CS 161
Autumn 2017

Pre-Lecture Exercise for Lecture 5
Date: October 6, 2017

Pre-lecture exercises will not be collected for credit. However, you will get more out of each lecture if you do them, and they will be referenced during lecture. We recommend **writing out** your answers to pre-lecture exercises before class. Pre-lecture exercises usually should not take you more than 20 minutes.

In this pre-lecture exercise, we will remember a little bit of probability!

1. Let X be a random variable which is 1 with probability 1/100 and 0 with probability 99/100.

   (a) What is the expected value $\mathbb{E}[X]$?

   (b) Suppose you draw n independent random variables, $X_1, X_2, \ldots, X_n$, distributed like X. What is the expected value $\mathbb{E}[\sum_{i=1}^{n} X_i]$?

   (c) Suppose I draw independent random variables $X_1, X_2, \ldots$ and I stop when I see the first "1". For example, if I draw

   $$X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 1$$

   then I would stop at X_4. Let N be the last index that we draw. (So in the previous example, $N = 4$). How big do you expect N to be?

   [**Note:** actually figuring out $\mathbb{E}[N]$ from scratch is a bit tricky, although you may have seen it in CS109. But even if you don't do it rigorously, intuitively how big do you expect N to be?]

2. Consider the following pseudocode, which is an in-place sorting algorithm for an array A.

```
def bogosort(A):
   while A is not sorted:
      A.shuffle() # this randomly permutes A
   return A
```

   (a) Let X_i be a random variable which is 1 if `A.shuffle()` is sorted after the i'th call, and 0 otherwise.

   (b) What is $\mathbb{E}[X_i]$?

   (c) What is the *expected* number of times that `bogosort` executes the `while` loop?