Reinforcement 2

CS109, Stanford University

Today

Quant Interview (with cupcakes)

Poker Information Theory

Magic Puppy Drugs

Linear Regression

Is that Poisson?

Quant Interview

Quant Interview

100 cupcakes.
Scenario 1: 51 are blue.
Scenario 2: 49 are blue.

a) You look at one cupcake. It is blue.

b) You look at three cupcakes (with replacement). Two are blue.

Quant Interview - Bonus

100 cupcakes.
Scenario 1: 51 are blue.
Scenario 2: 49 are blue.

a) You look at one cupcake. It is blue.

b) You look at three cupcakes (with replacement). Two are blue.

c) You look at three cupcakes (without replacement). Two are blue.

Quant Interview: Bonus

c. A hypergeometric is a random variable for the number of successes if you remove items **without** replacement from a fixed population. If $X \sim \text{Hypergeom}(t, k, n)$, the PMF is

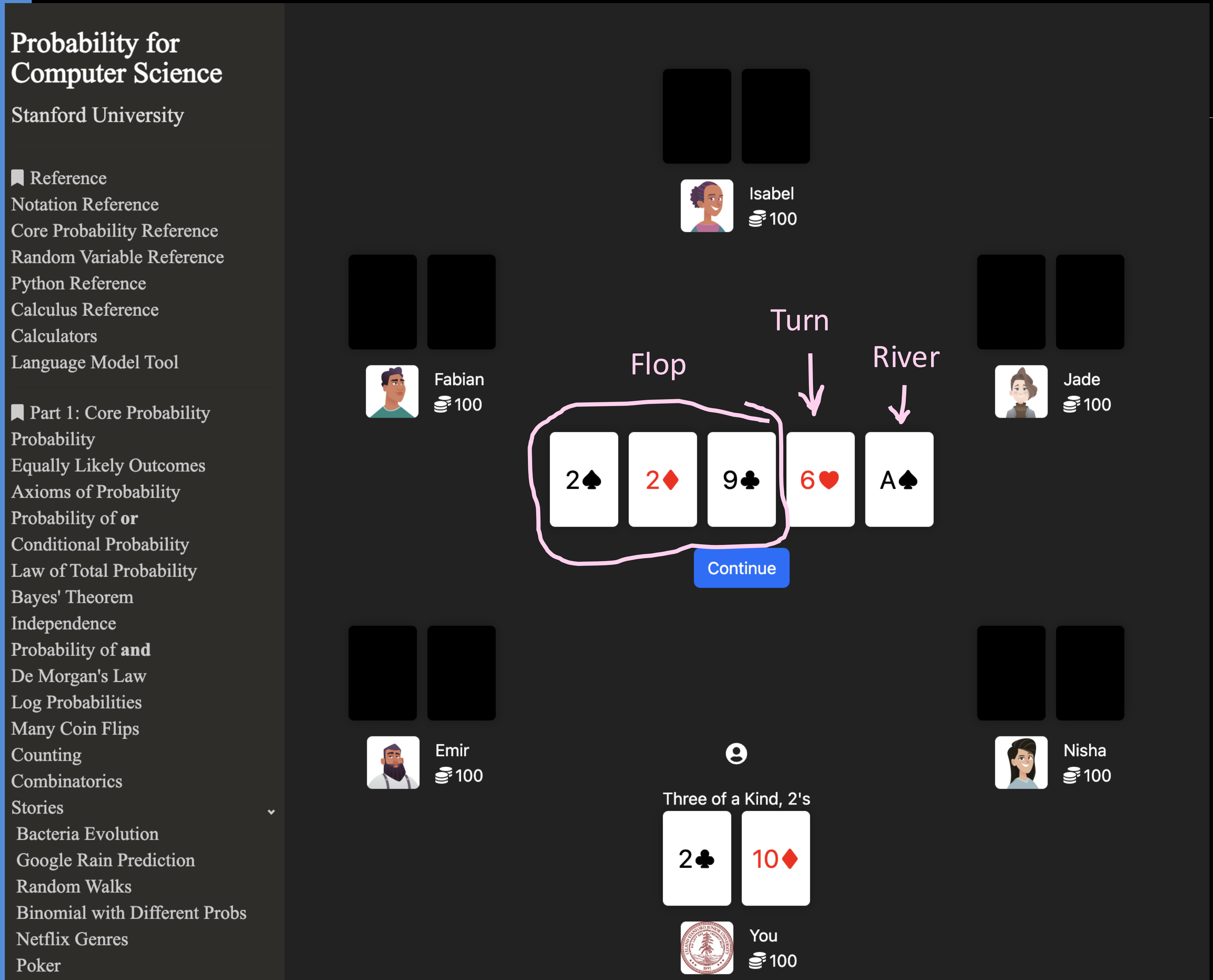
$$P(X = x) = \frac{\binom{k}{x}\binom{t-k}{n-x}}{\binom{t}{n}}$$

Where:

- t is the population size,
- k is the number of "success" items in the population,
- n is the number of draws (without replacement),
- X counts the number of successes observed in those n draws.

Three cupcakes are drown without replacement and two are blue. What is the probability that the majority were blue?

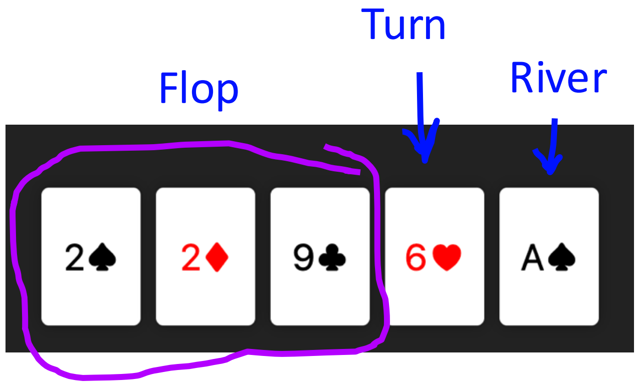
Poker Information Theory



Let X be a random variable for the set of 7 cards at the end. What is H(X) after each event?

There are 52 unique cards. Each card is equally likely.

Poker Entropy



$$H(\text{pre flop}) = 21.014$$

$$H(\text{pre turn}) = 10.078$$

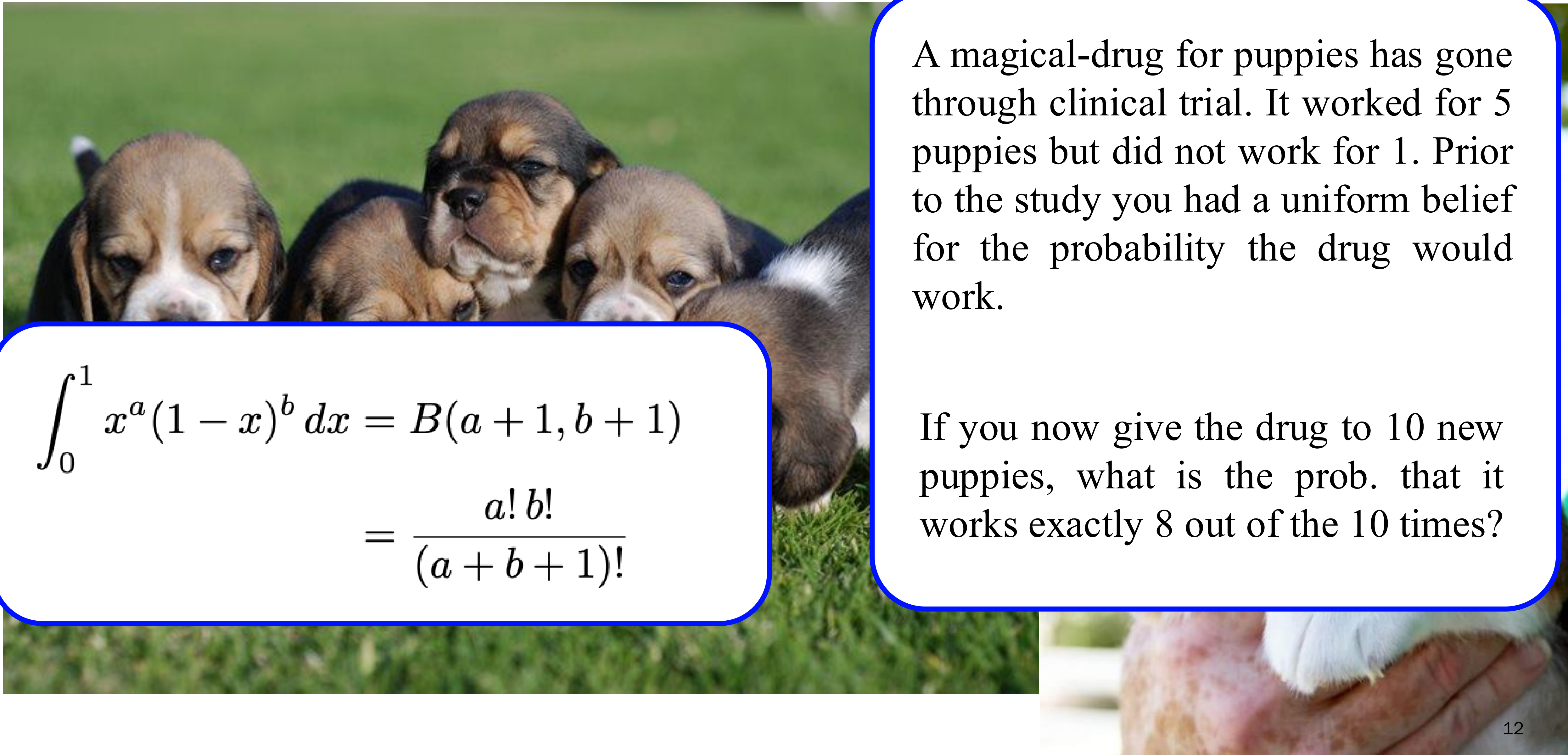
$$H(\text{pre river}) = 5.524$$

$$H(\text{post river}) = 0$$

Magical Puppy Drugs

Magical Puppy Drugs

Magical Puppy Drugs



A magical-drug for puppies has gone through clinical trial. It worked for 5 puppies but did not work for 1. Prior to the study you had a uniform belief for the probability the drug would work.

If you now give the drug to 10 new puppies, what is the prob. that it works exactly 8 out of the 10 times?

$$\int_0^1 x^a(1-x)^b\,dx = B(a+1,b+1) = \frac{a!\,b!}{(a+b+1)!}$$

Linear Regression P-Value

Linear Regression P-Value

Train a linear regression model and get parameters.

You have a set of 50 data points (x, y) where $x \in \mathbb{R}$ and $y \in \mathbb{R}$.

You fit a linear regression model which gives you a fit

$$\hat{y} = \theta_0 + \theta_1 \cdot x$$

You are surprised to see that $\theta_1 = 2$. You want to claim that this value of θ_1 is significantly high. What is the probability of seeing a value of $\theta_1 \geq 2$, given there is no relationship between x and y? Estimate your answer using sampling. Provide pseudo-code:

Linear Regression P-Value

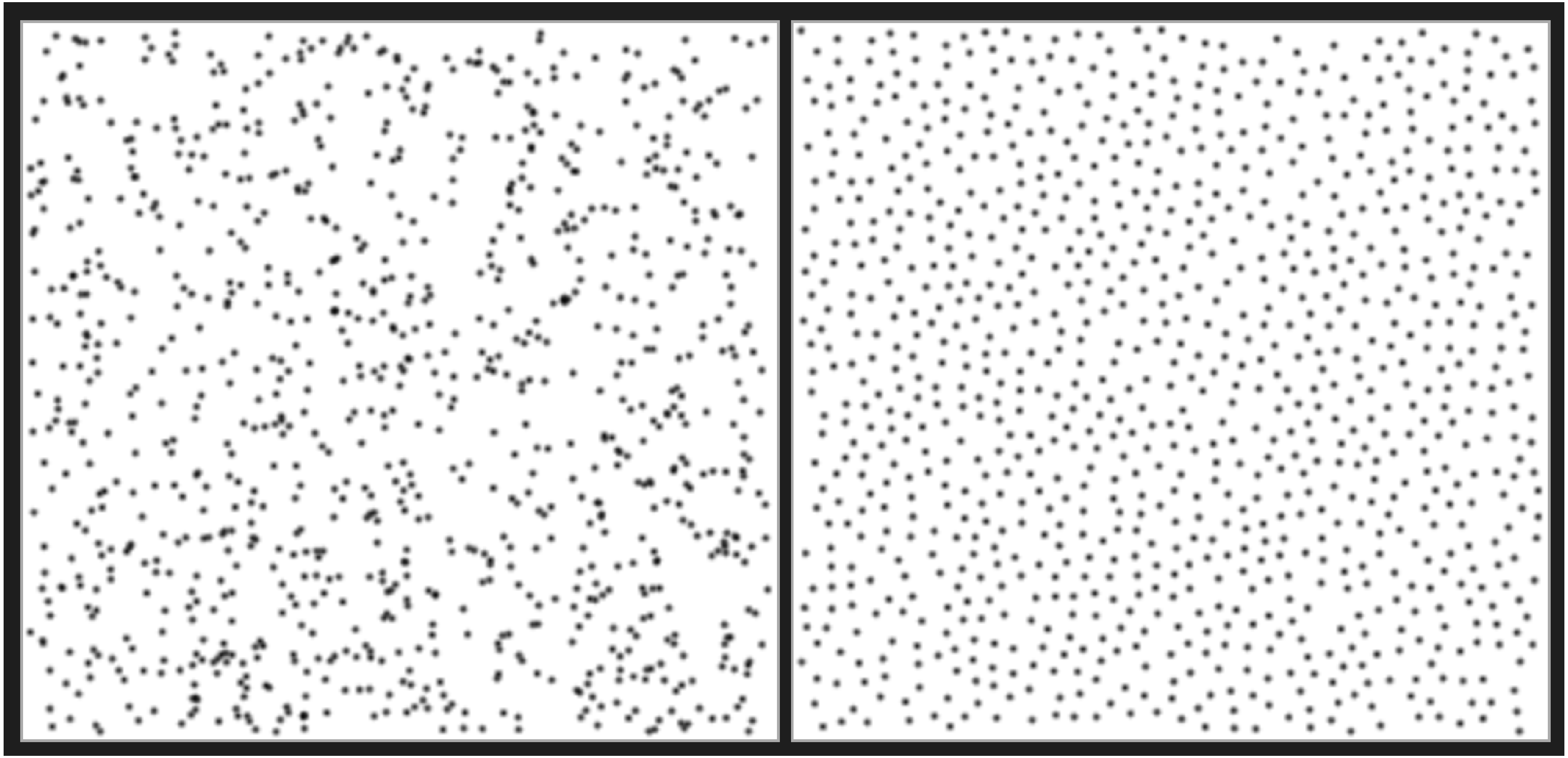
More sophisticated pseudocode than what we might expect on an exam.

```
THRESHOLD = 2.0

def bootstrapping(x, y):
    N = len(x)
    count_extreme = 0

    for i in range(10000):
        # resample independently
        x_boot = sample(x, 50, replace=True)
        y_boot = sample(y, 50, replace=True)

        # pseudo code
        theta0, theta1 = LinearRegression().fit(x_boot, y_boot)

        if theta1 >= THRESHOLD:
            count_extreme += 1

    return count_extreme / 10000
```

Very important !! Even on exam !!

Which One Is Poisson?

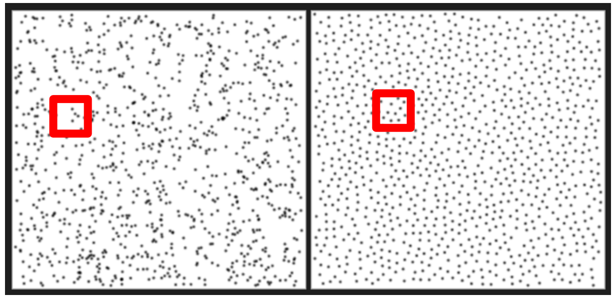
Which one is Poisson? Justify with Math…



Same number of points…

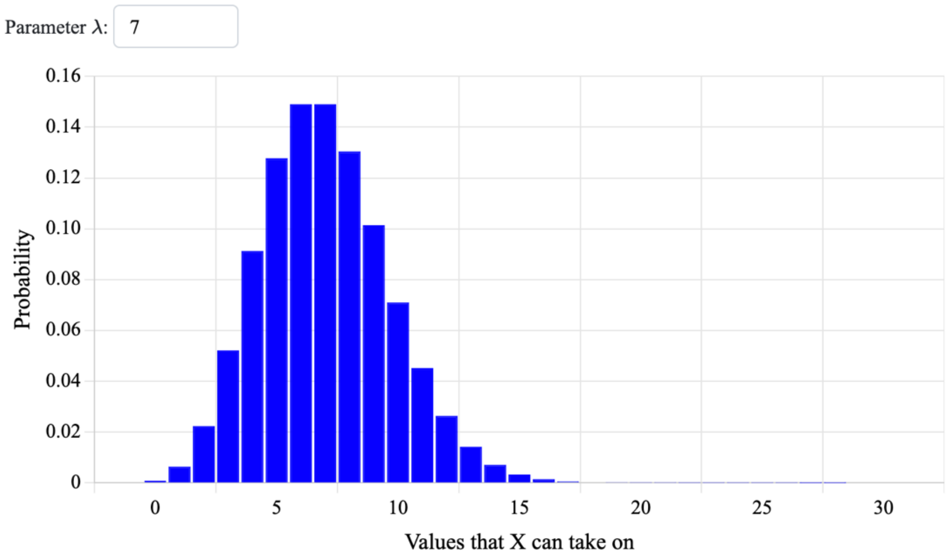
Which one is Poisson? Sampling Solution

Squares that are 1x1

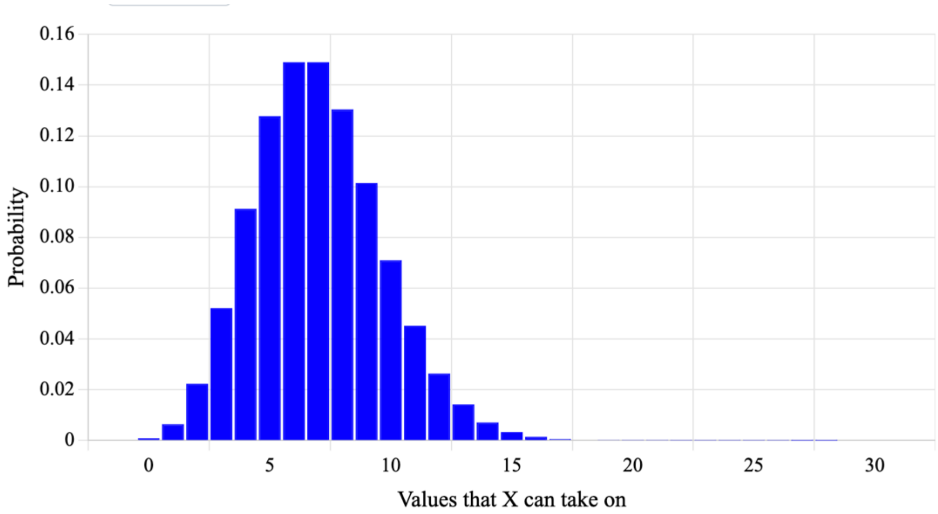


True rate = 7 points per unit size

Theory $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$

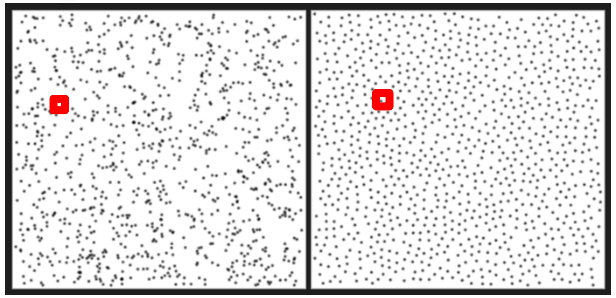


Observed (for both)



Which one is Poisson? Sampling Solution

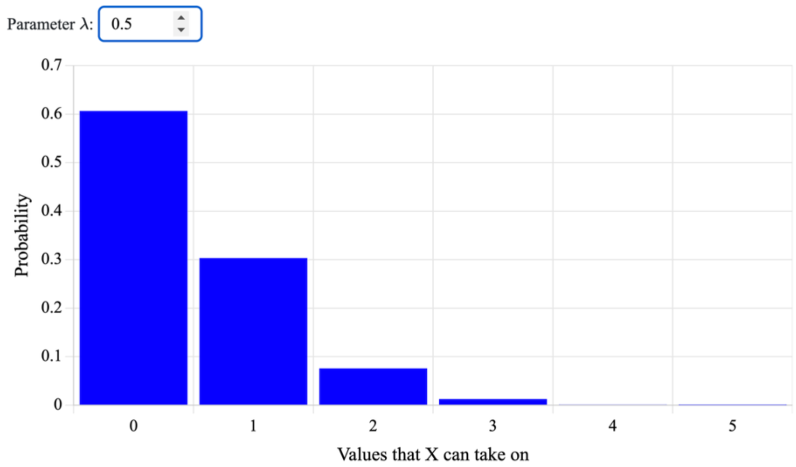
Squares that are .01 x .01

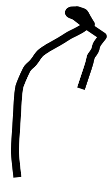


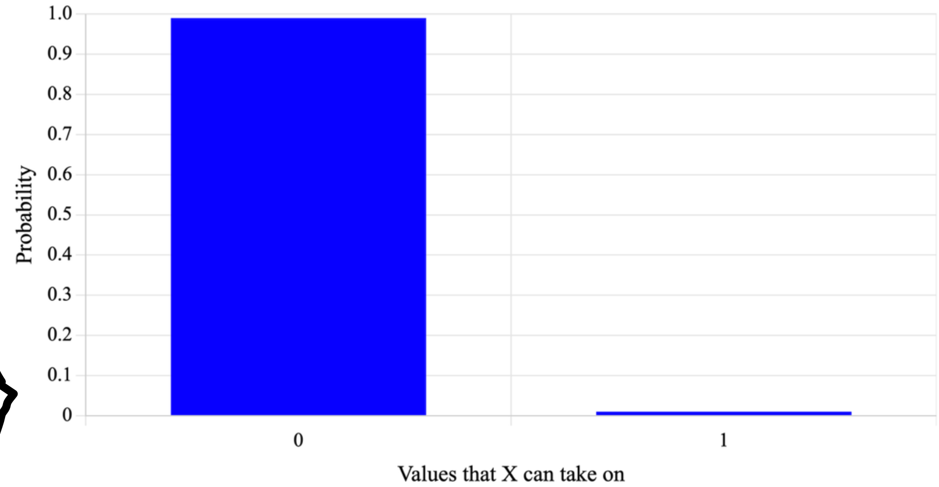
True rate = 7 points per unit size

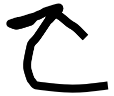
Theory $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$

Observed (for right)









Bonus, calculate the KL divergence between these two distributions

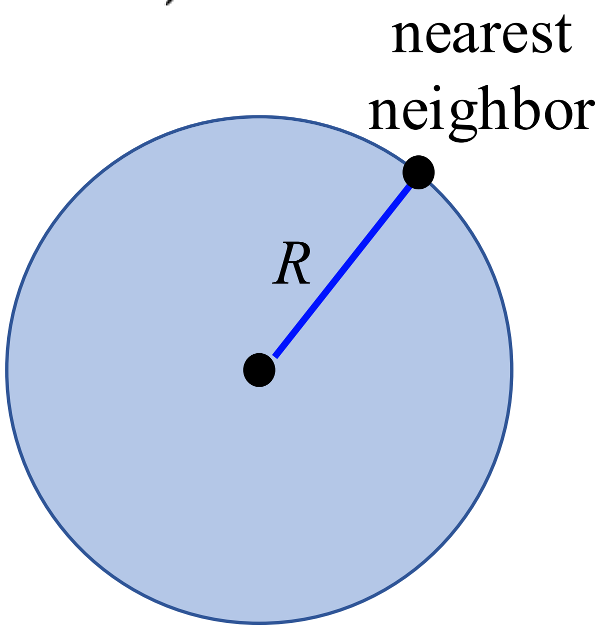
Which one is Poisson? Nearest Neighbor Distance

Let R be the distance to the nearest neighbor of a point

$$P(R < r) = 1 - P(\text{Zero points in radius } R)$$
$$= 1 - P(X = 0)$$

Let X be the number of points in radius R. $X \sim \text{Poi}(\lambda = 7 \cdot \pi \cdot r^2)$

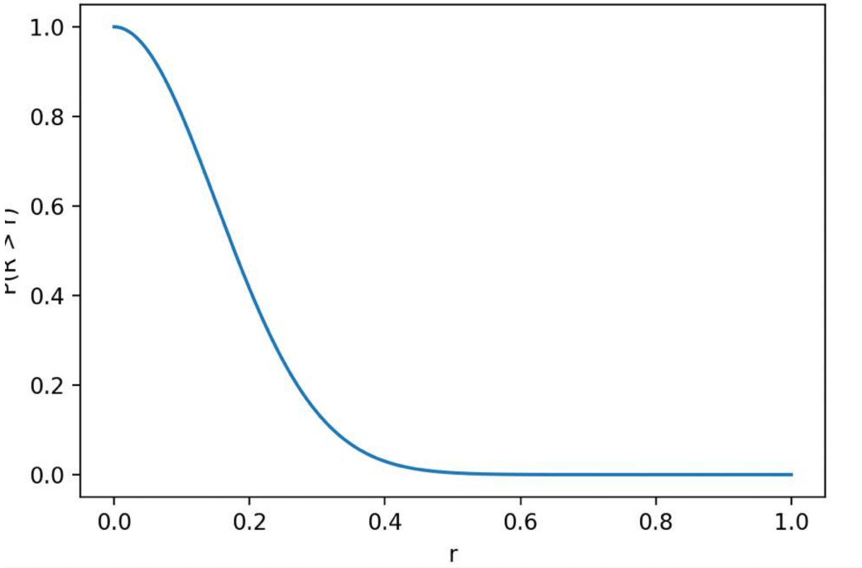
$$= 1 - \frac{(7\pi r^2)^0 e^{-7\pi r^2}}{0!}$$
$$= 1 - e^{7\pi r^2}$$



Which one is Poisson? Nearest Neighbor Distance

$$P(R > r) = 1 - e^{7\pi r^2}$$

Theory and picture on the left:



Picture on the right: