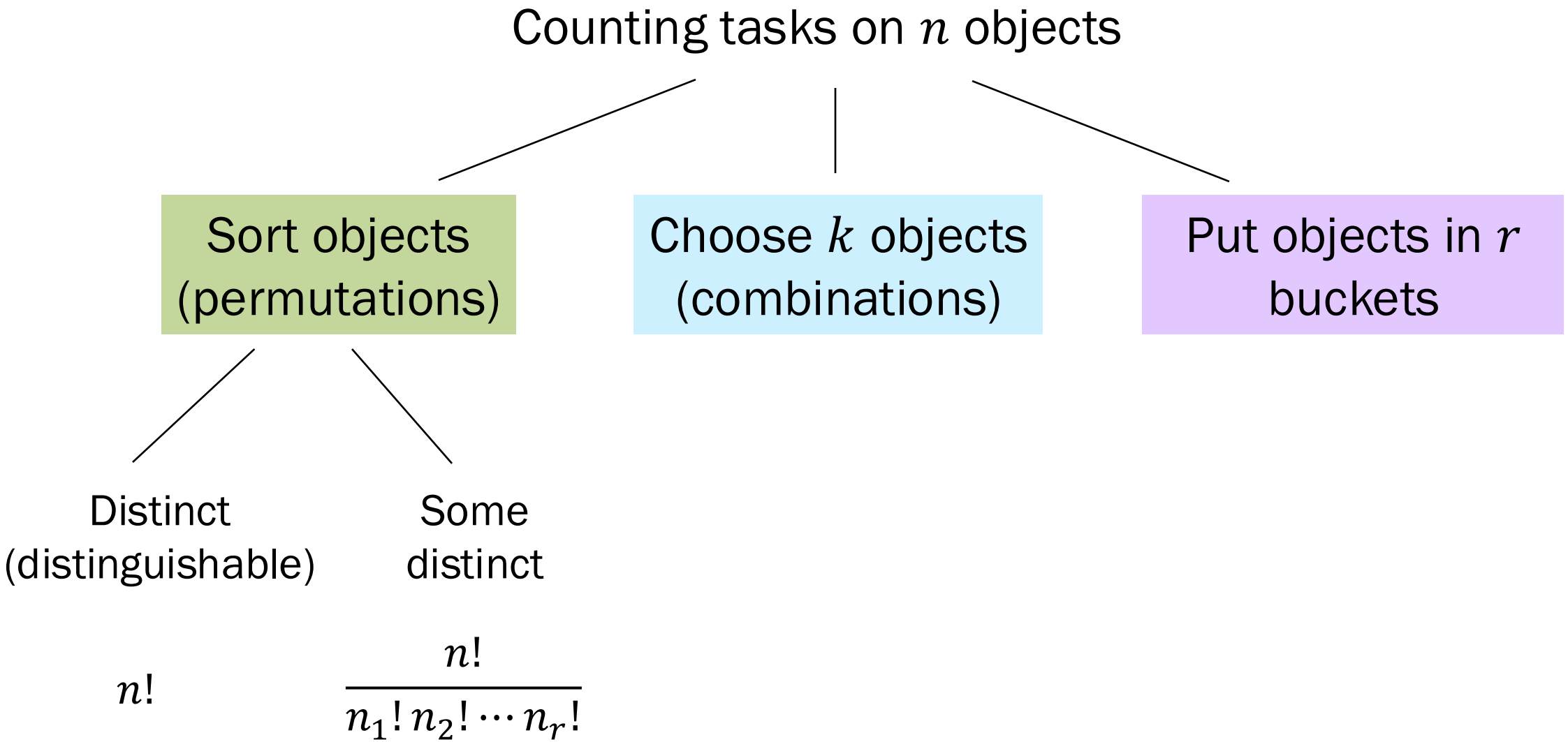
Steps:

1. Choose digit to repeat — 5 outcomes
2. Create passcode — (sort 6 digits: 4 distinct, 2 indistinct)

$$\text{Total} = 5 \times \frac{6!}{2!}$$

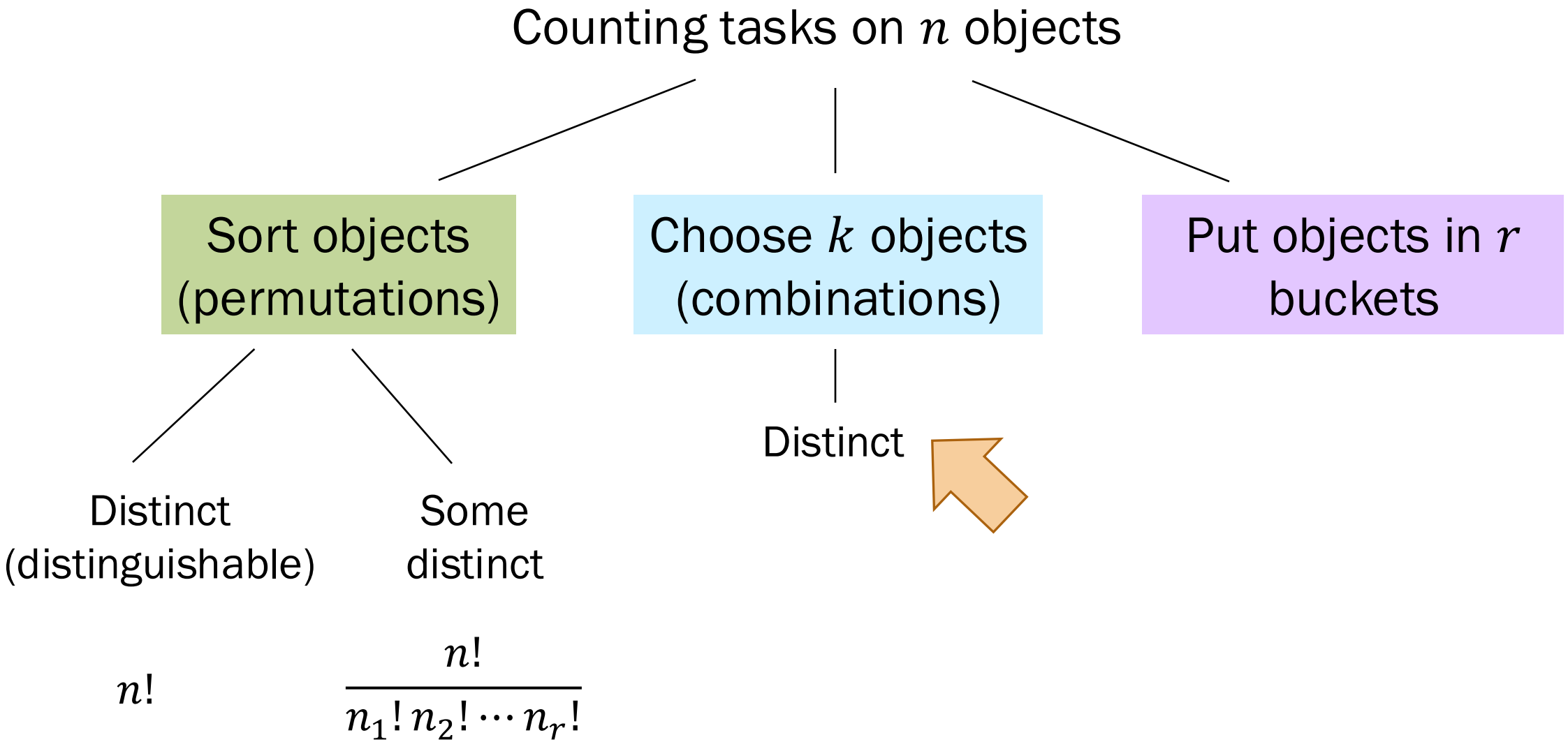
$$= 1{,}800 \text{ passcodes}$$

Summary of Combinatorics



Combinations I

Summary of Combinatorics



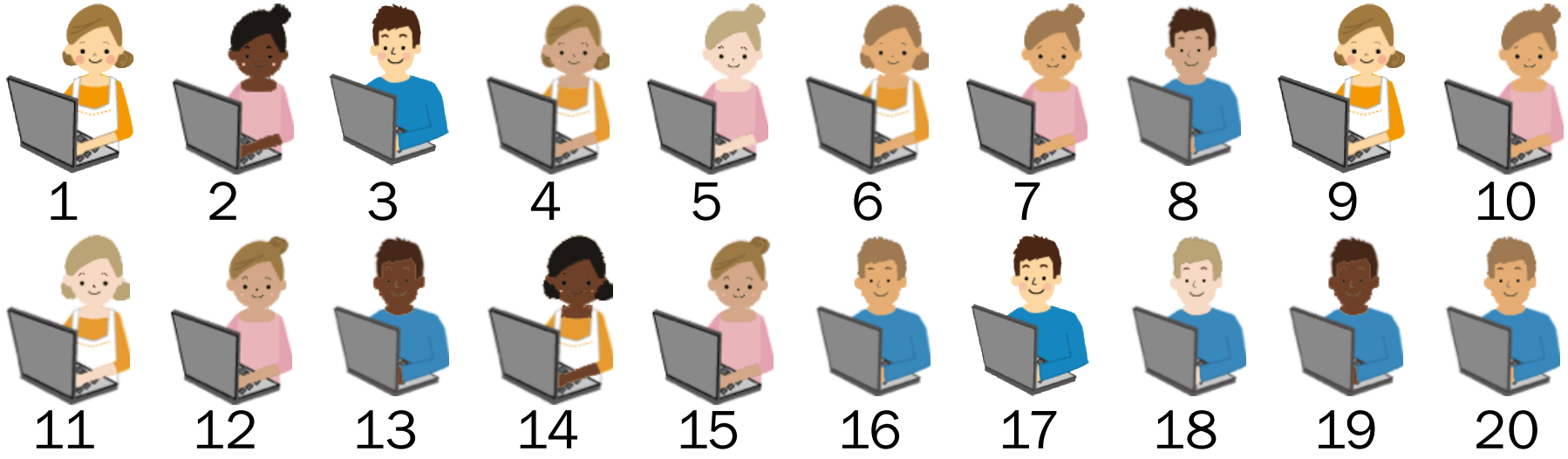
Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?

Consider the following generative process...

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?

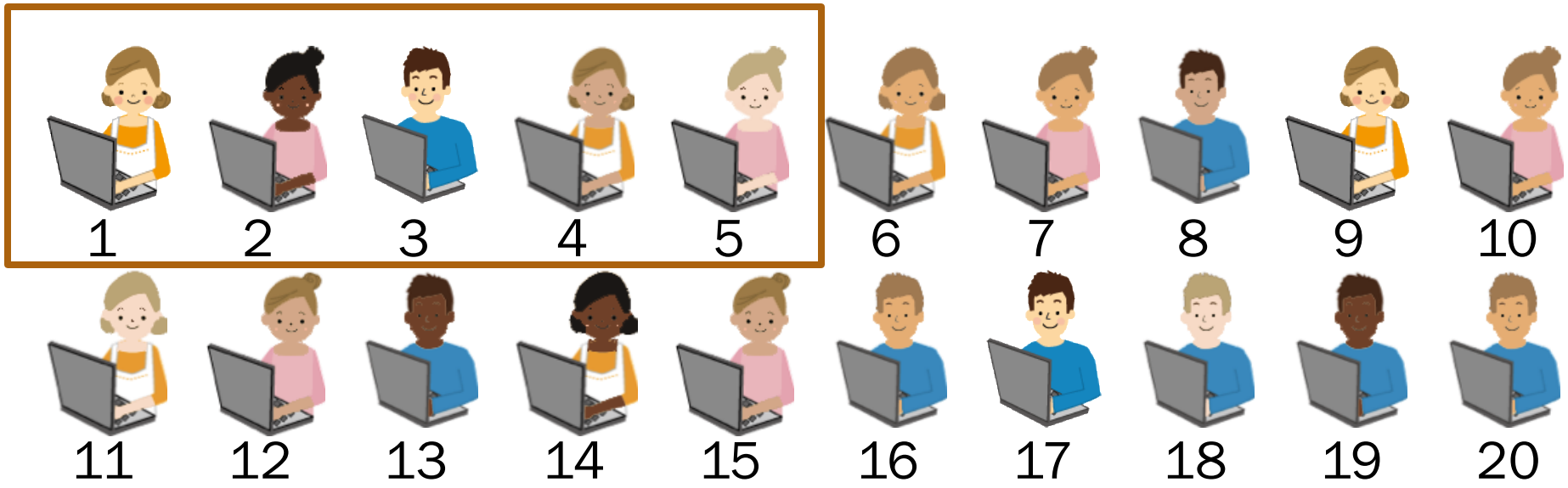


1. n people get in line

   $n!$ ways

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?



1. n people get in line

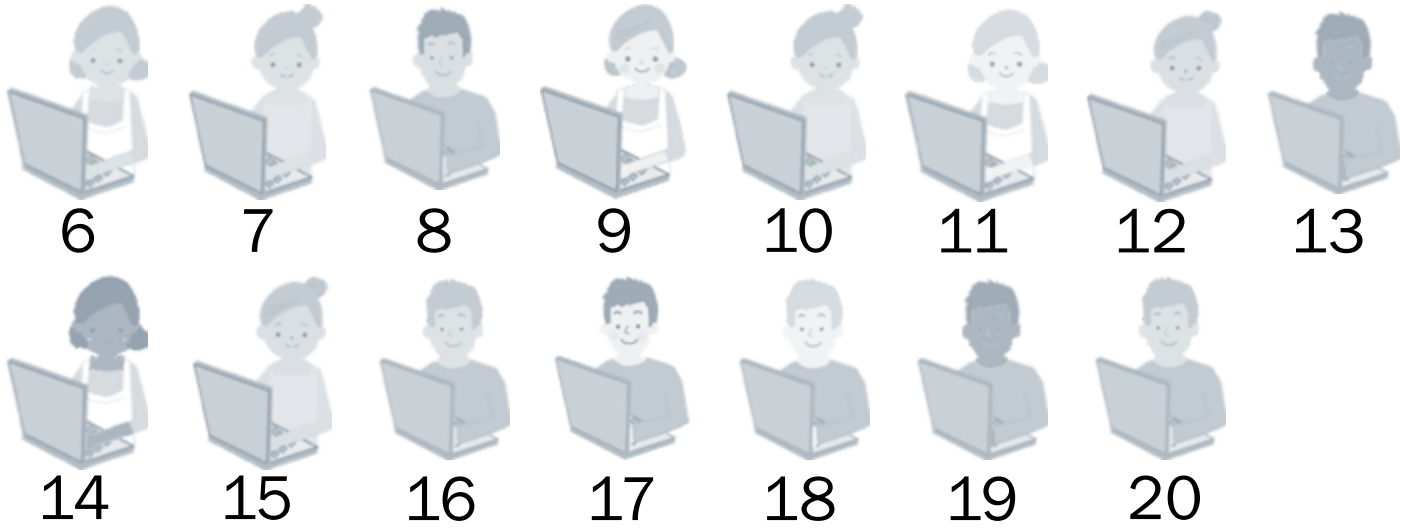
   $n!$ ways

2. Put first k in cake room

   1 way

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?



1. n people get in line

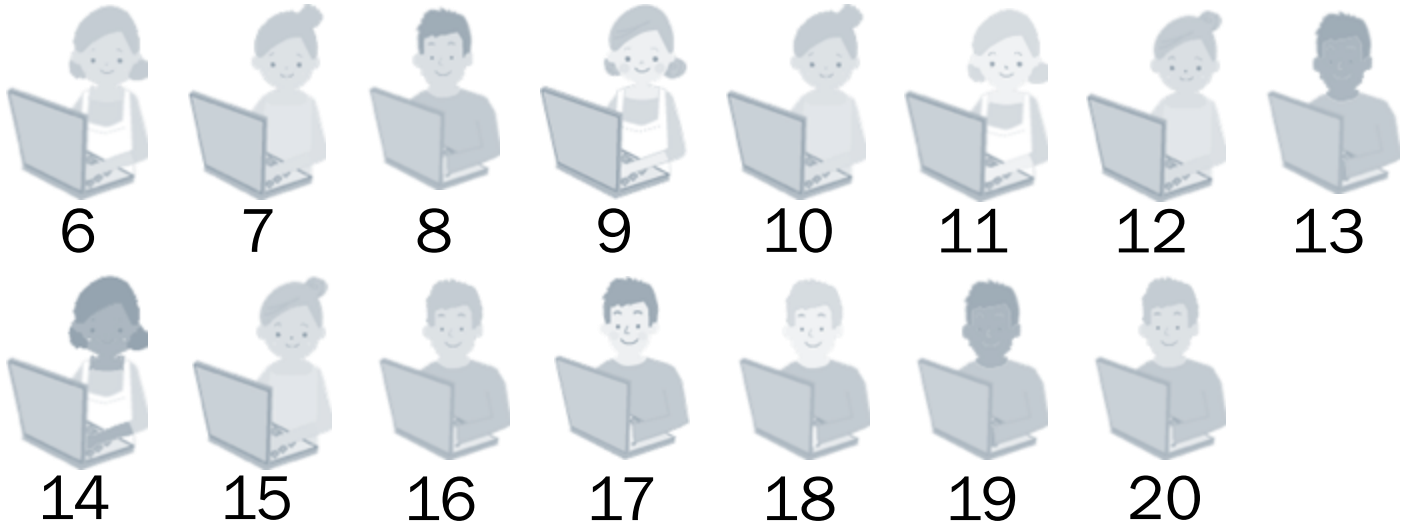
   $n!$ ways

2. Put first k in cake room

   1 way

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?



1. n people get in line

   $n!$ ways

2. Put first k in cake room

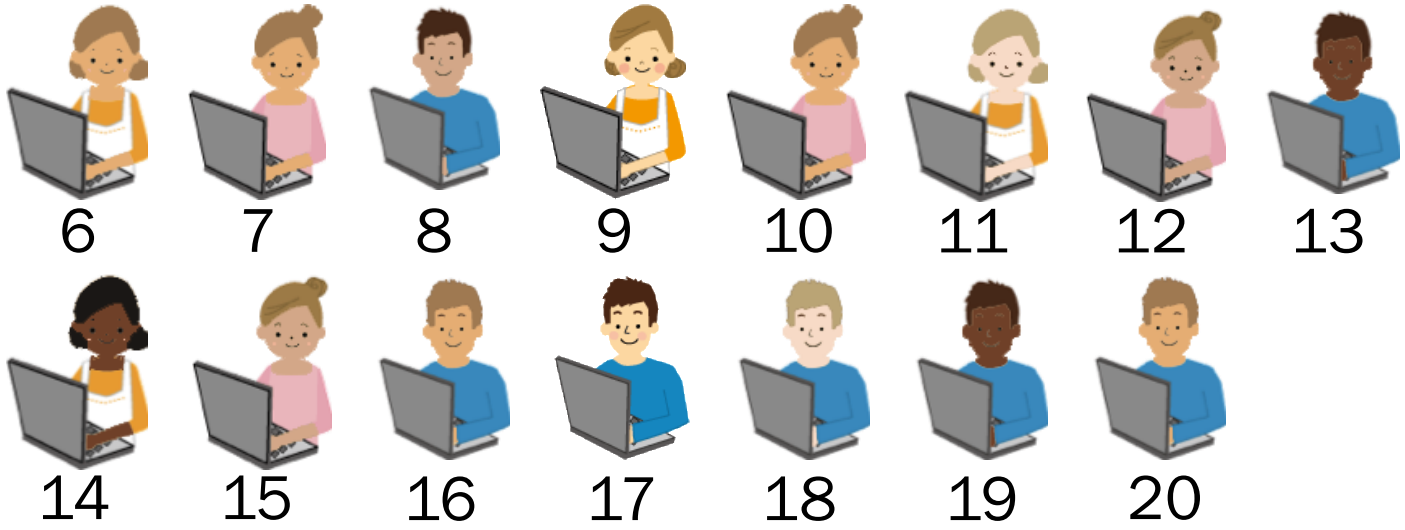
   1 way

3. Allow cake group to mingle

   $k!$ different permutations lead to the same mingle

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?



1. n people get in line

   $n!$ ways

2. Put first k in cake room

   1 way

3. Allow cake group to mingle

   $k!$ different permutations lead to the same mingle

4. Allow non-cake group to mingle

Combinations with cake

There are $n = 20$ people.
How many ways can we choose $k = 5$ people to get cake?

1. n people get in line

   $n!$ ways

2. Put first k in cake room

   1 way

3. Allow cake group to mingle

   $k!$ different permutations lead to the same mingle

4. Allow non-cake group to mingle

   $(n-k)!$ different permutations lead to the same mingle

Combinations

A combination is an unordered selection of k objects from a set of n distinct objects.

The number of ways of making this selection is

$$\frac{n!}{k!\,(n-k)!} = n! \times 1 \times \frac{1}{k!} \times \frac{1}{(n-k)!}$$

1. Order n distinct objects
2. Take first k as chosen
3. Overcounted: any ordering of chosen group is same choice
4. Overcounted: any ordering of unchosen group is same choice

Combinations

A combination is an unordered selection of k objects from a set of n distinct objects.

The number of ways of making this selection is

$$\frac{n!}{k!\,(n-k)!} = n! \times 1 \times \frac{1}{k!} \times \frac{1}{(n-k)!} = \binom{n}{k}$$

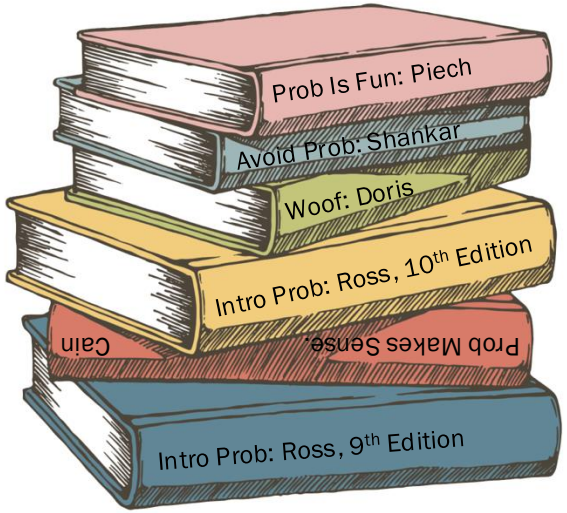
Binomial coefficient

Fun Fact: $\binom{n}{k} = \binom{n}{n-k}$

Probability textbooks

Choose k of n distinct objects $\binom{n}{k}$

How many ways are there to choose 3 books from a set of 6 distinct books?



$$\binom{6}{3} = \frac{6!}{3!\,3!} = 20 \text{ ways}$$

To the code!

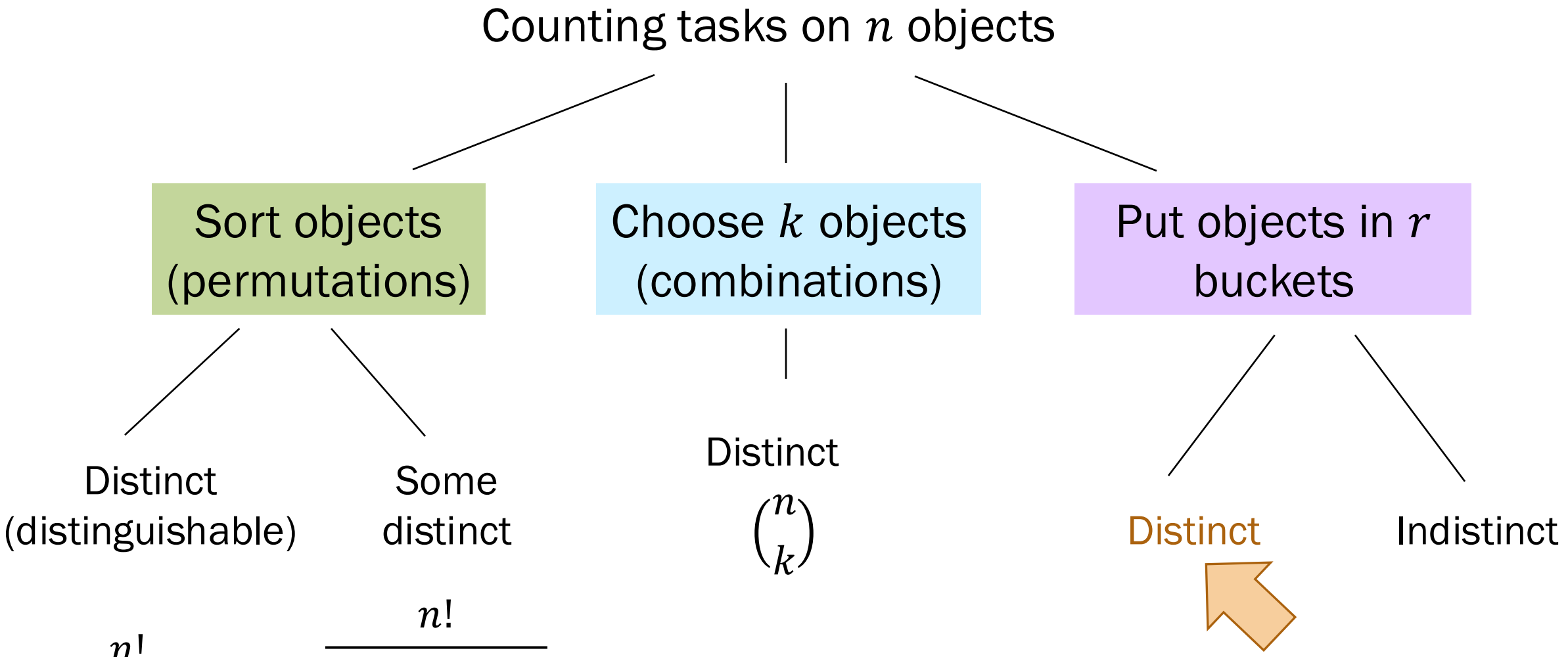
How many unique hands of 5 cards are there in a 52 card deck?

```
def main():
  cards = make_deck()
  all_hands = itertools.combinations(cards, 5)
  for hand in all_hands:
    print(hand)
```

```
def main():
  total = math.comb(52, 5)
  print(total)
```

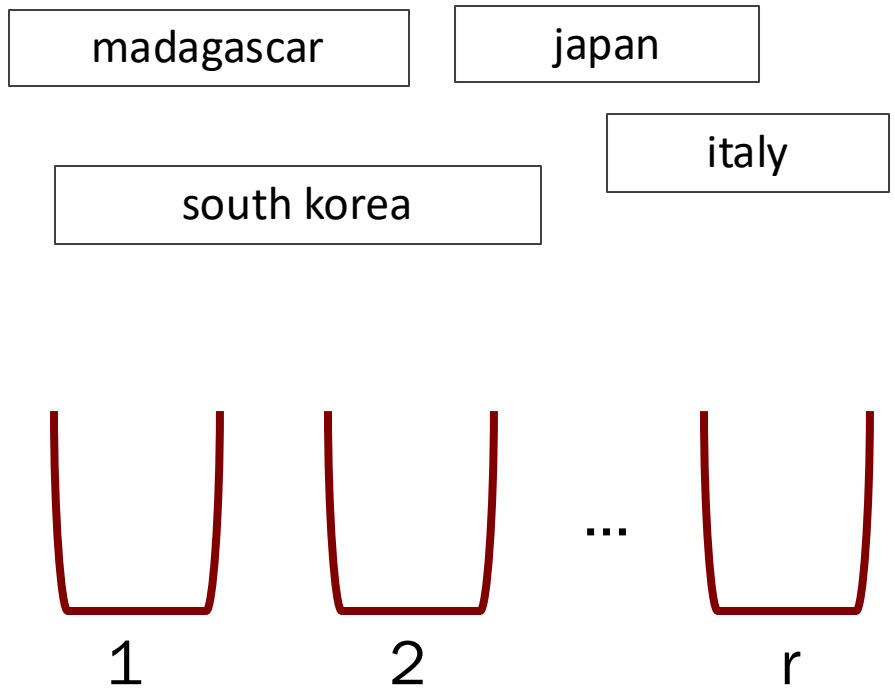
Buckets and The Divider Method

Summary of Combinatorics



~~Balls and urns~~ Hash tables and distinct strings

How many ways are there to hash n distinct strings to r buckets?



Steps:

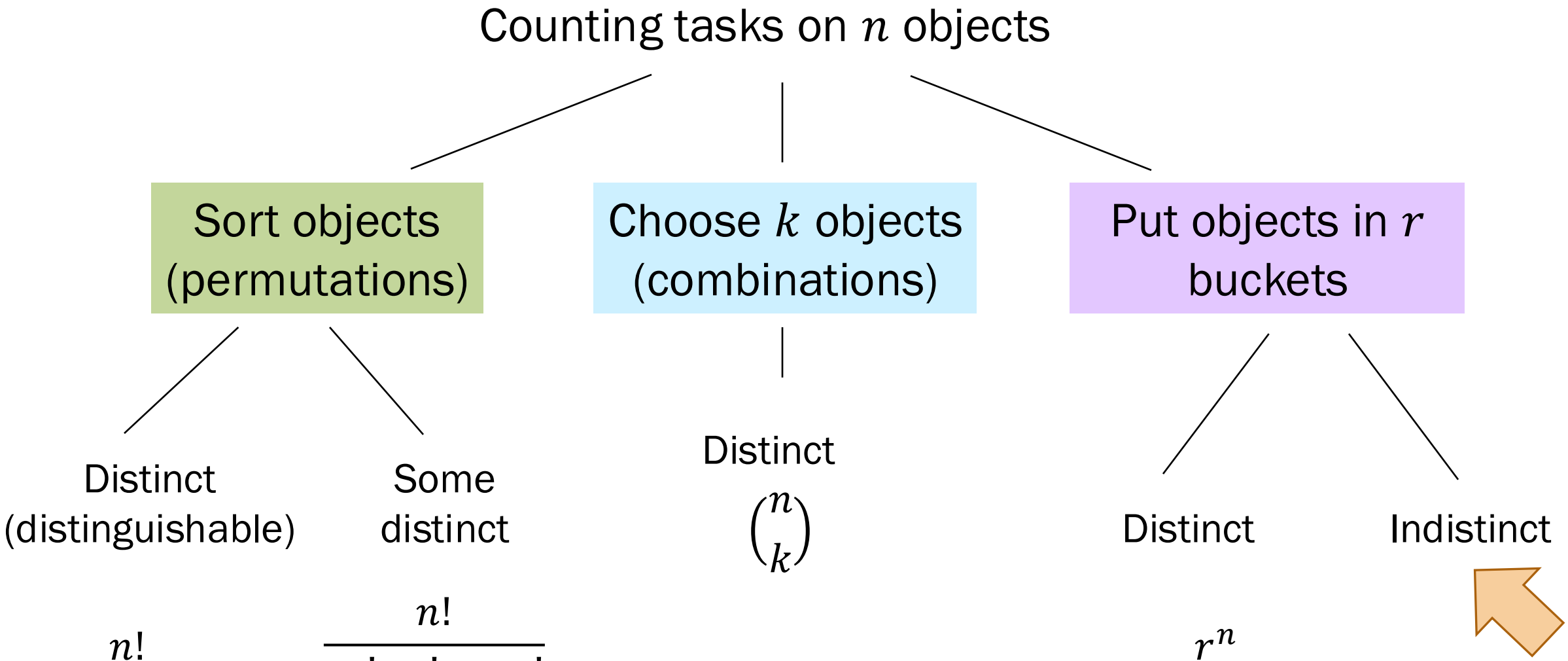
1. Bucket 1st string
2. Bucket 2nd string

...

n. Bucket n^{th} string

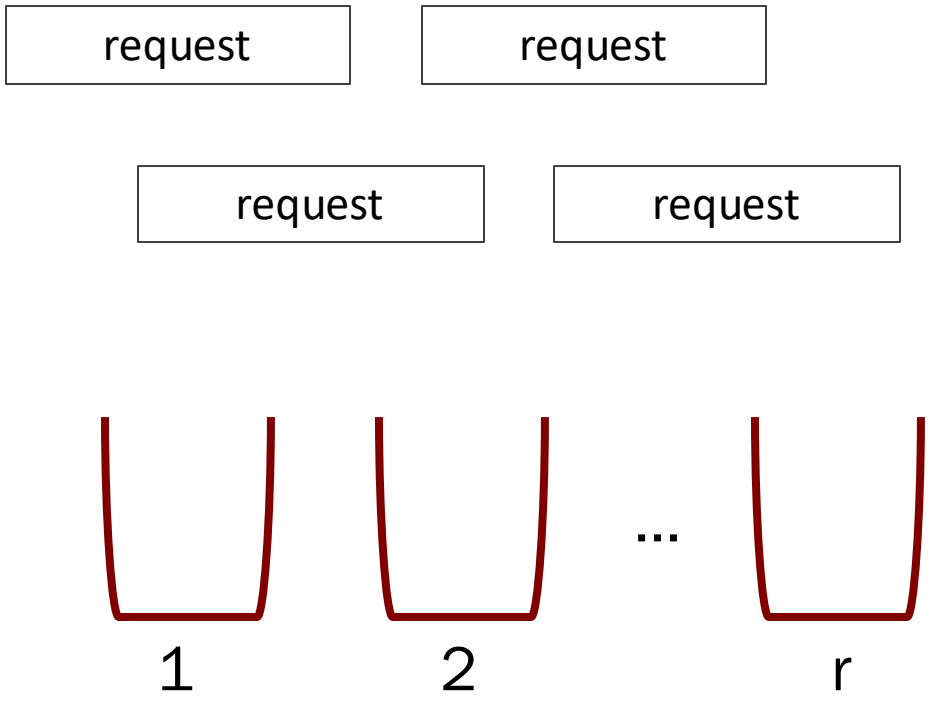
r^n outcomes

Summary of Combinatorics



Servers and indistinct requests

How many ways are there to distribute n indistinct web requests to r servers?



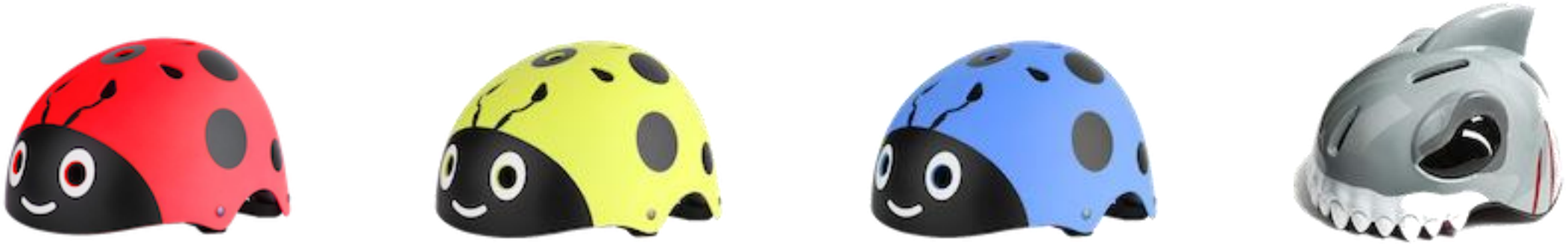
Goal

Server 1 has x_1 requests,
Server 2 has x_2 requests,
...
Server r has x_r requests

constraint: $\sum_{i=1}^{r} x_i = n$

Bicycle helmet sales

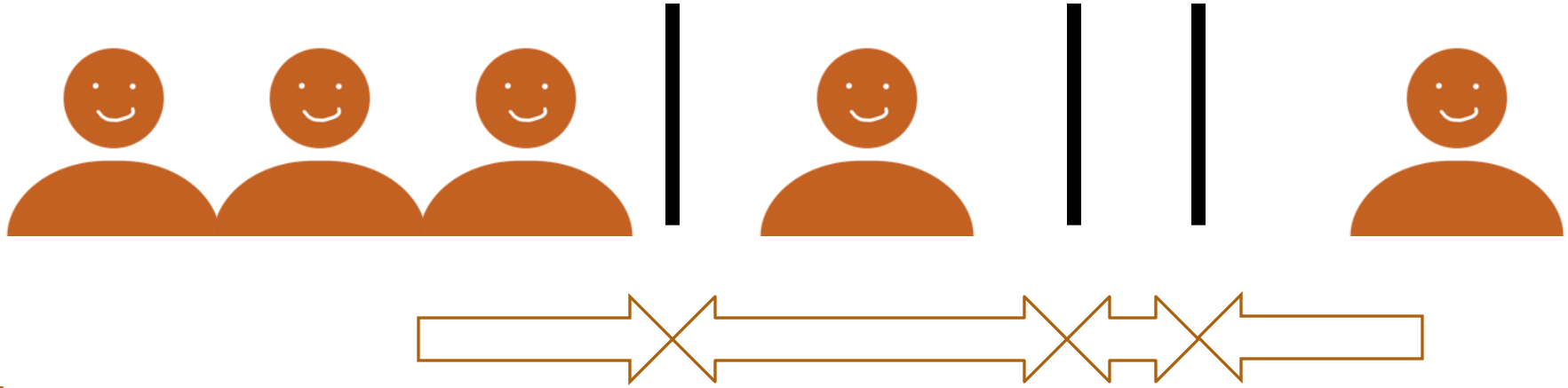
How many ways can we assign $n = 5$ indistinct children to $r = 4$ distinct bicycle helmet styles?



Bicycle helmet sales

1 possible assignment outcome:

Goal Order n indistinct objects and $r - 1$ indistinct dividers.



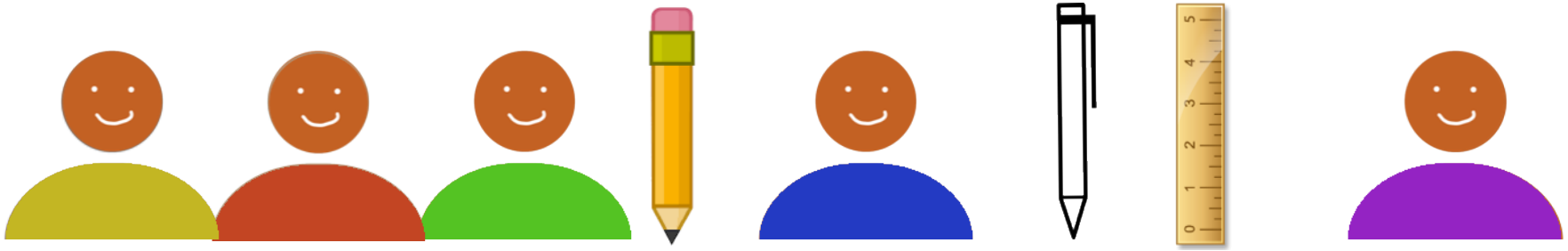
Consider the following generative process...

The divider method: A generative proof

How many ways can we assign $n = 5$ indistinct children to $r = 4$ distinct bicycle helmet styles?

Goal Order n indistinct objects and $r - 1$ indistinct dividers.

0. Make objects and dividers distinct



The divider method: A generative proof

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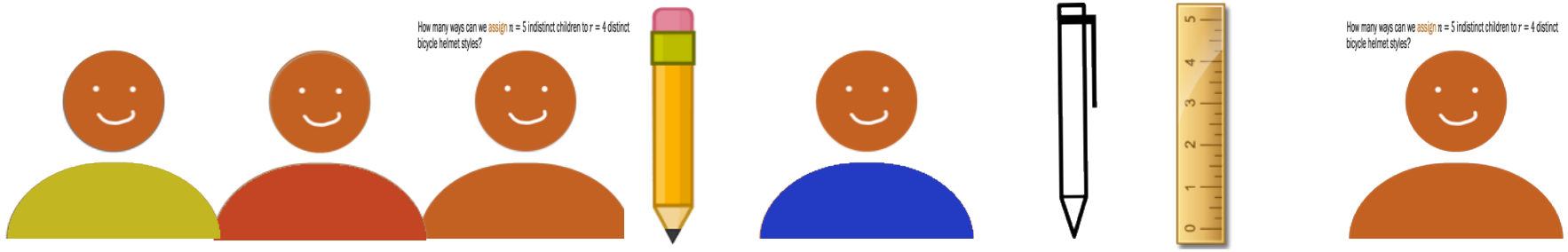

1. Order n distinct objects and $r - 1$ distinct dividers

$$(n + r - 1)!$$

The divider method: A generative proof

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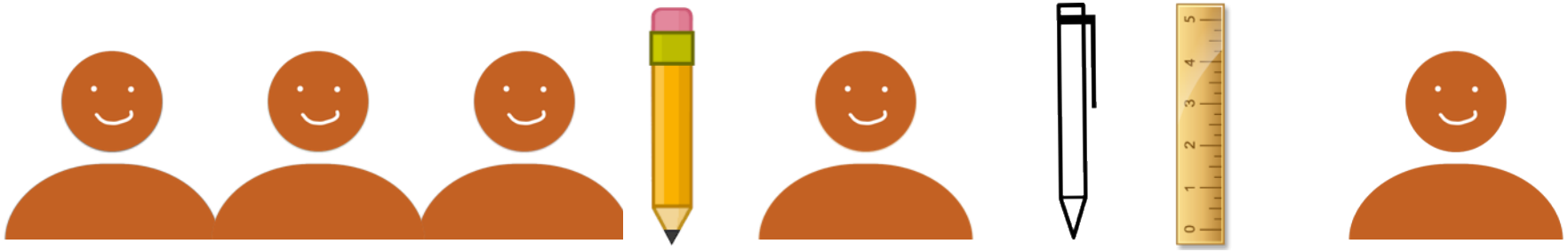


1. Order n distinct objects and $r - 1$ distinct dividers

$$(n + r - 1)!$$

2. Make n objects indistinct

$$\frac{1}{n!}$$

The divider method: A generative proof

How many ways can we assign $n = 5$ indistinct children to $r = 4$ distinct bicycle helmet styles?

Goal Order n indistinct objects and $r - 1$ indistinct dividers.

0. Make objects and dividers distinct

1. Order n distinct objects and $r - 1$ distinct dividers

$$(n + r - 1)!$$

2. Make n objects indistinct

$$\frac{1}{n!}$$

3. Make $r - 1$ dividers indistinct

$$\frac{1}{(r-1)!}$$

The divider method

The number of ways to distribute n indistinct objects into r buckets is

equivalent to the number of ways to permute $n + r - 1$ objects such that

n are indistinct objects, and

$r - 1$ are indistinct dividers:

$$\text{Total} = (n + r - 1)! \times \frac{1}{n!} \times \frac{1}{(r-1)!}$$

$$= \boxed{\binom{n + r - 1}{r - 1} \text{ outcomes}}$$

Integer solutions to equations

Divider method
(n indistinct objects, r buckets) $\binom{n+r-1}{r-1}$

How many integer solutions (assignments to each x) are there to the following equation:

$$x_1 + x_2 + \cdots + x_6 = 20$$

where for all i, x_i is an integer such that $0 \leq x_i \leq 20$?

Integer solutions to equations

Divider method (n indistinct objects, r buckets) $\binom{n+r-1}{r-1}$

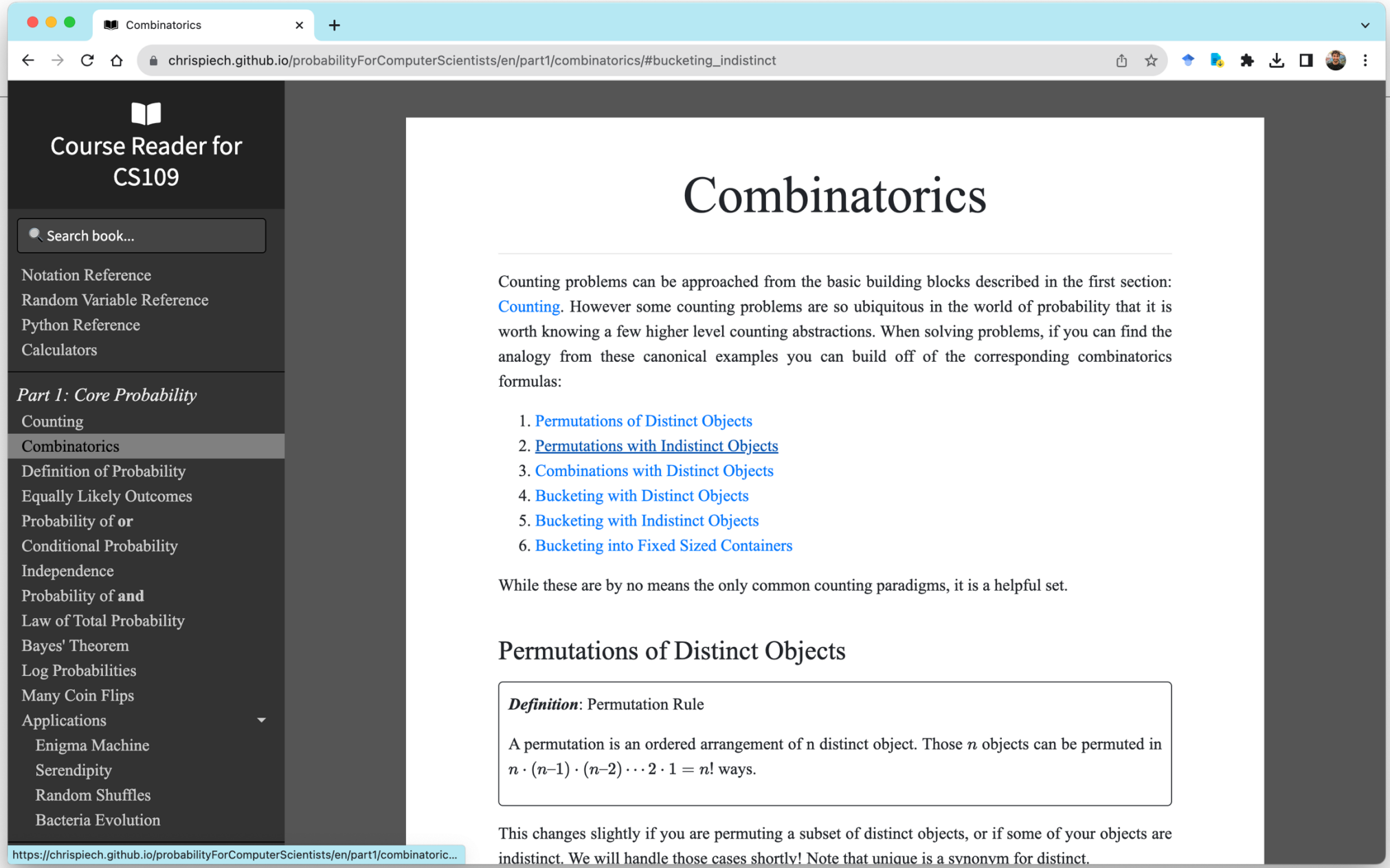
How many integer solutions (assignments to each x) are there to the following equation:

$$x_1 + x_2 + \cdots + x_6 = 20$$

where for all i, x_i is an integer such that $0 \leq x_i \leq 20$?

Example solution: 2 + 3 + 2 + 5 + 5 + 3

$$\binom{n+r-1}{r-1} = \binom{20+6-1}{6-1} = \binom{25}{5} = 53130$$


Combinatorics
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Conditional Probability
Independence
Probability of and
Law of Total Probability
Bayes' Theorem
Log Probabilities
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Applications
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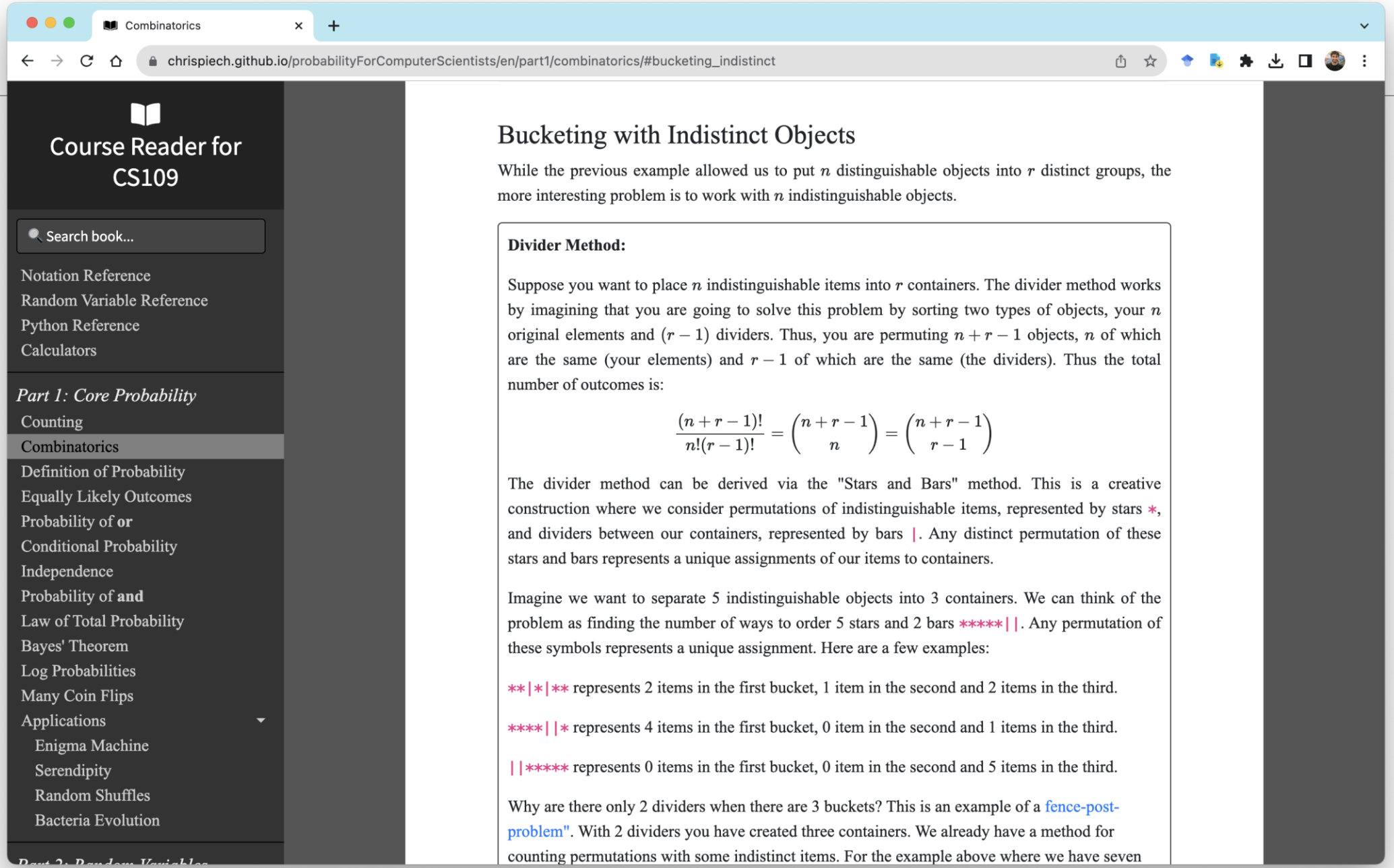
Combinatorics
Counting problems can be approached from the basic building blocks described in the first section: Counting. However some counting problems are so ubiquitous in the world of probability that it is worth knowing a few higher level counting abstractions. When solving problems, if you can find the analogy from these canonical examples you can build off of the corresponding combinatorics formulas:
1. Permutations of Distinct Objects
2. Permutations with Indistinct Objects
3. Combinations with Distinct Objects
4. Bucketing with Distinct Objects
5. Bucketing with Indistinct Objects
6. Bucketing into Fixed Sized Containers
While these are by no means the only common counting paradigms, it is a helpful set.
Permutations of Distinct Objects
Definition: Permutation Rule
A permutation is an ordered arrangement of n distinct object. Those n objects can be permuted in $n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1 = n!$ ways.
This changes slightly if you are permuting a subset of distinct objects, or if some of your objects are indistinct. We will handle those cases shortly! Note that unique is a synonym for distinct.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Conditional Probability
Independence
Probability of and
Law of Total Probability
Bayes' Theorem
Log Probabilities
Many Coin Flips
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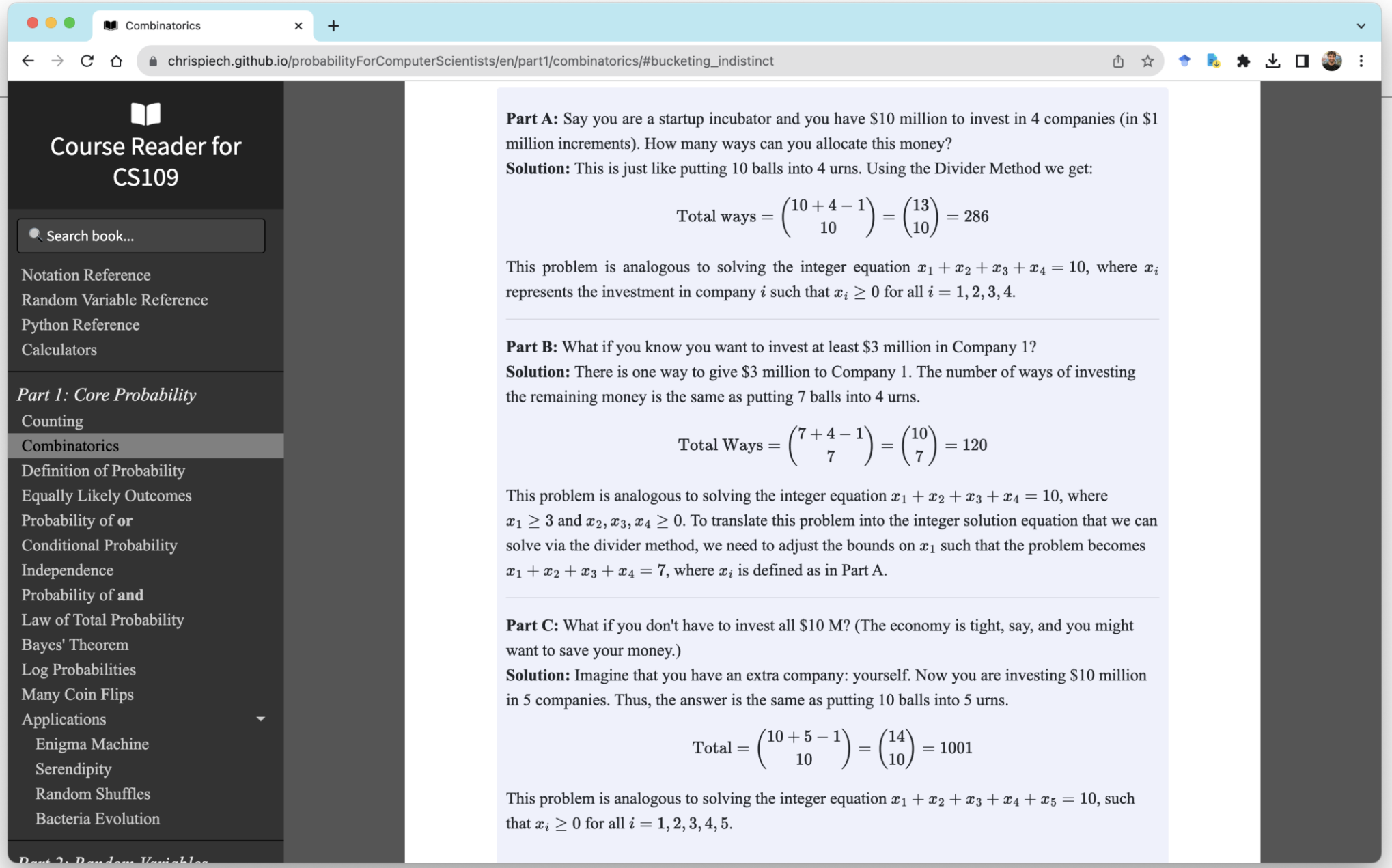
Bucketing with Indistinct Objects
While the previous example allowed us to put n distinguishable objects into r distinct groups, the more interesting problem is to work with n indistinguishable objects.
Divider Method:
Suppose you want to place n indistinguishable items into r containers. The divider method works by imagining that you are going to solve this problem by sorting two types of objects, your n original elements and $(r-1)$ dividers. Thus, you are permuting $n+r-1$ objects, n of which are the same (your elements) and $r-1$ of which are the same (the dividers). Thus the total number of outcomes is:
$$\frac{(n+r-1)!}{n!(r-1)!} = \binom{n+r-1}{n} = \binom{n+r-1}{r-1}$$
The divider method can be derived via the "Stars and Bars" method. This is a creative construction where we consider permutations of indistinguishable items, represented by stars *, and dividers between our containers, represented by bars |. Any distinct permutation of these stars and bars represents a unique assignments of our items to containers.
Imagine we want to separate 5 indistinguishable objects into 3 containers. We can think of the problem as finding the number of ways to order 5 stars and 2 bars *****||. Any permutation of these symbols represents a unique assignment. Here are a few examples:
|*| represents 2 items in the first bucket, 1 item in the second and 2 items in the third.
****||* represents 4 items in the first bucket, 0 item in the second and 1 items in the third.
||***** represents 0 items in the first bucket, 0 item in the second and 5 items in the third.
Why are there only 2 dividers when there are 3 buckets? This is an example of a fence-post-problem". With 2 dividers you have created three containers. We already have a method for counting permutations with some indistinct items. For the example above where we have seven



Part A: Say you are a startup incubator and you have \$10 million to invest in 4 companies (in \$1 million increments). How many ways can you allocate this money?
Solution: This is just like putting 10 balls into 4 urns. Using the Divider Method we get:

$$\text{Total ways} = \binom{10+4-1}{10} = \binom{13}{10} = 286$$

This problem is analogous to solving the integer equation $x_1 + x_2 + x_3 + x_4 = 10$, where x_i represents the investment in company i such that $x_i \geq 0$ for all $i = 1, 2, 3, 4$.

Part B: What if you know you want to invest at least \$3 million in Company 1?
Solution: There is one way to give \$3 million to Company 1. The number of ways of investing the remaining money is the same as putting 7 balls into 4 urns.

$$\text{Total Ways} = \binom{7+4-1}{7} = \binom{10}{7} = 120$$

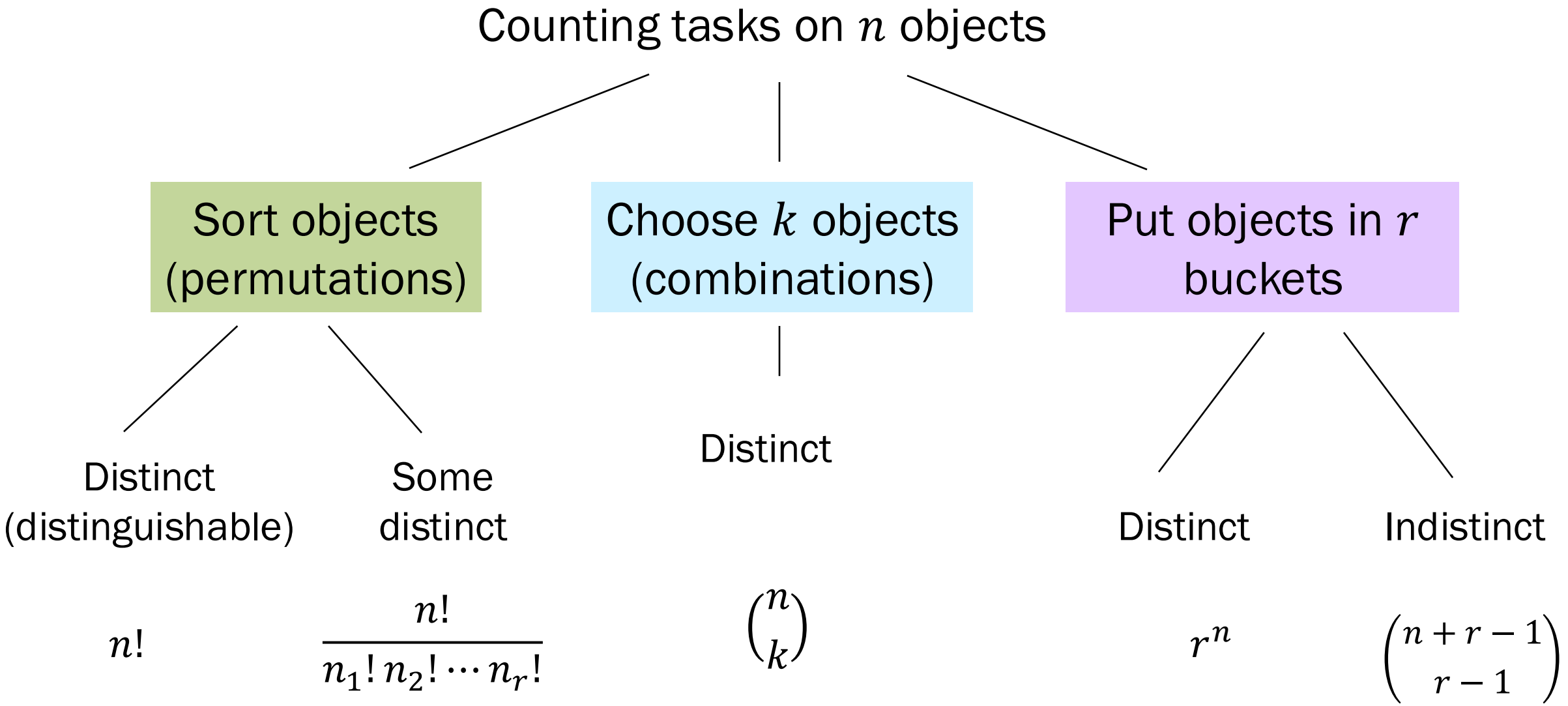
This problem is analogous to solving the integer equation $x_1 + x_2 + x_3 + x_4 = 10$, where $x_1 \geq 3$ and $x_2, x_3, x_4 \geq 0$. To translate this problem into the integer solution equation that we can solve via the divider method, we need to adjust the bounds on x_1 such that the problem becomes $x_1 + x_2 + x_3 + x_4 = 7$, where x_i is defined as in Part A.

Part C: What if you don't have to invest all \$10 M? (The economy is tight, say, and you might want to save your money.)
Solution: Imagine that you have an extra company: yourself. Now you are investing \$10 million in 5 companies. Thus, the answer is the same as putting 10 balls into 5 urns.

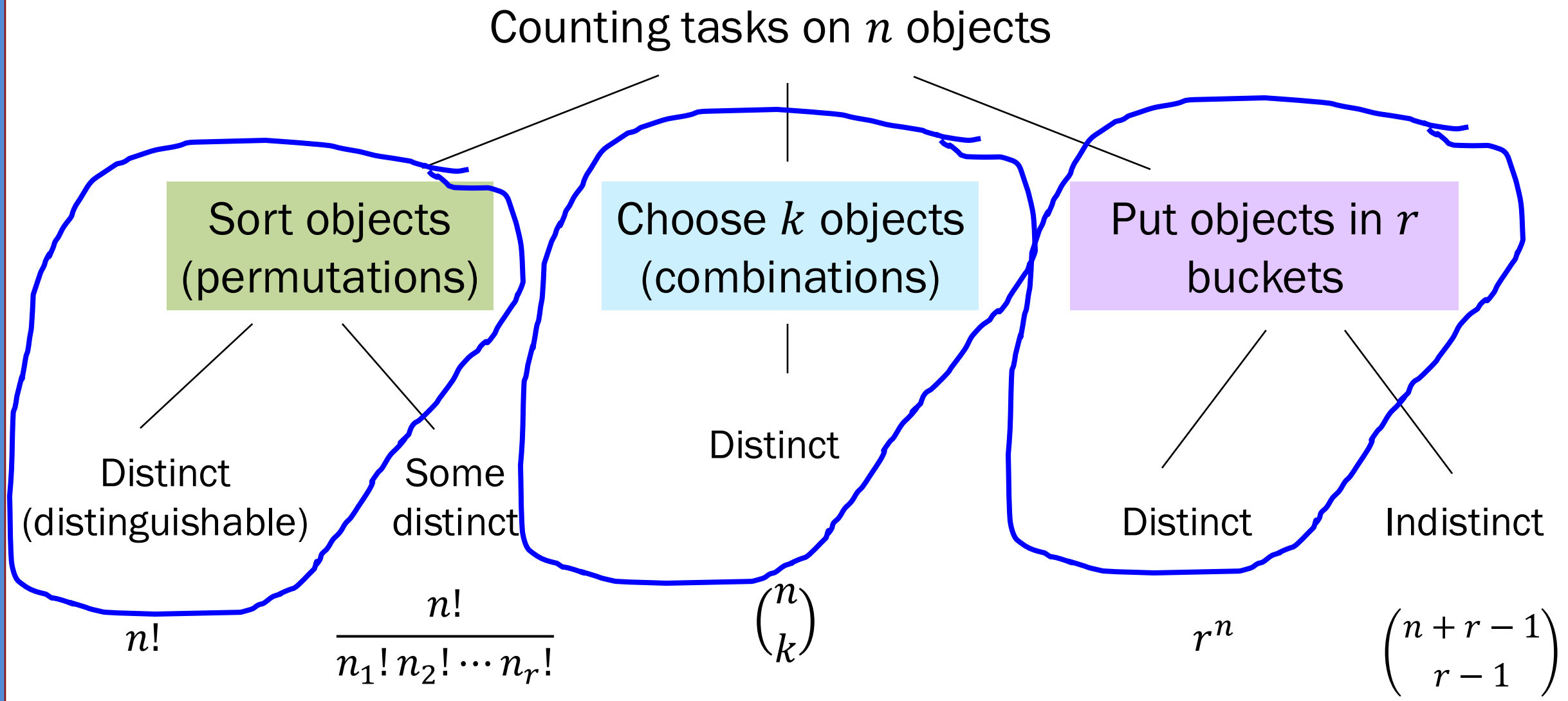
$$\text{Total} = \binom{10+5-1}{10} = \binom{14}{10} = 1001$$

This problem is analogous to solving the integer equation $x_1 + x_2 + x_3 + x_4 + x_5 = 10$, such that $x_i \geq 0$ for all $i = 1, 2, 3, 4, 5$.

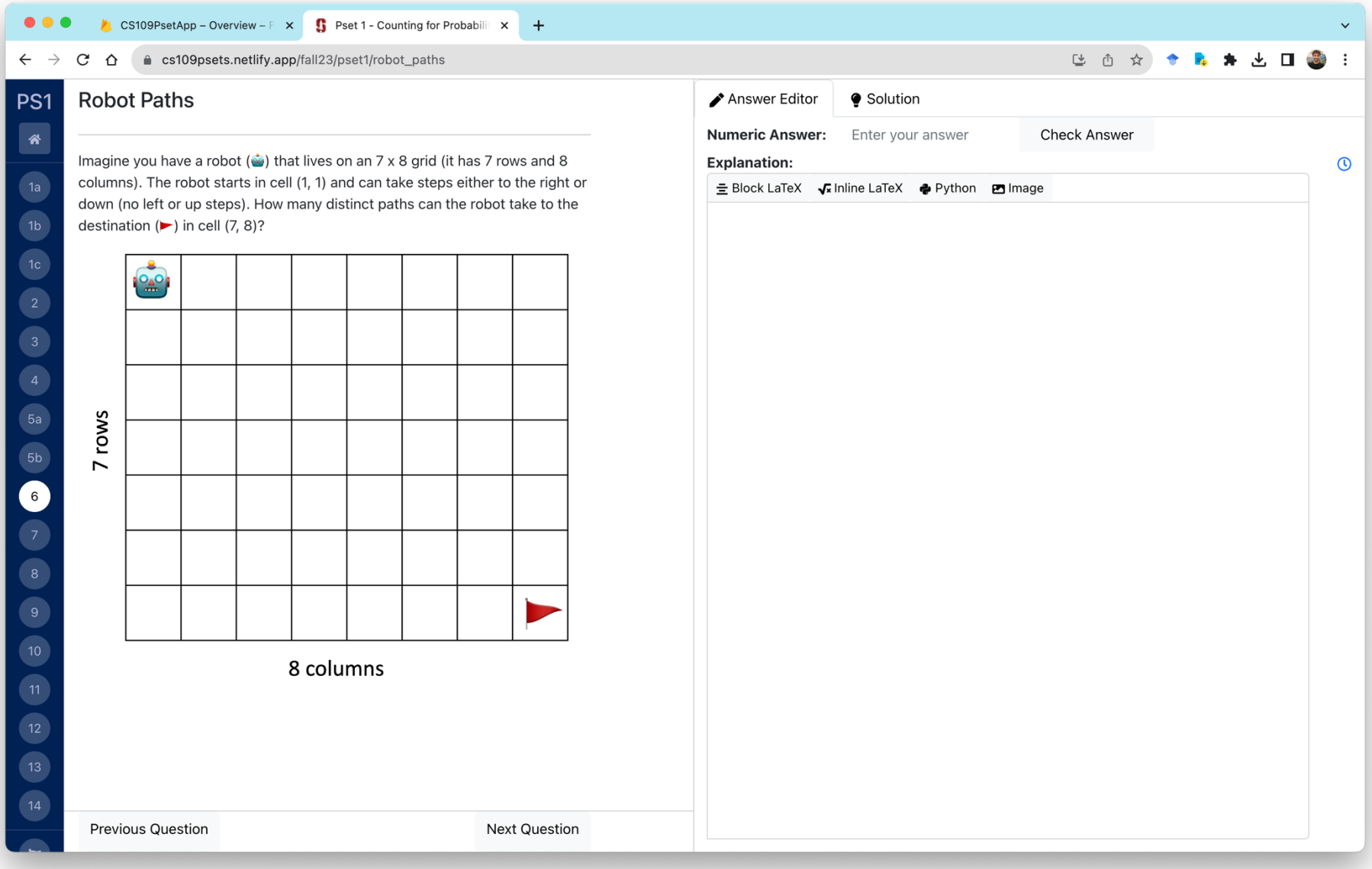
Summary of Combinatorics



Summary of Combinatorics



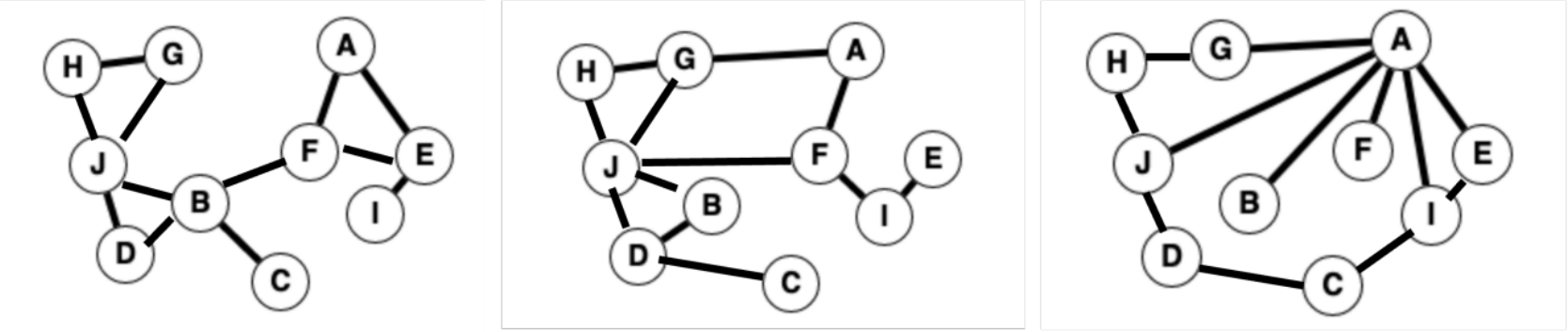
Ready for the first 6 problems (and the rest on Friday)



Formalizing a Word Problem is Hard

3 Random Edges [25 points]

Here are three different networks each with 10 nodes and 12 random edges:



a. (4 points) Consider a network with 10 nodes. Count the number of possible locations for undirected edges. An undirected edge from node A to node B is not distinct from an edge from node B to node A. You can assume that an edge does not connect a node to itself.

Want to go deep?