CS 161 | **Pre-Lecture Exercise for Lecture 4**
Autumn 2017 | **Date:** October 4, 2017

Pre-lecture exercises will not be collected for credit. However, you will get more out of each lecture if you do them, and they will be referenced during lecture. We recommend **writing out** your answers to pre-lecture exercises before class. Pre-lecture exercises usually should not take you more than 20 minutes.

In this pre-lecture exercise, you'll do a warm-up for the *substitution method,* where you will formally prove something that we already know is true.

In your previous pre-lecture exercise, you considered the recurrence relation

$$T(n) = 2T(n/2) + n, T(1) = 1$$

In that exercise (and in class at great length) we saw that when n is a power of 2, the solution was

$$T(n) = n(\log(n) + 1).$$

However, we technically never proved this formally. (and we certainly didn't prove anything formally for when n wasn't a power of 2). Below, we'll go through this example formally, via a proof by induction.

1. Suppose that $T(n) = 2T(\lfloor n/2 \rfloor) + n, T(0) = 0$. (Notice that we are changing up the form a little bit to be careful about what happens when n isn't a power of 2). Prove by induction, following the outline below, that $T(n) \leq n(\log(n) + 1)$, for all $n \geq 1$.

   - **Inductive Hypothesis:** For all k with $1 \leq k \leq n$, $T(k) \leq k(\log(k) + 1)$.
   - **Base case:** [You fill this in: show that the inductive hypothesis holds for $n = 1$]
   - **Inductive step:** [You fill this in: show that if the inductive hypothesis holds for $n - 1$, then it holds for n.]
   - **Conclusion:** [You've proven something by induction; what is it? Is it what you had hoped to prove?]