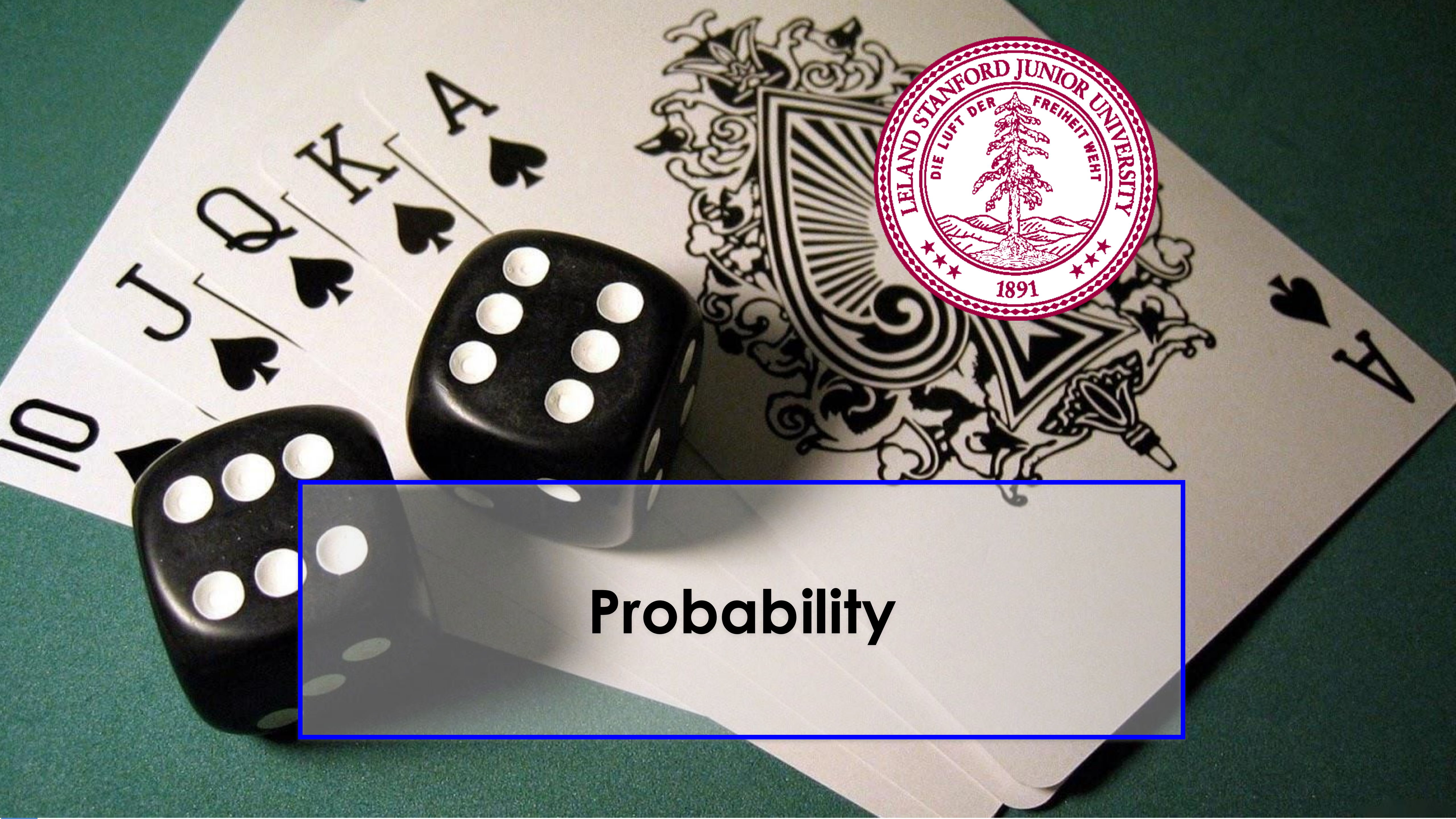
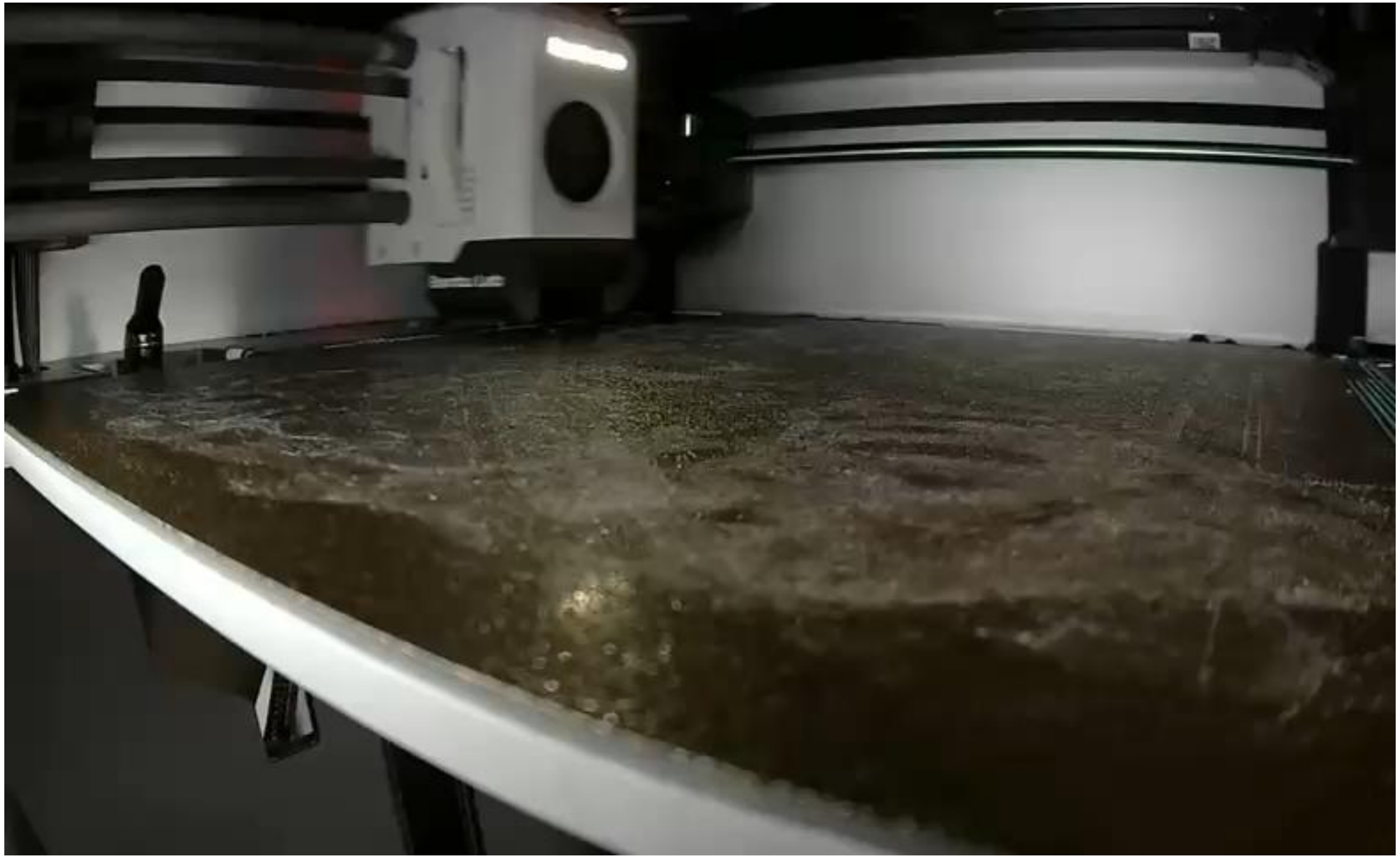


Probability

We are going to make history today

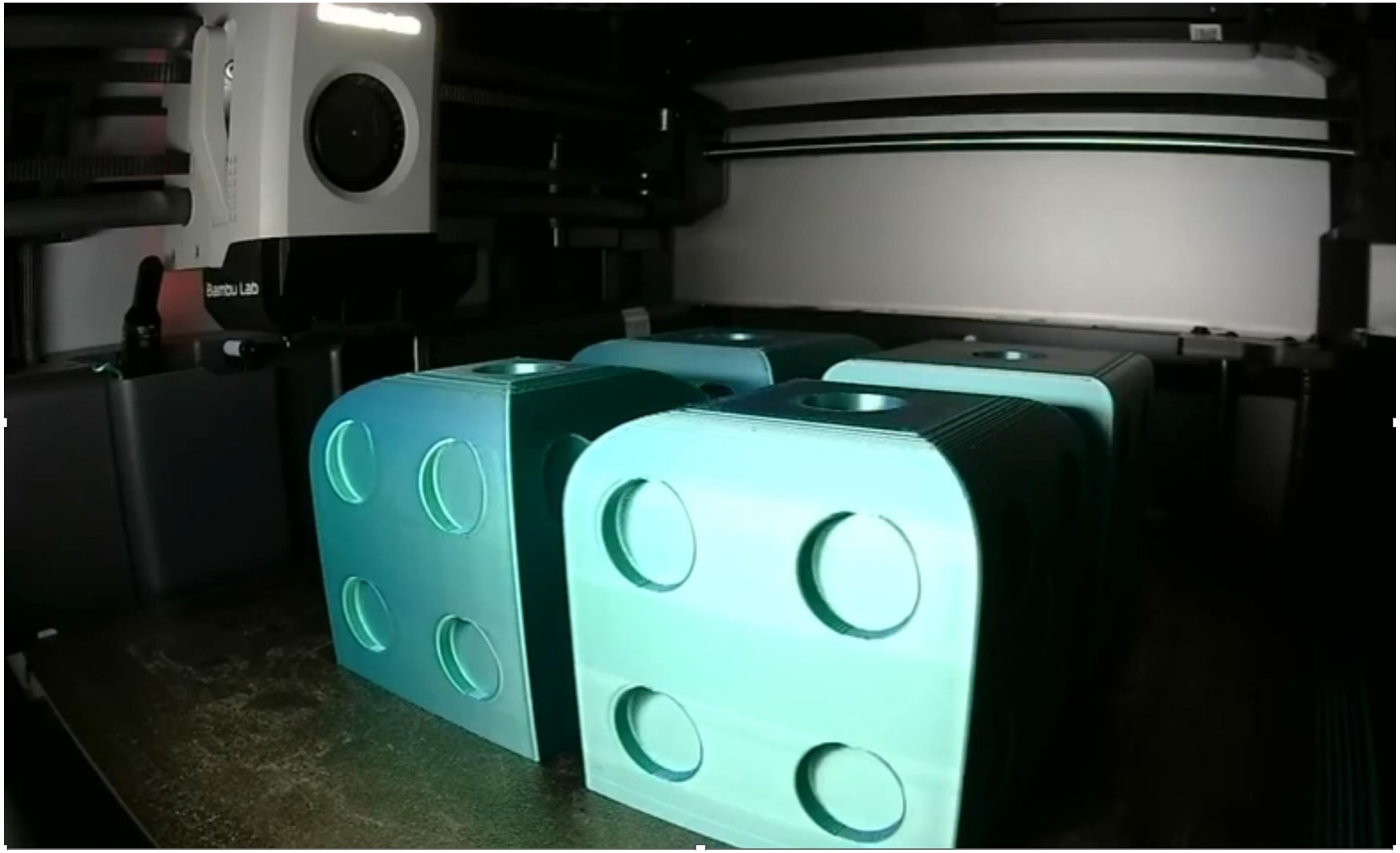
We are also going to learn about probability!

What is the Probability of a Six on This Dice?



I printed this last week

What is the Probability of a Six on This Dice?



I printed this last week

Great Start to Ed

Q: Can Students answer student questions?

A: Yes! **Thank You!**

Q: Do TAs work on the weekend?

A: Only on a voluntary basis. Especially great to help each other out over the weekends.

PSet FAQ

Q: Can I check my answers (many) times?

A: Yes. No penalty. We will know if you try all solutions.

Q: Will you just grade us on if the answers are correct?

A: No! You must explain your answer. Graded on style and correctness.

Q: How much explanation should we provide?

A: As detailed as the course reader worked examples

Q: Must I do all my work on the app?

A: No

PSet Example Answer

Enigma Machine

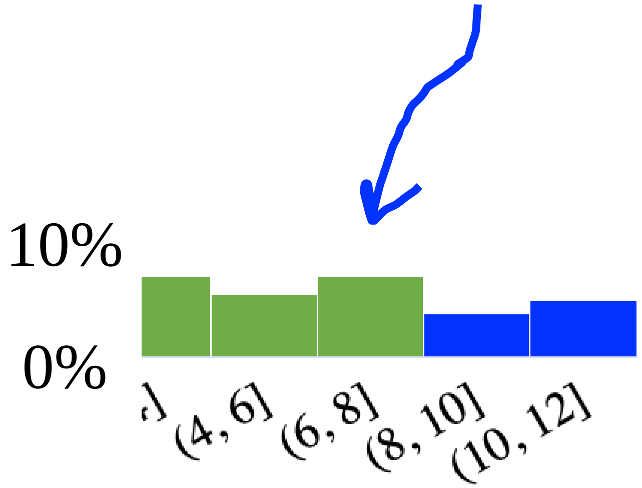
Two wires: How many ways are there to place exactly two wires? Recall that wires are not considered distinct. Each letter can have at most one wire connected to it, thus you couldn't have a wire connect 'K' to 'L' and another one connect 'L' to 'X'

There are $\binom{26}{2}$ ways to place the first wire and $\binom{24}{2}$ ways to place the second wire. However, since the wires are indistinct, we have double counted every possibility. Because every possibility is counted twice we should divide by 2:

$$\text{Total} = \frac{\binom{26}{2} \cdot \binom{24}{2}}{2} = 44,850$$

PSet Status

Try and be here

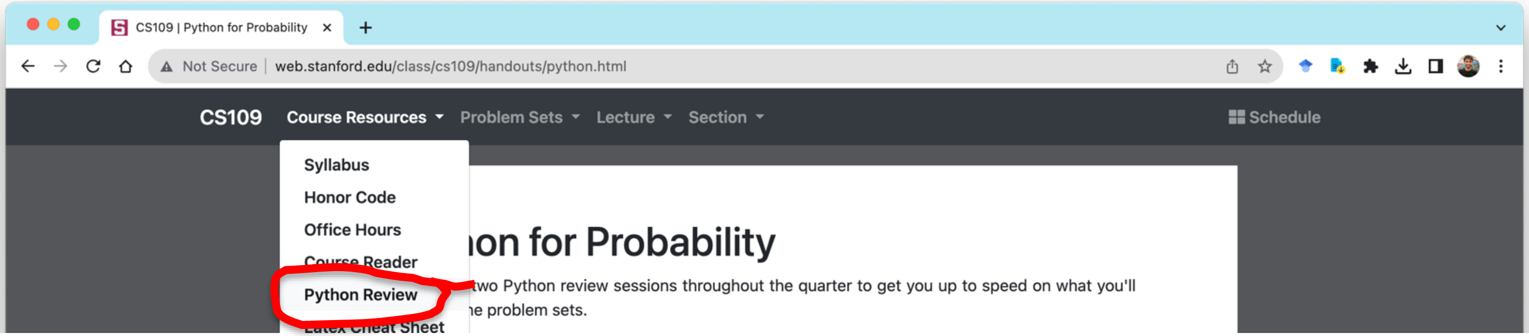


About half the class has started

Python Review Session

Today right after class, here!

Find links, recordings, and setup here



Review

Counting Rules

Counting operations on *n* objects

Sort, order matters {perms}

- Distinct: $n!$
- Some Distinct: $\dfrac{n!}{n_1!n_2!\dots}$

Choose *k* {combinations}

- Distinct: $\dbinom{n}{k} = \dfrac{n!}{k!(n-k)!}$

Put in *r* buckets

- Distinct: r^n
- None Distinct: $\dfrac{(n+r-1)!}{n!(r-1)!}$

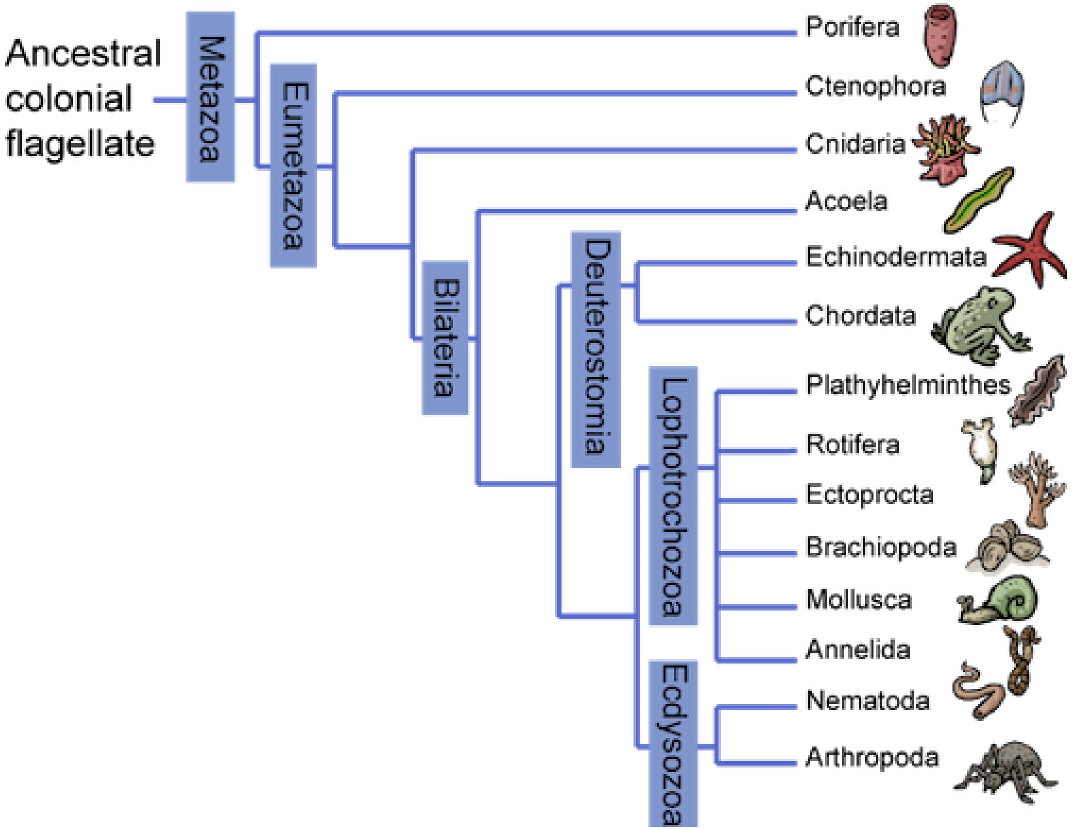
Counting Rules

Counting operations on *n* objects

- Sort, order matters {perms}
  - Distinct: $n!$
  - Some Distinct: $\frac{n!}{n_1!n_2!\dots}$
- Choose *k* {combinations}
  - Distinct: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- Put in *r* buckets
  - Distinct: r^n
  - None Distinct: $\frac{(n+r-1)!}{n!(r-1)!}$

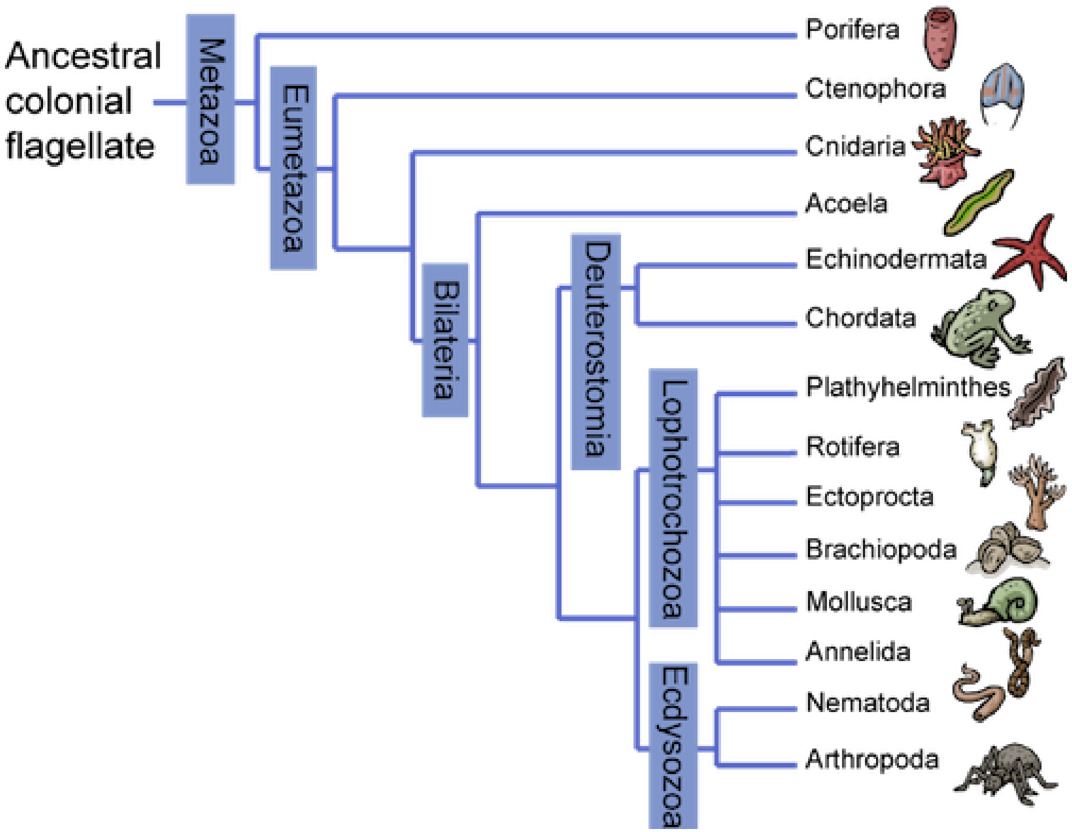
Counting Review

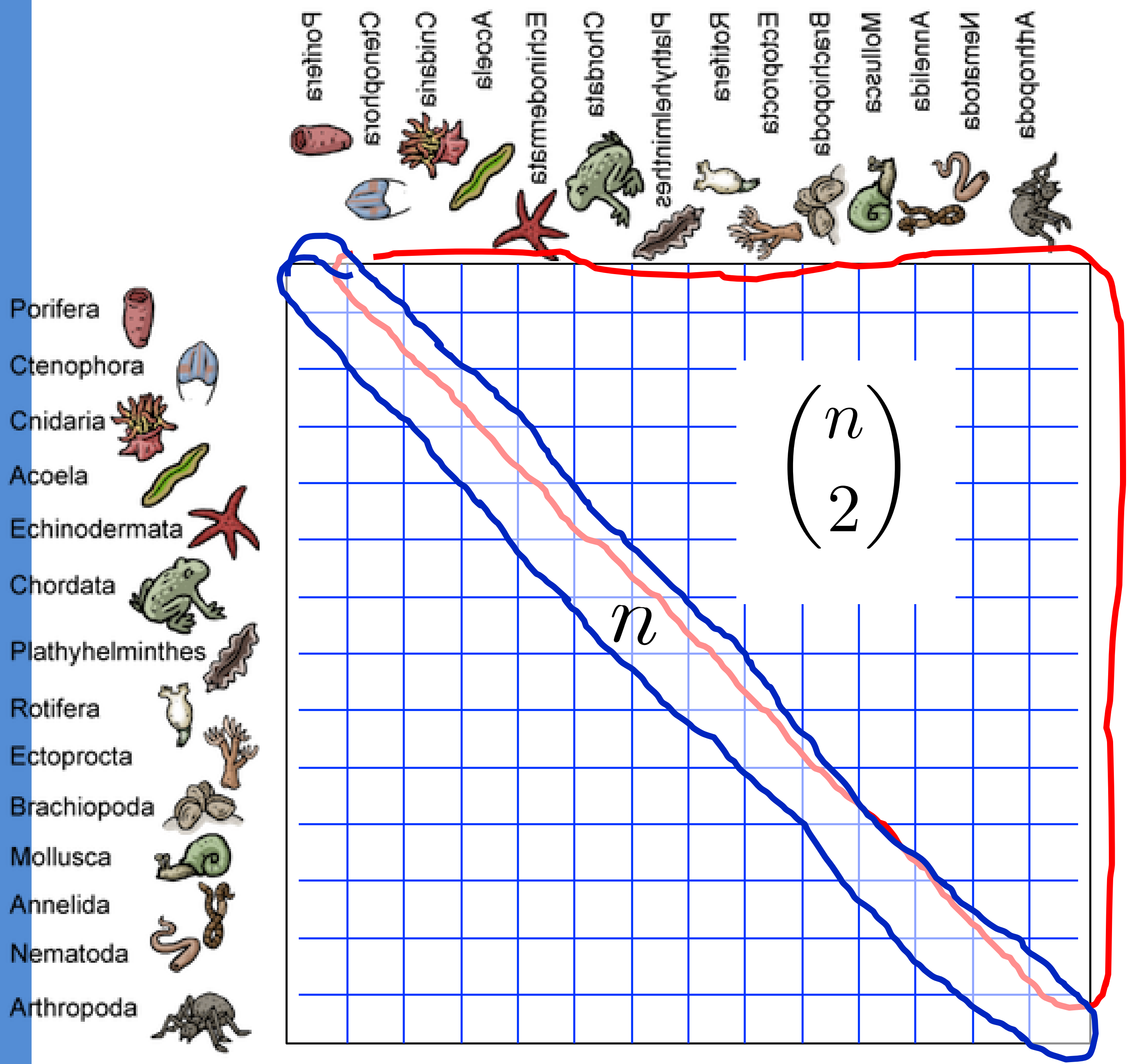
For a DNA tree we need to calculate the DNA distance between each pair of animals. How many calculations are needed?

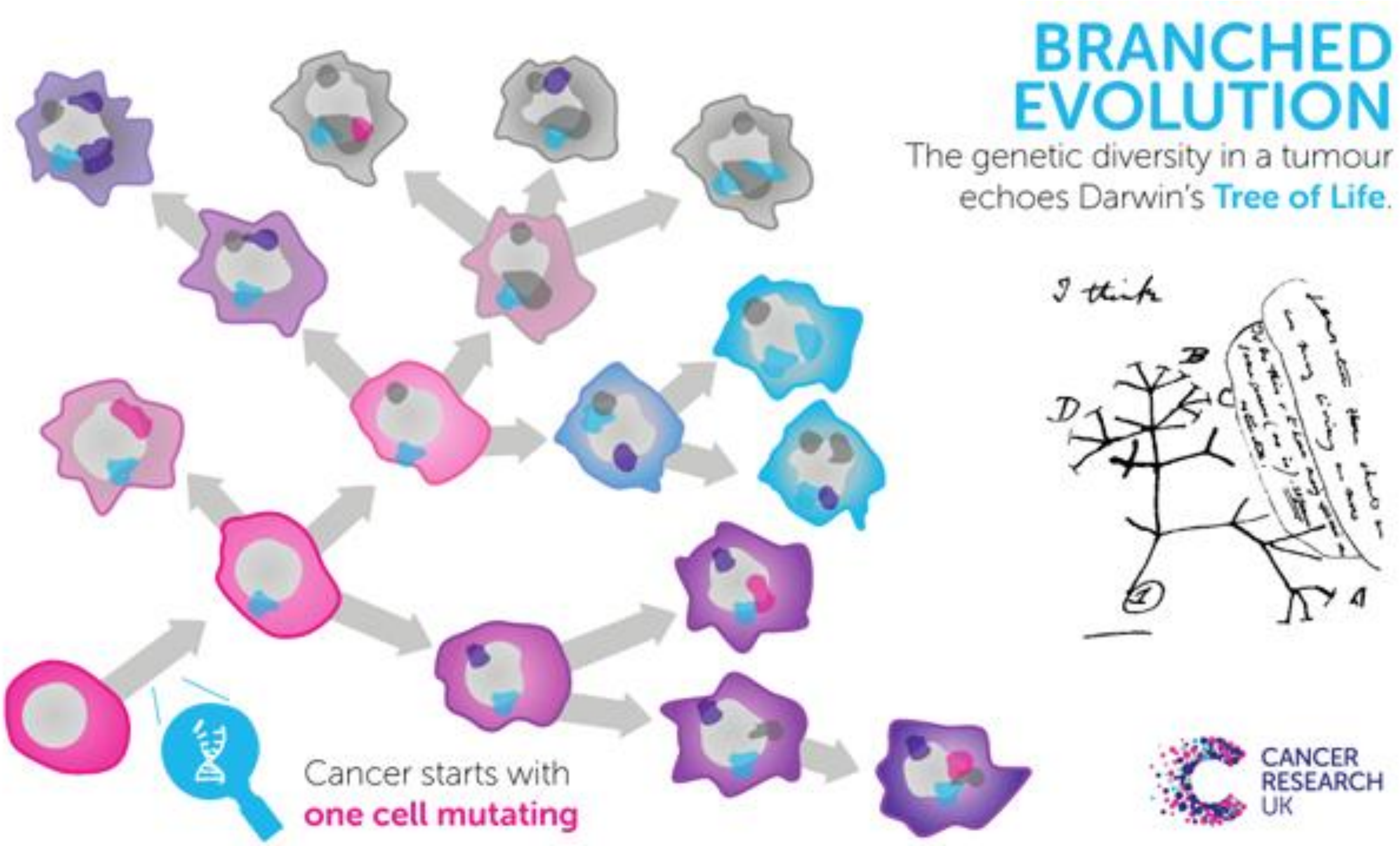


Counting Review

Q: There are *n* animals.
How many distinct pairs of animals are there?

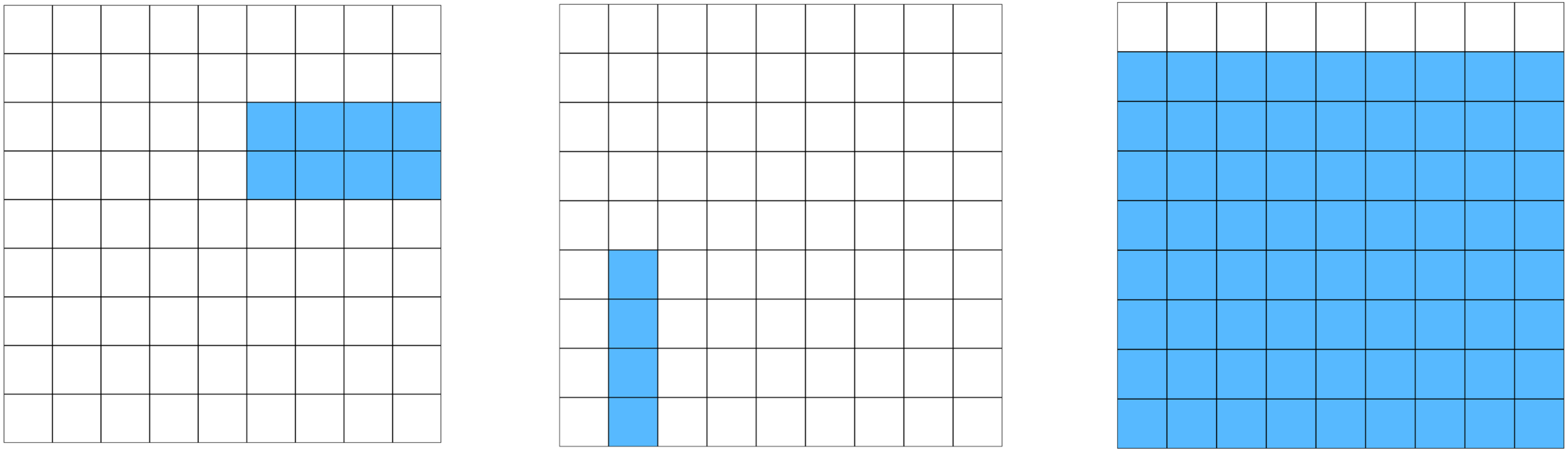



Porifera
Ctenophora
Cnidaria
Acoela
Echinodermata
Chordata
Plathyhelminthes
Rotifera
Ectoprocta
Brachiopoda
Mollusca
Annelida
Nematoda
Arthropoda
$\binom{n}{2}$
n


BRANCHED EVOLUTION
The genetic diversity in a tumour echoes Darwin's Tree of Life.
I think
Cancer starts with
one cell mutating
CANCER RESEARCH UK

Number of Rectangles in a 9x9 grid

Why is the answer 2025? Hint: $\binom{9}{2} = 45$



https://probabilityforcs.firebaseapp.com/book/rectangles_in_a_grid

End Review

Sample Space

- **<u>Sample space</u>**, S, is set of all possible outcomes of an experiment
  - Coin flip: S = {Head, Tails}
  - Flipping two coins: S = {[H, H], [H, T], [T, H], [T, T]}
  - Roll of 6-sided die: S = {1, 2, 3, 4, 5, 6}
  - # emails in a day: $S = \{x \mid x \in \mathbf{Z}, x \geq 0\}$ {non-neg. ints}
  - YouTube hrs. in day: $S = \{x \mid x \in \mathbf{R}, 0 \leq x \leq 24\}$

Event Space

- **<u>Event</u>**, E, is some subset of S $\{E \subseteq S\}$
  - Coin flip is heads: $E = \{\text{Head}\}$
  - ≥ 1 head on 2 coin flips: $E = \{[H, H], [H, T], [T, H]\}$
  - Roll of die is 3 or less: $E = \{1, 2, 3\}$
  - # emails in a day ≤ 20: $E = \{x \mid x \in \mathbf{Z}, 0 \leq x \leq 20\}$
  - Wasted day {≥ 5 YT hrs.}: $E = \{x \mid x \in \mathbf{R}, 5 \leq x \leq 24\}$

Note: When Ross uses: $\subset$, he really means: $\subseteq$

Event Space

Sample Space, S

- Coin flip
  $S = \{\text{Heads}, \text{Tails}\}$
- Flipping two coins
  $S = \{(\text{H,H}), (\text{H,T}), (\text{T,H}), (\text{T,T})\}$
- Roll of 6-sided die
  $S = \{1, 2, 3, 4, 5, 6\}$
- # emails in a day
  $S = \{x \mid x \in \mathbb{Z},\ x \geq 0\}$
- TikTok hours in a day
  $S = \{x \mid x \in \mathbb{R}, 0 \leq x \leq 24\}$

Event, E

- Flip lands heads
  $E = \{\text{Heads}\}$
- ≥ 1 head on 2 coin flips
  $E = \{(\text{H,H}), (\text{H,T}), (\text{T,H})\}$
- Roll is 3 or less:
  $E = \{1, 2, 3\}$
- Low email day (≤ 20 emails)
  $E = \{x \mid x \in \mathbb{Z},\ 0 \leq x \leq 20\}$
- Wasted day (≥ 5 TT hours):
  $E = \{x \mid x \in \mathbb{R}, 5 \leq x \leq 24\}$

What is a probability?

[suspense]

Number between 0 and 1

A number to which we ascribe meaning

$$P(E)$$

* Our belief that an event E occurs

A number to which we ascribe meaning

$$\Pr(E)$$

* Our belief that an event E occurs

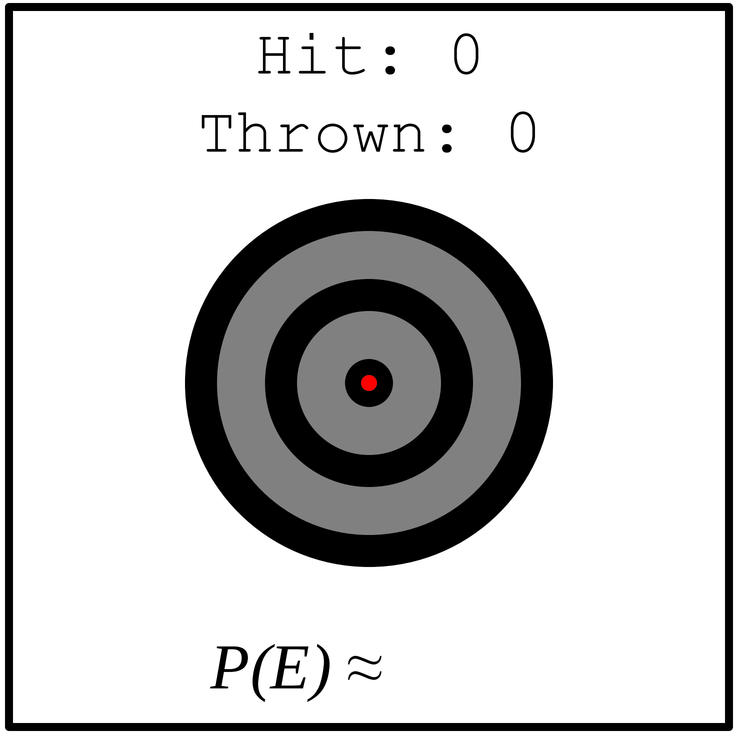
What is a Probability?

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

What is a Probability?

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

n is the number of trails

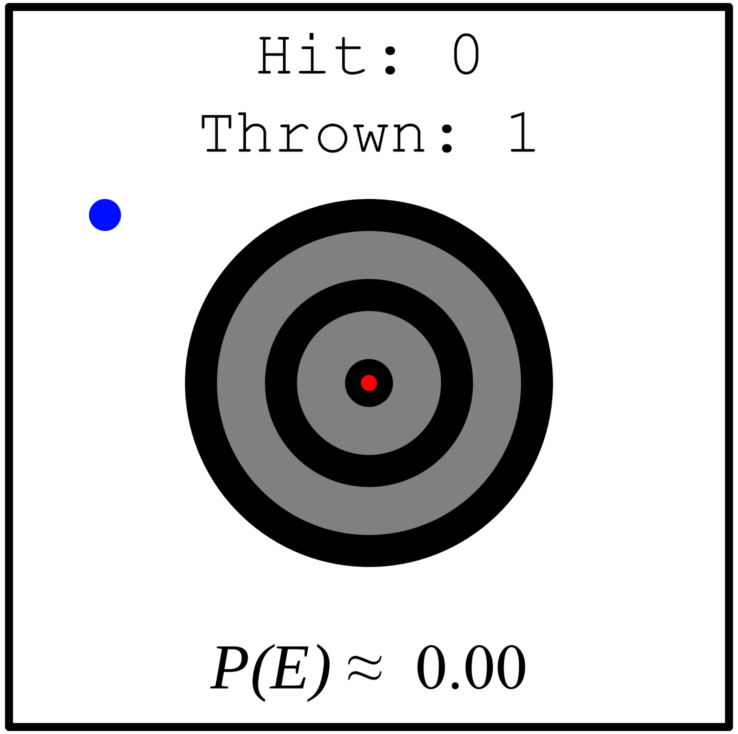


The "event" *E* is that you hit the target

What is a Probability?

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

n is the number of trails

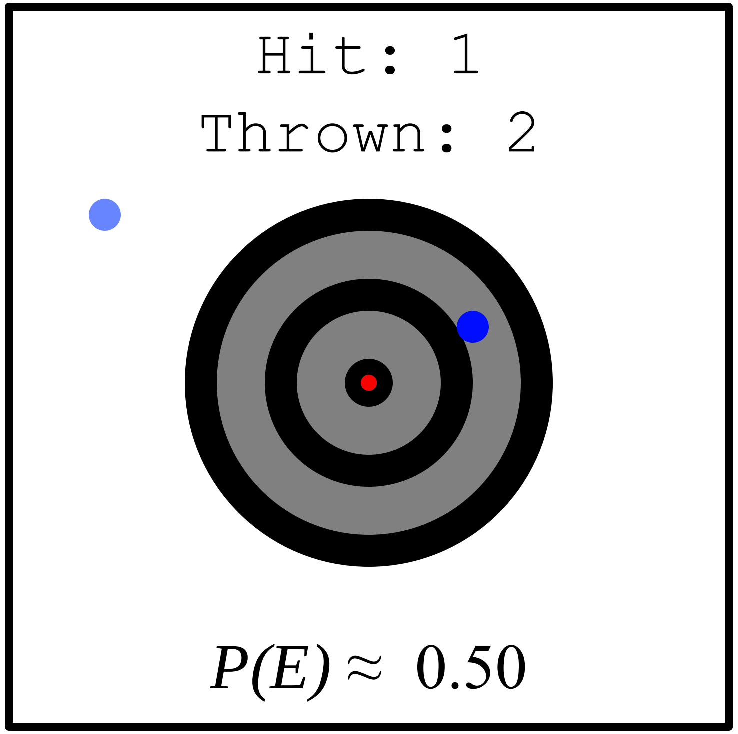


The “event” *E* is that you hit the target

What is a Probability?

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

n is the number of trails

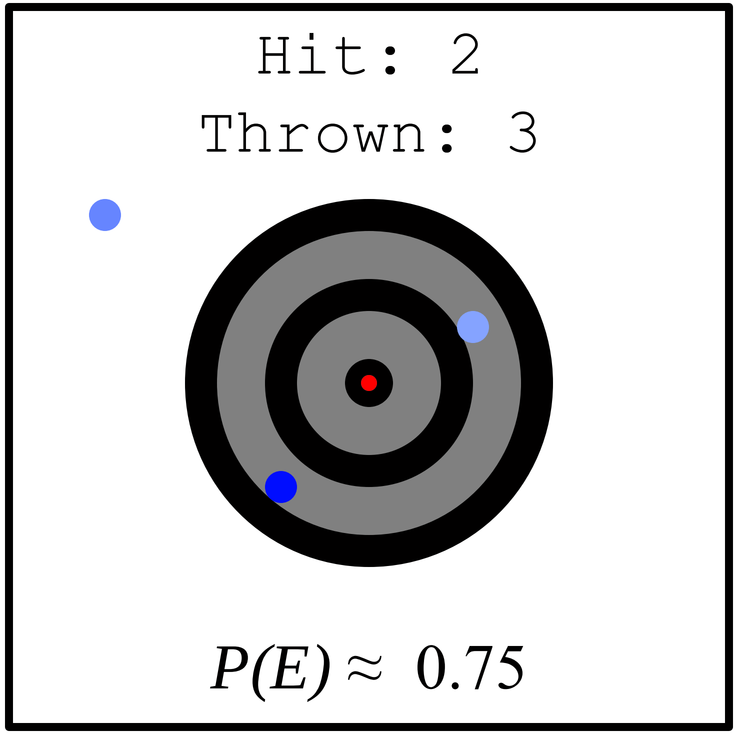


The "event" *E* is that you hit the target

What is a Probability?

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

n is the number of trails

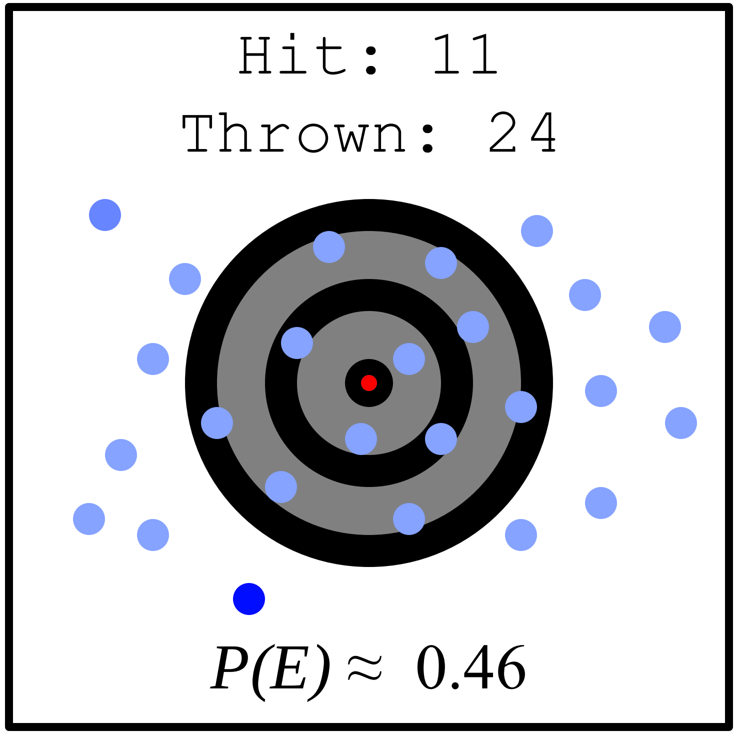


The "event" *E* is that you hit the target

What is a Probability?

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

n is the number of trails



The "event" *E* is that you hit the target

What is a Probability (in a Dataset)?

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

Dataset of weather

Trial	**Value**
1	Rainy
2	Sunny
3	Rainy
4	Cloudy
5	Rainy
6	Sunny
7	Sunny
8	Sunny
	…
10000	Cloudy

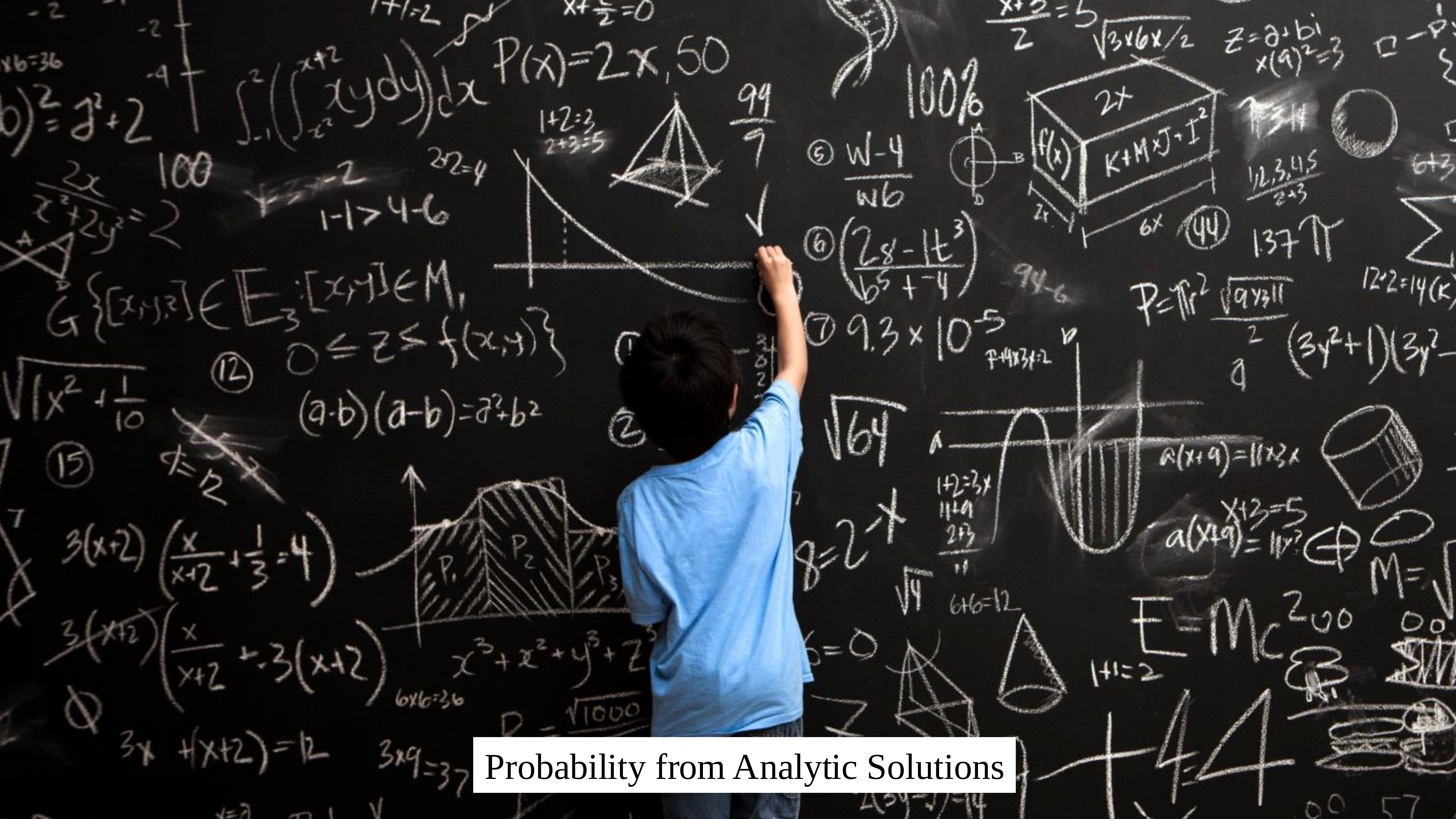
Let E be the event that it is **Sunny**

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

$$\approx \frac{\text{Count}(E)}{10000}$$

$$\approx \frac{3332}{10000} \approx 0.3332$$

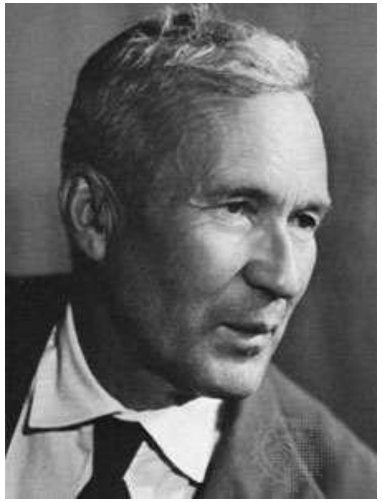

90
MLB 040



Probability from Analytic Solutions

Axioms of Probability

Kolmogorov



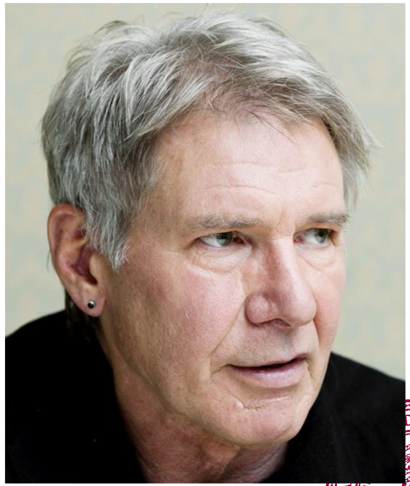
Recall: S = all possible outcomes. E = the event.

- Axiom 1: $0 \leq P(E) \leq 1$

- Axiom 2: $P(S) = 1$

- Axiom 3: If events E and F are mutually exclusive:

$$P(E \cup F) = P(E) + P(F)$$



Harrison Ford

Core Rules of Probability

Recall: S = all possible outcomes. E = the event.

- Axiom 1: $0 \leq P(E) \leq 1$
- Axiom 2: $P(S) = 1$
- Identity 3: $P(E^c) = 1 - P(E)$

Technically Identity 3 can be proved from the 3 axioms.

Special Case of
Analytic Probability

Equally Likely Outcomes

Equally Likely Outcomes

Some sample spaces have **equally likely outcomes**.

- Coin flip: S = {Head, Tails}
- Flipping two coins: S = {[H, H], [H, T], [T, H], [T, T]}
- Roll of 6-sided die: S = {1, 2, 3, 4, 5, 6}

If we have equally likely outcomes, then P(Each outcome) $= \frac{1}{|S|}$

Therefore $$P(E) = \frac{\text{\# outcomes in E}}{\text{\# outcomes in S}} = \frac{|E|}{|S|}$$ {by Axiom 3}

Not Everything is Equally Likely

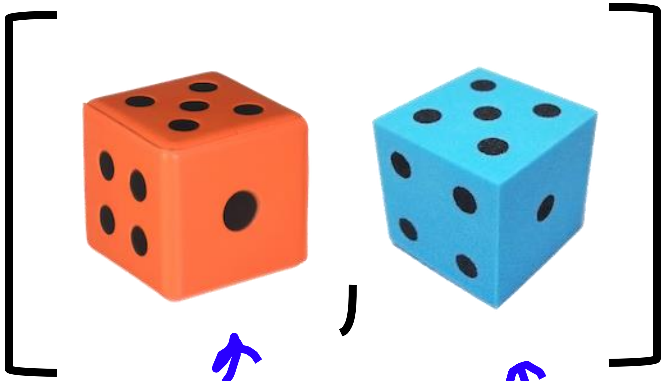
- Play lottery.
  - What is P(Win)?

- S = {Lose, Win}
- E = {Win}
- P(Win) = |E|/|S| = 1/2 = 50%

Sum of Two Die = 7?

Roll two 6-sidex dice. What is P[sum = 7]?

Each outcome



Value dice 1

Value dice 2

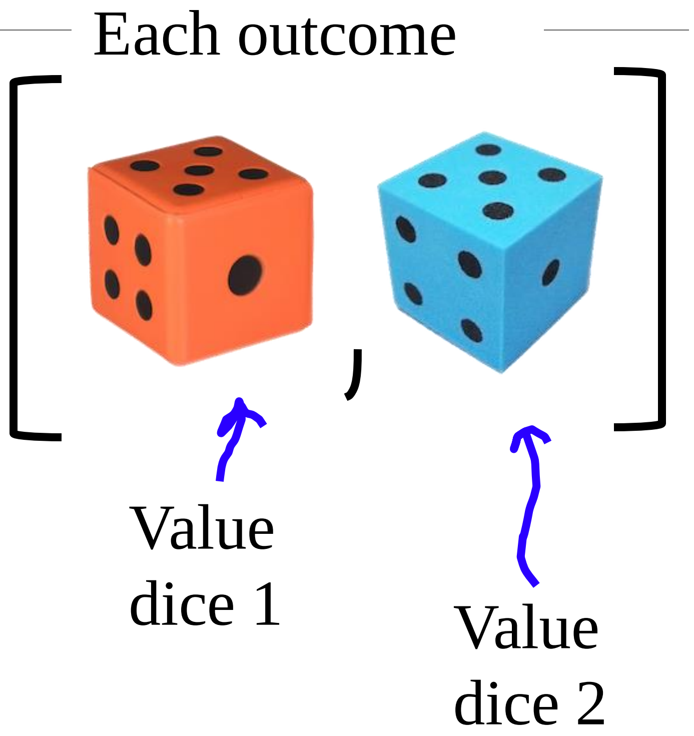
S = { [1,1] [1,2] [1,3] [1,4] [1,5] [1,6]
[2,1] [2,2] [2,3] [2,4] [2,5] [2,6]
[3,1] [3,2] [3,3] [3,4] [3,5] [3,6]
[4,1] [4,2] [4,3] [4,4] [4,5] [4,6]
[5,1] [5,2] [5,3] [5,4] [5,5] [5,6]
[6,1] [6,2] [6,3] [6,4] [6,5] [6,6] }

Sum of Two Die = 7?

Roll two 6-sidex dice. What is probability the sum = 7?
Let E be the event that the sum is 7

S = {

[1,1]	[1,2]	[1,3]	[1,4]	[1,5]	**[1,6]**
[2,1]	[2,2]	[2,3]	[2,4]	**[2,5]**	[2,6]
[3,1]	[3,2]	[3,3]	**[3,4]**	[3,5]	[3,6]
[4,1]	[4,2]	**[4,3]**	[4,4]	[4,5]	[4,6]
[5,1]	**[5,2]**	[5,3]	[5,4]	[5,5]	[5,6]
[6,1]	[6,2]	[6,3]	[6,4]	[6,5]	[6,6] }



E = *in blue*

$$P(E) = \frac{|E|}{|S|} = \frac{6}{36} = 0.16\bar{6}$$

Is it correct?

$$P(E) = \frac{|E|}{|S|} = \frac{6}{36} = 0.16\bar{6}$$

Sum of Two Die = 7?

```python
import random
from tqdm import tqdm

N_TRIALS = 10000000  # getting close to infinity
TARGET_SUM = 7       # do the two dice sum to 6?

def main():
    n_events = 0
    for i in tqdm(range(N_TRIALS)):
        dice_total = run_experiment()
        if dice_total == TARGET_SUM:
            n_events += 1
    pr_e = n_events / N_TRIALS
    print(f'after {N_TRIALS} trials')
    print('P(E) ≈ ', pr_e)

def run_experiment():
    d_1 = roll_dice()
    d_2 = roll_dice()
    return d_1 + d_2

def roll_dice():
    # give me a random dice roll
    # alternatively random.randint(1, 7)
    return random.choice([1,2,3,4,5,6])

if __name__ == '__main__':
    # this starts the program in main
    main()
```

$$P(E) = \frac{|E|}{|S|} = \frac{6}{36} = 0.16\bar{6}$$

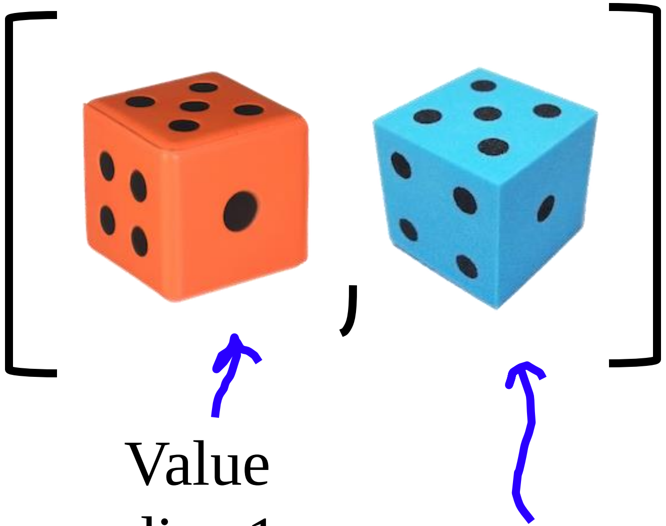
PROBLEMS   OUTPUT   DEBUG CONSOLE   TERMINAL

```
piech@Chriss-MBP-2 3 % python dice_soln.py
after 10000000 trials
P(E) =  0.1666913
```

Sum of Two Die = 2?

Roll two 6-sidex dice. What is probability the sum = 2?
Let E be the event that the sum is 2

S = { [1,1] [1,2] [1,3] [1,4] [1,5] [1,6]
[2,1] [2,2] [2,3] [2,4] [2,5] [2,6]
[3,1] [3,2] [3,3] [3,4] [3,5] [3,6]
[4,1] [4,2] [4,3] [4,4] [4,5] [4,6]
[5,1] [5,2] [5,3] [5,4] [5,5] [5,6]
[6,1] [6,2] [6,3] [6,4] [6,5] [6,6] }



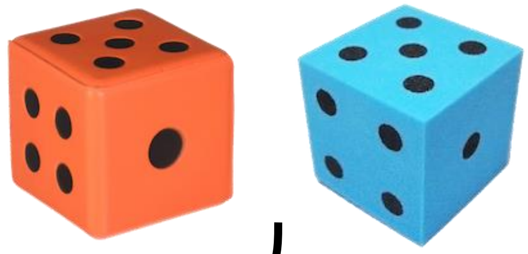
E =

Sum of Two Die = 2?

Roll two 6-sidex dice. What is probability the sum = 2?
Let E be the event that the sum is 2

S = { **[1,1]** [1,2] [1,3] [1,4] [1,5] [1,6]
[2,1] [2,2] [2,3] [2,4] [2,5] [2,6]
[3,1] [3,2] [3,3] [3,4] [3,5] [3,6]
[4,1] [4,2] [4,3] [4,4] [4,5] [4,6]
[5,1] [5,2] [5,3] [5,4] [5,5] [5,6]
[6,1] [6,2] [6,3] [6,4] [6,5] [6,6] }

Each outcome



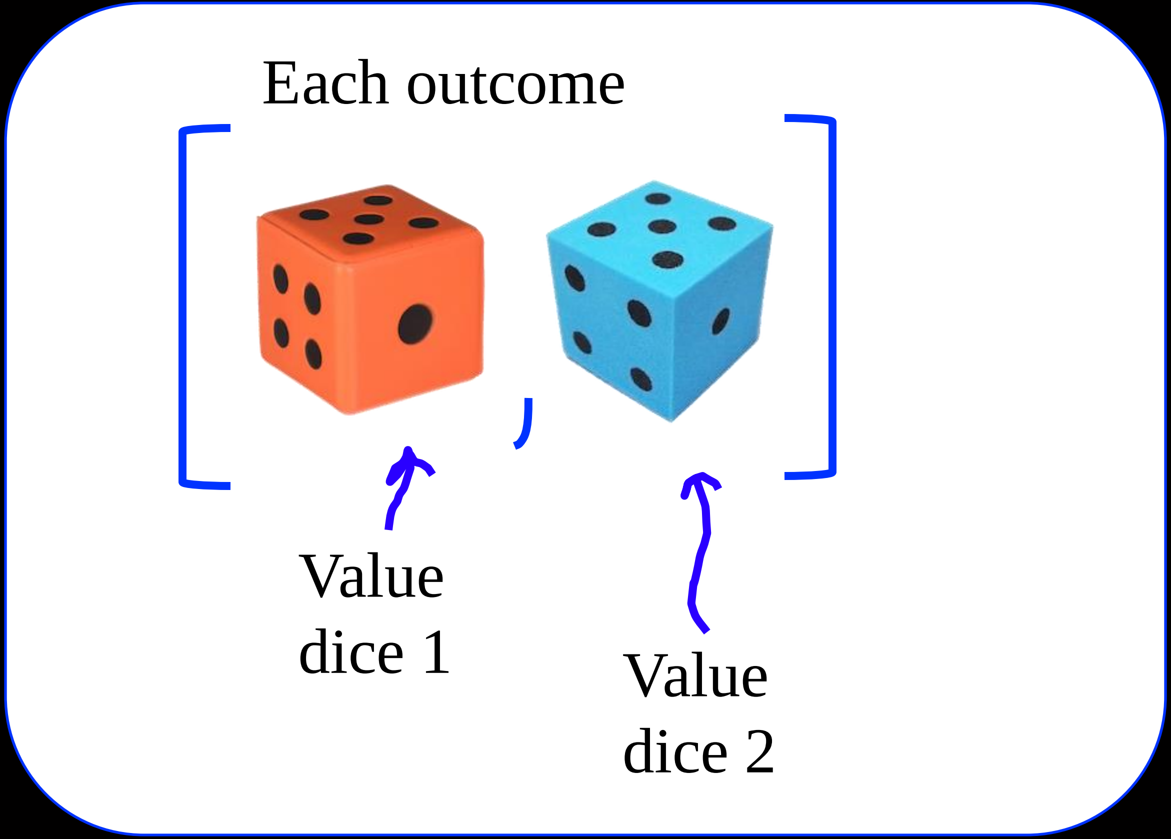
Value dice 1

Value dice 2

E = *in red*

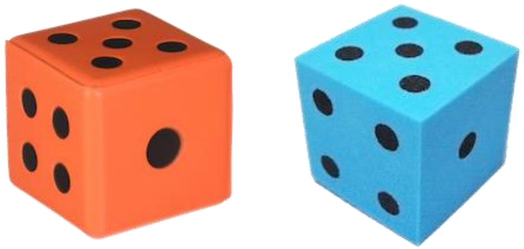
$$P(E) = \frac{|E|}{|S|} = \frac{1}{36} = 0.02\bar{7}$$

Other ways to make a Sample Space?



Sum of Two Die: Three options for the sample space

Value dice 1



Value dice 2

Think of the die as **distinct**

$[5, 5]$

Value dice 1

Value dice 2

Think of the die as **indistinct**

$\{5, 5\}$

Value of a dice

Value of a dice

Just look at the sum

10

Sum of Two Die = 7? Bug: Die are Indistinct

Each outcome

Just look at the sum

Roll two 6-sidex dice. What is probability the sum = 7?
Let E be the event that the sum is 7

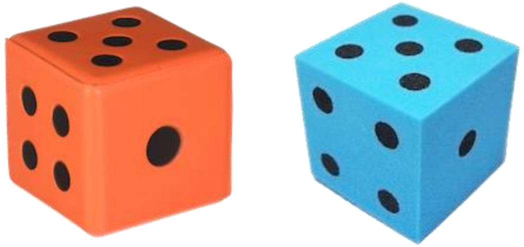
S = { 2 3 4 5 6
7 8 9 10 11 12 }

E = *in red*

$$P(E) = \frac{|E|}{|S|} = \frac{1}{11} = 0.0\overline{9}$$

Sum of Two Die: Three options for the sample space

Value
dice 1



Value
dice 2

Think of the die as **distinct**

[5 , 5]

Value
dice 1

Value
dice 2

Think of the die as **indistinct**

{ 5 , 5 }

Value of
a dice

Value of
a dice

Just look at the sum

10

Sum of Two Die = 7? Bug: Die are Indistinct

Roll two 6-sidex dice. What is P(sum = 7)?

S = { {1,1} {1,2} {1,3} {1,4} {1,5} **{1,6}**

{2,2} {2,3} {2,4} **{2,5}** {2,6}

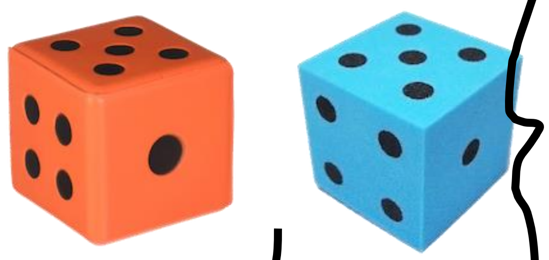
{3,3} **{3,4}** {3,5} {3,6}

{4,4} {4,5} {4,6}

{5,5} {5,6}

{6,6}}

Each outcome



Value dice 1

Value dice 2

E = *in blue*

$$P(E) = \frac{|E|}{|S|} = \frac{3}{20} = 0.15?$$

Sum of Two Die: Three options for the sample space

Value dice 1

Value dice 2

Think of the die as **distinct**

[5, 5]

Value dice 1

Value dice 2

Think of the die as **indistinct**

{5, 5}

Value of a dice

Value of a dice

Just look at the sum

10

Bunnies and Foxes

- 4 bunnies and 3 foxes in a toy box. 3 drawn.
  - What is P(1 bunny and 2 fox drawn)?

Equally likely sample space? Thought experiment

3 foxes

4 bunnies

The Choice of Sample Space is Yours!

	Distinct	Indistinct
Unordered	$\{F_1, B_2, B_3\}$ $\{F_1, F_2, F_3\}$	{2 foxes, 1 bunny} {3 foxes} {3 bunnies}
Ordered	$[F_1, B_2, B_3]$ $[F_1, F_2, F_3]$	[fox, bunny, fox] [fox, fox, fox] [bunny, bunny, bunny]

Which choice will lead to equally likely outcomes?

Bunnies and Foxes

- 4 bunnies and 3 foxes in a Bag. 3 drawn.
  - What is P(1 bunny and 2 foxes drawn)?
- **Ordered and Distinct**:
  - Pick 3 ordered items: |S| = 7 * 6 * 5 = 210
  - Pick bunny as either 1st, 2nd, or 3rd item:
    |E| = (4 * 3 * 2) + (3 * 4 * 2) + (3 * 2 * 4) = 72
  - P(1 bunny, 2 foxes) = 72/210 = 12/35
- **Unordered and Distinct**:
  - |S| = $\binom{7}{3}$ = 35
  - |E| = $\binom{4}{1}\binom{3}{2}$ = 12
  - P(1 bunny, 2 foxes) = 12/35

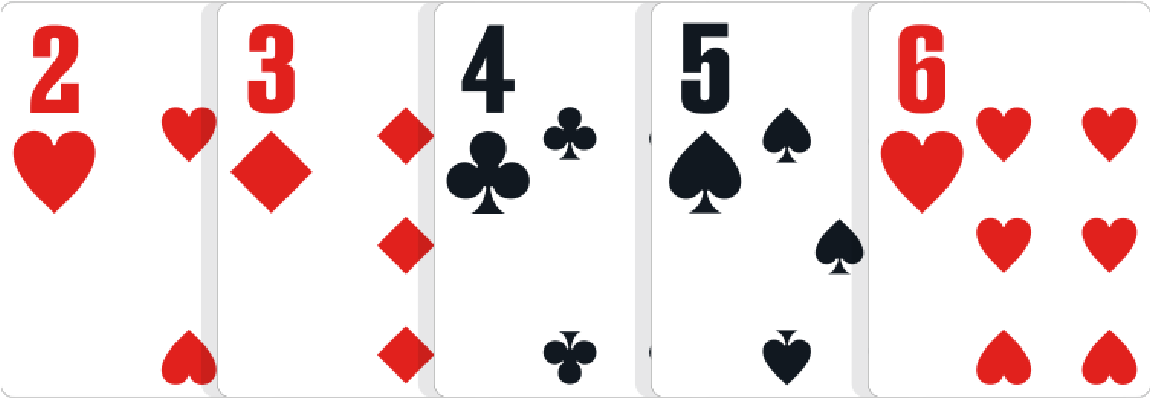
3 foxes

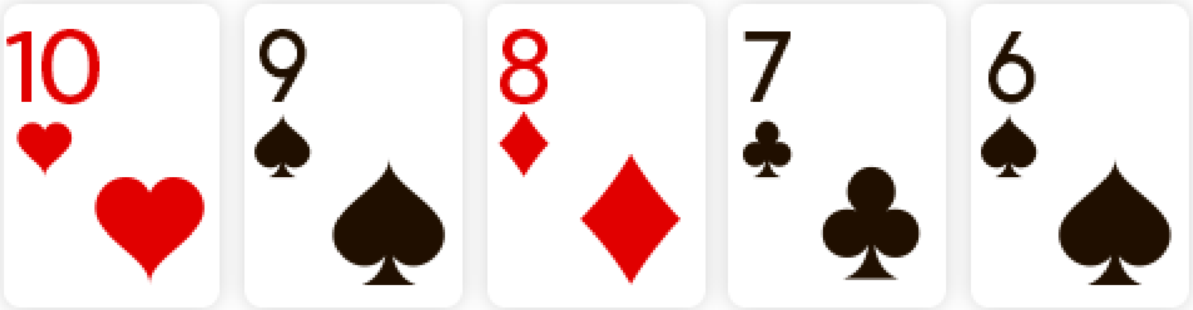
Make indistinct items **distinct** to get equally likely sample space outcomes

*You will need to use this “trick” with high probability

Straight Poker Hand

- Consider 5 card poker hands.
  - "straight" is 5 consecutive rank cards of any suit
  - What is P(straight)?





Straight Poker Hand

- Consider 5 card poker hands.
  - "straight" is 5 consecutive rank cards of any suit
  - What is P(straight)?

What is an example of one outcome?

Is each outcome equally likely?

$$|S| = \binom{52}{5}$$

$$|E| = 10 \cdot \binom{4}{1}^5$$

$$P(\text{straight}) = \frac{|E|}{|S|} = \frac{10 \cdot \binom{4}{1}^5}{\binom{52}{5}} \approx 0.00394$$

Straight Poker Hand

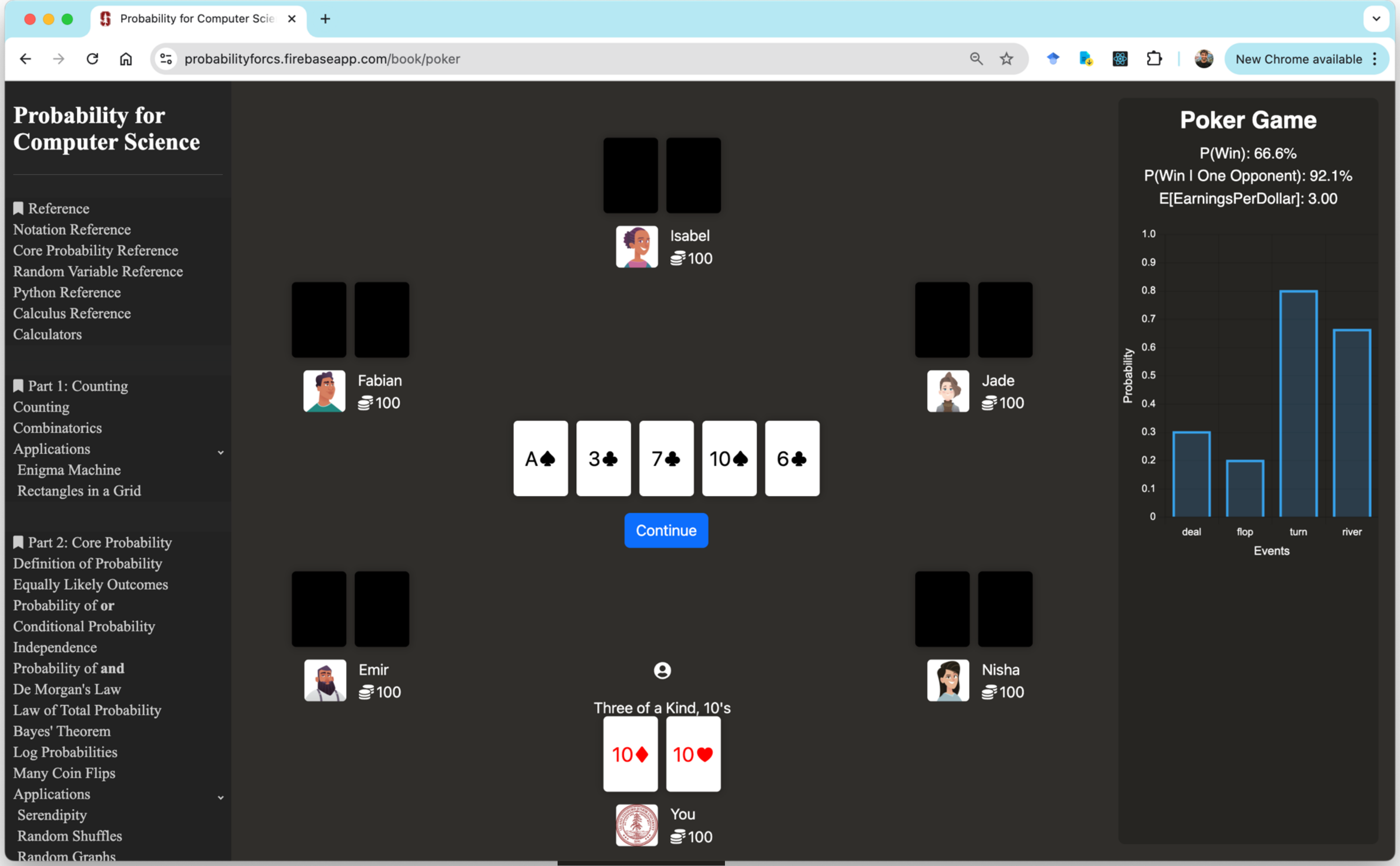
- Consider 5 card poker hands.
  - "straight" is 5 consecutive rank cards of any suit
  - "straight flush" is 5 consecutive rank cards of same suit
  - What is P(straight, but not straight flush)?

$$|S| = \binom{52}{5}$$

$$|E| = 10\binom{4}{1}^5 - 10\binom{4}{1}$$

$$P(\text{straight}) = \frac{|E|}{|S|} = \frac{10\binom{4}{1}^5 - 10\binom{4}{1}}{\binom{52}{5}} \approx 0.00392$$

When approaching an "**equally likely probability**" problem, start by defining **sample spaces** and **event spaces**.



Can enumerate all 990 remaining pairs of cards

Can check which ones you beat

Useful reference: Independent Sets

Lets discuss on ed!

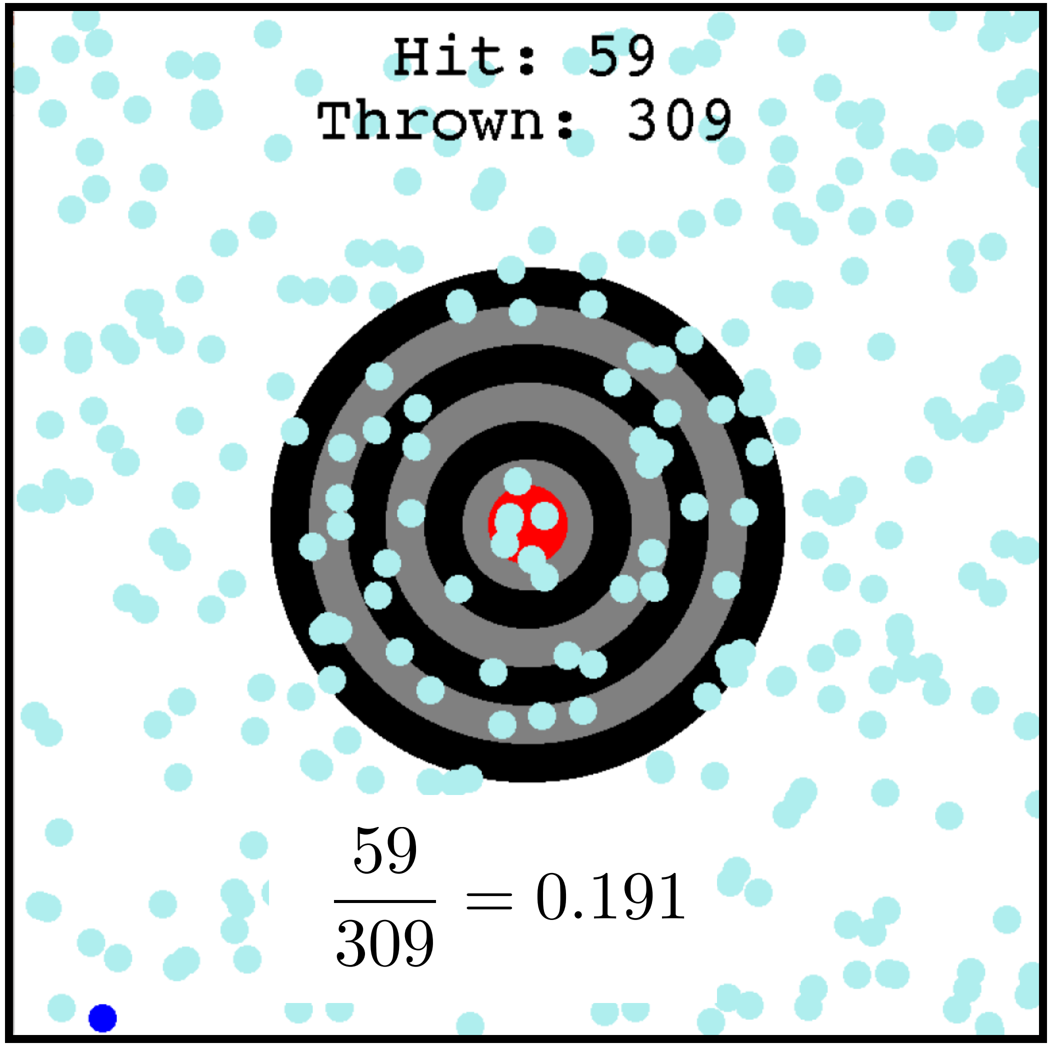
Challenge: how can you (efficiently) calculate the exact probability of winning?

Chip Defect Detection

- n chips manufactured, 1 of which is defective.
- k chips randomly selected from n for testing.
  - What is P(defective chip is in k selected chips)?
- $|S| = \binom{n}{k}$
- $|E| = \binom{1}{1}\binom{n-1}{k-1}$
- P(defective chip is in k selected chips)

$$= \frac{\binom{1}{1}\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{\frac{(n-1)!}{(k-1)!(n-k)!}}{\frac{n!}{k!(n-k)!}} = \frac{k}{n}$$

Target Revisited



Screen size = 800x800
Radius of target = 200

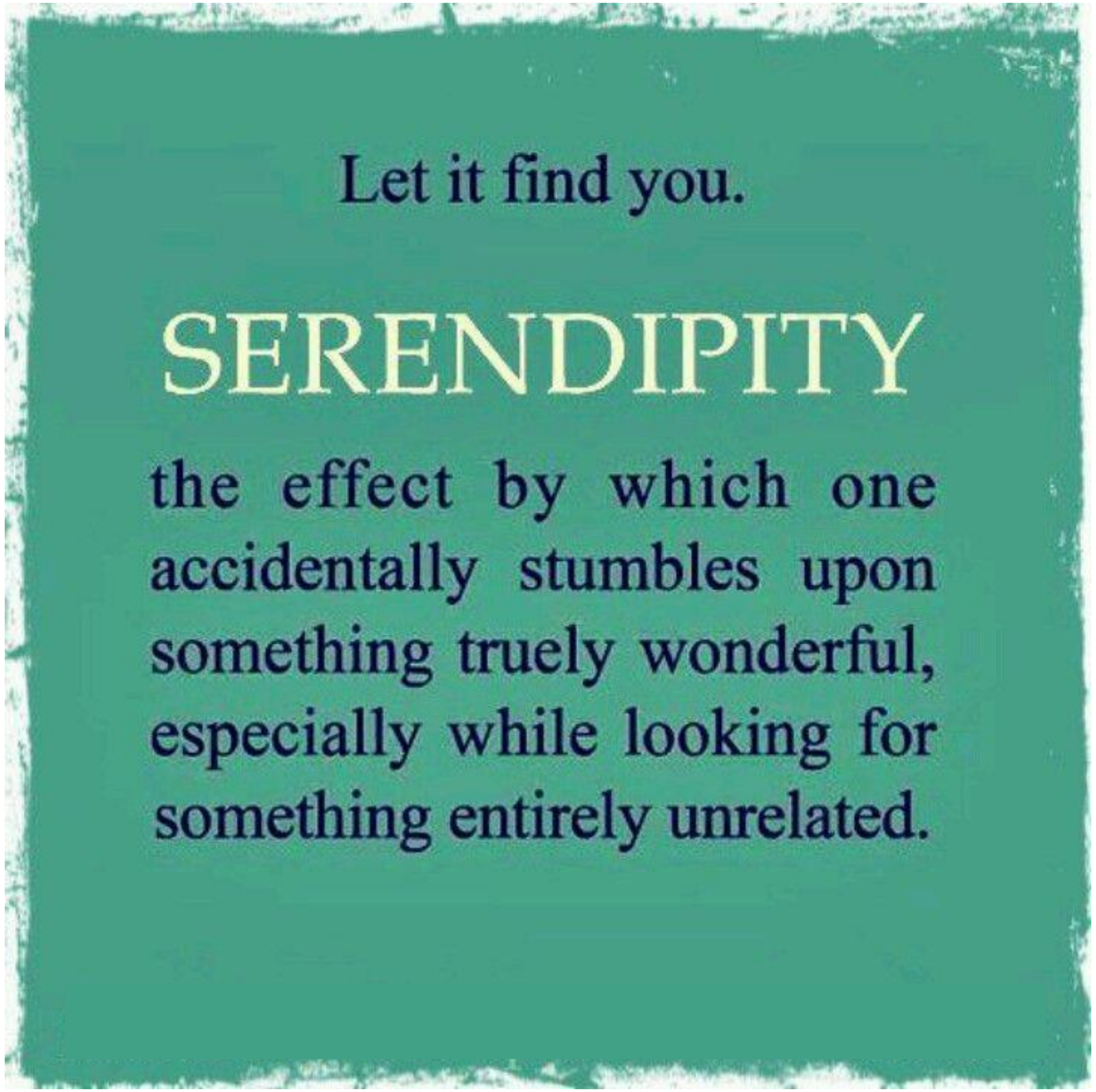
The dart is equally likely to land anywhere on the screen.

What is the probability of hitting the target?

$$|S| = 800^2$$

$$|E| = \pi 200^2$$

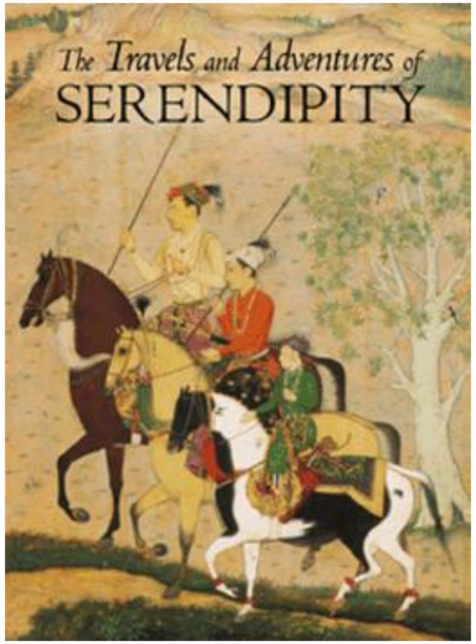
$$p(E) = \frac{\pi \cdot 200^2}{800^2} \approx 0.1963$$


Let it find you.
SERENDIPITY
the effect by which one accidentally stumbles upon something truely wonderful, especially while looking for something entirely unrelated.


WHEN YOU MEET YOUR BEST FRIEND
Somewhere you didn't expect to.

Serendipity

- Say the population of Stanford is 17,000 people
  - You are friends with ?
  - Walk into a room, see 450 random people.
  - What is the probability that you see someone you know?
  - Assume you are equally likely to see each person at Stanford



Many times it is easier to calculate $P(E^C)$.

Back to Axiom 3

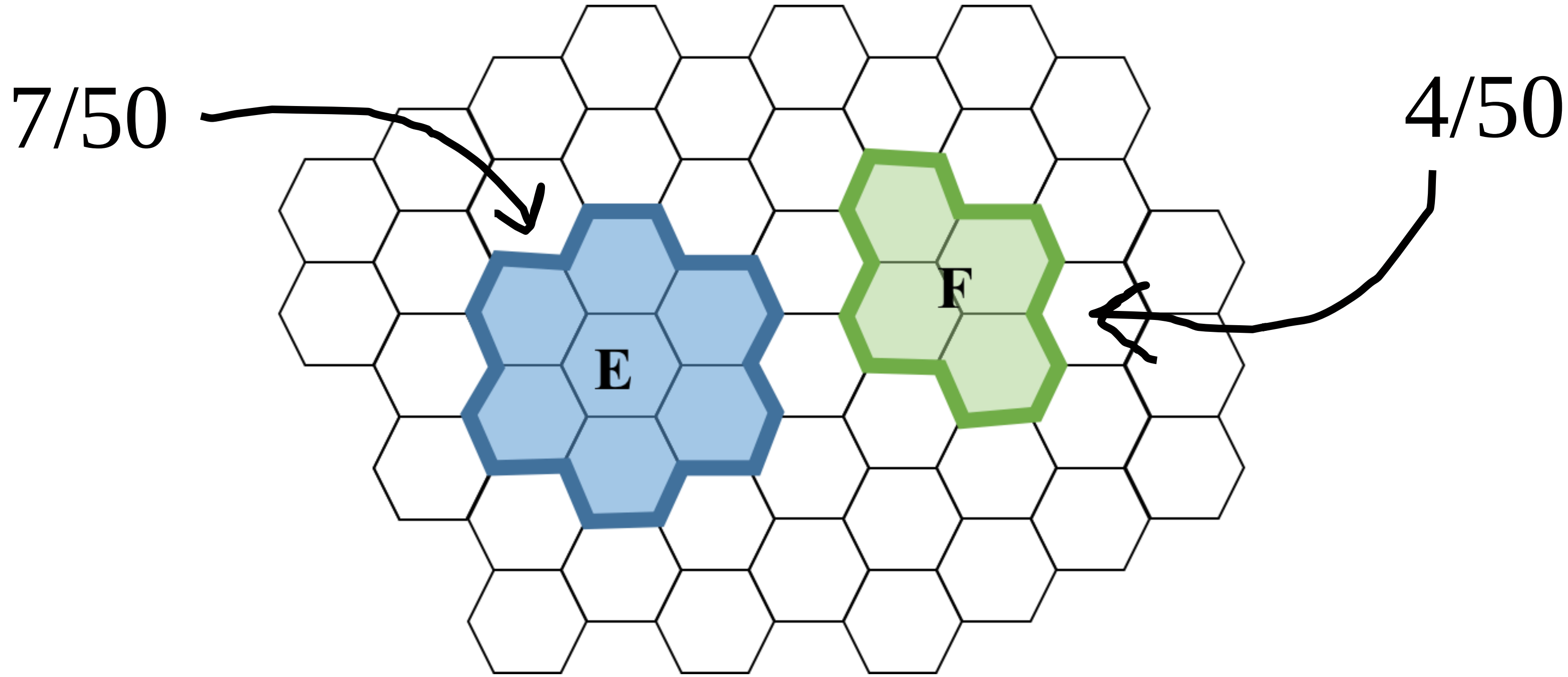
Axioms of Probability

Recall: S = all possible outcomes. E = the event.

- Axiom 1: $0 \leq P(E) \leq 1$
- Axiom 2: $P(S) = 1$
- Axiom 3: If events E and F are mutually exclusive:

$$P(E \cup F) = P(E) + P(F)$$

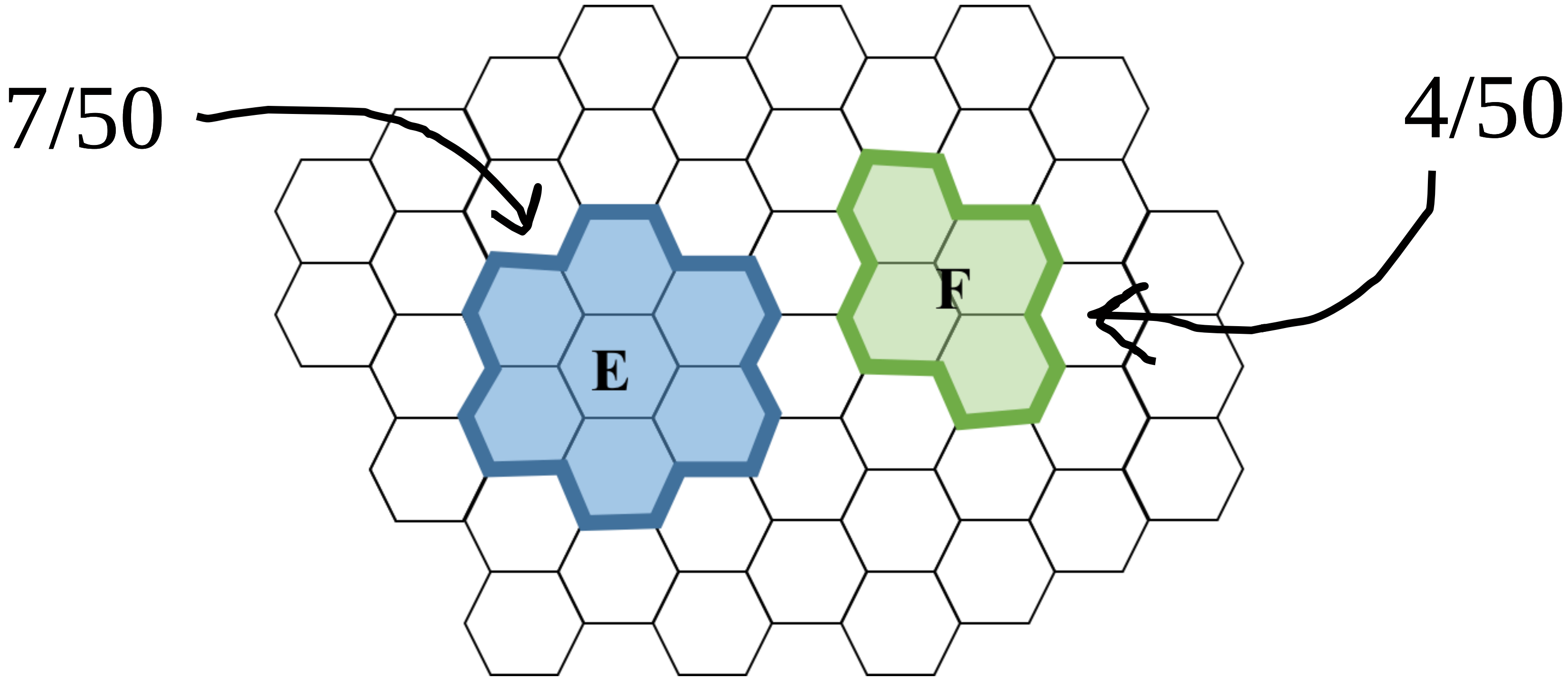
Mutually Exclusive Events



If events are mutually exclusive, probability of OR is simple:

$$P(E \cup F) = P(E) + P(F)$$

Mutually Exclusive Events

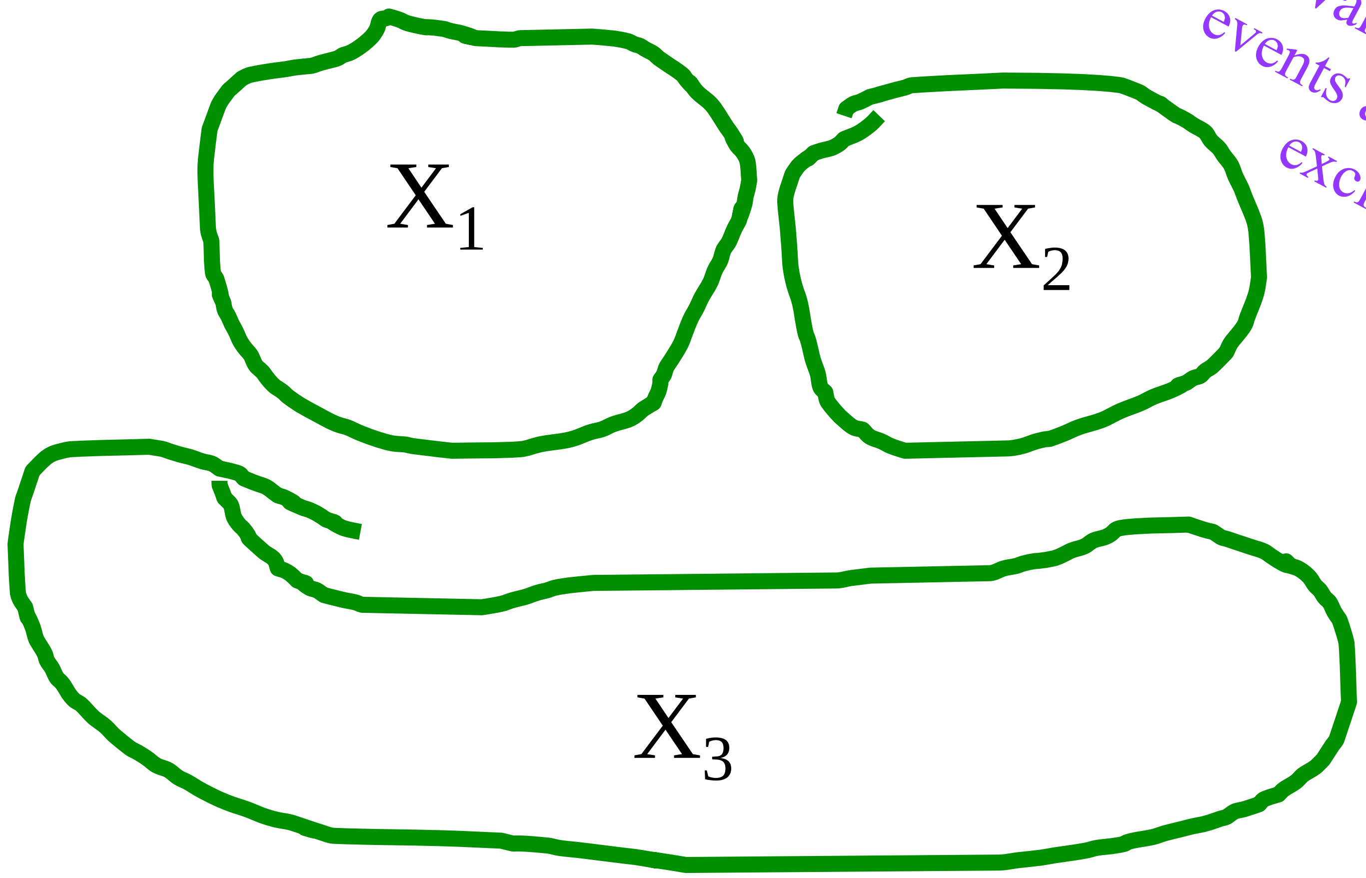


If events are mutually exclusive, probability of OR is simple:

$$P(E \cup F) = \frac{7}{50} + \frac{4}{5} = \frac{11}{50}$$

Probability of "or"

Wahoo! All my events are mutually exclusive



$$P(X_1 \cup X_2 \cup \cdots \cup X_n) = \sum_{i=1}^{n} P(X_i)$$

If events are *mutually exclusive* probability of OR is easy!

$$P(E^c) = 1 - P(E)?$$

$$P(E \text{ or } E^c) = P(E) + P(E^c)$$

Axiom 3. Since E and E^c are mutually exclusive

$$P(S) = P(E) + P(E^c)$$

Since everything must either be in E or E^c

$$1 = P(E) + P(E^c)$$

Axiom 2

$$P(E^c) = 1 - P(E)$$

Rearrange

Trailing the dovetail shuffle to it’s lair – Persi Diaconosis

Trailing the Dovetail Shuffle to Its Lair

The Annals of Applied Probability
1992, Vol. 2, No. 2, 294–313

TRAILING THE DOVETAIL SHUFFLE TO ITS LAIR

By Dave Bayer[1] and Persi Diaconis[2]

Columbia University and Harvard University

We analyze the most commonly used method for shuffling cards. The main result is a simple expression for the chance of any arrangement after any number of shuffles. This is used to give sharp bounds on the approach to randomness: $\frac{3}{2}\log_2 n + \theta$ shuffles are necessary and sufficient to mix up n cards.

Key ingredients are the analysis of a card trick and the determination of the idempotents of a natural commutative subalgebra in the symmetric group algebra.

1. Introduction. The dovetail, or riffle shuffle is the most commonly used method of shuffling cards. Roughly, a deck of cards is cut about in half and then the two halves are riffled together. Figure 1 gives an example of a riffle shuffle for a deck of 13 cards.

A mathematically precise model of shuffling was introduced by Gilbert and Shannon [see Gilbert (1955)] and independently by Reeds (1981). A deck of n cards is cut into two portions according to a binomial distribution; thus, the chance that k cards are cut off is $\binom{n}{k}/2^n$ for $0 \le k \le n$. The two packets are then riffled together in such a way that cards drop from the left or right heaps with probability proportional to the number of cards in each heap. Thus, if there are A and B cards remaining in the left and right heaps, then the chance that the next card will drop from the left heap is $A/(A+B)$. Such shuffles are easily described backwards: Each card has an equal and independent chance of being pulled back into the left or right heap. An inverse riffle shuffle is illustrated in Figure 2.

Experiments reported in Diaconis (1988) show that the Gilbert–Shannon–Reeds (GSR) model is a good description of the way real people shuffle real cards. It is natural to ask how many times a deck must be shuffled to mix it up. In Section 3 we prove:

Theorem 1. *If n cards are shuffled m times, then the chance that the deck is in arrangement π is $\binom{2^m+n-r}{n}/2^{mn}$, where r is the number of rising sequences in π.*

Rising sequences are defined and illustrated in Section 2 through the analysis of a card trick. Section 3 develops several equivalent interpretations of

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AMS 1980 *subject classifications*. 20B30, 60B15, 60C05, 60F99.

Key words and phrases. Card shuffling, symmetric group algebra, total variation distance.

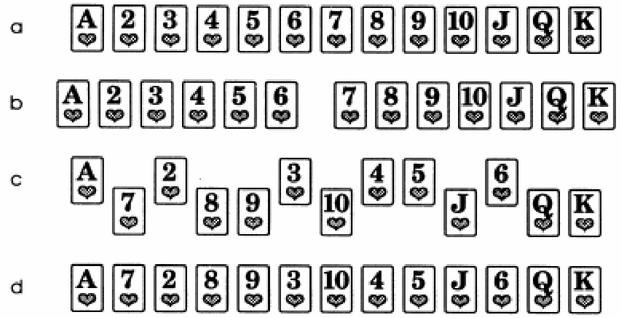

Fig. 1. *A riffle shuffle. (a) We begin with an ordered deck. (b) The deck is divided into two packets of similar size. (c) The two packets are riffled together. (d) The two packets can still be identified in the shuffled deck as two distinct "rising sequences" of face values.*

the GSR distribution for riffle shuffles, including a geometric description as the motion of n points dropped at random into the unit interval under the baker's transformation $x \to 2x \pmod 1$. This leads to a proof of Theorem 1.

Section 3 also relates shuffling to some developments in algebra. A permutation π has a descent at i if $\pi(i) > \pi(i+1)$. A permutation π has r rising sequences if and only if π^{-1} has $r-1$ descents. Let

$$A_k = \sum_{\pi \text{ has } k \text{ descents}} \pi$$

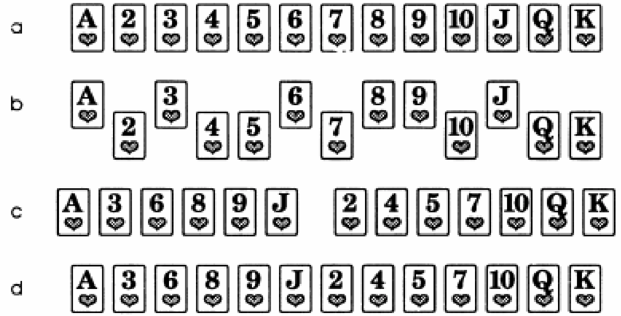

Fig. 2. *An inverse riffle shuffle. (a) We begin with a sorted deck. (b) Each card is moved one way or the other uniformly at random, to "pull apart" a riffle shuffle and retrieve two packets. (c) The two packets are placed in sequence. (d) The two packets can still be identified in the shuffled deck; they are separated by a "descent" in the face values. This shuffle is inverse to the shuffle diagrammed in Figure 1.*

Trailing the Dovetail Shuffle to Its Lair (Simple)

- What is the probability that a **single** shuffle is different from yours?
  - $|S| = 52!$
  - $|E| = 52! - 1$
  - $P(\text{different}) = (52!-1)/52!$

$P(\text{different}) > 0.9999999999\ldots$

$P(\text{same}) < 0.0000000\ldots$

About 67 “9”s

Trailing the Dovetail Shuffle to Its Lair (Medium)

- What is the probability that 2 shuffles are different from yours?
  - $|S| = (52!)^2$
  - $|E| = (52! - 1)^2$
  - P(no deck matching yours) = $(52!-1)^2/(52!)^2$

Trailing the Dovetail Shuffle to Its Lair (Full)

- What is the probability that in the *n* shuffles seen since the start of time, yours is unique?
  - $|S| = (52!)^n$
  - $|E| = (52! - 1)^n$
  - P(no deck matching yours) = $(52!-1)^n/(52!)^n$

- For $n = 10^{20}$,
  - P(deck matching yours) < 0.000000001

* Assume 7 billion people have been shuffling cards once a second since 52 deck cards were invented (see Topkapı deck)

Trailing the Dovetail Shuffle to Its Lair (Full)

```python
from decimal import *
import math

n = math.pow(10, 20)
card_perms = math.factorial(52)
denominator = card_perms
numerator = card_perms - 1

# decimal library because these are tiny numbers
getcontext().prec = 100 # increase precision
log_numer = Decimal(numerator).ln()
log_denom = Decimal(denominator).ln()
log_pr = log_numer - log_denom

# approximately -1.24E-68
print(log_pr)
```

$$\log\left(\frac{52!-1}{52!}\right)$$