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Section 6

With questions from Will Monroe and Julia Daniel

1   Beta Sum Warmup

What is the distribution of the sum of 100 IID Betas? Let X be the sum:

$$X = \sum_{i=1}^{100} X_i \quad \text{where each } X_i \sim \text{Beta}(a = 3, b = 4)$$

Note the expectation and variance of a Beta:

$$E[X_i] = \frac{a}{a+b} \qquad \text{Var}(X_i) = \frac{ab}{(a+b)^2(a+b+1)} \qquad \text{Where } X_i \sim \text{Beta}(a, b)$$

2   Food for Thought

Karel the dog eats an unpredictable amount of food. Every day, the dog is equally likely to eat an amount in the continuous range between 100g and 300g. How much Karel eats one day is independent of all other days.

You only have 6500g of food. What is the probability that 6500g will be enough for the next 30 days?

3   Sorted Random Values

We want to reason about the values produced by the following python code, which samples 10 random uniform values in the range [0, 1] and then sorts them from low to high:

```python
# generate 10 random uniform values
values = []
for i in range(10):
    value_i = random_uniform(0,1)   # sample from standard uniform
    values.append(value_i)

# sort all of the values ascending from low to high
sorted_values = sorted(values)
print(sorted_values)
```

Here is what the list could look like when printed. For clarity, each value is rounded to two decimal places, but these values really have near-infinite precision:

```
sorted_values = [0.03,0.13,0.45,0.51,0.52,0.63,0.69,0.82,0.88,0.91]
        index :   0    1    2    3    4    5    6    7    8    9
```

a. The first value produced by $\texttt{random\_uniform(0,1)}$ is 0.4. What is the probability it ends up at index 4 in the sorted list? *In other words: What is the probability that exactly 4 out of the 9 other values are less than 0.4?*

b. Let X_4 be the value at index 4 in the sorted list. What is the probability density of $X_4 = x$? *In other words: What is the probability density that $X_4 = x$, given that you know exactly 5 out of 9 numbers are greater than x?*