Pre-lecture exercises will not be collected for credit. However, you will get more out of each lecture if you do them, and they will be referenced during lecture. We recommend **writing out** your answers to pre-lecture exercises before class. Pre-lecture exercises usually should not take you more than 20 minutes.

On an island, there is a population of T toads, as well as a great deal of ice cream. Ice cream comes in two flavors, chocolate or vanilla. Each toad either likes or does not like each flavor of ice cream. For example, Tyrannus the toad may like chocolate but not vanilla. Consider the following two statements about the toads' preferences.

- **Statement 1.** Choose a random toad t out of the population. The probability that Toad t likes Vanilla is $1/2$. The probability that Toad t likes Chocolate is $1/2$.

- **Statement 2.** Choose a random toad t out of the population. The probability that toad t feels the same way about both chocolate and vanilla—that is, like them both or dislikes them both—is exactly $1/2$.

The question is:

1. Does **Statement 1** imply **Statement 2**?

2. Does **Statement 2** imply **Statement 1**?

For each question, try to give a proof if you think it's true or a counterexample if you think it's false.

SOLUTION: Neither of these statements are true.

1. Consider two toads, one of whom likes both flavors and one of whom dislikes both flavors. Then **Statement 1** is true, but **Statement 2** is not.

2. Consider two toads. One of them likes both chocolate and vanilla, and the other one likes chocolate but dislikes vanilla. Then **Statement 2** is true, but **Statement 1** is false.

   What does this have to do with hash families? Imagine that the toads are functions, the flavors of ice cream are items, and there are two buckets, "like" and "dislike." **Statement 1** says that the buckets are *balanced.* For each item, it has a equal chance of ending up in every bucket. This seems like it should be enough for a universal hash family (balance is good, right?) but it's not!!! Indeed, **Statement 2** is the statement of a universal hash family: the probability that chocolate and vanilla collide (land in the same bucket) should be $\leq 1/n = 1/2$. And the exercise above shows that **Statement 1** and **Statement 2** are *not* the same thing.