CS 161: Recitation 5 (Fall 2016)

Question 1

Given a directed graph $G = (V, E)$, let us call a node $v \in V$ a *viewpoint* if all other nodes are reachable from v (in other words, v has a path to all other nodes).

(a) Recall that any directed graph can be thought of as a meta-graph of its strongly connected components (SCCs), and that this meta-graph is acyclic. Briefly argue that if G has a viewpoint node, G must have only one *source* SCC, which is an SCC such that there are no edges from another SCC to it.

(b) Give an $O(|E| + |V|)$-time algorithm to find a viewpoint node in a directed graph, or report that none exists.

Question 2

We have a network of n nodes and m directed links between them. Each of the n nodes send/forward packets to each other along the m links. The links are not reliable and may fail due to a variety of reasons. Each link between node i and node j is available with probability $p_{i,j}$. We assume that each link fails independently, so the probability that a path, P, is available is simply the product of the probabilities that all of the links in the path are available, $\pi(P) = (p_{1,2})(p_{2,3}) \cdots (p_{k-1,k})$.

We can represent this network with a directed graph G where the edge weights are the probabilities. Give an efficient algorithm for finding the path with the greatest probability of successfully transmitting a packet from node i to node j.

Question 3

Consider a binary counter with b bits that has an initial value x that contains k 1-bits in its binary representation. Show that the number of bits flipped in the course of n increments is at most $2n + k$.