

CS 161: Recitation 6 (Fall 2016)

Question 1

Over the past few years, you have been keeping a jar of coins and you wish to cash them in for single dollar bills. You notice you have pennies, nickels, dimes, quarters, and 50-cent coins in the jar. Being the clever CS161 student that you are, you start thinking about how many different ways you can add up to 100 cents with the coins that you have. You can have any number of each coin, as long as the sum of all of the coins you choose is exactly 100.

- (a) Define the recurrence $C(n, K)$ where K is a list containing the values of coins (in this case, $[1, 5, 10, 25, 50]$), and n is the total cost of your coins you are aiming for (in this case, 100).
- (b) Design a dynamic programming algorithm `COUNT-CHANGE`(n, K) that runs in $O(n|K|)$.

Question 2

Given integers n and k , along with $p_1, \dots, p_n \in [0, 1]$, you want to determine the probability of obtaining exactly k heads when n biased coins are tossed independently at random, where p_i is the probability that the i th coin comes up heads. Give an $O(nk)$ algorithm for this task. Assume that you can multiply and add two numbers in $[0, 1]$ in $O(1)$ time.

Question 3

You are frantically trying to finish writing up your CS161 homework. Because the clock has ticked past midnight, drowsiness has started to affect your ability to type correctly and you start having many spelling errors. You start mistyping “*algorithm*” with “*algurytim*”. Luckily, your \LaTeX compiler has a spell-checker for you. However, it only suggests the correct spelling and does not replace the word for you. Being a good CS161 student, with algorithms on your mind, you start pondering what the minimum cost of insertions, deletions, or substitutions you need to fix your spelling mistake.

- “ a ” \rightarrow “ ab ” constitutes as an *insertion* of the letter b
- “ ab ” \rightarrow “ a ” constitutes as a *deletion* of the letter b
- “ ab ” \rightarrow “ ac ” constitutes as a *substitution* of the letter b with c .

Insertions and deletions have a cost of 1. Substitutions have a cost of $\text{COST}(c_i, c_j) = \text{dist}(c_i, c_j)$ as a function of the current letter, c_i and its substitute c_j . $\text{dist}(c_i, c_j)$ measures the distance between letter c_i and c_j on a standard QWERTY keyboard.

- (a) Given two spellings, $s1$ and $s2$, one incorrect and one correct respectively, we make the following observation: at the end of each our incorrect spelling, $s1$, we can make three possible corrections to our word to would lead to closer to the correct spelling. What are the three possible corrections?
- (b) Using $O(nm)$ space, where n is the length of $s1$ and m is the length of $s2$, design a dynamic programming algorithm that runs in time $O(nm)$.
- (c) Briefly outline a way to keep track of the exact corrections we need to correct our misspelled word.