CS109: Probability for Computer Scientists
Lecture 16 Worksheet — Algorithmic Analysis
Feb 13, 2026

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1 Quick Review

If X_i is an indicator RV where it is a 1 if event E_i occurs, show that $E[X_i] = P(E_i)$.

2 Differential Privacy

$X_1, \ldots, X_{100}$ are i.i.d. Bernoulli(p). For each i and Y_i is the output of the program in the slides.

(a) Compute $E[Y_i]$.

(b) Let $Z = \sum_{i=1}^{100} Y_i$. Compute $E[Z]$.

(c) How could you estimate p? Leave your answer in terms of Z.

(d) **Challenge**: what is the probability that your estimate is good?

3 Computer Cluster Utilization

A cluster has k servers. Each request independently goes to server i with probability p_i.

Let A_i be the event that server i receives no requests in the first n requests. Let

$$X = \#\{\text{events } A_1, \ldots, A_k \text{ that occur}\}.$$

(a) Can you express X in terms of the sum of another RV? Recall that A_i is an event and not a random variable.

(b) Compute $E[X]$.

4 Toy Collection

You are trying to collect n distinct toys. On each purchase, each toy is equally likely.

Let X be the number of purchases until you have at least one of each toy.

(a) Can you define X as the sum of some other RVs?

(b) What is the other RV? And what is its expectation?

(c) What is $E[X]$?

5   Expected Runtime (Netflix)

Let L be movie file location with:

Location ℓ	$E[\text{Runtime} \mid L = \ell]$	$P(L = \ell)$
Palo Alto	0.3s	0.10
SoCal	1.6s	0.50
Japan	300s	0.37
Space	7200s	0.03

Compute the expected runtime of `database.get_movie(movie_name)`.

6   Analyzing Recursive Code

```
int Recurse() {
   int x = randomInt(1, 3);  // equally likely
   if (x == 1) return 3;
   else if (x == 2) return 5 + Recurse();
   else return 7 + Recurse();
}
```

Let Y be the value returned by `Recurse()`. Compute $E[Y]$.