Continuous Variables

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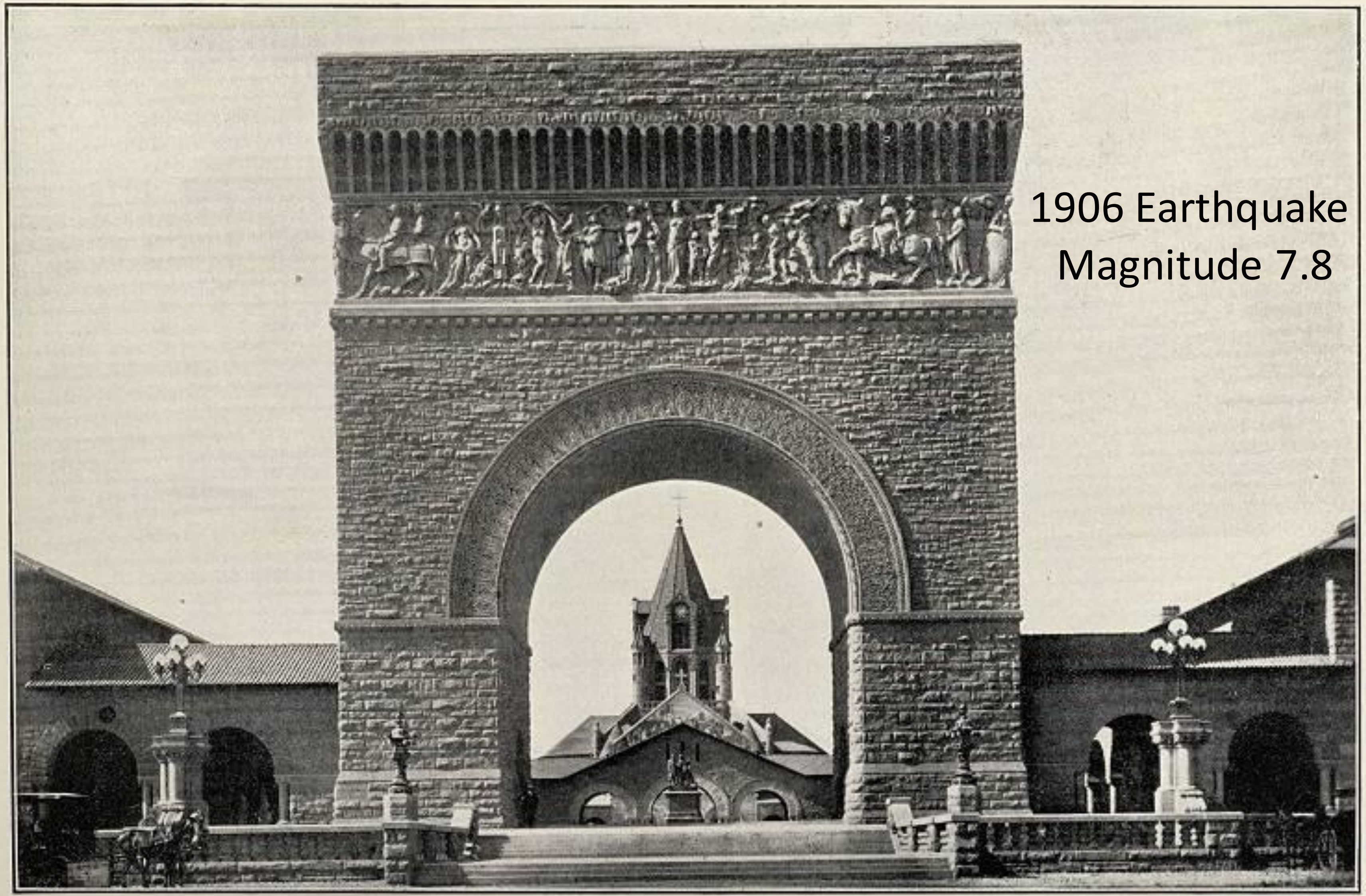

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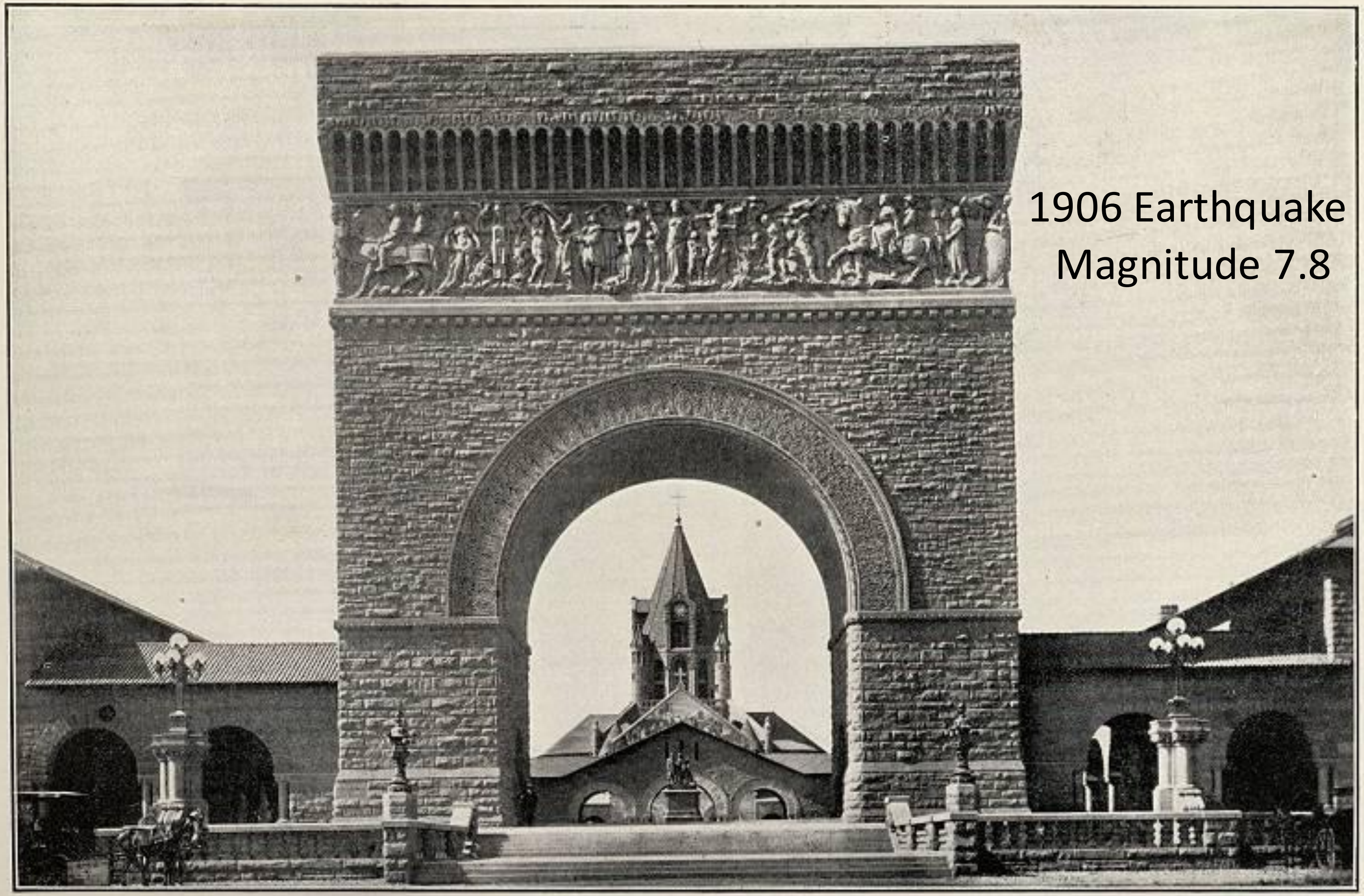

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How long until the next "big one"?

Review

Binomial Random Variable

```
(H, H, H, H, T, T, T, T, T, T)
(H, H, H, T, H, T, T, T, T, T)
(H, H, H, T, T, H, T, T, T, T)
(H, H, H, T, T, T, H, T, T, T)
(H, H, H, T, T, T, T, H, T, T)
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(H, H, H, T, T, T, T, T, T, H)
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(H, H, T, H, T, H, T, T, T, T)
(H, H, T, H, T, T, H, T, T, T)
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(H, H, T, H, T, T, T, T, H, T)
(H, H, T, H, T, T, T, T, T, H)
(H, H, T, T, H, H, T, T, T, T)
(H, H, T, T, H, T, H, T, T, T)
(H, H, T, T, H, T, T, H, T, T)
(H, H, T, T, H, T, T, T, H, T)
(H, H, T, T, H, T, T, T, T, H)
```
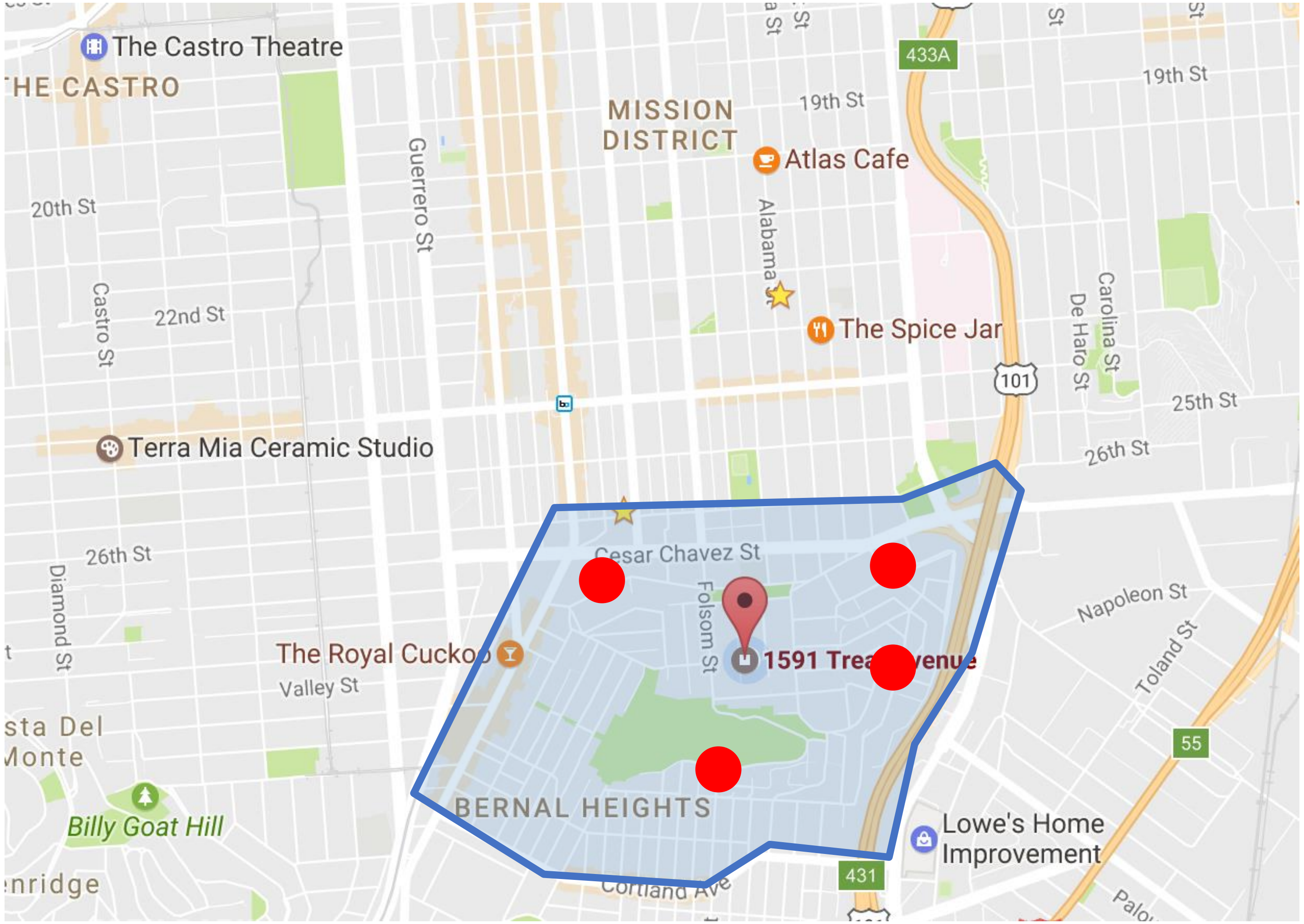

The number of **successes**, in *n* independent **trials**, where each **trial** is a **success** with probability p:

Poisson Random Variable

Probability of ***k* requests** from this area in the next 1 min

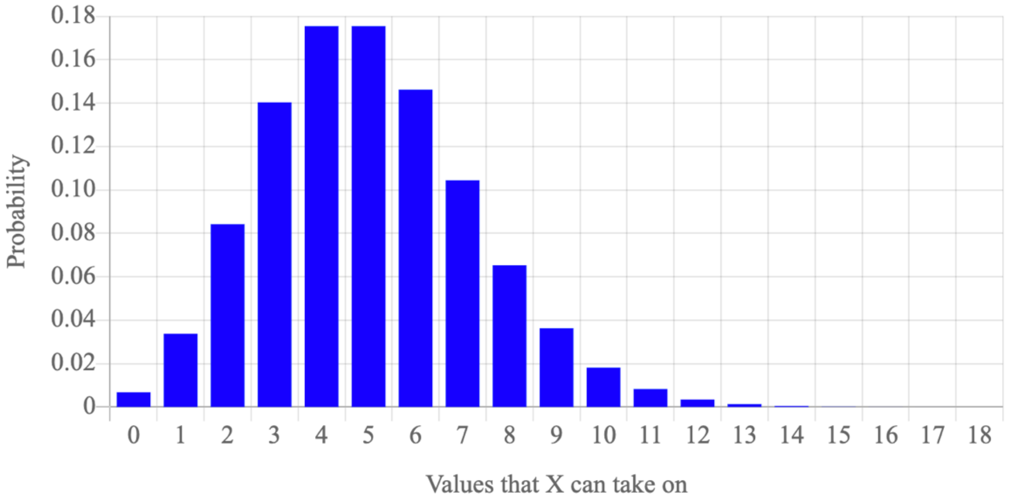


Poisson Random Variable

Poisson Random Variable

Notation:	$X \sim \text{Poi}(\lambda)$
Description:	Number of events in a fixed time frame if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.
Parameters:	$\lambda \in \{0, 1, \ldots\}$, the constant average rate.
Support:	$x \in \{0, 1, \ldots\}$
PMF equation:	$\Pr(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$
Expectation:	$\text{E}[X] = \lambda$
Variance:	$\text{Var}(X) = \lambda$
PMF graph:	

Parameter λ: 5



Discrete Random Variables

$X \sim \text{Bern}(p)$

Successes in one trial

$X \sim \text{Geo}(p)$

Trials until one success

$X \sim \text{Poi}(\lambda)$

Events in one time interval

$Y \sim \text{Bin}(n, p)$

Successes in n trials

$Y \sim \text{NegBin}(r, p)$

Trials until r success

Goal: Be Able to Use a New Random Variable

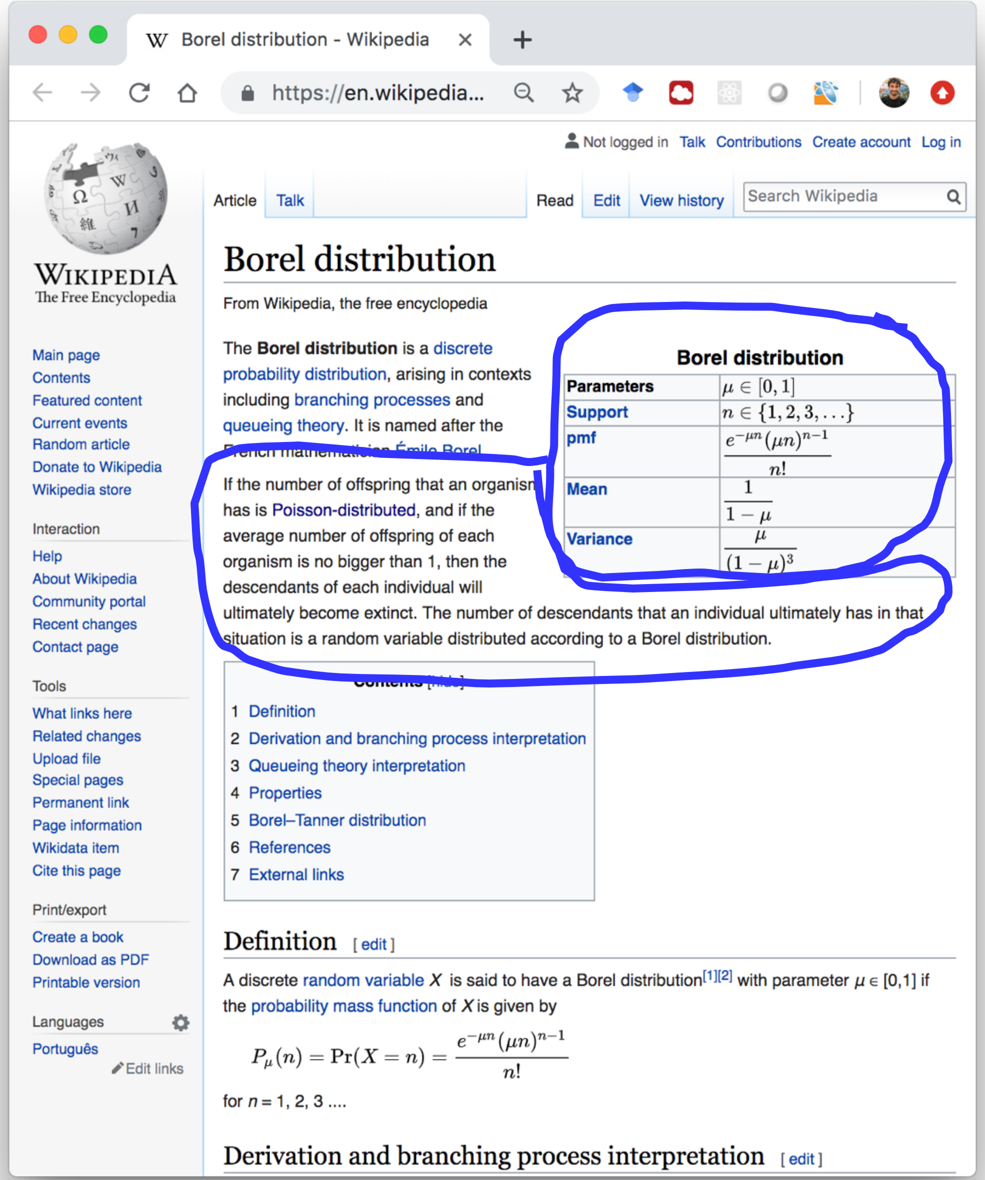
Can you use a novel random variable type if I give you its "fact sheet"?

Goal: Be Able to Use a New Random Variable

You are learning about servers…

You read about the MD1 queue…

You find a paper that says the length of a server "busy period" is distributed as a Borel with parameter μ = 0.2 …

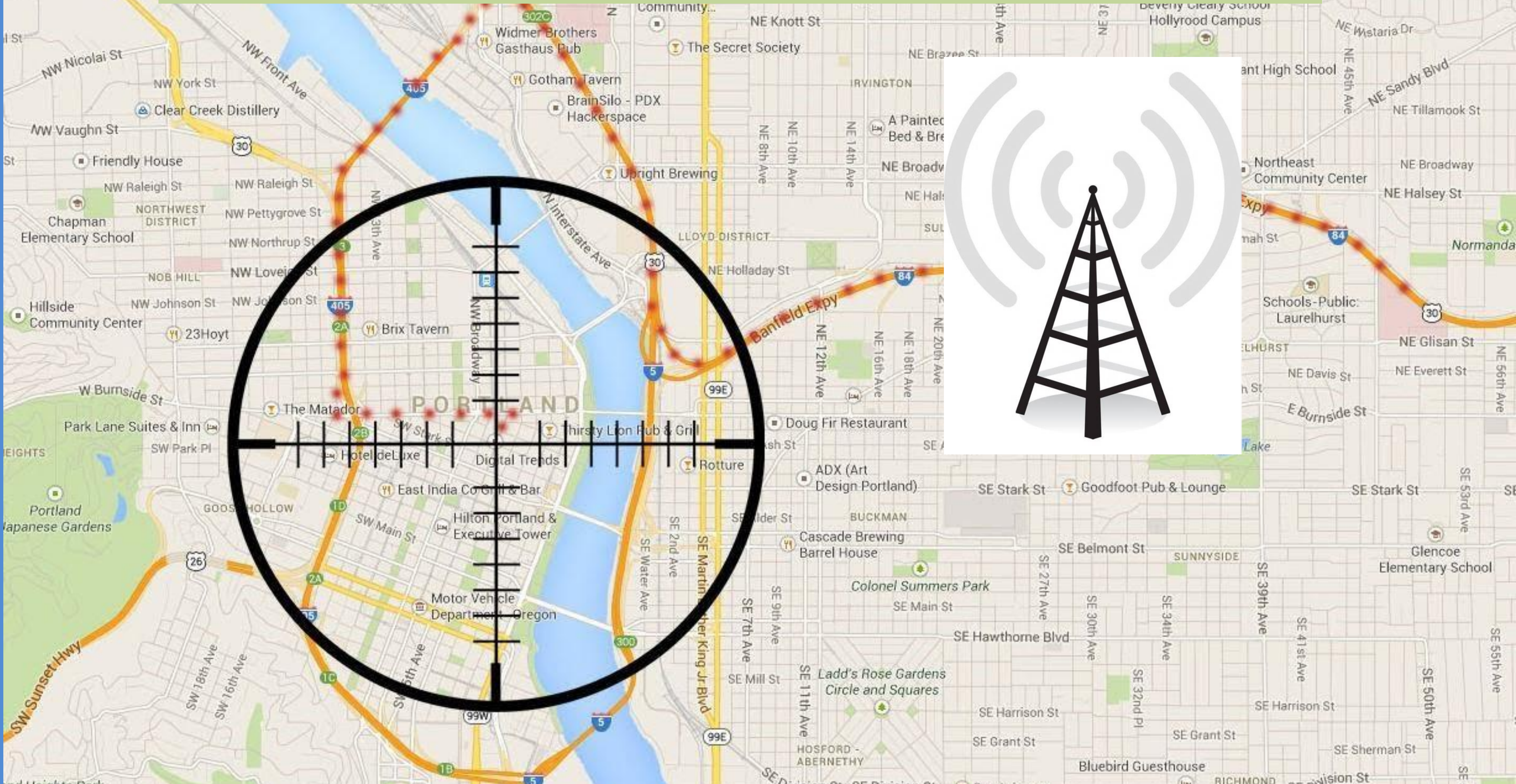


Learning Goals

1. Integrate a density function (PDF) to get a probability
2. Use a cumulative function (CDF) to get a probability

Big hole in our knowledge

Not all values are discrete



Can’t Talk About Continuous Values

Say the average rate of earthquakes is 1 every 100 years.

We **can** talk about the probability distribution of different numbers of earthquakes next year.

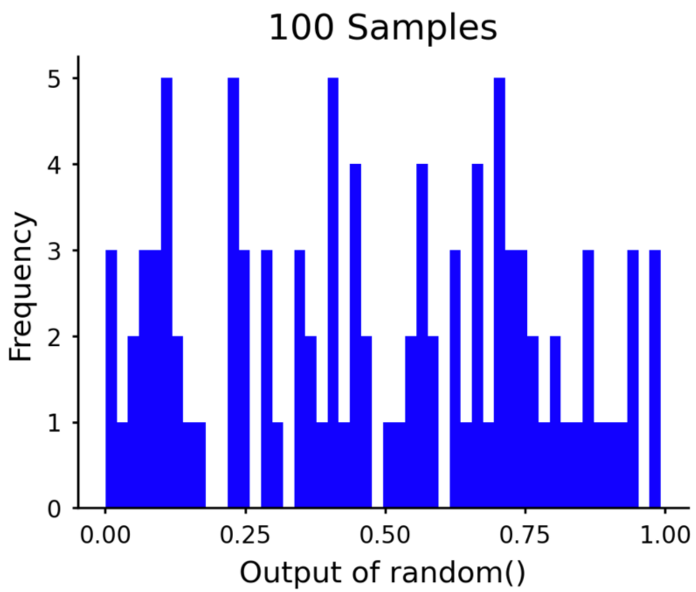
We **can’t** talk about the probability distribution of the amount of time until the next earthquake.

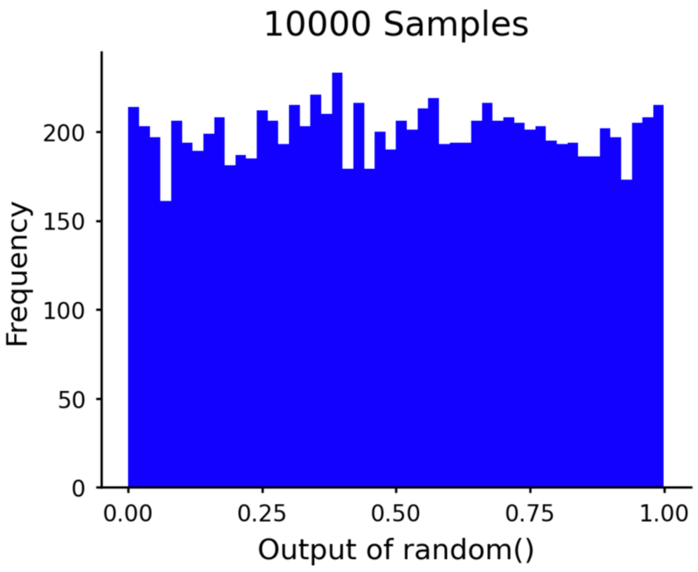
```
random()?
```

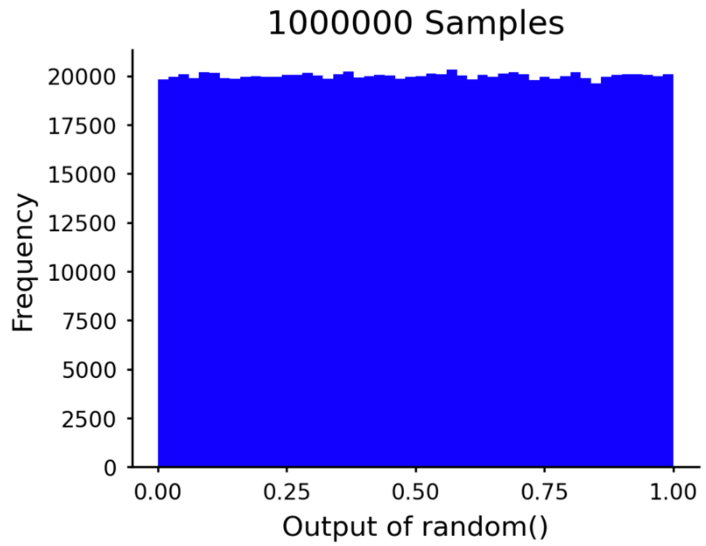
The `random()` Function

- Outputs values between 0 and 1
- All possible values are equally likely
- This is a continuous random variable!

```
import random

samples_small = []
for i in range(100):
    samples_small.append(random.random())

samples_medium = []
for i in range(10000):
    samples_medium.append(random.random())

samples_large = []
for i in range(1000000):
    samples_large.append(random.random())
```

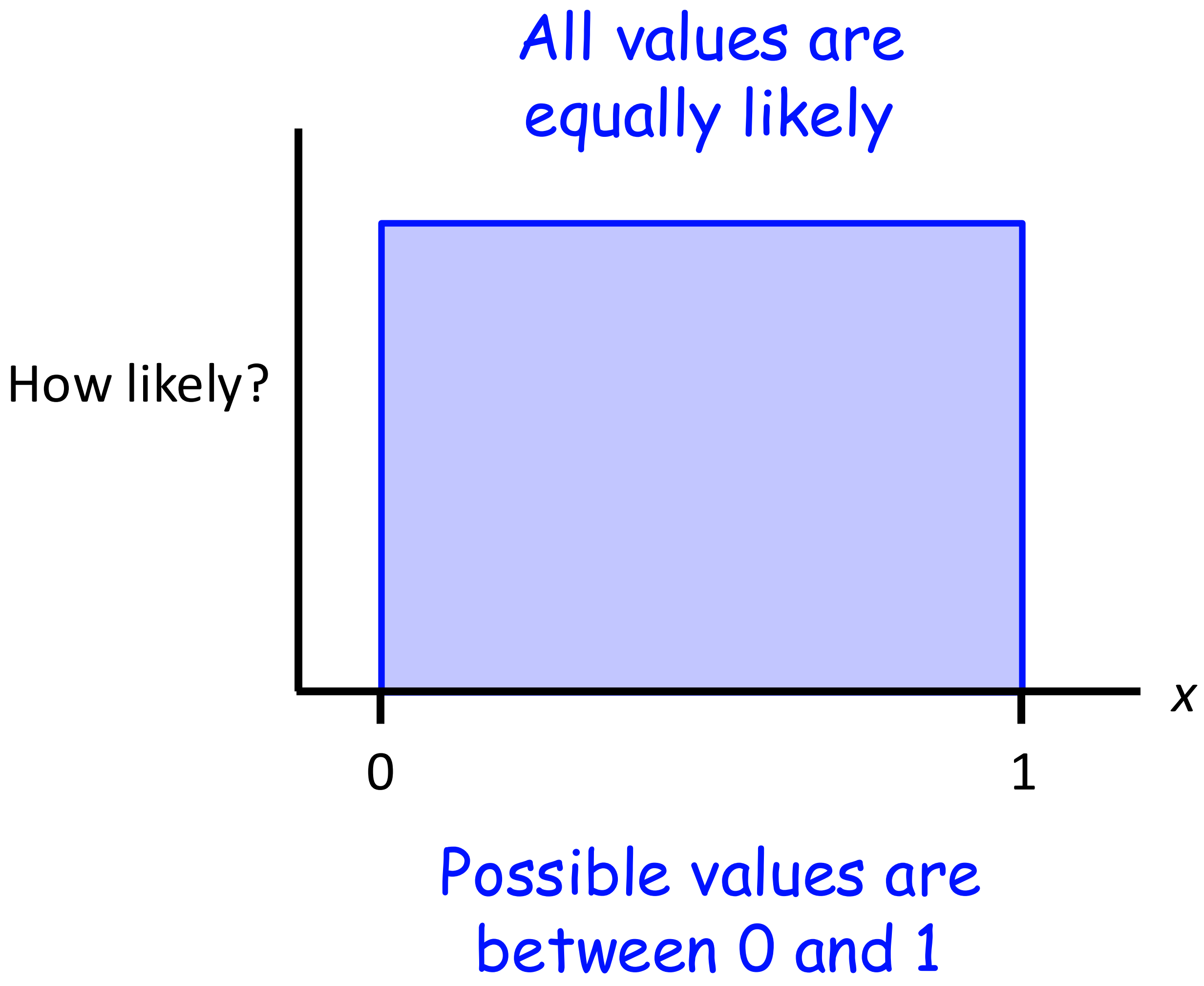






$X \sim \text{Uniform}(0,1)$: A Continuous Random Variable

$X \sim$ Uniform(0,1): A Continuous Random Variable



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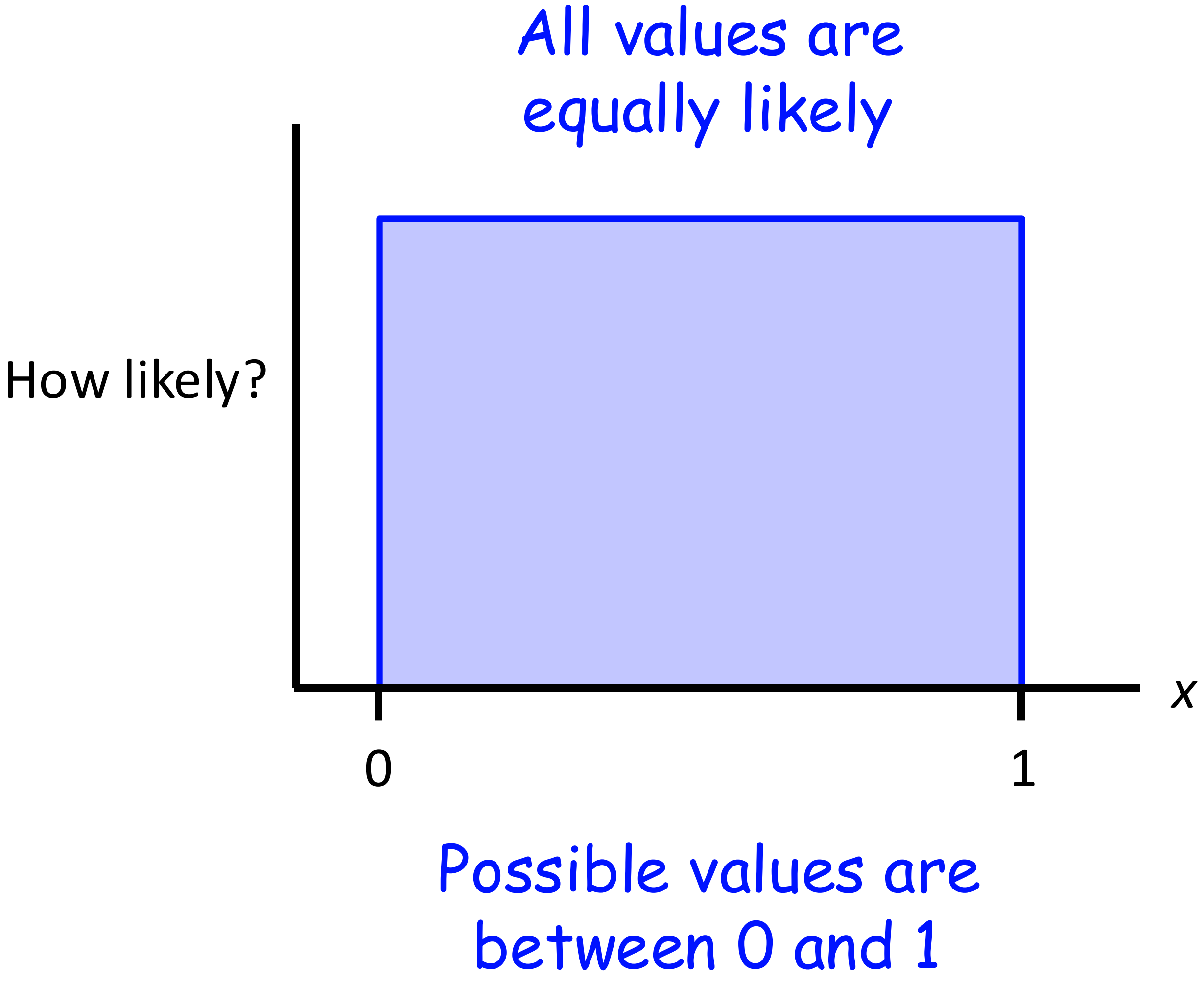
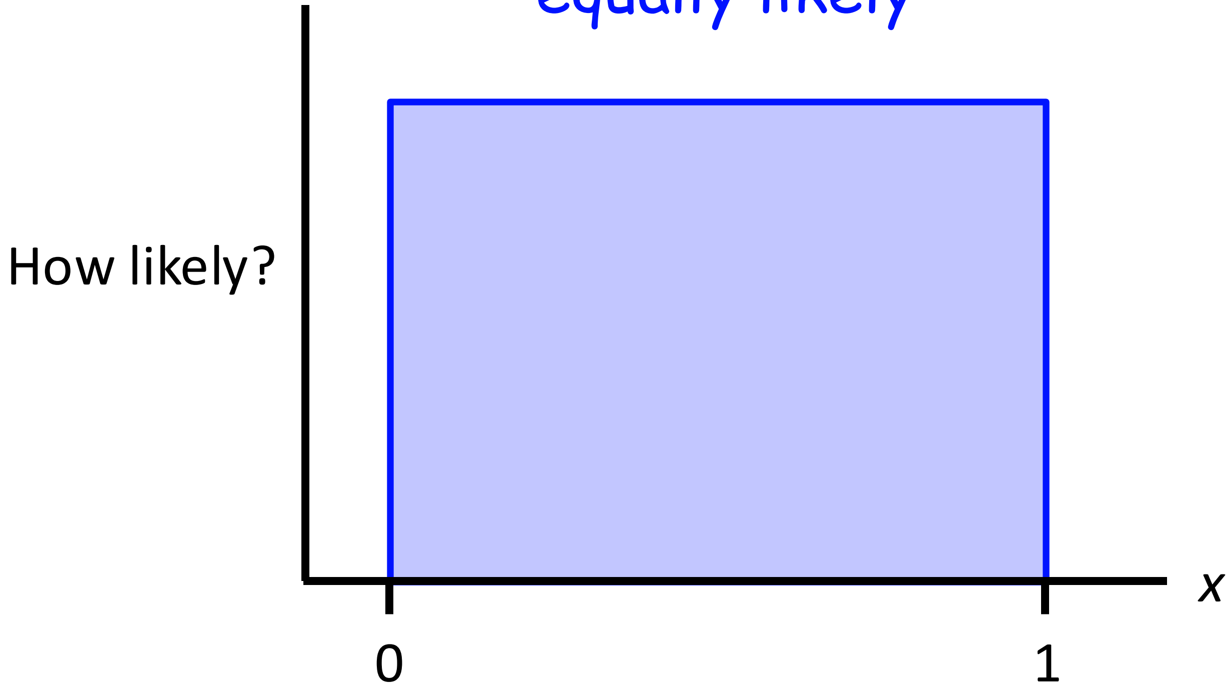


$$P(0 \leq X \leq 1) = \ ?$$

$X \sim \text{Uniform}(0,1)$: A Continuous Random Variable

All values are equally likely



Possible values are between 0 and 1

$$P(0 \leq X \leq 1) = 1$$

Probability of the whole sample space must equal 1

(Axiom 2)

$X \sim \text{Uniform}(0,1)$: A Continuous Random Variable

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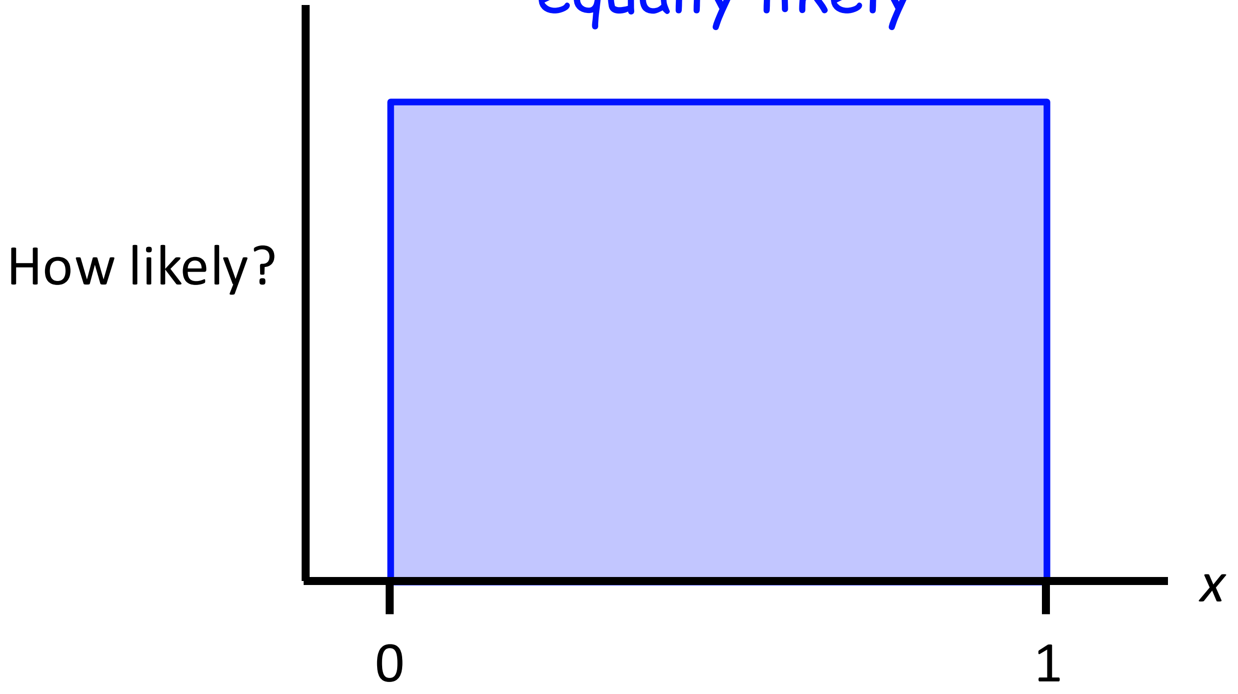

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$$P(0.5 \leq X \leq 1) = ?$$

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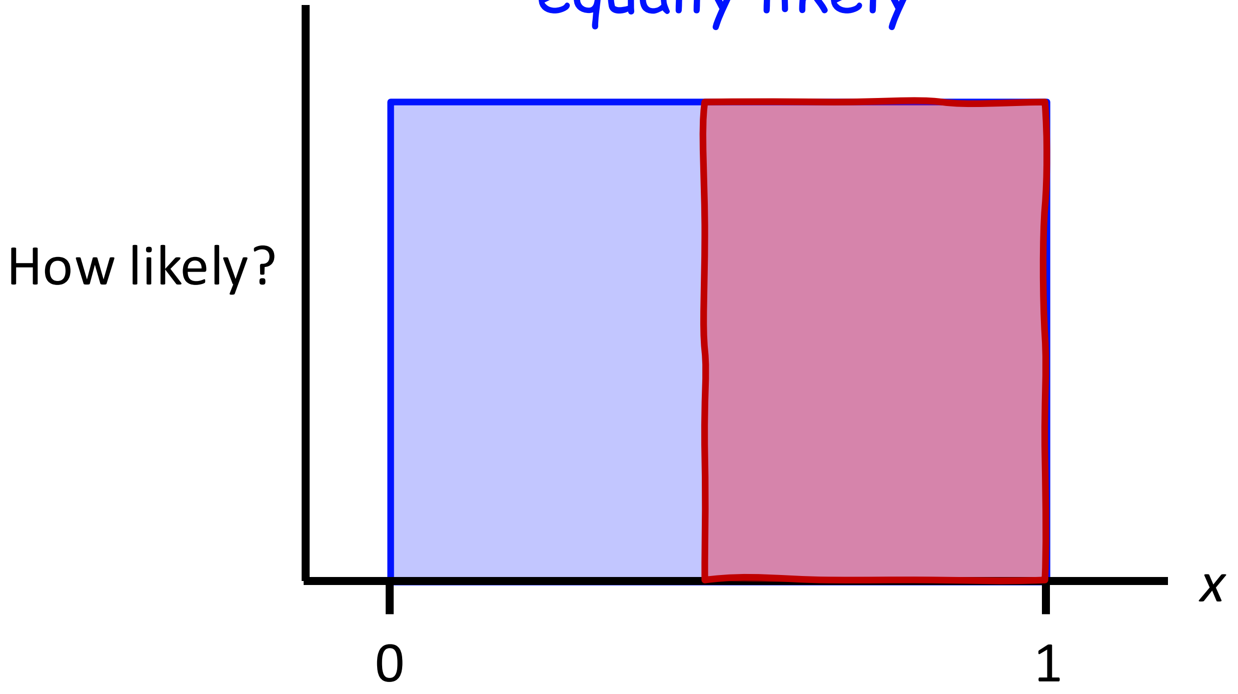


Possible values are between 0 and 1

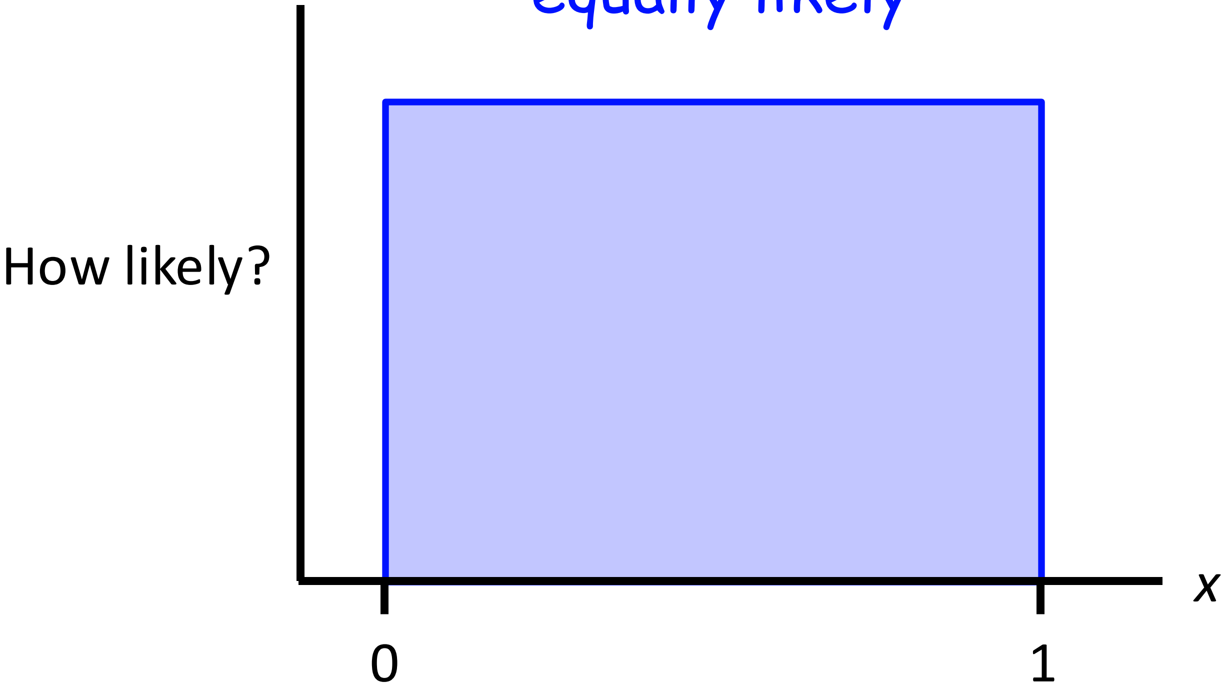
$$P(0 \leq X \leq 1) = 1$$

$$P(0.5 \leq X \leq 1) = 0.5$$

Half of all possible outcomes are between 0.5 and 1

$X \sim$ Uniform(0,1): A Continuous Random Variable

All values are equally likely



Possible values are between 0 and 1

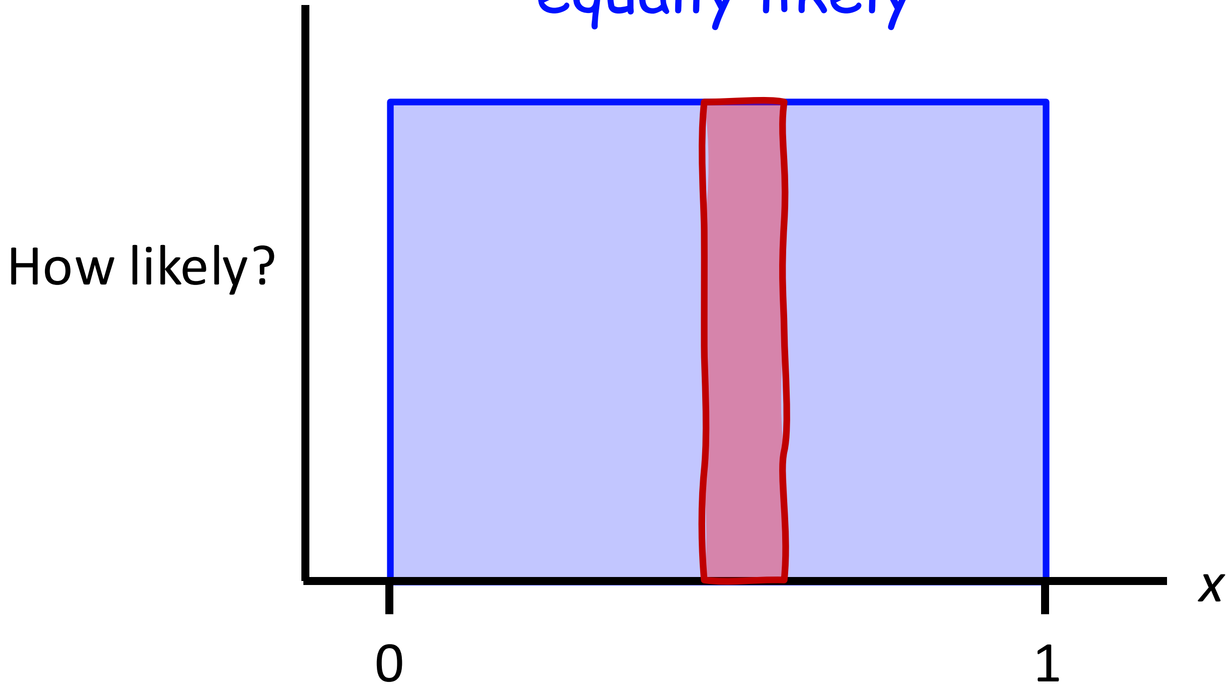
$$P(0 \leq X \leq 1) = 1$$

$$P(0.5 \leq X \leq 1) = 0.5$$

$$P(0.5 \leq X \leq 0.6) = ?$$

$X \sim$ Uniform(0,1): A Continuous Random Variable

All values are equally likely



Possible values are between 0 and 1

$$P(0 \leq X \leq 1) = 1$$

$$P(0.5 \leq X \leq 1) = 0.5$$

$$P(0.5 \leq X \leq 0.6) = 0.1$$

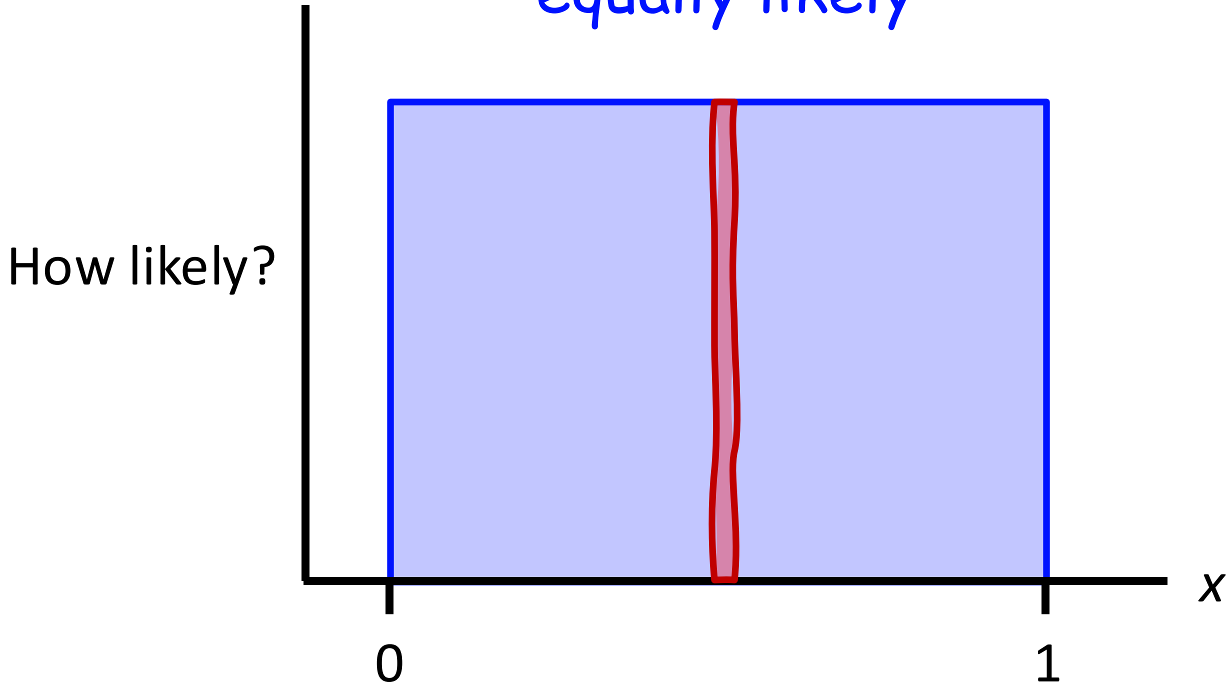
1/10 of all possible outcomes are between 0.5 and 0.6

So far, the pattern looks like:

P(start ≤ X ≤ end) = end - start

$X \sim$ Uniform(0,1): A Continuous Random Variable

All values are equally likely



Possible values are between 0 and 1

$$P(0 \le X \le 1) = 1$$

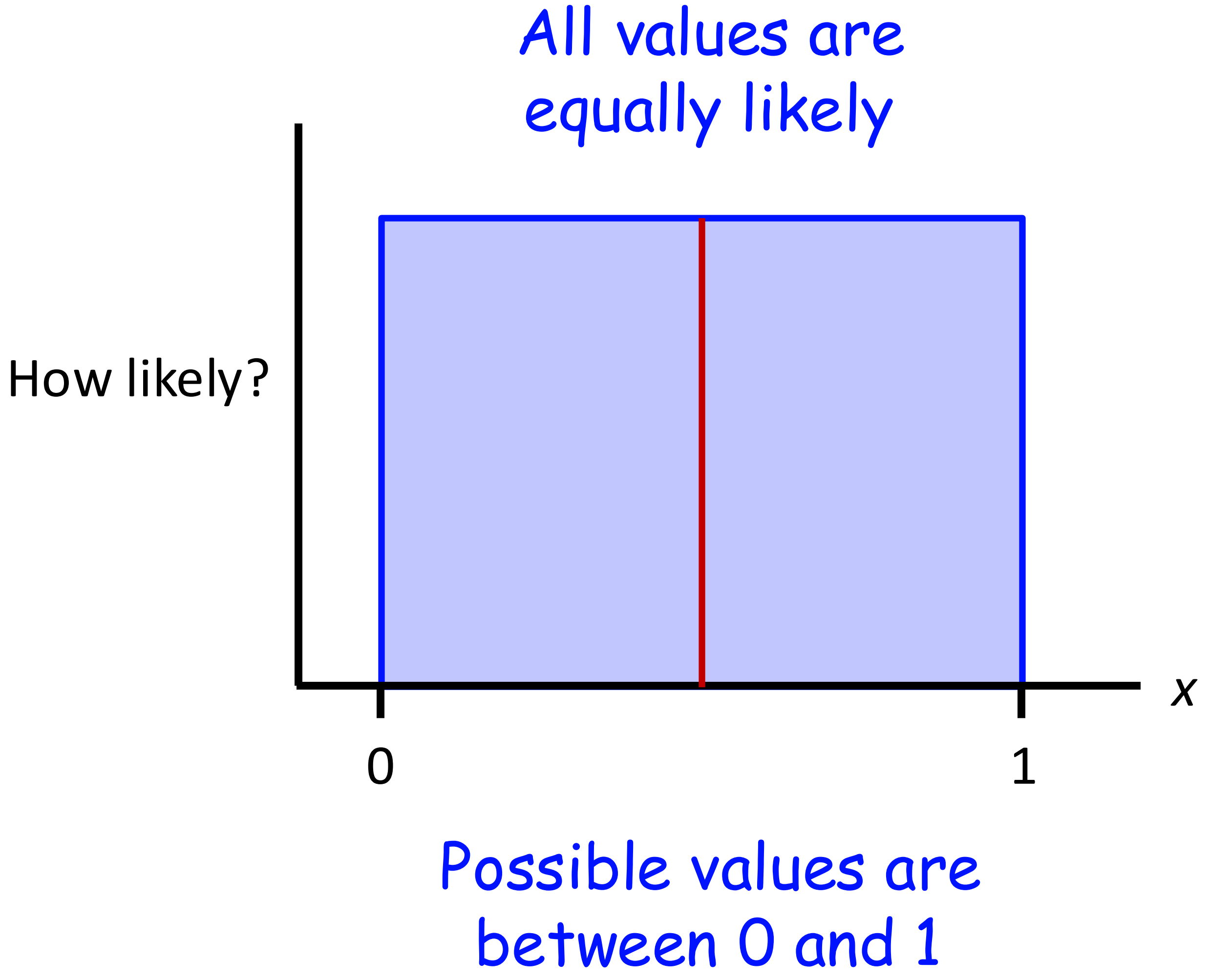
$$P(0.5 \le X \le 1) = 0.5$$

$$P(0.5 \le X \le 0.6) = 0.1$$

$$P(0.5 \le X \le 0.5001) = 0.0001$$

As we get more precise, probabilities keep shrinking…

$X \sim$ Uniform(0,1): A Continuous Random Variable



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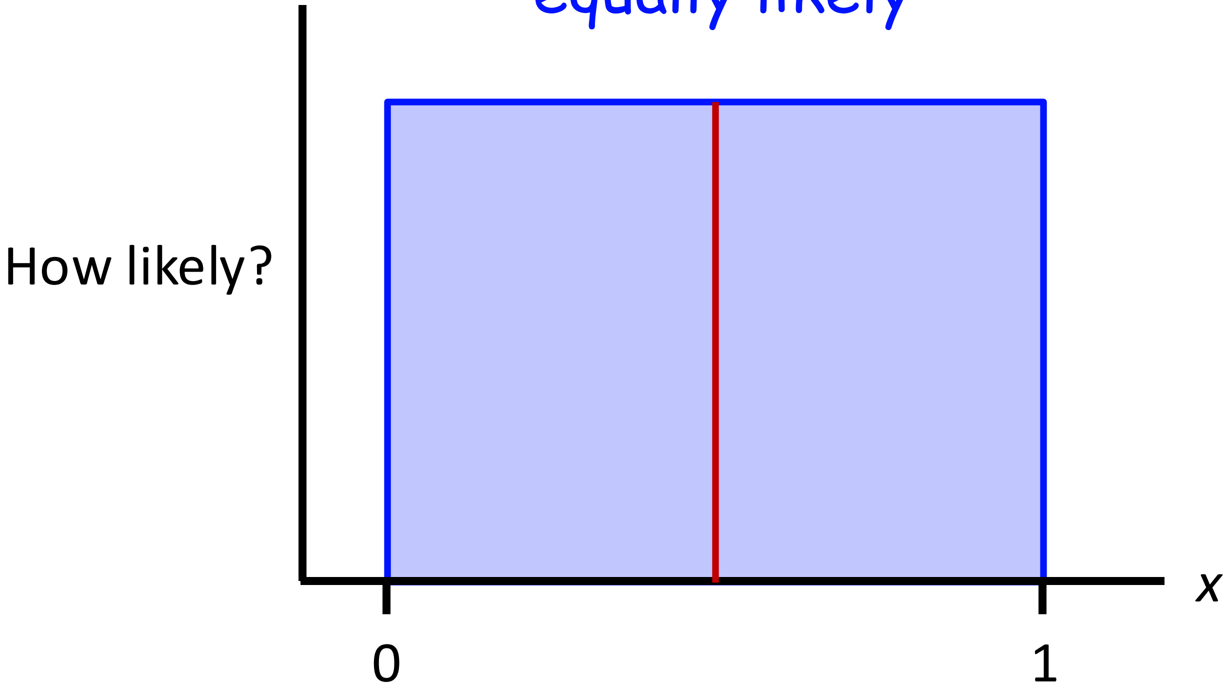
$$P(0.5 \leq X \leq 0.6) = 0.1$$

$$P(0.5 \leq X \leq 0.5001) = 0.0001$$

$$P(X = 0.5) = ?$$

$X \sim \text{Uniform}(0,1)$: A Continuous Random Variable

All values are equally likely



Possible values are between 0 and 1

$$P(0 \le X \le 1) = 1$$

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$$P(0.5 \le X \le 0.6) = 0.1$$

$$P(0.5 \le X \le 0.5001) = 0.0001$$

$$P(X = 0.5) = 0$$

The probability of any exact outcome, with infinite precision…is zero

$X \sim \text{Uniform}(0,1)$: A Continuous Random Variable

All values are equally likely

$$P(0 \leq X \leq 1) = 1$$

$$= 0.5$$

$$= 0.1$$

$$= 0.0001$$

$$P(X = 0.5) = 0$$

How likely?

x

0

1

The probability of any continuous random variable being exactly equal to any value is 0.

$$P(X = x) = 0, \text{ for all } x$$

Possible values are between 0 and 1

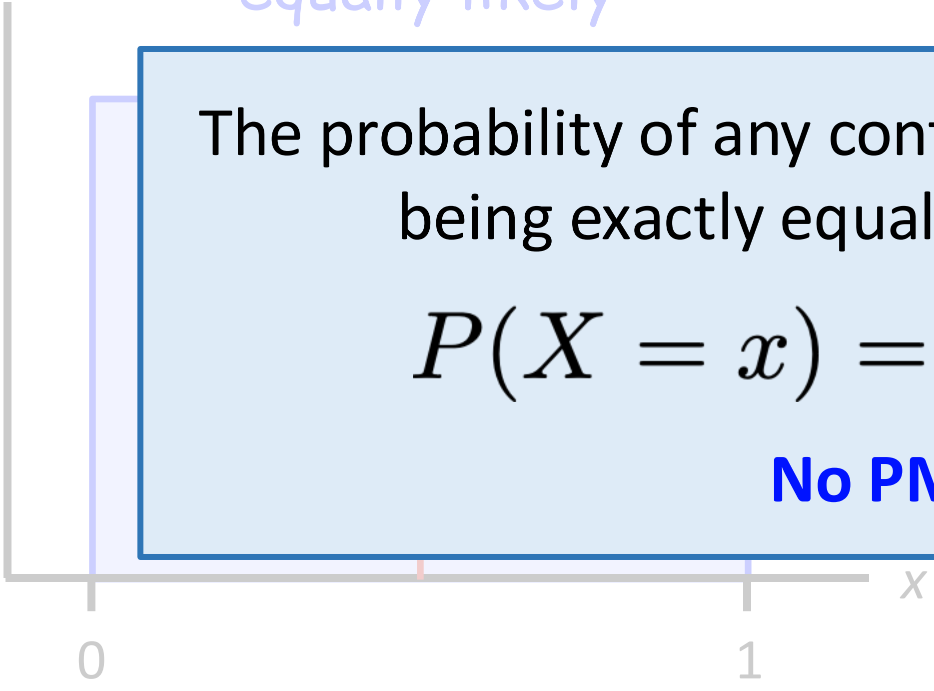
The probability of any exact outcome, with infinite precision…is zero

$X \sim$ Uniform(0,1): A Continuous Random Variable

All values are equally likely

$P(0 \leq X \leq 1) = 1$

How likely?



$= 0.5$

$= 0.1$

$= 0.0001$

$P(X = 0.5) = 0$

The probability of any continuous random variable being exactly equal to any value is 0.

$$P(X = x) = 0, \text{ for all } x$$

No PMFs!

Possible values are between 0 and 1

The probability of any exact outcome, with infinite precision…is zero

$X \sim$ Uniform(0,1): A Continuous Random Variable

$$P(0 \leq X \leq 1) = 1$$

$$P(0.5 \leq X \leq 1) = 0.5$$

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$$P(0.5 \leq X \leq 0.5001) = 0.0001$$

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ability of any exact outcome,
infinite precision…is zero

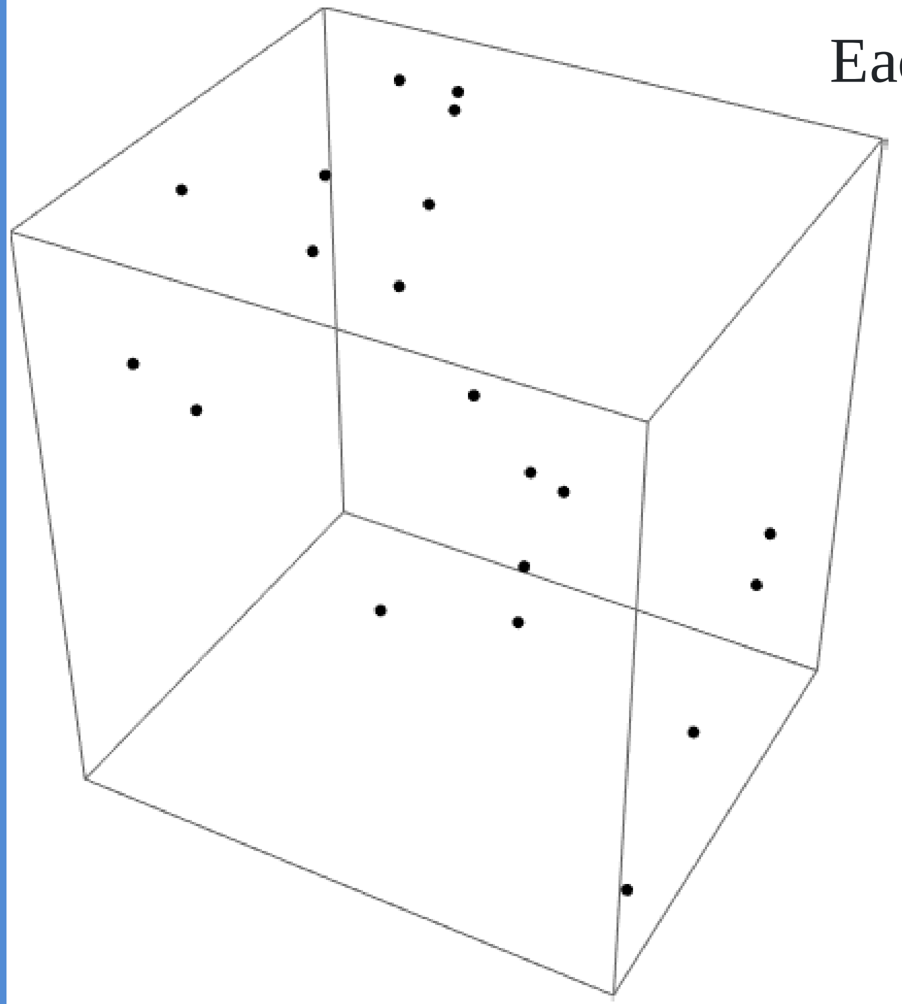
The only way to talk about probabilities of outcomes for *continuous* random variables is using ranges of possible values.

Curse of Dimensionality

A random *point* of dimension d is a list of *d* random values: $[X_1 \ldots X_d]$

$X_i \sim \text{Uni}(0, 1)$ for all i

Each value X_i is independent of other values



X_i is close to an edge if X_i is less than 0.01 **or** X_i is greater than 0.99. What is the probability that X_i is close to an edge?

Curse of Dimensionality

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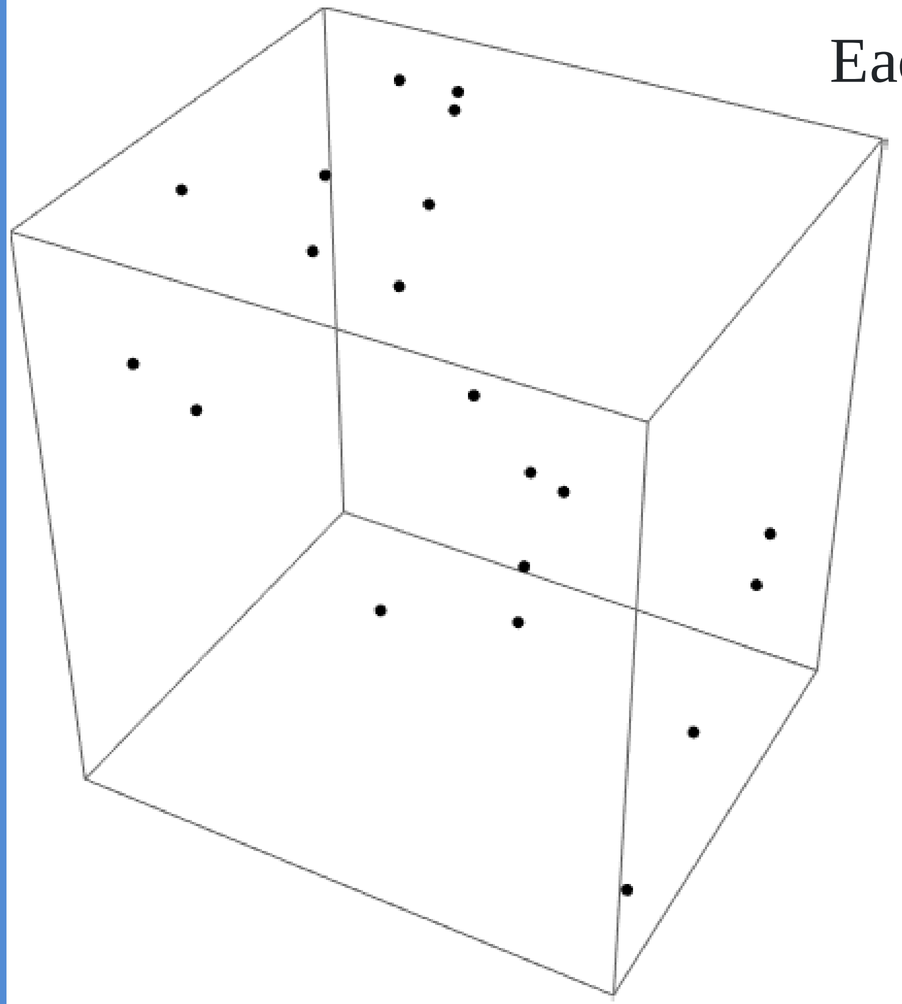
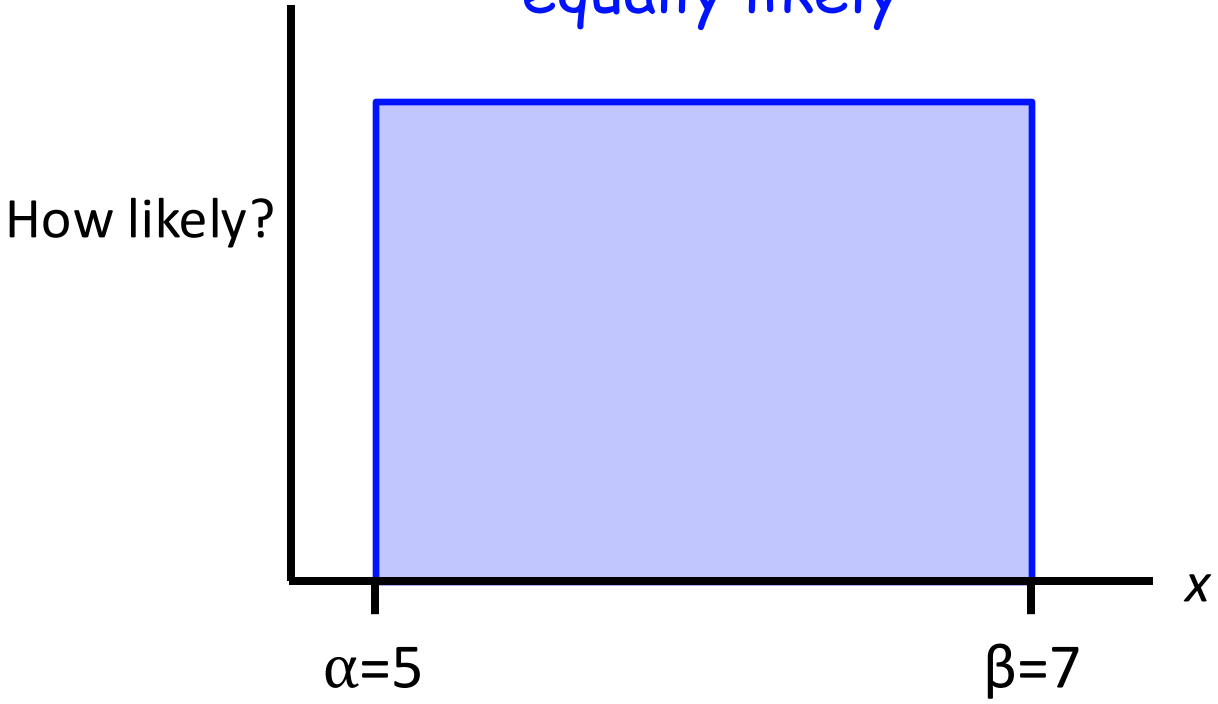


A random *point* $[X_1 \dots X_{100}]$ of dimension 100 is close to an edge if *any* of its values are close to an edge. What is the probability that a 100 dimensional point is close to an edge?

$X \sim$ Uniform(α, β): More General Case

All values are equally likely

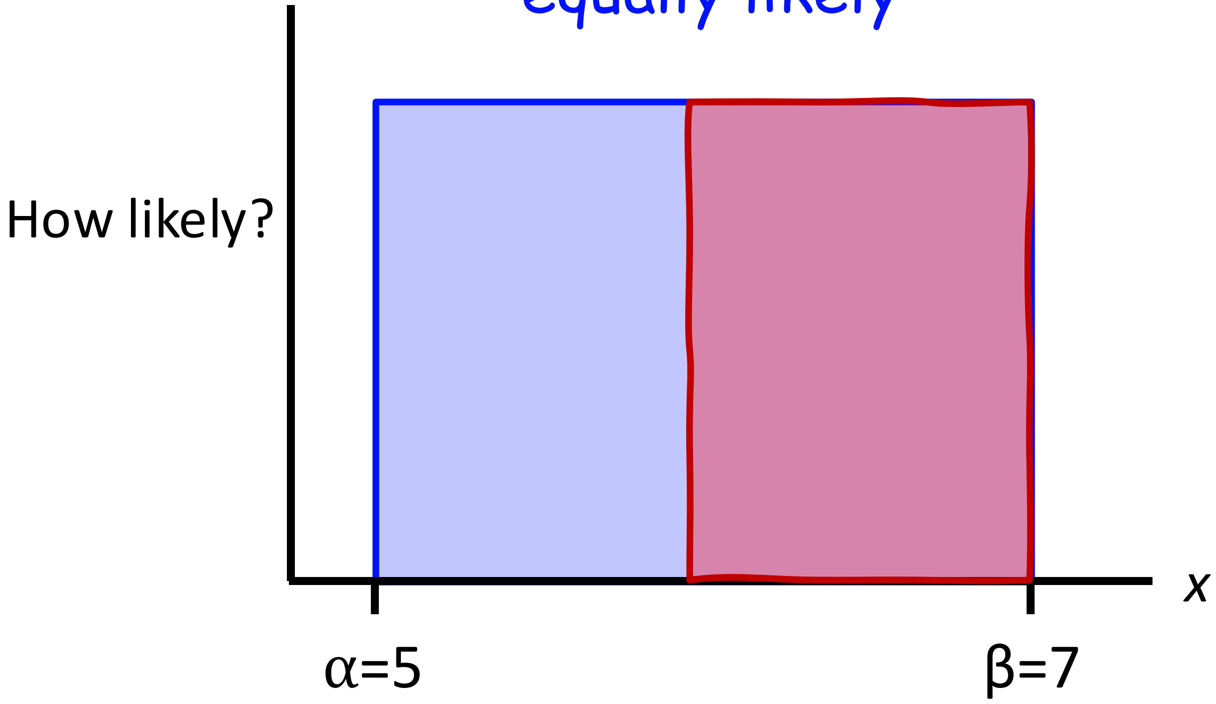


Possible values are now between α and β

$$P(5 \leq X \leq 7) = 1$$

$X \sim$ Uniform(α, β): More General Case

All values are equally likely

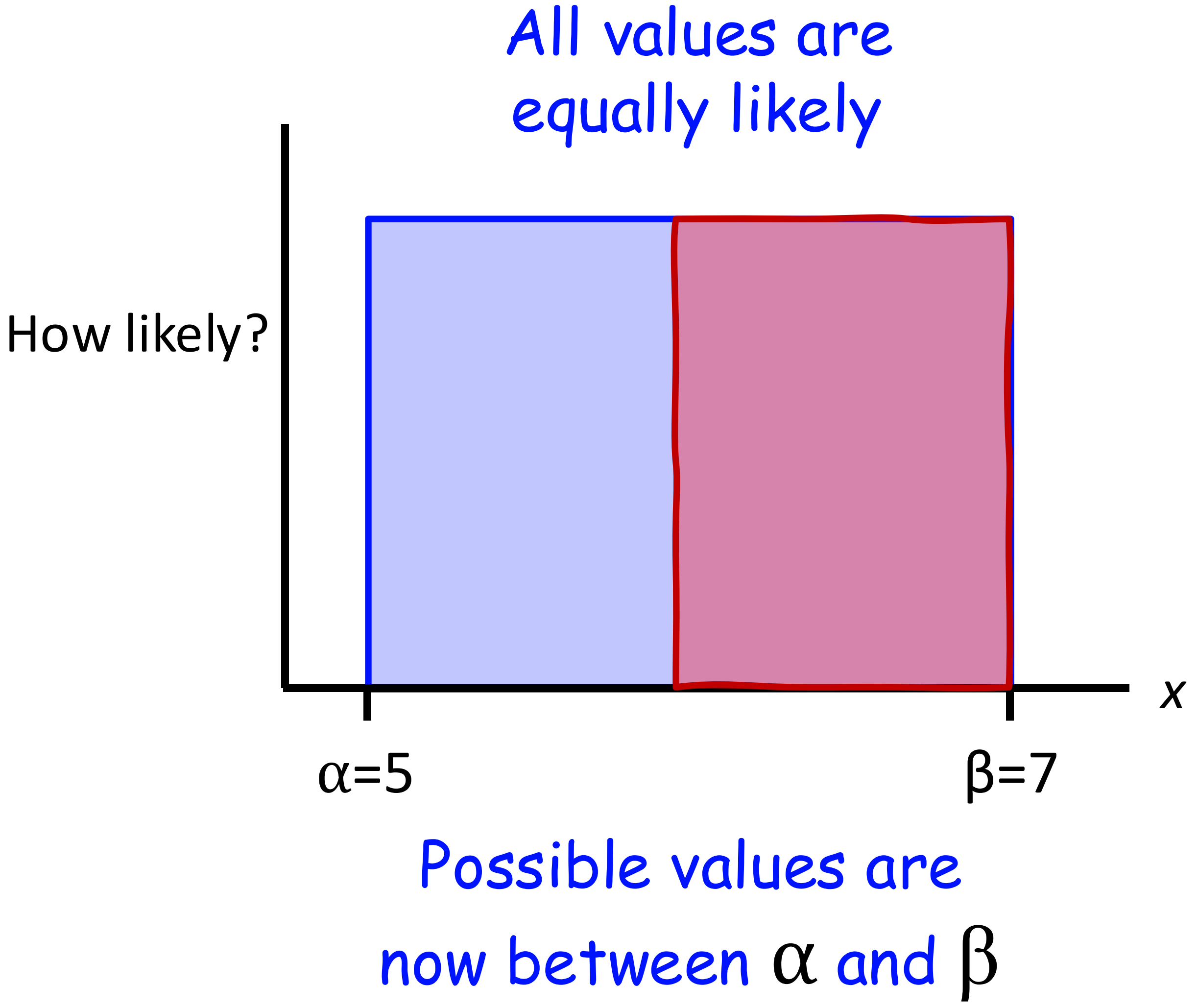


Possible values are now between α and β

$$P(5 \leq X \leq 7) = 1$$

$$P(6 \leq X \leq 7) = \ ?$$

$X \sim$ Uniform(α, β): More General Case



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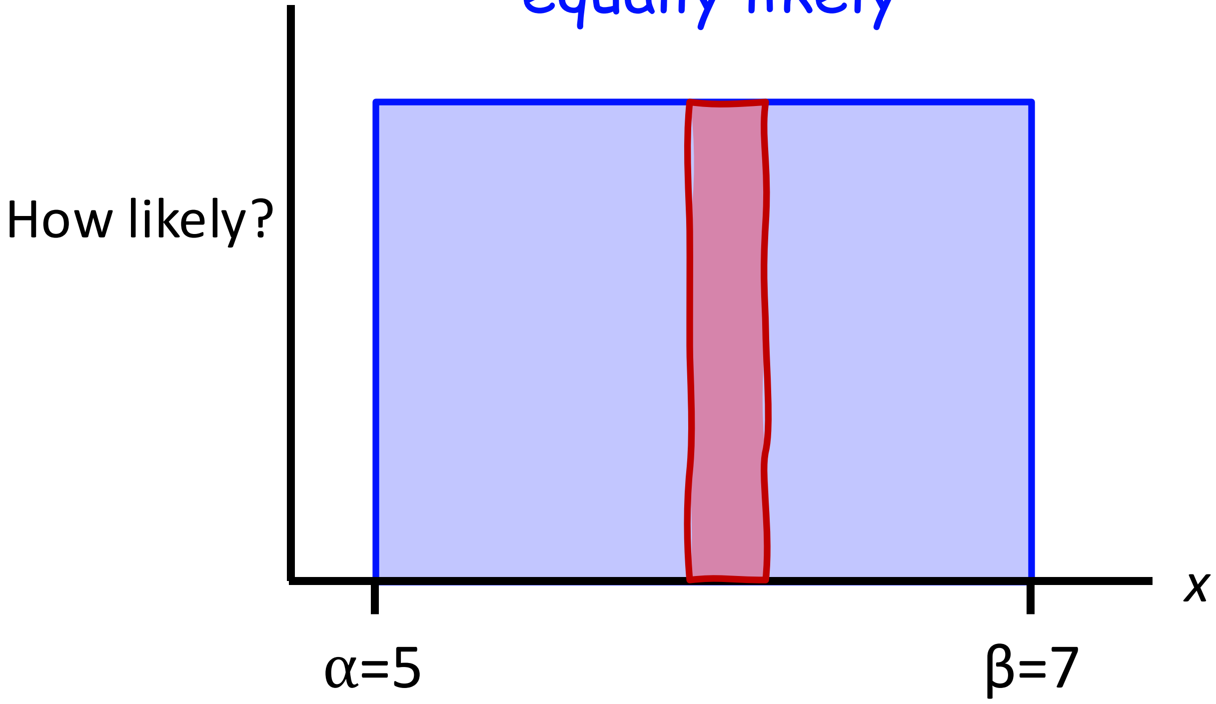
$X \sim$ Uniform(α, β): More General Case

All values are equally likely



Possible values are now between α and β

$$P(5 \leq X \leq 7) = 1$$

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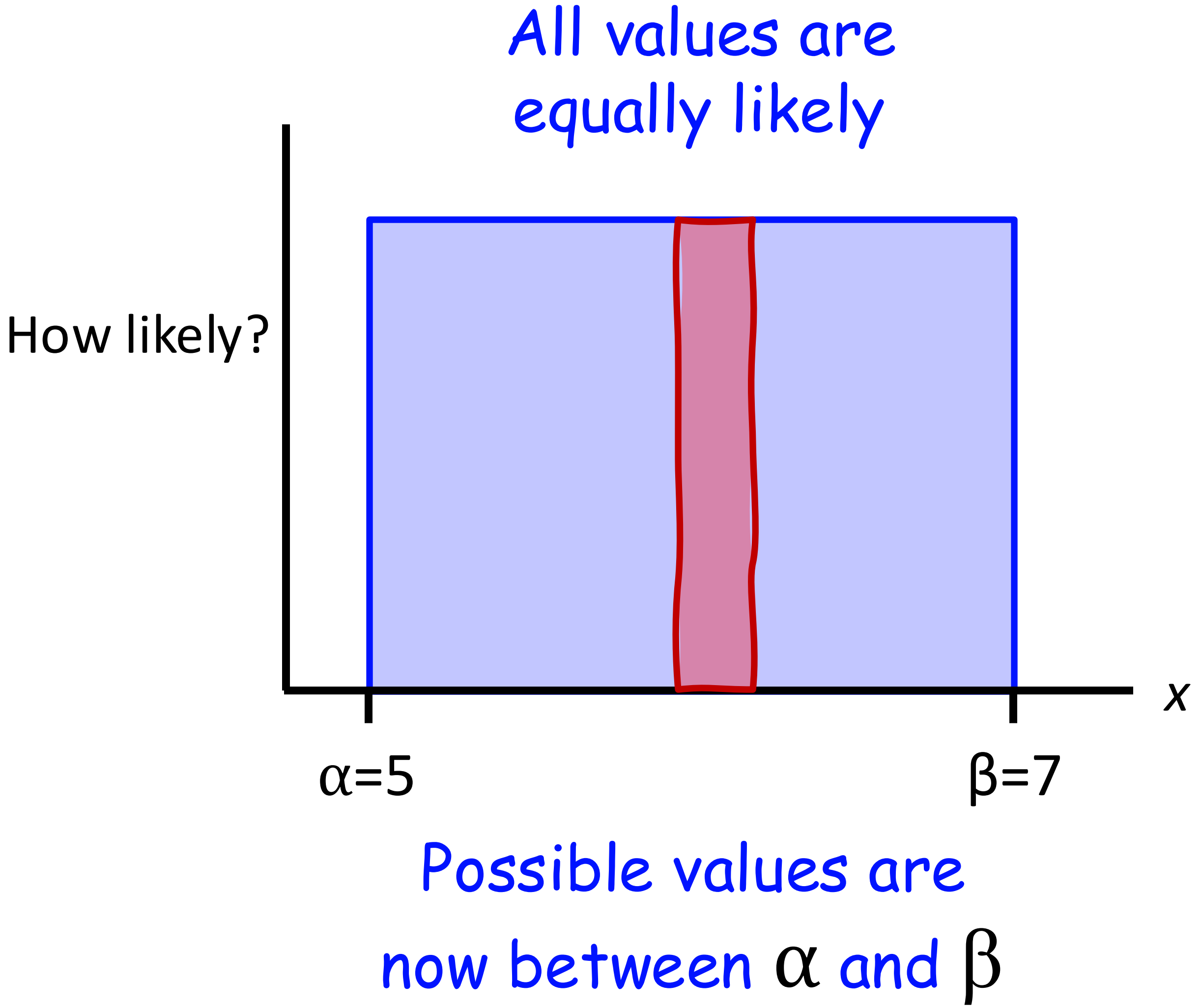
$$P(6 \leq X \leq 6.1) = \ ?$$

$X \sim$ Uniform(α, β): More General Case



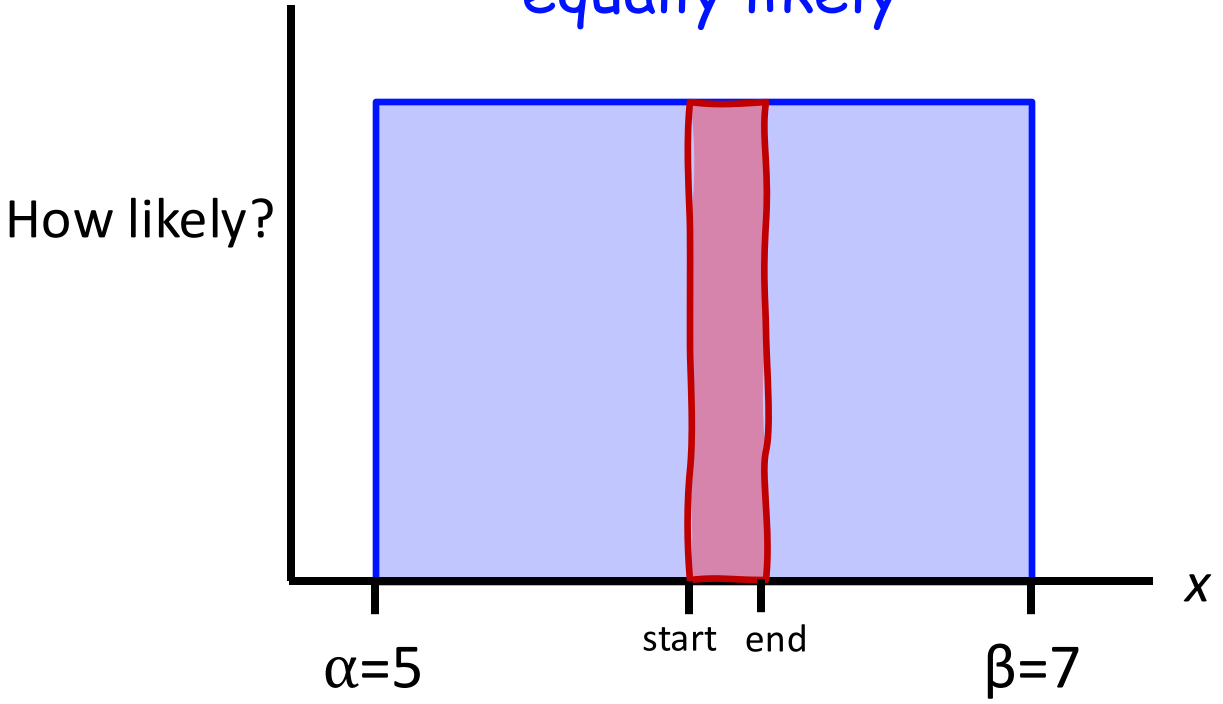
$$P(5 \leq X \leq 7) = 1$$

$$P(6 \leq X \leq 7) = 0.5$$

$$P(6 \leq X \leq 6.1) = 0.05$$

$X \sim \text{Uniform}(\alpha, \beta)$: More General Case

All values are equally likely



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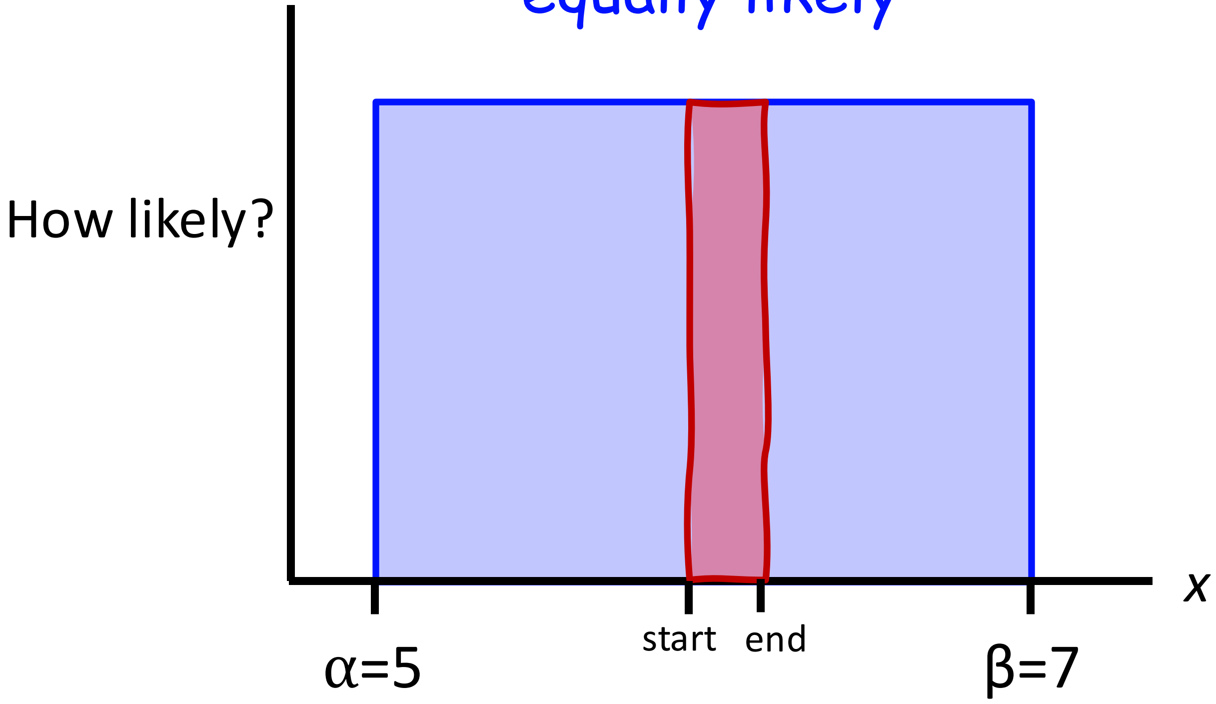
For Uniform(0,1):

P(start ≤ X ≤ end) = end - start

Does that still work?

$X \sim$ Uniform(α, β): More General Case

All values are equally likely



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For Uniform(0,1):

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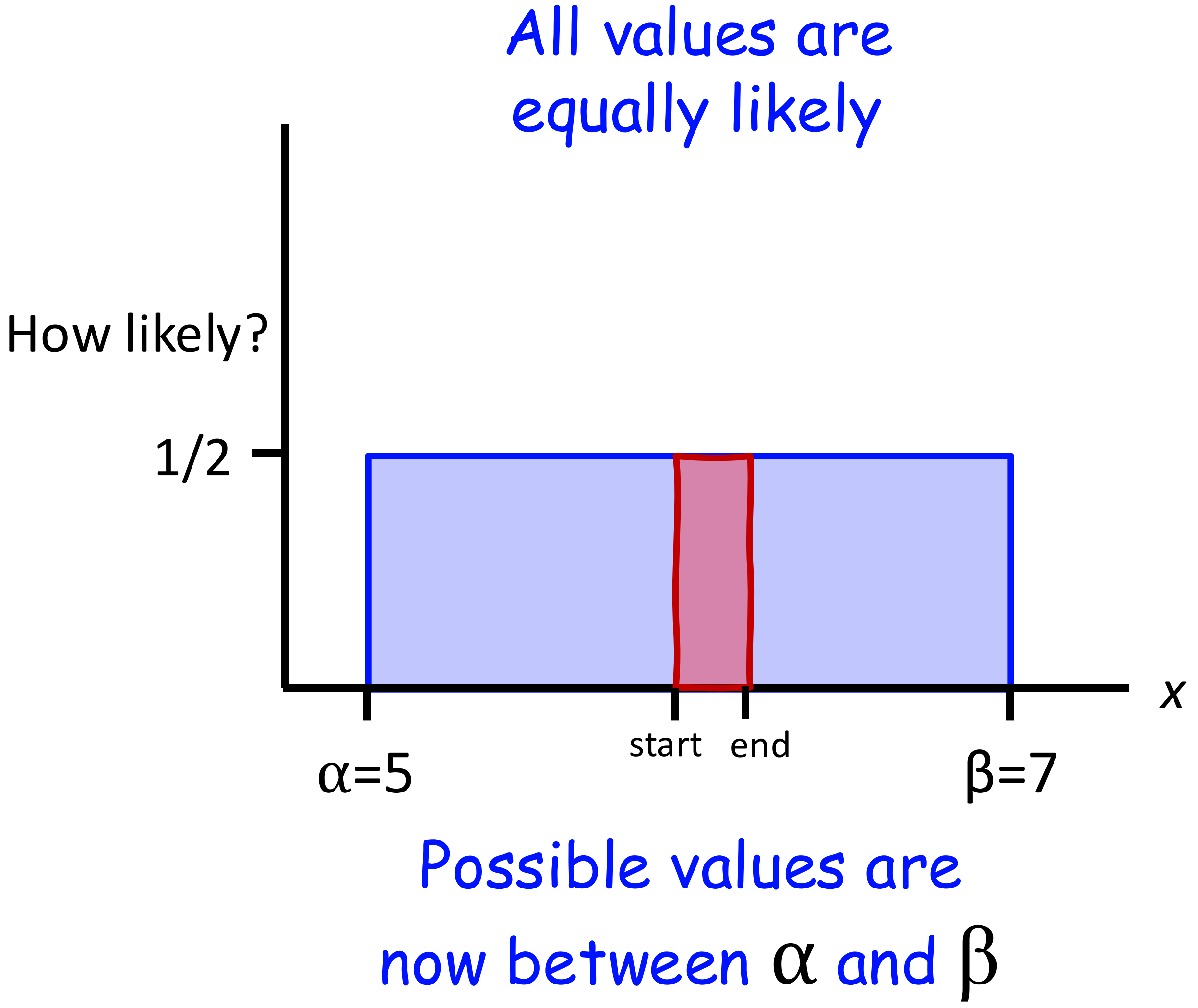
Does that still work? No!

Need to divide by 2?

$X \sim$ Uniform(α, β): More General Case



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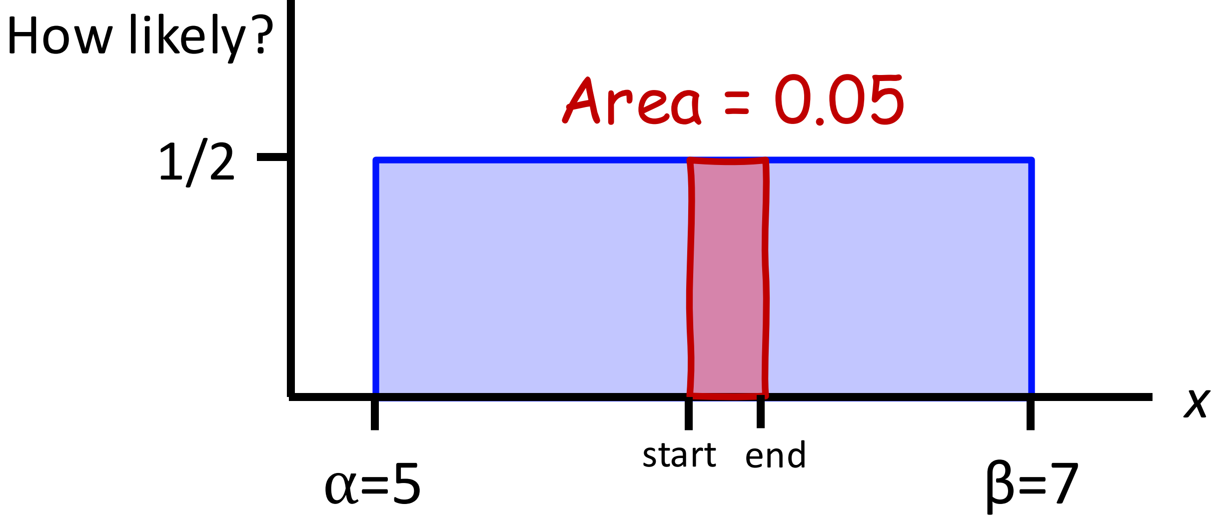
For Uniform(α, β):

$$P(\text{start} \le X \le \text{end}) = \frac{\text{end - start}}{\beta - \alpha}$$

$X \sim$ Uniform(α, β): More General Case

All values are equally likely



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For Uniform(α, β):

$$P(\text{start} \leq X \leq \text{end}) = \frac{\text{end - start}}{\beta - \alpha}$$

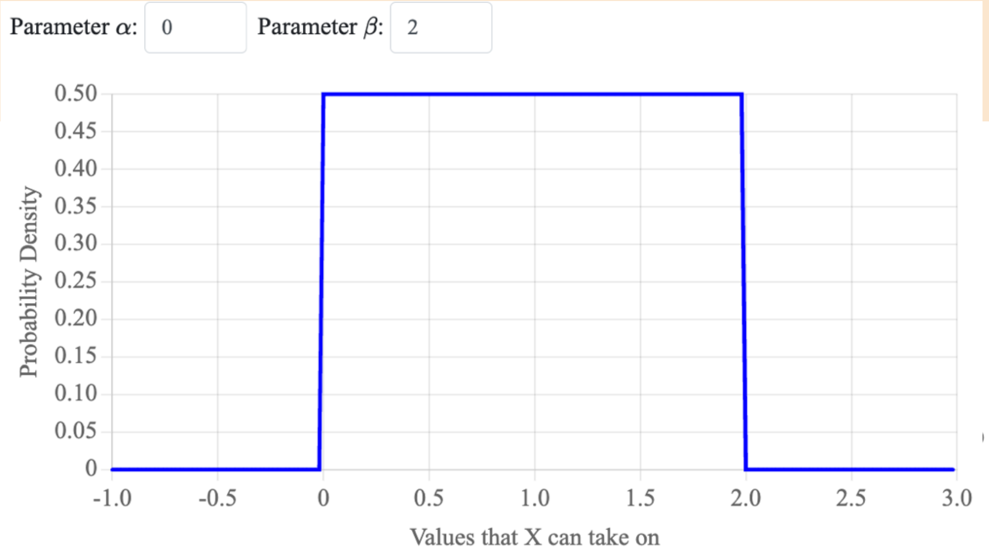
If we set y = 1/2 between α and β, then probabilities are "slices of the whole box"

Uniform Random Variable

A **Uniform** random variable X takes on a value, with equal likelihood between α and β.

$$X \sim \text{Uni}(\alpha, \beta) \qquad P(x_1 < X < x_2) = \frac{x_2 - x_1}{\beta - \alpha}$$

Support: $[\alpha, \beta]$



Examples:

- Result of python random()
- Random points

Can we generalize to other continuous random variables?

Riding the Marguerite

Riding the Marguerite

You're running to the bus stop. You don't know exactly when the bus arrives.

You have a probability distribution for bus arrival times -- some times are more likely than others.

You show up at 2:15pm. What is P(wait < 5 min)?

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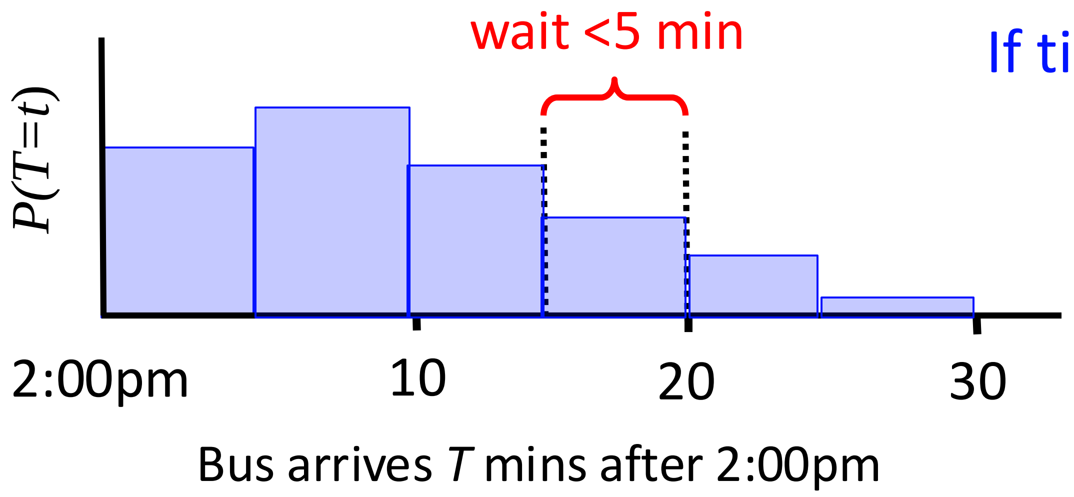

If time was discrete: a PMF could look like this.

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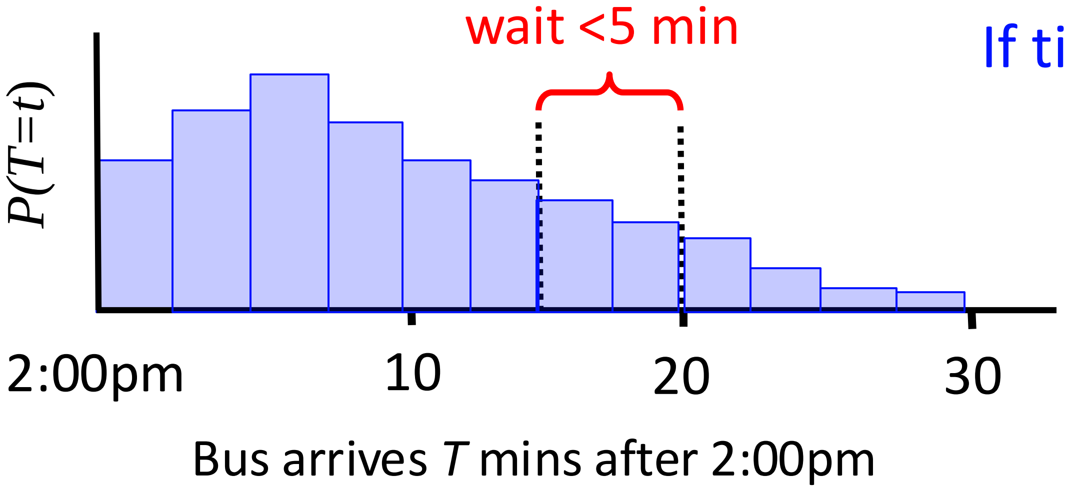

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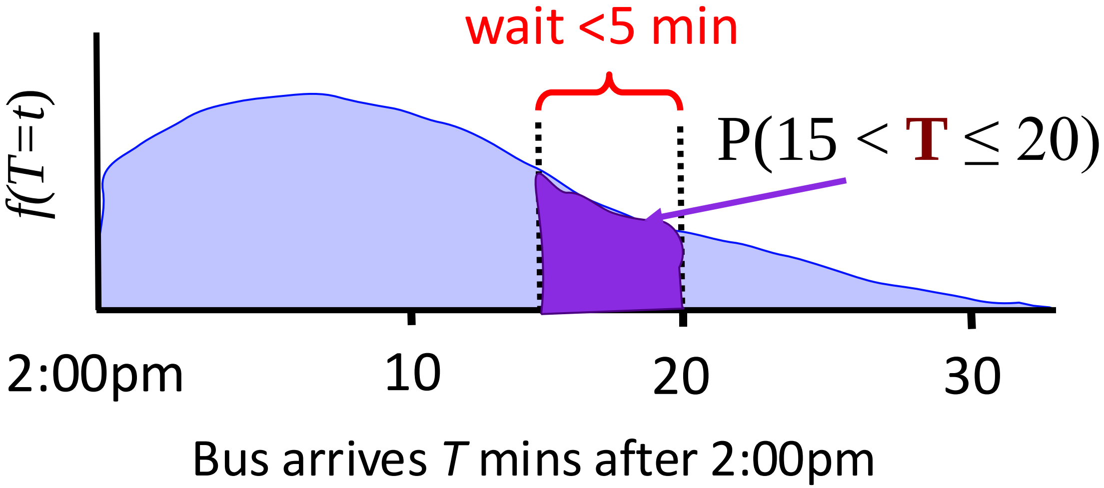

When interval sizes tend towards 0:

- Time is now a **continuous** variable
- The **probability mass function (PMF)** becomes a **derivative** called a **probability density function (PDF)**
- Probability are now calculated as **area under the curve**

Time For Integrals!!!!

Probability Density Function

The **probability density function** (PDF) of a continuous random variable represents the relative likelihood of various values.

Units of probability *divided by units of X*.
Integrate it to get probabilities!

$$P(a < X < b) = \int_{x=a}^{b} \boxed{f(X = x)}\, dx$$

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This is another way to write the PDF

Probability Density Function

The **probability density function** (PDF) of a continuous random variable represents the relative likelihood of various values.

Units of probability *divided by units of X*.
Integrate it to get probabilities!

$$P(a < X < b) = \int_{x=a}^{b} f(X = x)dx$$

PDFs like $f(X = x)$ vs. PMFs like $P(X = x)$

$$P(X = x)$$

"The probability that a **discrete** random variable X takes on the value *x*."

$$f(X = x)$$

"The ***derivative*** of the probability that a **continuous** random variable X takes at the value *x*."

They are both measures of how ***likely*** *X is to take on the value x.*
Sometimes called the ***distribution*** *function.*

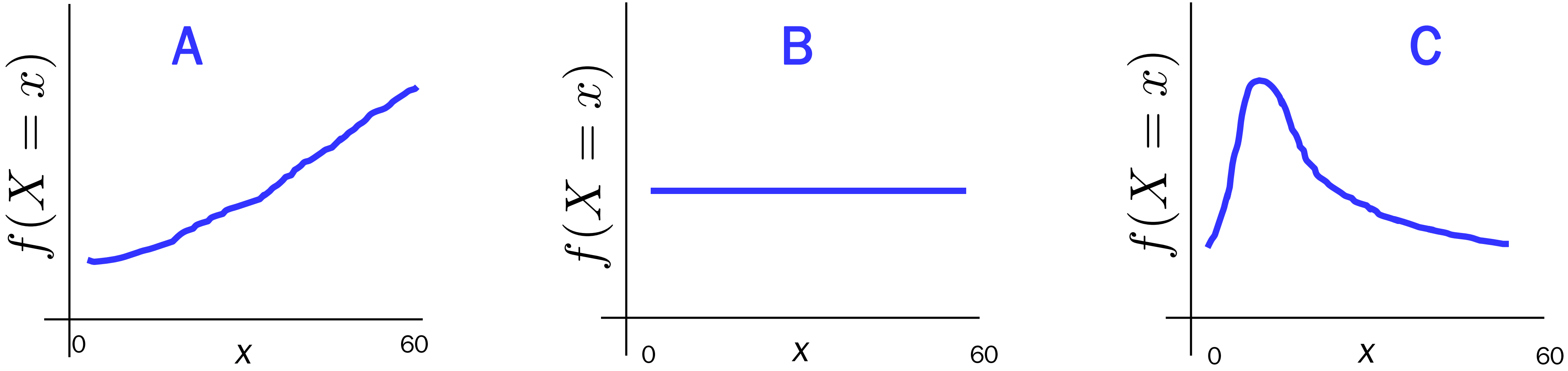
What do you get if you integrate over a probability *density* function?

A probability!

The Relative Values of PDFs Are Meaningful

Probability density functions are derivatives that articulate ***relative*** belief.

Let X be the # of minutes after 2pm that the bus arrives at a stop.



Which of these represent that the bus's arrival
is more likely to be close to 3:00pm?

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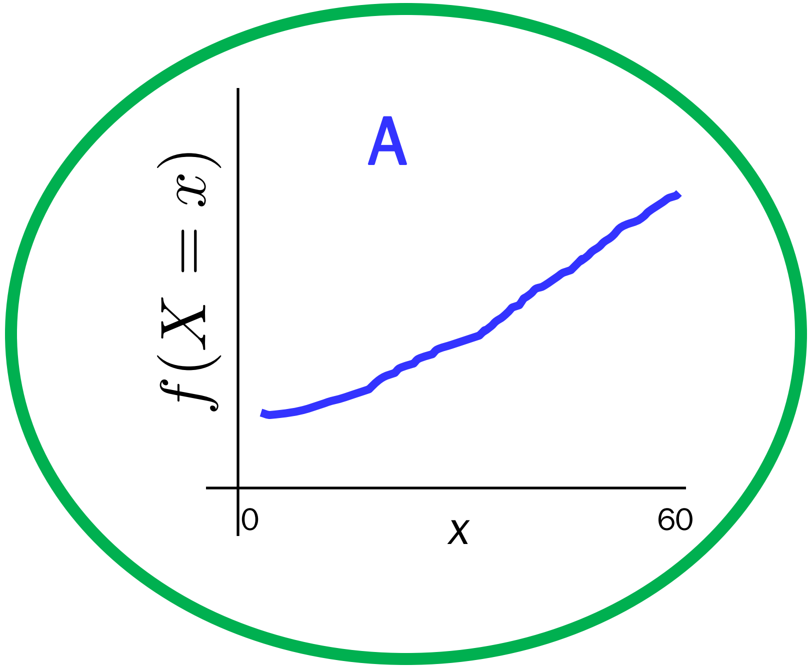

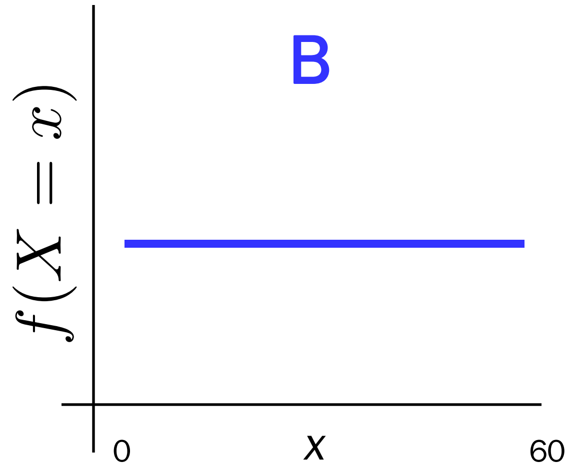

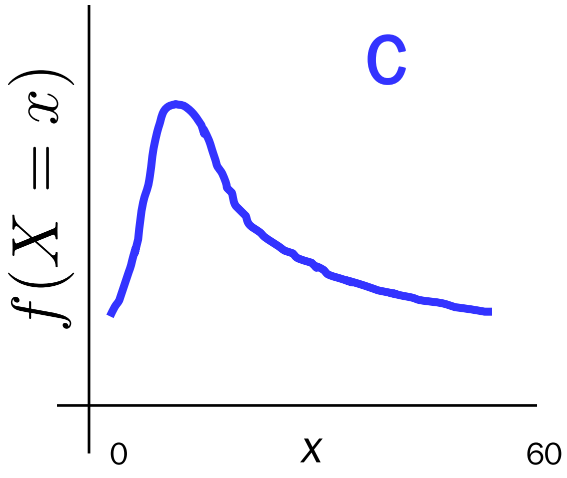

Which of these represent that the bus's arrival is more likely to be close to 3:00pm?

The relative value of probability densities is meaningful

Truths of Probability For Continuous Random Variables

Truth 1: $$P(a < X < b) = \int_{x=a}^{b} f(X = x)\ dx$$ Area under the curve!

Truths of Probability For Continuous Random Variables

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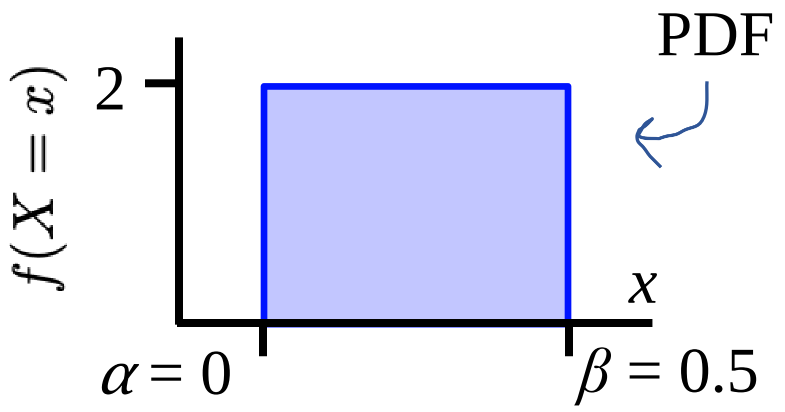
Truth 2: $$0 \leq \int_{x=a}^{b} f(X = x)\ dx \leq 1$$ Since the integral is a probability (Axiom 1)

Can a PDF ever have a value > 1?
Yes!



Truths of Probability For Continuous Random Variables

Truth 1:

$$P(a < X < b) = \int_{x=a}^{b} f(X = x)\ dx$$

Area under the curve!

Truth 2:

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Truth 3:

$$\int_{x=-\infty}^{\infty} f(X = x)\ dx = 1$$

That's all possible values (Axiom 2)

Truths of Probability For Continuous Random Variables

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Truth 3:

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That's all possible values (Axiom 2)

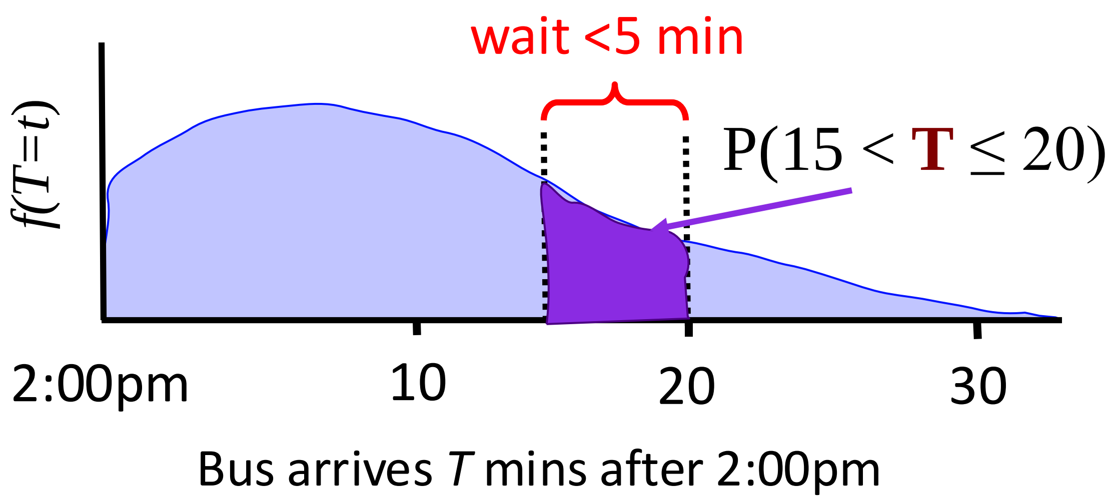
Truth 4:

$$P(X = x) = 0$$

What a time to be alive…

PDFs Need an Integral

What is the probability that the bus arrives at: 12.12332343234 mins after 2pm?



What do you get if you integrate over a probability *density* function?

A probability!

Pedagogic Pause

Quantum Example

Consider a random 5000x5000 matrix, where each element in the matrix is Uniform(0,1). What is the probability that a selected eigenvalue (λ) of the matrix is greater than 0?*

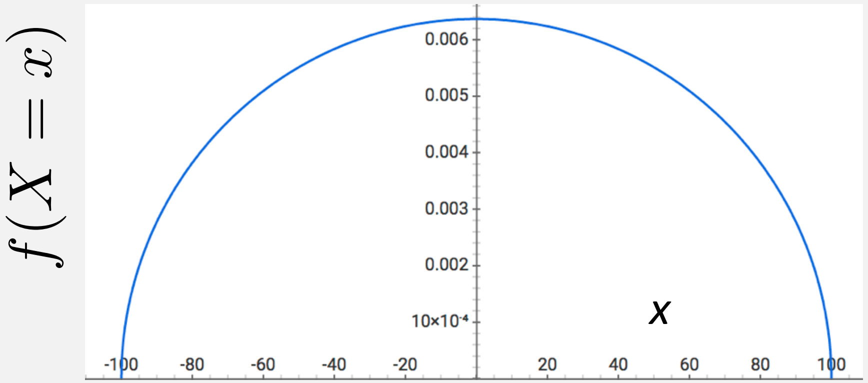
* With help from Wigner, Chris is rephrased this problem

Rephrased as a Standard Continuous Problem

Let X be a continuous random variable[1]

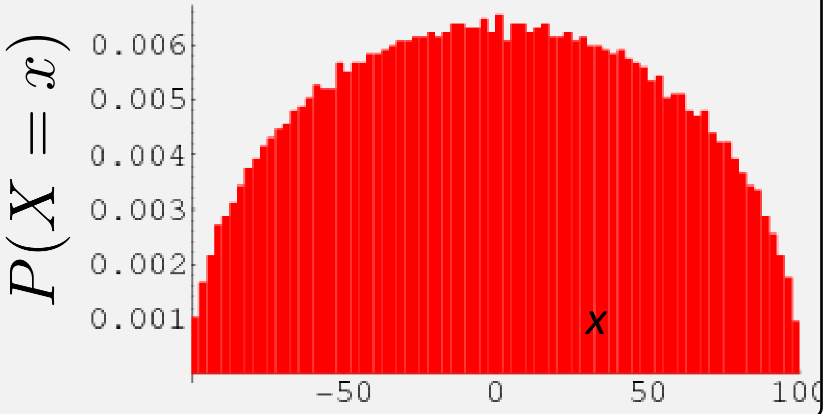
Theory

$$f(X = x) = \frac{1}{15708}\sqrt{100^2 - x^2}$$



Practice

From simulations



$$P(X > 0) = ?$$

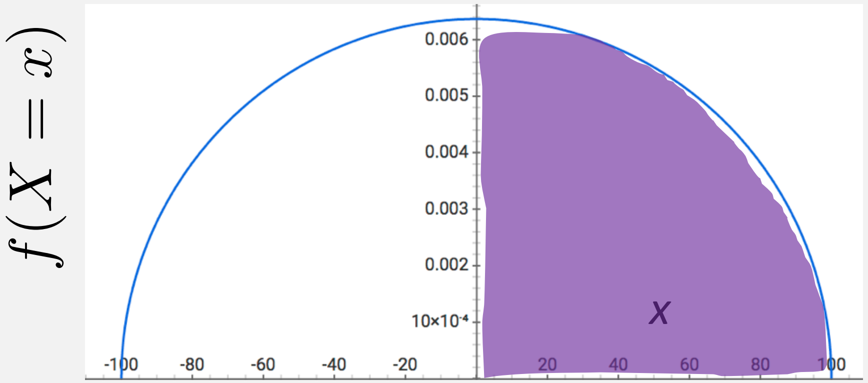
https://en.wikipedia.org/wiki/Wigner_semicircle_distribution

Simple Example from Quantum Physics

Let X be a continuous random variable[1]

Theory

$$f(X = x) = \frac{1}{15708}\sqrt{100^2 - x^2}$$



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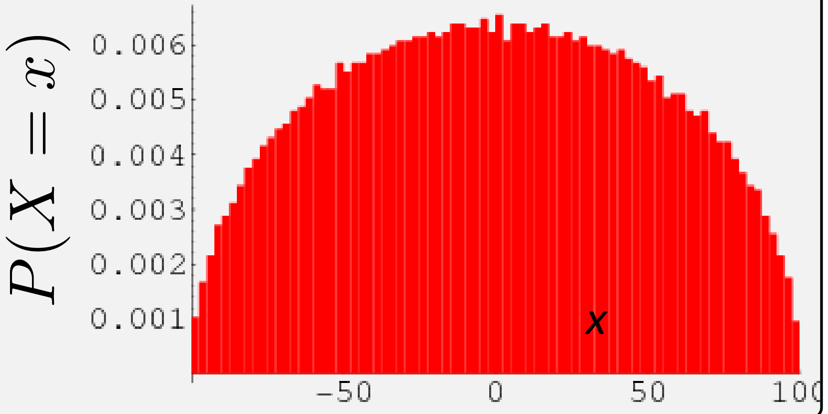

Approach #1: Integrate over the PDF

$$P(X > 0) = \int_{0}^{100} f(X = x)\, dx$$

Simple Example from Quantum Physics

Let X be a continuous random variable[1]

Theory

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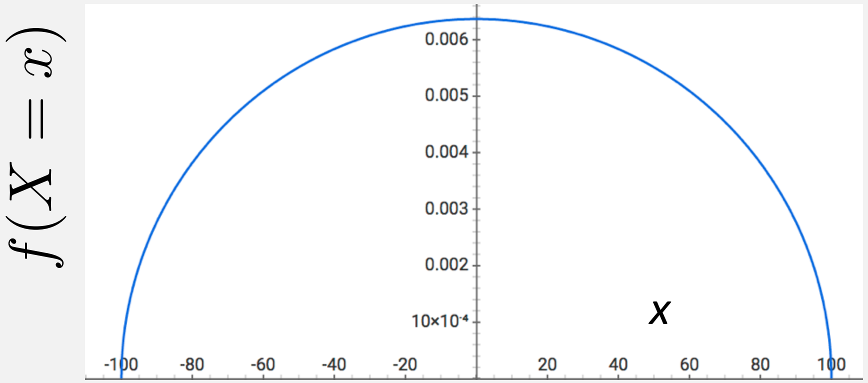
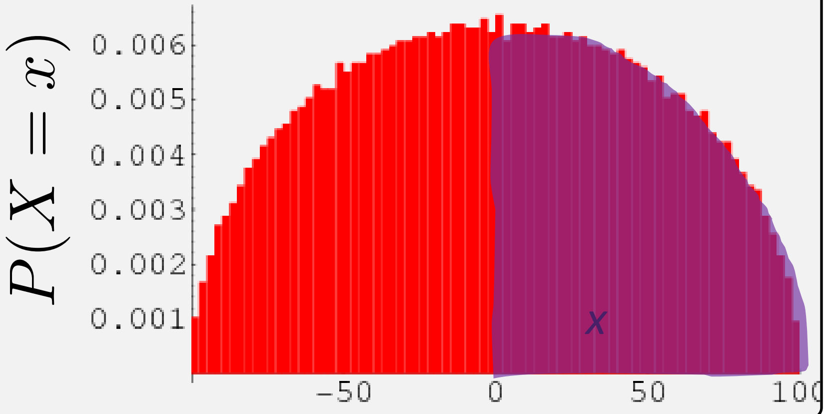


Practice

From simulations



Approach #2: Discrete Approximation

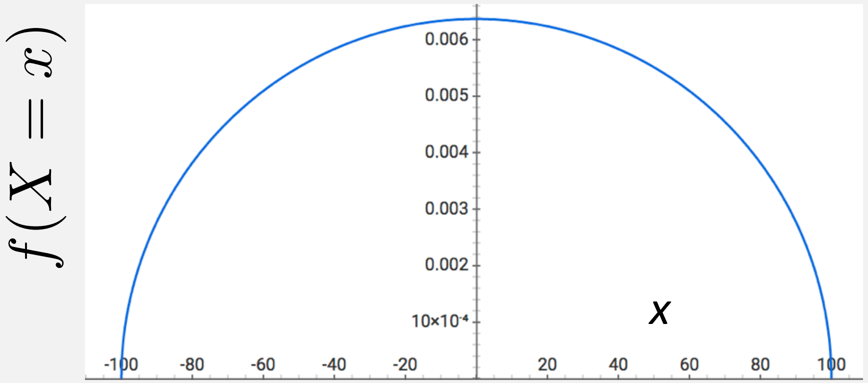
$$P(X > 0) \approx \sum_{i=0}^{100} P(X = i)$$

Simple Example from Quantum Physics

Let X be a continuous random variable[1]

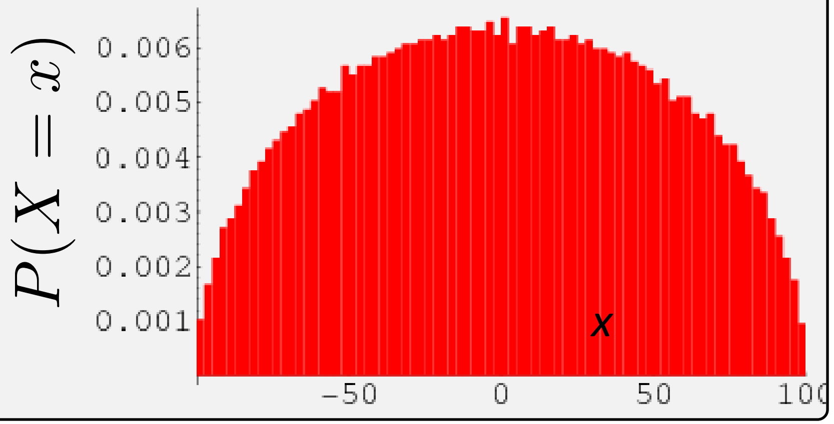
Theory

$$f(X = x) = \frac{1}{15708}\sqrt{100^2 - x^2}$$



Practice

From simulations



Approach #3: Know Semi-Circles

$$P(X > 0) = \frac{1}{2}$$

You are ready for the classic continuous random variables

Exponential Random Variable

Consider an experiment that lasts a duration of time until success occurs.

def An **Exponential** random variable X is the amount of time until success.

$X \sim \text{Exp}(\lambda)$

Support: $[0, \infty)$

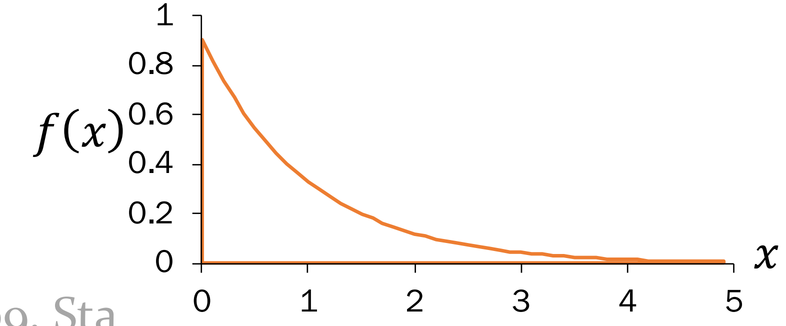
PDF $$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Expectation $$E[X] = \frac{1}{\lambda}$$

Variance $$\text{Var}(X) = \frac{1}{\lambda^2}$$

Examples:

- Time until next earthquake
- Time for request to reach web server
- Time until end of cell phone contract



The process for an Exponential and a Poisson are the same. So is the parameter λ. The question is different

1906 Earthquake
Magnitude 7.8

ILL. No. 65. MEMORIAL ARCH, WITH CHURCH IN BACKGROUND, STANFORD UNIVERSITY, SHOWING TYPES OF CARVED WORK WITH THE SANDSTONE

How Many Earthquakes

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the probability of **zero major earthquakes magnitude next year?**

$X =$ Number of major earthquakes next year

$$X \sim \text{Poi}(\lambda = 0.002)$$

$$P(X = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = \frac{0.002^0 e^{-0.002}}{0!} \approx 0.998$$

*In California, according to the USGS, 2015

ty

How Long Until the Next Earthquake

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the probability of **a major earthquake in the next 30 years?**

Y = Years until the next earthquake of magnitude 8.0+

$$Y \sim \text{Exp}(\lambda = 0.002)$$

$$f_Y(y) = \lambda e^{-\lambda y}$$
$$= 0.002 e^{-0.002y}$$

$$P(Y < 30) = \int_0^{30} 0.002 e^{-0.002y} \, dy$$

*In California, according to the USGS, 2015 ty

Integral Review

$$\int e^{cx}\ dx = \frac{1}{c}e^{cx}$$

How Long Until the Next Earthquake

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$$= 0.002^{-0.002y}$$

$$P(Y < 30) = \int_0^{30} 0.002 e^{-0.002y} \, dy$$
$$= 0.002 \Big[-500 e^{-0.002y} \Big]_0^{30}$$
$$= \frac{500}{500}(-e^{-0.06} + e^0) \quad \approx 0.058$$

How Long Until the Next Earthquake

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the **expected number of years until the next earthquake?**

Y = Years until the next earthquake of magnitude 8.0+

$$Y \sim \text{Exp}(\lambda = 0.002)$$

$$E[Y] = \frac{1}{\lambda} = \frac{1}{0.002} = 500$$

How Long Until the Next Earthquake

Based on historical data, major earthquakes (magnitude 8.0+) happen at a **rate of 0.002** per year*. What is the **standard deviation of years until the next earthquake?**

Y = Years until the next earthquake of magnitude 8.0+

$$Y \sim \text{Exp}(\lambda = 0.002)$$

$$\text{Var}(Y) = \frac{1}{\lambda^2} = \frac{1}{0.002^2} = 250,000 \text{ years}^2$$

$$\text{Std}(Y) = \sqrt{\text{Var}(X)} = 500 \text{ years}$$

*In California, according to the USGS, 2015

Is there a way to avoid integrals?

Cumulative Density Function

A cumulative density function (CDF) is a “closed form” equation for the probability that a random variable is less than a given value

$$F(x) = P(X < x)$$

If you learn how to use a cumulative density function, you can avoid integrals!

$$F_X(x)$$

This is also shorthand notation for the PMF

Cumulative Density Function

$$F(x) = P(X < x)$$

$$x = 2$$

$$0.03125$$

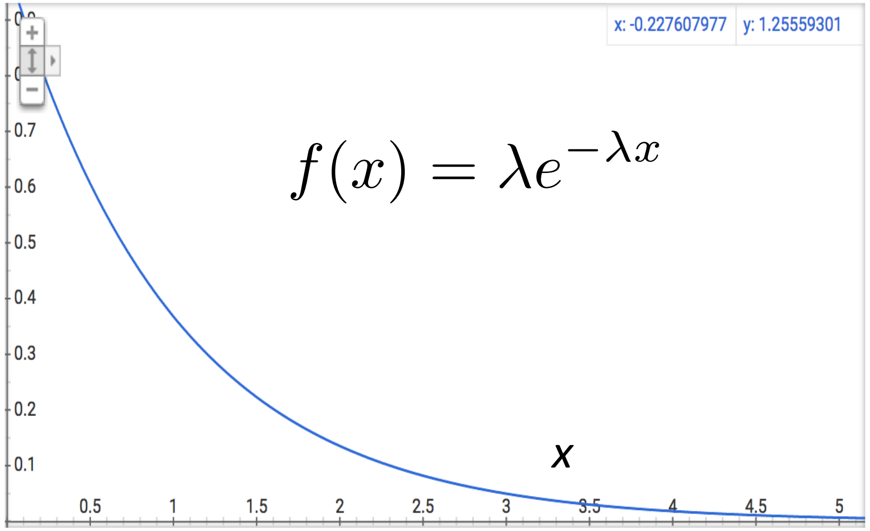
CDF of an Exponential

$$F_X(x) = 1 - e^{-\lambda x}$$

$$\begin{aligned}
P(X < x) &= \int_{y=-\infty}^{x} f(y)\, dy \\
&= \int_{y=0}^{x} \lambda e^{-\lambda y}\, dy \\
&= \frac{\lambda}{\lambda}\Big[-e^{-\lambda y}\Big]_0^x \\
&= [-e^{-\lambda x}] - [-e^{\lambda 0}] \\
&= 1 - e^{-\lambda x}
\end{aligned}$$

Using CDF Example. X is Exp(λ = 1)

Probability
density
function



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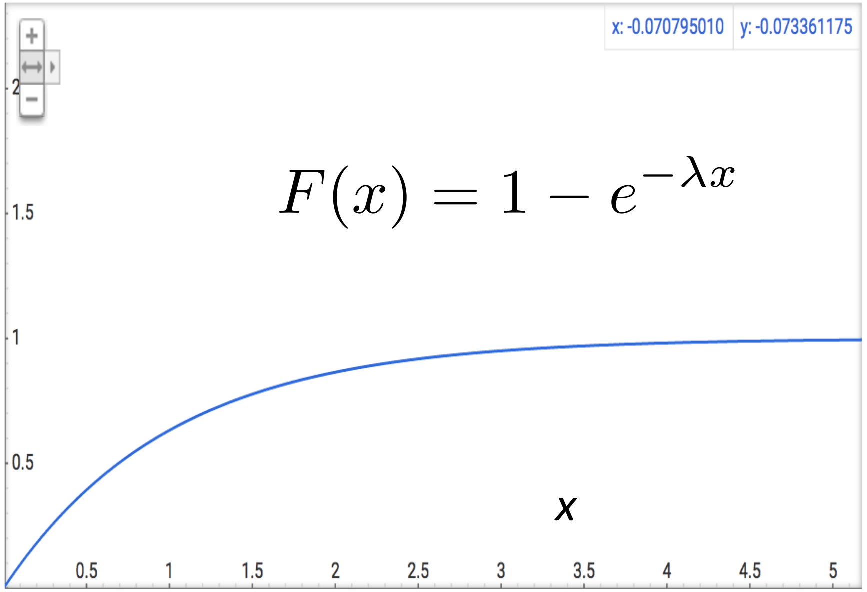
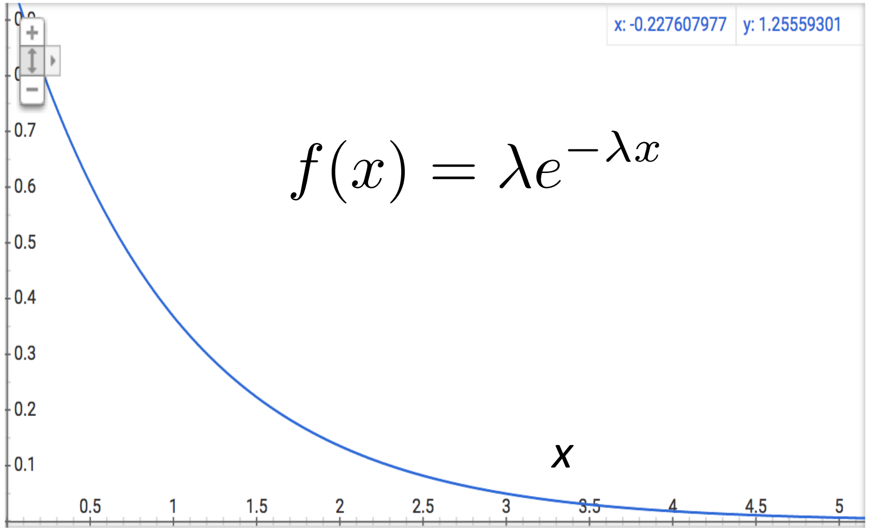


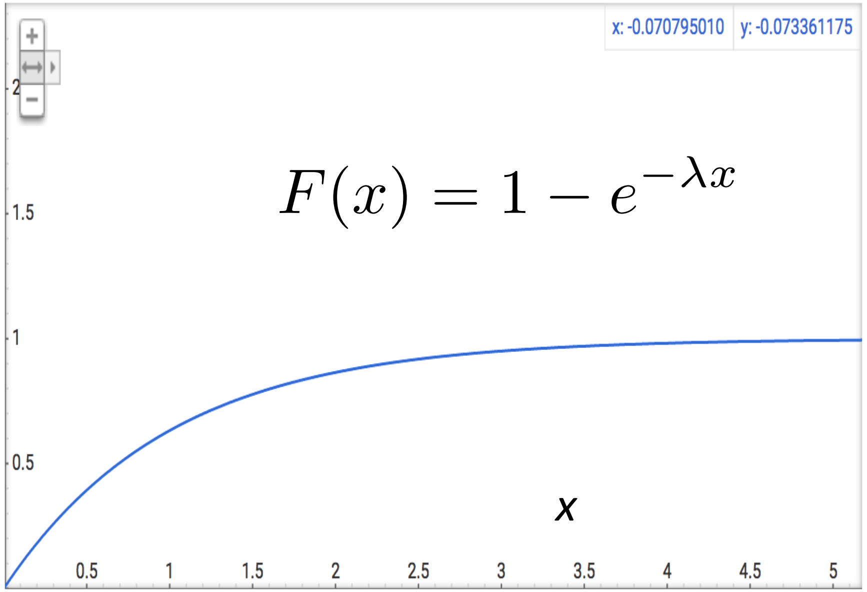
Using CDF Example. X is Exp(λ = 1)

Probability density function



P(X < 2)

Cumulative density function



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Using CDF Example. X is Exp(λ = 1)

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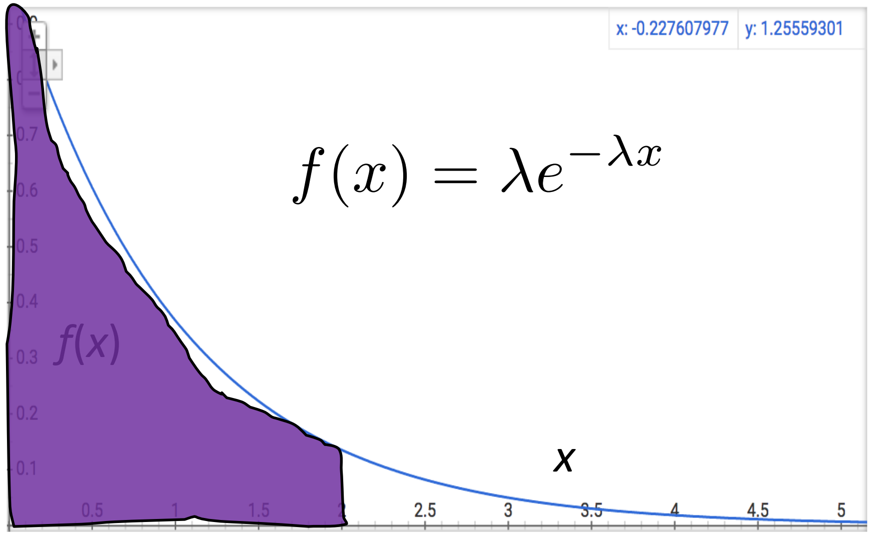

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Cumulative
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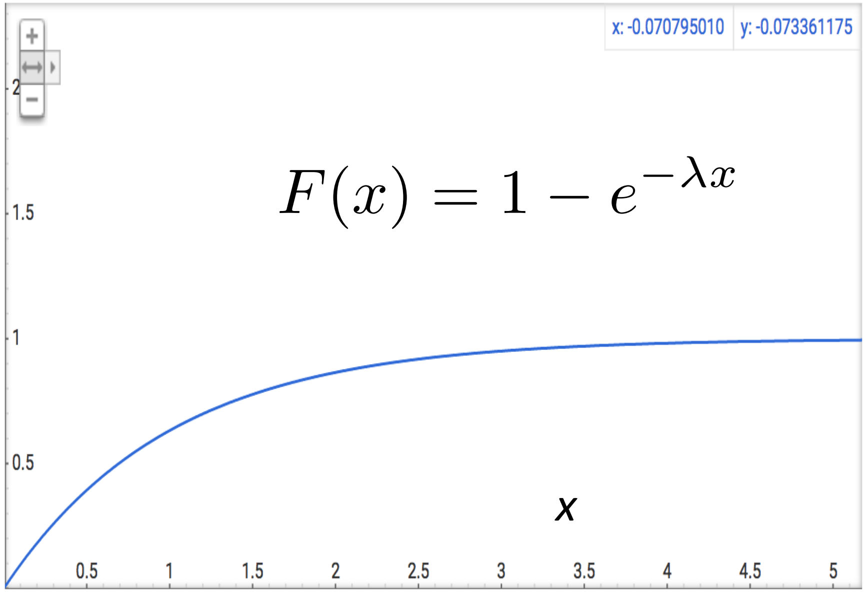
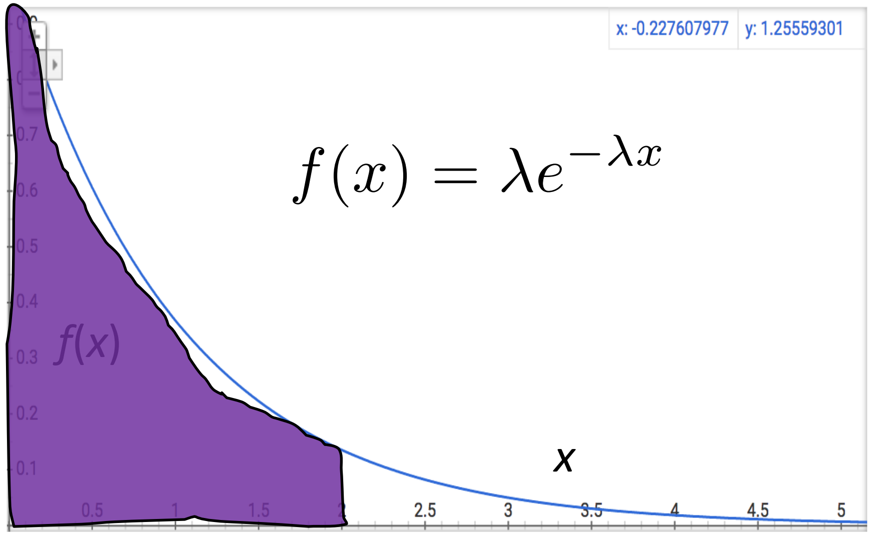


Using CDF Example. X is Exp(λ = 1)

Probability density function



P(X < 2)

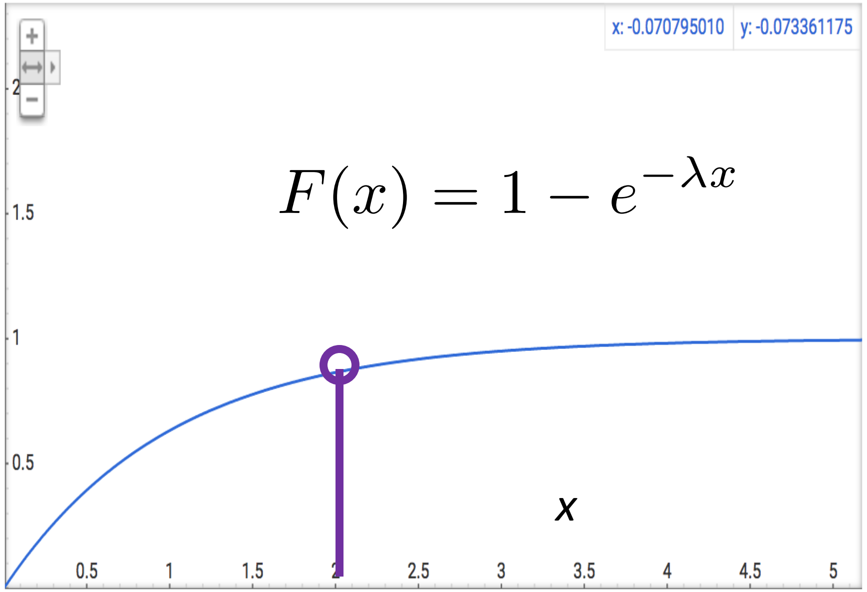
$$= \int_{x=-\infty}^{2} f(x)\, dx$$

or

Cumulative density function

$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



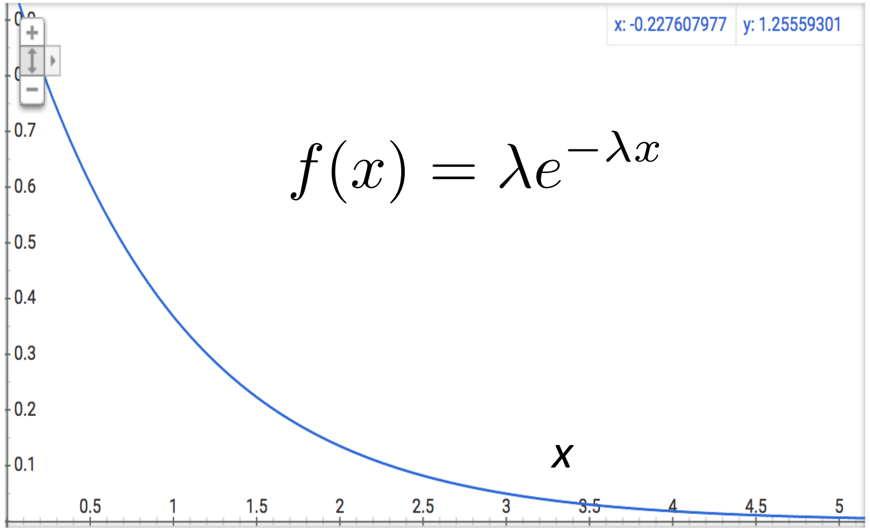
$$= F(2)$$

$$= 1 - e^{-2}$$

$$\approx 0.84$$

Using CDF Example. X is Exp(λ = 1)

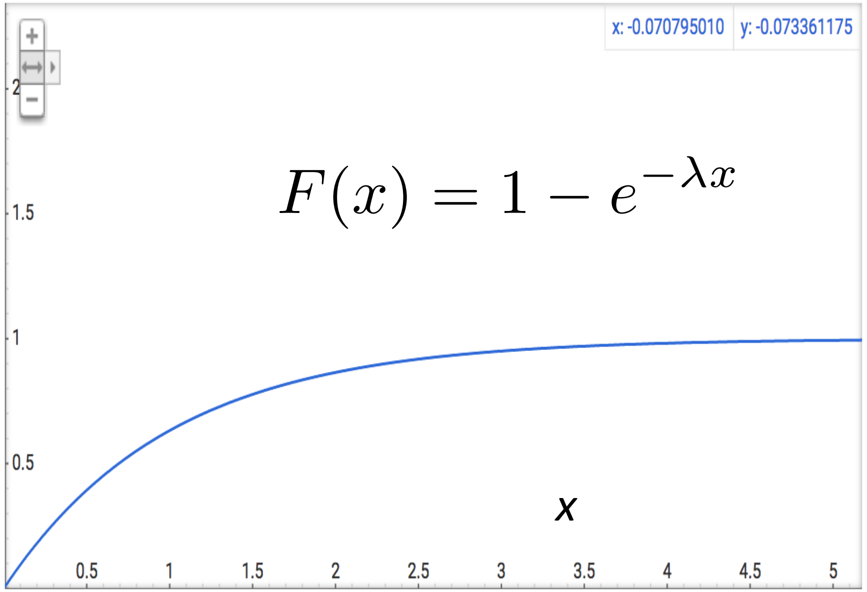
Probability density function



P(X > 1)

____

Cumulative density function

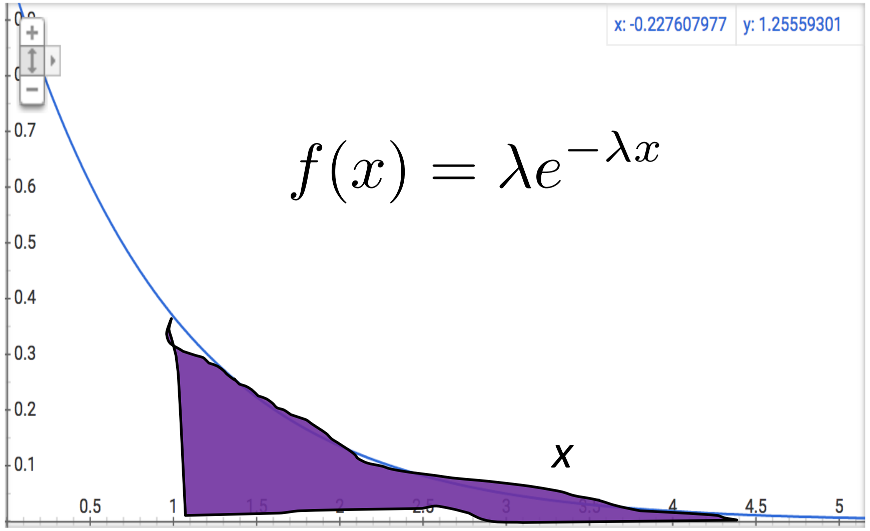


$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$

Using CDF Example. X is Exp($\lambda = 1$)

Probability density function



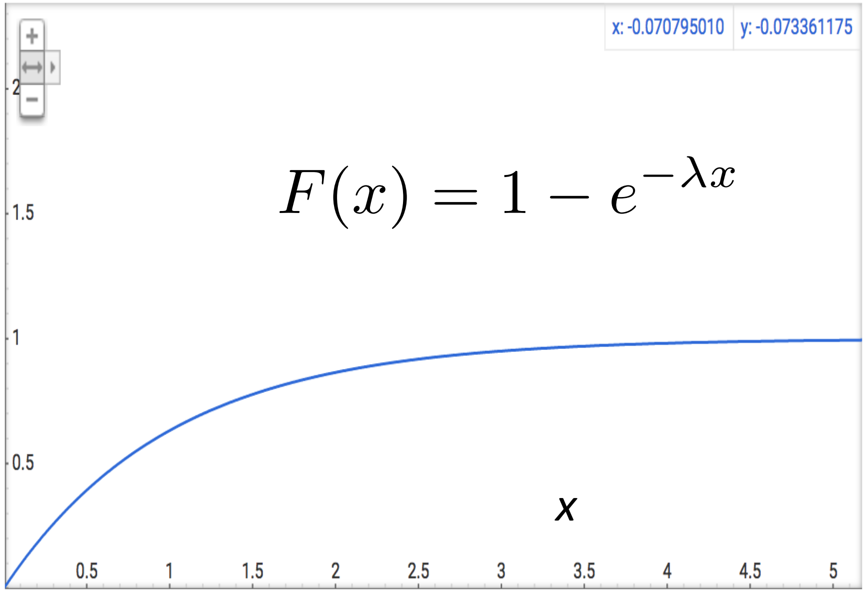
P(X > 1)

$$= \int_{x=1}^{\infty} f(x)\, dx$$

Cumulative density function

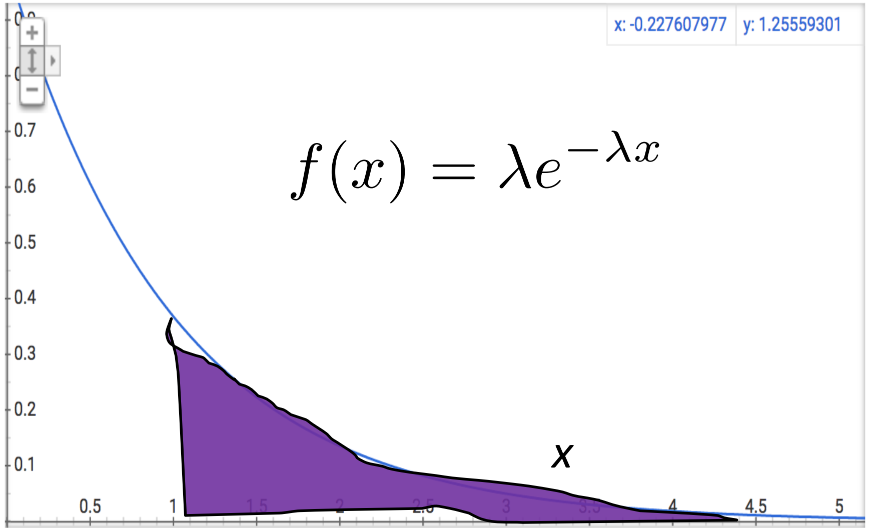
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp($\lambda = 1$)

Probability density function



P(X > 1)

$$= \int_{x=1}^{\infty} f(x)\, dx$$

or

$$= 1 - F(1)$$

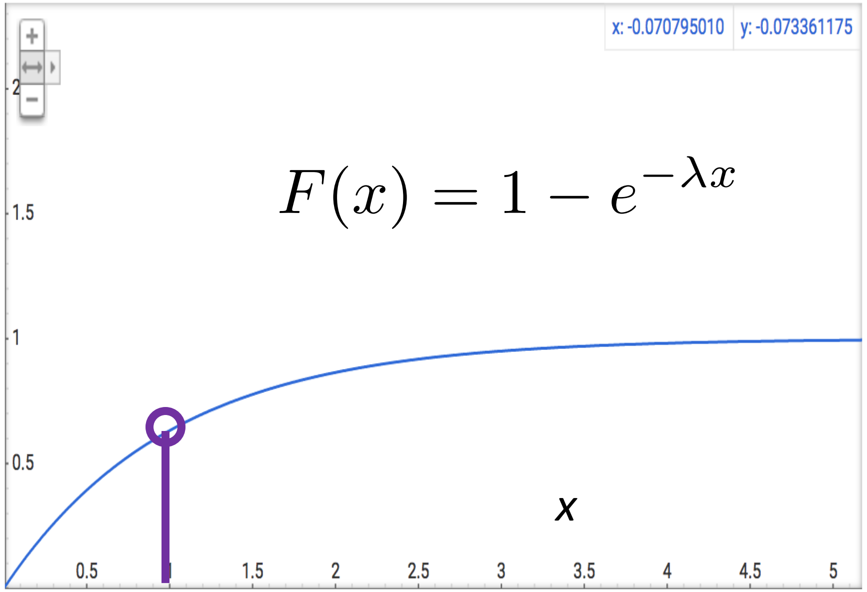
$$= e^{-1}$$

$$\approx 0.37$$

Cumulative density function

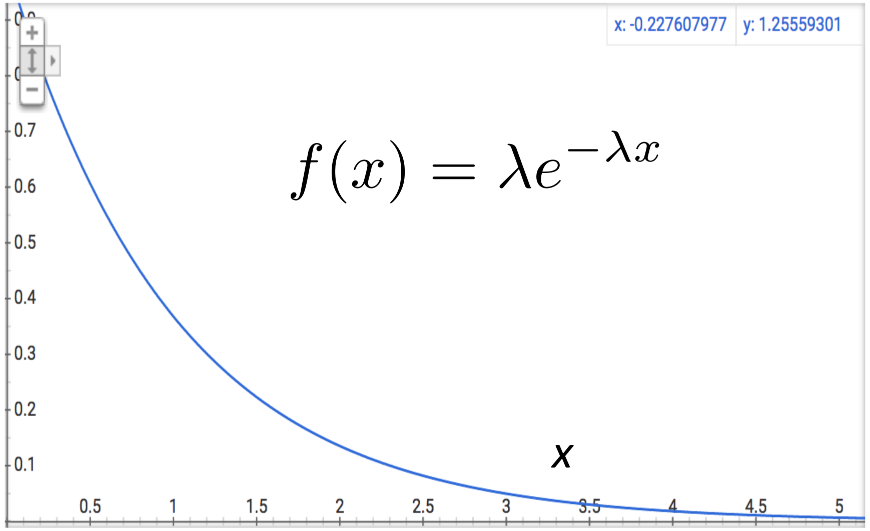
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp(λ = 1)

Probability density function

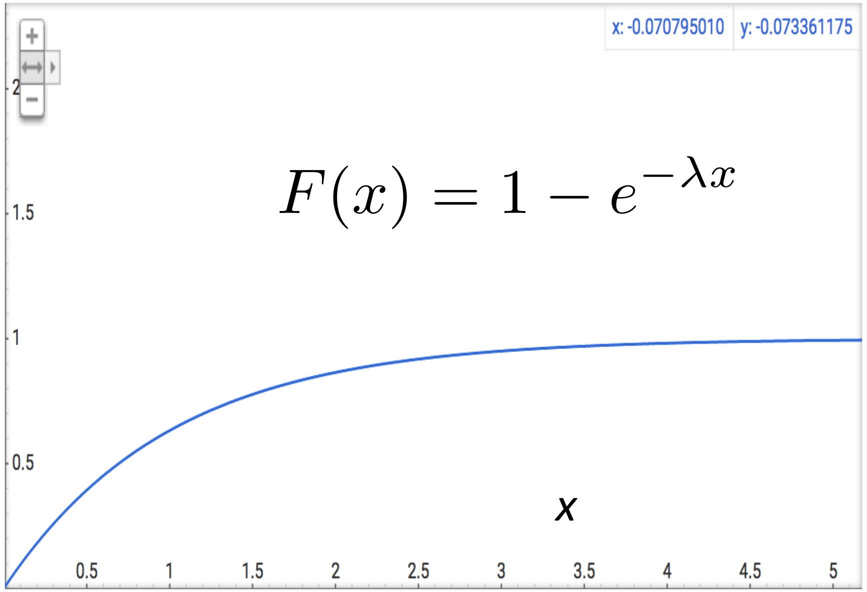


P(1 < X < 2)

Cumulative density function

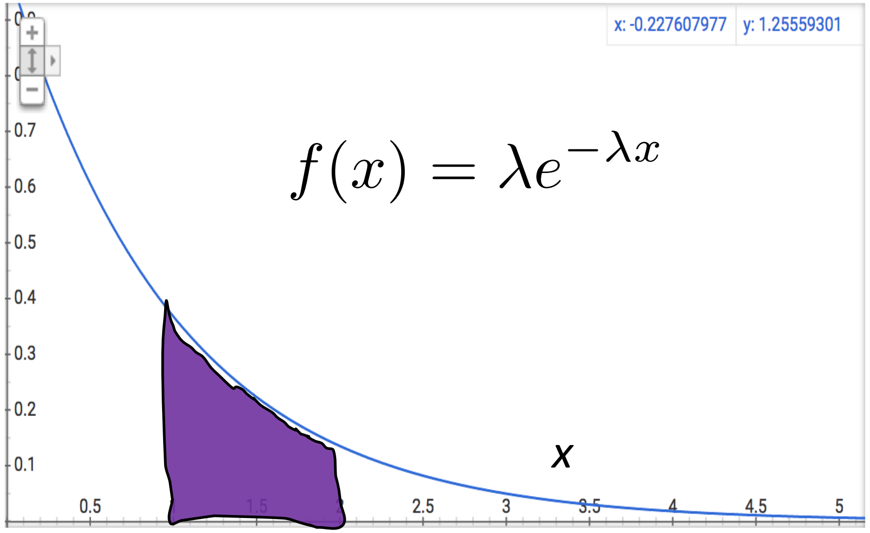
$$F_X(x) = P(X < x)$$

$$= \int_{y=-\infty}^{x} f(y)\, dy$$



Using CDF Example. X is Exp(λ = 1)

Probability density function



P(1 < X < 2)

$$= \int_{x=1}^{2} f(x)\, dx$$

Cumulative density function

$$F_X(x) = P(X < x)$$

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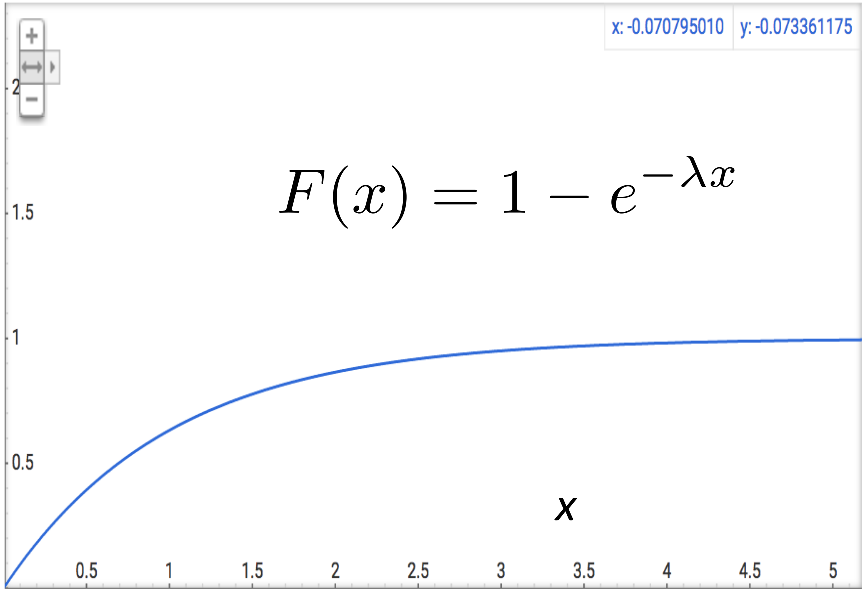
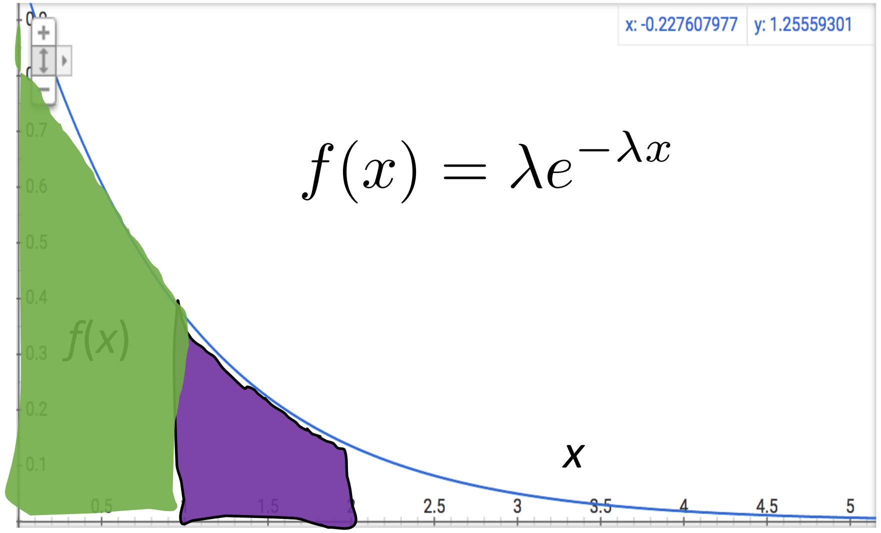


Using CDF Example. X is Exp(λ = 1)

Probability density function



P(1 < X < 2)

$$= \int_{x=1}^{2} f(x)\, dx$$

or

Cumulative density function

$$F_X(x) = P(X < x)$$

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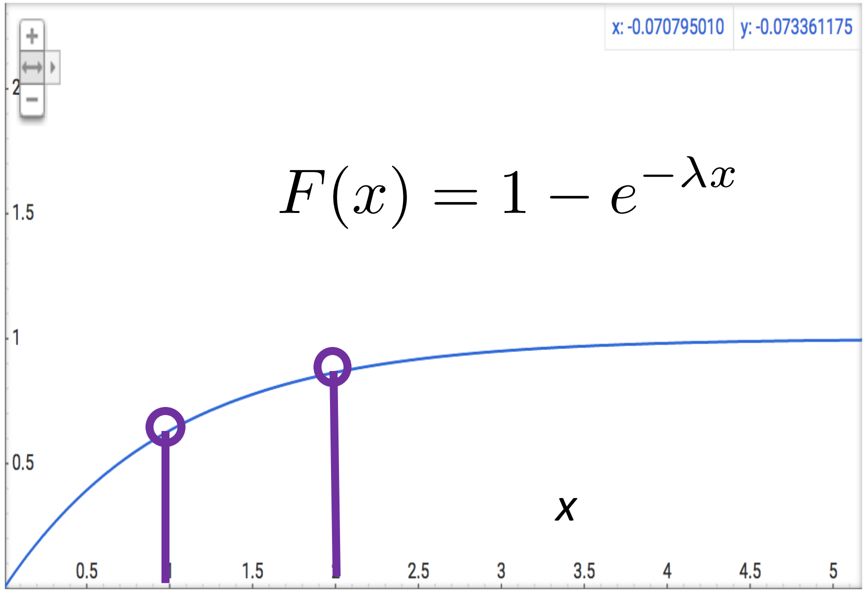


$$= F(2) - F(1)$$

$$= (1 - e^{-2}) - (1 - e^{-1})$$

$$\approx 0.23$$

Probability of Earthquake in Next 4 Years?

Based on historical data, earthquakes of magnitude 8.0+ happen at a **rate of 0.002** per year*. What is the probability of **an major earthquake in the next 4 years?**

Y = Years until the next earthquake of magnitude 8.0+

$$Y \sim \text{Exp}(\lambda = 0.002) \qquad F(y) = 1 - e^{-0.002y}$$

$$\begin{aligned} P(Y < 4) &= F(4) \\ &= 1 - e^{-0.002 \cdot 4} \\ &\approx 0.008 \end{aligned}$$

Feeling lucky?

*According to USGS, 2015

Two Classic Random Variables

Uniform Random Variable

Notation: $X \sim \text{Uni}(\alpha, \beta)$

Description: A continuous random variable that takes on values, with equal likelihood, between α and β

Parameters: $\alpha \in \mathbb{R}$, the minimum value of the variable.
$\beta \in \mathbb{R}$, $\beta > \alpha$, the maximum value of the variable.

Support: $x \in [\alpha, \beta]$

PDF equation: $f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{else} \end{cases}$

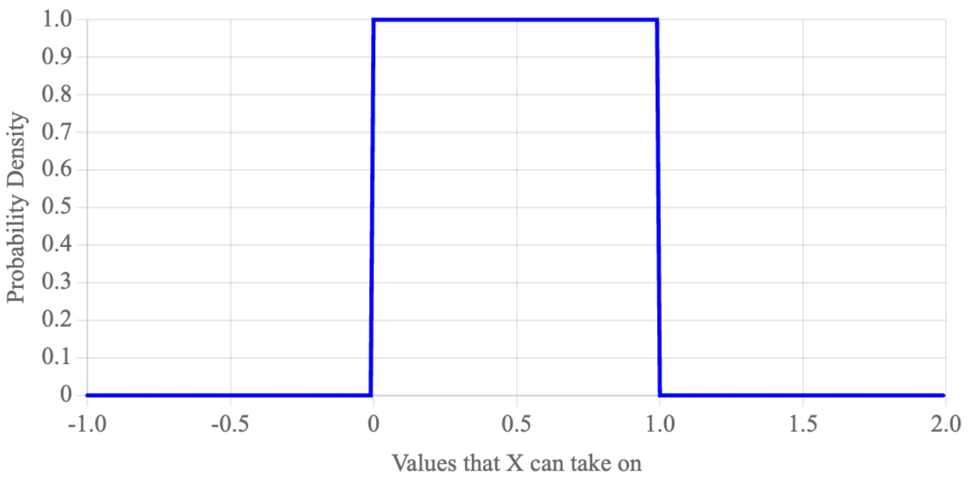
CDF equation: $F(x) = \begin{cases} \frac{x-\alpha}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{for } x < \alpha \\ 1 & \text{for } x > \beta \end{cases}$

Expectation: $\text{E}[X] = \frac{1}{2}(\alpha + \beta)$

Variance: $\text{Var}(X) = \frac{1}{12}(\beta - \alpha)^2$

PDF graph:

Parameter α: 0 Parameter β: 1



Exponential Random Variable

Notation: $X \sim \text{Exp}(\lambda)$

Description: Time until next events if (a) the events occur with a constant mean rate and (b) they occur independently of time since last event.

Parameters: $\lambda \in \{0, 1, \ldots\}$, the constant average rate.

Support: $x \in \mathbb{R}^+$

PDF equation: $f(x) = \lambda e^{-\lambda x}$

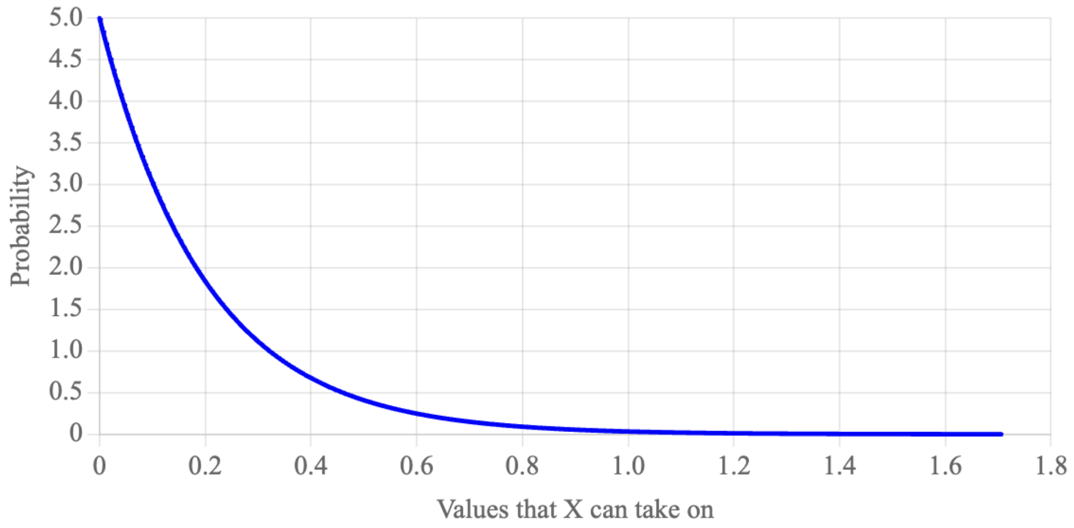
CDF equation: $F(x) = 1 - e^{-\lambda x}$

Expectation: $\text{E}[X] = 1/\lambda$

Variance: $\text{Var}(X) = 1/\lambda^2$

PDF graph:

Parameter λ: 5



Pset 1
Grades Sat at noon!

What is a check in CS109?

Functionality and style grades for the assignments use the following scale:

The modal grade in CS109

✓+	Satisfies all requirements of the assignment / full credit. An “A”
✓	Meets most requirements, but with some problems. An “A-” / “B+”
✓−	Has more serious problems, such as not explaining work.
−	Progress was made.

If your overall score is a ✓+ you rocked the PSet.

Small nit: Avoid answers that are just equations

Numeric Answer: Enter your answer | Check Answer

Explanation:

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$$|E| = \binom{f}{1}\binom{u-f}{s-1} = 50 \cdot \binom{11950}{249}$$

A small group of students made this mistake on pset1.

That is all folks!