Section 1: Analytic Probability

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1 The Birthday Problem

When solving a counting problem, it can often be useful to come up with a generative process, a series of steps that "generates" examples. A correct generative process to count the elements of set A will (1) *generate every element of* A and (2) *not generate any element of* A *more than once*. If our process has the added property that (3) *any given step always has the same number of possible outcomes*, then we can use the product rule of counting.

Assume that birthdays happen on any of the 365 days of the year with equal likelihood (we'll ignore leap years).

a. **Warmup:** How many ways can you choose birthdays for two (distinct) people?

b. What is the probability that of the n people in class, at least two people share the same birthday?

c. What is the probability that this class contains exactly one pair of people who share a birthday?

2 Conditional Probability Warmup

What is the difference between these two terms $P(B|A)$ and $P(A \cap B)$? Imagine that B is the event that a student "correctly answer a multiple choice question" and A is the event that the same student "guesses randomly". Provide an explanation as well as a mathematical relationship between the two.

3 Self-Driving Car

A self-driving car has a 60% belief that there is a motorcycle to its left based on all the information it has received up until this point in time. Then, it receives a new, independent report from its left camera. The camera reports that there is **no** motorcycle. What is the updated belief that there is a motorcycle to the left of the car? The camera is an imperfect instrument. When there is truly no motorcycle, the camera will report "no motorcycle" 90% of the time. When there actually is a motorcycle, the camera will report "no motorcycle" 5% of the time.

4 Extra Practice: Axioms of Probability

Decide whether each of the three statements below is true or false:

a. $P(A) + P(A^C) = 1$. Recall that A^C means A "complement" or "not" A

b. $P(A \cap B) + P(A \cap B^C) = 1$. Recall that $\cap$ means "and"

c. If $P(A) = 0.4$ and $P(B) = 0.6$ then it must be the case that $A = B^C$