Conditional Probability and Bayes

Announcements

- Sections start Wednesday. Times coming soon!

Review

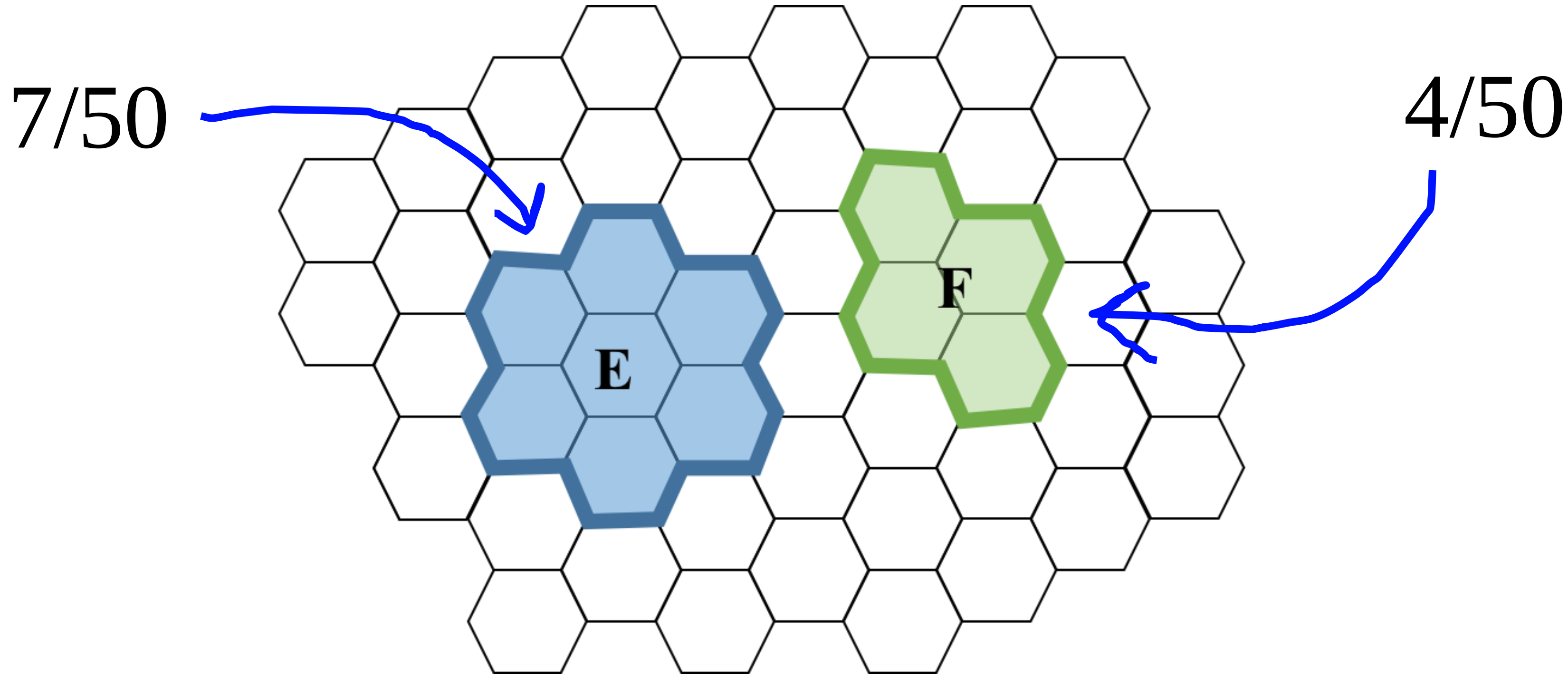
Review, Axioms of Probability

Recall: S = all possible outcomes. E = the event.

- Axiom 1: $0 \leq P(E) \leq 1$
- Axiom 2: $P(S) = 1$
- Axiom 3: If events E and F are mutually exclusive:

$$P(E \cup F) = P(E) + P(F)$$

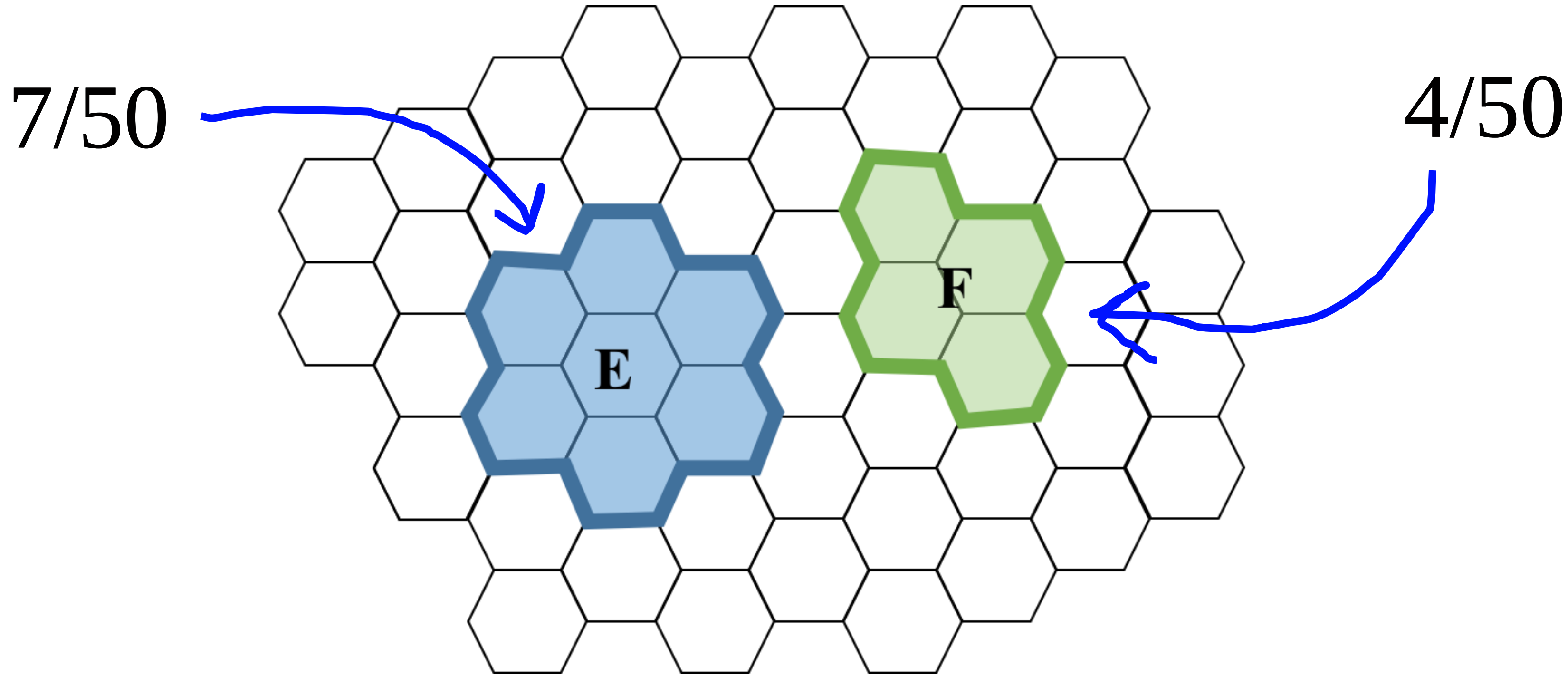
Mutually Exclusive Events



If events are mutually exclusive, probability of OR is simple:

$$P(E \cup F) = P(E) + P(F)$$

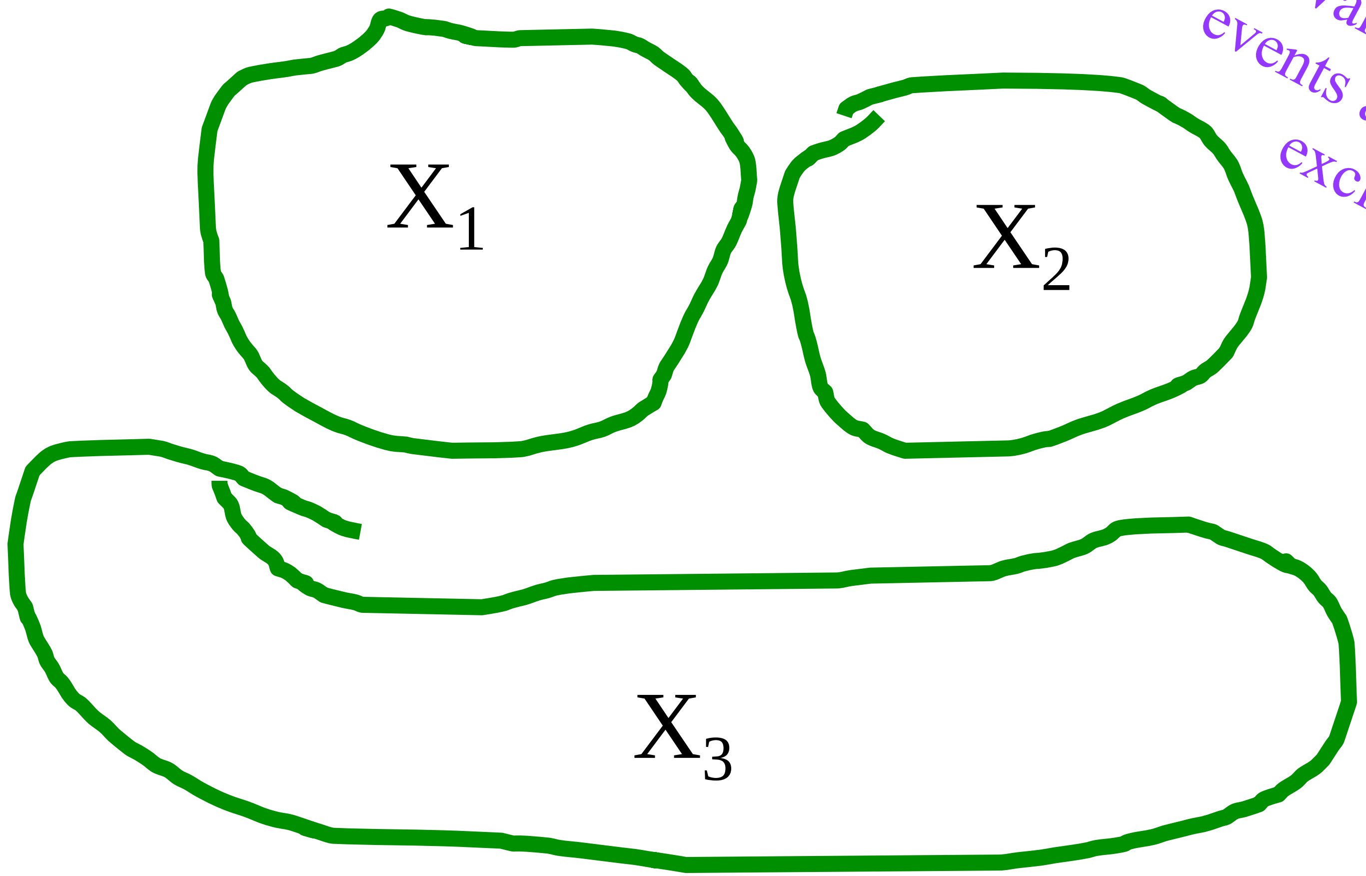
Mutually Exclusive Events



If events are mutually exclusive, probability of OR is simple:

$$P(E \cup F) = \frac{7}{50} + \frac{4}{5} = \frac{11}{50}$$

Mutually Exclusive Events



Wahoo! All my events are mutually exclusive

$$P(X_1 \cup X_2 \cup \cdots \cup X_n) = \sum_{i=1}^{n} P(X_i)$$

Review, Mutually Exclusive Events

If events are *mutually exclusive* probability of OR is easy!

$P(E^c) = 1 - P(E)$?

$$P(E \cup E^c) = P(E) + P(E^c)$$

Axiom 3. Since E and E^c are mutually exclusive

$$P(S) = P(E) + P(E^c)$$

Since everything must either be in E or E^c

$$1 = P(E) + P(E^c)$$

Axiom 2

$$P(E^c) = 1 - P(E)$$

Rearrange

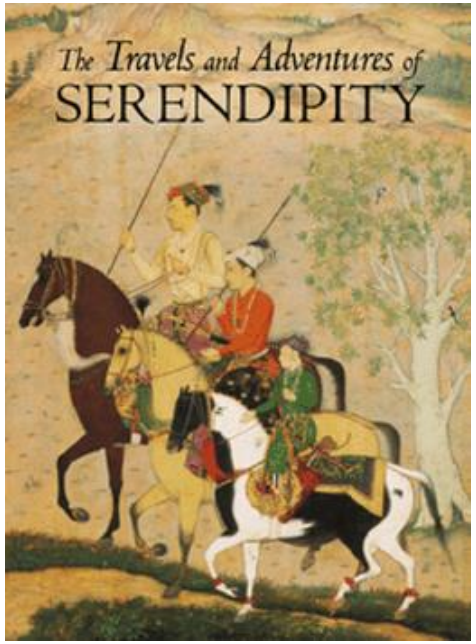
Let it find you.

SERENDIPITY

the effect by which one accidentally stumbles upon something truely wonderful, especially while looking for something entirely unrelated.

Serendipity Review

- Say the population of Stanford is ***p*** people
- You are friends with ***f*** people
- Walk into a room, see ***s*** random people.



Let ***E*** be the event that you see at least one friend

$$P(E^C) = \frac{\binom{p-f}{s}}{\binom{p}{s}}$$

$$P(E) = 1 - \frac{\binom{p-f}{s}}{\binom{p}{s}}$$

Many times it is easier to calculate $P(E^C)$.

End Review

Learning Goal for Today: Conditional Probability

$$P(E \text{ and } F)$$

Chain rule
(Product rule)

Definition of
conditional probability

$$P(E|F)$$

Law of Total
Probability

Bayes'
Theorem

$$P(E)$$

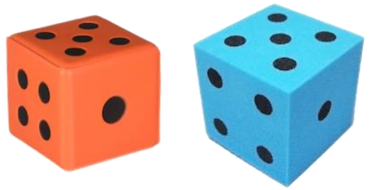
$$P(F|E)$$

Conditional Probability

Roll two dice

$P(E) = \frac{|E|}{|S|}$ Equally likely outcomes

Roll two 6-sided fair dice. What is P(sum = 7)?

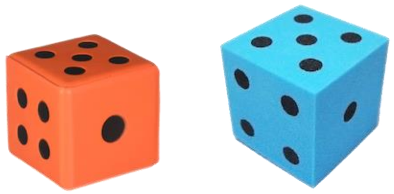


S = {(1,1) (1,2) (1,3) (1,4) (1,5) **(1,6)**
(2,1) (2,2) (2,3) (2,4) **(2,5)** (2,6)
(3,1) (3,2) (3,3) **(3,4)** (3,5) (3,6)
(4,1) (4,2) **(4,3)** (4,4) (4,5) (4,6)
(5,1) **(5,2)** (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

$E =$ *In blue*

Dice, our misunderstood friends

Roll two 6-sided dice, yielding values D_1 and D_2.
You want them to sum to 4.



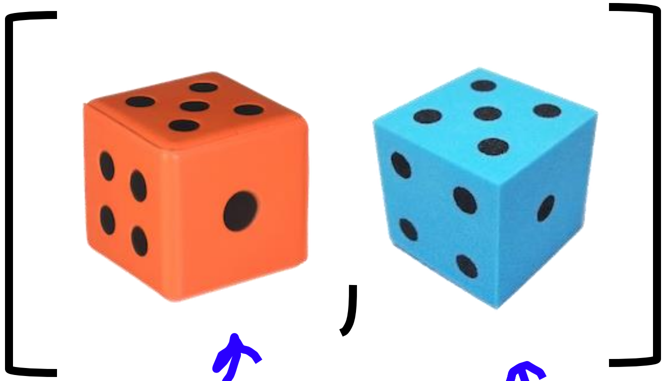
What is the best outcome for $P(D_1)$?

Your Choices:

- A. 1 and 3 tie for best
- B. 1, 2 and 3 tie for best
- C. 2 is the best
- D. Other/none/more than one

Sum of Two Die = 4?

Each outcome



Value
dice 1

Value
dice 2

Roll two 6-sidex dice. What is probability the sum = 4?
Let E be the event that the sum is 4

S = {	[1,1]	[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
	[2,1]	[2,2]	[2,3]	[2,4]	[2,5]	[2,6]
	[3,1]	[3,2]	[3,3]	[3,4]	[3,5]	[3,6]
	[4,1]	[4,2]	[4,3]	[4,4]	[4,5]	[4,6]
	[5,1]	[5,2]	[5,3]	[5,4]	[5,5]	[5,6]
	[6,1]	[6,2]	[6,3]	[6,4]	[6,5]	[6,6] }

E =

Dice, our misunderstood friends

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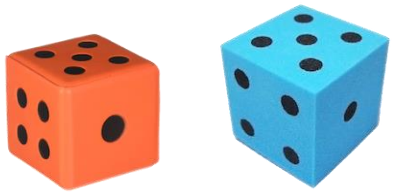


Let E be event: $D_1 + D_2 = 4$.

What is $P(E)$?

$|S| = 36$

$E = \{(1,3), (2,2), (3,1)\}$

$P(E) = 3/36 = 1/12$

Let F be event: $D_1 = 2$.

What is $P(E$, given F already observed$)$?

$S = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)\}$

$E = \{(2,2)\}$

$P(E) = 1/6$

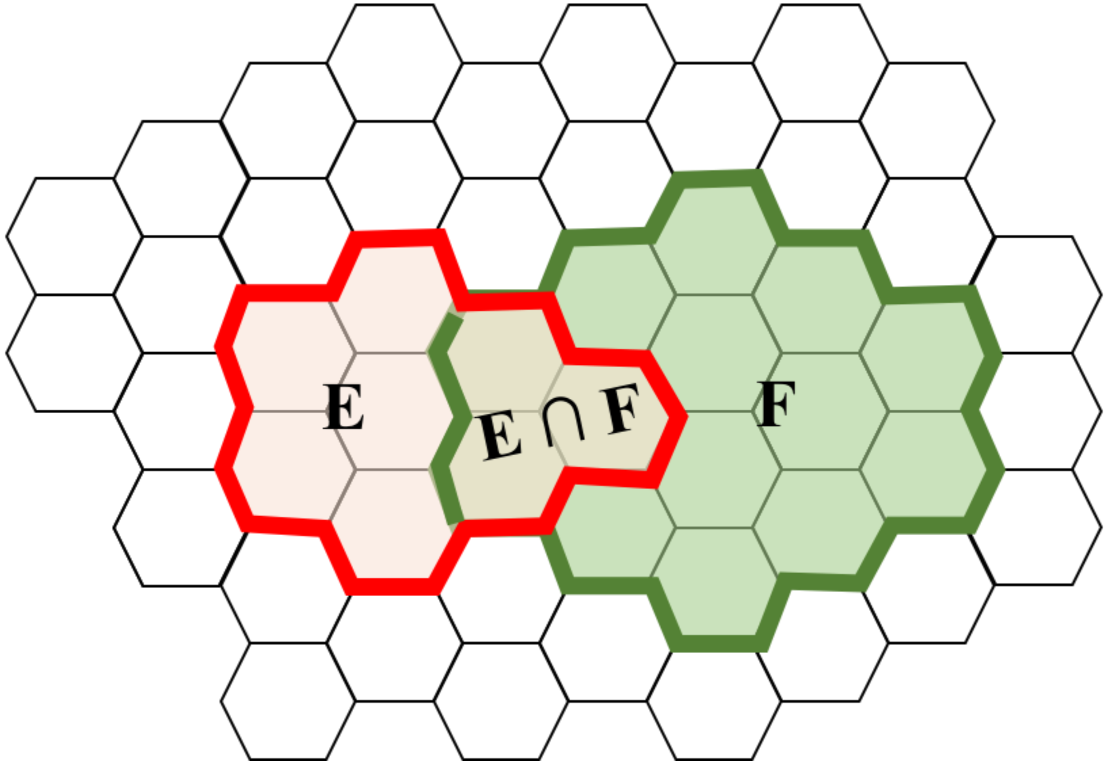
Conditional Probability

The conditional probability of E given F is the probability that E occurs given that F has already occurred. This is known as conditioning on F.

Written as:	$P(E\|F)$
Means:	"$P(E,$ given F already observed)"
Sample space →	all possible outcomes consistent with F (i.e. $S \cap F$)
Event →	all outcomes in E consistent with F (i.e. $E \cap F$)

Conditional Probability, visual intuition

The conditional probability of E given F is the probability that E occurs given that F has already occurred. This is known as conditioning on F.



$$P(E) = \frac{8}{50} \approx 0.16$$

$$P(E|F) =$$

Conditional Probability, visual intuition

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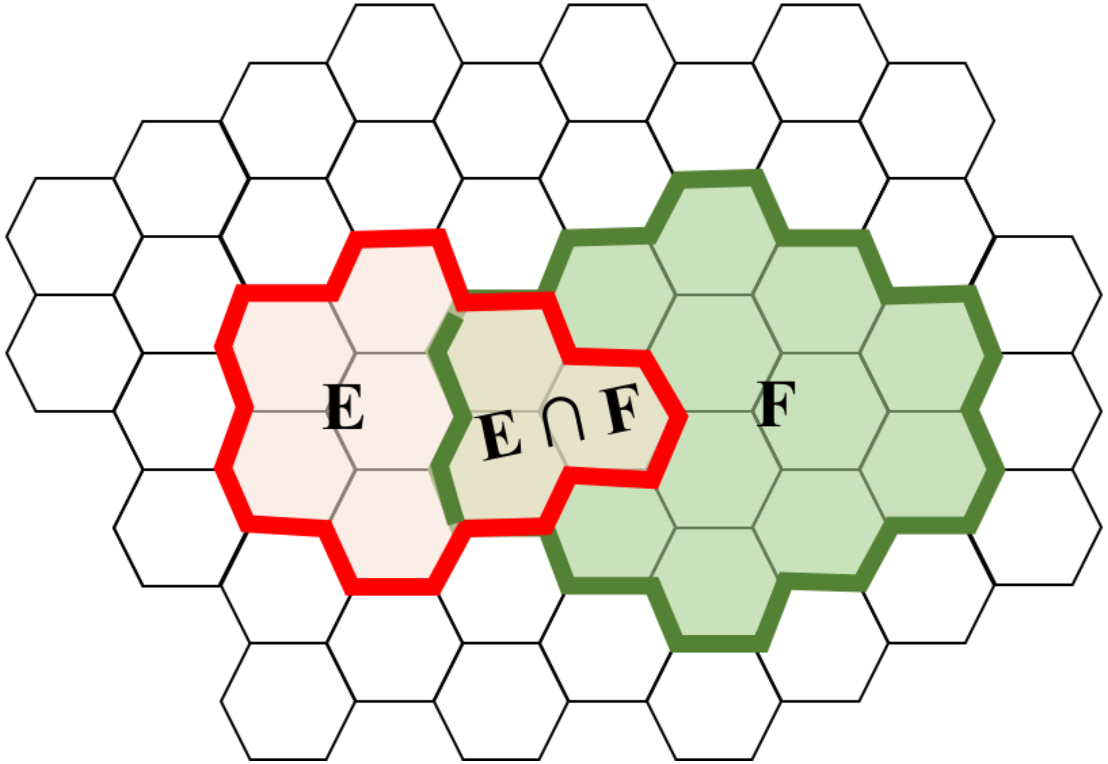

$$P(E) = \frac{8}{50} \approx 0.16$$

$$P(E|F) = \frac{3}{14} \approx 0.21$$

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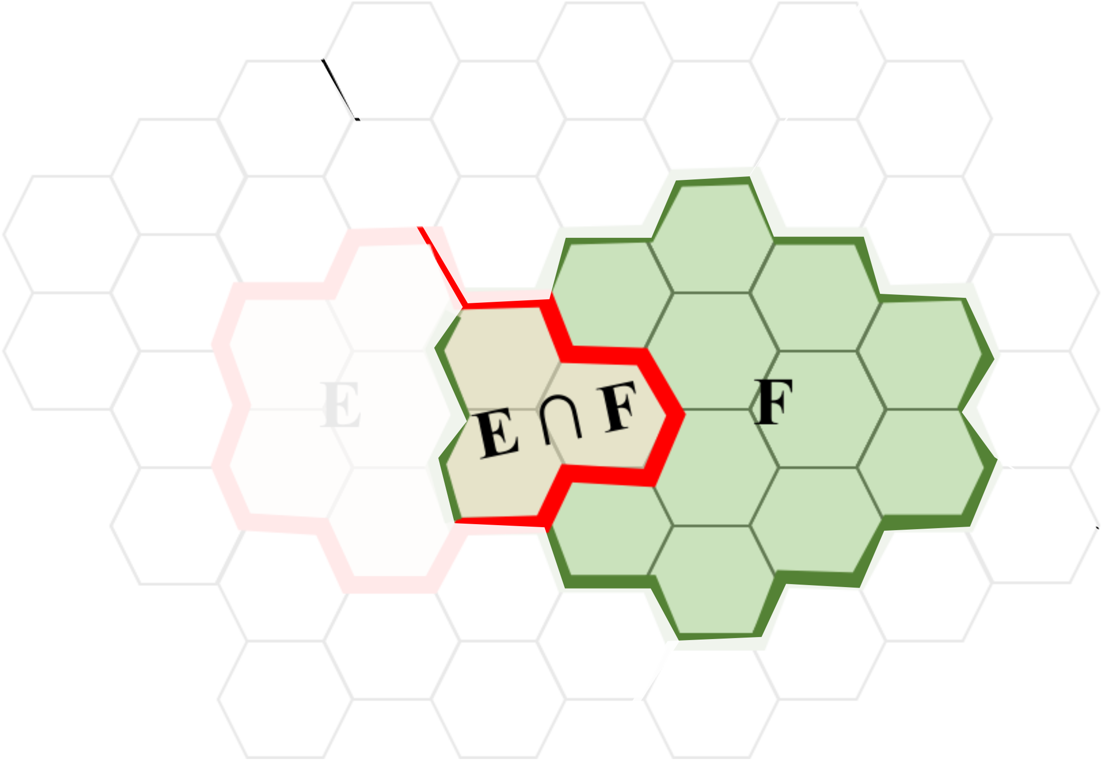

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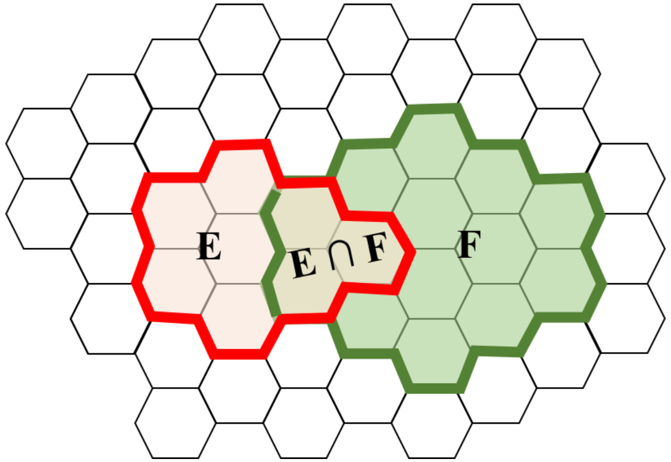
Conditional Probability, visual intuition

The conditional probability of E given F is the probability that E occurs given that F has already occurred. This is known as conditioning on F.

Shorthand notation for set intersection (aka set "and")

$$\Pr(E|F) = \frac{\text{\# of outcomes in } E \text{ consistent with } F}{\text{\# of outcomes in } S \text{ consistent with } F} = \frac{|EF|}{|SF|} = \frac{|EF|}{|F|}$$



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Conditional Probability, visual intuition

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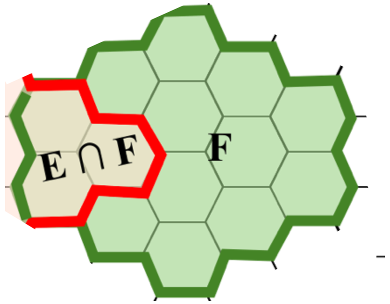

$$P(E) = \frac{8}{50} \approx 0.16$$

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Conditional probability in general

These properties hold even when outcomes are not equally likely.

General definition of conditional probability:

$$P(E|F) = \frac{P(EF)}{P(F)}$$

Shorthand for E and F

The Chain Rule (aka Product rule):

$$P(EF) = P(F)P(E|F)$$

What if P(F) = 0?

- P(E | F) undefined
- *Congratulations! Observed impossible*

Bye, land of equally likely outcomes

NETFLIX

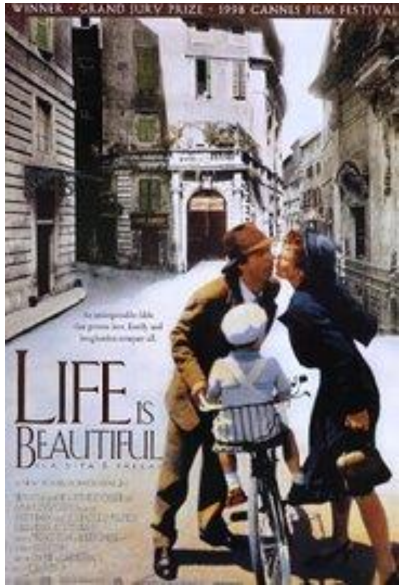
and Learn

Netflix and Learn

$$P(E|F) = \frac{P(EF)}{P(F)}$$ Definition of Cond. Probability

What is the probability that a user will watch Life is Beautiful?

$$P(E)$$



S = {Watch, Not Watch}

E = {Watch}

P(E) = ½ ?

Netflix and Learn

What is the probability that a user will watch Life is Beautiful?

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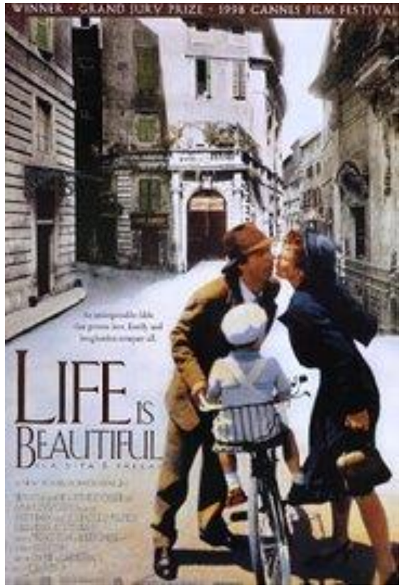

Netflix and Learn

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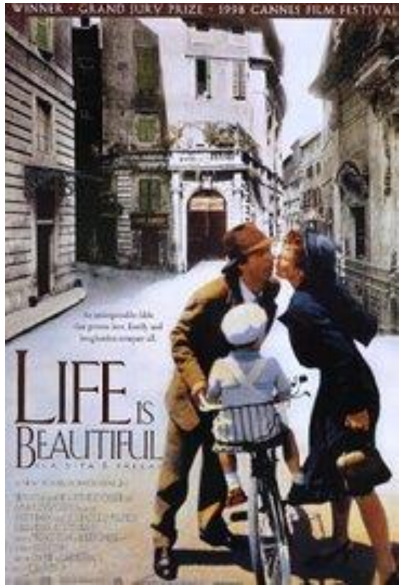


$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n} \approx \frac{\#\text{people who watched movie}}{\#\text{people on Netflix}}$$

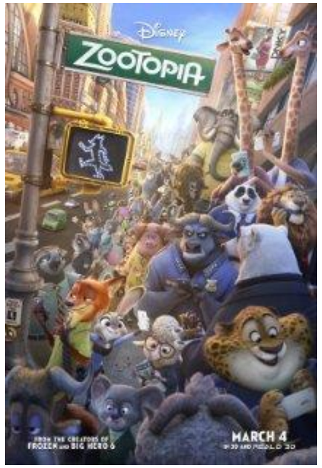
$P(E)$ = 10,234,231 / 50,923,123 = 0.20

Netflix and Learn

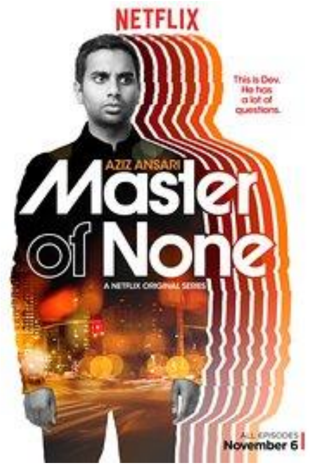
$$P(E|F) = \frac{P(EF)}{P(F)}$$ Definition of Cond. Probability

Let E be the event that a user watches the given movie.

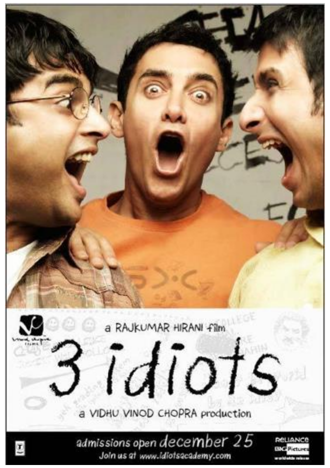
$P(E) = 0.19$



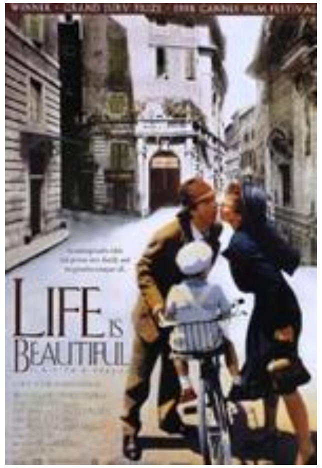
$P(E) = 0.32$



$P(E) = 0.20$



$P(E) = 0.09$



$P(E) = 0.20$

Netflix and Learn

$$P(E|F) = \frac{P(EF)}{P(F)}$$ Definition of Cond. Probability

Let E = a user watches Life is Beautiful.

Let F = a user watches CODA.

What is the probability that a user watches Life is Beautiful, given they watched CODA?

$$P(E|F)$$

$$P(E|F) = \frac{P(EF)}{P(F)} \approx \frac{\frac{\text{\# people who have watched both}}{\text{\# people on Netflix}}}{\frac{\text{\# people who have watched CODA}}{\text{\# people on Netflix}}}$$

$$\approx \frac{\text{\# people who have watched both}}{\text{\# people who have watched CODA}}$$

$$\approx 0.42$$

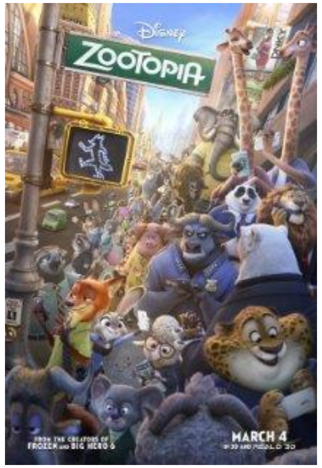
Netflix and Learn

$$P(E|F) = \frac{P(EF)}{P(F)}$$ Definition of Cond. Probability

Let E be the event that a user watches the given movie.
Let F be the event that the same user watches CODA (2021).

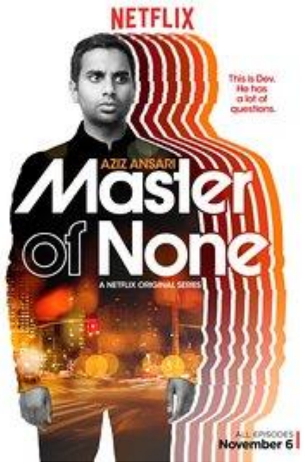
$P(E) = 0.19$

$P(E|F) = 0.14$



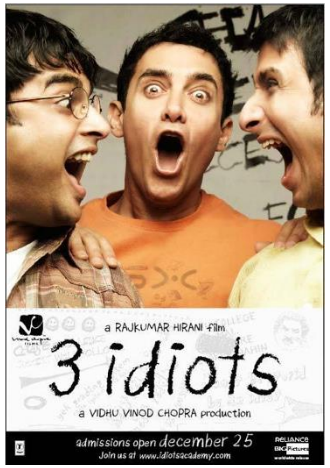
$P(E) = 0.32$

$P(E|F) = 0.35$



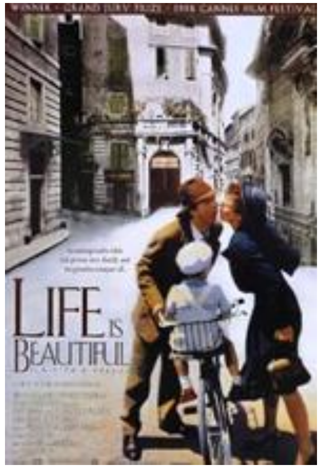
$P(E) = 0.20$

$P(E|F) = 0.20$



$P(E) = 0.09$

$P(E|F) = 0.72$



$P(E) = 0.20$

$P(E|F) = 0.42$

Machine Learning

Machine Learning is:
Probability + Data + Computers

Notation

And	Or	Given
$P(E \text{ and } F)$	$P(E \text{ or } F)$	$P(E\|F)$
$P(E, F)$	$P(E \cup F)$	$P(E\|F, G)$
$P(EF)$		
$P(E \cap F)$		

Probability of E given F and G

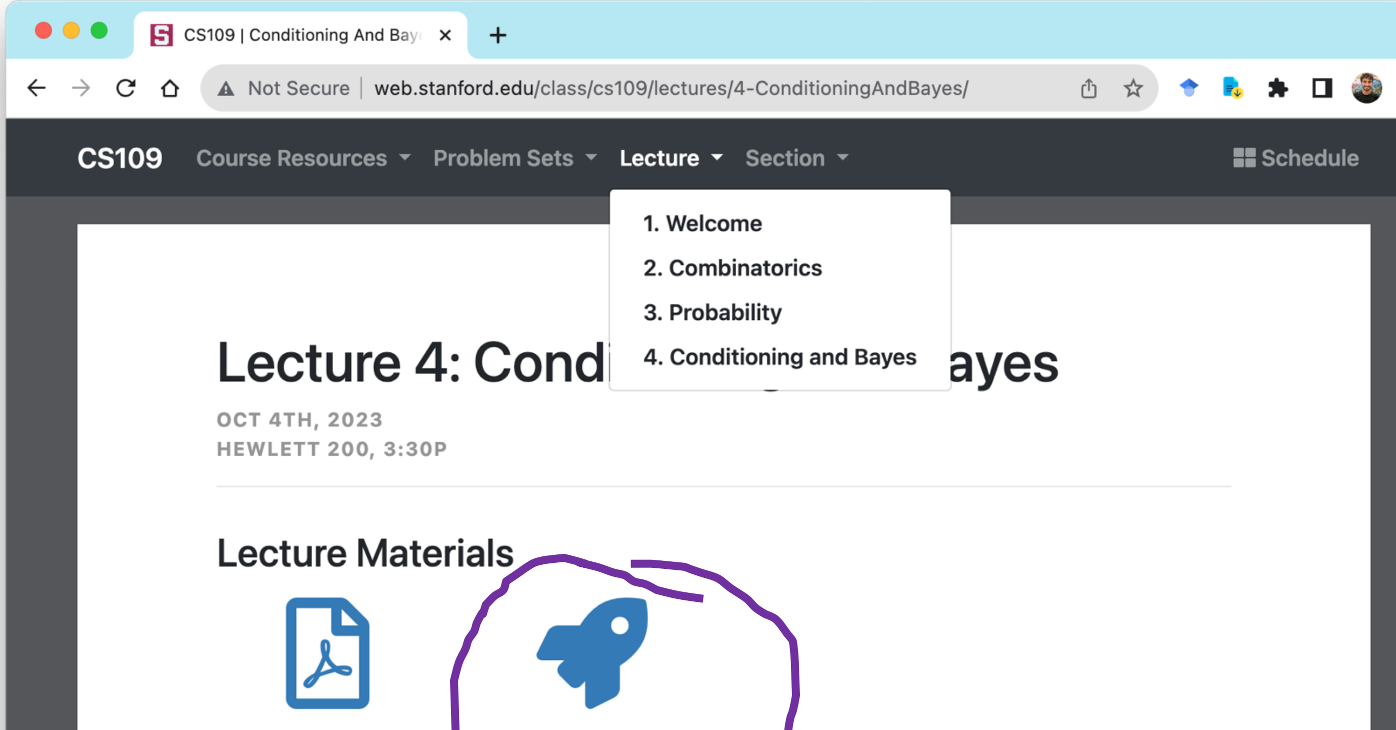
Chain Rule via Baby Poop

https://cs109psets.netlify.app/fall23/lecture4/poop

$$P(E|F) = \frac{P(EF)}{P(F)}$$

$$P(EF) = P(F)P(E|F)$$

In the morning when she wakes up, a baby has a 50% chance of having pooped. The chance that a baby cries **given** that she has pooped is 50%. What is the probability that a baby **has pooped, and cries**.



Generalized Chain Rule

$$\Pr(E_1 \text{ and } E_2 \text{ and } E_3 \text{ and } \ldots E_n)$$

$$= \Pr(E_1) \cdot \Pr(E_2|E_1) \cdot \Pr(E_3|E_1, E_2) \cdots \Pr(E_n|E_1, E_2 \ldots E_{n-1})$$

Law of Total Probability

Relationship Between Probabilities

$$P(E \text{ and } F)$$

Chain rule
(Product rule)

Definition of
conditional probability

$$P(E|F)$$

Law of Total
Probability

$$P(E)$$

Baby Poop Redux

In the morning when she wakes up, a baby has a 50% chance of having pooped.
The chance that a baby cries given that she has pooped is 50%.

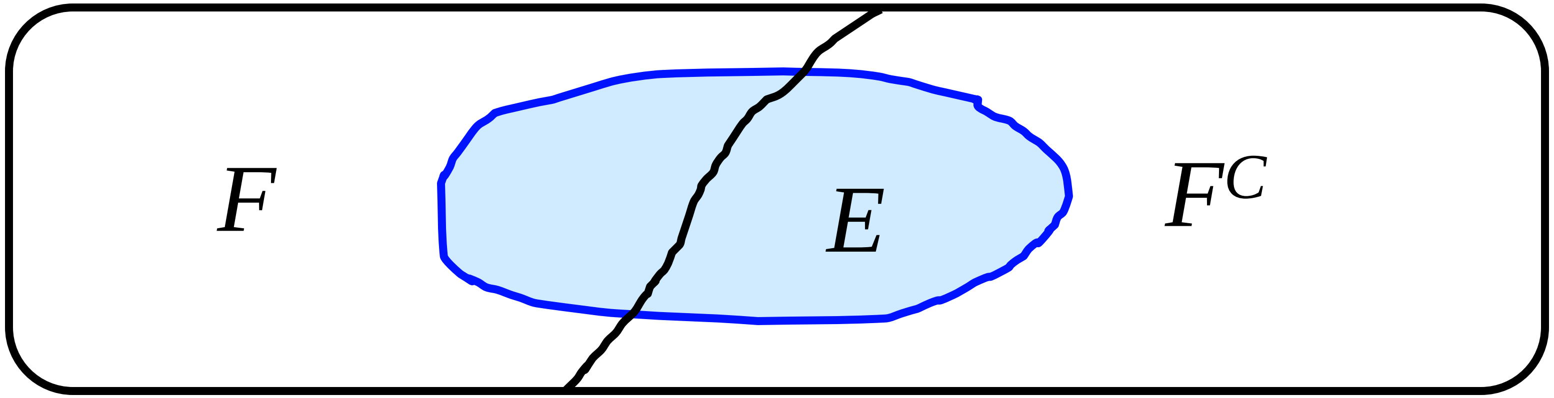
What is the probability of crying, unconditioned?

What information do you need?

Law of Total Probability

Say E and F are events in S

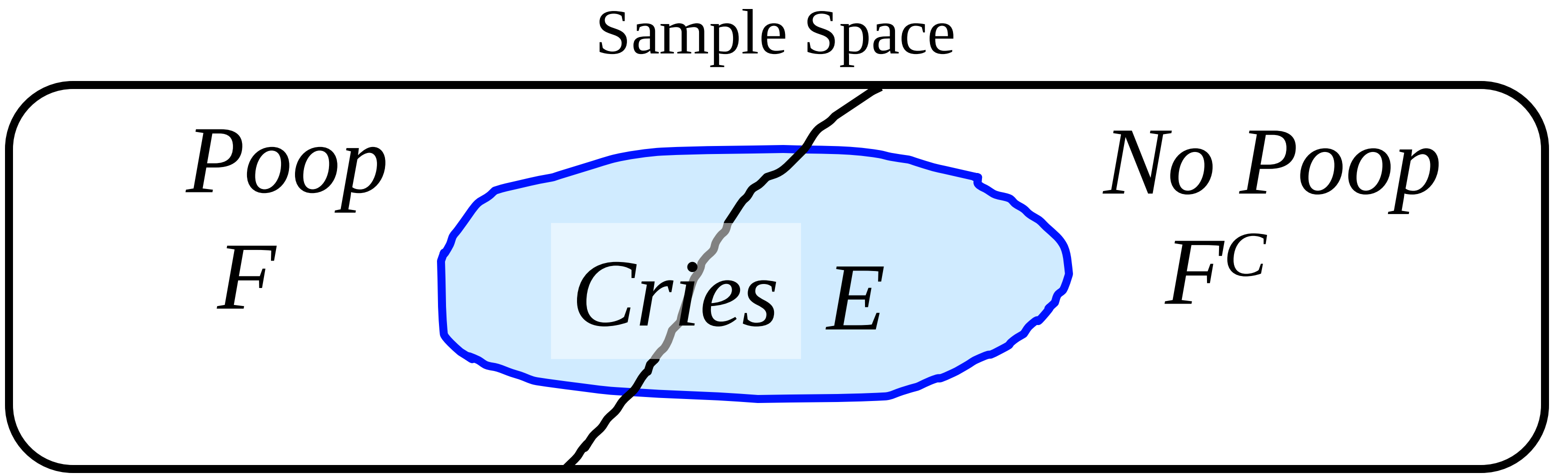
Sample Space



$$P(E) = P(EF) + P(EF^C)$$
$$= P(E|F)P(F) + P(E|F^C)P(F^C)$$

Law of Total Probability

Say E and F are events in S



$$P(E) = P(EF) + P(EF^C)$$
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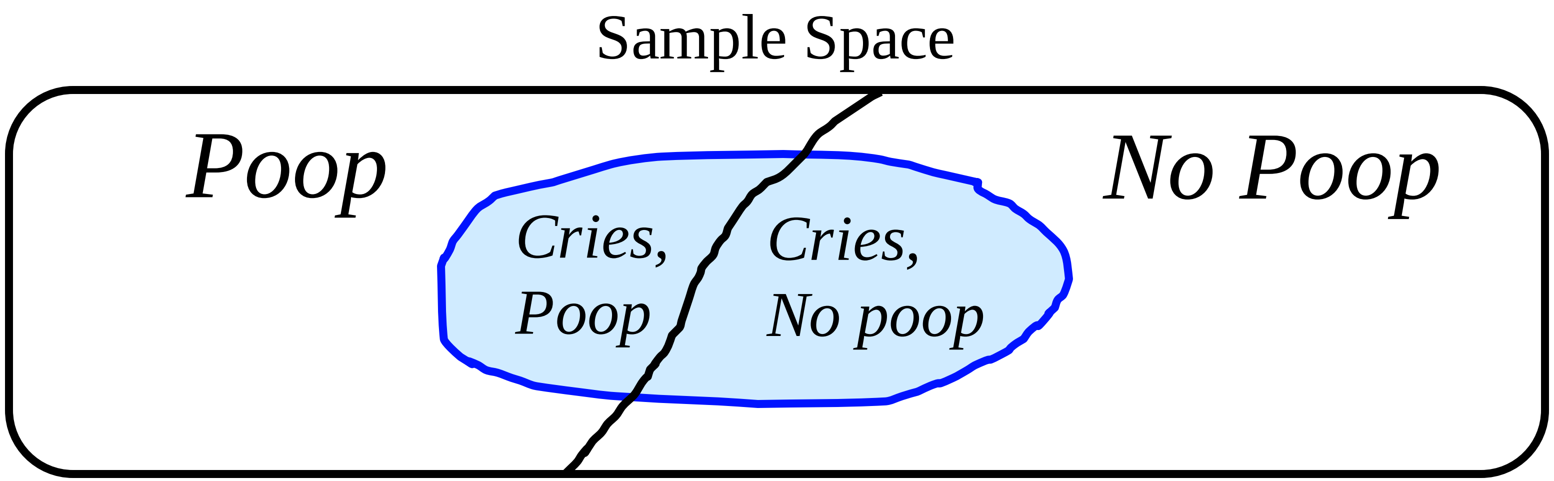

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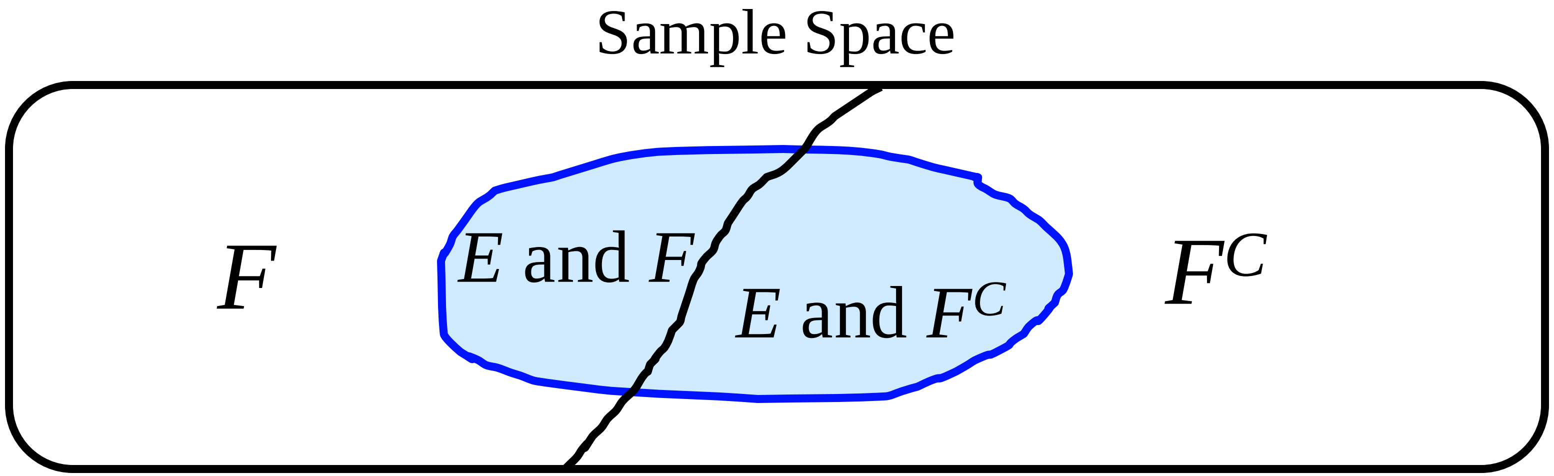
Law of Total Probability

Say E and F are events in S



$$
\begin{aligned}
P(E) &= P(EF) + P(EF^C) \\
&= P(E|F)P(F) + P(E|F^C)P(F^C)
\end{aligned}
$$

Law of Total Probability

Thm Let F be an event where $P(F) > 0$. For any event E,

$$P(E) = P(E|F)P(F) + P(E|F^C)P(F^C)$$

Proof

1. $E = (EF)$ or (EF^C) — Since F and F^C are disjoint
2. $P(E) = P(EF) + P(EF^C)$ — Probability of **or** for disjoint
3. $P(E) = P(E|F)P(F) + P(E|F^C)P(F^C)$ — Chain rule (product rule)

Baby Poop

In the morning when she wakes up, a baby has a 50% chance of having pooped.
The chance that a baby cries given that she has pooped is 50%.

Probability of crying (T)?

What information do you need?

Probability of crying given no poop.

Recall that T is crying and E is poop

$$P(T) = P(T|E)P(E) + P(T|E^C)P(E^C)$$

Evolution of Bacteria

$P(E) = P(E|F)P(F) + P(E|F^C)P(F^C)$ Law of Total Probability

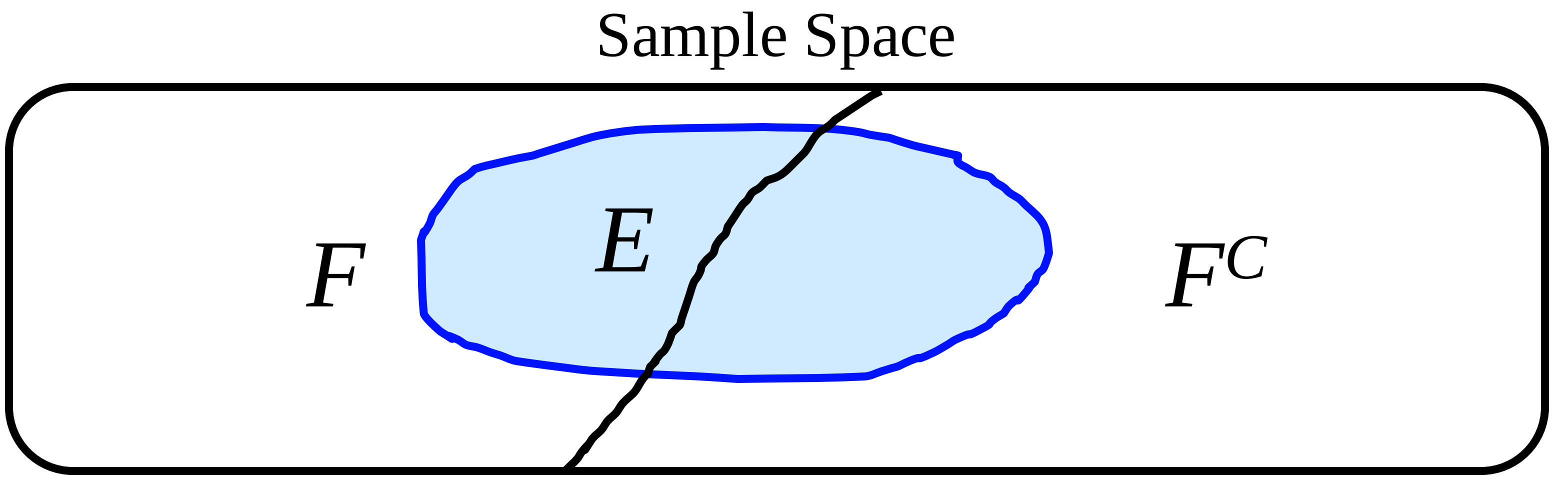
You have bacteria in your gut which is causing a disease.
10% have a mutation which makes them resistant to anti-biotics
You take half a course of anti-biotics...

Probability a bacteria survives given it has the mutation: 20%
Probability a bacteria survives given it doesn't have the mutation: 1%
What is the probability that a randomly chosen bacteria survives?

Let E be the event that a bacterium survives. Let M be the event that a bacteria has the mutation. By the By Law of Total Probability (LOTP):

$$\begin{aligned}
\Pr(E) &= \Pr(E \text{ and } M) + \Pr(E \text{ and } M^C) && \text{LOTP} \\
&= \Pr(E|M)\Pr(M) + \Pr(E|M^C)\Pr(M^C) && \text{Chain Rule} \\
&= 0.20 \cdot 0.10 + 0.01 \cdot 0.90 && \text{Substituting} \\
&= 0.029
\end{aligned}$$

Law of Total Probability



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P(E) &= P(EF) + P(EF^C) \\
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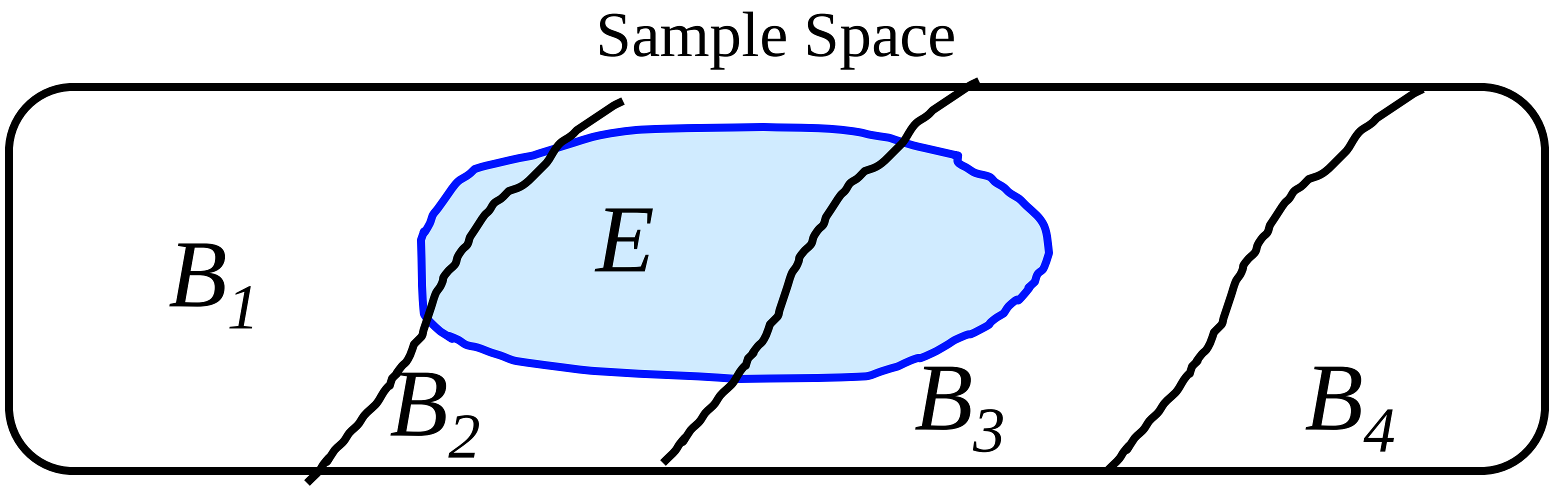
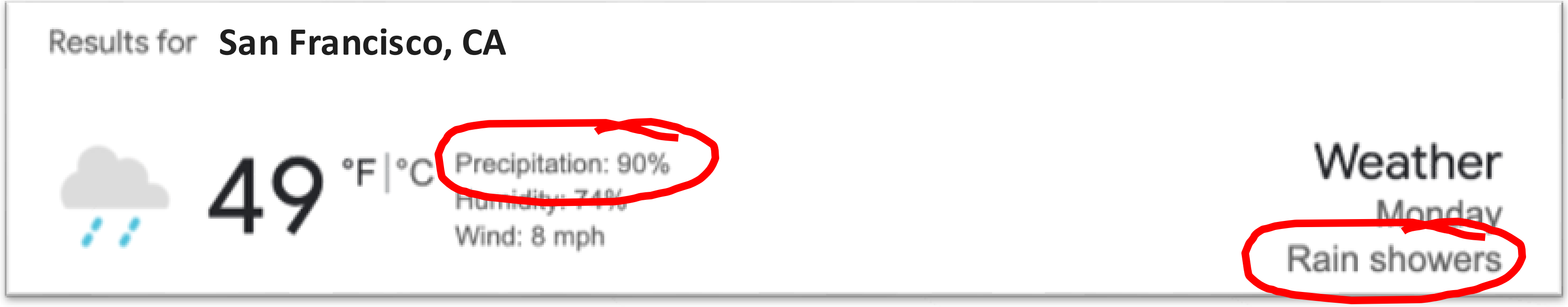
Law of Total Probability



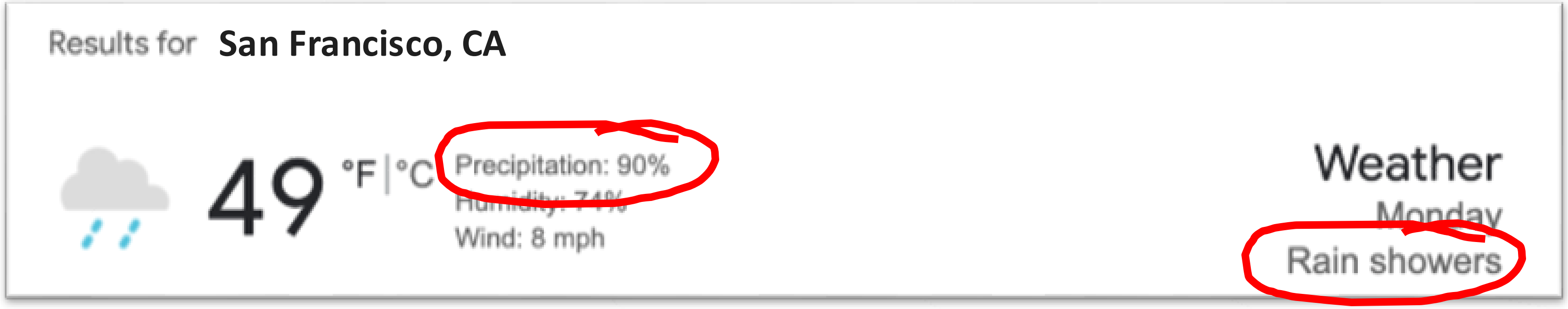
<u>Thm</u> For mutually exclusive events $B_1, B_2, \ldots, B_n$ s.t. $B_1 \cup B_2 \cup \cdots \cup B_n = S$,

$$P(E) = \sum_i P(B_i \cap E)$$

$$= \sum_i P(E|B_i)P(B_i)$$



Background event.
Where is the person in San Francisco?



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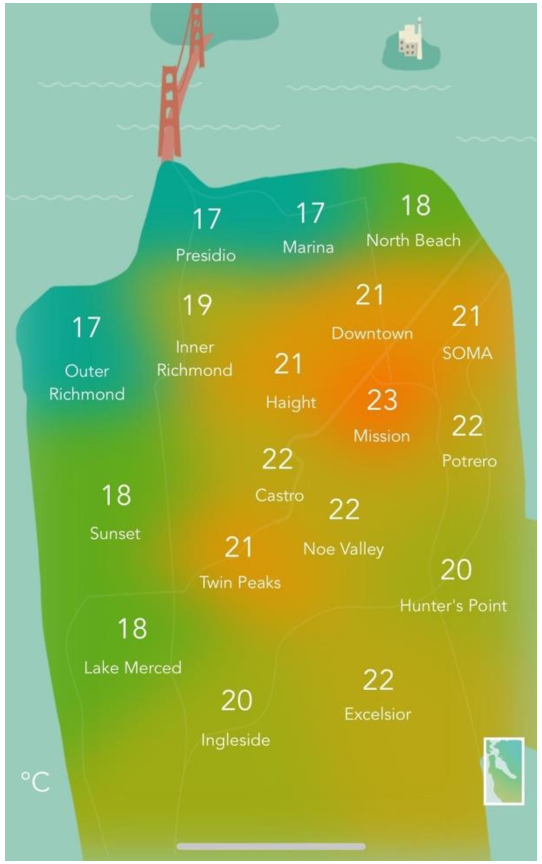


From Google's Perspective:
There are 18 different "districts" in San Francisco.

Know:

It rains tomorrow

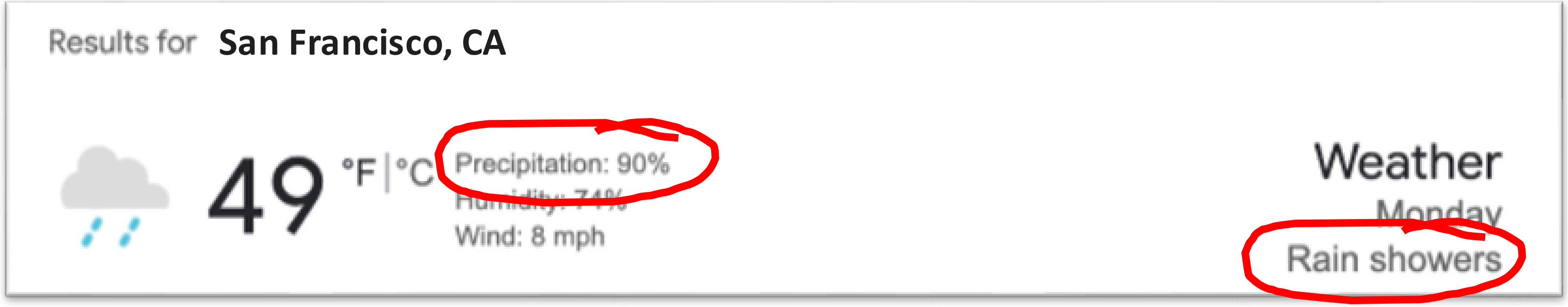
$$P(R|D_i)$$

Person is in district *i*

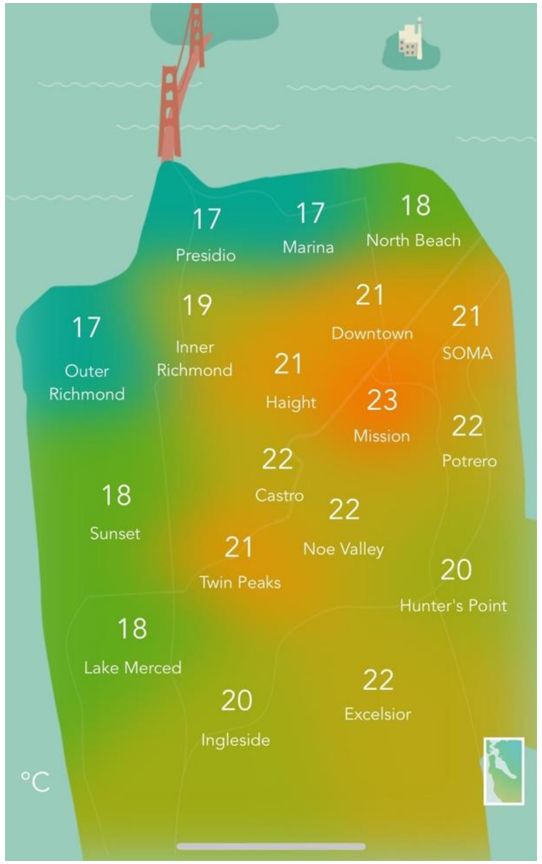
$$P(D_i)$$

Want:

$$P(R)$$



Background event.
Where is the person in San Francisco?



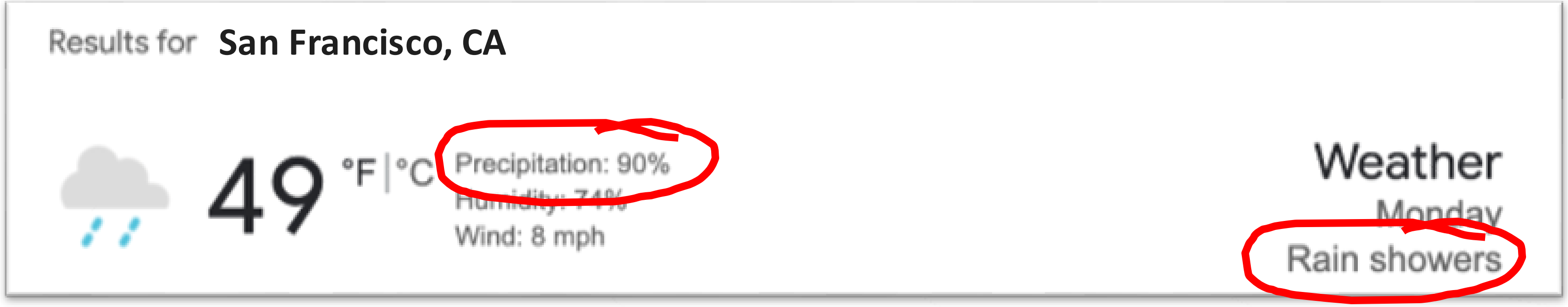
From Google's Perspective:
There are 18 different "districts" in San Francisco.

Know:

	Mission District	Presidio		SOMA
$P(R\|D_i)$	0.23	0.84	…	0.52
$P(D_i)$	0.15	0.02		0.24

Want:

$$P(R)$$



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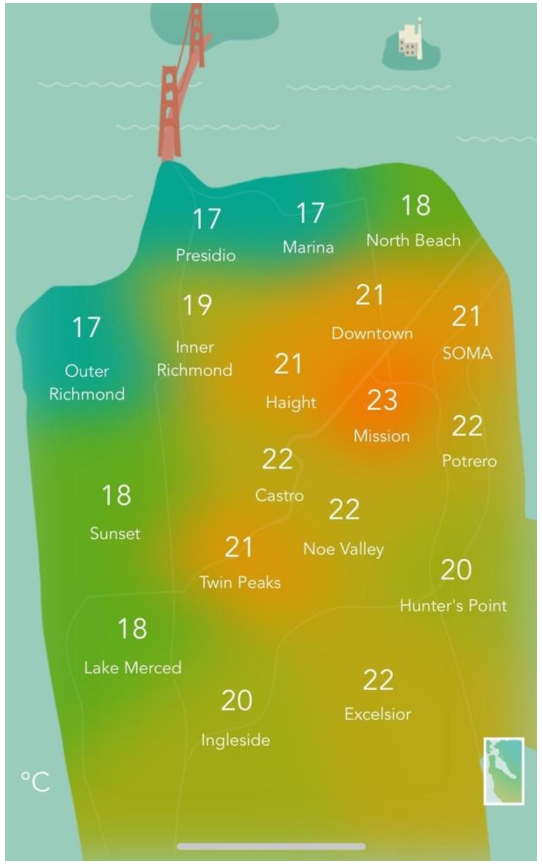


From Google's Perspective:
There are 18 different "districts" in San Francisco.

Know:

	Mission District	Presidio		SOMA
$P(R \mid D_i)$	0.23	0.84	…	0.52
$P(D_i)$	0.15	0.02		0.24

Want:

$$P(R) = \sum_{\text{district } i} P(R \text{ and } D_i) = \sum_{\text{district } i} P(R \mid D_i) \cdot P(D_i)$$

Know:

P(Cries | Poop), P(Cries), P(Poop)

Real question. What is the probability of poop given the baby cries…

P (Poop | Cries)

Relationship Between Probabilities

$$P(E \text{ and } F)$$

Chain rule
(Product rule)

Definition of
conditional probability

$$P(E|F)$$

Law of Total
Probability

$$P(E)$$

Relationship Between Probabilities

$$P(E \text{ and } F)$$

Chain rule
(Product rule)

Definition of
conditional probability

$$P(E|F)$$

Law of Total
Probability

Bayes'
Theorem

$$P(E)$$

$$P(F|E)$$

Bayes' Theorem

Thomas Bayes

Rev. Thomas Bayes (~1701-1761):
British mathematician and Presbyterian minister

He looked remarkably similar to Sean Astin

(but that's not important right now)

Thomas Bayes

$P(F \mid E)$

I want to calculate
P(State of the world F | Observation E)
It seems so tricky!…

The other way around is easy
P(Observation E | State of the world F)
What options to I have, chief?

$P(E \mid F)$

Thomas Bayes

Want $P(\,F\mid E\,)$. Know $P(\,E\mid F\,)$

$$P(F|E) = \frac{P(EF)}{P(E)} \quad \text{Def. of Conditional Prob.}$$

A little while later…

$$= \frac{P(E|F)P(F)}{P(E)} \quad \text{Chain Rule}$$

A little while later…

$$= \frac{P(E|F)P(F)P(F)}{P(E|F)P(F)P(F) + P(E|F)P(F^C)P(F^C)} \quad \text{LOTP}$$

Bayes' Theorem

$P(E|F) \Rightarrow P(F|E)$

Thm For any events E and F where $P(E) > 0$ and $P(F) > 0$,

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}$$

Proof

2 steps! See board

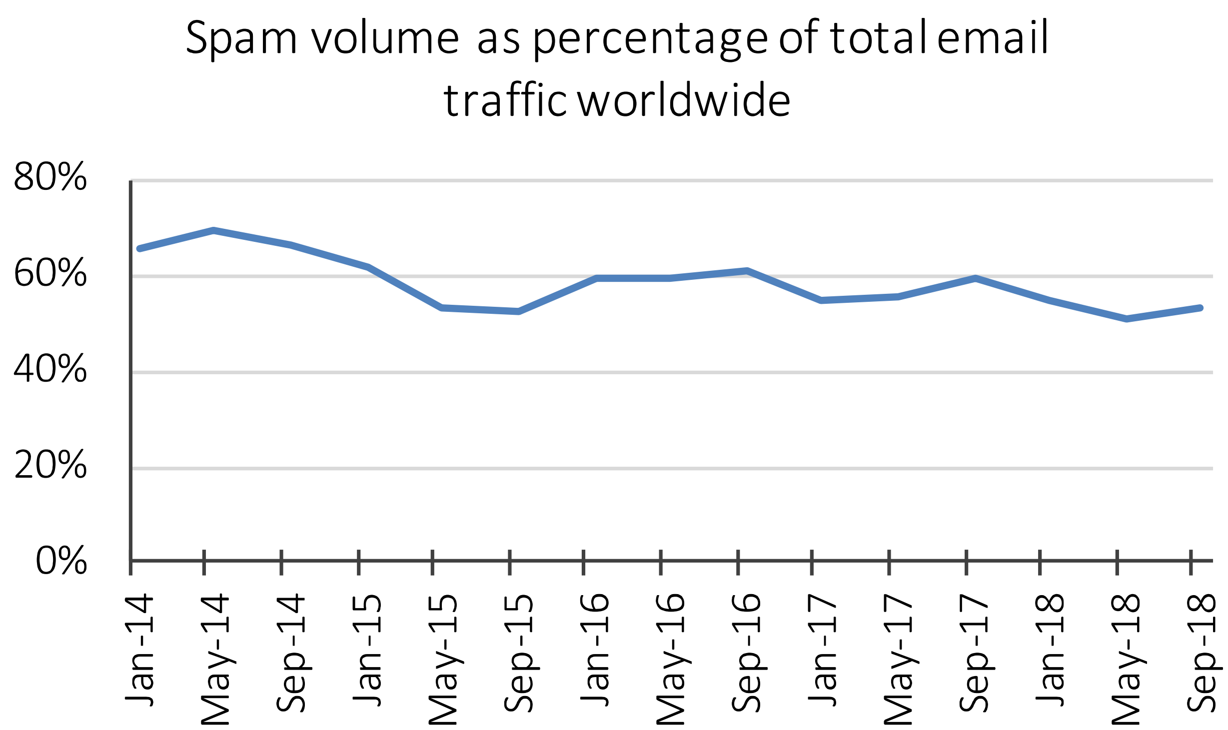
Expanded form:

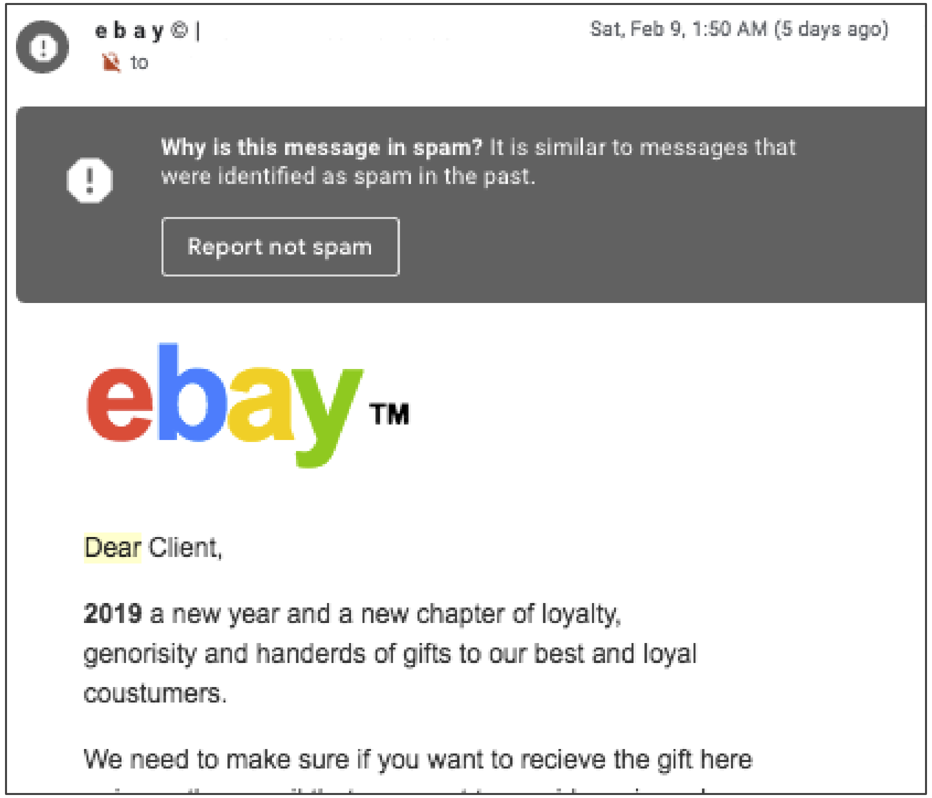
$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^C)P(F^C)}$$

Proof

1 more step! See board

Detecting spam email





We can easily calculate how many spam emails contain "Dear":

$$P(E|F) = P\left(\text{“Dear”} \,\middle|\, \begin{matrix}\text{Spam}\\ \text{email}\end{matrix}\right)$$

But what is the probability that an email containing "Dear" is spam?

$$P(F|E) = P\left(\begin{matrix}\text{Spam}\\ \text{email}\end{matrix} \,\middle|\, \text{“Dear”}\right)$$

(silent drumroll)

Detecting spam email

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^C)P(F^C)}$$ Bayes' Theorem

- 60% of all email in 2016 is spam.
- 20% of spam has the word "Dear"
- 1% of non-spam (aka ham) has the word "Dear"

You get an email with the word "Dear" in it.

What is the probability that the email is spam?

1. Define events & state goal
2. Identify known probabilities
3. Solve

Let: E: "Dear", F: spam

Want: P(spam | "Dear")

$= P(F|E)$

Bayes' Theorem terminology

- 60% of all email in 2016 is spam. $P(F)$
- 20% of spam has the word "Dear" $P(E|F)$
- 1% of non-spam (aka ham) has the word "Dear" $P(E|F^C)$

You get an email with the word "Dear" in it.

What is the probability that the email is spam? Want: $P(F|E)$

$$\underbrace{P(F|E)}^{\text{posterior}} = \frac{\overbrace{P(E|F)}^{\text{likelihood}}\,\overbrace{P(F)}^{\text{prior}}}{\underbrace{P(E)}_{\text{normalization constant}}}$$

SARS Virus Testing

A test is 98% effective at detecting SARS

- However, test has a "false positive" rate of 1%
- 0.5% of US population has SARS
- Let E = you test positive for SARS with this test
- Let F = you actually have SARS
- What is P(F | E)?

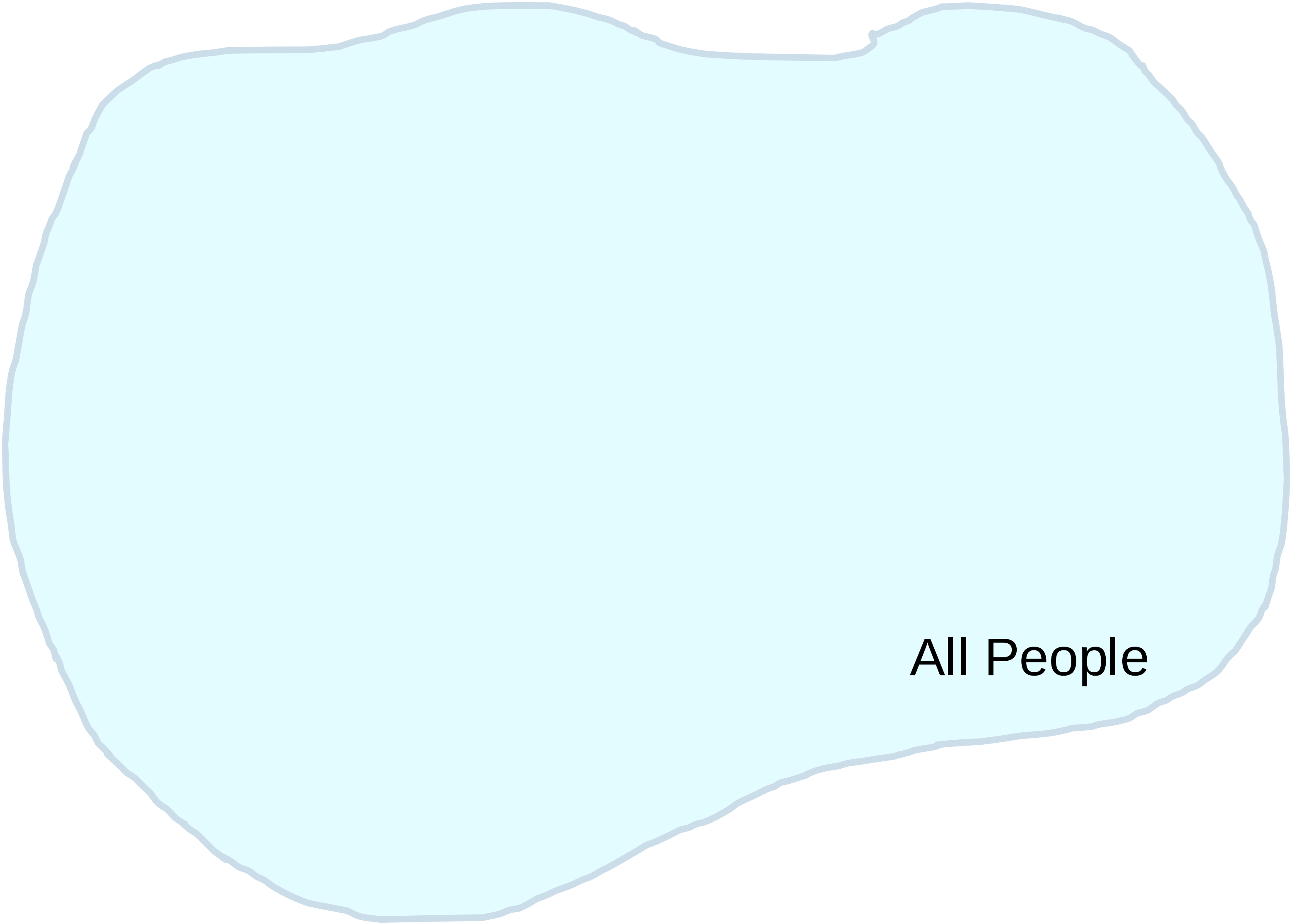
Solution:

$$P(F \mid E) = \frac{P(E \mid F)\, P(F)}{P(E \mid F)\, P(F) + P(E \mid F^c)\, P(F^c)}$$

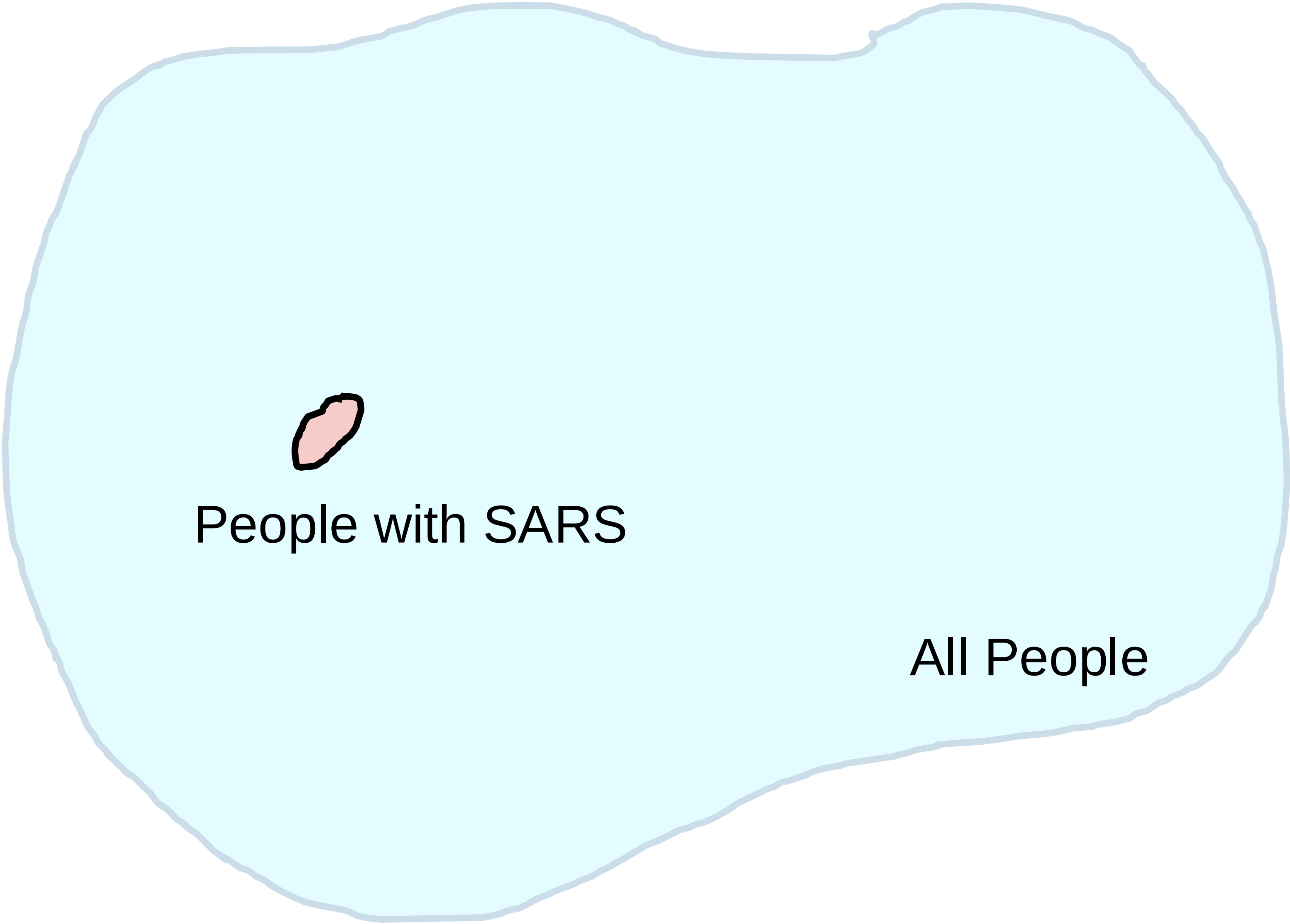
$$P(F \mid E) = \frac{(0.98)(0.005)}{(0.98)(0.005) + (0.01)(1 - 0.005)} \approx 0.330$$

Intuition Time

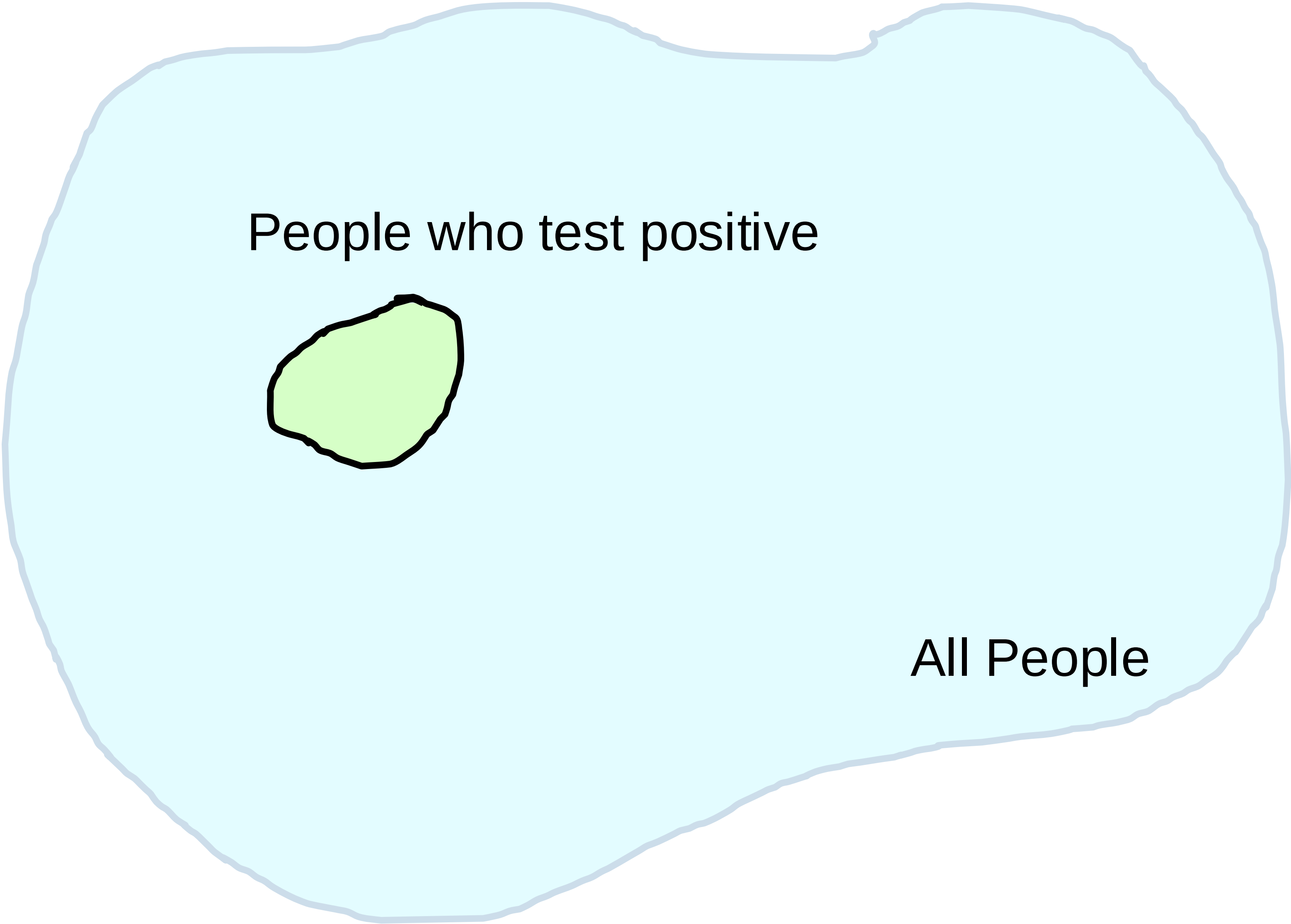
Bayes Thorem Intuition



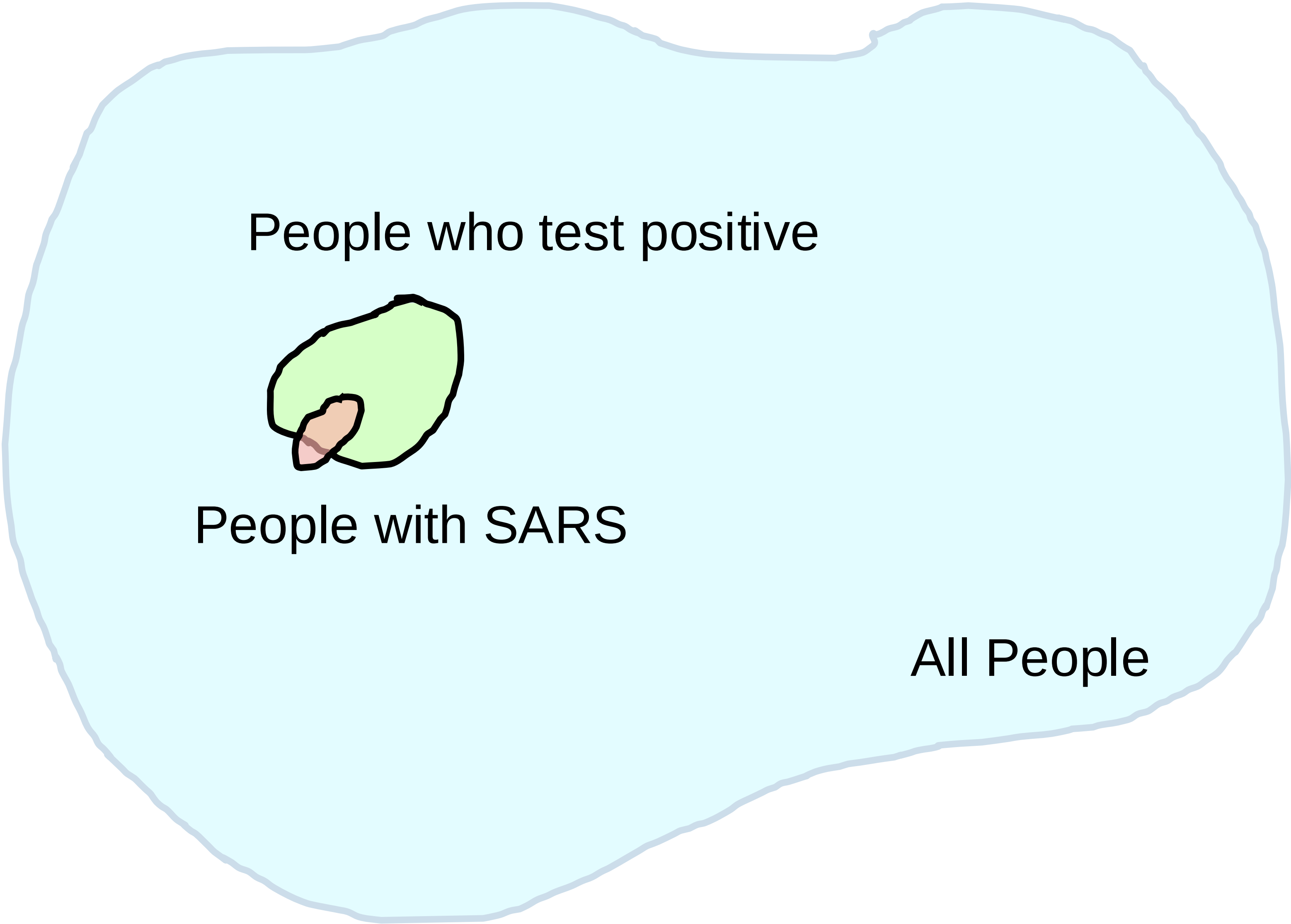
Bayes Thorem Intuition



Bayes Thorem Intuition

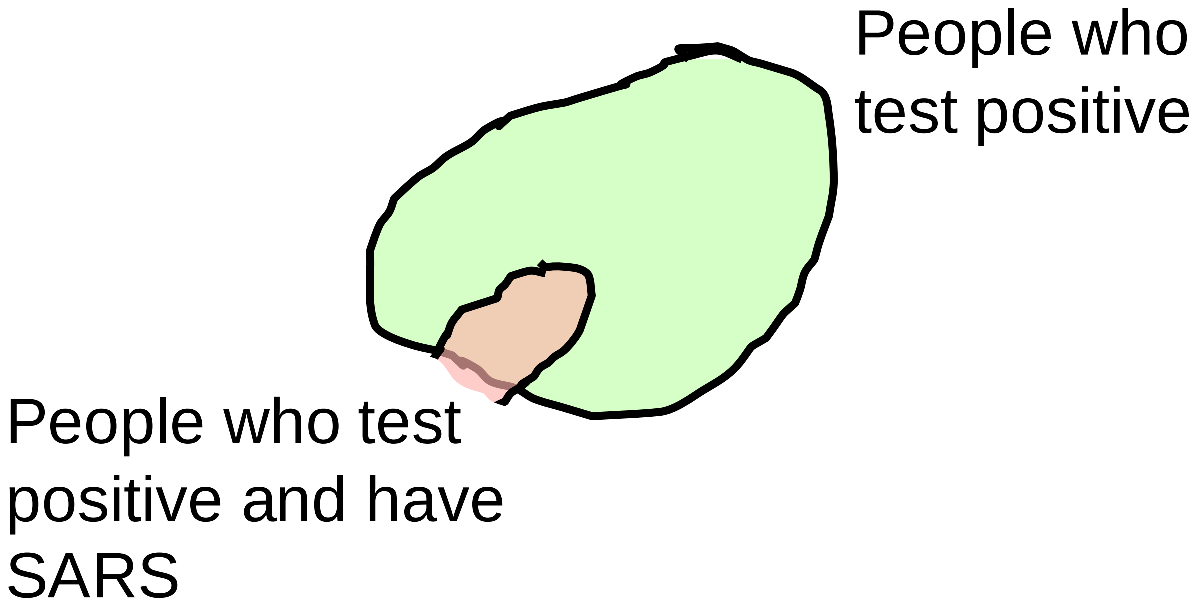


Bayes Thorem Intuition



Bayes Thorem Intuition

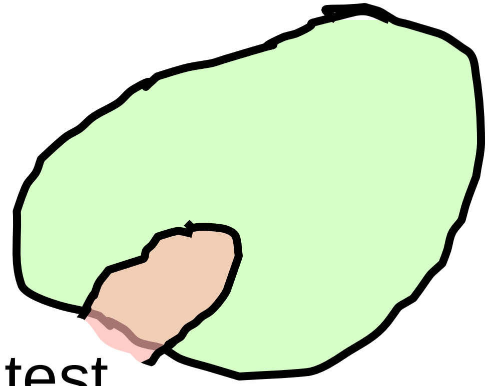
Conditioning on a positive result changes the sample space to this:



≈ 0.330

Bayes Thorem Intuition

Conditioning on a positive result changes the sample space to this:



People who test positive

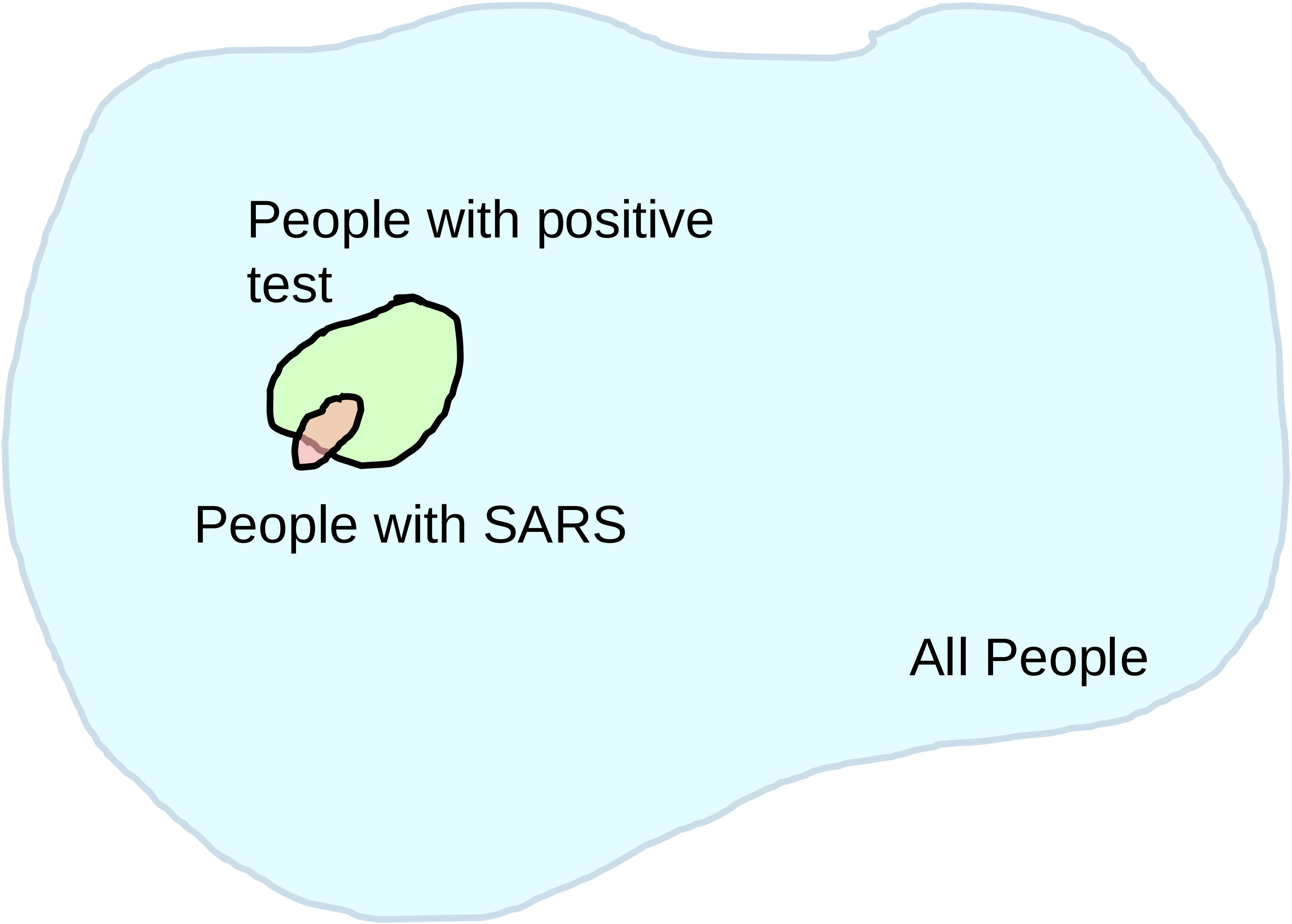
$$P(F)P(E|F) + P(F^c)P(E|F^c)$$

People who test positive and have SARS

$$P(F)P(E|F)$$

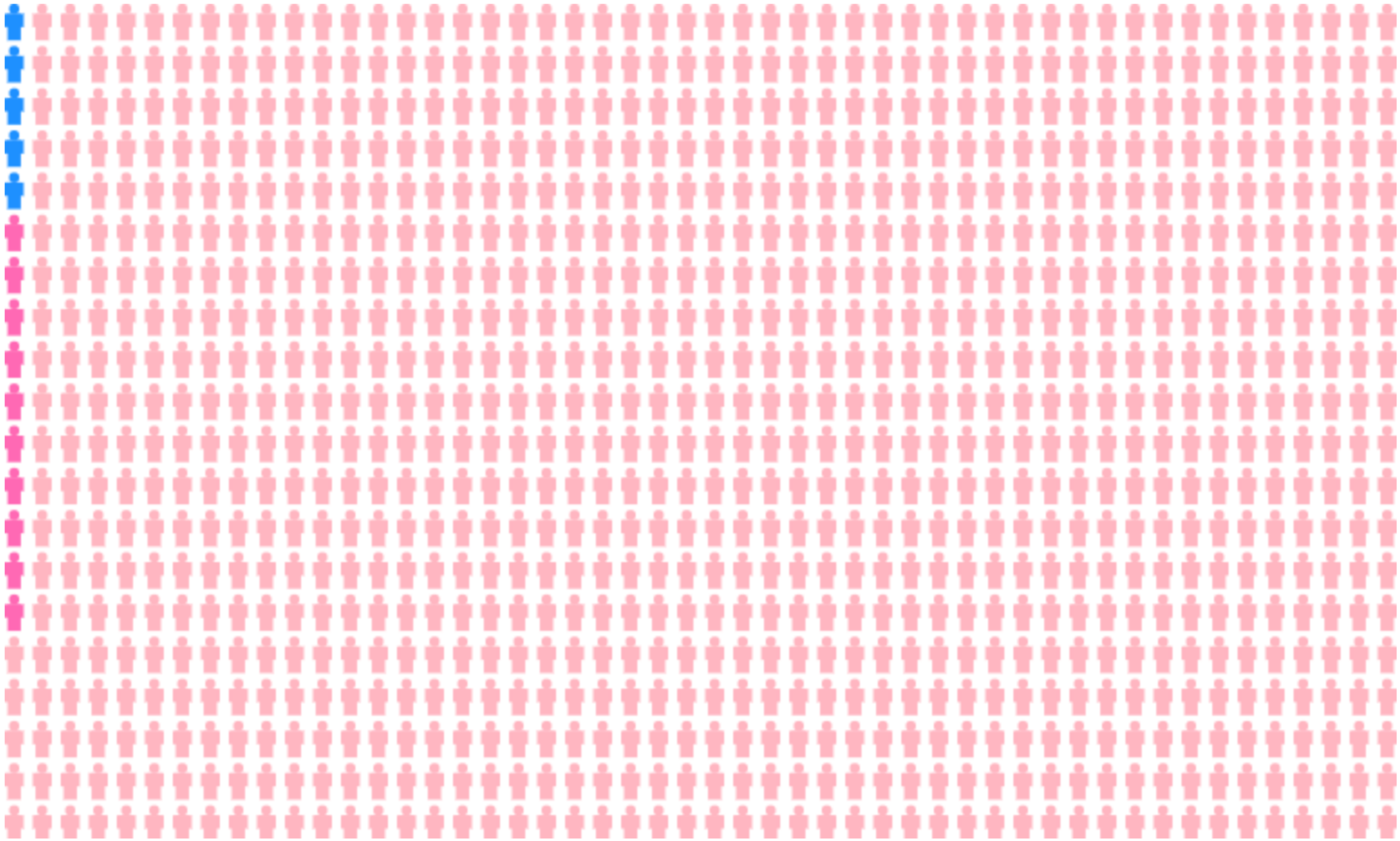
≈ 0.330

Bayes Thorem Intuition



Bayes Thorem Intuition

Say we have 1000 people:



5 have SARS and test positive, 985 **do not** have SARS and test negative.
10 **do not** have SARS and test positive. ≈ 0.333

Bayes Thorem Intuition

Conditioned on just those that test positive:

Notice that all the people with SARS are here,
but the group is still mainly folks without SARS

5 have SARS and test positive, 985 **do not** have SARS and test negative.
10 **do not** have SARS and test positive.

≈ 0.333

Why it is still good to get tested

	SARS +	**SARS –**
Test +	$0.98 = P(E \mid F)$	$0.01 = P(E \mid F^c)$
Test –	$0.02 = P(E^c \mid F)$	$0.99 = P(E^c \mid F^c)$

- Let E^c = you test <u>negative</u> for SARS with this test
- Let F = you actually have SARS
- What is $P(F \mid E^c)$?

$$P(F \mid E^c) = \frac{P(E^c \mid F)\, P(F)}{P(E^c \mid F)\, P(F) + P(E^c \mid F^c)\, P(F^c)}$$

$$P(F \mid E^c) = \frac{(0.02)(0.005)}{(0.02)(0.005) + (0.99)(1 - 0.005)} \approx 0.0001$$

Pedagogic Pause

Multiple Choice Theory

Let's consider the relationship between **knowing** the concepts used in a multiple choice midterm question, and getting the question correct, taking into account guessing and making silly mistakes.

Let 3/4 be the probability that a learner knows the concepts to a midterm question.

Let 1/4 be the probability that a learner gets the answer **correct** if they **don't** know the concepts.

Let 9/10 be the probability that a learner gets the question **correct** given they **do** know the concepts.

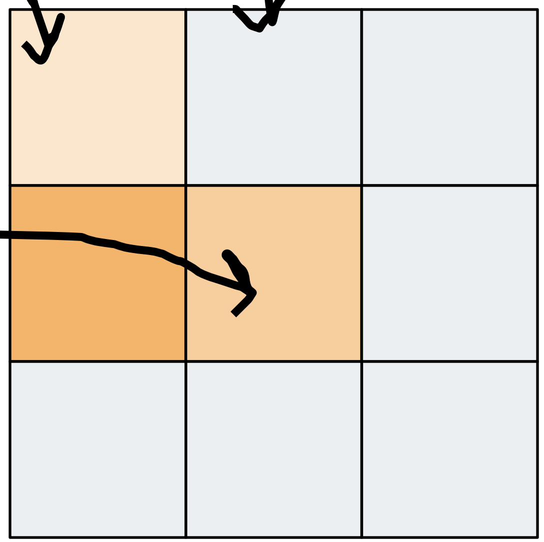
What is the probability they know the concept, given they answered correct?

Advanced

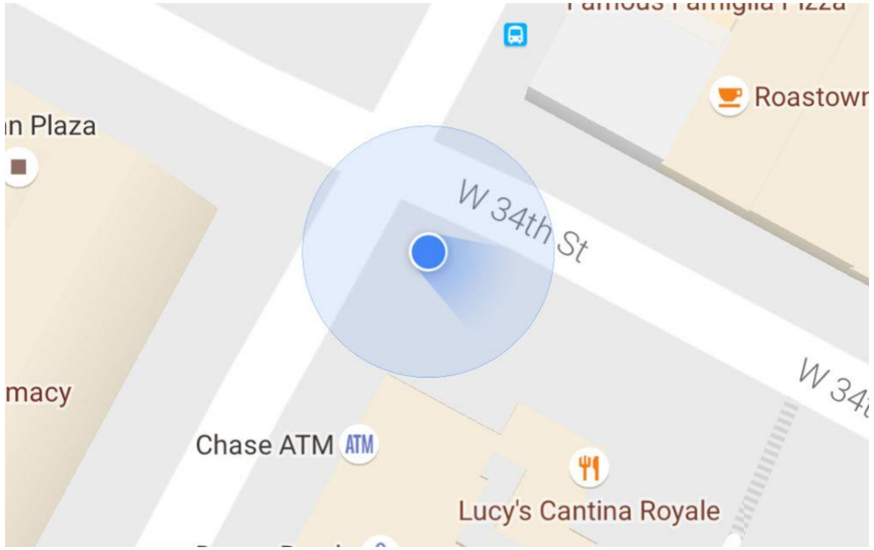
Bayes' Theorem and Location

$P(L_1)$ $P(L_2)$

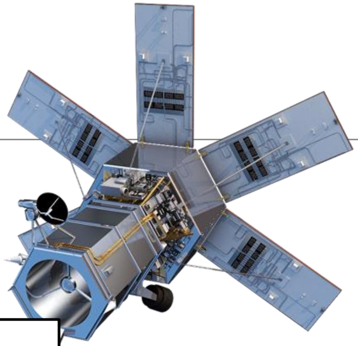
$P(L_5)$



Before Observation

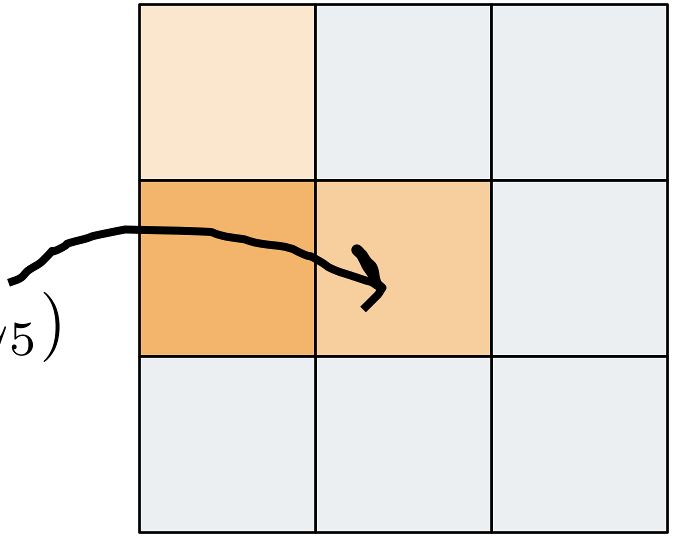


Bayes' Theorem and Location

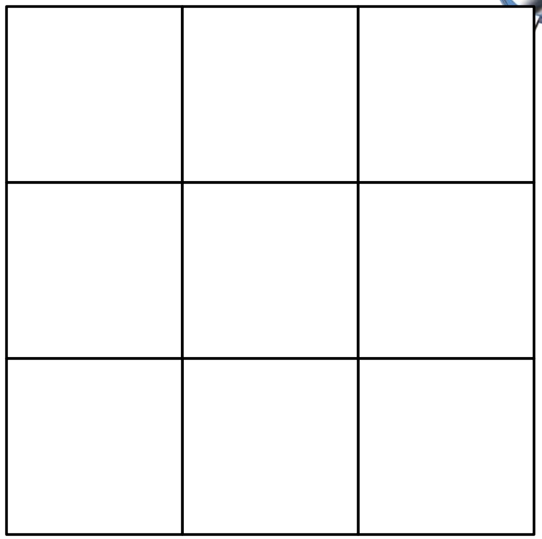


Know: $P(O|L_i)$

$P(L_5)$



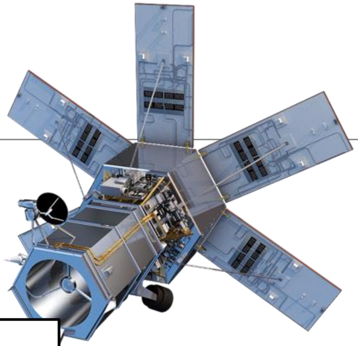
Before Observation



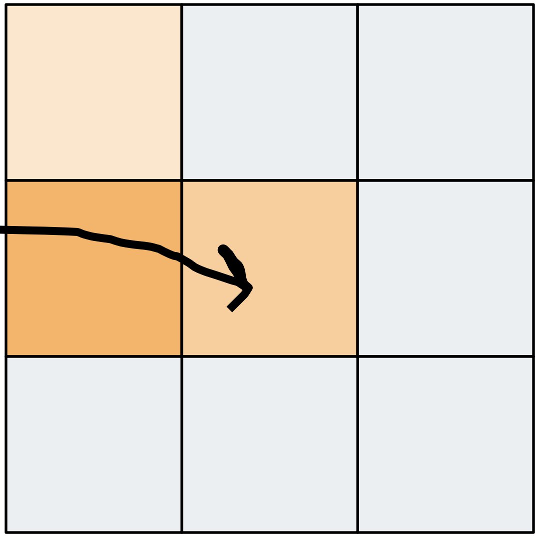
After Observation

Bayes' Theorem and Location

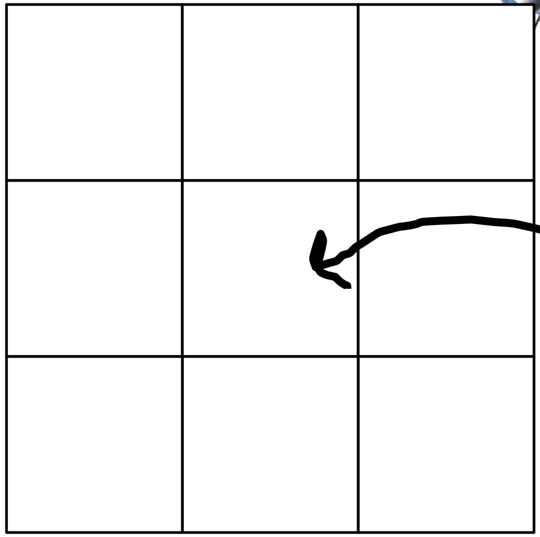
Know: $P(O|L_i)$



$P(L_5)$



Before Observation

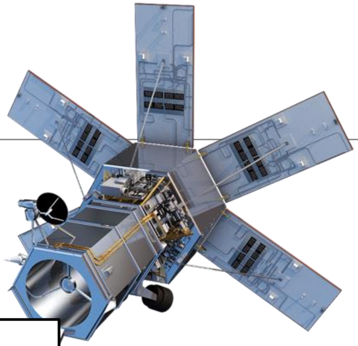


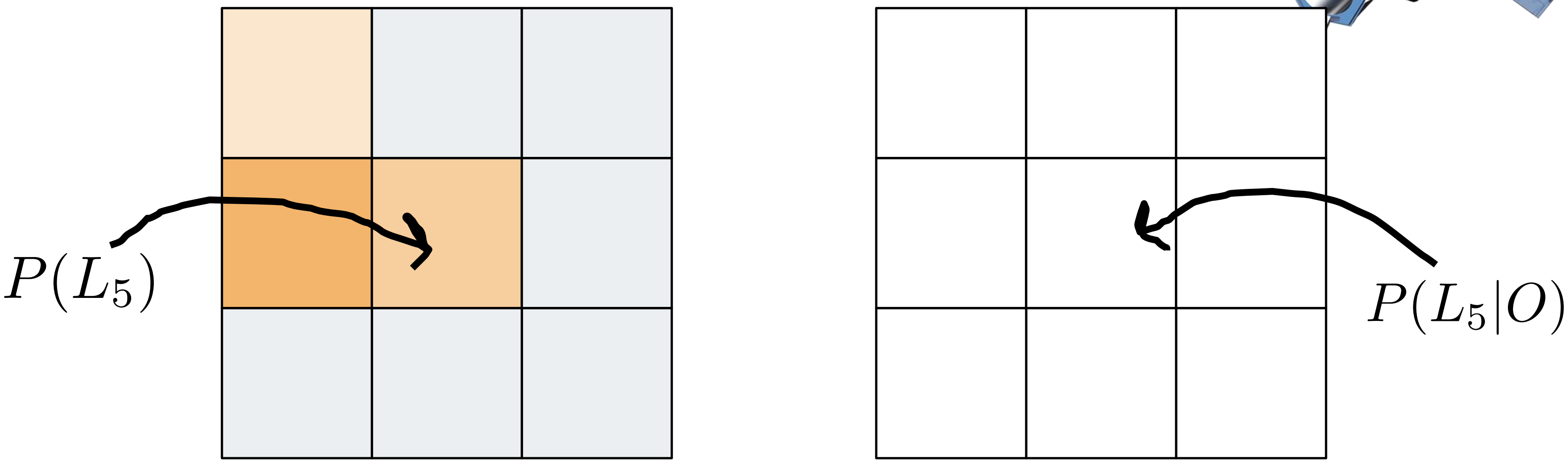
$P(L_5|O)$

After Observation

$$P(L_5|O) = \frac{P(O|L_5)P(L_5)}{P(O)}$$

Bayes' Theorem and Location



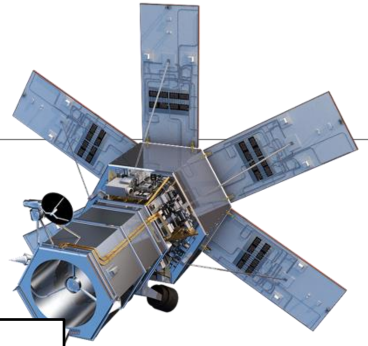


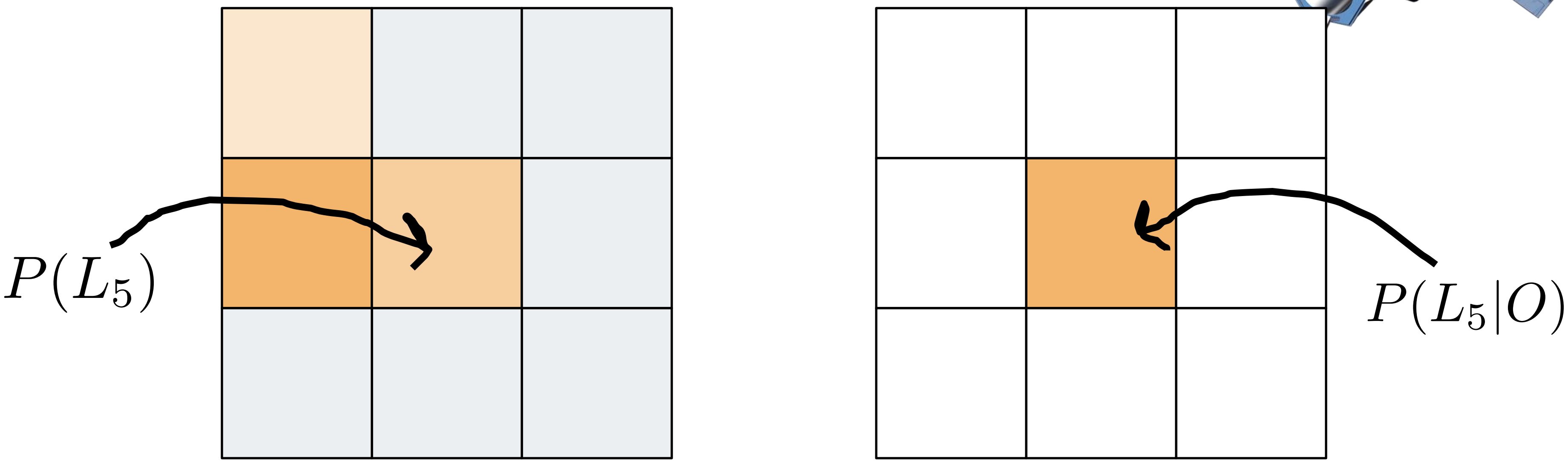
Before Observation

After Observation

$$P(L_5|O) = \frac{P(O|L_5)P(L_5)}{\sum_i P(O|L_i)P(L_i)}$$

Bayes' Theorem and Location





Before Observation

After Observation

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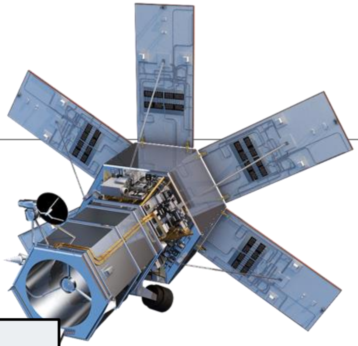
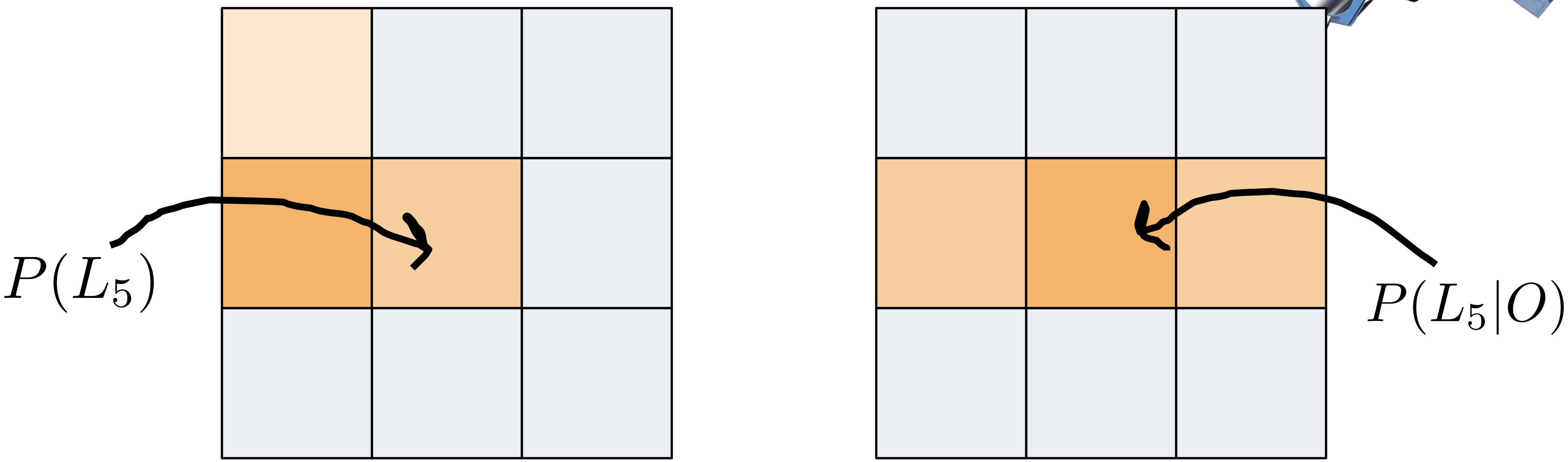
Bayes' Theorem and Location





Before Observation

After Observation

$$P(L_5|O) = \frac{P(O|L_5)P(L_5)}{\sum_i P(O|L_i)P(L_i)}$$

Let’s Gamble

Telling in Cards

Your opponent gets excited if they are winning…

Probability of a tell given they **have a winning** hand: **0.5**

Probability of a tell given they **don't have** a winning hand: **0.1**

The Tell Game

```
You are playing a game. Your opponent has two unseen cards. If either is an ace you lose...
The following cards are seen by you (ie are not in opponent hand):
3H
2S
3D
5S
9D
KD
AC
Opponent does *not* have an excited tell...
Do you choose to play? yes
```

Probability of a tell given they **have a winning** hand: 0.5

Probability of a tell given they **don't have** a winning hand: 0.1

End Review