# 1 MLE for Poisson

Suppose $X_1, \ldots, X_n$ are i.i.d. with $X_i \sim \text{Poisson}(\lambda)$.

(a) What is the likelihood of one $X_i$? What is the likelihood of all the data $x_1, \ldots, x_n$?

(b) What is the log-likelihood of all the data? (You may drop constants that do not depend on $\lambda$.)

(c) Find the value of $\lambda$ that maximizes the log-likelihood.

## 2   MLE for Pareto

Assume $X_1, \ldots, X_n$ are i.i.d. Pareto with scale $x_m = 1$ and shape $\alpha > 0$, so

$$f(x \mid \alpha) = \frac{\alpha}{x^{(\alpha+1)}}, \quad x \geq 1.$$

(a) What is the likelihood of one $X_i$? What is the likelihood of all the data $x_1, \ldots, x_n$?

(b) What is the log-likelihood of all the data?

(c) Find the value of $\alpha$ that maximizes the log-likelihood.

## 3   MLE for Erlang

Let $X_1, \ldots, X_n$ be i.i.d. Erlang with parameters $k$ and $\lambda$:

$$f(x \mid \lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)}, \quad x \geq 0.$$

(a) What is the likelihood of one $X_i$? What is the likelihood of all the data $x_1, \ldots, x_n$?

(b) What is the log-likelihood of all the data?

(c) Solve for the value of $\lambda$ that maximizes the likelihood.

(d) Solve for the value of k that maximizes the likelihood. What goes wrong here? And how would you solve for it?

# 4    MLE for Bernoulli

Suppose $X_1, \ldots, X_n$ are i.i.d. Bernoulli($p$), with $x_i \in \{0, 1\}$.

(a) What is the likelihood of one $X_i$? What is the likelihood of all the data?

(b) What is the log-likelihood of all the data?

(c) Find the value of $p$ that maximizes the log-likelihood.

# 5 MLE vs. Beta

The medicine is tried on 20 patients. It "works" for 14 and "doesn't work" for 6. What is your new belief that the drug works? In other words, we have 20 i.i.d. samples from a Bernoulli and want to estimate $p$:

$$[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0].$$

Assume for the Beta approach that before seeing this data, your prior belief is $p \sim \text{Beta}(8, 2)$.

(a) Compute the MLE estimate of $p$.

(b) Compute the Beta posterior distribution for $p$.

(c) Describe the difference between the MLE estimate of $p$ vs. the Beta estimate of $p$ in this case. When should we use one over the other?