

CS109: Probability for Computer Scientists
Lecture 15 Worksheet — Central Limit Theorem (TPS)
Feb 11, 2026

1 Quick Check: Are These i.i.d.?

For each case below, decide whether the random variables are i.i.d. and why.

- a) $X \sim \text{Ber}(0.5)$, $Y \sim \text{Ber}(0.5)$, and X, Y are independent.

- b) $X \sim \text{Ber}(0.3)$, $Y \sim \text{Ber}(0.7)$, and X, Y are independent.

- c) $X \sim \text{Ber}(p)$ and $Y = 1 - X$.

2 Convolution Intuition: Probability of a Tie

Let X be your score and Y your opponent's score in a game. Assume X and Y are independent and discrete with known PMFs $P(X = x)$ and $P(Y = y)$.

- a) Write an expression for $P(X = Y)$ in terms of the PMFs.

- b) Using the same setup, write an expression for $P(X + Y = 10)$.

3 Wildlife Disease Outbreak (Exact Distribution)

Population A has infections $A \sim \text{Bin}(5, 0.1)$ and population B has infections $B \sim \text{Bin}(8, 0.5)$, with A, B independent.

- a) Write a formula for $P(A + B = k)$. What are the possible values of k ?

4 Linear Transform (Bug Hunt)

Let $X \sim \mathcal{N}(\mu, \sigma^2)$ and $Y = X + X = 2X$. Here are two ways students might think to reason about Y :

Version A: Thinking of Y as a linear transform $Y = 2X \implies Y \sim \mathcal{N}(2\mu, 4\sigma^2)$.

Version B: Thinking of Y as a sum of independent normals

$$Y = X + X \implies Y \sim \mathcal{N}(\mu + \mu, \sigma^2 + \sigma^2) = \mathcal{N}(2\mu, 2\sigma^2).$$

Notice these two different framings lead to different variances. Which one of these framings is the correct one, and why?

5 ELO Model and Subtracting Gaussians

In an ELO-style model, team abilities are sampled as

$$A_W \sim \mathcal{N}(S_W, 200^2), \quad A_O \sim \mathcal{N}(S_O, 200^2),$$

independently. The Warriors win if $A_W > A_O$.

- a) Could you come up with a way to define $D = A_W - A_O$?

6 Virus Infections Revisited

Let $A \sim \text{Bin}(50, 0.1)$, and $B \sim \text{Bin}(100, 0.4)$ where A and B are independent. We want $P(A + B \geq 40)$. I want to solve this without using a for loop.

- a) What is the insight you would use to get you started solving this problem?

- b) Solve it here.

7 Sum of 100 Dice Rolls

Let X_1, \dots, X_{100} be i.i.d. fair six-sided dice and $S = \sum_{i=1}^{100} X_i$.

- a) How could you use the CLT to approximate S ?

8 Dice Game

You roll 10 fair six-sided dice such that $X = \sum_{i=1}^{10} X_i$. You win if $X \leq 25$ or $X \geq 45$.

- a) Use CLT to approximate $P(X \leq 25 \text{ or } X \geq 45)$.